Statistical Inference I - Lecture 1

Hanchao Zhang

Definition

Sample Set

Ω = Sample Space = Set of all possible outcomes

Example

Sample Set

$\Omega = \mathcal{R}^1$ or $\Omega = \mathcal{R}^2$ or $\Omega = \mathcal{R}^n$

It could be used to representing number of variables

Definition

σ-field

Let $\mathscr{F}$ = a collection of subsets of Ω

$\mathscr{F}$ is a σ-field **iff**

1. $\emptyset \in \mathscr{F}$
2. If $\mathcal{A} \in \mathscr{F} \Rightarrow \mathcal{A}^c \in \mathscr{F}$
3. If $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_1, \cdots, \in \mathscr{F} \Rightarrow \bigcup_{i=1}^{\infty} A_i \in \mathscr{F}$

Definition

Measure Space

Measure Space $(\Omega, \mathscr{F})$

Elements of $\mathscr{F}$ are called 'measurable' sets

Example

σ-field

Trivial σ-field $\mathscr{F} = \{\Omega, \emptyset\}$

Power set collection of all possinle subsets of Ω. Cardinality of power set $= |P(\mathcal{A})| = 2^n$

Example

σ-field

If $\mathcal{A} \in \Omega \Rightarrow \mathscr{F} = \{\emptyset, \Omega, \mathcal{A}, \mathcal{A}^c\}$ is a σ-field

Example

σ-field

If $\Omega = \{1, 2, 3, \ \ , n\}$ = Natural numbers

$\mathscr{F}$ = Power set = $\{\emptyset, \Omega, \{1\}, \{2\}, \cdots, \{n\}, \{1,2\}, \cdots, \{n-1, n\}\}$

$|\mathscr{F}| = 2^n$

Example

σ-field

Let $\mathcal{C} = \{\mathcal{A}_1, \mathcal{A}_2\}, \mathcal{A}_1, \mathcal{A}_2 \in \Omega$

The smallest σ-field $= \sigma(\mathcal{C}) = \{\emptyset, \Omega, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_1 \bigcup \mathcal{A}_2, (\mathcal{A}_1 \bigcup \mathcal{A}_2)^c, \mathcal{A}_1 \bigcup \mathcal{A}_2^c \mathcal{A}_1^c \bigcup \mathcal{A}_2, \cdots\}$

Definition

Borel σ-field

Used for most of the statistical applications

Generated by the **open sets**

Example

Borel σ-field

Borel σ-field on $\mathcal{R}^1$ contains all open and closed intervals

Definition

"Measure"

A notion of interval length or volume in a higher dimension

Let $(\Omega, \mathscr{F}$ denote a measurable space, ν = a set function on $\mathscr{F}$ is called a measure **iff**

1. $0 \leq \nu(A) \leq \infty$
2. $\nu(\emptyset) = 0$
3. If $\mathcal{A}_1, \mathcal{A}_2, \cdots, \in \mathscr{F}$ is disjointed $\Rightarrow \nu(\bigcup_{i=1}^{\infty} \mathcal{A}_i) = \sum_{i=1}^{\infty} \nu(\mathcal{A}_i)$

Example

Lebesgue Measure

Length:

A unique measure on $\mathcal{R}^1$ that associates an intervals length to its measure

volume:

Lebesgue measure ~ volume in higher dimension

If $\Omega = [0, 1] \Rightarrow$ Lebesgue measure ~ probability measure