Response to Reviewers' Comments

To PC co-chairs, meta reviewer, and reviewers:

We would like to thank the meta reviewer and reviewers for their thorough reviews and valuable comments, which have helped us to improve the quality of this paper. In this cover letter, we first summarize the major changes that we have made, and then provide detailed point-by-point responses to the concerns raised by each reviewer. For ease of reading, we have highlighted the major changes in the revised paper in blue.

SUMMARY OF CHANGES

- We have added a detailed exploration for the tuning and sensitivity analysis of hyperparameters in Section 4.3. More details can be found in our responses to Reviewer R#2 W1, D1, D2 and D3.
- We have added detailed theoretical analysis for each approximate algorithm of OBP in Section 3.2 and Section 3.3. The details can be found in the response of Reviewer R#2 W2, D4.
- We have added a detailed time complexity analysis for each approximate algorithm of OBP in Section 3.4. More details can be found in the response of Reviewer R#2 W3, D5.
- We have added experiments to evaluate the cost-accuracy trade-off for different hyperparameters in Section 4.4. More details can be found in our response to Reviewer R#4 D4.
- We have added more clarifications on the revised paper by addressing the comments in Reviewers R#3 D1, D4, D5, D6, D8 and R#4 D1. The revised sections with respect to the comments are located at:
  R#3 D1: Section 3.2 and Section 3.3
  R#3 D4-D6: Section 3.3
  R#3 D8: Section 3.5.2
  R#4 D1: Section 1
- We have also addressed other weak points and detailed comments of all three reviewers, including Reviewers R#3 D2, D3, D7 and R#4 D2, D3.
- To make space for the above changes, we have shortened Section 2.2, Section 3 and Section 4. We instead refer the readers to our online technical report [1] for this information.

RESPONSE TO META-REVIEWER

Comment W1: *Clarify remaining unclear technical details (e.g., comments by Reviewers R#3 and R#4).*
RESPONSE: In the revised manuscript, we have clarified the unclear technical details. Specifically, we have added more details about *QueClt* in Section 3.2. We have added more explanations about the intuition of the retention method in Section 3.3.1. We have added more explanations about the RAC (Reducing Affinity Computations) method in Section 3.5.2. We have improved the running example in Section 1, and we have added experiments about cost-accuracy trade-off, etc. Please refer to "Summary of Changes" and our responses to Reviewers R#3 and R#4 for more details.
Comment W2: *Conduct systematic experiments for hyper-parameter tuning (e.g., Bayesian optimization for the threshold parameters).*
RESPONSE: We have added detailed exploration and systematic experiments about hyper-parameter tuning in Section 4.3. Please refer to our response to R#2 W1 for further details.
Comment W3: *Improve the description of the intuition, analysis, and guarantees of the approximate algorithm.*
RESPONSE: We have significantly enhanced the description of the intuition, analysis, and guarantees of the approximate algorithm. Specifically, we have added more explanations about the intuition of the retention method in Section 3.3.1. Furthermore, we have added the theoretical analysis of each approximate algorithm of OBP, i.e., *QueClt* in Section 3.2 and *CltSplt* in Section 3.3. We also provided an analysis of the guarantee for bounded deviation from the optimal solution for each of the approximate algorithms. Please refer to our responses to R#2 W2 and R#2 W3 for more details.

RESPONSE TO REVIEWER #2

Comment W1: *Hyperparameter selection for the optimization problem lacks detailed exploration, particularly in terms of sensitivity and tuning.*
Comment D1: *The solution's effectiveness depends on several threshold parameters (e.g., question affinity threshold), with optimal values varying based on the characteristics of questions, demonstrations, and pairwise distance metrics. A more detailed sensitivity analysis would clarify the influence of these thresholds on performance.*
Comment D2: *The approach requires four pre-defined thresholds, but tuning them through grid search could be costly due to the expense of LLM inference. Guidance on effective tuning methods or heuristics would be valuable.*
Comment D3: *The optimization problem uses fixed thresholds to define the optimal grouping. This approach may lead to oversized or fragmented groups in cases where the embedding space of questions and demonstrations contains regions of varying density.*
RESPONSE: Thanks for the constructive comments. In the revised version, we have conducted an in-depth exploration of the tuning and sensitivity in response to **Comments W1, D1, D2**. For **Comment D3**, we have enabled an adaptive approach to calculate the thresholds, which accommodates varying densities across different tasks. The details are provided in Section 4.3.

Regarding **Comments W1, D1, D2**, we have provided a detailed explanation of guidance on tuning the four hyperparameters: *Question Group Affinity constraint*(τ_0), *Question Demonstration Affinity constraint*(τ_1), *Group Length constraint*(τ_2), and *Demonstration Coverage constraint*(τ_3), along with a comprehensive sensitivity analysis.

For τ_0, it measures the affinity between questions. τ_0 thus needs to distinguish between relevant and irrelevant questions, ensuring that questions within the same group are affinitive to each other. To determine the appropriate value for τ_0, we have the following guidance: we first calculate the affinity values of each question with all other questions and sort them in ascending order (with smaller values indicating higher relevance). Next, we identify the gap for each question, which is the largest difference between adjacent affinity values. Additionally, we denote the cutoff value as the larger value within the gap. Take the ANN dataset as an example. Figure 1(a)

shows the distribution of affinity values for a sample question (we take the 200-th question) from the ANN dataset, sorted in ascending order. We also show the gap and cutoff value for this sample question. Similarly, Figure 2(a) shows the distribution of affinity values for a sample question (we take the 21-th question) from the KBWT dataset. Finally, we determine the appropriate range of τ_0 through cutoff values across all questions. For example, we plot cutoff values for a batch of questions from the ANN and KBWT datasets in Figure 1(b) and Figure 2(b) respectively. As observed, in the ANN dataset, the cutoff values typically lie between 0.4 and 0.5. This indicates that the affinity values within this region can separate most of relevant questions from irrelevant ones. Thus, values in this region can be considered as a suitable choice for τ_0, and we denote this region as the cutoff region. To verify this guidance on determining appropriate τ_0, we conducted a sensitivity analysis on τ_0, as depicted in Figure 3. From the figure, we observe that τ_0 performs best in terms of accuracy within the cutoff region. For example, in the ANN dataset, the optimal accuracy is achieved when τ_0 lies within the cutoff region, i.e., between 0.4 and 0.5. Similarly, in the KBWT dataset, the optimal accuracy falls within the cutoff region, i.e. ranging from 0.06 to 0.15. Furthermore, we observe that τ_0 is not sensitive to accuracy within this region. For values smaller or larger than this region, the accuracy shows a significant decline. In the meanwhile, we observe that the cost decreases as τ_0 increases. This is because a smaller τ_0 implies fewer questions per group, resulting in a larger number of groups and thus increasing the task description cost. For a new dataset, users can first identify the appropriate range of τ_0 values based on the above guidance. If the users aim to reduce cost, they can choose the maximum value within the cutoff region as τ_0. Otherwise, they can tune τ_0 within this region, ultimately choosing the one that achieves the highest accuracy. In our implementation, we set τ_0 to the maximum value within the cutoff region to reduce cost.

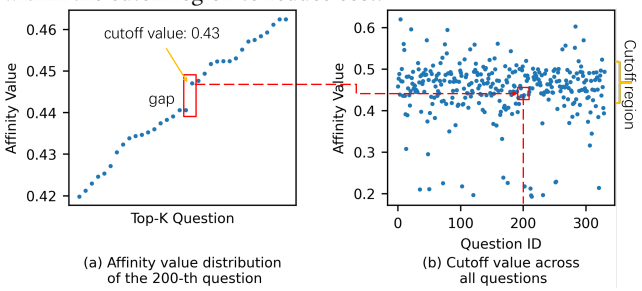

Figure 1: Affinity distribution of hyperparameter τ_0 on the ANN dataset.

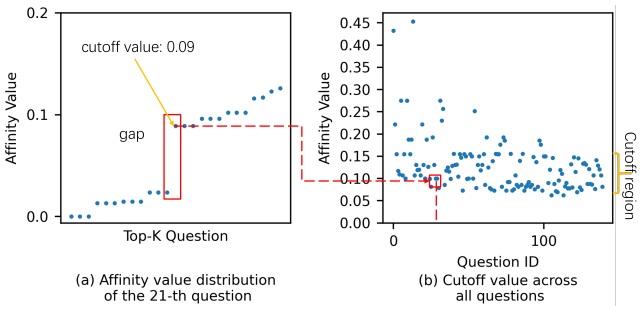

Figure 2: Affinity distribution of hyperparameter τ_0 on the KBWT dataset.

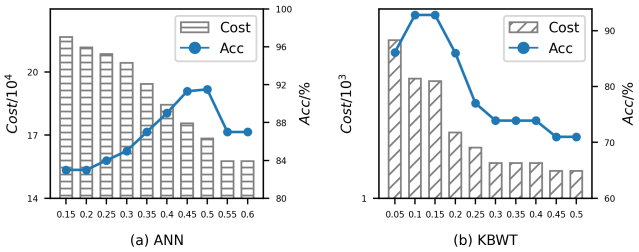

Figure 3: Sensitivity analysis of hyperparameter τ_0.

For τ_1, it measures the relationship between questions and demonstrations. Thus, τ_1 is used to distinguish between questions that are relevant and irrelevant to a demonstration. To identify the appropriate value for τ_1, we have the following guidance: we calculate the affinity scores of each demonstration with all questions and sort them in ascending order. We identify the gap within the first few affinity scores, as a demonstration cannot cover too many questions. Then, we determine the cutoff value for each demonstration. Figure 4(a) and Figure 5(a) illustrate a sample demonstration from the ANN and KBWT datasets respectively. Finally, similar to τ_0, we determine the appropriate range of τ_1 by inspecting cutoff values across all demonstrations, which are shown in Figure 4(b) and Figure 5(b). Note that when τ_1 becomes particularly small, it leads to a lack of relevant demonstrations, causing our framework to fail. For example, in the ANN dataset, when τ_1 drops below 0.1, the algorithm fails to return a result. (In Figure 4(b), the region below 0.09 is marked with a shadow to highlight this). From the figure, we observe that the cutoff values in the ANN dataset typically fall below 0.16, while those in the KBWT dataset generally fall below 0.1. This suggests that the affinity values within this region (i.e., the cutoff region) are effective in distinguishing relevant questions from irrelevant ones for most demonstrations. To verify this tuning guidance, we performed a sensitivity analysis on τ_1, as shown in Figure 6. We find that the best performance is achieved when τ_1 is set to values within the cutoff region. For instance, in the ANN dataset, when τ_1 is less than 0.16, the corresponding accuracy significantly outperforms that of other values. Similarly, in the KBWT dataset, when τ_1 is less than 0.1, the accuracy surpasses that of other values. In terms of costs, we find that cost decreases as τ_1 increases. This is because when τ_1 becomes larger, each demonstration can cover more questions, so fewer demonstrations are needed to cover all the questions within a group, leading to decreased costs. For a new dataset, if the users consider cost, they can select the maximum value within the cutoff region as τ_1. Otherwise, they can tune τ_1 within the cutoff region, selecting the one that achieves higher accuracy. In our implementation, we use the maximum value within the cutoff region as τ_1 in order to reduce cost.

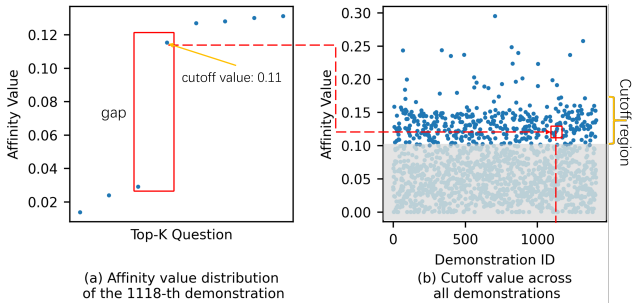

Figure 4: Affinity distribution of hyperparameter τ_1 on the ANN dataset.

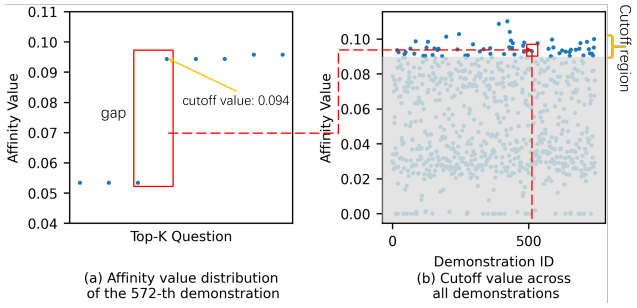

Figure 5: Affinity distribution of hyperparameter τ_1 on the KBWT dataset.

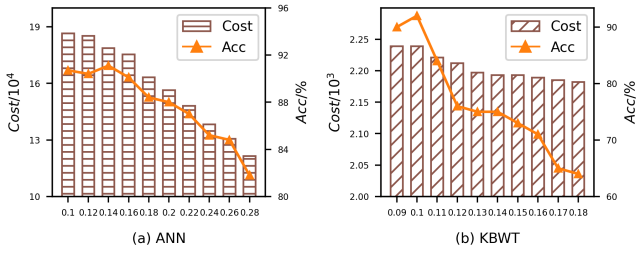

Figure 6: Sensitivity analysis of hyperparameter τ_1.

For τ_2, which constrains the group length, it directly affects the group size. In our setting, the group size includes both questions and demonstrations. Considering that the length of questions and demonstrations varies across tasks, we model τ_2 as $\alpha \times q_{len}$, where q_{len} is the average question length for each task, capturing task-specific characteristics. Then, we vary α to examine its impact on accuracy, the result of which is shown in Figure 7. From the figure, we find that when the value of α is around 15, it performs best. In terms of costs, we observe that when τ_2 becomes larger, the cost decreases. This is because a group can accommodate more questions as τ_2 becomes larger, thereby reducing the number of groups. For a new dataset, if users focus on cost, they can adjust τ_2 starting from $\alpha = 15$ and gradually increase it. Otherwise, they can tune τ_2 around $\alpha = 15$ and selecting the value that yields the best accuracy. In our implementation, we choose the first option to reduce cost.

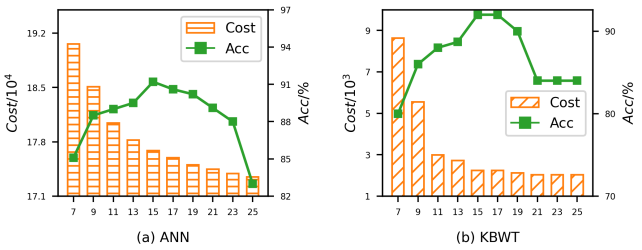

Figure 7: Sensitivity analysis of hyperparameter τ_2.

For τ_3, it limits the maximum number of questions that each demonstration can cover. This hyperparameter is typically set to a relatively small value. If it is set too large, it may lead to groups with many questions but few demonstrations, which negatively affects the reasoning capabilities of LLMs. The corresponding evaluation is shown in Figure 8. We observe that τ_3 performs best when it is set to 3 or 4. This suggests that the chosen values for τ_3 are general and can effectively adapt to different datasets. Taking cost into consideration, users can choose 4 as the value for τ_3. This is because when τ_3 is larger, one demonstration can cover more questions, thereby reducing the overall cost.

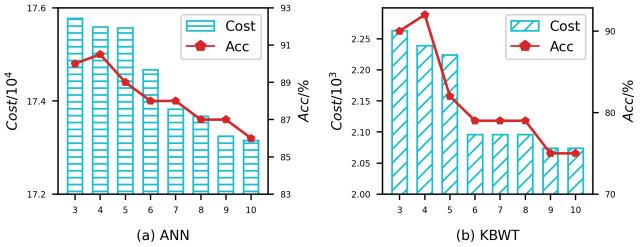

Figure 8: Sensitivity analysis of hyperparameter τ_3.

Regarding **Comment D3**, as described above, to determine the appropriate values of τ_0 and τ_1 which are related to embedding space, we calculate the affinity values through the embeddings for questions and demonstrations. For each question and demonstration, we calculate their gap and denote the cutoff value as the larger value within the gap. We determine the appropriate range based on the cutoff region, which is the region where most cutoff values fall. While the embedding space for questions and demonstrations exhibits varying density across different tasks, our guidance is based on the cutoff region derived from the embeddings, which can adaptively capture the characteristics of the embedding space. This enables dynamic determination of the optimal hyperparameters for each task.

Comment W2: *The approximate algorithm lacks theoretical justifications and a detailed analysis of its performance.*

Comment D4: *While OBP-Approx shows empirical effectiveness as in Table 6, a discussion on its potential optimality gap would be beneficial, as the algorithm includes multiple approximations and optimizations. Providing theoretical bounds or insights into its approximation quality would be interesting.*

RESPONSE: Thanks a lot for your valuable comments. In the revised manuscript, we have added the detailed theoretical analysis for each approximation algorithm of OBP in Section 3.2 and

Section 3.3. Specifically, our approximation method contains two steps, namely, *QueClt* and *CltSplt*.

For *QueClt*, we formalize it as the correlation clustering problem, which is NP-hard. We leverage the algorithm in [5] for implementation, and the approximation ratio of which is 3 [5].

After the *QueClt* step, there produces several clusters. For each cluster, it is input into the *CltSplt* step. This step has three stages. For Stage 1, we formalize it as the NP-hard weighted set cover problem, the approximation ratio of which is $0.72 \ln k$ [29], where k is the number of questions for each cluster. For Stage 2, we have added the theoretical analysis of the heuristic retention method in Algorithm 1 of the revised manuscript, and proved that $R(\mathcal{S}) \leq OPT(\mathcal{S}) + \frac{|o_{\arg\max_{i<n}(|s_i|-\frac{|o_i|}{2})}|}{2}$, where $R(\mathcal{S})$ denotes the number of questions in the largest set of the solution generated by Algorithm 1, and $OPT(\mathcal{S})$ denotes the number of questions in the largest set of the optimal solution. For stage 3, we formalize it as the NP-hard bin-packing problem, the approximation ratio of which is 1.7 [15].

Comment W3: *As efficiency is a primary goal, a detailed time complexity analysis for each stage would provide valuable insights.*
Comment D5: *Given that efficiency is the main objective, a time complexity analysis for the overall solution would be beneficial. This analysis would also help assess the solution's applicability to even larger real-world datasets than those presented in Table 2.*
RESPONSE: Our approximation method contains two steps, namely, *QueClt* and *CltSplt*.

For the *QueClt* step, given N questions, it runs in time $O(N^2)$, i.e., the runtime for the approximation algorithm of correlation clustering [8].

After the *QueClt* step, we assume that a number of C clusters are produced. For each cluster with k questions and m demonstrations, it is input into the *CltSplt* step. This step involves three stages. Stage 1 leverages weighted set cover to select demonstrations. Note that each demonstration corresponds to one set, and the total number of elements equals the number of questions in the cluster, i.e. k. Thus, its time complexity is $O(mk)$, i.e., the runtime for approximate set cover algorithm [29]. Note that n sets are produced by Stage 1, and subsequently input into Stage 2. Stage 2 includes a heuristic algorithm (i.e., Algorithm 1 in the revised manuscript) to balance the number of questions between sets. It first sorts n sets, then, for each iteration, it removes overlapping questions from sets. As described in the third paragraph in Section 3.4, the time complexity of the Stage 2 is $O(nlogn+n|o_i|)$. Note that $|o_i|$ indicates that number of overlapping questions in the each set. Then, the sets without overlapping questions input into Stage 3. Stage 3 exploits the first-fit bin packing [15] to complete the final grouping, whose runtime is $O(nlogn)$. Thus, the time complexity of the *CltSplt* step is $O(mk + n|o_i| + nlogn)$ for each cluster.

In conclusion, the time complexity of the overall algorithm is $O(N^2 + C(mk + n|o_i| + nlogn))$. We have added the above time complexity analysis in Section 3.4.

RESPONSE TO REVIEWER #3

Comment D1: *My major comment is that Section 3.2 and 3.3 are shallow. More details are necessary to understand the method and also convince that what's proposed makes sense.*

First: What is the overall algorithm for 3.2 that brings together different results. After partitioning the graph, each subgraph will have its own optimal result. Is it straightforward to bring the results from subgraphs into a single graph.
RESPONSE: We have added more details about OBP in Section 3. Specifically, we have added more details about *QueClt* in Section 3.2. We have added more explanations about the intuition of the retention method in the second paragraph of Section 3.3.1 and have added the time complexity analysis for each approximation algorithm of OBP in Section 3.4.

Section 3.2 (i.e., *QueClt*) partitions the questions into multiple clusters, making each cluster as independent as possible. For each cluster, we leverage MILP (or *CltSplt* described in Section 3.3) to produce groups. Finally, we combine these groups to give the final grouping, i.e., we bring together different results to produce the final grouping outcomes. We have added this explanation in Section 3.2.

The rationale for combining results of each cluster to form the final grouping is that we leverage correlation clustering to make each cluster as independent as possible, ensuring that they do not affect one another, and thus generate groups independently. Specifically, each question is represented as a node in the graph, with the affinity between questions corresponding to an edge between two nodes. This can be formalized as a correlation clustering problem, where the goal is to find an optimal clustering of nodes that minimizes the number of unrelated nodes within the same cluster and the number of related nodes across different clusters. After correlation clustering, unrelated questions are separated into different clusters, allowing each cluster to produce groups independently without being affected by others. We have added the above discussion in Section 3.2.

Comment D2: *Same section refers to that "there are several approximate correlation algorithm", but which one do you choose and why?*
Comment D7: *Section 3.3.2. "there are some approximation algorithm for it" –> which one? Again, an important detail is missing. You need to specify which one you use and give at least some explanation on why. Otherwise, the paper is simply not complete.*
RESPONSE: We have clarified the reason for the choice of approximate correlation clustering algorithms in the last paragraph of Section 3.2 and approximate bin packing algorithms in the last paragraph of Section 3.3.2.

Specifically, for correlation clustering, since the algorithm in [8] has the well-known optimality bound, which can help achieve a feasible solution within a reasonable time frame, we thus select it for implementation. For bin packing, we choose the first-fit bin packing algorithm proposed in [15], which guarantees a bounded deviation from the optimal solution.

Comment D3: *In Section 2.2, the authors refers to that "one can use BERT-based methods to compute affinity". What do you mean by "one"? Is this what you are using to compute affinity? Is there any other techniques? This needs to be more specific.*
RESPONSE: We have added more explanations about affinity computation in the fifth paragraph of Section 2.1. "one" here refers to the users. To compute the affinity, the first step is to generate representations (i.e. embeddings) for questions and demonstrations, and the second step is to calculate the affinity based on embedding.

In our framework, we do not restrict the specific implementation method of affinity computation, which is orthogonal to our method.

In our implementation, the BERT-based encoders [35] are exploited to tokenize questions into embeddings. Then, the affinity is calculated according to the distance metrics, i.e., Euclidean distance, between the embeddings. We have included the above implementation details in Section 2.1 and Section 4.2.

Comment D5: *Section 3.1.1: "apparently, this will result in groups with ..." –> I do not buy this sentence? There is nothing apparent in the made statement. Where is the data showing that? Furthermore, the authors follow up with a sentence saying "intuitively, we need to balance". This is not intuitive. I mean there is no intuition presented. This is the most problematic part of the paper. There should be an either thorough text or good empirical number supporting both claims.*

Comment D4: *The authors refer to that "they balance number of questions covered". Why? What is the intuition behind this? This is a strong design choice and practically not justified. It either rationally, or empirically needs to be justified. Otherwise, it is a random choice. In fact, what happens if we do a random choice? The experimental comparison between an imbalanced and balanced design could already answer this question.*

RESPONSE:

Regarding "apparently, this will result in groups with ...", after stage 1, a cluster produces a collection of sets. Each set consists of one demonstration and its associated questions. We need to combine as many sets as possible to generate the the minimum number of groups, while satisfying the *GL* constraint. However, it is possible to have some sets containing a large number of questions when a demonstration has the affinity relationship with many questions. When these sets are merged into groups, it results in groups with numerous questions but few demonstrations, which can impair the reasoning capacity of LLMs. We have added above explanations in the second paragraph of Section 3.3.1.

Regarding "they balance number of questions covered", after Stage 1 of *CltSplt*, there may be questions covered by multiple demonstrations across different sets (Figure 4(b) in the revised manuscript illustrates an example,), which is not permissible. We thus need a retention method for determining covering associations (i.e., each question can only be covered by a single demonstration) in Stage 2. If a random retention method is applied in this stage, it could result in one demonstration covering a large number of questions. When merging the sets into groups, the group that contains this demonstration has numerous questions but few demonstrations. Such imbalances lead to a limited number of demonstrations within the group, which may prevent LLMs from fully comprehending the questions, resulting in a decline in performance. To avoid this situation, we aim to balance the number of questions across sets as much as possible after the covering association determination. Thus, we have proposed a heuristic-based retention method to solve this challenge. We have added above explanations in the third paragraph of Section 3.3.

Furthermore, to compare the random method with the heuristic method, we have conducted corresponding experiments, the results of which are presented in Figure 9. Compared to the random method, we can observe that our method achieves the best accuracy on both ANN and KBWT datasets. This validates the idea of balancing the number of questions covered by each demonstration. We have added these experiments in Section 4.5.

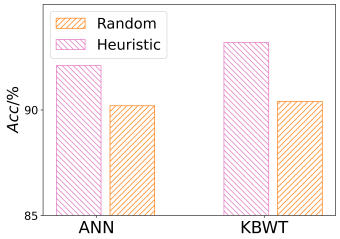

Figure 9: Comparison between random determination and our heuristic method.

Comment D6: *In the same section, why do you propose two retention method? Which one to choose and why and when? Is it case dependent, which I assume so? This is the second most problematic part of the paper.*

RESPONSE: We first devise a DP-based retention method. While it is effective, it is highly inefficient, the computation time of which is exponential for the number of overlapping questions and the number of sets. One set here represents a single demonstration along with its associated covering questions, which serves as the input for the retention method. Overlapping questions refer to those that are overlapped across sets. Consequently, we further propose an efficient heuristic method.

We select the retention methods based on the number of overlapping questions and the number of sets. Empirical results indicate that that when the number of sets exceeds 60 and the average number of overlapping questions per set is greater than 3, the runtime of the DP-based method exceeds one hour. In this case, the heuristic method is more efficient. Otherwise, the DP-based method is better, which yields more effective results. We have included the above discussion in the last paragraph of Section 3.3.1.

Comment D8: *Lemma 2: I didn't understand how triangle inequality relates to question affinity. N_T_0 seems to have nothing with triangles... Is it simply that some close-distance elements are removed?*

RESPONSE: We utilize the triangle inequality property to reduce the number of affinity computations (i.e., aff_p and aff_d). To illustrate the rationale of Lemma 2 and Algorithm 3, we give an example shown in the Figure 10 below. We first calculate the affinity between question q_0 and all other questions, and sort them by the affinity value, i.e., $s_1 < s_2 < s_3$. When calculating the affinity between a new question $q^{'}$ and other questions, we can reduce the number of calculations by means of Lemma 2. Specifically, we first calculate $s^{'}$ between q_0 and $q^{'}$. Note that questions q_0, q_1 and $q^{'}$ form a triangle. We then compare the values of $s_1 - s^{'}$ and τ_0. If $s_1 - s^{'} > \tau_0$, we can determine that the affinity value between q_1 and $q^{'}$ is greater than τ_0, so no further affinity calculation between $q^{'}$ and q_1 is needed. Subsequently, for questions q_2 and q_3, they both can form triangles with q_0 and $q^{'}$, if $s_2 - s^{'} > \tau_0$ and $s_3 - s^{'} > \tau_0$, we can avoid calculating the affinity between $q^{'}$ and q_2, $q^{'}$ and q_3. If $s_1 - s^{'} > \tau_0$, we can directly conclude that $s_2 - s^{'} > \tau_0$ and $s_3 - s^{'} > \tau_0$, because $s_1 < s_2 < s_3$. We have added the above explanations in Section 3.5.2 and included more details in the technical report [1].

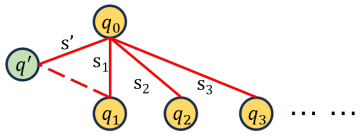

Figure 10: An illustration for Lemma 2.

RESPONSE TO REVIEWER #4

Comment D1: *Writing / examples is confusing. 1-2 consistent running examples would have been much nicer through the early part. For example, Figure 1, part 3 is confusing – what are the red circles. I would also generally try to improve the intro to be clearer.*

RESPONSE: We have updated and improved the running examples with corresponding explanations in Section 1.

In Figure 1 of the revised manuscript, the red circle in Question Grouping indicates that the first group of Simple Grouping (i.e., $\{q_1, q_2, q_3, q_4\}$) divides into two groups (i.e., $\{q_1, q_2\}$ and $\{q_3, q_4\}$). This corresponds to **Challenge 2**, i.e., we should limit the number of questions in the group. Furthermore, the red circle of Demonstration Selection in Figure 1 indicates that the first group of Simple Grouping adds more demonstrations (i.e., d_7 and d_8). This corresponds to **Challenge 3**, i.e., we should maintain sufficient demonstrations in the group. The above explanations are added in the fifth and sixth paragraphs of Section 1.

Furthermore, we have significantly enhanced the Section 1 (i.e., Introduction) in terms of presentation and structure. Specifically, we have highlighted the objectives and scope of the research, and have added more explanations about running examples to better understand our motivation and challenges.

Comment D2: *Also on writing: maybe some of the theory could be moved to an appendix for space, and re-use this space to have clearer examples, better figures etc. The theoretical statements make intuitive sense, but there is a lot of time spent on it.*

RESPONSE: D2 Thanks a lot for your suggestion. We have moved the proof of Theorem 1, Lemma 1 and Lemma 2 to our online technical report. Please refer to "Summary of Changes" for more details. Furthermore, we re-use this space to improve the running examples in Figure 1 of Section 1, add experiments for tuning and sensitivity analysis of hyperparameters in Section 4.3, etc.

Comment D3: *Experiments: whats the point of Table 4? We don't need to see the performance of this method with all the LLMs, or compare between them.*

RESPONSE: Thanks a lot for your suggestion. We have removed the content about the performance comparison of OBP with different LLMs, i.e., previously Section 4.2.3 and Figure 4.

Comment D4: *Much more interesting to ask questions like: adaptive grouping vs. fixed group sizes, using different clustering methods (k-means), cost-accuracy trade-off curves.*

RESPONSE: We have enhanced experiments about adaptive grouping vs. fixed group sizes and cost-accuracy trade-off curves.

For adaptive grouping vs. fixed group sizes, we have compared our method OBP which utilizes the adaptive grouping strategy with baseline Batcher which uses fixed group sizes, by varying the group sizes. The corresponding results are shown in Figure 5 of the revised manuscript. From these results, we find that OBP consistently outperforms Batcher in terms of accuracy regardless of the group sizes, demonstrating its superior performance. Meanwhile, OBP incurs relatively low cost. For Batcher, we can observe that group size significantly affects the accuracy, and the optimal group size varies for different datasets. Please refer to Section 4.2.3 for further details.

For the comparison with k-means, we think the comparison might not be appropriate. In k-means, selecting the optimal number of clusters, k, is crucial for obtaining good clustering results. However, the optimal value of k can vary across different datasets, which makes a fair comparison difficult.

For cost-accuracy trade-off curves, we analyze the cost-accuracy trade-off under varying parameters, with the results shown in Figures 3, 6, 7 and 8 of this cover letter.

For τ_0, Figure 3 illustrates the cost-accuracy trade-off as τ_0 varies. Regarding cost, it decreases as τ_0 increases. This is because a smaller τ_0 implies fewer questions per group, resulting in a larger number of groups and thus increasing the task description cost. Additionally, some questions that could share demonstrations cannot be grouped together, leading to more demonstrations. As for accuracy, it first increases and then decreases as τ_0 increases. When τ_0 is too small, most groups contain fewer demonstrations, which can impact the reasoning capability of LLMs. Conversely, when τ_0 is very large, the questions within each group tend to be randomly selected, leading to poor performance.

For τ_1, Figure 6 presents the cost-accuracy trade-off as τ_1 varies. Regarding cost, it decreases as τ_1 increases. This is because when τ_1 is relatively small, each demonstration may cover fewer questions, so more demonstrations are needed to cover all the questions within a group. Additionally, as τ_1 increases, the accuracy deteriorates. This is because the selected demonstration may differ significantly from the questions with a large τ_1, which do not effectively assist in the reasoning of LLMs.

For τ_2, the corresponding evaluations are shown in Figure 7. Regarding cost, it decreases when τ_2 becomes large. This is because a group can accommodate more questions as τ_2 becomes large, thereby reducing the number of groups, i.e., reducing the task description cost. As for accuracy, it first increases then decreases as τ_2 increases. For small τ_2, each group can accommodate few demonstrations, which can affect the reasoning capability of LLMs. When τ_2 becomes large, the LLM performance tends to degrade due to the larger number of questions within each group.

For τ_3, the experimental results are shown in Figure 8. For cost, when τ_3 is large, one demonstration may cover more questions, causing fewer costs. As for accuracy, when τ_3 is relatively large, it becomes worse. This is because one group may have fewer demonstrations, resulting in less domain knowledge LLMs obtained. LLMs consequently reason correctly for fewer questions.

Similar results can also be observed on other datasets. We have added the above experiments in Section 4.4 and also added more details in the technical report [1].

Optimized Batch Prompting for Cost-effective LLMs

Zhaoxuan Ji
Beijing Institute of Technology
jizhaoxuan@bit.edu.cn

Xinlu Wang
Beijing Institute of Technology
xinlu_wang@bit.edu.cn

Zhaojing Luo
Beijing Institute of Technology
zjluo@bit.edu.cn

Zhongle Xie
Zhejiang University
xiezl@zju.edu.cn

Meihui Zhang
Beijing Institute of Technology
meihui_zhang@bit.edu.cn

ABSTRACT

Large Language Models (LLMs) have recently demonstrated exceptional performance in various real-world data management tasks through in-context learning (ICL), which involves structuring prompts with task descriptions and several demonstrations. However, most LLMs are not free and charge based on the number of input tokens. Specifically, for data management tasks, there may be massive related questions, leading to high inference cost due to redundant prompt content (i.e., overlapping demonstrations and repeated task descriptions). In this paper, we investigate the idea of batch prompting in leveraging LLMs for data management, which leads to cost-effective LLMs by grouping questions and demonstrations to perform inferences in batches. Current studies on batch prompting are preliminary and mostly based on heuristics, making it difficult to generalize to various types of tasks and adapt to different grouping strategies. To address these challenges, in this work we first formalize the batch prompting problem in general setting. Then, we study the hardness of this problem and propose efficient algorithms for adaptive grouping. Finally, we conduct comprehensive experiments on 14 datasets. Extensive experimental results demonstrate that our solution consistently outperforms the state-of-the-art baselines while consuming lower cost.

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PVLDB Artifact Availability:
The source code, data, and/or other artifacts have been made available at https://github.com/jzx-bitdb/BatchPrompt.

1 INTRODUCTION

Large language models (LLMs) have demonstrated considerable effectiveness across a wide range of real-world applications, such as question answering, machine translation, context summarization, etc [42]. In particular, recent works [4, 16–18, 30, 41] delve into applying LLMs to data management tasks for improving accuracy, such as entity resolution [16] and data transformation [30]. These works typically employ in-context learning (ICL) [14], where several demonstrations (i.e., examples with corresponding answers from the same/similar task) are provided together with the task description and the target question in the prompt, see Single Prompting in Figure 1 for the illustration.

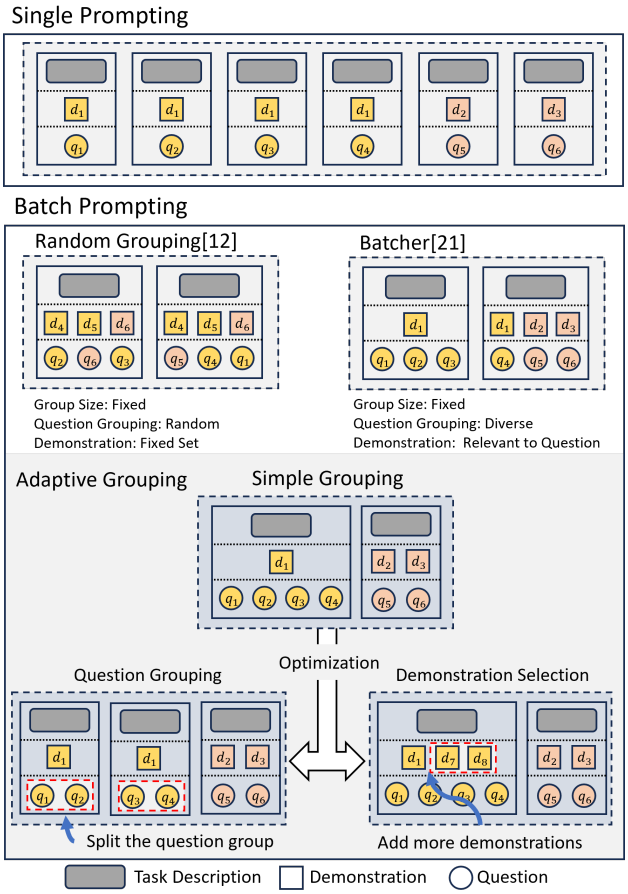

Figure 1: Comparison of different prompting strategies. Questions and demonstrations with the same color indicate they are affinitive (i.e., similar or diverse) to each other.

However, most closed-source LLMs are not free and charge based on the number of input tokens when calling the provided APIs. For data management tasks, there are usually a large number of questions, leading to prohibitively high monetary cost for using

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LLMs. Consider the data transformation [11] task as an example. Real-world datasets may contain a massive number of data, e.g., there are 165,236 data tables in the open dataset published by the U.S. government in March 2017 [31], each with tens of thousands of data rows on average. Transforming each row using LLMs, with around 100 tokens per row, would cost over $900k with GPT-4o ($5.00/1M tokens) [4, 33].

Batch prompting is an effective way for LLM cost reduction. To be specific, several questions can be consolidated in a single prompt for LLMs to perform inference, thereby eliminating redundant demonstrations and task descriptions. Batch prompting has been preliminarily explored in recent works [9, 16]. The method in [9] proposes to randomly combine questions into groups, each with the same number of questions and a fixed demonstration set. As Random Grouping in Figure 1 shows, this method randomly divides six questions into two groups, sharing the same demonstration set $\{d_4, d_5, d_6\}$. Batcher [16] targets only the entity resolution task. It clusters diverse questions into groups with a fixed group size. As shown in Figure 1, Batcher clusters questions with size 3. After the question grouping, it selects demonstrations that are the most relevant to questions within the group, e.g., it selects d_1 for the first group since it is the most relevant to all three questions $\{q_1, q_2, q_3\}$. While these works have preliminarily demonstrate the effectiveness of batch prompting on reducing LLM cost, several challenges remain unresolved.

Challenge 1: There lacks a general solution to the problem of LLM batch prompting. Existing works rely on fixed grouping strategies and are specifically designed for certain tasks. However, different tasks may require different grouping strategies. For instance, grouping similar questions works well for data transformation task since similar questions can help LLMs learn transformation rules for data of the same type. However, in entity resolution task, a better strategy is to group related questions (e.g., electronic products) but avoid putting questions with high similarity (e.g., iPhone 14 and iPhone 14 plus) in one group in order to reduce the ambiguity [16]. It therefore calls for an LLM batch prompting approach that is generally applicable to typical data management tasks.

Challenge 2: There is no adaptive question grouping solution for batch prompting. Existing methods use a fixed group size, i.e., the number of questions in all groups remains constant. However, it is always difficult or even infeasible to choose an appropriate group size. Further, a fixed size may degrade the performance (e.g., accuracy and cost) of LLMs. To be specific, if the number is set too large, unrelated questions may be grouped together, negatively affecting the accuracy of LLMs, e.g., the second group in the Batcher example in Figure 1. On the contrary, if the group size is small, the proliferation of groups would increase the overall cost. Therefore, a better strategy should be to adaptively group questions according to the number of related questions. A simple way is to form groups by clustering all related questions into one group. However, this may produce groups that have too many questions (e.g., group 1 in Simple Grouping in Figure 1), which could simply exceed the input token limit of LLMs, or result in low accuracy due to large input context [21, 23, 26, 27]. Thus, an optimized strategy should avoid large group sizes. For example in Figure 1, the first group in Simple Grouping should be further divided into two groups, i.e, $\{q_1, q_2\}$ and $\{q_3, q_4\}$ in the Question Grouping part. As a result, how to adaptively cluster related questions into groups while ensuring low cost and high accuracy is challenging.

Challenge 3: There is no effective strategy for demonstration selection. Existing methods either use a fixed demonstration set or select the most similar demonstrations to the questions for each group. Although the latter is regarded as more effective [16], it often results in groups containing an excessive number of questions with insufficient demonstrations, thereby impairing the reasoning ability of LLMs. For instance in Figure 1, the first group of Simple Grouping has only one demonstration, potentially compromising the accuracy of LLMs. It would be beneficial to add more demonstrations, such as d_7 and d_8 for better LLM accuracy, as outlined in the Demonstration Selection part. Thus, how to effectively select demonstrations in each group while balancing the LLM performance and the overall cost is the third challenge.

In this paper, we investigate batch prompting techniques for data management tasks that consist of numerous interdependent questions, with the goal of minimizing the overall LLM cost while maintaining the resulting accuracy. We first formalize this problem as a constrained optimization problem. Specifically, we consider four factors affecting the accuracy, including Question Group Affinity (relationship between questions), Question Demonstration Affinity (relationship between questions and demonstrations), Group Length (the number of input tokens in a group) and Demonstration Coverage (balance between questions and demonstrations). We further demonstrate that this overall problem is NP-hard. To resolve this problem, we propose a framework for data management tasks called **O**ptimized **B**atch **P**rompting (OBP) with exact and approximation solutions. In detail, we develop a question clustering method, and then introduce a three-staged approach for adaptively grouping questions and effectively selecting demonstrations. Last, we further propose two optimizations to enhance the efficiency of the framework, which filters out unnecessary demonstrations that are covered by others, and reduce question affinity computations by means of the triangle inequality property, respectively.

In summary, our main contributions are as follows:

- We investigate batch prompting for data management tasks, and formalize it as an optimization problem. We demonstrate it is NP-hard in general with a theoretical analysis.
- We propose the Optimized Batch Prompting (OBP) framework that adaptively groups questions and selects demonstrations, and generates results with low cost while ensuring the accuracy.
- We propose two optimizations to accelerate the computation, which filter unnecessary demonstrations and reduce the number of affinity calculations respectively.
- We conduct extensive experiments across three tasks with 14 datasets to evaluate the effectiveness and efficiency of our proposed method. Results show that our method reduces the cost by up to 35% compared to the state-of-the-art LLM and non-LLM based baselines. Meanwhile, our method outperforms the baselines on almost all datasets in terms of accuracy.

The rest of the paper is organized as follows. We formalize the batch prompting problem in Section 2. We then propose our framework in Section 3. In Section 4, we provide an extensive set of

experiments to validate the effectiveness and efficiency of our approach for the batch prompting. Finally, we discuss the related work in Section 5 and conclude the paper in Section 6.

2 PROBLEM FORMULATION

In this section, we first formulate the batch prompting problem. We then present a rigorous theoretical analysis of the problem and prove the hardness.

2.1 Problem Statement

For a batch of questions $Q = \{q_1, q_2, ..., q_N\}$ of certain data management tasks, we have a pool of demonstrations $\mathcal{D} = \{d_1, d_2, ..., d_M\}$. The goal is to combine these questions into groups, which are denoted as $\mathcal{G}$. Note that $\mathcal{G}$ covers all questions, and each question exactly belongs to one group. Assuming the optimal number of final groups is K, i.e., $\mathcal{G} = \{g_1, g_2, ...g_l, ..., g_K\}$, each group g_l consists of a task description $\mathcal{T}$, a subset of questions $Q_l \in Q$ and a subset of demonstrations $\mathcal{D}_l \in \mathcal{D}$. Thus, g_l can be written as $g_l = \{\mathcal{T}, \mathcal{D}_l, Q_l\}$.

Since LLMs charge based on the number of input tokens, we thus measure the cost by token counts. Each question and demonstration has its own cost, denoted by c_{q_i} and c_{d_j}. The cost of the task description is denoted as c_T. The cost of each group includes c_T and the cost of questions and demonstrations in the group.

Given a batch of questions Q and a pool of demonstrations $\mathcal{D}$, our goal is to cluster all questions in Q into groups with minimizing the overall cost of all groups. i.e., min $\sum_l c(g_l)$. However, merely minimizing the cost can result in low accuracy. Therefore, we need to minimize the cost without compromising the accuracy.

From a careful analysis, we observe the following four factors mostly affect the LLM accuracy in the batch prompting problem.
(1) Question Group Affinity (QGA). In the batch prompting setting, multiple questions are consolidated in a group. For questions in the group, unrelated ones often span distinct contexts or domains, creating noise and making it challenging for LLMs to focus on relevant information and make accurate inferences. Therefore, it is crucial to consider the relationships between questions in batch prompting, as related questions allow LLMs to draw on common patterns which leads to more coherent and accurate responses. Thus, the first factor we consider for enhancing accuracy is the relationship between questions within each group. We define a function aff_q to quantify this relationship for each group, formulated as follows:

$$aff_q(g_l) = \max_{\forall q_i, q_{i'} \in g_l} aff_p(q_i, q_{i'})$$

where $aff_p(q_i, q_{i'})$ represents the affinity between pair-wise questions q_i and $q_{i'}$. To compute the affinity, the first step is to generate representations (i.e. embeddings) for questions and demonstrations, and the second step is to calculate the affinity based on embedding. In our framework, we do not restrict the specific implementation method of affinity computation, which is orthogonal to our method. In our implementation, the BERT-based encoders [35] are exploited to tokenize questions into embeddings. Then, the affinity is calculated according to the distance metrics, i.e., Euclidean distance, between the embeddings.

(2) Question Demonstration Affinity (QDA). In the setting of ICL, when given a question, LLMs rely on demonstrations to perform few-shot learning. Therefore, demonstrations that are relevant to the question are crucial because they help LLMs better understand the question and provide accurate inferences. To guide our method to select suitable demonstrations to boost accuracy, we define a function $aff_d(q_i, d_j)$ to measure the relationship between question q_i and demonstration d_j. The process of calculating this affinity is akin to that used for aff_p.
(3) Group Length (GL). To ensure high accuracy of LLMs, the input length cannot be too large. First, LLMs have a token limit, and exceeding this limit can lead to incomplete information, which adversely affects accuracy. Second, overly long inputs make it difficult for LLMs to identify and extract relevant information. Therefore, it is important to consider input length when designing our batch prompting method for accurate responses. In our batch prompting scenario, the input for LLMs is a group containing a task description, multiple questions and demonstrations. We define the function $c(g_l)$ as the total token count of group g_l, so as to constrain the input length of our method.
(4) Demonstration Coverage (DC). As mentioned above, in the ICL setting, LLMs rely on demonstrations to generate accurate responses for questions. If the number of demonstrations is small, LLMs may struggle to comprehensively understand the question. Additionally, a limited number of demonstrations prevent LLMs from capturing nuances among questions within each group, thereby affecting the reasoning ability of LLMs. Thus, we define a function $cov(d_j)$ for each demonstration d_j in the group to represent questions it can cover. Here, a question covered by a demonstration d_j means that this question is affinitive to d_j. For each demonstration d_j in the group, its number of covered questions, i.e., $|cov(d_j)|$, cannot be too large to ensure sufficient demonstrations for better reasoning ability of LLMs.

After taking into consideration the above four factors, we propose the following constraints to ensure accuracy of our batch prompting method.

- *QGA constraint*: For each group $g_l \in \mathcal{G}$, the questions in g_l have to meet the affinity requirement constrained by the affinity threshold τ_0, i.e., $aff_q(g_l) \leq \tau_0$.
- *QDA constraint*: Under the setting of ICL, each question in the group should have one affinitive demonstration as its context, i.e., $\exists d_j \in \mathcal{D}_l, aff_d(q_i, d_j) \leq \tau_1, \forall q_i \in Q_l$.
- *GL constraint*: For each group $g_l \in \mathcal{G}$, the total token counts of the group, denoted as $c(g_l)$, cannot exceed the pre-defined maximum length, i.e., $c(g_l) \leq \tau_2$.
- *DC constraint*: For each demonstration d_j, the number of questions it covers cannot exceed a threshold, i.e., $|cov(d_j)| \leq \tau_3$.

To sum up, given a batch Q of questions and a pool $\mathcal{D}$ of demonstrations, the objective of batch prompting is to group all questions and the corresponding demonstrations into K groups, i.e., $\mathcal{G} = \{g_1, g_2, ...g_l, ..., g_K\}$ where $g_l = \{\mathcal{T}, \mathcal{D}_l, Q_l\}$, such that the overall cost of all groups is minimized, with the constraints of LLM accuracy satisfied, which is formulated as:

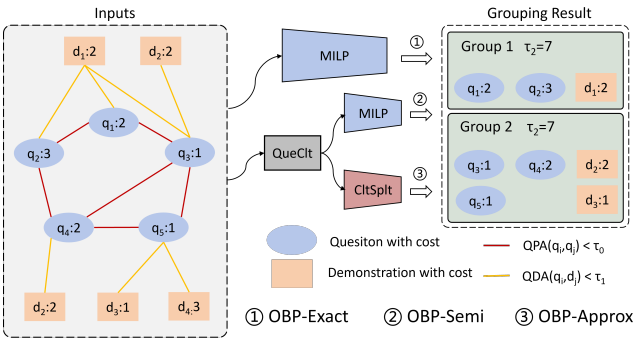


Figure 2: The framework for batch prompting.

$$
\begin{aligned}
\min \quad & \sum_{l}^{K} c(g_l) \\
s.t. \quad & aff_q(g_l) \le \tau_0, && \forall l \in [1, K] \\
& aff_d(q_i, d_j) \le \tau_1, && \forall q_i \in Q_l, \exists d_j \in \mathcal{D}_l \\
& c(g_l) \le \tau_2, && \forall l \in [1, K] \\
& |cov(d_j)| \le \tau_3, && \forall d_j \in \mathcal{D}_l
\end{aligned} \tag{1}
$$

2.2 Theoretical Analysis

Theorem 1. *The optimization problem of batch prompting shown in Equation 1 is NP-hard.*

3 THE OBP FRAMEWORK

In this section, we present our framework Optimized Batch Prompting (OBP) for adaptive grouping, which is shown in Figure 2. The input of our framework is a batch of questions Q and a demonstration pool $\mathcal{D}$. The OBP framework offers three versions of solutions. (1) OBP-Exact (Section 3.1): we transform the batch prompting problem into MILP (Mixed Integer Linear Programming) formulation, which can be solved via the MILP solver (e.g., GUROBI [22]) to get the exact solution. Several optimizations are also proposed to accelerate the computations of the MILP solver. (2) OBP-Semi (Section 3.2): Considering the expensive computation of the MILP solver, it is infeasible to have exact solution when the number of questions is large. To this end, we design question clustering method (*QueClt*), which first constructs a graph to represent the question relationship and then decompose it into clusters, so that each cluster can be solved by the MILP solver individually, which is significantly faster than directly solving the original problem. (3) OBP-Approx (Section 3.3): To further improve the efficiency, we design an approximation solution (*CltSplt*) to replace the MILP solver and produce grouping result for the clusters with a larger number of questions.

3.1 The OBP-Exact Solution

In this section, we discuss an MILP-based method, which can produce the exact solution to the batch prompting problem formalized in Section 2. Specifically, we transform the abstract format in Equation 1, such that it can be solved by MILP solvers. Let $x(g_l, i)$ and $y(g_l, j)$ denote a boolean decision variable of whether a question q_i and a demonstration d_j belong to group g_l or not, respectively. The following equation shows the transformation:

$$\min \quad \sum_{l}^{N}(\sum_{i}^{N} c_{q_i} x(g_l, i) + \sum_{j}^{M} c_{d_j} y(g_l, j) + u(l) c_T) \tag{2a}$$

$$s.t. \quad \sum_{l}^{N} x(g_l, i) = 1, \qquad \forall i \in [1, N] \tag{2b}$$

$$aff_q(q_i, q_{i'}) x(g_l, i) x(g_l, i') \le \tau_0, \qquad \forall l, i, i' \in [1, N] \tag{2c}$$

$$\sum_{l}^{N} \sum_{j}^{M} (aff_d(q_i, d_j) x(g_l, i) y(g_l, j)) = 1, \qquad \forall i \in [1, N] \tag{2d}$$

$$\sum_{i}^{N} c_{q_i} x(g_l, i) + \sum_{j}^{M} c_{d_j} y(g_l, j) + c_T \le \tau_2, \qquad \forall l \in [1, N] \tag{2e}$$

$$u(l) \le \sum_{i}^{N} x(g_l, i) + \sum_{j}^{M} y(g_l, j), \qquad \forall l \in [1, N] \tag{2f}$$

$$u(l) L \ge \sum_{i}^{N} x(g_l, i) + \sum_{j}^{M} y(g_l, j), \qquad \forall l \in [1, N] \tag{2g}$$

Equation 2a is the objective of our optimization problem, which minimizes the total cost of all groups. Equation 2b constrains that each question can only belong to one group. Equations 2c and 2d are the constraints about the *QGA* and *QDA*. To demonstrate the *QDA constraint*, we can intuitively express as: $\sum_{l}^{N} \sum_{j}^{M} \mathcal{I}(\tau_1 - aff_d(q_i, d_j) x(g_l, i) y(g_l, j)) \ge 1$. Note that $\mathcal{I}()$ denotes the indicator function, where $\mathcal{I}(x)$ returns true if $x \ge 0$, false otherwise. However, this format is challenging to be solved due to the indicator function. Fortunately, we can remove the indicator function by pre-processing it. Specifically, for each $aff_d(q_i, d_j)$, we rewrite it as 1 if it is less than τ_1, 0 otherwise. Thus, the format is shown in Equation 2d. Meanwhile, to maintain the *DC constraint*, for each demonstration d_j, we sort all $aff_d(q_i, d_j)$, and assign the first τ_3 questions as 1 (whose aff_d is less than τ_1), 0 otherwise. While this pre-processing is an approximation method[1], it exhibits excellent accuracy in practice [7]. Besides, the pre-processing can make the MILP solver more efficient. This is because all coefficients are changed from float numbers to boolean values. Note that we also change $aff_p(q_i, q_{i'})$ into 0 or 1. The *GL constraint* is shown in Equation 2e. Note that a total of K groups $(g_1, g_2, ..., g_K)$ would cover all questions, where $1 \le K \le N$ (i.e., at least one group, or at most N singleton groups could be formed). We cannot know the optimal group number K in advance, so we introduce an additional variable $u(l)$ with Equations 2a, 2f and 2g to identify the number of groups. Note that L in Equation 2g is a pre-defined positive number that can be considered as infinity.

[1]This constraint is enforced during the MILP solving, rather than before the data is input.

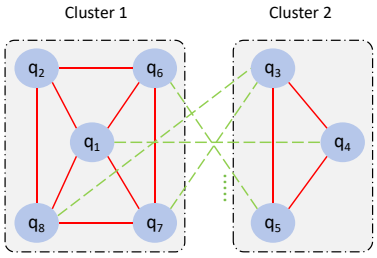


Figure 3: An example of Question Affinity Graph. The red lines denote positive edges while the green lines are negative edges. Note that we only draw partial green edges.

Once the MILP is formalized as shown in Equation 2, we exploit a general-purpose solver to solve it (e.g., GUROBI [22]).

3.2 The OBP-Semi Solution

The optimal solution presented in Section 3.1 may be expensive since the MILP-based solution has an exponential computation time in the worst case. To expedite this procedure, we propose further optimizations.

To allow the MILP-based solution to scale to a larger number of questions, we break down the original optimization problem into a sequence of small problems. To be specific, we divide the original question set Q into multiple clusters, making each cluster as independent as possible. After the splitting, unrelated questions are not grouped within the same cluster, allowing each cluster to generate groups independently without being affected by others. Then, combining these results gives the final grouping. Specifically, each cluster can formalize an optimization problem without the *QGA constraint* (i.e., the Equation 2c), so that the MILP solver can be exploited in each cluster independently. As it turns out, it is faster than the original MILP-based solution due to fewer constraints and smaller data volume.

Next, we describe the question clustering method named *QueClt*. To cluster original questions, we first build the question affinity graph to present the relationships between the questions. The affinity graph is defined as follows:

Definition 1 (Question Affinity Graph (QAG)). *Given an undirected (complete) graph $G_Q = (V_Q, E_Q)$, each node in V_Q refers to a question q_i, and each edge in E_Q refers to $aff_p(q_i, q_{i'})$ between q_i and $q_{i'}$, whose weight is either 1 (i.e., these pair-wise questions are affinitive to each other, denoted as the positive edge) or -1 (otherwise, denoted as the negative edge).*

Figure 3 presents an example *QAG*. The next step is to decompose the graph into subgraphs, which are as independent as possible. The goal is to find an optimal clustering of nodes, such that the number of unrelated nodes in the same cluster (connecting with negative edges) and the number of related nodes (connecting with positive edges) in different clusters are minimized, i.e., make each cluster as independent as possible. This is exactly the goal of correlation clustering [5]. To be specific, we give the formal definition of correlation clustering as follows:

Definition 2 (Correlation Clustering). *Let $G = (V, E)$ be an undirected (complete) graph, and the edge weights are 1 or -1. Let E^+ be the set of positive edges, and E^- be the set of negative edges. Intuitively, edge $e_{uv} \in E^+$ if u and v are related, and $e_{uv} \in E^-$ if u and v are unrelated. A clustering of G is a non-overlapping partition of its node set. The goal of the correlation clustering problem is to minimize: $|\{e_{uv} \in E^+ | C(u) \neq C(v)\}| + |\{e_{uv} \in E^- | C(u) = C(v)\}|$. Note that $C(u) = C(v)$ means nodes u and v are in the same cluster, and $C(u) \neq C(v)$ means that they are in different clusters.*

There are various approximation algorithms [3, 5] for correlation clustering. Among these, the algorithm proposed in [5] has the well-known optimality bound, which can help achieve a feasible solution within a reasonable time frame. We thus select it for implementation. The approximation ratio of it is 3 [5], which bounds deviation from the optimal solution. For example in Figure 3, after running the correlation clustering algorithm, two clusters are produced respectively: $\{q_1, q_2, q_6, q_7, q_8\}$ and $\{q_3, q_4, q_5\}$.

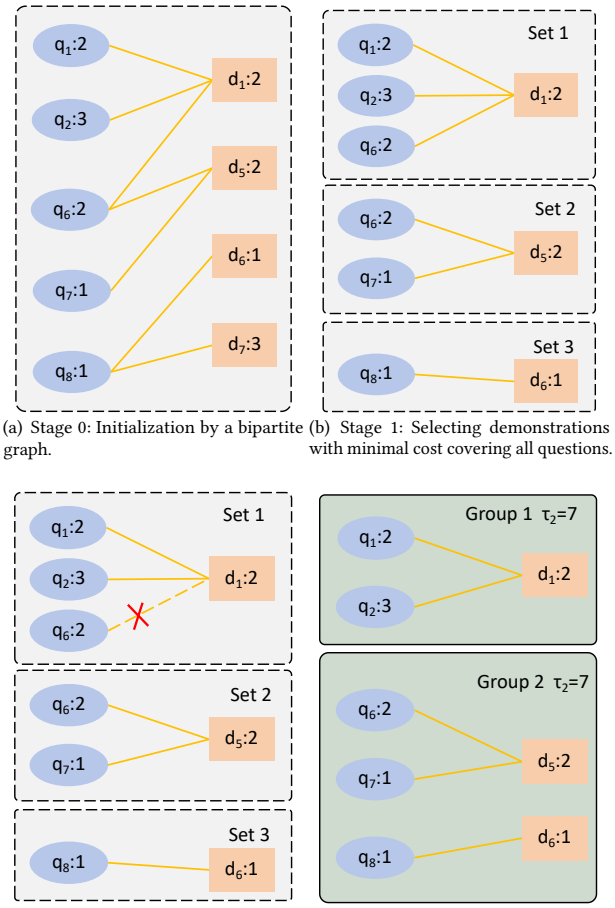
3.3 The OBP-Approx Solution

While we can use the MILP solver to obtain the optimization solution for each cluster, it is still intractable for large clusters with substantial questions. To further accelerate the computation, we design an efficient approximate method Cluster Splitting (*CltSplt*).

We illustrate the procedure of *CltSplt* with an example shown in Figure 4. Specifically, the cluster in the example has five questions and four demonstrations, which can be presented as a bipartite graph (Figure 4(a)). The blue nodes on the left represent the questions and the orange nodes on the right represent the demonstrations. The edge between a question and a demonstration indicates that they are affinitive. Next, we introduce the whole procedure of *CltSplt* based on this example.

Our solution includes three stages: (1) For each cluster, we select affinitive demonstrations with minimal cost to cover all questions. Each demonstration and its associated questions form a set. Intuitively, this problem is an instance of weighted set cover problem[2], so we can use the common-use approximation algorithm [29] to solve it, the approximation ratio of which is $0.72 \ln k$ [29]. Note that k indicates the number of questions in the cluster. As the example in Figure 4(b) shows, $\{d_1, d_5, d_6\}$ are the selected demonstrations that can cover all questions with minimal cost, and they together with the associated questions form three sets. (2) After stage 1, there may be questions that are covered by multiple demonstrations in different sets, e.g., in Figure 4(b), q_6 is covered by both d_1 in Set 1 and d_5 in Set 2. Since one question only belongs to one group in the final grouping, it needs to break the tie by retaining only one connecting demonstration for each question. However, a random method may lead to cases where a demonstration covers a large number of questions, resulting in groups with numerous questions but few demonstrations. As discussed in Section 2.1, this imbalance can affect the reasoning of LLMs. To this end, we propose a metric for retaining the covering associations between demonstrations and questions, and a retention algorithm in Section 3.3.1, with the aim to balance the number of questions covered by each demonstration. For example, in Figure 4(c), q_6 is removed from Set 1 and only retained in Set 2, making the number of questions covered

[2]The proof is similar to Section 2.2.



(a) Stage 0: Initialization by a bipartite graph.
(b) Stage 1: Selecting demonstrations with minimal cost covering all questions.
(c) Stage 2: Determining the covering associations to be retained.
(d) Stage 3: Combining sets into a minimal number of groups.

Figure 4: Illustration of the *CltSplt* procedure.

by these demonstrations more balanced; (3) The final stage is to combine as many sets as possible to generate the the minimum number of groups, while satisfying the *GL constraint*. This is to reduce the overall cost since for every additional group there will be an extra task description cost c_T. In Figure 4(d), three sets are finally combined into 2 groups. We shall elaborate on the combining strategy in Section 3.3.2.

3.3.1 Retention method. Recall that the retention method is applied when there are questions that are covered by multiple demonstrations in different sets in stage 2 of *CltSplt*. The aim is to retain the covering association that can lead to a high-quality grouping.

After stage 1, a cluster produces a collection of sets, denoted as $\mathcal{S}$, and each set $s_i \in \mathcal{S}$ consists of one demonstration and its associated questions, e.g., Set 1 in Figure 4(b) has $\{d_1\}$ and $\{q_1, q_2, q_6\}$. If a random retention method is applied, some sets may contain a large number of questions when a demonstration has the affinity relationship with many questions. The final group containing these sets will have numerous questions sharing few demonstrations. In the ICL setting, LLMs rely on demonstrations to generate accurate responses for questions. When the number of demonstrations is limited, LLMs may struggle to fully comprehend the questions, leading to a decline in performance. To avoid this situation, we shall balance the number of questions in each set as much as possible. A direct indicator of the balance level of $\mathcal{S}$ is the maximum number of questions in its all sets. Formally, we define the balance level of $\mathcal{S}$ as:

$$s_{max} = \max_{s_i} |s_i|, \forall s_i \in \mathcal{S} \tag{3}$$

Note that $|s_i|$ denotes the number of questions in set s_i. A smaller s_{max} value implies a more balanced set $\mathcal{S}$. Thus, our goal is to minimize s_{max} when determining which covering association to be retained. Consider the example in Figure 4(c). Removing q_6 from Set 1 results in a smaller s_{max}, which is better than removing q_6 from Set 2. To this end, we propose the following two retention methods.

The first method is a dynamic programming (DP) based algorithm. Specifically, we first sort the sets in $\mathcal{S}$ based on the number of questions, and define O_i as the overlapping questions in the first i sets. An overlapping question means that it appears in multiple sets. Then, we define $DP(O_i)$ as the retention result of the first i sets with minimizing s_{max} as the objective. Thus, the final retention result is $DP(O_n)$, assuming that the total number of sets in $\mathcal{S}$ is n. The recursive formula for dynamic programming can be computed as follows:

$$|DP(O_i)| = \min \begin{cases} |DP(O_i)| \\ \max(|DP(O_{i-1})|, |s_i \setminus o_i^{'}|), o_i^{'} \subset o_i \end{cases} \tag{4}$$

Note that o_i denotes set of the overlapping questions in s_i. We need to calculate $\max(|DP(O_{i-1})|, |s_i \setminus o_i^{'}|)$ for $perm(o_i)$ times and $perm(o_i)$ denotes the number of permutation of o_i. Thus, the time complexity of this DP algorithm is $O(perm(O_i)perm(o_i)n)$.

The computation time is exponential for the number of overlapping questions and the number of sets, making it challenging to obtain the optimal solution for large number of overlapping questions.

Version 1:

To this end, we propose a heuristic method which is shown in Algorithm 1. We begin by sorting $\mathcal{S}$ in descending order based on $|s_i|$. If two sets have the same number of questions, the set with a smaller $|o_i|$ is ranked higher (line 1). The rationale behind this is to prioritize removing overlapping questions from sets with larger $|s_i|$. When the number of questions in the sets (i.e., $|s_i|$) is equal, the set with fewer overlapping questions should be removed first. Then, we calculate the average number of non-overlapping questions in each set, denoted as d_{avg} (lines 2-7). Then, for each set, if the number of questions exceeds d_{avg}, we remove its overlapping questions using the function $RemoveOverlap_1(o_i)$(lines 8-12). Specifically, we assume that set s_i and h other sets have overlapping questions, denoted as $c_{i1}, c_{i2}, ..., c_{ih}$, such that $\sum_{j=0}^{h-1} |c_{ij}| = |o_i|$. Note that $|c_{ij}|$ denotes the number of overlapping questions between s_i and s_j, where $|o_i|$ refers to the number of overlapping questions in s_i. For the set s_j that has overlapping questions with s_i, if $|s_j| > d_{avg}$, the function $RemoveOverlap_1$ removes $\frac{|c_{ij}|}{2}$ overlapping questions from both s_i and s_j; otherwise, it removes $|c_{ij}|$

Algorithm 1 Retention Method

Input: Sets $\mathcal{S}$ after Stage 1.
Output: Retention Sets $\hat{\mathcal{S}}$.

1: $\hat{\mathcal{S}} = Sort(\mathcal{S})$, $Q_{uniq} = \{\}$.
2: **for** s_i in $\mathcal{S}$ **do**
3: **for** q_i in s_i **do**
4: $Q_{uniq}+ = \{q_i\}$.
5: **end for**
6: **end for**
7: $d_{avg} = \frac{|Q_{uniq}|}{|\mathcal{S}|}$.
8: **for** s_i in $\hat{\mathcal{S}}$ **do**
9: **while** $|s_i| \geq d_{avg}$ and Overlap(o_i) **do**
10: $RemoveOverlap_1(o_i)$.
11: **end while**
12: **end for**
13: **for** s_i in $\hat{\mathcal{S}}$ **do**
14: **while** $|s_i| \leq d_{avg}$ and Overlap(o_i) **do**
15: $RemoveOverlap_2(o_i)$.
16: **end while**
17: **end for**
18: Return $\hat{\mathcal{S}}$.

overlapping questions from s_i. The rationale behind this is to remove more overlapping questions from the set with a larger number of questions. Next, we iterate sets whose question count is below d_{avg} and apply $RemoveOverlap_2(o_i)$(lines 13-17). Unlike $RemoveOverlap_1(o_i)$, $RemoveOverlap_2(o_i)$ removes $\frac{|c_{ij}|}{2}$ overlapping questions from both s_i and s_j without any restrictions. Finally, the processed sets are returned (line 18). Next, we give a theoretical analysis about the heuristic algorithm.

Theorem 2. $R(\mathcal{S}) \leq OPT(\mathcal{S}) + \frac{|o_{\arg\max_{i<n}(|s_i|-\frac{|o_i|}{2})}|}{2}$

Proof. We use $R(\mathcal{S})$ to denote the number of questions in the largest set of the solution generated by Algorithm 1, and $OPT(\mathcal{S})$ denote the number of questions in the largest set of the optimal solution.

First, $R(\mathcal{S}) \leq \max_{i<n}(|s_i| - \sum_{|s_j|\geq d_{avg}} \frac{|c_{ij}|}{2} - \sum_{|s_j|<d_{avg}} |c_{ij}|)$. This is because $R(\mathcal{S})$ is guaranteed to be generated in sets with more than d_{avg} questions, so the result can be derived based on the $RemoveOverlap_1$ function. Thus, we have $R(\mathcal{S}) \leq \max_{i<n}(|s_i| - \sum \frac{|c_{ij}|}{2}) = \max_{i<n}(|s_i| - \frac{|o_i|}{2})$.

Second, we have $\max_{i<n}(|s_i| - |o_i|) \leq OPT(\mathcal{S})$. This is because $OPT(\mathcal{S})$ does not exceed the maximum value of these sets after all overlapping questions have been removed from each set. Therefore, we have:

Algorithm 2 Retention Method

Input: Sets $\mathcal{S}$ after Stage 1.
Output: Retention Sets $\hat{\mathcal{S}}$.

1: $\hat{\mathcal{S}} = Sort(\mathcal{S})$
2: **while** $Overlap(\hat{\mathcal{S}}) > 0$ **do**
3: **for** s_i in $\hat{\mathcal{S}}$ **do**
4: **if** Overlap(s_i) **then**
5: Randomly remove one overlapping question from s_i.
6: Break the loop.
7: **end if**
8: **end for**
9: **for** $s_j = s_{i+1}$; $j < |\hat{\mathcal{S}}|$; $j++$ **do**
10: **if** $|s_i| < |s_j|$ or $(|s_i| = |s_j|$ and $|o_i| > |o_j|)$ **then**
11: $Swap(s_i, s_j)$.
12: **end if**
13: **end for**
14: **end while**
15: Return $\hat{\mathcal{S}}$.

$$
\begin{aligned}
R(\mathcal{S}) &\leq \max_{i<n}(|s_i| - \frac{|o_i|}{2}) \\
&= \max_{i<n}(|s_i| - |o_i|) + \max_{i<n}(|s_i| - \frac{|o_i|}{2}) - \max_{i<n}(|s_i| - |o_i|) \\
&\leq OPT(\mathcal{S}) + \max_{i<n}(|s_i| - \frac{|o_i|}{2}) - \max_{i<n}(|s_i| - |o_i|) \\
&\leq OPT(\mathcal{S}) + (|s_{\arg\max_{i<n}(|s_i|-\frac{|o_i|}{2})}| - |\frac{o_{\arg\max_{i<n}(|s_i|-\frac{|o_i|}{2})}}{2}|) \\
&-(|s_{\arg\max_{i<n}(|s_i|-\frac{|o_i|}{2})}| - |o_{\arg\max_{i<n}(|s_i|-\frac{|o_i|}{2})}|) \\
&= OPT(\mathcal{S}) + \frac{|o_{\arg\max_{i<n}(|s_i|-\frac{|o_i|}{2})}|}{2}
\end{aligned}
\tag{5}
$$

□

Note that the selection of retention methods is determined by the number of overlapping questions and the number of sets. Empirical results indicate that that when the number of sets exceeds 60 and the average number of overlapping questions per set is greater than 3, the runtime of the DP-based method exceeds one hour. In this case, the heuristic method is more efficient. Otherwise, the DP-based method is better, which yields more effective results.

Version 2:

To this end, we propose a heuristic method which is shown in Algorithm 2. We first sort $\mathcal{S}$ in descending order based on $|s_i|$. If two sets have the same number of questions, the set with a smaller $|o_i|$ is ranked higher (line 1). The rationale is to prioritize removing overlapping questions from sets with larger $|s_i|$. When the number of questions in the sets (i.e., $|s_i|$) is equal, the set with fewer overlapping questions should be removed first. Next, we iteratively remove overlapping questions. In each iteration, we traverse each set s_i sequentially and remove one overlapping question from it (lines 3-8). Afterward, the sets are re-sorted (lines 9-13). The process repeats until no overlapping questions remain in $\mathcal{S}$, at which point the sets are returned (line 15). Finally, the time complexity is reduced

to $O(Overlap(\mathcal{S})n + nlogn)$, where $Overlap(\mathcal{S})$ indicates the total number of overlapping questions in $\mathcal{S}$.

Theorem 3. *The heuristic retention method is optimal when each set has overlapping questions with only one other set.*

3.3.2 Packing demonstrations and questions. In stage 3, we combine sets produced in stage 2 into groups with the objectives to minimize the number of groups and at the same time satisfy the *GL constraint*. This problem can be reduced to the well-known bin packing problem (BPP) [20], in which items of different sizes must be packed into a finite number of bins, each with a fixed given capacity. The goal is to minimize the number of bins used. We map each set s_i as the item, each group as the bin, and τ_2 as the capacity of the bin. Given this mapping, we can reduce BPP to this problem. While BPP is NP-hard, we utilize the first-fit bin packing algorithm [15] to implement it, which guarantees a bounded deviation from the optimal solution. The approximation ratio of this approximation algorithm is 1.7 [15].

3.4 Time complexity of the approximation method in OBP

In this section, we analyze the time complexity for the approximation method in OBP. Specifically, our approximation method contains two steps, namely, *QueClt* and *CltSplt*.

For the *QueClt* step, given N questions, it runs in time $O(N^2)$, i.e., the runtime for the approximation algorithm of correlation clustering [8].

After the *QueClt* step, we assume that a number of C clusters are produced. For each cluster with k questions and m demonstrations, it is input into the *CltSplt* step. This step involves three stages. Stage 1 leverages weighted set cover to select demonstrations. Note that each demonstration corresponds to one set, and the total number of elements equals the number of questions in the cluster, i.e. k. Thus, its time complexity is $O(mk)$, i.e., the runtime for approximate set cover algorithm [29]. Note that n sets are produced by Stage 1, and subsequently iare nput into Stage 2. Stage 2 includes a heuristic algorithm (i.e., Algorithm 1) to balance the number of questions between sets. This algorithm first sorts n sets, which takes $O(nlogn)$ time. Then, it iterates over all n sets. For each set, it remove $|o_i|$ questions. Therefore, the runtime for this step is $O(n|o_i|)$. As a result, the overall time complexity of Algorithm 1 is $O(nlogn+n|o_i|)$. Then, the sets without overlapping questions input into Stage 3. Stage 3 exploits the first-fit bin packing [15] to complete the final grouping, whose runtime is $O(nlogn)$. Thus, the time complexity of the *CltSplt* step is $O(mk + n|o_i| + nlogn)$ for each cluster.

In conclusion, the time complexity of the overall algorithm is $O(N^2 + C(mk + n|o_i| + nlogn))$.

3.5 Optimization for Efficient Computation

In this section, we propose two optimization strategies to further speed up the computation.

3.5.1 Pruning ineffective demonstrations . The demonstration pool may comprise a large number of demonstrations, causing expensive computations. Intuitively, we can prune the demonstrations that are not affinitive to any question in the batch $\mathcal{Q}$. Let the remaining demonstrations be referred as *valid* demonstrations. Meanwhile, we can rule out those valid demonstrations that are *dominated*, as given in the Lemma 1 below. Here, we define that a demonstration d_a is dominated by a demonstration d_b (denoted as $d_a \prec d_b$), if $cov(d_a) \subset cov(d_b)$ and $c_{d_a} \geq c_{d_b}$.

Lemma 1. *Pruning Dominated Demonstrations. Given two valid demonstrations d_a and d_b, d_a can be safely pruned if $d_a \prec d_b$ holds.*

3.5.2 Reducing affinity computations. Our method requires pair-wise aff_p computations for questions, and pair-wise aff_d computations for questions and demonstrations, which is time-consuming especially when the affinity calculation is performed over high-dimensional embeddings. In the following, we propose an optimization method to reduce the number of computations by means of triangle inequality. First, we give the formal definition of triangle inequality property:

Definition 3 (Triangle Inequality Property). *Let the distance metric between two points a and b be dist(a,b). For any three points a, b, and c, we call the distance metric satisfying the triangle inequality if $dist(a,c) \leq dist(a,b) + dist(b,c)$.*

Note that most common metrics, e.g., Euclidean distance and Cosine similarity, all satisfy this property. Through the computation of aff_p, for each question q_i, we need to obtain questions whose affinity with q_i is below τ_0, denoted by $N_{\tau_0}(q_i)$. We have the following lemma:

Lemma 2. *For any two questions $q_i, q_j \in \mathcal{Q}$, and any question q_r: $dist(q_i,q_r) - dist(q_j,q_r) > \tau_0 \Rightarrow q_i \notin N_{\tau_0}(q_j)$ and $q_j \notin N_{\tau_0}(q_i)$.*

We can use Lemma 2 to reduce some computations for aff_p. Specifically, according to Lemma 2, questions q_i, q_j and q_r form a triangle. If $dist(q_i,q_r) - dist(q_j,q_r) > \tau_0$, we can directly determine that $dist(q_i,q_j) > \tau_0$ without further affinity computations. The designed efficient method is shown in Algorithm 3. First, we select a pivot question q (e.g., the first question), and then calculate the affinity between q and all other questions (line 1). For each question q_i except q, we use the Lemma 2 and binary search to filter questions whose affinity with q_i is larger than τ_0 and then calculate the affinity for remaining questions (lines 2-6). For the computation of aff_d, the procedure is similar (lines 7-12).

4 EVALUATION

In this section, we present the experimental evaluation of our method and analyze its performance against the state-of-the-art baselines.

4.1 Experimental Setup

We implement the proposed framework in Python 3 and use GPT-3.5-Turbo, GPT-4o, GPT-4-Turbo of OpenAI [33], Claude 3 Opus of Anthropic [10] and Qwen-Long of Aliyun [34] as our backend LLMs. Meanwhile, our MILP solver is GUROBI [22]. For simplicity, we use OBP to represent the approximation method OBP-Approx when the context is clear.

Datasets. We evaluate our OBP framework using three representative tasks in data management, i.e., entity resolution (ER), data transformation (DT) and data generation (DG). In the following, we detail the corresponding datasets.

Algorithm 3 RAC

Input: A pivot question q, a batch of questions Q, all demonstrations $\mathcal{D}$, threshold τ_0 and τ_1.
Output: aff_p and aff_d.

Calculate $aff_p(q,\_)$=$aff_p(q,q_i), \forall q_i \in Q$ and sort them.
for q_i in Q **do**
  s=BinarySearch($aff_p(q,\_)$, $aff_p(q,q_i)+\tau_0$, >).
  e=BinarySearch($aff_p(q,\_)$, $aff_p(q,q_i)-\tau_0$, <).
  aff_p=$aff_p(Q_s^e, q_i)$.
end for
Calculate $aff_d(q,\_)$ =$aff_d(q,d_j), \forall d_j \in \mathcal{D}$ and sort them.
for d_j in $\mathcal{D}$ **do**
  s=BinarySearch($aff_d(q,\_)$,$aff_d(q,d_j)+\tau_1$, >)
  e=BinarySearch($aff_d(q,\_)$,$aff_d(q,d_j)-\tau_1$, <)
  aff_d=$aff_d(\mathcal{D}_s^e, d_j)$
end for
Return aff_p and aff_d.

(1) Entity Resolution (ER). We use the well-adopted datasets from Magellan [13], which are from a variety of domains, such as products, software, etc. They include eight datasets, and we split each dataset into train, validation, and test sets according to the existing ER studies [28, 36]. Each sample has two items with attributes and corresponding values. In this task, LLMs are used to determine whether these two items refer to the same entity.
(2) Data Transformation (DT). It is a critical task in the data management domain that involves converting data from its original format into another format suitable for further processing, such as data cleaning and data integration [11, 24]. We use the datasets introduced by [2, 43], which are collected from real-world tables. Each sample contains the source format and the target format, e.g., "Norm Adams" and "n.adams". In this task, LLMs are used to generate the target format given the source format.
(3) Data Generation (DG). We use the dataset from ATLAS [37] to evaluate our method, which is to generate code assertions for JAVA code. Each sample contains the context of a unit test followed by the instruction, and the LLMs need to generate a single assertion for the given test method. In this paper, we generate four datasets from ATLAS by the category of assertions, i.e., AssertTrue, AssertNotNull, AssertEquals, and AssertThat. For example, we put all the samples whose assertion method is AssertTrue into one dataset.

Baselines. We compare OBP with LLM-based baselines, which include batching baselines (i.e., Batcher and 1demo) and the non-batching baseline (i.e., Single). Furthermore, we also compare OBP with non-LLM state-of-the-arts (SOTAs) for each task. The brief introduction of these baselines is as follows:
LLM-based baselines: We compare with 3 LLM-based methods.
(1) Batcher [16]. They introduce a batch prompting framework that consists of two modules, question batching and demonstration selection. Specifically, they first cluster the questions, and then select one question from each cluster separately, to generate a group (in their experiments, all the group sizes are set to 8 by default). After generating groups, they leverage set cover to select demonstrations for each group.

Table 1: Comparison between OBP and baselines on 14 datasets. The LLM-based results are under GPT-3.5-Turbo. The unit of Accuracy is %, and the Cost refers to token counts (k). Note that the best results are bolded and the second best results are underlined.

Dataset	OBP		Batcher		1demo		Single		non-LLM
	Acc	Cost	Acc	Cost	Acc	Cost	Acc	Cost	Acc
FZ	**95.2**	**23.7**	**95.2**	30.2	90.5	38.2	90.0	53.2	91.7
Beer	**92.3**	**9.5**	88.9	11.9	85.7	14.2	88.0	20.3	90.4
iA	**94.1**	**7.9**	92.3	10.4	92.3	11.8	87.7	19.4	82.1
DA	93.0	**400.3**	**93.5**	522.7	93.0	612.5	91.3	766.9	87.5
WA	**83.7**	**223.7**	82.8	280.3	83.0	341.4	75.8	475.2	74.8
AB	**82.0**	**127.2**	79.5	171.2	79.4	208.0	80.7	323.1	73.1
AG	57.8	**175.5**	57.4	193.7	**59.4**	231.9	49.4	346.5	57.8
DS	**88.5**	**471.8**	78.5	501.6	77.2	588.5	78.3	778.7	85.7
WTS	**87.7**	**22.7**	87.6	32.7	83.7	51.9	82.1	136.5	61.3
KBWT	**97.2**	**2.2**	92.8	3.4	96.6	5.1	91.1	17.5	33.3
ATr	**88.8**	**87.4**	78.3	91.3	76.7	107.8	82.7	112.6	72.7
ANN	**92.1**	**174.6**	78.7	190.6	91.5	206.7	82.6	225.3	90.8
AE	**87.7**	**117.6**	79.7	133.8	87.4	175.2	79.4	176.8	73.2
ATh	**81.3**	**25.0**	61.3	35.6	74.7	44.7	78.7	46.4	63.9

(2) 1demo [16]. In this baseline, they use the same method in Batcher to group questions. Then, they select the most similar demonstration for each question in the group. If the most similar demonstration of a certain question has already been selected, they choose the next most similar demonstration. Thus, the number of questions is the same as the number of demonstrations in each group.
(3) Single [32]. This method asks LLMs to resolve one question at a time, with one demonstration that is most similar to the question.
Non-LLM Baselines. We compare against the non-LLM SOTA methods for each task.
(1) For entity resolution, we compare against Ditto [28], the current SOTA deep learning-based approach which finetunes BERT [12].
(2) For data transformation, we compare against DTT [11], a transformer based example-driven solution.
(3) For data generation, we compare against ATLAS [37], a neural machine translation-based approach.

Metrics. We conduct the evaluation on the following metrics.
(1) Cost. This metric measures how much an approach pays for calling the API of a certain LLM, which is the main metric we consider. It is worth noting that the LLM cost is charged by input tokens. Hence, we define the Cost metric as the number of input tokens in this paper.
(2) Accuracy. Based on the nature of the task, we adopt different accuracy metrics for ER, DT and DG. For ER, we use $F1$ score to measure the accuracy of an approach, which is a common metric for existing ER works [16, 28, 36]. For DT and DG, we use exact match accuracy, which denotes the percentage of numbers where the predicted output matches lexically with the expected output.
(3) Execution Time. We evaluate the efficiency of an approach by the execution time, which includes the time of processing (i.e., group generation) and calling LLMs.

4.2 Overall Efficacy Comparison

In this section, we compare our OBP framework with both LLM-based baselines and non-LLM SOTAs on 14 real-world datasets.

4.2.1 Comparison between OBP and LLM-based baselines. The comparison results are shown in Table 1. For Batcher, we use the default group size 8 in ER and DT, while the group size in DG is 2, which performs the best. Furthermore, when computing aff_q, we use the reciprocal of the Euclidean distance as the affinity metric for the ER task, which is consistent with Batcher [16]. Meanwhile, we use the Euclidean distance as the affinity metric for DT and DG tasks, as it yields the best performance.

Cost. Table 1 shows that our method incurs the least cost over all 14 datasets. Compared with Batcher, OBP can reduce the cost by up to 35% (i.e., KBWT). The cost of Batcher is the local optimum, and our method OBP can reach the global optimum approximately. To be specific, our method can adaptively adjust the number of groups and the number of questions per group based on data. Except for OBP, Batcher outperforms 1demo on cost. This is because for each group, the number of demonstrations of Batcher may be less than the number of questions due to demonstration sharing, and 1demo selects a fixed number of demonstrations, which is the same as the number of questions in the group.

Meanwhile, single prompting method (i.e., Single) incurs higher cost than batch prompting methods. Even for 1demo, the batch prompting method with the highest cost, it can reduce the cost by up to 30% compared to Single. While the demonstration number of Single and 1demo are the same (each question corresponds to a demonstration), Single treats each question as a group to query LLMs, resulting in higher task description cost.

Accuracy. From Table 1, we can observe that OBP performs the best for almost all datasets, and only performs the second best with only a slight difference in 2 datasets out of 14 datasets under GPT-3.5-Turbo, which indicates that our method can produce higher-quality results. Batcher and 1demo both use a fixed group size, which may result in some questions being put in the same group inappropriately. In contrast, our approach is able to perform adaptive grouping, leading to more accurate results.

Additionally, we observe that batch prompting methods (i.e., OBP, Batcher and 1demo) outperform single prompting method (i.e., Single), which validates the idea of batch prompting in leveraging LLMs for data management. It not only significantly reduces the cost, but also improves the result accuracy since more related context is provided in a batch to facilitate the reasoning ability of LLMs.

4.2.2 Comparison between OBP and non-LLM SOTAs. In this section, we compare OBP with non-LLM SOTA methods of each task, and the results are shown in the "non-LLM" column of Table 1. The results show that LLM-based methods outperform non-LLM SOTA methods on almost all datasets, and a significant performance gap can be observed on some datasets. For example, OBP under GPT-3.5-Turbo achieves 97.2% accuracy on dataset KBWT whereas the non-LLM SOTA only reaches 33.3%. We find that non-LLM baselines generally perform well on some datasets in simple tasks (e.g., the dataset FZ in entity resolution) but performs poorly on more challenging ones (e.g., the dataset KBWT in data transformation).

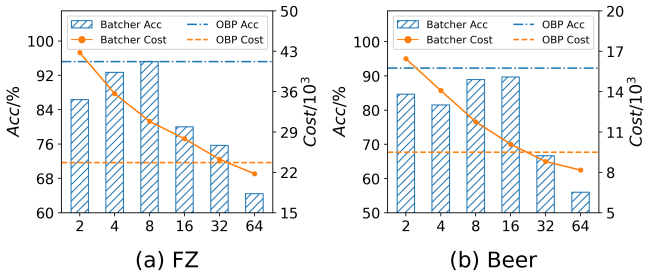


Figure 5: Comparison between OBP and Batcher with different group sizes on the entity resolution task. The group sizes of Batcher are varying from 2 to 64.

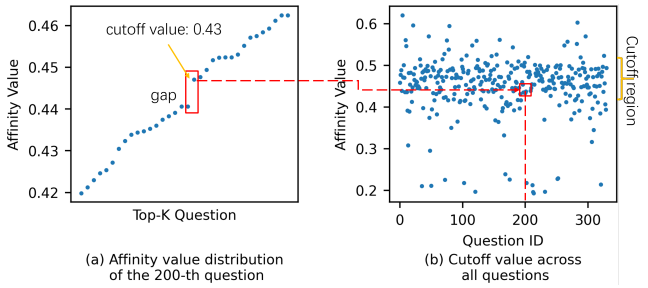


Figure 6: Affinity distribution of hyperparameter τ_0 on the ANN dataset.

Additionally, the LLM-based method is few-shot, requiring little training data (i.e., demonstrations).

4.2.3 Comparison between OBP and Batcher with different group sizes. Batcher clusters questions with a fixed group size (i.e., the number of questions in each group is constant). The default group size is suggested to be 8 [16]. In this section, we compare OBP with Batcher with group sizes varying from 2 to 64 to study the robustness of our method. We conduct experiments on entity resolution, which is the target task of Batcher. Due to space constraint, Figure 5 only shows the results on FZ and Beer datasets. We observe similar results on other datasets. We find that OBP consistently outperforms Batcher in terms of accuracy regardless of the group size, demonstrating its superior performance. Meanwhile, OBP incurs relatively low cost. For Batcher, we can observe that group size significantly affects the accuracy, and the optimal group size varies for different datasets. For instance, the optimal group size of dataset FZ is 8. But for dataset Beer, it is 16. In summary, OBP can adaptively cluster affinitive questions in one group, enhancing the overall accuracy while ensuring low cost.

4.3 Tuning and Sensitivity Analysis of Hyperparameters

Our framework involves four hyperparameters: $\tau_0, \tau_1, \tau_2, \tau_3$. In this section, we first present the guidance for tuning these hyperparameters, followed by a comprehensive sensitivity analysis for each one.

Hyperparameter τ_0. It measures the affinity between questions. Thus, τ_0 needs to distinguish between relevant and irrelevant questions, ensuring that questions within the same group are affinitive

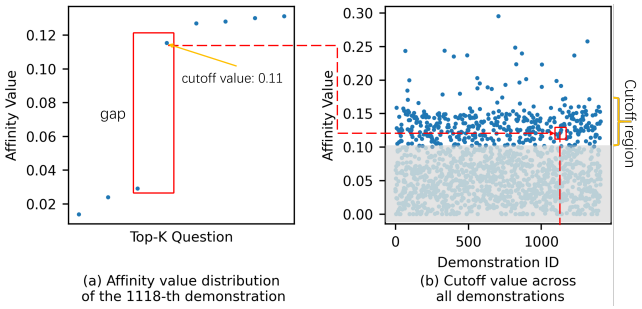


Figure 7: Affinity distribution of hyperparameter τ_1 on the ANN dataset.

to each other. To determine the appropriate value for τ_0, we have the following guidance: we first calculate the affinity values of each question with all other questions and sort them in ascending order Next, we identify the gap for each question, which is the largest difference between adjacent affinity values, and we denote the cutoff value as the bigger value within the gap. For example, Figure 6(a) shows the distribution of affinity values for a sample question from the ANN dataset in ascending order, along with gaps and cutoff values. Specifically, the cutoff value of the 200-th question is 0.43. Finally, we determine the appropriate range of τ_0 through cutoff values across all questions. For example, we plot cutoff values for a batch of questions from the ANN dataset in Figure 6(b). As observed, in the ANN dataset, the cutoff values typically lie between 0.4 and 0.5. This indicates that the affinity values within this region can separate most of relevant questions from irrelevant ones. Thus, values in this region can be considered as a suitable choice for τ_0, and we denote this region as the cutoff region. To verify this guidance on determining appropriate τ_0, we conducted a sensitivity analysis on τ_0, as depicted in Figure 8(a). From the figure, we observe that τ_0 performs best in terms of accuracy within the cutoff region. For example, in the ANN dataset, the optimal accuracy is achieved when τ_0 lies within the cutoff region, i.e., between 0.4 and 0.5. Furthermore, we observe that τ_0 is not sensitive to accuracy within this region. For values smaller or larger than this region, the accuracy shows a significant decline. In the meanwhile, we observe that the cost decreases as τ_0 increases. This is because a smaller τ_0 implies fewer questions per group, resulting in a larger number of groups and thus increasing the task description cost. For a new dataset, users first identify the appropriate range of τ_0 values based on the above guidance. If the users aim to reduce cost, they can choose the maximum value within the cutoff region as τ_0. Otherwise, they can tune τ_0 within this region, ultimately choosing the one that achieves the highest accuracy. In our implementation, we set τ_0 to the maximum value within the cutoff region to reduce cost.

Hyperparameter τ_1. It measures the relationship between questions and demonstrations. Thus, τ_1 is used to distinguish between questions that are relevant and irrelevant to a demonstration. To identify the appropriate value for τ_1, we have the following guidance: we calculate the affinity values of questions for each demonstration and sort them in ascending order. Then, we identify the gap within the first few affinity values, as a demonstration cannot cover too many questions. Then, we determine the cutoff value for each demonstration. Figure 7(a) illustrates a sample demonstration from the ANN dataset. Specifically, the cutoff score of 1118-th demonstration of the ANN dataset is 0.11. Finally, similar to τ_0, We determine the appropriate range of τ_1 by inspecting the distribution of cutoff values across all demonstrations, which is shown in Figure 7(b). Note that when τ_1 becomes particularly small, it leads to a lack of relevant demonstrations, causing our framework to fail. For example, in the ANN dataset, when τ_1 drops below 0.1, the algorithm fails to return a result, which is highlighted in the figure with a shadow. From the figure, we observe that the cutoff values in the ANN dataset typically fall below 0.16. This suggests that the affinity values within this region (i.e., the cutoff region) are effective in distinguishing relevant questions from irrelevant ones for most demonstrations. To verify this tuning guidance, we performed a sensitivity analysis on τ_1, as shown in Figure 8(b). We find that the best performance is achieved when τ_1 is set to values within the cutoff region. For instance, in the ANN dataset, when τ_1 is within the cutoff region, i.e., less than 0.16, the corresponding accuracy significantly outperforms that of other values. In terms of costs, we find that cost decreases as τ_1 increases. This is because when τ_1 becomes large, each demonstration can cover more questions, so fewer demonstrations are needed to cover all the questions within a group. Similar to τ_0, for a new dataset, if the users consider cost, they can select the maximum value within the cutoff region as τ_1. Otherwise, they can tune τ_1 within the cutoff region, selecting the one that achieves higher accuracy. In our implementation, we assign the maximum value within the cutoff region to τ_1 in order to reduce cost.

Hyperparameter τ_2. This hyperparameter constrains the group length, it directly affects the group size. In our setting, the group size includes both questions and demonstrations. Considering that the length of questions and demonstrations varies across tasks, we model τ_2 as $\alpha \times q_{len}$, where q_{len} is the average question length for each task, capturing task-specific characteristics. Then, we vary α to examine its impact on accuracy, the result of which is shown in Figure 8(c). From the figure, we find that when the value of α is around 15, it performs best. For example, the average question length in the ANN dataset is 200, making the optimal τ_2 value 3000. In terms of costs, we observe that when τ_2 becomes larger, the cost decreases. This is because a group can accommodate more questions as τ_2 becomes larger, thereby reducing the number of groups. For a new dataset, if users focus on cost, they can adjust τ_2 starting from $\alpha = 15$ and gradually increase it. Otherwise, they can tune τ_2 around $\alpha = 15$ and selecting the value that yields the best accuracy. In our implementation, we choose the first option to reduce cost.

Hyperparameter τ_3. It limits the maximum number of questions that each demonstration can cover. This hyperparameter is typically set to a relatively small value. If it is set too large, it may lead to groups with many questions but few demonstrations, which negatively affects the reasoning capabilities of LLMs. The corresponding evaluation is shown in Figure 8(d). We observe that τ_3 performs best when it is set to 3 or 4. This suggests that the chosen values for τ_3 are general and can effectively adapt to different datasets. Taking cost into consideration, users can choose 4 as the value for τ_3. This is because when τ_3 is larger, one demonstration can cover more questions, thereby reducing the overall cost.

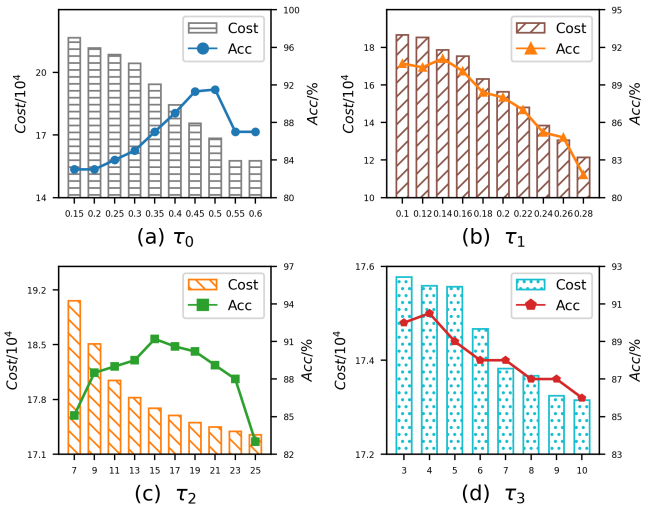


Figure 8: Sensitivity analysis of hyperparameters on the ANN dataset.

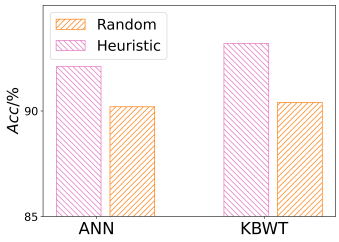


Figure 9: Comparison between random determination and our heuristic method.

4.4 Cost-accuracy trade-off analysis

In this section, we analyze the cost-accuracy trade-off under varying hyperparameters, with the relevant results shown in Figure 8.

For τ_0, Figure 8(a) shows that cost decreases as τ_0 increases, since bigger τ_0 leads to fewer groups, requiring fewer demonstrations. Accuracy initially improves, then declines as τ_0 increases. When τ_0 is too small, most groups contain fewer demonstrations, which can impact the reasoning capability of LLMs. Conversely, a large τ_0 leads to random groupings and poor performance. For τ_1, Figure 8(b) shows that cost decreases as τ_1 increases. This is because that smaller τ_1 requires more demonstrations. Accuracy drops with larger τ_1 due to fewer relevant demonstrations per group. For τ_2, Figure 8(c) shows that a larger τ_2 reduces cost by increasing number of questions per group. Accuracy initially increases, then decreases as τ_2 increases. Small τ_2 leads to fewer demonstrations per group, affecting LLM reasoning. Conversely, large τ_2 degrades performance due to the larger number of questions within each group. For τ_3, Figure 8(d) shows that cost decreases as τ_3 increases. As for accuracy, larger τ_3 reduces the number of demonstrations, limiting domain knowledge and affecting LLM performance.

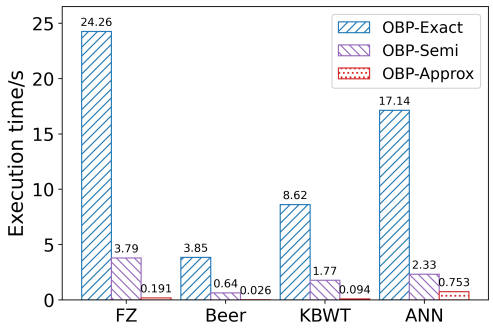


Figure 10: Evaluation on execution time of the three OBP variants.

4.5 Comparison of the random and heuristic retention methods

In this section, we compare the random retention approach with the heuristic method (i.e., Algorithm 1), the results of which are presented in Figure 9. Note that the random retention method refers to randomly removing overlapping questions from the sets. As shown, our method outperforms the random method, achieving the highest accuracy on both the ANN and KBWT datasets. This demonstrates the effectiveness of our approach and validates the idea of balancing the number of questions covered by each demonstration, ultimately leading to improved performance.

4.6 Ablation study on OBP

Recall that the OBP framework offers three versions of solutions, namely OBP-Exact, OBP-Semi and OBP-Approx. In this section, we conduct evaluation on the three OBP variants in terms of effectiveness and efficiency.

The comparison results on the execution time are shown in Figure 10. Note that after correlation clustering, the cluster splitting methods for each cluster can be executed concurrently. So the execution time for OBP-Semi and OBP-Approx in Figure 10 refers to the slowest time among all clusters. From the results, we can see that compared to OBP-Exact, the execution time of OBP-Semi can be reduced by around 85%. Take the dataset FZ as an example, it takes 24.26s with OBP-Exact, but only 3.79s with OBP-Semi. Generally, given n questions, OBP-Semi can eliminate n boolean variables and $O(n^2)$ constraints in Equation 2, so as to accelerate computation with the MILP solver. Furthermore, OBP-Approx, which replaces the computationally expensive MILP solver with our proposed approximation algorithm, achieves the shortest execution time. For example, the runtime of OBP-Approx is only 5% of OBP-Semi on the dataset FZ.

We summarize the comparison results on cost and accuracy of the three OBP variants in Table 2. From the results, we observe that OBP-Exact achieves the best performance for all datasets, due to its optimal exact solution. However, it incurs a severe execution delay as shown in Figure 10, making it impractical for real-world applications. On the contrary, OBP-Approx is efficient, and achieves approximately optimal performance in terms of accuracy and cost. Specifically, compared with OBP-Exact, the accuracy of OBP-Approx only decreases by up to 1.6%, and the increase of cost is

Table 2: Evaluation on cost and accuracy of the three OBP variants. Note that cost refers to token counts (k), and the best results are bolded.

Dataset	OBP-Exact		OBP-Semi		OBP-Approx	
	Acc (%)	Cost (k)	Acc (%)	Cost (k)	Acc (%)	Cost (k)
FZ	**94.1**	**12.8**	**94.1**	12.9	**94.1**	13.0
Beer	**91.4**	**24.4**	91.3	24.6	91.2	24.6
KBWT	**97.0**	**8.6**	96.3	8.8	**97.0**	8.8
ANN	**97.6**	**6.1**	96.5	6.3	96.0	6.4

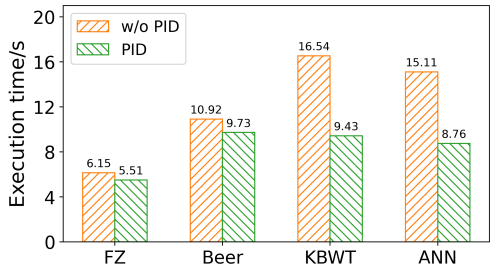
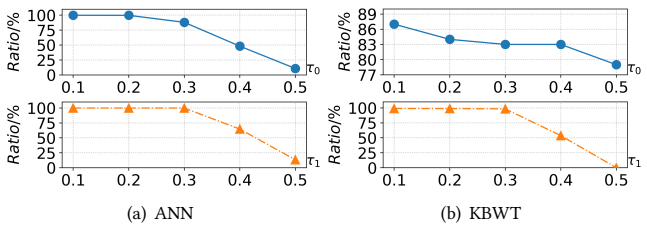


Figure 11: Evaluation on PID.



(a) ANN (b) KBWT

Figure 12: Evaluation on RAC.

nearly negligible. In summary, OBP-Approx is effective and efficient for LLM batch prompting in data management.

4.7 Evaluation on Computation Optimizations

In Section 3.5, we propose two pruning strategies to reduce unnecessary computations, namely Pruning Ineffective Demonstrations (PID) and Reducing Affinity Computations (RAC). In this section, we evaluate these two optimizations.

4.7.1 PID evaluation. In Figure 11, we compare the execution time with and without PID. The execution time here refers to the computation time for batch prompting, excluding the time of calling LLMs. We can observe that PID can reduce computations by up to 42%. This is because PID filters dominated demonstrations, causing Equation 2 to have fewer variables. In summary, PID can effectively filter out unnecessary demonstrations, thereby enhancing efficiency.

4.7.2 RAC evaluation. In this section, we evaluate the efficiency of RAC and present the experimental results in Figure 12.

In the figure, "Ratio" shows the percentage of computation reduction after applying RAC. Additionally, since RAC is influenced by the threshold τ_0 and τ_1, we thus evaluate the efficiency with respect to different values of τ_0 and τ_1. From the figure, we find that when τ_0 or τ_1 are small, this method can filter out the vast majority of computations, i.e., over 99.9%. This is because when these thresholds are small, only a small fraction of values of aff_p and aff_d are below the thresholds, so most of the values satisfy the Lemma 2. The reduction ratio decreases as the threshold increases. When the threshold is larger than 0.3, the ratio drops sharply. This is because the number of calculations for aff_p and aff_d below the thresholds is increasing. In conclusion, our optimization method helps reduce the computations of both aff_p and aff_d.

5 RELATED WORK

This work is related to two broad lines of research, i.e., LLMs prompting and LLMs for data management.

LLMs prompting. The main trend of using LLMs is through prompting [6, 38]. This approach has changed the research paradigm. LLMs only need to be given a suite of appropriate prompts. The major advantage of LLMs is that they do not need model training or fine-tuning, which is efficient for deployment and generally applicable to the majority of downstream tasks. Commonly, prompt can be divided into hard prompt [39] and soft prompt [40] respectively. Hard prompt means the same context is used for all questions, such as the task description and fixed demonstrations. Soft prompt typically refers to selecting or generating prompts dynamically with respect to the questions. For example, recent works try to retrieve demonstrations similar to the question for improving accuracy [16, 19, 32]. In this paper, we focus on batching questions under the soft prompt setting.

LLMs for data management. LLMs have recently achieved record-breaking results in various real-world applications. Recent works [17, 30] start to explore the possibility of applying LLMs in data management tasks, for example, entity resolution [30], data transformation [25], data generation [32], and so on. While LLMs demonstrate promising performance on these tasks, there are still some challenges. For example, for some data management tasks, they tend to process a large number of questions, which can be very costly for LLMs. It therefore calls for developing a cost-effective method. In this paper, we propose optimizations to reduce cost for LLMs through batching questions and demonstrations without compromising accuracy. We next elaborate more on three representative data management applications: entity resolution, data transformation and data generation.

In this paper, we focus on batch prompting for these applications in order to reduce the LLM cost while achieving higher accuracy.

6 CONCLUSION

In this paper, we study the problem of batch prompting in leveraging LLMs for data management. To this end, we develop a framework named Optimized Batch Prompting (OBP) aiming to find the optimal grouping of questions and demonstrations with accuracy guarantee and minimal cost. We first formalize the batch prompting problem as a constrained optimization problem in general setting. Then, we study the hardness of this problem and demonstrate that it is NP-hard. Finally, we design efficient methods for adaptive grouping. Extensive experiments on 14 real-world datasets from three representative data management tasks confirm the superiority of

our OBP compared to the state-of-the-art LLM and non-LLM based baselines in terms of both cost and accuracy.

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