Relative Efficiency of Pennsylvania Dairy Farms

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1 Data Desciption

In this note, we present an analysis of farms' production efficiencies using the DEA approach. The data we use is monthly data of a small set of Pennsylvania dairy farms. The data was collected from 53 farms that participated in the study. The three datasets can be summarized as:

	Data_set	Number_obs	Comments
1	Monthly Data	2303.00	IOFC data collected for each month 2013-2015, 53 farms, test date data included
2	Test Data	144.00	Test dates pooled such that for each farm there are 3-5 obs on different test dates
3	Annual Data	55.00	2013-2015 although 53 farms x 2yrs => 106 obs, many farms had missing data

Table 1: Summary of Datasets

2 Models

Now we define the input and output variables we use in the models. The inputs variates we choose are:

Number of Cows, Dry Matter, CP, Starch DM, pH
Purchased Feed, Feed Cost, Corn silage

Note inputs in last row below are only in annual data.

The outputs variates we choose are:

Milk per Milk Cow, Fat, Protein, (negative) MUN, (negative)Fecal Starch

2.1 Model 4-1 Test Data

Recall in test data, the test dates are pooled such that for each farm there are 3-5 obs on different test dates. For each of the output variables, we conduct DEA for all farms. The average efficiency scores can be summarized in table 2.

	Farm_ID	Milk_per_Milk_Cow	X__Fat	X__Pro	MUN	Fecal_Starch
1	3.00	0.93	0.92	0.93	0.92	0.92
2	4.00	0.99	0.91	0.91	0.91	0.91
3	5.00	0.99	0.91	0.91	0.91	0.97
4	9.00	0.93	0.95	0.94	0.94	0.97
5	10.00	0.87	0.90	0.87	0.87	0.87
6	14.00	0.91	0.90	0.90	0.90	0.91
7	18.00	0.92	0.89	0.89	0.89	0.89
8	21.00	0.93	0.93	0.93	0.93	0.95
9	22.00	0.93	0.92	0.92	0.92	0.92
10	23.00	1.00	1.00	1.00	1.00	1.00

11	24.00	0.92	0.90	0.91	0.90	0.92
12	25.00	0.97	0.95	0.95	0.96	0.99
13	31.00	0.94	0.93	0.92	0.91	0.94
14	37.00	0.97	0.96	0.96	0.96	0.97
15	38.00	0.91	0.90	0.90	0.91	0.94
16	51.00	1.00	1.00	1.00	1.00	1.00
17	60.00	0.95	0.91	0.91	0.93	0.92
18	61.00	0.93	0.92	0.92	0.94	0.92
19	62.00	0.94	0.94	0.93	0.93	0.93
20	63.00	0.98	0.93	0.93	0.94	0.94
21	65.00	0.88	0.86	0.86	0.86	0.86
22	66.00	0.89	0.86	0.86	0.88	0.86
23	67.00	0.94	0.86	0.86	0.86	0.86
24	69.00	0.98	0.94	0.94	0.95	0.95
25	70.00	0.96	0.92	0.92	0.92	0.93
26	93.00	1.00	0.91	0.91	1.00	0.94
27	95.00	0.90	0.92	0.89	0.90	0.92
28	106.00	0.99	0.96	0.96	0.97	0.99
29	107.00	0.97	0.96	0.96	0.98	0.96
30	111.00	0.87	0.90	0.88	0.87	0.87
31	113.00	0.93	0.91	0.91	0.92	0.96
32	115.00	0.99	0.96	0.96	0.97	0.99
33	129.00	0.89	0.90	0.89	0.87	0.89
34	130.00	0.98	0.96	0.96	0.96	0.98
35	133.00	0.93	0.89	0.89	0.89	0.93
36	135.00	0.91	0.88	0.88	0.88	0.88
37	144.00	0.97	0.99	0.99	0.97	0.99
38	146.00	0.93	0.96	0.93	0.92	0.97
39	149.00	0.95	0.92	0.92	0.92	0.92
40	150.00	0.96	0.92	0.92	0.95	0.92
41	153.00	0.97	1.00	0.97	0.97	0.97
42	159.00	1.00	0.94	0.94	0.95	0.97
43	162.00	0.95	0.95	0.95	0.96	0.95
44	163.00	0.93	0.96	0.93	0.93	0.93
45	164.00	0.99	1.00	0.99	1.00	1.00
46	170.00	1.00	0.99	0.99	1.00	0.99
47	179.00	0.95	0.92	0.92	0.93	0.97
48	180.00	0.99	0.98	0.98	1.00	1.00
49	194.00	0.94	0.94	0.94	0.94	0.95
50	195.00	0.90	0.88	0.88	0.91	1.00
51	196.00	0.90	0.89	0.89	0.89	1.00
52	198.00	0.92	0.90	0.90	0.93	0.90

Table 2: Average Test Data Efficiency Score by Farm

Each column is table 2 reports the average efficiency score based on that column

as output. We plot the distribution of the efficiency scores in figure 1 and figure 2.

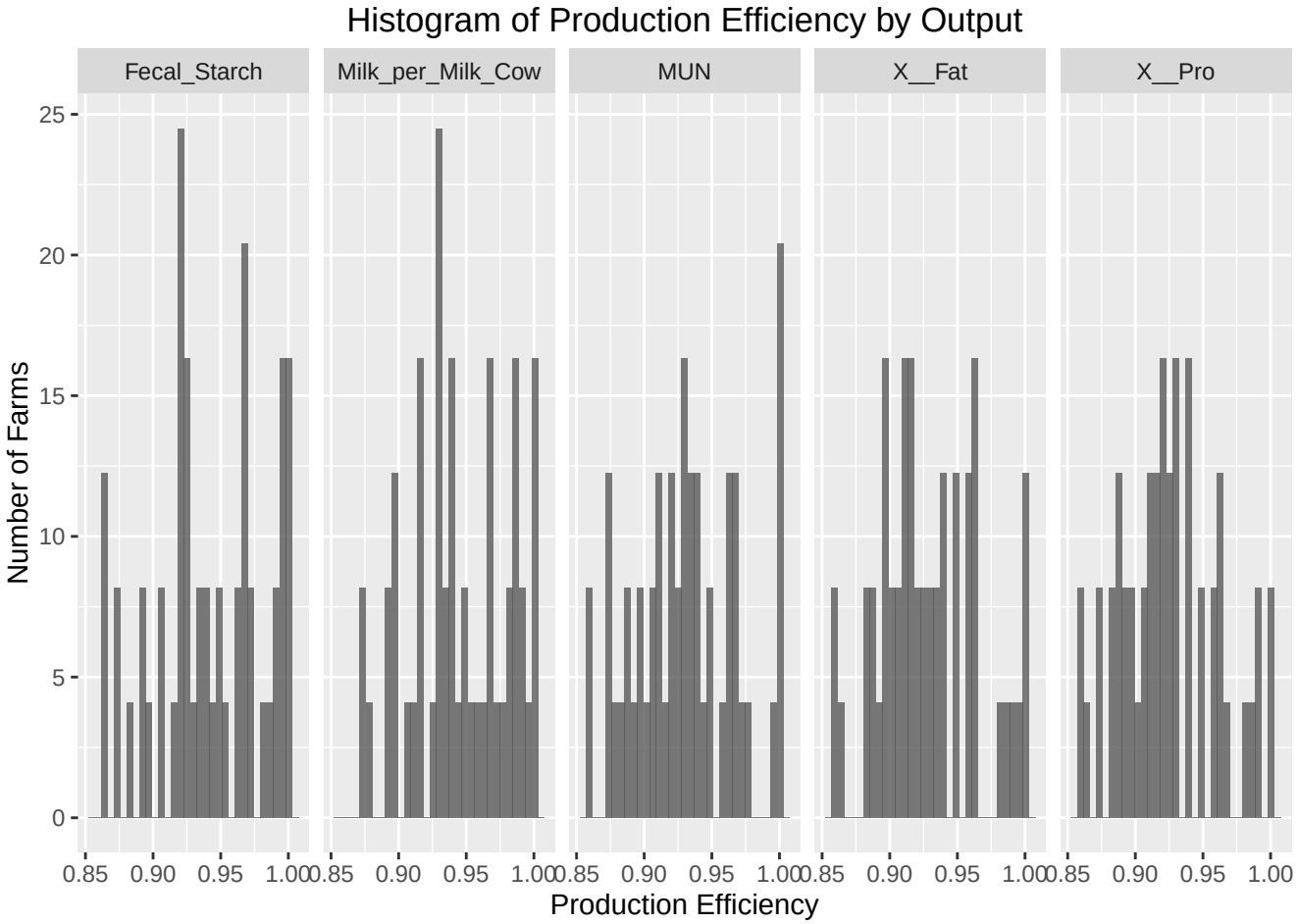

Figure 1: Histogram of Average Efficiency Score for each farm

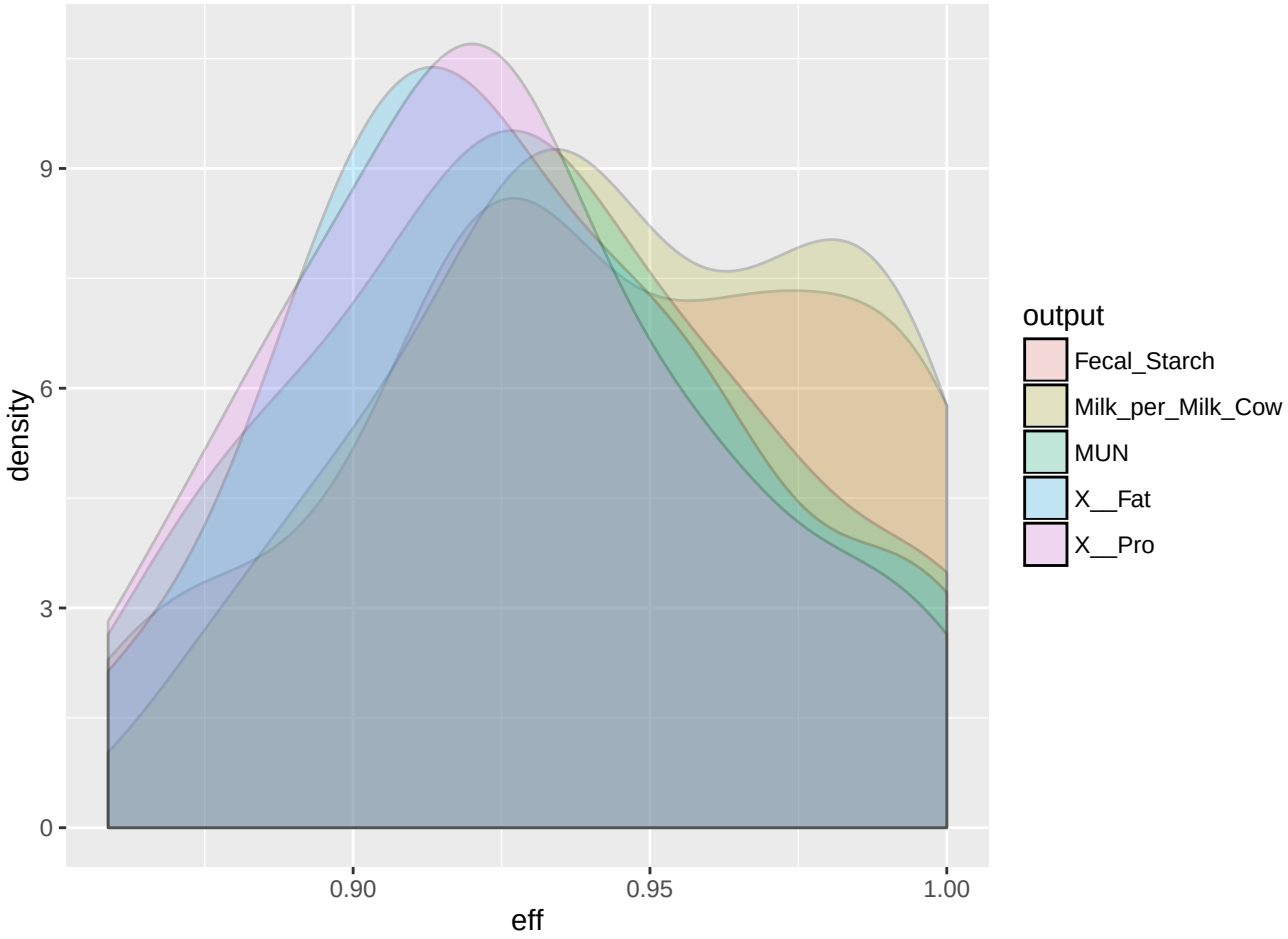

Figure 2: Distribution of Average Efficiency Score for each farm

2.2 Model 4-2 Test Data Seperated

In the section, we examine whether the efficiency of farms vary over time. Thus we partition the test data based on test date. Each test date considered a different sample.

The efficiency scores are reported in table 3.

	Farm_ID	Season_Year	Milk_per_Milk_Cow	X__Fat	X__Pro	MUN	Fecal_Starch
1	3.00	Fall 2013	0.93	0.93	0.94	0.93	0.93
2	3.00	Spring 2014	0.94	0.92	0.92	0.92	0.92
3	3.00	Spring 2015	1.00	1.00	1.00	1.00	1.00
4	3.00	Fall 2015	1.00	1.00	1.00	1.00	1.00
5	4.00	Fall 2013	1.00	0.96	0.96	0.96	0.96
6	4.00	Spring 2014	1.00	0.92	0.92	0.92	0.92
7	4.00	Fall 2014	1.00	1.00	1.00	1.00	1.00
8	5.00	Fall 2013	1.00	1.00	1.00	1.00	1.00
9	5.00	Fall 2014	0.99	0.94	0.94	0.98	0.98
10	5.00	Spring 2015	1.00	0.94	0.95	0.94	0.99
11	9.00	Fall 2013	0.98	1.00	1.00	1.00	0.98

12	9.00	Spring 2014	0.95	0.97	1.00	0.95	1.00
13	9.00	Fall 2014	0.99	1.00	0.99	1.00	1.00
14	9.00	Spring 2015	0.96	0.98	1.00	0.96	1.00
15	10.00	Fall 2014	0.89	0.90	0.89	0.89	0.89
16	14.00	Fall 2013	0.91	0.93	0.94	0.91	0.91
17	14.00	Spring 2014	0.95	0.92	0.92	0.95	0.92
18	14.00	Fall 2014	0.97	0.96	0.95	1.00	0.97
19	14.00	Spring 2015	0.96	0.92	0.92	0.92	0.94
20	18.00	Fall 2013	0.95	0.92	0.92	0.92	0.92
21	18.00	Spring 2014	0.96	0.92	0.93	0.92	0.94
22	18.00	Fall 2014	0.94	0.91	0.91	1.00	0.92
23	21.00	Fall 2013	0.96	0.97	0.97	0.96	0.96
24	21.00	Spring 2014	1.00	1.00	1.00	1.00	1.00
25	21.00	Fall 2014	0.98	0.98	0.98	1.00	0.98
26	21.00	Spring 2015	0.97	1.00	0.97	0.97	1.00
27	22.00	Fall 2013	0.93	0.93	0.93	0.93	0.93
28	22.00	Spring 2014	1.00	1.00	1.00	1.00	1.00
29	22.00	Fall 2014	0.95	0.93	0.93	0.95	0.93
30	22.00	Spring 2015	0.97	0.95	0.96	0.96	0.95
31	23.00	Fall 2013	1.00	1.00	1.00	1.00	1.00
32	23.00	Spring 2014	1.00	1.00	1.00	1.00	1.00
33	23.00	Fall 2014	1.00	1.00	1.00	1.00	1.00
34	23.00	Spring 2015	1.00	1.00	1.00	1.00	1.00
35	24.00	Fall 2013	0.93	0.93	0.95	0.93	0.93
36	24.00	Spring 2014	0.93	0.89	0.89	0.89	0.89
37	24.00	Spring 2014	0.96	0.92	0.92	0.92	0.99
38	25.00	Fall 2013	0.98	0.98	0.98	0.98	1.00
39	25.00	Spring 2014	0.98	0.94	0.95	1.00	1.00
40	25.00	Fall 2014	0.99	0.98	0.98	1.00	1.00
41	25.00	Spring 2015	1.00	1.00	1.00	1.00	1.00
42	31.00	Fall 2014	1.00	1.00	1.00	1.00	1.00
43	31.00	Spring 2015	0.96	0.96	1.00	0.96	0.96
44	37.00	Fall 2013	1.00	1.00	1.00	1.00	1.00
45	37.00	Spring 2014	1.00	1.00	1.00	1.00	1.00
46	37.00	Spring 2015	0.96	0.99	0.96	0.96	0.97
47	38.00	Fall 2013	0.96	0.96	0.96	0.96	1.00
48	38.00	Spring 2014	0.92	0.92	0.97	0.92	0.92
49	38.00	Fall 2014	0.95	0.95	0.95	1.00	0.97
50	38.00	Spring 2015	0.97	0.97	1.00	0.97	0.97
51	51.00	Spring 2014	1.00	1.00	1.00	1.00	1.00
52	60.00	Spring 2014	0.98	0.97	0.97	0.97	0.97
53	60.00	Fall 2014	0.91	0.90	0.90	0.95	0.91
54	60.00	Spring 2015	1.00	1.00	1.00	1.00	1.00
55	61.00	Spring 2014	0.93	0.92	1.00	0.94	0.92
56	62.00	Spring 2015	0.96	1.00	0.96	0.96	0.96
57	63.00	Spring 2014	1.00	0.97	0.97	0.97	0.97

58	63.00	Fall 2014	0.94	0.94	0.94	0.96	0.95
59	63.00	Spring 2015	1.00	1.00	0.99	0.99	0.99
60	65.00	Fall 2013	0.95	0.95	0.95	0.95	0.95
61	65.00	Spring 2014	0.91	0.89	0.90	0.88	0.88
62	65.00	Fall 2014	0.89	0.88	0.88	0.91	0.89
63	65.00	Spring 2015	0.87	0.88	0.95	0.87	0.87
64	66.00	Fall 2013	0.97	0.95	0.95	0.95	0.95
65	66.00	Spring 2014	0.92	0.87	0.87	1.00	0.88
66	66.00	Fall 2014	1.00	0.99	0.99	1.00	0.99
67	66.00	Spring 2015	0.95	0.96	0.94	0.94	0.94
68	67.00	Spring 2014	1.00	0.92	0.92	0.92	0.92
69	67.00	Fall 2014	0.91	0.85	0.85	0.87	0.85
70	67.00	Spring 2015	1.00	0.91	0.91	0.91	0.91
71	69.00	Fall 2013	1.00	1.00	1.00	1.00	1.00
72	69.00	Spring 2014	1.00	0.88	0.90	0.88	0.88
73	69.00	Fall 2014	1.00	0.98	0.98	1.00	0.98
74	69.00	Spring 2015	1.00	1.00	1.00	1.00	1.00
75	70.00	Fall 2013	1.00	0.95	0.95	0.95	0.95
76	70.00	Spring 2014	1.00	1.00	1.00	1.00	1.00
77	70.00	Fall 2014	1.00	0.97	0.97	0.99	0.97
78	70.00	Spring 2015	1.00	0.96	0.96	0.96	0.97
79	93.00	Spring 2015	1.00	1.00	1.00	1.00	1.00
80	95.00	Fall 2014	0.88	0.92	0.88	0.93	0.93
81	95.00	Spring 2015	0.97	1.00	0.97	0.97	0.97
82	106.00	Fall 2013	1.00	0.97	0.97	1.00	1.00
83	106.00	Spring 2014	0.99	0.97	0.97	1.00	0.97
84	106.00	Fall 2014	1.00	1.00	1.00	1.00	1.00
85	106.00	Spring 2015	1.00	1.00	1.00	1.00	1.00
86	107.00	Fall 2013	0.96	0.95	0.95	1.00	0.97
87	107.00	Spring 2014	0.96	0.96	0.96	0.97	0.96
88	107.00	Fall 2014	1.00	1.00	1.00	1.00	1.00
89	107.00	Spring 2015	1.00	1.00	1.00	1.00	1.00
90	111.00	Fall 2013	0.93	0.94	0.93	0.93	0.93
91	111.00	Spring 2014	0.88	0.90	0.91	0.88	0.88
92	113.00	Fall 2013	1.00	1.00	1.00	1.00	1.00
93	113.00	Spring 2014	0.95	0.92	0.94	0.92	1.00
94	113.00	Spring 2015	0.96	0.96	1.00	0.96	0.96
95	115.00	Fall 2014	1.00	0.98	0.98	1.00	1.00
96	115.00	Spring 2015	1.00	1.00	1.00	1.00	1.00
97	129.00	Fall 2013	0.92	1.00	1.00	0.92	0.92
98	129.00	Spring 2014	0.91	0.93	0.92	0.89	0.89
99	129.00	Fall 2014	0.96	0.94	0.93	0.96	0.95
100	129.00	Spring 2015	0.91	0.86	0.86	0.86	0.89
101	130.00	Fall 2013	1.00	1.00	1.00	1.00	1.00
102	130.00	Spring 2014	1.00	1.00	1.00	1.00	1.00
103	130.00	Fall 2014	1.00	0.97	0.98	0.98	0.97

104	130.00	Spring 2015	1.00	1.00	1.00	1.00	1.00
105	133.00	Fall 2013	1.00	0.98	0.98	0.98	1.00
106	133.00	Spring 2014	1.00	0.92	0.92	0.92	1.00
107	133.00	Fall 2014	0.99	0.94	0.94	0.94	0.97
108	133.00	Spring 2015	0.84	0.84	0.84	0.84	0.84
109	135.00	Fall 2013	0.96	0.95	0.95	0.95	0.95
110	135.00	Spring 2014	0.90	0.87	0.87	0.87	0.87
111	144.00	Fall 2013	0.99	1.00	1.00	0.99	0.99
112	144.00	Spring 2014	1.00	1.00	1.00	1.00	1.00
113	146.00	Fall 2013	1.00	1.00	1.00	1.00	1.00
114	146.00	Spring 2014	0.96	0.98	0.95	0.95	0.95
115	146.00	Fall 2014	1.00	1.00	1.00	1.00	1.00
116	146.00	Spring 2015	1.00	1.00	1.00	1.00	1.00
117	149.00	Spring 2014	0.95	0.92	0.92	0.92	0.92
118	150.00	Fall 2014	1.00	0.97	0.97	1.00	0.97
119	150.00	Spring 2015	1.00	0.98	0.98	0.99	0.98
120	153.00	Fall 2013	0.94	1.00	0.94	0.94	0.94
121	153.00	Spring 2014	1.00	1.00	1.00	1.00	1.00
122	153.00	Fall 2014	1.00	1.00	1.00	1.00	1.00
123	159.00	Spring 2014	1.00	0.96	0.96	1.00	0.99
124	159.00	Fall 2014	1.00	0.96	0.96	1.00	0.96
125	159.00	Fall 2014	1.00	0.96	0.96	0.97	1.00
126	162.00	Fall 2013	1.00	1.00	1.00	1.00	1.00
127	162.00	Spring 2014	0.93	0.93	0.96	0.93	0.93
128	162.00	Fall 2014	0.96	0.96	0.96	1.00	0.96
129	162.00	Spring 2015	1.00	1.00	1.00	1.00	1.00
130	163.00	Fall 2013	1.00	1.00	1.00	1.00	1.00
131	164.00	Fall 2013	1.00	1.00	1.00	1.00	1.00
132	164.00	Spring 2014	0.99	1.00	1.00	1.00	1.00
133	170.00	Fall 2014	1.00	1.00	1.00	1.00	1.00
134	170.00	Spring 2015	1.00	1.00	1.00	1.00	1.00
135	179.00	Fall 2014	0.93	0.89	0.89	0.91	1.00
136	179.00	Spring 2015	1.00	1.00	1.00	1.00	1.00
137	180.00	Fall 2014	1.00	1.00	1.00	1.00	1.00
138	180.00	Spring 2015	1.00	1.00	1.00	1.00	1.00
139	194.00	Fall 2014	1.00	1.00	1.00	1.00	1.00
140	194.00	Spring 2015	0.98	0.98	0.98	0.98	0.98
141	195.00	Spring 2015	0.97	0.99	1.00	0.97	1.00
142	196.00	Spring 2015	1.00	1.00	1.00	1.00	1.00
143	198.00	Fall 2014	0.96	0.94	0.94	1.00	0.94
144	198.00	Spring 2015	0.93	0.95	0.93	0.93	0.94

Table 3: Test Data Efficiency Score over Time

We plot these in the following figures:

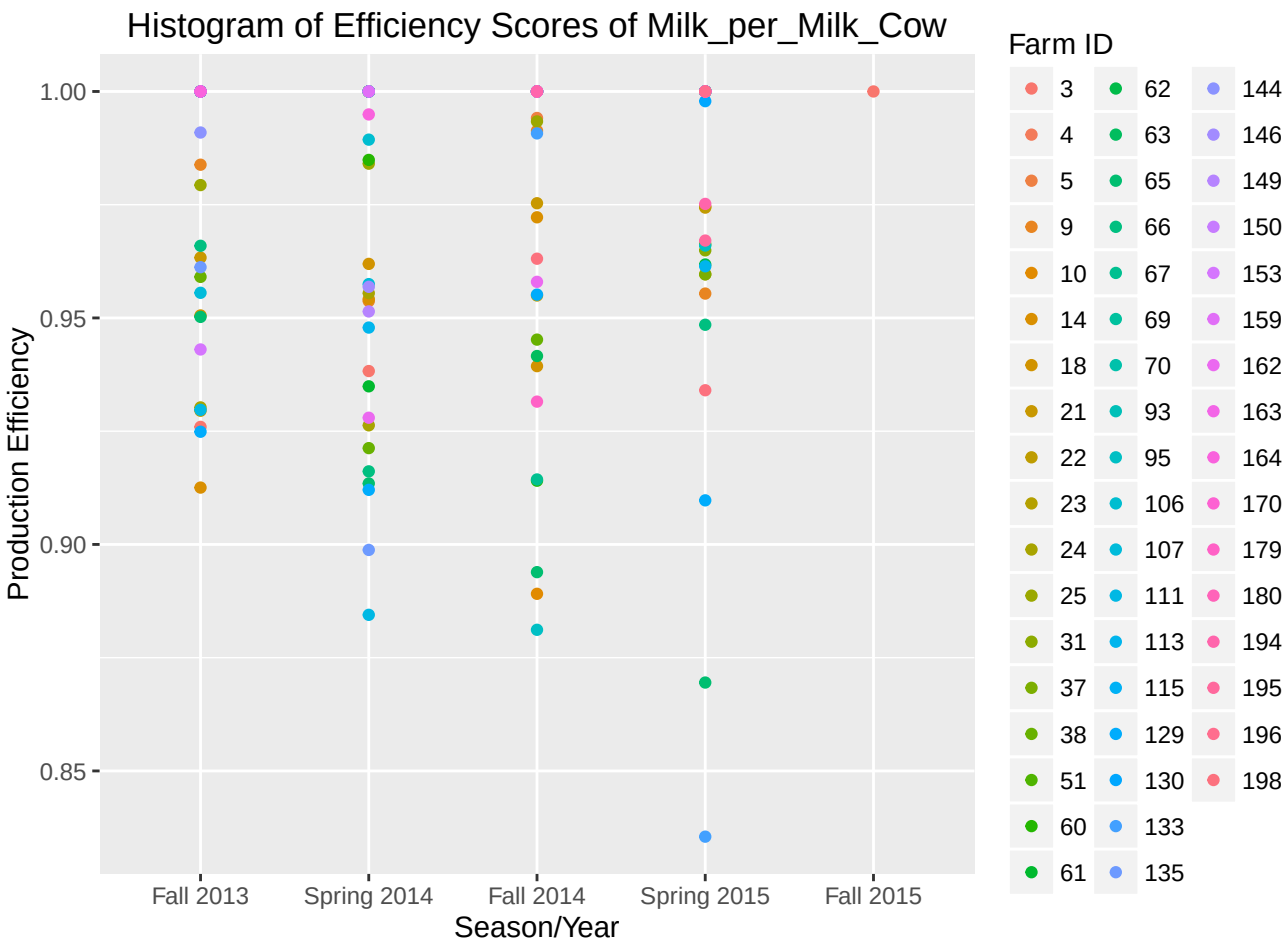

Figure 3: Test Data Efficiency Score over Time (output: Milk)

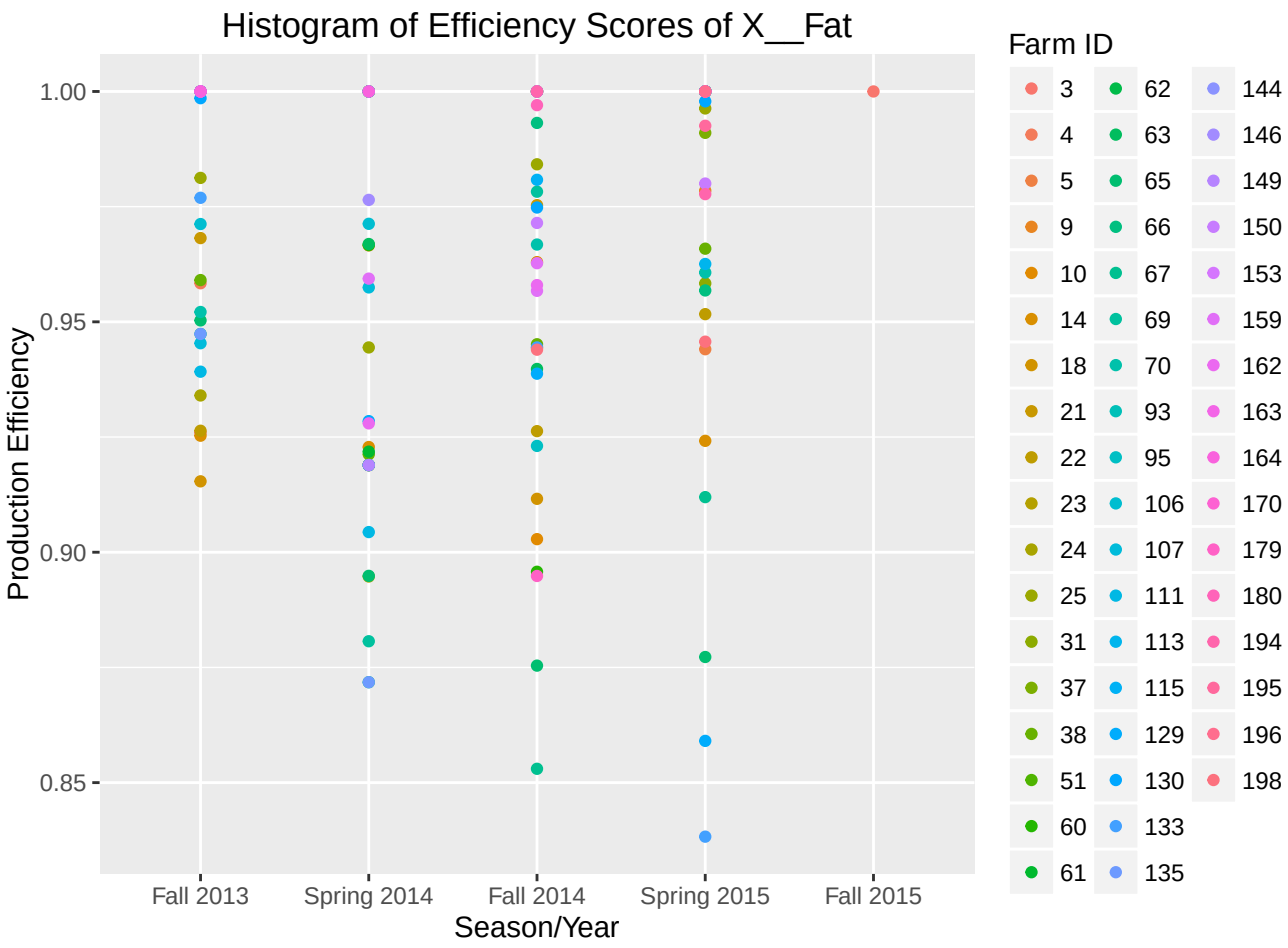

Figure 4: Test Data Efficiency Score over Time (output: Fat)

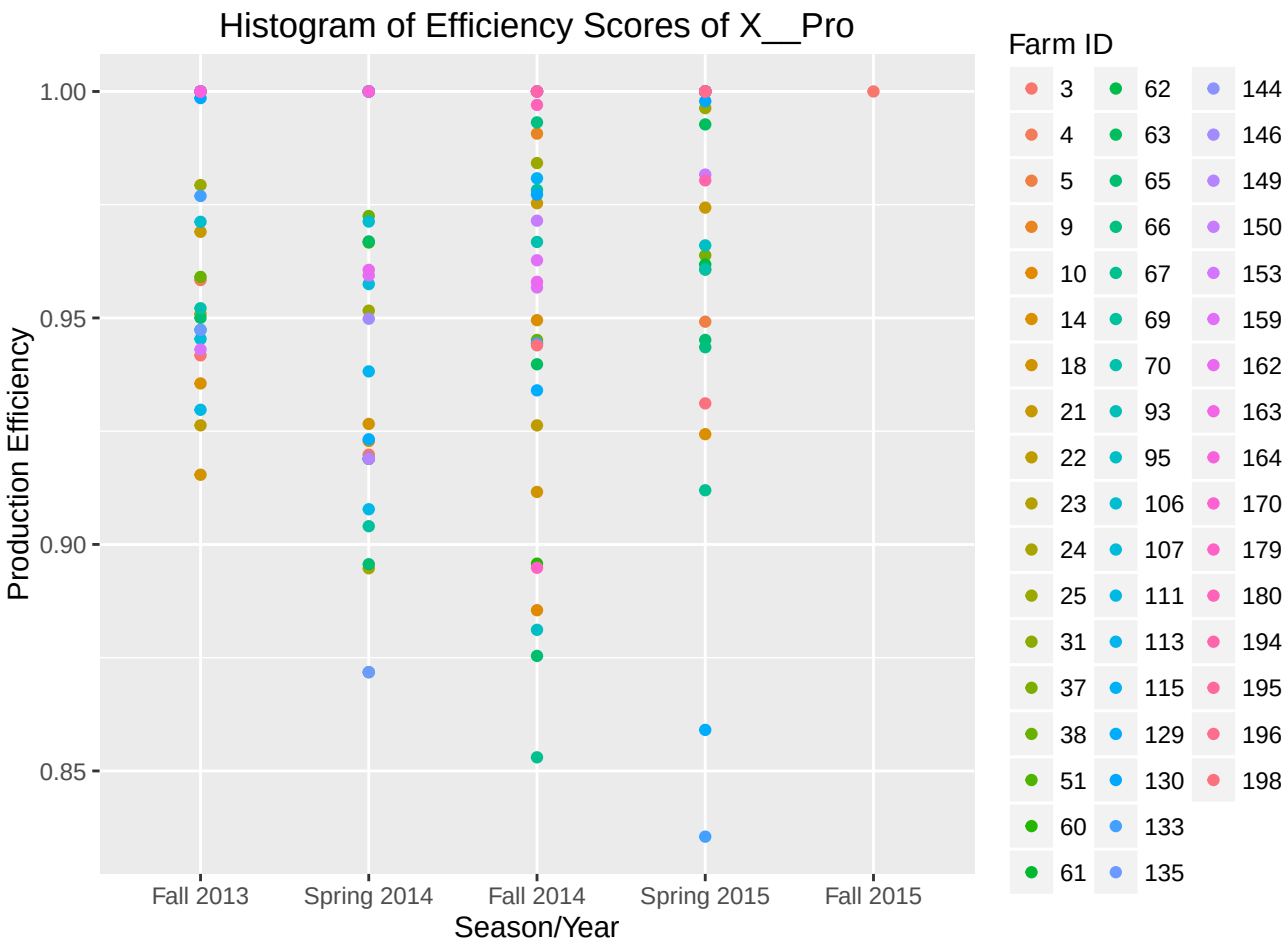

Figure 5: Test Data Efficiency Score over Time (output: Protein)

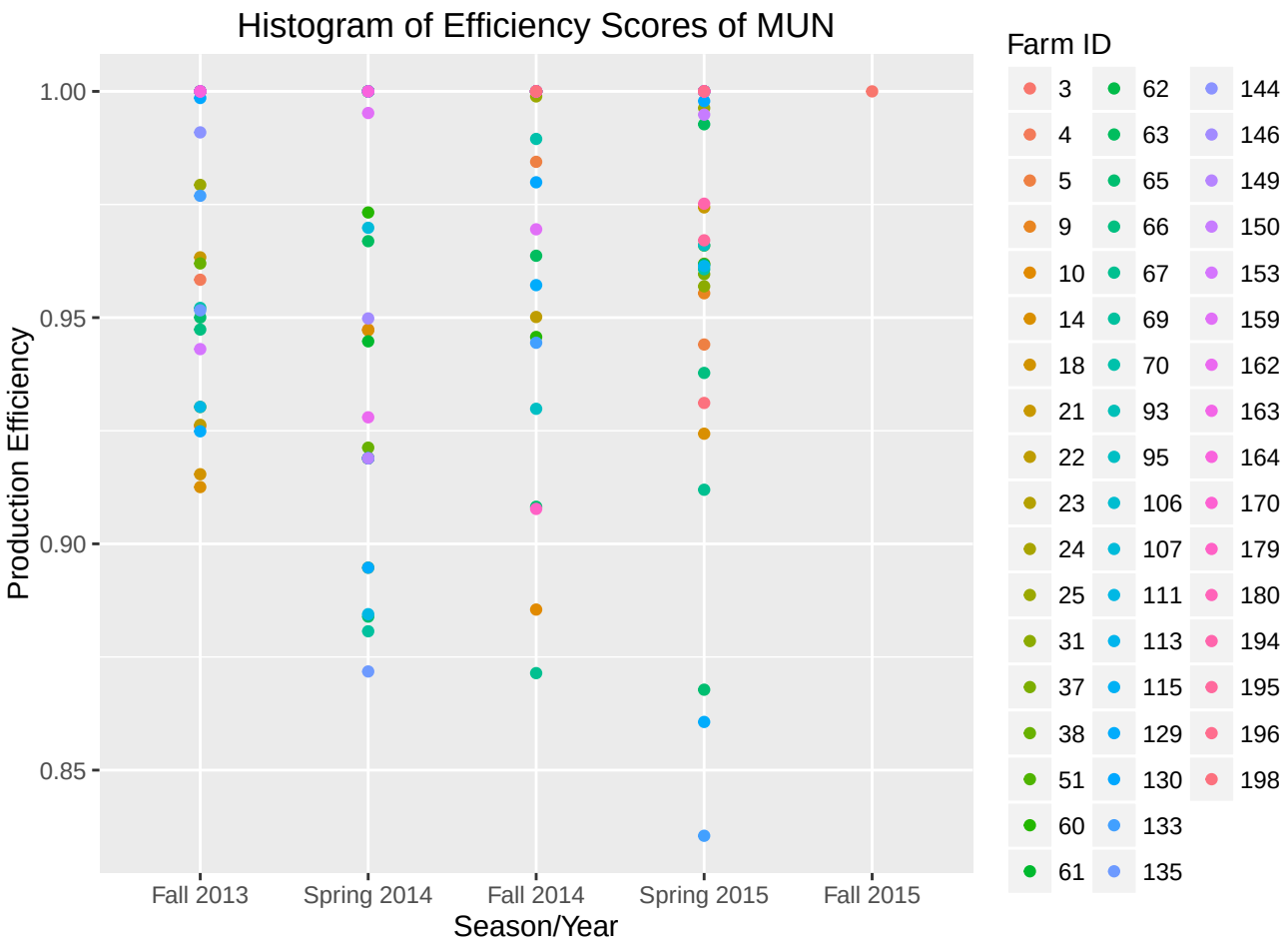

Figure 6: Test Data Efficiency Score over Time (output: MUN)

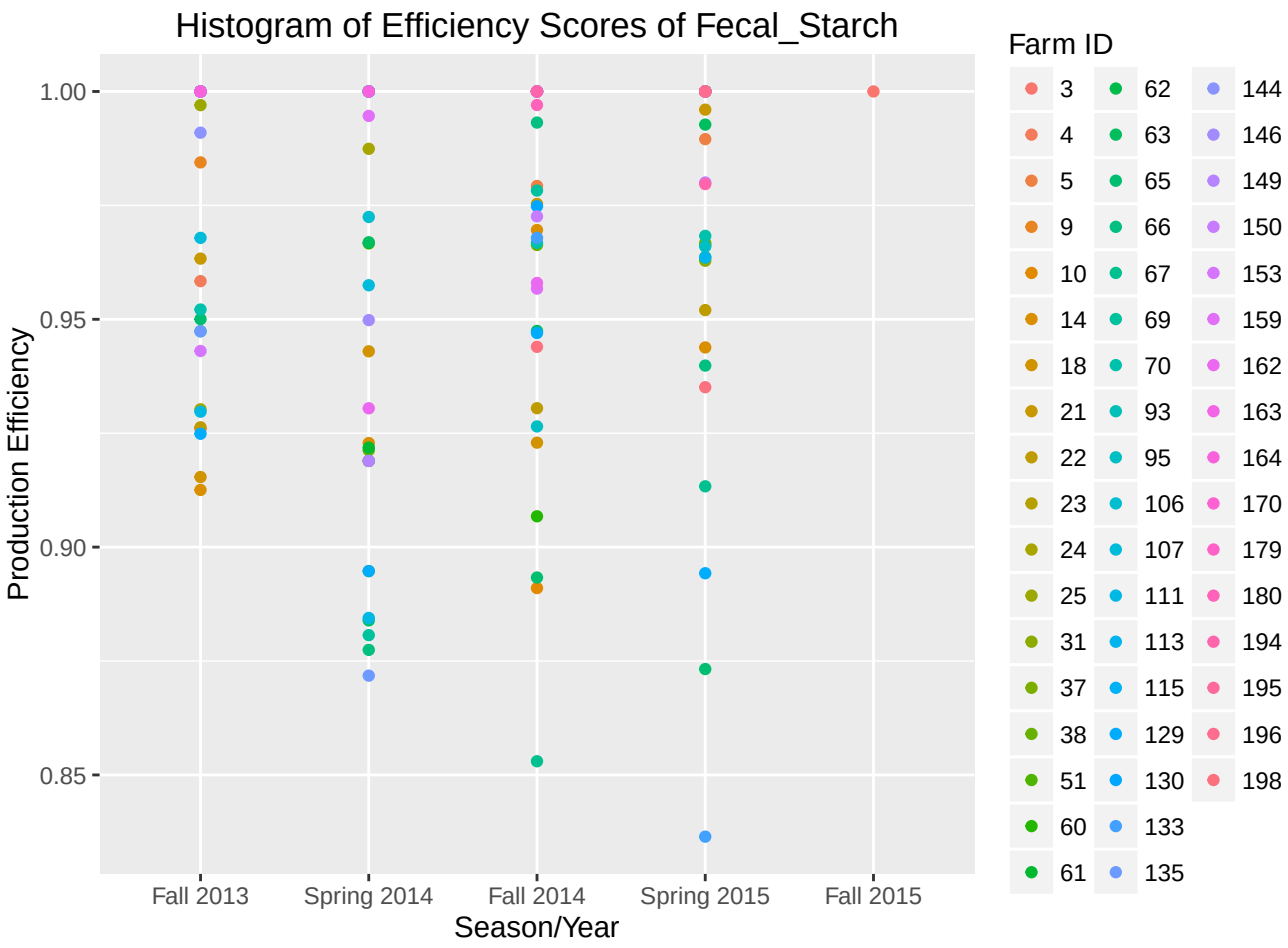

Figure 7: Test Data Efficiency Score over Time (output: Fecal Starch)

2.3 Model 5 Annual Data

In this section, we merge annual data into pooled test date data. We have 55 avaible observations in total. With annual data, we are able to use Purchased Feed, Feed Cost, Corn silage as input variables. For each of these output variables, we conduct DEA for all farms. The average efficiency scores can be summarized in table 4.

	Farm_ID	Milk_per_Milk_Cow	X__Fat	X__Pro	MUN	Fecal_Starch
1	3.00	0.94	0.93	0.93	0.93	0.93
2	4.00	1.00	1.00	1.00	1.00	1.00
3	5.00	1.00	1.00	1.00	1.00	1.00
4	9.00	1.00	1.00	1.00	1.00	1.00
5	10.00	0.89	0.90	0.94	0.89	0.89
6	14.00	0.96	0.95	0.95	0.97	0.96
7	18.00	0.99	0.98	0.98	0.98	0.98
8	21.00	0.99	1.00	1.00	1.00	0.99
9	22.00	0.98	0.97	0.97	0.97	0.97
10	23.00	1.00	1.00	1.00	1.00	1.00
11	24.00	0.94	0.93	0.93	0.93	0.96

12	25.00	0.99	0.97	0.98	0.99	1.00
13	37.00	1.00	1.00	1.00	1.00	1.00
14	38.00	0.98	0.98	0.98	0.99	0.99
15	63.00	1.00	1.00	1.00	1.00	1.00
16	65.00	1.00	1.00	1.00	1.00	1.00
17	66.00	0.93	0.92	0.93	0.95	0.93
18	67.00	1.00	1.00	1.00	1.00	1.00
19	69.00	0.98	0.95	0.95	0.95	0.95
20	70.00	0.99	0.99	1.00	0.99	0.99
21	106.00	1.00	0.99	0.99	0.99	1.00
22	107.00	1.00	1.00	1.00	1.00	1.00
23	113.00	0.99	0.98	0.98	0.98	1.00
24	129.00	0.96	0.97	0.98	0.96	0.96
25	130.00	1.00	0.99	0.99	0.99	0.99
26	133.00	1.00	0.95	0.95	0.95	0.98
27	135.00	0.90	0.89	0.89	0.89	0.90
28	146.00	1.00	1.00	1.00	1.00	1.00
29	149.00	0.95	0.93	0.93	0.93	0.95
30	159.00	1.00	0.95	0.95	0.95	0.98
31	162.00	0.99	0.99	1.00	0.99	0.99

Table 4: Average Annual Data Efficiency Score by Farm

Each column is table 4 reports the average efficiency score based on that column as output. We plot the distribution of the efficiency scores in figure 8 and figure 9.

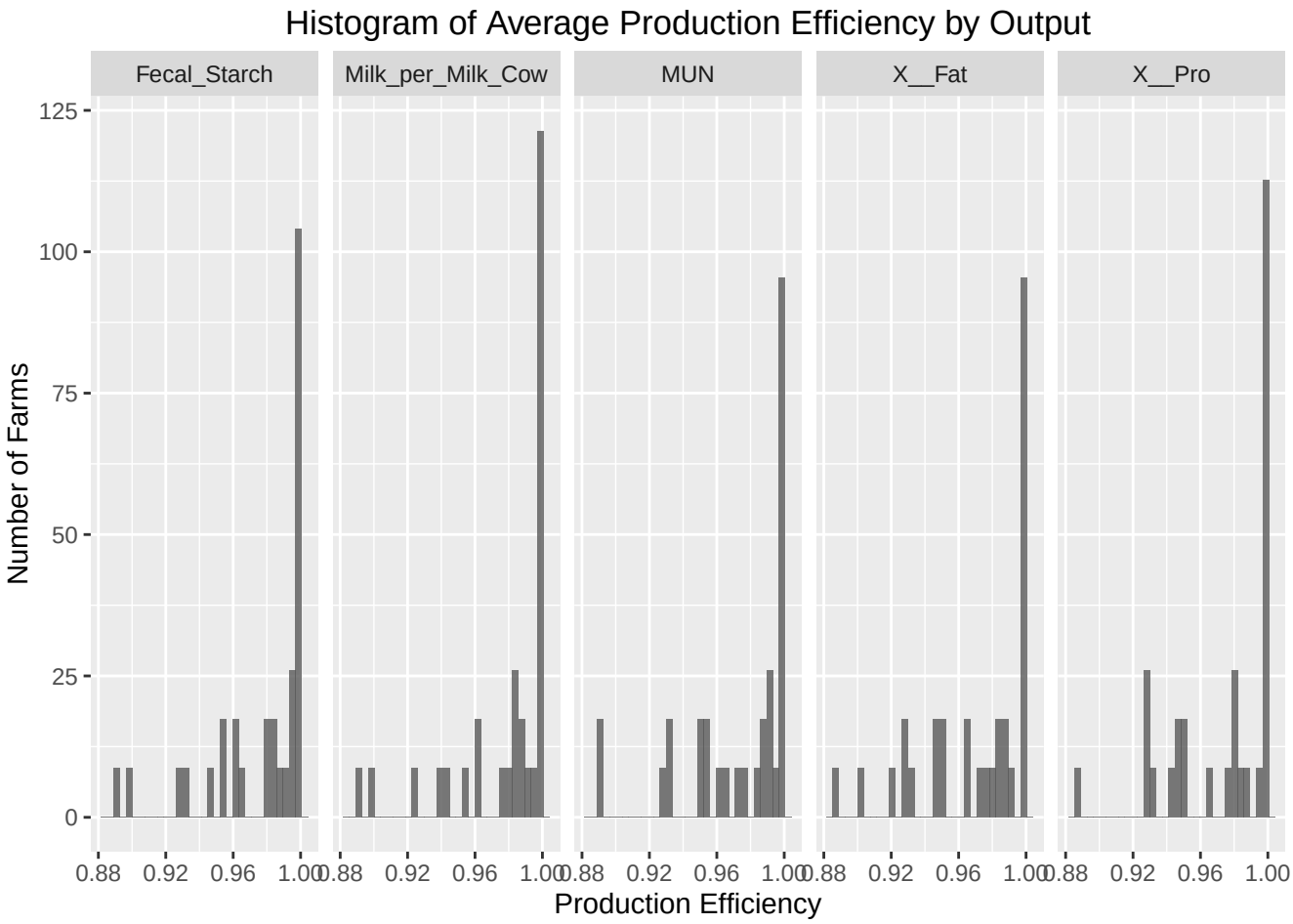

Figure 8: Histogram of Average Efficiency Score for each farm

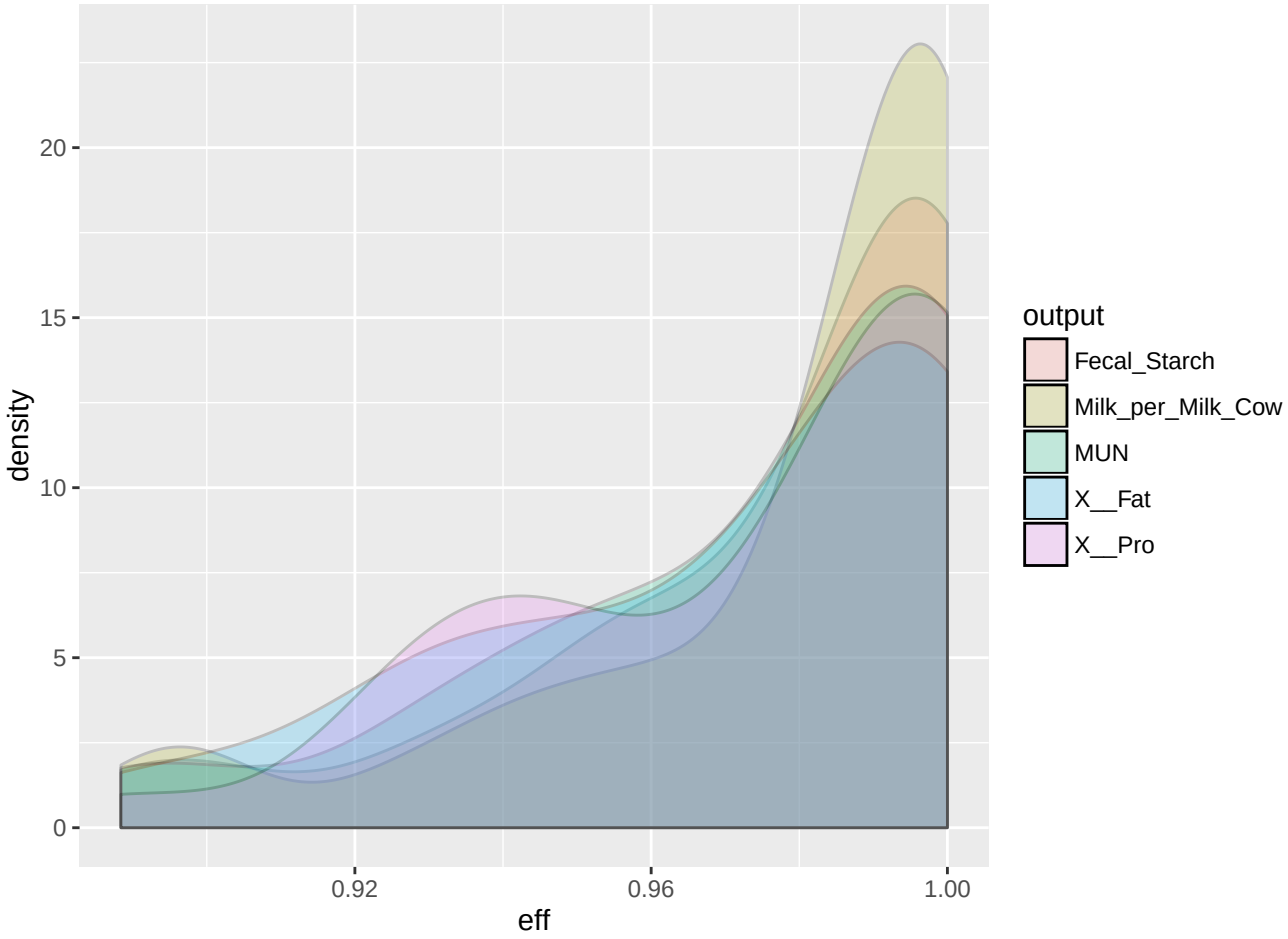

Figure 9: Distribution of Average Efficiency Score for each farm

3 New Addition 03/31/2016

3.1 Plot efficiency versus Variates

In this section we plot the efficiency score we got in section 2.1 (Model 4-1) versus fecal starch, starch digestibility, and MUN.

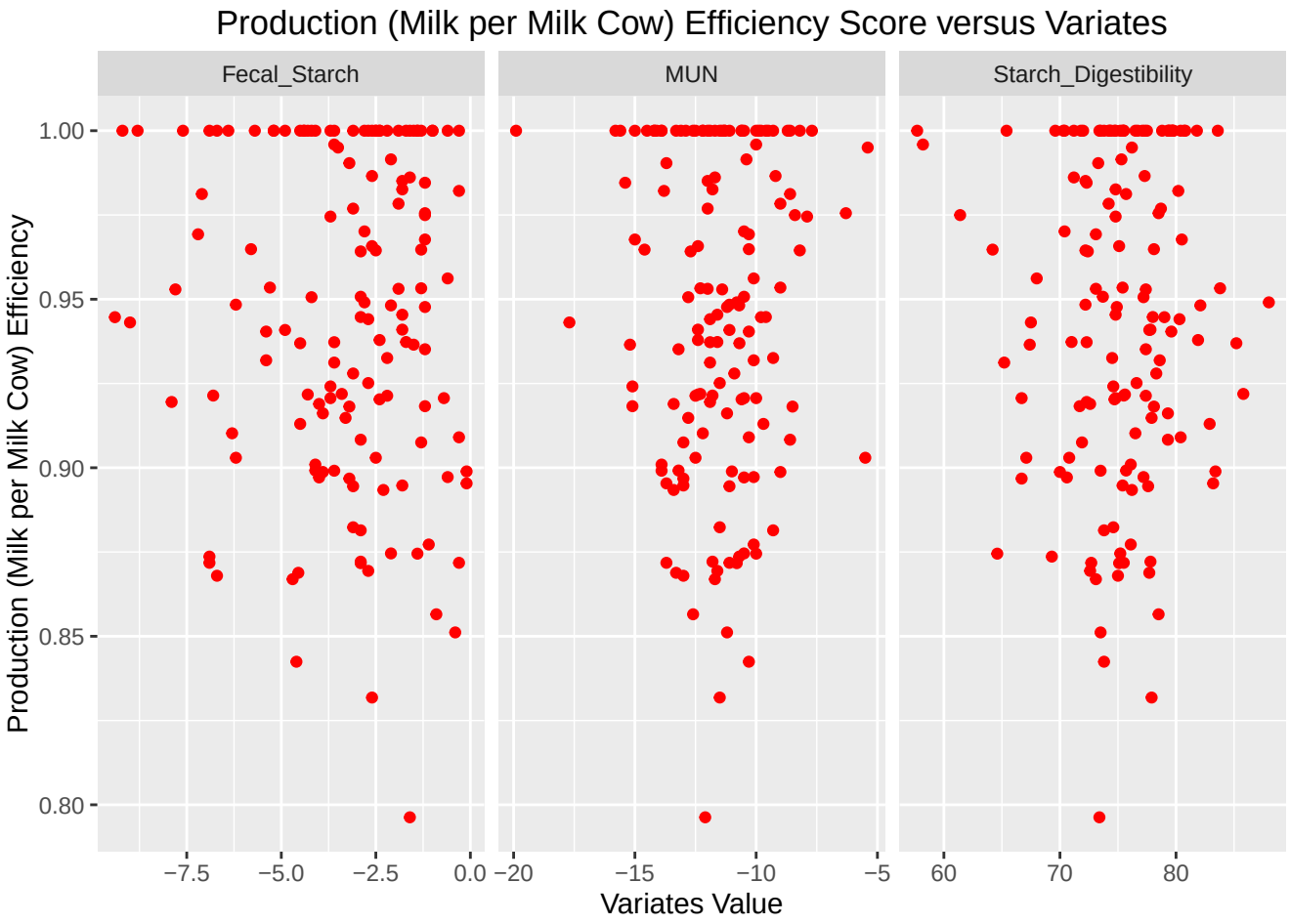

Figure 10: Production (Milk per Milk Cow) Efficiency Score versus Variates

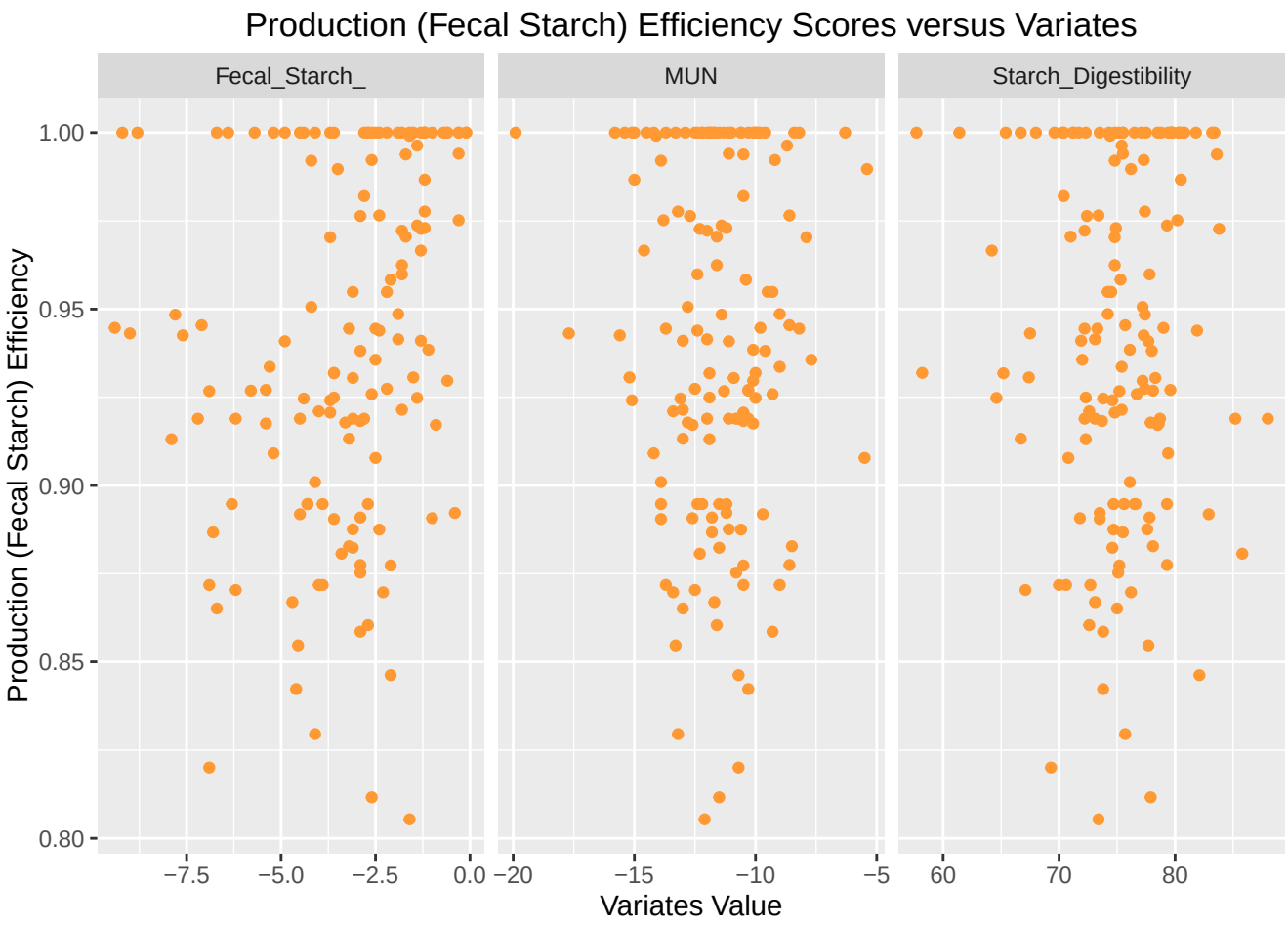

Figure 11: Production (Fecal Starch) Efficiency Scores versus Variates

3.2 Plot efficiency versus Variates by Group (0-100 cows, 100-275 cows)

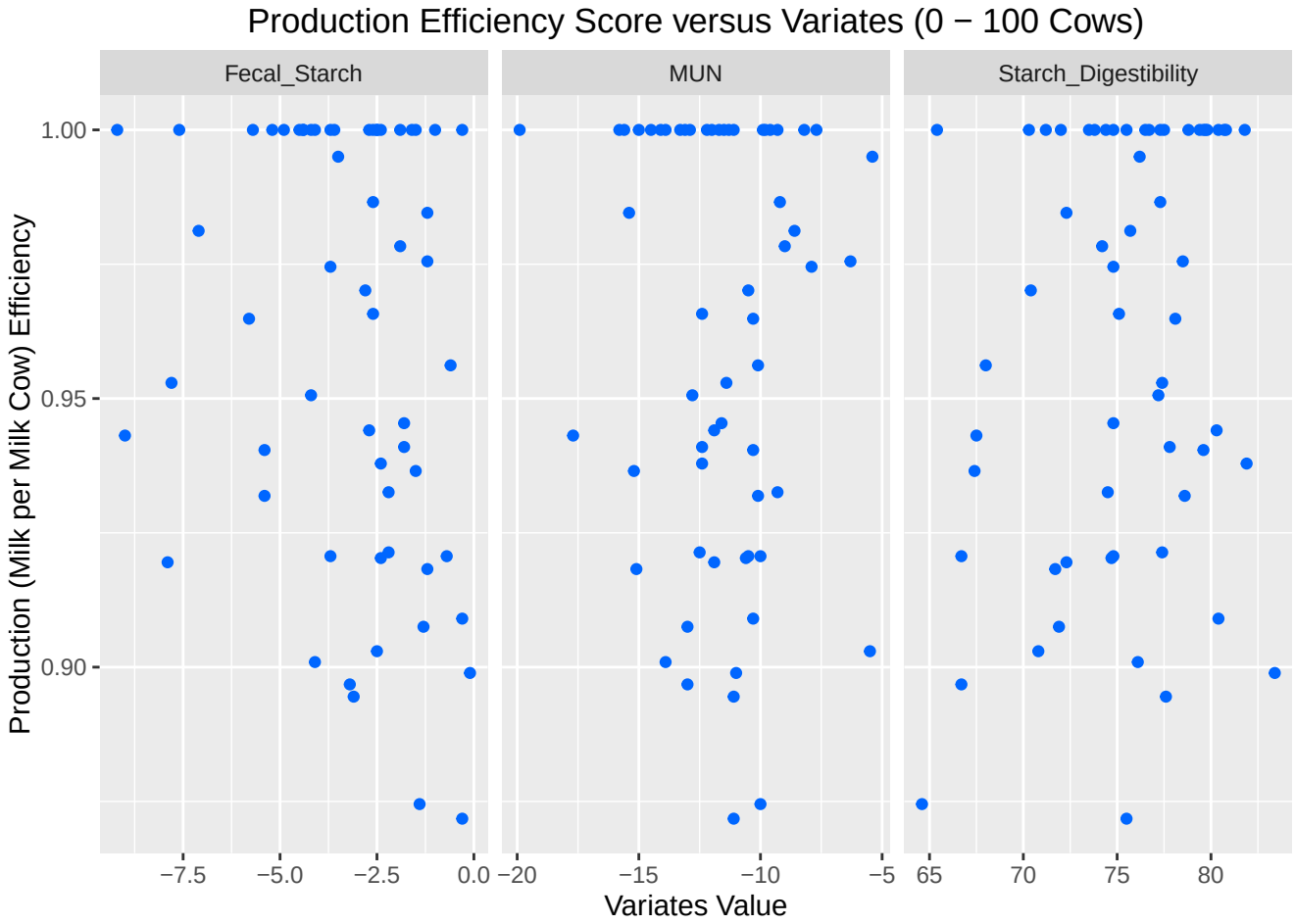

Figure 12: Production Efficiency Score versus Variates (0 - 100 Cows)

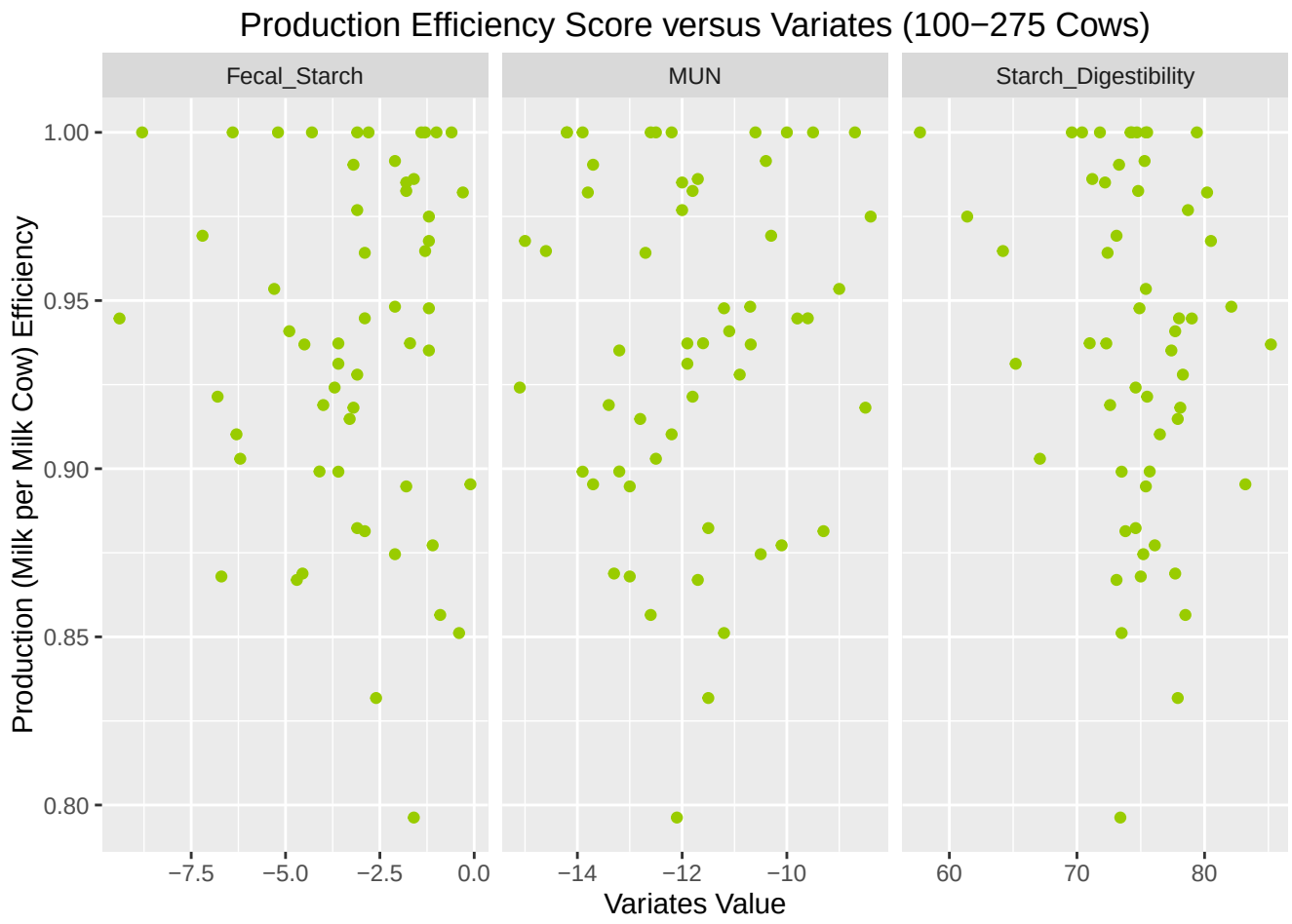

Figure 13: Production Efficiency Score versus Variates (100-275 Cows)

3.3 Two-Step DEA

In this section we run two step DEA on the Annual Data. **Step 1**: For the whole data, we estimate efficiency score using DEA. **Step 2**: We run a tobit regression of efficiency score on four variates (daily IOFC Surplus Group, Crop Cost, Purchased Feed and Corn Silage). The results are as follows.

```
Call:
censReg(formula = Annual_eff$eff_Milk_per_Milk_Cow ~ Annual_eff$Pur_Feed_Calc +
    Annual_eff$Corn_silage + Annual_eff$Fecal_Starch + Annual_eff$MUN,
    left = 0, right = 1)

Observations:
         Total  Left-censored     Uncensored Right-censored
            55              0             23             32

Coefficients:
                          Estimate Std. error t value Pr(> t)
(Intercept)              1.0082852  0.0686582  14.686  <2e-16 ***
Annual_eff$Pur_Feed_Calc 0.0012297  0.0049023   0.251  0.8019
Annual_eff$Corn_silage  -0.0012545  0.0007313  -1.715  0.0863 .
Annual_eff$Fecal_Starch  0.0030734  0.0043589   0.705  0.4808
```

```
Annual_eff$MUN            -0.0064460  0.0045548  -1.415  0.1570
logSigma                  -2.9976705  0.1673256 -17.915  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Newton-Raphson maximisation, 13 iterations
Return code 2: successive function values within tolerance limit
Log-likelihood: 17.87233 on 6 Df
```

The coefficients of the tobit regression are:

```
             (Intercept) Annual_eff$Pur_Feed_Calc   Annual_eff$Corn_silage
             1.008285189              0.001229718             -0.001254456
 Annual_eff$Fecal_Starch           Annual_eff$MUN                 logSigma
             0.003073360             -0.006445951             -2.997670468
```

The marginal effects $\frac{\partial \mathbb{E}(Y|X)}{\partial x_j}$ are

```
Annual_eff$Pur_Feed_Calc   Annual_eff$Corn_silage  Annual_eff$Fecal_Starch
            0.0005122647             -0.0005225698             0.0012802721
          Annual_eff$MUN
           -0.0026851951
```