

Statistical Inference I - Lecture 1

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Definition

Sample Set

Ω = Sample Space = Set of all possible outcomes

Example

Sample Set

$\Omega = \mathcal{R}^1$ or $\Omega = \mathcal{R}^2$

Definition

σ -field

Let \mathcal{F} = a collection of subsets of Ω

\mathcal{F} is a σ -field iff

1. $\emptyset \in \mathcal{F}$
2. If $\mathcal{A} \in \mathcal{F} \Rightarrow \mathcal{A}^c \in \mathcal{F}$
3. If $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_1, \dots, \in \mathcal{F} \Rightarrow \bigcup_{i=1}^{\infty} \mathcal{A}_i \in \mathcal{F}$

Definition

Measure Space

Measure Space (Ω, \mathcal{F})

Elements of \mathcal{F} are called 'measurable' sets

Example

σ -field

Trivial σ -field $\mathcal{F} = \{\Omega, \emptyset\}$

Power set collection of all possible subsets of Ω . Cardinality of power set = $|P(\mathcal{A})| = 2^n$

Example

σ -field

If $\mathcal{A} \in \Omega \Rightarrow \mathcal{F} = \{\emptyset, \Omega, \mathcal{A}, \mathcal{A}^c\}$ is a σ -field

Example

σ -field

If $\Omega = \{1, 2, 3, \dots, n\} = \text{Natural numbers}$

$\mathcal{F} = \text{Power set} = \{\emptyset, \Omega, \{1\}, \{2\}, \dots, \{n\}, \{1, 2\}, \dots, \{n-1, n\}\}$

$$|\mathcal{F}| = 2^n$$

Example

σ -field

Let $\mathcal{C} = \{\mathcal{A}_1, \mathcal{A}_2\}, \mathcal{A}_1, \mathcal{A}_2 \in \Omega$

The smallest σ -field $= \sigma(\mathcal{C}) = \{\emptyset, \Omega, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_1 \cup \mathcal{A}_2, (\mathcal{A}_1 \cup \mathcal{A}_2)^c, \mathcal{A}_1 \cup \mathcal{A}_2^c \mathcal{A}_1^c \cup \mathcal{A}_2, \dots\}$

Definition

Borel σ -field

Used for most of the statistical applications

Generated by the **open sets**

Example

Borel σ -field

Borel σ -field on \mathcal{R}^1 contains all open and closed intervals

Definition

“Measure”

A notion of interval length or volume in a higher dimension

Let (Ω, \mathcal{F}) denote a measurable space, $\nu =$ a set function on \mathcal{F} is called a measure **iff**

1. $0 \leq \nu(A) \leq \infty$
2. $\nu(\emptyset) = 0$
3. If $\mathcal{A}_1, \mathcal{A}_2, \dots \in \mathcal{F}$ is disjointed $\Rightarrow \nu(\bigcup_{i=1}^{\infty} \mathcal{A}_i) = \sum_{i=1}^{\infty} \nu(\mathcal{A}_i)$

Example

Lebesgue Measure

Length:

A unique measure on \mathcal{R}^1 that associates an intervals length to its measure

volume:

Lebesgue measure \sim volume in higher dimension

If $\Omega = [0, 1] \Rightarrow$ Lebesgue measure \sim probability measure