MATH 217 W24 - LINEAR ALGEBRA, Section 001 (Dr. Paul Kessenich)
Homework Set Part B due ??? at 11:59pm
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1. Question

   (a) Prove that F is alternating if and only if $F(\vec{u},\vec{v}) = -F(\vec{v},\vec{u})$ for all $\vec{u},\vec{v} \in \mathbb{R}^2$.

   Solution: By bilinearity, we know

$$\begin{aligned} F(u+v,v+u) &= 0 \\ F(u,v+u)+F(v,v+u) &= 0 \\ F(u,v)+F(u,u)+F(v,v)+F(v,u) &= 0 \\ F(u,v)+0+0+F(v,u) &= 0 \\ F(u,v)+F(v,u) &= 0 \\ F(u,v) &= -F(v,u) \end{aligned}$$

   (b) Prove that if F is alternating and $F(\vec{e}_1,\vec{e}_2) = 1$, then $F(\vec{u},\vec{v}) = \det[\vec{u}\ \vec{v}]$ for all $\vec{u},\vec{v} \in \mathbb{R}^2$.

   Solution: Express $\vec{u}$ and $\vec{v}$ as linear combinations of e_1, e_2 :

$$\vec{u} = u_1\vec{e}_1 + u_2\vec{e}_2 \text{ and } \vec{v} = v_1\vec{e}_1 + v_2\vec{e}_2$$

   Then

$$F(\vec{u},\vec{v}) = F(u_1\vec{e}_1 + u_2\vec{e}_2, \vec{v} = v_1\vec{e}_1 + v_2\vec{e}_2)$$