$\langle \hat{\theta}, \hat{\phi} \rangle = \arg\max KL(q_\phi(z|x) \| P_\theta(z|x))$

$= \arg\max \text{ELBO}$

$= \arg\max E_{q_\phi(z|x)}[\log P_\theta(x,z)] - H[q_\phi]$

$= \arg\max E_{q_\phi(z|x)}[\log P_\theta(x|z) + \log P_\theta(z)] - H[q_\phi]$

$= \arg\max E_{q_\phi(z|x)}[\log P_\theta(x|z)] + E_{q_\phi(z|x)} \log P_\theta(z)$

$- E_{q_\phi(z|x)} \log q_\phi(z|x)$

$= \arg\max E_q[\log P_\theta(x|z)] - KL(q_\phi(z|x) \| P(z))$

$q_\phi$ 熵最大.

(三+三) Normalizing Flow 流模型

$$\begin{tikzcd}
z_0 \arrow[r, "f_0"] \arrow[d] & z_1 \arrow[r, "f_1"] \arrow[d] & \cdots \arrow[r, "f_{k-1}"] & z_k \arrow[r, "f_k"] \arrow[d] & x \arrow[d] \\
P_0(z_0) & P_{z_1}(z_1) & & P_{z_k}(z_k) & P_x(x)
\end{tikzcd}$$

$\hookrightarrow N(0, I)$

change of variable theorem.

$x = f(z),\ z, x \in \mathbb{R}^P$

$z \sim P_z(z),\quad x \sim P_x(x)$

$f$ is continuous, invertible.

$\int_z P_z(z)\, dz = 1 = \int_x P_x(x)\, dx \Rightarrow |P_z(z)\, dz| = |P_x(x)\, dx|$

$P_x(x) = \left|\frac{dz}{dx}\right| \cdot P_z(z)$

$x = f(z)$, $f$ is invertible

$z = f^{-1}(x)$

$P_x(x) = \left|\frac{\partial f^{-1}(x)}{\partial x}\right| \cdot P_z(z)$ $\quad \frac{\partial f^{-1}(x)}{\partial x}$ Jacobian Matrix

$P_x(x) = \left|\det\left(\frac{\partial f^{-1}(x)}{\partial x}\right)\right| \cdot P_z(z)$ $\quad \det\left(\frac{\partial f^{-1}(x)}{\partial x}\right)$ Jacobian Det

$= \left|\det\left(\frac{\partial f(z)}{\ldots}\right)\right|^{-1} P_z(\ldots)$