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CS 310

Assignment 0904

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Problem 1 — Use the definitions to prove or disprove the statement $2n^2 + 3 \in O(n^3)$, and illustrate this graphically.

Answer: The definition requires us to find c and n_0 so that $2n^2 + 3 \leq cn^3$ when $n \geq n_0$. Based on the definition n and c are both non-negative constants. Since we have:

do not use \\ in paragraph mode

\$n\$ \$c\$

$$2n^2 + 3 \leq cn^3$$

formatting -2

$$\frac{2n^2 + 3}{n^3} \leq c$$

\$n\$

And raise the degree of all other terms as the highest one including constants with n.

$$\frac{2n^2 + 3}{n^3} \leq \frac{2n^3 + 3n^3}{n^3} \text{ (for all } n_0 \geq 1)$$

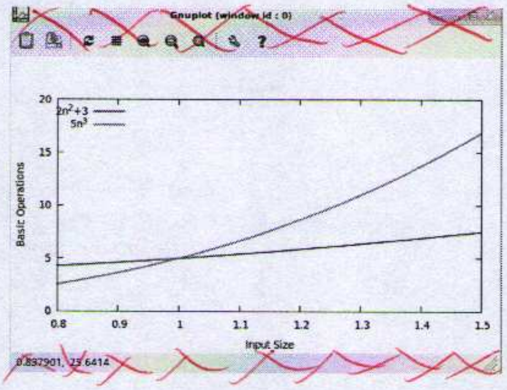
Obviously, the right-hand will be definitely higher or equal to the left-side for all $n_0 \geq 1$. Then, cancel all the terms and end with a constant:

$$\frac{2n^2 + 3}{n^3} \leq 5 \text{ (for all } n_0 \geq 1)$$

$$c = 5 \text{ (for all } n_0 \geq 1)$$

Therefore, we conclude that $2n^2 + 3 \in O(n^3)$ is true since we have $2n^2 + 3 \leq 5n^3$.

This is graphically illustrated by the following plot that shows $2n^2 + 3$ along with the standard function n^3 scaled by the constant coefficient $c = 5$, for all n $\geq$ 1.



Just the plot

Figure 1: The graph shows y = $2n^2$ + 3 and y = $5n^3$

Problem 2 — Either prove the following assertion using the definitions or disprove it with a specific counterexample:

$$\text{if } T(n) \in O(S(n)) \text{ then } S(n) \in \Omega(T(n))$$

don't say it twice

Answer: If T(n) ∈ O(S(n)) then there is a constant c which is great and equal to zero (c ≥ 0), and a constant n_0 which that n is great or equal to n_0(n ≥ n_0), where we can conclude:

$$\text{T(n)} \leq c \times S(n)$$

why is this in text mode?

We can divide both sides of the expression by the constant c, then we have:

$$\frac{1}{c} \times T(n) \leq S(n) \text{ (for } \frac{1}{c} > 0 \text{ and n} \geq n_0.)$$

Sometimes T(n) is in text mode, some-times in math mode. why? You must be consistent. T(n) is a math element.

\$k\$

Therefore, there is a constant $\frac{1}{c}$. which can be replaced by constant k. Then we can conclude:

$$S(n) \geq k \times T(n)$$

for k > 0 and n ≥ n_0, which is exact definition of S(n) ∈ Ω (T(n)).

Problem 3 — For the following algorithm, explain what it computes, state what the input size for analysis is, state what basic operations should be counted for analyzing it, state exactly how many operations are executed as a function of the input size, and state the efficiency class to which it belongs.

```
1  void foo(vector<unsigned>& array)
2  {
3    size_t n = array.size();
4    for (size_t pass_indx = 0; pass_indx < n - 1; pass_indx++)
5    {
6      size_t min_position = pass_indx;
7      for (size_t compare_indx = pass_indx + 1; compare_indx < n;
8           compare_indx++)
9      {
10       if (array.at(compare_indx) < array.at(min_position))
11       {
12         min_position = compare_indx;
13       }
14     }
15
16     if (min_position != pass_indx)
17     {
18       swap(array.at(pass_indx), array.at(min_position));
19     }
20   }
21 }
```

Answer: This algorithm is used to sort the elements in the vector from minimum to maximum. Consider line 3, which defines `n`. This allows us to see that the input size is the size of the vector.

For the worst cases (Big-O), the operations that are counted are:

- the assignment on line 3, we count as 1 operation,
- the for loop condition on line 4, we count as 2n operations,

\$2n\$

$n-1$

these are math elmts

- the `min_position` assignment on line 5, we count as (n-1) operations,
- the second for loop condition on line 7 and 8 will are counted as (n)+(n-1)+(n-2)..+2 number of times, which is $\frac{(n+2)(n-1)}{2}$, and have two operations each time, so we count (n+2)(n-1) number of operation,
- the if condition on line which is comparison on line 10, we count as (n-1)+(n-2)+...1, which is $\frac{n^2-n}{2}$,
- the assignment of `min_position` inside the if statement on line 12, we count as (n-1)+(n-2)+...1, which is also $\frac{n^2-n}{2}$,
- the `min_position` comparison in if condition on line 16, we count as (n-1) operations,
- and the swap function on line 18, we count as 2(n-1).

After sum up all, the number of times of all operations that are executed is:

$$2n^2 + 6n - 5$$

For the best cases (Big-Ω), the operations that are counted are:

- the assignment on line 3, we count as 1 operation,
- the for loop condition on line 4, we count as 2n operations,
- the `min_position` assignment on line 5, we count as (n-1) operations,
- the second for loop condition on line 7 and 8 will are counted as (n)+(n-1)+(n-2)..+2 number of times, which is $\frac{(n+2)(n-1)}{2}$, and have two operations each time, so we count (n+2)(n-1) number of operation
- the if condition on line which is comparison on line 10, we count as (n-1)+(n-2) +...1, which is $\frac{n^2-n}{2}$,
- the assignment of `min_position` inside the if statement on line 12, we count as 0 based on best case.
- the `min_position` comparison in if condition on line 16, we count as (n-1) operations,
- and the swap function on line 18, we count as 0 times, since this is best case.

After sum up all, the number of times of all operations that are executed is:

$$n^2 + 5n + \frac{n^2 - n}{2} - 1$$

Since there are two if conditions which could cause the best and the worst cases. Therefore, we can find out that this algorithm belongs to Big-Oh and Big-Omega efficiency class.

$$T(n) \in O(n^2) \text{ for c = 4, all n} \geq 0$$
$$T(n) \in \Omega(n^2) \text{ for c = 1, all n} \geq 0$$

or equivalently,

$$T(n) \in \Theta(n^2)$$

Problem 4 — Write a C++ program that implements the algorithm in problem 3, counts the number of basic operations, and outputs the input size and the count of basic operations to the cerr stream.

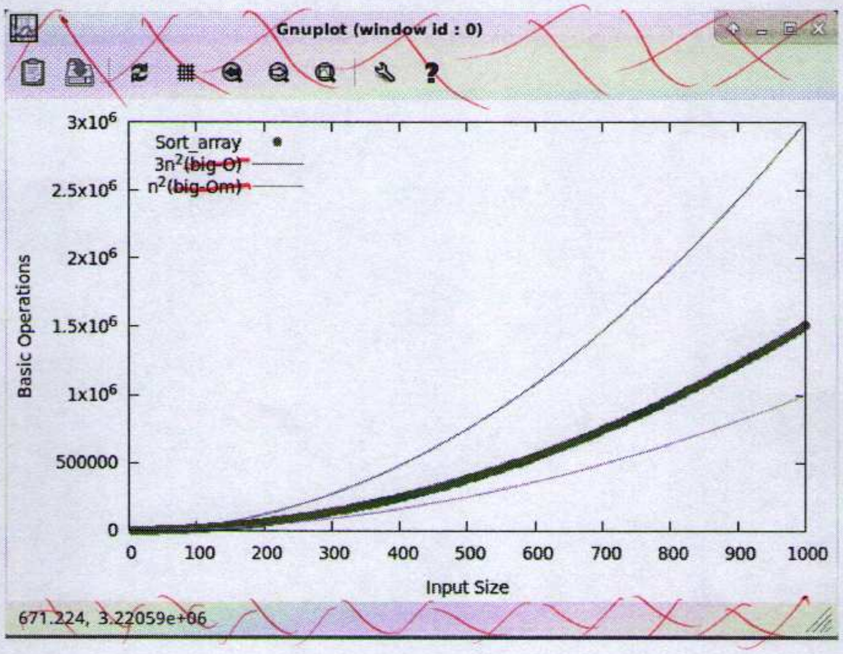
Answer: See the submitted file HM.cpp

Problem 5 — Run your program from problem 4 many times with many different inputs and capture the results. Using the output of the, create a plot of input size vs. basic operations, along with one or more standard functions properly scaled, to illustrate the analysis you obtained in Problem 3.

Answer: When the program is run with the command

```
for n in `seq 10 10 1000`
do
    ./program $n > /dev/null
    ./program $n > /dev/null
done 2> results.dat
```

and the resulting data file is plotted, we get the following. Also plotted on the same axes are the scaled standard functions $3n^2$ and n^2 which illustrate that the algorithm is big-theta.



```cpp
/**
 * A program that will sort a vector and calculate the number of operations in
 * the algorithm.
 * @author Bill Jin
 * @version 2 September 2020
 */
#include <iostream>
#include <chrono>
#include <random>
#include <vector>

using namespace std;

/**
 * sort all the elements in a vector in an ascending order
 * @param array is the vector to be ordered
 */
void foo(vector<unsigned>& array);

int main(int argc, char** argv)
{
  //Get the random elements to assign to vector.
  const unsigned MAX_RANDOM_VALUE = 1000000;
  default_random_engine get_next_value(static_cast <unsigned>
                  (chrono::system_clock::now().time_since_epoch().count()));
  uniform_int_distribution<unsigned> uniform(0, MAX_RANDOM_VALUE);

  //Use dummy to avoid warning
  int dummy = argc;
  dummy++;

  //The replacement for magic number.
  const unsigned INPUT_SIZE_INDEX = 1;
```

X this makes program harder to read

```cpp
  unsigned long input_array_size = stoul(argv[INPUT_SIZE_INDEX]);
```

never use long. Use size_t

```cpp
  //Declare a vector.
  vector<unsigned> random_array(input_array_size);

  //Use for loop to assign each element in vector.
  for (size_t i = 0; i < input_array_size; i++)
  {
    unsigned random_value = uniform(get_next_value);

    random_array.at(i) = random_value;
  }

  //Output the elements.
  for (unsigned& element : random_array)
  {
    cout << element << " ";
  }
  cout << endl;

  foo(random_array);

  return 0;
}

void foo(vector<unsigned>& array)
{
  //Magic number replacement.
  const unsigned ZERO = 0;
  const unsigned TWO_INCREMENT = 2;
```

X this makes code harder to read

```cpp
  //Declare counter variable to count.
  unsigned counter = ZERO;
```

supposed to be uint64_t

```cpp
  size_t n = array.size();
  counter++;
  for (size_t pass_indx = 0; pass_indx < n - 1; pass_indx++)
  {
    counter += TWO_INCREMENT;
```

```cpp
    size_t min_position = pass_indx;
```

missing counter

```cpp
    for (size_t compare_indx = pass_indx + 1; compare_indx < n;
        compare_indx++)
    {
      counter += TWO_INCREMENT; //count for loop condition twice a time.

      counter++;
      if (array.at(compare_indx) < array.at(min_position))
      {
        counter++;
        min_position = compare_indx;
      }
    }
    counter += TWO_INCREMENT; //The for loop is count two more time as it ends

    counter++; // Count for if condition.
    if (min_position != pass_indx)
    {
      counter~~++~~; //count for swap function.
      swap(array.at(pass_indx), array.at(min_position));
    }
  }
  counter += TWO_INCREMENT; //The for loop is count two more time.

  cerr << array.size() << " " << counter << endl;
}
```

+= 2

counters -1