$$\sum_{i=1}^{N} P(z_i = C_k \mid x_i, \theta^t) + P_k \cdot \lambda = 0$$

$$\Rightarrow \underbrace{\sum_{i=1}^{N} \sum_{k=1}^{K} P(z_i = C_k \mid x_i, \theta^t)}_{1} + \sum_{k=1}^{K} P_k \lambda = 0$$

$$N + \lambda = 0$$
$$\lambda = -N$$

$$\sum_{i=1}^{N} P(z_i = C_k \mid x_i, \theta^t) + P_k (-N) = 0$$

$$P_k = \frac{1}{N} \sum_{i=1}^{N} P(z_i = C_k \mid x_i, \theta^t)$$

$$\theta = \{P_1, P_2, \sim P_k, \mu_1, \mu_2, \sim \mu_k, \Sigma_1, \Sigma_2, \sim \Sigma_k\}$$

E-step:

$$Q(\theta, \theta^t) = \int_z \log P(x, z \mid \theta) P(z \mid x, \theta^t)\, dz$$

$$= \sum_{i=1}^{N} \sum_{z_i} \log P_{z_i} \cdot N(x_i \mid \mu_{z_i}, \Sigma_{z_i}) \; \frac{P_{z_i} \cdot N(x_i \mid \mu_{z_i}^t, \Sigma_{z_i}^t)}{\sum_{k=1}^{k} P_k^t \cdot N(x_i \mid \mu_k^t, \Sigma_k^t)}$$

$$= \sum_{i=1}^{N} \sum_{z_i} \log \left[P_{z_i} \cdot N(x_i \mid \mu_{z_i}, \Sigma_{z_i}) \right] P(z_i \mid x_i, \theta^t)$$

$$= \sum_{k=1}^{K} \sum_{i=1}^{N} \log \left[P_k \cdot N(x_i \mid \mu_k, \Sigma_k) \right] \cdot P(z_i = C_k \mid x_i, \theta^t)$$

$$= \sum_{k=1}^{K} \sum_{i=1}^{N} \left[\log P_k + \log N(x_i \mid \mu_k, \Sigma_k) \right] \cdot P(z_i = C_k \mid x_i, \theta^t)$$

M-step:

$$P^{t+1} = (P_1^{t+1}, P_2^{t+1}, \sim, P_k^{t+1})$$

$$\begin{cases} \max\limits_{P} \sum_{k=1}^{K} \sum_{i=1}^{N} \log P_k \cdot P(z_i = C_k \mid x_i, \theta^{t+1}) \\ s.t. \sum_{k=1}^{K} P_k = 1 \end{cases}$$

$$L(P, \lambda) = \sum_{k=1}^{K} \sum_{i=1}^{N} \log P_k \cdot P(z_i = C_k \mid x_i, \theta^t) + \lambda \left(\sum_{k=1}^{K} P_k - 1 \right)$$