南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

DDL: 14:00 Thursday of the sixteenth academic week (June 5th) .

The homework contains 4 questions and the score is 100 in total.

1. Sketch a number of trajectories corresponding to the following autonomous systems, and indicate the direction of motion for increasing t. Identify and classify any rest points as being stable, asymptotically stable, or unstable.
   (a) $\frac{dx}{dt} = x, \frac{dy}{dt} = y$
   (b) $\frac{dx}{dt} = -x, \frac{dy}{dt} = 2y$
   (c) $\frac{dx}{dt} = y, \frac{dy}{dt} = -2x$
   (d) $\frac{dx}{dt} = -x + 1, \frac{dy}{dt} = -2y$

2. Consider the following economic model: Let P be the price of a single item on the market. Let Q be the quantity of the item available on the market. Both P and Q are functions of time. If we consider price and quantity as two interacting species, the following model might be proposed:

$$\frac{dP}{dt} = aP\left(\frac{b}{Q} - P\right), \qquad \frac{dQ}{dt} = cQ(fP - Q),$$

   where a, b, c, and f are positive constants. Justify and discuss the adequacy of the model.
   (a) If $a = 1$, $b = 20{,}000$, $c = 1$, and $f = 30$, find the equilibrium points of this system. Classify each equilibrium point with respect to its stability, if possible. If a point cannot be readily classified, explain why.
   (b) Perform a graphical stability analysis to determine what will happen to the levels of P and Q as time increases.
   (c) Give an economic interpretation of the curves that determine the equilibrium points.

3. Consider two species whose survival depends on their mutual cooperation. Let's take as an example a species of bee that feeds primarily on the nectar of one plant species and simultaneously pollinates that plant. One simple model of this *mutualism* is given by the autonomous system

$$\frac{dx}{dt} = -ax + bxy, \qquad \frac{dy}{dt} = -my + nxy.$$

   (a) What assumptions are implicitly being made about the growth of each species in the absence of cooperation?

   (b) Interpret the constants a, b, m, and n in terms of the physical problem.

   (c) What are the equilibrium levels?

   (d) Perform a graphical analysis and indicate the trajectory directions in the phase plane.

   (e) Find an analytic solution and sketch typical trajectories in the phase plane.

   (f) Interpret the outcomes predicted by your graphical analysis. Do you believe the model is realistic? Why?

4. Let X denote a guerrilla force and Y denote a conventional force. The autonomous system

$$\frac{dx}{dt} = -gxy, \qquad \frac{dy}{dt} = -bx$$

   is a Lanchestrian model for conventional–guerrilla combat in which there are no operational loss rates and no reinforcements.

   (a) Discuss the assumptions and relationships necessary to justify the model. Does the model seem reasonable?

   (b) Solve the system and obtain the *parabolic law*

$$g\,y^2 = 2b\,x + M, \qquad \text{where } M = g\,y_0^2 - 2b\,x_0.$$

   (c) What condition must be satisfied by the initial force levels x_0 and y_0 for the conventional Y force to win? If the Y force does win, how many survivors will there be?

Hint: You can reasonably use any AI tools to assist you in completing your homework. Attention: Please submit ONLY the PDF of your homework to

jzlisustc@gmail.com to keep record.