

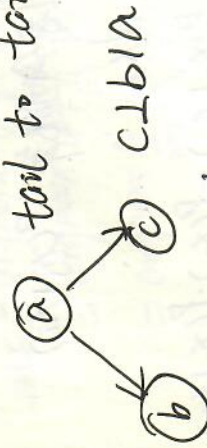
Bayesian Network

Vazyme

因子分解 $P(x_1, x_2, \dots, x_p) = \prod_{i=1}^p P(x_i | \text{pa}(x_i))$

子节点集合

tail to tail



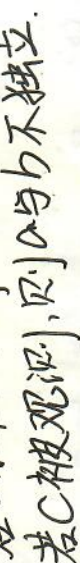
$c \perp b | a$

head to tail



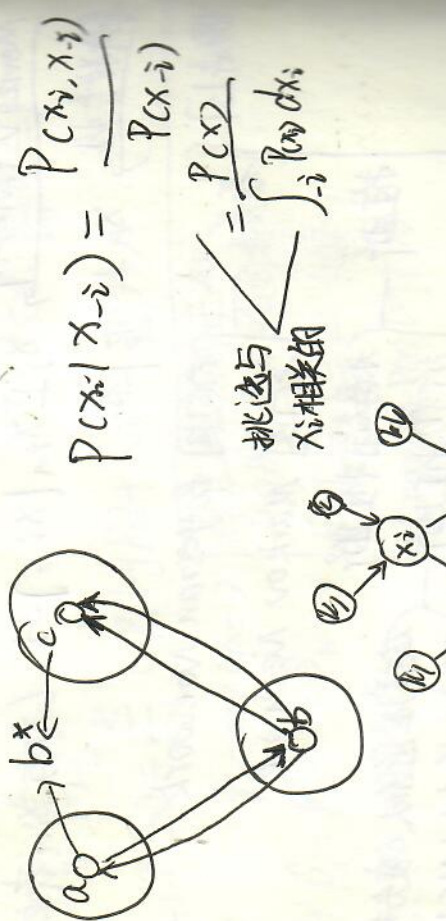
$a \rightarrow b \rightarrow c$

head to head



若 C 未被观测, $a \perp b$
若 C 被观测, a, b 不独立

D-separation



EM (Expectation Maximum) 期望最大

Vazyme

MLE: $P(x|\theta)$ $\theta_{MLE} = \arg \max_{\theta} \log P(x|\theta)$

$\theta^{t+1} = \arg \max_{\theta} \int_z \log P(x, z|\theta) \cdot P(z|x, \theta^t) dz$

$E_{z|x, \theta^t} [\log P(x, z|\theta)]$

$\theta^t \rightarrow \theta^{t+1}$

$\log P(x|\theta^t) \leq \log P(x|\theta^{t+1})$

$\log P(x|\theta) = \log P(x, z|\theta) - \log P(z|x, \theta)$

$\theta = \log P(x|\theta) \int_z P(z|x, \theta^t) dz = \log P(x|\theta)$

$\theta = \int_z \underbrace{P(z|x, \theta^t) \cdot \log P(x, z|\theta)}_{Q(\theta, \theta^t)} dz - \underbrace{\int_z P(z|x, \theta^t) \cdot \log P(z|x, \theta)}_{H(\theta, \theta^t)} dz$

$Q(\theta^t, \theta^t) \geq Q(\theta^t, \theta^t)$

$H(\theta^t, \theta^t) \leq H(\theta^t, \theta^t)$

$H(\theta^t, \theta^t) - H(\theta^t, \theta^t) \leq 0$

收敛性: $\log P(x|\theta^{t+1}) \geq \log P(x|\theta^t)$