EECS 203: Discrete Mathematics
Winter 2024
Homework 1

Due **Thursday, January 25th**, 10:00 pm

No late homework accepted past midnight.

Number of Problems: $7 + 1$ Total Points: $100 + 20$

- **Match your pages!** Your submission time is when you upload the file, so the time you take to match pages doesn't count against you.
- Submit this assignment (and any regrade requests later) on Gradescope.
- Justify your answers and show your work (unless a question says otherwise).
- By submitting this homework, you agree that you are in compliance with the Engineering Honor Code and the Course Policies for 203, and that you are submitting your own work.
- Check the syllabus for full details.

Individual Portion

1. Collaboration and Support [3 points]

(a) Give the names and uniqnames of 2 of your EECS 203 classmates (these could be members of your homework group or other classmates).

(b) When you have questions about the course content, where can you ask them? Where are *you* most likely to ask questions?

(c) Name one self-care action you plan to do this semester to maintain your overall well-being.

Solution:

(a) Will Guo, guowill
Hannah Hoang-Pham, hoangpha

(b) I would most likely ask them in person at the lectures, or through text.

(c) I want to take a lot of walks, I find them a nice source of stress relief.

2. Rock the Vote [12 points]

Let p and q be the following propositions:

- p: The election has been decided.
- q: The votes have been counted.

Express each of these propositions as an English statement:

(a) $\neg p \to \neg q$

(b) $\neg q \vee (\neg p \wedge q)$

Solution:

(a) "If the election has not been decided, then the votes have not been counted."

(b) "Either the votes have not been counted or both the election has not been decided and the votes have been counted."
This can be simplified to "Either the votes have not been counted or the election has not been decided."

3. Negation Station [16 points]

For each of the following propositions, give their negation in natural English. Your answer should not contain the original proposition. That is, you shouldn't negate it as "It is not the case that ..." or something similar. **Note:** You do not need to show work besides your translation, but you may earn partial credit if you do.

(a) a is greater than 6 or at most 2.

(b) b is a perfect square, odd, and not divisible by 7.

(c) c is odd whenever it is prime and greater than 3.

(d) If d is divisible by 2, then it is even.

Solution:

(a) a is at most 6 and greater than 2.

(b) b is not a perfect square, is not odd, or is divisible by 7.

(c) c is not odd whenever it is prime and greater than 3.

(d) d is divisible by 2 and not even.

4. Lying and Politics [16 points]

Imagine a world with two kinds of people: knights and knaves, where knights always tell the truth and knaves always lie. There are three people A, B, and C, and one of them is the city mayor.

- A says "I am not the city mayor."
- B says "The city mayor is a knave."
- C says "All three of us are knaves."

Is the city mayor a knight or a knave? As part of your solution, determine everything you can about whether A, B, and C are knights or knaves.

Solution:

C's statement cannot be true, since he calls himself a knave with his statement. Thus, he is a knave, and at most 2 of the people are knaves. Hence only one of A or B is a

knave.
If A is a knave, they are the city mayor. B is then telling the truth. If A is telling the truth, B cannot be a knave because B and C would both be knaves, and have the city mayor among them. This would make B's statement true, causing a contradiction. Thus B is a knight, making C the knavish city mayor.
In conclusion, the city mayor is a knave, C is always a knave, and B is always a knight.

5. Is Equivalence Equivalent to Equality? [15 points]

Show that $(b \to a) \land (c \to a)$ is logically equivalent to $\neg(b \lor c) \lor a$. If you use a truth table, be sure to state why the table tells you what you claim. If you use logical equivalences, be sure to cite each law you use.

Solution:

$$
\begin{aligned}
(b \to a) \land (c \to a) &\equiv (\neg b \lor a) \land (\neg c \lor a) && \text{implication breakout} \\
&\equiv (\neg b \lor a \land \neg c) \lor (\neg b \lor a \land a) && \text{distributive property} \\
&\equiv (\neg b \lor a \land \neg c) \lor (\neg b \lor a) && a \land a \equiv a \\
&\equiv (\neg b \lor a) \lor (\neg b \lor a \land \neg c) && \text{commutativity of or} \\
&\equiv (a \lor \neg b) \lor (\neg b \lor a \land \neg c) && \text{commutativity of or} \\
&\equiv a \lor \neg b \lor a \land \neg c && \neg b \lor \neg b \equiv \neg b \\
&\equiv a \lor \neg b \lor a \land \neg c && \neg b \lor \neg b \equiv \neg b \\
&\equiv a \lor \neg b \land \neg c && a \lor a \equiv a \\
&\equiv a \lor \neg(b \lor c) && \text{DeMorgan's Law}
\end{aligned}
$$

6. Deduce, Reuse, Recycle [20 points]

Given that the following statements are **true**:

$$(p \land r) \to q \qquad \neg q \qquad r \lor s \qquad q \lor r$$

For each of the propositions, p, q, r, and s, state its truth value and explain. If it cannot be found, briefly explain why.

Solution:

q: False, since $\neg q$ is true.
r: True, since q is false, so for $q \vee r$ to be true, r must be true.
s: Unknown, r is true so $r \vee s$ will be true no matter what value s has.
p: False. Since q is false, the condition of the first statement must be false. r is true, so p must make the $\wedge$ statement false.

7. Preposterous Propositions [18 points]

Consider the following truth table, where s, t, and w are unknown propositions.

p	q	r	s	t	w
T	T	T	F	T	F
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	F	T	T
F	T	F	F	F	F
F	F	T	F	T	T
F	F	F	F	T	F

Use the above truth table to answer the following questions. For each unknown proposition, s, t, and w:

- Find an equivalent compound proposition using p, q, and/or r.
- You may use **only** $\wedge$, $\vee$, $\neg$, and parentheses in each of your answers.
- You may use p, q, and r **at most once** in each of your answers.

Solution:

s: $p \wedge q \wedge \neg r$
t: $\neg q \vee r$
w: $r \wedge \neg(p \wedge q)$