CSCI 2300 HW #2

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Question 1 (2.23):

a) If A has a majority element v, v must also be a majority element of A_1 or A_2 or both. To find v, recursivly compute the maroity elemtns, if any, of A_1 and A_2 check whether one of these is a majority element of A.

Run Time: $T(n) = 2T(n/2)\, O(n) = O(n \log n\,)$.

b) After the procedure, there are at most $\left(\frac{n}{2}\right)$ elements left since at lease one element in the pairs is discarded. If these remaining elements have a majority, v exists and appears at least $\left(\frac{n}{4}\right)$ times so v must be paired with itself at least $\left(\frac{n}{4}\right)$ times which shows that A contains at least $\left(\frac{n}{2}\right)$ copies of v.

Run Time: $T(n) = T\left(\frac{n}{2}\right) + O(n) = O(n)$

Question 2 (2.24):

a) function quicksort(array)

if length(array) <= 1

return array

select and remove a pivot element *piv* from array

create empty lists *less* and *greater*

for each x in array:

if x <= pivot then append x to less

else append x to greater

return concatenate(quicksort(less),list(pivot),quicksort(greater))

b) $T(n) = T(k) - T(n-k) + \alpha n$

Worst Case:

$T(n) = T(1) + T(n-1) + \alpha n$

$T(n) = T(n-1) + T(n) + \alpha n$

$T(n) = [T(n-2) + T(1)\, \alpha(n-1)] + T(1) + \alpha n$

$T(n) = T(n-2) + 2T(1) + \alpha(n-1+n)$

$T(n) = [T(n-3) + T(1) + \alpha(n-2)] + 2T(1) + \alpha(n-1+n)$

$T(n) = T(n-3) + 3T(1) + \alpha((n-2) + (n-1) + n)$

$T(n) = [T(n-4) + 4T(1) + \alpha(n-3)] + 3T(1) + \alpha((n-2) + (n-1) + n)$

$T(n) = T(n-4) + 4T(1) + \alpha((n-3) + (n-2) + (n-1) + n)$

$T(n) = T(n-i) + \mathrm{iT}(1) + \alpha((n-i+1+\ldots+n-2+n-2+n))$

sub $i = n-1$ since n can$'t$ be less than 1. $\sum\, (n-j)$

$\mathrm{nT}(1) + \alpha\left(\frac{n}{n-2}\right) - \frac{(n-2)(n-1)}{2} = O(n^2)$

c) $T(n) \leq O(n) + \frac{1}{n}\sum\ (T(i) + T(n))$

$T(n) = 2T\left(\frac{n}{2}\right) + \alpha n$

$T(n) = 2\left(2T\left(\frac{n}{2}\right) + \alpha\frac{n}{2}\right) + \alpha n$

$T(n) = 2^2T\left(\frac{n}{4}\right) + 2\alpha n$

$T(n) = 2^2\left(2T\left(\frac{n}{8}\right) + \alpha\frac{n}{4}\right) + 2\alpha n$

$T(n) = 2^3T\left(\frac{n}{8}\right) + 3\alpha n$

$T(n) = 2^kT\left(\frac{n}{2^k}\right) + k\alpha n$

recurrence will continue until $k = 2^k$ otherwise $\frac{n}{2^k} < i$

sub, $k = \log n$

$T(n) = \mathrm{nT}(1) + \alpha\, n \log n$ Therefore $O(n \log n)$

Question 3 (3.3):

a)

A (1, 14)	E (11, 12)
B (15,16)	F (4, 9)
C (2, 13)	G (5, 6)
D (3, 10)	H (7, 8)

b) A and B are sources while G and H are sinks

c) Topological Sort: Decreasing order of postnum

B, A, C, E, D, F, H, G

d) {A,B} , C , {D,E} , F, {G, H} where pairs in {} can appear in any order there are a total of $2^3 = 8$ orderings.

Question 4 (3.4):

i. {C, D, F, G, H, I, J} is found first and is the Source SCC.

{A, B, E} is found second and is the sink SCC.

By adding one edge in the sink to any vertex in the source would make the entire graph SCC.

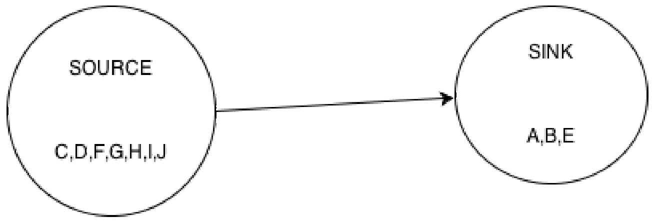
ii. {D, F, G, H, I} is found first and is the sink SCC.

{C} is found second

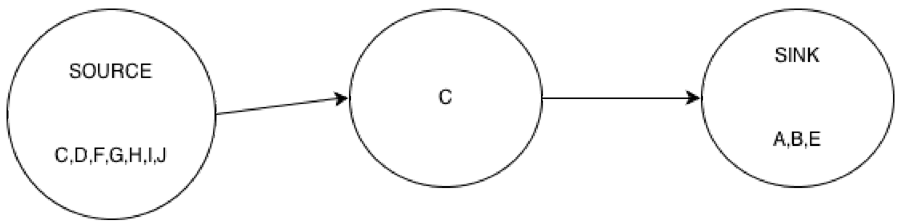
{A, B, D} is found third and is the source SCC.

By adding one edge in the sink to any vertex in the source would make the entire graph SCC.

i.)



ii.)



Question 5 (3.23):

Linearize the DAG;

Any path from s to t will only pass through the linearized vertices from s to t.

$S = V_0, V_1, V_2, ..., V_{k=t}$ (The vertices from s -> t linearized)

for each i:

count paths from s to V_i as n_i

each path to vertex i and an edge (i, j), gives a path vertex j

for all edges in E:

$n_j = \sum n_i$

i < j for all (i, j) in E:

compute in increasing order.

Question 6 (4.1):

NODE	0	1	2	3	4	5	6	7
A	0	0	0	0	0	0	0	0
B	∞	1	1	1	1	1	1	1
C	∞	∞	3	3	3	3	3	3
D	∞	∞	∞	4	4	4	4	4
E	∞	4	4	4	4	4	4	4
F	∞	8	7	7	7	7	6	6
G	∞	∞	7	5	5	5	5	5
H	∞	∞	∞	∞	8	8	6	6

Table 1.

Question 7 (4.2)

NODE	0	1	2	3	4	5	6
S	0	0	0	0	0	0	0
A	∞	7	7	7	7	7	7
B	∞	∞	11	11	11	11	11
C	∞	6	5	5	5	5	5
D	∞	∞	8	7	7	7	7
E	∞	6	6	6	6	6	6
F	∞	5	4	4	4	4	4
G	∞	∞	∞	9	8	8	8
H	∞	∞	9	7	7	7	7
I	∞	∞	∞	∞	8	7	7

Table 2.

Question 9 (5.2)

EDGE	COST	KNOWN
(A,B)	1	{A,B}
(B,C)	2	{A,B,C}
(C,D)	3	{A,B,C,D}
(D,G)	2	{A,B,C,D,G}
(G,F)	1	{A,B,C,D,G,F}
(G,H)	1	{A,B,C,D,G,F,H}
(A,E)	4	{A,B,C,D,G,F,H,E}
TOTAL	13	

Table 3.