均方根差：MSE $E(y-\hat{f}(x))^2$

方差：Var $Var(\hat{f}(x)) = E[\hat{f}(x)^2] - E[\hat{f}(x)]^2$

偏差：Bias $Bias(\hat{f}(x)) = E(\hat{f}(x)) - f(x)$

$Var[X] = E[X^2] - (E[X])^2$

$$Var[X]\ \frac{\sum_{i=1}^{m}(X-\bar{X})^2}{m} = \frac{1}{m}\sum_{i=1}^{m}(X^2+\bar{X}^2-2X\bar{X})$$

$$= \frac{\sum_{i=1}^{m}X^2}{m} + \frac{\sum_{i=1}^{m}\bar{X}^2}{m} - \frac{\sum_{i=1}^{m}2X\bar{X}}{m}$$

$$E(X^2) + \bar{X}^2 - 2\bar{X}E(X)$$

$$= E(X^2) + (E(X))^2 - 2(E(X))^2$$

$$= E(X^2) - (E(X))^2$$

Vazyme

$y = f + \varepsilon$, ε (noise, zero mean and variance σ^2)

$$E[(y-\hat{f})^2] = MSE$$

$$= E[(f+\varepsilon-\hat{f})^2]$$

$$= E[(f+\varepsilon-\hat{f}+E(\hat{f})-E(\hat{f}))^2]$$

$$= E[(f-E(\hat{f}))^2] + E[(E(\hat{f})-\hat{f})^2]$$

$$+ E(\varepsilon^2) + 2E[(f-E(\hat{f}))\cdot(E(\hat{f})-\hat{f})]$$

$$+ 2E[(f-E(\hat{f}))\varepsilon] + 2E[(E(\hat{f})-\hat{f})\cdot\varepsilon]$$

$$= (f-E(\hat{f}))^2 + E(\varepsilon^2) + E[(E(\hat{f})-\hat{f})^2]$$

$$= (f-E(\hat{f}))^2 + Var(\varepsilon) + Var(\hat{f})$$

$$= Bias(\hat{f})^2 + \sigma^2 + Var(\hat{f})$$