MATH 215 FALL 2023
Homework Set 8: §16.1 – 16.3

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1. Compute $\int_C x^2 y\, ds$, where C is the segment of the helix of radius 1 about the z-axis, oriented counter-clockwise in the xy-plane, starting at (1, 0, 0) and ending at $(0, 1, \frac{\pi}{2})$.

2. Compute $\int_C x^2\, dx + y^2\, dy$, where C is the circular arc starting at (2, 0) and ending at (0, 2) followed by the straight line segment from (0, 2) to (-1, 1).

3. Do Exercise 53 of §16.2 in $Stewart's Multivariable Calculus$.

4. A wire has the shape of a helix with parametrization $x = t, y = 2\cos t, z = 2\sin t$ for $0 \le t \le 6\pi$, where distances are measured in cm. Find the mass and the center of mass of the wire if the density (in grams/cm) of the wire at any point is equal to four times the square of the distance from the origin to the point.

5. Let $\overrightarrow{F} = \nabla f$ where $f(x, y) - \frac{y^{2002}}{1+x^{200002}+y^{2002}}$. Can you find a (smooth, simple, but not necessarily closed) curve C with the following property:
   (a) $\int_C \overrightarrow{F} \cdot d\overrightarrow{r} = \frac{1}{2}$
   (b) $\int_C \overrightarrow{F} \cdot d\overrightarrow{r} = 1$

6. Do Exercises 11, 31, and 32 of §16.3 in $Stewart's Multivariable Calculus$.

7. (a) Calculate $\oint_C \overrightarrow{F} \cdot d\overrightarrow{r}$ where

$$\overrightarrow{F}(x, y) = \langle \frac{2xy}{(x^2 + y^2)^2}, \frac{y^2 - x^2}{(x^2 + y^2)^2} \rangle$$

   (b) Repeat the previous part, only this time take the curve C to be the ellipse defined by $4x^2 + 9y^2 = 36$, oriented counterclockwise. Hint: It may be possible to do this integration without parametrizing the ellipse.