Statistical Inference I - Expectations and Transformations

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Definition

Statistics

$(\Omega, \mathscr{F}, P)$

collect data, representing by a sequence of R.V.s $X_1, X_2, \cdots, X_n$

Define if $\boldsymbol{X} = (X_1, \cdots, X_n)'$ R.V.'s

A function $T(\boldsymbol{X})$ is called a Statistic if we can compute $T(\boldsymbol{X})$ without knowing value of parameters

Example

Statistics

$T_1(\boldsymbol{X}) = \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i =$ sample mean

$T_2(\boldsymbol{X}) = \frac{1}{n-1}\sum_{i=1}^{n} |X_i - \bar{X}| =$ mean absolute deviation

$T_3(\boldsymbol{X}) = \frac{1}{n-1}\sum_{i=1}^{n} (X_i - \bar{X})^2 =$ sample variance

$T_4(\boldsymbol{X}) =$ sample skewness

Definition

Expected value

$X \sim$ R.V. in $(\Omega, \mathscr{F}, P)$

$E(X) = \int_{\Omega} X(\omega) dP(\omega)$

If X has CDF $F(x) \Rightarrow E(X) = \int_{\mathbb{R}} X dF(x)$

If X is discrete $\int$ becomes $\sum$