MATH 217 W24 - LINEAR ALGEBRA, Section 001 (Dr. Paul Kessenich)
Homework Set Part B due Thursday, January 25 at 11:59pm

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1. In parts (a) - (d) below, determine whether the given function is injective, surjective, both, or neither. Justify your answers.
   (a) the function $f : [0,4]ß[0,18]$ defined by $f(x) = x^2 + 2$;

   Solution: Solution

   (b) the function $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = 2x - 5$;

   (c) the function $h : \mathbb{R}^2 \to \mathbb{R}$ defined by $h(x, y) = 2x^2 + 5y^2$;

   (d) the function q : N → N defined by $q(n) = \begin{cases} n, & \text{if n is odd} \\ n/2 & \text{if n is even.} \end{cases}$

2. Determine whether each statement is true or false. If it is true, prove it. If it is false, prove this by giving a counterexample.
   (a) For every function $f : X \to Y$ and all $A, B \subseteq X$, if $A \cap B = \emptyset$, then $f[A] \cap f[B] = \emptyset$.

   (b) For every function $f : X \to Y$ and all $A, B \subseteq X$, if $f[A] \cap f[B] = \emptyset$, then $A \cap B = \emptyset$.

   (c) For every function $f : X \to Y$ and all $A \subseteq X$, we have $f^{-1}[f[A]] = A$.

   (d) For every function $f : X \to Y$ and all $A \subseteq X$, we have $f[X \backslash A] = Y \backslash f[A]$.

   (e) For every bijective function $f : X \to Y$ and all $A, B \subseteq X$, we have $f[A \cap B] = f[A] \cap f[B]$.