Computational Intelligence & Adversarial Machine Learning:

Particle Swarm Optimization

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Particle Swarm Optimization

- Particle Swarm Optimization (PSO) applies to concept of social interaction to problem solving.
- It was developed in 1995 by James Kennedy and Russ Eberhart [Kennedy, J. and Eberhart, R. (1995). "Particle Swarm Optimization", *Proceedings of the 1995 IEEE International Conference on Neural Networks*, pp. 1942-1948, IEEE Press.] (http://dsp.jpl.nasa.gov/members/payman/swarm/kennedy95-ijcnn.pdf)
- It has been applied successfully to a wide variety of search and optimization problems.
- In PSO, a swarm of n individuals communicate either directly or indirectly with one another search directions (gradients).
- PSO is a simple but powerful search technique.

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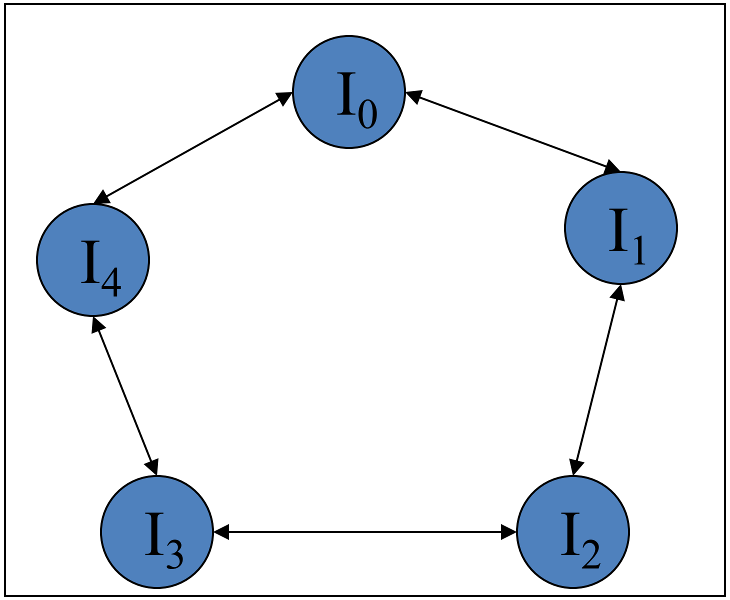
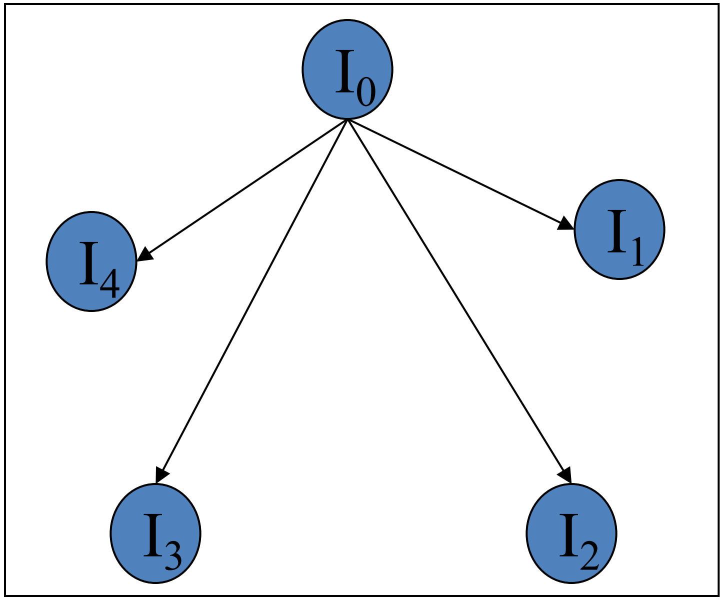
Particle Swarm Optimization:
Swarm Topology

- In PSO, there have been two basic topologies used in the literature
  - Ring Topology (neighborhood of 3)
  - Star Topology (global neighborhood)





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Particle Swarm Optimization:

The Anatomy of a Particle

- A particle (individual) is composed of:
  - Three vectors:
    - The **x-vector** records the current position (location) of the particle in the search space,
    - The **p-vector** records the location of the best solution found so far by the particle, and
    - The **v-vector** contains a gradient (direction) for which particle will travel in if undisturbed.
  - Two fitness values:
    - The **x-fitness** records the fitness of the x-vector, and
    - The **p-fitness** records the fitness of the p-vector.

$$\mathbf{I_k}$$

$$X = \langle x_{k0}, x_{k1}, \ldots, x_{kn-1} \rangle$$

$$P = \langle p_{k0}, p_{k1}, \ldots, p_{kn-1} \rangle$$

$$V = \langle v_{k0}, v_{k1}, \ldots, v_{kn-1} \rangle$$

$$x\_fitness = ?$$

$$p\_fitness = ?$$

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Particle Swarm Optimization: Swarm Search

- In PSO, particles never die!
- Particles can be seen as simple agents that fly through the search space and record (and possibly communicate) the best solution that they have discovered.
- So the question now is, “How does a particle move from on location in the search space to another?”
- This is done by simply adding the v-vector to the x-vector to get another x-vector ($X_i = X_i + V_i$).
- Once the particle computes the new Xi it then evaluates its new location. If x-fitness is better than p-fitness, then $P_i = X_i$ and p-fitness = x-fitness.

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Particle Swarm Optimization:

Swarm Search

- Actually, we must adjust the v-vector before adding it to the x-vector as follows:
  - $v_{id} = v_{id} + \varphi 1 * rnd() * (p_{id} - x_{id}) + \varphi 2 * rnd() * (p_{gd} - x_{id});$
  - $x_{id} = x_{id} + v_{id};$
- Where **i** is the particle,
- $\varphi 1, \varphi 2$ are learning rates governing the **cognition** and **social** components
- Where **g** represents the index of the particle with the best p-fitness, and
- Where **d** is the d^{th} dimension.

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Particle Swarm Optimization: Swarm Search

- Intially the values of the velocity vectors are randomly generated with the range [-Vmax, Vmax] where Vmax is the maximum value that can be assigned to any v_{id}.

Particle Swarm Optimization:

Swarm Types

- In his paper, [Kennedy, J. (1997), "The Particle Swarm: Social Adaptation of Knowledge", Proceedings of the 1997 International Conference on Evolutionary Computation, pp. 303-308, IEEE Press.]
- Kennedy identifies 4 types of PSO based on φ1 and φ2 .
- Given: $v_{id} = v_{id} + \varphi 1 * rnd() * (p_{id} - x_{id}) + \varphi 2 * rnd() * (p_{gd} - x_{id});$

$$x_{id} = x_{id} + v_{id};$$

  - Full Model (φ1, φ2 > 0)
  - Cognition Only (φ1 > 0 and φ2 = 0),
  - Social Only (φ1 = 0 and φ2 > 0)
  - Selfless (φ1 = 0, φ2 > 0, and g ≠ i)

Particle Swarm Optimization:
Related Issues

- There are a number of related issues concerning PSO:
  - Controlling velocities (determining the best value for Vmax),
  - Swarm Size,
  - Neighborhood Size,
  - Updating X and Velocity Vectors,
  - Robust Settings for ($\varphi 1$ and $\varphi 2$),
  - An Off-The-Shelf PSO

- Carlisle, A. and Dozier, G. (2001). “An Off-The-Shelf PSO”, *Proceedings of the 2001 Workshop on Particle Swarm Optimization*, pp. 1-6, Indianapolis, IN.

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Particle Swarm Optimization: Controlling Velocities

- When using PSO, it is possible for the magnitude of the velocities to become very large.
- Performance can suffer if Vmax is inappropriately set.
- Two methods were developed for controlling the growth of velocities:
  - A dynamically adjusted inertia factor, and
  - A constriction coefficient.

Particle Swarm Optimization:

The Inertia Factor

- When the inertia factor is used, the equation for updating velocities is changed to:

$$v_{id} = \omega * v_{id} + \varphi 1 * rnd() * (p_{id} - x_{id}) + \varphi 2 * rnd() * (p_{gd} - x_{id});$$

- Where ω is initialized to 1.0 and is gradually reduced over time (measured by cycles through the algorithm).

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Particle Swarm Optimization:
The Constriction Coefficient

- In 1999, Maurice Clerc developed a constriction Coefficient for PSO.
  - $v_{id} = K[v_{id} + \varphi1*rnd()*(p_{id}-x_{id}) + \varphi2*rnd()*(p_{gd}-x_{id})];$
  - Where K = 2/|2 - φ - sqrt(φ^2 - 4φ)|,
  - $\varphi = \varphi1 + \varphi2$, and
  - $\varphi > 4$.

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Particle Swarm Optimization:

Swarm and Neighborhood Size

- Concerning the swarm size for PSO, as with other ECs there is a trade-off between solution quality and cost (in terms of function evaluations).
- Global neighborhoods seem to be better in terms of computational costs. The performance is similar to the ring topology (or neighborhoods greater than 3).

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Particle Swarm Optimization:

Particle Update Methods

- There are two ways that particles can be updated:
  - Synchronously
  - Asynchronously
- Asynchronous update allows for newly discovered solutions to be used more quickly
- The asynchronous update method is similar to ____.