

$$\langle \hat{\theta}, \hat{\phi} \rangle = \arg \min_{\theta, \phi} KL(q_{\phi}(z|x) || p_{\theta}(z|x))$$

$$= \arg \max_{\theta, \phi} ELBO$$

$$= \arg \max_{\theta, \phi} E_{q_{\phi}(z|x)} [\log p_{\theta}(x, z)] + H[q_{\phi}]$$

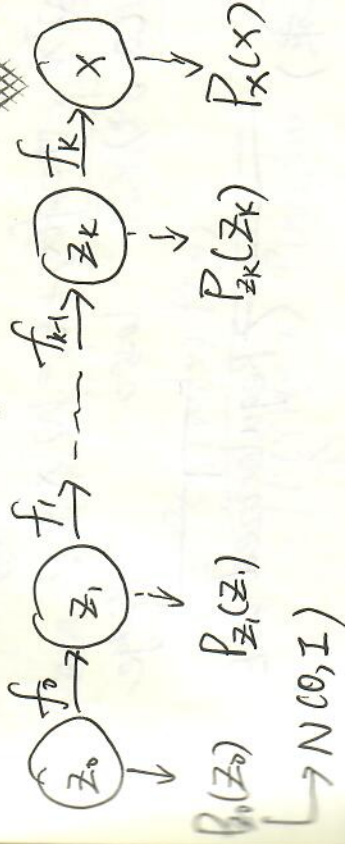
$$= \arg \max_{\theta, \phi} E_{q_{\phi}(z|x)} [\log p_{\theta}(x|z)] + \log p_{\theta}(z) + H[q_{\phi}]$$

$$= \arg \max_{\theta, \phi} E_{q_{\phi}(z|x)} [\log p_{\theta}(x|z)] + E_{q_{\phi}(z|x)} [\log p_{\theta}(z)]$$

$$= \arg \max_{\theta, \phi} E_{q_{\phi}(z|x)} [\log p_{\theta}(x|z)] - KL(q_{\phi}(z|x) || p_{\theta}(z))$$

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(Z+X) Normalizing Flow 流模型 Vazyme



change of variable theorem.

$$x = f(z), z, x \in \mathbb{R}^p$$

$$z \sim p_z(z), x \sim p_x(x)$$

f is continuous, invertible.

$$\int_z p_z(z) dz = 1 = \int_x p_x(x) dx \Rightarrow \left| p_z(z) dz \right| = \left| p_x(x) dx \right|$$

$$p_x(x) = \left| \frac{dz}{dx} \right| \cdot p_z(z)$$

$x = f(z)$, f is invertible

$$z = f^{-1}(x)$$

$$p_x(x) = \left| \frac{\partial f^{-1}(x)}{\partial x} \right| \cdot p_z(z) \frac{\partial f^{-1}(x)}{\partial x} \quad \text{Jacobian Matrix}$$

$$p_x(x) = \left| \det \left(\frac{\partial f^{-1}(x)}{\partial x} \right) \right| \cdot p_z(z) \det \left(\frac{\partial f^{-1}(x)}{\partial x} \right) \quad \text{Jacobian Det}$$

$$= \left| \det \left(\frac{\partial f(z)}{\partial z} \right) \right| \cdot p_z(z)$$