Nexus between Energy and Commodity Prices: Jump Transmission

Robert D.Weaver and Jiachuan Tian

April 15, 2015

1 Introduction

While structural break, threshold and asymmetric cointegration models can allow us to characterize the linear and nonlinear dynamics in price transmission in level, it is of equal interest to differentiate across the type of price change to consider what might be thought of as typical price changes versus extreme price changes associated with either temporary structural change or mean reverting change as in what we call price jumps. In particular, while a "structural break" a permanent and long-run structural shifts in DGM, a "jump" in a series represents a sudden temporary change in the pattern of the series. Such change is temporary in a sense that its effect usually diminishes rather quickly (usually in relatively few periods). That means, intuitively, in relatively short time span after a jump, the price series will revert to its "mean" or its long-run smooth pattern, which we call the trend of the series.

Jumps in prices are usually modelled as the jump components in the jump-diffusion model in high frequency data. On the other hand, in low frequency data, the mechanism behind a price "jump" or what we will define as a price "spike" is not well defined. And there could be several different structures in DGMs that could cause jumps or spikes. This paper focuses on modelling of jump occurrence and jump transmission in energy and commodity prices. In the paper, I present a model to predict the probability of jumps and jump size conditioning on a set of exogenous variables. In particular, if I choose the exogenous variables to be the indicators of presence of jumps in other price series, then such model can be interpreted as characterizing jump transmission.

A strand of energy and commodity price literature related to jumps (transmission) looks into volatility (transmission). Aizenman and Pinto (2005) point out that volatility in energy prices introduces risk to the economic system and high volatility leads to an overall welfare loss. Serra and Zilberman (2011) such volatility may also spill over directly to soft commodities markets (energy as input in soft commodity), and indirectly through ethanol markets. Thus, the increasing volatility in energy price (and thus in soft commodities price) is a major concern for agricultural producer and agents along the food chain (Balcombe, 2009). Motivated by the need to mitigate the effect of volatility, this paper seeks to study volatility and its transmission between energy and soft commodities prices.

While volatility plays an important role in the economy, the concept of volatility is not well defined in literature. Volatility by its meaning should include two kinds of effects: systematic variance changes (VC) and nonsystematic oscillations that cannot be characterized by VC, like jumps and spikes. While systematic variance changes can explain particular types of oscillations or volatility, they cannot represent any long-run or short-run structural changes in the price series. Thus, to fully understand volatility, one has to consider all kinds of oscillation effects, especially jumps.

On the other hand, in many works of literature, the word "volatility" refers to systematic variance changes. In particular, price volatility is usually characterized as the volatility term in continuous time models (for example, high-frequency stock prices as geometric Brownian motion in Black-Scholes model), or heteroskedasticity error structure in discrete time models (for example, low-frequency energy prices as in time series models). In particular, to characterize "volatility" (variance change effect) transmission in energy and commodity prices (low-frequency data), most existing literature use multivariate general auto-regressive conditional heteroskedasticity (MGARCH) model. Harri and Hudson (2009) employ a MGARCH and Chung and Ng test to investigate "volatility" transmission between crude oil futures and a set of soft commodities. Serra and Zilberman (2009) employ Baba-Engle-Kraft-Kroner (BEKK) MGARCH to model price "volatility" in ethanol market. Zhang et al.(2008) also employs BEKK model to investigate "volatility" of US gasoline prices. Essentially, the MGARCH approach models variance change (VC) effects. As discussed above, variance changes are neither jumps nor the only source of volatility. Thus, the MGARCH models cannot be used to model jumps, and since jumps contribute heavily to volatility, these models cannot efficiently characterize volatility or its transmission with the presence of jumps.

To model jumps and spikes, one approach is to use the autoregressive conditional hazard (ACH) function. Christensen et al. (2012) employ this model to forecast spikes in electricity prices. This approach provides a reasonable modelling of the probability of jumps given a set of exogenous variables. On the other hand, it cannot model jump size conditioning on that there is a jump.

Another approach is to use a Markov regime switching model. Higgs and Worthington (2008) use this method to model the Australian wholesale spot electricity markets. They decompose price at time t into deterministic and stochastic components. Here they characterize stochastic components by Markov regime switching model: state 0 is denoted as the "normal" behavioural status, state 1 as a jump and state -1 as a mean reverting process after a jump. Such process is assumed to a Markov process.

It is worth noting that the Markov regime switching approach assumes that the transition probability is fixed, thus the probability of the presence of a jump is homogeneous for all t. This is not a realistic assumption because the dynamics of price would most probably change over the sample period and hence the probability of a jump will change. It is also not easy to use exogenous (economic) variables to predict the probability of a jump in this scheme since the probability of a jump is assumed to be fixed. If we are interested in predicting the probability of a jump given a set of exogenous economic variables, we can model $\pi^i(0,0)$ as a function of these variables. Since most economic variables are not stationary, $\pi^i(0,0)$ as a function of these variables and is not stationary. That directly violates the definition of $\pi^i(0,0)$ as a transition probability in a Markov Chain. Thus, the Markov regime switching approach, because of its Markov Chain structure, is inherently unable to predict probability of a jump conditioning on a set of exogenous variables.

2 Types of Outlier Effects

In time series literature, the concept of "jumps" is related to outlier effects. As Fox(1972), Hillmer et al. (1983), Tsay (1988), Chen and Tiao (1990), Chen and Liu (1993) discussed, there are five types of outlier effects: innovational outliers (IO), additive outliers (AO), level shifts (LS), temporary changes (TC) and seasonal level shift (SLS). Innovational outliers (IO) refer to those outliers that have effect not only on the particular observation but also subsequent observations, while additive outliers (AO) refer to those that may be mean reverting (zero expectation) and affecting a single observation. Level shifts (LS) represents structure changes. Temporary change (TC) and seasonal level shift (SLS)correspond to temporary level shift.

To formally define these outlier effects, we follow Chen and Liu and define a price series p_t subject to the influence of non-repetitive events as:

$$p_t = {p_t}^* + \sum_{j=1}^{N} \omega_j L_j(B) \mathbf{1}_O(t_j) \tag{1}$$

$$\text{where} \quad p_t^* = \frac{\theta(B)}{\alpha(B)\phi(B)} Z_t \tag{2}$$

where $Z_t \sim WN(0, \sigma^2)$; p_t^* represents the smooth trend of the price and follows a ARIMA process; $\mathbf{1}_O(t)$ is an indicator variable that equals to 1 when there is outliers effect at time t $(t \in O)$and 0 $(t \notin O)$ elsewhere; O is the set of possible latent time locations of outliers; and ω^i represents the magnitude of the outlier effects; and N is the total number of outliers. Note here B is the backward shifter: $Bp_t = p_t - 1$, and $\theta(B), \phi(B), \alpha(B)$ are polynomials of B; all roots of $\theta(B)$ and $\phi(B)$ are outside unit circle; and all roots of $\alpha(B)$ are on the unit circle. Note here $L(B)$ is also a polynomial of B, which denotes the dynamic pattern of the outlier effects.Then the five types of outlier effects can be defined as:

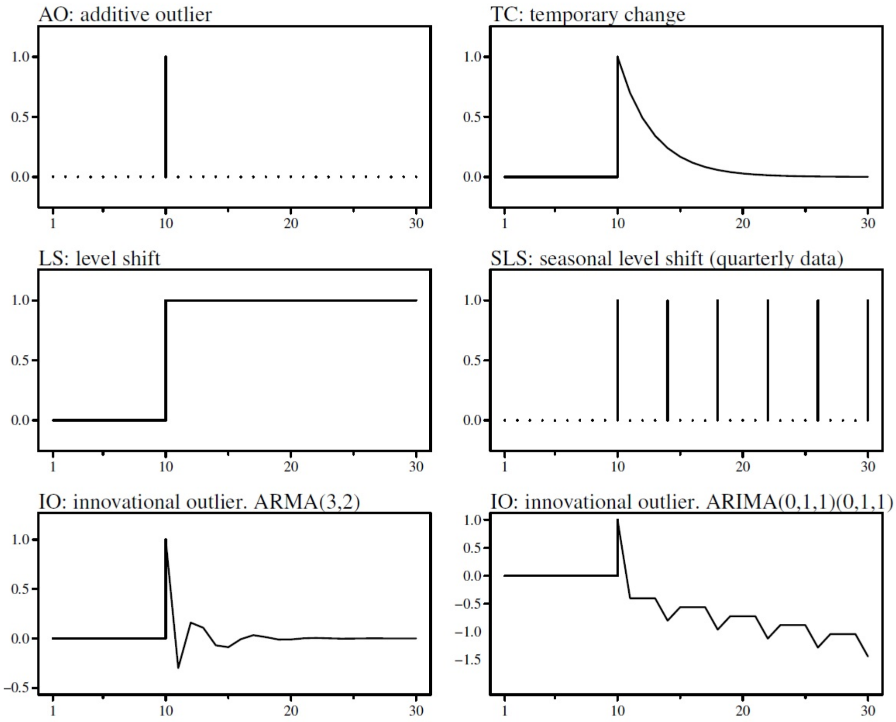
$$\begin{aligned} &\text{IO: } L(B) = \frac{\theta(B)}{\alpha(B)\phi(B)}; \quad \text{LS: } L(B) = \frac{1}{(1-B)}; \quad \text{SLS: } L(B) = \frac{1}{1-B^s}; \\ &\text{AO: } L(B) = 1; \qquad\qquad\quad \text{TC: } L(B) = \frac{1}{1-\delta B}. \end{aligned} \tag{3}$$

where s is the periodicity of the data and the value δ is usually equal to 0.7. Figure 1 shows a unit impulse for each type of outlier at time point at $t = 10$. Intuitively, jumps or spikes refer to those temporary and permanent-structural-change-free outliers that have effect on subsequent observations. Thus, jumps should most appropriately be modelled as innovational outliers (IO). In this case, the dynamic pattern is $\theta(B)/\alpha(B)\phi(B)$. And since we are only interested in innovational outliers (jumps), we change the set O to J which denotes the set of time location of jumps.

3 Methodology

In this section we present the modelling of this paper. In particular, this paper employs a two-step method. The first step is to identify the time when there are jumps using test by Chen and Liu (1993). With jump time specified, the second step is to model and predict jumps conditioning on a set of economic variables. I first present the time series model of the price series. Since we are only interested

Figure 1: Unit impulse for different types of outliers



in innovational outliers, then combining (1) and (3) together, we have that the observed price series can be written as:

$$\begin{aligned} p_t &= {p_t}^* + \sum_{j=1}^{m} \omega_j \frac{\theta(B)}{\alpha(B)\phi(B)} \mathbf{1}_J(t_j) \\ &= \frac{\theta(B)}{\alpha(B)\phi(B)} (Z_t + \sum_{j=1}^{m} \mathbf{1}_J(t_j)) \end{aligned} \tag{4}$$

where the set J is the set of time location of all jumps, and m is the number of jumps ($|J|$). Then the estimated residuals can be written as:

$$\pi(B)p_t \equiv \hat{e}_t = Z_t + \sum_{j=1}^{m} \mathbf{1}_J(t_j) \tag{5}$$

where the coefficients of the power series expansion $\pi(B) = \sum_{i=0}^{\infty} \pi_i B^i$ can be determined by:

$$\pi(B) = \frac{\alpha(B)\phi(B)}{\theta(B)} \tag{6}$$

Since all the roots of $\theta(B)$ lie outside the unit circle, the coefficients $\{\pi_i\}_{i=0}^{\infty}$ is an absolutely convergent series ($\sum_{i=0}^{\infty} |\pi_i| < \infty$). The $t-$statistic of $\hat{e}_t$ is:

$$\hat{\tau(t)} = \frac{\hat{e_t}}{\hat{\sigma_e}} \tag{7}$$

$$\text{where} \quad \hat{\sigma_e} = 1.483 \times \text{median}\{|\hat{e_t} - \tilde{e_t}|\} \tag{8}$$

where $\tilde{e_t}$ is the median of the estimate residuals. Now we can start with the detection procedure of jumps.

3.1 Detection Procedure

The Chen and Liu procedure of detecting jumps can be described as following:

Step 1: Locate outliers. Given an ARIMA model fitted to the data, identify an observed price point at time location t as jump if $\hat{\tau(t)}$ as defined in equation (7) is bigger than 95% critical value of t distribution.

Step 2: Remove outliers. Given a set of potential outliers, an ARIMA model is chosen and fitted according to equation (4). The significance of the outliers is reassessed in the new fitted model. Those outliers that are not significant are removed from the set of potential outliers.

Step 3: Iterate stage 1 and 2, for both the original series and adjusted series.

With the set of jumps J defined, we can start to model and predict jump conditioning an chosen exogenous set of economic variables. In particular, if I choose the exogenous variables to be the indicators of presence of jumps in other price series, then such model can be interpreted as characterizing jump transmission.

3.2 Econometric Model

It is worth noting that there could be many factors that cause jumps in prices. In stock markets, the usual drivers are liquidity (common measures are bid-ask spread, time between two sequential transaction, etc.), macroeconomics news and firm news. It is worth noting that here liquidity measures the ease of making a transaction in stock market. Such concepts of liquidity can be generalized to accommodate our case of low-frequency energy and commodity price data. In our case, the measure of liquidity is inventory. When the inventory level of some particular commodity becomes critically low, producers are facing a high risk of stock out. Facing such risk of stock-out, producer would pay much more (than usual) up to the cost when the production is forced to stop. As a result, the low inventory level results in a jump increase in prices. Thus, the process of jumps$\mathbf{1}_J(t_j)$ is driven by a set of exogenous variables Y_t. Following Heckman (1979), define latent process J_t as $\mathbf{1}_J(t_j) = 1 \text{if} J_t >= 0$. Note J_t is conditional on a set of variables. Then the selection equation is defined as:

$$E(J_t|Y_t) = Y_t^T \beta + \nu_{1t} \tag{9}$$

where $\nu_{1t} \sim N(0,1)$. Then the probability of a jump can be modelled as a probit equation:

$$P(\mathbf{1}_J(t_j) = 1|Y_t) = P(J_t >= 0|Y_t) = \Phi(Y_t^T \beta) \tag{10}$$

If we choose $\mathbf{1}_J(t_j)^e$ the jump process of another series to be one of Y_t, then equation (10) can be used to model jump transmission. Now to model the jump size ω_j, consider a latent variable $\omega_j{}^*$ for $omega_j$:

$$\omega_j = \omega_j * \times \mathbf{1}_J(t_j) \tag{11}$$

That is, we only observe jump size ω_j if there is a jump. Such $\omega_j{}^*$ is depending on another set of exogenous variables W_t:

$$E(\omega_t{}^*|W_t) = W_t^T \beta + \nu_{2t} \tag{12}$$

where $\nu_{2t} \sim N(0, \sigma_\nu^2)$ Then the jump size can be modelled as the outcome equation of Heckman selection model:

$$E(\omega_t|W_t, \mathbf{1}_J(t) = 1) = W_t^T \beta + E(\nu_{2t}|\mathbf{1}_J(t) = 1) \tag{13}$$

Note there is interdependence between ω_j and W_t, and hence interdependence between the two latent process $\omega_j{}^*$ and J_t. Such interdependence is represented by the correlation in the joint normal distribution of (ν_{1t}, ν_{2t}) in the Heckman model. In particular, we assume they following a multivariate normal distribution with correlation ρ. Then equation (13) can be written as:

$$E(\omega_t|W_t, \mathbf{1}_J(t) = 1) = W_t^T \beta + \frac{\rho \phi(Y_t^T \beta)}{\sigma_\nu \Phi(Y_t^T \beta)} \tag{14}$$

This finishes the modelling of jump size ω_j.

4 Data Requirement

We have not started with the actual estimation work using this model. However, we notice some potential problem if we have low frequency data with relative limited number of jumps. Here I present an example.

For an illustration purpose of the potential problem with relative limited number of jumps, I choose weekly data from January 2000 through December 2010 for a corn, wheat, soybeans a subset of "softs" commodity prices for Italian and US, and European Brent blend. The Italian prices of maize and wheat are obtained from DATIMA provided by ISMEA. The US prices of wheat, corn and soybeans are provided by FAO sourced from USDA. Data are weekly prices monitored for a length of time that started in February 2005 and ended in February 2010. The prices in $\$/ton$ are converted in e/ton using the official $\$/e$ exchange rate. Missing values are replaced by using an imputation algorithm and the corresponding R-package AMELIA II (King et al., 2001; Honaker et al., 2009). For the fuel prices, the weekly United States spot prices and weekly Europe (UK) Brent blend spot price are converted to e per barrel. We analyze the natural logarithms of these prices. Table 1 shows the jump detected by Chen and Liu procedure.

As seen from Table 1, the series US Corn US wheat have only 2 jumps detected. This is because we have a weekly dataset and we are averaging over a week. Thus we have a limited number of jumps detected. This in further means we have only limited "observations" regarding to jumps and we will not have enough degrees of freedom to estimate our model in 3.2. Thus a most appropriate dataset for jump and jump transmission transmission could be at least daily frequency if not higher.

5 Current Progress and Work Plan

As shown above, for chapter 3, we have finished the modelling part. Compared to Christensen et al.(2012), this approach models jumps as a component of the price series rather than a standalone generalized Poisson process. This gives us the flexibility to model the jump size conditioning on the presence of jumps through the consideration of the selection problem inherent in the study of jumps. That is, intuitively, we can only estimate the jump size if there is a jump. The next steps are to first identify the set of exogenous variables, and then estimate the model using data. A possible concern is that this model uses a two-step method, thus the estimation methods of the selection model may need some modifications. A most appropriate candidate is to use Bayesian methods to estimate this hierarchical model. Research to complete this chapter will consider this alternative.

Table 1: Jumps Detection using Chen and Liu (1993) Procedure

EU_corn	EU_wheat	EU_soybean	US_corn	US_wheat	US_soybean	Brent Blend
29	24	28	468	72	78	49
31	26	29	563	74	219	51
34	76	31			220	90
134	77	32			237	99
135	78	36			295	155
137	129	37			296	469
140	130	73			297	471
186	182	77			298	
191	191	78			367	
192	192	134			427	
233	201	135			457	
237	202	136			505	
243	203	137				
244	230	186				
297	236	188				
298	237	189				
299	288	190				
300	338	191				
345	340	193				
346	343	231				
395	347	232				
396	353	235				
397	354	236				
398	355	238				
399	388	295				
402	396	297				
451	397	345				
452	400	401				
453	435	441				
454	436	449				
489	474	453				
490	498	473				
493	445	499				
502	451	500				
503	459	505				
550	460	506				
554	470	542				
	493	543				
	494	544				
	554	549				
	555	551				
	572	555				
	573					