对偶性的几何解释

$$\begin{cases} \min\limits_{x \in R^p} f(x) \\ s.t.\ m_1(x) \le 0 \end{cases}$$

D: 定义域

$D = \mathrm{dom} f \cap \mathrm{dom}\ m_1$

$L(x,\lambda) = f(x) + \lambda m_1(x)$

$p^* = \min f(x)$ (原问题最优解)

$d^* = \max\limits_{\lambda} \min\limits_{x} L(x,\lambda)$ (对偶问题最优解)

集合G: $G = \{(m_1(x), f(x)) \mid x \in D\}$

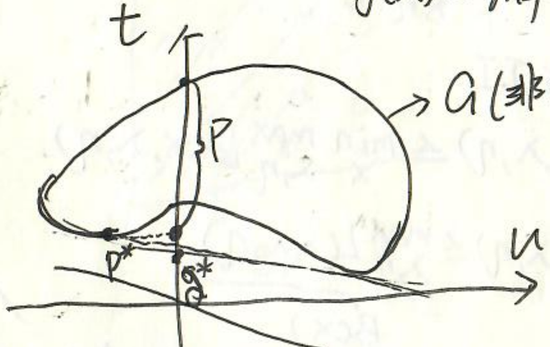
$= \{(u, t) \mid x \in D\}$

$p^* = \inf \{t \mid (u,t) \in G, u \le 0\}$ inf: 下确界

$d^* = \max\limits_{\lambda} \min\limits_{x} L(x,\lambda) = \max\limits_{\lambda} \underbrace{\min\limits_{x} (t + \lambda u)}_{g(\lambda)}$

$= \max\limits_{\lambda} g(\lambda)$

$g(\lambda) = \inf \{t + \lambda u \mid (u,t) \in G\}$



$\to G$(非凸集) $\searrow$ $d^* < p^*$

G(凸集) $d^* = p^*$

Slater condition

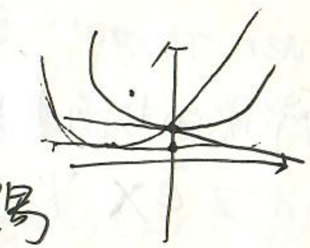
无边界的中部

$\exists \hat{x} \in \mathrm{relint}\ D$ (relative interior)

s.t. $\forall i = 1, \dots, m\ m_i(\hat{x}) < 0$

① 对于大多数凸优化, slater成立

限制了G集合



KKT condition

凸优化 + slater $\Rightarrow$ 强对偶

$\Updownarrow$

KKT.

KKT:

$$\begin{cases} \text{可行条件}: \begin{cases} m_i(x^*) \le 0 \\ n_j(x^*) = 0 \\ \lambda^* \ge 0 \end{cases} \\ \text{互补松弛}: \lambda_i m_i = 0 \\ \text{梯度为}0: \left.\dfrac{\partial f(x, \lambda^*, \eta^*)}{\partial x}\right|_{x=x^*} \end{cases}$$

$d^* = \max\limits_{\lambda,\eta} g(\lambda,\eta) = g(\lambda^*, \eta^*)$

$= \min\limits_{x} L(x, \lambda^*, \eta^*)$

$\underset{\Delta}{\le} L(x^*, \lambda^*, \eta^*)$

$= f(x^*) + \underbrace{\sum_i \lambda_i^* m_i}_{\le 0} + \underbrace{\sum_j \eta_j^* n_j}_{= 0}$

$\underset{\Delta}{\le} f(x^*)$

$= p^*$