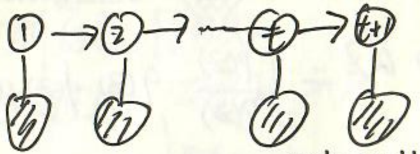
(十四) HMM 隐马尔可夫模型



$\lambda = (\pi, A, B)$ — A: 状态转移, B: 发散矩阵, π: 初始 prob dist

观测变量：$o_1, o_2 \cdots o_t \rightarrow V = \{v_1, v_2, \cdots, v_m\}$ (观测的值域)

隐状态变量：$i_1, i_2 \cdots i_t \rightarrow Q = \{q_1, q_2, \cdots, q_N\}$ (隐状态的值域)

$A = [a_{ij}]$, $a_{ij} = P(i_{t+1} = q_j \mid i_t = q_i)$ 状态转移矩阵.

$B = [b_j^{(k)}]$ $\quad b_j(k) = P(O_t = V_k \mid i_t = q_j)$ 发射矩阵, 状态 j 到观测 k

两个假设：① 齐次 Markov 假设. ② 观察独立.

$$P(i_{t+1} \mid i_t, i_{t-1}, \cdots i_1, O_t, O_{t-1}, \cdots O_1) = P(i_{t+1} \mid i_t)$$

$$P(O_t \mid i_t, i_{t-1}, \cdots i_1, O_{t-1}, \cdots O_1) = P(O_t \mid i_t)$$

三个问题：

① Evaluation $\rightarrow P(O \mid \lambda)$ 前向后向

② Learning $\rightarrow$ EM

③ Decoding $\quad I = \arg\max P(I \mid O)$

预测 $\rightarrow P(i_{t+1} \mid O_1, O_2, \cdots O_t)$

Evaluation 问题：Given λ, 求 $P(O \mid \lambda)$

$$P(O \mid \lambda) = \sum_I P(I, O \mid \lambda) = \sum_I P(O \mid I, \lambda) \cdot P(I \mid \lambda)$$

$$P(I \mid \lambda) = P(i_1, i_2, \cdots i_T \mid \lambda) = P(i_T \mid i_1, i_2, \cdots i_{T-1}, \lambda) \cdot P(i_1, i_2, \cdots i_{T-1}, \lambda)$$

$$\underbrace{P(i_T \mid i_{T-1})}_{a_{i_{T-1} i_T}}$$

$$= \pi(a_{i_1}) \prod_{t=2}^{T} a_{i_{t-1}, i_t}.$$

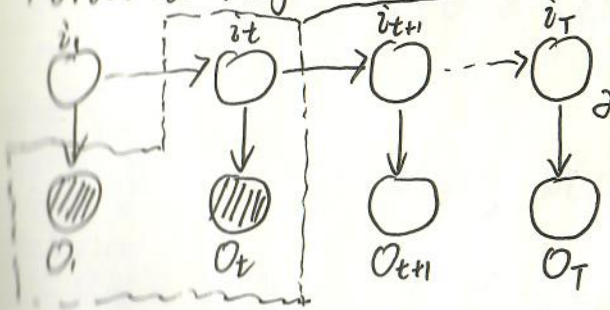
$$P(O \mid I, \lambda) = \prod_{t=1}^{T} b_{i_t}(O_t)$$

$$P(O \mid \lambda) = \sum_I \pi(a_{i_1}) \prod_{t=2}^{T} a_{i_{t-1}, i_t} \prod_{t=1}^{T} b_{i_t}(O_t)$$

$\sum$ I 的可能的种类为 $\sum_{i_1} \cdot \sum_{i_2} \cdots \sum_{i_t}$.

$$= \underbrace{\sum_{i_1} \sum_{i_2} \cdots \sum_{i_t}}_{O(N^t)} \pi(a_{i_1}) \prod_{t=2}^{T} a_{i_{t-1}, i_t} \prod_{t=1}^{T} b_{i_t}(O_t)$$

Forward Algorithm.



联合概率

$\alpha_t(i) = P(O_1, \cdots O_t, i_t = q_i \mid \lambda)$

$\alpha_T(i) = P(O, i_T = q_i \mid \lambda)$

$$P(O \mid \lambda) = \sum_{i=1}^{N} P(O, i_t = q_i \mid \lambda)$$

$$= \sum_{i=1}^{N} \alpha_T(i)$$