Secant and Newton Method about Finding Black-Scholes Implied Volatility

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1 Introduction

In numerical analysis, the secant method is a root-finding algorithm that uses a succession of roots of secant lines to better approximate a root of a function f.The secant method is a finite difference approximation of Newton's method.Black–Scholes model uses a variety of inputs to derive a theoretical value for an option. the implied volatility of an option contract is that value of the volatility of the underlying instrument which, when input in Black–Scholes will return a theoretical value equal to the current market price of the option. The value of an option depends on its implied volatility.

2 Functions Used

2.1 Required functions.

The assignment specifications require the following 6 functions:

- bs(callput, S0, K, r, T, sigma, q=0.) for using callput type, underlying option price, strike, risk-free rate , time to maturity, implied volatility and Annual Dividend Yield on black-scholes model. This function returns three variables: option price, delta and vega.
- secantsolve(target, targetfunction, start=None,bounds=None, tols=[0.01,0.01], maxiter=100)for using secant method to get root(implied volatility).This function returns an array: bounds,start point and root.
- newton(target, targetfunction, start=None,bounds=None, tols=[0.01,0.01], maxiter=100)for using Newton method to get root(implied volatility).This function returns an array: bounds,start point and root.

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- bsimpvolsec(callput, S0, K, r, T, price, q=0.,priceTolerance=0.01, reportCalls=False) for using secant method to find black-scholes implied volatility.This function returns an array: bounds, start and implied volatilities.
- bsimpvolnewton(callput, S0, K, r, T, price, q=0.,priceTolerance=0.01, reportCalls=False) for using newton method to find black-scholes implied volatility.This function returns an array: bounds, start and implied volatilities.

2.2 Helper Functions

f(sigma)is a new function which only the volatility is an unknown variable,the other variable become known. This function return the option price. We use this new function to find it's root,which is the implied volatility.

3 Test Suite

In Question 1, I defined two test using matlab black-scholes caculator's result to my BS formula program. It all went through well.

In Question 2, I used $test_secant.py$ to test my Secant method,all seven test pass,which means my $secantsolve(target, targetfunction, start = None, bounds = None, tols = [0.01, 0.01], maxiter = 100)$ is correct.

In the Question3, see the figure 1 on the top of this page.I captured the result of line 51-100 to compare the volatilities.Comparing the implied volatilities that I computed to the CSVfile's volatilities, they are quite close.

4 Range Of Inputs

I set up bounds as$[-0.5, 2]$, start at 0.5 and make the tolerance as $[0.0, priceTolerance]$ in both secand and newton method to find the implied volatility.

	A	B	C	D	E
1			Secant method	Newton method	
2	Call	36.93935	(43.6667106923751 05, 6)	(43.6381673727103 06, 5)	
3	Call	36.45443	(42.1935985715723 23, 6)	(42.1556261923959 71, 5)	
4	Call	34.55272	(38.8765939522707 67, 7)	(38.9140644760305 42, 5)	
5	Call	34.50927	(39.7058554282969 64, 6)	(39.6671348403512 95, 5)	
6	Call	34.69501	(39.2562099245437 04, 6)	(39.2301945636096 86, 5)	
7	Call	33.01292	(37.918519198865 29, 6)	(37.8971199508186 13, 5)	
8	Call	35.03238	(37.2787413350446 07, 6)	(37.2661861315038 51, 5)	
9	Call	33.6817	(35.8493595924305 38, 6)	(35.8413697004258 44, 5)	
10	Call	34.53408	(34.7420311219987 14, 6)	(34.7383747028438 28, 5)	
11	Call	33.26348	(34.3245189490766 83, 6)	(34.3235912914515 14, 5)	
12	Call	33.82242	(33.7911272530663 64, 6)	(33.7910994110060 23, 5)	
13	Call	33.19453	(33.3350407509193 12, 6)	(33.3351422217970 64, 5)	
14	Call	33.16488	(33.5130984656755 12, 5)	(33.4951789296103 54, 5)	
15	Call	32.83799	(32.7012575984850 64, 5)	(32.6967726205231 2, 5)	
16	Call	32.74414	(32.2759054338634 93, 5)	(32.2947745739887 04, 4)	
17	Call	32.54714	(32.2046717949594 38, 4)	(32.1768797908128 83, 4)	
18	Call	32.29975	(32.0661672437550 68, 5)	(32.0728998333003 62, 4)	
19	Call	32.10896	(31.9127772386685 14, 5)	(31.9098269151675 6, 5)	
20	Call	32.14469	(31.7419077326268 28, 5)	(31.7255598759213 73, 5)	
21	Call	32.11252	(31.7142898922671 68, 6)	(31.7144430027472 88, 5)	
22	Call	32.05	(31.7159974134792 19, 6)	(31.7162395070268 95, 5)	
23	Call	31.93858	(31.5463195223129 2, 6)	(31.5460522413246 34, 5)	
24	Call	31.92193	(31.5687261332965 86, 6)	(31.5658461362308 92, 5)	
25	Call	32.05179	(31.6668287738129 77, 6)	(31.6570153975351 86, 5)	
26	Call	32.0827	(31.6766389237022 81, 7)	(31.7170617524091 58, 5)	
27	Call	32.16443	(31.7714215144871 59, 7)	(31.7692061934497 05, 6)	

	A	B	C	D
28	Call	32.29364	(31.9172476727916 39, 7)	(31.9119076602296 73, 6)
29	Call	32.01359	(32.1799145187792 56, 7)	(32.1691814269173 62, 6)
30	Call	32.57051	(32.3206410470102 55, 7)	(32.2999320798537 33, 6)
31	Call	32.43449	(32.5250437654218 27, 7)	(32.4893992814353 47, 6)
32	Call	32.97719	(32.8651174328908 13, 7)	(32.8112272231731 48, 6)
33	Call	33.33853	(33.4554466942945 44, 7)	(33.3459718253485 99, 6)
34	Call	33.64762	(33.8284576470947 23, 8)	(33.8390899104601 81, 6)
35	Call	34.56532	(34.8618399533328 26, 8)	(34.8731760768933 58, 6)
36	Call	35.56525	(35.8553816734681 02, 8)	(35.8663126840273 92, 6)
37	Call	36.10959	(36.3158697626964 23, 8)	(36.3287372688894 48, 6)
38	Call	36.09336	(38.0448085770304 02, 7)	(37.7254991088197 83, 6)
39	Call	36.78809	(39.1922013421529 41, 7)	(38.9103212593896 12, 6)
40	Call	39.83114	(40.5955408996975 61, 7)	(40.3956585069461 06, 6)
41	Call	41.43212	(44.2668521477802 34, 6)	(43.9981992248546 87, 5)
42	Call	41.09762	(43.0752206931151 21, 7)	(43.2435540121250 1, 5)
43	Call	50.00593	(49.2086593049388 7, 5)	(49.2095370505283 26, 4)
44	Call	46.60835	(46.0560834045160 8, 6)	(45.9007132333385 85, 5)
45	Call	45.04365	(47.2940151355948 33, 5)	(47.3008700352276 11, 4)
46	Call	56.71939	(55.6922566475561 5, 7)	(56.0175446575410 52, 5)
47	Call	57.1808	(56.0760429818015 64, 7)	(56.6485220045975 38, 5)
48	Call	58.99926	(58.0945368226339 55, 8)	(58.0887914410856 33, 6)
49	Call	60.77925	(60.2103088271181 83, 8)	(60.1313534340842 68, 6)
50	Call	62.53238	(61.1785123687397 6, 9)	(61.5579440134883 74, 7)
51	Call	63.69106	(62.4211251830467 76, 10)	(62.8975443652950 12, 7)
52				
53				
54				

5 Analysis

Compare these two method,secant and newton,secant always make 1-3 more calls than the newton method to the function bsformula.That means secant method takes more time. It is reasonable. Using secant method,we need two points in each step while using newton method,we only need one point.So newton method converages faster.

The implied volatilities are really closed using these two method.That indicate that both these two method get correct answer or they both wrong. Giving the comparision of the result that we compute to the result of the csvfile,we can see they are quite close to the exact volatilities.That means both method are correct.But there still remain question that I do not understand.

6 Comparison of results

Using Secant method and Newton method get close volatilities.

7 Futher Investigation

These two method both have advantages and disadvantages.Secant method need more known variables but it is much easier to compute the function. Newton method,on the other hand, is much faster and require less known variables, but it needs derivative. We need to use computer tools to help us get function derivatives.The secant method can be thought of as a finite difference approximation of Newton's method.The recurrence formula of the secant method can be derived from the formula for Newton's method.

8 Qusetion

1.When I first finished my BS.py. I can go through my test. But when I finished the last question, add and edit two definition of secantsolve() and newton(), BS.py can not went through my test, I can not figure out why.
2.Although the result in the middle is quite close to csvfile result,there still a lot of results is not good. They are far away from the real one.What's the reason for that?

9 Conclusion

Both secant and newton method are good ways to find the black-scholes implied volatilities.If we have our computer by hand,we can use both these two method,otherwise, we'd better use secant method since it is much more easier to calculate.