$$L(w,b,\lambda)=\frac{1}{2}w^Tw+\underbrace{\sum_{i=1}^{N}\lambda_i(1-y_i(w^Tx_i+b))}_{\text{拉格朗日项}}$$

$\lambda_i \geq 0$ (直接定 $\lambda_i \geq 0$)

$$\underbrace{1-y_i(w^Tx_i+b)}_{\Delta}\leq 0$$

$$\max_{\lambda} L(w,b,\lambda)=\frac{1}{2}w^Tw+0$$

$$\begin{cases}\min_{w,b}\frac{1}{2}w^Tw=\min_{w,b}\max_{\lambda}L(w,b,\lambda)\\ s.t.\ \lambda_i\geq 0\end{cases}$$

$\Delta > 0$ 丢掉 | $\Delta \leq 0$

对偶问题 $\begin{cases}\max_{\lambda}\min_{w,b}L(w,b,\lambda)\\ s.t.\ \lambda_i\geq 0\end{cases}$

定理：$\min\max f(x) \geq \max\min f(x)$ (弱对偶)

$\min\max f(x)=\max\min f(x)$ 强对偶

这里的条件满足强对偶.

$\frac{\partial L}{\partial b}=0 \Rightarrow \sum_{i=1}^{N}\lambda_i y_i=0$, 代入 $L(w,b,\lambda)$

$\frac{\partial L}{\partial w}=0 \Rightarrow w=\sum_{i=1}^{N}\lambda_i y_i x_i$. 代入

$$\min_{w,b} L(w,b,\lambda)=-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\lambda_i\lambda_j y_i y_j x_i^T x_j+\sum_{i=1}^{N}\lambda_i$$

(max 式

KKT条件

$$\begin{cases}\frac{\partial L}{\partial w}=0\quad \frac{\partial L}{\partial b}=0\quad \frac{\partial L}{\partial \lambda}=0\\ \lambda_i(1-y_i(w^Tx_i+b))=0\\ \lambda_i\geq 0\\ 1-y_i(w^Tx_i+b)\leq 0\end{cases}$$

(只有 $w^Tx_i+b=1$ 或 -1, $\lambda_i\neq 0$)

$$w^*=\sum_{i=0}^{N}\lambda_i y_i x_i$$

$\exists\,(x_k,y_k)$, $w^Tx+b=1 \Rightarrow 1-y_k(w^Tx_k+b)=0$

$y_k(w^Tx_k+b)=1$

$y_k^2(w^Tx_k+b)=y_k\quad (y_k^2=1)$

$$b^*=y_k-w^Tx_k=y_k-\sum_{i=0}^{N}\lambda_i y_i x_i^T x_k$$

$w^*=\sum_{i=0}^{N}\lambda_i y_i x_i$ (data 线性组合, 由支持向量决定)

$b^*=y_k-\sum_{i=0}^{N}\lambda_i y_i x_i^T x_k$

$f(x)=\mathrm{sign}(w^{*T}x+b^*)$