MATH 217 - LINEAR ALGEBRA
Homework 1 Part B, DUE Thursday, January 18 at 11:59pm
Zhengyu James Pan (jzpan@umich.edu)

1. Decide whether the following statements are true or false. Briefly justify your answers.
   (a) 2 is even or 3 is odd.

   Solution: $\boxed{\text{True,}}$ both P and Q are true, so the "or" statement is also true.

   (b) If the Riemann Hypothesis is true, then 217 is not a prime number.

   Solution: $\boxed{\text{True,}}$ Q is true. "If" propositions can only be false when Q is false.

   (c) $\frac{d}{dx}(x^2) = 2x$ if and only if $tan(\pi/6) = \sqrt{3}$.

   Solution: $\boxed{\text{True,}}$ both P and Q are true, so $P \implies Q$ and $Q \implies P$ are true.

   (d) If the set of even prime numbers is infinite, then 10 is even and 10^10 is odd.

   Solution: $\boxed{\text{True,}}$ P is false.

   (e) If every right triangle in $\mathbb{R}^2$ has two acute angles, then every real number has a positive cube root.

   Solution: $\boxed{\text{False,}}$ P is true but Q is false.

2. (a) Let P (x) be a statement whose truth value depends on x. An example is a value of x that makes $P(x)$ true, and a counterexample is a value of x that makes P (x) false. Fill in the blank spaces with "is true", "is false", or "nothing" as appropriate:

   Solution:

	"$\forall x, P(x)$"	"$\exists x$ s.t. $P(x)$"
An example proves	nothing	is true
A counterexample proves	is false	nothing

   (b) Every prime number is even or odd.

   Solution: True, prime numbers are all integers, which are all either even or odd.

   (c) Every prime number is even or every prime number is odd.

   Solution: False, 3 and 2 are counterexamples respectively.

   (d) There exists $n \in \mathbb{Z}$ such that for every $x \in \mathbb{R}, n < x$.

   Solution: True,