Statistical Thinking for Forensic Practitioners

Quiz on Part 8: Analyzing & Interpreting Forensic Evidence

We will practice applying the two methods for assessing forensic evidence discussed in the Part 8 lecture notes: the two-stage and likelihood ratio approaches. We will consider a study performed by Carriquiry, Daniels, and Stern from 2000 in which five trace element concentrations (antimony (Sb), copper (Cu), arsenic (As), bismuth (Bi), and silver (Ag)) were measured for 200 bullets manufactured by Cascade Cartridge Industries (CCI). The 200 bullets were obtained from four boxes, two of which had the same packaging date. The goal of the study was to quantify the evidence that a bullet fragment found at a crime scene came from the same box (i.e., same source) as a bullet found on a suspect using their trace element concentrations. Let x and y represent the trace element concentrations for the bullet and bullet fragment, respectively (meaning x, y together represent the "evidence", E).

1. Formulate two competing hypotheses regarding the box of origin for the bullet fragment and bullet similar to those given on slide 8 of the lecture slides.

1 The likelihood approach

2. Using the notation you introduced in question 1, re-express the odds in favor of the same source hypothesis given the evidence x, y in terms of the likelihood ratio and prior odds using Bayes' Theorem (similar to slide 55).

3. As the forensic evidence domain expert, identify which term(s) in the expression given in question 2 you are accountable for. If you were a member of the jury, which term(s) would you be accountable for? Explain.

4. Identify what must be true about the prior odds for the likelihood ratio to be equal to the odds in favor of the same source hypothesis given the evidence. What does this imply about the prior probabilities on the two hypotheses $P(S)$ and $P(\bar{S})$?

5. Given your answer to question 3 (i.e., who is accountable for the prior odds), why might the condition you identified in question 4 not be favorable under the "presumption of innocence" legal principle?

A law of probability (specifically, the definition of conditional probability/likelihood) says that we can express the likelihood ratio as

$$LR = \frac{p(x,y|S)}{p(x,y|\bar{S})} = \frac{p(y|x,S)}{p(y|x,\bar{S})} \frac{p(x|S)}{p(x|\bar{S})}$$

6. Interpret $\frac{p(y|x,S)}{p(y|x,\bar{S})}$ in the context of the problem and explain why it might be useful to determine which hypothesis is more supported by the evidence. (Hint: Slide 68 might be useful.)

7. Why might it be plausible to assume $p(x|S) = p(x|\bar{S})$?

Assuming $p(x|S) = p(x|\bar{S})$ means that $\frac{p(x|S)}{p(x|\bar{S})} = 1$ and the likelihood ratio can be expressed as

$$LR = \frac{p(y|x, S)}{p(y|x, \bar{S})}.$$

Carriquiry, Daniels, and Stern (2000) point out that, due to common manufacturing practices, bullets manufactured from different vats of molten lead may end up in the same box. These vats are assumed to have differing levels of the 5 elements measured, meaning bullets manufactured from the same vat tend to be more similar than bullets manufactured from different vats. This complicates the problem of identifying bullets coming from the same box. Using the Law of Total Probability, they re-express the likelihood ratio as

$$LR = \frac{p(y|x,\ \text{same vat})p(\text{same vat}|S) + p(y|x,\ \text{different vat})p(\text{different vat}|S)}{p(y|x,\ \text{same vat})p(\text{same vat}|\bar{S}) + p(y|x,\ \text{different vat})p(\text{different vat}|\bar{S})}$$

where now "same vat" and "different vat" refer to the hypothesis that the bullet fragment and bullet came from the same and different vat of molten lead, respectively. These are now likelihoods that can be estimated using the data and knowledge of the manufacturer (e.g., how many vats they use).

8. Explain the difference between $p(y|x,\ \text{same vat})$ and $p(y|x,\ \text{different vat})$.

9. Interpret $p(\text{same vat}|S), p(\text{different vat}|S), p(\text{same vat}|\bar{S})$, and $p(\text{different vat}|\bar{S})$ within the context of the problem.

Carriquiry, Daniels, and Stern (2000) calculate all of these likelihood "pieces" for a bullet with trace element concentrations $x = (10.221, 5.202, 4.571, 5.157, 4.032)$ and a bullet fragment with trace element concentrations $y = (10.219, 5.195, 4.536, 5.157, 4.065)$. While we will skip over precisely how these are calculated (we're more interested in the interpretation), suppose that the likelihood pieces are given by

$$\begin{aligned}
p(x|y,\ \text{same vat}) &= 172223 \\
p(x|y,\ \text{different vat}) &= 196.4 \\
p(\text{same vat}|S) &= .67 \\
p(\text{different vat}|S) &= .33 \\
p(\text{same vat}|\bar{S}) &= .27 \\
p(\text{different vat}|\bar{S}) &= .73
\end{aligned}$$

10. Calculate the likelihood ratio using the values given above. Using the table on slide 86 of the lecture slides, provide a verbal summary of how strongly the evidence support the same source hypothesis.

11. A juror is confused by the summary your provided in question 10 and asks: "So you're saying that there is at least some support that the bullet fragment and bullet come from the same box based on this evidence?" How would you respond to this juror?

2 The two-stage approach

We now consider the two-stage approach. As stage 2 of the two-stage approach is very difficult to implement in practice (e.g., requires a large reference database of the appropriate population), we will only cover some potential issues that we might need to consider if we were to actually perform stage 2.

For simplicity, we consider the following slightly altered situation. Suppose now that we will base our conclusions by only comparing the average antimony concentration from 5 measurements taken from the bullet fragment and bullet. We would like to determine whether the *true* average antimony concentration for the bullet fragment, μ_f say, is different from that of the bullet, μ_b. We might conclude that the two bullets came from the same vat if a difference wasn't found. The 5 measurements from the bullet fragment are

$$1.2145, 1.5049, 2.1582, 2.3771, 0.6584$$

and the 5 measurements from the bullet are

$$2.4529, 0.5609, 2.8233, 0.7805, 5.8222.$$

The sample standard deviations of these two data sets are .670 and 2.113, respectively.

12. Use the information provided above to conduct a hypothesis test at the $\alpha = .05$ level to answer our research question (are the mean antimony concentrations different?). Your answer should include null and alternative hypotheses for the population parameters of interest, a test statistic, a determination of a critical value or p-value (or both), and a conclusion.

13. Based on the results of the hypothesis in question 12, should we proceed to the second stage of the two-stage approach? Explain

14. How would you explain to a juror the purpose of stage 2 of the two-stage approach?

15. Suppose the crime from which the bullet fragment was obtained happened in a small, rural town. The only ammo vendor in this town is connected to a local ammo manufacturer and thus sells only this manufacturer's product. Explain how this would affect the population of interest on which you would base stage 2 compared to, say, a crime that occurred in a large city.

16. The local manufacturer is constantly receiving new shipments of raw materials to manufacture bullets. How might this fact affect how you construct your representative sample of the population of interest for stage 2?