Implicit Schemes For the Vasicek Model

Jingjie Zhou

1.Introduction

In this project, I use Vasicek model to price a bond with an embedded call owned by its issuer. Bond holdings often have a short option position, where the contract specifies the issuer may call the bond away if interest rates move in their favor. I will treat early exercise through a version of policy iteration.

2.Function Used

The assignment specifications require the following 6 functions:

- def VasicekLimits(r0, sigma, kappa, theta,T,prob=1e-6) return a 2-tuple (rMin, rMax) of the r levels for the given tail probability.

- def VasicekParams(r0, M,sigma, kappa, theta,T,prob=1e-6): return a 4-tuple (rMin, dr, N, dtau) of parameters such that the structure constant does not exceed 0.25 and the boundaries for r at time T are at the prob level.
- def VasicekDiagonals(sigma, kappa, theta, rMin, dr, N, dtau): %,boundaryConditions='Neumann'): return a 3-tuple (subdiagonal, diagonal, superdiagonal) suitable for solving in the

tridiagonal solver

- def CheckExercise(V,eex): return a numpy array of logical (True or False) values with True at the indices where early exercise is desirable.

- def CallExercise(R, ratio, tau): return a number with the strike value K.

- def VasicekPolicyDiagonals(subdiagonal,diagonal, superdiagonal,vOld, vNew, eex): return a 3-tuple (policySubdiagonal, policyDiagonal, policySuperdiagonal) such that the sub- and superdiagonals have been set to zero and the diagonal has been set to vm+1(n)/vm(n) for indexes n where early exercise is desirable, and the diagonal and sub- and superdiagonals have been set to their original values otherwise.

- def Iterate(subdiagonal, diagonal,superdiagonal,vOld, eex,maxPolicyIterations=10): return 2-tuple vNew, numP consisting of a new set of contingent claim values vNew and the number numP of policy iterations used.

- def VasicekCallableZCBVals(r0, R, ratio, T,sigma, kappa, theta, M,prob=1e-6,maxPolicyIterations=10): return a 2-tuple (r, v) of t = 0 short rates r and call option prices v.

- def VasicekCallableZCB(r0, R, ratio, T,sigma, kappa, theta, M,prob=1e-6,maxPolicyIterations=10): return a value V corresponding to the linearly interpolated call value from VasicekCallableZCBVals at the current short rate r0.

3.Helper Function

- def TridiagonolSolve(subd,d,superd,old):return the new value.
- def exact(r0,kappa,theta,sigma,T):return the exact value.

3.Test Suite

I pass all the tests.

4.Range of Input

```
r0 = 0.05
sigma = 0.03
kappa = 0.5
theta = 0.02
T = 5
prob = 1e-6
M = 250
ratio = 1.0
R = 0.02
```

I tried these M

```
M=(100,500,1000,3000,5000,7000,9000,13000,20000)
```

I increased the strike price and make bond out of money, which is a nonecallable bond.

The strike price is $x = \rho e^{-R\tau}$. So there are two ways to increase it: increase ratio or increase –R.

1. Change R to make strike price higher to make bond out of money: set the ratio 1, R=-1000
2. Change R to make strike price higher to make bond out of money : set the ratio 100,R=0.02

For two ways, the values are similar.

For R=-1000:

Callable zero coupon bond value=

[0.86019977 0.85997918 0.85996323 0.85994525 0.85994137 0
.85994078 0.85993999 0.85993889 0.85993842]

For ratio 100:

Callable zero coupon bond value=

[0.86019977 0.85997918 0.85996323 0.85994525 0.85994137 0
.85994078 0.85993999 0.85993889 0.85993842]

For exact bond values, I used

THEOREM 4.4 (Zero-coupon bond in the Vasicek model). *In the Vasicek model, the price of a zero-coupon bond with maturity T at time $t \in [0,T]$ is given by*

$$P(t,T) = A(t,T)e^{-r(t)B(t,T)},$$

where

$$B(t,T) = \frac{1-e^{-k(T-t)}}{k}$$

and

$$A(t,T) = \exp\left\{\left(\theta - \frac{\sigma^2}{2k^2}\right)(B(t,T) - T + t) - \frac{\sigma^2}{4k}B^2(t,T)\right\}.$$

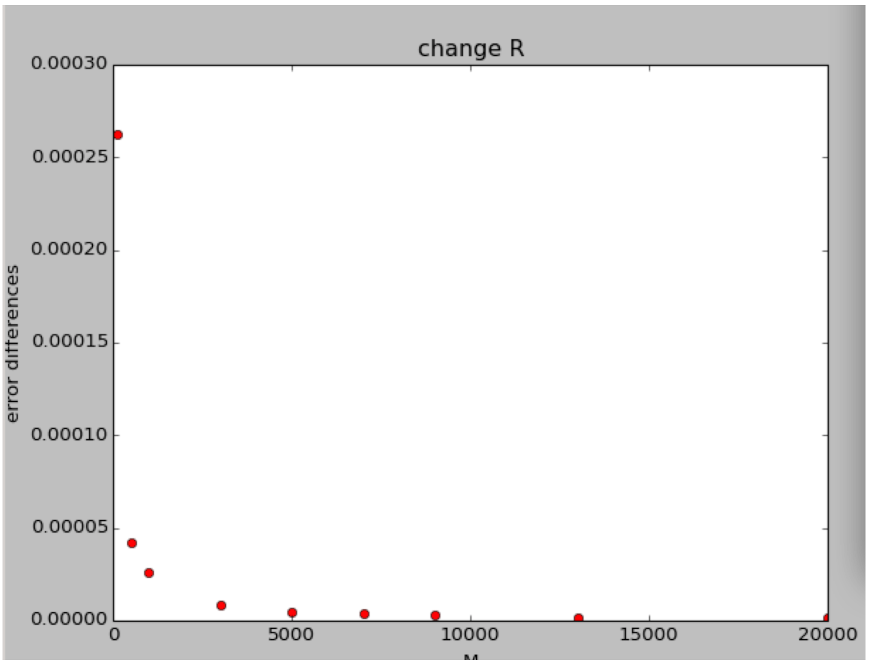
And I got exact value= 0.85993711337.

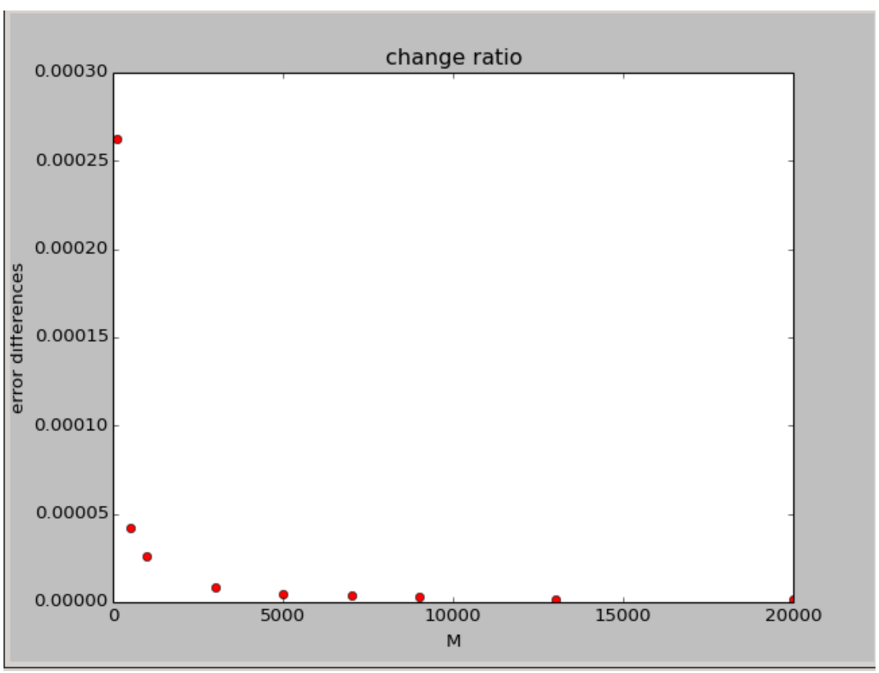
5.Analysis

Plot the graph of the error between the exact bond values and the scheme values.

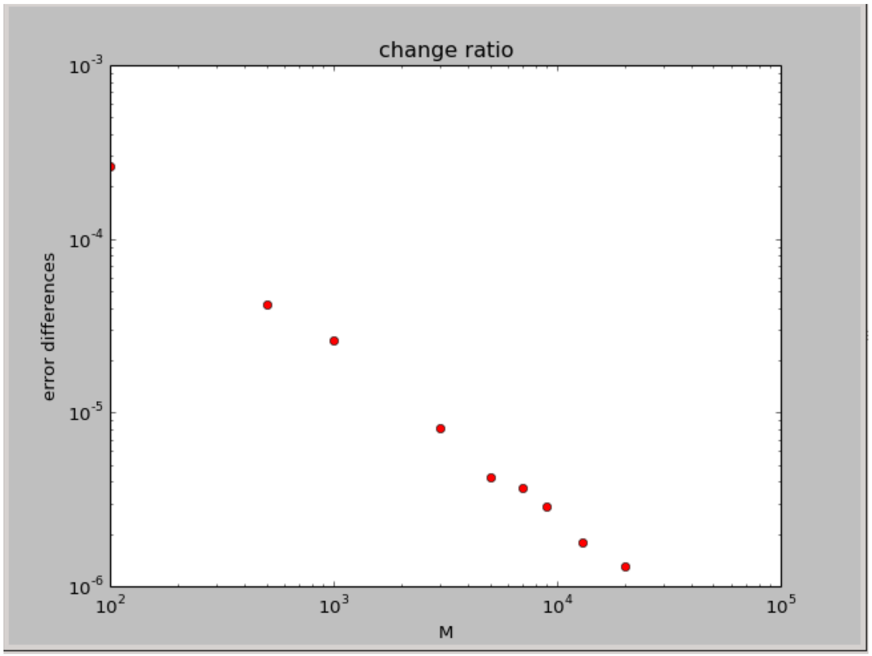
I plotted two graphs (original, Log-Log scale plot) for each way of increasing the strike price.

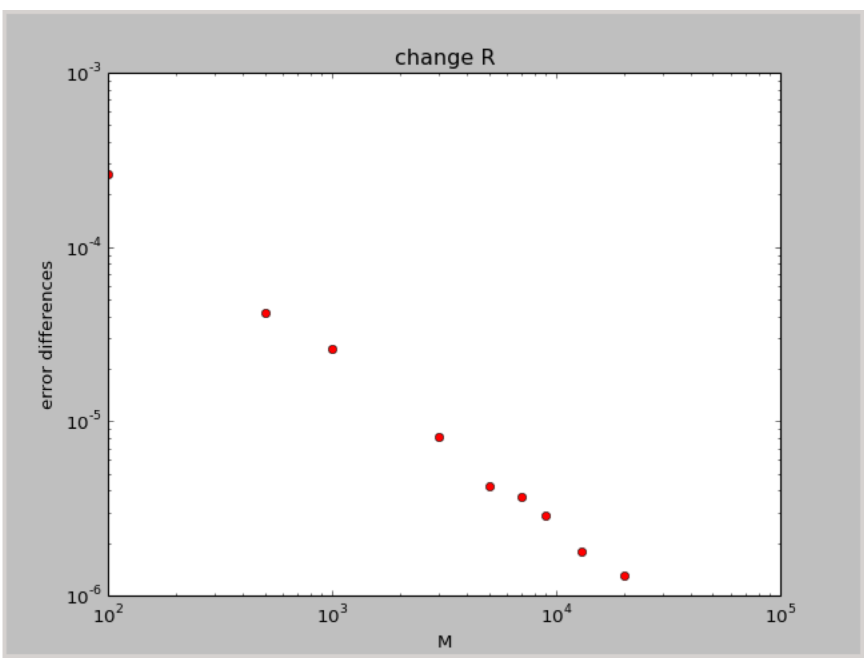
1.original





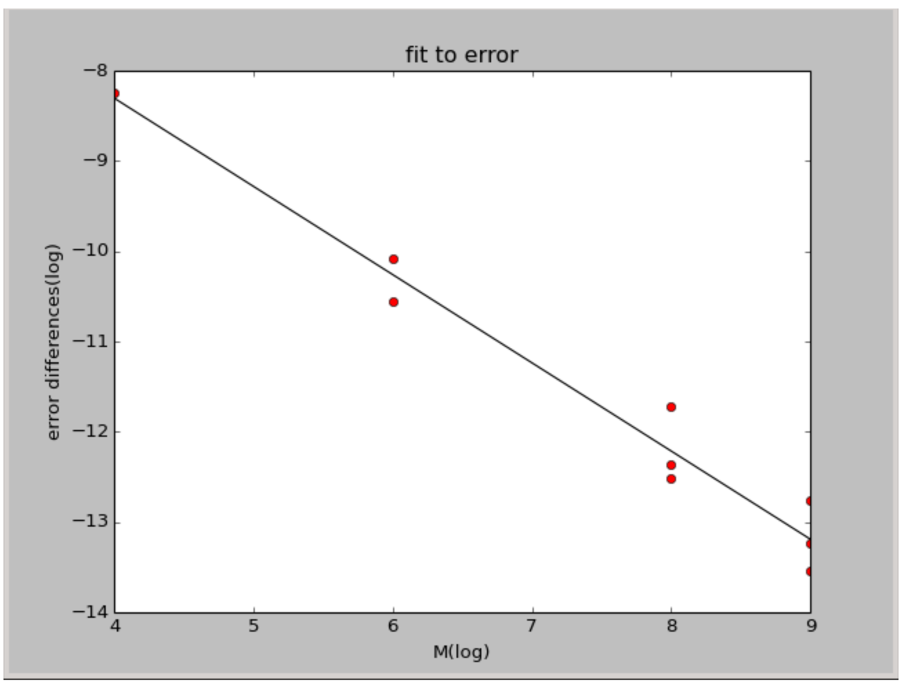
2. Log-Log scale plot





Setting a strike so that the call is far out of the money, scheme values against exact bond values are almost the same. It is similar to change ratio.

I plotted the fit to error graph and compute the slope of this line. Slope= -0.976680485079, which is close to -1. So the error approximation is close to O (1/M), which matches theoretical expectations.



Set the maximum policy iteration count to zero, using no policy iteration results, I tried theta=-10, the pricing error is significant, which is 2.08719925607 and I tried theta=-100, the pricing error is 2.0290790643e+14.

To experiment with the probability width, I tried several probability(1e-6,0.1,0.3,0.4,0.5), and the pricing scheme start to break down between 0.4435 and 0.4436.

To experiment with extreme values of r0, I tried several extreme r0, even 1, the pricing scheme did not break down.

6.Conclusion

Implicit Schemes For the Vasicek Model is a good way to price the bond value.