TestProblems

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Contents

1 One dimensional smooth problems

1.1 DensityWave

1.1.1 Problem description

This problem sets up a smooth density wave advecting with a uniform velocity and periodic boundaries. This is probably the easiest problem for a code which solves the Euler equations, as all the non-linear features are absent, and the behavior is only that of the scalar conservation law $\dot{\rho} + u_0\rho' = 0$. The problem is described by the following parameters:

1. sound speed: c_s
2. advection velocity: u_0
3. background density: ρ_0
4. wave amplitude: ρ_1
5. background pressure: $p_0 = \rho_0 c_s^2/\Gamma$
6. wave-number: $k_0 = 2\pi/L$

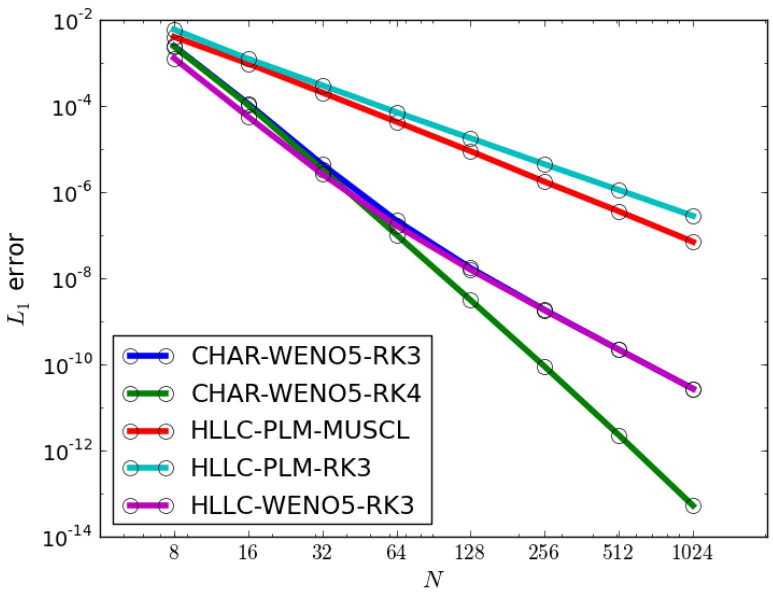
$$\begin{pmatrix} \rho(x,t) \\ p(x,t) \\ u(x,t) \end{pmatrix} = \begin{pmatrix} \rho_0 + \rho_1 \cos k_0(x - u_0 t) \\ p_0 \\ 0 \end{pmatrix}$$

Because of its smoothness and simplicity, this problem is ideal for gauging the convergence order of a scheme. Any scheme claiming to converge at order n must absolutely pass this test before any other. For the convergence test, the problem is run for half of the domain-crossing time at resolutions 2^n for n between 3 and 10. Five different reference schemes are used:

7. `HLLC-PLM-MUSCL`
8. `HLLC-PLM-RK3`
9. `HLLC-WENO5-RK3`
10. `CHAR-WENO5-RK3`
11. `CHAR-WENO5-RK4`

1.1.2 Performance of the schemes

For the formally 5th order `WENO5` reconstruction schemes, it is evident that the truncation error introduced by the 3rd order temporal update comes to dominate the error by resolution 64^3. At higher resolutions, the classic 4th order Runge-Kutta update preserves 5th order convergence. Since this problem does not involve any non-linear waves, it is not surprising that the Godunov scheme `HLLC-WENO5-RK3` accomplishes the same convergence order as the characteristic decomposition `CHAR-WENO5-RK3`.



N	`HLLC-PLM-MUSCL`	`HLLC-PLM-RK3`	`HLLC-WENO5-RK3`	`CHAR-WENO5-RK3`	`CHAR-WENO5-RK4`
16	2.07	2.27	4.52	4.46	4.53
32	2.23	2.04	4.42	4.69	4.95
64	2.25	2.08	3.97	4.33	5.04
128	2.27	2.00	3.40	3.66	5.04
256	2.36	2.01	3.13	3.23	5.13
512	2.28	1.98	3.04	3.06	5.28
1024	2.35	2.00	3.01	3.02	5.44

1.2 SoundWave

1.2.1 Problem description

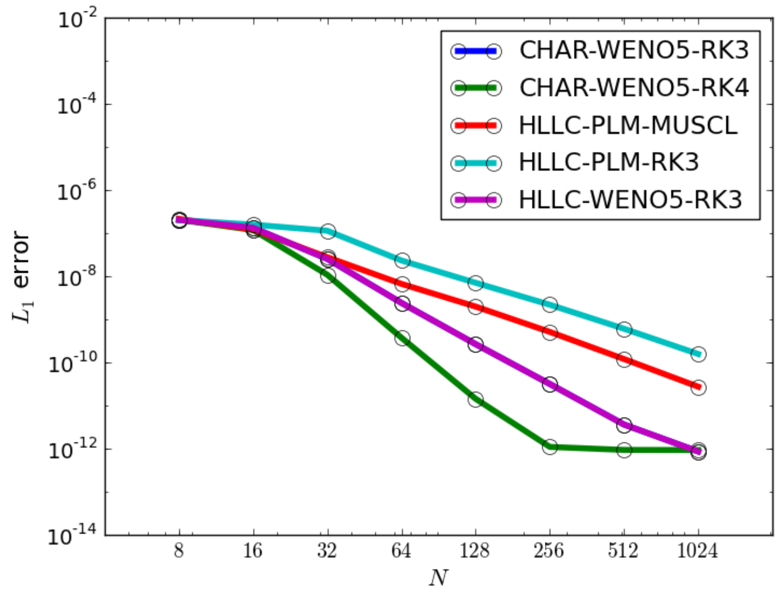
This problem sets up a smooth and small-amplitude perturbation to create a left-going sound wave. It is only slightly more challenging as a convergence test than the density wave. The problem is described by the following parameters:

1. sound speed: c_s
2. background density: ρ_0
3. background pressure: $p_0 = \rho_0 c_s^2/\Gamma$
4. density wave amplitude: $\rho_1 = 10^{-6}\rho_0$
5. pressure wave amplitude: $p_1 = c_s^2\rho_1$
6. velocity wave amplitude: $u_1 = c_s\rho_1/\rho_0$
7. wave-number: $k_0 = 8\pi/L$
8. frequence: $\omega_0 = c_s k_0$

$$\begin{pmatrix} \rho(x,t) \\ p(x,t) \\ u(x,t) \end{pmatrix} = \begin{pmatrix} \rho_0 + \rho_1 \cos(k_0 x - \omega_0 t) \\ p_0 + p_1 \cos(k_0 x - \omega_0 t) \\ u_0 + u_1 \cos(k_0 x - \omega_0 t) \end{pmatrix}$$

1.2.2 Performance of the schemes

The need for higher temporal order is highlighted by this problem. While RK4 accomplishes 5th order convergence, there is no resolution for which RK3 converges faster than 3rd order. Since the problem uses a very small-purturbation (\$$10^{-6}$) the absolute error is small, and round-off error exceeds the truncation error quickly.



N	HLLC-PLM-MUSCL	HLLC-PLM-RK3	HLLC-WENO5-RK3	CHAR-WENO5-RK3	CHAR-WENO5-RK4
16	0.81	0.36	0.61	0.61	0.78
32	2.14	0.47	2.40	2.40	3.45
64	2.06	2.29	3.39	3.39	4.85
128	1.75	1.72	3.18	3.18	4.71
256	1.95	1.69	3.07	3.07	3.71
512	2.09	1.85	3.12	3.12	0.23
1024	2.13	1.95	2.08	2.08	0.01

1.3 Collapse1d

2 One dimensional two-state problems

2.1 Shocktube1

ρ	1.000000	0.125000
p	1.000000	0.100000
v_x	0.000000	0.000000
v_y	0.000000	0.000000
v_z	0.000000	0.000000

2.2 Shocktube2

ρ	1.000000	1.000000
p	0.400000	0.400000
v_x	-2.000000	2.000000
v_y	0.000000	0.000000
v_z	0.000000	0.000000

2.3 Shocktube3

ρ	1.000000	1.000000
p	1000.000000	0.010000
v_x	0.000000	0.000000
v_y	0.000000	0.000000
v_z	0.000000	0.000000

2.4 Shocktube4

ρ	1.000000	1.000000
p	0.010000	100.000000
v_x	0.000000	0.000000
v_y	0.000000	0.000000
v_z	0.000000	0.000000

2.5 Shocktube5

ρ	5.999240	5.999240
p	460.894000	46.095000
v_x	19.597500	-6.196330
v_y	0.000000	0.000000
v_z	0.000000	0.000000

2.6 ContactWave

ρ	1.000000	0.100000
p	1.000000	1.000000
v_x	0.000000	0.000000
v_y	0.700000	0.700000
v_z	0.200000	0.200000

2.7 SrhdCase1DFIM98

ρ	10.000000	1.000000
p	13.300000	0.000001
v_x	0.000000	0.000000
v_y	0.000000	0.000000
v_z	0.000000	0.000000

2.8 SrhdCase2DFIM98

ρ	1.000000	1.000000
p	1000.000000	0.010000
v_x	0.000000	0.000000
v_y	0.000000	0.000000
v_z	0.000000	0.000000

2.9 SrhdHardTransverseRAM

ρ	1.000000	1.000000
p	1000.000000	0.010000
v_x	0.000000	0.000000
v_y	0.900000	0.900000
v_z	0.000000	0.000000

3 Credits