MATH 217 W24 - LINEAR ALGEBRA, Section 001 (Dr. Paul Kessenich)
Homework 3 Part B due Thursday, February 1 at 11:59pm
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1. Determine whether the following statements are true or false, and justify your answer with a proof or a counterexample.
   (a) For all 2×2 matrices A and B, $(AB)^T = A^T B^T$.

   Solution:

   $$(AB)^\top = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

   (b) For all 2×2 matrices A and B, $(AB)^\top \neq A^\top B^\top$.
   (c) For all matrices A and B such that the matrix product AB exists, $(AB)^\top = B^\top A^\top$.
   (d) If A is a symmetric matrix, then for all $n \in \mathbb{N}$, An is also symmetric.
   (e) If A is a square matrix and A^2 is symmetric, then so is A.