已知 W, Y, 求 O, P.

$$\underbrace{Y^T Y}_{K\times K} = (y_1, y_2, \cdots, y_N)\begin{pmatrix} y_1^T \\ y_2^T \\ \vdots \\ y_N^T \end{pmatrix} = \sum_{i=1}^{N} y_i y_i^T$$

$$= \begin{pmatrix} N_1 & & & \\ & N_2 & & \\ & & \ddots & \\ & & & N_k \end{pmatrix}_{K\times K} = \begin{pmatrix} \sum_{i\in A_1} 1 & & \\ & \ddots & \\ & & \sum_{i\in A_k} 1 \end{pmatrix}_{K\times K}$$

N_k: 在N个样本中,属于K的样本个数.

$$\sum_{k=1}^{K} N_k = N, \quad N_k = |A_k| = \sum_{i\in A_k} 1$$

$$\sum_{i=1}^{N} y_i y_i^T d_i = \sum_{i=1}^{N} y_i d_i y_i^T$$

$$P = \begin{pmatrix} \sum_{i\in A_1} d_i & & \\ & \ddots & \\ & & \sum_{i\in A_k} d_i \end{pmatrix} \qquad D = \begin{pmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_N \end{pmatrix}$$

$$= Y^T \cdot D \cdot Y \qquad\qquad = diag(W \cdot 1_N)$$

Vazyme

$$O = \begin{pmatrix} W(A_1, \bar{A}_1) & & \\ & \ddots & \\ & & W(A_k, \bar{A}_k) \end{pmatrix}$$

$$W(A_1, \bar{A}_1) = \underbrace{W(A_1, V)}_{\sum_{i\in A_1} d_i} - \underbrace{W(A_1, A_1)}_{\sum_{i\in A_1}\sum_{j\in A_1} w_{ij}}$$

$$O = \begin{pmatrix} \sum_{i\in A_1} d_i & & \\ & \ddots & \\ & & \sum_{i\in A_k} d_i \end{pmatrix} - \begin{pmatrix} W(A_1, A_1) & & \\ & \ddots & \\ & & W(A_k, A_k) \end{pmatrix}$$

$$O = Y^T D Y - Y^T W Y$$

$$Y^T W Y = (y_1 \cdots y_N)\begin{pmatrix} w_{11} & \cdots & w_{1N} \\ \vdots & & \vdots \\ w_{N1} & \cdots & w_{NN} \end{pmatrix}\begin{pmatrix} y_1^T \\ \vdots \\ y_N^T \end{pmatrix}$$

$$= \left(\sum_{i=1}^{N} y_i w_{i1} \cdots \sum_{i=1}^{N} y_i w_{iN}\right)\begin{pmatrix} y_1^T \\ \vdots \\ y_N^T \end{pmatrix}$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N} y_i y_j^T w_{ij}$$

$$= \begin{pmatrix} \sum_{i\in A_1}\sum_{j\in A_1} w_{ij} & \cdots & \sum_{i\in A_1}\sum_{j\in A_k} w_{ij} \\ \vdots & \ddots & \vdots \\ \sum\sum w_{ij} & \cdots & \sum_{i\in A_k}\sum_{j\in A_k} w_{ij} \end{pmatrix}$$