Zhang_Jeffrey_code4

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1 AMSC 460 HW 4 Part 2

1.1 Jeffrey Zhang

Find the discrete trigonometric approximation of order n for a general function $f(x)$ on a general segment [a, b]. Test the code on $f(x) = e^x - e^{-2x}$ on $[-3, 3]$ with $n = 2, 5, 10$.

The general discrete least squares trigonometric polynomial $S_n(x)$ is defined as:

$$S_n(x) = \frac{a_0}{2} + a_n \cos(nx) + \sum_{k=1}^{n-1} (a_k \cos(kx) + b_k \sin(kx)) \tag{1}$$

The discrete least squares approximation for $2m$ equally-spaced x_i on $[a, b]$ for a_k and b_k is defined as:

$$\begin{array}{ll} a_k = \frac{1}{m} \sum_{i=0}^{2m-1} f(x_i) \cos(kx_i) & b_k = \frac{1}{m} \sum_{i=0}^{2m-1} f(x_i) \sin(kx_i) \\ \quad k = 0 \ldots n & k = 1 \ldots n-1 \end{array} \tag{2}$$

$x_{i+1} - x_i = \dfrac{b-a}{2m}$ for $i = 0 \ldots 2m-1$

In the context of this problem, m is not given, so it is assumed that it is something the programmer assigns

[157]:
```
import math
import numpy as np
from sympy import *
from scipy.integrate import quad
import scipy.special as special
from sympy.plotting import plot

def f(x):
    return math.exp(x) - math.exp(-2*x)

def variable_summation_ak(difference_x, m, a, k):
    variable = 0
    x_i = a
    for i in range(2*m):
        variable += f(x_i)*math.cos(k*x_i)
        x_i += difference_x
```

```python
    return variable/m

def variable_summation_bk(difference_x, m, a, k):
    variable = 0
    x_i = a
    for i in range(2*m):
        variable += f(x_i)*math.sin(k*x_i)
        x_i += difference_x
    return variable/m

def S_n(n,m,a,b):
    difference_x = float((b-a)/(2*m))
    a_0 = variable_summation_ak(difference_x, m, a, 0)
    a_n = variable_summation_ak(difference_x, m, a, n)
    x = symbols('x')
    S_n = (a_0/2) + a_n*cos(n*x)
    for k in range(1,n):
        a_k = variable_summation_ak(difference_x, m, a, k)
        b_k = variable_summation_bk(difference_x, m, a, k)
        S_n += a_k*cos(k*x) + b_k*sin(k*x)
    return S_n

def print_to_compare(Sn):
    x = symbols('x')
    p1 = plot(Sn, (x,-3,3))
    p2 = plot(exp(x) - exp(-2*x),(x,-3,3))
```

Let's assign $m = 1000$ and see what happens with the results

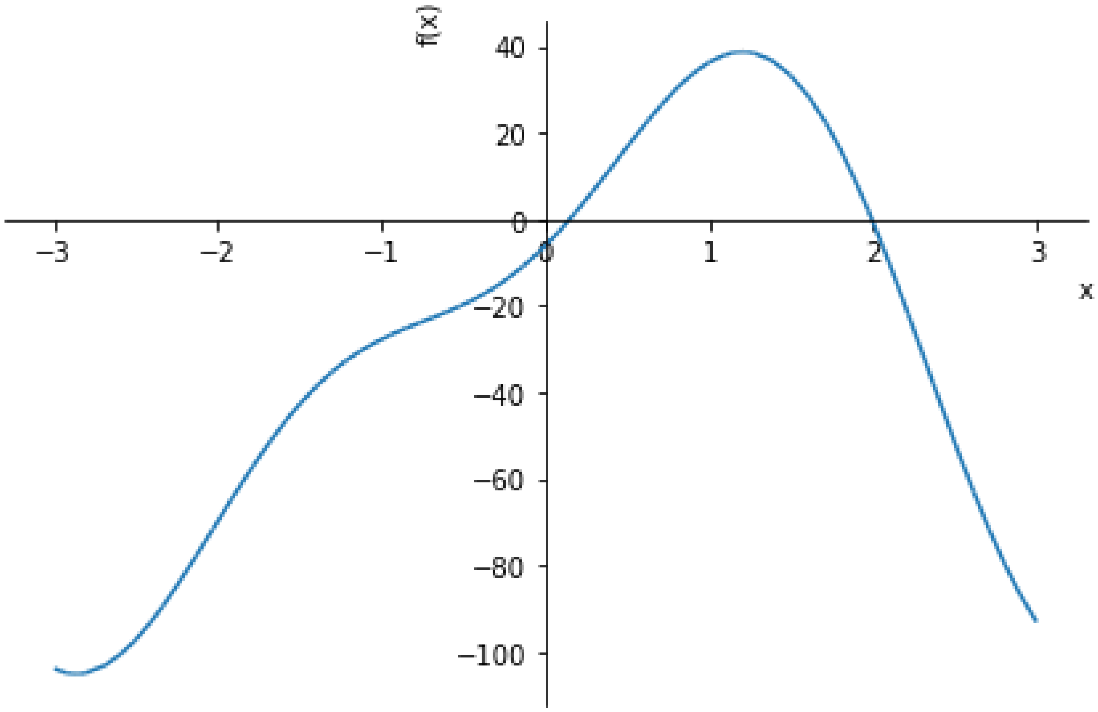
1.1.1 n = 2

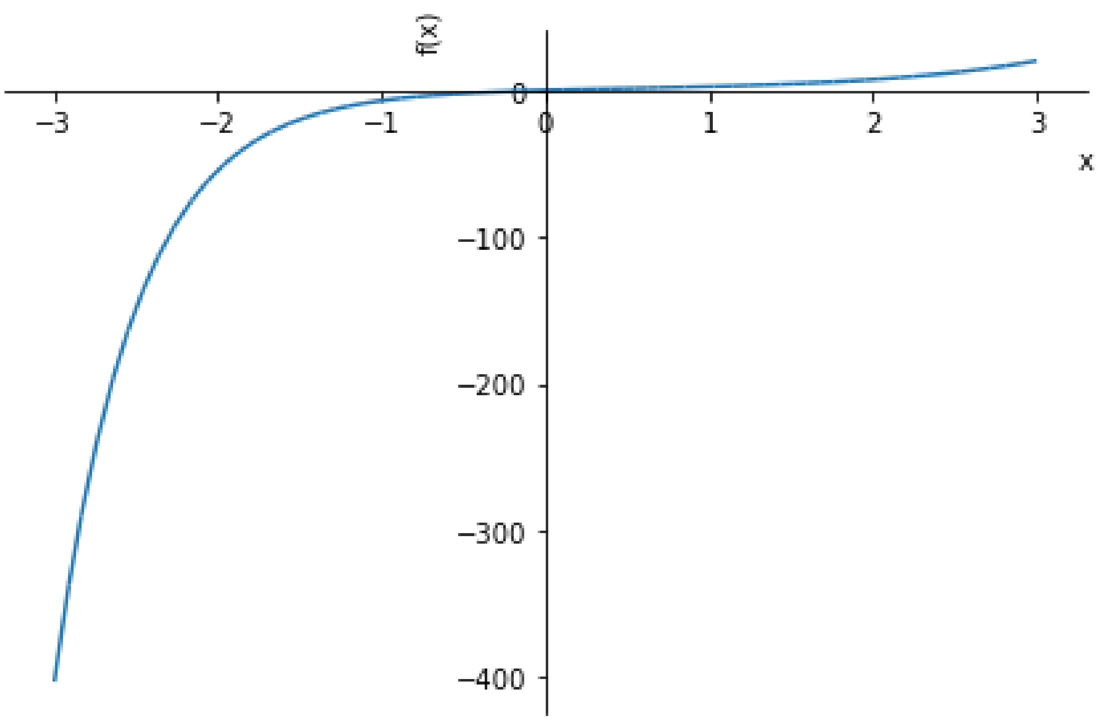
[158]:
```python
S_2 = S_n(2,1000,-3,3)
S_2
```

[158]:

$$38.0233244719654 \sin (x) + 46.8339178794455 \cos (x) - 22.5572585595501 \cos (2x) - 30.385531787441$$

[159]:
```python
print_to_compare(S_2)
```


f(x)
40
20
0
−20
−40
−60
−80
−100
−3
−2
−1
0
1
2
3
x


f(x)
0
−100
−200
−300
−400
−3
−2
−1
0
1
2
3
x

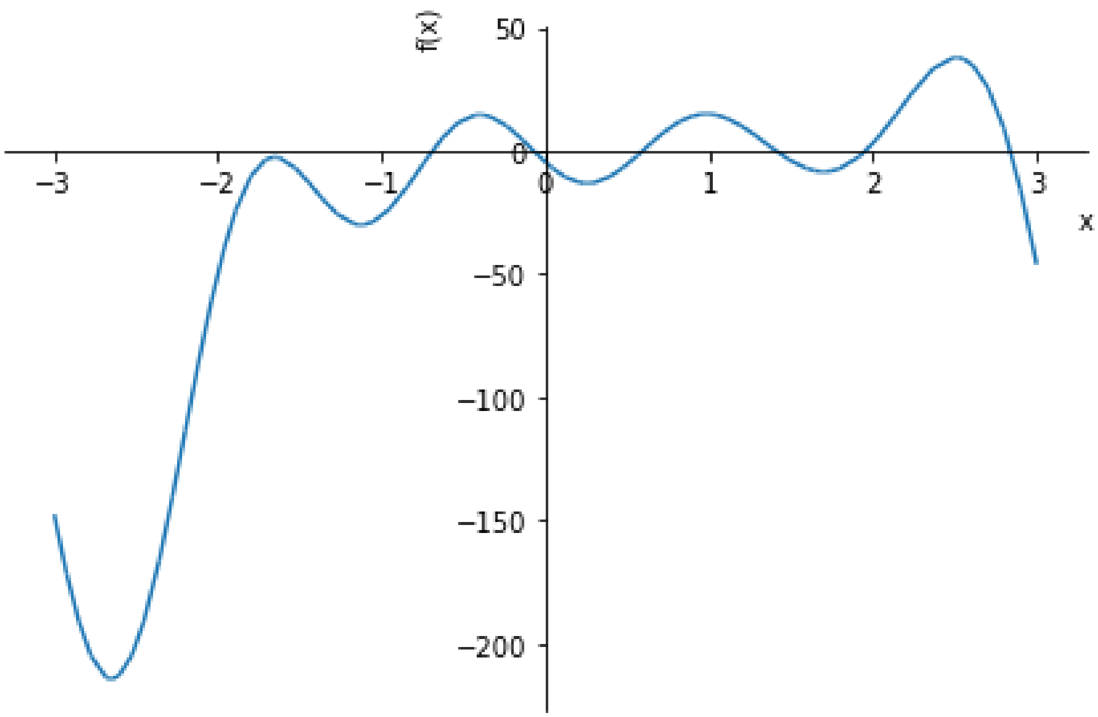
1.1.2 n = 5

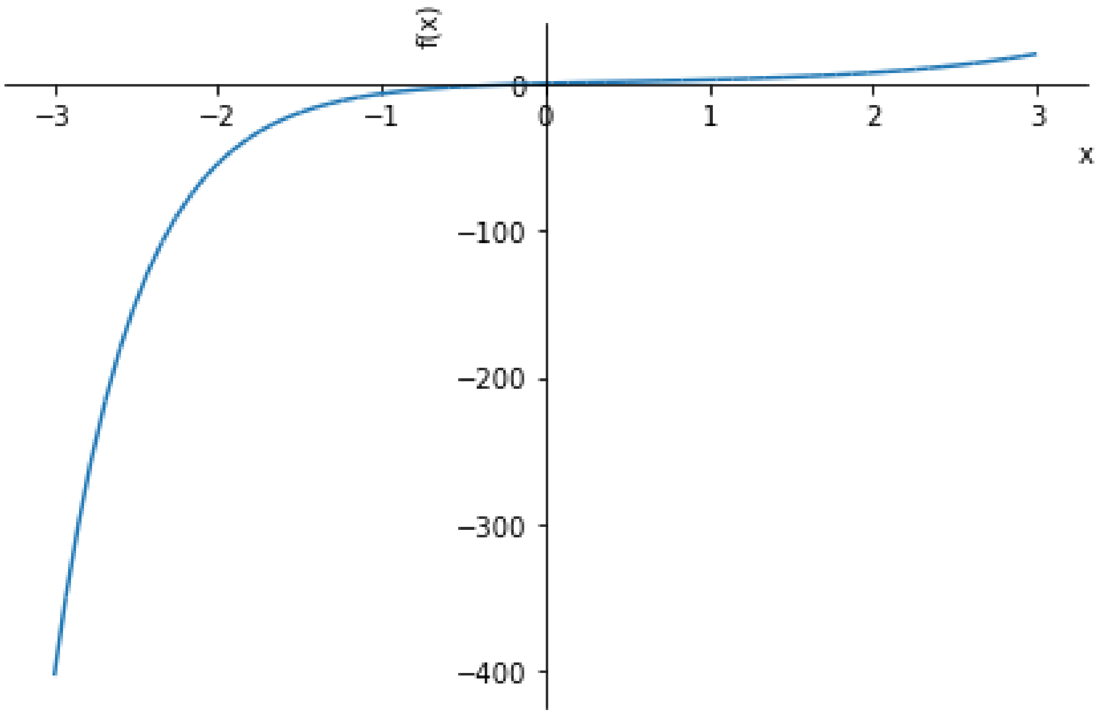
[160]:
```
S_5 = S_n(5,1000,-3,3)
S_5
```

[160]:

$38.0233244719654\sin(x) - 44.667085397949\sin(2x) + 38.9820471546976\sin(3x) - 31.551692467262\sin(4x) + 46.8339178794455\cos(x) - 22.5572585595501\cos(2x) + 6.47512006011178\cos(3x) + 2.38868481876482\cos(4x) - 7.22639487165301\cos(5x) - 30.385531787441$

[161]:
```
print_to_compare(S_5)
```





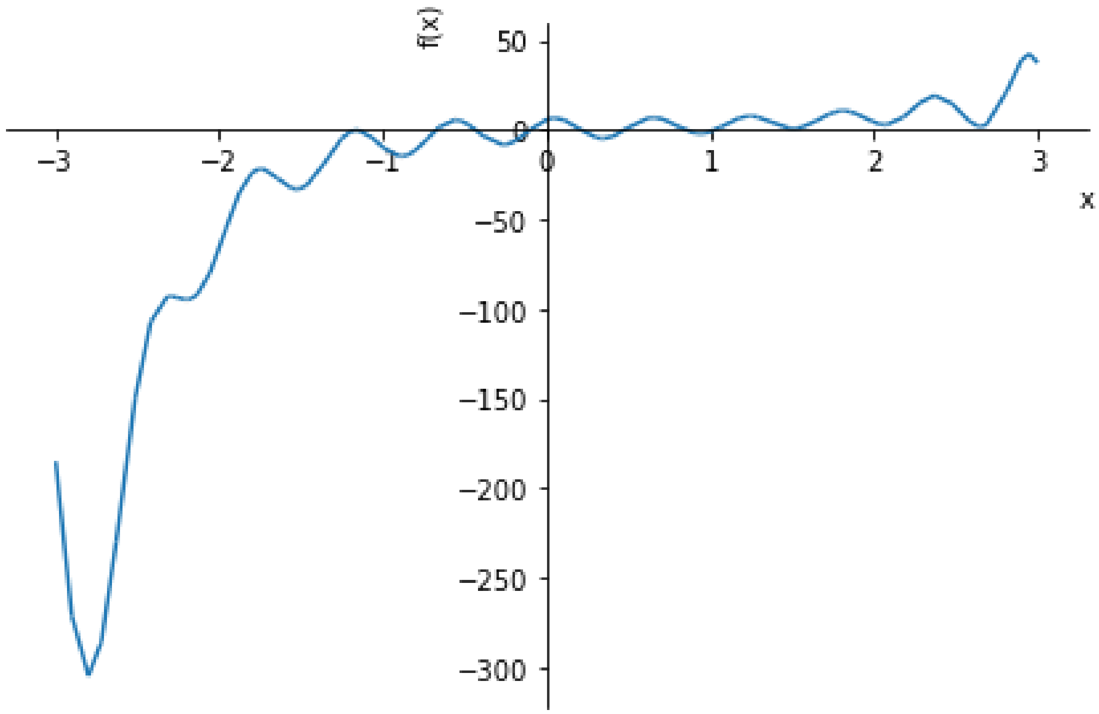
1.1.3 n = 10

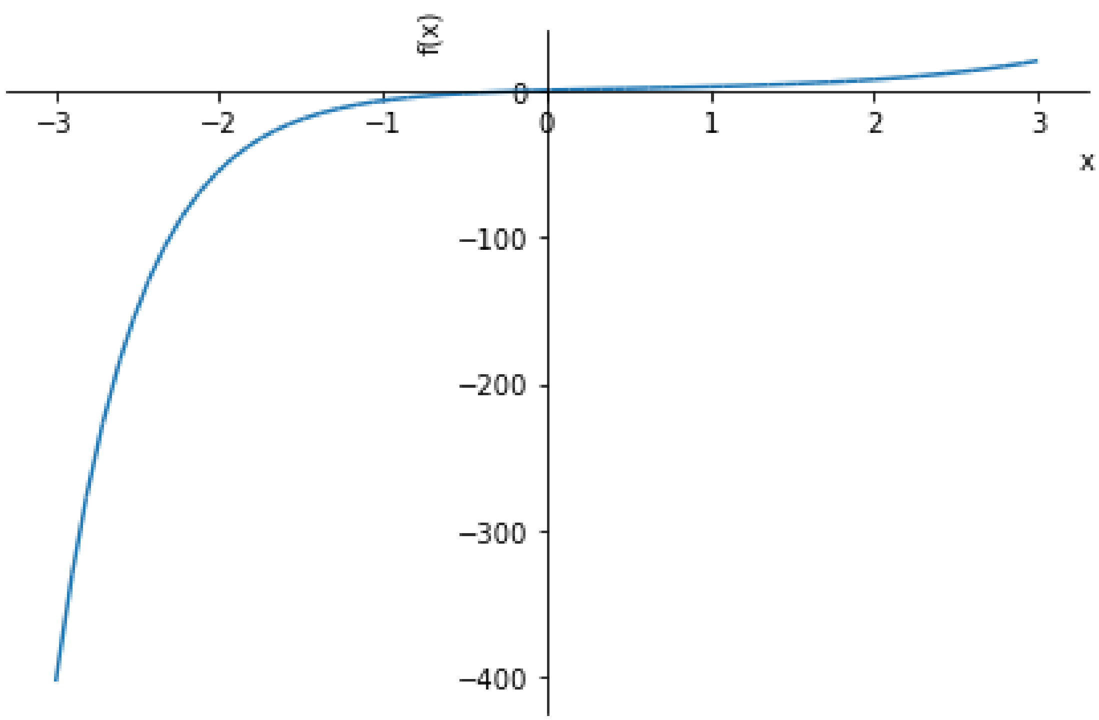
[152]:
```
S_10 = S_n(10,1000,-3,3)
S_10
```

[152]:

$$38.0233244719654 \sin(x) - 44.667085397949 \sin(2x) + 38.9820471546976 \sin(3x) - 31.551692467262 \sin(4x) + 24.9125636609138 \sin(5x) - 19.3640073234597 \sin(6x) + 14.7584142318553 \sin(7x) - 10.9080895217708 \sin(8x) + 7.66140291269082 \sin(9x) + 46.8339178794455 \cos(x) - 22.5572585595501 \cos(2x) + 6.47512006011178 \cos(3x) + 2.38868481876482 \cos(4x) - 7.22639487165301 \cos(5x) + 9.87021409790035 \cos(6x) - 11.2508275524605 \cos(7x) + 11.8542368923221 \cos(8x) - 11.9497682207099 \cos(9x) + 11.6968121808432 \cos(10x) - 30.385531787441$$

[163]:
```
print_to_compare(S_10)
```


f(x)
50
0
−50
−100
−150
−200
−250
−300
−3
−2
−1
0
1
2
3
x


f(x)
0
−100
−200
−300
−400
−3
−2
−1
0
1
2
3
x

As the order n gets larger, the discrete trigonometric approximation interpolant fits the function better