(十三) MCMC (采样方法，为了近似后验).

推断后验 $P(\theta|X)$

近似推断 | 确定性 → VI
　　　　 | 随机性 → MCMC.

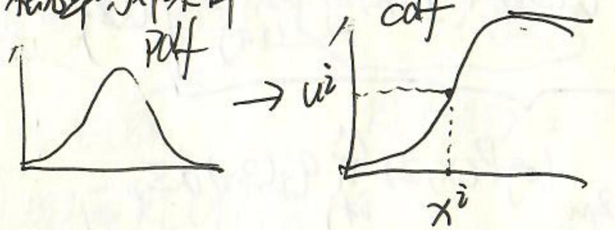
Monte Carlo method：基于采样的随机近似法.

latent, observed data

$P(Z|X) \longrightarrow E_{Z|X}[f(Z)] = \int p(Z|X) \cdot f(Z)\, dZ \approx \frac{1}{N}\sum_{i=1}^{N} f(Z_i)$ ← MCMC

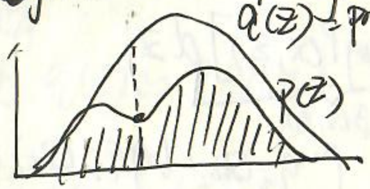
$Z^1, Z^2, \cdots Z^N \sim P(Z|X)$

① 概率分布采样



$x^i = cdf^{-1}(u^i)$

② Rejection Sampling



$q(Z)$: proposed distribution.

$\forall Z_i,\ Mq(Z_i) \geq P(Z_i)$

取样，阴影部分接收，α：接收率 $\alpha = \frac{P(Z^i)}{Mq(Z^i)}$, $0 \leq \alpha \leq 1$

1. $Z^i \sim q(Z)$

③ Importance Sampling

$E_{p(z)}[f(Z)] = \int_Z p(z) f(z)\, dZ = \int \frac{P(Z)}{q(Z)} \cdot q(Z) \cdot f(Z)\, dZ$

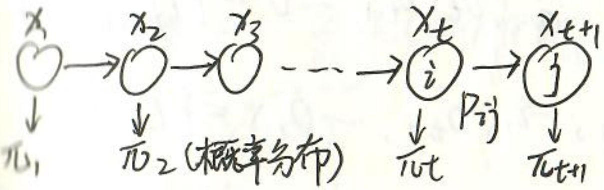
$= \int f(z) \cdot \frac{P(Z)}{q(Z)} \cdot q(Z)\, dZ$

$\approx \frac{1}{N}\sum_{i=1}^{N} f(Z^i) \frac{P(Z^i)}{q(Z^i)}$ → weight　　$Z_i \sim q(Z)$ $i=1,2,\cdots N$

一阶 (齐次) Markov chain. $P(X_{t+1}=x \mid x_1, x_2, \cdots x_t) = P(X_{t+1}=x \mid X_t)$

P → 转移矩阵 $[P_{ij}]$

$P_{ij} = P(X_{t+1}=j \mid X_t = i)$



$\pi_{t+1}(x^*) = \int \pi_t(x) \cdot P(x \to x^*)\, dx$

$\begin{cases} \pi = [\pi^{(1)}, \pi^{2}, \cdots \pi^{N} \cdots] \\ \sum_{i=1}^{\infty} \pi(i) = 1 \end{cases}$

平稳分布：π^k 是 x_t 的平稳分布