Unit 3 Lecture 4: Lasso regression

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In this R demo, we will learn about the `glmnet` and `glmnetUtils` packages and how to run a cross-validated lasso and elastic net regressions using the `cv.glmnet()` and `cva.glmnet()` functions, respectively.

Let's load the `glmnetUtils` package:

```
library(glmnetUtils)
```

Let's also source a file called `plot_glmnet` with some helper functions.

```
source("../../functions/plot_glmnet.R")
```

We will apply lasso regression to study the effect of 97 socioeconomic factors on violent crimes per capita based on data from 90 communities in Florida:

```
crime_data = read_csv("../../data/CrimeData_FL.csv")
crime_data
```

```
## # A tibble: 90 x 98
##    population household.size race.pctblack race.pctwhite race.pctasian
##         <dbl>          <dbl>         <dbl>         <dbl>         <dbl>
##  1      16023           2.63         13.8           83.9          1.42
##  2      29721           2.34          3.52          95.1          1.03
##  3      10205           2.46          1.06          97.4          1.04
##  4     124773           2.47         29.1           68.2          1.75
##  5      13024           2.25         31.3           67.2          0.5
##  6     280015           2.44         25.0           70.9          1.35
##  7      79443           2.94          3.48          93.1          2.12
##  8      16444           2.57          5.38          91.2          1.96
##  9      46194           2.28         20.1           77.7          0.63
## 10      14044           2.17          0.48          98.3          0.58
## # ... with 80 more rows, and 93 more variables: race.pcthisp <dbl>,
## #   age.pct12to21 <dbl>, age.pct12to29 <dbl>, age.pct16to24 <dbl>,
## #   age.pct65up <dbl>, pct.urban <dbl>, med.income <dbl>, pct.wage.inc <dbl>,
## #   pct.farmself.inc <dbl>, pct.inv.inc <dbl>, pct.socsec.inc <dbl>,
## #   pct.pubasst.inc <dbl>, pct.retire <dbl>, med.family.inc <dbl>,
## #   percap.inc <dbl>, white.percap <dbl>, black.percap <dbl>,
## #   indian.percap <dbl>, asian.percap <dbl>, hisp.percap <dbl>, ...
```

Let's split the data into training and testing, as usual:

```
set.seed(471)
train_samples = sample(1:nrow(crime_data), 0.8*nrow(crime_data))
crime_data_train = crime_data %>% filter(row_number() %in% train_samples)
crime_data_test = crime_data %>% filter(!(row_number() %in% train_samples))
```

Running a cross-validated lasso regression

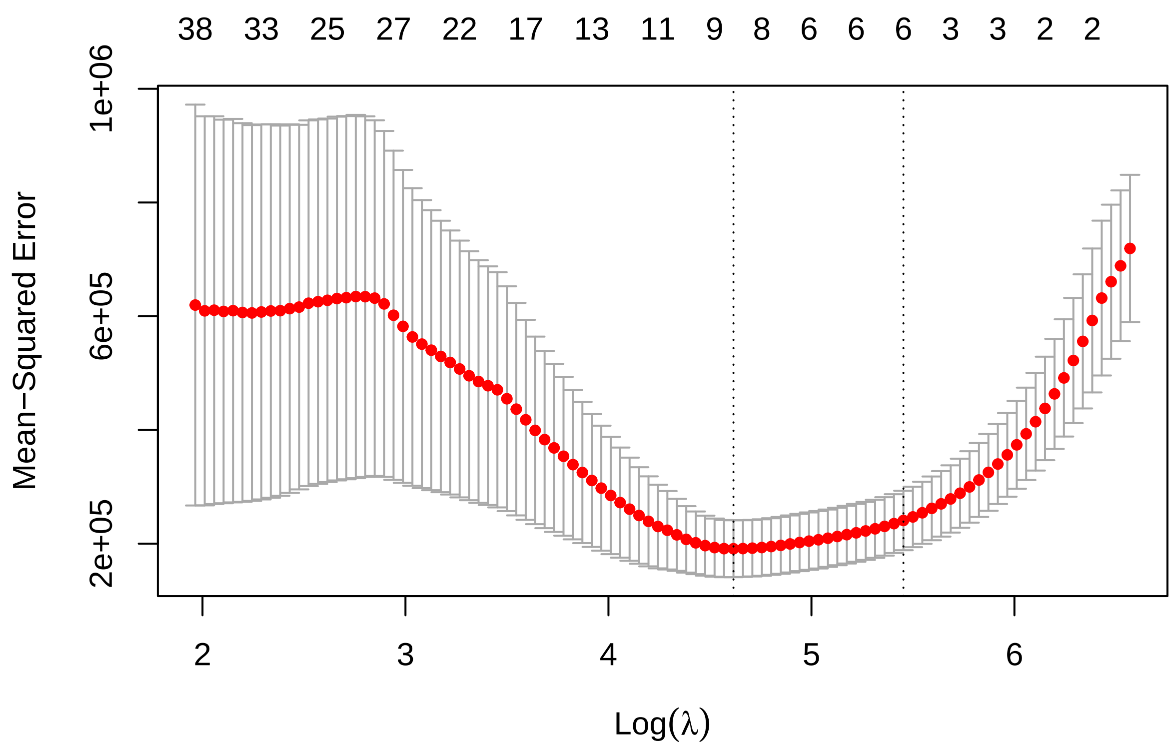
We call `cv.glmnet` on `crime_data_train`:

```
lasso_fit = cv.glmnet(violentcrimes.perpop ~ .,  # formula notation, as usual
                      alpha = 1,                 # alpha = 1 for lasso
                      nfolds = 10,               # number of folds
                      data = crime_data_train)   # data to run lasso on
```

Inspecting the results

The `glmnet` package has a very nice `plot` function to produce the CV plot:

```
plot(lasso_fit)
```



The `lasso_fit` object has several fields with information about the fit:

```
# lambda sequence
head(lasso_fit$lambda)
```

```
## [1] 713.0629 680.6531 649.7163 620.1857 591.9973 565.0901
```

```
# number of nonzero coefficients
head(lasso_fit$nzero)
```

```
## s0 s1 s2 s3 s4 s5
##  0  1  1  2  2  2
```

```r
# CV estimates
head(lasso_fit$cvm)
```

```
## [1] 719195.4 688634.8 660698.6 631908.0 592507.2 555672.5
```

```r
# CV standard errors
head(lasso_fit$cvsd)
```

```
## [1] 129486.1 132612.6 135495.0 136165.2 126567.6 117928.5
```

```r
# lambda achieving minimum CV error
lasso_fit$lambda.min
```

```
## [1] 101.0748
```

```r
# lambda based on one-standard-error rule
lasso_fit$lambda.1se
```

```
## [1] 233.4959
```

To get the fitted coefficients at the selected value of lambda:

```r
coef(lasso_fit, s = "lambda.1se") %>% head()
```

```
## 6 x 1 sparse Matrix of class "dgCMatrix"
##                        s1
## (Intercept)    1138.536
## population        .
## household.size    .
## race.pctblack     .
## race.pctwhite     .
## race.pctasian     .
```

```r
coef(lasso_fit, s = "lambda.min") %>% head()
```

```
## 6 x 1 sparse Matrix of class "dgCMatrix"
##                        s1
## (Intercept)    7695.849
## population        .
## household.size    .
## race.pctblack     .
## race.pctwhite     .
## race.pctasian     .
```

Note that these coefficient vectors are sparse. We can get a list of the nonzero standardized coefficients as follows:

```r
beta_hat_std = extract_std_coefs(lasso_fit, crime_data_train)
beta_hat_std
```

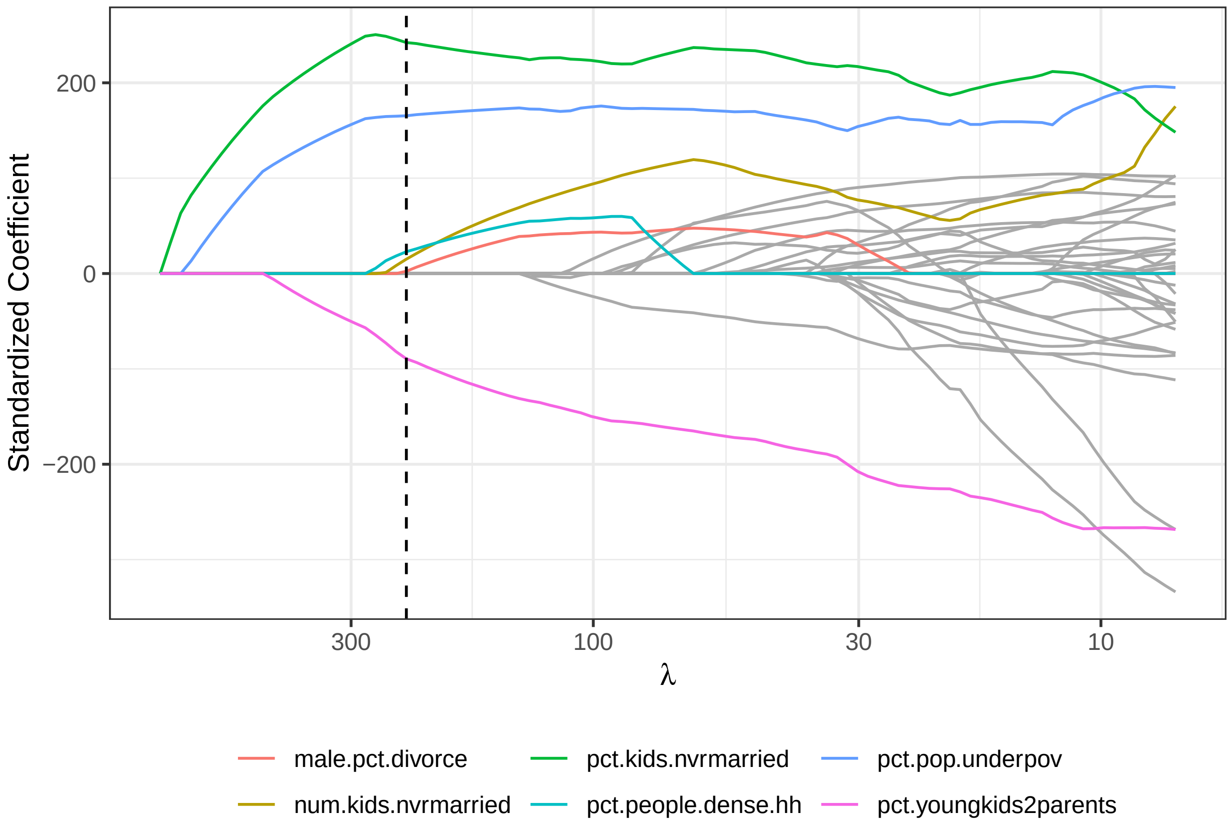
```
## # A tibble: 97 x 2
##    feature        coefficient
##    <chr>                <dbl>
##  1 population               0
##  2 household.size           0
##  3 race.pctblack            0
##  4 race.pctwhite            0
##  5 race.pctasian            0
##  6 race.pcthisp             0
##  7 age.pct12to21            0
##  8 age.pct12to29            0
##  9 age.pct16to24            0
## 10 age.pct65up              0
## # ... with 87 more rows
```

```r
beta_hat_std %>% filter(coefficient != 0)
```

```
## # A tibble: 6 x 2
##   feature               coefficient
##   <chr>                       <dbl>
## 1 pct.pop.underpov           165.
## 2 male.pct.divorce             2.37
## 3 pct.youngkids2parents      -89.4
## 4 num.kids.nvrmarried         15.1
## 5 pct.kids.nvrmarried        242.
## 6 pct.people.dense.hh         22.9
```

To visualize the fitted coefficients as a function of lambda, we can make a plot of the coefficients like we saw in class. To do this, we can use the `plot_glmnet` function, which by default shows a dashed line at the lambda value chosen using the one-standard-error rule:

```r
plot_glmnet(lasso_fit, crime_data_train)
```



By default, `plot_glmnet` annotates the features with nonzero coefficients. To interpret these coefficient estimates, recall that they are for the *standardized* features.

Making predictions

To make predictions on the test data, we can use the `predict` function (which we've seen before):

```
lasso_predictions = predict(lasso_fit,
                            newdata = crime_data_test,
                            s = "lambda.1se") %>% as.numeric()
lasso_predictions
```

```
##  [1] 1901.3904 1331.1849  950.1509  809.1830  757.2346  756.9001  864.4810
##  [8]  786.1132  820.8253 1063.3772  647.6317 1343.7530 1163.0144 1078.2869
## [15] 1366.4746  741.4528  883.7059  820.3576
```

We can evaluate the root-mean-squared-error as before:

```
RMSE = sqrt(mean((lasso_predictions - crime_data_test$violentcrimes.perpop)^2))
RMSE
```

```
## [1] 400.7576
```

Elastic net regression

Next, let's run an elastic net regression. We can do this via the `cva.glmnet()` function:

```
elnet_fit = cva.glmnet(violentcrimes.perpop ~ .,   # formula notation, as usual
                       nfolds = 10,                # number of folds
                       data = crime_data_train)    # data to run on
```

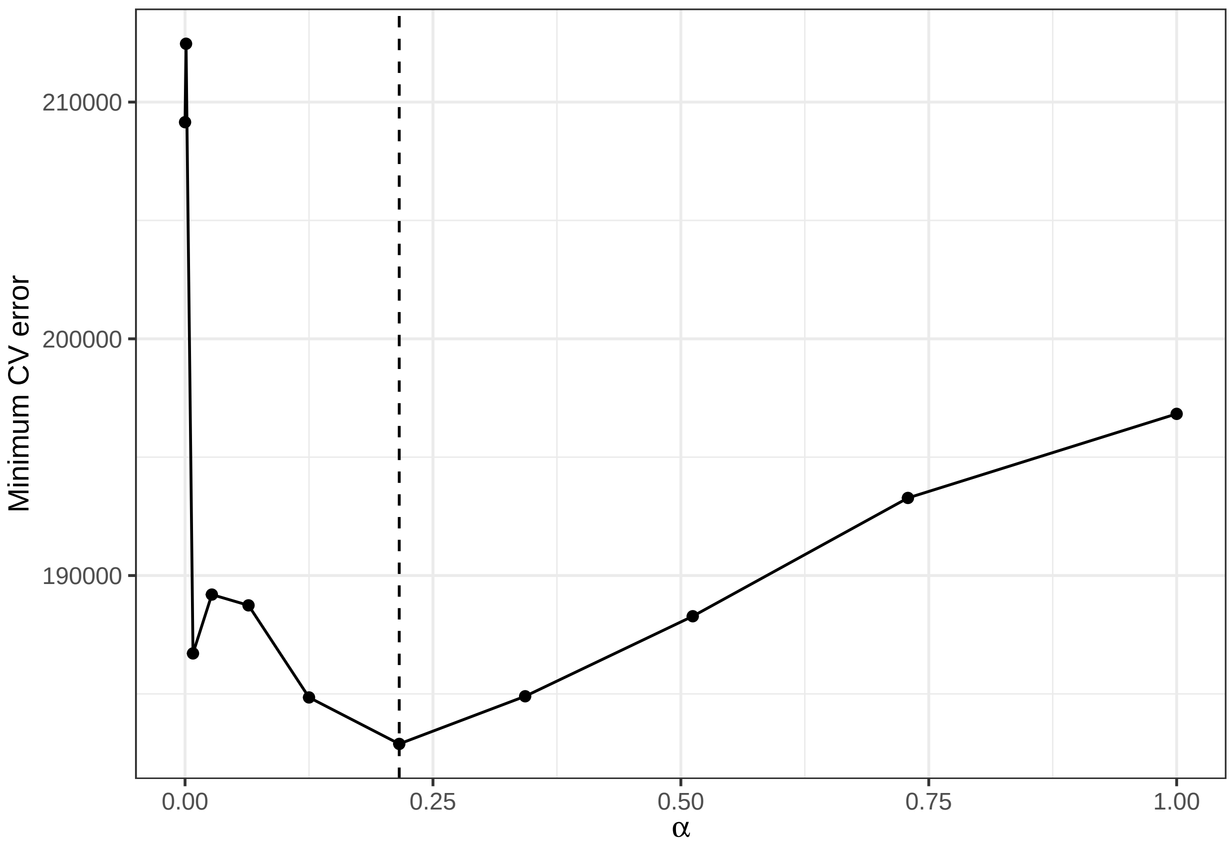
The following are the values of `alpha` that were used:

```
elnet_fit$alpha
```

```
##  [1] 0.000 0.001 0.008 0.027 0.064 0.125 0.216 0.343 0.512 0.729 1.000
```

We can plot the minimum CV error for each value of alpha using the helper function `plot_cva_glmnet()` from `plot_glmnet.R`:

```
plot_cva_glmnet(elnet_fit)
```



We can then extract the `cv.glmnet` fit object based on the optimal `alpha` using `extract_best_elnet` from `plot_glmnet.R`:

```
elnet_fit_best = extract_best_elnet(elnet_fit)
```

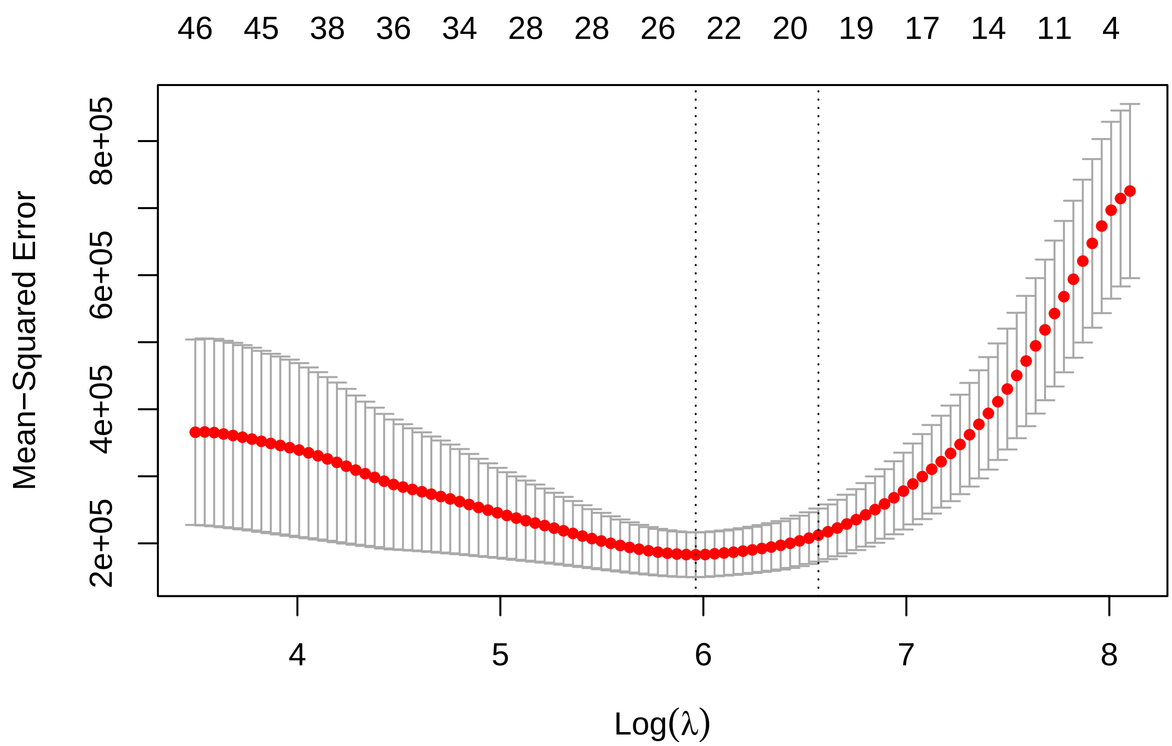
The `elnet_fit_best` object is a usual `glmnet` fit object, with an additional field called `alpha` specifying which value of `alpha` was used:

```
elnet_fit_best$alpha
```

```
## [1] 0.216
```

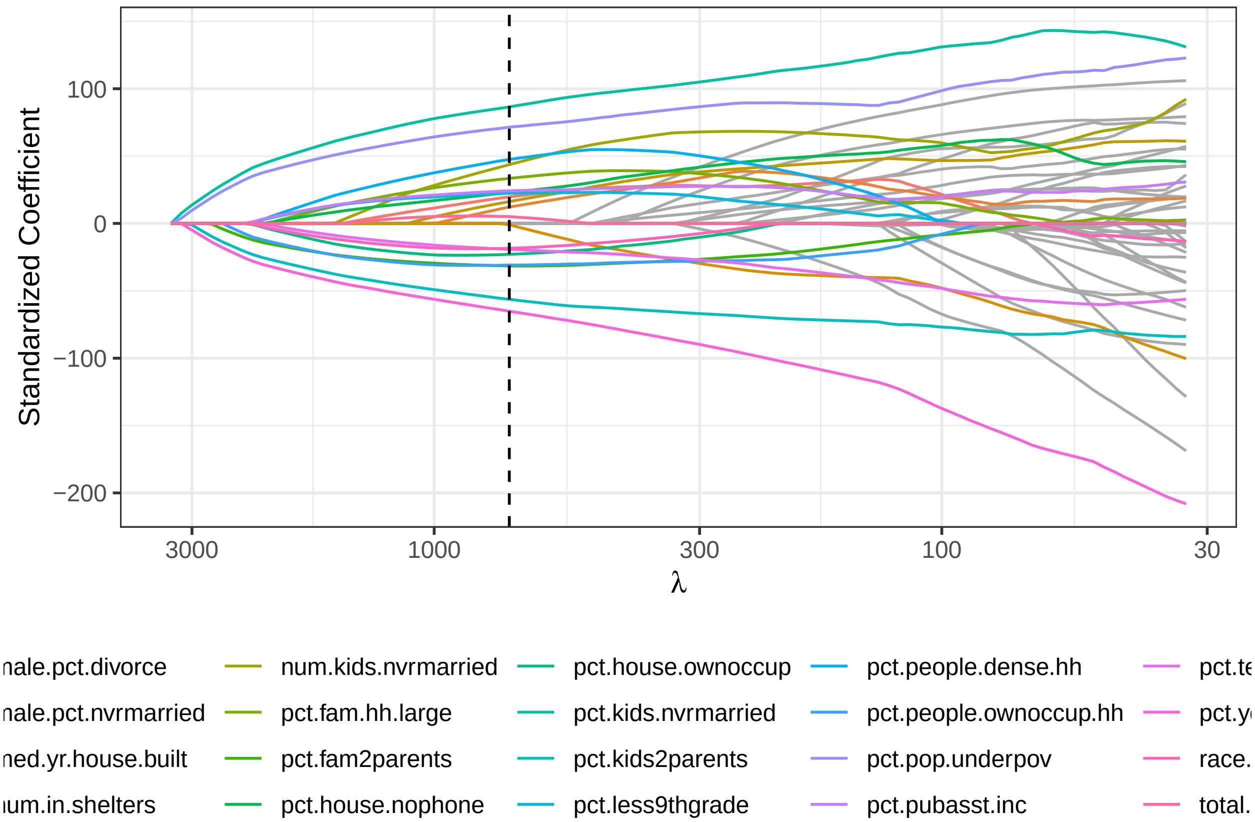
We can make a CV plot to select `lambda` as usual:

```
plot(elnet_fit_best)
```



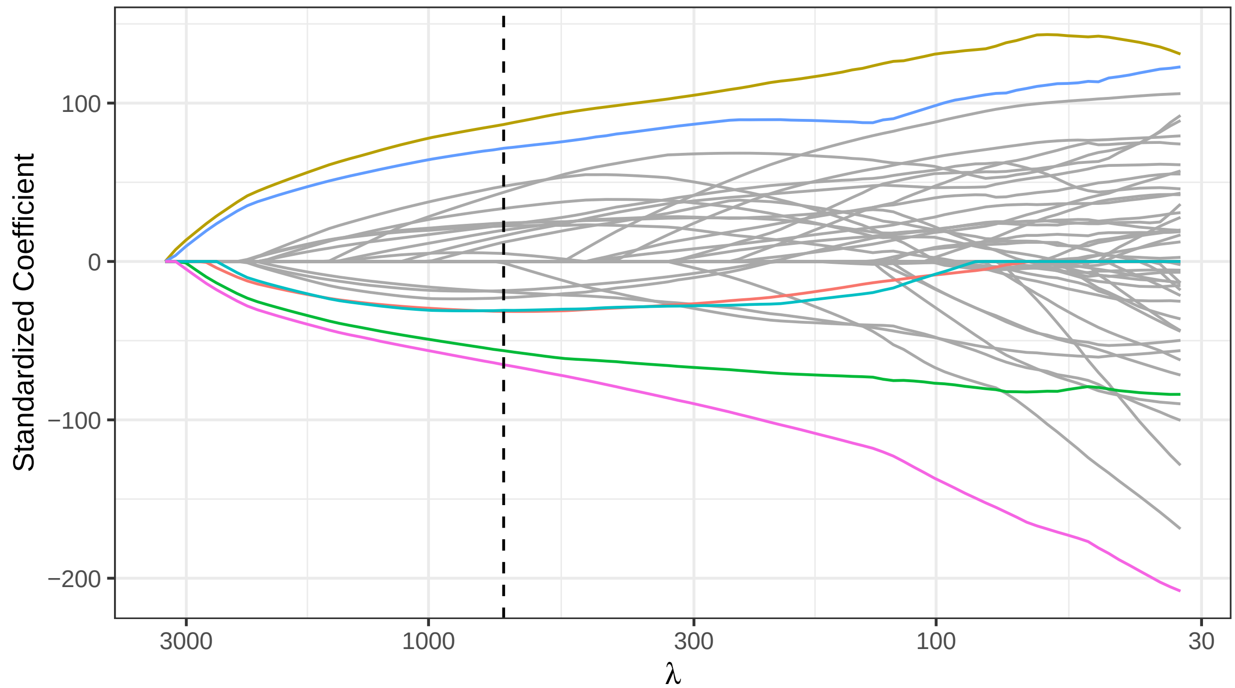
And we can make a trace plot for this optimal value of `alpha`:

```
plot_glmnet(elnet_fit_best, crime_data_train)
```



This is too many features to highlight, so let's choose a smaller number:

```
plot_glmnet(elnet_fit_best, crime_data_train, features_to_plot = 6)
```



We can make predictions and evaluate test error using the `elnet_fit_best` object:

```
elnet_predictions = predict(elnet_fit,
                            alpha = elnet_fit$alpha,
                            newdata = crime_data_test,
                            s = "lambda.1se") %>% as.numeric()
elnet_predictions
```

```
##  [1] 1737.2357 1344.0327 1036.0706  762.1868  671.2751  690.3775  975.6859
##  [8]  807.5988  833.6900  744.3694  545.3223 1384.0474 1250.4805 1258.0862
## [15] 1445.2206  702.8391  693.6531  701.8896
```

```
RMSE = sqrt(mean((elnet_predictions - crime_data_test$violentcrimes.perpop)^2))
RMSE
```

```
## [1] 392.1493
```

Ridge logistic regression

We can also run a lasso-penalized logistic regression. Let's try it out on a binarized version of the crime data:

```
# redefine response based on whether violentcrimes.perpop is above the median
crime_data_binary_train = crime_data_train %>%
  mutate(violentcrimes.perpop =
           as.numeric(violentcrimes.perpop > median(violentcrimes.perpop)))
```

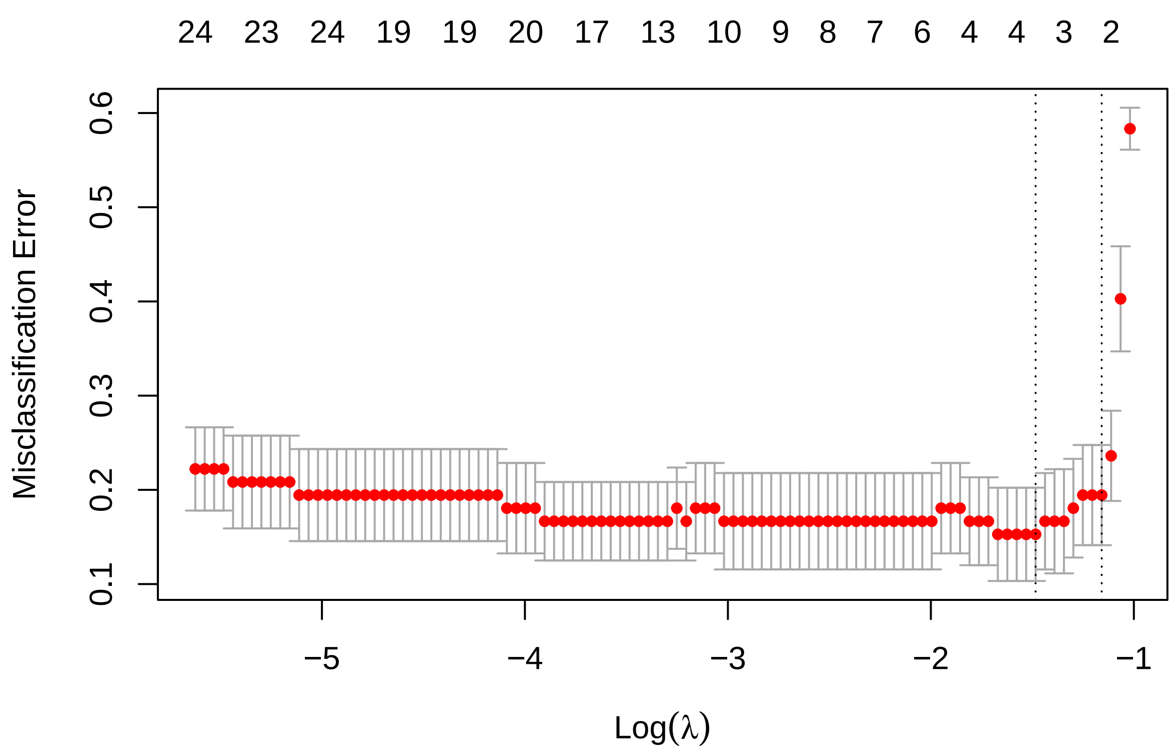
```r
crime_data_binary_test = crime_data_test %>%
  mutate(violentcrimes.perpop =
           as.numeric(violentcrimes.perpop > median(violentcrimes.perpop)))
```

To run the logistic lasso regression, we call `cv.glmnet` as before, adding the argument `family = binomial` to specify that we want to do a logistic regression and the argument `type.measure = "class`: to specify that we want to use the misclassification error during cross-validation.

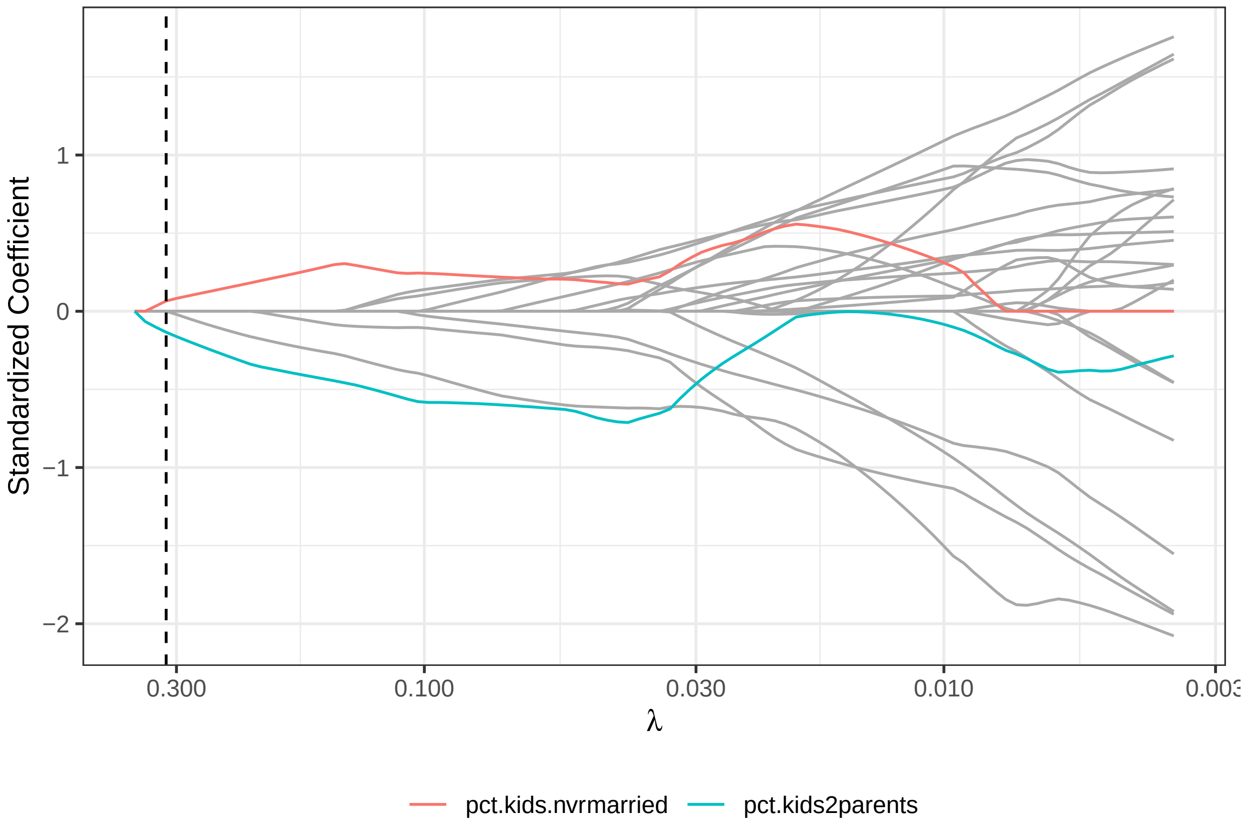
```r
lasso_fit = cv.glmnet(violentcrimes.perpop ~ .,        # formula notation, as usual
                      alpha = 1,                       # alpha = 0 means lasso
                      nfolds = 10,                     # number of CV folds
                      family = "binomial",             # logistic regression
                      type.measure = "class",          # use misclassification error
                      data = crime_data_binary_train) # train on default_train data
```

We can then take a look at the CV plot and the trace plot as before:

```r
plot(lasso_fit)
```




```r
plot_glmnet(lasso_fit, crime_data_binary_train)
```



To predict using the fitted model, we can use the `predict` function again, this time specifying `type = "response"` to get the predictions on the probability scale (as opposed to the log-odds scale).

```
probabilities = predict(lasso_fit,                          # fit object
                        newdata = crime_data_binary_test, # new data to test on
                        s = "lambda.1se",                 # value of lambda to use
                        type = "response") %>%            # output probabilities
  as.numeric()                                            # convert to vector
head(probabilities)
```

```
## [1] 0.5750745 0.5179763 0.4617836 0.4392386 0.4409267 0.4460029
```

We can threshold the probabilities to get binary predictions as we did with regular logistic regression.