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THREE ESSAYS ON THE NEXUS BETWEEN ENERGY AND SOFT COMMODITY

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Abstract

CHAPTER 1 (with Robert Weaver) : Energy Price Dynamics in the Real Economy: Causal Evidence

The variation of energy prices has been a traditional source of shocks to the real economy. In many cases, this variation has manifested in jumps in energy prices that were characterized by some persistence. From another perspective, energy price volatility has historically been noted and its effects on real economy debated. Historically, the importance of the shocks to the real economy has led them to be labeled as energy crises, as they were argued to have resulted in substantial changes in real prices that induced changes in behavior on the demand and supply sides of the many markets. This paper re-examines evidence of such a linkage by considering the transmission of energy prices into soft commodity prices. This nexus lies within the core of any real effects as softs include food-related commodities. The paper contributes to the literature by re-examining this linkage with a close eye on the role played by structural breaks within a time series and by considering the question of causality within a nonlinear framework. The paper finds that functional form is a critical specification that conditions inference. Using linear forms, we find no cointegration between energy and food in the full sample under the maintained hypothesis that there are no structural breaks. Using linear nonparametric methods, we examine the series for structural breaks and find evidence of their importance. Based on subdivisions of the sample period as suggested by the structural break examination, within the structural break intervals identified we find evidence of co-integration. We next reconsider the issue within the context of nonlinear functional forms posing the question of whether evidence of structural breaks based on linear methods follow from underlying nonlinearity. Our results confirm the importance of functional form specification and we find evidence of nonlinear causality between energy and soft commodity prices.

CHAPTER 2 (with Robert Weaver) : Macroeconomics and the Nexus between Energy and Agricultural Commodities Prices

The variation of energy prices has been a traditional source of shocks to the real economy. In many cases, this variation has manifested in jumps in energy prices that were characterized by some persistence. From another perspective, energy price volatility has historically been noted and its effects on real economy debated. Historically, the importance of the shocks to the real economy has led them to be labeled as energy crises, as they were argued to have resulted in substantial changes in real prices that induced changes in behavior on the demand and supply sides of the many markets. However, empirical studies of transmission of energy prices into the real economy have been challenged by a number of significant specification issues that have resulted in substantial variation in inference drawn from results. Among these issues is the question of completeness of model specification. This paper examines the question of whether such models need to incorporate macroeconomic indicators. Clearly, macroeconomic factors such as interest rates and exchange rates play a role in the determination of energy and commodity prices, however, considerable specification uncertainty characterizes the question of which macro metrics to incorporate. This paper examines this issue from the perspective of weak exogeneity and finds evidence that the parameter estimates associated with time series models that exclude consideration of macro indicators are not compromised by their exclusion. We examine this issue using Italian, U.S. grain, and Brent crude oil prices.

Table of Contents

List of Figures ix

List of Tables xi

Acknowledgments xii

Chapter 1
Energy Price Dynamics in the Real Economy: Causal Effects 1
1.1 Introduction 1
1.2 Past Literature 6
1.3 Methodology 8
1.3.1 Notations 8
1.3.2 Models 9
1.3.2.1 Case l: Bivariate VAR(p) in the original price . . . 9
1.3.2.2 Case 2: Bivariate VAR(p) in the first differenced prices 10
1.3.2.3 Case 3: Bivariate VECM 10
1.3.2.4 Case 4: Time-Varying Cointegration Model 11
1.3.2.5 Case 5: General Nonlinear Vector Models 11
1.3.3 Tests 12
1.3.3.1 Augmented Dickey–Fuller Test for Stationarity . . 12
1.3.3.2 Granger Causality Test 13
1.3.3.3 CUSUM Test for Structural Breaks 15
1.4 Data 16
1.5 Empirical Results 18
1.5.1 Stationarity and Cointegration Results 18
1.5.2 Structural Breaks 20
1.5.3 Linear and Nonlinear Granger Causality Test Results 21
1.6 Conclusions 23

Chapter 2
Macroeconomics and the Nexus between Energy and Agricultural Commodities Prices **25**
2.1 Introduction . . . 25
2.2 Past Literature . . . 28
2.3 Methodology . . . 32
2.4 Empirical Results . . . 35
2.5 Concluding Remarks . . . 40

Chapter 3
Nexus between Energy and Commodity Prices: Jump Identification and Transmission **41**
3.1 Introduction . . . 41
3.2 Types of Outlier Effects . . . 44
3.3 Methodology . . . 49
3.3.1 Detection Procedure . . . 51
3.3.2 Econometric Model . . . 52
3.4 Empirical Results . . . 53
3.4.1 Data . . . 53
3.4.2 Identified Jumps in Observed Price Series . . . 54
3.5 Concluding Remarks . . . 59

Chapter 4
A Simple Model of Price Jumps **60**
4.1 Introduction . . . 60
4.2 Model Basics . . . 64
4.3 Notation . . . 71
4.3.1 Notation on Uncertainty . . . 71
4.3.2 Notation on Ordering and Indices . . . 72
4.3.3 Buyer Side Notation . . . 73
4.3.4 Supplier Side Notation . . . 74
4.3.5 Market Condition Notation . . . 75
4.3.6 Agent's Information, Learning, and Estimate Notation . . . 75
4.4 Model . . . 78
4.4.1 Buyer Side . . . 78
4.4.1.1 Buyer's Search Cost and Searching Rule . . . 79
4.4.1.2 Buyers' Learning Rule . . . 81
4.4.1.3 Buyers' Offer Rule . . . 87
4.4.1.4 Relation with Auction Model . . . 88
4.4.1.5 Deviation from the Optimal Bid Price . . . 89

4.4.2 Supplier Side . . . 91
4.4.2.1 Waiting Cost and Acceptance Rule . . . 92
4.4.3 Transaction Price . . . 100
4.4.4 Market conditions . . . 100
4.5 Analytical Solutions . . . 101
4.5.1 A First Example (SAC1) . . . 101
4.5.1.1 Supplier Side . . . 101
4.5.1.2 Buyer Side . . . 102
4.5.2 A Second Example (SAC4) . . . 104
4.5.2.1 Supplier's Side . . . 104
4.5.2.2 Buyer Side . . . 104
4.6 Extensions to Other Types of Trading Sessions . . . 106
4.6.1 Extensions to TS2 . . . 106
4.6.2 Extensions to TS3 and TS4 . . . 108
4.7 Simulation . . . 109
4.7.1 Single Trading Session Simulation . . . 110
4.7.2 Multiple Trading Session Simulation . . . 111
4.7.2.1 Simulation of TS1BOB2SAC4 . . . 112
4.7.2.2 The Effect of Storage: Simulation of TS3BOB2SAC5 130
4.8 Concluding Remarks . . . 136

Appendix A
Graphics for Chapter 4 **149**
A.1 Comparison of CandleStick chart . . . 149
A.2 Comparison of Jump Size . . . 153
A.3 Comparison of the Number of Encounters and Transactions . . . 156

Appendix B
A Short Review of Nonlinear Time Series Methods: Structural Breaks, Outliers, Thresholds **160**
B.1 Introduction . . . 160
B.2 Structural Break . . . 162
B.2.1 Motivation . . . 162
B.2.2 Core Topics in structural breaks . . . 162
B.2.3 Estimation and inference about break dates . . . 162
B.2.3.1 Model and Assumptions . . . 162
B.2.3.2 Consistency and Asymptotic Distribution of the break date estimators $\left(\widehat{T_1}, ..., \widehat{T_m}\right)$ . . . 163
B.2.4 Tests for a single structural break and multiple structural breaks . . . 164

B.2.4.1 CUSUM Test [Brown, Durbin and Evans, 1975] . . 164
B.2.4.2 CUSSQ Test [Brown, Durbin and Evans, 1975] . . 164
B.2.4.3 *sup-LR* test 165
B.2.4.4 *sup-Wald* test 165
B.2.4.5 Tests for unit root in the presence of structural breaks 166
B.3 Nonlinear Time Series Models 167
B.3.1 Test for nonlinearity 167
B.3.2 Markov Switching Model (Hamilton, 1989) 168
B.3.3 TAR and SETAR 168
B.3.4 STAR 169
B.4 Nonlinear Granger Causality 169
B.4.1 Linear Granger causality test 169
B.4.2 Nonlinear Granger causality test 170
B.4.3 DP critique on HJ test 172

List of Figures

1.1 US and Italy Soft Commodities and Energy Weekly Prices 18

3.1 Unit impulse for different types of outliers 50
3.2 Jumps in Italy Corn Price Series 56
3.3 Jumps in Italy Wheat Price Series 56
3.4 Jumps in Italy Soybeans Price Series 57
3.5 Jumps in US Corn Price Series 57
3.6 Jumps in US Wheat Price Series 58
3.7 Jumps in US Soybean Price Series 58
3.8 Jumps in Brent Blend Price Series 59

4.0 Jumps in Italy Wheat Price Series 60
4.1 A Timeline of a *realized* sequence of encounters 68
4.2 Simulated Personal Optimal Bid Price under Imperfect and Perfect Information about Market Condition 90
4.3 Single Buyer Average Optimal Bids to Suppliers versus Excess Demand 112
4.4 Excess Demand Realization used to Initialize Trading Sessions . . . 113
4.5 Bid Price across Trading Sessions ($c_i \sim N(100, 25)$ and $\beta = 100$) . . 115
4.6 Open, Close, High and Low Prices across Trading Sessions 116
4.7 Jump Size γ_t across Trading Sessions Measured by Closing Price . . 117
4.8 Unfulfilled Demand Ratio U_t across Trading Sessions 118
4.9 Unsold Supply Ratio across Trading Sessions 119
4.10 Closing and Equilibrium Prices with Jump indicated 120
4.11 Number of Encounters and Transactions ($\beta = 100$ High Search Cost) 122
4.12 Number of Encounters and Transactions ($\beta = 300$ Low Search Cost) 123
4.13 Examples of Distribution of Initial Supply Allocation of a Trading Session 124
4.14 Number of Jumps Detected over Different HHIs 126
4.15 Bid Price in TS1 with High Penalty Cost Variation ($c_i \sim N(100, 100)$ and $\beta = 100$) 127

4.16 Comparison of Average Bid Price in TS1 vs. TS3 131
4.17 Bid Price in a Type 3 Trading Sessions ($c_i \sim N(100, 25)$ and $\beta = 100$) 132
4.18 Comparison of Close Price in TS1 vs. TS3 133
4.19 Comparison of Jump Size in TS1 vs. TS3 134
4.21 Number of Encounters in TS3 134
4.20 Comparison of Jump Size in TS1 vs. TS3 135
4.22 Unfulfilled Demand Ratio in TS3 135

A.1 Unfulfilled Demand Ratio across Trading Sessions 149
A.2 Bid Price in TS1 ($c_i \sim N(100, 25)$ and $\beta = 100$) 150
A.3 Bid Price in TS1 with Low Seach Cost ($c_i \sim N(100, 25)$ and $\beta = 300$) 150
A.4 Bid Price in TS1 with High Concentration ($c_i \sim N(100, 25)$ and $\beta = 100$) 151
A.5 Bid Price in TS1 with High Waiting Cost ($c_i \sim N(100, 25)$ and $\beta = 100$) 151
A.6 Bid Price in TS1 with High Penalty Cost Variation ($c_i \sim N(100, 100)$) 152
A.7 Bid Price in TS3 ($c_i \sim N(100, 25)$ and $\beta = 100$) 152
A.8 Jump Size Measured by Closing Price 153
A.9 Jump Size with Low Search Cost 154
A.10 Jump Size in TS1 with High Concentration 154
A.11 Jump Size in TS1 with High Waiting Cost 155
A.12 Jump Size in TS1 with High Penalty Cost Variation 155
A.13 Jump Size in TS3 156
A.14 Number of Encounters and Transactions in TS1 (Base Case) 157
A.15 Number of Encounters and Transactions in TS1 with Low Search Cost 157
A.16 Number of Encounters and Transactions in TS1 with High Concentration 158
A.17 Number of Encounters and Transactions in TS1 with High Waiting Cost 158
A.18 Number of Encounters and Transactions in TS1 with High Penalty Cost Variation 159
A.19 Number of Encounters and Transactions in TS3 159

List of Tables

1.1 Bivariate Correlation (Weekly Data January 2000 to December 2010) 17
1.2 Augmented Dickey Fuller unit root test 19
1.3 Johansen Trace Test for Bivariate Cointegration of the Full Sample 20
1.4 Dates of Identified Structural Breaks 21
1.5 Cointegration test results within subsamples (Null: No cointegration) 21
1.6 Linear Granger causality tests on the first differenced VAR (Full Sample) 22
1.7 Granger causality tests on the first-differenced VAR with subsamples 22
1.8 Nonlinear Granger causality tests 23

2.1 Bivariate Pair and Macro Factors (M2, 10 year maturity rate and US/EU exchange rate) Weak Exogeneity Tests 36
2.2 Cointegration Rank of Bivariate Pair with Macro Factors 37
2.3 Cointegration Rank of each bivariate pair without Macro factors . . 38
2.4 Bivariate Pair and Macro Factors (3-month Maturity Rate and US/EU exchange rate) Weak Exogeneity Test 39

3.1 Jumps Detection using Chen and Liu (1993) Procedure 55

4.1 Types of Trading Sessions 66
4.2 Summary of Specifications: Base Case 111
4.3 Jump Frequency, Jump Size and Realized Volatility 124
4.4 Jump Frequency (10K Trading Sessions) 126
4.5 Jump Frequency, Jump Size, Realized Volatility and Trading Volume 128
4.6 Jump Frequency, Jump Size, Realized Volatility and Trading Volume: 10000 Trading Sessions 129

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Chapter 1 Energy Price Dynamics in the Real Economy: Causal Effects

1.1 Introduction

The variation of energy prices has been viewed as an important source of shocks to the real economy. In many cases, this variation has manifested in jumps in energy prices that were characterized by some persistence. From another perspective, energy price volatility has historically been noted and its effects on real economy debated. Historically, the importance of the energy price shocks to the real economy has led them to be labeled as energy crises, as they were argued to have resulted in substantial changes in real effects on productivity, output, employment, and prices that induced changes in behavior on the demand and supply sides of the many markets. Indeed, Hamilton's (1983) suggestion that oil price shocks were the predominant cause of recessions in the post-War period continues to be examined and debated. For several decades, the sheer magnitude of oil price shocks has understandably peaked interest in establishing their real effects (see [Ham83]). Most notable among these crises were those in 1973 following an OPEC embargo, those following political and social disruptions and war include those 1979 following the Iranian revolution, the global oil excess supply that followed as energy demand responded to high prices, the 1990 oil price shock coincident with the Iraqi invasion of Kuwait, a decade long crisis at the start of new millennium associated with a series of unanticipated events that led to a nearly five-fold increase in oil prices.

In each case, these periods were associated with unanticipated perceived shifts in supply availability, procurement panics, and dramatic oil and energy price increases that were often cited as causing periods of more general price increases, inflation, expansion in national deficits in oil importing countries, recession, reduced productivity, and associated exchange rate depreciation, see e.g. Barsky and Kilian (2004) ([BK04]), Hamilton (1996, 2003) (see [Ham96] [Ham03]). In fact, early work to establish the empirical relations included a series of papers finding a negative relation between energy prices and various indicatiors of macroeconomic performance, see e.g. Hamilton (2003) for a review. The strong conclusion of Hamilton (1983) the oil price shocks played a major role in causing recessions continues to motivate reconsideration.

As Segal and others have noted, both policy and behavioral response to oil and energy shocks has often been substantial and this may explain the stream of empirical results that find a weak to no relationship between such shocks and the real economy. From the fiscal perspective, recognition of potential real effects has led to strong support for increased exploration (e.g. Alaskan oil from Prudhoe bay), R& D for improved energy efficiency, for alternative energy sources, and for new extraction technologies. From a monetary perspective, recognition of potential real effects has led to monetary policy actions, see e.g. [Seg11]. At the agent level, evidence of adaption and mitigation is strong. Together, these economic responses suggest the relationship between energy prices and the real economy may be weaker than first thought and at least it would be dependent on the time frame analyzed. As early work by Hooker (1996) showed in [Hoo96], the time frame analyzed is important given variation in initial economic conditions as well as policy and economic behavioral response to oil price shocks.

Consistent with this intuition, a stream of papers have re-examined the real effects and found inconclusive results in support of negative effects. Within this stream, Darrat et al. (1996) and Hooker (1996) noted the importance of sample period and specification issues such as functional form (see [DGM96] [Hoo96]). Barsky and Kilian (2002) noted the need to establish causal ordering of any relationship, while Kilian (2009) showed results that highlight the differential roles of demand and supply side shocks (see [Kil08]). A series of more recent papers have considered nonlinearity and asymmetry, see Mork (1989), Hamilton (2003), LeBlanc and Chinn (2004), van den Noord and Andre (2007) and Gregorio et al. (2007) ([De +07], [Ham03], [LC04], [Mor89], [VA07]) each found evidence of nonlinearity and a diminution of energy price pass-through to inflation. Hamilton (2003) specifically focused on nonlinearity as well as asymmetry in response of GDP and using a parametric test of linearity rejected linearity as a null. Hooker (2002) reconsidered Mork's interpretation of nonlinearity and asymmetry, highlighting the presence of structural breaks in the relationships. Consistent with these results, Chen (2009) reported findings that indicate that currency depreciation, active monetary policy, and greater openness in international trade has led to diminution of oil price pass-through to inflation (see [Che09]). Chen also reports evidence of variation of Phillips curve relationships over time.

While the aggregate effects of oil price shocks have received continued attention, the specific nature of these effects on prices requires a finer lens. Within this context the impacts of oil shocks on food commodities, as well as wholesale and retail prices is of interest. The nexus of food and energy prices has persistently raised concerns as energy price transmission into food prices can be expected to directly impact consumer welfare. Clearly, from the supply side, the energy intensiveness of the crop and animal production, processing and logistics suggests a strong linkage should exist. However, more recently, attention has been drawn to demand side dynamics with the emergence of biofuel which would seem to have amplified this nexus as crop land has shifted to biofuel crops reducing food supply, see Chen et al. 2010, McCalla (2009), Abbott et al. (2008), FAO (2008), Mitchell (2008), and OECD (2011) ([AHT08], [CKC10], [Mit08], [OCE11]). Despite this intuition, a majority of studies have found no evidence of causation between oil and agricultural commodity prices, e.g. see Yu et al. (2006), Zhang and Reed (2008), Kaltalioglu and Soytas (2009), Gilbert (2010), Mutuc et al. (2011), Lombardi et al. (2012), Nazlioglu and Soytas (2012) ([Gil10], [KS09], [LOS12], [MPH11], [NS12], [Y+06], [Z+08]). Nonetheless, several studies have reported unidirectional causation from oil to food commodities, see Hameed and Arshad (2008), Arshad and Hameed (2009), Cooke and Robles (2009), and more recently, Zhang et al.(2010) who find biofuel crop prices (sugar) cause oil prices ([AH09], [C+09], [HA08], [Zha+10]). To re-examine the stability of any relationship between energy and food prices, a very limited number of studies have considered the issues of nonlinearity and structural breaks. Nazlioglu (2011) in [Naz11] reconsiders linear causality using the Toda and Yamamoto (1995) nonparametric approach (see [TY95]) and nonlinear causality using the Diks and Panchenko (2006) method (see [DP06]). Results for the linear case show no evidence of causality while results for nonlinear causality show evidence of causality from oil to maize and soy commodity prices. Nazlioglu and Soytas (2012) use monthly prices ranging from January 1980 to February 2010 and a panel of twenty four agricultural products to examine panel cointegration and Granger causality ([NS12]). In a model of the world crude oil and agricultural product prices, and real effective US dollar exchange rate, they find strong evidence of impact of world oil price changes on agricultural commodity prices.

Given the results from Phillips curve studies already cited (see Hamilton (2010) for a summary), it is not surprising that functional form might constitute an important specification for study of energy price transmission to other prices. However, these past studies have not addressed the question of structural change in the relationships as noted in the macro real effects literature. Campiche et al. (2007) note the need to consider parameter variation within the context of cointegrated VAR models (see [Cam+07]), perhaps using tests introduced by Hansen and Johansen (1999) ([HJ99]). Penaranda and Micola (2009) in [FA09] use the Bai and Perron (2003) method (see [BP03]) to identify structural breaks in the relationship between oil and non-energy (grains, softs, livestock, and metals) commodity futures prices. Using daily data for 1990-June 2011, they find univariate evidence based on a linear specification of structural breaks and changes in relationships within identified intervals. Chen (2009) examined oil price pass-through to inflation in a country-level panel data set using a time-varying parameter VEC model based on one-time structural breaks using the method of Andrews (1993) and Andrews and Ploberger (1994) (see [And93], [AP94]). Avalos (2014) in [Ava14] examines evidence in US data of a discrete break following 2006 US biofuel policy implementation and rejects structural stability and finds after that break point an increased strength of transmission of oil price innovations to corn, feedback from corn to oil and soybean prices, and existence of cointegration between oil and corn prices. Baumeister and Kilian (2014) use a similar approach based on a discrete time of break (May 2006) (see [BK14]).

Methods to identify and estimate time points of change in underlying parameters have been adopted in a limited number of studies, e.g. Du et al. (2011), Qui et al. (2012) use the Bai et al. (1998) and Bai and Perron (2003) method of estimating discrete changes in parameters of multivariate representations such as VARs or VECs (see [BLS98], [BP03], [DCH11], [Qiu+12]). In the tradition of the long literature on detection of structural breaks as parametric changes, these approaches follow the early work of Chow (1960) and Quandt (1960) using F statistics to evaluate parametric metrics of change (see [Cho60] [Qua60]). However, this approach requires the underlying DGM to be stable such that observed realizations of time series are stationary. Further, it incorporates a specific alternative hypothesis that specifies a single point of shift or change. At this point of change, parameters are in the alternative specified to change. Thus, by testing the null of no change vs. a specific point-based change, the F-statistic provides a basis for inference. Despite this shortcoming, recent work has continued to explore these methods using continuously varying parameter specifications, see Enders and Holt (2012) ([EH12]). However, while smooth dynamics in parameters characterizes structural change, it is of interest to identify discrete points of change in the underlying data generating mechanisms. This type of change must be distinguished from structural breaks focused on parameter shifts. A second approach is that of generalized fluctuation tests that focus on the properties of the time series process associated with a highly generalized change hypothesis that all parameters change each time period, see Zeileis (2005) ([Zei05]). Methods such as the CUSUM method are in this second class.

This paper re-examines evidence of such a linkage by considering the transmission of energy prices into soft commodity prices. This nexus lies within the core of any real effects as softs include food-related commodities. The paper contributes to the literature by re-examining this linkage with a close eye on the role played by structural breaks within a time series and by considering the question of causality within a nonlinear framework. This builds on initial work considering inference relative to nonlinearity and structural breaks, see Koop et al. (2000). Koop et al. note that structural breaks interpreted as shifting parametric structure may well reflect underlying nonlinearity in relationships.

The paper finds that functional form is a critical specification that conditions inference. Using linear forms, we find no cointegration between energy and food in the full sample under the maintained hypothesis that there are no structural breaks. Using linear nonparametric methods, we examine the series for structural breaks and find evidence of their importance. Based on subdivisions of the sample period as suggested by the structural break examination, within the structural break intervals identified we find evidence of co-integration relying on the CUSUM method. We next reconsider the issue within the context of nonlinear functional forms posing the question of whether evidence of structural breaks based on linear methods follow from underlying nonlinearity. Our results confirm the importance of functional form specification and we find evidence of nonlinear causality between energy and soft commodity prices.

1.2 Past Literature

Past literature has examined both short- and long-run relationships among soft commodity prices and in some cases among selected soft commodities prices and crude oil price, see the recent reviews by Serra and Zilberman (2013), and Zhang et al. (2010) ([SZ13] [Zha+10]). Early work has focused on correlation, e.g. Malliaris and Urrutia (1996) who highlighted substantial agricultural future contracts of corn, wheat, oats, soybeans, soybean oil and conclude that they are correlated (see [M+96]). More recent work on co-movement continues to rely on correlation analysis. The relationship between energy prices and agricultural commodities has been addressed at a variety of levels, see Tyner and Taheripour (2008) discussed origins of linkages as does more recent work by Chavas et al. (2012) (see [CD12] [TT08]). Gilbert (2010) suggests that all agricultural markets are affected by the change in oil prices ([Gil10]); namely by increasing the production and logistics costs or by using food as an input for biofuel production. Baumeister and Kieler (2014) refute that argument noting salient characteristics of the role the energy in the food system ([BK14]). The predominant stream of empirical work has typically focused on vector error correction (VEC) models given underlying nonstationarity of price data. Campiche et al. (2007) found no conclusive evidence of a relationship between crude oil prices and corn, sorghum, sugar, soybeans, soybean oil, and palm oil prices during the 2003-2005 period whereas evidence was reported that suggested corn prices and soybean prices were cointegrated with oil prices during the 2006-2007 time period ([Cam+07]). Yu et al. (2006) results support the inference of long-run independence between major edible oil and crude oil prices (no cointegration) and conclude that shocks in crude oil prices do not have a significant influence on the variation of edible oil prices ([Y+06]). Zhang and Reed (2008) examined the impact of the crude oil price on feed grain (corn and soybeans) and pork prices in China and report no evidence to support cointegration ([Z+08]). Similar results are found by Nazlioglu and Soytas (2011) for Turkey ([Naz11]). Harri et al. (2009)

examined the relationship between agricultural commodity goods and the oil price and concluded that there is an increasing connection between corn and oil ([H+09]). They suggested this is because of the growing use of corn for ethanol and the greater use of petroleum-based inputs in both corn and cotton markets. Muruc et al. (2011) investigated the relationship between cotton and oil price and concluded that they are not cointegrated and that the response of cotton price to fluctuation in oil price is greatly different depending on whether the fluctuation is demand-driven or supply-driven ([MPH11]). Moving to the relations oil – biofuel - crops, Serra et al. (2011a) find that the prices of oil, ethanol and corn for the US to be positively correlated, and the existence of a long term equilibrium relationship between these prices, with ethanol ([SZG11]). In Brazil, using the sugar as feedstock Serra et al. (2011b) demonstrate that sugar and oil prices are exogenously determined ([Ser+11]); by focusing their attention on the response of ethanol prices to changes in these two exogenous drivers, these authors conclude that ethanol prices respond relatively quickly to sugar price changes, but more slowly to oil prices. Serra and Gil (2012) explain the price volatility of agricultural commodities affected by the energy prices, corn stocks and global economic conditions in [SG12]. Their findings support evidence of price volatility transmission between ethanol and corn markets. While the impacts of stocks in the very short-run are very high relative to the effects of energy price and macroeconomic instability, in the long-run the ethanol price and interest rate volatility are found to have the strongest impacts.

A second stream in atheoretic time series approaches has considered nonlinearity in processes. Nazlioglu (2011) by examining the relationships between oil and ag-commodities corn, soybeans, and wheat has found evidence of nonlinear causality between oil and agricultural commodity prices ([Naz11]); in another work, Nazlioglu and Soytas (2012) have found strong evidence of the world oil price changes on agricultural commodity prices using the panel cointegration and Granger causality methods (see [NS12]).

An interesting new stream has begun with work by Bastianin et al. (2013) that examined the linkages between the distributions of biofuels and commodity food prices, instead of the sample means and volatilities (see [BGM13]). They concluded that the distribution of the ethanol returns can be predicted using field crops returns, but not vice versa.

A further stream has various policy effects on transmission processes. Considering now the role of trade policy intervention, Esposti and Listorti (2013) in [EL13] consider the role of trade policy regimes on price transmission mechanisms outlined by Listorti (2008), Stigler (2011) (see [Lis08] [S+11]). They note that the 2007–2008 price bubble led the EU to adopt suspend import duties for cereals even though they were already set at very low levels due to the high world prices. The authors analyze agricultural price transmission during price bubbles, in particular, considering Italian and international weekly spot (cash) price data over the years 2006–2010. Their results suggest that the bubble had only a slight impact on the price spread and the temporary trade-policy measures, when effective, have limited this impact.

Paralleling the time series stream has been a series of partial and general equilibrium simulation studies using various calibration approaches. Gohin and Chantret (2010) using a CGE model examined the role of macro-economic linkages in analyzing the relationship between food and energy prices (see [GC10]). They concluded that although food and energy prices are mostly positively correlated, the correlation could be negative when the real income effect is considered.

1.3 Methodology

1.3.1 Notations

In this paper, we treated prices as stochastic variates that vary over time. Let $(\Omega, \mathbb{P})$ be the underlying probability space on which the random variables are defined. Here Ω is the set of all possible scenarios (that correspond to all possible realized values of the prices) and ω a particular such scenario corresponding to a specific realized price. $\mathbb{P}$ is a probability measure on Ω.

We denote $P_i^o(t, \omega)$ as the price of price i at time t, for $i \in \{1, ..., n\}$ and $t \in \{1, ..., T\}$. In our application, we set $n = 7$ and $T = 574$. Denote the log transformation of the prices as $P_i(t, \omega) = log(P_i^o(t, \omega))$. Note that for $i = 1, \dots, 7$, for a fixed $T_0 \in \mathrm{M}, P_i(t_0, \omega)$ is a random variable in $\omega \in \Omega$, where Ω is the underlying sample space. We label $P_i(t_0, \omega)$ as an observation in a sequence that we define as a realization ω. That is, for a given n, and for a fixed $\omega_0 \in \Omega, P_i(t, \omega_0)$ is a realization of $P_i(t, \omega)$ which we label as an underlying data generating mechanism (DGM). We assume that the realization has finite variance, i.e. $\int_\Omega P_i(t_0,\ \omega)^2 d\mathbb{P}(\omega) < \infty$. We note that a sample is a part of a realization drawn from a data generating mechanism. Later in this paper, we will abbreviate $P_i(t,\omega)$ as P_t^i because the underlying probability space is fixed.

1.3.2 Models

The functional form specification is of importance when empirically examining the price transmission. Hamilton (2003) illustrated in [Ham03] illustrated the importance of functional form specification when examining the spillover effect of oil price and suggested that a flexible nonlinear function specification could be more appropriate. In the section we discuss different specifications to model price transmission between crude oil price to soft commodity prices. In particular, we consider a family of linear multivariate models (e.g. Vector Autoregressive (VAR thereafter), Vector Error Correction Model (VECM thereafter)) as well as nonlinear models (e.g. Time-Varying Cointegration Model). In this section we focus on bivariate models since of interest is how the crude oil price transmits to another soft commodity. Conditioning on the test results in later sections, only one or some of these bivariate specifications are appropriate for the data.

1.3.2.1 Case l: Bivariate VAR(p) in the original price

this article, we use bivariate VAR to test linear and nonlinear Granger causality in comparison with a bivariate VECM. In particular, if P_t^i is stationary for $\forall i = 1, \ldots, 7$, we can construct a bivariate VAR(p) on $\begin{bmatrix} P_t^i \\ P_t^j \end{bmatrix}$ as follows:

$$\begin{bmatrix} P_t^i \\ P_t^j \end{bmatrix} = \begin{bmatrix} \mu_i \\ \mu_j \end{bmatrix} + \sum_{k=1}^{p} A_k \begin{bmatrix} P_{t-k}^i \\ P_{t-k}^j \end{bmatrix} + \begin{bmatrix} u_t^i \\ u_t^j \end{bmatrix} \tag{1.1}$$

where A_k are coefficient matrix. A more general VAR(p) model is $\vec{P}_t = \vec{\mu} + \sum_{k=1}^{p} A_k \vec{P}_{t-k} + \vec{u}_t$ where $\vec{P}_t = [P_t^1, P_t^2, ..., P_t^7]^t$.

1.3.2.2 Case 2: Bivariate VAR(p) in the first differenced prices

If ΔP_t^i is stationary for $\forall i = 1, \ldots, 7$, we can construct a bivariate VAR(p) on $\begin{bmatrix} \Delta P_t^i \\ \Delta P_t^j \end{bmatrix}$ as follows:

$$\begin{bmatrix} \Delta P_t^i \\ \Delta P_t^j \end{bmatrix} = \begin{bmatrix} \mu_i \\ \mu_j \end{bmatrix} + \sum_{k=1}^{p} B_k \begin{bmatrix} \Delta P_{t-k}^i \\ \Delta P_{t-k}^j \end{bmatrix} + \begin{bmatrix} v_t^i \\ v_t^j \end{bmatrix} \tag{1.2}$$

where B_k are coefficient matrix.

1.3.2.3 Case 3: Bivariate VECM

If P_t^i is not stationary for some $i \in \{1, \ \ldots, 7\}$ but ΔP_t^i is stationary for any $i \in \{1, \ \ldots, 7\}$, then rewrite VAR(p) as VECM model as follows

$$\begin{bmatrix} \Delta P_t^i \\ \Delta P_t^j \end{bmatrix} = \begin{bmatrix} \mu_i \\ \mu_j \end{bmatrix} + \Pi \begin{bmatrix} P_{t-1}^i \\ P_{t-1}^j \end{bmatrix} + \sum_{k=1}^{p-1} \Gamma_k \begin{bmatrix} \Delta P_{t-k}^i \\ \Delta P_{t-k}^j \end{bmatrix} + \begin{bmatrix} e_t^i \\ e_t^j \end{bmatrix} \tag{1.3}$$

where $\Pi = -(I_k - A_1 - \cdots - A_p)$ and $\Gamma_k = -(A_{i+1} + \cdots + A_p)$ for A_i's as in Case 1. Then depending on the rank of the matrix Π, we have three subcases:

- Case 3-1 rank(Π) = 0: In this case, there is no linear combination of P_t^i and P_t^j that is stationary[1]. The coefficient matrix Π of rank 0 (hence of all 0 entries) implies the second term in the (rhs) of equation (1.3) is zero. As a result, equation (1.3) suggests a first differenced VAR as in case 2.

- Case 3-2 rank(Π) = 2: A full rank Π suggests both P_t^i and P_t^j are stationary. As a result, equation (1.3) suggests a VAR on P_t^i, P_t^j as in case 1[2].

---

[1]More accurately, it says there is no linear combination of P_t^i and P_t^j that is *of the same integrating order* of their first difference (ΔP_t^i or ΔP_t^j). Since most economic variates including the price series of interest in this paper are of integrating order at most 1 (I(1)), we assume the first differences are stationary.

[2]More accurately, it says there exist two (linearly) independent linear combinations of P_t^i and P_t^j that is stationary (of the same order of their first difference), which further means any linear combination of P_t^i and P_t^j is stationary. In particular, $P_t^i = 1 \times P_t^i + 0 \times P_t^j$ is stationary. From another perspective, if Π has full rank and hence invertible, we can write:

$$\begin{bmatrix} P_{t-1}^i \\ P_{t-1}^j \end{bmatrix} = \Pi^{-1} \Big\{ \begin{bmatrix} \mu_i + e_t^i - \Delta P_t^i \\ \mu_j + e_t^j - \Delta P_t^j \end{bmatrix} + \sum_{k=1}^{p-1} \Gamma_k \begin{bmatrix} \Delta P_{t-k}^i \\ \Delta P_{t-k}^j \end{bmatrix} \Big\} \tag{1.4}$$

- Case 3-3 rank(Π) = 1: In this case, there exist a linear combination of P_t^i and P_t^j that is stationary. Such linear combination represents long-run relationship between P_t^i and P_t^j. To see this, for any Π with rank 1, we can decompose Π as $\Pi = \begin{bmatrix} \alpha_i \\ \alpha_j \end{bmatrix} [1 \;\; -\beta_{ij}]$ Then equation 1.3 can be rewritten as:

$$\begin{aligned} \begin{bmatrix} \Delta P_t^i \\ \Delta P_t^j \end{bmatrix} &= \begin{bmatrix} \mu_i \\ \mu_j \end{bmatrix} + \begin{bmatrix} \alpha_i \\ \alpha_j \end{bmatrix} [1 \;\; -\beta_{ij}] \begin{bmatrix} P_{t-1}^i \\ P_{t-1}^j \end{bmatrix} + \sum_{k=1}^{p-1} \Gamma_k \begin{bmatrix} \Delta P_{t-k}^i \\ \Delta P_{t-k}^j \end{bmatrix} + \begin{bmatrix} e_t^i \\ e_t^j \end{bmatrix} \\ &= \begin{bmatrix} \mu_i \\ \mu_j \end{bmatrix} + \begin{bmatrix} \alpha_i \\ \alpha_j \end{bmatrix} (P_{t-1}^i - \beta_{ij} P_{t-1}^j) + \sum_{k=1}^{p-1} \Gamma_k \begin{bmatrix} \Delta P_{t-k}^i \\ \Delta P_{t-k}^j \end{bmatrix} + \begin{bmatrix} e_t^i \\ e_t^j \end{bmatrix} \end{aligned} \tag{1.5}$$

  This says $(P_{t-1}^i - \beta_{ij} P_{t-1}^j)$ must be of the same integration order of their first difference. In the case when P_t^i and P_t^j are $I(1)$, $(P_{t-1}^i - \beta_{ij} P_{t-1}^j)$ is stationary. Thus the *cointegrating* vector $[1 \;\; -\beta_{ji}]$ represents the long relationship between P_t^i and P_t^j.

1.3.2.4 Case 4: Time-Varying Cointegration Model

To consider the nonlinear effect of price transmission, in a simple case, we allow the parameters in VECM (Case 3) to vary over time. Then we the so-called time-varying cointegration model.

$$\begin{bmatrix} \Delta P_t^i \\ \Delta P_t^j \end{bmatrix} = \begin{bmatrix} \mu_i \\ \mu_j \end{bmatrix} + \begin{bmatrix} \alpha_i \\ \alpha_j \end{bmatrix} [1 \;\; -\beta_{ijt}] \begin{bmatrix} P_{t-1}^i \\ P_{t-1}^j \end{bmatrix} + \sum_{k=1}^{p-1} \Gamma_k \begin{bmatrix} \Delta P_{t-k}^i \\ \Delta P_{t-k}^j \end{bmatrix} + \begin{bmatrix} e_t^i \\ e_t^j \end{bmatrix} \tag{1.6}$$

Here the change of β_{ijt} with time characterizes the change in the long run relationship between P_t^i and P_t^j.

1.3.2.5 Case 5: General Nonlinear Vector Models

If the conditions of case 1 hold but there exists nonlinearity, we can use a Bivariate nonlinear model, such as

$$\begin{bmatrix} P_t^i \\ P_t^j \end{bmatrix} = f\left(\begin{bmatrix} P_{t-1}^i \\ P_{t-1}^j \end{bmatrix}, \; \ldots, \; \begin{bmatrix} P_{t-k}^i \\ P_{t-k}^j \end{bmatrix} \right) + \begin{bmatrix} \epsilon_t^i \\ \epsilon_t^i \end{bmatrix} \tag{1.7}$$

And thus P_t^i and P_t^j are stationary since μ, e_t, and the first differenced prices are stationary.

where f is a function from $\mathbb{R}^2$ to $\mathbb{R}^2$ for fixed $t \in$ M, and $\begin{bmatrix} \epsilon_t^i \\ \epsilon_t^i \end{bmatrix}$ is stationary. In this article we do not specify the function f. But we assume it is the underlying DGM in nonlinear Granger causality testing.

1.3.3 Tests

1.3.3.1 Augmented Dickey–Fuller Test for Stationarity

We note that a time series process, P_t^i is by definition a DGM. A time series data set can be defined as a chronologically ordered set of values. A key issue that researchers must resolve is to determine whether a data set represents a sample of realization (from a single DGM). If it does, then the sample may be useful in characterizing the features of the underlying DGM. A key method we use to infer whether a sample is drawn from a single DGM is to test for stationarity. In general, an autoregressive (AR) time series process P_t^i (for $i = 1, ..., 7$) can be written as:

$$P_t^i = \alpha_1^i P_{t-1}^i + \alpha_2^i P_{t-2}^i + \cdots + \alpha_p^i P_{t-p}^i + \epsilon^i \tag{1.8}$$

where $\epsilon^i \sim WN(0, \sigma_\epsilon^2)$. If the coefficients in the AR process satisfy $|1 - \alpha_1^i z - \alpha_2^i z^2 - \cdots - \alpha_p^i z^p| \neq 0$ for $|z| \leq 1$, then we call such process P_t^i an (asymptotically) stationary process.

The stationarity test we use in this paper is augmented Dickey-Fuller (ADF) test. The ADF test usually takes form of the first difference. Rewrite equation (1) in its first difference form:

$$\Delta P_t^i = (\rho^i - 1) P_{t-1}^i + \sum_{j=2}^{p} \beta_j^i P_{t-j}^i + e^i \tag{1.9}$$

where $\rho^i = \sum_{j=1}^{p} \alpha_j^i$ and $\beta_j^i = -\sum_{k=j+1}^{p} \alpha_k^i$ (see Appendix). Let $\hat{\rho}_i$ be the least square estimator of ρ^i. The null hypothesis of ADF test is $\rho^i = 1$. Under the null hypothesis, the ADF test statistic is $\tau_{\hat{\rho}_i - 1}$ converges to standard normal distribution, i.e.

$$\tau_{\hat{\rho}_i - 1} = \frac{\hat{\rho}_i - 1}{\hat{\sigma_{\rho_i}}} \xrightarrow{d} N(0, 1). \tag{1.10}$$

1.3.3.2 Granger Causality Test

We are also interest in testing the Granger-Causality between the (log) *levels* of the prices (if they are stationary). By the definition of Granger Causality, P_t^i does not (Granger) cause P_t^j if

$$P_{t+1}^j | P_t^j, P_{t-1}^j, ..., P_{t-ly+1}^j, P_t^i, ..., P_{t-lx+1}^i \sim P_{t+1}^j | P_t^j, P_{t-1}^j, ..., P_{t-ly+1}^j \tag{1.11}$$

for any lx and ly finite lags. Intuitively, equation (3) says P_t^i does not (Granger) cause P_t^j if information about P_i^t does not help to forecast P_t^j.

Linear Case

The classical Granger-Causality test focuses on the linear specification of the DGM. In such case, there is a bivariate time series $\begin{bmatrix} P_t^i \\ P_t^j \end{bmatrix}$ which is stationary and has a vector autoregessive (VAR) representation, i.e.

$$\begin{bmatrix} P_t^i \\ P_t^j \end{bmatrix} = \begin{bmatrix} \mu^i \\ \mu^j \end{bmatrix} + \begin{bmatrix} \alpha_1^{ii} & \alpha_1^{ij} \\ \alpha_1^{ji} & \alpha_1^{jj} \end{bmatrix} \begin{bmatrix} P_{t-1}^i \\ P_{t-1}^j \end{bmatrix} + \cdots + \begin{bmatrix} \alpha_p^{ii} & \alpha_p^{ij} \\ \alpha_p^{ji} & \alpha_p^{jj} \end{bmatrix} \begin{bmatrix} P_{t-p}^i \\ P_{t-p}^j \end{bmatrix} + \begin{bmatrix} \epsilon^i \\ \epsilon^j \end{bmatrix} \tag{1.12}$$

The null hypothesis that P_t^i does not Granger cause P_t^j can be written as:

$$\boldsymbol{H_0} : \alpha_k^{ij} = 0 \text{ for all } k = 1, \ldots, p \tag{1.13}$$

The test statistic is

$$\hat{F} = \frac{(RSS_1 - RSS_2)/p}{RSS_2/(n - 4p - 2)} \tag{1.14}$$

where RSS_1 and RSS_2 are the residual sum of squares of restricted model where $\alpha_k^{ij} = 0$ for $\forall k$ and the unrestricted model respectively. Under the null hypothesis, the test statistic $\hat{F} \sim F(p, n - 4p - 2)$.

Nonlinear Case

In this section I present a Granger-causality test developed by Dicks and Panchenko (2006) in [DP06] (DP test thereafter). DP test is a nonparametric test that is robust under nonlinear specification. To test the null hypothesis P_t^i is not Granger causing

P_t^j, which is defined in equation (3), DP test states an equivalent condition to that in equation (3):

$$\frac{f(P_{t+1}^j, P_t^j, ..., P_{t-ly+1}^j, P_t^i, ..., P_{t-lx+1}^i)}{f(P_t^j, ..., P_{t-ly+1}^j, P_t^i, ..., P_{t-lx+1}^i)} = \frac{f(P_{t+1}^j, P_t^j, ..., P_{t-ly+1}^j)}{f(P_t^j, ..., P_{t-ly+1}^j)} \tag{1.15}$$

For simplicity, define

$$\mathbf{X} = [P_t^i, ..., P_{t-lx+1}^i],\ \mathbf{Y} = [P_t^j, ..., P_{t-ly+1}^j],\ Z = P_{t+1}^j \tag{1.16}$$

Then equation (7) can be written as:

$$\frac{f_{\mathbf{X},\mathbf{Y},Z}(\mathbf{x},\mathbf{y},z)}{f_{\mathbf{Y}}(\mathbf{y})} = \frac{f_{\mathbf{X},\mathbf{Y}}(\mathbf{x},\mathbf{y})}{f_{\mathbf{Y}}(\mathbf{y})} \frac{f_{\mathbf{Y},Z}(\mathbf{y},z)}{f_{\mathbf{Y}}(\mathbf{y})} \tag{1.17}$$

This implies

$$q = \mathbb{E}\Big[\big(\frac{f_{\mathbf{X},\mathbf{Y},Z}(\mathbf{x},\mathbf{y},z)}{f_{\mathbf{Y}}(\mathbf{y})} - \frac{f_{\mathbf{X},\mathbf{Y}}(\mathbf{x},\mathbf{y})}{f_{\mathbf{Y}}(\mathbf{y})} \frac{f_{\mathbf{Y},Z}(\mathbf{y},z)}{f_{\mathbf{Y}}(\mathbf{y})}\big) g(\mathbf{x},\mathbf{y},z)\Big] = 0 \tag{1.18}$$

for any weight function $g(\mathbf{x},\mathbf{y},z) \geq 0$. In particular, if we choose $g_0(\mathbf{x},\mathbf{y},z) = f_{\mathbf{Y}}^2(\mathbf{y})$, then we have:

$$q_0 = \mathrm{E}[f_{X,Y,Z}(x,y,z) f_Y(y) - f_{X,Y}(x,y) f_{Y,Z}(y,z)]. \tag{1.19}$$

Thus to test the null hypothesis that P_t^i is not Granger causing P_t^j is to test equation (3), which is equivalent to testing equation (10). DP test by testing the condition in equation (11) provides evidence for accepting or rejecting the null hypothesis: if equation (11) fails, then equation (10) must fail and so do equation (7) and (3), then one can reject the null hypothesis that there is no Grange causality. To derive empirical evidence of equation (11), a natural estimator of q_0 is:

$$T_n(\epsilon) = \frac{(2\epsilon)^{-d\mathbf{x}-2d\mathbf{y}-dz}}{n(n-1)(n-2)} \sum_i [\sum_{k\neq i} \sum_{j \neq i} (I_{ik}^{\mathbf{xy}z} I_{ij}^{\mathbf{y}} - I_{ik}^{\mathbf{xy}} I_{ij}^{\mathbf{y}z})] \tag{1.20}$$

where $I_{ij}^V = I(||V_i - V_j||_{\sup} < \epsilon)$. If let $\hat{f}_{\mathbf{w}}(w_i) = \frac{(2\epsilon^{-d\mathbf{w}})}{n-1} \sum_{j\neq i} I_{ij}^{\mathbf{w}}$, then we can rewrite

$T_n(\epsilon)$ as

$$T_n(\epsilon) = \frac{(n-1)}{n(n-2)} \sum_i [\hat{f}_{\mathbf{X},\mathbf{Y},Z}(\mathbf{x}_i, \mathbf{y}_i, z_i)\hat{f}_Y(\mathbf{y}_i) - \hat{f}_{\mathbf{X},\mathbf{Y}}(\mathbf{x}_i, \mathbf{y}_i)\hat{f}_{\mathbf{Y},Z}(\mathbf{y}_i, z_i)] \tag{1.21}$$

Under the null hypothesis that the P_t^i is not Granger causing P_t^j, the DP test statistic is:

$$\sqrt{n}\frac{(Tn\sqrt{\epsilon_n} - q)}{Sn} \xrightarrow{d} N(0,1) \tag{1.22}$$

where $\epsilon_n = Cn^{-\beta}$ a sequence of bandwidths with $C > 0$ and $\beta \in (\frac{1}{4}, \frac{1}{3})$, and S_n^2 is a robust estimator of $Var(T_n(\epsilon))$. Dicks and Panchenko (2006) suggest that the optimal bandwidth is $\epsilon_n^* = C^* n^{\frac{-2}{7}}$, where $C^* = \left(\frac{18 \cdot 3q_2}{4(E[s(W)])^2}\right)^{1/7}$.

1.3.3.3 CUSUM Test for Structural Breaks

We next turn to the empirical identification of structural breaks in the underlying DGM. We first recall that the question of interest is not whether there exists dynamic transition in the parametric representation of the DGM as is considered in tests proposed by Bai and Perron (1998). That type of test presents information about systematic parametric structural change in a stable dynamic relationship (Hansen, 2001) ([Han01]). This approach may be useful for testing whether change occurred at a specific point in history. Here, we attempt to examine empirical evidence of change in the underlying DGM, i.e. structural breaks using a fluctuation-type approach.

If there is a long-run relationship between P_t^i and P_t^j, then we can represent as an approximation of this relationship in linear form:

$$P_t^i = \beta_{ij} P_t^j + v_t^{ij} \tag{1.23}$$

If there are parametric structural changes, then the parameters β_{ij}'s vary over time. In that case, we can write $\beta_{ij} = \beta_{ijt}$ and equation (16) becomes to

$$P_t^i = \beta_{ijt} P_t^j + v_t^{ij} \tag{1.24}$$

Structural changes in this equation means that β's change over time. Hence to test the null hypothesis that there are structural breaks is to test whether β_{ijt} indeed changes over time. A structural break test we use in this paper is the CUSUM test (Brown, Durbin & Evans, 1975; Xiao & Phillips, 2002) ([BDE75] [XP02]). The CUSUM test statistics is:

$$T_{CUSUM} = \max_{K \leq r \leq T} \left| \frac{\sum_{t=K+1}^{r} \widetilde{v_t^{ij}}}{\hat{\sigma}\sqrt{T-K}} \right| / (1 + 2\frac{r-K}{T-K}) \tag{1.25}$$

where $\widetilde{v_t^{ij}}$'s are recursive residuals defined as $\widetilde{v_t^{ij}} = P_t^i - \hat{\beta_{ijt}} P_t^j / f_t^j$, in which $\hat{\sigma}^2$ is a consistent estimator of the variance of v_t^{ij}; $\hat{\beta_{ijt}}$ is the least square estimator of β_{ijt} in equation (17) and f_t^j is define as follows:

$$f_t^j = \left\{ 1 + (P_t^j)^2 [P_1^i, ..., P_{t-1}^i] \begin{bmatrix} P_1^j \\ \vdots \\ P_{t-1}^j \end{bmatrix} \right\}^{1/2} \tag{1.26}$$

Under the null hypothesis that there are no structural changes,

$$T_{CUSUM} \xrightarrow{d} \sup_{0 \leq r \leq 1} \{ | \frac{W(r)}{1+2r} | \} \tag{1.27}$$

where W is a Brownian bridge. We reject the null hypothesis when T_{CUSUM} is large.

To identify structural breaks, we use an iterative algorithm as follows. We test for structural breaks over the full sample using OLS-CUSUM, MOSUM, supF, and expF tests. Conditional on test results that support the presence of structural breaks, we identify structural breaks based on a Schwarz criterion (BIC) and RSS with maximum of 5 structural breaks followed by re-application of the approach within identified subsamples to identify the presence of further structural breaks. After the subsamples are identified, cointegration tests are conducted within subsamples and conditional on those results estimate appropriate models as indicated in the previous section.

1.4 Data

We employ weekly data from January 2000 through December 2010 for a corn, wheat, soybeans a subset of softs" commodity prices for Italian and US, and European Brent blend. The Italian prices of maize and wheat are obtained from DATIMA provided by ISMEA. The US prices of wheat, corn and soybeans are provided by FAO sourced from USDA. Data are weekly prices monitored for a length of time that started in February 2005 and ended in February 2010. The prices in \$/ton are converted in €/ton using the official\$/ϵi exchange rate. Missing values are replaced by using an imputation algorithm and the corresponding R-package AMELIA II (King et al., 2001; Honaker et al., 2011) ([H+11] [Kin+01]. For the fuel prices, the weekly United States spot prices and weekly Europe (UK) Brent blend spot price are converted to €per barrel.

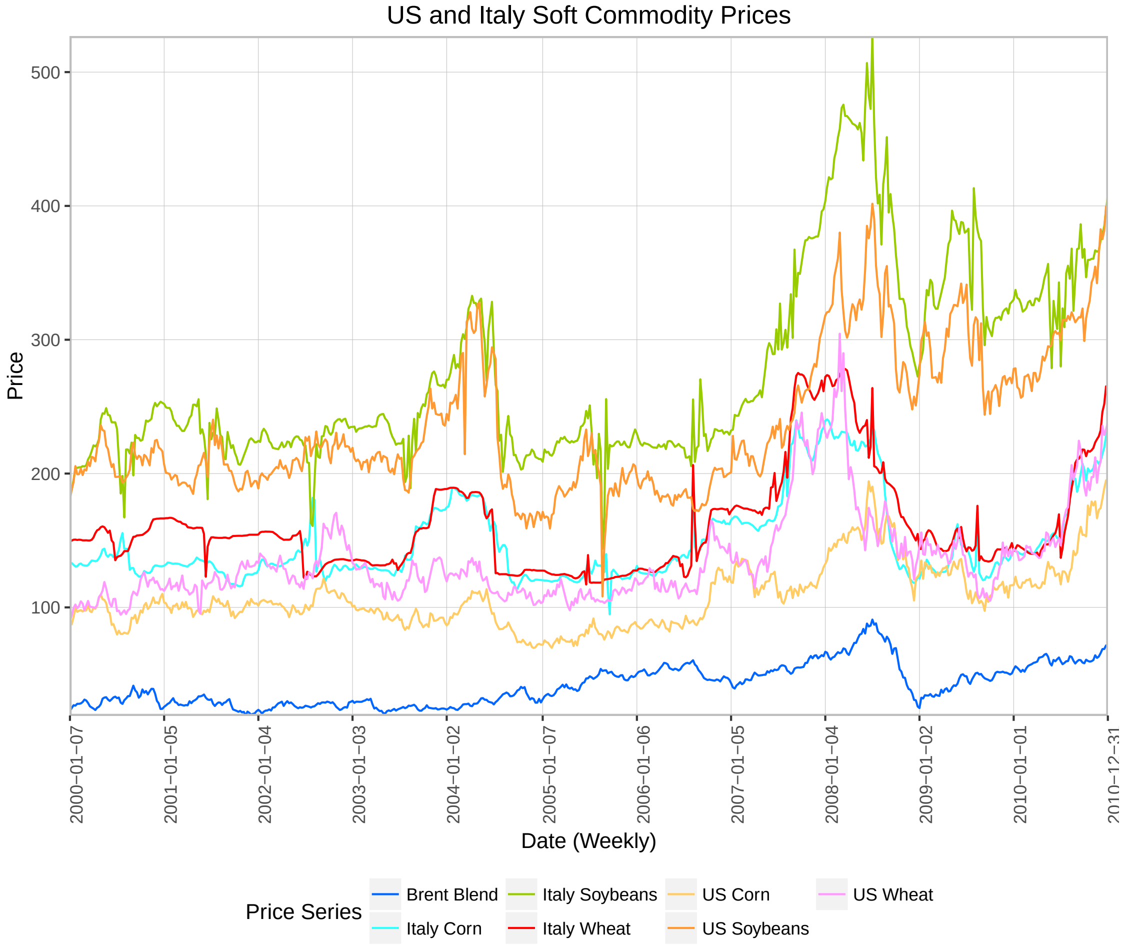
We analyze the natural logarithms of these prices. Table 1 shows the high bivariate correlations across these softs as well as between EU and US prices for the same commodity. In contrast, the correlation between softs and the Brent blend price is less striking. have some positive correlation, indicating there maybe cointegration between them. The causal direction, if we can construct a stable system, is not clear. This motivates our following causality analysis. Figure 1 presents graphic evidence of this correlation in price levels.

Table 1.1. Bivariate Correlation (Weekly Data January 2000 to December 2010)

	IT corn	IT wheat	IT soybean	US corn	US wheat	US soybean
IT corn	1.000					
IT wheat	0.910	1.000				
IT soybean	0.720	0.731	1.000			
US corn	0.650	0.679	0.804	1.000		
US wheat	0.788	0.835	0.726	0.721	1.000	
US soybean	0.664	0.630	0.911	0.832	0.666	1.000
Brent Blend	0.572	0.501	0.692	0.609	0.566	0.556

Note: This table reports correlation between each pair of prices (weekly price levels for the period January 2000 to December 2010). Since the correlation matrix is systematic, only the lower triangle table is reported.

Figure 1.1. US and Italy Soft Commodities and Energy Weekly Prices



Note: Figure 1 plots US and Italy soft commodities and Brent blend prices (weekly price levels for the period January 2000 to December 2010)

1.5 Empirical Results

1.5.1 Stationarity and Cointegration Results

To examine the characteristics of any underlying univariate data generating mechanisms (DGM) using Augmented Dickey-Fuller test (ADF test) to test the stationarity of both original and the first difference of the data, see Table 2.

Table 1.2. Augmented Dickey Fuller unit root test

	Level	First Difference
IT corn	-2.371643	-7.876299^{***}
IT wheat	-1.723259	-8.777176^{***}
IT soybean	-2.45791	-10.147263^{***}
US corn	-1.87636	-10.346361^{***}
US wheat	-2.627275	-8.847174^{***}
US soybean	-2.493837	-10.511357^{***}
Brent Blend	-2.775068	-9.743351^{***}

Note: This table reports the t-statistics of the Augmented Dickey Fuller test (H_0: the series is not stationary.) of weekly price levels for the period January 2000 to December 2010. The critical values are-3.96, -3.41 and-3.12 for 1%,5% and 10% respectively. ***, ** and * denote statistical significance at 1%,5% and 10% level of significance, respectively.

The results indicate that no price series is stationary in level while rejection of the hypothesis of unit roots in the first differences support the inference that these series are stationary implying each series is integrated by order (1) and hence may share a common stochastic trend. If a linear system of all level variables can be constructed such that it is stationary, then a vector error-correlation model (VECM) can be used to represent the underlying joint DGM. Accordingly, we next examine cointegration of pairs of the differences series using the Johansen trace test (Johansen, 1998) is employed. For each bivariate pair, the null hypothesis is that the series are cointegrated to order r where $r \leq 0$. We note that for each cointegrating regression estimated to fill Table 3, we retained the estimated rank as it is informative with respect to what type of further modeling might be appropriate. For example, for those cases where the estimated rank is close to 1, a VEC representation would be appropriate, while for an estimated rank of 2 a VAR would be appropriate. When the null hypothesis is rejected, we conclude that the bivariate pair is cointegrated and infer the existence of a long run relationship between them. Results reported in Table 3 indicate that: 1) all pairs of Italian agricultural commodities prices are cointegrated; 2) US wheat and US soybean are cointegrated with most other commodities; 3) the Brent Blend price is not cointegrated with any the softs prices. The first and second of these conclusions are consistent with the observation of high correlation noted above and the fact that agricultural markets are highly integrated by arbitrage. However, our finding of no evidence for cointegration between the Brent blend and commodities prices for the full sample does not agree with the positive correlation between many of them. Although similar results have been reported (see e.g. Yu et al., 2006; Zhang and Reed, 2008; Kaltalioglu and Soytas, 2009; Gilbert, 2010; Lombardi et al., 2010; Mutuc et al., 2010), we note this finding is also consistent with the existence of structural breaks or nonlinearity.

Table 1.3. Johansen Trace Test for Bivariate Cointegration of the Full Sample

	IT corn	IT wheat	IT soybean	US corn	US wheat	US soybean
IT corn						
IT wheat	35.247***					
IT soybean	36.241***	33.658***				
US corn	21.956	17.328	22.946*			
US wheat	30.500***	33.571***	35.222***	23.352*		
US soybean	27.011**	23.624*	35.454***	20.559	24.258*	
Brent Blend	18.453	13.182	16.882	13.374	16.139	16.007

Note: This table reports the test statistics of Johansen bivariate cointegration test (H_0: there is no cointegration) for each pair of prices (weekly price levels for the period January 2000 to December 2010). The critical values are 30.45, 25.32 and 22.76 for 1%, 5% and 10% respectively. $***$, $**$ and $*$ denote statistical significance at 1%, 5% and 10% level of significance respectively.

1.5.2 Structural Breaks

We next examine these possibilities using cointegration tests with structural breaks. Evidence reported above of no cointegration between Brent blend and agricultural commodities in the full sample does not reject the hypothesis that structural breaks exist that define time intervals within which cointegration exists. To identify such intervals we implement a nonparametric approach to examine linear relationships for each bivariate pair within identified structural breaks. Based on these results, we partition the full sample into subsamples according to these structural breaks and examine cointegration within each subsample.

Structural break dates identification and test results of bivariate cointegration between Brent blend and commodity within subsamples are reported in Table 4 and Table 5 based on the CUSUM approach. By allowing for structural breaks, we find stronger evidence of cointegration within subsamples. In contrast of no cointegration between Brent blend and commodities prices in the whole sample, evidence supports the inference that the Brent blend price is cointegrated with Italian soybeans price from week 1 2000 to week 36 2003, Italian wheat prices from week 54 2004 to week

33 2006 and US wheat price from week 8 2008 at significant levels of 1%, 5% and 10% respectively.

Table 1.4. Dates of Identified Structural Breaks

	IT corn	IT wheat	IT soybean	US corn	US wheat	US soybean
Brent Blend		2004w53	2003w36	2007w34	2008w8	
		2006w33				

Note: This table reports the identified dates of structural breaks based on CUSUM test (H_0: there is no structural break in the bivariate pair) for each pair of prices (weekly price levels for the period January 2000 to December 2010).

Table 1.5. Cointegration test results within subsamples (Null: No cointegration)

	IT corn	IT wheat	IT soybean	US corn	US wheat	US soybean
Brent Blend	18.453	14.45	35.60**	12.15	10.35	16.007
		26.53**	10.66	6.46	24.10*	
		14.73				

Note:This table reports the test statistics of Johansen bivariate cointegration test (H_0: there is no cointegration) for each pair of prices (weekly price levels for the period January 2000 to December 2010) in the subsample. Subsample results are stacked chronologically, corresponding to periods noted in Table 4. The critical values are 30.45, 25.32 and 22.76 for 1%, 5% and 10% respectively. $***, **$ and $*$ denote statistical significance at 1%, 5% and 10% level of significance respectively.

1.5.3 Linear and Nonlinear Granger Causality Test Results

We next examine evidence of nonlinearity in relationships between each bivariate pair of commodities and Brent blend prices given that linear causality tests, such as the Granger test (1969) (see [Gra69]), can fail to reveal nonlinear relationships (see e.g. Baek and Brock, 1992 ([BB92]); Hiemstra and Jones, 1994 ([HJ94])). Based on Baek and Brock (1992), we implement a nonparametric approach based on the correlation integral to detect nonlinear causal relations between the time series. Before moving to the nonlinear Granger causality test, a linear Granger causality test based on a VAR of first differenced series is conducted for comparison. It is expected the results should be with consistent with the cointegration test results. Linear Granger causality test results are reported in Tables 6. We base our linear Granger consideration on VARs across first differenced data as that is consistent with our estimates of cointegration ranks. We exploit our results with respect to structural breaks and report results for within subsample, linear Granger causality. In table 6, we again find absence of relationship across the full sample. This is consistent with our finding of an absence of cointegration in the full sample. Also, consistent with cointegration results from Table 5, there is some evidence of linear Granger causality within the subsamples. Note that only causality from Brent to grains is reported.

Table 1.6. Linear Granger causality tests on the first differenced VAR (Full Sample)

	IT corn	IT wheat	IT soybean	US corn	US wheat	US soybean
Brent Blend	0.544	0.686	0.402	0.114	0.017**	0.315

Note: This table reports the p-values (probability of first type error) of the linear Granger causality test (H_0: there is no Granger causality of Brent Blend to grain) for each pair of prices (weekly price levels for the period January 2000 to December 2010). The critical values are 30.45, 25.32 and 22.76 for 1%, 5% and 10% respectively. $***$, $**$ and $*$ denote statistical significance at 1%, 5% and 10% level of significance respectively.

Table 1.7. Granger causality tests on the first-differenced VAR with subsamples

	IT corn	IT wheat	IT soybean	US corn	US wheat	US soybean
Brent Blend	0.544	0.599	0.068*	0.270	0.369	0.315
		0.397	0.890	0.211	0.733	
		0.099*				

Note: This table reports the p-values (probability of first type error) of the linear Granger causality test (H_0: there is no Granger causality of Brent Blend to grain) for each pair of prices (weekly price levels for the period January 2000 to December 2010) in the subsample. Subsample results are stacked chronologically, corresponding to periods noted in Table 4. The critical values are 30.45, 25.32 and 22.76 for 1%, 5% and 10% respectively. $***$, $**$ and $*$ denote statistical significance at 1%, 5% and 10% level of significance respectively.

For the full sample, we also find evidence supporting the inference that Brent blend price Granger causes US wheat prices, however, for the subsamples no such evidence is apparent. These inferences are not consistent with the cointegration test results in the full sample, though cointegration found in the subsample may account this conclusion.

Next, we turn to nonlinear Granger causality test results presented in Table 8. The results suggest that causal relationships are more extensive than those suggested by linear causality results. Our logic for considering the full sample nonlinear

Granger results is to allow consideration of the relative roles of nonlinearity vs. structural breaks. Evidence supports the inference that the Brent blend price causes the price of Italian corn, US corn and US soybean and that each US commodity price causes the price of Brent blend. These results are dramatically different results than those found with cointegration and linear causality tests. In particular, results suggest causality in both direction exists between the Brent Blend and US corn and US soybean prices. Further, the results indicate spatial dimensions of causal relations exist across US and Italian prices, and within region across products.

Table 1.8. Nonlinear Granger causality tests

	IT corn	IT wheat	IT soyb	US corn	US wheat	US soyb	Brent
IT corn		0.001***	0.004***	0.006***	0.041**	0.023**	0291
IT wheat	0.294		0.002***	0.004***	0.297	0.005***	0.402
IT soybean	0.017**	0.007***		0.005***	0.007***	0.011**	0.604
US corn	0.041**	0.318	0.081		0.008***	0.005**	0.003***
US wheat	0.482	0.044**	0.500	0.042**		0.129	0.033**
US soybean	0.055*	0.020**	0.066**	0.003***	0.172		0.032**
Brent Blend	0.061*	0.173	0.384	0.001***	0.118	0.010***	

Note: This table reports the p-values (probability of first type error) of the linear Granger causality test (H_0: there is no Granger causality of Column variable to row variable) for each pair of prices (weekly price levels for the period January 2000 to December 2010). The critical values are 30.45, 25.32 and 22.76 for 1%, 5% and 10% respectively. $***$, $**$ and $*$ denote statistical significance at 1%, 5% and 10% level of significance respectively.

1.6 Conclusions

By comparison to existing results, we reconsider the role of structural breaks and find that by identifying break points, we are enabled to consider short- and long-run properties within identified intervals of stability. We find clear evidence of structural breaks in the DGMs that underlie the sample as evidenced by evidence of cointegration within subsamples. To place our results in context, Nazlioglu found that allowing for endogenous (but common across series) structural breaks led to evidence of cointegration between corn & oil prices, and some evidence for wheat & oil prices. Nazlioglu found no evidence for linear Granger causality using residuals from the cointegration models, however, by relaxing linear functional form he found evidence of nonlinear feedbacks from oil to corn and soy prices, and two-way causality between oil and wheat prices. Our results suggest that Nazlioglu's results may be based on a sample period during which structural breaks exist. Our results also find no evidence of cointegration in the full (albeit arbitrarily defined) sample. By allowing for structural breaks, we find the oil price is cointegrated with several agricultural commodity prices. We find similar results with respect to linear Granger causality, we find no evidence of linear causality when considering the full sample, however, within subsamples identified by break points we find some, though limited evidence of causality from Brent to IT wheat and soy. We find no linear causal effects on US grains. Finally, our finding of more extensive evidence of causation based on nonlinear Granger tests must be interpreted as evidence of the crucial role that functional form plays in understanding causal structure using sample data.

This paper presents the basis for two solid conclusions. As we always find, specification decisions affect the empirical bases for inference. In this study we see strong evidence that sample specification is crucial. While methods for identification of parametric structural change have long been available, they seem less useful when attempting to identify unknown points in time at which the underlying DGM might change. We find that the CUSUM approach offers a useful approach in this regard. Second, the work illustrates the crucial role functional form plays. In this case, absent priors on linearity would seem to support use of nonlinear approaches to examining Granger causality. Our results show that inference will differ across linear and nonlinear methods.

Chapter 2 Macroeconomics and the Nexus between Energy and Agricultural Commodities Prices

2.1 Introduction

Over the past decades, the nexus of food and energy systems has received periodic though persistent attention as unanticipated events have resulted in short-lived jumps or persistent shifts in levels, trends, or volatility.The prices of agricultural commodities have increased dramatically from 2005 to 2008 (see [Gil10] [HF08]). The prices of a wide range of agricultural commodities more than doubled in this period ([Gil10]). The 2005-2008 food crisis has received large attention and a wide of range of explanation including demand growth, monetary expansion, exchange rate and energy price transmission. Yet the causation of these prices increases remains controversial. Several streams of literature has considered food price dynamics.More recently, energy prices have plummeted joined by substantial decreases in most commodity prices. Piesse and Thirtle (2009) argues that low stock to utilization ratio is “the key variable” to explain food price increases (see [PT09]). Trostle (2008) confirms Thirtle’s argument as well as reviews other factors on agricultural commodities demand and supply (see [Tro+08]). Another stream has argued energy price shocks are transmitted into the real economy and have induced response in other commodity prices including food prices. Mitchell (2008) notes that food production costs increase due to higher energy prices (see [Mit08]). He also concludes that the large increase in biofuel production has reduced field crop production directed toward food. Headey and Fan (2008) conclude that high oil prices and the use of biofuel are crucial factors.Other literature has considered the causal effects of macroeconomy performance and policy on commodity prices such as food and energy (see [HF08]). Gilbert (2010) confirms the effect of GDP growth on the rising demand of food commodities ([Gil10]). Trostle (2008) also mention that rapid global economic growth continues to put upward pressure on food commodity prices through increases in food demand (see [Tro+08]). Piesse and Thirtle (2009) argues that policy changes to reduce supply and stocks (decreasing subsidies, cutting agricultural R&D investments)and to increase demand (promoting use of biofuels in US and EU) are factors resulting the price increase ([PT09]). Frankel (2006) examines the effect of monetary policy on real commodity prices and concludes that low real interest rates lead to high real commodity prices (see [Fra06]).

On the one hand, the importance of the energy price shocks to the real economy has led them to be labeled as energy crises, as they were argued to have resulted in substantial changes in real effects on productivity,output, employment, and prices that induced changes in behavior on the demand and supply sides of the many markets. Thus, these arguments focus on the causal role of energy price shocks on the macro economy. Indeed, Hamilton's (1983) suggestion that oil price shocks were the predominant cause of recessions in the post-War period continues to be examined and debated ([Ham83]). For several decades, the sheer magnitude of oil price shocks has understandably peaked interest in establishing their real effects. Most notable among these crises were those in 1973 following an OPEC embargo, those following political and social disruptions and war include those 1979 following the Iranian revolution, the global oil excess supply that followed as energy demand responded to high prices, the 1990 oil price shock coincident with the Iraqi invasion of Kuwait, a decade long crisis at the start of new millennium associated with a series of unanticipated events that led to a nearly five-fold increase in oil prices. Most recent experience include the rapid decline in crude oil prices as alternative sources of petroleum fuels were developed since the start of the new millennium. However, these crude oil price changes were accompanied by a rapid decrease in Asian economic growth that resulted in a general decrease in demand for energy and other commodities.

To explain changes in food prices that may have been induced by energy prices, just looking at their empirical evidence of price transmission may not be adequate. There are a wide range of factors (including macroeconomic factors) that could affect food and energy prices. In the food price literature, Serra and Zilberman (2013) pointed out that macroeconomic conditions are very important because of their impact on agricultural commodity price levels and volatility (see [SZ13]). Headey and Fan (2008) conclude that export restrictions, depreciation of USD, weather shocks and low interest rates are important factors of food prices showing evidence of causality as well as oil prices and biofuels ([HF08]). Gilbert (2010) investigates exchange rates, monetary factors, future markets and oil prices as potential causes of food prices ([Gil10]). He concludes that world GDP growth, monetary expansion and exchanges rate are significant determinants of changes in world agricultural prices over a 38-year period. On the energy prices side, Hamilton (2008) argues that exchange rate, real income and GDP significantly affect crude oil prices ([Ham08]). Anzuiniet al. (2012) estimate a VAR to explore the the effects of US monetary policy on commodity prices and find a strong positive relationship ([ALP12]). Roache (2010) finds evidence of transmission from US inflation and the US dollar exchange rate into food commodity prices ([Roa10]). Doenmez and Magrini (2013) estimate the link between macroeconomic variables and commodity price volatility and find inclusion of macroeconomic variables improves explanation of price volatility (see [DM13]). Within the context of the nexus between energy and food prices, the necessity of considering the role of macroeconomic indicators has also been noted by Harri et al. (2009),Cooke (2009), Balcombe (2011), Gohin (2010), and Wright (2011) (see [Bal09], [C+09], [GC10], [H+09], [Wri11]).

Despite these past studies, the question must be raised whether the nexus between energy and food commodity prices can be explored independent of consideration of macroeconomic factors. This paper focuses on the cointegration of energy and food prices and within this context takes on this question by reconsidering the weak exogeneity of macroeconomic indicators. The paper is organized as follows. In the next section, we briefly summarize key literature relevant to our research question. We highlight three specification issues that have varied substantially across the literature: 1) inclusion of variables, 2) nonlinearity in functional form, and 3) frequency of data employed. We next present our approach, followed by our results and conclusions.

2.2 Past Literature

Hamilton (2008) summarized previous studies of the role of macroeconomic factors and oil prices and concludes that income and price elasticity of the demand of petroleum are both below unity at least in developed, industrialized countries (see [Ham08]). Such price inelasticity suggests oil price changes can be readily transmitted into real economy including commodity prices. As oil prices increase, if consumption does not respond, the cost effects will be transmitted throughout the vertical chains in which petroleum products are critical inputs. Estimates of the income inelasticity of petroleum demand suggest insensitivity of oil prices to macroeconomic. Hamilton further notes that U.S. data suggests that as GDP per capita increases, the income elasticity declines. Nontheless, Hamilton notes that a large range of macro-economic factors and conditions could affect crude oil price including commodity price speculation, strong world demand, time delays or geological limitation on increasing production, OPEC monopoly pricing and scarcity rent. However, he concludes the three most important factors to explain the high price of oil in 2008 are low price elasticity of demand in industrialized countries, the strong growth of demand for oil in newly industrialized economies where income elasticity of demand is positive,and an absence of global supply response to price increases.Gohin and Chantret (2010) employ a world Computable General Equilibrium (CGE) model to investigate macro-economic linkages in the long-run impart of energy prices on world agricultural markets ([GC10]). They conclude that the introduction of the real income may imply a negative relationship between world food and energy prices. They also conclude that both supply and demand factors are important in the long-run evolution of energy prices. As for the effect of economic growth on food commodity prices, Gilbert (2010) presents evidence of a strong effect of GDP growth on the rising demand of food commodities (see [Gil10]). Trostle (2010) also mention that rapid global economic growth continues to put upward pressure on food commodity prices through increases in food demand (see [Tro+08]).

Many studies have investigated the role of exchange rate dynamics and oil prices.Chen and Chen (2007) investigate the long-run relationship between real oil prices and real exchange rates using panel data (see [CC07]). They test the cointegration of real exchange rates and oil prices using monthly data on the G7 countries from 1972:1 to 2005:10. They show that the real oil prices may have been the dominant cause of real exchange rate movements during that period. They also conclude real oil prices have significant forecasting power for exchange rates. In contrast, Ferraro et al. (2012) investigate whether oil prices have a reliable and stable out-of-sample relationship with the Canadian/U.S dollar nominal exchange rate (see [FRR12]). They find little systematic relation between oil prices and this exchange rate at the monthly and quarterly frequencies. However, they find the existence of such relationship at daily frequency. Beckman and Czudaj(2012) employ a multivariate Markov-Switching vector error correction model (MS-VECM) and test for bidirectional causality between oil prices and exchanges rates (see [BC13]). They contribute to literature in two aspects. First, they investigate the relation between oil prices and nominal exchange rates. Secondly, they are able to discriminate between long-run and short-run dynamics by using MS-VECM. They conclude different and also time-varying causalities results across different countries. In general, their results support the existence of a bidirectional dynamic relation between oil price and exchange rates.

On the other hand, only a very limited literature has investigated the role of exchange rates in the energy-food price transmission. Baek and Koo (2009) investigated the short-and long-run effects of market factors (prices of energy, agricultural commodities and exchange rate) on U.S. food commodity prices using a cointegration analysis (see [BK10]). They conclude that agricultural commodity prices play a key role in affecting the short-and long-run behavior of U.S. food prices. They also conclude that energy prices and exchange rate have been significant factor influencing U.S. food prices in recent years in both the short-and long-run. Harri et al. (2009) investigate the relationship between oil price, exchange rates and commodity prices including agricultural commodities (see [H+09]). Using time series VECM model, they conclude that oil price, corn price and exchange rates are interrelated. They also identified long-run relationships (cointegrations) between oil prices and all agricultural prices except wheat. Nazlioglu and Soytas (2011) use panel cointegration method to model exchange rate, energy and commodity prices (see [NS12]). They conclude that considering the exchange rate there is strong evidence of transmission from world oil prices to several agricultural commodity prices. They also conclude that the exchange rate has an impact on agricultural prices. Serra and Zilberman (2013) noted that a spectrum of macroeconomic conditions may impact on agricultural commodity price levels and volatility (see [SZ13]). The need to consider multiple macroeconomic indicators when investigating the nexus energy-food price levels and volatility transmission was also highlighted by Harri et al. (2009), Cooke (2009), Balcombe (2011), Nazilouglu (2011) and Wright (2011) (see [Bal09] [C+09] [H+09] [NS12] [Wri11]). Summarizing, evidence of roles of multiple macroeconomic indicators in affecting the nexus between energy and food prices has been reported including real and nominal exchange rates (Chen and Chen, 2007; Baek and Koo, 2009; Harri et al. 2009; Hamilton, 2009; Beckman and Czudaj, 2012), real income (Gohin and Chantret, 2009; Hamilton, 2009) and GDP (Hamilton, 2009).

Another stream of literature has considered existence of a direct effect of exchange rates on agricultural commodity prices. Almost all international commodities are traded in US dollars. It follows that dollar depreciation results in an increase in commodities prices dominated in US dollars. Over 2005-2008, the dollar depreciated by around 25% against the euro but much less against other currencies. The likely dollar depreciation effect was therefore small but should be evident (Gilbert, 2008). Rilder and Yandle (1972) investigate the impact of exchange rates on commodity prices and develop a model: $dlogP = -(1 - v_1)\theta$ where θ is dollar depreciation rate and v_1 is the price elasticity of the agricultural commodity (see [RY72]). Thus,they conclude that dollar price rise in proportion to the dollar depreciation by one minus elasticity. Gilbert later (1989) shows this result to be robust in general cases and concludes dollar depreciation increases commodity prices with an elasticity of between 0.5 and 1.0 (see [Gil89]). Mitchell (2008)suggested that the depreciation of the dollar has increased food prices by around 20%, assuming an elasticity of 0.75 (see [Mit08]). Abbott et al. (2008) also show that in the current crisis the divergence between the dollar and many other currencies is quite stark compared to previous increases in nominal dollar denominated food price increases (see [AHT08]). He also shows that agricultural commodity exports (particular grain and oil seeds) increase when dollar depreciates.

Monetary explanations of changes in price levels and relative prices have received wide support in 1970-1980s (Gilbert, 2008). Bordo (1980) consider the impact of monetary growth on agricultural prices and concludes that monetary expansion could raise agricultural prices relative to a more general price deflator (see [Bor80]). Chambers and Just(1982) confirm Bordo's finding. However, Gilbert (2008) notes that 2005-2008 boom in agricultural prices took place contemporaneous with booms in other commodities prices as well as with equity and real estate price booms (see [Gil10]). He concludes that monetary growth may spill over into asset prices as well as commodity prices. Browne and Cronin (2010) also investigate the relationship between commodity prices, money and inflation (see [BC10]). Using a cointegating VAR approach and US data, they conclude that both commodity and consumer prices rise proportionally to the money supply growth in the long run.

Literature has also considered the possible effects of both nominal and real interest rates on commodity prices. Some literature argues that low real interest rates have an effect on the commodities prices. Frankel (2006) argues that low real interest rate caused a general price increase in a wide range of commodities during the period of 1950-2005 (see [Fra06]). He indicates that low interest rates reduce storage cost thereby increasing the demand for storable commodities, and encourage speculators to shift out of treasury bills and into commodity contracts. Each results in an increase in commodities prices. Frankel (2006) also investigates such relationships and concludes that low real interest rates lead to high real commodity prices. However, Headey and Fan (2008) argue that low interest rates may be a less convincing factor than the depreciation of USD (see [HF08]). Akram (2009) investigate the recent (2005-2008) fluctuations in commodity prices based on structural VAR models estimated on quarterly data over 1990q1-2007q4 (see [Akr09]). They show that commodity prices increase significantly in response to reduction in real interest rates.

With respect to methods, three issues have been addressed with respect to examination of the nexus between energy and food prices. First, a multivariate approach would seem to be motivated by evidence of a relationship between macroeconomic indicator sand the agricultural commodities and energy price nexus. However, considerable specification uncertainty exists with respect which macro indicators should be considered. Second, the functional form of any relationships is not resolved by theory and introduces further specification uncertainty. Given the results from Phillips curve studies (see Hamilton (2010) for a summary), it is not surprising that functional form might constitute an important specification for study of energy price transmission to other prices. Our past research also shows that functional form is a critical specification that conditions inference. Using weekly agriculture commodity prices and weekly UK Brent Blend spot price from January 2000 through December 2010, we found very different causality inferences using linear and nonlinear causality tests. Thus,in considering the macroeconomic conditions in the nexus of food and energy prices, one has to investigate the problem of nonlinearity (this is also noted by Beckman and Czujaj (2012) in [BC13]). In our case, this includes testing for structural breaks using a series of nonparametric tests (CUSUM, MOSUM, sup-F, and exp-F)as well as testing nonlinear causality using TY test (Toda and Yamaoto, 1995 see [TY95]) and DP test (Diks and Panchenko, 2006 see [DP06]). Third, the frequency of data used would seem a critical specification issue that will condition inference. Most past studies in this area use low frequency data (daily, weekly, monthly). This is mainly because of the relative stability of macroeconomic conditions in short run as well as the availability of data.

2.3 Methodology

Fundamentally, a key area of uncertainty in modeling transmission across prices is the specification of the determinants of the impacted price. That is, in a vertical chain, the downstream price can be related to the upstream price, however, other factors must also be considered, at least theoretically. In the current context, to explore the nexus between energy and food commodity prices would seem to demand a full specification of the determinants of food commodity prices. Economic systems often include substantial numbers of variables and motivate interest in specification and estimation of parts of such systems. Indeed, where specification of the overall system is uncertain, or where interest focuses only on a subsystem such partial systems are both of interest and may allow explicit consideration of the scope of an uncertain system. This opens the question of under what conditions estimation of a subsystem provides useful information (consistency and efficiency) concerning parts of a larger system. Johansen (1992) introduced the concept of weak exogeneity to respond to this question. Our approach is based on exploiting this concept of weak exogeneity to examine existence of conditions which would obviate the need to consider macro indicators while exploring the nexus between energy and food commodity prices.

In this paper, we treated prices and macroeconomic factors of interest as stochastic variates that vary over time. We denote these variates as a vector X_t of dimension p. Consider a full linear system representing the transmission of cointegrated prices.

We can write such a system in the form of a vector error correction model (VECM):

$$\Delta X_t = \alpha\beta' X_{t-k} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \phi D_t + \mu + \epsilon_t,\ t = 1,\ ..,\ T \tag{2.1}$$

where $X_{-k+1}, .., X_0$ are fixed, α and β are $p \times m$ matrix, and $\epsilon_1, \dots, \epsilon_t$ are independent p-dimensional Gaussian variable with mean zero and variance matrix Λ. The vector D_t denotes seasonal dummies centered at zero. The parameters in the model are short-run effects $\Gamma_1, \dots, \Gamma_{k-1}$, the seasonal coefficients ϕ, the constant term μ, the covariance matrix Λ, the $p \times r$ matrices α (the adjustment coefficients) and β (the cointegrating relations). It is often the case that the econometrician is unable to specify or measure all elements of the full system represented by the VECM. Such model uncertainty leads to estimation of what we could call a "partial system" and implicitly defines a †† residual system" that is left out of consideration. Granger's definition of weak exogeneity allows us to consider implications of estimation of the partial system when the partial system excludes variables in the residual system. Formally, let a be a $p \times m$ full rank selection matrix by which we define the partial system given the full system. Let $b = a_\perp$ be a $p \times (p - m)$ full rank matrix of vectors orthogonal to a by which we define the residual system. To investigate the weak exogeneity of the residual system $b'\Delta X_t$, relative to the partial system, we need to consider the conditional expectation of the partial system outcomes $a'\Delta X_t$ conditional on those of the residual system $b'\Delta X_t$. To derive this, by pre-multiplication of the full VECM, we have the conditional expectation $a'\Delta X_t$:

$$\begin{aligned} &\mathbb{E}(a'\Delta X_t | b'\Delta X_t,\ \Delta X_{t-1},\ \dots,\ \Delta X_{t-k}) \\ =&(a - b\Lambda_{bb}^{-1}\Lambda_{ba})'(\sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \alpha'\beta X_{t-k} + \phi D_t + \mu) + \Lambda_{ab}\Lambda_{bb}^{-1} b'\Delta X_t \end{aligned} \tag{2.2}$$

where $\Lambda_{aa} \equiv a'\Lambda a$, $\Lambda_{ab} \equiv a'\Lambda b$, $\Lambda_{ba} \equiv b'\Lambda a$,$\Lambda_{bb} \equiv b'\Lambda b$. We further denote $\Lambda_{aa} - \Lambda_{ab}\Lambda_{bb}^{-1}\Lambda_{ba}$ as $\Lambda_{aa,b}$. Then we have:

$$Var(a'\Delta X_t | b'\Delta X_t,\ \Delta X_{t-1},\ \dots,\ \Delta X_{t-k}) = \Lambda_{aa} - \Lambda_{ab}\Lambda_{bb}^{-1}\Lambda_{ba} = \Lambda_{aa,b} \tag{2.3}$$

Thus, by adding a random error, we can write such an error in terms of that of the partial and residual system parameters, i.e.

$$
\begin{aligned}
&a'\Delta X_t \\
=&(a - b\Lambda_{bb}^{-1}\Lambda_{ba})'(\sum_{i=1}^{k-1}\Gamma_i\Delta X_{t-i} + \alpha'\beta X_{t-k} + \phi D_t + \mu) + \Lambda_{ab}\Lambda_{bb}^{-1}b'\Delta X_t + u_t
\end{aligned} \tag{2.4}
$$

where $t = 1, ..., T$ and $u_\tau = (a - b\Lambda_{bb}^{-1}\Lambda_{ba})'\epsilon_t$ are independent Gaussian variables with zero mean and variance matrix $\Lambda_{aa,b}$. Granger called such model a partial VAR model or a conditional model for $a'\Delta X_t$ given $b'\Delta X_t$ and past information. Using this notation, Johansen defines weak exogeneity of $b'\Delta X_t$ for α and β as the parameters of the distribution of $a'\Delta X_t$ given $b'\Delta X_t$ and the past information $\Delta X_{t-1}, \ldots, \Delta X_{t-k+1}, \Delta X_{t-k}$ are variation independent of parameters of the distribution of $\Delta X_{t-1}, \ldots, \Delta X_{t-k+1}, \Delta X_{t-k}$ (Tohansen, 1992). In particular, note we can write $b'\Delta X_t$ as

$$
b'\Delta X_t = \sum_{i=1}^{k-1} b'\Gamma_i\Delta X_{t-i} + b'\alpha\beta' X_{t-k} + b'\phi D_t + b'\mu + b'\epsilon_t,\ t = 1,\ ..,\ T \tag{2.5}
$$

If b'a $= 0$, then $b'\Delta X_t = \sum_{i=1}^{k-1} b'\Gamma_i\Delta X_{t-i} + b'\phi D_t + b'\mu + b'\epsilon_t, t = 1, .., T$. This means $b'\Delta X_t$ does not contain information on the cointegration relations (vector β). Following the definition by Engle et.al (1983) and Johansen (1992), this indicates $b'\Delta X_t$ is weakly exogenous for the parameter vector (α, β). It has been shown that weak exogeneity of $b'\Delta X_t$ means the conditional distribution of ΔX_t given $b'\Delta X_t$ and $\Delta X_{t-1}, \ldots, \Delta X_{t-k+1}, \Delta X_{t-k}$ contains parameters α and β, whereas the (un)distribution of $b'\Delta X_t$ given $\Delta X_{t-1}, \ldots, \Delta X_{t-k+1}, \Delta X_{t-k}$ does not contain α and β Johansen and Juselius, 1992). Intuitively, this property provides a basis for a nested parameter test of weak exogeneity.

Indeed, Johansen (1992) provides a parameter restriction that is implied by the condition of weak exogeneity. He shows that if $b'\alpha = 0$, then $b'\Delta X_t$ is weakly exogenous for the parameter (α, β) . Hence the maximum likelihood estimator for (α, β) in the full model is the same as that in partial model. Equivalently, Johansen (1992) shows that the hypothesis of weak exogeneity of $Z_t = b'X_t$ for α and β can be formulated as

$$H_0 : \alpha_Z = 0$$

From above, it is clear that Z may be of smaller dimension than X. We define α_Z as composed of the rows of α corresponding to Z_t. Then, under the null hypothesis H, the maximum likelihood estimation of the parameters could be performed by reduced rank regression, and the rest of H in H_r:($\Pi = \alpha'\beta$ where a and β are $p \times r$ matrices) consists in comparing the eigenvalues $\hat{\lambda}_l$ and $\overline{\lambda_l}$, where $\hat{\lambda}_l$ is the eigenvalue without the restriction and $\overline{\lambda_l}$ is the eigenvalue with the restriction respectively. The test statistic is

$$T = \sum_{i=1}^{r} \ln\{\frac{(1-\overline{\lambda_l})}{(1-\hat{\lambda}_l)}\} \tag{2.6}$$

where p_Z is the dimension of Z_t. Under the null hypothesis, $T \rightarrow d\chi^2(rp_Z)$.

2.4 Empirical Results

We suspect macroeconomic factors may play a role in each bivariate VECM. To motivate the choice of the macroeconomic factors, we restrict our interest on those macroeconomic factors that can be directly adjusted by policies (e.g. interest rate, money supply, etc.) rather than macro performance indicators such as GDP growth or unemployment rates. We begin by consideration of money supply, exchange rate and interest rate to be our macroeconomic factors to consider.

As Table 1 shows, money supply (M2) is not weakly exogenous in every pair of commodity and energy pair. In addition, the US/Euro exchange rate is found to be not exogenous in most pairs. These results imply that the money supply and exchange rate are endogenous within the system defined as commodity and energy prices and hence cannot be omitted models considering the cointegration of energy and commodity prices. The significant role of M2 plays in the price systems is consistent with monetary theories (Bordo 1980) and evidence from the agricultural commodity markets literature, e.g. Chambers and Just (1982), Gilbert (2008). Further, the result of exchange rate being endogenous is also consistent with findings reported in previous literature (see e.g. Baek and Koo 2009, Gilbert 2008,

Table 2.1. Bivariate Pair and Macro Factors (M2, 10 year maturity rate and US/EU exchange rate) Weak Exogeneity Tests

It_corn	0.001	It_corn	0.731	It_corn	0.340	It_corn	0.000	It_corn	0.628	It_corn	0.541
It_wheat	0.039	It_soybean	0.000	US_corn	0.992	US_wheat	0.012	US_soybean	0.000	Brent_Blend	0.267
M2	0.000	M2	0.000	M2	0.000	M2	0.000	M2	0.000	M2	0.000
X10.year	0.012	X10.year	0.001	X10.year	0.145	X10.year	0.108	X10.year	0.000	X10.year	0.621
US.Euro	0.076	US.Euro	0.075	US.Euro	0.016	US.Euro	0.013	US.Euro	0.081	US.Euro	0.025
		It_wheat	0.280	It_wheat	0.062	It_wheat	0.000	It_wheat	0.184	It_wheat	0.175
		It_soybean	0.000	US_corn	0.698	US_wheat	0.014	US_soybean	0.000	Brent_Blend	0.382
		M2	0.000	M2	0.000	M2	0.000	M2	0.000	M2	0.000
		X10.year	0.026	X10.year	0.154	X10.year	0.727	X10.year	0.019	X10.year	0.884
		US.Euro	0.071	US.Euro	0.017	US.Euro	0.014	US.Euro	0.074	US.Euro	0.024
				It_soybean	0.830	It_soybean	0.002	It_soybean	0.001	It_soybean	0.292
				US_corn	0.947	US_wheat	0.510	US_soybean	0.077	Brent_Blend	0.333
				M2	0.000	M2	0.000	M2	0.000	M2	0.000
				X10.year	0.079	X10.year	0.198	X10.year	0.370	X10.year	0.248
				US.Euro	0.023	US.Euro	0.117	US.Euro	0.106	US.Euro	0.040
						US_corn	0.625	US_corn	0.862	US_corn	0.716
						US_wheat	0.387	US_soybean	0.729	Brent_Blend	0.519
						M2	0.000	M2	0.000	M2	0.000
						X10.year	0.195	X10.year	0.124	X10.year	0.206
						US.Euro	0.041	US.Euro	0.017	US.Euro	0.014
								US_wheat	0.553	US_wheat	0.651
								US_soybean	0.020	Brent_Blend	0.350
								M2	0.000	M2	0.000
								X10.year	0.133	X10.year	0.914
								US.Euro	0.221	US.Euro	0.124
										US_soybean	0.318
										Brent_Blend	0.307
										M2	0.000
										X10.year	0.342
										US.Euro	0.045

Note: Each block in the table corresponds to a linear system of interest. In a block, there are five variates that constructs a VECM of the variates. For each of these variables, we test its weak exogeneity to the system in the block. The p-value's are reported for each of the five tests.

Harri et al. 2009, Nazioglu and Soytas 2011). Given the results in Table 1, we examine robustness of these results with respect to the set of macroeconomic variables included and reconsider weak exogeneity of exchange rate and interest rate play in the price transmission in a model that excludes M2. The logic for this exclusion follows from the intuition that M2 is key policy control target that may directly impact exchange and interest rates. Thus, if we restrict our interest to exchange and interest rates, the role of M2 may already be considered by variation of exchange and interest rates. In addition, we also reconsider the maturity length for interest rates following the intuition that short-run rates may be central to inventory decisions. From this perspective, we consider the role of short-run interest rates using a 3 month maturity.

Table 2.2. Cointegration Rank of Bivariate Pair with Macro Factors

	IT corn	IT wheat	IT soyb	US corn	US wheat	US soyb
IT corn						
IT wheat	1					
IT soyb	1	1				
US corn	0	0	0			
US wheat	0	0	0	0		
US soyb	1	1	1	0	0	
Brent blend	0	0	0	0	0	0

Note: This table reports cointegration rank of the VECM system consisting each pair of prices (weekly price levels for the period January 2000 to December 2010) and marco-factors. In this table, each ij-th element represents the cointegration rank of the VECM system consisting of i-th row variable, j-th column variable, 3 month maturity rate and exchange rate. Since the matrix is systematic, only the lower triangle table is reported . The cointegration rank test is conducted in R *using function'ca.jo'. The test results are based on* 10% *significance level.*

Table 3 reports cointegration ranks of bivariate VECM consisting of two price variables, while table 2 reports cointegration ranks of multivariate VECM consisting of the exact two price variables together with two macro factors (3 month maturity rate and exchange rate). Comparing cointegration ranks reported in two tables, we find the cointegration rank is not increasing as we add in macro-factors. This impies that the maro-factors are not significant in the bivariate VECM system and hence we may exclude them from estimation of bivariate VECMs consisting price variables only. To verify this inference, we conduct weak exogeneity test in the VECM.

For those VECMs with strictly positive cointegration rank as reported in Table

Table 2.3. Cointegration Rank of each bivariate pair without Macro factors

	IT corn	IT wheat	IT soyb	US corn	US wheat	US soyb
IT corn						
IT wheat	1					
IT soyb	1	1				
US corn	0	0	0			
US wheat	1	1	1	0		
US soyb	1	0	1	0	0	
Brent blend	0	0	0	0	0	0

Note: This table reports cointegration rank of the VECM system consisting each pair of prices (weekly price levels for the period January 2000 to December 2010). In this table, each ij-th element represents the cointegration rank of the VECM system consisting of i-th row variable and j-th column variable. Since the matrix is systematic, only the lower triangle table is reported . The cointegration rank test is conducted in R *using function'ca.jo'. The test results are based on* 10% *significance level.*

3, we test weak exogeneity of each variable in the four-variable VECMs. The results are reported in Table 4. As shown in Table 4, the short-run interest rate is not (weakly) exogenous in four of the bivariate pairs, while interest rate is (weakly) exogenous in every pair. This result basically agrees with the previous conclusion that we drew from the comparison of Table 2 and Table 3, that is we can exclude macroeconomic factors in bivariate VECM for further analysis.

Table 2.4. Bivariate Pair and Macro Factors (3-month Maturity Rate and US/EU exchange rate) Weak Exogeneity Test

It_corn	0.008	It_corn	0.803	It_corn	0.966	It_corn	—	It_corn	0.622	It_corn	—
It_wheat	0.003	It_soybean	0.000	US_corn	0.000	US_wheat	—	US_soybean	0.000	Brent_Blend	—
X10.year	0.044	X10.year	0.044	X10.year	0.142	X10.year	—	X10.year	0.002	X10.year	—
US.Euro	0.211	US.Euro	0.422	US.Euro	0.002	US.Euro	—	US.Euro	0.120	US.Euro	—
		It_wheat	0.709	It_wheat	—	It_wheat	—	It_wheat	0.165	It_wheat	—
		It_soybean	0.000	US_corn	—	US_wheat	—	US_soybean	0.000	Brent_Blend	—
		X10.year	0.019	X10.year	—	X10.year	—	X10.year	0.052	X10.year	—
		US.Euro	0.714	US.Euro	—	US.Euro	—	US.Euro	0.341	US.Euro	—
				It_soybean	—	It_soybean	0.000	It_soybean	0.000	It_soybean	—
				US_corn	—	US_wheat	0.430	US_soybean	0.030	Brent_Blend	—
				X10.year	—	X10.year	0.032	X10.year	0.890	X10.year	—
				US.Euro	—	US.Euro	0.252	US.Euro	0.449	US.Euro	—
						US_corn	—	US_corn	—	US_corn	—
						US_wheat	—	US_soybean	—	Brent_Blend	—
						X10.year	—	X10.year	—	X10.year	—
						US.Euro	—	US.Euro	—	US.Euro	—
								US_wheat	—	US_wheat	—
								US_soybean	—	Brent_Blend	—
								X10.year	—	X10.year	—
								US.Euro	—	US.Euro	—
										US_soybean	—
										Brent_Blend	—
										X10.year	—
										US.Euro	—

Note: Each block in the table corresponds to a linear system of cointegration rank at least one as reported in table 2.2. In a block, there are four variates that constructs a VECM of the variates. For each of these variables, we test its weak exogeneity to the system in the block. The p-value's are reported for each of the five tests.

2.5 Concluding Remarks

This paper examines the question of whether empirical conditions exist that would suggest exclusion of key macroeconomic factors from models of price transmission will not compromise estimates based on simple models that restrict consideration of price transmission models to include prices only. The relevance of this issue is motivated by a now vast literature in both the agricultural economics literature and in the economics literature that examines price transmission using time series models that exclude other key determinants of prices. Clearly, structural modeling approaches could be used with the inclusion of all variables hypothesized by researchers as both exogenous as well as those endogenous variables that causally determine prices of interest. However, such approaches suffer ultimately from the specification uncertainty encompassing which variables to include, as well as functional form, etc. In this paper, we show that key macro economic variables such as interest rates and exchange rates appear to be weakly exogenous to crude and food commodity prices. Following Granger's work, this condition allows their exclusion from linear price transmission models without compromising desirable properties of the resulting estimators.

Chapter 3
Nexus between Energy and Commodity Prices: Jump Identification and Transmission

3.1 Introduction

While structural break, threshold and asymmetric cointegration models can allow us to characterize the linear and nonlinear dynamics in price transmission in level, it is of equal interest to differentiate across the type of price change to consider what might be thought of as typical price changes versus extreme price changes associated with either temporary structural change or mean reverting change as in what we call price jumps. In particular, while a "structural break" is a permanent and long-run structural shift in DGM, a "jump" in a series represents a sudden temporary change in the pattern of the observations generated. Such change is temporary in a sense that its effect usually diminishes rather quickly (usually in relatively few periods). That means, intuitively, in relatively short time span after a jump, the price series will revert to its "mean" or its long-run smooth pattern which we call the trend of the series. In this chapter, we present a detailed discussion of the proper representation of such price jumps and show that there are price jumps in the real-world economic price series.

Jumps in prices are usually modelled as the jump components in a jump-diffusion model of high frequency data. On the other hand, in low frequency data, the mechanism behind a price "jump" or what we will define as a price "spike" is not well-defined. A number of different structures in DGMs could cause jumps or spikes. This paper focuses on modelling of jump occurrence and jump transmission in energy and commodity prices. In the paper, we present a model to predict the probability of jumps and jump size conditioning on a set of exogenous variables.

A strand of energy and commodity price literature related to jumps (transmission) looks into volatility (transmission). Aizenman and Pinto (2005) point out that volatility in energy prices introduces risk to the economic system and high volatility leads to an overall welfare loss (see [AP05]). Serra and Zilberman (2011) in [SZG11] note such volatility may also spill over directly to soft commodities markets (energy as input in soft commodity), and indirectly through ethanol markets . Thus, the increasing volatility in energy price (and thus in soft commodities price) is a major concern for agricultural producer and agents along the food chain (Balcombe, 2009 see [Bal09]). Motivated by the need to mitigate the effects of volatility, this paper seeks to study volatility and its transmission between energy and soft commodities prices.

While volatility plays an important role in the economy, the concept of volatility is not well-defined in literature. Volatility should include two kinds of effects: systematic variance changes (VC) and nonsystematic changes that cannot be characterized by VC, like jumps and spikes. While systematic variance changes can explain particular types of oscillations or volatility, they cannot represent any long-run or short-run structural changes in the price series. Thus, to fully understand volatility, one has to consider all kinds of oscillation effects, especially jumps.

In many works of literature, the word "volatility" refers to systematic variance changes. In particular, price volatility is usually characterized as the volatility term in continuous time models (for example, high-frequency stock prices as geometric Brownian motion in a Black-Scholes model), or heteroskedasticity error structure in discrete time models (for example, low-frequency energy prices as in time series models). In particular, to characterize "volatility" (variance change effect) transmission in energy and commodity prices (low-frequency data), most existing literature use a multivariate general auto-regressive conditional heteroskedasticity (MGARCH) model. Harri and Hudson (2009) employ a MGARCH and Chung and Ng test to investigate "volatility" transmission between crude oil futures and a set of soft commodities. Serra and Zilberman (2009) employ Baba-Engle-Kraft-Kroner (BEKK) MGARCH to model price "volatility" in ethanol market. Zhang et al.(2008) also employs BEKK model to investigate "volatility" of US gasoline prices. Essentially, the MGARCH approach models variance change (VC) effects. As discussed above, variance changes are neither jumps nor the only source of volatility. Thus, the MGARCH models cannot be used to model jumps, and since jumps contribute heavily to volatility, these models cannot efficiently characterize volatility or its transmission with the presence of jumps.

To model jumps and spikes, one approach is to use the autoregressive conditional hazard (ACH) function. Christensen et al. (2012) employ this model to forecast spikes in electricity prices. This approach provides reasonable modelling of the probability of jumps given a set of exogenous variables. On the other hand, it cannot model jump size conditioning on one occurrence of a jump.

Another approach is to use a Markov regime switching model. Higgs and Worthington (2008) use this method to model the Australian wholesale spot electricity markets. They decompose price at time t into deterministic and stochastic components. Here they characterize stochastic components by Markov regime switching model: state 0 is denoted as the "normal" behavioural status, state 1 as a jump and state -1 as a mean reverting process after a jump. Such a process is assumed to be a Markov process.

It is worth noting that the Markov regime switching approach assumes that the transition probability is fixed, thus the probability of the presence of a jump $1 - \pi^t(0,0)$[1] is identical for all t. This is not a realistic assumption because the dynamics of price would most probably change over the sample period and hence the probability of a jump will change. It is also not of interest to use exogenous (economic) variables to predict the probability of a jump in this scheme since the probability of a jump is assumed to be fixed. If we are interested in predicting the probability of a jump given a set of exogenous economic variables, we can model $\pi^i(0,0)$ as a function of these variables. Since most economic variables are not stationary, $\pi^t(0,0)$ as a function of these variables and is not stationary. That directly violates the definition of $\pi^t(0,0)$ as a transition probability in a Markov Chain. Thus, the Markov regime switching approach, because of its Markov Chain structure, is inherently unable to predict probability of a jump conditioning on a set of exogenous variables.

[1]Here "$0''$ corresponds to the normal state (no jumps). Thus $\pi^t(0,0)$ reads the probability of no jumps at time t given there is no jumps at time $t-1$.

3.2 Types of Outlier Effects

In time series literature, the concept of "jumps" is related to outlier effects. As Fox(1972), Hillmer et al. (1983), Tsay (1988), Chen and Tiao (1990), Chen and Liu (1993) (see [Fox72], [HBT83], [Tsa88], [CT90], [CL93]) discussed, they note five types of outlier effects: innovational outliers (IO), additive outliers (AO), level shifts (LS), temporary changes (TC) and seasonal level shift (SLS). Innovational outliers (IO) refer to those outliers that have effect not only on the particular observation but also subsequent observations, while additive outliers (AO) refer to those that may be mean reverting (zero expectation) and affecting a single observation. Level shifts (LS) represents structure changes. Temporary change (TC) and seasonal level shift (SLS)correspond to temporary level shift.

The theory of price movement in perfect competitive markets (see e.g. [Mut61], [HW77], [WW82]) indicates that the contemporaneous price P_t at time t can be written as the (weighted) rational expected price P_t^e formed at time $t-1$ in the Muthian sense plus a shock in supply ([Mut61]) or in both supply and demand sides ([HW77]). To see this, consider a model (see, [Mut61], [Mas69], [HW77]) of perfect competitive market:

$$\begin{aligned} D_t(P_t) &= \alpha_0(Z_t) - \alpha_1 P_t \\ S_t(P_t^e) &= \beta_0(Z_t) + \beta_1 P_t^e + V_t \end{aligned} \tag{3.1}$$

where D_t is the demand quantity and S_t is the supply. P_t is the market price at time t and P_t^e is the rational expectation of P_t formed at time $t-1$. Z_t is a vector of exogenous factors that shift demand and supply curves. α_0 and β_0 are parameters that depend on Z_t, V_t's are identical distributed random variates with 0 mean and variance σ_v^2 which represent supply shocks (for example, grain harvest shortage due to extreme weather). Then

$$P_t = \frac{\alpha_0 - \beta_0}{\alpha_1} - \frac{\beta_1}{\alpha_1} P_t^e - \frac{V_t}{\alpha_1} \tag{3.2}$$

Define $e_t \equiv -\frac{V_t}{\alpha_1}$, then e_t has zero mean the equation above can be written as

$$P_t = a_t + b_t P_t^e + e_t \tag{3.3}$$

where $a_t = \frac{\alpha_0 - \beta_0}{\alpha_1}$ and $b_t = \frac{\beta_1}{\alpha_1}$. Thus the contemporaneous price P_t at time t can be written as weighted rational expected price P_t^e plus some random shocks. In the case when the shocks V_t (and hence e_t) have serial or intertemporal correlation[2], the shocks can be written as a moving average of white noise:

$$V_t = \sum_{i=0}^{p^V} w_i^V \epsilon_{t-i}^V \tag{3.4}$$

where $\epsilon_t^V \sim WN(0, \sigma^2_{\epsilon_V})$. Then rational expected price can be solved for using equation (3.2) conditioning on the information set I_{t-1} at $t-1$, i.e

$$\begin{aligned}
P_t^e \; &= \mathbb{E}(P_t|I_{t-1}) = \mathbb{E}(\frac{\alpha_0 - \beta_0}{\alpha_1} - \frac{\beta_1}{\alpha_1}P_t^e - \frac{V_t}{\alpha_1}|I_{t-1}) \\
&= \frac{\alpha_0 - \beta_0}{\alpha_1} - \frac{\beta_1}{\alpha_1}P_t^e - \frac{1}{\alpha_1}\mathbb{E}(\sum_{i=0}^{p^V} w_i^V \epsilon_{t-i}^V|I_{t-1}) \\
&= \frac{\alpha_0 - \beta_0}{\alpha_1} - \frac{\beta_1}{\alpha_1}P_t^e - \frac{1}{\alpha_1}\sum_{i=1}^{p^V} w_i^V \epsilon_{t-i}^V - \frac{1}{\alpha_1}\mathbb{E}(w_0^V \epsilon_t^V|I_{t-1}) \\
&= \frac{\alpha_0 - \beta_0}{\alpha_1} - \frac{\beta_1}{\alpha_1}P_t^e - \frac{1}{\alpha_1}\sum_{i=1}^{p^V} w_i^V \epsilon_{t-i}^V
\end{aligned} \tag{3.5}$$

Then we have:

$$P_t^e = \frac{\alpha_0 - \beta_0}{\alpha_1 + \beta_1} - \frac{1}{\alpha_1 + \beta_1}\sum_{i=1}^{p^V} w_i^V \epsilon_{t-i}^V \tag{3.6}$$

Plug this into equation (3.2), we can solve for the contemporaneous price P_t:

$$\begin{aligned}
P_t \; &= \frac{\alpha_0 - \beta_0}{\alpha_1} - \frac{\beta_1}{\alpha_1}P_t^e - \frac{V_t}{\alpha_1} \\
&= \frac{\alpha_0 - \beta_0}{\alpha_1 + \beta_1} - \frac{\epsilon_t^V}{\alpha_1} - \frac{1}{\alpha_1 + \beta_1}\sum_{i=1}^{p^V} w_i^V \epsilon_{t-i}^V
\end{aligned} \tag{3.7}$$

[2]Here the serial correlation between shocks ensures the auto-correlation structure and hence the time series representation of contemporaneous price series. Other cases such as perfect competitive market with storage effect (see [HW77] can also generate such auto-correlation.

If we define:

$$\mu \equiv \frac{\alpha_0 - \beta_0}{\alpha_1 + \beta_1}$$
$$\theta_i \equiv \begin{cases} \frac{1}{\alpha_1} \text{ for } i = 0 \\ -\frac{w_i^V}{\alpha_1+\beta_1} \text{ for } i = 1, 2, \ldots \end{cases} \tag{3.8}$$

then P_t can be written as:

$$P_t = \mu + \sum_{i=0}^{p} \theta_i \epsilon_{t-i} \tag{3.9}$$

where $p = p^V$ and $\epsilon_t = \epsilon_t^V$. That is, the contemporaneous price can be written a moving average of white noises[3].

These theoretical results suggest contemporaneous prices can be written a moving average of white noise. This motivates the time series representation of perfect competitive market price. That is, if the price P_t can be written as a single time

[3]That case with both supply and demand shock is more complicated. In particular, consider the model in [Mas69] and [HW77]) of perfect competitive market:

$$\begin{aligned} D_t(P_t) &= \alpha_0(Z_t) - \alpha_1 P_t + U_t \\ S_t(P_t^*) &= \beta_0(Z_t) + \beta_1 P_t^e + V_t \end{aligned} \tag{3.10}$$

where $D_t, S_t, P_t, P_t^e, Z_t, \alpha_0, \beta_0$ are as defined above. U_t and V_t are identical distributed random shock with 0 mean and variance σ_u^2 and σ_v^2. U_t and V_t represent demand (for example, unexpected electricity usage peak due to unanticipated events) and supply shocks (for example, grain harvest shortage due to extreme weather), respectively. Then

$$P_t = \frac{\alpha_0 - \beta_0}{\alpha_1} - \frac{\beta_1}{\alpha_1} P_t^e + \frac{U_t - V_t}{\alpha_1} \tag{3.11}$$

In the case when the shocks U_t and V_t (and hence e_t) have serial or intertemporal correlation, the shocks can be written as a moving average of white noise:

$$\begin{aligned} U_t &= \sum_{i=0}^{p^U} w_i^U \epsilon_{t-i}^U \\ V_t &= \sum_{i=0}^{p^V} w_i^V \epsilon_{t-i}^V \end{aligned} \tag{3.12}$$

where $\epsilon_t^U \sim WN(0, \sigma_{\epsilon_U}^2)$ and $\epsilon_t^V \sim WN(0, \sigma_{\epsilon_V}^2)$. Then rational expected price can be solved for series DGM that is stationary[4], then it must be some moving average of white noises by Wold Decomposition.

On the other hand, when there are extreme price movements which we label as price jumps that may not be fully characterized by perfect competitive market, there must be some structural deviation (or structural changes) in the "white noise" terms. Such structural deviation, in simple cases, can be modelled as an instantaneous shift in the mean of the "white noise" terms [5]. That is, for a single intervention at time $t^* \leq t$, we have an add-on term of some weight at the time of the jump to the "white noise" term, i.e.

$$P_t = \mu + \sum_{i=0}^{p} \theta_i(\epsilon_{t-i} + \omega J_{t^*-i}) \tag{3.16}$$

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using equation (3.2) conditioning on the information set I_{t_1} at $t-1$, i.e

$$\begin{aligned} P_t^e \quad &= \mathbb{E}(P_t|I_{t-1}) = \mathbb{E}(\frac{\alpha_0-\beta_0}{\alpha_1} - \frac{\beta_1}{\alpha_1}P_t^e + \frac{U_t - V_t}{\alpha_1}|I_{t-1}) \\ &= \frac{\alpha_0-\beta_0}{\alpha_1} - \frac{\beta_1}{\alpha_1}P_t^e + \frac{1}{\alpha_1}\mathbb{E}(\sum_{i=0}^{p^U} w_i^U \epsilon_{t-i}^U - \sum_{i=0}^{p^V} w_i^V \epsilon_{t-i}^V|I_{t-1}) \\ &= \frac{\alpha_0-\beta_0}{\alpha_1} - \frac{\beta_1}{\alpha_1}P_t^e + \frac{1}{\alpha_1}(\sum_{i=1}^{p^U} w_i^U \epsilon_{t-i}^U - \sum_{i=1}^{p^V} w_i^V \epsilon_{t-i}^V) + \frac{1}{\alpha_1}\mathbb{E}(w_t^U \epsilon_t^U - w_t^V \epsilon_t^V|I_{t-1}) \\ &= \frac{\alpha_0-\beta_0}{\alpha_1} - \frac{\beta_1}{\alpha_1}P_t^e + \frac{1}{\alpha_1}(\sum_{i=1}^{p^U} w_i^U \epsilon_{t-i}^U - \sum_{i=1}^{p^V} w_i^V \epsilon_{t-i}^V) \end{aligned} \tag{3.13}$$

Then we have:

$$P_t^e = \frac{\alpha_0-\beta_0}{\alpha_1+\beta_1} + \frac{1}{\alpha_1+\beta_1}(\sum_{i=1}^{p^U} w_i^U \epsilon_{t-i}^U - \sum_{i=1}^{p^V} w_i^V \epsilon_{t-i}^V) \tag{3.14}$$

Plug this into equation (3.2), we can solve for the contemporaneous price P_t:

$$\begin{aligned} P_t \quad &= \frac{\alpha_0-\beta_0}{\alpha_1} - \frac{\beta_1}{\alpha_1}P_t^e + \frac{U_t - V_t}{\alpha_1} \\ &= \frac{\alpha_0-\beta_0}{\alpha_1+\beta_1} + \frac{\epsilon_t^U - \epsilon_t^V}{\alpha_1} + \frac{1}{\alpha_1+\beta_1}(\sum_{i=1}^{p^U} w_i^U \epsilon_{t-i}^U - \sum_{i=1}^{p^V} w_i^V \epsilon_{t-i}^V) \end{aligned} \tag{3.15}$$

Then unless $w_i^V = w_i^U$ for $\forall i$, we do not have a simple moving average form.

[4]That is, there are no structural breaks in P_t.

[5]In other cases, these changes can be in both the first and second moments in the "white noise" terms. Then we can refer to GARCH with jumps (see e.g. [FG99], [ABD07], [LLP11]). This paper only considered the former cases.

where J_{t^*-i} equals to 1 if there is a jump at time $t^* - i$ (that is, when $i = 0$) and 0 otherwise, and ω is the corresponding weight of jumps which characterizes the jump size (or, the deviation of the actual price series from its smooth trend).

A natural extension is to consider nonstationary prices. In this case, we can apply a filter (e.g. first difference operator) to the level prices so that the filtered series is stationary. And the discussion of the stationary prices stated as above can be naturally carried over. Later in the context we see this gives the exact form of innovational outliers defined as in Chen and Liu (1993) for single outlier.

Formally, we follow Chen and Liu (1993) and define a price series P_t subject to the influence of multiple non-repetitive events (or outlier effects) as:

$$P_t = P_t{}^* + \sum_{j=1}^{m} \omega_j L_j(B) J_{t_j} \tag{3.17}$$

$$\text{where} \quad P_t^* = \frac{\theta(B)}{\alpha(B)\phi(B)} \epsilon_t \tag{3.18}$$

where $\epsilon_t \sim WN(0, \sigma^2)$; P_t^* represents the smooth trend of the price and follows a ARIMA process; J_t is an indicator variable that equals to 1 when there is outliers effect at time t and 0 otherwise; ; and ω_j represents the magnitude of the outlier effects, or, jump sizes; and m is the total number of outliers. Note here B is the backward shifter: $BP_t = P_{t-1}$, and $\theta(B), \phi(B), \alpha(B)$ are polynomials of B; all roots of $\theta(B)$ and $\phi(B)$ are outside unit circle; and all roots of $\alpha(B)$ are on the unit circle. Note here $L_j(B)$ is also a polynomial of B, which denotes the dynamic pattern of the outlier effects.Then the five types of outlier effects can be defined as:

$$\begin{aligned} &\text{IO: } L_j(B) = \frac{\theta(B)}{\alpha(B)\phi(B)}; \quad \text{LS: } L_j(B) = \frac{1}{(1-B)}; \quad \text{SLS: } L_j(B) = \frac{1}{1-B^s}; \\ &\text{AO: } L_j(B) = 1; \qquad\qquad\qquad \text{TC: } L_j(B) = \frac{1}{1-\delta B}. \end{aligned} \tag{3.19}$$

where s is the periodicity of the data and the value δ is usually equal to 0.7. Figure 3.1 shows a unit impulse for each type of outlier at time point at $t = 10$. Intuitively, jumps or spikes refer to those temporary and permanent-structural-change-free outliers that have effect on subsequent observations. Thus, jumps should most appropriately be modelled as innovational outliers (IO). In this case, the dynamic pattern is $\theta(B)/\alpha(B)\phi(B)$.

From another perspective, we show earlier in this chapter that the equilibrium price of perfect competitive market can be represented by an ARIMA series P_t^*. Then the jump effect ω's can be interpreted as the difference between the observed price series P_t and its smooth trend P_t^* which can be characterized by the perfect competitive market models. Such difference reflects that the perfect competitive market models can not fully characterize the observed price series. Later in Chapter 4 we further show that perfect competitive market models can not fully capture the frequency and magnitude of observed price jumps.

Following such interpretation, we can estimate the total contemporaneous outlier effects $\sum_{j=1}^{m} \omega_j L_j(B) J_{t_j}$ at time t using the deviation of the observed price series from the (relatively smooth) price that can be characterized by the perfect competitive market model. That is, if we define $\gamma_t \equiv \sum_{j=1}^{m} \omega_j L_j(B) J_{t_j}$ as the total outlier effects at time t, then we can estimate γ_t using the following equation:

$$\hat{\gamma}_t = P_t - P_t^e \tag{3.20}$$

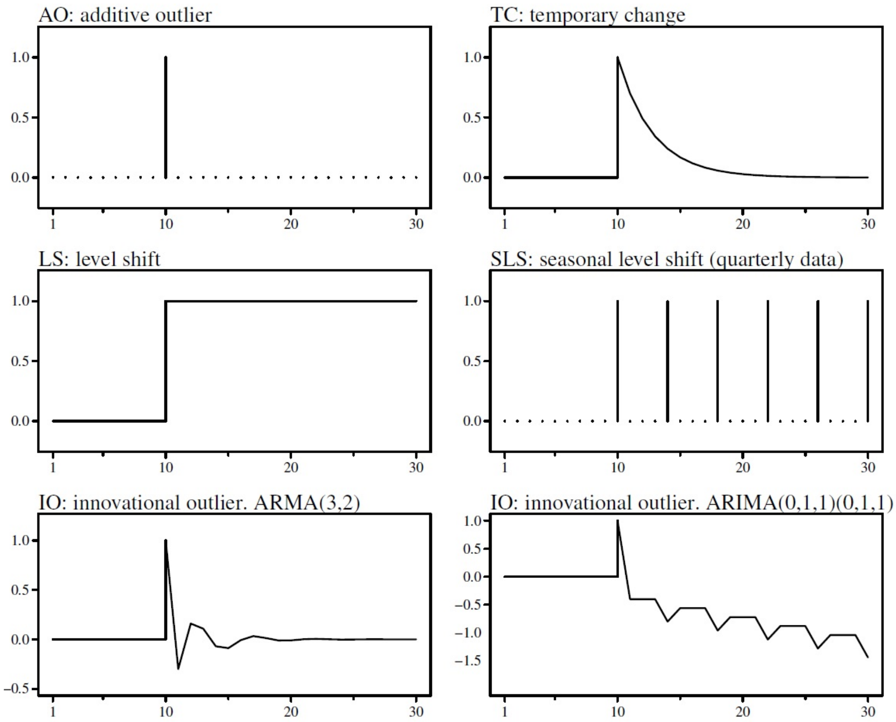
where P_t^e is the equilibrium price of perfect competitive market model. Later in chapter 4 we use the normalized version[6] of (3.20) to estimate the contemporaneous jump effects at time t in the synthetic data.

3.3 Methodology

In this section we present the modelling of this paper. In particular, this paper employs a two-step method. The first step is to identify the time when there are jumps using test by Chen and Liu (1993). With jump time specified, the second step is to model and predict jumps conditioning on a set of economic variables. I first present the time series model of the price series. Since we are only interested in innovational outliers, then combining (1) and (3) together, we have that the

[6] For the purpose of comparison over different periods (trading sessions as we define in Chapter 4), we normalize $\hat{\gamma}_t$ by P_t^e and use the ratio (or, the percentage that is not characterized by P_t^e) as a measure of jump effects in Chapter 4.

Figure 3.1. Unit impulse for different types of outliers



observed price series can be written as:

$$
\begin{aligned}
P_t &= {P_t}^* + \sum_{j=1}^{m} \omega_j \frac{\theta(B)}{\alpha(B)\phi(B)} J_{t_j} \\
&= \frac{\theta(B)}{\alpha(B)\phi(B)} (\epsilon_t + \sum_{j=1}^{m} \omega_j J_{t_j})
\end{aligned}
\tag{3.21}
$$

where the set J_{t_j} is the indicators of jumps at time t_j, and m is the total number of jumps. Then the estimated residuals can be written as:

$$
\pi(B) P_t \equiv \hat{e_t} = \epsilon_t + \sum_{j=1}^{m} \omega_j J_{t_j} \tag{3.22}
$$

where the coefficients of the power series expansion $\pi(B) = \sum_{i=0}^{\infty} \pi_i B^i$ can be determined by:

$$\pi(B) = \frac{\alpha(B)\phi(B)}{\theta(B)} \tag{3.23}$$

Since all the roots of $\theta(B)$ lie outside the unit circle, the coefficients $\{\pi_i\}_{i=0}^{\infty}$ is an absolutely convergent series ($\sum_{i=0}^{\infty} |\pi_i| < \infty$). If there were no jump at time t, the $\hat{e}_t$ should behave like the white noise ϵ_t. Thus to test whether there is jump at time t, we test whether w_j's are not zero. If w_j is statistically significant, then an jump is identified at time t_j. In particular, the t-test statistics is:

$$\hat{\tau}_t = \frac{\hat{e}_t}{\hat{\sigma}_e} \tag{3.24}$$

$$\text{where} \quad \hat{\sigma}_e = 1.483 \times \text{median}\{|\hat{e}_t - \tilde{e}_t|\} \tag{3.25}$$

where $\tilde{e}_t$ is the median of the estimate residuals. Now we can start with the detection procedure of jumps.

3.3.1 Detection Procedure

The Chen and Liu procedure of detecting jumps can be described as following:

Step 1: Locate outliers. Given an ARIMA model fitted to the data, identify an observed price point at time location t as jump if $\hat{\tau}_t$ as defined in equation (7) is bigger than 95% critical value of t distribution.

Step 2: Remove outliers. Given a set of potential outliers, an ARIMA model is chosen and fitted according to equation (4). The significance of the outliers is reassessed in the new fitted model. Those outliers that are not significant are removed from the set of potential outliers.

Step 3: Iterate stage 1 and 2, for both the original series and adjusted series.

With the set of jumps defined, we can start to model and predict jump conditioning a chosen exogenous set of economic variables. In particular, if I choose the exogenous variables to be the indicators of presence of jumps in other price series, then such model can be interpreted as characterizing jump transmission.

3.3.2 Econometric Model

It is worth noting that there could be many factors that cause jumps in prices. In stock markets, the usual drivers are liquidity (common measures are bid-ask spread, time between two sequential transaction, etc.), macroeconomics news and firm news. It is worth noting that here liquidity measures the ease of making a transaction in stock market. Such concepts of liquidity can be generalized to accommodate our case of low-frequency energy and commodity price data. In our case, the measure of liquidity is inventory. When the inventory level of some particular commodity becomes critically low, producers are facing a high risk of stock out. Facing such risk of stock-out, producers may be willing to pay much more (than usual) up to the cost when the production is forced to stop. As a result, we hypothesize that the low inventory level results in a jump increase in prices. Thus, the process of jumps J_t is driven by a set of exogenous variables Y_t. Following Heckman (1979), suppose there is a latent continuous variable J_t^* that controls J_t such that $J_t = 1$ if and only if $J_t^* \geq 0$. the selection of J_t is conditional on a set of variables. Then the selection equation is defined as:

$$E(J_t^*|Y_t) = Y_t^T\beta + \nu_{1t} \tag{3.26}$$

where $\nu_{1t} \sim N(0,1)$. Then the probability of a jump can be modelled as a probit equation:

$$\mathbb{P}(J_t = 1|Y_t) = \mathbb{P}(J_t^* \geq 0|Y_t) = \Phi(Y_t^T\beta) \tag{3.27}$$

Now to model the jump size ω_j, consider a latent variable $\omega_{t_j}^*$ for ω_j:

$$\omega_j = \omega_{t_j}^* \times J_{t_j} \tag{3.28}$$

That is, we only observe jump size ω_j if there is a jump. Such $\omega_{t_j}^*$ is depending on another set of exogenous variables W_t:

$$E(\omega_{t_j}^*|W_t) = W_t^T\beta + \nu_{2t} \tag{3.29}$$

where $\nu_{2t} \sim N(0, \sigma_\nu^2)$ Then the jump size can be modelled as the outcome equation of Heckman selection model:

$$E(\omega_t | W_t, J_t = 1) = W_t^T \beta + E(\nu_{2t} | J_t = 1) \tag{3.30}$$

Note there is interdependence between ω_j and W_t, and hence interdependence between the two latent process $\omega_j{}^*$ and J_t. Such interdependence is represented by the correlation in the joint normal distribution of (ν_{1t}, ν_{2t}) in the Heckman model. In particular, we assume they follow a multivariate normal distribution:

$$\begin{pmatrix} \nu_{1t} \\ \nu_{2t} \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho\sigma_\nu \\ \rho\sigma_\nu & \sigma_\nu^2 \end{bmatrix} \right) \tag{3.31}$$

Then equation (13) can be written as:

$$E(\omega_t | W_t, J_t = 1) = W_t^T \beta + \frac{\rho\phi(Y_t^T \beta)}{\sigma_\nu \Phi(Y_t^T \beta)} \tag{3.32}$$

This finishes the modelling of jump size ω_j.

3.4 Empirical Results

3.4.1 Data

We use weekly data from January 2000 through December 2010 for a corn, wheat, soybeans a subset of softs" commodity prices for Italian and US, and Brent blend. The Italian prices of maize and wheat are obtained from DATIMA provided by ISMEA. The US prices of wheat, corn and soybeans are provided by FAO sourced from USDA. Data are weekly prices monitored for a length of time that started in February 2005 and ended in February 2010. The prices in \$/ton are converted in €/ton using the official \$/€exchange rate. Missing values are replaced by using an imputation algorithm and the corresponding R-package AMELIA II (King et al., 2001; Honaker et al., 2011) ([H+11] [Kin+01]. For the fuel prices, the weekly United States spot prices and weekly Europe (UK) Brent blend spot price are converted to €/barrel.

We are interested in applying our method to this particular dataset for several reasons:

- Italy commodity prices can serve as a comparison to (US) domestic prices since Italy is a typical small open economy in which commodity prices are subject to being affected by those in the large economies such as US. This is also the rationale for our previous cointegration analysis.

- The dataset includes some of main storable commodities prices. This ensures us to examine price movements of main storable commodities. In chapter 4, we shows storage effects may to some extent mitigate extreme price movements.

3.4.2 Identified Jumps in Observed Price Series

In this section, we apply the method proposed in previous sections to the dataset for modelling jumps transmission. The estimation strategy is as follows:

1. Estimate price jumps and jump sizes using [CL93] procedure.

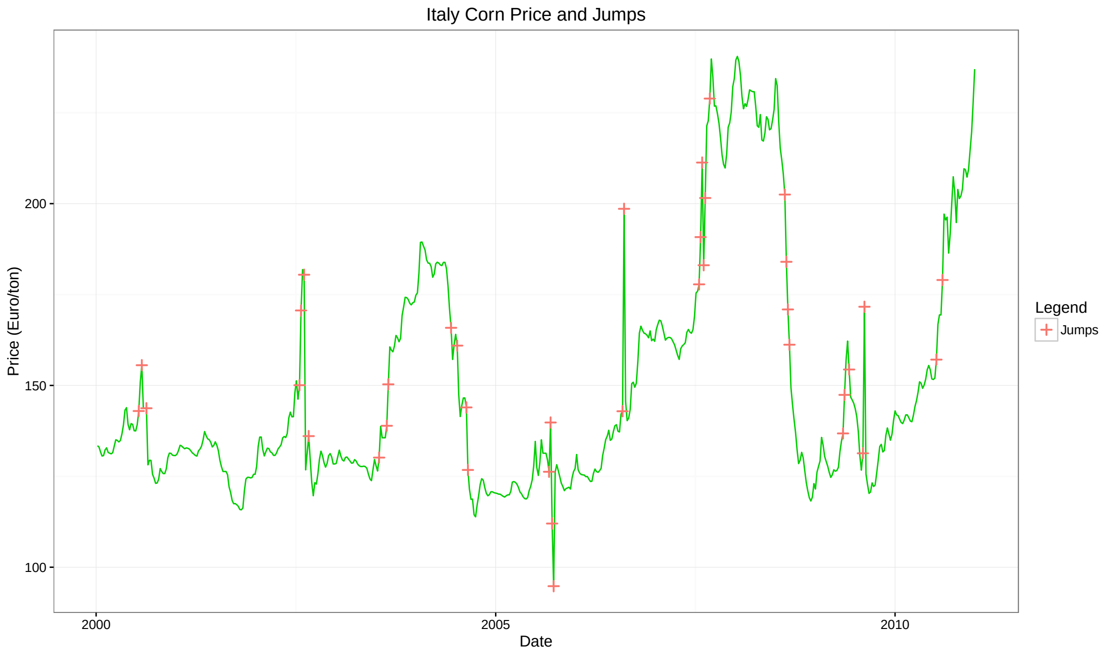
2. Using probit and Heckman selection model to predict the probability of jumps and corresponding jump sizes.

We report the results of the first step in table 1. We plot the dates of jumps in figure 1-7. As seen from Table 1, the series US Corn US wheat have only 2 jumps detected. We suspect this is due to the weekly sampling frequency that averages out short-run price movements. Such limited number of jumps results in an inadequate degree of freedom to estimate our model. To apply the method of detecting jump and modelling jump transmission, we need a dataset of either higher sampling frequency or larger time span in which more price jumps are present.

Table 3.1. Jumps Detection using Chen and Liu (1993) Procedure

EU corn	EU wheat	EU Soybean	US corn	US wheat	US soybean	Brent Blend
29	24	28	468	72	78	49
31	26	29	563	74	219	51
34	76	31			220	90
134	77	32			237	99
135	78	36			295	155
137	129	37			296	469
140	130	73			297	471
186	182	77			298	
191	191	78			367	
192	192	134			427	
233	201	135			457	
237	202	136			505	
243	203	137				
244	230	186				
297	236	188				
298	237	189				
299	288	190				
300	338	191				
345	340	193				
346	343	231				
395	347	232				
396	353	235				
397	354	236				
398	355	238				
399	388	295				
402	396	297				
451	397	345				
452	400	401				
453	435	441				
454	436	449				
489	474	453				
490	498	473				
493	445	499				
502	451	500				
503	459	505				
550	460	506				
554	470	542				
	493	543				
	494	544				
	554	549				
	555	551				
	572	555				
	573					

Figure 3.2. Jumps in Italy Corn Price Series

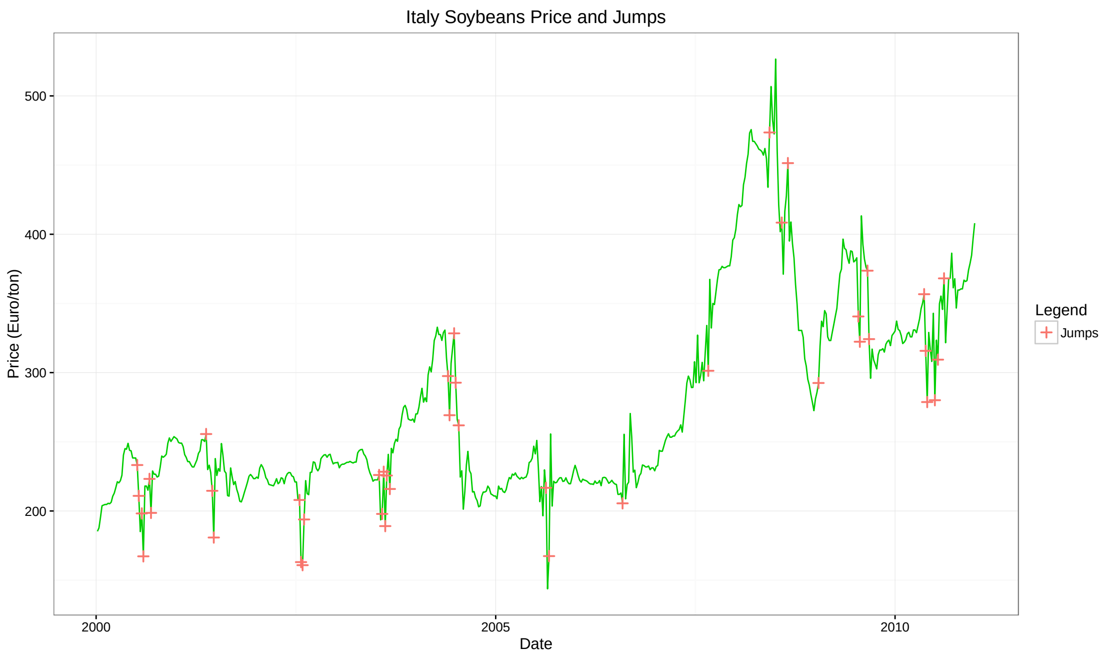


Note: In this figure, a cross indicates time at which jump is detected.

Figure 3.3. Jumps in Italy Wheat Price Series

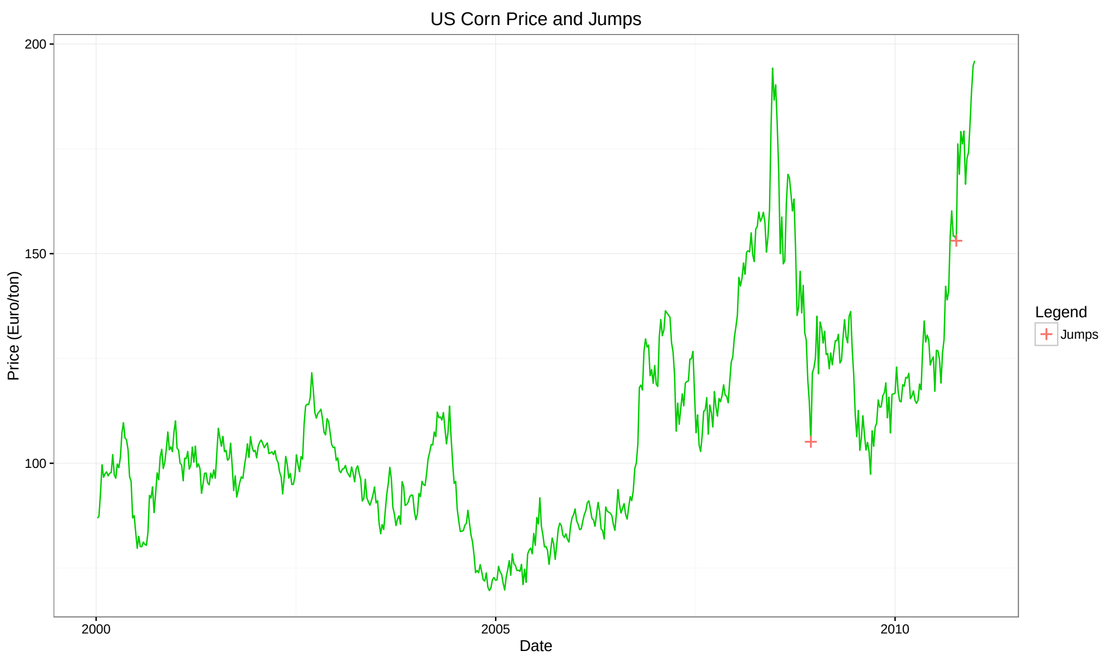
Note: In this figure, a cross indicates time at which jump is detected.

Figure 3.4. Jumps in Italy Soybeans Price Series



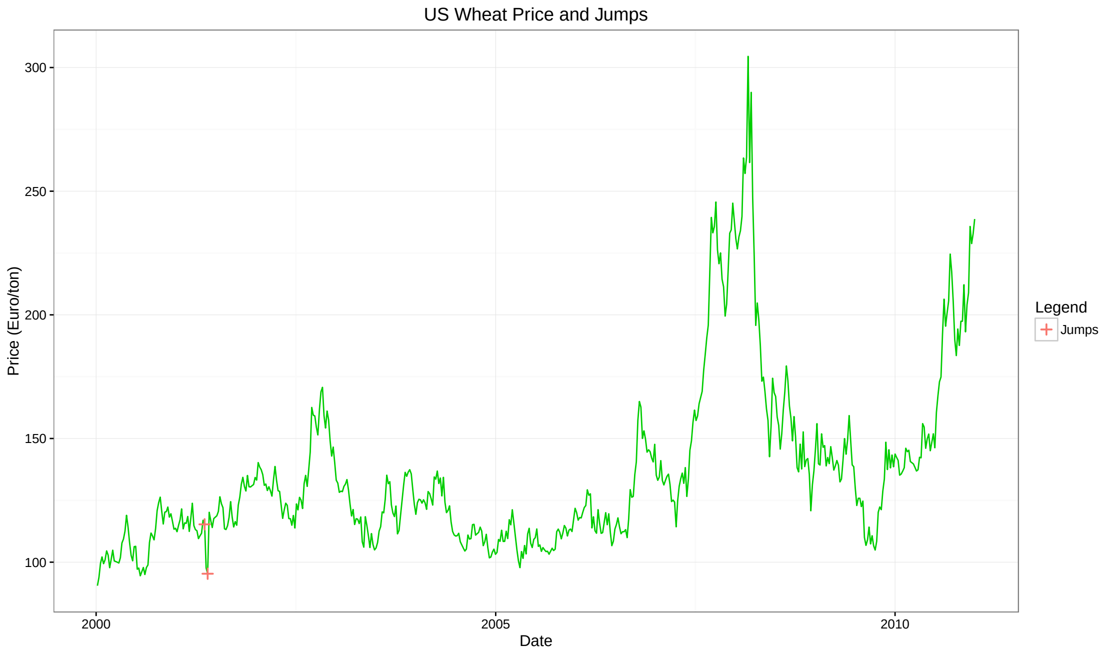
Note: In this figure, a cross indicates time at which jump is detected.

Figure 3.5. Jumps in US Corn Price Series



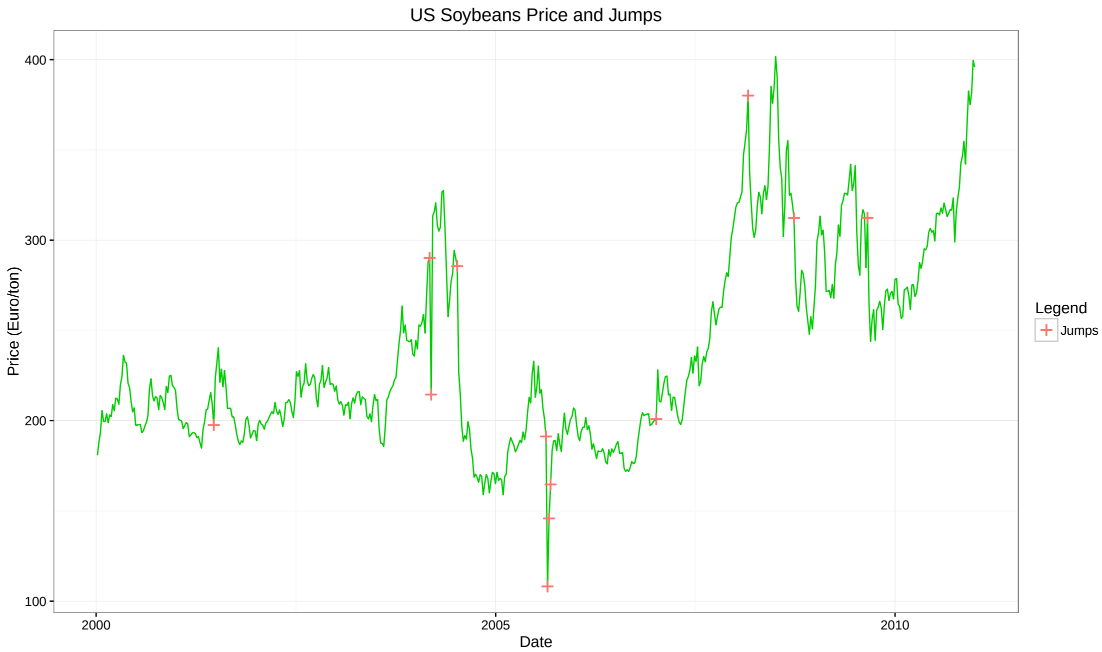
Note: In this figure, a cross indicates time at which jump is detected.

Figure 3.6. Jumps in US Wheat Price Series



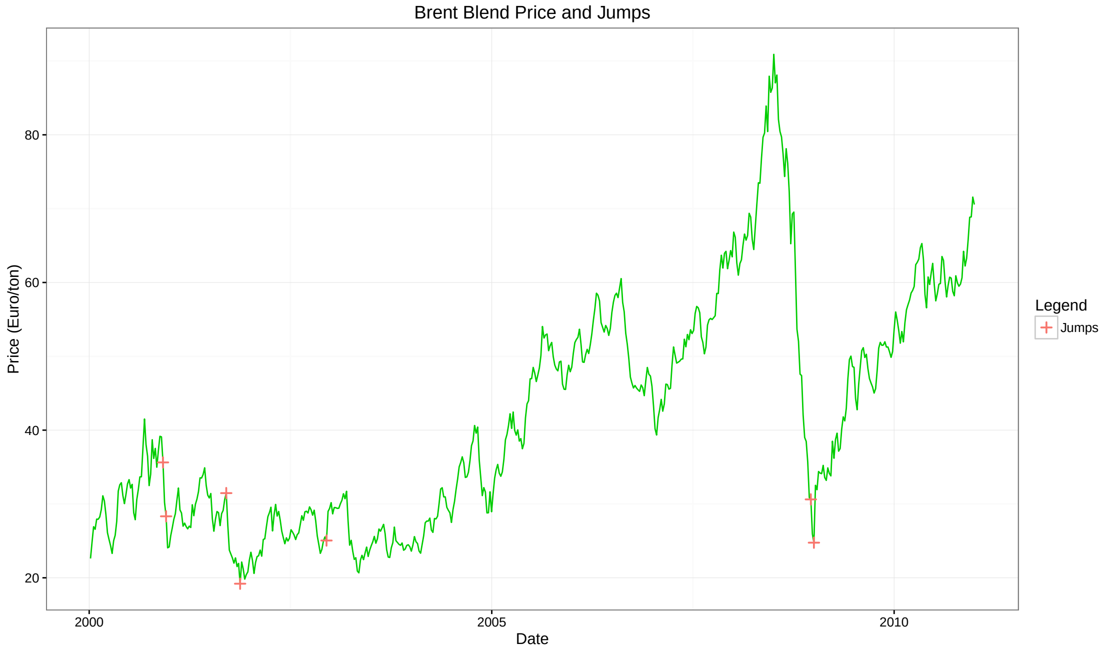
Note: In this figure, a cross indicates time at which jump is detected.

Figure 3.7. Jumps in US Soybean Price Series



Note: In this figure, a cross indicates time at which jump is detected.

Figure 3.8. Jumps in Brent Blend Price Series



Note: In this figure, a cross indicates time at which jump is detected.

3.5 Concluding Remarks

In this paper, we discuss the proper mathematical representation of jumps in low-frequency price series. Compared to Christensen et al.(2012), this approach models jumps as a component of the price series rather than a standalone generalized Poisson process. This gives us the flexibility to model the jump size conditioning on the presence of jumps through the consideration of the selection problem inherent in the study of jumps. That is, intuitively, we can only estimate the jump size if there is a jump. Using [CL93] procedure, we show that there are price jumps in observed price series. This motivates a closer look at the source and origin of the price jumps.

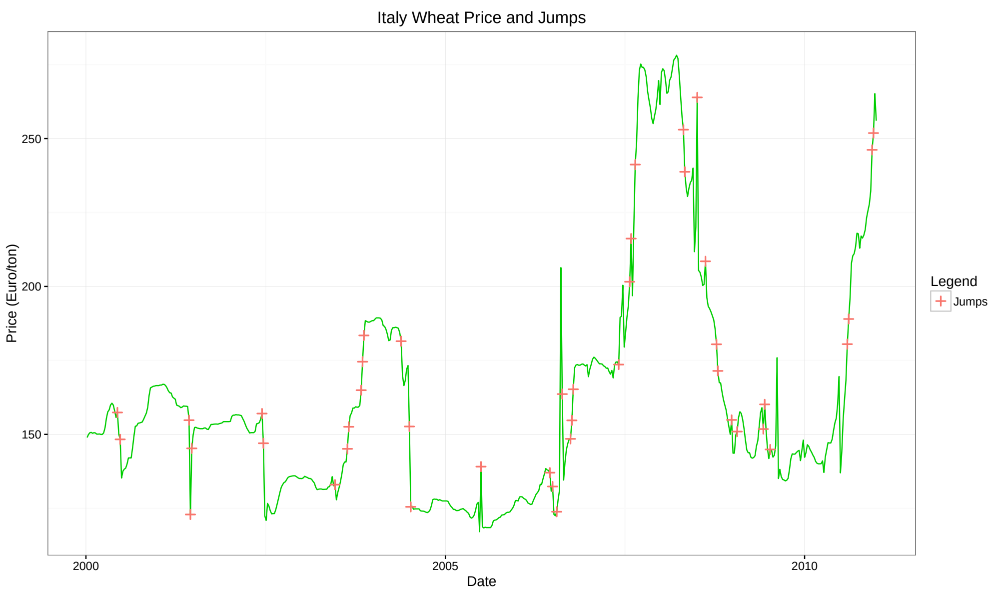
We also note because of the number of observation of our data, we have limited number of jumps which do not give us enough degrees of freedom to estimate our model. On the other hand, in chapter 4, we will simulate data of enough many observations and apply the method to the synthetic data.

Chapter 4
A Simple Model of Price Jumps

4.1 Introduction

As shown in the chapter three, economic price series often present short-run sudden changes of noticeable magnitude which we label as price jumps. They differ from structural changes which represent permanent level shifts or variance changes yet have a substantial effect in the economy. As shown in figure 4.0 is an example of Italy wheat price. Using Chen and Liu (1993) (see [CL93]) procedure, we show there are jumps of rather high frequency in weekly Italy wheat price from 2000 to 2010.

Figure 4.0. Jumps in Italy Wheat Price Series



The importance of price jumps to the economy has been noted by many researchers and their effects on real economy examined. Hamilton (2003) investigated the effect of shocks[1] in oil prices on the U.S. GDP growth and concluded that GDP growth responds nonlinearly and asymmetrically to oil price shocks (see [Ham03]). Kilian (2007) (see [Kil07]) had a detailed discussion about the linkage between energy price shocks and economic performance.

Price jumps have received continuing attention in the literature. From the source of price jumps to their (mathematical) representation have been investigated in the literature. For soft commodity markets, Trostle et al. (2008) discussed the importance of price jumps in explaining volatile soft commodity prices and discussed possible factors that lead to price jumps (see [Tro+08]). Sumner (2009) in [Sum09] also noted such importance in explaining soft commodity price movements. For the electricity market, Huisman, Ronald, and Ronald Mahieu (2003) proposed a model of price jumps series resulting from regime switching processes (see [HM03]). Barlow (2002) in his paper [Bar02] used a diffusion model to represent jumps in electricity prices. Deng in his book [Den00] proposed stochastic models of different energy commodity prices to integrate price jumps. Shafiee and Topal in [ST10] reviewed global gold market and suggested a jump-diffusion modelling of gold prices.

In a perfectly competitive market of a product of interest, the equilibrium price will fluctuate as exogenous factors vary. In a simple model (see, [Mas69], [HW77]) of perfect competitive market:

$$\begin{aligned} D_t(P_t) &= \alpha_0(Z_t) - \alpha_1 P_t + U_t \\ S_t(P_t^*) &= \beta_0(Z_t) + \beta_1 P_t^* + V_t \end{aligned} \tag{$*$}$$

where D_t is the demand quantity and S_t is the supply. P_t is the market price at time t. Z_t is a vector of exogenous factors that shift demand and supply curves. α_0 and β_0 are parameters that depend on Z_t, U_t and V_t are independent identical distributed (i.i.d. thereafter) random variable with 0 mean and variance σ_u^2 and σ_v^2. U_t and V_t represent independent demand (for example, unexpected electricity usage peak due to unanticipated events) and supply shocks (for example, grain harvest

[1] Note “shock" may seem a bit vague. Here it indicates some kind of short-run volatility which includes but is not limited to the case of price jumps. Recall to what extent a current price differs from its past so that such difference can be called a jump is explicitly defined in the previous chapter.

shortage due to extreme weather), respectively. P_t^* is the rational expectation of equilibrium price of time t formed at time of $t-1$ in the Muthian sense ([Mut61]). That is, $P_t^* \equiv E_{t-1}P_t \equiv \mathbb{E}(P_t|\Phi_{t-1})$, where E_{t-1} denotes the conditional expectation of price at time $(t-1)$ as in [Mut61] and Φ_{t-1} is the information available to the agent at time $(t-1)$ which includes the past history of observed Z, U and V up to time $t-1$. Then without storage[2], $D_t(P_t) = S_t(P_t^*)$ and the system $(*)$ can be easily solved algebraically by taking conditional expectation on both sides of the equation. Given U_t and V_t are i.i.d. over time, $\mathbb{E}(U_t|\Phi_{t-1}) = \mathbb{E}U_t = 0$. The rational expected price can be written as:

$$P_t^* = \frac{\alpha_0(Z_t) - \beta_0(Z_t)}{\alpha_1 + \beta_1}$$

This says the rational expectation is stable over time since the agents can not anticipate future shocks and P_t^* is affected only by Z_t. Then we have $S_t = \frac{\alpha_1\beta_0(Z)+\beta_1\alpha_0(Z)}{\alpha_1+\beta_1} + V_1$. Plug this back into the linear system $(*)$, we have

$$P_t = P_t^* + \frac{U_t - V_t}{\alpha_1} \tag{$**$}$$

Thus the market price can be decomposed into a stable variable P_t^* and a random shock $\frac{U_1-V_1}{\alpha_1}$. That says, the actual market price (at time t) equals the expected price (formed at time *t-1*) plus a random shock due to contemporaneous demand or supply shock. In particular, either supply or demand shocks impact prices, e.g. a negative V_t in $(**)$ leads to an increase in the market price by $|\frac{V_t}{\alpha_1}|$. Importantly, given linearity, the price effect is determined by the magnitude of the shocks. Further, if demand or supply factors are consider as determinants of α_0 and β_0, $(**)$ clarifies their effects on price.

In any case, such market price variation induced by shocks can hardly be labeled as price jumps mainly for two reasons:

- Magnitude: We see from the model that the market equilibrium price P_t responds to supply shocks linearly (by some constant determined by demand

[2]The case with storage is more complex. Weaver and Helmberger (1977) discussed in detail different situations when the market price falls into different price intervals. On the other hand, these interval are bounded by linear functions of demand and supply shocks. Thus the argument that the competitive market model can not generally characterize price jumps is valid for the case with storage.

price elasticity). This is inconsistent with the observation that price jumps usually exhibit nonlinearity and/or discontinuity of noticeable magnitude as shown in previous chapters.

- Frequency: In the competitive market model with no inventories, a large negative supply shock (or positive demand shock) generates a large contemporaneous increase in the market price. On the other hand, if in the next period $(t+1)$ the supply restores to its normal level, the market price P_{t+1} will immediately revert to its mean level. This implies contemporaneous demand or supply have no effect on subsequent prices. This is inconsistent with the observation that the response of market price to a single supply shock usually lasts over several periods before the effects of shock completely vanishes[3]. In a competitive market model with inventories, the equilibrium prices could show wider fluctuations ([HW77]). But whether these fluctuations can be labelled as jumps is still subject to their magnitude, though as [HW77] shows stock out could induce nonlinearity. With the linearity, it is likely that these fluctuations are not large enough to be characterized as jumps. In that case, the fluctuations are more likely to be interpreted as high conditional variance rather than outlier (jump) effects.

Thus, the perfect competitive market model does not fully capture key features of jumps in observed economic price series. This motivates a closer look into the mechanism that induces price jumps of the magnitude and frequency as observed in reality.

This paper is concerned with the micro-structure specification to identify origins of price jumps that can not be generally characterized by the competitive market models. In particular we propose a rather general model of procurement process where imperfectly informed buyers search for and place bids to suppliers to fulfill procurement demand. We show that in this process, search cost, market structure and market condition are crucial factors in generating price jumps. Later in the simulation part we show that the model proposed in this paper can generate jumps that resemble those in the observed economic price series.

[3]In some particular market, for example electricity market, price jumps exhibits mean reverting pattern. On the other hand, in general, prices do not immediately revert to their mean level after jumps.

Another strand of literature relating to our model focuses on the procurement process. The procurement problem in energy markets has received consistent attention in the literature. [BW01] studied the procurement problem for electrical utilities under stochastic demand and market prices. The procurement plan balances contract and spot market purchases and is modeled as a stochastic programming problem and solved by a two-phase procedure, which performs better than a rolling-horizon, stochastic-programming heuristic. [SW07] proposed a methodology for profit maximized bidding under price uncertainty in a multi-unit and pay-as-bid procurement auction for power systems reserve. The model we propose in this paper resembles a multi-unit procurement auction (see [SW07], [Wol97]) but is extended to accommodate different cases of buyers' learning and offer rule and suppliers' acceptance rule.

The paper is organized as follows. Section II presents an overview of basic settings of this model. Section III and section III introduces the notations used in the model. Section IV is the main body of the model. Section V presents two exemplary analytical solutions to the model under two specific cases. Section VI discusses extensions of the model. Section VII presents some simulation results. The paper ends with some concluding remarks.

4.2 Model Basics

In this model, we assume the product of interest is homogeneous, continuously divisible. We suppose there is an unordered finite set of risk neutral [4] buyers I and an unordered finite set of risk neutral suppliers K who compose what we label as

[4]Risk premiums in the case of risk aversion are ignored for the simplicity of the argument. That is, we assume that an agent does not react to uncertainty in his payoff and his utility is a linear function of his expected "payoff", where payoff can be thought as *negative* cost in the case of buyers or profit in the case of supplier. On the other hand, since later in the context we assume that the search cost is a nonlinear function of the market condition, even with the linear utility function, the model characterizes the buyers' risk aversion in the uncertainties in market condition. Later in equation (5) search cost satisfies

$$\frac{\partial \widetilde{SC_{ik}}^{i}}{\partial \widetilde{Z}^{i}} > 0, \frac{\partial^2 \widetilde{SC_{ik}}^{i}}{\partial \widetilde{Z}^{i2}} > 0$$

That means that the search cost is a convex function in the market condition. Hence the payoff is a concave function in the market condition and so is the buyer's utility function. Thus the buyer is risk averse in market condition.

a market of a particular (homogeneous) product. Each of these two sets does not possess natural ordering.

We assume buyers and suppliers are heterogeneous and imperfectly informed. We assume buyers do not know at what price suppliers are willing to sell. Further, we assume suppliers do not know what buyers might be willing to pay. Each buyer $i \in I$ has an initial fixed demand that buyer seeks to fulfill at lowest possible cost. To fulfill such demand, he will search for the product at each supplier $k \in K$ (or, location k) until some stopping time[5] determined by what we define as buyers' *optimal search stopping rule*. At each supplier k, the buyer offers a bid, which is a price-quantity pair. If the supplier k accepts his bid, the price in bid becomes (part of) the transaction price, and the supplier's accepted bid quantity can be subtracted from the buyer's demand to be fulfilled decreases by .We note this price is a bilateral transaction price between buyer willingness to pay and supplier willingness to sell. It is not a market transaction price.

To characterize the time scope of this model, we define a trading session as a relatively complete time interval during which there is at most one transaction between each buyer and each supplier. Each trading session starts with an initial supply allocation to suppliers and demand allocation to buyers, and ends when either all buyers or all suppliers are out of market[6]. We assume there are four types of trading sessions:

- In a *type 1 trading session* (TS1), there is no *direct* interaction between buyers. They arrive at suppliers sequentially. Thus, suppliers receive one bid at a time and buyers can not observe or react to other buyer bids. Further, we assume there is no intertemporal dependence across trading sessions. That is, the demand or supply in current session will not be carried over to the next one. We assume in this type of trading session an agent (a buyer or supplier) has only *short memory* in a sense that he can only remember the history of that single trading session (although his unfulfilled demand or unsold supply can be carried over to the next trading session). And agents are *myopic* in that they fail to consider implications of history for future sessions.

[5]Here the term "stopping time" is a general term and not to be confused with the use in stochastic processes.

[6]In a trading session, a buyer is out-of-market if he has fulfilled all his procurement demand; and a supplier is out of market if he has sold all his supply capacity.

- In a *type 2 trading session* (TS2), buyers are allowed to have *direct* interaction. Multiple buyers may arrive at a supplier at the same time (*coincidentally*) and may submit multiple bids to one supplier in a single trading session. However, we continue to assume that there is no intertemporal dependence across sessions. That is, there is no carry-over inventory across sessions, and any agent is myopic and has short memory as in *type 1 trading session*.

- In a *type 3 trading session* (TS3), there is no *direct* interaction between buyers as in TS1. We add intertemporal dependence between sessions. Buyers' unfulfilled demand and suppliers' inventory can be carried over to the next one. In this type of trading session, we assume buyers are myopic and have short memory. But we assume that suppliers can look into future and make tradeoff between selling in current session or in future sessions. We call such suppliers as "*far-sighted*". We assume suppliers have "long memory" in a sense that they can recall what happened in all past sessions. In particular, suppliers know the equilibrium price and supply capacities in all past trading sessions.

- In a *type 4 trading session* (TS4), there is direct interaction between buyers as in TS2 and intertemporal dependence between trading sessions as in TS3.

Table 4.1. Types of Trading Sessions

Assumptions	No Intertemporal Dependence across Trading Sessions	Intertemporal Dependence across Trading Sessions
No Buyers' Coincidental Arrival	TS1	TS3
With Buyers' Coincidental Arrival	TS2	TS4

These four types of trading sessions are summarized in table 1. It is worth noting that in each of these trading session cases, we assume the agents behave exactly the same in each single trading session. That is, they make decisions following the same set of rules (buyer's optimal bid rule, supplier's acceptance rule, etc.) in every session. Thus for each type of these trading sessions, we only concern with a single trading session in the model. We first present the model of agents' behaviour in a type 1 trading session as a simple case. Extensions to other types of trading sessions are discussed later in section VI.

The buyers visit the sellers in a random order, in a sense that a particular visiting order is a *realization* of some data generating mechanism (DGM). Each buyer i has a different and random order of visiting sellers. The DGM determines the arrival order of buyers at supplier k. Since the visiting order is random, the buyers' arrival order at supplier k is also random.

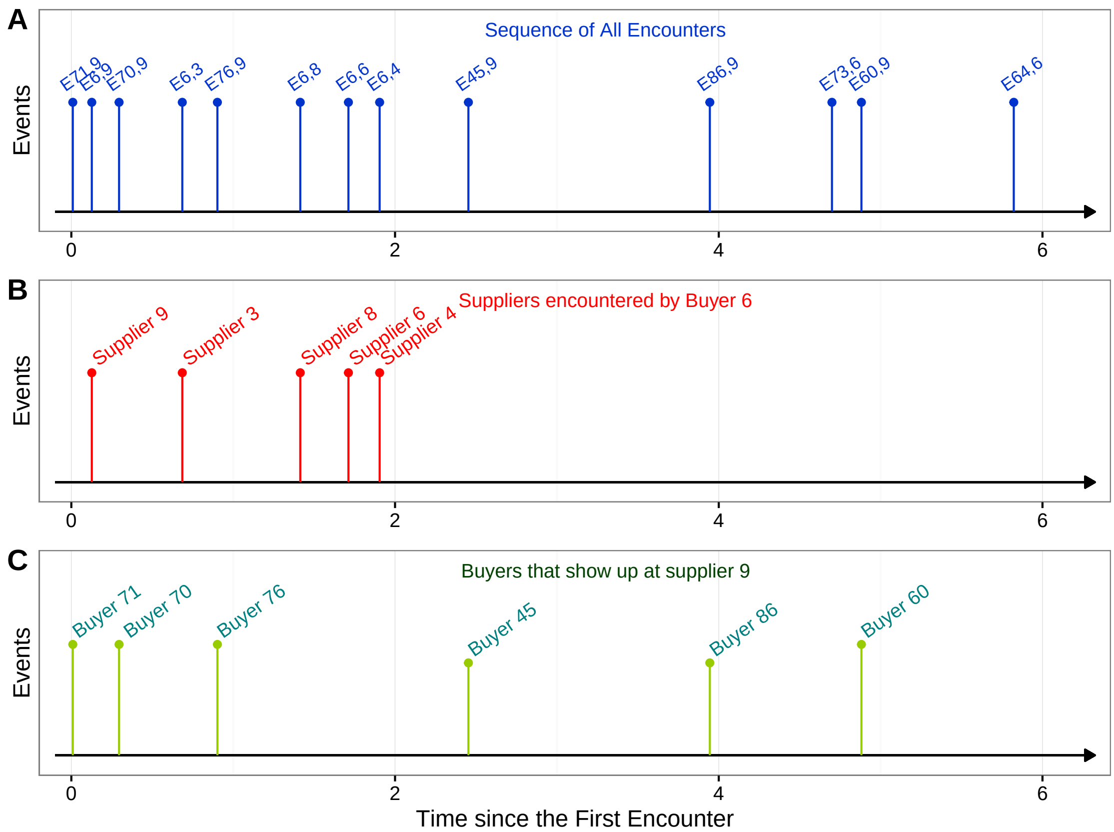
From another perspective, we can define each meeting between a buyer i and a supplier k as an *encounter*, which is a pair of a buyer and a supplier (i, k). Each encounter generates a bid. The arrival pairs (encounters) occurs in an order which we see as a *realization* of some random generating mechanism. A partition of these encounters with respect to a buyer i is the buyer i's visiting order. A partition of these encounters with respect to a supplier k can be seen as the sequence of buyers' arrivals to the supplier k. Here of interest is not the continuous time of arrivals, but the order of the arrivals. In Figure 1 is an illustration of an realized encounter sequence.

In figure 1, the timeline is plotted with respect to the time of first encounter, i.e., the time of first encounter is labeled as time 0. Since we are not concerned with the actual continuous time of arrivals but the order of the arrival, we index encounters by order position in the sequence. A (partial) particular *realized* sequence of encounters is plotted in Figure-1-A. If we partition these encounters with respect to buyers, then a particular buyer (here buyer 6[7]) partition represents a buyer's visiting sequence (to suppliers), which is plotted in Figure-1-B. If we partition these encounters with respect to suppliers, then a particular supplier (here supplier 9) partition is the buyers' arrival sequence of this particular supplier, which is plotted in Figure-1-C. Since the sequence of encounters is random generated, both buyers' visiting sequences and suppliers' buyer arrival sequences as partitions of the encounters sequence are random.

Given a particular *realized* sequence of all encounters, we can define an ordering of the buyer set with respect to a supplier k by the buyers' arriving at the supplier k. We denote the ordered set of buyers as I_k. Such ordering is k-specific since buyers'

[7]Here the number "6" in "buyer 6" does not indicate any order of the buyer set (i.e., buyer 6 is not the sixth buyer). It only serves as a convenient label of that buyers. It is no different from "buyer Z" or "buyer Alex", etc. Such label can be given since the buyer set I is at most countable. On the other hand, later in the context, we see, with respect to a particular realized sequence of encounters, at supplier k, there is a natural ordering of buyer set I_k defined by the arrival order of buyers. For example, in Figure 1, at supplier 9, the first buyer is buyer 10, and the last buyer is buyer 74.

Figure 4.1. A Timeline of a *realized* sequence of encounters



Note: Figure 1 plots an exemplary realized sequence of events. The x-axis is the time since the first encounter. Since we are not concerned with the actual continuous time of arrivals but the order of the arrival, we index encounters by order position in the sequence. An event in subplot Figure-1-A represents an arbitrary encounter. For example, "E73,6" represents the encounter between buyer 73 and seller 6. Events in subplot Figure-1-B (as a partition of the encounters in Figure-1-A) represent suppliers encountered by buyer 6. Events in subplot Figure-1-C correspond to buyers that show up at supplier 9.

arrivals sequence at different suppliers are different and in its nature random as a partition of a realization of encounters generating mechanism. Similarly, we can also define an ordering of the supplier set with respect a buyer i by the buyer's visiting sequence to the suppliers. We define the ordered supplier set as K_i. It processes the same property the ordered buyer set.

At the beginning of each trading session, each supplier k has an initial endowment or allocation of supply S_k^* which is the maximal quantity he can supply to the market.

His goal is to maximize profit[8] via market. As buyers come to the supplier one at a time, the supplier has no information concerning which buyer will arrive next, so the supplier views arrival of buyers as random. The supplier needs to choose whether he sells to each bid as it is received or wait for a potential higher bid with waiting cost. Thus, to maximize profit, the supplier k must use some criteria to make decision. In the model, such criteria is defined as the *supplier's acceptance rule*. Since there is uncertainty in arrival order and bids of buyers (and in other factors such as deterioration), the waiting cost as the cost of deterioration is treated as a random variate. Such uncertainty implies randomness in the supplier's acceptance and implies acceptance must be represented with a probability driven acceptance rule.

Each buyer i has an initial demand D_i^* to fulfill at the beginning of each trading session. At each encounter, if the buyer does not fulfill the procurement requirement, there exists an encounter-specific or an instantaneous penalty cost. For the simplicity of analysis, we assume the marginal penalty cost is constant, but buyer-specific. The penalty cost motivates the buyer to search for the product at each supplier k until some stopping time determined by buyers' stopping rule. We assume the buyer i perceives that there is randomness in the supplier's acceptance. He knows that whether his bid will be accepted or not depends on the supplier's probabilistic acceptance rule. If accepted, the buyer pays the price of products; if rejected, the buyer faces *search costs* which we assume are a function of the buyer's unfulfilled demand and what we will define as the buyer's perception of market conditions. Thus if the buyer bids high, he will have a higher probability of acceptance and securing the product. This means he can search for less time or even stop searching (when all his procurement demand is fulfilled). In this case, his expected search cost is determined. In contrast, if the bid (b_{ik}, q_{ik}) is not accepted, he will search this quantity q_{ik} again and search cost will remain uncertain. However, if he bids high, and his bid is accepted, the cost of buying the product will be high. Since the total procurement cost includes both search cost and the cost of products, a buyer will be assumed to compose bids such as to minimize the total procurement cost, making a tradeoff between the search cost and the cost of products. We assume buyers have information about the function form of suppliers' acceptance probability but not

[8]Later we see in a type 3 or type 4 trading sessions that the maximal quantity equals to s_k^* (new allocation) plus carry-over storage.

other buyers' bids (which is a part of acceptance rule 1). The buyer makes this tradeoff by choosing an optimal bid price conditioned on his information about the supplier's acceptance rule.

Buyers who have imperfect information about market must search for the products in their procurement process and hence face search costs. Stigler (1961) modeled search cost as a constant in each additional search (i.e.constant marginal search cost) (see [Sti61]). De los Santos et al. (2013) more generally specified search cost as an stochastic variable (see [DHW13]). In this paper, we model *marginal* search cost as an increasing function of market conditions defined by market excess demand. That is, we assume as market becomes tighter, it is more difficult for buyers to find an additional unit of product. Such an effect could be nonlinear for search cost can be rather high with low total supply as noted in [EW12]. Thus, we model search cost as a nonlinear increasing function of market conditions.

If the distribution of random variates (e.g. search cost) are unobservable, imperfectly informed buyers can only make their decision based on their personal estimate of the economic variates. Such personal estimates are made based on buyers' subjective belief (priors) in the market uncertainties. In later text, we show that buyers who make (personal) optimal decision based on the expectation of their personal estimate of variates are actually optimizing over their subjective expectation of the real variates. On the other hand, in the process of searching for suppliers, the buyer usually makes decisions based on information collected from past encounters. This means, an agent may learn as he searches. In our model, the buyer will learn (update his subjective belief) about the market's supply capacity (i.e. market condition) by Bayesian updating their belief on the supply capacity (and hence market condition).

The market condition plays a central role in this model. It affects the final transaction price through buyers' search costs (as stated above), suppliers' waiting costs, and their acceptance probabilities. The supplier's acceptance decision is also based on their interpretation of market conditions. If there is a large excess demand that is perceivable to both sides in the market, the buyers would bid high to secure product (since search cost is high). Suppliers who perceive such information will become reluctant to accepts bids. Hence, the probability of acceptance will decrease, and the bid(s) that supplier accepts will be high. As a result, the transaction price will be high. As shown later in this paper, if search cost increases nonlinearly in excess demand, a large excess demand yields a huge increase in search cost. Such high search cost motivates buyers to bid high to win (otherwise, they will have to face high search costs). The increases in bid prices should be about proportional to (usually higher than, since if there is shortage in supply, buyers tend to observe low stock in the first several suppliers and form a biased belief that supply is extremely short) those in search costs. This means the final transaction price as the price in the accepted bid will increase nonlinearly in excess demand. This will provide our explanation of jump in prices.

4.3 Notation

We first introduce the convention on the notations. In this paper, unless otherwise stated, we use a capital English letter X to denote random variable. We use a small English letter x to denote a scalar or a realization of a random variable. We use calligraphic letter $\mathscr{X}$ to denote a general set. We use capital Greek letters to denote a parameter set. We use a small Greek letter to denote a particular parameter. We use (math) bold letters (e.g. $\mathbb{P}$, $\mathbb{E}$) to denote a probability measure, a probabilistic distribution or an expectation.

4.3.1 Notation on Uncertainty

As stated above, there is uncertainty in the supplier's waiting cost and in the supplier's supply capacity. To account for such uncertainty, we define:
ω: a scenario corresponding to a particular waiting cost and supply capacity pair[9];
Ω: the set of scenarios that generates all possible waiting cost and supply capacity pairs (all ω's).

If both suppliers' waiting cost and supply capacity admit finite values, then it is easy to enumerate all possible scenarios (2^{Ω}). However, in our settings, both these random variates admit continuous values ($WC \in \mathbb{R}_+$, and $S^* \in \mathbb{R}_+$) and we can not enumerate all possible scenarios. In this case, to measure the possibility of a particular event (for example, $S^* \leq 6$ or $WC \in (2, 10]$), we need to define

[9]Note since we assume suppliers' waiting cost and supply capacity are identically distributed, such ω's are not k-specific. Similarly, we do not write waiting cost and supply capacity DGM (WC and S) as k-specific.

a $\sigma-$algebra. Each set in this $\sigma-$algebra is a measurable[10] event (for example, $\{\omega|WC(\omega) \in (2, 10]\}$ is an event).
$\mathscr{F}$: the $\sigma-$algebra generated by Ω; each set in $\mathscr{F}$ is an event, on which we can assign a probability measure[11];
$\mathbb{P}$: an objective probability measure defined on the measurable space $(\Omega, \mathscr{F})$.

Then the triple $(\Omega, \mathscr{F}, \mathbb{P})$ is the underlying probability space on which all random variables are defined. Let $\mathbb{E}$ be the expectation taken with respect to $\mathbb{P}$. Now we can define variates of both sides in the market. We further assume all random variables are defined on $C_b^1(\Omega)$ (the set of continuous, differentiable and bounded functions).[12]

4.3.2 Notation on Ordering and Indices

As discussed in section 2, the set of buyers I and the set of sellers K are both unordered. On the other hand, with respect to a particular seller k, we can define an order of the buyer set by their (*realized*) arrival sequence to supplier k. We denote this ordered set of buyers as I_k. In this ordered set, each buyer $i \in I$ has a unique ordered (integer) index $\tau_k \in I_k$ such that any other buyer i' with index $\tau_k' > \tau_k$ means that buyer i' come to the supplier k earlier than buyer i. It is worth noting that there is one-to-one mapping between i and τ_k. And the index set $\{\tau_k\}_{\tau=1}^{|I|}$ is k-specific. Every other index set $\{\tau_{k'}\}$ (for supplier k') is a permutation of $\{\tau_k\}$.

Similarly, with respect to a particular buyer i, we can define an order of the supplier set by the buyer's visiting order. We denote this ordered set of suppliers as K_i. Then each supplier $k \in K$ has a unique ordered index $\tau_i \in K_i$ such that for any other supplier k' with index τ_i', the buyer i visits supplier k' prior to supplier k if $\tau_i' < \tau_i$. The set of index $\{\tau_i\}_{\tau=1}^{|K|}$ has similar properties as $\{\tau_k\}_{\tau=1}^{|I|}$.

In the context where supplier k is definite (for example, some variate $X_{\tau_k k}$ is

[10]"Measurable" in a sense that one can assess the possibility of the event, objectively or subjectively.

[11]Here, we assume all the uncertainties in the model are those generated from waiting cost and supply capacity. A more technical formulation is: for some underlying metric space (Ω, ρ), where each ω represents some randomness, define two continuous and bounded mappings waiting cost WC and (local) supply capacity S_k^* from (Ω, ρ) to $(\mathbb{R}_+, \lambda)$, where λ is the Lebesgue measure. Denote $\sigma_{WC} = \{WC^{-1}(B) \text{ for } \forall B \in \mathscr{B}\}$ be the σ-algebra generated by waiting cost WC, and σ_{S^*} the σ-algebra generated by supply capacity. Let $\mathscr{F} = \sigma(\sigma_{WC} \cup \sigma_{S^*})$. Then $(\Omega, \mathscr{F})$ is a measurable space and both WC and S^* are random variables on this space.

[12]It is worth noting that continuity and differentiability requires some metric on Ω. Indeed, if WC and S^* admit values in $\mathbb{R}_+$, then $\Omega = \mathbb{R}_+^2$. Let ρ be the euclidean distance on $\mathbb{R}_+^2$, then (Ω, ρ) is a metric space. And thus continuity and differentiability are properly defined.

clearly related to supplier k), for simplicity, we will write τ instead of τ_k (in this case, we write $X_{\tau k}$ instead of $X_{\tau_k k}$). Similar rule applies to τ_i.

In the case when both the set I and the buyer are numerically labeled from $i = 1$ to $i = |I|$ where $|\cdot|$ is the cardinality, the one-to-one mapping between I and I_k (for any k) is a permutation matrix and hence an isometry. Then it is essentially the same to write i or τ_k. Same thing applies to k and τ_i. On the other hand, for consistency, we will keep write τ_k and τ_i when ordering is involved.

4.3.3 Buyer Side Notation

There is an finite unordered set of buyers I. For each buyer $i \in I$, his characteristics are:

D_i^*: initial procurement demand that buyer i seeks to fulfill at the beginning of each trading session; if he fails to fulfill such demand, he will face a cost (or penalty);

(i, k): a buyer and supplier pair to denote there is an encounter between buyer i and supplier k (or, when the buyer i visit to supplier k); every encounter generates a price-quantity bid pair;

(b_{ik}, q_{ik}): the price-quantity bid pair he offers to supplier k, where q_{ik} is the bid price, and b_{ik} is the bid quantity;

c_i: per-unit instantaneous penalty cost of unsatisfied demand;

D_i^{*k}: the unfulfilled procurement demand (the amount he needs to search) before he bids at supplier k, that is the total demand minus the amount of product he already procured ($D_i^{*k} = D_i^* - \sum_{\tau' < \tau} q^a_{i\tau'}$), where τ is the index of supplier $k \in K$ in the ordered set K_i ;

$SC_{ik}(\omega, D_i^{*k}, \text{ market conditions}, \text{ current supplier's supply})$: his search cost given k-th supplier which is unknown before hand and is an increasing function of quantity needed D_i^{*k} and the market conditions; it is a decreasing function of current supplier's supply;

$\mathbb{Q}_i$: his subjective probability measure on $(\Omega, \mathscr{F})$; it reflects the buyer's personal belief in the uncertainty in the system (waiting cost and supplier's capacity); let $\mathbb{E}_{\mathbb{Q}_i}$ be the expectation taken with respect to $\mathbb{Q}_i$. The buyer i generates his estimate of supplier's waiting cost and supply capacity based on his personal belief.

Thus at the beginning of each trading session, a buyer has an initial demand D_i^* to fulfill. At supplier k, his offers a bid of (b_{ik}, q_{ik}) a price-quantity pair. He holds his own subjective belief in the unknowns, which is characterized by his subjective probability $\mathbb{Q}_i$.

4.3.4 Supplier Side Notation

There is an finite unordered set of suppliers K. For each supplier $k \in K$, his characteristics are:
$S_k^*(\omega)$: the supplier k's supply capacity; we assume this supply capacity is random. Supply is identically distributed across different suppliers (locations) by some data generating mechanism $S^*(\omega)$. In this model each supplier can be seen to have a unique location, and thus later in this article, S_k^* is also referred as local supply.
s_k^*: the *realized* suppler k's capacity (or, realized local supply) and *observed* by buyers;
$WC_{ik}(\omega)$: his waiting cost before the buyer i offers a bid; it is a random variable that depends on market conditions;
$B_{ik}(\omega)$: B_{ik} is the supplier's choice or control. We define it as a binary indicator based on acceptance of the buyer i's bid (b_{ik}, q_{ik}); that is, B_{ik} is 1 if the bid (b_{ik}, q_{ik}) is accepted by supplier k and 0 if the bid is not accepted; it is a random variable since the acceptance rule is probability-driven and conditioned by market condition;
P_{ik}(market conditions): as B_{ik} is binary, we define P_{ik} as the probability that a bid is accepted at the encounter between i and k;
$q_{ik}^a(\omega)$: the quantity of fulfillment associated with the buyer i's bid *accepted* by supplier k; in a simple case, $q_{ik}^a \equiv B_{ik} q_{ik}$;
$\mathbb{C}_k$: the supplier k's optimal set of accepted bids, i.e. $\mathbb{C}_k = \{i \in I | B_{ik}^* = 1\}$;
$\mathbb{Q}_k$: his subjective probability measure on $(\Omega, \mathscr{F})$; it reflects the supplier k's personal belief in the uncertainty in the system (waiting cost and supplier's capacity); let $\mathbb{E}_{\mathbb{Q}_k}$ be the expectation taken with respect to $\mathbb{Q}_k$.

The sequence of buyers' arrivals to supplier k is generated from a random data generating mechanism. The sequence of buyer's arrivals at a supplier (or, encounter) generates a sequence of bids. Thus the supplier k faces a *realized*, ordered sequence of bids $\{(q_{\tau k}, b_{\tau k})\}_{\tau \in I_k}$. The supplier k only observes a partially realized ordered sequence of bids. Upon the arrival of a particular buyer i with index $\tau \in I_k$, the supplier k only knows past arrivals and their bids prior to buyer i, i.e, $\{q_{\tau' k}, b_{\tau' k}\}_{\tau' < \tau}$. If we define the set of buyers that comes earlier than i (or, with index strictly smaller than τ) as $I_{<\tau}$[13], the supplier k's information about past bids can be written as $\{(q_{ik}, b_{ik})\}_{i\in I_{<\tau}}$. The supplier uses this information to assess whether there is a potential higher bid in the future and decide whether to accept the current bid.

4.3.5 Market Condition Notation

For the market conditions, we define:
$\sum_{i\in I} D_i^*$: total initial market demand at the beginning of each trading session;
$\sum_{k\in K} S_k^*(\omega)$: total market supply; this is a random variate since there is shock in supply, as discussed above;
$Z(\omega) = \sum_{i\in I} D_i^* - \sum_{k\in K} S_k^*(\omega)$: actual excess demand in market at the start of each trading session, unobservable to buyers.

4.3.6 Agent's Information, Learning, and Estimate Notation

In this model, agents on neither side of the market have perfect information. Their information evolves as they have encounters. That is, a buyer has a sequence of increasingly richer information set as he visits more suppliers; and a supplier has richer information as more buyers visit him.
Φ_i^k: buyer i's information set when he is at supplier k. Note each supplier k announce his supply capacity before the buyer offers a bid, thus his information set before offering bid to supplier k (with index τ_i). Φ_i^k includes:

1. all the supply from previous supplier including supplier k, $\{S_{1_i}, S_{2_i}, ..., S_{\tau_i}\}$;
2. the sequence of past accepted quantity: all the accepted quantity by previous suppliers excluding supplier k, that is, $\{q_{i1}^a, q_{i2}^a, ..., q_{i(\tau-1)}^a\}$;
3. the function form of supplier's acceptance probability.

After offering the bid to supplier k, the supplier announces whether the bid will be accepted or not (or, accepted quantity). That is, the acceptance experience signals market conditions. Then the buyer's information set (after the supplier's announcement of accepted quantity) becomes to $\Phi_i^k \cup \{q_{i\tau}^a\}$.

[13] Similarly we denote the set of suppliers visited by buyer i earlier than supplier k (with index $\tau \in K_i$) as $K_{<\tau}$

To define the relation between this information set and personal belief, we note a perfectly informed agent must have his belief $\mathbb{Q}_i$ agreeing with the true (objective) probability $\mathbb{P}$. That is, he knows all the waiting cost and supplier capacity pairs and is able to assess any event with correct probability. For an imperfectly informed individual, in a simple case, suppose the buyer i has partial but correct information, then for two sets with different $\mathbb{P}$ probability, $\mathbb{Q}_i$ may admit the same probability[14]. In a more general case, buyer i may have some misinformation, which means $\mathbb{Q}_i$ and $\mathbb{P}$ do not agree for any set in $\mathscr{F}$ (the buyer hold a "wrong" subjective belief for an event, "wrong" in sense it is different than the *true* probability).

Since buyers do not have perfect information about market condition, waiting cost and other buyers' bids, he must estimate these variates, on which he forms his best *personal* optimal bid price.

$\widetilde{X^i}(\omega)$: for a random variable $X(\omega)$, we use $\widetilde{X^i}(\omega)$ to denote the buyer i's estimate of this random variable $X(\omega)$; here $\widetilde{X^i}(\omega)$ is a random variable which is (almost surely) defined by the distribution function $\mathbb{Q}_i \circ X^{-1}$, where X^{-1} is the inverse of X and $\circ$ is composition[15]. That is, the cumulative density function $F_{\widetilde{X}_i}(t) = \mathbb{Q}_i(X^{*-1}((-\infty, t]))$, for $\forall t \in \mathbb{R}$.[16]

$\widetilde{S_k}^i(\omega)$: for buyer i and his visiting sequence K_i, at the time he visits supplier k with index $\tau \in K_i$, he only knows about past suppliers' supply $\{S^*_{\tau'}\}_{\tau'<\tau}$; for the suppliers he has not visited, he must estimate the supply capacity; here we use $\widetilde{S_k}^i(\omega)$ to denote the buyer i's estimate of the k-th supplier's capacity;

$\widetilde{Z}^i(\omega) = \sum_{i\in I} D^*_i - \sum_{k\in K} \widetilde{S_k}^i(\omega)$: at the time of encounter ik, the buyer i estimates the available supply capacity in the market based on his information about past visit $\{S^*_{\tau'}\}_{\tau'<\tau}$; we use $\widetilde{Z}^i(\omega)$ to denote the buyer i's estimate of market condition (excess demand Z).

Similarly, a supplier k has richer information set as more buyers arrive. To characterize his information and personal estimate, we define:

Ψ^i_k: supplier k's information set when the buyer i (with index $\tau_k \in I_k$) arrives and makes a bid to him. In general Ψ^i_k includes:

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[14]That is, $\sigma(\{\mathbb{Q}_i^{-1}(a) : \text{ for } \forall a \in [0,1]\}$, which is a strict subset of $\sigma(\{\mathbb{P}^{-1}(b) : \text{ for } \forall b \in [0,1]\}) = \mathscr{F}$.

[15]To simplify notation, I will write $\mathbb{P} \circ X^{-1}$ as $\mathbb{P}X^{-1}$ to denote the probability distribution of X on the probability space $(\Omega, \mathscr{F}, \mathbb{P})$ in the following text.

[16]More accurately, a buyer makes his estimate based on the *observed* value of random variates. Thus the estimate must be measurable with respect to the filtration $\mathscr{F}_k$, where $\mathscr{F}_k$ is the sigma algebra generated by supplier waiting cost and supply capacity up to supplier k.

1. all the past buyers' bids received by supplier k prior to (and including) buyer i, $\{(q_{ik}, b_{ik})\}_{i \in I_{\leq \tau_k}} = \{(q_{\tau' k}, b_{\tau' k})\}_{\tau' \leq \tau_k}$, where $I_{\leq \tau_k}$ is defined as the set of buyers that comes to the supplier k earlier than (including) buyer i;

2. the waiting costs for the most recently encountered buyers' waiting costs $\{WC_{ik}\}_{i \in I_{\leq \tau_k}}$, where $I_{\leq \tau_k}$ is defined as above.

When suppliers can be assumed to be able to observe other supplier decisions and their available supply (e.g. with a small set of suppliers), supplier k will have rich information about market conditions. It follows the supplier information set Ψ_k^i can be assumed to include (in addition to all information defined as above) : market condition Z, other suppliers' supply $\{S_j^*\}_{j \neq k}$ and market equilibrium price b^e.We use $\widetilde{X}^k(\omega)$ to denote the supplier k's estimate of a random variate $X(\omega)$ such that its distribution is defined as $\mathbb{Q}_k \circ X^{-1}$, where $\mathbb{Q}_k$ is his personal belief.

The reason to use such formulation to model buyer's personal estimate of supply is three fold. First, there are no subjective random variables. To represent a buyer's personal estimate of some economic (random) variate, we need a proper definition. Second, a person does not estimate or predict economic variates independently; instead, he holds a fundamental belief in the market uncertainties, on which he forms his estimates of random variable. Last, but most importantly, this is a consistent notation, in a sense that the objective (true) expectation of the estimate equals to the personal expectation of the true random variable. To see this, for any random variable X on $(\Omega, \mathscr{F})$, define $\tilde{X}$ by the distribution $\mathbb{Q}X^{-1}$, then $\mathbb{P}\tilde{X}^{-1} = \mathbb{Q}X^{-1}$, since they both are the distribution function (equivalent measure on $(\mathbb{R}, \mathscr{B})$). In further, for any continuous function f such that $f(X)$ has finite first moment, we have:

$$\begin{aligned} \mathbb{E}_{\mathbb{P}} f(\tilde{X}) = \int_\Omega f(\tilde{X}(\omega))\mathbb{P}(d\omega) = & \int_{\mathbb{R}} f(x)\mathbb{P}\tilde{X}^{-1}(dx) \\ = & \int_{\mathbb{R}} f(x)\mathbb{Q}X^{-1}(dx) = \int_\Omega f(X(\omega))\mathbb{Q}(d\omega) = \mathbb{E}_{\mathbb{Q}} f(X) \end{aligned} \tag{4.1}$$

That is, the expectation of the personal estimate of the variate equals to the personal expectation of the real variate. Thus buyers who make optimal decisions based on the expectation of their estimate of variates are actually optimizing over their personal expectation of real variates[17].

[17] $\max_c \mathbb{E}_{\mathbb{P}} f(c; \widetilde{X}) = \max_c \mathbb{E}_{\mathbb{Q}_i} f(c; X)$ where X is a random variate, $\widetilde{X}$ is the estimate of X and

4.4 Model

4.4.1 Buyer Side

In this section we present a model of the buyer's choice problem. As stated in section 3, each buyer i visits suppliers in a random order which can be seen as a realization of some data generating mechanism. A particular *realized* visiting sequence gives the unordered supplier set K an ordering K_i. Then for a supplier k, suppose he is the τ-th supplier that buyer i encounters, then we say his index is τ in the ordered set K_i. At the time that buyer i visits supplier k, his information set Φ_i^k includes his price-quantity bid offered to previous suppliers, the suppliers' accepted quantity and their supply capacity., i.e. $\{(b_{i\tau'}, q_{i\tau'}), S^*_{\tau'}, q^a_{i\tau'}\}_{\tau'<\tau}$. Recall, we label the set of suppliers visited by buyer i earlier than supplier k (with index $\tau \in K_i$) as $K_{<\tau}$. Thus, the buyer's information set Φ_i^k includes $\{(b_{ik'}, q_{ik'}), S^*_{k'}, q^a_{ik'}\}_{k'\in K_{<\tau}}$[18]. At each supplier k, the buyer i must choose the optimal price-quantity bid to minimize his total cost conditioned [19]on his information set Φ_i^k, i.e. :

$$\begin{aligned} \min_{(q_{ik}, b_{ik})} \quad & \mathbb{E}_{\mathbb{Q}_i}\Big\{q^a_{ik}b_{ik} + SC_{ik} + c_i(D_i^{*k} - q^a_{ik})^+\Big\} \text{ for } \forall k \\ \text{subject to} \quad & q^a_{ik} \leq S^*_k \quad \forall k \in K \end{aligned} \tag{4.2}$$

where D_i^{*k} is the unsatisfied procurement demand (the amount he needs to search) at the ik-th encounter, i.e. before he bids at supplier k, that is the total demand minus the amount of product he already procured (recall $D_i^{*k} = D_i^* - \sum_{\tau'<\tau} q^a_{i\tau'}$). $q^a_{i\tau}$ is the accepted quantity by the τ-th supplier (in the ordered set K_i), i.e. $q^a_{ik} = B_{ik}q_{ik}$. If the buyer i has full information about supplier k's acceptance rule and the supply S^* (and hence $\mathbb{Q}_i$ and $\mathbb{P}$ agree)[20], the expected accepted quantity $\mathbb{E}_{\mathbb{Q}_i}(q^a_{ik}) =$

c is some deterministic variate.

[18]Similarly, we write set of supplier encountered by buyer i later than (including) supplier k as $K_{\geq\tau}$.

[19]Intuitively, the objective of problem (2) is to minimize the expectation conditioned on his current information $\mathbb{E}_{\mathbb{Q}_i}\Big\{q^a_{ik}b_{ik} + SC_{ik} + c_i(D_i^* - q^a_{ik})^+ | \Phi_i^k\Big\}$. Since buyer i's personal belief $\mathbb{Q}_i$ evolves as his information set Φ_i^k becomes richer and hence has integrated the effect of information, we do not use the conditional expectation notation.

[20]Actually, in this case, the buyer i forms a "correct" expectation, in a sense that for $\forall \tilde{X}$ defined by $\mathbb{Q}_i X^{-1}$, and for $\forall$ continuous function f, $\mathbb{E}_{\mathbb{P}} f(\tilde{X}) = \int_\Omega f(\tilde{X}(\omega))\mathbb{P}(d\omega) = \int_{\mathbb{R}} f(x)\mathbb{P}\tilde{X}^{-1}(dx) = \int_{\mathbb{R}} f(x)\mathbb{Q}X^{-1}(dx) = \int_\Omega f(X(\omega))\mathbb{Q}(d\omega) = \int_\Omega f(X(\omega))\mathbb{P}(d\omega) = \mathbb{E}_{\mathbb{P}} f(X)$. In particular, the subjective expectation in the objective function in problem (2) becomes to objective expectation,

$\mathbb{E}_{\mathbb{Q}_i}\Big\{B_{ik}q_{ik}\Big\} = \mathbb{E}\Big\{B_{ik}q_{ik}\Big\} = P_{ik}q_{ik}$ and hence the buyer best *subjective* response is indeed the optimal one . In a more general case, the buyer knows only partially about supplier k's acceptance rule and market condition. Then $\mathbb{E}_i f(\tilde{X}) \neq \mathbb{E}f(X)$. Thus the buyer's best response shifts away from the objectively best one, to what extent his subjective belief is biased.

Problem (4.2) says at every supplier k, buyer i tries to minimize the total expected cost. Such cost includes the money he pays for the bid $q^a_{ik}b_{ik}$, search cost for supplier k and instantaneous penalty cost $c_i(D^*_i - q^a_{ik})^+$. Note $c_i(D^*_i - q^a_{ik})^+ \equiv c_i \max(0, D^*_i - q^a_{ik})$ in the penalty cost means we only count the positive part of penalty cost. On the other hand, since once $D^*_i - q^a_{ik}$ hits zero the buyer will be out of market, we will drop + sign from now on.

To solve the problem (4.2), at a particular supplier k^*, the risk neutral buyer i needs to decide whether to bid or not at supplier k^*. If he does not bid at supplier k^*, his procurement cost minimization problem becomes to:

$$\begin{aligned} \min_{(q_{ik},b_{ik})} \quad & \mathbb{E}_{\mathbb{Q}_i}\Big\{q^a_{ik}b_{ik} + SC_{ik} + c_i(D^{*k}_i - q^a_{ik})^+\Big\} \text{ for } \forall k \\ \text{subject to} \quad & q_{ik^*} = 0 \\ & q^a_{ik} \leq S^*_k \quad \forall k \in K \setminus \{k^*\} \end{aligned} \tag{4.3}$$

Note problem (4.3) is equivalent to problem (4.2) with an additional constraints $q_{ik^*} = 0$. Then the solution to problem (4.3) must be no lower than that to problem (4.2). Thus, to minimize his procurement cost, the buyer places a bid at each supplier k.

4.4.1.1 Buyer's Search Cost and Searching Rule

Each buyer must search for suppliers and products, in what process he faces search cost (SC_{ik} in equation (4.2)). Intuitively, a buyer's search cost depends on market condition and the amount of products he is searching. On the other hand, a buyer does not know his search cost beforehand and hence he has to estimate it. Such estimate of search cost should also depend on the supply capacity at current encounter. If the buyer observes a low supply at current supplier (say, suppose the buyer i is now at supplier k), he will expect a higher search cost in the future. Thus

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and hence the buyer's best subjective response is indeed the best objective one.

the buyer's *estimated* search cost is a function of his estimate of market condition, the amount of products for search, and *observed* supplier k's supply s_k^*:

$$\widetilde{SC_{ik}}^{i} = f(D_i^{*k} - q_{ik}^a, \widetilde{Z}^i, s_k^*) \tag{4.4}$$

where $\widetilde{SC_{ik}}^{i}$ is the buyer i's estimated search cost, D_i^{*k} and q_{ik}^a are as defined above, $\widetilde{Z}^i$ is the buyer's estimate of market condition (we will specify this later in context) and f is a nonlinear function. To derive some properties of the functional form of f, we assume f is smooth function that is twice differentiable with respect to Z and s_k^* to characterize the curvature (nonlinearity). We note that search cost should be an increasing function of the quantity to be searched after current encounter $(D_i^{*k} - q_{ik}^a)$ and the market excess demand. In addition, as discussed in section II, search cost should increase nonlinearly in market excess demand. Thus, given the differentiability assumption of search cost, the first- and second-order derivatives of search cost with respect to market condition and the first-order derivative of search cost with respect to the quantity to be search should be positive.

Further, as discussed above, search costs should be able to characterize the effect of current supplier's supply on the buyer's personal estimate of search cost. That is, if the buyer i observes that the current supplier k's stock is “surprisingly" low, then the buyer's estimate of search cost will increase nonlinearly. These assumptions imply that buyer search cost is a decreasing function of supply capacity at current encounter with a decreasing second derivative[21]. In summary, these assumptions regarding search cost can be summarized as follows:

$$\frac{\partial \widetilde{SC_{ik}}^{i}}{\partial \widetilde{Z}^i} > 0, \frac{\partial \widetilde{SC_{ik}}^{i}}{\partial (D_i^{*k} - q_{ik}^a)} > 0, \frac{\partial \widetilde{SC_{ik}}^{i}}{\partial s_k^*} < 0, \frac{\partial^2 \widetilde{SC_{ik}}^{i}}{\partial \widetilde{Z}^{i2}} > 0, \frac{\partial^2 \widetilde{SC_{ik}}^{i}}{\partial s_k^{*2}} > 0 \tag{4.5}$$

An exemplary specification of the search cost which we use later for analytical solution is

$$\widetilde{SC_{ik}}^{i} = (D_i^{*k} - q_{ik}^a)(\widetilde{Z}^i)^2/\beta s_k^*. \tag{4.6}$$

where β is the parameter specified later in the simulation section. It is easy to check that this formulation agrees with the desired properties of search cost specified in

[21]That is, as $s_k^* \to 0$, $\frac{\partial \widetilde{SC_{ik}}^{i}}{\partial s_k^*}$ increases.

equation (5).

A risk neutral buyer will keep searching as long as his estimated search cost does not exceed his expected marginal benefits from search. Recall in this model, a buyer is assumed to remember only what happened in a single trading session ("short memory") and concerns only what will happen in that session, which we label as "myopic". Since a myopic buyer has an instantaneous penalty cost at every encounter with supplier in current trading session, every unit of unfulfilled demand admits an penalty cost for every encounter left in that trading session. As a result, he will not stop searching as long as his estimated search cost does not exceed the corresponding penalty cost[22] [23]. These results can be summarized as the buyer's optimal stopping rule.

Optimal Stopping Rule: A risk neutral buyer will stop searching if and only if his expected marginal benefits from search is smaller than his estimated search cost. In our settings (equation 2), a myopic (and risk neutral) buyer will stop searching if and if only his estimated search cost exceeds the corresponding penalty cost.

4.4.1.2 Buyers' Learning Rule

A buyer has imperfect information about market conditions. As the buyer i encounters suppliers, the buyer usually makes decisions based on prior information, learning information and past search information. This means, an agent learns as he searches. In this setting, the buyer will learn (updates his subjective belief) about the supplier's individual supply capacity S_k^* (and hence market condition[24]) by Bayesian updating their belief on the supply capacity (and hence market condition).

Most "search with learning" models assume the agent has complete knowledge about the distribution of prices. With such an assumption, it can be shown that there exists an optimal stopping rule that controls and truncates further sampling from the distribution. Such a stopping rule is myopic and defines a reservation price.

[22]For a buyer that is not myopic, he will not stop searching until all his demand is fulfilled since every unit of unfulfilled demand admits an infinite penalty cost in the infinite remote future. If we in further allows for buyers' hoarding, then he will never stop search another case since every *potential* unit of unfulfilled demand admits an infinite penalty cost in the infinite remote future.

[23]In another case, if a myopic buyer does not have instantaneous penalty cost but penalty cost at the end of each trading session, then the buyer will not stop search as long as his marginal penalty cost at the end of each session does not exceed his estimated search cost

[24]This can be done since supply is assumed to be identically distributed.

The existence of optimal stopping rule which is myopic and admits an reservation price was extended to the case in which an agent does not have complete information of the distribution of prices. Rothschild (1974) assumed price admits only finite values (multinomial Dirichlet distribution) and showed that the optimal stopping rule also has the reservation price under an unknown distribution (see [Rot74]). Bikhchandani and Sharma (1996) in [BS96] further relaxed the assumption and assumed prices can be any real number. Using a Dirichlet process, they provided sufficient conditions for the existence of optimal stopping rules with the reservation property. Santos et al. (2013) applied such method to the case where the unknown opportunity is a utility function instead of price (see [DHW13]).

In our model, the buyer is searching for products (or, supply). It is too restrictive to assume the quantity of products for search can take only finite values following a multinomial Dirichlet distribution. In particular, we allow for supply to take any positive integer value. Since Dirichlet process is conjugate to any prior distribution (of supply), we follow [BS96] and [DHW13] and use a Dirichlet process to model the buyer's Bayesian learning rule.

Formally, let $(\Omega^{\ddagger}, \mathscr{F}^{\ddagger}, \mathbb{P}^{\ddagger})$ be an underlying probability space representing the randomness in the random measure. Suppose the random variable $S^{*}(\omega)$ admits any value in $\mathbb{R}_{+}$ (suppliers do not have negative stock and cannot sell short). Let $\mathscr{B}$ be the Borel σ-algebra of $\mathbb{R}_{+}$. Then $(\mathbb{R}_{+}, \mathscr{B})$ is a measurable space. A mapping $P(\omega^{\ddagger}, B)$ from $\Omega^{\ddagger} \times \mathscr{B}$ to [0,1] is called a *random probability measure* on $(R_{+}, \mathscr{B})$ if:
(1) for fixed $\omega^{\ddagger} \in \Omega^{\ddagger}$, P is a probability measure on $(R_{+}, \mathscr{B})$;
(2) for fixed $B \in \mathscr{B}$, P is a random variable on $(\Omega^{\ddagger}, \mathscr{F})$.
A random measure D is distributed according to a Dirichlet process with base distribution $\mathbb{G}$ and concentration parameter α if for any finite measurable partition $(A_1, A_2, ..., A_n)$ of $\mathbb{R}_{+}$, the random vector

$$(D(A_1), D(A_2), ..., D(A_n)) \sim \mathrm{Dir}(\alpha\mathbb{G}(A_1), \alpha\mathbb{G}(A_2), ..., \alpha\mathbb{G}(A_n)), \tag{4.7}$$

where $\mathrm{Dir}(\alpha\mathbb{G}(A_1), \alpha\mathbb{G}(A_2), ..., \alpha\mathbb{G}(A_n))$ is a Dirichlet distribution with parameters $\alpha\mathbb{G}(A_1), \alpha\mathbb{G}(A_2), ..., \alpha\mathbb{G}(A_n)$, denoted as $D \sim DP(\alpha, \mathbb{G})$ (see [Fer73]).

A Dirichlet process prior is conjugate for any distribution. That is, a Dirichlet process is the conjugate prior for any infinite, nonparametric discrete distributions. That means we do not need to assume any particular distribution of the observed sample. In our setting, we assume the prior that each supplier's capacity $S^*_k \sim D$. A buyer who has observed $s^*_1, s^*_2, ..., s^*_\tau$ [25] that are not assumed to follow any particular distribution. Then the buyer who observation has a posterior distribution

$$D|s^*_1, s^*_2, ..., s^*_\tau \sim DP(\alpha + \tau, \frac{\alpha}{\alpha+\tau}\mathbb{G} + \frac{\tau}{\alpha+\tau}\frac{\sum_{j=1}^{\tau}\delta_{s^*_j}}{\tau}) \tag{4.8}$$

where $\delta_{s^*_j}$ is the point mass at s^*_j and τ is the number of suppliers that buyer i has already encountered. That is, the buyer i is currently bidding at supplier k with index $\tau \in K_i$. Note that the posterior base distribution is a convex combination of the prior base distribution $\mathbb{G}$ and the empirical distribution $\frac{1}{\tau}\sum_{k=1}^{\tau}\delta_{S^*_j}$. The posterior base distribution is also the predictive distribution of S^*_{n+1} given $s^*_1, s^*_2, ..., s^*_\tau$ and can be written as[26]:

$$S^*_{n+1}|s^*_1, s^*_2, ..., s^*_\tau \sim \frac{\alpha}{\alpha+\tau}\mathbb{G} + \frac{\tau}{\alpha+\tau}\frac{\sum_{j=1}^{\tau}\delta_{s^*_j}}{\tau} \tag{4.9}$$

Thus, under perfect information, the best (objective) estimator of $S^*_{\tau+1}$ conditioning on $S^*_1, ..., S^*_\tau$ is its posterior mean:

$$\widehat{S^*_{\tau+1}}|S^*_1, S^*_2, ..., S^*_\tau = \mathbb{E}(S^*_{\tau+1}|S^*_1, S^*_2, ..., S^*_\tau) = \frac{\alpha}{\alpha+\tau}\mathbb{E}\mathbb{G} + \frac{\tau}{\alpha+\tau}\frac{\sum_j^{\tau} S^*_j}{\tau} \tag{4.11}$$

where $\mathbb{E}\mathbb{G}$ denotes the expectation taken with respect to $\mathbb{G}$.[27]

On the other, an imperfect informed buyer does not know the true base distribution $\mathbb{G}$ and the concentration parameter α. Suppose his personal estimate of $\mathbb{G}$ is $\widetilde{\mathbb{G}}^i$ and his estimate of α is $\tilde{\alpha}^i$. Then his best personal estimator of $S^*_{\tau+1}$ given his

[25]Note here the indices $\{1, 2, ..., n\}$ in $s^*_1, s^*_2, ..., s^*_\tau$ are indices of s_τ in the ordered set K_i

[26]More accurately, the $S^*_{\tau+1}|s^*_1, s^*_2, ..., s^*_\tau$ follows the base distribution of the posterior distribution of Dirichlet Process.

$$\begin{aligned} \mathbb{P}(S^*_{\tau+1}(\omega) \in B|s^*_1, s^*_2, ..., s^*_\tau) &= \mathbb{E}^\ddagger\{D(B)|s^*_1, s^*_2, ..., s^*_\tau\} \\ &= \frac{\alpha}{\alpha+\tau}\mathbb{G}(B) + \frac{\tau}{\alpha+\tau}\frac{\sum_{j=1}^{\tau}\delta_{s^*_j}(B)}{\tau}, \ \forall B \in \mathscr{B} \end{aligned} \tag{4.10}$$

where $\mathbb{E}^\ddagger$ is the expectation taken with respect to $P^\ddagger$ on $\Omega^\ddagger$.

[27]To simplify notation, define $\mathbb{E}\mathbb{G} \equiv \mathbb{E}X$, for X a random variable distributed as $\mathbb{G}$ (or $\mathbb{P}X^{-1}((-\infty, x]) = \mathbb{G}((-\infty, x])$, for $\forall x \in \mathbb{R}_+$).

observation $S_1^*, ..., s_\tau^*$ is:

$$\widetilde{S_{\tau+1}^*}|S_1^*, S_2^*, ..., S_\tau^* = \mathbb{E}_{\mathbb{Q}_i}(S_{\tau+1}^*|S_1^*, S_2^*, ..., S_\tau^*) = \frac{\widetilde{\alpha}^i}{\widetilde{\alpha}^i + \tau}\mathbb{E}\widetilde{\mathbb{G}}^i + \frac{\tau}{\widetilde{\alpha}^i + \tau}\frac{\sum_i^\tau S_j^*}{\tau} \tag{4.12}$$

To verify this learning process is consistent with our model, we need to examine the asymptotic behaviour of $\widetilde{S}^*$. In order to do so, suppose we have countable infinite suppliers ($|K| = \infty$), then we note as the observations $\tau \to \infty$ the total number of suppliers, in equation (7) the first part of (rhs) $\mathbb{E}\mathbb{G}$ converges pointwise to 0, the second part of (rhs) converges to S^* in distribution (see [Lo83]). As a result, $\widehat{S_{\tau+1}^*} \xrightarrow{d} S^*$ as $\tau \to \infty$. By similar argument, we have

$$\widetilde{S_{\tau+1}^*} \xrightarrow{d} S^* \text{ as } \tau \to \infty \tag{4.13}$$

This means for whatever initial belief of α and $\mathbb{G}$ the buyer holds, after he observes enough suppliers, he will have a good estimate of each supplier's supply (or, local supply). We also note (10) is equivalent to $P\widetilde{S_{\tau+1}^*}^{-1} \Rightarrow PS^{*-1}$, where $\Rightarrow$ is *weak convergence* of measures on $(\mathbb{R}_+, \mathscr{B})$. On the other hand, since we constructed the buyer i's estimate $\widetilde{S}^*$ by the distribution $\mathbb{Q}_i S^{-1}$, then $\mathbb{Q}_i S^{*-1}$ and $\mathbb{P}(\widetilde{S}^i)^{-1}$ must coincide. Thus we have:

$$\mathbb{Q}_i^\tau S^{*-1} \Rightarrow \mathbb{P}S^{*-1} \text{ on } (\mathbb{R}_+, \mathscr{B}). \tag{4.14}$$

where $\mathbb{Q}_i^\tau \equiv \mathbb{Q}_i$ when buyer i is at supplier τ. This says given enough observations, the buyer's estimate of (the distribution of) S_k^* will become rather accurate (say). In further, if there is no uncertainty in the waiting cost, then the buyer's belief $\mathbb{Q}_i$ weakly converges to the objective measure $\mathbb{P}$[28]. This means $\mathbb{E}_{\mathbb{Q}_i}X = \mathbb{E}_{\mathbb{P}}X$, for any continuous bounded X. This means the optimization problem (2) admits the true optimal bid strategy.

These results can be summarized as the buyer's learning rule.

Learning Rule of Local Supply: Recall the local supply is each supplier's indi-

[28] *Proof*: For $\forall\mathscr{F}$-measurable set C_Ω closed in (Ω, ρ), since $\mathscr{F} \subseteq \sigma_S^*$, $C_\Omega \in \sigma_S^*$. Then there exist a set $C_{\mathbb{R}_+} \in \mathscr{B}$ such that $S^{*-1}(C_{\mathbb{R}_+}) = C_\Omega$. By S^{*-1} is continuous, such set $C_\mathbb{R}$ must be closed in $(\mathbb{R}_+, \rho)$. By $\mathbb{Q}_i^\tau Z^{-1} \Rightarrow \mathbb{P}Z^{-1}$, we know $\limsup_\tau \mathbb{Q}_i^\tau Z^{-1}(C_\mathbb{R}) \leq \mathbb{P}Z^{-1}(C_\mathbb{R})$. Thus $\limsup_\tau \mathbb{Q}_i^\tau(C_\Omega) \leq \mathbb{P}(C_\Omega)$, i.e. $\mathbb{Q}_i \Rightarrow \mathbb{P}$ weakly.

vidual supply S_k^* and it follows the random distribution D. The random distribution is distributed as Dirichlet process $DP(\alpha, \mathbb{G})$, where $\mathbb{G}$ is prior base distribution which represents prior knowledge or belief about the supply capacity and the random distribution. A imperfectly informed buyer i holds personal estimates $\widetilde{\alpha}^i$ and $\widetilde{\mathbb{G}}^i$. Thus, the buyer's estimate of D is $\widetilde{D}^i \sim DP(\widetilde{\alpha}^i, \widetilde{\mathbb{G}}^i)$. After the buyer observes a sequence of supply capacities $s_1^*, s_2^*, ..., s_\tau^*$, his posterior estimate about the distribution D will become to $\widetilde{D}^i | s_1^*, s_2^*, ..., s_\tau^* \sim DP(\widetilde{\alpha}^i + \tau, \frac{\widetilde{\alpha}^i}{\widetilde{\alpha}^i+\tau}\widetilde{\mathbb{G}}^i + \frac{\tau}{\widetilde{\alpha}^i+\tau}\frac{\sum_{j=1}^{\tau}\delta_{S_j^*}}{\tau})$. His (subjective) prediction about $S_{\tau+1}$ is distributed as the posterior base distribution of the posterior Dirichlet process. That is:

$$\widetilde{S_{\tau+1}^*} | s_1^*, s_2^*, ..., s_\tau^* = \frac{\widetilde{\alpha}^i}{\widetilde{\alpha}^i + \tau}\mathbb{E}\widetilde{\mathbb{G}}^i + \frac{\tau}{\widetilde{\alpha}^i + \tau}\frac{\sum_j^\tau s_j^*}{\tau} \tag{4.15}$$

Further, as the number of encounters go to infinity, the buyer's personal estimate converges to the true supply S^* in distribution, no matter what initial belief the buyer holds.

We are also interested in the buyer's estimate of the market condition Z (excess demand). To derive the buyer i's best estimate of Z after he encounters supplier k[29], we note

$$\begin{aligned}
\mathbb{E}_{\mathbb{Q}_i}(Z|S_1^*, S_2^*, ..., S_\tau^*) &= \mathbb{E}_{\mathbb{Q}_i}\Big\{(\sum_{i\in I} D_i^* - \sum_{\tau=1}^{|K|} S_\tau^*)|S_1^*, S_2^*, ..., S_\tau^*\Big\} \\
&= \sum_{i\in I} D_i^* - \mathbb{E}_{\mathbb{Q}_i}(\sum_{\tau=1}^{|K|} S_k^* | S_1^*, S_2^*, ..., S_\tau^*) \\
&= \sum_{i\in I} D_i^* - (S_1^* + S_2^* + ... + S_\tau^*) - \sum_{j=k+1}^{|K|} \mathbb{E}_{\mathbb{Q}_i}(S_j^*|S_1^*, S_2^*, ..., S_\tau^*)
\end{aligned} \tag{4.16}$$

It can be shown[30] that $\mathbb{E}_{\mathbb{Q}_i}(S_j|S_1^*, S_2^*, ..., S_\tau^*) = \mathbb{E}_{\mathbb{Q}_i}(S_{j+h}|S_1^*, S_2^*, ..., S_\tau^*)$ for $\forall j \geq \tau+1$

[29] Recall we assume the supplier k announces his supply before buyer i place the bid.

and $\forall h \geq 0$, thus the best personal estimator of Z given observation $s_1^*, s_2^*, ..., s_\tau^*$ is

$$\begin{aligned}
\widetilde{Z}^i|S_1^*, S_2^*, ..., S_\tau^* &= \mathbb{E}_{\mathbb{Q}_i}(Z|S_1^*, S_2^*, ..., S_\tau^*) \\
&= \sum_{i\in I} D_i^* - (S_1^* + S_2^* + ... + S_\tau^*) - (|K| - \tau)\mathbb{E}_{\mathbb{Q}_i}(S_{k+1}^*|S_1^*, S_2^*, ..., S_\tau^*) \\
&= \sum_{i\in I} D_i^* - \sum_{j=1}^{\tau} S_j^* - (|K| - \tau)\frac{\widetilde{\alpha}^i \mathbb{E}\widetilde{\mathbb{G}}^i + \sum_{j=1}^{\tau} S_j^*}{\widetilde{\alpha}^i + \tau} \\
&= \sum_{i\in I} D_i^* - \frac{\widetilde{\alpha}^i(|K| - \tau)}{\widetilde{\alpha}^i + \tau}\mathbb{E}\widetilde{\mathbb{G}}^i - \frac{|K| + \widetilde{\alpha}^i}{\widetilde{\alpha}^i + \tau}\sum_{j=1}^{\tau} S_j^*
\end{aligned} \tag{4.17}$$

We note as $\tau \to |K|$, $(\widetilde{Z}^i|S_1^*, S_2^*, ..., S_\tau^*) \xrightarrow{d} Z$ in distribution. This says after enough many observations on suppliers, the buyer i will form a good estimate of the total market condition. These results can be summarized as the buyer's learning rule.

Learning Rule of Market Condition: Under assumptions of previous learning rule, at supplier k with index $\tau \in K_i$, the buyer i has observed $s_1^*, s_2^*, ..., s_\tau^*$ (since s_τ^* is announced right before he bid, but no earlier), his best personal estimate of

[30] *Proof*: (By induction) It suffices to show $\mathbb{E}_{\mathbb{Q}_i}(S_{j+h}|S_1^*, S_2^*, ..., S_\tau^*) = \mathbb{E}_{\mathbb{Q}_i}(S_{\tau+1}^*|S_1^*, S_2^*, ..., S_\tau^*)$ for $\forall j \geq \tau + 1$ and $\forall h \geq 0$. For $j = \tau + 1$, the conclusion trivially holds. Suppose for $j = \tau + h + 1$, the conclusion holds. Then it is must be true that

$$\begin{aligned}
&\int ... \int (S_{\tau+1}^* + S_{\tau+2}^* + ... + S_{\tau+h}^*) f(S_{\tau+h}^*|S_1^*, \quad S_2^*, ..., S_{\tau+h-1}^*) f(S_{\tau+h-1}^*|S_1^*, S_2^*, ..., S_{\tau+h-2}^*) \\
&\qquad ... f(S_{\tau+1}^*|S_1^*, ..., S_\tau^*) dS_{\tau+h}^* dS_{\tau+h-1}^* ... dS_{\tau+1}^* \\
=& h\mathbb{E}_{\mathbb{Q}_i}(S_{\tau+1}^*|S_1^*, S_2^*, ..., S_\tau^*)
\end{aligned}$$

Then for $j = \tau + h + 2$, we have,

$$\begin{aligned}
&\mathbb{E}_{\mathbb{Q}_i}(S_{\tau+h+2}^*|S_1^*, S_2^*, ..., S_\tau^*) \\
=& \int ... \int \mathbb{E}_{\mathbb{Q}_i}(S_{\tau+h+2}^*|S_1^*, S_2^*, ..., S_\tau^*, S_{\tau+1}^*, ..., S_{\tau+h+1}^*) f(S_{\tau+h+1}^*|S_1^*, S_2^*, ..., S_{\tau+h}^*) \\
&... f(S_{\tau+1}^*|S_1^*, ..., S_\tau^*) dS_{\tau+h+1}^* dS_{\tau+h}^* ... dS_{\tau+1}^* \\
=& \Big\{ \frac{\widetilde{\alpha}^i + \tau}{\widetilde{\alpha}^i + \tau + h + 1} + \frac{1}{\widetilde{\alpha}^i + \tau + h + 1}\Big(\frac{\widetilde{\alpha}^i + \tau}{\widetilde{\alpha}^i + \tau + h} + \frac{(\widetilde{\alpha}^i + \tau + h + 1)h}{\widetilde{\alpha}^i + \tau + h}\Big)\Big\} \mathbb{E}_{\mathbb{Q}_i}(S_{\tau+1}^*|S_1^*, S_2^*, ..., S_\tau^*) \\
=& \mathbb{E}_{\mathbb{Q}_i}(S_{\tau+1}^*|S_1^*, S_2^*, ..., S_\tau^*)
\end{aligned}$$

Thus the conclusion holds for $j = \tau + h + 2$.

excess demand is

$$\widetilde{Z}^i|s_1^*, s_2^*, ..., s_\tau^* = \sum_{i\in I} D_i^* - \frac{\widetilde{\alpha}^i(|K|-\tau)}{\widetilde{\alpha}^i+\tau}\mathbb{E}\widetilde{\mathbb{G}}^i - \frac{|K|+\widetilde{\alpha}^i}{\widetilde{\alpha}^i+\tau}\sum_{j=1}^{\tau} s_j^* \tag{4.18}$$

In further, as observations goes to $|K|$, the buyer's personal estimate of excess demand $\widetilde{Z}^i$ converges to the real demand Z in distribution, no matter what initial belief the buyer holds.

4.4.1.3 Buyers' Offer Rule

To determine buyer's optimal bid price, we first note that in this bidding framework, the constraints $q_{ik} \leq S_k^*$ $\forall k \in K$ are automatically satisfied. Thus we drop the constraints in problem (2). Then at each encounter k, conditioning on his information Φ_i^k, the buyer's optimization problem becomes to:

$$\min_{(q_{ik},b_{ik})} \mathbb{E}_{\mathbb{Q}_i}\left\{q_{ik}^a b_{ik} + SC_{ik} + c_i(D_i^{*k} - q_{ik}^a)\right\} \tag{4.19}$$

To minimize the procurement cost based on his information, the buyer's personal optimal bid price must satisfy[31]:

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial b_{ik}} =& \mathbb{E}_{\mathbb{Q}_i}\left\{\frac{\partial q_{ik}^a}{\partial b_{ik}}b_{ik} + q_{ik}^a + \frac{\partial SC_{ik}}{\partial q_{ik}^a}\frac{\partial q_{ik}^a}{\partial b_{ik}} - c_i\frac{\partial q_{ik}^a}{\partial b_{ik}}\right\} \\ =& \mathbb{E}_{\mathbb{Q}_i}\left\{\frac{\partial q_{ik}^a}{\partial b_{ik}}(b_{ik} - c_i + \frac{\partial SC_{ik}}{\partial q_{ik}^a}) + q_{ik}^a\right\} = 0\end{aligned} \tag{4.20}$$

To determine the bid quantity q_{ik}, we first note that bid quantity does not affect supplier's acceptance which only depends on bid price. On the other hand, if there is shortage in supply, the buyer i might bid up to S_k^*. Since the supplier can determine q_{ik}^a on his own, the buyer's bid quantity generally does not affect the acceptance probability[32]. These results can be summarized into the following rule:

***Offer Rule*:** To minimize procurement cost, an imperfect informed buyer i will offer a bid (q_{ik}, b_{ik}) to some supplier k. The buyer's personal optimal bid price

[31] Assume search cost and accepted bid quantity are independent and all variates are continuously differentiable as stated in section 3.1, then apply Leibniz integral rule.

[32] On the other hand, in some special cases, q_{ik} does have an effect on the acceptance probability. For example, if $q_{ik}^a = B_{ik}q_{ik}$ and $q_{ik} = S_k^*$, then in order for the bid to be accepted, buyer i's bid price must be higher than all other buyers' bid prices.

satisfies the following equation:

$$\mathbb{E}_{\mathbb{Q}_i}\Big\{\frac{\partial q^a_{ik}}{\partial b_{ik}}(b_{ik} - c_i + \frac{\partial SC_{ik}}{\partial q^a_{ik}}) + q^a_{ik}\Big\} = 0 \tag{4.21}$$

4.4.1.4 Relation with Auction Model

In this section, we will show if a buyer i bids at supplier k, then under some conditions his optimization problem (16) resembles an first-price sealed-price auction model. Suppose there are finite suppliers ($|K| < \infty$), and each supplier's accepted bid quantity follows $q^a_{ik} = B_{ik}q_{ik}$. In addition, suppose there is shortage in supply and each buyer bid quantity is s^*_k[33] (now $q^a_{ik} = B_{ik}q_{ik}$). Then at each supplier k, the buyer i's benefit from winning the auction is

$$c_i q^a_{ik} + \mathbb{E}_{\mathbb{Q}_i} SC(q^a_{ik}) \tag{4.22}$$

where c_i is the marginal penalty cost and $SC(q^a_{ik}) \equiv f(q^a_{ik}, \widetilde{Z}^i, s^*_k)$ with f defined as in equation (4). Equation (22) says, if he wins the auction, he will benefit from a reduced penalty cost of q^a_{ik}. In addition, he will not need to search for q_{ik} at a cost of $\mathbb{E}_{\mathbb{Q}_i} SC(q^a_{ik})$ in this case. The buyer i's per unit cost of wining the auction is b_{ik}. Then his optimization problem can be written as:

$$\max_{b_{ik}} \mathbb{E}_{\mathbb{Q}_i}\Big\{(c_i q_{ik} + SC(q^a_{ik}) - b_{ik}q_{ik})\mathbb{1}(B_{ik} = 1)\Big\} \tag{4.23}$$

If we further assume search cost and acceptance rule are independent as in (16), then problem (21) becomes to

$$\max_{b_{ik}} \Big(c_i q_{ik} + \mathbb{E}_{\mathbb{Q}_i} SC(q^a_{ik}) - b_{ik}q_{ik}\Big)\mathbb{Q}_i(B_{ik} = 1) \tag{4.24}$$

In the first or second sealed price auction model, $\mathbb{Q}_i(B_{ik} = 1) = \mathbb{P}(B_{ik} = 1) = F^{n-1}(b^{-1}(b_{ik}))$ depending the distribution of the largest order statistics, where b is the optimal bidding function. In our model, who wins auction is not determined by the bids' relative ranking, but by the supplier's acceptance rule (as described in next section). In the supplier's acceptance case 1 (SAC1), the supplier's acceptance rule reflects bids' ranking as well as their arriving time which translates to supplier's

[33]Note in this here the auction is essentially a single-unit auction.

waiting cost.

4.4.1.5 Deviation from the Optimal Bid Price

Buyer's Optimal Bid Case 1

If a buyer is perfectly informed of the actual distributions of waiting cost WC and suppliers' supply capacity S^*, then his personal optimal bid price agrees with the objective optimal one, which satisfies

$$\mathbb{E}_{\mathbb{P}}\Big\{\frac{\partial q_{ik}^a}{\partial b_{ik}}(b_{ik}-c_i+\frac{\partial SC_{ik}}{\partial q_{ik}^a})+q_{ik}^a\Big\}=0 \tag{4.25}$$

Later in this article it is shown if $SC=\frac{Z^2(D_i^{*k}-q_{ik}^a)}{\beta s_k^*}$ increases nonlinearly in excess demand, and suppliers take logit acceptance rule ($\mathbb{P}(B_{ik}=1)=\frac{e^{b_i*}}{\sum_{j\in I}e^{b_j}}$), then the (objective) optimal bid price is

$$b_{ik}^*=c_i+\frac{\mathbb{E}_{\mathbb{P}}Z^2}{\beta s_k^*}-1-\mathscr{W}(\frac{e^{c_i+\mathbb{E}_{\mathbb{P}}\frac{Z^2}{\beta S_k^*}-1}}{K}) \tag{4.26}$$

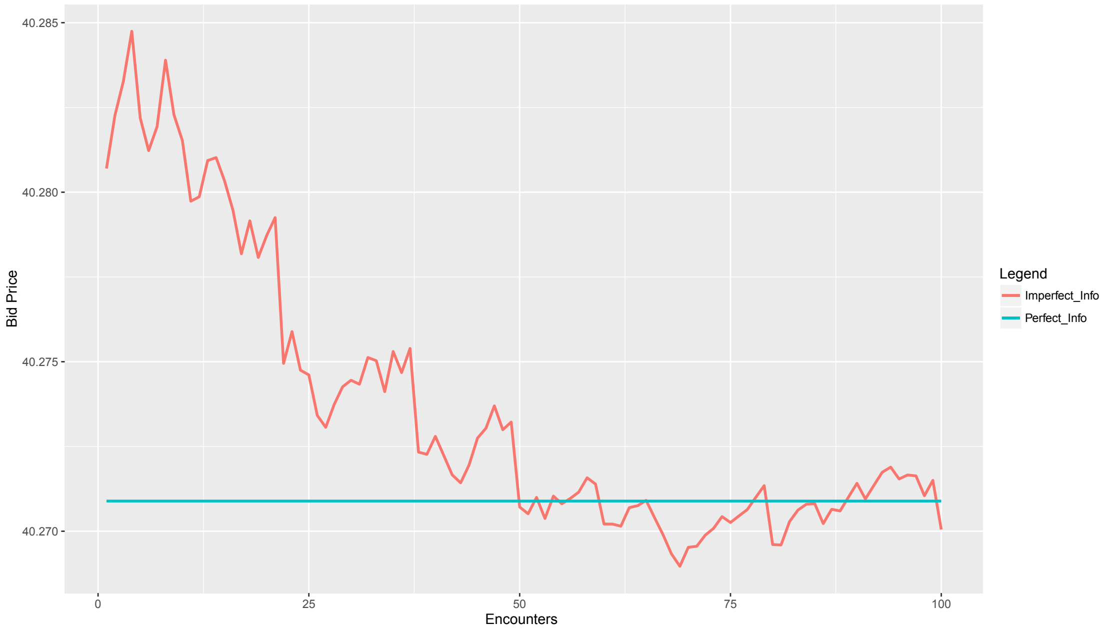
where $K=\sum_{j\neq i}e^{\widetilde{b_j}}$ and $\mathscr{W}$ is Lambert W function: for $f(z)=ze^z$, we have $z=f^{-1}(ze^z)=\mathscr{W}(ze^z)$.
Similarly, a buyer i with imperfect information has his personal bid price:

$$\widetilde{b_{ik}^*}^i=c_i+\frac{\mathbb{E}_{\mathbb{Q}_i}Z^2}{\beta s_k^*}-1-\mathscr{W}(\frac{e^{c_i+\frac{\mathbb{E}_{\mathbb{Q}_i}Z^2}{\beta s_k^*}-1}}{\widetilde{K^i}}) \tag{4.27}$$

where $\widetilde{K^i}=\sum_{j\neq i}e^{b_j}$ and $\mathscr{W}$ is defined as above. We know from above that when the buyer i has enough many observations, his estimate of market condition converges to the real one. Thus, as $k\to|K|$, $\mathbb{E}_{\mathbb{Q}_i}Z^2=\mathbb{E}_{\mathbb{P}}\widetilde{Z^2}^i\to\mathbb{E}_{\mathbb{P}}Z^2$ (pointwise). If a buyer has perfect information about other bidders' price, then the deviation between personal optimal bid price and the true optimal one is

$$\triangle_{b_{ik}}=\widetilde{b_{ik}^*}^i-b_{ik}^*=\frac{\mathbb{E}_{\mathbb{Q}_i}Z^2}{\beta s_k^*}-\frac{\mathbb{E}_{\mathbb{P}}Z^2}{\beta s_k^*}-\mathscr{W}(\frac{e^{c_i+\frac{\mathbb{E}_{\mathbb{P}}Z^2}{\beta s_k^*}-1}}{K})-\mathscr{W}(\frac{e^{c_i+\frac{\mathbb{E}_{\mathbb{Q}_i}Z^2}{\beta s_k^*}-1}}{K}) \tag{4.28}$$

Figure 4.2. Simulated Personal Optimal Bid Price under Imperfect and Perfect Information about Market Condition



Note: Figure 2 shows how an imperfectly informed buyer's personal (subjective) optimal bid price converges to the true (objective) optimal one. The x-axis is the number of suppliers the buyer has encountered and observed. The buyer's subjective optimal bid price is plotted in red. The true optimal bid price is plotted in blue.

A simulation result is plotted in Figure 2. It shows, if buyer has perfect information about others' bid prices, his personal optimal bid price will converge to the real optimal one as the number of encounters and observations of supplier available suppliers goes up (since his estimate of market condition becomes more accurate). This is consistent with analytical result that $\triangle_{b_{ik}} \to 0$ pointwise as number of observations goes to $|K|$.

Buyer's Optimal Bid Case 2

In another case, if the buyer does not know others' bids, and he estimates others' bids using his own bid price $\widetilde{b_j}^i = b_i$, then his best personal bid price is

$$\begin{aligned}\widetilde{b_{ik}^*}^i =& \frac{(N-1)(c_i + \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2}{\beta s_k^*} - 1) - 1}{N-1} \\ =& c_i + \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2}{\beta s_k^*} + \frac{N-2}{N-1}\end{aligned} \tag{4.29}$$

And its deviation from the optimal bid pirce is:

$$\triangle_{b_{ik}} = \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2}{\beta s_k^*} - \frac{\mathbb{E}_{\mathbb{P}} Z^2}{\beta s_k^*} - \mathscr{W}\left(\frac{e^{c_i + \frac{\mathbb{E}_{\mathbb{P}} Z^2}{\beta s_k^*} - 1}}{K}\right) + \frac{1}{N} \tag{4.30}$$

Note, in this case, as observations increase in number, the deviations can be not eliminated ($\triangle_{b_{ik}} \to -\mathscr{W}(\frac{e^{c_i + \frac{\mathbb{E}_{\mathbb{P}} Z^2}{\beta s_k^*} - 1}}{K}) + \frac{1}{N} \neq 0$). The deviation in the limit case reflects buyer i's biased estimate of other buyers' bids.

4.4.2 Supplier Side

Similar to buyer's optimization problem, to maximize his profit, at encounter (i, k) each supplier k chooses to accept whether to accept or reject a bid. Thus, his (optimal) decision is a sequence of 0 and 1's (indicators B_{ik}) indexed by the buyer's arrival. His optimal *choice* problem can be written in a 0-1 integer programming form as [34] [35]:

$$\begin{aligned} \underset{\{B_{ik}\}_{i\in I}}{\text{maximize}} \quad & \mathbb{E}_{\mathbb{Q}_k}\Big\{\sum_{i\in I} B_{ik} q_{ik}(b_{ik} - WC_{ik})\Big\} \\ \text{subject to} \quad & \sum_{i\in I} B_{ik} q_{ik} \leq S_k^* \end{aligned} \tag{4.31}$$

Problem (4.31) indicates each supplier k maximizes his profit by choosing the best bids offered until his capacity is sold out. Not accepting the bid (q_{i^*k}, b_{i^*k}) means the supplier may miss a good offer. Thus the supplier must compare the

[34] On the other, we will use discrete choice methods to approach this problem. Thus equation (31) is more of a general conceptual formulation.

[35] It is worth noting that in equation (31), the decision variables $\{B_{ik}\}_{i\in I}$ are deterministic; however, the optimal solutions $\{B_{ik}\}^*_{i\in I}$ depend on the uncertainty ω, which means the solutions to (29), $\{B_{ik}\}^*_{i\ inI}$, are random variables

profit $q_{i^*k}b_{i^*k}$ made from accepting the bid (q_{i^*k}, b_{i^*k}) and the expected maximal profit[36].

If each supplier k has perfect information about what bid prices will be received and their time of arrival, he can have an explicit value ranking of all bids based on the ranking $b_{ik} - WC_{ik}$. Then to maximize profit as in equation (31), he will accept the bids with highest values $(b_{ik} - WC_{ik})$ until his supply is sold out. This is a deterministic acceptance rule.

On the other hand, if the supplier k does not have perfect information on either the bid values or their time of arrival or both, then his acceptance must be conditioned on uncertain waiting time. It follows his acceptance rule is stochastic. For example, if the supplier k knows the values of the bid but does not know their arrival timing, each supplier k only accepts highest bids until his supply capacity is reached. Let $\mathbb{C}_k$ denote the supplier k's set of chosen bids. That is, $\mathbb{C}_k = \{i \in I | B^*_{ik} = 1\}$, where B^*_{ik} is the optimal solution to his profit maximization problem (4.31). Then the supplier k will choose to accept the bid (q_{ik}, b_{ik}) if and only if

$$b_{ik} - WC_{ik} > b_{jk} - WC_{jk}, \text{ for } \forall j \notin \mathbb{C}_k \tag{4.32}$$

Note since waiting cost is a random variate, the supplier's acceptance rule in (4.32) is a stochastic rule.

As a special case of rule (4.32), if there is an extreme shortage in supply (that is, $D^*_i > S^*_k$, for all i, k.), the supplier will only accept the highest bid with consideration of waiting cost (the accepted set $\mathbb{C}_k$ is a singleton). Then the supplier will choose to accept the bid (q_{ik}, b_{ik}) if

$$b_{ik} - WC_{ik} > b_{jk} - WC_{jk}, \text{ for } \forall j \neq i \tag{4.33}$$

Note here the acceptance rule is also stochastic since waiting cost is uncertain.

4.4.2.1 Waiting Cost and Acceptance Rule

The waiting cost WC_{ik} is the k-th supplier's future and uncertain waiting cost before the buyer i makes a bid. As stated above, waiting cost is in its nature stochastic

[36] In a bid with a recall framework, not accepting the bid immediately only means he will compare the profit with the waiting cost WC_{ik}. Here we use a bid without recall setting.

since there is uncertainty in the time when buyers bid and other factors. Further, it also depends on the market conditions Z.

The supplier is viewed as static, like a shop keeper who waits for the random arrival of buyers. As a buyer arrives, a bid is received. The supplier must decide whether to accept or reject the bid. If accepted, the volume of the sale must be determined by the supplier. In this specification we assume the buyer offers a pair (b_{ik}, q_{ik}) that must be accepted or rejected. Thus an acceptance rule must be specified. Here we focus on specifications of waiting cost. That is, if the supplier rejects the bid, the supplier must wait for the next buyer's bid to be received. Waiting cost involves both an expectation of feasible bids that might arrive as well as time cost of waiting (WC). Here, we focus on the former as a metric of the outside option of the supplier when faced with a bid from buyer i. We view the information available to the supplier in forming the WC as including perceived or true market conditions (Z), and the sequence of past bids received, $\{b_{\tau' k}\}_{\tau' \in I_{\leq \tau}}$. If only Z is considered, we specify Z as either determining WC as a simple linear projection, or conditioning a distribution from which a draw is taken for WC. A third alternative is to consider past bids. Suppliers use Bayesian learning (or, in a simple case, *adaptive expectation* as in [Mut61]) to estimate future bid prices, based on which they form their acceptance rule. A final specification of interest would be to consider an observable, small set of suppliers. In this case, suppliers base their acceptance rule on their rational expectation of future bids that would clear the market.

Supplier's Waiting Cost and Acceptance Rule Case 1

A possible way to model WC_{ik} is to decompose WC_{ik} as sum of a deterministic component and a stochastic component:

$$WC_{ik} = \delta_k + e_{ik} \tag{4.34}$$

where δ_k is the deterministic component representing the systematic part of waiting cost, and e_{ik} is the stochastic component representing the uncertainty faced by the supplier. Since the waiting cost is stochastic and depends on market conditions Z, we assume WC_{ik} follows a distribution f_Z depending on Z the market conditions. A simple example is when WC_{ik} is distributed as a scale location family with only its expectation depending on Z, and e_{ik} is zero mean disturbance term, then only the deterministic (systematic) part δ_k depends on Z ($\delta_k = \delta_k(Z)$. A more general case is to assume e_{ik} follows a distribution f_Z.

In the case when supplier knows bid price values but not their arrival time, based on the decomposition on WC_{ik} in equation (34), the supplier problem (32) is equivalent to:

$$e_{ik} - e_{jk} < b_{ik} - b_{jk} + \delta_k - \delta_k = b_{ik} - b_{jk}, \text{ for } \forall j \notin \mathbb{C}_k \tag{4.35}$$

where $\mathbb{C}_k$ is the supplier's optimal choice set. Note in this case, the acceptance rule is dependent of the market condition Z.

Recall B_{ik} is the indicator of the supplier accepting the bid (q_{ik}, b_{ik}). Then, the probability of supplier accepting the bid (q_{ik}, b_{ik}) is:

$$\mathbb{P}(\text{supplier k accepts bid } i|Z) = \mathbb{P}(B_{ik} = 1|Z) \tag{4.36}$$

Equation (4.36) says the probability of acceptance depends on market conditions Z. An example is if the excess demand increases, then there exists a higher probability that someone bids high price. If such increasing excess demand is perceived by supplier k, then he will expect to get a high price offer and hence will be reluctant to sell at the bid price levels previously accepted.

In the case of severe supply shortage ($D_i^* > S_k^*$, we can derive an analytical form of acceptance probability. Under the first specification of waiting cost $WC_{ik} = \delta_k + e_{ik}$, the acceptance probability $\mathbb{P}(B_{i^*k} = 1|Z)$ can be written as:

$$\mathbb{P}(B_{ik} = 1|Z) = \int \mathbb{1}(e_{ik} - e_{jk} < b_{ik} - b_{jk}, \forall j \neq i^*) f_Z(\mathbf{e}_k) d\mathbf{e}_k \tag{4.37}$$

where $\mathbf{e}_k = (e_{1k}, e_{2k}, ..., e_{Ik})$ is an I by 1 vector. In the case of $\mathbf{e}_k \sim MN(\vec{0}, \Sigma)$, the analytical form resembles a multinomial probit problem. Note in this case, $\delta_k = \delta_k(Z) = \mathbb{E}(WC_{ik}|Z)$ is a function of Z.

***Acceptance Rule 1*:** Each supplier k chooses whether to accept a bid or not based on his interpretation of market conditions Z. The supplier will only accept the highest bid(s) i with consideration of cost. If the supplier k is informed about what the bids will be but does not know their arrival time, then under specification of equation (34), he will accept bid i if and only if

$$e_{ik} - e_{jk} < b_{ik} - b_{jk}, \text{ for } \forall j \notin \mathbb{C}_k \tag{4.38}$$

where $\mathbb{C}_k$ is the supplier k's optimal choice set defined as before. We label this acceptance rule as SAC1.

Supplier's Waiting Cost and Acceptance Rule Case 2

In case 1, the waiting time is calculated from time 0 [37]. Thus, at time when buyer i came and bid at supplier k, the waiting time of next (potential higher) bid is the difference between two waiting time (note the buyer i's waiting time is already known, or realized random variable). Hence, waiting cost is defined as the difference of the two waiting cost. In this case, we assume past waiting time has no influence on supplier's acceptance of buyer i's bid. To determine whether to accept bid i^* or not, the supplier k only considers the waiting cost from the time when the bid i^* is received. In this case, the waiting time is calculated from the time when the bid i^* is received [38]. In particular, we assume that the waiting time (from the current bid) follows an exponential distribution [39]as a function of market condition Z, i.e.

$$WC_{ik} \sim exp(f(Z)) \tag{4.39}$$

Under the second specification of waiting cost where $WC_{ik} \sim exp(f(Z))$, the

[37]For each supplier k, we define time 0 can be seen as the time when the first buyer came to the supplier k. Thus time 0 is k specific.

[38]Or, we can think in this case the time 0 is the time when the buyer i submits his bid to the supplier k. Thus in this case, time 0 is i^* and k specific.

[39]This is different from assuming buyers' arrivals follow Poisson process. It is worth noting that since in this model buyers come to the supplier randomly, a Poisson process modelling seems natural. However, such a formulation has two drawbacks. First, in Poisson process, the order of each arrival is exogenously determined (since Poisson process assumes a natural order). In contrast, in this model, the set of buyers is an unordered set and the order of arrival is *endogenously* determined by their "waiting time" (more accurately, the Poisson process modelling of customer arrivals assume every customer is homogeneous, thus any ordering does not matter). Secondly, the Poisson process modelling does not admit a general analytical solution. To see this, we assume the set of buyers can be indexed by their arrival to supplier k and the interval time between two arrivals (the interval time between buyer 1 and buyer 2 is denoted as T_1) is distributed as exponential random variable with parameter Z (note as Z increases, the interval time has a higher probability to be a small number). That is, the interval time $T_1, T_2, ...T_N \sim i.i.d.\ exp(Z)$. Since a Poisson process is memory-less, we denote T_i as the waiting time of next buyer from buyer i. Suppose the waiting cost is proportional to the waiting time ($WC_i = c\sum_{i'<i} T_{i'}$), then the acceptance rule

supplier k will accept the bid τ if $b_{\tau k} > \widetilde{b_{\tau' k}}^k - \widetilde{WC_{\tau' k}}^k$ for future bid $\tau' > \tau$, where τ is the index for current bid in the ordered set I_k, and τ' is in the index for some future bid j in the ordered set I_k. $\widetilde{b_{\tau' k}}^k$ is his estimate of some future bid $b_{\tau' k}$ and $\widetilde{WC_{\tau' k}}^k$ is corresponding estimated waiting cost. In this case, the acceptance probability can be written as:

$$\begin{aligned}\mathbb{P}(B_{\tau k} = 1|Z) =& \mathbb{P}(\widetilde{WC_{\tau' k}}^k > \widetilde{b_{\tau' k}}^k - b_{\tau k} \text{ for } \forall \tau' > \tau|Z) \\ =& \prod_{\tau' > \tau}^{N} e^{-z(\widetilde{b_{\tau' k}}^k - b_{\tau k})} = e^{-f(Z)\sum_{\tau' > \tau}^{N}(\widetilde{b_{\tau' k}}^k - b_{\tau k})}\end{aligned} \tag{4.40}$$

These results can be summarised as acceptance rule 2.

***Acceptance Rule 2*:** Let waiting cost be specified as in case 2 (equation (39)). Under the same condition as in acceptance rule 1, the supplier will accept bid i with index $\tau \in I_k$ if the following conditions is satisfied:

$$b_{\tau k} > \widetilde{b_{\tau' k}}^k - \widetilde{WC_{\tau' k}}^k \text{ for } \forall \tau' > \tau \tag{4.41}$$

---

becomes: the supplier only accepts the i^* bid if

$$b_{i^*} > b_j - c \sum_{k=i^*+1}^{j} T_k, \text{for } \forall j \geq i^*$$

Then the probability of supplier accepting the bid i^* $\mathbb{P}(\text{supplier k accepts bid } i^*|Z)$ is

$$\mathbb{P}(b_{i^*} > b_{i^*+1} - cT_{i^*}, b_i^* > b_{i^*+2} - c(T_{i^*} + T_{i^*+1}), \ldots, b_{i^*} > b_N - c \sum_{k=i^*+1}^{N} T_k|Z)$$

For $i^* = N-1$, the acceptance probability in the equation above can be written as $\mathbb{P}(T_{N-1} > \frac{1}{c}(b_N - b_{N-1})) = e^{-\frac{Z(b_N - b_{N-1})}{c}}$ for $b_N > b_{N-1}$.
For $i^* = N-2$, such probability becomes to $\mathbb{P}(T_{N-1} > \frac{1}{c}(b_N - b_{N-1}), T_{N-1} + T_{N-2} > \frac{1}{c}(b_N - b_{N-2})) = \int_{T_{N-1}} \mathbb{1}(T_{N-1} > \frac{1}{c}(b_N - b_{N-1}) \int_{T_{N-2}} \mathbb{1}(T_{N-2} > \frac{1}{c}(b_N - b_{N-2}) - T_{N-1}) dT_{T_{N-2}} dT_{T_{N-1}} = e^{-\frac{Z(b_N - b_{N-2})}{c}} \left(\frac{Z(b_N - b_{N-1})}{c} + 1\right)$ for $b_N > b_{N-1} > b_{N-2}$ (otherwise it is a degenerate case similar to $i^* = N-1$).
For $i^* = N-3$, the acceptance probability becomes to $\mathbb{P}(T_{N-1} > \frac{1}{c}(b_N - b_{N-1}), T_{N-1} + T_{N-2} > \frac{1}{c}(b_N - b_{N-2}), T_{N-1} + T_{N-2} + T_{N-3} > \frac{1}{c}(b_N - b_{N-3})) = e^{-\frac{Z(b_N - b_{N-3})}{c}} \left(\frac{Z^2}{2c^2}\left((b_N - b_{N-2})^2 + (b_{N-1} - b_{N-2})^2\right) + Z(b_B - b_{N-2}) + 1\right)$ for $b_N > b_{N-1} > b_{N-2} > b_{N-3}$ (otherwise it is a degenerate case similar to previous cases).

Hence we see there is no clear formula for a general i^*. This reason together with the endogenous ordering problem make the Poisson process formulation an inappropriate one in the context of this paper.

where τ, τ', $\widetilde{b_{\tau' k}}^k$ and $\widetilde{WC_{\tau' k}}^k$ are defined as above. We label this acceptance rule as SAC2.

Supplier's Waiting Cost and Acceptance Rule Case 3

In the case when the suppliers are not informed about the bid values, the supplier needs to estimate future bid price (and their arrival time). When past bids are considered, we use Bayesian learning or adaptive rule based learning on bid prices.

In the setting of the supplier k Bayesian learning on bid price, we assume the supplier has some prior information about bid price. Suppose that his prior mean of the bid price is $\widetilde{b_k}$. Then, after observing a sequence of bids $(b_{1k}, b_{2k}, \ldots, b_{\tau k})$ in the ordered set I_k, his best estimate of bid price is $\frac{\alpha}{\alpha+\tau}\widetilde{b_k} + \frac{\tau}{\alpha+\tau}\bar{b}$, where α is the concentration factor as defined in section 3, τ is the number of bids before and including the bid $b_{\tau k}$ and $\bar{b}$ is the average bid price received before (and including) the bid $b_{\tau k}$. Thus, at the time buyer τ offers his bid to supplier k, supplier k's best estimate of future price is

$$\widetilde{b_{\tau' k}}^k = \frac{\alpha}{\alpha+\tau}\widetilde{b_k} + \frac{\tau}{\alpha+\tau}\bar{b} \text{ for } \forall \tau' > \tau \tag{4.42}$$

where $\widetilde{b_k}$ is the prior mean of bid price and $\bar{b}$ is the average bid receive before (and including) the bid $b_{\tau k}$.

As a special case of Bayesian learning on bid price, the supplier k has no prior information on bid price and only uses the last bid to estimate future bid price. That is,

$$\widetilde{b_{\tau' k}}^k = b_{\tau k} \text{ for } \forall \tau' > \tau \tag{4.43}$$

This is an adaptive rule based on learning the bid price where the meaning of "adaptive" is consistent with that in adaptive expectation ([Mut61])[40].

[40] In the adaptive expectation, $\widetilde{b_{jk}}^k$ is written as $\mathbb{E}_{k,i^*}(b_j)$ which equals to b_{i^*}. If the supplier k's information set at the time of receiving bid i^* I_{i^*k} is the singleton $\{b_{i-1,k}\}$, the such estimate can be seen as supplier k's rational expectation given the information set $I_{i^*k} = \{b_{i-1,k}\}$. Based on this estimated bid price, supplier can form their acceptance rule as described in SAC1 or SAC2. We label this case as SAC3.

Supplier's Waiting Cost and Acceptance Rule Case 4

A fourth alternative is to consider a rational expectation of future bids that would clear the market. In this case we assume an observable and small set of suppliers. Each supplier k has rich information about market conditions and other suppliers. To make decisions, the supplier k considers his competitors reaction to market conditions. In particular, we assume supplier k's information set $\Psi_k^{i^*}$ at the time of receiving bid i^* includes:

- market condition Z;
- other suppliers' supply $\{S_j^*\}_{j \neq k}$;
- other suppliers' acceptance rules are the same as his own one;
- buyers know about suppliers' capacity and acceptance rule.

With such information, the supplier k can calculate the equilibrium price b^e that clears the market [41], i.e.

$$\sum_{k \in K} S_K^* = \sum_{i \in I} D_i(b^e) \tag{4.44}$$

Then the suppliers can use this equilibrium price to estimate the future bid price, based on which he constructs his acceptance rule.

There are several noteworthy properties of the market equilibrium price b^e in this setting. b^e is the supplier k's rational expectation of the future bid price. If we assume each supplier observes (4.44), then other suppliers use this b^e to estimate future bid price for the acceptance rule (supplier j accepts the bid i if $b_{ij} > b^e - WC_j$, for $\forall j \neq k$), and supplier k uses the same acceptance rule:

$$\text{Accept bid } i \text{ if } b_{ik} > b^e - WC_k \tag{4.45}$$

It follows that this acceptance rule is the most profitable strategy of supplier k and it guarantees he can sell all his products. To see this, we note if supplier k chose a higher threshold $b^t > b^e$, then there is a positive probability that he can not sell all

[41] Note here D_i as a function of price, which will be defined later in the text, is different from the initial demand the procurement requirement D_i^* to fulfill at the beginning of each trading session. Also note that here S_k^* does not change over price.

his products [42]. On the other hand, supplier k does not have an incentive to choose a threshold price b^e that is lower than b^e since at b^e since all his products can been sold (at b^e, he does not need to lower price to compete for demand). As a result, using b^e to estimate future bid price in the acceptance rule is a reasonable choice for suppliers.

We note in the absence of waiting cost, such a setting is essentially a Bertrand-Edgeworth oligopoly game. In such a game, there is no pure strategy Nash equilibrium [AH86]. In particular, given all other suppliers use b^e in their acceptance rule, supplier k may want to choose a threshold price that is higher than b^e [43]; in this case, every supplier uses b^e as thresholds in acceptance rule is not a Nash Equilibrium.

Since we assume buyers know about suppliers' acceptance rule and they will not bid higher than the per unit penalty adjusted by per unit search cost (otherwise, they will not search for the product). In this case, buyers' demand D_i can be written as

$$D_i = \begin{cases} 0 \text{ if } b^e > c_i \\ D_i^* \text{ if } b^e \leq c_i \end{cases} \tag{4.46}$$

Then $Z(b^e) = \sum_i D_i - \sum_k S_k^* = \sum_i D_i^* \mathbb{1}(b^e \leq c_i) - \sum_k S_k^*$. Since b^e is the equilibrium price, we must have

$$Z(b^e) = 0 \tag{4.47}$$

The equation (46) is easily solvable numerically. Then with the solved b^e, the seller k's acceptance probability is

$$\begin{aligned} \mathbb{P}(B_{ik} = 1|Z) =& \mathbb{P}(b_{ik} > b^e - WC_{ik}|Z) \\ =& e^{-f(Z)(b_{ik}-b^e)} \end{aligned} \tag{4.48}$$

The content in this section can be summarized in the following acceptance rule.

***Acceptance Rule 3*:** In order to make a decision, each supplier k needs to estimate future bid prices. In the settings where there is only a small group[44] of relatively

[42] In the absence of search cost, the buyers can compare suppliers with no cost, then the supplier k with higher threshold price can not sell all his products almost surely (with probability 1).

[43] He will not choose a threshold that is lower that e^b since his capacity is reached.

[44] Since in this settings we require each supplier has rich information about the other suppliers,

well informed suppliers, the suppliers can use market equilibrium price b^e to estimate future bid prices. Then the supplier will accepts bid i if

$$b_{ik} > b^e - WC_k \tag{4.49}$$

where b^e and WC_k are defined as above. We label such acceptance rule as SAC4.

4.4.3 Transaction Price

In this model, each bidder i comes to the supplier k and submits a bid (q_{ik}, b_{ik}) to the supplier. The supplier k then determines whether to accept this bid based on his interpretation of market conditions. We label the accepted bid prices as transaction prices. They reflect the supplier's estimate of market conditions through the probability of acceptance which depends on the market conditions. For example, if there exists a large excess demand that is perceivable to both sides in the market, then buyers will bid high prices to secure (at least partial) products to fulfill his fixed procurement demand. Suppliers who perceive the large excess demand expect such situation (buyers bid high) and become reluctant to accept the lower bids. They would like wait for higher prices (and there will actually be high prices since buyers who learn about supply shortage through their searching process will bid high to secure their products). As a result, the transaction prices, as those in the accepted bids, will increase. Thus, the market conditions play an essential role in determining transaction price. This motivates a closer look at the market conditions.

4.4.4 Market conditions

Case 1: $\sum_k S_k^* \gg \sum_i D_i^*$ $(Z \ll 0)$

In this case, supply is sufficient (much larger than demand). With such large excess supply that is perceivable to both buyers and suppliers, buyers who know this will bid low prices and suppliers may have to accept lower price to clear supply.

---

we assume in this case there is only a small group of suppliers for the requirement to be realistic. On the other hand, such assumption can be relaxed as long as each supplier has rich information about the market and other suppliers.

Case 2: $\sum_k S_k^* \sim \sum_i D_i^*$ $(Z \sim 0)$

In this case, demand and supply are about the same. The market has no noticeable excess demand or supply to both sides in the market. Since there is no large excess supply, suppliers are confident about selling their product out and the this is no need to sell at a lower price. And no noticeable excess demand means most buyers will not be panic about (not) being able to fulfill their procurement requirement.

Case 3: $\sum_k S_k^* \ll \sum_i D_i^*$ $(Z \gg 0)$

In this case, there is a large noticeable shortage in supply. Buyers who learn about this would like to pay more to guarantee supply. Thus limited supply force buyer to bid high to guarantee fulfillment. And suppliers are confident about selling their supply at high prices and would like to wait until there is one. As a result, the transaction price will increase. This is an essential driving source of price jumps.

4.5 Analytical Solutions

In this section, I present two exemplary cases with analytical solutions.

4.5.1 A First Example (SAC1)

In this first example, I first investigate the analytical solution corresponding to supplier's acceptance rule 1 (SAC1).

4.5.1.1 Supplier Side

In particular, to derive an analytical solution, I assume e_{ik} is the negative Gumbel random variable, i.e.

$$f(e_{ik}) = e^{e_{ik} - e^{e_{ik}}} = e^{e_{ik}} e^{-e^{e_{ik}}} \tag{4.50}$$

and the cumulative distribution is

$$F(e_{ik}) = 1 - e^{-e^{e_{ik}}} \tag{4.51}$$

With such assumption, the probability of the supplier accepting a bid (q_{ik}, b_{ik}) (conditioning on Z) in equation (14) can be written as:

$$
\begin{aligned}
& \mathbb{P}(\text{supplier } k \text{ accepts bids } i^*|Z) = \mathbb{P}(B_{i^*k} = 1|Z) \\
= & \int \mathbb{1}(e_{i^*k} - e_{ik} < b_{i^*k} - b_{ik} \ \forall j \neq i^*) f_Z(e_k) de_k \\
= & \frac{e^{b_{i^*k}}}{\sum_{j \in I} e^{b_{jk}}}
\end{aligned}
\tag{4.52}
$$

Note here the market dependent variate δ_k is cancelled. Thus the acceptance rule depends all buyers' bid prices, but not on market condition.

4.5.1.2 Buyer Side

Now let us consider the buyer side. Recall (4.21) where the buyer's optimal bidding price satisfies:

$$
\mathbb{E}_{\mathbb{Q}_i}\Big\{\frac{\partial q_{ik}^a}{\partial b_{ik}}(b_{ik} - c_i + \frac{\partial SC_{ik}}{\partial q_{ik}^a}) + q_{ik}^a\Big\} = 0 \tag{4.53}
$$

In the case when $SC = \frac{Z^2(D_i^{*k} - q_{ik}^a)}{\beta s_k^*}$ and $q_{ik}^a = B_{ik} q_{ik}$, the equation above can be written as:

$$
\begin{aligned}
(lhs) &= \frac{\partial \mathbb{Q}_i(B_{ik} = 1) s_k^*}{\partial b_{ik}}(b_{ik} - c_i - \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2}{\beta s_k^*}) + \mathbb{Q}_i(B_{ik} = 1) q_{ik} \\
&= q_{ik}\Big\{\frac{\partial \mathbb{Q}_i(B_{ik} = 1)}{\partial b_{ik}}(b_{ik} - c_i - \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2}{\beta s_k^*}) + \mathbb{Q}_i(B_{ik} = 1)\Big\} = 0
\end{aligned}
\tag{4.54}
$$

where *(lhs)* is the left hand side of equation (41). As specified above, the supplier's probability of acceptance is

$$
\mathbb{P}(B_{ik} = 1) = \frac{e^{b_{i^*}}}{\sum_{j \in I} e^{b_j}} \tag{4.55}
$$

Since a buyer does not know other buyers' bid prices, he can only estimate it. Denote his estimate of buyer j's bid price b_{jk} is $\widetilde{b_{jk}}^i$, then his estimate of supplier k's probability of acceptance is:

$$
\mathbb{Q}_i(B_{ik} = 1) = \frac{e^{b_{i^*k}}}{\sum_{j \in I} e^{\widetilde{b_{jk}}^i}} \tag{4.56}
$$

In this case, his personal optimal bid price $\widetilde{b_{ik}^*}^i$ must satisfies

$$e^{\widetilde{b_{ik}^*}^i} + \widetilde{b_{ik}^*}^i \widetilde{K}^i - \widetilde{K}^i (c_i + \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2}{\beta s_k^*} - 1) = 0 \tag{4.57}$$

where $\widetilde{K}^i = \sum_{j \neq i} e^{b_j}$. Solve equation (56) for b_{ik}, and we have :

$$\widetilde{b_{ik}^*}^i = c_i + \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2}{\beta s_k^*} - 1 - \mathscr{W}(\frac{e^{c_i + \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2}{\beta s_k^*} - 1}}{\widetilde{K}^i}) \tag{4.58}$$

as the buyer i's best personal bid price to supplier k, where $\mathscr{W}$ is Lambert W function: for $f(z) = ze^z$, we have $z = f^{-1}(ze^z) = \mathscr{W}(ze^z)$.

Recall in a simple case when buyer i estimates others' bids using his own bid price $\widetilde{b_j}^i = b_i$, then his best personal bid price is

$$\begin{aligned} \widetilde{b_{ik}^*}^i =& \frac{(N-1)(c_i + \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2}{\beta s_k^*} - 1) - 1}{N-1} \\ =& c_i + \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2}{\beta s_k^*} + \frac{N-2}{N-1} \end{aligned} \tag{4.59}$$

There are some good analytical properties of this formulation. In particular, we are interested in how a buyer's personal optimal bid price reacts to market condition and other buyers' bid. To see this, by implicit function theorem, we note [45]:

$$\frac{\partial \widetilde{b_{ik}^*}^i}{\partial \mathbb{E}_{\mathbb{Q}_i} Z^2} = -\frac{\partial (lhs)/\partial \mathbb{E}_{\mathbb{Q}_i} Z^2}{\partial (lhs)/\partial \widetilde{b_{ik}^*}^i} = \frac{\widetilde{K}^i}{\widetilde{K}^i + e^{\widetilde{b_{ik}^*}^i}} > 0 \tag{4.60}$$

$$\frac{\partial \widetilde{b_{ik}^*}^i}{\partial \widetilde{K}^i} == -\frac{\partial (lhs)/\partial \widetilde{K}^i}{\partial (lhs)/\partial \widetilde{b_{ik}^*}^i} = -\frac{\widetilde{b_{ik}^*}^i - c_i - \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2}{\beta s_k^*} + 1}{\widetilde{K}^i + e^{\widetilde{b_{ik}^*}^i}} > 0 \tag{4.61}$$

where *(lhs)* is the left hand side of equation (52). Equation (59) and (60) show that buyer i's personal optimal bid price increases as excess demand increases, and as his estimate of other buyers' bid prices increases.

[45] *Proof* of (60): Let $N = c_i + \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2}{\beta s_k^*} - 1$. It suffices to $N > \widetilde{b_{ik}}^i$. Plug N in (56), we have $e^N + \widetilde{K}^i N - \widetilde{K}^i N = e^N > 0$. By $f(x) = e^x + \widetilde{K}^i x - \widetilde{K}^i N$ strictly monotonically increases in x, we know $N > \widetilde{b_{ik}}^i$.

4.5.2 A Second Example (SAC4)

In this section, we consider an analytical solution corresponding to supplier acceptance case 4 (SAC4).

4.5.2.1 Supplier's Side

Recall in SAC4, we have that suppliers accept an offer if i^* if $b_{i^*k} > b^e - WC_k$, where b^e is the equilibrium price and WC_k the waiting cost. Following the specification of SAC4, we assume

$$WC_k \sim exp(f(Z)) \tag{4.62}$$

where $f(\cdot)$ is an increasing function. Note this specification means that the excess demand Z increases, the average waiting time $\frac{1}{f(Z)}$ will decrease. Then the probability of the supplier k accepting a bid (q_{ik}, b_{ik}) (conditioning on Z) can be written as:

$$\begin{aligned} & \mathbb{P}(\text{supplier } k \text{ accepts bids } i^*|Z) = \mathbb{P}(B_{i^*k} = 1|Z) \\ = \ & \mathbb{P}(WC_k > b^e - b_{i^*k}) \\ = \ & e^{f(Z)(b_{i^*k} - b^e)} \end{aligned} \tag{4.63}$$

Note $\frac{\partial e^{f(Z)(b_{i^*k} - b^e)}}{\partial b_{ik}} = e^{f(Z)} > 0$ agrees with the fact that as bid price increases it has a higher probability to be accepted.

4.5.2.2 Buyer Side

Now let us consider the buyer side. Recall each buyer i's optimization problem can be written as:

$$\begin{aligned} \max_{b_{ik}} \ & \mathbb{E}_{\mathbb{Q}_i}\Big\{(c_i q_{ik} + SC(q_{ik}) - b_{ik}q_{ik})\mathbb{1}(B_{ik} = 1)\Big\} \\ = & \mathbb{E}_Z\Big\{\mathbb{E}_{\mathbb{Q}_i}\Big\{(c_i q_{ik} + SC(q_{ik}) - b_{ik}q_{ik})\mathbb{1}(B_{ik} = 1)\Big\}|Z\Big\} \end{aligned} \tag{4.64}$$

If we further assume search cost and acceptance rule are conditional independent (conditioning on Z) and $SC(q_{ik}) = Z^2 q_{ik}/\beta s^*_{ik}$ as in the first example, then we have:

$$
\begin{aligned}
(lhs) =& \mathbb{E}_Z\Big\{\mathbb{E}_{\mathbb{Q}_i}\Big\{(c_i q_{ik} + SC(q_{ik}) - b_{ik}q_{ik})|Z\Big\}\mathbb{Q}_i(B_{ik}=1|Z)\Big\} \\
=& \mathbb{E}_Z\Big\{\mathbb{Q}_i(B_{ik}=1|Z)(c_i q_{ik} + \frac{Z^2 q_{ik}}{\beta s_k^*} - b_{ik}q_{ik})\Big\}
\end{aligned}
\tag{4.65}
$$

Since $q_{ik} > 0$ (as the buyer is still in the market), the first order condition can be written as:

$$
\begin{aligned}
\frac{\partial \mathscr{L}}{\partial b_{ik}} =& \frac{\partial \mathbb{E}_Z\Big\{\mathbb{Q}_i(B_{ik}=1|Z)(c_i q_{ik} + \frac{Z^2 q_{ik}}{\beta s_k^*} - b_{ik}q_{ik})\Big\}}{\partial b_{ik}} \\
=& q_{ik}\mathbb{E}_Z\{(c_i f(Z) - b_i f(Z) - 1)e^{f(Z)(b_i - b^e)} + f(Z)\frac{Z^2}{\beta s_k^*}e^{f(Z)(b_i-b^e)}\}
\end{aligned}
\tag{4.66}
$$

Then the buyer's personal optimal bid price satisfies:

$$
\widetilde{b_{ik}^*}^i = c_i - \frac{\mathbb{E}_{\mathbb{Q}_i} e^{f(Z)(b_{ik}-b^e)}}{\mathbb{E}_{\mathbb{Q}_i} f(Z) e^{f(Z)(b_{ik}-b^e)}} + \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2 f(Z) e^{f(Z)(b_{ik}-b^e)}}{\beta s_k^* \mathbb{E}_{\mathbb{Q}_i} f(Z) e^{f(Z)(b_{ik}-b^e)}} \tag{4.67}
$$

In a simple case where $WC \sim exp(\lambda)$ is independent of Z the market conditions, then we have

$$
\widetilde{b_{ik}^*}^i = c_i + \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2}{\beta s_k^*} - \frac{1}{\lambda} \tag{4.68}
$$

which only differs from the optimal solution in the first example by a constant. Hence this solution remains analytical as in the first example (equation (59) and (60)).

These two examples show that if seller's acceptance probability is independent of market condition (and known to buyers), each buyer i's optimal bid price is determined by the marginal instantaneous cost of not meeting requirement c_i plus the marginal search cost $\frac{\partial SC_{ik}}{\partial q_{ik}}$. Thus, in the case when search cost is a nonlinear function of the excess demand Z, a shock in supply will trigger a nonlinear increase in the optimal bid price. Such an effect will be further augmented by the bias of buyer's personal belief (e.g. that Z is close to 0).

4.6 Extensions to Other Types of Trading Sessions

In previous sections we presented the model of a single type 1 trading session (TS1). That is, there is no direct interaction between buyers and intertemporal dependence across trading sessions. In this section we generalize the model to other types of trading sessions as shown in table 1. We extend the model to accommodate buyers' coincidental arrival (as in TS2 and TS4) and intertemporal dependence (as in TS3 and TS4). By comparison, we showed TS1 can generate price jumps due to search cost, market structure and market condition. In the following extensions we show price jumps can also be generated due to buyer interaction, and can be mitigated by the storage effect.

4.6.1 Extensions to TS2

In a type 2 trading session, we introduce the possibility that multiple buyers arrive coincidentally. This setting could allow for only one-time, simultaneous bids, or repeated bidding, i.e. buyers submit multiple bids at the same encounter. Formally, depending on whether buyers can submit repeated bids and what information they are specified as having, there are three sub-cases in a type 2 trading session.

- TS2-1: Buyers can submit only one bid at the encounter. They submit sealed bids simultaneously and hence can not observe each other's bid. In this case, the only information available to buyers is the number of people showing up.
- TS2-2: Buyers can submit one bid at the encounter as in TS2-1, however they can observe bids of buyers that coincidentally arrive at the supplier.
- TS2-3: Buyers can submit multiple bids at the same encounter which results in repeated gaming. They can observe others' bids as in TS2-2 and are allowed to bid against each other to secure supply. Buyers are *fast reactors* in a sense that they can react to other buyers and submit multiple bids within the same encounter.

These assumptions lead to richer information sets of buyers (than in TS1). In particular, in the first case (TS2-1), the buyer's information set becomes $\Phi_k^i \cup \{n_k\}$, where Φ_k^i is the buyer information set at supplier k as in TS1 and n_k is the number of buyers that coincidentally come to the supplier. In the other cases (TS2-2 and TS2-3), the buyer's information includes that in TS2-1 plus the set of other buyers' bids.

In each of these cases, the buyers' optimal bids are determined by whether and how they evaluate the new information. If a buyer ignores the new information, his estimate of market condition and hence his optimal bid price will be exactly the same as those in TS1. As a result, given supplier's acceptance rule and waiting cost, the transaction prices will be the same as those in TS1. If the buyer uses the additional information to revise his estimate of market condition under some rule, his optimal bid price will generally deviate from that in TS1. The deviation depends on how buyers integrate the information observed at the encounter into his learning rule to form his estimate of market condition.

A possible approach to model the interaction between buyers (as in TS2-3) is to consider the herd behaviour among buyers. In past literature, buyers (traders) are usually categorized into two types: *rational traders* and *noise traders* (see [De +90], [SS90]). Rational traders base their expectation on economic fundamentals while noise traders are subject to erroneous stochastic beliefs and have irrational expectations. Within rational traders, we label those that are independently well-informed ones as *informed traders* and those that derive their information from informed trader choices as *followers*. Followers can be viewed as quasi-rational as they mimic decisions of informed, rational traders. Herd behaviour can be defined as follower behaviour resulting from noise traders shifting follow and thereby amplifying the share of transactions by follower or informed traders.

Shiller (1980) employed volatility tests and found stock market volatility to be far greater than described in efficient market hypothesis (see [Shi80]). Pindyck and Rotemberg (1988) argued that extreme movements of commodity prices are partially induced by "herd behaviour" of traders ([PR88]). That is, noise traders are either bullish or bearish across all commodity markets without justification from economic fundamentals. Bouchaud and Cont (2000) discussed the relationship between herd behaviour and the fluctuation in financial markets and showed that herd behaviour tends to induce extreme (heavy tail distributed) stock market returns (see [BC00]). Thus we suspect the transaction price tends to be higher through herd behaviour.

The settings of our model are not compatible with common herd effect models for two reasons. First, since early models of herd behaviour ([Ban92], [BHW92]), most herd behaviour literature concerns with discrete action and signal spaces while in our model, a buyer's action space (the space of his bid), signal space (the space of others' bids), and the state space (the space of true market conditions) are continuous. Although it is not impossible to model herd effect and information cascades with continuous action space (see [HO98], [ER09], [ER10]), it is hard to capture the continuous state space in this context.

More importantly, agents' information acquisition process in our model is rather different from the information cascade described in the herd effect models. In particular, in our model the buyers' learning rule is based on nonparametric Bayesian learning on the market condition through the searching process rather than from the signals of some other pioneer buyers. That there is another buyer i' bidding at the same supplier as well as the bid price this buyer i' offers are signals to the current buyer i, since the other buyer i' bid price based on his estimate of market condition. Such signals are hard to be integrated into buyer i's own learning process through observations of suppliers' capacity. In addition, buyer i could hardly assess the quality of the signal since a buyer's estimate of market condition is not revealed from his optimal bid (e.g. in equation (57) and (67), a buyer i can not infer another buyer i' estimate of market condition from buyer i''s optimal bid $\widetilde{b_{i'k}}^{i}$ since buyer i does not know buyer i''s penalty cost $c_{i'}$.). It is of great interest to integrate herd behaviour into the context of this model. On the other hand, the construction could be rather complicated.

4.6.2 Extensions to TS3 and TS4

In a type 3 or type 4 trading session, we allow for "*far-sighted*" suppliers. That is, a supplier is allowed to consider selling in the current session versus selling in the future. If a supplier k thinks the equilibrium in the current trading session is relatively "low" compared to normal (the products are currently undervalued), he can choose to put some of his supply quantity into storage and sell in the future.

To model such storage behaviour, we add an expectation of the future equilibrium price (in future trading sessions) to the current acceptance rule[46]. Under acceptance

[46]There are actually two possible ways to model such storage behaviour. Another way is to view storage as a special type of buyer (inventory speculator). The estimated bid price of the storage buyer is the estimated equilibrium price of the next session. The waiting cost of the buyer is storage cost (here we assume the inter-trading session waiting costs are much larger than the

rule 3 (SAC4), for a bid b_{ik}, the supplier accepts the bid if $b_{ik} > b_t^e - WC_k$ where b_t^e is the market equilibrium price in current session. With consideration of storage, the supplier will only accept the bid if it is higher than current equilibrium price and less than future equilibrium price adjusted by storage cost. These results can be summarized as the following rule:

Acceptance Rule in a Type 3 Trading Session: In a type 3 trading session, each supplier k can make tradeoff between selling in the current session and selling in the future. The supplier's acceptance rule is as follows:

- For a bid $b_{ik} > b_t^e - WC_k$, a *myopic* supplier k will accept the bid and ignore future sessions.
- For a bid $b_{ik} > \widetilde{b_{t+1}^e}^k - stc_k - WC_k \geq b_t^e - WC_k$ or $b_{ik} > b_t^e - WC_k > \widetilde{b_{t+1}^e}^k - stc_k - WC_k$, where we define $\widetilde{b_{t+1}^e}^k$ as the supplier k's estimated future equilibrium price in the next trading session and str_k is supplier k's storage cost, b_{ik} is higher than both equilibrium prices that he is guaranteed to sell in current and future session, and a *far-sighted* supplier k will accept the bid.
- For a bid $\widetilde{b_{t+1}^e}^k - stc_k - WC_k > b_{ik} > b_t^e - WC_k$, it is more profitable for the supplier k to sell in the future since it is guaranteed that he can sell at a price $\widetilde{b_{t+1}^e}^k - stc_k - WC_k$ that is higher than b_{ik}. In this case, a *far-sighted* supplier k will not accept the bid.
- For a bid $\widetilde{b_{t+1}^e}^k - stc_k - WC_k < b_{ik} < b_t^e - WC_k$, a *far-sighted* supplier k will reject the bid and choose to sell in the future (equivalently, he puts q_{ik}^a of his supply into storage).

We label such acceptance rule as SAC5.

4.7 Simulation

In this section we present numerical simulation to show that the model proposed in this model can generate price jumps that can not be characterized by competitive

inter-trading session ones). Then if the storage price is accepted based on some acceptance rule (for example, acceptance rule 4 in TS3), then the supplier chooses to store his supply to sell in the future.

market model and resemble those in reality. As shown in the previous sections, the fundamental driving forces to price jumps are:

1. Nonlinear effect of market condition in buyers' search cost;

2. Buyer's personal belief bias in market condition;

We also suspect given the same total supply, different allocation/concentration of the total supply across suppliers give rise to different transaction price distributions. In this section, we will simulate each of these three effects and show how these effects can drive up the transaction price when there is shortage in supply.

In the simulation, we assume there are 100 buyers and 10 suppliers in the market. We simulate 100 trading sessions. In each trading session, the supply and demand are generated from a normal distribution with parameter $(n_s, 1)$ and $(10, 9)$ respectively. Across the trading session, the parameter n_s is generated from Weibull distribution $H(10, 10)$ (with scale and location parameters equalling to 10) to characterize extreme events (e.g. severe shortage in supply). By using the hierarchical method to generate supplies, we are able to capture the circumstances that supplies across all suppliers co-move with an presence of extreme events (e.g. all farmer's harvests of a grain product decrease with extreme weather). As a result of this setting, in most cases, the total demand is about the same as total supply. In contrast, under extreme events, there could be rather large excess demand in the market. The generated excess demand across trading sessions are plotted in figure 4. The parameter β in search costs is chosen to be 100 so that search cost and penalty cost are comparable (note we also simulate the case $\beta = 300$ in section 7.1). We generate penalty costs based on normal distribution $N(100, 25)$ to capture heterogeneity in buyers' penalty cost. The sequence of encounters as in figure 1 is determined by the order of the time of the encounters generated from a Weibull distribution $H(1, 1)$. The buyers' learning rule, offer rule, and suppliers' acceptance rules are as specified in previous sections. All the specifications for the base case are summarized in table 4.2.

4.7.1 Single Trading Session Simulation

In this section we simulate the market in single session. We first simulate the nonlinear effect of market condition in buyers' search cost and hence their optimal

Table 4.2. Summary of Specifications: Base Case

Parameter / Random Variable	Value / Distribution
$\lvert I \rvert$	100
$\lvert K \rvert$	10
S_k^*	$N(n_s, 1)$
n_s	$H(10, 10)$
D_i^*	$N(10, 9)$
stc_k	$N(10, 25)$
β	100
c_i	$N(100, \sigma_c^2)$
σ_c	5
Encounter Time	$H(1, 1)$

bid price using simulation. Recall the buyer i's *personal* optimal bid price based on acceptance rule 1 in equation (58) is

$$\widetilde{b_{ik}^*}^i = c_i + \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2}{\beta s_k^*} - 1 - \mathscr{W}\left(\frac{e^{c_i + \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2}{\beta s_k^*} - 1}}{\widetilde{K}^i}\right) \tag{4.58}$$

Similarly based on acceptance rule 4, the buyer i's *personal* optimal bid price is

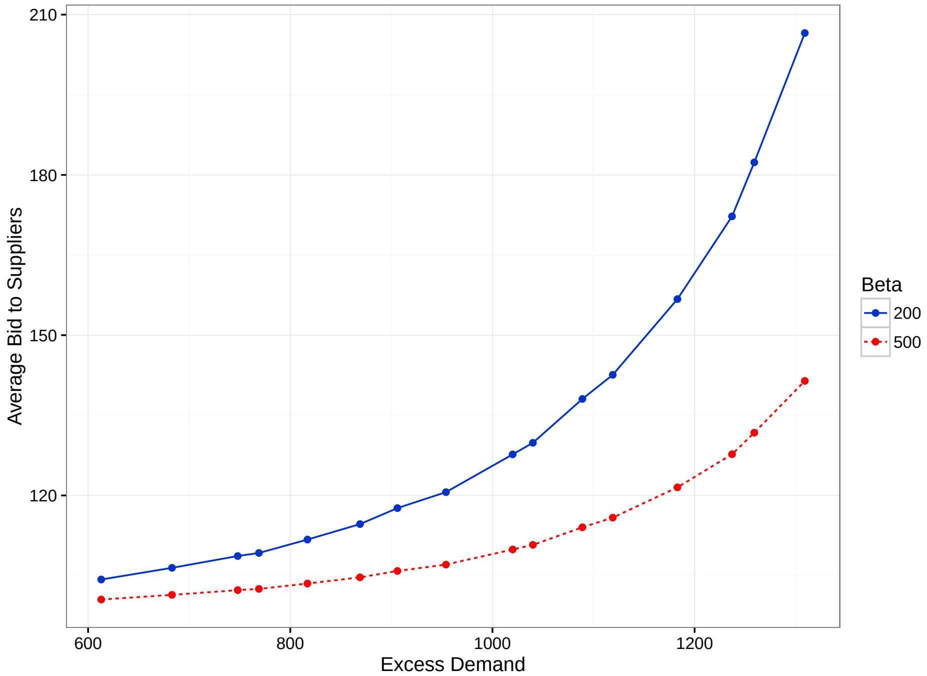
$$\widetilde{b_{ik}^*}^i = c_i - \frac{\mathbb{E}_{\mathbb{Q}_i} e^{f(Z)(b_{ik} - b^e)}}{\mathbb{E}_{\mathbb{Q}_i} f(Z) e^{f(Z)(b_{ik} - b^e)}} + \frac{\mathbb{E}_{\mathbb{Q}_i} Z^2 f(Z) e^{f(Z)(b_{ik} - b^e)}}{\beta s_k^* \mathbb{E}_{\mathbb{Q}_i} f(Z) e^{f(Z)(b_{ik} - b^e)}} \tag{4.67}$$

In either case, the buyer i's *personal* optimal bid price is a nonlinear function of market condition Z. Figure 3 shows that the buyer's optimal bids reacts to excess demand nonlinearly. The solid line the optimal bid price versus different market condition under the case $\beta = 200$. This is the parameter value we later use. In comparison, we also simulate the case when $\beta = 500$. c_i are generated from $N(100, 10)$. The figure shows that higher β gives lower optimal bid price. This is consistent with our model since search cost decreases with β. We also note, in both case, the optimal bid price increase nonlinearly in excess demand.

4.7.2 Multiple Trading Session Simulation

To proceed in this section, we first illustrate price behaviours graphically. That is generated over a small number (100) of trading sessions. This ensures graphical

Figure 4.3. Single Buyer Average Optimal Bids to Suppliers versus Excess Demand

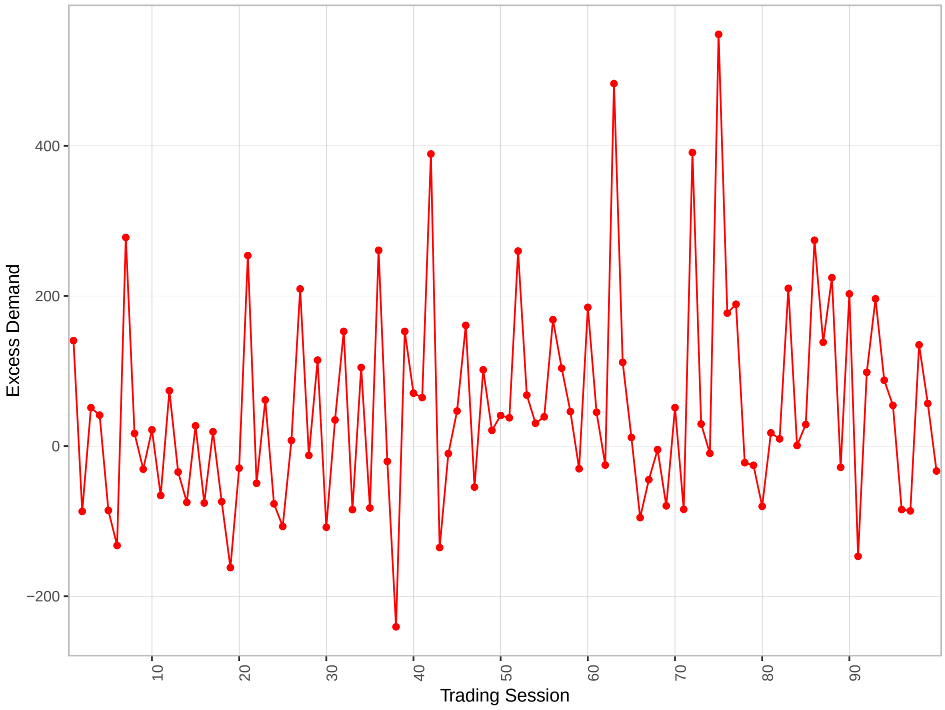


presentation. Next, to evaluate frequency and other characteristics of jumps, we present summary statistics based on 10000 trading sessions. We simulate different trading session cases under different assumptions on buyers and suppliers. We first simulate a type 1 trading session (TS1) with assumption buyer's optimal bid case 2 (BOB2 thereafter) and supplier's waiting cost and acceptance rule case 4 (SAC4 thereafter). We label this case as TS1BOB2SAC4. We also simulate prices in a type 3 trading session (TS3) with the same assumption on buyers and suppliers. We label this second case as TS3BOB2SAC4.

4.7.2.1 Simulation of TS1BOB2SAC4

In this section, we simulate a case under the assumption trading session case 1, buyer's optimal bid case 2 and supplier's waiting cost and acceptance rule case 4 (or, acceptance rule 3). We start with simulating a sequence of supply capacity across trading sessions. The suppliers supply are generated in the procedure specified at the beginning of section 6. Figure 4 reports the excess demand in each session.

Figure 4.4. Excess Demand Realization used to Initialize Trading Sessions



Under the market condition shows in Figure 4, in each single session, the buyers make optimal bid prices and the supplier accept or reject bids based on acceptance rule 4 as specified in the model. Figure 5 reports accepted bid prices (transaction prices) and quantities in each session using candlestick chart. In each trading session, the top of the vertical line reports the maximum transactional price, and the bottom of the vertical line reports the minimum transactional price. The opening price and the closing price in one trading session are plotted as the top and bottom of the candle bar. If the opening price is lower than the closing price, then we mark the trading session as a price rise session. Otherwise, it is a price fall session. The equilibrium prices across trading sessions are plotted as blue dots and connected with dashed line. The trading volume of each trading session is represented as the width of candle stick.

As seen in Figure 5, when there is a large excess demand in a trading session, the transaction prices are significantly higher than the equilibrium price. Such difference reflects that buyers' search cost increase nonlinearly as market becomes tight and suppliers becomes more reluctant to accept bids in tight market. This figure shows that the model proposed in this paper can generate price jumps that resemble those in reality but are not characterized by the market equilibrium price.

Figure 5 shows in the session with large excess demand, the transaction prices are significantly higher than the market equilibrium price. Such price jumps reflect that buyers are difficult to search and that suppliers are reluctant to acceptance offers in tight market. These effects are characterized as buyers' search cost nonlinear increases and suppliers' acceptance probability nonlinear decreases in tight market by our model.

We also note there is relatively large variation in transaction prices in each trading session. We suspect that this is due to the specification of penalty cost c_i's and search cost. This motivates a closer look into these factors.

In order to measure the jump size across trading sessions in figure 5, we define some jump metric as percentage difference between some representation of the sequence of transaction prices in a trading session (e.g. weighted average price, closing price, or minimum price of that session) and market equilibrium price in that trading session. We first plot open, close, high and low prices over trading sessions in Figure 6. They all show similar movements over time. This indicates we may use either one as a representation of the transaction prices. Here we choose closing price[47] as such representation in that closing price characterizes the most up-to-date status of the market within a trading session[48] . Formally, we define jump size as the difference between contemporaneous closing bid price and equilibrium price normalized by equilibrium price. Following Chapter 3, we denote contemporaneous jump size by γ_t, then we have

$$\gamma_t \equiv \frac{\text{(Closing Bid Price - Equilibrium Price)}}{\text{Equilibrium Price}} \times 100\% \tag{4.69}$$

[47]We assume there is no transaction or encounter outside a trading session (no *after-hours trading*). Thus the closing price is defined as the last transaction price in a trading session.

[48]Some other jump metrics can be used here, for example:

- *Jump Metric 1* $\equiv \frac{\text{(Weighted Average Bid Price - Equilibrium Price)}}{\text{Equilibrium Price}} \times 100\%$
- *Jump Metric 2* $\equiv \frac{\text{(Minimum Bid Price - Equilibrium Price)}}{\text{Equilibrium Price}} \times 100\%$

The plots of jump size using these metrics are in appendix. On the other hand, the jump size measured by these jump metrics basically shows the similar pattern and reflects jumps shown in figure 5 as that measured by closing price. Thus we will stick to use the jump metric by closing price.

Figure 4.5. Bid Price across Trading Sessions ($c_i \sim N(100, 25)$ and $\beta = 100$)

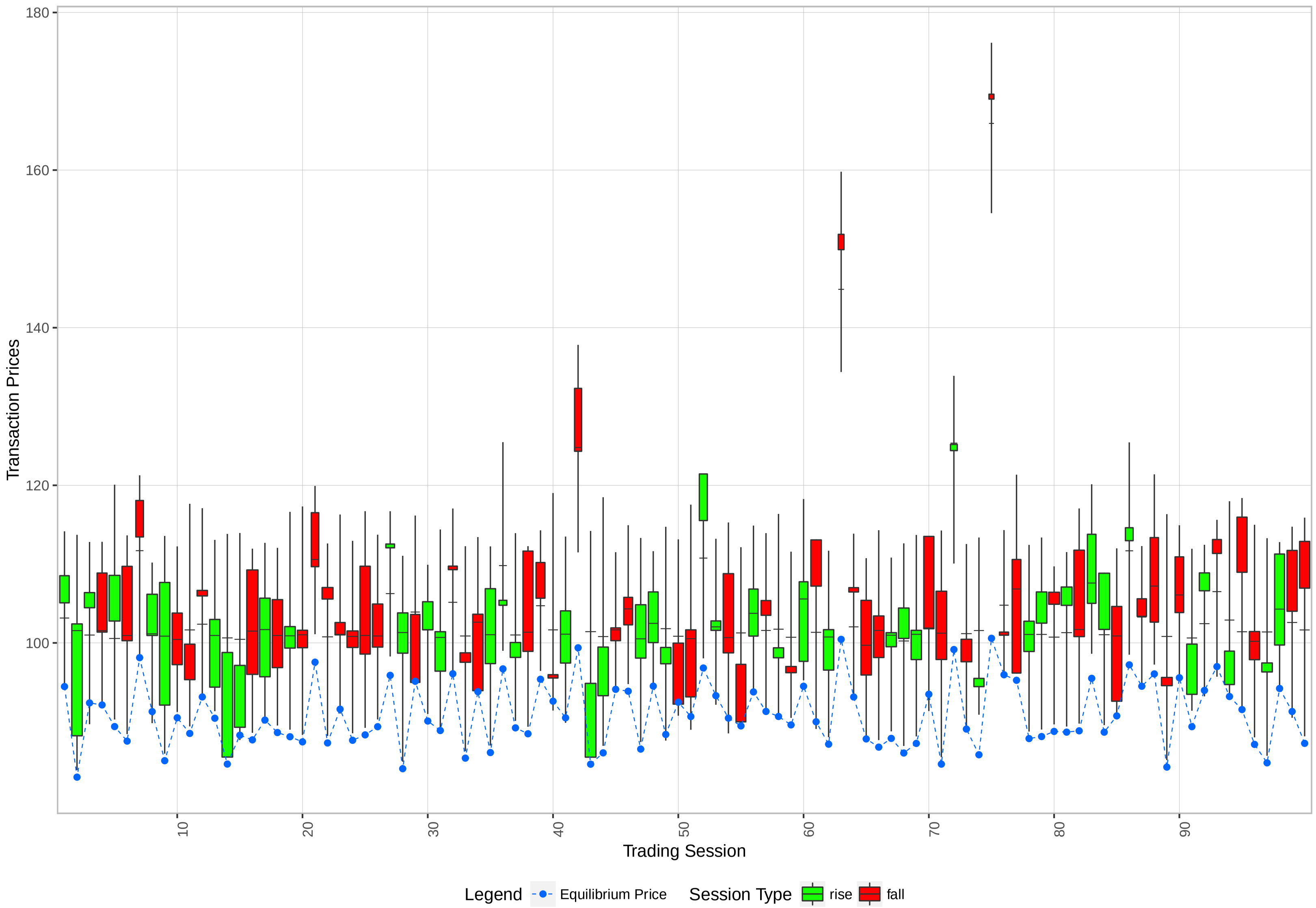
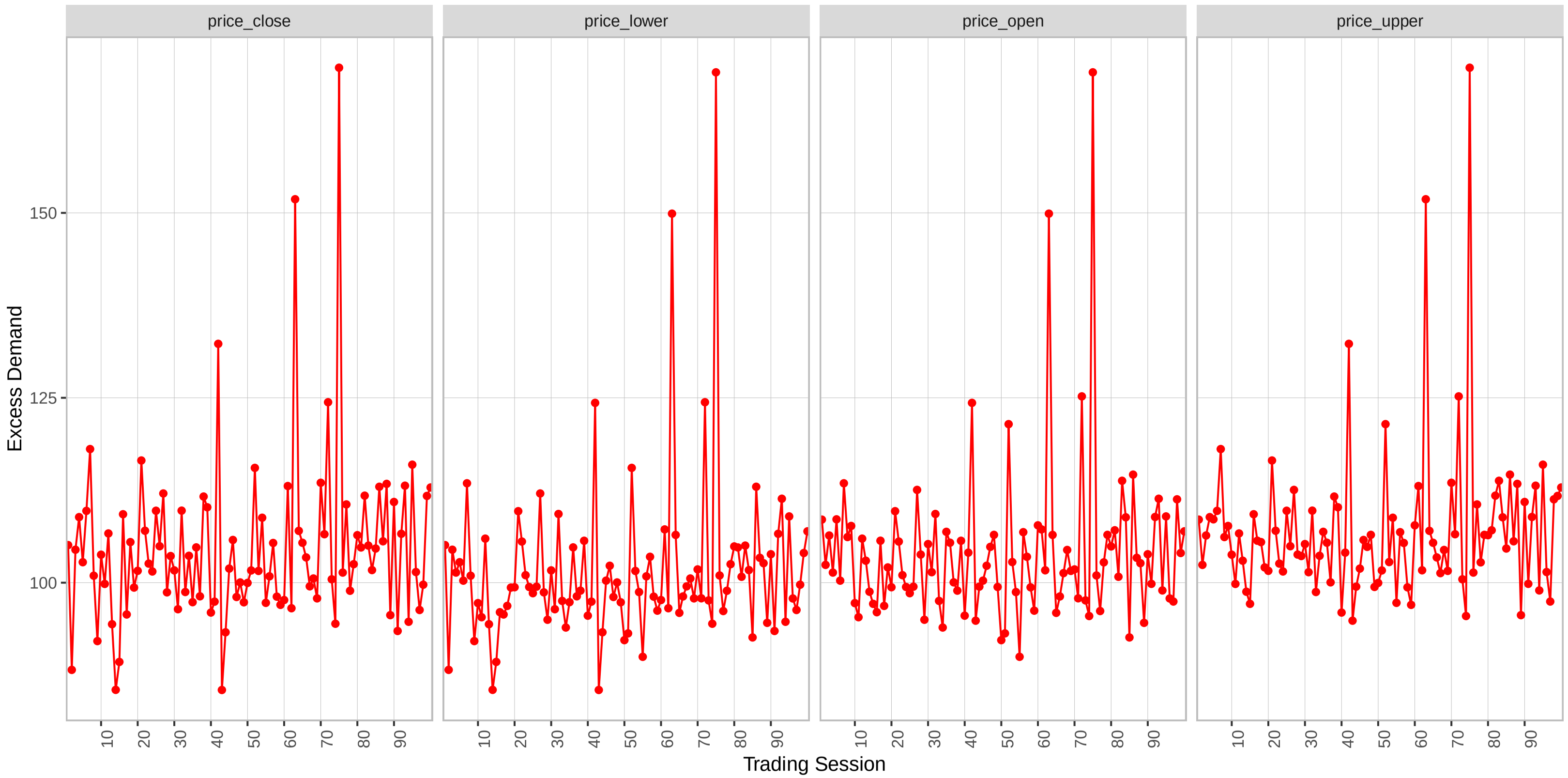
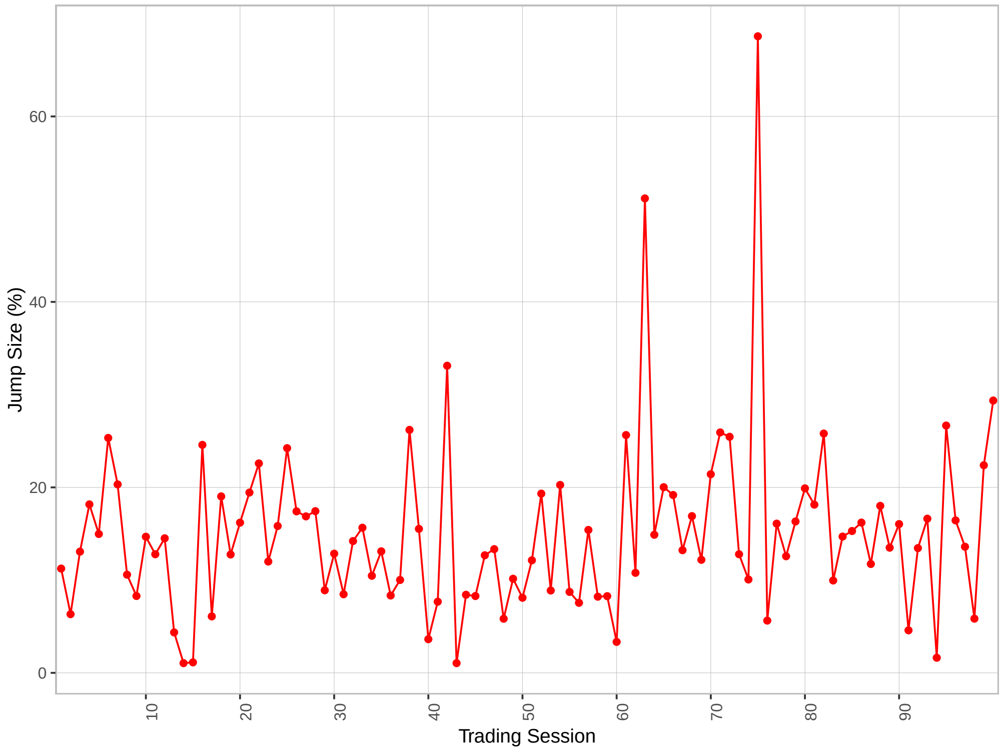


Figure 4.6. Open, Close, High and Low Prices across Trading Sessions



We plot price jump size measured such jump metric in Figure 7. We see that in most trading sessions, jump size fluctuates around 10%. Such difference reflects the search cost under normal market condition (demand and supply are approximately the same). In cases when supply is in severe shortage (for example, trading session 63, trading session 75), the jump size exceeds 50%.

Figure 4.7. Jump Size γ_t across Trading Sessions Measured by Closing Price

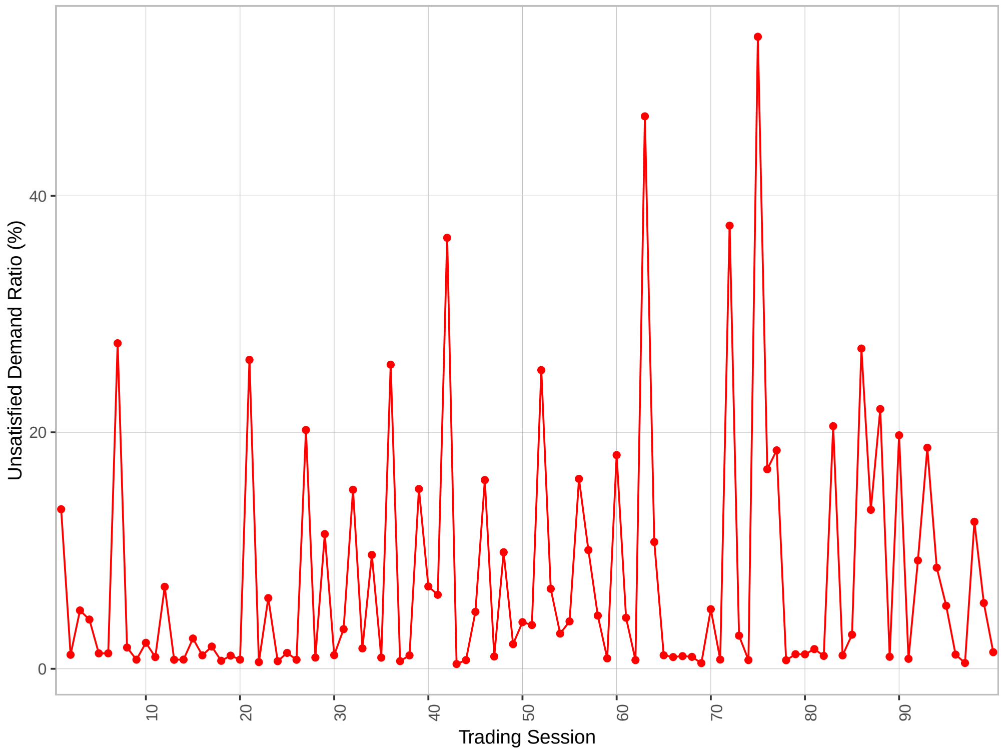


In this type of trading session (TS1), buyers' unfulfilled demand in current trading session can not be carried over to the next session. Thus it is of importance to know the unfulfilled demand at the end of each trading session. We define *unfulfilled demand ratio* as the ratio of unfulfilled demand at the end of each trading session and excess demand at the beginning of each trading session. We denote unfulfilled demand ratio as U_t, i.e.

$$U_t \equiv \frac{\text{Unfulfilled Demand at the End of Each Trading Session}}{\text{Excess Demand at the Beginning of Each Trading Session}} \times 100\%$$

The unfulfilled demand ratio's across trading sessions are plot in Figure 8. We see from the figure that the peaks unfulfilled demand ratio directly corresponds to the peaks in the excess demand plotted in figure 4. This reflects the fact that the unfulfilled demand ratio is the positive part of excess demand scaled by itself (i.e. unfulfilled demand ratio $\approx \frac{max(0,Z)}{Z}$). We also plotted the unsold supply ratio across trading sessions in Figure 9. We see in Figure 9 there are general unsold supply capacity. This implies the trading volume as plotted in figure 5 does not generally equal to the total allocated supply in trading sessions.

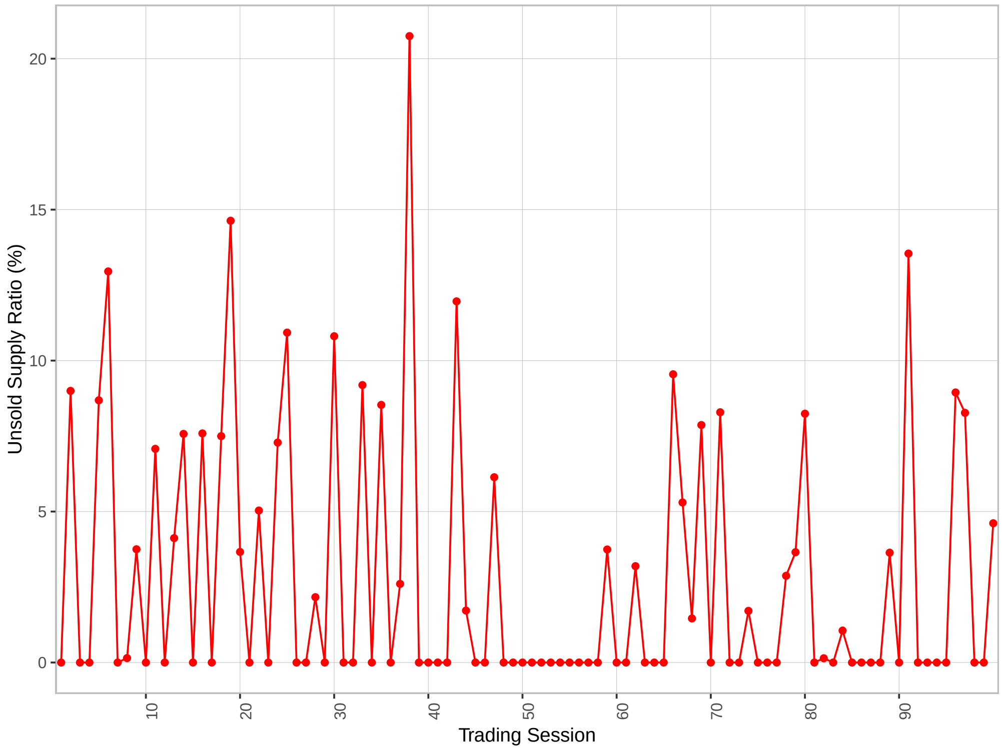
Figure 4.8. Unfulfilled Demand Ratio U_t across Trading Sessions



To compare the frequency of jumps in closing prices (as in our model) and equilibrium prices (as in competitive model), we use the empirical outliers test proposed by in [CL93] (CL test thereafter). [CD05] extended the method to GARCH model and applied it to examine the effects of outliers in three daily stock market indexes. The outlier identification procedure can be described as follows. Suppose the simulated closing prices can be thought as a univariate time series indexed by trading sessions t and hence can be written as:

$$P_t = \frac{\theta(B)}{\alpha(B)\phi(B)} e_t + \sum_{j=1}^{T_o} \omega_j L_j(B) \mathbf{1}_O(t_j) \tag{4.70}$$

Figure 4.9. Unsold Supply Ratio across Trading Sessions



where $e_t \sim WN(0, \sigma^2)$; B is the backward shifter: $BP_t = P_{t-1}$. $\theta(B)$ is the moving average polynomial, $\phi(B)$ is the autoregressive polynomial, and $\alpha(B)$ is the integration polynomial. All roots of $\theta(B)$ and $\phi(B)$ are outside unit circle; and all roots of $\alpha(B)$ are on the unit circle. Thus, $\frac{\theta(B)}{\alpha(B)\phi(B)}e_t$ represents the smooth trend of the price and follows a ARIMA process. $L(B)$ is a polynomial of B representing the dynamic pattern of the outlier effects. $\mathbf{1}_O(t)$ is an indicator variable that equals to 1 when there is outliers effect at time t ($t \in O$)and 0 ($t \notin O$) elsewhere; O is the set of possible latent time locations of outliers; and ω_j represents the magnitude of the outlier effects; and T_o is the total number of outliers.

If there are no outlier effects, the second term in equation (72) is 0 and hence

$$\tilde{e}_t \equiv \frac{\alpha(B)\phi(B)}{\theta(B)} P_t \tag{4.71}$$

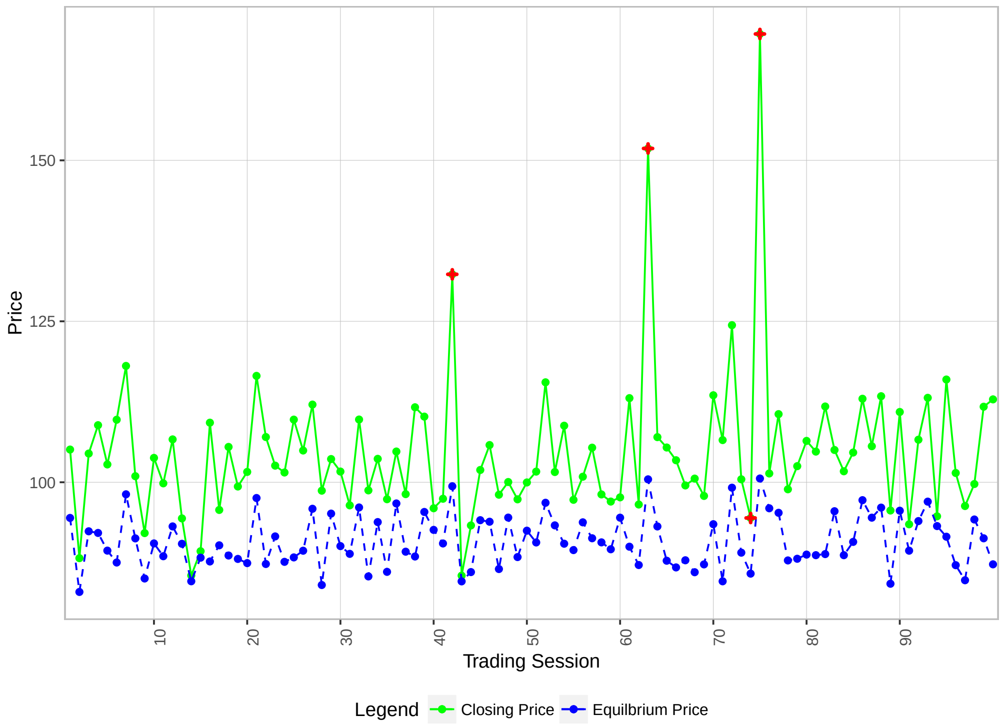
should be sufficiently close to the white noise e_t. Thus to test the presence of outliers, CL test examines to what extent $\tilde{e}_t$ is not zero. The test statistic is:

$$\hat{\tau}_t = \frac{\tilde{e}_t}{\hat{\sigma_e}} \tag{4.72}$$

where $\hat{\sigma_e} = 1.483 \times \text{median}\{|\hat{e_t} - \bar{e_t}|\}$ and $\bar{e_t}$ is the median of the estimate residuals.

With CL test presented, we can empirically examine jump frequency of a set of simulated prices. We first examine the number jumps in closing and equilibrium prices simulated in the first model (with $\beta = 100$). The closing and equilibrium prices are plotted in Figure 10 and jumps are marked by cross. As shown in Figure 10, there are four jumps in closing prices. These jumps correspond to the positive peaks of the excess demand. In contrast, there are no jumps in equilibrium prices under the same market conditions. This shows our model captures more price jumps than the competitive market model.

Figure 4.10. Closing and Equilibrium Prices with Jump indicated



Note: In this figure, a cross indicates time at which jump is detected.

- **The Effect of Search Cost**

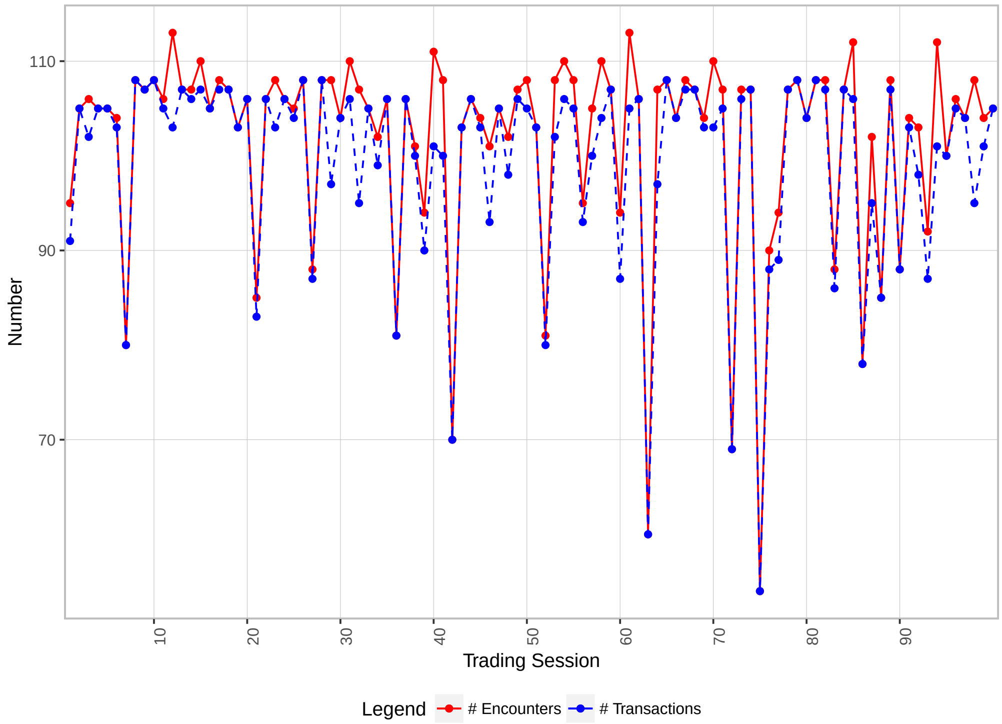
In figure 5 and Figure 7, we have seen that the search cost is a key driver in forming prices and inducing price jumps. From another perspective, search cost can be thought as a type of transaction cost inducing market *frictions* which by definition is a type of market incompleteness that prevents a trade from being executed smoothly (see [Coh+83], [DR07]). Here we suspect search cost introduces frictions in the buyers' search process: buyers are reluctant to search under high search cost. To examine this, we simulate the number of *actual* encounters and transactions across trading sessions. We consider the difference between them as a measure of frictions. That is, a small difference means buyers are reluctant to search due to high frictions while a big difference shows that the market encourages buyers to search. We see in Figure 11, the number of actual encounters and transactions moves in a similar way across trading sessions. Thus, we conclude that in general (*actual*) encounters lead to transactions. We suspect the small difference between them is due to the specification of relative high search cost, since under high search cost ($\beta = 100$), buyers are less willing to search and forced to bid high (otherwise they need to face high search cost). Then, suppliers tend to accept such high bids (given the same waiting cost) and as a result, encounters are more likely to lead to transactions. In comparison, low search cost ($\beta = 300$) as plotted in Figure 12 encourages buyers to search more. As a result, the difference between the number of actual encounters and that of transaction becomes bigger, showing small frictions in the search process.

Since search cost is a key driver to price jumps, we more closely examine the effect of search cost. We examine jump frequencies under different search cost specifications in the same way as we did for the first model. We are also interested in the volatility of the closing prices under different cases. We suspect that with high search cost (low β) the prices become more volatile since a low β will exaggerate the difference in buyers' belief of the market condition. Formally, we use *realized volatility* (see [AB98]) to measure the variation in the closing prices, denoted as RV:

$$RV \equiv \sqrt{\sum_{t=2}^{T}(\log P_t - \log P_{t-1})^2} \tag{4.73}$$

where T is the number of trading sessions (here $T = 100$) and P_t is the closing price. The jump frequency, size and realized volatility under difference search cost

Figure 4.11. Number of Encounters and Transactions ($\beta = 100$ High Search Cost)



Note: The dash line plots the number of transactions and the solid line plots that of encounters.

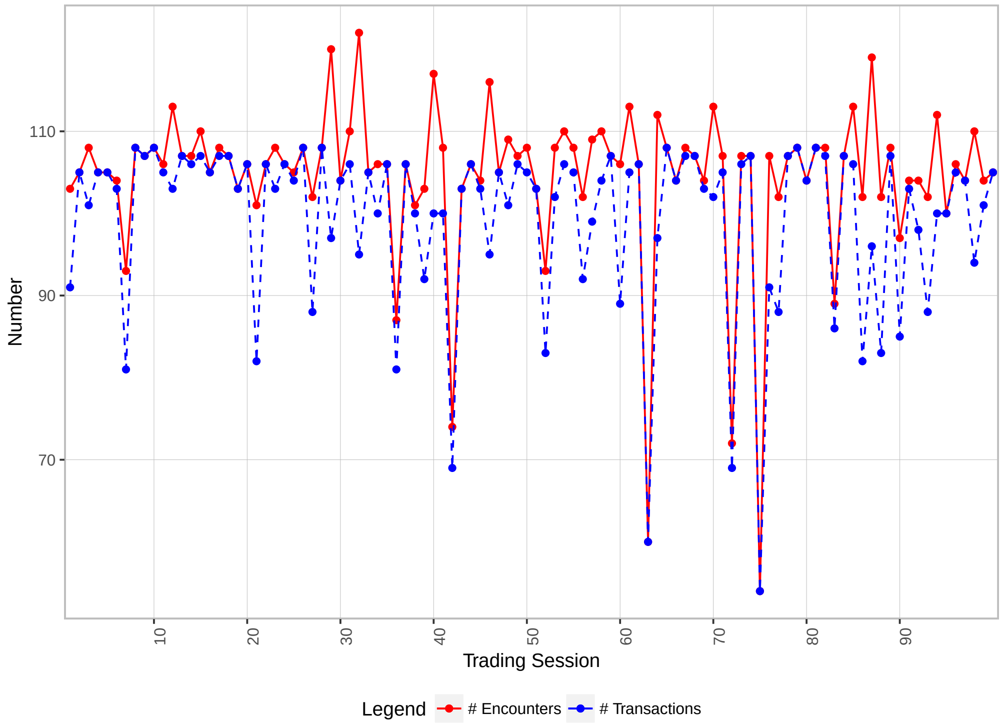
specifications are reported in table 4.5 and table 4.6.

As seen in table 4.3, as search cost increases (β decreases), there are more jumps in closing price. In contrast, the number of jumps in equilibrium price stays the same (all 0) since they do not have the search cost component. We also note, as search cost increases, closing prices becomes more volatile. This proves our previous conjecture. A small β will exaggerate buyers' belief in market condition and generate more extreme bid prices.

- **The Effect of Concentration of Supply**

We next examine the role of concentration of *total* supply in a trading session when supply varies substantially across suppliers. That is, observed supply at any encounter with a supplier provides the buyer with one observation that may be useful for predicting total supply in the market. For example, if only a few suppliers hold most supply available in the market, a buyer is more likely to observe supply

Figure 4.12. Number of Encounters and Transactions ($\beta = 300$ Low Search Cost)



Note: The dash line plots the number of transactions and the solid line plots that of encounters.

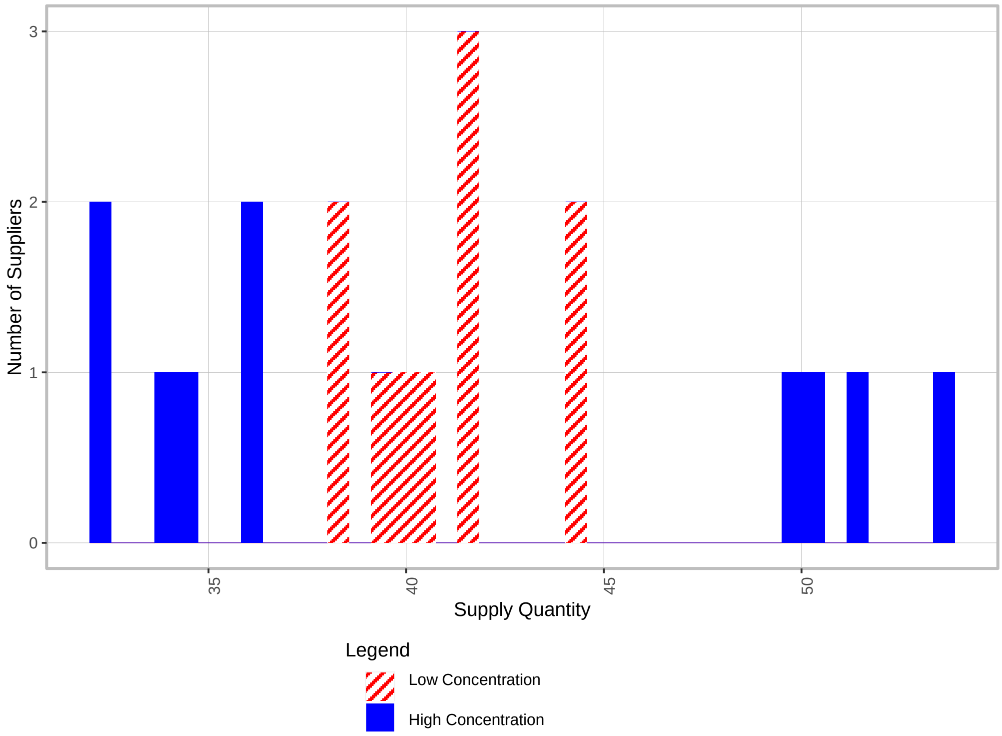
availability that is not representative of market level total supply condition. As a result, he will think the market is very tight (tighter than it will already be in supply shortage), i.e., his $\widetilde{Z}^i$ could be very low. In contrast, with a more dispersed distribution of supply across suppliers, a buyer is more likely to observe available supply that reflects market conditions. As a result, his personal estimate of the market is more accurate and the accuracy (unbiasedness) increases more rapidly across encounters. Figure 13 plots two supply allocations. The red allocation corresponds to the case when every supplier holds roughly the same supply. The blue histogram shows the case with high concentration of supply.

Table 4.3. Jump Frequency, Jump Size and Realized Volatility

β	Jumps (Closing Price)	Jumps (b^e)	Max Jump Size	Realized Volatility
50	10	0	133.7769 %	2.086653
75	5	0	90.35319 %	1.665363
100	4	0	68.64135 %	1.440626
150	2	0	46.92952 %	1.217818
200	2	0	36.07360 %	1.102902
250	2	0	29.56005 %	1.028019
300	1	0	29.3648 %	0.9517208

Note: In this table, the second and third columns are the number of jumps in closing prices and equilibrium prices (b^e) in 100 trading sessions. The fourth column reports the maximum of jump size using jump metric defined as above. The last column reports the realized volatility of closing prices defined in equation 74.

Figure 4.13. Examples of Distribution of Initial Supply Allocation of a Trading Session



To measure market concentration, we use Herfindahl–Hirschman Index (see [Hir64], [JB79]), which is denoted by HHI. Formally HHI can be calculated as follows:

$$HHI \equiv \sum_{k=1}^{|K|} \left(\frac{S_k^*}{\sum_{j=1}^{|K|} S_j^*} \right)^2 \tag{4.74}$$

where $\sum_{j=1}^{|K|} S_j^*$ is the total market supply and hence $\frac{S_k^*}{\sum_{j=1}^{|K|} S_j^*}$ is the market share of supplier k in the market. A higher HHI means that the market is more concentrated. Note HHI is bounded from below by $\frac{1}{|K|}$ when every supplier holds exactly the same market share. And HHI is bounded (from above) by 1 when one supplier is the only one with stock and holds all supply in the market.

Note in the first simulation of TS1BOB2SAC4, each supplier's initial allocation of supply is drawn from a normal distribution. In this case the average score of the concentration index equals to 0.3162. In this section, we simulate the market with higher concentration. The result is reported in the third row of table 4.5. As seen from the table, with high concentration of the supply with $HHI = 0.1042273$ in contrast to 0.1000105, there are 6 price jumps compared to 4. We also note maximal jump size is significantly higher than that in the first case. These results show that (even slight) higher concentration of supply can induce price jumps. We conclude that the distribution of supply is a key determinant in price jumps. This result is not so trivial from analytical solutions (in contrast, the effect of search cost in price jumps is much easier to identify from analytical solutions). Transaction prices and jump size in the case of high concentration are plotted in figure 23 and 29 respectively.

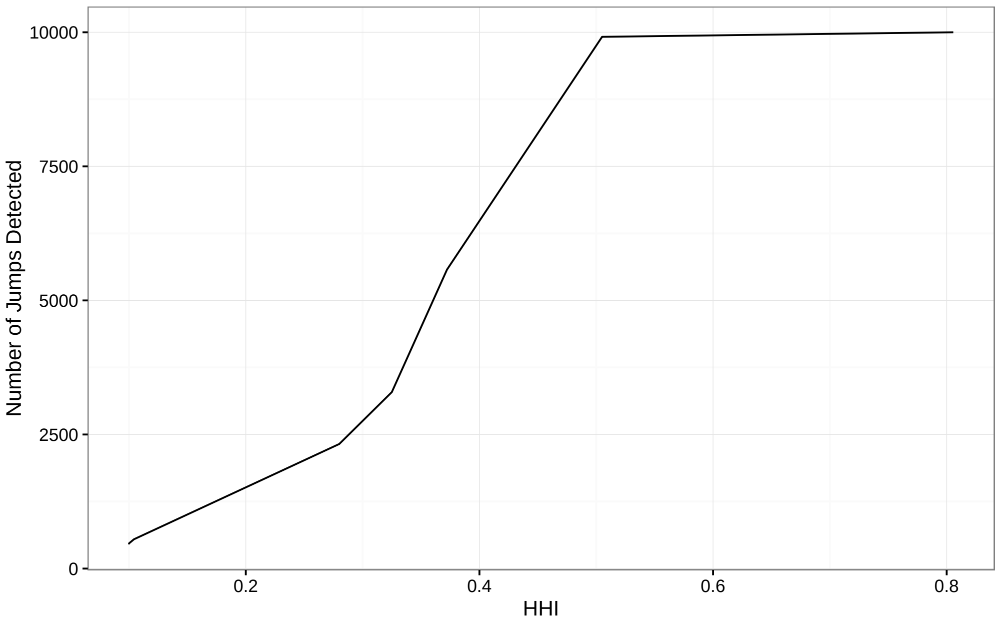
To better estimate jump frequency and examine its change due to higher concentration, we simulate 10000 trading sessions under the two cases. The results are reported in table 4.4. As shown in the table, the number of detected jumps increases from 469 to 546 as concentration index increases slightly from 0.10001 to 0.10423. More generally, we see as concentration increases, both the number of jumps and realized volatility increases in a nonlinearly fast manner. We plot the results in figure 4.14. This is consistent with our previous results of 100 trading sessions and suggests that high concentration of supply is a key source of price jumps.

Table 4.4. Jump Frequency (10K Trading Sessions)

HHI	Jumps	Max Jump Size	*RV*	Mean Volume	Max Volume
0.10000	451	762.7608	12.604	937.611	1105.919
0.10001	469	691.6276	12.713	937.611	1105.919
0.10423	546	902.1253	15.424	937.274	1101.769
0.28000	2323	1709.11	38.181	937.629	1105.919
0.32500	3290	2947.372	40.679	937.309	1123.802
0.37225	5575	1037.715	46.679	937.281	1111.703
0.50500	9915	3720.9	85.151	938.434	1112.502
0.80567	10000	94135.65	290.658	937.630	1105.919

Note: In this table, the first column reports HHI of the case. The second column reports the number of jumps in closing prices in 10000 trading sessions. The third column reports the maximum of jump size using jump metric defined as above. The fourth column reports the realized volatility of closing prices defined in equation 74. The last two columns report mean trading volume and maximal trading volume respectively.

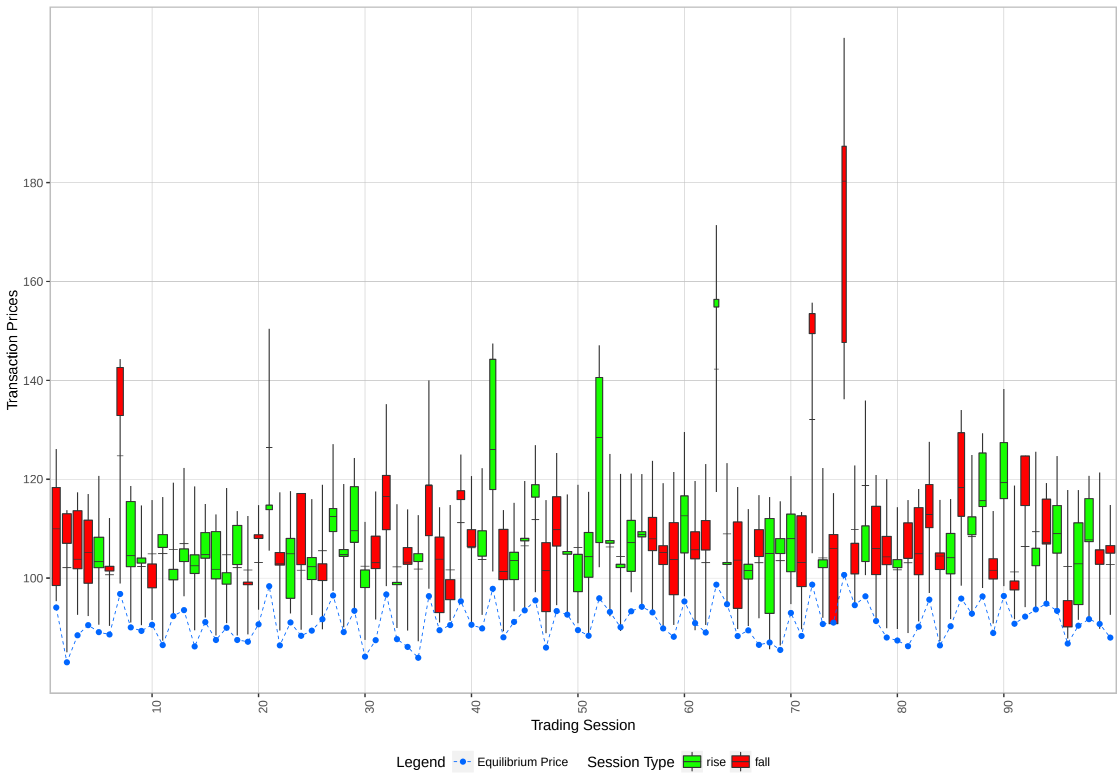
Figure 4.14. Number of Jumps Detected over Different HHIs



- **The Effect of Penalty Cost Variation**

Intuitively, if penalty cost has larger variation, the transaction prices should be more volatile since high variance encourages the presence of extreme values. As a result, there could be more jumps in the transaction prices. In this part, we simulate the case when $\sigma_c^2 = 100$. Results presented in figure 4.15 can be compared to those for $\sigma_c^2 = 25$ presented in figure 4.5. The result is reported in the fourth row of table 4.5 and table 4.6. We see although prices becomes significantly more volatile (realized volatility is rather high compared to other cases), the numbers of jumps in both closing and market equilibrium prices decreases from 469 to 223 for 10000 trading sessions. At the first glance, the result is rather surprising since price volatility and the presence of price jumps are closely related. A possible explanation is that the price fluctuations in this case are better characterized by the conditional variance model (for example, GARCH) than by the jump models.

Figure 4.15. Bid Price in TS1 with High Penalty Cost Variation ($c_i \sim N(100, 100)$ and $\beta = 100$)



Note: This figure plots the simulated prices with $\sigma_c^2 = 100$. The results presented in this figure can be compared to those for $\sigma_c^2 = 25$ presented in figure 4.5.

Table 4.5. Jump Frequency, Jump Size, Realized Volatility and Trading Volume

Case	Jumps (Closing Price)	Jumps (b^e)	Max Jump Size	RV	Mean Volume	Max Volume
Base Case TS1BOB2SAC4 $\beta = 100, \sigma_c = 5,$ $HHI = 0.1000105$	4	0	68.641%	1.441	940.2868	1072.494
High Search Cost TS1BOB2SAC4 $\beta = 300, \sigma_c = 5,$ $HHI = 0.1000105$	1	0	29.365%	0.952	940.2868	1072.494
High Concentration TS1BOB2SAC4 $\beta = 100, \sigma_c = 5,$ $HHI = 0.1042273$	6	0	86.163%	1.463	942.3243	1094.446
High Penalty Cost Variation TS1BOB2SAC4 $\beta = 100, \sigma_c = 10,$ $HHI = 0.1000105$	2	0	70.850%	2.008	940.1837	1072.494
Base Case Storage TS3BOB2SAC5 $\beta = 100, \sigma_c = 5,$ $HHI = 0.1000105$	3	0	63.913%	1.354	970.6786	1081.269

Note: This table corresponds to the cases for graphical illustration that uses 100 trading sessions. In this table, the first column indicates the case and parameter we simulate. The second and third columns are the number of jumps in closing prices and equilibrium prices (b^e) in 100 trading sessions. The fourth column reports the maximum of jump size using jump metric defined as above. The fifth column reports the realized volatility of closing prices defined in equation 74. The last two columns report mean trading volume and maximal trading volume respectively.

Table 4.6. Jump Frequency, Jump Size, Realized Volatility and Trading Volume: 10000 Trading Sessions

Case	Jumps (Closing Price)	Jumps (b^e)	Max Jump Size	Realized Volatility	Mean Volume	Max Volume
Base Case TS1BOB2SAC4 $\beta = 100, \sigma_c = 5,$ $HHI = 0.1000105$	469	0	691.6276%	12.7131	937.611	1105.919
High Search Cost TS1BOB2SAC4 $\beta = 300, \sigma_c = 5,$ $HHI = 0.1000105$	126	0	222.9008%	8.212194	937.611	1105.919
High Concentration TS1BOB2SAC4 $\beta = 100, \sigma_c = 5,$ $HHI = 0.1042273$	546	0	902.1253%	15.42406	937.2741	1101.769
High Penalty Cost Variation TS1BOB2SAC4 $\beta = 100, \sigma_c = 10,$ $HHI = 0.1000105$	223	0	644.4719%	18.22781	937.41	1105.919
Base Case Storage TS3BOB2SAC5 $\beta = 100, \sigma_c = 5,$ $HHI = 0.102851$	399	39	459.9445%	11.40037	967.3564	1116.094

Note: In this table, the first column indicates the case and parameter we simulate. The second and third columns are the number of jumps in closing prices and equilibrium prices (b^e) in 10000 trading sessions. The fourth column reports the maximum of jump size using jump metric defined as above. The fifth column reports the realized volatility of closing prices defined in equation 74. The last two columns report mean trading volume and maximal trading volume respectively.

We take a closer look into this case. Figure 15 plots the transaction prices across 100 trading sessions. We notice that transaction prices are more volatile than previous case. The presence of extreme prices are also high. A possible reason that our statistics do not capture these jumps is that we use closing prices to represent the transaction prices within a whole trading session. For example, in trading session 75, the large jump in transaction prices does not appear in the closing price. This shows although the closing price is a good representation of the prices in a trading session, it might lead to some misinformation. Combining the results from Figure 15 and table 4.4, we conclude that high penalty cost variation do induce price jumps and higher volatility in transaction prices.

4.7.2.2 The Effect of Storage: Simulation of TS3BOB2SAC5

In the section, we present the simulation results of type 3 trading. The new allocated demand and supply will be the same as in TS1BOB2SAC4 and hence the excess demand is the same as in Figure 4. We assume suppliers use acceptance rule 5 to determine whether to accept or not. Storage cost is drawn from $N(10, 25)$ to characterize suppliers' heterogeneity.

For the purpose of comparison, we first simulate transaction prices and trading volume as in type 1 trading sessions. Figure 17 reports the result in the same fashion of Figure 5.

We note that in Figure 5, the two biggest price jumps (at trading session 63 and 75) directly correspond to the large excess demand in Figure 4. This is consistent with that each trading session is assumed to be independent. In contrast, Figure 15 shows that in type 3 trading sessions the transaction price jumps do not directly to supply shortage. This can be caused by the supply storage. That is, price jumps can be mitigated by storage (for example, trading session 75). On the other hand, the carryover of unfulfilled demand can generate price jumps that are not induced by new generated excess demand (for example, trading session 90).

To further compare the transaction price in type 1 trading session and type 3 trading session, we plot the weighted average price and close price in each type of trading sessions in Figure 16 and Figure 18. We see from the figures that both close price and weighted average price in type 1 trading sessions are more volatile than those in type 3 trading sessions. This is consistent with previous results that suppliers' storage can mitigate the price oscillations. We further plot jump size (using jump size 1 and jump size 3) in Figure 19 and Figure 20. We see from the figures that jump sizes in type 1 trading sessions and type 3 trading sessions have roughly the same magnitude. On the other hand, we see type 3 trading sessions do no produce extreme price jumps as type 1 trading sessions do when there is large supply shortage (for example, trading session 75 in Figure 19). This says suppliers' storage behaviour helps mitigating extreme price jumps.

Figure 4.16. Comparison of Average Bid Price in TS1 vs. TS3

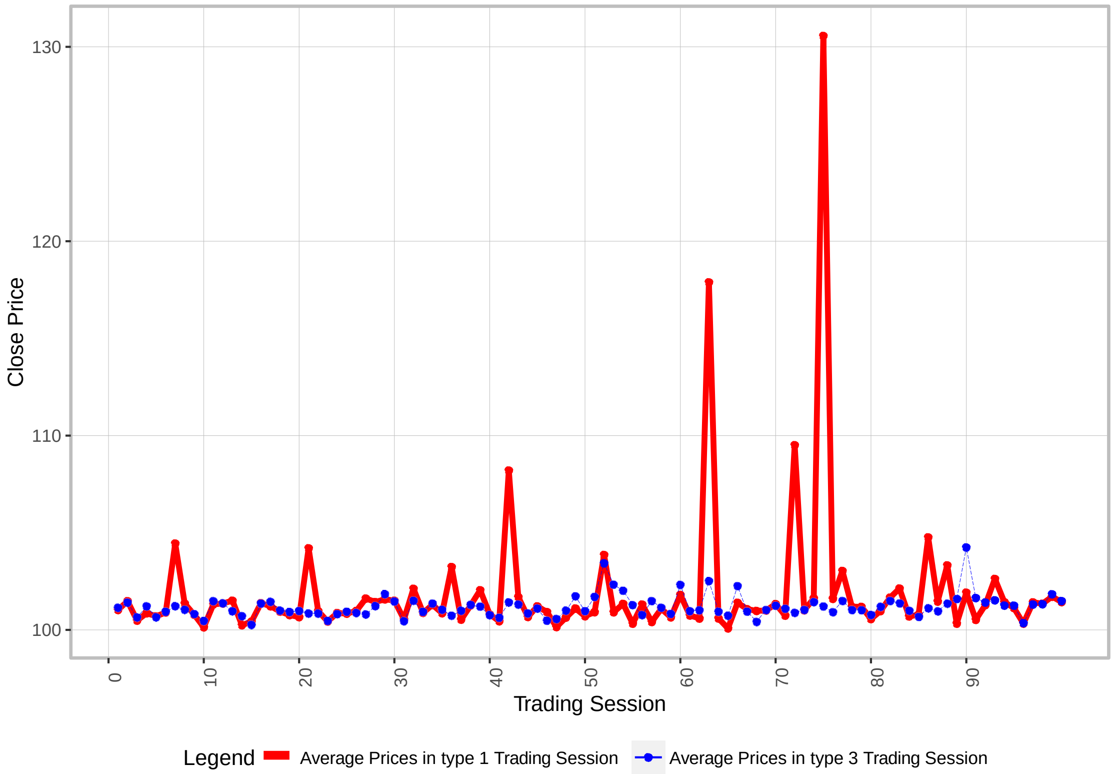
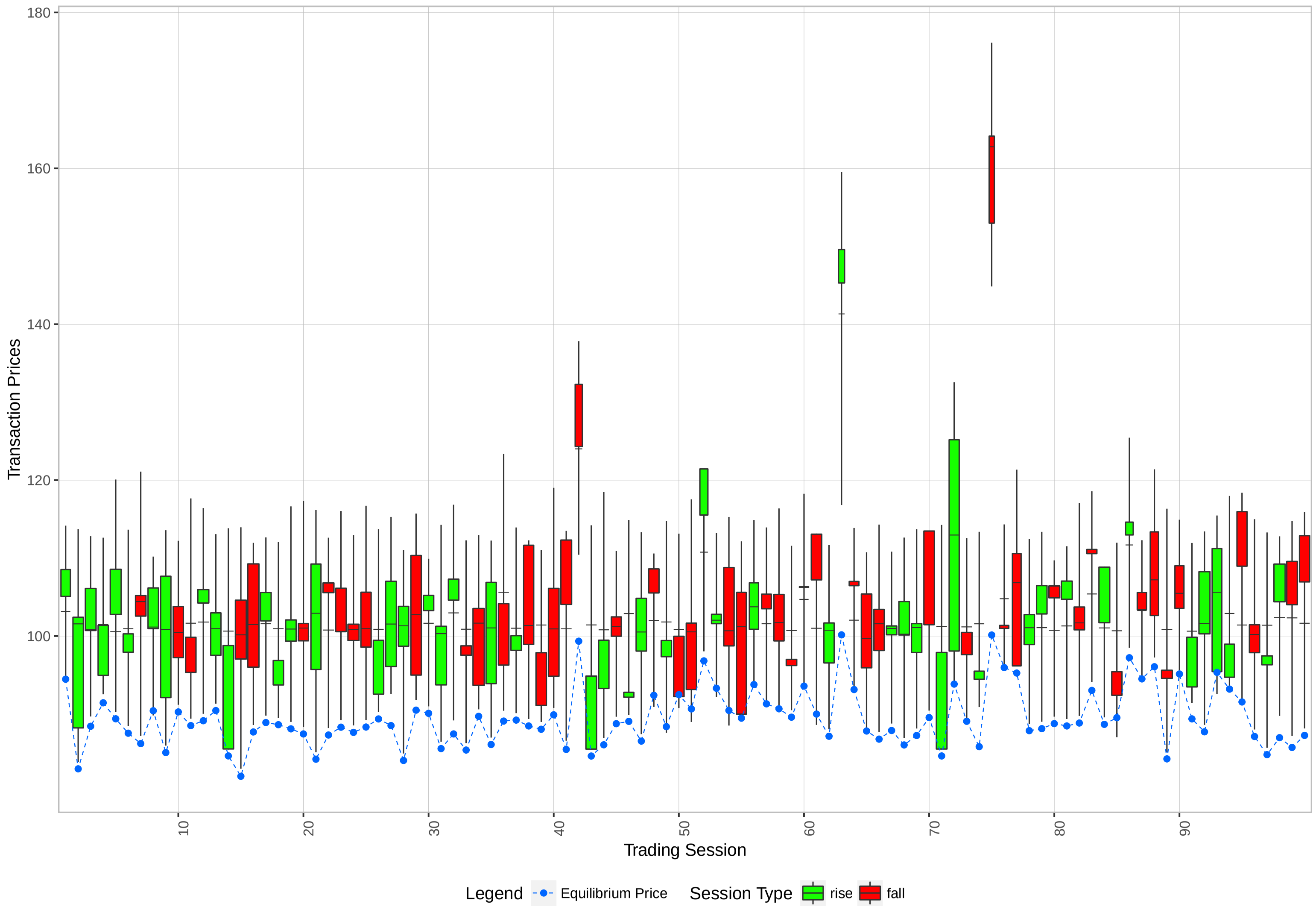
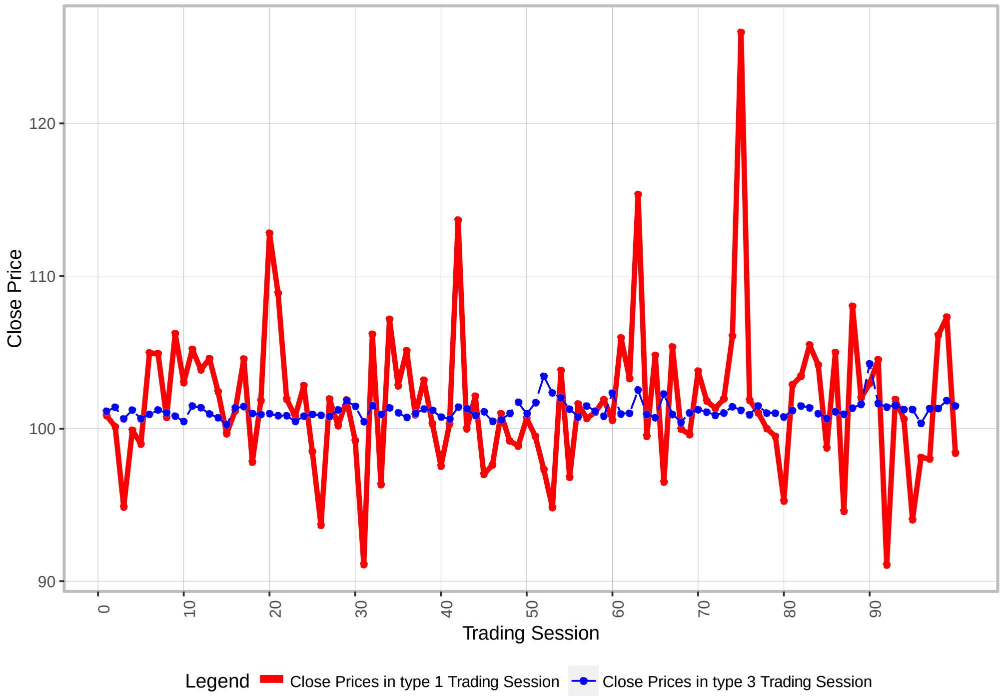


Figure 4.17. Bid Price in a Type 3 Trading Sessions ($c_i \sim N(100, 25)$ and $\beta = 100$)



Note: The figure plots the prices with storage effect. The results presented can be compared to those with no storage effect in figure 4.5.

Figure 4.18. Comparison of Close Price in TS1 vs. TS3



We further plot the number of transactions and encounters in type 3 trading sessions in Figure 21. We see that in general *actual* encounters lead to transactions. Similar to trading session 1, this is due to the relative high search costs. Since in the case of TS1BOB2SAC4 we already examine the effect of search cost on number of transactions. We do not re-simulate the effect here again.

In Figure 22, we plot buyers' unfulfilled demand ratio in current trading session. Such demand can be carried over to the next trading session. In Figure 22, we see that in the most cases when there is no large excess demand, the unfulfilled demand ratio is smaller than that in TS1. Such observation is consistent with the result that total realized volatility in TS3 is smaller than that in TS1 (as reported in Table 4.4). On the other hand, when there is large excess demand (stock-out), the unfilled demand ratio is much higher than that in TS1. This creates more price fluctuations in the session with large excess demand. This result is consistent with [HW77].

Figure 4.19. Comparison of Jump Size in TS1 vs. TS3

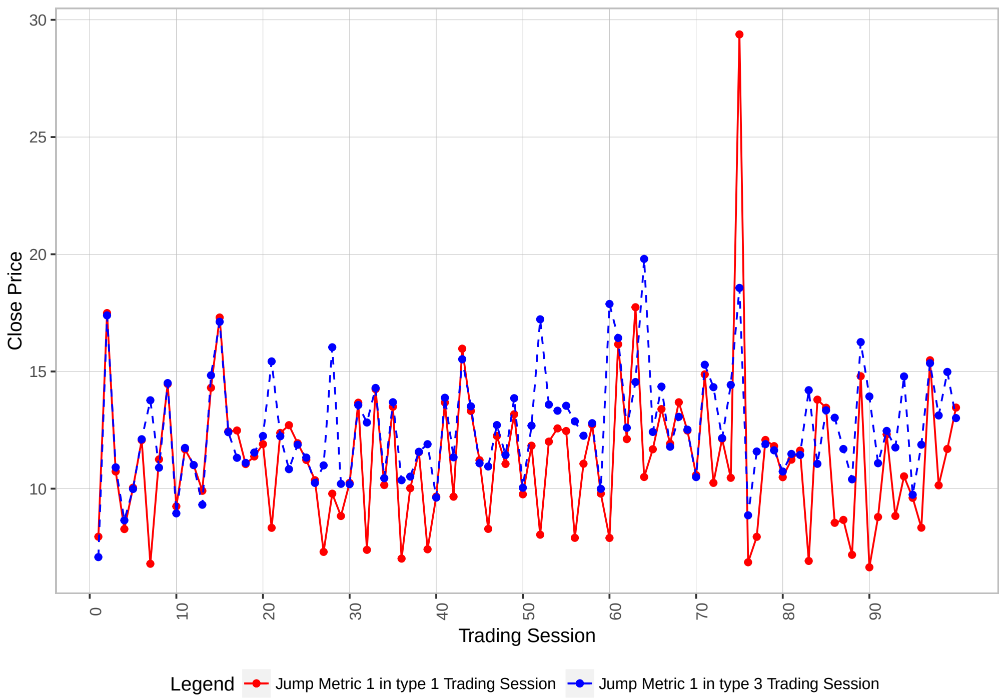
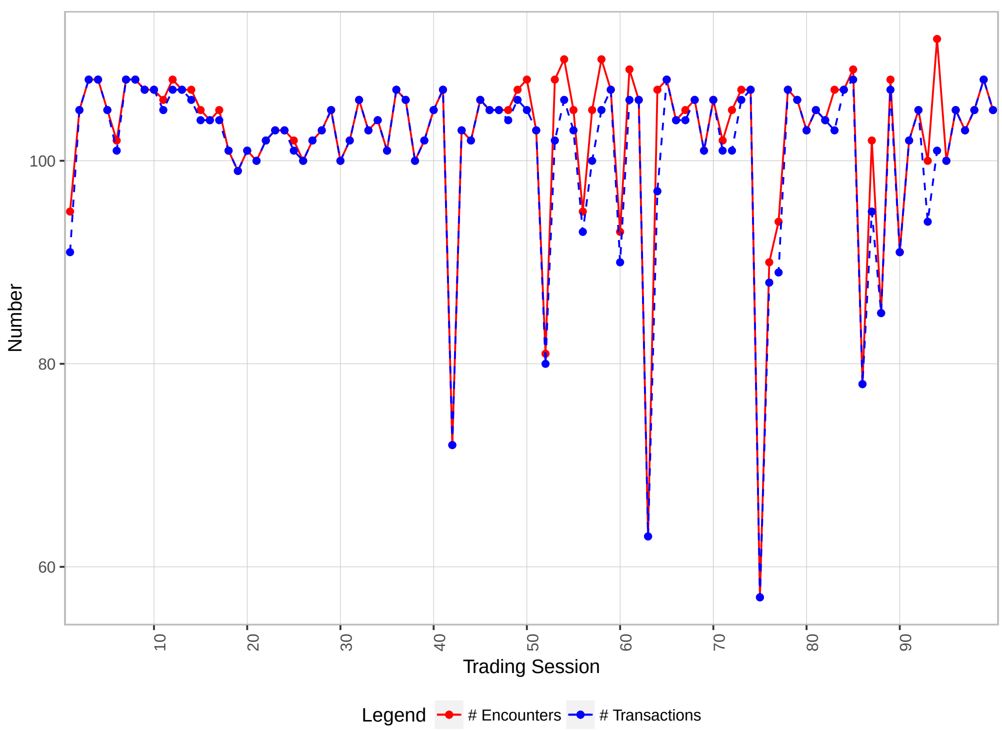


Figure 4.21. Number of Encounters in TS3



Note: The dash line plots the number of transactions and the solid line plots that of encounters.

Figure 4.20. Comparison of Jump Size in TS1 vs. TS3

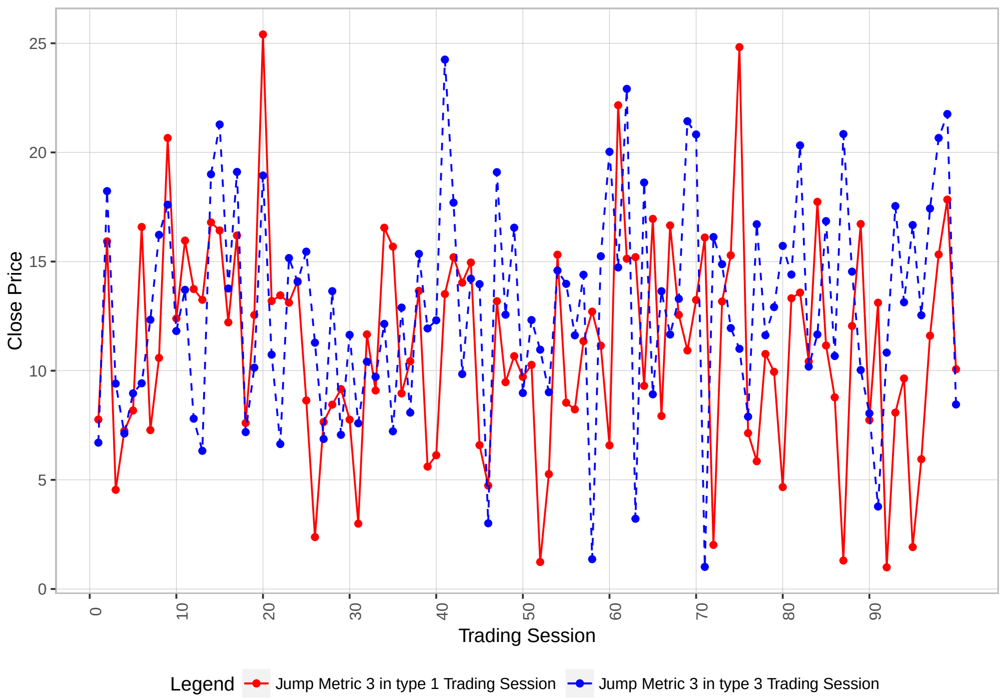
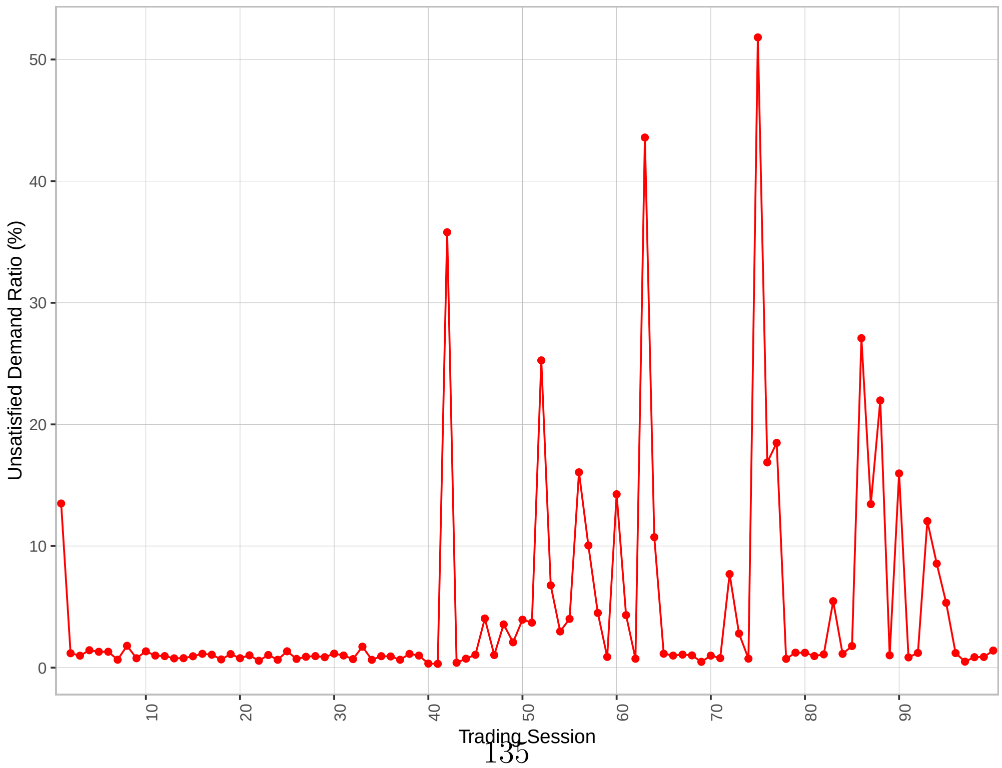


Figure 4.22. Unfulfilled Demand Ratio in TS3



The results corresponding to Figure 16-19 are reported in table 4.5 (100 trading sessions). We further simulate 10000 trading sessions to more accurately estimate the jump frequency. The result is reported in the last row of table 4.6. An interesting observation from table 4.6 is that the number of jumps in equilibrium price (b^e) increases from 0 to 39 in 10000 trading session. This is consistent with the results of [HW77] suggesting that in competitive market models, storage behaviour can induce price jumps when there is stock-out. On the other hand, when our model with search cost and imperfect information is considered, the number of detected jumps decreases when storage effect is considered. This suggests that the extreme price movements can be mitigated by storage behaviour, to what extent the carry over storage exceeds zero. We think such discrepancy is due to the settings of respective models. In the competitive market model, usually only total supply is considered; while in our model, not only the total supply but also its distribution across suppliers plays an significant role in prices through the search cost of imperfectly informed buyers.

In summary, we see from the simulation of case TS1BOB2SAC4 that the model proposed in this paper can generate price jumps that resemble those in reality. We show the number of jumps and volatility are sensitive to the search cost specification, concentration of supply and penalty cost. We also show that high search costs and high concentration of supply can result into more volatile price series in which there are more price jumps.

In the simulation of case TS3BOB2SAC4, we show in this section that the storage behaviour (or inventory speculation) in TS3 can mitigate shocks in supply. They also induce intertemporal correlation in price (of trading sessions). We show that if storage behaviour is allowed, the extreme price jumps can be smoothed to what extent the carry over storage exceeds zero. This result is consistent with our intuition that carry over demand or supply storage generate serial (auto)correlatiion of prices. It also agrees with the results from previous literature (for example, [DL96], [Mut61]).

4.8 Concluding Remarks

In this paper we proposed a rather general model of procurement with a set of different assumptions of the market. We discussed different cases of buyers, suppliers, and trading sessions. In particular, we present two exemplary analytical solutions to the model in two cases (labelled as TS1BOB2SAC1 and TS3BOB2SAC5). Using simulation, we show that buyers' search cost and their personal belief, suppliers' acceptance rule and the allocation of total supply (across suppliers) are the key driving forces of price jumps. As a result, our model generates jumps of similar magnitude and frequency as those in observed economic price series that generally can not be characterized by the perfect competitive market models.

Although the linkage between agents' risk aversion and price volitity has been noted in literature (see [NS79], [DL92]), most literature investigating commodity price dynamics assume risk-neutral agents (see [HW77], [WW82], [DL92], [DL96], [CB96]). A key feature of our model is that we integrate buyers' risk aversion in market conditions (though they are risk neutral in payoffs) through their personal belief and search costs. That is, buyers' risk aversion in market condition affects their *estimated* search costs through their personal belief, which leads to large price movements. Buyers' risk aversion increases their sensitivity to market conditions, which exaggerates price movements with presence of supply shocks.

With discussions of type 3 trading sessions, we extend the work of [HW77] in examining the effect of storage in commodity price dynamics by allowing for extreme events and risk aversion. Simulation results show although storage behaviour might introduce some fluctuation in general cases (this agrees with [HW77]), it to some extent mitigates severe price movements with presence of extreme events.

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Appendix A
Graphics for Chapter 4

A.1 Comparison of CandleStick chart

Figure A.1. Unfulfilled Demand Ratio across Trading Sessions

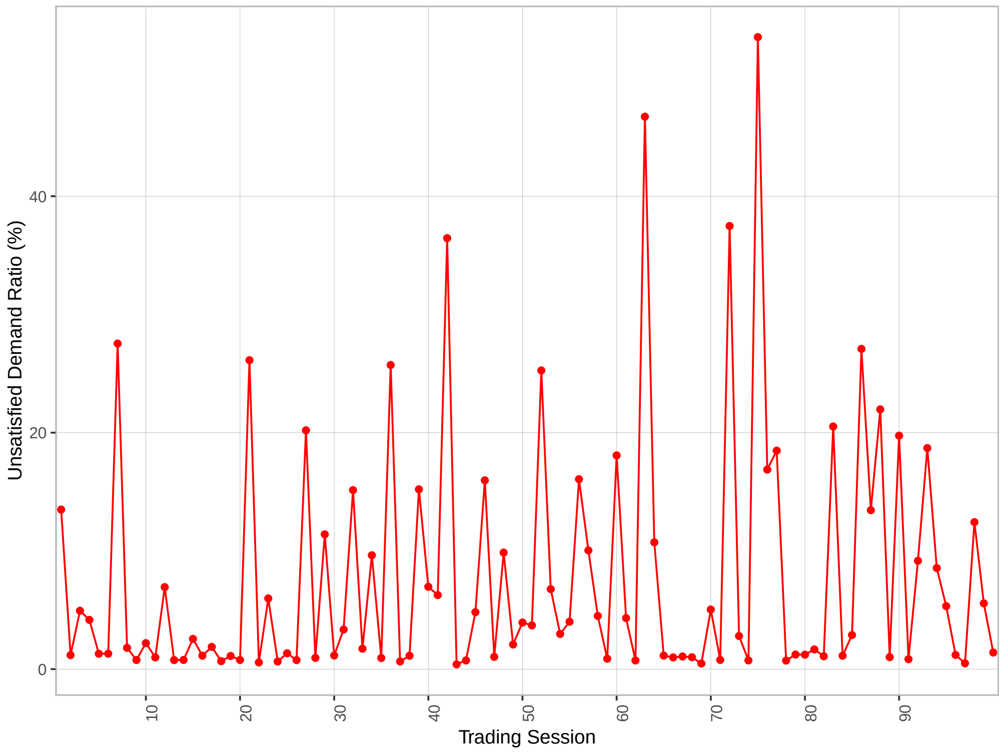


Figure A.2. Bid Price in TS1 ($c_i \sim N(100, 25)$ and $\beta = 100$)

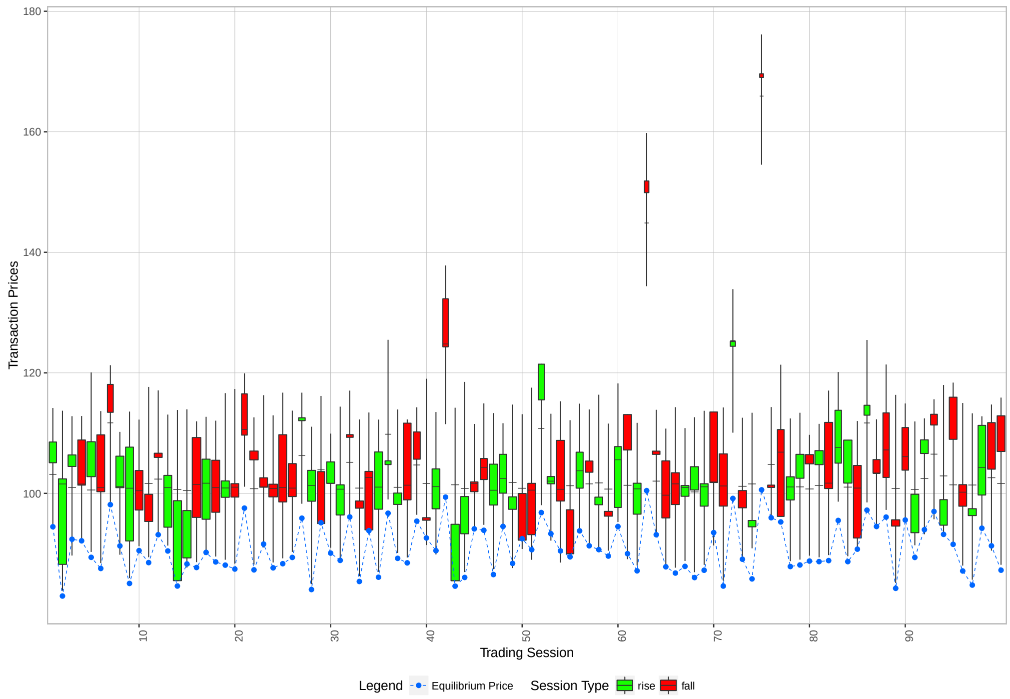


Figure A.3. Bid Price in TS1 with Low Seach Cost ($c_i \sim N(100, 25)$ and $\beta = 300$)

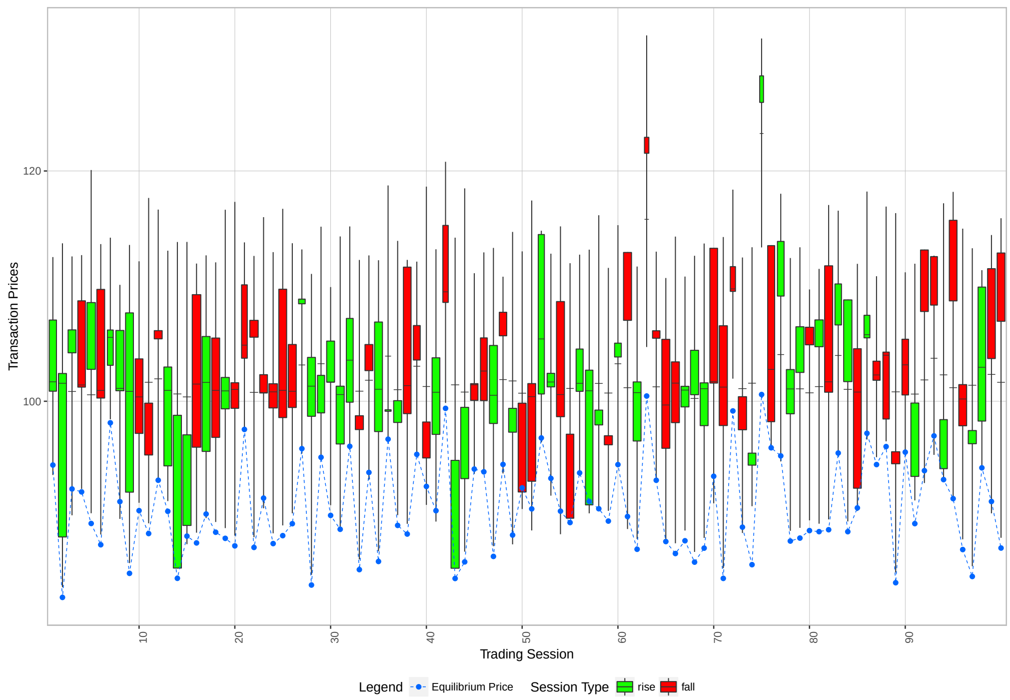


Figure A.4. Bid Price in TS1 with High Concentration ($c_i \sim N(100, 25)$ and $\beta = 100$)

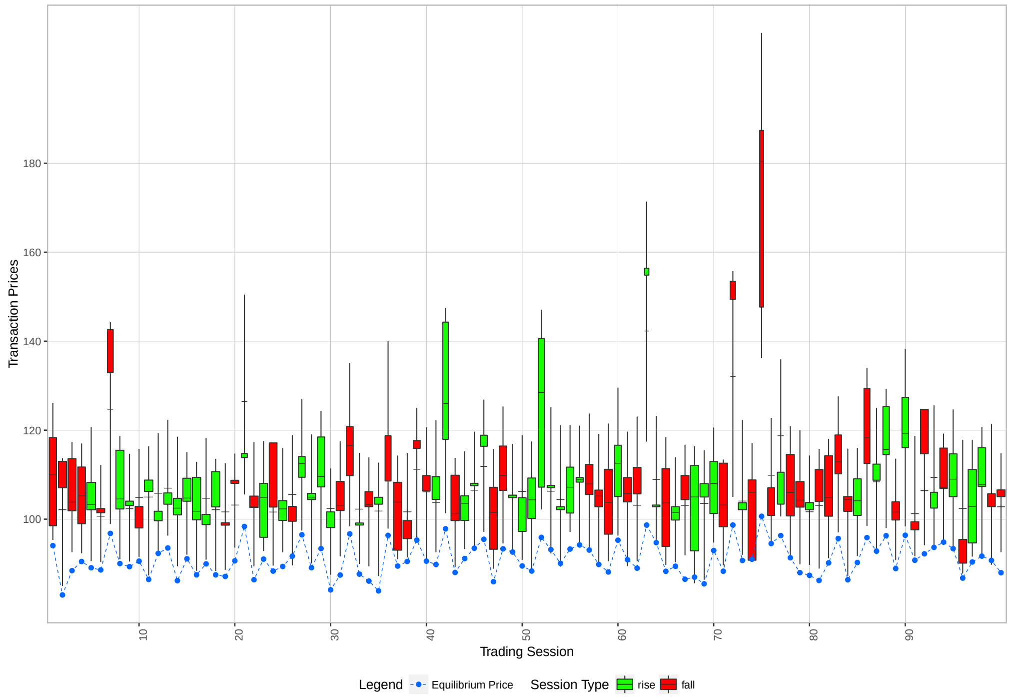


Figure A.5. Bid Price in TS1 with High Waiting Cost ($c_i \sim N(100, 25)$ and $\beta = 100$)

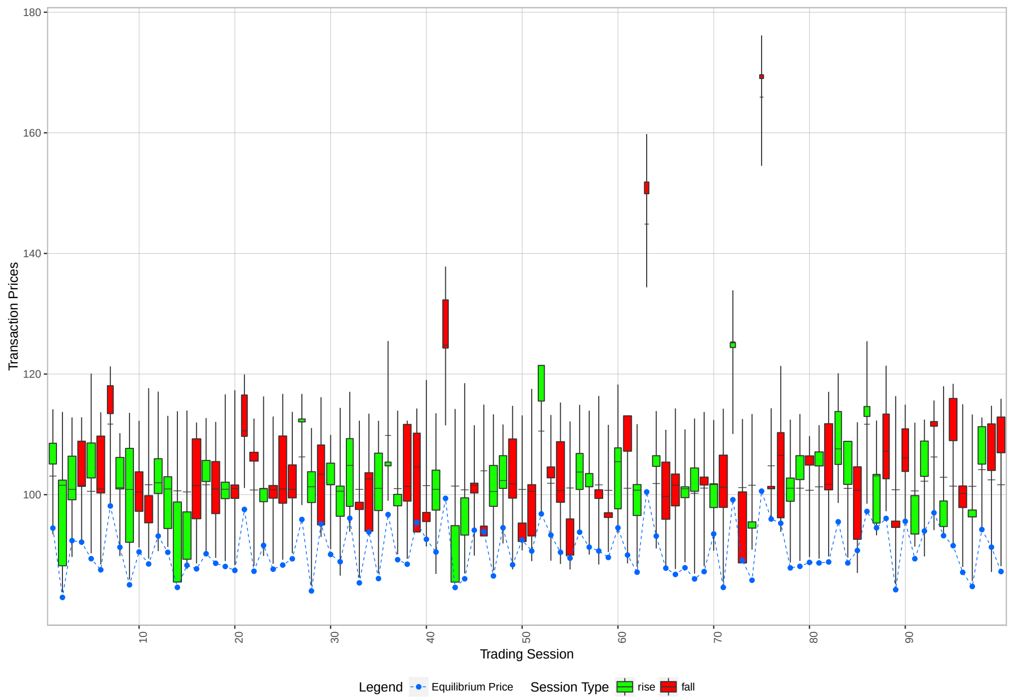


Figure A.6. Bid Price in TS1 with High Penalty Cost Variation ($c_i \sim N(100, 100)$)

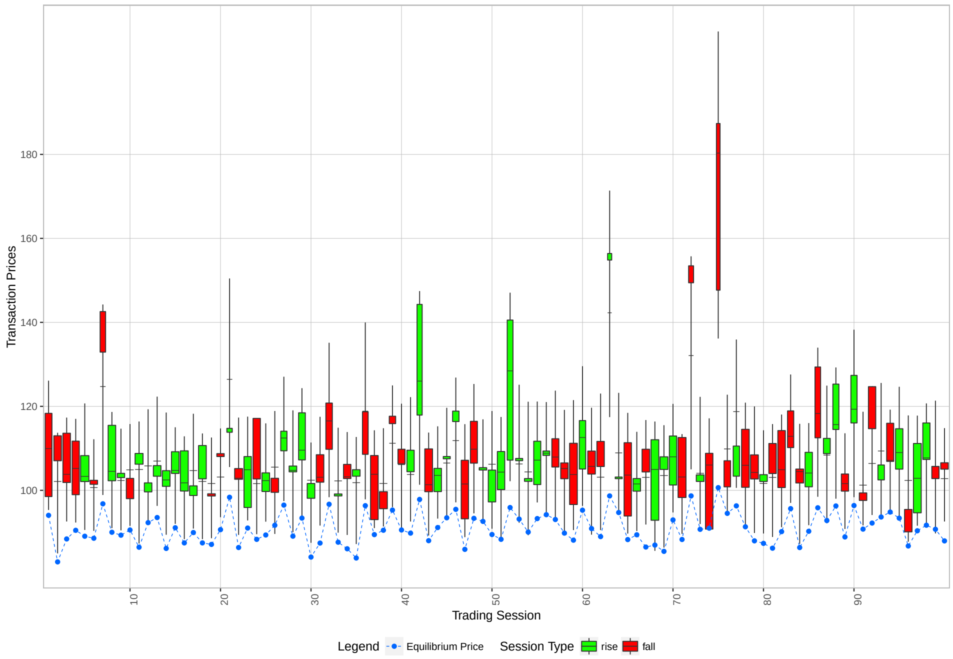
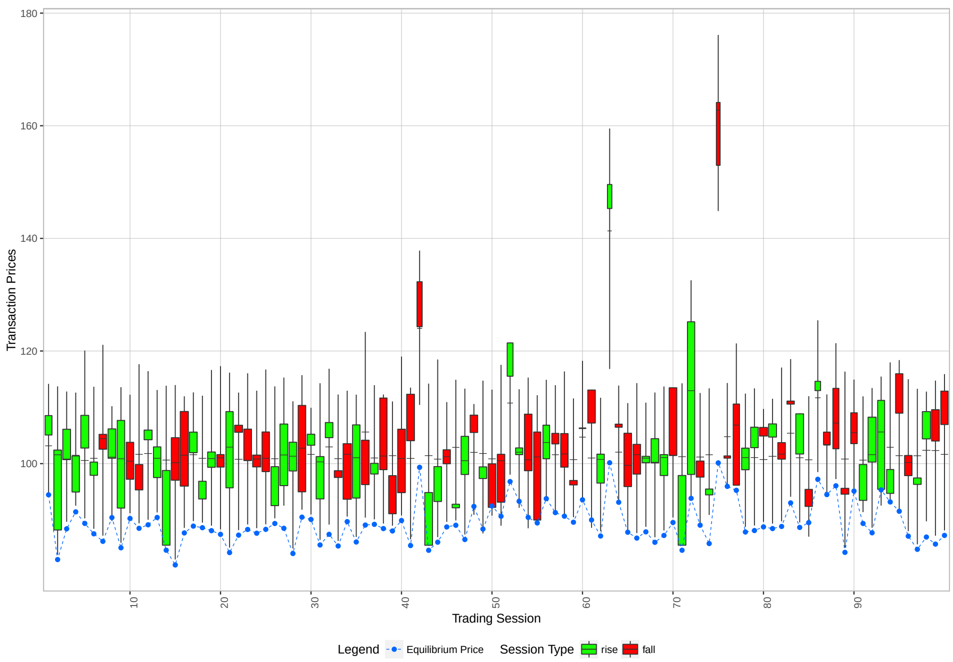


Figure A.7. Bid Price in TS3 ($c_i \sim N(100, 25)$ and $\beta = 100$)



A.2 Comparison of Jump Size

Figure A.8. Jump Size Measured by Closing Price

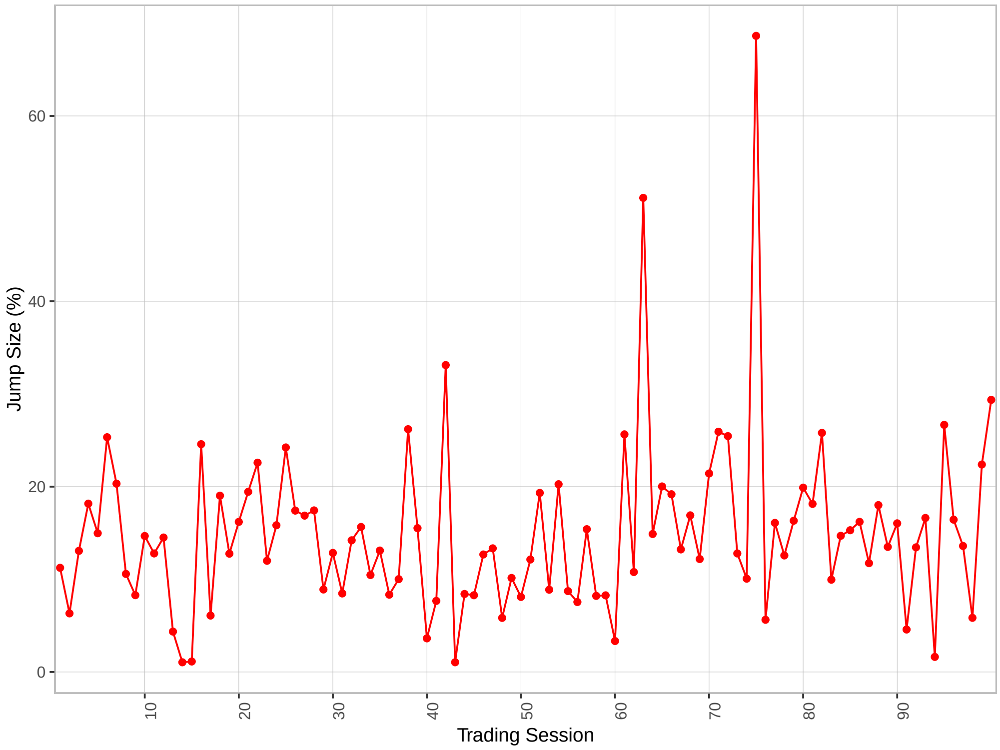


Figure A.9. Jump Size with Low Search Cost

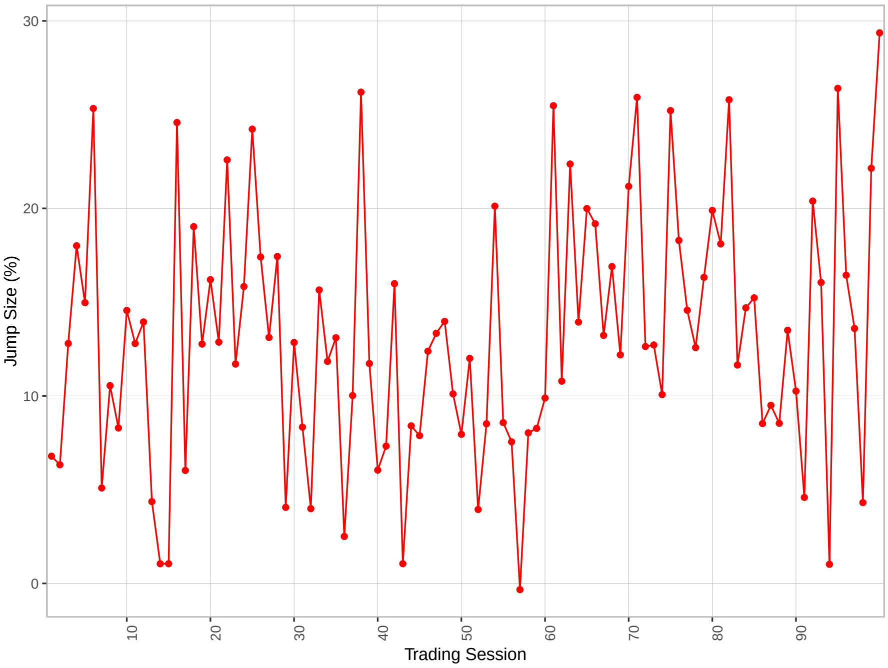


Figure A.10. Jump Size in TS1 with High Concentration

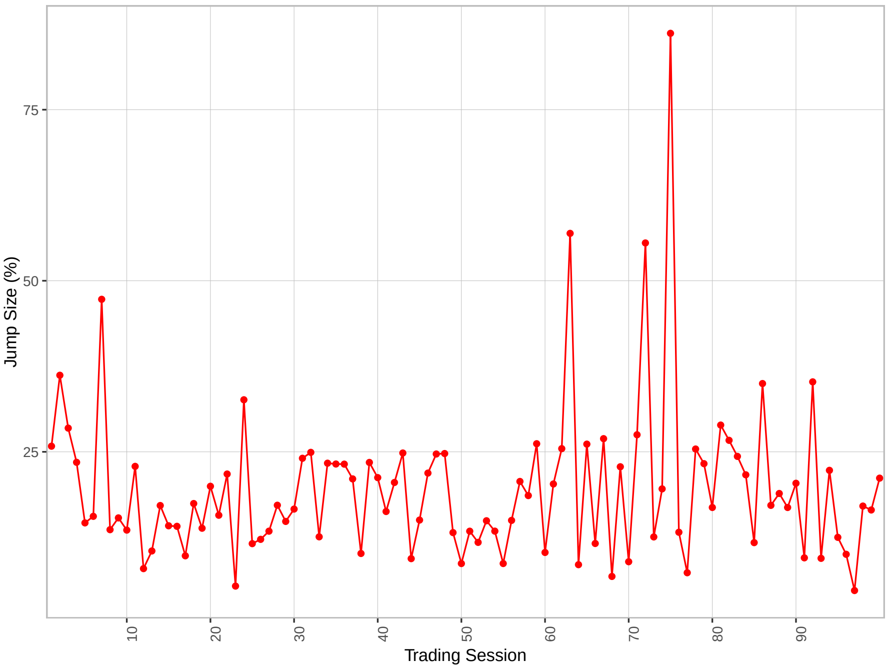


Figure A.11. Jump Size in TS1 with High Waiting Cost

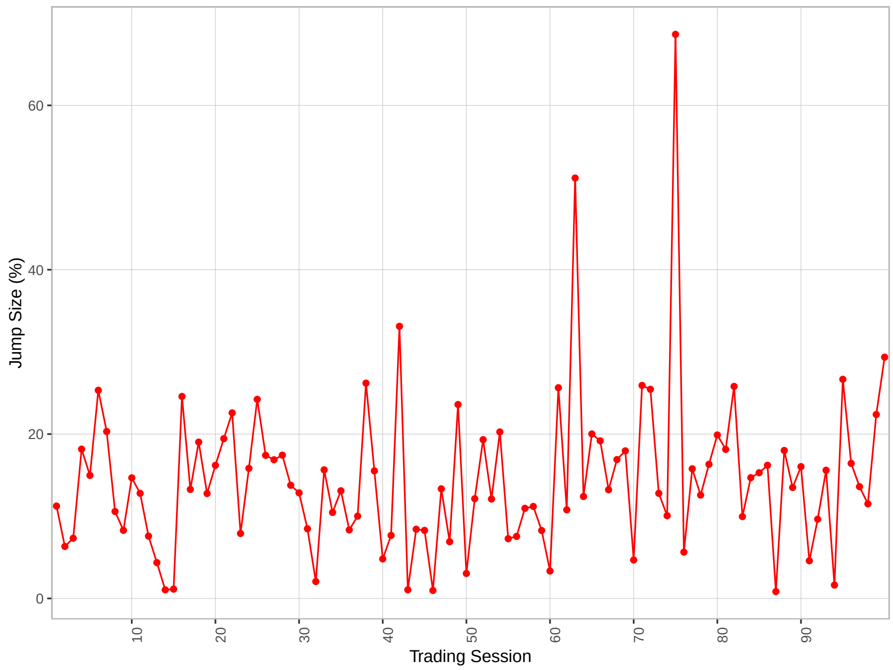


Figure A.12. Jump Size in TS1 with High Penalty Cost Variation

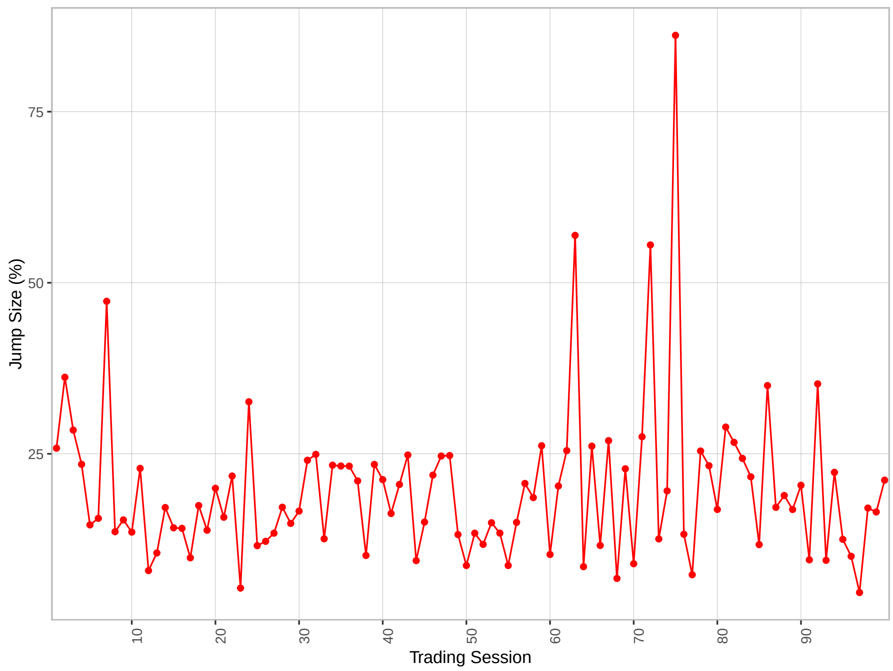
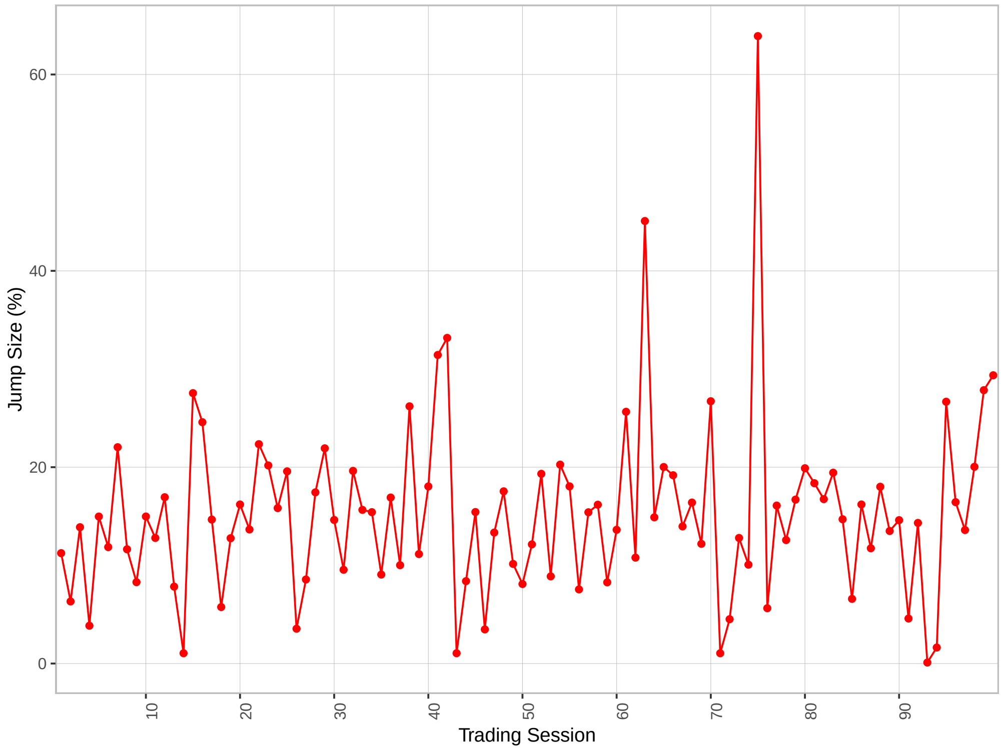


Figure A.13. Jump Size in TS3



A.3 Comparison of the Number of Encounters and Transactions

Figure A.14. Number of Encounters and Transactions in TS1 (Base Case)

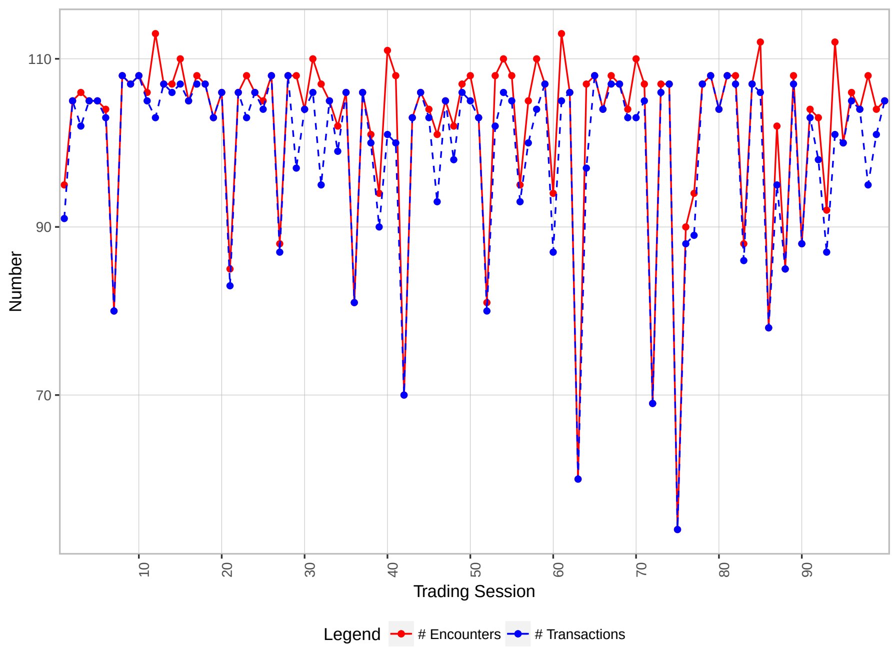


Figure A.15. Number of Encounters and Transactions in TS1 with Low Search Cost

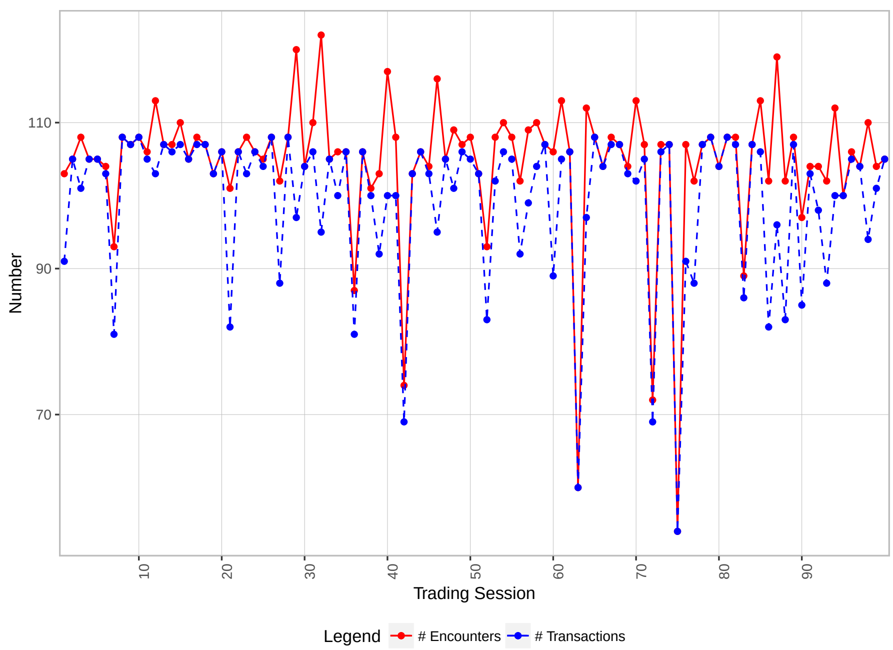


Figure A.16. Number of Encounters and Transactions in TS1 with High Concentration

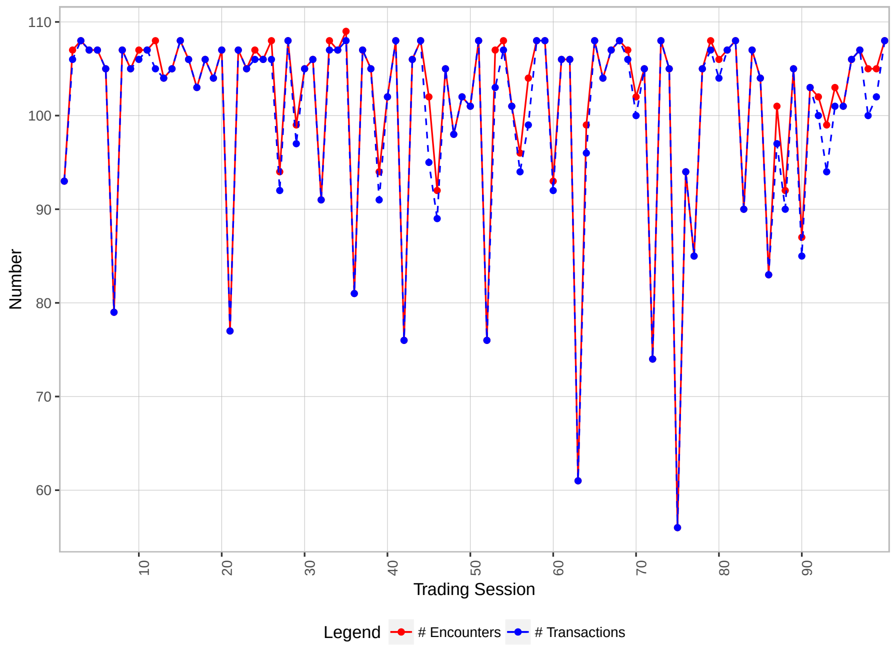


Figure A.17. Number of Encounters and Transactions in TS1 with High Waiting Cost

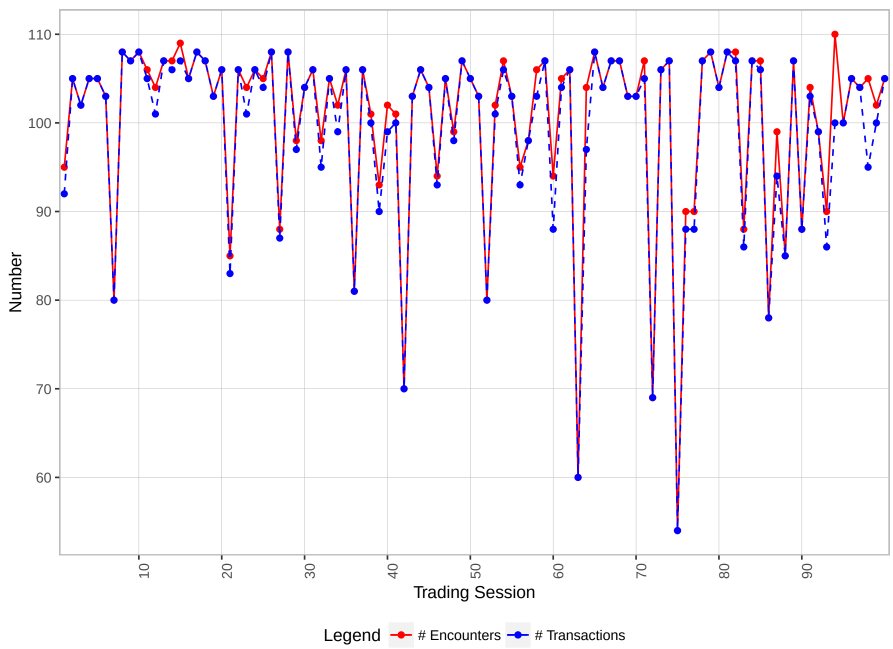


Figure A.18. Number of Encounters and Transactions in TS1 with High Penalty Cost Variation

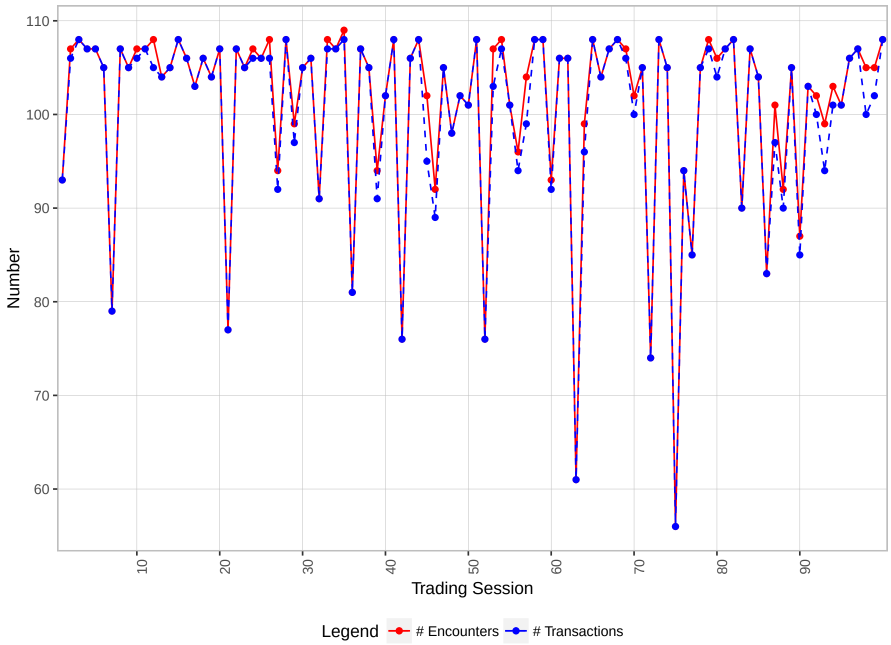
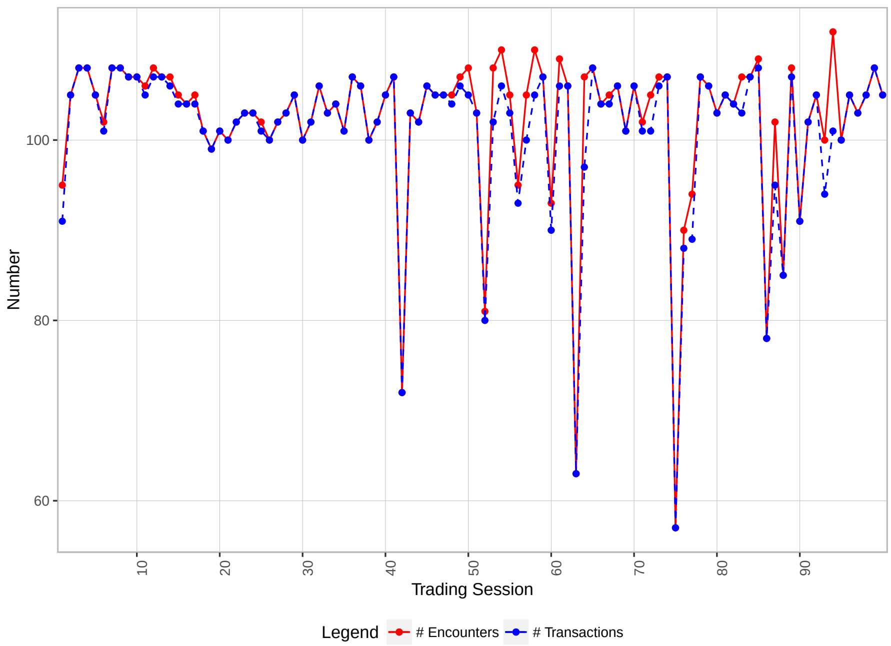


Figure A.19. Number of Encounters and Transactions in TS3



Appendix B
A Short Review of Nonlinear Time Series Methods: Structural Breaks, Outliers, Thresholds

1. **Introduction**
2. **Structural Break**
3. **Nonlinear Time Series Models**
4. **Nonlinear Granger Causality**
5. **Extended Granger Causality**

B.1 Introduction

Structural Break, Nonlinearity and Outliers happen frequently in economic time series and they all in a sense appear to be Nonlinearity. By definition (Koop, 2002), a model where the dynamics change permanently in a way that cannot be predicted by the history of the series is considered as structural break. Alternatively, it is possible that dynamic properties can vary over the business cycle. And we refer to models which allow for dynamics which vary over the business cycle in a predictable way as ?nonlinear" models. Still another possibility is that apparent departures from linearity are due to unpredictable large shocks which have only temporary effects, which is ?outlier?. Consider a three regime switching specification:

$$Y_t = \left\{ \begin{array}{ll} \alpha_{00} + \beta_{0p}(L) Y_{t-1} + \varepsilon_{0t} & if\ I_t = 0 \\ \alpha_{10} + \beta_{1p}(L) Y_{t-1} + \varepsilon_{1t} & if\ I_t = 1 \\ \alpha_{20} + \beta_{2p}(L) Y_{t-1} + \varepsilon_{2t} & if\ I_t = 2 \end{array} \right\}$$

$$where\ \ I_t\ an\ indicator\ for\ the\ regimes\ and\ \beta_{ip}(L)\ \ a\ polynomial\ of\ order\ p\ in$$
$$the\ lag\ operator.$$

We consider four ways of defining I_t

1. **A linear AR model** is obtained if $I_t = 0\ for\ \forall t$.

2. **A nonlinear model (TAR)** is obtained if

$$\left\{ \begin{array}{c} I_t = 0 \quad if\ r_2 \leq Y_{t-d} \leq r_1 \\ I_t = 1 \quad\quad if\ Y_{t-d} > r_1 \\ I_t = 2 \quad\quad if\ Y_{t-d} < r_2 \end{array} \right.$$

3. **A structural Break model** is obtained if

$$\left\{ \begin{array}{c} I_t = 0 \quad\quad if\ t < \tau_1 \\ I_t = 1 \quad\quad if \tau_1 \leq t < \tau_2 \\ I_t = 2 \quad if\ t \geq \tau_2 \end{array} \right.$$

4. **A outlier model** is obtained if

$$\left\{ \begin{array}{c} I_t \neq 0 \quad\quad if\ t = \tau_1,\ \tau_2 \\ I_t = 0\ \ otherwise \end{array} \right.$$

$$and$$

$$\left\{ \begin{array}{c} \beta_{0p}(L) = \beta_{1p}(L) = \beta_{2p}(L) \\ \varepsilon_{0t} = \varepsilon_{1t} = \varepsilon_{2t} \\ \alpha_{00} \neq \alpha_{10} \neq \alpha_{20} \end{array} \right.$$

In specifying model 1-4, Koop (2000) suggest a Bayesian method, while Giordani, Kohn and Dijk suggest a State-space framework, among many others. Now we will give a closer look on each model and methods above.

B.2 Structural Break

B.2.1 Motivation

"Parameter instability for economic models is a common phenomenon. And this is particularly true for time series data covering an extended period, as it is more likely for the underlying data-generating mechanism to be disturbed over a longer horizon by various factors such as policy-regime shift."

B.2.2 Core Topics in structural breaks

a. Estimation and inference about break dates for single equations and multiple equations;
b. Tests for a single structural break and multiple structural breaks ;
c. Tests for unit root in the presence of structural breaks;
d. Tests for cointegration in the presence of structural breaks.

B.2.3 Estimation and inference about break dates

Bai gave the estimation and inference about one single break point(1995). Perron and Bai (1998) later introduced the estimation and inference about multiple break points. Here we only talk about the multiple break points situation.

B.2.3.1 Model and Assumptions

For a linear regression,

$$y_t = \mathrm{x}_\mathrm{t}' + \mathrm{z}_\mathrm{tj}' + _\mathrm{t} \quad t = T_{j-1} + 1?.T_j$$

for *j=1...(m+1)* denoting there are m structural breaks (or *m+1* regimes). $y_t$denotes the observation at time t, $x_t$and z_t are vectors of regressors. And such regression can be rewritten in matrix form as

$$Y = X\beta + Z_0\delta + \epsilon$$

For each m partition ($T1 \ldots Tm$), the associated least-squares estimates of and j are obtained by minimizing the sum of squared residuals ϵ'^{ϵ}. Let $\widehat{\beta}(\{Tj\})$ and $\widehat{\delta}(\{Tj\})$ denote the estimates based on the given m partition *(T1, ..., Tm)* denoted {Tj}. Substituting these in the objective function and denoting the resulting sum of squared residuals as *ST (T1, ..., Tm)*, the estimated break points are

$$\left(\widehat{T_1}, ?\widehat{T_m}\right) = argmin_{(T_1,?T_m)} S_T(T_1?T_m)$$

The assumptions of such model relax from iid models up to a shift [Yao, 1987], [Bhattacharya, 1987] to a mean shift for a Gaussian AR proceess [Picard, 1994], and further to multiple regressions [Bai,1995]. The model has assumptions on regressors, errors, break dates, and the minimization procedure. The specific assumptions refer to *Change Point Estimation in multiple regression models* [Bai,1995] and *Dealing with structural break* [Perron, 2005].

B.2.3.2 Consistency and Asymptotic Distribution of the break date estimators $\left(\widehat{T_1}, ..., \widehat{T_m}\right)$

"With the assumptions on the regressors, the errors and given the asymptotic framework adopted, the limit distributions of the estimates of the break dates are independent of each other. Hence, for each break date, the analysis becomes exactly the same as if a single break has occurred." This reduces the study of asymptotic distribution in multiple break points to single one, which has well been stated by Bai (1995).
Under the assumptions,

$$\widehat{T_1} = T_1 + O_p(\|\delta_T\|^{-2})$$

And with the results, together with $\epsilon_t$being uncorrelated and $E\epsilon_t^2$=σ^2 for all t, then

$$\begin{bmatrix} \sqrt{T}\left(\widehat{\beta} - \beta\right) \\ \sqrt{T}\left(\widehat{\delta} - \delta_T\right) \end{bmatrix} \xrightarrow{d} N(0, \sigma^2 V^{-1})$$

where $V = plim\frac{1}{T}\begin{bmatrix} \sum_{t=1}^{T} x_t x_t' & \sum_{t=T1}^{T} x_t z_t' \\ \sum_{t=T1}^{T} z_t x_t' & \sum_{t=T1}^{T} z_t z_t' \end{bmatrix}$.

And the under assumptions,

$$\hat{T} - T \xrightarrow{d} arg\max_{m} W^{*}(m)$$

where $W^{*}(m) = \begin{cases} 0, \ m = 0 \\ W_1(m), \ m < 0 \\ W_2(m), \ m > 0 \end{cases}$, a two sided random walk with (stochastic) drift.

B.2.4 Tests for a single structural break and multiple structural breaks

B.2.4.1 CUSUM Test [Brown, Durbin and Evans, 1975]

For a linear regression with k regressors

$$y_t = x_t^{'\beta + \epsilon_t}$$

The CUSUM statistic is defined as

$$CUSUM = \max_{k+1<r\leq T} \left| \frac{\sum_{t=k+1}^{r} \widetilde{v}_t}{\widehat{\sigma}\sqrt{T-k}} \right| /(1 + 2\frac{r-k}{T-k})$$

Where $\widehat{\sigma}^2$ is a consistent estimate of the variance of ϵ_t.
$\widetilde{v}_t = \frac{y_t - x_t^{'}\widehat{\beta_{t-1}}}{f_t}$ and $f_t = (1 + x_t^{'(X_{t-1}^{'}X_{t-1})x_t)}1/2$
The asymptotic distribution of CUSUM is:

$$CUSUM \xrightarrow{d} sup_{0\leq r\leq 1} \left| \frac{W(r)}{1+2r} \right|$$

Where *W(r)* is a unit Wiener process defined on (0,1). [Sen, 1982]

B.2.4.2 CUSSQ Test [Brown, Durbin and Evans, 1975]

$$CUSSQ = \max_{k+1<r\leq T} \left| S_T^{(r)} - \frac{r-k}{T-k} \right|$$

Where $S_T^{(r)} = \left(\sum_{t=k+1}^{r} \widetilde{v}_t^{\,2}\right) /(\sum_{t=k+1}^{T} \widetilde{v}_t^{\,2})$

Ploberger and Kramer (1990) considered the local power functions of the CUSUM and CUSUM of squares. The former has non-trivial local asymptotic power unless the mean regressor is orthogonal to all structural changes. On the other hand, the latter has only trivial local power (i.e., power equal to size) for local changes that specify a one-time change in the coefficients (see also Deshayes and Picard, 1986). This suggests that the CUSUM test should be preferred, a conclusion we shall revisit below.

B.2.4.3 *sup-LR* test

The *sup-LR* test statistic is:

$$sup_{\lambda_1 \in \Lambda_\epsilon} LR_T(\lambda_1)$$

Where $LR_T(\lambda_1)$ denotes the value of likelihood ratio evaluated at some break point $T_1 = [T\lambda_1]$ and maximization is restricted over break fractions that are in $\Lambda_\epsilon = [\epsilon_1, 1 - \epsilon_2]$.
The asymptotic distribution of *sup-LR* test is:

$$sup_{\lambda_1 \in \Lambda_\epsilon} LR_T(\lambda_1) \xrightarrow{d} sup_{\lambda_1 \in \Lambda_\epsilon} G_q(\lambda_1)$$

Where $G_q(\lambda_1) = \frac{[\lambda_1 W_q(1) - W_q(\lambda_1)]' [\lambda_1 W_q(1) - W_q(\lambda_1)]}{\lambda_1(1-\lambda_1)}$
$W_q(\lambda)$ a vector of independent Wiener processes of dimension q, the number of coefficients that are allowed to change.

B.2.4.4 *sup-Wald* test

The *sup-Wald* test statistic is defined as

$$sup_{\lambda_1 \in \Lambda_\epsilon} W_T(\lambda_1; q)$$

and $W_T(\lambda_1) = [SSR(1,T) - SSR(1,T_1) - SSR(T_1+1,T)] / \{[SSR(1,T_1) + SSR(T_1+1,T)]/T\}$
where *SSR(i,j)* is the sum of squared residuals from regressing y_t on a constant using data from data i to date j, *i.e.*

$$SSR(i,j) = \sum_{t=i}^{j} \left(y_t - \frac{1}{j-i} \sum_{t=i}^{j} y_t \right) = \sum_{t=i}^{j} (e_t - \sum_{t=i}^{j} e_t)$$

The asymptotic distribution of *sup-Wald* test statistic is:

$$W_T(\lambda_1)\xrightarrow{d}\frac{1}{\lambda_1(1-\lambda_1)}[\lambda_1 W\left(1\right)-\lambda_1 W\left(\lambda_1\right)-(1-\lambda_1)W(\lambda_1)]2$$

Note here $W_T\left(k\right)$ is monotonic transformation of $S_T\left(k\right)$. So it follows that

$$\left(\widehat{T_1},?\widehat{T_m}\right)=argmin_{(T_1,?T_m)}S_T\left(T_1?T_m\right)=argmin_{(T_1,?T_m)}W_T(T_1?T_m)$$

Hence, the estimator obtained by minimizing the sum of squared residuals is the same as maximizing Wald-type statistics.

B.2.4.5 Tests for unit root in the presence of structural breaks

Consider a linear regression model in which we allow shifts in both intercept and slope,

$$y_t=\mu_1+\beta_1 t+\left(\mu_2-\mu_1\right)DU_t+\left(\beta_2-\beta_1\right)DT_t^*+\epsilon_t$$

where $DU_t=1,\ DT_t^*=t-T_1\ if\ t>T_1\ and\ 0\ otherwise$
(Such model is call model AO-C by Perron)
Rewriting the model AO-C

$$y_t=\mu+\theta DU_t+\beta t+\gamma DT_t^*+\alpha y_{t-1}+\sum_{i=1}^{k}c_i\Delta y_{t-i}+e_t$$

Then the test statistic for a unit root allowing for changes at unknown dates is:

$$t_\alpha^*=inf_{\lambda_1\in[\epsilon,1-\epsilon]}t_\alpha(\lambda_1)$$

where $t_\alpha(\lambda_1)$ is the t-statistic for testing $\alpha=1$
The asymptotic distribution of such statistic is:

$$t_\alpha^*\xrightarrow{d}inf_{\lambda_1\in[\epsilon,1-\epsilon]}\frac{\int_0^1 W^*\left(r,\lambda_1\right)dW(r)}{\left[\int_0^1 W^*\left(r,\lambda_1\right)^2dr\right]^{1/2}}$$

$W^*\left(r,\lambda_1\right)$ is the residual function from a projection of a Wiener process *W(r)* on the relevant continuous time versions of the deterministic components.

B.3 Nonlinear Time Series Models

B.3.1 Test for nonlinearity

The BDS test developed by Brock, Dechert and Scheinkman (1987) is arguably the most popular test for nonlinearity. When applied to the residuals of a fitted linear time series model, the BDS test can be used to detect remaining dependence and the presence of omitted nonlinear structure (Zivot, 2005). The null hypothesis of BDS test is that the linear model specification is appropriate (residual is i.i. distributed).

Consider a time series $x_t,\ t = 1, 2, ?, T$ and define its m-lag history as $x_t^m = (x_t, x_{t-1}, ?x_{t-m+1})$. The correlation integral at embedding dimension m can be estimated by:

$$C_{m,\varepsilon} = \frac{2}{(T-m+1)(T-m)} \sum_{m \le s < t \le T} \sum I\left(x_t^m, x_s^m; \varepsilon\right)$$

$$where\ I\left(x_t^m, x_s^m; \varepsilon\right) = \begin{cases} |x_{t-i} - x_{s-i}| < \varepsilon & for\ i = 0, 1, ?, m-1 \\ 0 & otherwise \end{cases}$$

The correlation integral estimates the probability that any two m-dimensional points are within a distance of ε *of each other*, i.e.

$$\widetilde{\Pr}\left(|x_t - x_s| < \varepsilon, |x_{t-1} - x_{s-1}| < \varepsilon\ , ?, |x_{t-m+1} - x_{s-m+1}| < \varepsilon\ \right) = C_{m,\varepsilon}$$

If $x_t \sim iid$,

$$C_{1,\varepsilon}^m = \Pr(|x_t - x_s| < \varepsilon)^m$$

Then the BDS statistic is defined as follows:

$$V_{m,\varepsilon} = \sqrt{T} \frac{C_{m,\varepsilon} - C_{1,\varepsilon}^m}{S_{m,\varepsilon}}$$

$$where\ S_{m,\varepsilon} = SD(\sqrt{T}(C_{m,\varepsilon} - C_{1,\varepsilon}^m))$$

$$Then\ V_{m,\varepsilon} \xrightarrow{d} N(0,\ 1)$$

B.3.2 Markov Switching Model (Hamilton, 1989)

Consider a general VAR time series model:

$$z_t = \mu + D_t + S_t + \sum_{i=1}^{p} \phi_i z_{t-i} + \varepsilon_t$$

where D_t is the trend component and S_t the seasonal component. D_t obeys a Markov trend in levels if

$$D_t = \alpha_1 x_t + D_{t-1} \tag{B.1}$$

where $x_t = 0$ or 1 depending on the unobserved state of the system. We assume the transition between states is governed by a first-order Markov process:

$$\Pr\left(X_t = 1 \mid X_t = 1\right) = p,$$

$$\Pr\left(X_t = 0 \mid X_t = 1\right) = 1 - p,$$

$$\Pr\left(X_t = 0 \mid X_t = 0\right) = q,$$

$$\Pr\left(X_t = 1 \mid X_t = 0\right) = 1 - q.$$

Then the model can be called a VAR with Markov trend in levels. Further., Hamilton (1989) develop a time series model with Markov trend in lags and develop estimation algorithm and filter for such model.

B.3.3 TAR and SETAR

A Threshold Autoregressive model (TAR) (Potter,1995) is defined as that in Part I:

$$Y_t = \left\{ \begin{array}{ll} \alpha_{00} + \beta_{0p}\left(L\right) Y_{t-1} + \varepsilon_{0t} & if\ I_t = 0 \\ \alpha_{10} + \beta_{1p}\left(L\right) Y_{t-1} + \varepsilon_{1t} & if\ I_t = 1 \\ \alpha_{20} + \beta_{2p}\left(L\right) Y_{t-1} + \varepsilon_{2t} & if\ I_t = 2 \end{array} \right\}$$

where I_t is an indicator for the regimes and $\beta_{ip}L)$ is a polynomial of order p in the lag operator. And I_t is defined as

$$\begin{cases} I_t = 0 \quad if\ r_2 \leq z_t \leq r_1 \\ I_t = 1 \quad if\ z_t > r_1 \\ I_t = 2 \quad if\ z_t < r_2 \end{cases}$$

where z_t is a weakly exogenous threshold variable. Thus, in each regime, the time series Y_t follows a different AR(*p)* model.
When the threshold variable $z_t = Y_{t-d}\ with\ the\ delay\ parameter\ d\ being\ a\ positive\ integrer$, then the dynamics or regime of Y_t is determined by its own lagged value Y_{t-d} is called a self-exciting **TAR (SETAR)** model.

B.3.4 STAR

In the TAR models, a regime switch happens when the threshold variable crosses a certain threshold. However, it suggests the regime switch is discontinuous. If the discontinuity of the thresholds is replaced by a smooth transition function, TAR models can be generalized to **smooth transition autoregressive (STAR)** model. Take exponential STAR **(ESTAR)** model as an example. Replace z_t by *G(*z_t*),* a smooth transition function 0< *G(*z_t*)*<1 which depends on a transtition variable z_t (like the threshold variable in TAR model), the model becomes a smooth transition model:

$$Y_t = \mu + \beta_{0p}(L)\, Y_{t-1}\,(1 - G(z_t)) + \beta_{1p}(L)\, Y_{t-1} G(z_t) + \varepsilon_t$$

where $G(z_t; \gamma, c) =\ 1 - e^{-\gamma(z_t - c)^2}$, $\gamma > 0$ (exponential). To avoid issues caused by the unidentified STAR model parameters under the null hypothesis of a linear AR model, Luukkonen, Saikkonen and Terasvirta (1988) propose to replace the transition function G(z$_t$; , c) by a suitable Taylor series approximation around = 0.Then the test for ESTAR nonlinear can be conducted within the linear context.

B.4 Nonlinear Granger Causality

B.4.1 Linear Granger causality test

Granger causality test (Granger, 1969) is designed to detect causal direction in a bivariate linear time series. It tests on the correlation between the current value of one variable and the past values of the other one:

Consider a bivariate VAR(p):

$$\begin{bmatrix} x_t^1 \\ x_t^2 \end{bmatrix} = \begin{bmatrix} a_{10} \\ \vdots \\ a_{(k+l)0} \end{bmatrix} + \sum_{h=1}^{p} \begin{bmatrix} a_{11,t-h} & ? & a_{1(k+l),t-h} \\ \vdots & \ddots & \vdots \\ a_{(k+l)1,t-h} & ? & a_{(k+l)(k+l),t-h} \end{bmatrix} \begin{bmatrix} x_{t-h}^1 \\ x_{t-h}^2 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \vdots \\ \varepsilon_{(k+l),t} \end{bmatrix}$$

$$where\ \Sigma = \begin{bmatrix} \sigma_{1,t}^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_{(k+l),t}^2 \end{bmatrix}.$$

To test Ho:$[x_t^2]_j$does not Granger-cause $[x_t^1]_i$, we need to jointly test on $a_{ij,h} = 0\ for\ h = 1, 2, ?, p$. And the alternative is: $\exists a_{21,i}\ ne\ 0\ for\ some\ i$. The test statistic is distributed as $F(p, (k+l)\,, (k+l)?p - A)$, where A equals to the total number of parameters in the above VAR (p) including the deterministic regressors.

B.4.2 Nonlinear Granger causality test

The linear causality tests, such as the Granger test (1969), can fail to uncover nonlinear predictive power (Baek and Brock, 1992; Hiemstra and Jones, 1994). Hiemstra and Jones (1994), based on Baek and Brock (1992), proposed a nonparametric statistical method based on the correlation integral to detect nonlinear causal relations between time series. Consider two strictly stationary and weakly dependent time series $\begin{bmatrix} x_t \\ y_t \end{bmatrix}$:

$$x_t^m = (x_t, x_{t+1}, ?, x_{t+m-1})\,,\ m = 1, 2, ?,\ t = 1, 2, ?$$

$$x_{t-lx}^{lx} = (x_{t-lx}, x_{t-lx+1}, ?, x_{t-1})\,,\ lx = 1, 2, ?,\ t = lx+1, lx+2, ?$$

$$y_{t-ly}^{ly} = (y_{t-ly}, y_{t-ly+1}, ?, y_{t-1})\,,\ ly = 1, 2, ?,\ t = ly+1, ly+2, ?$$

The then null hypothesis is that Y does not strictly Granger cause X, which by definition means:

$$\Pr\left(\|x_t^m - x_s^m\| < e \;\middle|\; \left\|x_{t-lx}^{lx} - x_{s-lx}^{lx}\right\| < e,\ \left\|y_{t-ly}^{ly} - y_{s-ly}^{ly}\right\| < e\right) = \Pr\left(\|x_t^m - x_s^m\| < e \;\middle|\; \left\|x_{t-lx}^{lx} - x_{s-lx}^{lx}\right.\right.$$

Rewrite this condition,

$$\frac{C_1(m+lx,ly,e)}{C_2(lx,ly,e)}=\frac{C_3(m+lx,e)}{C_4(lx,e)}$$

$$where$$

$$C_1(m+lx,ly,e)=\Pr(\left\|x_{t-lx}^{m+lx}-x_{s-lx}^{m+lx}\right\|<e,\ \left\|y_{t-ly}^{ly}-y_{s-ly}^{ly}\right\|<e)$$

$$C_2(lx,ly,e)=\Pr(\left\|x_{t-lx}^{lx}-x_{s-lx}^{lx}\right\|<e,\ \left\|y_{t-ly}^{ly}-y_{s-ly}^{ly}\right\|<e)$$

$$C_3(m+lx,e)=\Pr(\left\|x_{t-lx}^{m+lx}-x_{s-lx}^{m+lx}\right\|<e)$$

$$C_4(lx,e)=\Pr\left(\left\|x_{t-lx}^{lx}-x_{s-lx}^{lx}\right\|<e\right)$$

As in BDS test, we use correlation-integral to estimate these probabilities:

$$C_1(m+lx,ly,e,n)=\frac{2}{n(n-1)}\sum\sum I(x_{t-lx}^{m+lx},x_{s-lx}^{m+lx},e)\bullet I(y_{t-ly}^{ly},\ y_{s-ly}^{ly},e)$$

$$C_2(lx,ly,e,n)=\frac{2}{n(n-1)}\sum\sum I(x_{t-lx}^{lx},x_{s-lx}^{lx},e)\bullet I(y_{t-ly}^{ly},\ y_{s-ly}^{ly},e)$$

$$C_3(m+lx,e)=\frac{2}{n(n-1)}\sum\sum I(x_{t-lx}^{m+lx},x_{s-lx}^{m+lx},e)$$

$$C_4(lx,e)=\frac{2}{n(n-1)}\sum\sum I(x_{t-lx}^{lx},x_{s-lx}^{lx},e)$$

$$where\ t,s=\max(lx,ly)\ +1,?,T-m+1;$$

$$n=T-\max(lx,ly)\ -m+1$$

For given values$m,\ lx,ly\geq 1\ and\ e>0$, under the assumptions that $x_t\ and\ y_t$ are strictly stationary, weakly dependent and ergodic, if y_t does not strictly Granger cause x_t, then

$$\sqrt{n}\left[\frac{C_1(m+lx,ly,e)}{C_2(lx,ly,e)}-\frac{C_3(m+lx,e)}{C_4(lx,e)}\right]\sim N\left(0,\sigma^2(m,lx,ly,e)\right)$$

The statistics here will be applied to the residuals from the VAR models. And the null hypothesis will be rejected at 5% if the statistic is bigger than 1.96, as that in

BDS test.

B.4.3 DP critique on HJ test

Diks and Panchenko (2004) address a problem in HJ test. DP indicates that

$$
\begin{aligned}
&Pr(\text{Ę}x_t^m - x_s^m\text{Ę} < e\text{Ę}x_{(}t-lx)^l x - x_{(}s-lx)^l x\text{Ę} < e, \text{Ę}y_{(}t-ly)^l y - y_{(}s-ly)^l y\text{Ę} < e)\\
=&Pr(\text{Ę}x_t^m - x_s^m\text{Ę} < e\text{Ę}x_{(}t-lx)^l x - x_{(}s-lx)^l x\text{Ę} < e)
\end{aligned}
\tag{B.2}
$$

does not follow the null hypothesis. The difference is, in the case of $L_y = 1$,

$$
D = Cov(I(|X_t - X_s| < e, I(|Y_{(}t-1) - Y_{(}s-1)| < e|\text{Ę}x_{(}t-lx)^l x - x_{(}s-lx)^l x\text{Ę} < e)
\tag{B.3}
$$

rewriting this, we obtain

$$
D = Cov(r(e/V), r(e/W))
$$

Depending on the joint distribution of V and W, D can be either negative, zero or positive. If either V or W is degenerate, D=0. Otherwise, D is most likely bigger than 0, as the one-sided HJ test tend to over-reject.