目的是：类内小，类间大.

目标函数：$J(w)=\frac{(\bar{z}_1-\bar{z}_2)^2}{s_1+s_2}$

$\hat{w}=\arg\max_w J(w)$

$(\bar{z}_1-\bar{z}_2)^2=\left[w^T\left(\frac{1}{N_1}\sum_{i=1}^{N_1}x_i-\frac{1}{N_2}\sum_{i=1}^{N_2}x_i\right)\right]^2$

$=[w^T(\bar{X}_{c1}-\bar{X}_{c2})]^2$

$s_1+s_2=\frac{1}{N_1}\sum_{i=1}^{N_1}(w^Tx_i-\bar{z}_1)(w^Tx_i-\bar{z}_1)^T+\frac{1}{N_2}\sum_{i=1}^{N_2}(w^Tx_i-\bar{z}_2)(w^Tx_i-\bar{z}_2)^T$

$=\frac{1}{N_1}\sum_{i=1}^{N_1}(w^Tx_i-\frac{1}{N_1}\sum_{j=1}^{N_1}w^Tx_j)(w^Tx_i-\frac{1}{N_1}\sum_{j=1}^{N_1}w^Tx_j)^T+\cdots$

$=\frac{1}{N_1}\sum_{i=1}^{N_1}w^T(x_i-\bar{X}_{c1})(x_i-\bar{X}_{c1})^Tw$

$=w^T\frac{1}{N_1}\sum_{i=1}^{N_1}(x_i-\bar{X}_{c1})(x_i-\bar{X}_{c1})^Tw$

$=w^TS_{c1}w+w^TS_{c2}w.$

$J(w)=\frac{[w^T(\bar{X}_{c1}-\bar{X}_{c2})]^2}{w^TS_{c1}w+w^TS_{c2}w}$

$=\frac{w^T(\bar{X}_{c1}-\bar{X}_{c2})(\bar{X}_{c1}-\bar{X}_{c2})^Tw}{w^T(S_{c1}+S_{c2})w}$

S_b：between-class 类间方差 $(\bar{X}_{c1}-\bar{X}_{c2})(\bar{X}_{c1}-\bar{X}_{c2})^T$

$J(w)=\frac{w^TS_bw}{w^TS_ww}=w^TS_bw(w^TS_ww)^{-1}$

$\frac{\partial J(w)}{\partial w}=2S_bw(w^TS_ww)^{-1}+w^TS_bw\cdot(-1)\cdot(w^TS_ww)^{-2}\cdot 2w$

$=0$

$S_bw(w^TS_ww)^{-1}=w^TS_bw(w^TS_ww)^{-2}\cdot w$

$S_bw(w^TS_ww)=\underbrace{w^TS_bw}_{\in\mathbb{R}}S_ww$

$w=p\times 1$

$w^T=1\times p$

$S_w=p\times p$

$\hat{w}=\frac{w^TS_ww}{w^TS_bw}S_w^{-1}\cdot S_b\cdot w$

(正比于) $\propto S_w^{-1}\underbrace{S_b\cdot w}$

$(\bar{X}_{c1}-\bar{X}_{c2})\underbrace{(\bar{X}_{c1}-\bar{X}_{c2})^T\cdot w}_{\in\mathbb{R}}$

$\propto S_w^{-1}(\bar{X}_{c1}-\bar{X}_{c2})$