Can Chance-Constrained Contact Uncertainty Quantification Improve Feasibility of Robust Trajectory Optimization?

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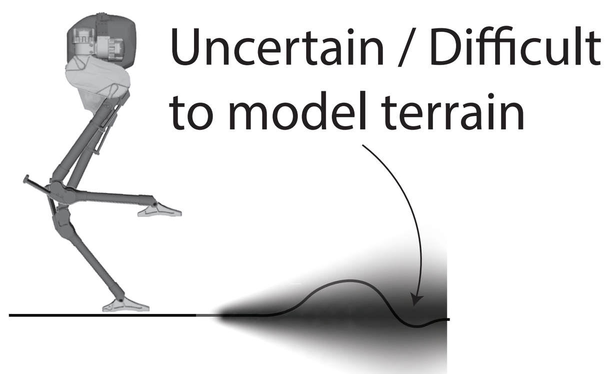
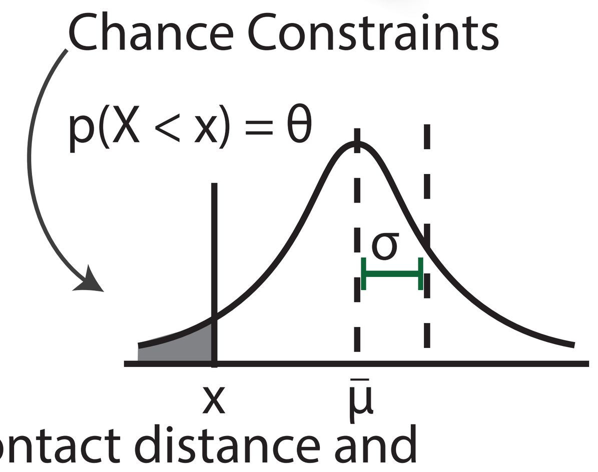
Contact Robust Trajectory Optimization

For dynamic robot locomotion, planning under uncertainty with intermittent contact is critical, as failure to make and maintain contact can be catastrophic [1].



Current robust trajectory optimization method [2] produces dynamically infeasible trajectories under high uncertainty.

Mediating between trajectory robustness and feasibility can help generate motion plans that are both satisfactory to model constraints and robust to environment uncertainty.



Objectives

1) **Design chance constraints for contact with uncertainty** in contact distance and friction coefficient.

2) Demonstrate that chance constraints, combined with a contact-sensitive objective, can **control the trade-off between robustness to contact uncertainty and contact constraint satisfaction** at fixed values of uncertainty.

Modeling Contact Uncertainty

Standard Complementarity Problem for Contact:

Normal Distance Constraint

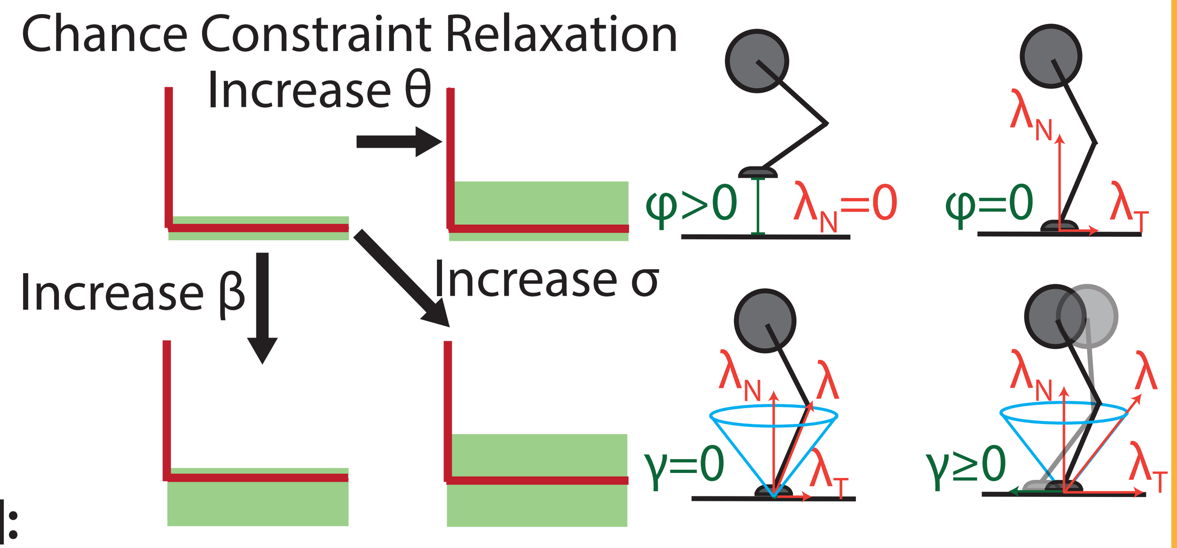
$$\phi(q) \geq 0, \quad \lambda_N \geq 0, \quad \phi(q)^\top \lambda_N = 0$$

Friction Cone and Sliding Constraints

$$\lambda_T \geq 0, \quad \gamma + J_T^\top \dot{q} \geq 0, \quad \lambda_T^\top(\gamma + J_T^\top \dot{q}) = 0$$

$$\gamma \geq 0, \quad \mu\lambda_N - \lambda_T \geq 0, \quad \gamma^\top(\mu\lambda_N - \lambda_T) = 0$$

Chance Constraint Relaxation



Chance Complementarity Constraints [3]:
Assume one of the variables is normally distributed.

$$x \perp F(x), \qquad F(x) \sim \mathcal{N}(\bar{\mu}, \sigma)$$

$$x \geq 0, F(x) \geq -\sqrt{2}\sigma\, \mathrm{erf}^{-1}(2\beta - 1),$$

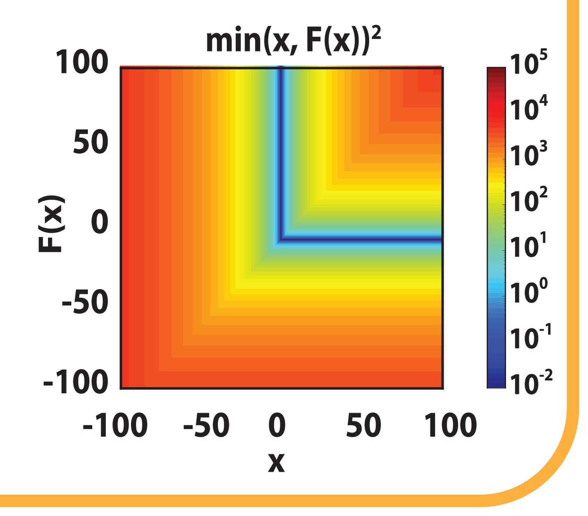
$$xF(x) \leq -\sqrt{2}\sigma\, \mathrm{erf}^{-1}(1 - 2\theta)$$

β, θ are **risk-bounded** failure probabilities between 0.5 and 1

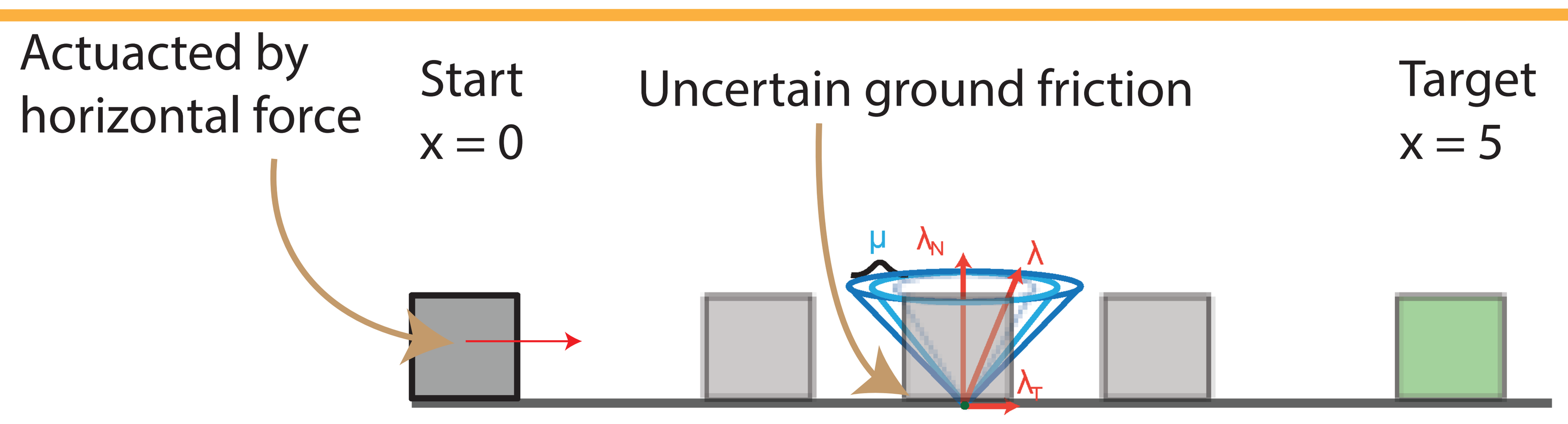
Robust Objective for Complementarity with Uncertain Parameters

Expected Residual Minimization [4]:

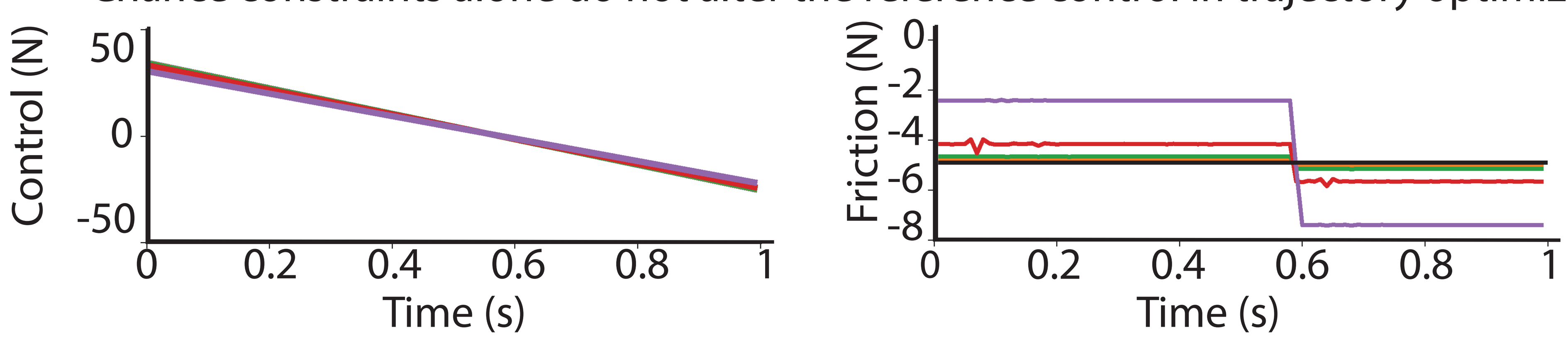
$$\mathbb{E}[\min(x, F(x))^2] = x^2 - \sigma^2(x + \bar{\mu})p(x) + (\sigma^2 + \bar{\mu}^2 - x^2)P(x)$$



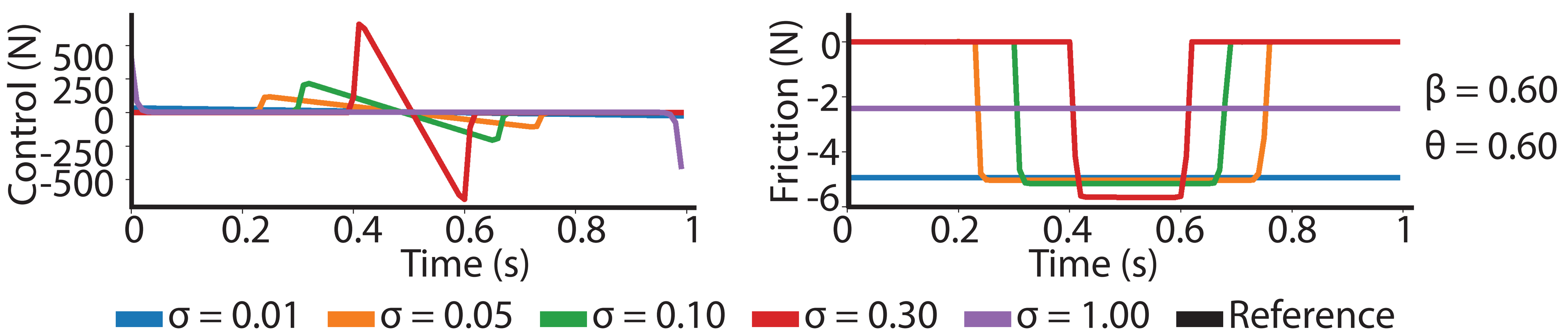
Sliding Block with Uncertain Friction



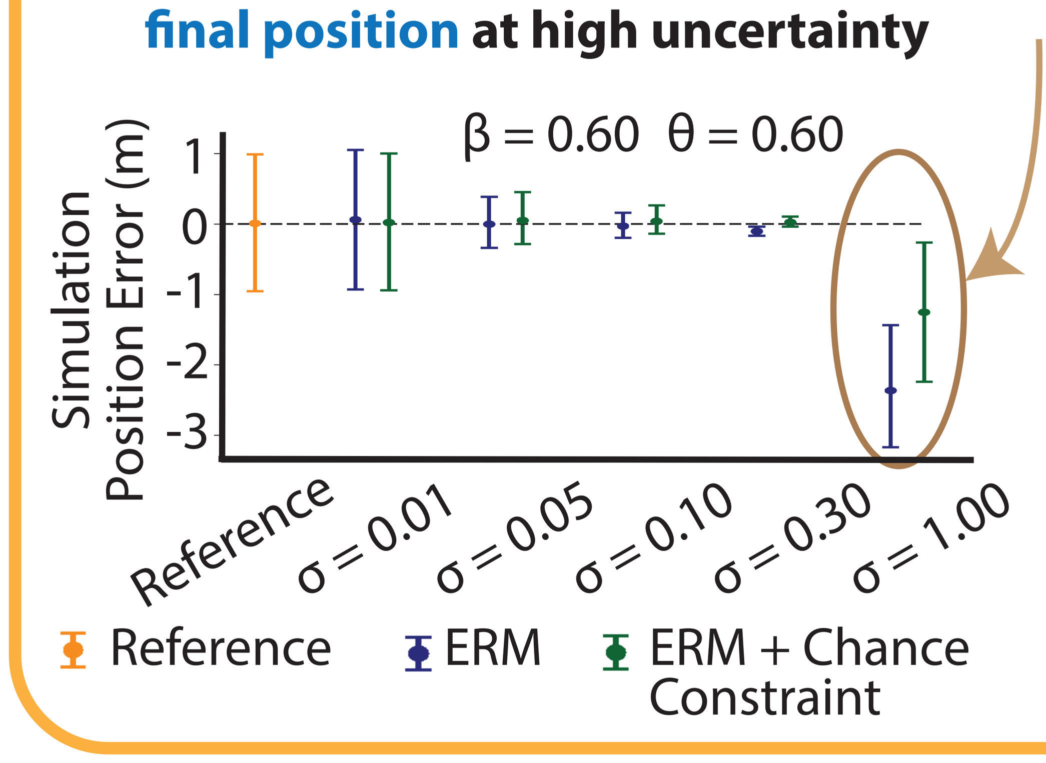
Chance constraints alone do not alter the reference control in trajectory optimization.



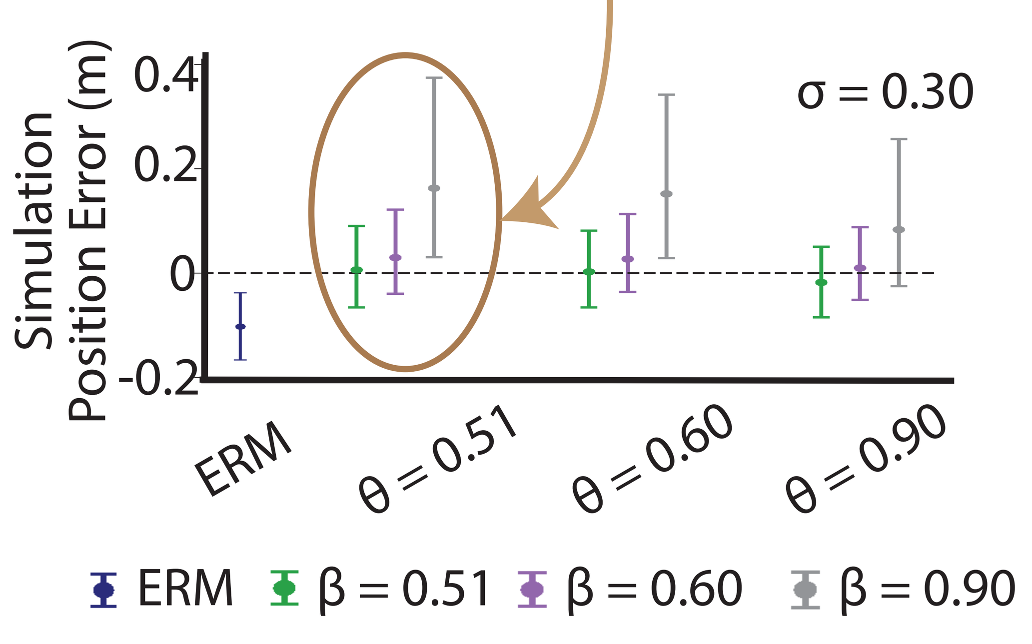
Chance constraints **combined** with ERM generates robust control trajectories.



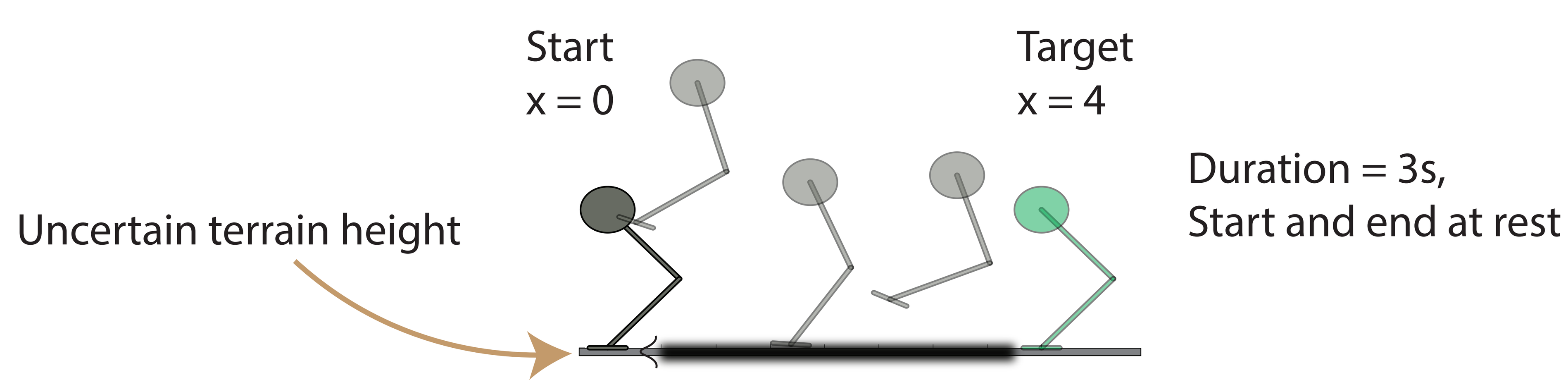
Chance constraints improve the average final position at high uncertainty

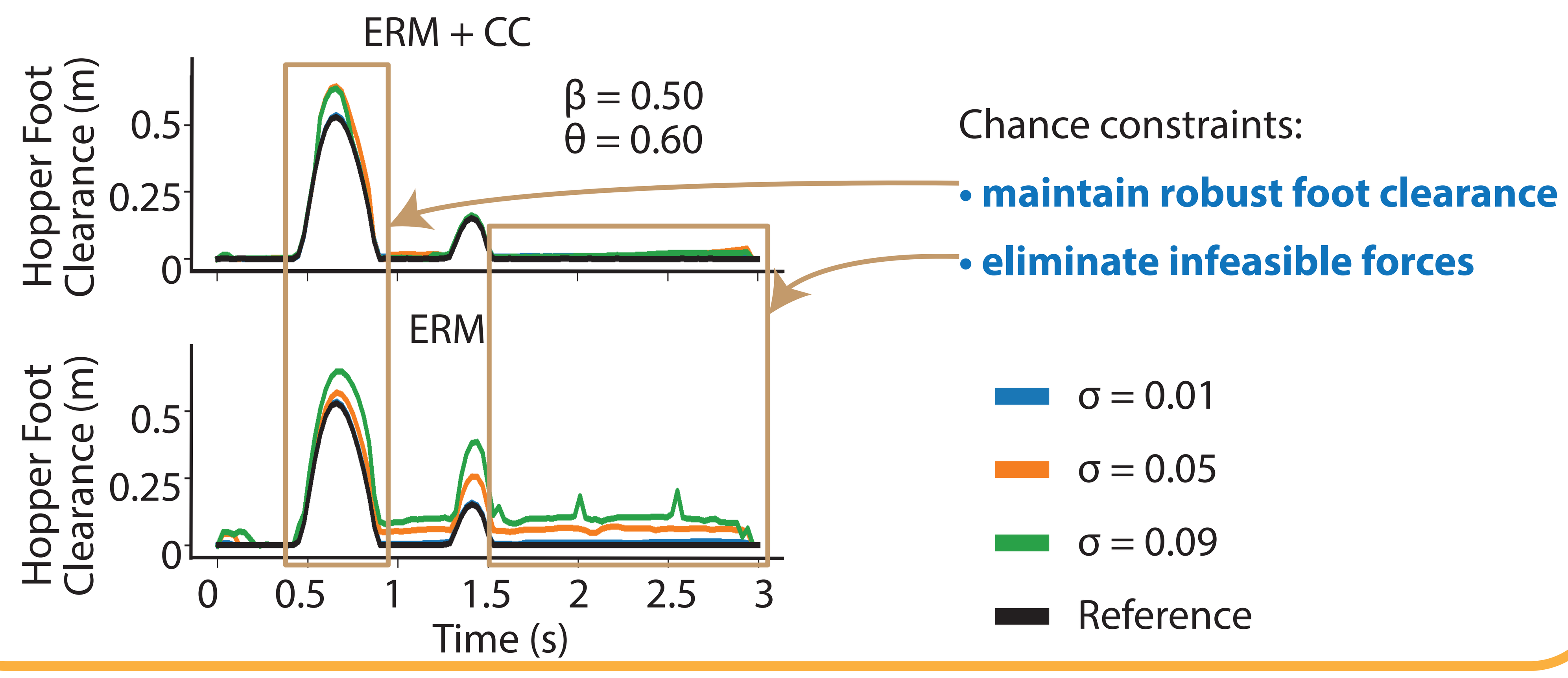


As β increases, both mean and range of the error increases.



Single Legged Hopper Over Uncertain Terrain





Discussion and Future Works

1) Designed **chance constraints for contact** with uncertainty in contact distance and friction coefficient.

2) Provided a **risk-bounded interpretation** to the relaxed chance complementarity constraints.

3) Demonstrated that the **trade-off between robustness to contact uncertainty and constraint satisfaction can be controled** using chance constraints.

Future works could include scaling up our optimization to a full-body robot in 3-D and experimentally validate our methods with physical robots. More investigations on improving algorithm efficiency could bring our approach closer to real world applications.

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References: [1] M. Posa, C. Cantu, and R. Tedrake (2014). [2] L. Drnach and Y. Zhao (2021). [3] A. Gazer, et al. (2020). [4] Y. Tassa and E. Todorov (2010).