Week 4, Lectures 9,10,11

Game plan for this week: Functions of multiple variables!

Review:

Velocity, speed and acceleration: Centripetal Force: Ballistic motion: Normal and tangential acceleration:

Functions:
Domain and range:

Independent and dependent variables:

Example: My dialing in recipe for espresso:

Example: What are the domain and range of $f(x) = \sqrt{9 - x^2 - y^2}$?

Graphs:

Level sets: also called countour lines

Functions of 3 variables:

Notation: In general, it is cumbersome to write (x, y, z) or $(x_1, x_2, \cdots, x_n)$.

A collage of beautiful contour lines:

Limits: (briefly) **Continuity:**

Partial derivatives:

Example: What are the partial derivatives of $\sin x \cos y$?

Example: If we have an implicitly defined function for $z = f(x, y)$ according to the equation $x^3 + y^3 + z^3 + 6xyz = 1$, find the partials of f.

Higher order partials:

Clairaut's theorem:

A glance at PDE:

Nodal sets: (for fun)

Tangent planes: Equation of the tangent plane: Linearization: (really affine)

Continuity: If derivatives exist and are continuous near a point, then the function is differentiable there.

Total differential:

Example: If $f(x, y) = 3x^2 + 2xy + y^4$, find the differential df.