定义:
$$A \subset V,\ B \subset V,\ A\cap B = \emptyset$$

$$W(A,B) = \sum_{\substack{i\in A \\ j\in B}} w_{ij}$$

(W(A,B) 是两个相连通的节点之后的和可以看出是
将连各空节到的以后的根本)

假如一共 K 个类别

$$cut(V) = cut(A_1, A_2, \cdots, A_K)$$ (将V分成点区个子集)

$$V = \bigcup_{k=1}^{K} A_k \qquad A_i \cap A_j = \emptyset,\ \forall i,j \Rightarrow \{1,2,\cdots,K\}$$

$$cut(V) = \sum_{k=1}^{K} W(A_k, \bar{A_k}) = \sum_{k=1}^{K} W(A_k, V) - W(A_k, A_k)$$

目标: min $Cut(V)$ $\{A_k\}_{k=1}^{K}$

$$Ncut = \sum_{k=1}^{K} \frac{w(A_k, \bar{A_k})}{\Delta} \qquad \Delta = degree(A_k)$$

$$d_i = \sum_{j=1}^{N} w_{ij} \qquad = \sum_{i\in A_k} d_i$$

$$Ncut = \sum_{k=1}^{K} \frac{w(A_k, \bar{A_k})}{\sum_{i\in A_k} d_i}$$

Model:
$$\{\hat{A_k}\}_{k=1}^{K} = argmin\ Ncut(V)$$
$$\{A_k\}_{k=1}^{K}$$

indicator vector:
$$y_i \in \{0,1\}^K \qquad y_{ij} = 1 \Leftarrow 第i个样本属于j个类别j$$

$$\sum_{j=1}^{K} y_{ij} = 1 \qquad \begin{array}{l} 1\leq i \leq N \\ 1\leq j \leq K \end{array}$$

$$Y = (y_1, y_2, \cdots, y_N)^T \quad _{N\times K} \qquad y_i = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$$

$$\hat{Y} = argmin\ Ncut(V)$$

$$Ncut(V) = \sum_{k=1}^{K} \frac{w(A_k, \bar{A_k})}{\sum_{i\in A_k} d_i}$$

$$= tr\left(\begin{pmatrix} \dfrac{w(A_1, \bar{A_1})}{\sum_{i\in A_1} d_i} & & 0 \\ & \ddots & \\ 0 & & \dfrac{w(A_K, \bar{A_K})}{\sum_{i\in A_K} d_i} \end{pmatrix} \right)$$

$$= tr\left(\begin{pmatrix} w(A_1, \bar{A_1}) \\ \\ w(A_K, \bar{A_K}) \end{pmatrix} \cdot \begin{pmatrix} \sum_{i\in A_1} d_i \\ \\ \sum_{i\in A_k} d_i \end{pmatrix}^{-1} \right)$$