SVD for PCA

$HX = U\Sigma V^T$ $\begin{cases} U^TU = I \\ V^TV = VV^T = I \\ \Sigma \text{ (对角)} \end{cases}$

H: 中心化

* $\underset{(p\times p)}{S} = X^THX = X^TH^THX = V\Sigma U^TU\Sigma V^T = V\Sigma^2V^T$

$\underset{(N\times N)}{T} = HXX^TH = U\Sigma V^TV\Sigma U^T = U\Sigma^2U^T$

S: 特征分解，得到方向(主成份)，然后 $HX\cdot V \to$ 坐标，

T: 特征分解，直接得到坐标 $HX = U\Sigma V$

$HX\cdot V = U\Sigma V^TV = U\Sigma \to$ 坐标矩阵.

$$X = (x_1, x_2, \cdots x_N)^T = \begin{pmatrix} x_1^T \\ \vdots \\ x_n^T \end{pmatrix} = \begin{array}{c|c} & p \to \text{feature} \\ n \downarrow \text{Sample} & \begin{pmatrix} x_{11} & \cdots & x_{1p} \\ x_{21} & \cdots & x_{2p} \\ \vdots & & \\ x_{n1} & \cdots & x_{np} \end{pmatrix} \end{array}$$

$$HX = \underbrace{U\Sigma}_{\text{坐标矩阵}} \underbrace{V}_{\text{主成分方向}}$$

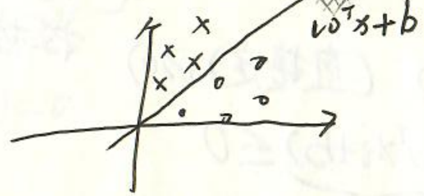
(六). SVM

SVM：间隔，对偶，核方法.

hard-margin SVM

soft-margin SVM

Kernel SVM



最大间隔分类器

max margin(w,b)

s.t. $y_i(w^Tx_i+b) > 0$

(for $\forall\, i=1,2,\cdots N$)

点到直线的距离公式：

$\min\limits_{\substack{w,b,x_i \\ i=1\cdots N}} \frac{1}{\|w\|}|w^Tx_i+b|$

最大间隔 $\Rightarrow \max\limits_{w,b}\min\limits_{x_i} \frac{1}{\|w\|} y_i(w^Tx_i+b)$

$$\begin{cases} = \max\limits_{w,b} \frac{1}{\|w\|} \min\limits_{x_i} y_i(w^Tx_i+b) \\ \text{s.t. } y_i(w^Tx_i+b) > 0 \Rightarrow \exists\, r>0,\ \min\limits_{x_i,y_i} y_i(w^Tx_i+b) = r \end{cases}$$

由于 w^Tx_i+b 可以任意缩放 $n\cdot w^Tx_i + n\cdot b$，设 $r=1$

$$\Rightarrow \begin{cases} \max\limits_{w,b} \frac{1}{\|w\|} \\ \text{s.t. } \min y_i(w^Tx_i+b) = 1 \to y_i(w^Tx_i+b) \geq 1,\ i=1,2,\cdots N. \end{cases}$$

$$\Rightarrow \begin{cases} \min\limits_{w,b} \frac{1}{2}w^Tw \\ \text{s.t. } y_i(w^Tx_i+b) \geq 1 \text{ for } \forall\, i=1,2,\cdots N. \end{cases}$$

(凸优化问题)

找到对偶问题 dual problem.