一个中心：原始特征空间的重构
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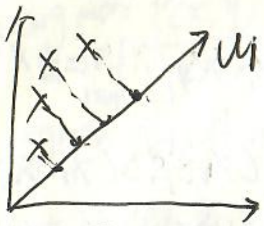
两个基本点：最大投影方差
最小重构距离



u_1 最大化方差的投影向量

$$J = \frac{1}{N}\sum_{i=1}^{N} (\underbrace{(x_i - \bar{x})}_{\text{中心化}} u_1)^2 \quad \text{s.t. } u_1^T u = 1$$

→（中心化后均值已经为0）.

$$= \sum_{i=1}^{N} \frac{1}{N} u_1^T (x_i - \bar{x})(x_i - \bar{x})^T u_1$$

$$= u_1^T S u_1$$

$$\begin{cases} \hat{u}_1 = \arg\max u_1^T S u_1 \\ \text{s.t. } u_1^T u = 1 \end{cases}$$

拉格朗日项 $\lambda(1 - u_1^T u)$

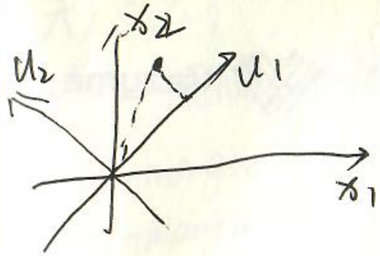
$$L(u, \lambda) = u_1^T S u_1 + \lambda(1 - u_1^T u)$$

$$\frac{\partial L}{\partial u_1} = 2Su_1 + \lambda(-2u) = 0$$

$$Su_1 = \lambda u_1$$

u_1 为 S 的 eigen-vector

λ 为 S 的 eigen-value



原数据 $x_i = \sum_{k=1}^{P} \underbrace{(x_i^T u_1)}_{\text{坐标值}} \underbrace{u_1}_{\text{向量}}$

$\hat{x}_i = \sum_{k=1}^{q} (x_i^T u_k) u_k$ （只用了前 q 组 $q < P$

$$J = \frac{1}{N}\sum_{i=1}^{N} \|x_i - \hat{x}_i\|^2$$ （前 q 组相同）.

$$= \frac{1}{N}\sum_{i=1}^{N} \left\| \sum_{k=q+1}^{P} (x_i^T u_k) u_k \right\|^2$$

$$= \frac{1}{N}\sum_{i=1}^{N} \sum_{k=q+1}^{P} \left[\underbrace{(x_i^T u_k)}_{\text{坐标值}} \underbrace{u_k}_{\text{坐标}} \right]^2$$

$$\triangleq \frac{1}{N}\sum_{i=1}^{N} \sum_{k=q+1}^{P} ((x_i - \bar{x})^T u_k)^2$$

$$= \sum_{i=1}^{N} \frac{1}{N} \sum_{k=q+1}^{P} ((x_i - \bar{x})^T u_k)^2$$

$$= \sum_{k=q+1}^{P} \underbrace{\sum_{i=1}^{N} \frac{1}{N} ((x_i - \bar{x})^T u_k)^2}_{u_k^T S u_k}$$

$$= \sum_{k=q+1}^{P} u_k^T S u_k \quad \text{s.t. } u_k^T u_k = 1$$

$$\rightarrow \begin{cases} u_k = \arg\min \sum_{k=q+1}^{P} u_k^T S u_k \\ \text{s.t. } u_k^T u_k = 1 \end{cases}$$

减去的特征值最小化 = 留下的特征值最大