IE523: Financial Computing
Fall, 2018
Mid-Term Programming Assignment: Portfolio Optimization Software in C++ under Parallel Shifts in Term Structure
Due Date: 19 October 2018

Instructions

1. **If you worked with someone on a specific problem, make sure you tell me about it. If you do not do this and I notice a verbatim reproduction of material, I consider it as plagiarism. There can be serious consequences to this.**
   (a) I will randomly pick ≈ 6 students in the class, who will be asked to explain all aspects of their submitted code to me. Do not be disturbed if your name appears in this list – I just need to go through it to ensure the grading is fair.
2. **If you picked-up the solution from some source, make sure you mention it. If you do not do this and I notice a verbatim reproduction of material, I consider it as plagiarism. There can be serious consequences to this.**
3. **Each submitted document should be personalized. I expect to see**
   (a) C++ code and header files (if any), the comments in the code should pretty much tell me what you are trying to do
   (b) A PDF version of sample-runs that accompanies each submission, have to be uploaded on Compass by the due date.
4. **The written In-Class Exam will be held during class hours on 17 October 2018.**

I want you to write a comprehensive, readable (i.e. put a lot of comments; have meaningful variable names; etc. etc.) *Portfolio Optimization Software Package* in C++ assuming you have parallel shifts in the term structure.

Input Specification: The input file to your software will have the following format:

1st-line:	#CFs									
2nd-line:	CF-1's PV	Maturity	cf_1^1	cf_2^1	cf_3^1	cf_4^1	$\cdots$	$cf_{n_1}^1$		
3rd-line:	CF-2's PV	Maturity	cf_1^2	cf_2^2	cf_3^2	cf_4^2	$\cdots$	$\cdots$	$cf_{n_2}^2$	
4th-line:	CF-3's PV	Maturity	cf_1^2	cf_2^2	cf_3^2	$\cdots$	$cf_{n_3}^2$			
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	
kth-line:	CF-k's PV	Maturity	cf_1^k	cf_2^k	cf_3^k	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$cf_{n_k}^k$
$(k+1)$*th-line:*	FV of user's debt obligation	due date								

(i.e. k-many Bonds; the 1st number in each row is the PV of the bond, followed by the bond's cash-flows; not all of them have the same maturity; followed by the FV of user's debt obligation and the time when its is due).

Output Specification 1: We do not assume a flat term structure, but we assume the term structure changes are *parallel* (i.e. the yield changes by the same %-age amount for all maturities).

1. Compute the *YTM* for each cash flow.

2. Compute the *duration* for each cash flow.

3. Compute the *convexity* for each cash flow.

Output Specification 2 Assuming no changes in term structure, compute the %-age of the face value of each cash flow you would have to purchase to meet the future debt obligation.

Output Specification 3: Pick the bond-portfolio that will meet this obligation when it is due, and has the largest convexity among all possible portfolio choices. You should pose this as a Linear Programming Problem and use lp_solve to find the optimal answer. Your final answer will say that we need to buy λ_1% of first cash flow, λ_2% of second cash flow, λ_3% of third cash flow, etc. No short-selling is permitted (i.e. all your λ's have to be non-negative).

1. In your write-up, present an explanation of the strategy/method that you used in picking the best portfolio.

2. Make sure your program will handle the case when there is no portfolio that will meet the debt obligation if we are worried about (parallel) movements in the term structure (cf. figure 2).

I also want to see a couple of sample runs (cook-up your own data; or use lesson 4 of my notes).

Partial Credit Information: This should help you decide how your efforts should be spent in the next two weeks.

1. (*20 points*) Making sure you catch every possible error/infeasibility that might occur in the general setting.
2. (*20 points*) Soundness of your theoretical arguments for the design-methodology that you adopt for the portfolio design.
3. (*60 points*) The correct functioning of your code.

A sample screen shot is shown in figure 1.

```
[MacBook-Air-2:Debug sreenivas$ ./Bond\ Immunization input1
 Input File: input1
 We owe 1790.85 in 10 years
 Number of Cash Flows: 5
 ----------------------------------------------------------------------
 Cash Flow #1
 Price = 1131.27
 Maturity = 10
 Percentage of Face Value that would meet the obligation = 0.892239
 Yield to Maturity = 0.0499999
 Duration = 7.7587
 Duration (to be used in LP-formulation below) = 8.69576
 (Note) 7.7587 = 8.69576 x 0.892239
 Convexity  = 70.4264
 Convexity (to be used in LP-formulation below) = 78.9322
 (Note) 70.4264 = 78.9322 x 0.892239
 ----------------------------------------------------------------------
 Cash Flow #2
 Price = 1069.88
 Maturity = 15
 Percentage of Face Value that would meet the obligation = 0.943436
 Yield to Maturity = 0.0625639
 Duration = 9.93582
 Duration (to be used in LP-formulation below) = 10.5315
 (Note) 9.93582 = 10.5315 x 0.943436
 Convexity  = 119.831
 Convexity (to be used in LP-formulation below) = 127.016
 (Note) 119.831 = 127.016 x 0.943436
 ----------------------------------------------------------------------
 Cash Flow #3
 Price = 863.5
 Maturity = 30
 Percentage of Face Value that would meet the obligation = 1.16892
 Yield to Maturity = 0.07
 Duration = 13.6774
 Duration (to be used in LP-formulation below) = 11.7009
 (Note) 13.6774 = 11.7009 x 1.16892
 Convexity  = 262.769
 Convexity (to be used in LP-formulation below) = 224.796
 (Note) 262.769 = 224.796 x 1.16892
 ----------------------------------------------------------------------
 Cash Flow #4
 Price = 1148.75
 Maturity = 12
 Percentage of Face Value that would meet the obligation = 0.878662
 Yield to Maturity = 0.0574999
 Duration = 8.58082
 Duration (to be used in LP-formulation below) = 9.76578
 (Note) 8.58082 = 9.76578 x 0.878662
 Convexity  = 87.6798
 Convexity (to be used in LP-formulation below) = 99.7879
 (Note) 87.6798 = 99.7879 x 0.878662
 ----------------------------------------------------------------------
 Cash Flow #5
 Price = 1121.39
 Maturity = 11
 Percentage of Face Value that would meet the obligation = 0.9001
 Yield to Maturity = 0.0549998
 Duration = 8.20531
 Duration (to be used in LP-formulation below) = 9.116
 (Note) 8.20531 = 9.116 x 0.9001
 Convexity  = 79.1966
 Convexity (to be used in LP-formulation below) = 87.9864
 (Note) 79.1966 = 87.9864 x 0.9001
 **********************************************************************
 Average YTM (which I use to compute PV of Debt) = 0.0590127
 Present value of debt = 1009.36
 **********************************************************************
 Model name:
                 C1       C2       C3       C4       C5
 Minimize  -78.9322 -127.016 -224.796 -99.7879 -87.9864
 R1         1131.27  1069.88    863.5  1148.75  1121.39  =  1009.36
 R2         8.69576  10.5315  11.7009  9.76578    9.116  =       10
 Type          Real     Real     Real     Real     Real
 upbo           Inf      Inf      Inf      Inf      Inf
 lowbo            0        0        0        0        0
 Largest Convexity we can get is: 143.262
 Optimal portfolio:
 %Cash Flow:1  0.554367
 %Cash Flow:2  0
 %Cash Flow:3  0.442645
 %Cash Flow:4  0
 %Cash Flow:5  0

 That is, buy
 $627.139 of Cash Flow#1
 $382.224 of Cash Flow#3
 MacBook-Air-2:Debug sreenivas$
```

Figure 1: A sample screen shot.

```
MacBook-Air-2:Debug sreenivas$ ./Bond\ Immunization input2
Input File: input2
We owe 1790.85 in 10 years
Number of Cash Flows: 3
------------------------------------------------------------------
Cash Flow #1
Price = 1131.27
Maturity = 10
Percentage of Face Value that would meet the obligation = 0.934116
Yield to Maturity = 0.0499999
Duration = 7.7587
Duration (to be used in LP-formulation below) = 8.30593
(Note) 7.7587 = 8.30593 x 0.934116
Convexity  = 70.4264
Convexity (to be used in LP-formulation below) = 75.3936
(Note) 70.4264 = 75.3936 x 0.934116
------------------------------------------------------------------
Cash Flow #2
Price = 1121.39
Maturity = 11
Percentage of Face Value that would meet the obligation = 0.942346
Yield to Maturity = 0.0549998
Duration = 8.20531
Duration (to be used in LP-formulation below) = 8.70733
(Note) 8.20531 = 8.70733 x 0.942346
Convexity  = 79.1966
Convexity (to be used in LP-formulation below) = 84.042
(Note) 79.1966 = 84.042 x 0.942346
------------------------------------------------------------------
Cash Flow #3
Price = 1148.75
Maturity = 12
Percentage of Face Value that would meet the obligation = 0.919902
Yield to Maturity = 0.0574999
Duration = 8.58082
Duration (to be used in LP-formulation below) = 9.32798
(Note) 8.58082 = 9.32798 x 0.919902
Convexity  = 87.6798
Convexity (to be used in LP-formulation below) = 95.3144
(Note) 87.6798 = 95.3144 x 0.919902
******************************************************************
Average YTM (which I use to compute PV of Debt) = 0.0541665
Present value of debt = 1056.74
******************************************************************
Model name:
                C1        C2        C3
Minimize  -75.3936  -84.042 -95.3144
R1         1131.27  1121.39  1148.75  =  1056.74
R2         8.30593  8.70733  9.32798  =       10
Type          Real     Real     Real
upbo           Inf      Inf      Inf
lowbo            0        0        0
There is no portfolio that meets the duration constraint of 10 years
MacBook-Air-2:Debug sreenivas$
```

Figure 2: A sample screen shot of an infeasible set of cashflows.

```
MacBook-Air-2:Debug sreenivas$ ./Bond\ Immunization input3
Input File: input3
We owe 1790.85 in 10 years
Number of Cash Flows: 3
----------------------------------------------------------------
Cash Flow #1
Price = 1051.52
Maturity = 10
Percentage of Face Value that would meet the obligation = 0.951007
Yield to Maturity = 0.0600001
Duration = 7.6655
Duration (to be used in LP-formulation below) = 8.0604
(Note) 7.6655 = 8.0604 x 0.951007
Convexity  = 67.9958
Convexity (to be used in LP-formulation below) = 71.4987
(Note) 67.9958 = 71.4987 x 0.951007
----------------------------------------------------------------
Cash Flow #2
Price = 1095.96
Maturity = 15
Percentage of Face Value that would meet the obligation = 0.912445
Yield to Maturity = 0.0599997
Duration = 10
Duration (to be used in LP-formulation below) = 10.9596
(Note) 10 = 10.9596 x 0.912445
Convexity  = 121.484
Convexity (to be used in LP-formulation below) = 133.142
(Note) 121.484 = 133.142 x 0.912445
----------------------------------------------------------------
Cash Flow #3
Price = 986.24
Maturity = 30
Percentage of Face Value that would meet the obligation = 1.01396
Yield to Maturity = 0.0599996
Duration = 14.6361
Duration (to be used in LP-formulation below) = 14.4347
(Note) 14.6361 = 14.4347 x 1.01396
Convexity  = 296.143
Convexity (to be used in LP-formulation below) = 292.067
(Note) 296.143 = 292.067 x 1.01396
****************************************************************
Average YTM (which I use to compute PV of Debt) = 0.0599998
Present value of debt = 1000
****************************************************************
Model name:
                C1       C2       C3
Minimize  -71.4987 -133.142 -292.067
R1         1051.52  1095.96   986.24  =     1000
R2          8.0604  10.9596  14.4347  =       10
Type          Real     Real     Real
upbo           Inf      Inf      Inf
lowbo            0        0        0
Largest Convexity we can get is: 144.404
Optimal portfolio:
%Cash Flow:1  0.632508
%Cash Flow:2  0
%Cash Flow:3  0.339581

That is, buy
$665.095 of Cash Flow#1
$334.908 of Cash Flow#3
MacBook-Air-2:Debug sreenivas$
```

Figure 3: A sample screen shot for the example that is done in lesson 4. You can compare these results with those in the Brute-Force Excel Spreadsheets provided on Compass with Lesson 4.