Statistical Thinking for Forensic Practitioners

Quiz on Part 8: Analyzing & Interpreting Forensic Evidence

We will practice applying the two methods for assessing forensic evidence discussed in the Part 8 lecture notes: the two-stage and likelihood ratio approaches. We will consider a study performed by Carriquiry, Daniels, and Stern from 2000 in which five trace element concentrations (antimony (Sb), copper (Cu), arsenic (As), bismuth (Bi), and silver (Ag)) were measured for 200 bullets manufactured by Cascade Cartridge Industries (CCI). The 200 bullets were obtained from four boxes, two of which had the same packaging date. The goal of the study was to quantify the evidence that a bullet fragment found at a crime scene came from the same box (i.e., same source) as a bullet found on a suspect using their trace element concentrations. Let x and y represent the trace element concentrations for the bullet and bullet fragment, respectively (meaning x, y together represent the "evidence", E).

1. Formulate two competing hypotheses regarding the box of origin for the bullet fragment and bullet similar to those given on slide 8 of the lecture slides.

   The same source hypothesis S would mean that x and y are two sets of measurements originating from a single box. The different source hypothesis $\bar{S}$ would mean that x and y are two sets of measurements originating from different boxes.

1 The likelihood approach

2. Using the notation you introduced in question 1, re-express the odds in favor of the same source hypothesis given the evidence x, y in terms of the likelihood ratio and prior odds using Bayes' Theorem (similar to slide 55).

   $\frac{p(S|x,y)}{p(\bar{S}|x,y)} = \frac{p(x,y|S)}{p(x,y|\bar{S})} \frac{p(S)}{p(\bar{S})}$

3. As the forensic evidence domain expert, identify which term(s) in the expression given in question 2 you are accountable for. If you were a member of the jury, which term(s) would you be accountable for? Explain.

   It is the job of the forensic evidence domain expert to quantify how likely a piece of evidence under the same source and different source hypotheses. That is, an expert testimony should establish the likelihood ratio $\frac{p(x,y|S)}{p(x,y|\bar{S})}$. In contrast, it is the job of a juror to come to a decision regarding the same source and different source hypotheses (choose between one of them). The prior odds $\frac{p(S)}{p(\bar{S})}$ quantifies one's belief regarding the validity of the same source hypothesis over the different source hypothesis before observing any evidence. Similarly, the "posterior odds" $\frac{p(S|x,y)}{p(\bar{S}|x,y)}$ quantifies one's belief regarding the validity of the same source hypothesis over the different source hypothesis after observing the evidence (i.e., learning about the likelihood ratio from the expert testimony).

4. Identify what must be true about the prior odds for the likelihood ratio to be equal to the odds in favor of the same source hypothesis given the evidence. What does this imply about the prior probabilities on the two hypotheses $P(S)$ and $P(\bar{S})$?

   It would need to be true that $\frac{P(S)}{P(\bar{S})} = 1$. This would imply that $P(S) = P(\bar{S})$.

5. Given your answer to question 3 (i.e., who is accountable for the prior odds), why might the condition you identified in question 4 not be favorable under the "presumption of innocence" legal principle?

   Defendants are assumed "innocent until proven guilty" under the presumption of innocence principle. Assuming that the same source hypothesis at least supports the claim that a defendant is guilty (i.e., if a bullet fragment found at a crime scene is determined to match a bullet found on a defendant, then there's greater evidence that the defendant is guilty), then we would prefer that a juror's prior probability that the same source hypothesis is true be less than that of the different source hypothesis. In other words, before observing any supporting or contradicting evidence, a juror believes stronger that the bullet fragment and bullet do not share the same source than that they do. This would better align with the presumption of innocence principle.

A law of probability (specifically, the definition of conditional probability/likelihood) says that we can express the likelihood ratio as

$$LR = \frac{p(x, y|S)}{p(x, y|\bar{S})} = \frac{p(y|x, S)}{p(y|x, \bar{S})}\frac{p(x|S)}{p(x|\bar{S})}$$

6. Interpret $\frac{p(y|x,S)}{p(y|x,\bar{S})}$ in the context of the problem and explain why it might be useful to determine which hypothesis is more supported by the evidence. (Hint: Slide 68 might be useful.)

   The numerator quantifies how likely it is to observe the trace element concentration of the bullet fragment given it is from the same box as the bullet. The denominator quantifies how likely it is to observe the trace element concentration given they are not from the same box as the bullet. This ratio captures our question of interest: are the bullet fragment and bullet from the same box? If this value is greater than 1, then there the evidence supports the same source hypothesis. If it is less than 1, then the evidence supports the different source hypothesis.

7. Why might it be plausible to assume $p(x|S) = p(x|\bar{S})$?

   The trace element concentration obtained from the suspect's bullet doesn't (or at least shouldn't) depend on whether the suspect was the source of the bullet or not. For example, it's safe to assume that the suspect hasn't done anything to the bullet to alter its trace element concentration.

Assuming $p(x|S) = p(x|\bar{S})$ means that $\frac{p(x|S)}{p(x|\bar{S})} = 1$ and the likelihood ratio can be expressed as

$$LR = \frac{p(y|x, S)}{p(y|x, \bar{S})}.$$

Carriquiry, Daniels, and Stern (2000) point out that, due to common manufacturing practices, bullets manufactured from different vats of molten lead may end up in the same box. These vats are assumed to have differing levels of the 5 elements measured, meaning bullets manufactured from the same vat tend to be more similar than bullets manufactured from different vats. This complicates the problem of identifying bullets coming from the same box. Using the Law of Total Probability, they re-express the likelihood ratio as

$$LR = \frac{p(y|x, \text{ same vat})p(\text{same vat}|S) + p(y|x, \text{ different vat})p(\text{different vat}|S)}{p(y|x, \text{ same vat})p(\text{same vat}|\bar{S}) + p(y|x, \text{ different vat})p(\text{different vat}|\bar{S})}$$

where now "same vat" and "different vat" refer to the hypothesis that the bullet fragment and bullet came from the same and different vat of molten lead, respectively. These are now likelihoods that can be estimated using the data and knowledge of the manufacturer (e.g., how many vats they use).

8. Explain the difference between $p(y|x, \text{ same vat})$ and $p(y|x, \text{ different vat})$.

   $p(y|x,$ **same vat) quantifies how likely it is to observe the trace element concentration of the bullet fragment given it is from the same vat as the bullet.** $p(y|x,$ **different vat) is the same except if the bullet fragment and bullet are from different vats.**

9. Interpret $p(\text{same vat}|S), p(\text{different vat}|S), p(\text{same vat}|\bar{S})$, and $p(\text{different vat}|\bar{S})$ within the context of the problem.

   p(same vat$|S$) is the probability that the bullet fragment and bullet came from the same vat given they are from the same box.

   p(different vat$|S$) is the probability that the bullet fragment and bullet came from different vats given they are from the same box.

   p(same vat$|\bar{S}$) is the probability that the bullet fragment and bullet came from the same vat given they are from different boxes.

   p(different vat$|\bar{S}$) is the probability that the bullet fragment and bullet came from different vats given they are from the different boxes.

Carriquiry, Daniels, and Stern (2000) calculate all of these likelihood "pieces" for a bullet with trace element concentrations $x = (10.221, 5.202, 4.571, 5.157, 4.032)$ and a bullet fragment with trace element concentrations $y = (10.219, 5.195, 4.536, 5.157, 4.065)$. While we will skip over precisely how these are calculated (we're more interested in the interpretation), suppose that the likelihood pieces are given by

$$\begin{aligned}
p(x|y, \text{ same vat}) &= 172223 \\
p(x|y, \text{ different vat}) &= 196.4 \\
p(\text{same vat}|S) &= .67 \\
p(\text{different vat}|S) &= .33 \\
p(\text{same vat}|\bar{S}) &= .27 \\
p(\text{different vat}|\bar{S}) &= .73
\end{aligned}$$

10. Calculate the likelihood ratio using the values given above. Using the table on slide 86 of the lecture slides, provide a verbal summary of how strongly the evidence support the same source hypothesis.

    $LR = \frac{.67(172223)+.33(196.4)}{.27(172223)+.73(196.4)} = 2.48$**. Summary: "The forensic findings provide weak support for the same source proposition relative to the different source proposition."**

11. A juror is confused by the summary your provided in question 10 and asks: "So you're saying that there is at least some support that the bullet fragment and bullet come from the same box based on this evidence?" How would you respond to this juror?

    The juror's question is framed as a judgement on the validity of the two hypotheses. This is not the job of the expert witness, who instead helps the jury understand how probative a particular piece of evidence is towards one of the two hypotheses (e.g., even if the bullet fragment and bullet were found to be indistinguishable, how common is it to measure two indistinguishable trace element concentrations?). A different way to phrase the above summary might be: "The similarity between the two pieces of evidence is explained slightly better by the same source hypothesis than the different source hypothesis."

2 The two-stage approach

We now consider the two-stage approach. As stage 2 of the two-stage approach is very difficult to implement in practice (e.g., requires a large reference database of the appropriate population), we will only cover some potential issues that we might need to consider if we were to actually perform stage 2.

For simplicity, we consider the following slightly altered situation. Suppose now that we will base our conclusions by only comparing the average antimony concentration from 5 measurements taken from the bullet fragment and bullet. We would like to determine whether the *true* average antimony concentration for the bullet fragment, μ_f say, is different from that of the bullet, μ_b. We might conclude that the two bullets came from the same vat if a difference wasn't found. The 5 measurements from the bullet fragment are

$$1.2145, 1.5049, 2.1582, 2.3771, 0.6584$$

and the 5 measurements from the bullet are

$$2.4529, 0.5609, 2.8233, 0.7805, 5.8222.$$

The sample standard deviations of these two data sets are .670 and 2.113, respectively.

12. Use the information provided above to conduct a hypothesis test at the $\alpha = .05$ level to answer our research question (are the mean antimony concentrations different?). Your answer should include null and alternative hypotheses for the population parameters of interest, a test statistic, a determination of a critical value or p-value (or both), and a conclusion.

    The hypotheses are $H_0 : \mu_f = \mu_b$ vs. $H_a : \mu_f \neq \mu_b$. The sample means for the two groups are $\bar{x}_f = 1.814$ and $\bar{x}_b = 2.488$. The standard errors associated with these two sample means are $s_f^2 = \sqrt{\frac{.67}{5}} = .366$ and $s_b^2 = \sqrt{\frac{2.113}{5}} = .650$. Then the t statistic is $t = \frac{1.814-2.488}{\sqrt{.366^2+.65^2}} = -.904$. Under H_0 we compare this to a t distribution with $n_1 + n_2 - 2 = 8$ degrees of freedom, which has a critical value (using the `T.INV` function) of 2.306. Because $-.904$ is within the "fail-to-reject" region of $[-2.306, 2.306]$, we conclude that there is insufficient evidence to conclude that the two mean antimony concentrations are different. The two-sided p-value associated with this is $p = .50$, which is above $\alpha = .05$ which yields the same conclusion.

13. Based on the results of the hypothesis in question 12, should we proceed to the second stage of the two-stage approach? Explain

    Yes, the hypothesis test carried out in question 12 provides evidence that the bullet fragment and bullet are matches (i.e., come from the same source). Thus, we will want to move on to stage 2.

14. How would you explain to a juror the purpose of stage 2 of the two-stage approach?

    Answers will obviously vary. The key thing to note here is that, although we found evidence that the bullet fragment and bullet match, we did so on (1) a small data set and (2) using only one variable, the antimony concentration, to make our conclusion. These are two considerations (among others) we should make when considering the *probative value* of the evidence found in question 12 (i.e., are the findings practically significant?). Stage 2 would give us a way to quantify how likely it is to observe a coincidental match in the larger population of interest (presumably under the same conditions as in question 12).

15. Suppose the crime from which the bullet fragment was obtained happened in a small, rural town. The only ammo vendor in this town is connected to a local ammo manufacturer and thus sells only this manufacturer's product. Explain how this would affect the population of interest on which you would base stage 2 compared to, say, a crime that occurred in a large city.

    The population of interest for the small-town crime would presumably be bullets from the local manufacturer. It would be much harder to pin-down the population of interest in a larger city, where we might have to consider a much larger set of manufacturers, suspects, etc.

16. The local manufacturer is constantly receiving new shipments of raw materials to manufacture bullets. How might this fact affect how you construct your representative sample of the population of interest for stage 2?

    Optimally, we would like to only consider bullets that were manufactured close to when the suspect's bullet (and bullet fragment) were manufactured. The probability of a coincidental match may change over time if the manufacturer regularly receives new shipments of raw materials.