A Note of Search Cost

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1 Introduction

In this note, I present some specifications of search cost. All the specifications assume single (homogeneous) agent and there is no bidding across agents in these models.

2 General Setting of the Problem

Suppose there are N agents (producers) and T sampling periods. For agent $i \in \{1, ..., N\}$, at some time $t \in \{1, ..., T\}$, define,
y_{it} : a scalar output;
p_{it} : the price scalar for y_{it};
$\vec{x_{it}} = \{x_{ikt}\}_{k=1}^{K}$: a $K \times 1$ vector of perfectly elastic inputs;
$\vec{r_{it}} = \{r_{ikt}\}_{k=1}^{K}$: the $K \times 1$ price vector for $\vec{x_{it}}$;
z_{it} : a scalar input with imperfect elastic supply;
$\bar{z_{it}}$: a threshold value for z_{it} ($z_{it} \leq \bar{z_{it}}$, supply is elastic; $z_{it} > \bar{z_{it}}$, supply is inelastic);
$\Delta z_{it} = z_{it} - \bar{z_{it}}$: the part of input when supply is inelastic;
r_{it}^{z} : the price scalar for z_{it} when $z_{it} \leq \bar{z_{it}}$;
$\bar{C}(\Delta z_{it})$: the cost of procuring Δz_{it} including search cost.
The technology of agent i can be written as:

$$y_{it} = g_i(\vec{x_{it}}, z_{it}) \tag{1}$$

And the profit maximization problem of agent i can be written as:

$$\max_{\vec{x},z} \pi_{it} = \max_{\vec{x},z} \; p_{it}y_{it} - \vec{r_{it}}\vec{x_{it}} - r_{it}^{z}z_{it}\mathbb{1}(z_{it} \leq \bar{z_{it}}) - (r_{it}^{z}\bar{z_{it}} + \bar{C}(\Delta z_{it}))\mathbb{1}(z_{it} > \bar{z_{it}}) \tag{2}$$

where $\mathbb{1}(\cdot)$ is the indicator function. It is worth noting that there is no price in $\bar{C}(\Delta z_{it})$, and I will specify this quantity in the following section. The first order condition of (1) with respect to x_{ikt} is

$$\frac{\partial \pi_{it}}{\partial x_{ikt}} = p_{it}\frac{\partial g_i}{\partial x_{ikt}} - r_{ikt} = 0, \; \forall k \in \{1, ..., K\} \tag{3}$$

By x_{ikt}'s are elastic, condition (3) are binding at $\{x_{ikt}^{*}\}_{k=1}^{K}$. Following the notation above, denote $\{x_{ikt}^{*}\}_{k=1}^{K}$ as $\vec{x_{it}}^{*}$. Plug the optimal $\vec{x_{it}}^{*}$ into (2), then π_{it}^{*} is a function of z_{it}. And problem (1) becomes to

$$\max_{z} \pi_{it}^{*} = \max_{z} \; p_{it}y_{it}^{*} - \vec{r_{it}}\vec{x_{it}}^{*} - r_{it}^{z}z_{it}\mathbb{1}(z_{it} \leq \bar{z_{it}}) - (r_{it}^{z}\bar{z_{it}} + \bar{C}(\Delta z_{it}))\mathbb{1}(z_{it} > \bar{z_{it}}) \tag{4}$$

where $y_{it}^{*} = g_i(\vec{x_{it}}^{*}, z_{it})$. Then the willingness to pay for z_{it} is $\frac{\partial \pi_{it}^{*}}{\partial z_{it}}$. In particular, under the threshold value $\bar{z_{it}}$, the WTP for z_{it} is

$$\frac{\partial {\pi_{it}^{*}}^{-}}{\partial z_{it}} = \left.\frac{\partial \pi_{it}^{*}}{\partial z_{it}}\right|_{z_{it}\uparrow\bar{z_{it}}} = p_{it}\frac{\partial g_i(\vec{x_{it}}^{*}, z_{it})}{\partial z_{it}} - r_{it}^{z} \tag{5}$$

And when z_{it} exceeds $\bar{z_{it}}$, the WTP for z_{it} is

$$\frac{\partial {\pi_{it}^{*}}^{+}}{\partial z_{it}} = \left.\frac{\partial \pi_{it}^{*}}{\partial z_{it}}\right|_{z_{it}\downarrow\bar{z_{it}}} = p_{it}\frac{\partial g_i(\vec{x_{it}}^{*}, z_{it})}{\partial z_{it}} - r_{it}^{z} - \left.\frac{\partial \bar{C}(\Delta z_{it})}{\partial \Delta z_{it}}\right|_{\Delta z_{it}\downarrow 0} \tag{6}$$

It is worth noting that π^*_{it} (and hence π_{it}) is not differentiable at $\bar{z_{it}}$: $\frac{\partial \pi^*_{it}}{\partial z_{it}}^+ > \frac{\partial \pi^*_{it}}{\partial z_{it}}^-$. This says that agent i's WTP for z_{it} must be higher if he needs more input z_{it} than threshold quantity $\bar{z_{it}}$.

Another remark is that by doing (2) - (4), we are doing optimization stepwise: we first optimize π_{it} over $\vec{x_{it}}$ and then optimize over z_{it} and get an optimum. Such optimum is a global optimum only if the profit function π_{it} is well-shaped (e.g., globally convex). Note π_{it} is linear in x_{ikt} and z_{it} when $z_{it} \leq \bar{z_{it}}$, thus the only potential problem is in $C(\cdot)$, which we will discuss in details in following sections.

3 A First Specification of Search Cost

The maximization problem (2) (or the problem (4) equivalently if π_{it} is well-shaped as discussed above) is well defined and can be solved if we can specify the cost of procuring Δz_{it}. In this section, I present a specification of search cost follows that in [DlS08]. In particular, I show that the specification in [DlS08] does not fit in our problem. A key difference is that in our model there is a strict positive probability that agent i can not find Δz_{it} at any location. To be specific, to specify the procurement cost of Δz_{it}, in addition to the general settings, I further assume:

- There are J locations of finding Δz_{it}; in addition, with loss with generality, assume the order of these locations are the same as agent i's order to try these locations [otherwise, we can permute the ordering of these locations].
- Agent i's search cost for each location $j \in \{1, ..., J\}$ is $\vec{v^i} = (v^i_1, ..., v^i_J)$.
- There is an objective probability p^i_j of finding Δz_{it} at location j. And such location is unknown to agent i before he tries. Define $\vec{p^i} = (p^i_1, ..., p^i_J)$.
- Let $\vec{r^z} = (r^z_1, ..., r^z_J)$ be the prices of z at location $\{1, ..., J\}$. These prices are unknown to agent i beforehand.

In this specification, agent i beforehand (so there is no learning) optimally chooses the numbers of locations h ($h < J$) to check for availability and price. Note such h depends on i. If he finds only one location where he can buy Δz_{it}, the he will buy at the price of that location. If he finds availability of Δz_{it} at multiple locations, he will buy from the location with lowest price. Formally, define r^h be the lowest price of all locations with availability:

$$r^h = min_{j \in A}(r^z_j) \tag{7}$$

where $A = \{j = 1, ..., J | \text{there is } \Delta z_{it} \text{ at location } j.\}$. Note if agent i does not find Δz_{it} at the number n of locations which he chooses optimally beforehand, $A = \emptyset$ and hence $r^h = \infty$. With the specification above, the cost of procuring Δz_{it} can be written as

$$\bar{C}(\Delta z_{it}) = \sum_{i=1}^{h} v^i_1 + r^h \Delta z_{it} \tag{8}$$

Note such $\bar{C}(\Delta z_{it})$ is a random variable. Thus we can only optimize over its expectation:

$$\mathbb{E}(\bar{C}(\Delta z_{it})) = \sum_{i=1}^{h} v_1^i + \Delta z_{it}\mathbb{E}(r^h) \tag{9}$$

And the original problem (2) and (4) become to

$$\max_z \mathbb{E}\pi_{it}^* = \max_z \ p_{it}y_{it}^* - \vec{r_{it}}\vec{x_{it}}^* - r_{it}^z z_{it}\mathbb{1}(z_{it} \leq \bar{z_{it}}) - (r_{it}^z \bar{z_{it}} + \mathbb{E}\bar{C}(\Delta z_{it}))\mathbb{1}(z_{it} > \bar{z_{it}}) \tag{10}$$

On the other hand, note

$$\begin{aligned}\mathbb{E}(r^h) = \mathbb{P}(\exists\ \Delta z_{it} \text{ for some location})r + \mathbb{P}(\nexists\ \Delta z_{it} \text{ for all locations})\infty \\ > \mathbb{P}(\nexists\ \Delta z_{it} \text{ for all locations})\infty \to \infty \ \ \forall h\end{aligned}$$

This means $\mathbb{E}(\bar{C}(\Delta z_{it})) \to \infty$. As a result,

$$\max_z \mathbb{E}\pi_{it}^* = \max_z \ p_{it}y_{it}^* - \vec{r_{it}}\vec{x_{it}}^* - r_{it}^z z_{it}\mathbb{1}(z_{it} \leq \bar{z_{it}}) - \infty\mathbb{1}(z_{it} > \bar{z_{it}}) \tag{11}$$

Since the last term goes to infinity if z_{it} is bigger than $\bar{z_{it}}$, problem (11) has optimal solution:

$$z_{it}^* < \bar{z_{it}} \tag{12}$$

That is, agent i will always choose some optimal z_{it}^* smaller than $\bar{z_{it}}$, which means he will never search.

While this specification is mathematically correct, it does not provide much economic insight into the search cost problem in this setting. Reasons of such failure at least includes the following aspects:

- The agent i chooses h the numbers of location beforehand and hence there is no chance of learning (same as in [DlS08]);
- There is no sure guarantee that agent i can find Δz_{it} at at least one location. Thus each under this risk neutral case, the expected procurement cost for Δz_{it} explodes to infinity and the agent i chooses to not to do any searching (this is different from [DlS08]).

Thus to better model the searching process in our scheme, we either eliminate randomness (we can surely find Δz_{it} somewhere) or add in 'learning in searching'.

References

[DlS08] Babur De los Santos. Consumer search on the internet. *Available at SSRN 1285773*, 2008.