$$\min_G V(D_G^*, G)$$

$$= \min_G E_{x\sim P_d}\left[\log \frac{P_d}{P_d+P_g}\right] + E_{x\sim P_g}\left[\log \frac{P_g}{P_d+P_g}\right]$$

$$= \min_G E_{x\sim P_d}\left[\log \left(\frac{P_d}{(P_d+P_g)/2}\cdot\frac{1}{2}\right)\right] +$$

$$E_{x\sim P_g}\left[\log \left(\frac{P_g}{(P_d+P_g)/2}\cdot\frac{1}{2}\right)\right]$$

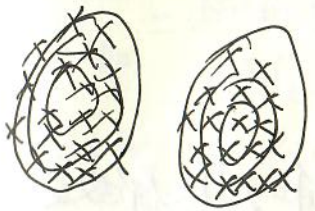
$$= \min_G KL\left(P_d \,\|\, \frac{P_d+P_g}{2}\right) + KL\left(P_g \,\|\, \frac{P_d+P_g}{2}\right) - \log 4$$

$$\geq \log 4$$

当 $P_d = \frac{P_d+P_g}{2} = P_g$ 时，等号成立

$$P_g^* = P_d,\quad D_G^* = \frac{P_d}{P_d+P_g} = \frac{1}{2}.$$

(二十二) 谱聚类 Spectral Clustering.



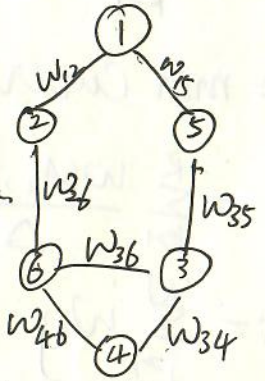
⇒ compactness:
K-means GMM.

⇒ Connectivity:
spectral clustering.

Graph-based (带权重的无向图)

$G = \{V, E\}$

$V = \{1, 2, \cdots, N\}$ (顶点集合)
一个节点一个样本



E: $W = [w_{ij}]$, $1 \leq i,j \leq N$
(N个顶点)

W: Similarity matrix (affinity matrix)

$$w_{ij} = \begin{cases} K(x_i, x_j) = \exp\left(-\frac{\|x_i - x_j\|^2}{2\sigma^2}\right), & (i,j) \in E \\ \end{cases}$$