MATH 217 - LINEAR ALGEBRA
Homework 1 Part B, DUE Thursday, January 18 at 11:59pm
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1. Decide whether the following statements are true or false. Briefly justify your answers.

   (a) 2 is even or 3 is odd.

   Solution: $\boxed{\text{True,}}$ both P and Q are true, so the "or" statement is also true.

   (b) If the Riemann Hypothesis is true, then 217 is not a prime number.

   Solution: $\boxed{\text{True,}}$ Q is true. "If" propositions can only be false when Q is false.

   (c) $\frac{d}{dx}(x^2) = 2x$ if and only if $tan(\pi/6) = \sqrt{3}$.

   Solution: $\boxed{\text{True,}}$ both P and Q are true, so $P \implies Q$ and $Q \implies P$ are true.

   (d) If the set of even prime numbers is infinite, then 10 is even and 10^10 is odd.

   Solution: $\boxed{\text{True,}}$ P is false.

   (e) If every right triangle in $\mathbb{R}^2$ has two acute angles, then every real number has a positive cube root.

   Solution: $\boxed{\text{False,}}$ P is true but Q is false.

2. (a) Let P (x) be a statement whose truth value depends on x. An example is a value of x that makes $P(x)$ true, and a counterexample is a value of x that makes P (x) false. Fill in the blank spaces with "is true", "is false", or "nothing" as appropriate:

   Solution:

	"$\forall x, P(x)$"	"$\exists x$ s.t. $P(x)$"
An example proves	nothing	is true
A counterexample proves	is false	nothing

   (b) Every prime number is even or odd.

   Solution: True, prime numbers are all integers, which are all either even or odd.

   (c) Every prime number is even or every prime number is odd.

   Solution: False, 3 and 2 are counterexamples respectively.

   (d) There exists $n \in \mathbb{Z}$ such that for every $x \in \mathbb{R}, n < x$.

   Solution: False, if you fix such a n, $n \not< n-1$ which is a contradiction.

   (e) For every $x \in \mathbb{R}$ there exists $n \in \mathbb{Z}$ such that $n < x$.

   Solution: True, $n = \lfloor x \rfloor < x$ by definition.

   (f) Some squares are rectangles.

   Solution: True, all squares are rectangles, so some squares are also rectangles.

   (g) For every nonnegative real number a, there exists a unique real number x such that $x^2 = a$.

   Solution: False, $4 = 2^2 = (-2)^2$.

3. Formulate the negation of each of the statements below in a meaningful way (these statements have been recycled from Problems 1 and 2). Note: just writing "It is not the case that . . . " before each statement will not receive credit, as that does not help the reader understand the meaning of the negation. (No justification is needed – you may just write the negation).

   (a) 2 is even or 3 is odd.

   Solution: 2 is not even and 3 is not odd.

   (b) If the Riemann Hypothesis is true, then 217 is not a prime number.

   Solution: The Riemann Hypothesis is true and 217 is a prime number.

   (c) $\frac{d}{dx}(x^2) = 2x$ if and only if $tan(\pi/6) = \sqrt{3}$.

   Solution: $\frac{d}{dx}(x^2) = 2x$ if and only if $tan(\pi/6) \neq \sqrt{3}$.

   (d) If the set of even prime numbers is infinite, then 10 is even and 10^10 is odd.

Solution: The set of even prime numbers is infinite, and either 10 is not even or 10^10 is not odd.

(e) If every right triangle in $\mathbb{R}^2$ has two acute angles, then every real number has a positive cube root.

Solution: Every right triangle in $\mathbb{R}^2$ has two acute angles, and every real number does not have a positive cube root.

(f) There exists $n \in \mathbb{Z}$ such that for every $x \in \mathbb{R}, n < x$.

Solution: For every $x \in \mathbb{R}$, no $n \in \mathbb{Z}$ exists such that $n < x$.

(g) Some squares are rectangles.

Solution: All squares are not rectangles.

4. Write both the converse and the contrapositive of the following "if-then" statements.
   (a) If something can think, then it exists.
   (b) If p is an irrational number, then p^2 is an irrational number.
   (c) If $n > 2$ is a natural number such that the Collatz sequence beginning with n does not eventually reach 1, then $n^2 + 1$ is prime.