Contact-Implicit Trajectory Optimization with Chance Constraints

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I. Summary

Recent advancements in trajectory optimization in robotics have allowed researchers to generate diverse locomotion trajectories without predetermined terrain contact schedules. However, these approaches require exact models of the dynamics and the terrain, which makes executing the planned trajectory susceptible to failure due to modeling errors and uncertainties. In this work, we explicitly model uncertainties in terrain geometries as chance constraints to relax the complementarity constraints in the optimization problem. We demonstrate in simulation that our methods generate similar trajectories to the strict complementarity interpretations. Additionally, as uncertainty approaches zero, our trajectories converge to those generated with traditional contact-implicit trajectory optimization methods with exact dynamics parameters. Our study could be a step towards reasoning about terrain uncertainty in trajectory optimization.

II. Introduction

Designing safe and robust locomotion behaviors for legged robots is an important and challenging problem. Contact-implicit trajectory optimization recently gained attention for its ability to generate diverse locomotion behaviors [1]. However this method requires exact models of the terrain and the robot, which is extremely difficult to measure in real world environments. Modeling uncertainty has been approached in previous work by perturbing individual model parameters to generate an ensemble of reference trajectories [2]. In this work, we explicitly investigate uncertainty resulting from the terrain parameters and aim for following contributions:

- Develop a relaxed version of the traditional linear complementarity problem using chance constraints.
- Demonstrate chance constraints are a generalization for relaxed complementarity constraints (Figure 1a).
- Show that trajectories from our method converge to those of the traditional method as uncertainty decreases.

III. Methods

Our study assumes a normal distribution over terrain parameters and develops a corresponding complementarity problem. By developing linear Gaussian chance constraints and treating a single contact parameter as normally distributed, we recover a set of relaxed complementarity constraints. We evaluate this approach on two toy examples with uncertain terrain height: a sliding block and a five-link walker. We compare trajectories generated using various levels of terrain uncertainty to one generated with traditional contact-implicit approach.

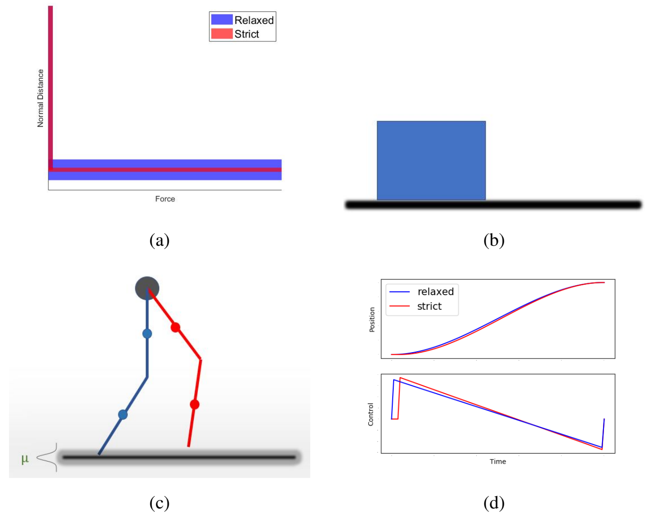

Fig. 1: Overview of the study. (a) The feasible region for normal-force and contact distance under chance constraints (blue) compared to strict constraints (red). (b,c) Example systems with contact on which the proposed method is evaluated. (d) Example solution trajectories from the block example with chance constraints (blue) and strict complementarity constraints (red).

IV. Results and Discussion

We characterized the feasibility regions of the chance complementarity constraints (Figure 1a). Chance constraints generalized the relaxed constraints in [1] and allow for terrain penetration under some circumstances. So far we have shown that chance constraints produce similar trajectories compared to strict complementarity constraints (Figure 1d). Our work represents the first time chance constraints have been incorporated in the contact model in contact-implicit trajectory optimization. We linked the relaxed complementarity problem [1] to probability theory, which could open up more robust problem modeling in trajectory optimization, as was done with state constraints [3]. In the future, we could focus on testing the robustness to variations in terrain parameters of stochastic MPC with chance constraints against traditional robust MPC approach without chance constraints.

References

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[2] I. Mordatch, K. Lowrey, and E. Todorov, "Ensemble-CIO: Full-Body Dynamic Motion Planning that Transfers to Physical Humanoids," in *Proceedings of the International Conference on Robotics and Automation (ICRA)*, 2015.

[3] A. Gazer, M. Khadiv, A. Del Prete, and L. Righetti, "Stochastic and robust mpc for bipedal locomotion: A comparative study on robustness and performance."