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Review

A survey of image processing techniques and statistics for ballistic specimens in forensic science

George Gerules [a], Sanjiv K. Bhatia [a,*], Daniel E. Jackson [b]

[a] Department of Math & Computer Science, University of Missouri-St. Louis, St. Louis, MO 63121, United States
[b] Police Crime Laboratory, St. Louis County Police, Clayton, MO 63105, United States

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ABSTRACT

This paper provides a review of recent investigations on the image processing techniques used to match spent bullets and cartridge cases. It is also, to a lesser extent, a review of the statistical methods that are used to judge the uniqueness of fired bullets and spent cartridge cases. We review 2D and 3D imaging techniques as well as many of the algorithms used to match these images. We also provide a discussion of the strengths and weaknesses of these methods for both image matching and statistical uniqueness. The goal of this paper is to be a reference for investigators and scientists working in this field.

Contents

* Corresponding author. Tel.: +1 314 516 6520; fax: +1 314 516 5400.
E-mail addresses: georgegerules@umsl.edu (G. Gerules), sanjiv@acm.org (S.K. Bhatia), djackson@stlouisco.com (D.E. Jackson).

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1. Introduction

The use of computers in forensic investigation is well-established. We routinely see the computers being used in a number of forensic tasks such as fingerprint matching, shoe print matching, DNA-based evidence, and matching of ballistic samples [1]. None of these tools gives us a complete match, even when they are certified by human experts. They merely provide a statistically viable match that may be used to apprehend and convict a criminal, or point to a suspect's innocence. The traditional techniques based on a human expert's judgment of match may introduce the expert's bias into the case as well. The relatively new field of computational forensics attempts to overcome the human bias angle by objective analysis and statistical probability that can withstand the arguments in a court of law. Our main focus in this paper is to survey one of those computational forensic techniques — the automated association of ballistic samples. The computer images in systems such as IBIS are presented as correlation lists against a given specimen. This association provides an investigative lead that allows a forensic examiner to microscopically examine the evidence to determine if there is indeed an identification or if the image association resulted from just a trick of shadow and light.

The ballistic samples – spent bullets and cartridge cases – provide important clues to solve criminal cases involving the use of a firearm. These samples are based on the characteristic markings produced on the cartridge as a result of firing a gun. There are more than thirty characteristic markings that can be distinguished to identify a firearm [2]. The markings on bullets are a result of *rifling*, or the process of making grooves and lands in the barrel of a gun. There is also hexagonal or octagonal rifling. The rifling causes the projectile to twist at a rate determined by the manufacturer to be ballistically effective for a given caliber and firearm. Thus, rifling helps in improving the accuracy and range of the bullet as it is fired. Since the rifling is unique for every firearm, the markings due to the rifling, known as *striated marks* or *striae*, can be analyzed to identify the firearm used in the firing.

The striated marks have been under the lens of inspection by forensic scientists for almost a century. However, the techniques dealing with image processing and the statistical methods used in identifying the uniqueness of ballistic specimens have been documented only recently. Bachrach [3] has noted that even under the best of conditions, the striae on the bullets fired even from the same gun may not be the same. Therefore, the forensic experts look for *significant similarities* or regions of similarities between two bullets. This observation has introduced a measure of confidence into the match, leading to legal challenges to the matching process.

Challenges to existing manual matching techniques in the light of the *Daubert v. Merrell Dow Pharmaceuticals, Inc.* decision in 1993 have raised the bar on what is admissible evidence in a court of law [4]. The challenges have appeared due to the courts noticing that the forensic sciences, including finger print and ballistic matching, have not been rigorously tested using a scientific method [4]. This paper summarizes some of these important results on image matching and statistical analysis techniques to describe the uniqueness of spent bullets and cartridge cases from the last few decades.

This paper is organized as follows. In Section 2, we provide information on the structure and nomenclature of the cartridge components as related to forensics. We present the traditional way of matching the marks on the bullets in Section 3. Section 4 examines the use of imaging technology in matching an evidence bullet with a test bullet. In Sections 5 and 6, we take a look at the 2D imaging techniques in the spatial and frequency domains, respectively. In Section 7, we discuss 3D surface analysis techniques. This is followed by image and shape matching in Section 8. We then round out this survey with Section 9 on statistical uniqueness, a Section 10 on some of the debates in the literature on uniqueness, and a conclusion in Section 11.

2. The nomenclature of forensic ballistics

In this section, we'll present the terminology needed to discuss the cartridges as well as other terms in forensic analysis as related to firearms.

A cartridge is made up of several parts illustrated in Fig. 1. The top of the cartridge is the projectile known as the bullet. It is usually made of lead, or lead covered with another metal such as copper or some metal alloy, leading to the appellation *copper-jacketed bullet*. Next, moving down the cartridge is the *cartridge case*. The cartridge case is usually made of some brass alloy and holds together all the parts of the cartridge. It has room to hold the gunpowder that is to be used as the propellant for the projectile. The bottom part of the cartridge case is called the *rim*. The rim facilitates the extraction of the case. It is also used to seat the cartridge in the chamber. At the very bottom of the cartridge case is the primer. The firing pin of the gun strikes the *primer cap* which contains a small explosive charge to ignite the gun powder resulting in the release of the bullet.

The cartridges are classified into center-fire cartridges and rim-fire cartridges based on how the firing pin strikes the primer. In center-fire cartridges, the firing pin strikes the center of the primer cap which is a small disk of metal enclosing an ignition source [5]. That small disk is inserted into the end of the cartridge case. For rim fire primers, the ignition source is inside of the cartridge case along the rim at the end of the cartridge case. After firing the bullet, the cartridge case is ejected from the gun through an ejector mechanism. The ejector mechanism can be either automatic or manual. This split-second process of firing leaves the characteristic marks on the bullet because of rifling on the gun barrel, as well as marks on the

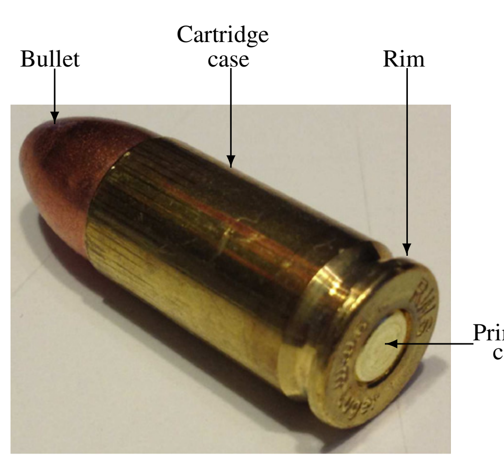

Fig. 1. Different parts of a bullet.

cartridge case because of the firing pin impression, ejector marks and breech face impressions [6,7].

Davis [5] has investigated the effects of hardness, grain size, composition, and thickness of the primer cap on breech face marks. His research is important in correlating the type of cartridge to the type of gun used to fire the bullet. This gives an investigator a starting point in the criminal investigation by narrowing down the possible combinations by looking at the links between the physical properties of the primer cap and the marks. But, even with this starting point there is some debate in the literature on how these properties affect breech face marks [8–10]. The bullets fired from the same gun may get different marks due to the barrel getting worn by repeated use. Bachrach studied the effects of wear of the barrel on the fired bullets and concluded that there is no significant effect of barrel wear on the correct identification of bullets [11].

In the following two subsections, we'll examine the striae of interest on fired bullets and the cartridge cases.

2.1. Striae on fired bullets

The inside of a gun barrel, with the exception of shotguns and other smooth bore weapons, contains rifling that helps to spin the projectile when the bullet is fired. The rifling helps to stabilize the bullet along its flight trajectory. Rifling generally consists of *lands* and *grooves*. Lands are raised areas inside the gun barrel while grooves are the areas between the lands as can be seen in Fig. 2.

The *land engraved areas* (LEA) are the engraved portions on the fired bullet which correspond to the rifled lands inside the barrel. The lands pressing into the bullet, as a result of firing, cause the bullet to spin in flight and create the LEAs. These class characteristics can be used to correlate the bullet to a class or range of firearms, but are not individuating in themselves.

The class characteristics, such as the number of lands and grooves, direction of twist, and widths of the lands and grooves, can be used for class identification. That is, they can narrow down a bullet as having been fired by a particular type of firearm and can eliminate the possibility of others. For example, a .357 caliber bullet with five lands and grooves with a right-hand twist cannot have been fired by a .44 caliber weapon, neither can it have been fired by a weapon that has six lands and grooves with a left-hand twist.

The individual striae and other marks are dependent on the imperfections of the tooling process or the general wear-and-tear of the weapon over time. The class characteristics help to narrow down the investigation of the type and manufacturer of the gun while the individual marks point to the specific weapon used in firing.

Bachrach has commented on the effects of barrel quality on the striated marks and identifiability of bullets [11]. He identified three

Fig. 2. Lands and grooves in a pistol barrel.

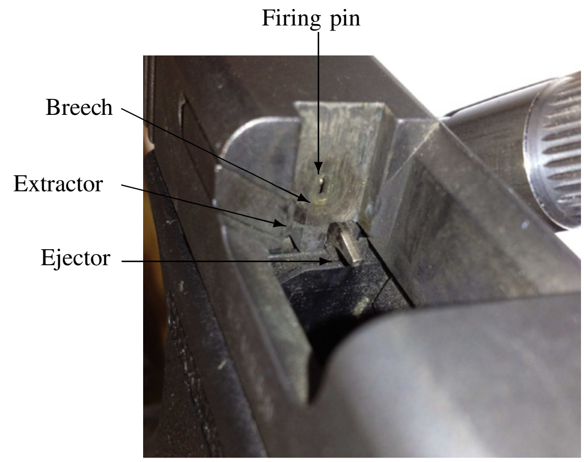

Fig. 3. Breech, firing pin, extractor, and ejector.

manufacturing characteristics for barrels: dimensional tolerance, general dimensional quality, and finishing of barrel interior surface. He observed that the cheaper materials and processes in manufacture result in lower dimensional tolerance and quality that makes it harder to look for striae on spent bullets. The high quality finishing in interior surface also makes it harder to examine the marks. He concluded that these limitations can be overcome by more sensitive instrumentation.

Forensic firearm and toolmark examiners individually identify firearms by correspondence of individual, random characteristics such as the above-mentioned striae. Firearm and toolmark examiners may count consecutively matching striae (CMS) marks to help match one fired bullet to another [12]. In some cases, the striae may be optically straightened to enable the counting of CMS marks. Some researchers have investigated the straightening process for the deformed marks for such a match [13,14]. Some authors have also investigated striae made by flat-bladed screw drivers on soft materials [15,16]. Since a firearm is essentially a tool which imparts class and individual marks on softer materials, such striae are treated in a similar manner as those found on spent bullets.

2.2. Marks on cartridge cases

The marks on cartridge cases may come from a number of sources including the firing pin, the ejector, the extractor, the breech face, chambering, and the magazine feed. An example of a breech, firing pin, extractor, and ejector mechanism can be seen in Fig. 3.

There have been some attempts to classify these marks. Terms such as circular, elliptical, ring, rectangular, ax head, and square, have been used as classification terms to identify correlations among images of firing pin marks on cartridge cases [2,17,18]. These may be used as fiduciary marks in high magnification images. Though the normal optical microscope is sufficient for comparison of marks on the cartridges, the low-angle backscattered electron techniques provide good results, especially in the case of firing pin impressions [6]. Other marks, such as breech face marks, firing pin drag,[1] ejector, extractor, magazine, and chamber marks, have been extracted as features from images for matching purposes [19,20]. Some researchers have argued for the use of scanning electron microscopes to examine cartridge cases [2,6]. In the case of automatic firearms, we can also

[1] Firing pin drag is a special kind of mark left by a firing pin on the primer cap. Normally, when a firing pin lands on the primer of a center-fired cartridge, it leaves a relatively well-defined impression with clean margins. However, in certain semiautomatic firearms, during the extraction–ejection cycle after firing, the breech will unlock and the breech end of the barrel will drop before the firing pin is fully retracted into the breech face of the bolt or slide. This causes the firing pin to drag across the primer, causing a distinctive type of mark (referred to by some authors as a *tongue mark*). These can be individuating marks.

find the location of extractor marks, and the ejector mark in a fixed relation to the extractor marks [2].

In addition to the above individual firearm marks on cartridge cases, there are also class characteristics on the cartridges that include the manufacturer's stamp on the head, and the type and caliber of ammunition. In the case of rim-fired cartridges, we can also find the shape of the firing pin impression useful. The shape marks of firing pin have been classified using terms such as circular, elliptical, rectangular, square, double, and ringed [2]. It should be noted that such characteristics lead to a class of firearms (such as a revolver by a certain manufacturer) rather than an individual specific instance of the firearm.

To give another example of non-individuating marks used by some researchers, headstamps are made with tools called bunters, which cannot be used for individual identification. A bunter may stamp many tens of thousands of cartridge case heads before it is replaced. Thus, the headstamps can match an instance back to the tool, but that does not help to identify the firearm.

In the next section, we'll limit our discussion to examine the traditional methods that involve the manual matching of striae on bullets and cartridges.

3. Matching bullets and cartridge cases

When a crime is committed with a gun, the firearm and toolmark examiners may have to match fired bullets and cartridge cases from the scene with each other, and with a firearm if one is recovered. In the event that a firearm is recovered, test shots will first be fired, using a recovery medium (such as a water tank, as illustrated in Fig. 4) which allows the retrieval of fired bullets in relatively pristine condition. A minimum of two test shots are fired as a standard practice, in order to allow the examiner to compare test shot to test shot, identifying reproducible unique and individual marks before beginning to compare evidence to firearm.

The process of matching a bullet or cartridge to a gun is a multi-step process. Assuming that there is agreement of class characteristics, an evidence bullet or cartridge case will be mounted on one stage of a comparison microscope (Fig. 5). Another evidence or test-fired bullet or cartridge case will be mounted on the other stage. The examiner then manipulates the evidence and stages in order to obtain alignment of corresponding markings, attempting to identify or eliminate the use of a common firearm for the two specimens.

In the case of bullet comparisons, the marks generally used for identification or elimination of a common source are the microscopic individual, unique striae imparted on the bullet, principally by the barrel, during the firing process.

In the case of cartridge case comparisons, a wide variety of marks can be useful. The firing pin impressions, breech face impressions and (in the case of semiautomatic and automatic firearms) ejector, extractor, chambering and feed marks, and occasionally other marks unique

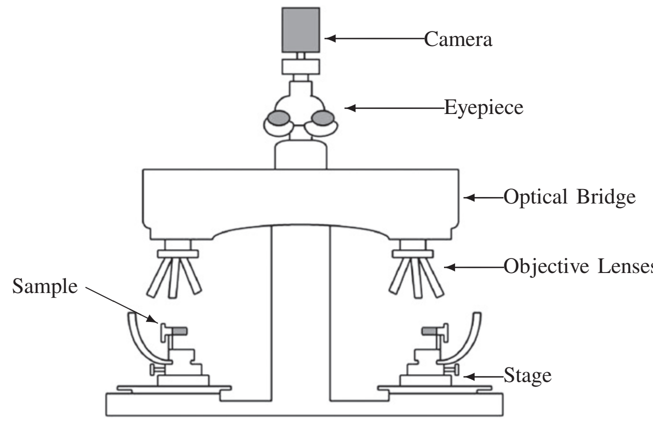

Fig. 5. Comparison microscope.

to the cycling action of the particular weapon, may all be useful. These marks may be impressed toolmarks (for example, parallel breech face marks or irregular, nonlinear marks such as firing pin impressions), or striated marks (for example, chambering/feed marks). The examiner must be aware of strengths and shortcomings regarding the use of such marks (for example, differentiate between machined and molded gun parts).

A camera, mounted on the microscope, may be used to take a picture of the matching marks for later use in pending related court proceedings. An example of matching marks between evidence and reference cartridge cases and bullets is shown in Fig. 6.

As this paper is principally concerned with the identification of striae, identification of two bullets to a common source will be used as a first example. As the two bullets are aligned, the investigator looks for how well the striae are aligned and the similarity in their width and amplitude. The test is considered a match if two firearms examiners concur that it is a match, as specified in the Association of Firearm and Toolmark Examiners "Theory of Identification" [21].

The process to match cartridge cases is similar, placing cartridge cases instead of bullets on the comparison stages of the microscope. Firearms and toolmark examiners look for many kinds of matching marks towards the rear of a cartridge case where the primer is located. For example, they look for ejector marks along the rim of the end of the cartridge case. They also look for breech face striae along the surface end of the cartridge case. This type of mark is imprinted on the rim from the breech of the cartridge case as a result of the bullet being discharged from the other end of the cartridge case during the firing process. They also look for marks left by the firing pin on the primer cap or the rim of the cartridge case. The firing pin shape, size, and depth can lead to a determination of the type of gun used. In addition, they also look for extractor marks, chambering marks, and feed and magazine marks. Many imaging systems use this

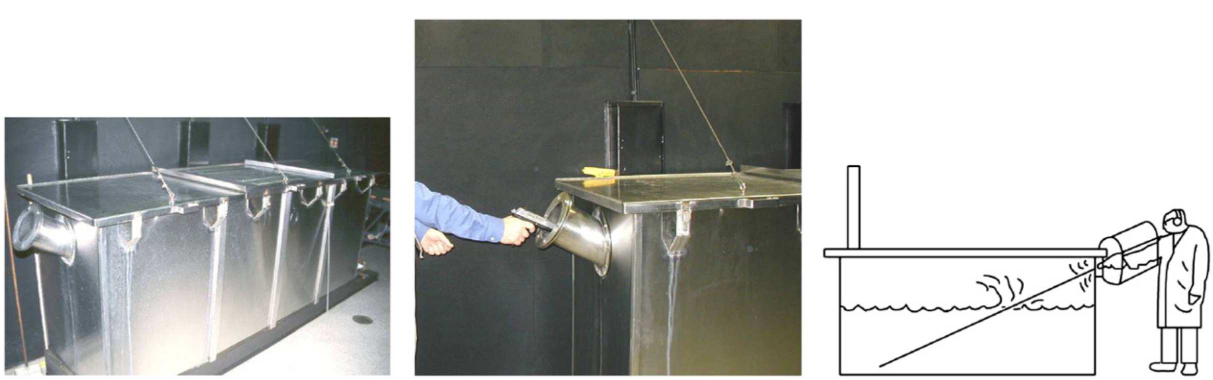

Fig. 4. Ballistic bullet recovery water tank.

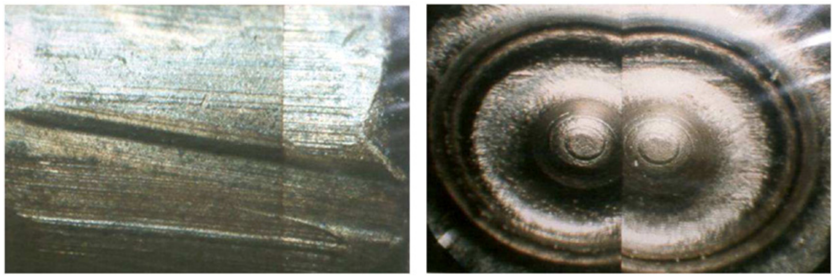

Fig. 6. Matching striae on bullets and firing pin marks on cartridge cases.

information to help identify matches between the reference and suspect cartridges. Leloglu et al. designed a technique that uses the surface height information on the base of the cartridge case to segment the sections [23]. This technique then detects three circles corresponding to primer, primer pocket gap, and cartridge base case using the Hough transform. The three circles are assumed to be approximately concentric to facilitate detection.

When using optical microscopes, some systems experience focusing and lighting issues that arise due to the depth and steeply sloping walls of firing pin impressions and tool marks. Randich et al. have described the use of backscattered electron (BSE) signal in a scattered electron microscope (SEM) to overcome those problems [6]. They reported that SEM/EDX (SEM coupled with energy dispersive X-ray analysis) has been widely used for detection and identification of gunshot residue (GSR). These days, various 2D and 3D imaging systems are used to record the matching marks on bullets. The details of these systems are discussed in [3,22,11,15].

This background on the characteristics of evidence and the matching process leads us into the automated techniques to match the evidence with reference objects using digital image processing.

4. Imaging in forensic ballistics

The use of digital imaging for ballistics first appeared in the literature in the 1970s with the publication of Gardner's paper on computer identification of bullets [24]. Gardner described the use of a scanning electron microscope to examine the striae on a spent bullet. In the mid 1980s, Catterick and Taylor used an imaging system linked to a PET Commodore 4032 computer to capture striae on bullets. They used a plotter in addition to the video images to print out the height information of the striae [25]. The turn of the millennium saw the use of automated search and retrieval techniques as an acquisition component to acquire data from the sample bullet or cartridge case and to prepare it for analysis as well as a correlation component to compare sets of normalized data [22].

During the 1990s, the first patents relating to imaging and ballistic specimens were filed with various government patent agencies around the world starting with the patent application by Baldur and Barret [26]. This patent and a number of others from the 1990s through early 2000s describe the initial commercial work using imaging techniques to acquire and process ballistic specimens [27–35].

In the mid-1990s, the number of publications related to imaging of ballistic specimens started to increase, starting with an article by Smith and Cross [2]. That paper describes a ballistic image acquisition and storage system, called *Fireball*, attached to a forensic microscope. After 1995, a number of papers have appeared on a regular basis. A further detailed discussion of articles and patents will take place in subsequent sections in this article. However, prior to doing that, we need to examine the traditional methods of viewing details on spent bullets and cartridge cases.

4.1. Use of computers and computerized devices in matching bullets and cartridge cases

A large amount of work has been performed over the past 30 years in using computers to capture the images of ballistic specimens, analyze those images, and speed up the search for a match with those specimens. In this subsection, we provide a brief overview of systems developed by various law enforcement agencies, universities, and corporations.

In the United States, the Federal Bureau of Investigation (FBI), commissioned the creation of a system, known as *Drugfire*, to capture the images of bullets and cartridge cases in the early 1990s [36]. Around the same time, the Bureau of Alcohol, Tobacco, and Firearms (ATF) contracted with a Canadian company, Forensic Technology, Inc. (FTI), to develop a similar system. The ATF system is known as *Integrated Ballistic Identification System* (IBIS). Both *Drugfire* and IBIS contained a large database of digital images of bullets recovered from victims or crime scenes. The IBIS database is made up of two modules. The first module, *Bulletproof*, contains digital images of bullets recovered from crime scenes. The second module, *Brasscatcher*, contains images of cartridge cases [37]. In 1999, FBI phased out *Drugfire* and joined ATF in standardizing IBIS under the National Integrated Ballistic Identification Network (NIBIN). In early 2009, an Interpol press release announced signing an agreement to use FTI's IBIS system for its forensic ballistic needs.

In Europe, there were similar efforts underway, with TRAX in the Netherlands [38]. TRAX contained images and administrative data about the tool marks. Some other systems include Condor in Russia, the GE/2 system in Germany [14], and the Forensic Expert Assistance System Abal Labview (FEASABLE) in Belgium [34].

There have been a number of publications from the academic circles, especially in Australia. One of the early references is the *Fireball* system in Australia [39,2]. *Fireball* performs the imaging and information extraction from 2D images [40]. As the Internet usage grew, Chase and Li explored a web-based approach to store and retrieve ballistics data over the web as a front-end to *Fireball* [41]. In their system, they have a web-based form interface linked to a back-end relational database server. This database is used to store images and text fields of data. The system finds a match for a ballistic specimen by forming a proper SQL query for the database. This query is in the form of text fields describing aspects of the ballistics specimen [41]. Other systems originating in academia include the POLIVIEW system in India [42] and the *Balistika* system from Turkey [43]. *Balistika* uses photometric stereo images to build a 3D surface of a cartridge case.

In addition to the above-mentioned systems, some other notable commercial systems include the Advanced Ballistic Analysis System (ALIAS) from Pyramidal Technologies, Canada, the Evofinder (evidence finder) system from ScannBi Technology, Germany, and the Arsenal system from Papillon Systems in Russia.

In 2000, Bachrach demonstrated a prototype system, SCICLOPS, that captures and stores the data on bullets in 3D and performs the matching of striae in 3D [22,3]. SCICLOPS was later commercialized as BulletTRAX™-3D which was designed to be fully compatible with IBIS [11,44].

De Kinder et al. have looked at the performance of a reference ballistic image database (RBID) using IBIS [9]. They have suggested the use of RBID to reduce the number of cartridges that must be optically compared. They identified two criteria to measure the performance of an RBID: the efficiency of the matching algorithm and the reproducibility of the striae and impressions on the cartridge components. Their study shows that there is an overlap in class characteristics of firearms from different manufacturers. For example, there may be more than one manufacturer with the same orientation of the extractor and/or ejector.

4.2. Imaging systems for ballistic specimens

Imaging of ballistic specimens has followed in lock step the progress in imaging technology since the mid-1970s. Such imaging has made use of numerous sensing devices, including scanning electron microscopes, 1D line scan CCD cameras, 2D CCD cameras, lasers, and ultra-sound. A number of different imaging systems have used these devices to extract data to investigate ballistic specimens. In this subsection, we will first take a look at the devices and then, examine the use of those devices. Then, we will look at some of the lighting problems introduced by these systems when imaging specimens.

4.2.1. Scanning electron microscopes

The early use of scanning electron microscope (SEM) to examine the striae on spent bullets has been reported by Gardner [24]. An SEM forms an image that may be considered similar to that formed by a high quality optical microscope. However, instead of light being focused by optical lenses, an SEM streams the electrons in a fine beam towards the target. This beam of electrons is focused by magnetic fields and scans the target surface in a raster fashion [24]. Electrons interfere with the electrons on the surface of the target and are bounced back to detectors where they are transformed into visible light images. The resolution of such images can get down to the micrometer range [45,6]. SEMs are also excellent in extracting the topographical information from the surface of bullets in an effective manner [24].

Banno et al. have used a confocal microscope to obtain 3D data of striature marks by measuring observed intensity at different points while moving the object [46]. This data is used to reconstruct the surface topography in the form of range images, representing the elevation by a gray scale value at each point. They use Gaussian filter and median filter to reduce the influence of noise and outliers.

4.2.2. Charge couple devices

A charge couple device (CCD) is a computer chip with two layers. One layer has a capacitive region that converts light energy into a stored charge. The second layer reads this charge by a special register on the chip that converts the charge to a voltage that can be interpreted as visible signal [47]. The cells that make up the CCD can be arranged into a line, known as a one-dimensional (1D) line scan configuration. The cells may also be arranged into a two-dimensional (2D) array of CCD cells [48].

When taking a picture with a 1D CCD device, the picture is captured as a 1D strip of pixels as the bullet is rotated along its long axis. These strips are then joined together to form a surface image of the bullet as though it was unwrapped [49,17,50,51,2,52,18].

The 2D CCD array captures an entire field of view under the sensor and converts it into a digital picture [48]. Another way to obtain a 1D image from a 2D CCD device can be capturing the middle row or column of each image as the bullet is rotated [53].

4.3. Effect of lighting on image acquisition and matching

The type and position of light source during image acquisition plays a major role during the matching of evidence and reference material. There have been several articles and patents that address the difficulties in matching images of bullets and cartridge cases [54,38,13,6,43]. Images of evidence and reference bullets may be sensed under different lighting conditions and angles. For example, the object may be illuminated by a single light source or diffused lighting. In such a case, the marks left on bullets or cartridge cases can appear completely different depending on lighting angle, intensity, and color [22,55,10,33].

A single light source produces directional lighting that results in different intensities in different areas of the object, based on the position of the light source. However, the diffused lighting results in a homogeneous appearance with high contrast attenuation, leading to suboptimal results [14].

A number of methods have been proposed to work around these lighting issues. Some of those attempts include differing wavelengths of light [56], infra-red light [20], use of gray scale images [16], side and ring lights [57,58,39,43], co-axial lighting [54] and diffused lighting [57,59]. Also, different types of microscopes using laser light, such as laser confocal microscopes, have been used where lighting direction may not be a consideration [46,12,60].

The GE/2 system varies the illumination direction such that the grooves are always perpendicular to the incident light, maximizing surface contrast. It may take a series of frames in different wavelengths and combine them using image fusion to get enhanced images [14].

Some researchers have advocated the use of side lighting and ring lighting on ballistic specimens to match images [28,38,61]. The side light facilitates the visualization of fine marks. In addition, there have been some suggestions to use diffused lighting [59]. Geradts has used Lambert's Law to standardize the use of fluorescent light with an angle of 45° to capture the images in TRAX [38]. He also indicated that methods in current literature [of the time] use gray scaled images to account for the differences in lighting. He notes that in practice, the image preprocessing steps such as histogram equalization, take care of many of the lighting variables. Another technique suggested by Geradts is based on co-axial lighting, using a beam splitter to uniformly light and acquire a specimen image. This involves the projection of light a beam splitter where it is reflected onto the surface of the object. The imaging device then takes a picture of the light reflected from the surface. This technique removes glare and unwanted shadows [54]. Geradts has also used this technique to capture 3D images of specimens and called it *coded light approach* (CLA) [38]. Lajeunesses et al. have solved the problem by placing two light sources perpendicular to each other and pointing at a user definable angle to the back surface of the cartridge case [33]. Brein noted that a combination of spot and ring lighting is good to detect letters while a combination of ring and diffused lighting is good to detect circles [57].

Huai and Dongguang [56] have investigated the use of different wavelengths of light in *Fireball*. They have used an optical microscope with a ring light and a CCD camera. They used seven different wavelengths of light spread across the visible light portion of the electromagnetic spectrum, from blue (400 nm) to red (700 nm). They examined the images of a bullet in each of the seven wavelengths of light and concluded that the longer wavelengths were better for the purpose of comparison.

Some researchers have investigated the use of infra-red light to observe the bullets [20,18]. Prokoski used the same set of bullets imaged under visible and infrared lights and concluded that the infrared light performed better than visible light in detecting features on cartridge cases [20]. She also indicated the need to model emissivity changes because of alloy, optics, spectral response, sensitivity, and type of IR camera. Zographos et al. noticed that the monochromatic quality of the light worked better in their camera's peak sensitivity in the near infrared region [18]. They also notice that lighting angle had a major impact on the image quality.

Zographos et al. also investigated the difference between a line scan camera and a 2D camera to acquire the bullet images [18]. They noticed a reduction in reflection off a surface when an image was acquired via a 1D line scan camera. However, in the 2D sensor, the high image intensity obscured details of the bullet surface.

Another technique to capture fine details of bullet surface involves the use of white-light profilometry [10]. This involves shining a light through a semi-transparent mirror onto an object. Light is reflected off the surface, bouncing off a semi-transparent mirror and making its way back to the sensor. This technique exploits the chromatic properties of lenses so that different wavelengths of light get refracted at slightly different angles when going from light source to the sensor. This technique allows the recording of surface differences of 1 μm.

4.4. Use of lasers

A number of systems have used lasers to capture the images of ballistic specimens. Lasers have also been used to monitor and adjust the position of a bullet under optical observation for forensic purposes [27]. Most of the use of lasers has concentrated on building profile information of specimens in 3D.

Lasers have been used to help focus a microscope's lens when taking an image [30], and to obtain the z-depth to build a 3D surface [62]. As an example, Breitmeier and Schmid use a laser to project a 1 micron dot onto the surface of a cartridge case. A photodiode helps position the objective lens keeping the 1 micron dot in focus. This setup allows the observation of height information as a result of the movement of the objective lens [63].

Lasers have been used in confocal microscopes to obtain bullet and cartridge images with a high resolution (Fig. 7). Laser confocal microscopy uses a combination of lenses, pin holes, and beam splitters to filter in-focus portions of the image from out-of-focus portions of an image. These systems use a laser instead of a conventional bulb light source to illuminate a specimen, using a large range of wavelengths, from 200 nm ultraviolet to far infrared 1000 nm. Still, in practice, the confocal microscopes use the visible wavelengths (400–700 nm) to image the specimens [54,12,55].

Some researchers have used laser conoscopic holography to build prototype systems [64]. This system uses an off-the-shelf laser sensor to measure the depth of a laser point projected on a ballistic specimen. A laser conoscopic holography sensor projects a laser beam to a specimen's surface. The beam is then reflected along the same path and is passed through a conoscopic crystal. This crystal produces a diffraction pattern that can be analyzed to determine the z height of the surface under consideration. An xy translation table is used to move the specimen to measure other depths. This method results in a highly detailed surface map. The technique tries to compare a 3D surface map with another map to create a "virtual" 3D comparison.

A 3D surface profile can also be built using a laser displacement sensor [51]. This is achieved by projecting a finely focused laser through a lens onto a specimen's surface. Light is refracted off the surface at various angles and then focused through a receiver lens onto a CCD sensor. The angle between the laser source and CCD pixel location is used to infer height. Yet another technique to determine the z height of specimens relies on a laser focal spot [11].

In this section, we looked at some basic image matching and other techniques to acquire images of ballistic specimens. In the next two sections, we'll examine the processing of those images in spatial and frequency domains.

5. 2D image processing — spatial domain

Images are commonly preprocessed to enhance certain attributes for later processing by other algorithms. The sensing process may introduce some noise in the acquired signal that needs to be corrected. In addition, the preprocessing algorithms are used to compensate for any changes in lighting during image acquisition. Most of the researchers have used more than one image processing step for the purpose, with an eye on improvement through different combinations of operators in the form of an image processing pipeline.

In the following subsections, we'll describe the use of image processing to enhance an image to prepare it for comparison and some operations used to compare images of specimens.

5.1. Elementary image processing operations

The noise in the images can be reduced by using a smoothing operator. A smoothing operator performs *blurring* on the image that reduces the noise through defocusing. It is typically achieved by convolving the image with a blurring kernel. One of the popular kernels to smooth out noisy images is the Gaussian kernel [65,12,60]. Others have used a median filter that works well for bipolar and unipolar impulse noise generated by some kinds of sensors [66].

The sensed image is processed to extract the objects of interest and remove the image background. Some developers have used manual processes to extract the relevant portion of the image using interactive masks [38]. The Belgian system FEASABLE asks its users to manually draw the contour of the relevant regions in the image after the image has been acquired [34]. The automated techniques to draw a boundary around the object of interest have been based on edge detection. There are many methods to find edges within an image and most of those center around the detection of sharp changes in image intensity between adjacent pixels. The rate of change in pixel intensities is mathematically modeled using the first- and second-order partial derivatives on adjacent pixels. Two of the most popular edge detectors are the Sobel operator and the

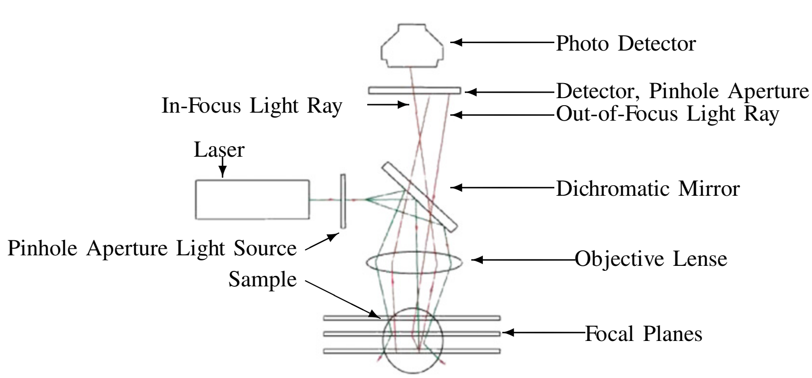

Fig. 7. Laser scanning confocal microscope.

Canny edge detector. These operators have been used to extract edges to identify the object from contrast-enhanced images under the presence of excessive noise [57,53,39]. Sobel masks have also been used to find striae edges in line-scanned images of bullets [67]. Chu et al. used Canny edge detectors to extract striae from images created with a confocal microscope. They used 4- and 8-connectivity to help get the striae [12,60].

In other cases, we may want to isolate an object completely from its background, retaining the texture and other properties of the specimen while ignoring other properties of the background. This extraction relies on an image segmentation operation which itself relies on image thresholding. The most elementary thresholding assumes the pixels corresponding to the specimen to have a certain intensity range that is distinct from the background pixels. Threshold has been used to find a dominant angle for an orientation tensor computed for individual pixel values. This angle is then used to help build a mask to identify regions of interest to match ballistic specimens [34]. Chu et al. have used thresholding after the application of Canny edge detector to find striae on bullets, using a hysteresis threshold operation to find connectivity of edges around striae [12,60]. Hysteresis thresholding aims at getting connectivity of pixels specified by a lower and upper value in the range.

The thresholding operation typically results in an image where the pixels are characterized by one of the two values — black or white. Such images are known as *binary images* and may be used to analyze the shape of specimens.

The extracted objects need to be aligned for further processing, both in terms of spatial orientation and image characteristics. For example, the two images to be compared need to be presented under similar contrast and brightness settings. This can be achieved by histogram-based analysis. A histogram is a structure to represent the probability distribution function (PDF) of pixel intensities in an image. Using the value at a location in the PDF and adding up all the preceding values gives us the Cumulative Probability Function of the value at that location [66]. The cumulative probability distribution was one of the first techniques to help make matches between striature marks for images acquired from a scanning electron microscope [24].

The probability distribution in histograms can be manipulated to improve the contrast of images, using a technique known as *histogram equalization* [66]. For example, if an image is dark the pixel intensity levels can be grouped in the darker range of the image histogram. When equalization is performed, the contrast is stretched so the image appears lighter to expose more detail. The converse is also true for images that are too light or overexposed. For those images, the contrast will be stretched and the image appears darker. Histogram equalization is used to compensate for different lighting conditions of firing pin marks [38,68,58,61]. Histogram equalization is also used to ensure equal distribution of colors over a zoomed image of the height map of a cartridge case [10]. This allows to expand the contrast of the zoomed area that helps make various surface features more visible. Some systems have used the contrast enhancement transform to increase the dynamic range of the gray scale levels in the image [53].

Histogram of a reference image can be used to modify the intensity distribution of another image such that the contrast of the other image is similar to the contrast in reference image. This is achieved by an image processing technique known as *histogram equalization* [66]. In other words, it is a way of taking the contrast description of one image and using it to specify the contrast of another image. It is commonly used as a way to make one image look like another while keeping the subject of each image the same. This technique is also used to account for minute lighting variations between the separate smaller images when they are stitched together to create a larger high resolution image of a cartridge case [69].

Another popular technique to perform analysis on images has been through wavelets [38]. Wavelets have been demonstrated to be useful in multiresolution analysis and allow for filtering properties of objects in the images at different scales from coarse to fine. Geradts has used the *à trous* wavelet based on B3 splines to examine the striature marks at different resolutions [38]. He has applied histogram equalization to the first four levels of the pyramid images used to compute the à trous wavelet of ballistic images.

5.2. Comparing images of ballistic specimens

After processing the images to remove noise and extract the objects of interest, we need to compare the images for a match. The simplest operation to perform a match is image subtraction to observe the difference between two images. Image subtraction involves taking a difference of corresponding pixels in two images to obtain a *difference image* [66]. The image to be compared against a reference image can be slightly rotated by a small angle until a minimal difference is detected [68,61].

The histograms of two images are also compared to look for similarities. Murthy et al. used the difference-of-histogram technique to compute the change between frames in L^k metric [42]. Essentially, their technique examines the differences of histogram values between two images to find the overall minimum of these values.

There has also been an effort to use orientation tensor to compute an orientation image after smoothing the image to remove noise. The tensors yield an orientation image where each pixel is replaced with the orientation angle of the pixel. The range of the orientation angle is in $[-90°, +90°]$. The histogram of the orientation image is then used to determine the alignment angle for the image [34]. The histogram of marks on cartridge cases and primer caps is also used to describe the images [59]. The minimum distance between descriptors is used to find matches between images.

5.3. Higher-order image processing operations

Most of the operations in the last subsections are pixel-level operations that take into account a single pixel, or its very small neighborhood, to enhance the image. This works well up to a point. The quality of processing can be improved by accounting for larger neighborhoods that may be able to intelligently separate the foreground objects from background objects and other clutter in the image. In this subsection, we'll examine some of the image processing techniques for this purpose, starting with image segmentation.

5.3.1. Image segmentation

Image segmentation is used to extract regions of interest from an image. These regions of interest may include points, lines, edges, or regions. Some of these techniques are based on the elementary operations discussed in the previous section.

A simple segmentation scheme attempts to create regions on the image corresponding to background and foreground objects based on color invariant properties [70]. Another technique creates a model-based segmentation based on $2\frac{1}{2}$D height maps obtained from a depth-from-focus method to discriminate between the impression marks on the cartridge case from the manufacturing marks [57,71]. This technique uses a histogram of a cartridge case to find the proper minimum and maximum range of $2\frac{1}{2}$D depth data from surface data generated from a depth from focus technique [57]. Other researchers have used segmentation based on partial derivatives of pixel intensity values to help identify features on cartridge cases [23].

The segmentation process is typically followed by the labeling of connected components that form a segment. A connected component is a set of adjacent pixels that are all related by some criteria, for example a shared or close pixel intensity value. The set of pixels that meet the criteria, typically specified by a predicate, can be assigned a label [66]. Brein has used connected component labeling to help identify the outer edge and letters on images of cartridge cases [57]. This algorithm links together pixels that are either 4-connected or 8-connected to generate shapes [72].

5.3.2. Image registration

Image registration is a technique to align two similar images for the purpose of comparison such that pixels in the corresponding location describe the same object. It is used to optimize the position of an image for a closest fit to the reference image. There are many ways to accomplish this for forensic ballistic matching.

One of those techniques moves a slice of the 3D profile of a bullet so that it would match another potential image [55]. This involves computing a rough radius of the cylindrical bullet profile and moving one image by an x or y amount so that the object centers would roughly coincide. Another technique to register images involves the use of a planar beam laser to monitor and adjust the position of a bullet under optical observation [27].

De Smet et al., while computing the seating depth of primer caps, experimented with registration techniques from several off-the-shelf packages such as PSReg, Scanalize, and ITK [10]. However, these tools did not contribute to positive results for their study.

Some researchers have tried to combine many high resolution images of small areas of a cartridge case into one large high resolution image. They have used scale invariant feature transform (SIFT) [73] to get an initial seam match for the images [69]. This algorithm still leaves some mismatches where a sub-image is not lined up properly with the overall larger image. This is fixed through Graph Transform Matching (GTM) [74]. GTM finds matching points between two images and can be used to build mosaic images from many smaller images.

Some researchers have tried to bypass the registration issue by using invariant image descriptors instead of the original images. For example, Geradts has used log polar transform of the Fourier magnitude to avoid the effects of rotation and uniform scaling into shifts in orthogonal directions [38]. He suggested the use of log polar transform for a faster selection of images to be compared further; however, the computation of the transform on the images in the database is expensive (seven days for a database of 4900 images).

Chu et al. have used the Newton–Raphson integration method in conjunction with a cross correlation function to find the optimal alignment of an image to a reference image [75].

5.3.3. Analysis of geometric moments

Geometric moments are known by several names, such as invariant moments and statistical moments. It is a branch of mathematics that grew out of group theory and algebraic invariants which was studied in the early 1900s by Gordon and Hilbert. Invariant moments are scalar quantities used to describe sets of data and are used as shape descriptors. These have been used by statisticians in the form of mean (first moment), variance (second moment), skew (third moment) and kurtosis (fourth moment). Other higher-order moments help with rotation invariance and deformation of text on a deformed surface in an image. One of the weakness of using moments is that they are global in nature and do not work well for occluded objects within an image [76].

Modern study of geometric moments was first derived for image processing by Hu in 1962. He generated seven invariants for an image translated, rotated, and scaled in-plane about an axis [77]. Flusser and others generalized Hu's work to higher order moments. Moment invariance has also been used in the 3D realm with projective geometry and implicit invariance to elastic transformations [76].

In forensic analysis, Hu's seven translation, rotation, and scaling invariant moments have been used to help classify impressions the shoe impressions found at the scene of crime [38]. Other researchers have used moments up to sixth order to help extract features from images of firing pin impressions on spent cartridge cases [19].

5.3.4. Hough transform

The Hough transform was created to help detect patterns in bubble chambers from particle accelerators [78]. Duda and Hart used it to derive an efficient parameterization of the equation of a line to help identify lines from a random set of images for a mobile robot [79]. The Hough transform has been extended to recognize other shapes, such as circles [79,78].

In forensic analysis, Hough transform has been used to identify the location of primer caps on a cartridge case. Hough transform has also been used to find the perimeter of a cartridge case head [57,59,23,80]. Some researchers have used the Hough transform to extract the firing pin marks for use in a similarity measure when comparing two images [81]. Hough transform has also been used to locate the inner edge of the primer cap and the outside edge of the cartridge [82].

Some researchers have used Hough transform on a binary image, generated by the application of Canny edge detector, to locate the firing pin and cartridge outline from an image [57]. Others have used it to find circles in different resolutions of images generated from wavelet analysis [80]. Leloglu et al. have used a Hough transform as one of several methods to help match images of cartridge case heads [23]. Others have combined image segmentation with the Hough transform to help match color images [70].

5.3.5. Top hat transform

The top hat transform is an operator found in morphological image processing to extract features from an image while concentrating on the form and structure of the region extracted. Morphological operations were originally performed on binary images, but have been extended to include gray scale images. Common operations include erosion and dilation that are concerned with shrinking or expanding the contours of a region through image masks called structuring elements. The top-hat transform is the result of including image subtraction with erosion and dilation [66].

Chu et al. have used the top hat transform with image thresholding to remove areas that were not considered to be striature marks [12]. Li has used structuring elements and morphological operators to help extract features in cartridge cases [83].

5.4. Neural networks

Neural networks are a mathematical model, inspired by biology, of how neurons activate and fire [66]. *Self organizing feature maps* (SOFM), proposed by Teuvo Kohonen, are a kind of neural network useful in pattern classification and recognition. An SOFM neural network is organized into layers. In the learning or training phase of an SOFM neural network, a wide variety of similar data is fed to the neural network. Weights are adjusted until training is finished. After the training phase and the network is put into use, data can be fed to the network for identification. During the identification phase a neuron will “fire” when a similar pattern is fed to the network and a match is made [84].

Li has proposed a system using neural networks based on Kohonen's SOFM algorithm. This system uses the morphological gradient of the image for features. It uses an SOFM neural network to help find matches for firing pin marks made on rim fire cartridge cases [83]. Geradts has used a single layer feed forward back propagation neural network to find matches between shoe prints [38].

6. 2D image processing — frequency domain

Images are nothing but signals to describe the intensities in two dimensions. Thus, we can manipulate the images using signal processing techniques. The most common of those techniques utilizes Fourier analysis to convert the set of pixels into a set of frequencies, and the inverse process.

A Fourier transform relies on the fact that any complex signal can be decomposed into simpler frequencies. The process is reversible in the sense that the simple frequencies can be added to recreate the original signal. The frequencies constituting the signal can be manipulated to perform operations such as filtering and smoothing. Thus, an image can be transformed into frequency domain, changing the set of pixel intensities into a catalog of frequencies. These frequencies are described by the coefficients of sine or cosine functions. Filtering out certain frequencies tends to blur (or sharpen) an image and can be an important image pre-processing tool [66,53].

The classical Fourier transform is slow to compute, even with fast computers. In terms of its asymptotic analysis, it is computed with a time complexity of $O(n^4)$ which puts it into a class of algorithms that takes a very long time as the number of data points increases. Most of the practical applications are based on fast Fourier transform methods. Many forensic researchers have use Fourier transform methods on ballistic specimens [38,59,67,20,64,85,86,62,82].

Several researchers have used filters in the frequency domain to remove noise from CCD images of ballistic specimens [55,83]. Noise tends to be uneven pixel intensity values leading to higher frequencies. An appropriate low-pass filter will result in only the identifying marks in the image of a bullet or cartridge case.

Some researchers have used Fourier analysis to correlate the magnitude of frequencies in two images being matched. The Fourier magnitude coefficients of an image are immune to rotation and translation of the image object [68,58]. Vorburger et al. have used three filters to apply cross correlation functions on bullets and cartridge cases. One of these filters works in frequency domain to help gather data from images of 3D surface profiles [62].

Others have used finite impulse response systems to find matches between cartridge cases uses Fourier coefficients. These coefficients are used as an input to the FIR algorithm [86]. The same author has also explored the use of Fourier coefficients and Fisher's linear discriminant function to find matches between cartridge cases [85].

When using the Fourier transform alone, the images, or magnitudes, produced may show rotation and scaling information. Therefore, when comparing two images in the frequency domain, a log-polar transform can be applied to the Fourier magnitude image to get another image. This new image is the Fourier–Mellin transform of the original image. The data produced by this transform is invariant to translation, rotation, and scaling, and hence, more helpful in image matching [87,64,85,86].

Frequency domain analysis has also been used to describe the shape of a region of pixels in an image. A simple descriptor in an image might be a circle in an image. In this case, the shape descriptor would be the mathematical description in the form of the radius, the perimeter, and the center point of that circle. Smith and Cross used shape descriptors to help discriminate the marks left by firing pins on cartridge cases. The descriptors used are the Fourier coefficients of the image transformed into frequency domain [2]. Legra et al. also used Fourier coefficients of the image of a cartridge case to help distinguish marks left on a cartridge case by a firing pin [59].

6.1. Wavelets

In the late 1980s, wavelet techniques emerged as a powerful methodology to investigate frequency-based data. Wavelets are based on sub-band coding, quadrature mirror filtering, and pyramidal image processing. This is collectively referred to as multiresolution theory. In general, wavelets are used to investigate various amounts of detail that might be apparent at differing image resolutions [66].

From an algorithmic view point, an image pyramid is developed from an image by sampling it at different resolutions. For example, if there is a 512×512 image, the next layer in the pyramid would be a 256×256 pixel image. Smaller and smaller images are made until a minimum image size is reached. Sub-band coding is another technique within multiresolution theory where, as a process of building the image pyramid, information on how to rebuild the original resolution image from the smaller image in the pyramid is computed and stored. This may be achieved using finite impulse response filters. In such a case, we can use filter banks to upsample or downsample the image from higher to lower resolutions and vice versa. An important point to remember is that input data is divided into different scales of resolution rather than different frequencies as in Fourier processing.

Wavelets have been used to match images in different resolutions [68,58,61]. Wavelets have also been used to help identify lines on striae of fired bullets [80].

7. Analysis in 3D

More recently, we have seen the use of 3D surface modeling techniques in the surface construction of ballistic specimens, both cartridge and bullet. Most of the current techniques involve analysis of an image of a specimen's surface. Such an image is essentially a 2D representation of a 3D surface, with the third dimension being implied by the pixel intensity values and guessed by shading. However, the pixel values may get affected by the lighting quality. 3D surface reconstruction aims to rectify this situation by measuring the surface of a specimen and getting a true 3D representation of the surface thereby making the surface potentially easier to match. This surface reconstruction is based on a combination of scanner and software methods [88].

Forensic researchers have investigated many methods to extract fine detail from the surface of bullets and cartridge cases. Bachrach has argued that 3D surface scanning is better at representing surface striae on bullets than other image based methods [22].

The 3D surface plots can be generated by many methods, for example, by measuring the point where the tip of a stylus makes contact on a surface. The object under consideration is placed on a stage where the object is moved in relation to the stylus. The stylus is moved in the z direction to make contact with the surface where the (x,y,z) coordinates are read. This data can yield highly detailed surface maps, down to the μm level [62].

3D modeling can also be achieved using photometric stereo techniques. Photometric stereo is based on inferring the surface height and direction by changing the light reflected from the surface of the specimen. This method has two variations on obtaining surface geometry from images. In the first method, the object is kept fixed relative to the camera and the light source direction is varied by direction. In the second method, the lighting is kept fixed and the object is rotated relative to the light and camera. In both methods, the light is reflected from a portion of the object surface at different angles. The resulting observation is affected by the portion of the surface relative to the light source.

The surface height is calculated through data known a priori. These techniques rely on a surface normal to help calculate the direction of facets on the surface. If the surface normal points towards the light source, the resulting facet is brighter. If it points away from the light source, the surface facet is darker. These normals can be grouped together to form surfaces relative to other light and dark areas [89]. In the case of a single image, this technique is considered shape from shading [90,91]. Photometric stereo has been explored to obtain surface data from cartridge cases by using Lambert's equations for diffuse light reflectance for a surface [43].

Lambert's equations are used to model diffuse light hitting a surface and being reflected back to the user. In the Lambertian model, objects reflect light uniformly. This property cannot be achieved in real-world scenario. This led to a closely related technique, known as Phong lighting model, that is used to model various reflectance properties of a surface. Phong lighting model also handles specular reflection, the type of reflection caused by shiny surfaces which can include cartridge cases and bullets. This model accounts for the properties of the surface including the shine as well as the viewing angle for the sensor [90]. Phong model has been used to compute surface height information from surface data generated by a confocal microscope [46].

The surface depth has also been modeled using optical focus-defocus techniques. These techniques exploit the highly precise nature of servo motors for focusing a lens in a camera. Positional information on Z depth can be read from these motors when sensing several images of a subject. An estimation of the blur in the image is used to help find when the surface features are in and out of focus. This information can be used to determine the height of a surface [92]. This technique has been used to build the $2\frac{1}{2}$D surface model of cartridge cases [57].

Another technique to model the 3D surface is based on structured light. Here, a black and white pattern of lines is projected onto an object to be scanned. The camera and angle of the projector are kept constant and different frames of varying width lines are projected onto the object one after the other. An image is sensed after projecting each unique line width pattern. The structured pattern appears to be deformed when projected on the surface being scanned. These deformations and the widths of the lines being deformed provide enough information to calculate surface points and the relation of those surface points to each other. This method has been used to recover 3D striae from a surface marked with 6 different screwdrivers. In this approach, thin lines are projected by an LCD projector onto a surface using commercial off the shelf equipment [93].

The filters described in Section 5 can be tweaked to work in 3D as well. For example, the median filter has been used to remove the gross outliers of height data in 3D for a scanned cartridge case [71].

8. Image and shape matching

The preprocessed images can be compared with the previously captured ballistics specimens for potential match. Such a match is typically achieved by using statistical techniques that will be discussed in this section.

A simple quantification for the measure of similarity between two toolmark striae is provided by standard deviation. If the standard deviation between two user-selected striae samples is less than a specified minimum, it is assumed to be a match [38,68,58].

A slightly better method for match is based on Bayesian techniques that are used to determine the likelihood of one hypothesis winning out over another hypothesis. For example, in the world of fingerprints Francis Galton who is credited with first working on individuality of fingerprints gathered 8000 fingerprints as a basis for a proof individuality of a person's fingerprint. However, having a sample of 8000 fingerprints does not prove that *all* fingerprints are unique. It just means that we have 8000 unique fingerprints from a sample size of 8000. Bayesian methods help determine whether a given fingerprint matches the one in the database [94]. These methods use likelihood ratios of prior odds to create posterior odds based on two competing hypotheses with corresponding evidence [95].

Let H_p be a hypothesis that the marks on two samples were made with the same gun and let H_d be the hypothesis that the marks were made with a different gun. Let E indicate the available evidence and I provide the background information. Then, we can denote the a priori odds by $Pr(H_p)/Pr(H_d)$. The likelihood ratio is given by $\frac{\Pr(E|H_p,I)}{\Pr(E|H_d,I)}$, and the posterior odds are given by $\frac{\Pr(H_p|E,I)}{\Pr(H_d|E,I)}$. These are used to compute the Bayesian probability [96,94] as

$$\frac{Pr\left(H_p|E,I\right)}{Pr(H_d|E,I)} = \frac{Pr\left(E|H_p,I\right)}{Pr(E|H_d,I)} \times \frac{Pr\left(H_p\right)}{Pr(H_d)}.$$

Bayesian likelihood ratio has been used to compare consecutively matching striae (CMS) marks on bullets [97,98]. Bunch has discussed the strengths and weaknesses of consecutively matching striae on fired bullets. He gives a good discussion of the problems of subjectivity by the firearms examiner when calculating the Bayesian likelihood ratios for matching striae. He includes a detailed analysis of how Bayesian statistics might be used by a firearms examiner as it relates to a case in a court of law. Also he raises some theoretical questions on barrel wear and how they might affect Bayesian statistics. He goes on to say that the likelihood ratios should be presented to juries instead of posterior odds, and let the jury derive posterior odds, if needed [98]. Curran has described the application of Bayesian likelihood ratios to glass and DNA evidence as well as fingerprints [94].

Aitken and Taroni provide an in depth discussion of Bayesian inference and the likelihood ratio [96]. They provide a good treatise on the pitfalls and fallacies of applying statistical methods to evidence generated from a crime scene. Champod, et al., defend the Bayesian approach for ballistic specimens and consecutively matching striae [95].

Another prominent statistical technique used in pattern classification and analysis is known as Fisher's linear discriminant analysis. It is one of many techniques used to reduce high dimensional data to lower dimensions. The basic idea is to take data in an n dimensional space and project it onto a lower dimensional line and perform an analysis to verify that the data separates into groups. One of the main focuses of this type of analysis is to reduce the time complexity of analyzing data to find patterns [84]. This technique has been used by Thumwarin et al. to help make matches for cartridge case images [85,86].

Groups of data inside an entire data set can also be identified by using cluster analysis techniques [99]. This type of analysis finds the minimum Euclidean distance between a group of points, as exemplified by k-means clustering. The goal of this type of clustering is to partition the data into a set of k clusters such that the sum of squared error over all the k clusters is minimized. This is a computationally expensive operation and known in computer science as an NP hard problem. Multi-dimensional cluster analysis methods have been used by Smith to examine several parameters related to the grouping of ballistics data. He has used many parameters to perform classification such as caliber, direction of twist, and land mark width. He has shown the existence of groupings of data between various ballistic specimens [17].

In some cases, we may be able to get better information on parts of the specimens. These parts can be matched in different ballistic specimen images by using correlation coefficients, especially to verify the match between portions of images or striae patterns [15]. This can be achieved by using a moving window along the sample image. If the window matches another signal in another image, the window may be moved by a random amount along both the images. If the portions still match, then there is a higher probability that the rest of the signal matches as well.

A correlation coefficient ρ is the covariance of two random variables Y_1 and Y_2, divided by the standard deviation σ_1 and σ_2 of the two random variables, respectively [100].

$$\rho = \frac{Cov(Y_1, Y_2)}{\sigma_1 \sigma_2}$$

where the covariance is given by

$$Cov(Y_1, Y_2) = E[(Y_1 - \mu_1)(Y_2 - \mu_2)]$$

and μ is the average of the sample. $E[Y]$ is the expected value of Y.

De Kinder and Bonfanti have calculated correlation coefficients to find matches between 3D scanned profiles of bullet striae [55]. They performed a study with 151 scans and reported correlation coefficients of 70% on 90 of those scans. In a closely related field, Faden et al. used a Pearson correlation method to study marks left by consecutively manufactured screw drivers and found a high correlation value amongst those [101]. However, they noted that other methods should be used in conjunction with this method to find striae matches.

If the correlation coefficient is defined over a set of data it is called a *cross correlation function* (CCF) [102]. Cross correlation functions have been used by many researchers to align cartridge case primers in images [23]. This was performed by generating circles using Hough transform to outline the primer and overall cartridge head, which were then aligned using CCF. CCFs have also been used to find matches for cartridge case images [38,16,68,58,20]. CCFs have also been used to help find similarities between surface profiles of NIST standard reference bullets [103,104,62]. Zhou et al. have used a correlation measure based on the angular similarity of two images of cartridge cases [82].

One of the variations of CCFs is an auto correlation function, computed using a window over a signal's data [102]. Auto correlation has been used to compare the striae profiles of bullets in the NIST standard reference bullets [104,105]. This allows the comparison of profiles of two bullets through a moving window. If the windows match there is a correlation of 1.0 for that portion of the profile. As the window moves the correlation will be much less than 1.

Another statistical technique to compare two independent groups based on a hypothesis is the Mann–Whitney *U* test. It is a non-parametric rank-sum significance test and is significant because it does not assume anything about the shape of the distribution [106,107]. It has been used to compare the correlation coefficients calculated from a sliding window of two images [15].

The shapes can be registered against each other in three dimensions by using a technique known as Iterative Closest Point (ICP) matching. This method chooses pairs of points on the two surfaces. It involves the rotation of one image against another until the overall minimum distance is found using a threshold value [46].

All the techniques described thus far can be utilized to match the bullets and cartridges. Next, we'll examine the use of statistics to show the uniqueness of such matches.

9. Statistics and uniqueness in forensic science

The initial investigations on consecutive matching striae in fired bullets relied on basic statistical techniques [108]. The first such study was based on gathering data on total line count and total matching line or groove counts on 24 .38 caliber Smith and Wesson revolvers. In addition, this study gathered data on the frequency of sets of matching consecutive striae. The analysis focused on computing standard deviation, variance, arithmetic mean, and probability estimates on the data gathered. The study concluded that it could not find more than three consecutive matching striae for bullets fired from different revolvers [108]. Howitt et al. revisited the striae issue and calculated the uniqueness probabilities for striae on fired bullets [109].

Another early study was based on creating a randomized computer model of comparisons of striae on bullets. This study observed that the model is tedious and not of immediate use to examiners [110].

There have been several recent publications, in book form, devoted to the use of statistics in the forensic sciences. However, there is a notable lack of references to cartridge cases or firing pin impressions in these books [111,96].

Bachrach observed that two bullets fired by the same gun may not match with a high confidence. He suggested that the evidence bullet should be compared with a number of control bullets from a gun to establish the fact that the bullet was fired by a suspect gun. Furthermore, the measure of similarity should be defined between a bullet and a gun, as opposed to two bullets fired from the same gun. He recommended the use of two similarity measures: an average similarity of the evidence bullet against all the bullets fired from the same gun, and a peak similarity between the evidence bullet and all the bullets fired from the same gun. However, the problem may be harder because the bullets made of different materials may get a different signature from the same gun [22].

Striature marks are uniformly distributed across a section on the surface of the bullet. If two bullets are fired from the same gun, the probability of error quantifies the fact that the striature marks matching could occur at random. Let n_i give the number of striae on bullet *i*, with *n* indicating the number of striae that match in two bullets within a distance δ. For two given bullets, let $n_1 \leq n_2$. Since the striae match with distance δ, divide the section of the bullet into $m = 1/(2\delta)$ rectangular segments of equal interval. Then, the probability that P_k of k striae, $(1 \leq k \leq n_1)$, match in their position on two bullets is computed as [24]

$$P_k = \frac{\binom{n_2}{k}\binom{m-n_2}{n_1-k}}{\binom{m}{n_1}}.$$

He also emphasized the important property of P_k that $\sum_{k=0}^{n_1} P_k = 1$ indicating that we are certain to match between 0 and n_1 striae on the two bullets. This probability of position of match can be used to compute the probability of error in matching two bullets, P_{error}, based just on position, as

$$P_{\mathrm{error}} = 1 - \sum_{k=0}^{n-1} P_k.$$

The uniqueness of striae using mathematical and computer models for identification of consecutively matched striae has also been reviewed in [112,113]. Some of the statistical analysis for match can also use the techniques developed to match other forensic data such as shoe prints [114] and face recognition using principal component analysis (PCA) and maximum likelihood Gaussian Linear Classification Analysis [115].

The ideas from shoe print identification [114] were extended to other surfaces including impressions left by firing pins on cartridge cases [116]. There has been the use of a grid superimposed on a bullet image as is usually done for shoe print identification. Stone has advocated a grid of 1 mm squares for recording and ultimate calculation of uniqueness. Collins pointed out that 1 mm may be too coarse for uniqueness calculations in the case of bullets and advocates the use of grid squares of size 0.001 in. or 0.0254 mm. A grid with squares of this size can be imposed on the image of the firing pin and recording of unique marks can take place with uniqueness calculations [116].

Another set of similarity measurements is based on a combinatoric counting of the land engraved areas, as in SCICLOPS using a 3D surface profiling system [11]. This system uses calculations and thresholds for hypothesis testing and has inferred that the marks left on bullets were unique for the measurement system used.

In addition to the above techniques, there has been some research on the classification of striae [117,116,70,17,50,51,2,118,52]. Collins modified Stone's original probabilistic model to calculate the uniqueness of random marks on a surface. The marks include points, lines, enclosing shapes, repetitive shapes, and geometric shapes. The criteria for assigning these labels by the investigator to various attributes of an image, as defined for these marks, is outlined in [116].

10. Comments on identification of ballistic and cartridge case

There has been some debate on the merits of matching fired bullets, casings, and the guns used to fire them. First, we'll examine the weaknesses and then the strengths discussed in the literature. Brinck has suggested that the identification of cartridges has not performed up to a high standard of expectation of firearms examiners in systems like IBIS though the cartridge case comparison has done well [44]. At the same time, the 3D imaging is expected to lead to better results. Brinck's study on matching striae in the IBIS using BulletTRAX-3D systems concluded that the performance on copper-jacketed bullets was significantly better than the lead bullets [44]. In fact, the study achieved a 100% match on copper-jacketed bullets.

An investigation on the effects of hardness, grain size, composition, and thickness of the primer cap on breech face marks concluded that there can be a wide variety of repeatability among the impressions left by the same firearm on different primer caps [5]. In a report to the California legislature, De Kinder noted that primer cap seating depth and the pressure generated for this discharge of the primer may affect the quality of marks on the primer cap [8]. However, a later study noted that the seating depth of the primer cap did not have any noticeable impact on the striae left from the breech face [10].

De Kinder et al. have pointed out unacceptable miss rates for the IBIS system during the early parts of the last decade [9]. More recent studies have looked into the hardness of primer cap [5].

Another study noticed that after a barrel is rifled for the first time, the marks left on a bullet change drastically from the first through fifth bullet discharge [119]. The change of land impressions is less perceptible between consecutive firings after this, but it does change to where after the 50th shot, lead deposits drastically reduce positive comparisons. Cartridge cases vary to a lesser extent. This observation leads to the fact that comparing ballistic marks is not like comparing DNA samples because the ballistic marks change over the lifetime of a given weapon [9]. In the laboratory setting, some specimens were created by altering the firing pin between collections of bullet casings over time. However, these specimens led to a positive match in casings over time due to other marks left by the breech on the cartridge case [8].

At present, there is no information on production numbers for every kind of ammunition ever made. That information would be needed to help develop likelihood ratios for CMS of fired bullets as well as the firearms. The barrel wear, over time, may also affect the matches to various guns. At present, the assignment of numbers for likelihood ratios remains a subjective process [98]. The change in bullet signatures over time is also discussed in [104] in addition to an emphasis on the need to create standardized bullet striae used to calibrate IBIS systems across the country.

There has been some work on probabilistic models to compute the uniqueness of identification marks based on an image of the surface [116]. Interpretation of statistics from forensic data may lead to individualization fallacies. This is important in proving the claim of unique individuality in a court of law [120]. There has been some recent research in the validity of the question whether a particular forensic sample is statistically unique [121]. This work asserts that finding statistical uniqueness is a mathematical philosophy and does not prove that the sample is truly unique. It also argues that many of the population sizes are assumed in the uniqueness calculations. It presents uniqueness as a philosophical ideal that may never be obtained [121].

Nichols has defended the role of statistics in forensic ballistic matching [122]. He notes that while some cite the fallacious uses of statistics, for example the “cause to effect” of statistics, there has been work in the use of Bayesian likelihood ratios when referring to forensic evidence [122]. Others have reported positive results using Bayesian methods and likelihood ratios on a test database of known and unknown matches of CMS for fired bullets [97].

Champod et al. outlined a defense of Bayesian methods and the statistics of consecutively matching striae. One of their key points is that the use of probabilities in Bayesian methods is not the same as objectivity. Bayesian methods have been portrayed as comparing two competing hypothesis [95]. In another report, the AFTE examiners performed better than a computerized algorithm for positive matches on flat blade screw driver striature marks left on test surfaces [15].

There has been some effort to create a standard reference for bullets and cartridges. This is partly based on the observation of variability of ballistic specimens fired from the same gun over a period of time. The U.S. National Institute of Standards and Testing (NIST), at the request of the (ATF), created a set of 40 reference bullets that could be used for quality control of the IBIS installations around the United States. This standard is known as SRM 2460 [104]. The standard was created by using a CNC machine with a diamond-tipped cutter to put striae marks on bullets. They have been designed to have a high success rate for matches in the ibis system. There is also a companion standard called SRM 2461 to be used as a standard for cartridge cases [75,123,105,104].

11. Conclusion

In this paper, we have consolidated the information from many different sources of information for image processing, image matching, and the uniqueness of marks on ballistics specimens. Image processing for examining ballistic specimens is clearly at an early stage and there is still some debate in the literature on the validity of paths to take when evaluating evidence. In addition, we have presented the current work on statistical analysis of ballistic evidence with an eye on its suitability in a court of law.

Appendix A. Daubert v Merrell Pharmaceuticals

In 1993, the U.S. Supreme Court handed down a ruling on expert testimony. This ruling states that in order for an expert's testimony to be admitted in court, it has to satisfy four criteria. First, has the opinion by the expert been tested? Second, is there an error rate for the technique in the opinion and is this error rate acceptable by other scientists in the literature? Third, has the technique been published in a peer reviewed journal? And fourth, is the technique generally accepted by other scientists in the field? The threshold for such testimony can be whether the scientific knowledge expressed by an expert witness has been subject to peer review in scientific publications. This is also referred to as the Daubert standard [4].

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