Nonlinearity in Cointegration

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Introduction

Engel and Granger (1987) introduced the concept of cointegraion to characterize the situation when some linear combination of a set of nonstationary variables is stationary. Such cointegration can be interpreted as a long-run (linear) relationship between those nonstationary variables. Granger representation theorem (see [En87]) assures that cointegrated variables can be representated in a vector error correction model (VECM). While the concept of (linear) cointegration and its VECM representation have been widely applied because most economic variables are differenced stationary (I(1)) and tend to have a common stochastic trend (i.e. cointegrated), they have a very strong implicit assumption that the adjustment of the deviations towards the long-run equilibrium is made instantaneously and symmetrically at each period ([St10]). Such assumption may be often violated in the real world. For example, in the presence of sticky prices and transactional costs ([St10]), the system may not adjust to the long-run equilibrium instantaneously. Another example would be market power. In the presence of market power (over space or along a vertical chain), such adjustment (in this case, that is spatial or vertical price transmission) is often asymmetric. Varva and Goodwin (2005) examined the price transmission along the food chain and concluded that there are significant asymmetries in responses to negative and positive price shocks in terms of both speed and magnitude of the adjustment (see [Va05]).

Balka and Fomby (1997) introduced threshold cointegration to characterize the nonlinear adjustment to long-run equilibrium. By allowing the short-run adjustment coefficient to vary in different regimes, the threshold cointegration method is able to test and characterize nonlinearity in short-run dynamics using univariate tests. Hansen and Seo (2002) extended Balka and Fomby's work to the multivariate (only bivariate) case and proposed an algorithm to obtain MLE of the threshold cointegration model for the bivariate case. They also proposed a test for the presence of a threshold and derived its asymptotic distribution under the null hypothesis.

Another kind of nonlinearity in the short-run adjustment is asymmetry. Granger and Lee (1989) point out that a cointegration system may respond asymmetrically to the deviation term or changes in variables (see [Gr89]). Such asymmetry is often caused by market power or menu cost. Frey and Manera (2007) summarized all common types of asymmetric price transmission including the asymmetric adjustment in VECM and interpreted their economic meaning.

While threshold cointegration and asymmetric adjustment mainly focus on short-run dynamics, there would be nonlinearity in long run. If some (non-trivial) nonlinear function of variables is stationary, then these variables can been seen as in a long-run nonlinear cointegration. An analogue to that is the nonlinear regression in cross-section data. A special case is when such function is a piecewise linear function. That means, variables form different linear combinations (with stationary residuals) across different subsamples. In such case, we call there is a structural break (fundamental change) in the linear combination (long-run equilibrium). Then relationship between variables may not be called as a long-run equilibrium in a classical linear cointegration sense, but it can be seen as a set of different long-run equilibriums. If the time span of the sample is large, it is a reasonable assumption that the long-run linear equilibrium could change. Note such structural shift problem can be examined in the VECM context.

Methodology

In this section, I will focus nonlinearity in cointegration. In particular, I discuss several different models: traditional linear cointegration model, threshold cointegration model, structural break models and asymmetric price transmission in the cointegration context.

Model 1: Linear Cointegration

Engel and Granger (1987) first introduced the concept of cointegraion to characterize a set of nonstationary variables with common stochastic trend. They assumed that the system will adjust in a linear, symmetric and instantaneous way to short-run deviation from its long-run equilibrium. Note by Granger representation theorem ([En87]), such cointegerated linear system has a VECM representation. Now, consider such full cointegrated linear system in the form of a vector error correction model (VECM):

$$\Delta X_t = \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \alpha\beta' X_{t-1} + \phi D_t + \mu + \varepsilon_t, t = 1,..,T$$

where $X_{-k+1},..,X_0$ are fixed and $\varepsilon_1,\dots,\varepsilon_T$ are independent p-dimensional Gaussian variable with mean zero and variance matrix Λ. The vector D_t denotes seasonal dummies centered at zero. The parameters in the model are short-run effects $\Gamma_1,\dots,\Gamma_{k-1}$, the seasonal coefficients ϕ, the constant term μ, the covariance matrix Λ, the $p \times r$ matrices α (the adjustment coefficients) that define effects of deviation from long-run equilibrium, and β (the cointegrating relations). That is, if we define the model prediction of the long run equilibrium as

$$ECT_{t-1} = \beta' X_{t-1}$$

then α defines the effect of $|ECT_{t-1}|$ on change in long-run equilibrium. In further, following notations of [Ha02], we define

$$Y_{t-1}(\beta) \equiv \begin{pmatrix} \iota_p \\ D_t \\ ECT_{t-1} \\ \Delta X_{t-1} \\ \Delta X_{t-2} \\ \vdots \\ \Delta X_{t-k+1} \end{pmatrix}_{[(k+1)p+r]\times 1}$$

and

$$A' \equiv [\mu \times I_p \quad \phi \times I_p \quad \alpha \quad \Gamma_1 \quad \Gamma_2 \quad \dots \quad \Gamma_{k-1}]_{p\times[(k+1)p+r]}$$

where $\iota_p = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}_{p\times 1}$. Then the original VECM can be written in a compact way:

$$\Delta X_t = A' Y_{t-1}(\beta) + \varepsilon_t.$$

Note this specification has three strong assumptions:
(A1) The effect of $|\Delta X_{t-i}|$ is the same regardless of the sign of ΔX_{t-i}, and similarly the effect of $|ECT_{t-1}|$is the same regardless of the sign of ECT. (Symmetric assumption)
(A2) The coefficient matrix A remains the same over the full sample. (No threshold assumption)
(A3) The cointegration matrix β stays the same over the full sample. (No structural break assumption)
Note A1 assumes that there is no asymmetry in the model. If A1 is violated, one needs to consider asymmetric adjustment (asymmetric price transmission) model. A2 assumes that the short run adjustment term is perfectly linear and hence there is no time-varying parameter in the adjustment matrix. When A2 is violated, we need to employ the threshold cointegration model. Cointegration means that one or some particular linear combinations of the nonlinear variables is stationary. By assuming the cointegration relationship staying the same, A3 indicates such linear combinations does not change over time.

However, as Granger and Lee (1989) point out, it may matter whether the variable ΔX_{t-i} is positive or negative or whether ECT is positive or negative (see [Gr89]). That is, the system position defined by ΔX_{t-i} may respond asymmetrically to changes in ΔX_{t-i} or ECT. We exclude consideration of asymmetry in response to seasonal effects as these are modelled descriptively. Following the notation of [Gr89], for a general (random) variable$x$, let $x^+ = \max\{x,0\}\, and\ x^- = \min\{x,0\} = -\max\{-x,0\}$, then we can write $x = x^+ + x^-$. In this way, we can write

$$\Delta X_{t-i} = \Delta X_{t-i}{}^+ + \Delta X_{t-i}{}^-, where\ \Delta X_{t-i}{}^+ = \max\{\Delta X_{t-i},0\}, \Delta X_{t-i}{}^- = \min\{\Delta X_{t-i},0\}$$

and

$$ECT = ECT^+ + ECT^-, where\ ECT^+ = \max\{ECT,0\}, ECT^- = \min\{ECT,0\}$$

It follows that if we do not impose the symmetry constraints, then the original VECM can be written as:

$$\Delta X_t = \sum_{i=1}^{s} \Gamma_i{}^+ \Delta X_{t-i}{}^+ + \sum_{i=1}^{q} \Gamma_i{}^- \Delta X_{t-i}{}^- + \alpha^+ ECT^+ + \alpha^- ECT^- + \phi D_t + \mu + \varepsilon_t, t = 1,..,T.$$

If $\Gamma_i{}^+ = \Gamma_i{}^-, \alpha^+ = \alpha^-$ and $s = q$, then the VECM can be considered as symmetric. If $\Gamma_i{}^+ = \Gamma_i{}^-$ and $s = q$ but $\alpha^+ \neq \alpha^-$, then the VECM shows the existence of asymmetry in Reaction time and Equilibrium adjustment path (RTA and EAPA, see [Fr07]). That is, in this case of asymmetry, the system position change would respond differently to positive vs. negative changes in the predicted long-run equilibrium. If $\Gamma_i{}^+ \neq \Gamma_i{}^-$, then the VECM shows the existence of asymmetry in Contemporaneous impact (COIA) of past system position changes defined by ΔX_{t-i}. If $\Gamma_i{}^+ \neq \Gamma_i{}^-\ and\ s \neq q$, then the presence of asymmetry exists in distributed lag effect and cumulated impact (DLEA and CUIA). Note, contemporaneous impact asymmetry is short-run asymmetry, while other types of asymmetries are long-run asymmetries (see [Fr07]).

Software Implementation and Results

This paper employs the R-package ‘apt by Sun ([Su12]) to test price asymmetry. Note while the package supports each type of asymmetry mentioned above, we only test $\alpha^+ = \alpha^-$. This approach is consistent with our interest in asymmetric response to change in long-run equilibrium position of the system, i.e. asymmetry in equilibrium adjustment path (EAPA). Thus the model we examine is:

$$\Delta X_t = \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \alpha^+ ECT^+ + \alpha^- ECT^- + \phi D_t + \mu + \varepsilon_t, t = 1, .., T.$$

To test the asymmetry in equilibrium adjustment path, we test the null: $H_0: \alpha^+ = \alpha^-$.

<**Table 1.** Test for asymmetry in equilibrium adjustment path>

As shown in Table 1, only four bivariate pairs show asymmetry in equilibrium adjustment path. That is, for most bivariate VECMs the null of asymmetry in equilibrium adjustment path can be rejected.

As a result, we choose to proceed to analyze structural breaks in bivariate VECM without modelling structural break with asymmetric equilibrium adjustment path VECM.

Table 1. Test for asymmetry in equilibrium adjustment path

	EU_corn	EU_wheat	EU_soybean	US_corn	US_wheat	US_soybean	Brent Blend
EU_corn	NA	0.469	0.008	0.981	0.874	0.206	0.012
EU_wheat	0.294	NA	0.266	0.267	0.789	0.024	0.034
EU_soybean	0.507	0.053	NA	0.482	0.153	0.805	0.15
US_corn	0.151	0.967	0.246	NA	0.395	0.538	0.755
US_wheat	0.119	0.315	0.247	0.607	NA	0.056	0.44
US_soybean	0.859	0.317	0.692	0.351	0.692	NA	0.406
Brent Blend	0.131	0.647	0.294	0.611	0.605	0.757	NA

Note: The asymmetry test is conducted in R using package 'apt' (see [Su12]). The null hypothesis is H_0: $\alpha^+ = \alpha^-$ (there is no asymmetry in equilibrium adjustment path). The 5% significance test statistic is marked by red color, which means there is price asymmetry in equilibrium adjustment path.

Table 2. Test for linear cointegration specification versus threshold cointegration specification

	EU_corn	EU_wheat	EU_soybean	US_corn	US_wheat	US_soybean	Brent Blend
EU_corn	NA	0.25	0.01	0.48	0.45	0	0.01
EU_wheat	0.21	NA	0.02	0.55	0.55	0.1	0.22
EU_soybean	0	0.04	NA	0.01	0.7	0.1	0.13
US_corn	0.47	0.57	0.03	NA	0.69	0.03	0.65
US_wheat	0.51	0.56	0.77	0.62	NA	0.46	0.4
US_soybean	0	0.1	0.03	0	0.49	NA	0.47
Brent Blend	0.06	0.16	0.14	0.48	0.43	0.48	NA

Note: This test is first introduced by Hansen and Seo (2002) (see [Ha02]) and conducted in R using package 'tsDyn' (see [St10]). The null hypothesis is H_0: the bivariate pair has a linear cointegration. H_a: the bivariate repair has a threshold cointegration The 5% significance test statistic is marked by red color, which means a threshold cointegration model would be more appropriate in this case.

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