soft-margin SVM.

$\min \frac{1}{2}w^Tw + loss$

loss用距离表示

如果 $y_i(w^Tx_i+b) \geq 1$, $loss=0$

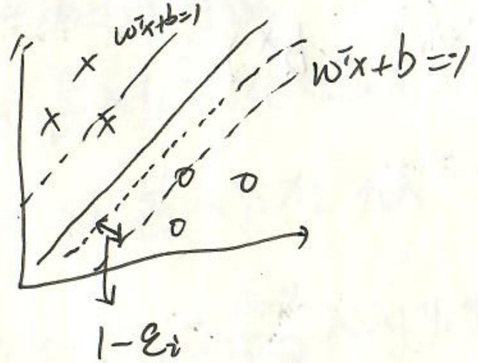
如果 $y_i(w^Tx_i+b) < 1$, $loss = 1 - y_i(w^Tx_i+b)$

$loss = \max\{0, 1-y_i(w^Tx_i+b)\}$

$$\begin{cases} \min\limits_{w,b} \frac{1}{2}w^Tw + C\sum\limits_{i=1}^{N}\max\{0, 1-y_i(w^Tx_i+b)\} \\ s.t.\ y_i(w^Tx_i+b) \geq 1-\varepsilon_i \end{cases}$$

$$\begin{cases} \min\limits_{w,b} \frac{1}{2}w^Tw + C\sum\limits_{i=1}^{N}\varepsilon_i \\ s.t.\ y_i(w^Tx_i+b) \geq 1-\varepsilon_i \\ \quad\ \varepsilon_i \geq 0 \end{cases}$$

$\varepsilon_i = \max(0, 1-y_i(w^Tx_i+b))$




约束优化问题. (原问题) I

$$\begin{cases} \min f(x) \quad x \in \mathbb{R}^p \\ s.t.\ m_i(x) \leq 0 \quad i=1,\cdots,m \\ \quad\ \ n_j(x) = 0 \quad j=1,\cdots,n \end{cases} \quad ①$$

拉格朗日函数 (原问题拉格朗日化) II

$L(x,\lambda,\eta) = f(x) + \sum\limits_{i=1}^{M}\lambda_i m_i + \sum\limits_{j=1}^{N}\eta_j n_j$

$$\begin{cases} \min\limits_{x} \max\limits_{\lambda,\eta} L(x,\lambda,\eta) \\ s.t.\ \lambda_i \geq 0 \end{cases} \quad ②$$

①与②等价

如果x违反约束$m_i(x)$, $m_i(x)>0$, $\max\limits_{\lambda} L \to \infty$

如果x符合$m_i(x) \leq 0$, $\max\limits_{\lambda} L \neq +\infty$

$\min\limits_{x}\max\limits_{\lambda} L = \min\limits_{x}\{\max\limits_{\lambda} L, +\infty\} = \min\limits_{x}\max\limits_{\lambda} L$

对偶问题: III

$$\begin{cases} \max\limits_{\lambda,\eta}\min\limits_{x} L(x,\lambda,\eta) \\ s.t.\ \lambda_i \geq 0 \end{cases}$$

$\max\limits_{\lambda,\eta}\min\limits_{x} L(x,\lambda,\eta) \leq \min\limits_{x}\max\limits_{\lambda,\eta} L(x,\lambda,\eta)$

弱对偶性: III ≤ II

证明: $\max\limits_{\lambda,\eta}\min\limits_{x} L(x,\lambda,\eta) \leq \min\limits_{x}\max\limits_{\lambda,\eta} L(x,\lambda,\eta)$

$\min\limits_{x} L(x,\lambda,\eta) \leq L(x,\lambda,\eta) \leq \max\limits_{\lambda,\eta} L(x,\lambda,\eta)$