A Historiography of the Geometric Analysis of Muqarnas

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The Muqarnas Form and History

The muqarnas is a three dimensional architectural form unique and widespread to the Islamic world. It emerged around the 11th century AD. Its origins have been debated, with various theories about points of emergence and whether these origins are interrelated or independent. Proposed centers of origin include 10th century Iran, 11th century North Africa, and 11th century Baghdad.[1] Regardless, by the 12th century, muqarnas had become a wide spread architectural form adopted by the Islamic world and it continued to evolve in complexity and application in the centuries onwards. Different styles of muqarnas, classifiable through the geometry of their patterns and the complexity of their designs, reached peaks in certain monuments, such as how the square style[2] or 45 degree muqarnas block family[3] was carried out in the 14th century Alhambra ceilings. Yet new styles would emerge and continue to evolve elsewhere, such as the Shirazi style muqarnas with stellate and pentagonal designs[4], as construction techniques and design techniques created different organizing principles and geometries. The breadth and spread of muqarnas demonstrates a common architectural language that the Islamic world could design in and yet the transmittance of its construction and the evolution of these geometric understandings around the Islamic world remain a mystery to scholars. Muqarnas, an architectural form unique and birthed utterly from the Islamic world, still contain many mysteries in origins, etymology, and typological characteristics which many scholars have tried to answer in their studies.

My paper looks to how a variety of scholars, from mathematicians to art historians to computational architects, have dealt with the question of the geometry and formal qualities of the muqarnas. With little historical material left on their existence, creation, conception, and

[1] Al-Asad, 349
[2] Takahashi
[3] Sakkal, 21
[4] Al-Kashi via Dold-Samplonius 1992, 226

design, like many Islamic architectural monuments, we are left with puzzling over the buildings without written design records or the plans with little to no annotation. I will trace our changing and emerging geometric understanding of muqarnas in hopes of illuminating a little more about this important architectural form.

Two Dimensional into Three Dimensional: The Muqarnas Form and Definition

In order to speak about how different scholars conceptualized muqarnas, it is important to look at a number of examples of what muqarnas are considered to be. The enormous variety in material, application, location, and style makes it a difficult form to pin down. Muqarnas have been built in brick, plaster, tile, wood, and stone, and have been found in vaults, domes, column capitals, archways, niches, and so forth. Left with such breadth, it is the geometry of the muqarnas that scholars turn to in order to give it a specific definition. There have been descriptions that call the muqarnas "stalactite vaults"[5], "stairlike arrangments"[6], and "repeated units with a concave surface"[7].

For the moment, for the conception of muqarnas, we will provide an alternate, more formal definition, as given by Oleg Grabar in the book, "Alhambra", and still widely referenced among scholars in the field[8]:

> "…from the moment of its first appearance, the muqarnas acquired four characteristic attributes, whose evolution and characteristics form its history: it was three dimensional and therefore provided volume wherever it was used, the nature and depth of the volume was left to the discretion of the maker; it could be used both as an architectonic form, because of its relationship to vaults, and as an applied ornament, because its depth could be controlled; it had no intrinsic limits, since not one of its elements is a finite unit of composition and there is no logical or mathematical limitation to the scale of any one composition; and it is a volume which could be solid or void, a projecting mass of complex

[5] Dold-Samplonius 2004, 709
[6] Al-Asad, 349
[7] Sakkal, 21
[8] Yaghan, 27, although he takes issue with some of Grabar's definitions, such as the ideas of no limits and that the muqarnas form can possibly be a void.

shapes or a complex outline – a three dimensional unit which could be resolved into a two-dimensional outline."[9]

Perhaps the most important line in the geometric study of muqarnas over the last fifty years was the last one, where muqarnas can be resolved from their three dimensional form into a two dimensional plan. After all, muqarnas embed in their plans almost all the knowledge requisite to build them for the artisan skilled and experienced in their construction. Muqarnas are planar projections and it is through the understanding of both drawings and object that scholars have managed to draw their geometric analysis today.

Orientalist First Studies, the Survey Methodology, and Major Discoveries

In the beginnings of the study of muqarnas by Western scholars, much of the early writing was mainly concerned with questions of origin, historical value, or aesthetic decorative value. Examples include the general surveys that Owen Jones completed in his The Grammar of Ornament in which he documented the muqarnas ceilings for the Alhambra in 1856[10] or the focused surveys done by Rosintal's *Pendentifs, trompes et stalactites dans l'architecture oriental* published 1928 to argue how muqarnas were formed and spread. These surveys were based on field research or previous surveys with often little to no reference to each other and studied muqarnas in the context of individual ornamental phenomena. At best, early scholars sometimes looked at the units used to construct the muqarnas as a manner to understand their geometric complexity, as Jones broke down the muqarnas in the Alhambra into wooden unit blocks.

Even the term muqarnas has clouded origins in relation to the forms it describes. Muqarnas as a technical and architectural term first arises in the 11th century, arguably after some of its earliest incarnations. Yet muqarnas was not a word associated with its root, *qrns,* in dictionaries of medieval Arabic, because these dictionaries mainly recorded the Arabic language of pre- and early Islamic times[11]. Western scholars often inserted their own terms to explain

[9] Grabar, 178
[10] Jones, See Figure 1.
[11] Al-Asad, 349

these forms they had never encountered, with descriptions that demonstrated their own difficulties in rationalizing the form, such as stalactites, honeycombs, or cell-like squinches.

The reason I posit for their lack of unified theory is both a dearth of written primary sources as well as a lack of methodology from which muqarnas could be analyzed and studied. In terms of contemporary written accounts of historical muqarnas, there is but one extant example of discussing muqarnas in a direct manner in al-Kashi's Miftah al-Hisab, Key to Arithmetic. This text remained inaccessible to Western scholars until the 20th century translations into Russian and English. On the other hand, the lack of methodology is in part a result of the difficulty in rationalizing muqarnas from only its architectural three dimensional form.

While the three dimensional form of muqarnas is the crucial gestalt of the form, the inability to study the system and therefore muqarnas on a large scale like any architectural monument necessitates two dimensional portrayals in the form of plans. The lack of historical plans from which muqarnas could be analyzed and studied played into this dearth of theory. These aforementioned difficulties hint a little at the lack of proper scholarly attention to other avenues of study outside the ahistorical, orientalist, and ornamental perspective. For example, Paccard endeavored to show through an anthropological methodology in the 80s the lineage of historical muqarnas construction through the practices of contemporary Moroccan craftsmen.[12] These craftsmen have pattern books that have passed down muqarnas plans into the modern day. The masters have enormous amounts of tacit and practical construction knowledge about how to create muqarnas vaults in a style descended from the *mudejar,* similar to the Alhambra.

Paccard is but one example of the diversification of methodology that emerges in the 20th century and employed by scholars interested in muqarnas. The interdisciplinary breadth of research that thus emerges out of the 20th century onward has allowed for more

[12] Necipoglu, 20. Of interest, Necipoglu says this on the subject of contemporary muqarnas building about Clarke, who brought the Mirza Akbar scrolls, contemporary plans for master masons in Iran, back to the Victoria and Albert research: "Although an "anthropological" approach whereby historical building techniques are deduced from contemporary practices may be methodologically suspect, the modern eyewitness accounts.. do help us visualize how the radial muqarnas patterns of the Topikapi and Tashkent scrolls, which were for the most part intended to be cast in plaster, would have been translated into spatial forms." (46)

comprehensive historic and geometric understandings of *muqarnas* in comparison to the field in its earlier days.

Survey and Formal Classification

One scholar who attempted to rectify the academic survey methodology with analysis and sheer scope in the last forty years was Shiro Takahashi. From the Tama Art University, in 1978, he had documented over a thousand muqarnas and over 600 plans, taken all from existing muqarnas in the Islamic world and has placed these plans on an online database. His written scholarship remains a mystery as he has only one documented article in the Tama Art Bulletin, published in 1978, and an enormous annotated online database. He inspired a large line of academic research especially in the computational field, who have used his plans in order to create systems of constructing 3D muqarnas forms out of 2D plans[13]. Departing from the more piecemeal survey done by scholars before him, Takahashi created three large style categorizations based on the geometry of these hundreds of plans in order to classify muqarnas on an unprecedented scale. His scholarship can be connected with the Necipolglu's creation of a chronology of evolving muqarnas styles, even though she does not mention nor seem to have known of Takahashi's research.

The first is the square style, which utilizes 45 degree rhomboids, squares, and deconstructions of these two shapes. This style was by far the most wide-spread, Takahashi claims, and it peaked in the 14th century, reaching its zenith with the Alhambra's muqarnas ceilings. Square style muqarnas were built with whatever the local materials were and have been seen in brick, stone, and plaster. Its restricted pool of geometric pieces made such craftsmanship adaptions possible. The geometry of its plan could be characterized as "a nonperiodic tiling tessellation… [with] fourfold rotational symmetry"[14], due to its square module basis.

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[13] Yaghan received guidance and grant from the Tama Art University to write his thesis on Muqarnas and in his book cites Shiro Takahashi as one of the scholars involved in his research from the university.
[14] Takahashi

The second style is pole table style, which was centered in the geographic middle east of modern day Iran, Iraq, and Afghanistan. It rose to prominence in the 15th century under the developmental auspices of the Timurid Period, and continued to expand into 17th century across the region. Its key features include stellate polygons arranged at "at the concentric circle coordinates. The star shapes consist not only of 4, 5, 6, and 8 point stars, but also of 7 and 9 point stars, among others." The primary materials were plaster, stucco, and mosaic tiles, which reflects the local construction practices of the region. Necipolglu claims the flexibility of material like plaster to be carved in fact influenced the ability for such stellate based designs and non-square module plans to be produced in these regions[15]. The pole table style, Takahashi said, brought new levels of geometric complexity with different features of false arches, characteristics of dripping, and a departure from orthogonal organizational symmetries.

His final category was a nebulous miscellaneous group that included other muqarnas-like forms such as those with diamond tiling, mixed geometry, and designs that he had difficulty placing in either of his two categories. The category is mainly of interest to demonstrate the applicability to muqarnas to architecture outside of vaults and domes as well as the broad possibility of muqarnas-like surfaces departing from more canonical, historical accepted forms.

Takahashi's scholarship is perhaps the largest public access to muqarnas plans. His attempt to categorize muqarnas plans in broad temporal-spatial strokes, based all on survey data, is the only one I found. As one of the last large studies with a survey based methodology in the 20th century, its focus on looking at muqarnas plans and using the two dimensional representations as a way to quickly sketch out understandings of regional and temporal evolutions of muqarnas geometry is similar to the plan based methodologies that would follow.

Tracing the Present Lineage of Study

The current movement of English written scholarship has not favored the survey and field research method to study muqarnas and has split into two lineages that feed into one another. The first is concerned with the drawings and plans of the muqarnas. Scholars analyzed

[15] Necipoglu, 44

muqarnas through decoding and writing about the historical plans from sources like the Topkapi scrolls by Necipoglu and the Tashkent Scrolls by Notkin. The juxtaposition of these drawings upon historical knowledge as well as geometric analysis allow for larger understandings of historical muqarnas: how their designs are achieved, and then possibilities for their transmittance. These drawings were not specific to existing built muqarnas but act as tools for their design and construction like pattern-books for master craftsmen. The lack of elevation drawings, annotations, and physical three dimensional counterparts demonstrate how these drawings' audience were for those already familiar and comfortable with reading muqarnas plans and translating them into finished architectural forms[16]. This methodology of study has been facilitated by the new discoveries and writings on 2D plans such as those in the Topkapi Scroll, the Tashkent Scrolls, the Takht Sulayman plate, and others.

The second is a computationally based methodology in which computation and architects are creating software based systems out of historical research and their own theories. The creation of muqarnas models in CAD and other computer representations aims to create new definitions and analyze geometry through the process of creating explicit representations. These computational architects often have formal interest in muqarnas as a complex geometric surface and look for ways to parameterize the perceived form. Through generating theories of making and borrowing from other fields, these digital muqarnas designers hope to use new technological tools to better understand the geometry of muqarnas overall.

The two methodologies borrow from one another heavily through their interdisciplinary scholastic collaborations, as demonstrated by historical scholars like Al-Asad that create computational models of muqarnas or computational architects like Yaghan who compile extensive historical research to develop the argument of their muqarnas building programs. What led the post 1970s era to be so fruitful and interdisciplinary include a number of factors, but mainly the increased material scholars have had to work with, both in primary and secondary sources. The discovery of the Takht-I Sulayman plate, a gypsum plate with the earliest known muqarnas plan inscribed, can attributed to Harb in a German excavation in

[16] Notkin, 153

1968[17]. Dold-Samplonius who translated the relevant portions of al-Kashi's Key to Arithmetic, was the first to write about its contents relating to muqarnas in 1992[18]. An entire line of Uzbekistan scholarship out of the Soviet era interested in the muqarnas drawings of the Tashkent Scroll and Islamic monuments in Central Asia was brought to Western scholars with the publishing of Notkin's paper on decoding the drawings in English in 1995[19]. The Topkapi Scroll which had never before been published was written, analyzed, and documented extensively in Necipoglu's book in 1995.

These key discoveries led to a changing approach in studying muqarnas from ornamental historical three-dimensional objects to a formal, geometric analysis with the aid of plans. Much of the late 20th and 21st century scholarly works have been using plans as platforms to revise, understand, and come to new systems of analyzing muqarnas. It is only in the latter half of the 20th century did the focused geometric study of muqarnas could actually move past its scattered, individual survey focus.

Drawing Based Methodology: Interpreting Al-Kashi and Decoding Plans

One of the definitions that has been most pervasive and referenced in studies of geometric analysis was given by the 15th century mathematician, al-Kashi. His book, published in Samarkand on March 2nd, 1427, Miftah al-hisab (Key to Arithmetic), contains the only contemporary written account about muqarnas as an architectural form, found at the end of a chapter. The book was intended to be a practical handbook for calculation, a companion treatise, for artisans already familiar with the muqarnas form. Al-Kashi wrote about muqarnas in its idealized geometric design of which there would be great interest for artisans to be able to understand mathematically, demonstrating a key relationship between mathematicians and artisans working in contemporary of each other.

[17] Dold-Samplonius 2005, 88, See Figure 2.
[18] Ibid, 87
[19] Necipoglu, 9. The work done of the Tashkent scrolls demonstrated a number of claims in the studies of muqarnas by Notkin and his predecessor, Bulatov, tying the style to the forms of the Timurid designers. Suited for the medium of plaster, the movement away from orthogonal grids to radial ones with star units was echoed in the Timurid design sensibilities. See Figure 3.

The intertwining of mathematicians with artisans has a long history of precedence in the world of geometric design, of which muqarnas can be considered a large subject of. Sarhangi analyzes how Abu al-Wafa Buzjani, a mathematician in the 10th century, wrote a famous treatise, *On Those Parts of Geometery Needed by Craftsmen,* in order to demonstrate how a craftsman would tessellate and tile a dome. Buzjani, like al-Kashi, writes mathematical processes and logic to demonstrate how to create spherical Platonic and Archimedean solids, with the implication that the learned artisan would already be familiar the side of construction and making and need only to now translate that mathematical abstract knowledge to their skills. In Buzjani's treatise, the spherical solids and their divisions are drawn in two dimensional plan, despite their complexity as three dimensional subdivided objects. Both Sarhangi and Necipoglu assert the ability to read two dimensional plans and understand them as a complete three dimensional form was the norm for any craftsman that was to be a master. The ability was critical for a master working in architectural projects in the medieval Islamic world.[20] While Buzjani does not write about muqarnas directly, Sarhangi uses Buzjani to demonstrate a methodology in the study of architecturally based Islamic geometric designs. He says that, "Even though observers today may focus on harmony and symmetry, the above explanations present evidence for the direct involvement of mathematicians, based on scientific approaches that were available at that time for the creation of such works."[21] Sarhangi's paper, published in 2008, makes a claim to tie the mathematical proofs and understandings to the development of geometric design, of which we will examine further with al-Kashi and muqarnas.

Of note, Necipoglu builds her argument on the importance of muqarnas plans and the interwoven nature of mathematicians and artisans by bringing up the historical method in which artisans were paid. Artisans were paid through calculating the material used in construction and the manner it was employed in work, such that, Necipoglu theorizes, an extremely practical mutual relationship could be established between these mathematicians

[20] Shelby, quoted by Necipoglu, 45. The quote was originally about European medieval stonemasons and their relationship to two dimensional plans, that given a fixed design language about projection from two dimensions, medieval architects worked primarily with plans, unlike their modern or Hellenistic counterparts. Necipoglu asserts a similarity in the medieval manner of working in Europe with the world of medieval Islamic architecture.

[21] Sarhangi, 523

and these artisans. The calculations of payment, like the mathematical proofs in the treatises, were done to idealized models of an object's geometry, as the improvisation of artisans at the construction site due to imprecise building or changed plans is noted by both mathematician and historian.[22]

Muqarnas, being a form that can be mathematically rationalized and written in a two dimensional form, must also be subject to the worlds of interrogation as a 2D design construction as well as an architectural form. Like his predecessor, Buzjani, al-Kashi also wrote in close relation to the artisans and craftspeople his treatise was for. His geometrically engaged definition is less than intuitive and mimics the practice of masons, whose ability to read plans and construct muqarnas were passed down through experience as opposed to written introductions and learnt from book practice. Translated here into English by Yvonne Dold-Samplonius, his definition is:

"The muqarnas is a roofed [vault] like a staircase with facets and a flat roof. Every facet intersects the adjacent one at either a right angle, or half a right angle, or their sum, or another than these two. The two facets can be thought of as standing on a plane parallel to the horizon. Above them is built either a flat surface, not parallel to the horizon, or two surfaces, either flat or curved, that constitute their roof. Both facets together with their roof are called one cell. Adjacent cells, which have their bases on one and the same surface parallel to the horizon are called one tier. The measure of the base of the largest facet is called the module of the muqarnas."[23]

This definition is one that belies the geometric complexity of the muqarnas and the intentions of al-Kashi in discussing them. Al-Kashi spends the rest of the chapter describing the four typological categories of muqarnas, which are based on the variations of facets and roofs, his names for the measurements in a muqarnas unit, and the mathematics behind the existing practices of the masons. His diagrams are all plan based except for one diagrammatic section of

[22] Both Al-Kashi and Necipoglu state this in reference to medieval architectural projects.
[23] Dold Samplonius 1992, 226, quoting al-Kashi, fol. 83v, 1.8

the curvature of a muqarnas cell.[24] He provides four categories of muqarnas, based entirely on a muqarnas cells' geometry.

The first is simple, in which the muqarnas cells' interior facets and roofs are made of planes. The second is clay-plastered in which the height of the tiers might differ, and that some tiers may only have roof and no facets.[25] The third category is called curved muqarnas. While curved muqarnas have the same square based geometry like simple muqarnas, the roofs of the cells are curved. Between the roofs of two adjacent cells, a curved surface can either be a triangle or two triangles. Finally the last is the Shirazi Muqarnas which employs more geometric forms in its plans than the basic forms defined by al-Kashi. He called their measures "innumerable" and the possibility for roofs include "triangles, squares, pentagons, hexagons, star polygons, and so forth, both flat and curved"[26]. It is as calculable as any other muqarnas, however. Takahashi's Pole Style Muqarnas is a rudimentary definition that correlates with al-Kashi's definition and most scholars henceforth refer to these stellate styles, 15th century onward designs as Shirazi, such as al-Asad, Notkin, and others.

First, let's discuss a little of the exact geometric definitions al-Kashi gives in the speaking of the "unit" of the muqarnas. Al-Kashi identified a set of basic muqarnas elements within his definition. Dold Samplonius translates it as thus: A muqarnas element can be either a cell or an intermediate element. A cell is two facets and a roof placed together. The facets and roofs refer to the walls of each cell and the variations of geometric forms in the plan lead to different cells and intermediate elements. These geometric forms include basic elements such as the square, rhombus, half rhombus, and also self-named shapes like a deltoid, a small biped, a jug, a large biped, and barley kernels.[27] Every shape is a derivation from the square or rhomboid excepting the barley kernels which are upper vault filling elements that stretch to accommodate the constrictions of each vault. Each of these geometric forms is related to each other

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[24] See Figure 4.

[25] Of note, al-Kashi himself says the clay-plastered typology is mostly found in ancient muqarnas examples in Isfahan, and notes them as different but does not provide more mathematic measurement or proof for the style, saying it is analogous to the simple roof muqarnas.

[26] Al-Kashi, provided by Dold-Samplonius 1992, 230

[27] See Figure 4.

proportionally through the module of the muqarnas which is the length of the side of the square.[28]

Dold-Samplonius has used his treatise as a manner to decode and interpret muqarnas plans like the Takht-I Sulayman Plan. She also provides her own insight into al-Kashi's conclusion on muqarnas, stating that the four types he outlines were all just derivations of a singular form that already tended to encourage artisan improvisation, resisting categorization of larger styles. The curvature of the roof and its possible geometric forms was but the variable that separated the types. A particular interesting quote by her is that "photogrammetry measures the object as it really is, whereas al-Kashi measures the object as it should be, slight irregularities are not important here."[29] From the type of research Dold-Samplonius as well as other plan based historians conducted on understanding muqarnas, we can conclude both some limitations and some freedoms of a plan based geometric analysis.

In what ways are the third dimension and the physical objects important in this analysis? If the cells were adjusted and squashed and lost their perfect angles during the construction of the physical muqarnas, such knowledge would never be apparent through only the consultation of the plans. The question of the overwhelming importance of the plan also becomes suspect. Despite Sarhangi and Dold-Samplonius' assertions, the lack of historical evidence to characterize the nature of these plans as design beginnings, middles, or ends leave them in a liminal space regarding their relationship to the built muqarnas. Given the nature in decoding these plans required a body of shared graphic and architectural knowledge and codification did these muqarnas drawn plans reflect the transmittances of muqarnas themselves? Necipoglu supports the primacy of the plan through the ties to different medieval ways building and knowing and also asserts three dimensionality of the form was but an obvious point for al-Kashi and his contemporaries. One of the important issues in translating the plans to three dimensions remains the height of the cells and the curve of the cell. Al-Kashi provides a drawing called the "method of the masons"[30] where its height is twice its width and a certain factor

[28] Dold-Samplonius 2005, 85-86
[29] Dold-Samplonius 1992, 225
[30] Dold-Samplonius 2005, 87

curve. Dold-Samplonius concludes it is taken from practice and therefore flexible to the needs of the artisan.

Al-Asad acknowledges the difficulty of both interpreting and concluding muqarnas from just the plan. Working with the Topkapi Scroll, he analyzes a plan to construct a possible Shirazi style muqarnas form as a 3D model[31]. The primary geometric elements include four, five, and eight pointed stars over a radially arranged composition and given the drawings, he can demonstrate the steps in both how to draw and construct based on such a muqarnas plan. He views al-Kashi's analysis as guidelines for which the modern day scholar must still make their own inferences and geometric interventions in order to achieve a coherent muqarnas, such as resolving the questions of irregularity in plans and inconsistencies in symmetry. Al-Asad tries to look at the muqarnas plan on a "different level of perception, one in which the gestalt is ignored and the individual unit becomes the primary focus of attention."[32] In the process of translating the muqarnas plan into a three dimensional form, Al-Asad is able to reach a new understanding of radially organized, symmetrical Shirazi muqarnas, their geometric properties and constraints, as guided by al-Kashi.

However, in response to all these systems of decoding and reading based on al-Kashi, Yaghan finds al-Kashi's definition affected by shortcomings such as the lack of definition of the intermediate elements he names and his lack of elaborations on Shirazi style units. Yaghan in general criticizes many previous scholars on their unit analysis for missing elements, a vague definition, and a conflation of geometric units versus erection units. These critiques in part come from his definitions of muqarnas units meant to best facilitate his computational universal muqarnas model and play a large part in his computationally based methodology.

Computational Based Methodology: A Question of Units

The history of computationally based methodology in scholarship is intertwined with the desire to design and make muqarnas forms. The nature of this methodology places a large burden on the ideas of muqarnas unit definition and parameterization. In order to accomplish

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[31] See Figure 5.
[32] Al-Asad, 350

these ends, these scholars have conducted their own analysis of muqarnas geometry and create their own theories about the form. Yaghan, published in 2001, follows a long lineage of those interested in generating muqarnas, especially through surface modeling and digital models upon the arrival in the information age. Yaghan's rejection of al-Kashi opens up a new avenue of how computational avenues can create different kinds of geometric analysis. The geometry in a three dimensional modeling environment can now be interrelated with construction- different than the questions of historical or real world construction. Freed from those immediate constraints, it is possible to come up with different geometric and algorithmic lenses to understand muqarnas.

In 1988, Mamoun Sakkal created his own definitions for a geometric analysis. His paper on the geometry of muqarnas domes does a number of services at once. It attempts to explain how domes would be designed in plan, what are the units that define the construction of a muqarnas, and consider new geometric definitions of the purpose of a muqarnas. He would later on create the FormWriter, a software that tries to computationally generate muqarnas algorithmically, mostly as enclosed domes. This perhaps demonstrate a different focus on the three dimensional form in his research. For Sakkal, muqarnas are said to be small niche-like structures joining together to create the structure, as a manner of enclosing and dividing space.

The manner in which he creates a unit is to define a muqarnas block[33]. The curvature is mostly irrelevant and he simplifies it into 45, 90, and 135 degree angles for simulation, not unlike the appearance of simple muqarnas as defined by al-Kashi. Sakkal views muqarnas as a geometric tool to enclose space and the patterns are arrived at by a composition within the constraints of the dome. Through its division on paper[34], the plan of the muqarnas emerges. The blocks are neither a unit of exact construction nor a single unit in the plan, which is different than all the readings of the unit we have thus seen. The geometric subdivisions here relate best to understanding the muqarnas as a three dimensional continuous form. As Sakkal puts it:

[33] See Figure 6.
[34] See Figure 7. It is interesting to note how intermediate elements as defined by al-Kashi are but an after- note in a plan generation hinging mostly on the niches.

"The blocks used in a composition are related to each other in precise geometry. In regular compositions, the angle of the block's base is increased by increments of one full angle in each successive block. For example, the 45 degrees family of muqarnas blocks has three blocks with 45, 90, and 135 degree…"[35]

From this frame work, the interior surfaces of the block now have multiple possibilities for single curvature, double curvature, planes, and such and parameterizing these is just an additional step. In all of these computational systems, we see analysis in the form of making, based on the writings of those who decoded the historical plans. The interweaving of these disciplines seems necessitated to move into a comprehensive understanding of muqarnas.

Yaghan criticizes the conflation of construction or erection units and geometric units when analyzing muqarnas. He creates an entire new system at different grouping tiers. The most basic element is called the "unit-surfaces" and "roof-patches". These elements have different parameterization qualities depending on their direction and curvature, however, the most important aspect is, as their name suggests, they are the basic surfaces, planes and curves, broken down as elementary units, that make up the muqarnas surface. In grouping these elements together, one can create a "visual unit". "Visual-units" are "unit-surfaces" together that create a form described as a visual niche, squinch, or, by al-Kashi's descriptions, the roofs as well as the two facets. Finally, a "layer-group" is composed of "visual units" composed along their side edges to form a layer or a tier on the same horizontal plane. Finally the combinations of "layer-groups" on top of one another create the "muqarnas-group" or form.[36]

Yaghan's definitions allow him to extract and create a program that, through the subdivisions of layer lines in the plan, a three dimensional profile curve, and a given height of the structure, can automatically generate a three dimensional muqarnas model. The extreme parameterization of muqarnas units as surfaces in a computational model is what allows for this ease of translation. As a visual representation and a formal exercise in analyzing and breaking

[35] Sakkal, from his notes on Computational Geometry in Islamic Architecture, a course he taught at the University of Washington, 2001
[36] Yaghan, 34-35. See Figure 8.

apart the gestalt of muqarnas, the use of making can provide new systems to understand geometrically, survey, and re-examine existing historical plans. Yet, computational and making-based scholarship in isolation suffers from its difficulties in relating to the realities of history. Once the programmer creates a parametrization for the muqarnas system, the system remains often inflexible else prone to reductionist maneuvers. The model is a visual representation, an end point of reconstruction, with little information embedded about the construction techniques and what informed these forms to begin with, without additional research surrounding the topic.

Concluding Remarks

Geometric analysis and the process of geometric making are not necessarily a one to one relationship. But when speaking about muqarnas, the importance of speaking about both in order to never divorce the historical context of Islamic artisans from the properties of the three dimensional remain paramount. The interdisciplinary methods of these scholars make these principles more pertinent to muqarnas than any other art object discussed in this class. We look not just at art, architecture, or ornament, but an entire system of mathematics, artistic making, and design, at a level of complexity which Notkin says is “the manifestations of a culture of unsurpassed synthesizing order and sophisticated imagination”[37]. The modern day scholarship as it evolves further to better analyze the muqarnas form in some way reflects the interdisciplinary nature of historical muqarnas. As only through the crucial network of connection between the mathematicians and the craftspeople, the historians and the architects, can the creation and understanding of muqarnas be realized.

[37] Notkin, 153

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Appendix

Figure 1: (left to right) Muqarnas Pendentive (Goury and Jones, 1836-45, vol. 1, Plate 10). The plan is drawn as understood by construction units to an observer. Notice the inscribed curvature within the plan.

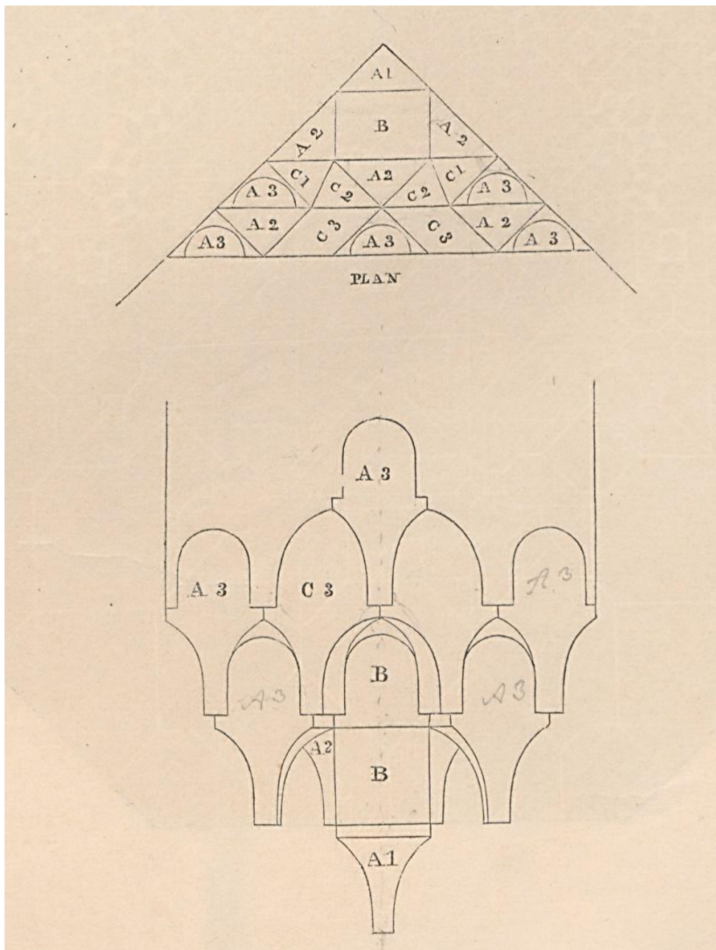
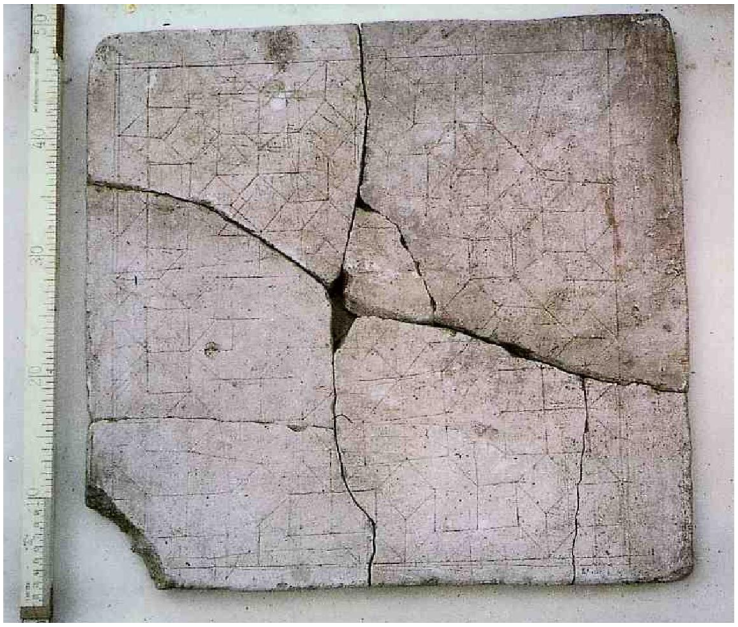


Figure 2. Takht-I Sulayman Plate. Below are a sample of interpretations by Harb and Dold-Samplonius.



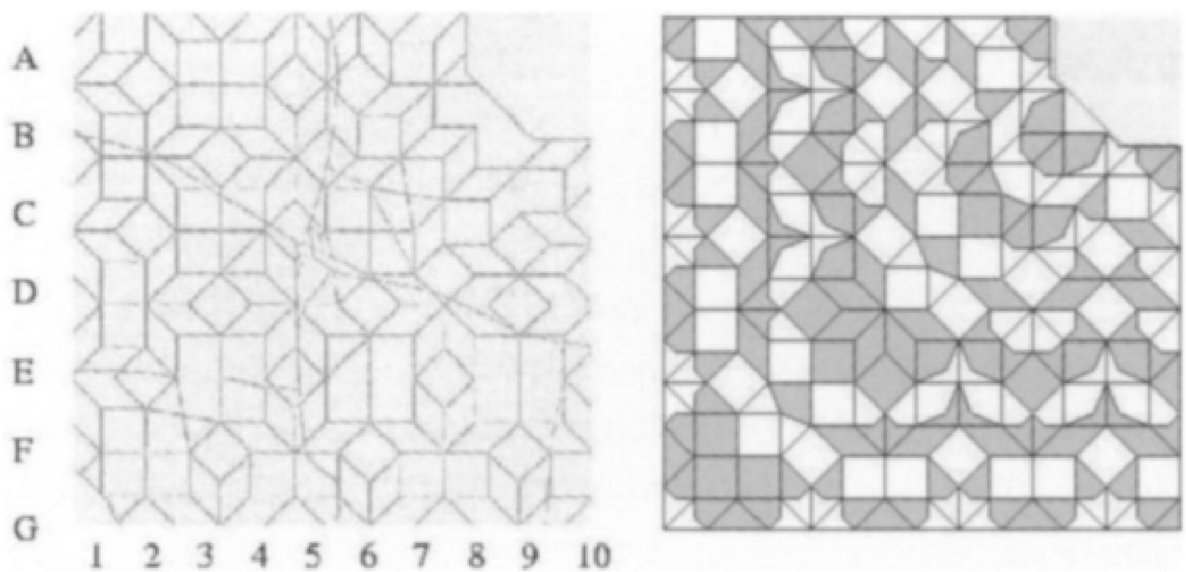


Fig. 9. Left: design of the plate, as read by Harb. Right: our interpretation.

Figure 3. Tashkent Scrolls, from 16th century. From Notkin, 161. Demonstrates the manner in which the plans were decoded and modeled. The alternating color schema on the original muqarnas drawings refer to an architectural drawing and graphic convention well established in and after the Timurid Period.

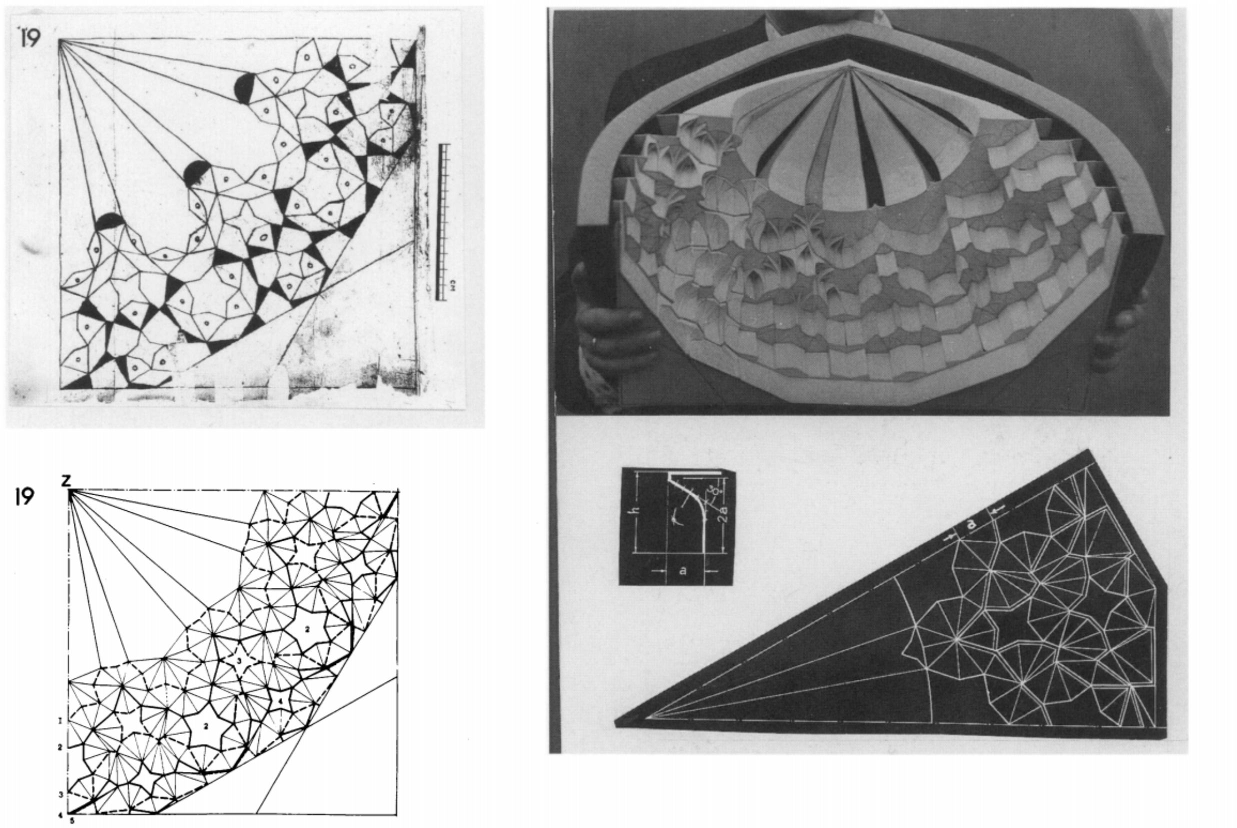
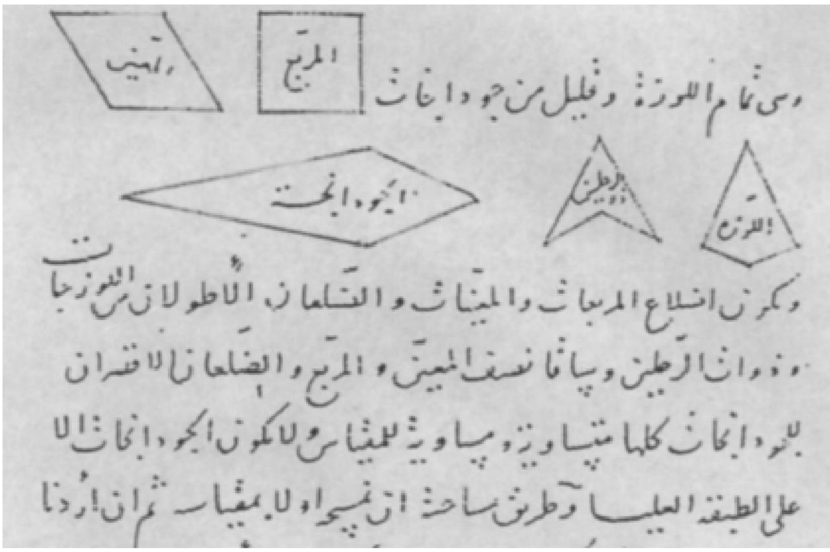
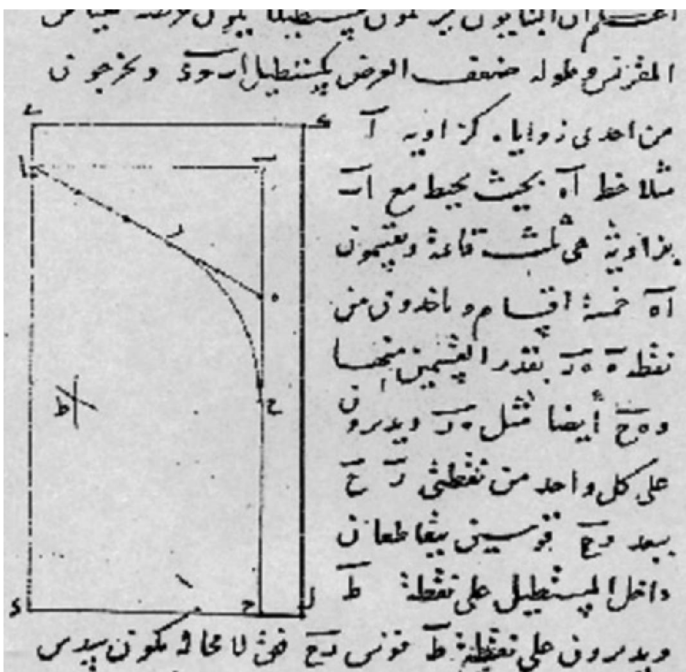


Fig. 15. One quarter of a five-row muqarnas vault on intersecting arches (drawing 19). Alongside are a more elaborate working sketch and a paper and cardboard model of a half-dome, illustrating the various phases of the preparation of the stalactites.

Figure 4. (left to right) (1) Plane projections of the Cells, as given by al-Kashi in *Key to Arithmetic.* From left to right, top to bottom, the rhombus, square, barley kernel, small biped, and almond shame. (2) Curve, “Method of the Masons”, described by al-Kashi in Key of Arithmetic. Taken from Dold-Samplonius and Harmsen’s Practical Arabic Mathematics. (Malek Library in Tehran, ms. 3180/1)



(1)



(2)

Figure 5. Mamoun Sakkal's Muqarnas Blocks from the 45 degree block family and variations, generated in FormWrite. Sakkal's blocks transcend the 'tier' based unit analysis being done by other scholars, and each block in 2D thus self-completes into the rhombus or square form. See the colored plan for details.

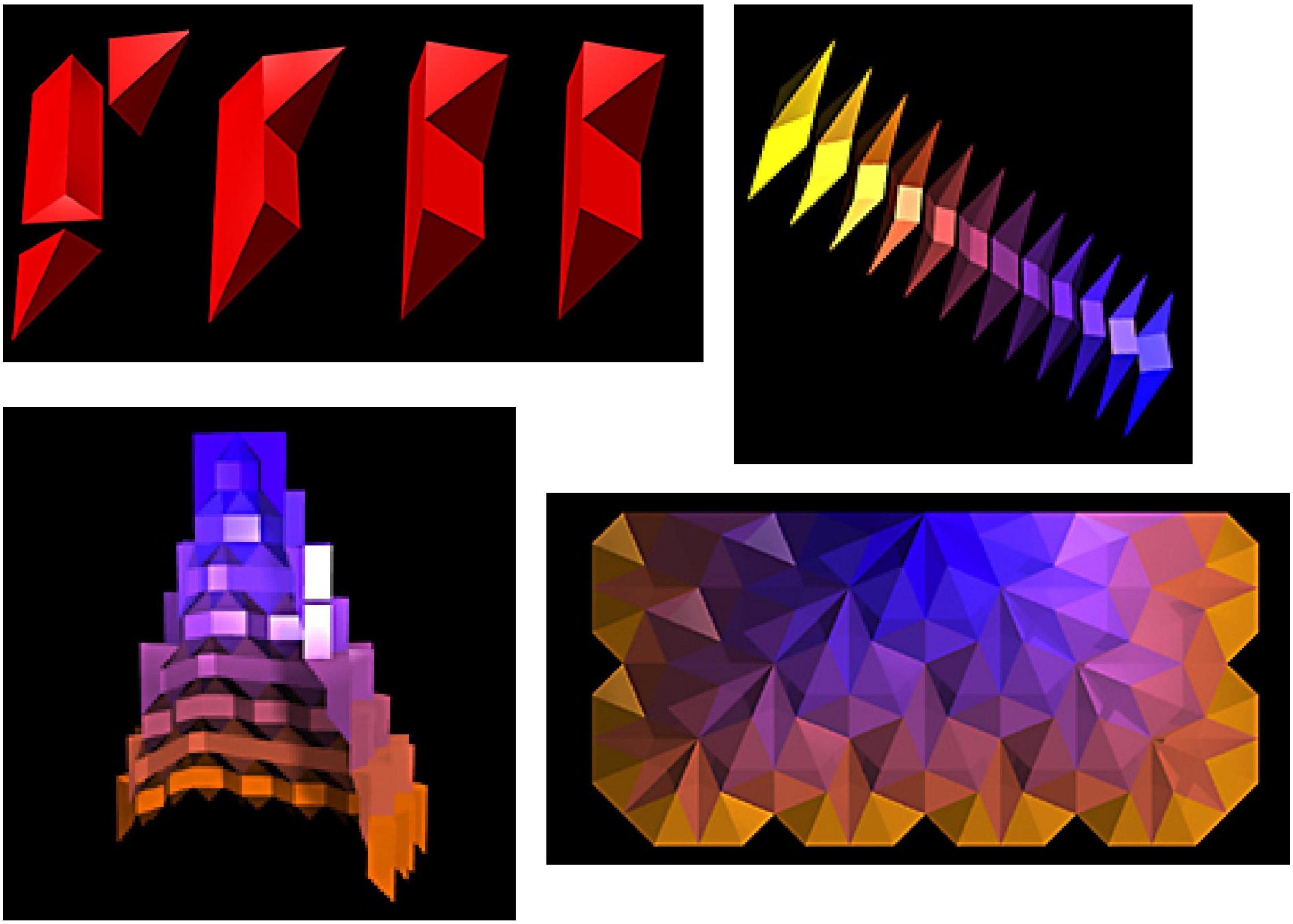
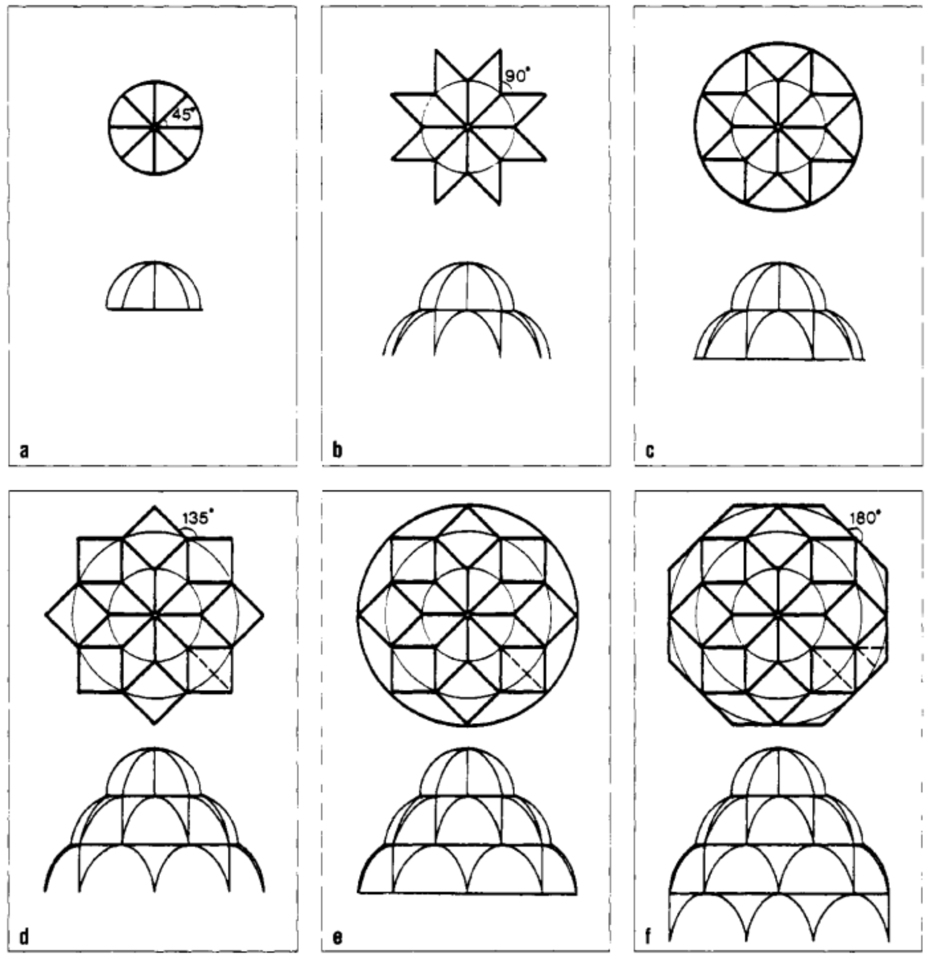


Figure 6. Sakkal's Dome Dividing Method, 25.

FIGURE 4

Generating muqarnas blocks by Dome-dividing method.

Méthode de division des dômes pour engendrer des blocs muqarnas.



* Les blocs de muqarnas réguliers sont ceux engendrés par la méthode de division des dômes. En ce qui concerne les blocs semiréguliers et irréguliers, on pourra consulter (Sakkal 1982) p. 100–113.

Figure 7. (top to bottom, left to right) Yaghan's Unit Subdivision System. (1) Unit-Surfaces. Three types, Edge to point (ETP), Point to Edge (PTE), and Edge to Edge (ETE). (2) Roof patches. Defined by being mostly flat and analogue to their appearance on the plan. (3) Layer lines. (4) A Layer-Group, a set of visual-units composed of unit-surfaces in order to form a layer resting on the same horizontal plane.