MATH 217 W24 - LINEAR ALGEBRA, Section 001 (Dr. Paul Kessenich)
Homework Set Part B due SUNDAY, MARCH 31 at 11:59pm

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1. Consider the four points (2, 4, 6), (1, 3, 2), (1, 1, 0) and (1, 2, 3) in $\mathbb{R}^3$.

   (a) Write a matrix equation that, if it were consistent, could be used to find the coefficients A, B, C in the equation of a plane of the form $z = Ax + By + C$ that contains all four points.

   Solution: Solution

   (b) Show that the matrix equation from (a) is, in fact, inconsistent.

   (c) Now write a matrix equation that can be used to find the least-squares solution to the equation you wrote in (a). Fully simplify any matrix products that occur in your equation, but do not (yet) attempt to solve the equation.

   (d) Now, solve your equation using methods taught in this course. (You can use a matrix calculator to check your answer, but you must be able to solve this problem by hand.)

2. (a) Which of the following is an inner product in P2? Explain.
    i. $\langle f, g\rangle = f(1)g(2) + f(2)g(1) + f(3)g(3)$
    ii. $\langle f, g\rangle = f(1)g(1) + f(2)g(2) + f(3)g(3)$

   (b) Let $V = C^\infty[-1, 1]$, the vector space of smooth functions on the interval $[-1, 1]$. Which of the following is an inner product in V? Explain.
    i. $\langle f, g\rangle = \int_{-1}^{1} x f(x)g(x)dx$
    ii. $\langle f, g\rangle = \int_{-1}^{1} x^2 f(x)g(x)dx$

3. Let $V = C^\infty\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the vector space of smooth functions on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, and consider the inner product defined by $\langle f, g\rangle = \int_{-\pi/2}^{\pi/2} f(x)g(x)\sin^2(x)dx$. (You do not need to show that this is an inner product, but make sure that you would be able to do so if it were an exam question!) Let $W = \operatorname{span}(1, x, x^2)$.

   In what follows, you may feel free to use an online integral calculator (e.g. Wolfram Alpha) to evaluate any difficult integrals, but make sure that your work shows clearly what integrals you are computing, and how you are making use of the results. Results may be expressed using either exact expressions (e.g., $\pi/\sqrt{2}$) or decimal approximations (e.g., 2.2214), but if you use decimal approximations, please retain at least four digits' worth of precision.

   (a) Compute each of the following.
      i. $\langle 1, x\rangle$
      ii. $||1||$
      iii. $||x||$

   (b) Find a basis $\mathcal{U}$ for the subspace W that is orthonormal relative to the given inner product.

   (c) Let $h \in C^\infty\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ be the function defined by $h(x) = e^x$ for all $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Compute $\operatorname{proj}_W h$.