MATH 215 FALL 2023
Homework Set 4: §14.1 – 14.5
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1. Do Exercise 32 of §14.1 of Stewart's Multivariable Calculus.

2. Do Exercises 61-66 of §14.1 of Stewart's Multivariable Calculus.

3. Do Exercise 6 of §14.3 of Stewart's Multivariable Calculus.

4. (a) Suppose $g(x, y) = \sqrt{9 - 9x^2 - y^2}$. Draw a contour map for g and then sketch the graph of g.

   (b) Draw a contour map of the function $m(x, y) = \frac{x}{(x2+3y2)}$, showing and labelling several level curves. curves.

5. (a) Use a linear approximation to estimate $(0.99)3 + (2.01)3 - 6(0.99)(2.01)$.

   (b) Let $f(x, y) = xe^{y^2} - ye^{x^2}$ and find the equation for the tangent plane to the graph of f at (1, 2).

   (c) What point on the surface $z = x^2 - y^2$ has a tangent plane parallel to the plane found in the previous part?

6. The wave heights h in the open sea depend on the speed v of the wind and the length of time t that the wind has been blowing at that speed. Values of the function $h = f(v, t)$ are recorded in feet in the following table:

Duration (hours)

Wind speed (knots) v \ t	5	10	15	20	30	40	50
10	2	2	2	2	2	2	2
15	4	4	5	5	5	5	5
20	5	7	8	8	9	9	9
30	9	13	16	17	18	19	19
40	14	21	25	28	31	33	33
50	19	29	36	40	45	48	50
60	24	37	47	54	62	67	69

   (a) What are the meanings of the partial derivatives $\frac{\delta h}{\delta v}$ and $\frac{\delta h}{\delta t}$?

(b) Estimate the values of $f_v(40, 15)$ and $f_t(40, 15)$. What are the practical interpretations of these values?

(c) Estimate the values of $f_{vv}(30, 20)$, $f_{tt}(30, 20)$, $f_{vt}(30, 20)$, and $f_{tv}(30, 20)$. Are your answers for f_{tv} the same as for f_{vt}? Should they be? Explain. Hint: This problem might be trickier than it looks.

7. Determine which of the following functions is a solution to Laplace's equation uxx + uyy = 0:

(a)