Journal of Monetary Economics 106 (2019) 16–26

Contents lists available at ScienceDirect

Journal of Monetary Economics

journal homepage: www.elsevier.com/locate/jmoneco

Some simple bitcoin economics[☆]

Linda Schilling[a], Harald Uhlig[b,*]

[a] École Polytechnique, CREST, France
[b] Kenneth C. Griffin Department of Economics, University of Chicago, USA

ARTICLE INFO

Article history:
Received 14 December 2018
Revised 29 June 2019
Accepted 5 July 2019
Available online 8 July 2019

Keywords:
Cryptocurrency
Bitcoin
Exchange rates
Currency competition

ABSTRACT

We provide a model of an endowment economy with two competing, but intrinsically worthless currencies (Dollar, Bitcoin). Dollars are supplied by a central bank to achieve its inflation target, while the Bitcoin supply grows deterministically. Our fundamental pricing equation implies in its simplest form that Bitcoin prices form a martingale. "Mutual impatience" implies absence of speculation. Price volatility therefore does not invalidate the medium-of-exchange function. Bitcoin block rewards are not a tax on Bitcoin holders: they are financed with a Dollar tax. We discuss monetary policy implications, Bitcoin production, taxation, welfare and entry, and characterize the range of equilibria.

1. Introduction

In December 2018, the market capitalization of cryptocurrencies reached nearly 400 Billion U.S. Dollars, equivalent to 11% of M1 in the U.S.A. While central banks seek to control the price level or inflation of their traditional fiat currencies, Cryptocurrencies, however, are not necessarily controlled by central banks or central institutions. As a result, the price of cryptocurrencies such as Bitcoin has fluctuated wildly. In this paper, we consider a possibly future world, where cryptocurrencies such as Bitcoin may be increasingly used as a medium of exchange. Can they serve this role, despite their price volatility? What determines the price of cryptocurrencies, how can their fluctuations arise and what are the consequences for the monetary policy of the traditional central bank? This paper sheds light on these questions.

For our analysis, we construct a simple model of currency competition. Two types of infinitely-lived agents alternate between consumption and production. This lack of the double-coincidence of wants provides a role for a medium of exchange. There are two intrinsically worthless currencies, which can both be used for transactions: Bitcoins and Dollars. A central bank targets a stochastic Dollar inflation via appropriate Dollar injections, while the Bitcoin quantity grows deterministically. While we use the label "Bitcoin", our analysis applies more broadly. Instead of Bitcoin one may imagine any other cryptocurrency or intrinsically worthless object, which is storable, pays no dividend, whose price is not stabilized by some institution and which may be used as a medium of exchange. Our analysis applies just as much to certain cryptocurrencies such as Litecoin as it applies to objects such as pebbles or seashells. The analysis does not apply to gold, sugar, utility tokens such as ether and binance coin, equity tokens or stablecoins.

☆ We thank our discussants Aleksander Berentsen, Alex Cukierman, Pablo Kurlat, Aleh Tsivinski and Randy Wright. We thank Pierpaolo Benigno, Bruno Biais, Gur Huberman, Todd Keister, Ricardo Reis and many participants in conferences and seminars for many insightful comments.

* Correspondence to: Kenneth C. Griffin Department of Economics, University of Chicago, 1126 East 59th Street, Chicago, IL 60637, USA.
E-mail address: huhlig@uchicago.edu (H. Uhlig).

https://doi.org/10.1016/j.jmoneco.2019.07.002
0304-3932/

Three key results are Propositions 1 and 2 and Theorem 1 in Section 3. Proposition 1 provides what we call a fundamental pricing equation, which has to hold in the fundamental case, where both currencies are simultaneously in use. In its most simple form, this equation says that the Bitcoin price expressed in Dollar follows a martingale, i.e., that the expected future Bitcoin price equals its current price. Proposition 2, on the other hand, shows that the Bitcoin price has to rise in expectation and Bitcoins have to earn a real interest, if not all Bitcoins are spent on transactions and buyers ("hodlers") hold them back, speculating on their appreciation. Under Assumption 3, Theorem 1 shows that this speculative condition cannot hold and that therefore the fundamental pricing equation has to apply. Price volatility therefore does not invalidate the medium-of-exchange function of Bitcoin. Section 4 examines some key implications. We show that Bitcoin gradually must disappear as a medium of exchange, if their total supply is bounded above and if Dollar inflation is strictly positive, see Proposition 3.

Section 5 discusses monetary policy implications of Theorem 1, i.e. all monies are spent every period and sum to the total nominal value of consumption. This implies intriguing interplays between the Bitcoin price and the Dollar supply. We show that the increases in the quantity of Bitcoin or block rewards are not a tax on Bitcoin holders. Instead, they are financed by Dollar taxes imposed by the Dollar central bank, see Theorem 2. We examine a "conventional scenario", where the exogenous Bitcoin price fluctuations dictate central bank dollar supply decisions, as well as an "unconventional scenario", where the central bank controls the Bitcoin price.

While these block rewards are assumed to be received as lump sums in the baseline version of the model, we examine costly effort to "mine" additional Bitcoins in Section 6. We show an irrelevance theorem. Any equilibrium in the baseline version is also an equilibrium in the costly-effort version: the only difference is the provision of effort. We further discuss welfare implications and the taxation of Bitcoin production. Free entry of cryptocurrencies and its implications for our results are examined in Section 7. The online appendix provides further background material and literature discussion, contains details, proofs and some extensions as well as characterizes and constructs equilibria.

Our analysis is related to a substantial body of the literature. We provide a more in-depth review in Appendix A, and focus on a few key related papers here. The idea of the "denationalisation of money" goes back to Hayek (1976). Our model can be thought of as a simplified version of Bewley (1977), Townsend (1980) and Lagos and Wright (2005). Our aim here is decidedly not to provide a new micro foundation for the use of money, but to provide a simple starting point for our analysis. The key perspective for much of the analysis is the celebrated exchange-rate indeterminacy result in Kareken and Wallace (1981) and its stochastic counterpart in Manuelli and Peck (1990). There are three key differences to the latter. First, our agents are infinitely lived, allowing for the possibility of speculative holdings of currencies. Second, the supply of Bitcoin and Dollar evolves over time here, thus generating several novel and crucial insights. Third, we analyze Bitcoin production. Garratt and Wallace (2017) is closely related in spirit to our analysis, as they also draw on Kareken and Wallace (1981). They utilize a deterministic two-period OLG model, focus on a fixed stock of Bitcoin and assume a carrying cost for Dollars, among some of the differences to us. In the spirit of our exercise here, Fernández-Villaverde and Sanches (2016) as well as Zhu and Hendry (2018) examine the scope of currency competition in an extended (Lagos and Wright, 2005) model. Huberman et al. (2017) examine congestion effects in Bitcoin transactions. The Bitcoin-disappearance result is reminiscent of Section 8.12 of Ljungvist and Sargent (2018).

2. The model

Time is discrete, $t = 0, 1, \ldots$. In each period, a publicly observable, aggregate random shock $\theta_t \in \Theta \subset \mathbf{R}$ is realized. All random variables in period t are assumed to be functions of the history $\theta^t = (\theta_0, \ldots, \theta_t)$ of these shocks, i.e. measurable with respect to the filtration generated by the stochastic sequence $(\theta_t)_{t\in\{0,1,\ldots\}}$ and thus known to all participants at the beginning of the period. Note that the length of the vector θ^t encodes the period t: therefore, functions of θ^t are allowed to be deterministic functions of t.

There is a consumption good which is not storable across periods. There is a unit interval each of two types of agents. We shall call the first type of agents "red", and the other type "green". Both types of agents j enjoy utility from consumption $c_{t,j} \geq 0$ at time t per $u(c_{t,j})$, as well as loathe providing effort $e_{t,j} \geq 0$, where effort may be necessary to produce Bitcoins, see Section 6. The consumption-utility function $u(\cdot)$ is strictly increasing and concave. The utility-loss-from-effort function $h(\cdot)$ is strictly increasing and weakly convex. We assume that both functions are twice differentiable.

Red and green agents alternate in consuming and producing the consumption good, see Fig. G.1. We assume that red agents only enjoy consuming the good in odd periods, while green agents only enjoy consuming in even periods. Red agents $j \in [0, 1)$ inelastically produce (or: are endowed with) y_t units of the consumption good in even periods t, while green agents $j \in [1, 2]$ do so in odd periods. This creates the absence of the double-coincidence of wants, and thereby reasons to trade. The endowment $y_t = y(\theta^t)$ is stochastic with support $y_t \in [\underline{y}, \bar{y}]$, where $0 < \underline{y} \leq \bar{y}$. As a special case, we consider the case, where y_t is constant, $\underline{y} = \bar{y}$ and $y_t \equiv \bar{y}$ for all t. Let the discount rate β satisfy $0 < \beta < 1$. Life-time utility is given by $U = E[\sum_{t=0}^{\infty} \beta^t (\xi_{t,j} u(c_{t,j}) - h(e_{t,j}))]$. Alternating periods of utility from consumption are imposed per $\xi_{t,j} = 1_{t \text{ is odd}}$ for $j \in [0, 1)$ and $\xi_{t,j} = 1_{t \text{ is even}}$ for $j \in [1, 2]$.

There are two forms of money. The first shall be called Bitcoins and its aggregate stock at time t shall be denoted with B_t. The second shall be called Dollar and its aggregate stock at time t shall be denoted with D_t. Both currencies can be used equally well for purchases. An extension of differentiation across goods in terms of ease of trade per currency is examined

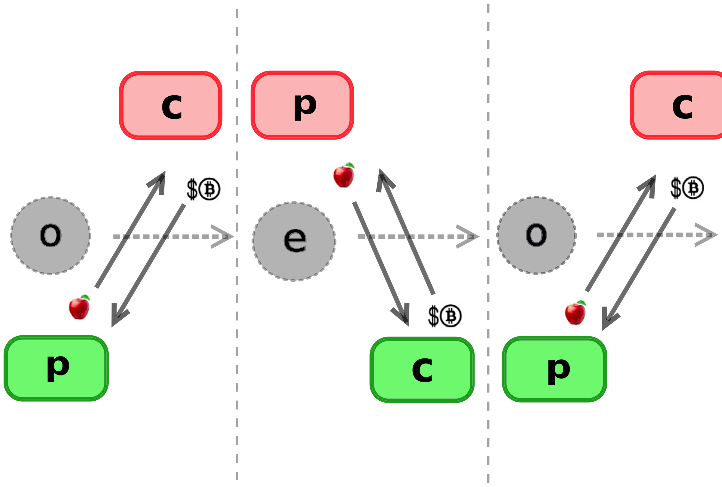

Fig. G.1. Alternation of production and consumption. In odd periods, green agents produce and red agents consume. In even periods, red agents produce and green agents consume. Alternation and the fact that the consumption good is perishable gives rise to the necessity to trade using fiat money..

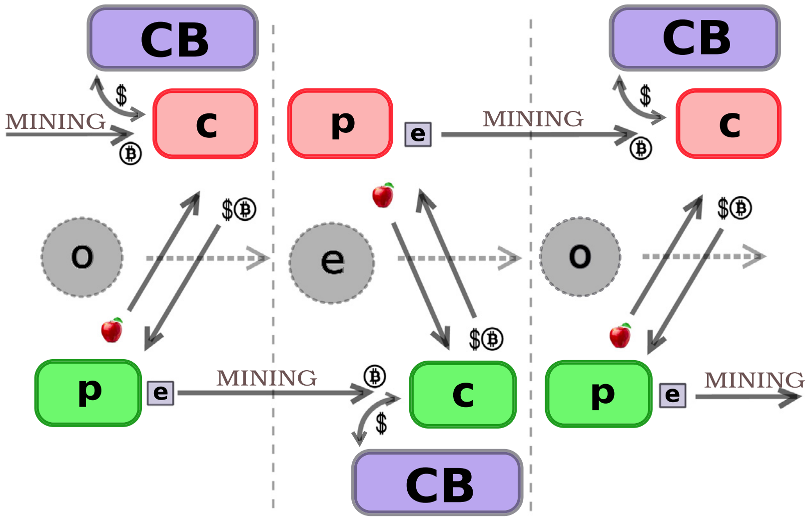

Fig. G.2. Transfers: In each period, a central bank injects to or withdraws Dollars from agents, before they consume, to target a certain Dollar inflation level. By this, the Dollar supply may increase or decrease. Across periods, agents can put effort to mine Bitcoins. By this, the Bitcoin supply can only increase.

in Schilling and Uhlig (2019), using a transaction costs approach. There is a central bank, which governs the aggregate stock of Dollars D_t, while Bitcoins can be produced privately.

The sequence of events in each period is as follows, see Fig. G.2. First, θ_t is drawn. Given the information on θ^t, the central bank issues or withdraws Dollars, per "helicopter drops" or lump-sum transfers and taxes on the agents ready to consume in that particular period. The central bank produces Dollars at zero cost. Consider a green agent j entering an even period t, holding some Dollar amount $\tilde{D}_{t,j}$ from the previous period. The agent will receive a Dollar transfer $\tau_t = \tau(\theta^t)$ from (or, if negative, pay taxes to) the central bank, resulting in $D_{t,j} = \tilde{D}_{t,j} + \tau_t$. Red agents do not receive (or pay) τ_t in even period. Conversely, the receive transfers (or pay taxes) in odd periods, while green agents do not. The aggregate stock of Dollars changes to $D_t = D_{t-1} + \tau_t$.

The green agent j then enters the consumption good market holding $B_{t,j}$ Bitcoins from the previous period and $D_{t,j}$ Dollars. The green agent will seek to purchase the consumption good from red agents. Let $P_t = P(\theta^t)$ be the price of the consumption good in terms of Dollars and let $\pi_t = \frac{P_t}{P_{t-1}}$ denote the resulting inflation. We could likewise express the price of goods in terms of Bitcoins, but it is more appealing to let $Q_t = Q(\theta^t)$ denote the price of Bitcoins in terms of Dollars. The price of one unit of the good in terms of Bitcoins is then P_t/Q_t. Let $b_{t,j}$ be the amount of the consumption good purchased with Bitcoins and $d_{t,j}$ be the amount of the consumption good purchased with Dollars. The green agent cannot spend more of each money than she owns but may choose not to spend all of it. This implies the constraints $0 \le P_t b_{t,j} \le Q_t B_{t,j}$ and $0 \le P_t d_{t,j} \le D_{t,j}$.

The green agent then consumes $c_{t,j} = b_{t,j} + d_{t,j}$ and leaves the even period, carrying $B_{t+1,j} = B_{t,j} - \frac{P_t}{Q_t} b_{t,j} \geq 0$ Bitcoins and $D_{t+1,j} = D_{t,j} - P_t d_{t,j} \geq 0$ Dollars into the next and odd period $t+1$.

At the beginning of that odd period $t+1$, the aggregate shock θ_{t+1} is drawn and added to the history θ^{t+1}. The green agent produces y_{t+1} units of the consumption good. Green agents produce $A_{t+1,j} \geq 0$ additional Bitcoins per agent, which they can spend in the subsequent period. For the baseline version of the model, we assume that the production of these additional Bitcoins takes the form of an endowment and is the same for all agents of the same type, $A_{t+1,j} = A_{t+1}$, and that no effort is involved, $e_{t,j} \equiv 0$. In Section 6, we examine the case, where nonzero effort is required to receive or "mine" Bitcoins. In both cases, we assume that the aggregate quantity $A_{t+1} \geq 0$ of additional Bitcoins or Bitcoin block rewards is a deterministic function of time. Thus, the aggregate stock of Bitcoin B_t is a deterministic and weakly increasing function of time. We will occasionally impose that the Bitcoin quantity is bounded from above, $B_t \leq \bar{B}$ for all t: this is a feature of Bitcoin in practice. In odd periods, only green agents may produce Bitcoins, while only red agents get to produce Bitcoins in even periods.

The green agent sells the consumption goods to red agents. Given market prices Q_{t+1} and P_{t+1}, he decides on the amount of output $x_{t+1,j} \geq 0$ sold for Bitcoins and $z_{t+1,j} \geq 0$ sold for Dollars, where $x_{t+1,j} + z_{t+1,j} = y_{t+1}$, as the green agent has no other use for the good. After these transactions, the green agent holds $\tilde{D}_{t+2,j} = D_{t+1,j} + P_{t+1} z_{t+1,j}$ Dollars, which then may be augmented per central bank lump-sum transfers at the beginning of the next period $t+2$ as described above. As for the Bitcoins, the green agent carries the total of $B_{t+2,j} = A_{t+1,j} + B_{t+1,j} + \frac{P_{t+1}}{Q_{t+1}} x_{t+1,j}$ to the next period.

The role of red agents and their budget constraints is entirely symmetric to green agents, per swapping the role of even and odd periods. There is one difference, though, and it concerns the initial endowments with money. Since green agents are first in period $t = 0$ to purchase goods from red agents, we assume that green agents initially have all the Dollars and all the Bitcoins and red agents have none.

While there is a single and central consumption good market in each period, payments can be made with the two different currencies. We therefore get the two market clearing conditions $\int_{j=0}^{2} b_{t,j} dj = \int_{j=0}^{2} x_{t,j} dj$ and $\int_{j=0}^{2} d_{t,j} dj = \int_{j=0}^{2} z_{t,j} dj$, where we adopt the convention that $x_{t,j} = z_{t,j} = 0$ for green agents in even periods and red agents in odd periods as well as $b_{t,j} = d_{t,j} = 0$ for red agents in even periods and green agents in odd periods.

The central bank picks transfer payments τ_t, which are itself a function of the publicly observable random shock history θ^t, and thus already known to all agents at the beginning of the period t. In particular, the transfers do not additionally reveal information otherwise only available to the central bank. We will impose that the central bank is targeting some Dollar price path P_t, using the transfers as its policy tool, while there is no corresponding institution worrying about the Bitcoin price Q_t.

We restrict attention to symmetric equilibria, in which all agents of the same type end up making the same choice. Thus, instead of subscript j and with a slight abuse of notation, we shall use subscript g to indicate a choice by a green agent and r to indicate a choice by a red agent. Section Appendix B provides the formal definition of an equilibrium.

3. Analysis

For the analysis, proofs not included in the main text can be found in Appendix D. The equilibrium definition implies that $c_t = y_t$, that $B_{t+1} = B_t + A_t$ and that $D_t = D_{t-1} + \tau_t$. Bitcoin production is analyzed in Section 6. We restrict attention to equilibria and central bank target Dollar price paths P_t and strictly positive Dollar supplies and prices, where inflation is always larger than unity:

Assumption 1. $D_t > 0$, $0 < P_t < \infty$ and $\pi_t = \frac{P_t}{P_{t-1}} \geq 1$ for all t.

Throughout, we shall assume that no matter how many units of the consumption good an agent consumes today she will always prefer consuming an additional marginal unit of the consumption good now as opposed to consuming it at the next opportunity two periods later:

Assumption 2. For all t, $u'(y_t) - \beta^2 E_t[u'(y_{t+2})] > 0$.

The following lemmata are a consequence of a central bank policy aiming at nonnegative inflation, see Assumption 1, together with the opportunity cost for holding money, see Assumption 2. This is in contrast to the literature concerning the implementation of the Friedman rule, where that opportunity cost is absent. We examine the welfare consequences in Section 6.1.

Lemma 1. ***(All Dollars are spent:)*** *Agents will always spend all Dollars and $z_t > 0$ for all t. Thus, $D_t = D_{t,g}$ and $D_{t,r} = 0$ in even periods and $D_t = D_{t,r}$ and $D_{t,g} = 0$ in odd periods.*

Lemma 2. ***(Dollar Injections:)*** *In equilibrium, the post-transfer amount of total Dollars is $D_t = P_t z_t$, and the transfers are $\tau_t = P_t z_t - P_{t-1} z_{t-1}$.*

3.1. The fundamental pricing equation

The following proposition establishes properties of the Bitcoin price Q_t in the “fundamental” case, where Bitcoins are used in transactions.

Proposition 1. *(**Fundamental pricing equation**[1]:)*
Suppose that sales happen both in the Bitcoin-denominated consumption market as well as the Dollar-denominated consumption market at time t as well as at time $t+1$, i.e. suppose that $x_t > 0$ and $x_{t+1} > 0$. Then

$$E_t\left[u'(c_{t+1})\,\frac{P_t}{P_{t+1}}\right] = E_t\left[u'(c_{t+1})\,\frac{(Q_{t+1}/P_{t+1})}{(Q_t/P_t)}\right]. \tag{1}$$

If $u'(c_{t+1})/P_{t+1}$ is uncorrelated with Q_{t+1}, conditional on time-t information, then the stochastic Bitcoin price process $\{Q_t\}_{t\geq 0}$ is a martingale

$$Q_t = \mathbb{E}_t[Q_{t+1}]. \tag{2}$$

Eq. (1) can be understood from a standard asset pricing perspective. The goods seller will sell against the currency which offers a higher risk-preference adjusted return. Due to the currency competition, sales against both Bitcoins and Dollars implies seller indifference. The risk-adjusted returns, therefore, have to be the same. If zero Bitcoins are traded, the fundamental pricing equation becomes an inequality, see lemma 3 in the online appendix.

The result is a version of the celebrated result in Kareken and Wallace (1981). These authors did not consider stochastic fluctuations. Our martingale result then reduces to a constant Bitcoin price, $Q_t = Q_{t+1}$, and thus their “exchange rate indeterminacy result” for time $t = 0$, that any Q_0 is consistent with some equilibrium, provided the Bitcoin price stays constant afterwards, see proposition 6 for further details. Our result here reveals that this indeterminacy result amounts to a potentially risk-adjusted martingale condition in the stochastic case. Our result furthermore corresponds to equation (14’) in Manuelli and Peck (1990), who provide a stochastic generalization of the 2-period OLG model in Kareken and Wallace (1981). Manuelli and Peck (1990) derive their results from considering intertemporal savings decisions, which then, in turn, imply the indifference between currencies. While we agree with the latter, we do not insist on the former. Indeed, it may be empirically problematic to base currency demand on savings decisions without considering interest-bearing assets. By contrast, we obtain the indifference condition directly from the willingness of sellers to accept either currency as a means of payment.

Finally, our result relates to the literature on uncovered interest parity. In that literature, it is assumed that agents trade safe bonds, denominated in either currency. That literature derives the uncovered interest parity condition, which states that the expected exchange rate change equals the return differences on the two nominal bonds. This result is reminiscent of our equation above. Note, however, that we do not consider bond trading here: rates of returns, therefore, do not feature in our results. Instead, they are driven entirely by cash use considerations.

3.2. Equilibrium conditions for (the absence of) speculation

The next proposition establishes properties of the Bitcoin price Q_t, if potential goods buyers prefer to keep some or all of their Bitcoins in possession, rather than using them in a transaction, effectively speculating on lower Bitcoin goods prices or, equivalently, higher Dollar prices for a Bitcoin in the future. This condition establishes an essential difference to Kareken and Wallace (1981) and Manuelli and Peck (1990). In their models, agents live for two periods and thus splurge all their cash in their final period of life. Our infinitely-lived agents may instead consider to speculate on currency appreciation.

Proposition 2. ***(Speculative price bound:)***
Suppose that $B_t > 0$, $Q_t > 0$ and that not all Bitcoins are spent in t, $b_t < (Q_t/P_t)B_t$. Then,

$$u'(c_t) \leq \beta^2 E_t\left[u'(c_{t+2})\,\frac{(Q_{t+2}/P_{t+2})}{(Q_t/P_t)}\right], \tag{3}$$

where this equation furthermore holds with equality, if $x_t > 0$ and $x_{t+2} > 0$.

The juxtaposition of Propositions 1 and 2 is instructive. Heuristically, assume agents sacrifice consumption today to keep some Bitcoins as an investment in order to increase consumption the day after tomorrow, as envisioned in Proposition 2. Tomorrow, these agents produce goods which they will need to sell. Since all Dollars change hands in every period per Lemma 1, sellers must always weakly prefer receiving Dollars over Bitcoins as payment in equilibrium, see Proposition 1. The Bitcoin price tomorrow can therefore not be too low. However, with a high Bitcoin price tomorrow, sellers today will weakly prefer receiving Dollars only if the Bitcoin price today is high as well. But at such a high Bitcoin price today, it cannot be worth it for buyers today to hold back Bitcoins for speculative purposes, a contradiction.

[1] The proof for this proposition as well as most other results are in Appendix D.

One can see this immediately, if the economy is non-stochastic. The two conditions of the two Propositions 1 and 2 would require $Q_{t+1} \le Q_t \le \beta^2 Q_{t+2}/(\pi_{t+2}\pi_{t+1})$, a contradiction, since $\beta < 1$ and $\pi_t \ge 1$ in all t per Assumption 1. The logic can formalized in the general case, provided we impose a slightly sharper version of Assumption 2:

Assumption 3. (Mutual Impatience:) $u'(y_t) - \beta E_t[u'(y_{t+1})] > 0$ for all t.

Note that Assumption 3 compares marginal utilities across different agent types. We discuss this issue further in Appendix F. Mutual impatience together with the law of iterated expectations imply Assumption 2. Conversely, if Assumption 2 holds, then the condition in Assumption 3 cannot be violated two periods in a row.

Theorem 1. ***(No-Bitcoin-Speculation:)*** *Suppose that $B_t > 0$ and $Q_t > 0$ for all t. Impose Assumption 3. Then all Bitcoins are spent and (1) holds in every period t.*

With Assumption 3, Proposition 1 therefore applies. Consequently, price volatility does not invalidate the medium-of-exchange function of Bitcoin.

4. Price properties and equilibrium construction

Define the nominal pricing kernel m_t per $m_t = \frac{u'(c_t)}{P_t}$. We can then equivalently rewrite Eq. (1) as

$$Q_t = \mathbb{E}_t[Q_{t+1}] + \frac{\text{cov}_t(Q_{t+1}, m_{t+1})}{\mathbb{E}_t[m_{t+1}]}. \tag{4}$$

Note that one could equivalently replace the pricing kernel m_{t+1} in this formula with the nominal stochastic discount factor of a red agent or a green agent, given by $M_{t+1} := \beta^2(u'(c_{t+1})/u'(c_{t-1}))/(P_{t+1}/P_{t-1})$.

Corollary 1. ***(Equilibrium Bitcoin Pricing Formula:)***

Suppose that $B_t > 0$ and $Q_t > 0$ for all t. Impose Assumption 3. In equilibrium, the Dollar-denominated Bitcoin price satisfies

$$Q_t = E_t[Q_{t+1}] + \kappa_t \cdot \text{corr}_t(m_{t+1}, Q_{t+1}), \tag{5}$$

where $\kappa_t = (\sigma_{m_{t+1}|t} \cdot \sigma_{Q_{t+1}|t})/\mathbb{E}_t[m_{t+1}] > 0$, where $\sigma_{m_{t+1}|t}$ is the standard deviation of the pricing kernel, $\sigma_{Q_{t+1}|t}$ is the standard deviation of the Bitcoin price and $\text{corr}_t(m_{t+1}, Q_{t+1})$ is the correlation between the Bitcoin price and the pricing kernel, all conditional on time t information.

One immediate implication of Corollary 1 is that the Dollar denominated Bitcoin price process is a supermartingale (falls in expectation) if and only if in equilibrium the pricing kernel and the Bitcoin price are positively correlated for all $t+1$ conditional on time t-information. Likewise, under negative correlation, the Bitcoin price process is a submartingale and increases in expectation. We construct examples for these in Appendix E.2. In the special case that in equilibrium the pricing kernel is uncorrelated with the Bitcoin price, the Bitcoin price process is a martingale.

Consider $P_t \equiv 1$ across time. Then the Bitcoin price can be all, a super- or a submartingale, depending on its correlation with marginal utility of consumption. Conversely, if agents are risk-neutral, then $u'(c_{t+1})$ is constant and the correlation of the inverse of inflation with the Bitcoin price determines, whether Bitcoin prices fall or rise in expectations.

The following result is specific to cryptocurrencies such as Bitcoin,which have an upper limit on their quantity.

Proposition 3. ***(Real Bitcoin Disappearance:)***

Suppose that the quantity of Bitcoin is bounded above, $B_t \le \bar{B}$ and let $\pi_t \ge \underline{\pi}$ for all $t \ge 0$ and some $\underline{\pi} > 1$. If marginal consumption is positively correlated or uncorrelated with the exchange rate $\frac{Q_{t+1}}{P_{t+1}}$, i.e. if $\text{cov}_t(u'(c_{t+1}), \frac{Q_{t+1}}{P_{t+1}}) \ge 0$, then $E_0\left[\frac{Q_t}{P_t}B_t\right] \to 0$, as $t \to \infty$. In words, the purchasing power of the entire stock of Bitcoin shrinks to zero over time, if inflation is bounded below by a number strictly above one.

This is reminiscent of section 8.12 of Ljungqvist and Sargent (2018). The assumption on the covariance is satisfied in particular when agents are risk-neutral or if output is constant, $c_t \equiv y_t \equiv \bar{y}$. In that case,

$$\frac{Q_t}{P_t} = \mathbb{E}_t\left[\frac{Q_{t+1}}{P_{t+1}}\right] \cdot \mathbb{E}_t\left[\frac{1}{\pi_{t+1}}\right]^{-1} \ge \mathbb{E}_t\left[\frac{Q_{t+1}}{P_{t+1}}\right], \tag{6}$$

by Eq. (1): the real value of Bitcoin falls in expectation.

Another way to understand Proposition 3 is to rewrite the fundamental pricing Eq. (1) for the case of a constant Bitcoin stock $B_t \equiv B$ and a constant inflation $\pi_t \equiv \pi$ as

$$E_t\left[\frac{v_{t+1}}{v_t}\right] = -\frac{\text{cov}_t\left(u'(c_{t+1}), \frac{v_{t+1}}{v_t}\right)}{E_t[u'(c_{t+1})]} + \frac{1}{\pi}, \tag{7}$$

where $v_t = \frac{Q_t}{P_t}B_t$ is the real value of the Bitcoin stock at time t. The left-hand side of (7) is the growth of the real value of the Bitcoin stock. The first term on the right-hand side (with minus sign) is the risk premium for holding Bitcoins. With a constant Bitcoin amount and inflation, the equation says that the expected increase of the real value of the stock of Bitcoins is (approximately) equal to the risk premium minus the inflation rate on Dollars. Only a sufficiently high risk premium can avoid the Bitcoin disappearance.

Corollary 2. ***(Real Bitcoin price bound:)*** *Suppose that $B_t > 0$ and $Q_t, P_t > 0$ for all t. The real Bitcoin price is bounded by $\frac{Q_t}{P_t} \in (0, \frac{\bar{y}}{B_0})$.*

The upper bound on the Bitcoin price is established by two traits of the model. First and per assumption, the Bitcoin supply may only increase. Second, by assumption, we bound production fluctuations: this can be generalized.[2]

The challenge in explicitly constructing equilibria (and thereby showing their existence) lies in the zero-dividend properties of currencies. In asset pricing, one usually proceeds from a given future stream dividend process D_t and prices these, using the stochastic discount factor. This approach will not work here, since dividends of fiat currencies are identical to zero.

We therefore provide an alternative approach, and provide a sketch here: the details and explicit examples can be found in Appendix E. The solution rests in rewriting (4) as

$$Q_{t+1} = Q_t + \epsilon_{t+1} - \frac{\mathrm{cov}_t(\epsilon_{t+1}, m_{t+1})}{\mathbb{E}_t[m_{t+1}]}. \tag{8}$$

Let a sequence of Dollar goods prices P_t and pricing kernels m_t be given. Pick the initial Bitcoin price Q_0 and pick some sequence ϵ_t with $E_t[\epsilon_{t+1}] = 0$ and some chosen conditional covariance with the pricing kernels. Use Eq. (8) to recursively construct the sequence Q_{t+1}, going forward in time.

5. Implications for monetary policy

Throughout, we impose Assumption 3. Theorem 1 therefore applies. Since all Dollars and Bitcoins are spent once in every period, the velocity of both currencies equals one. In a world with only one currency, classical quantity theory yields $y_t = \frac{D_t}{P_t}$. Depending on output y_t, the central bank adjusts the dollar quantity D_t such that the desired Dollar price level P_t realizes. In our model instead, equilibrium market clearing implies

$$y_t = \frac{D_t}{P_t} + \frac{Q_t}{P_t} B_t. \tag{9}$$

There are now two endogenous variables, the Dollar quantity D_t and the Bitcoin price Q_t. This generates crucial differences to the one-currency world. Holding P_t, y_t and Q_t constant, a deterministic increase in B_t must be compensated by a corresponding decrease in D_t in equilibrium:

Theorem 2. ***(Bitcoin block rewards are financed by Dollar taxes:)*** *Any no-speculation equilibrium price sequence (Q_t, P_t) with $Q_t > 0$ for some given path $B_t > 0$ can also be supported as an equilibrium price sequence by an alternative path B'_t, provided that $0 < B'_t < (P_t/Q_t)y_t$, per appropriately adjusting the Dollar quantity D_t. For given Dollar lump-sum transfers τ_t in the original equilibrium, the adjusted Dollar lump-sum transfers τ'_t are given by $\tau'_t = \tau_t - (Q_t - Q_{t-1})(B'_t - B_t) - Q_{t-1}\left((B'_t - B'_{t-1}) - (B_t - B_{t-1})\right)$*

Suppose the economy is deterministic and that the Bitcoin quantity in the original economy is constant $B_t \equiv B$, while the Bitcoin stock in the new economy is increasing period-per-period per the Bitcoin block rewards A'_t, i.e. $B'_{t+1} = A'_t + B'_t$. Theorem 1 with Proposition 1 implies that Q_t is constant, $Q_t \equiv Q$. Theorem 2 then states that $\tau'_t = \tau_t - Q(B'_t - B'_{t-1})$, i.e. the additional Dollar lump-sum taxes $Q(B'_t - B'_{t-1})$ pay for the block rewards.

Theorem 2 states that this logic applies generally. The fundamental pricing Eq. (1) holds irrespective of the evolution of the Bitcoin quantity. The block rewards to miners, which are provided as lump-sum payments in the baseline version and are earned through mining effort in Section 6, are financed not by deflating the Bitcoin currency but by the Dollar central bank, which has to accordingly decrease its Dollar supply. It does so per imposing Dollar lump-sum taxes on the population. The block rewards are not a tax on Bitcoin holders but are financed through Dollar taxes imposed by the Dollar central bank.

To examine the consequences for monetary policy more deeply, consider the price level P_t to be endogenous as well. Think of the Dollar quantity as a policy choice, while treating $y_t = y$ and $B_t = B$ as exogenous. Consider two different Dollar quantities $D_t = D$ and $D_t = D'$. For each Dollar quantity, there is now a set or line $L = \{(Q, P) \mid P = \frac{D}{y} + Q\frac{B}{y}\}$, $L' = \{(Q, P) \mid P = \frac{D'}{y} + Q\frac{B}{y}\}$ of equilibrium values for $P = P_t$ and $Q_t = Q$ satisfying (9). These two lines are shown in the left panel of Fig. G.3. Suppose we start from the equilibrium at point A for the Dollar quantity D. What happens, as the central bank issues the Dollar quantity D' instead? Without further assumptions, any (Q, P)-pair on line L' may constitute the new equilibrium. Some additional equilibrium selection criterion would be required. One possibility, which we label the "conventional scenario", is to think of the Bitcoin price as moving exogenously: in the figure, we fix it at $Q = \bar{Q}$. In that case, we get a version of the classic relationship in that the increase in the dollar quantity from D to D' leads to a higher price level, moving the equilibrium from point A to point B. Another possibility though, which we label the "unconventional scenario", is to instead fix the price level at some exogenously given level $P = \bar{P}$: now, increasing the Dollar quantity reduces the Bitcoin price, moving the equilibrium from point A to point C. Many other equilibrium selection criteria can be introduced, in principle.

[2] Assume, we allow the support of production y_t to grow or shrink in t: $y_t \in [\underline{y}, \bar{y}_t]$. The result would then be $\frac{Q_t}{P_t} \leq \frac{\bar{y}_t}{B_0}$.

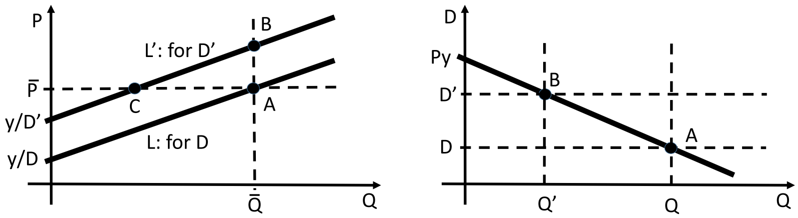

Fig. G.3. Examining equation (9), $y = \frac{D}{P} + \frac{Q}{P}B$, taking y and B as given.

In the model, we have side-stepped these equilibrium selection issues per fixing π_t as an exogenous stochastic process along with P_0[3] The relevant equilibrium relationship is then given by the right panel of Fig. G.3, where we have fixed the price level P. The "conventional" scenario now amounts to assuming exogenous fluctuations in Q, moving, say, from Q to Q', to which the central bank has to react per moving the Dollar quantity from D to D', see (9). The equilibrium then moves from point A to point B. Conversely, and for the "unconventional" scenario, one may wish to think of the central bank as picking the Dollar quantity as D or D' and thereby picking the Bitcoin price to be Q or Q'.

For more structure, introduce two new random variables $P_t^* > 0$ and $Q_t^* > 0$, drawn alongside y_t as a function of the stochastic state history θ^t at the beginning of the period[4]: they will serve as targets for the central bank. Introduce a central bank objective via the loss function $\mathcal{L}(P_t, Q_t) = -(P_t - P_t^*)^2 - (Q_t - Q_t^*)^2$. The central bank seeks to minimize this loss per a suitable choice of the dollar supply D_t, thereby stabilizing the Dollar price level as well as the Bitcoin price near their target outcomes. The two scenarios result from imposing two different additional exogeneity assumptions.

Assumption 4. (**Conventional Scenario:**) Assume that Q_t is drawn alongside P_t^*, Q_t^*, y_t at the beginning of the period as a function of θ^t.

Corollary 3. ***(Bitcoin-price-driven Policy:)***
Under Assumption 4, the loss-minimizing central bank picks the Dollar quantity $D_t = y_t P_t^* - Q_t B_t$, to uniquely achieve the Dollar price level $P_t = P_t^*$.

Corollary 3 and the fundamental pricing equation then provide a straightforward receipt for forecasting the dollar supply, given the correlation $\text{corr}_t(m_{t+1}, Q_{t+1})$ and κ_t of Corollary 1, see Appendix G for details.

Assumption 5. (Unconventional Scenario:) Assume that P_t is drawn alongside $\bar{P}_t, Q_t^*, y_t$ at the beginning of the period as a function of θ^t.

Assumption 5 implies that the central bank can maintain the inflation level π_t independently of the transfers she sets.

Corollary 4. ***(Policy-driven Bitcoin price:)***
Under Assumption 5, the loss-minimizing central bank picks the Dollar quantity $D_t = y_t P_t - Q_t^* B_t$, to uniquely achieve the Bitcoin price $Q_t = Q_t^*$

Intuitively, the causality is in reverse compared to the conventional scenario: now the central bank policy drives Bitcoin prices. Section 6.1 discusses welfare and optimal monetary policy. Further implications are in Appendix G.

6. Bitcoin production

So far, we have assumed that Bitcoin production takes the form of an exogenous endowment. The purpose of this section is to examine what happens when it takes effort $e_{t,j}$ for agent j to produce or "mine" additional Bitcoins (or: "receive Bitcoin block rewards") $A_{t,j}$. Effort $e_{t,j}$ causes disutility $h(e_{t,j})$, where $h(\cdot)$ is strictly increasing and weakly convex. We have chosen the effort formulation rather than a resource-cost formulation, in order to avoid some tedious terms of reducing overall goods consumption, due to the diversion of output into Bitcoin production[5]. In light of that practice, one may wish to interpret effort $e_{t,j}$ as the individually provided hash rate resulting from consuming electricity or a combination of electricity and the appropriate utilization of hardware. If one views electricity is the only input and production is viewed as scalable, then $h(\cdot)$ is a linear function with a positive slope. If there is an additional capacity constraint arising from available hardware, then $h(\cdot)$ could be chosen to be piecewise linear, with a low slope up to that constraint and a much higher slope beyond. This leaves the issue of hardware acquisition and investment aside, see Prat and Walter (2018).

[3] There is an extensive literature on the capability of central banks to influence the Dollar price level, and we have nothing new to contribute to that: our assumption about the exogeneity of the price level encodes that literature. The question of interest here is to what degree the central bank can influence the Bitcoin price.

[4] Recall that the Bitcoin quantity B_t is a deterministic function of time.

[5] To the degree that the resource costs of producing Bitcoins in practice are still minor compared to global output, this seems appropriate for the analysis.

Define the aggregate effort level for mining Bitcoin at time t per $\bar{e}_t = \int_{j\in[0,1]} e_{t,j}\, dj$. Recall that A_t are the block rewards, i.e. is the deterministic quantity of additional Bitcoins added in period t in the aggregate. We assume that an individual agent expanding effort $e_{t,j} \geq 0$ will then receive additional Bitcoins according to the production function $A_{t,j} = A_t \frac{e_{t,j}}{\bar{e}_t}$. As a consequence, the total number of newly mined coins per period indeed satisfies $A_t = \int_{j\in[0,1]} A_{t,j}\, dj$, and is independent of the aggregate effort level. This modeling choice captures the real-life feature, that the increase in the hash rate has no impact on the total Bitcoin production,[6] see Huberman et al. (2017). In comparison to the practice of Bitcoin mining, one might wish to interpret $\frac{e_{t+1,j}}{\int_{j\in[0,1]} e_{t+1,j}}$ as the probability for an individual miner to win the proof-of-work competition. Typically, individual miners associate in large mining pools, thereby eliminating the idiosynchratic risk of mining success, see Cong et al. (2018), consistent with our formulation of $A_{t+1,j}$ as a deterministic function. The equilibrium definition needs to be suitably adjusted: see Appendix B for the formal details.

Proposition 4. ***(Bitcoin Production Condition:)*** *Suppose that the aggregate quantity A_t of additionally mined Bitcoins in period t is strictly positive. Given the processes*[7] *for $Q_{t+1}, P_{t+1}, y_{t+1}$, the equilibrium Bitcoin production effort $e_{t,j} = \bar{e}_t > 0$ is the unique solution to*

$$h'(\bar{e}_t) = \beta E_t\left[u'(c_{t+1})\frac{Q_{t+1}}{P_{t+1}}\right]\frac{A_t}{\bar{e}_t}. \tag{10}$$

The Proposition implies that the equilibrium mining effort $\bar{e}_t$ is always strictly positive, provided that the price $Q_{t+1} > 0$ and the aggregate quantity A_t are strictly positive. This is due to the externality in mining: if other agents were to put zero mining effort, then an agent can ensure herself the entire new block reward but providing only a small amount. Eq. (10) further tells us that the individual and therefore also the aggregate mining effort increases as expectations on the real Bitcoin price increase.

We can now show, that introducing Bitcoin production does not otherwise change the equilibrium. This may be a bit of a surprise: one might have thought that costly Bitcoin production should somehow lead to a tax on Bitcoin transactions, for example[8] or alter the outcomes in some other way.

Theorem 3. ***(Irrelevance of Mining Effort:)*** *Consider an equilibrium of the baseline economy with exogenous (effort-less) Bitcoin production, and that Bitcoin production is always strictly positive. Then there exists an equilibrium of the economy with effort needed to produce Bitcoins, where all variables are the same except that $e_t = \bar{e}_t > 0$ is the unique solution to Eq. (10) rather than $e_t = \bar{e}_t = 0$.*

6.1. Optimal monetary policy and welfare

Analyzing welfare in the baseline model or in the baseline model extended with effort-driven Bitcoin production is straightforward.[9] Let $0 < \lambda < 1$ be the welfare weight on green agents and $1 - \lambda$ be the welfare weight on red agents, assuming that agents of the same type all receive the same weight, see also Appendix F. The overall welfare function is then given by $W = \lambda U_g + (1-\lambda)U_r$, where U_g and U_r are given in equations (B.1) and (B.11). Welfare is maximized if all output of the consumption good is always consumed and no Bitcoins are ever produced. Note that the social planner cannot redistribute consumption goods across periods, i.e., changing the welfare weight λ does not change the welfare-maximal allocation.

In all our equilibria, all output of the consumption good is always consumed. Therefore, the welfare arising from goods consumption is unaffected by monetary policy or the price path for Bitcoins. This implies that low inflation as well as high inflation all achieve the same goods consumption welfare. The only part of utility possibly affected is the disutility from producing Bitcoin.

Any equilibrium in the baseline model, where additional Bitcoins are received as endowment without the supply of effort, is welfare optimal. Any production of Bitcoin requiring effort is wasteful in terms of welfare, since anything that can be done with Bitcoins can be done with Dollars. “Only Dollars” is preferable. Note, though, that this policy conclusion is reached straight from the assumption, that the government is better at doing something than the private sector. Also note that Bitcoin mining increases GNP, since the mined Bitcoins would need to be evaluated at their market price. More Bitcoin production means more GNP, but less welfare. These results are unlikely to remain true in generalizations of our model.

6.2. Taxing bitcoin production

Could a government capable of imposing a tax on Bitcoin production change the equilibrium outcome and how? As a practical matter, it may be easiest to impose a proportional tax on the energy consumption of miners or, in the language

6 This is achieved by regular adaption of the difficulty level of the proof-of-work competition.

7 Recall that we focus on equilibria in which $Q_{t+1} > 0$. If $Q_{t+1} \equiv 0$ instead, then mining effort in t must be zero.

8 One reads this argument frequently in popular postings on Bitcoins.

9 A full welfare analysis becomes a bit involved if we allow for effort-driven Bitcoin production as well as versions of the unconventional scenario and thereby endogeneity of Q_t.

here, on the effort of miners. If one reads $h(e_{t,j})$ as the energy costs of mining effort $e_{t,j}$, these costs will then increase to $h(e_{t,j})/(1-\tau)$, where $\tau \in [0, 1)$ is that tax rate.

Proposition 5. ***(Irrelevance of Mining Taxation:)*** *Consider an equilibrium of the original economy without imposing taxes on Bitcoin production, and that Bitcoin production is always strictly positive. There is then an equilibrium of the economy with imposing taxes* $0 < \tau < 1$ *on Bitcoin production, where all variables are the same except that* $e_t = \bar{e}_t$ *is the unique solution to* $\frac{h'(\bar{e}_t)}{1-\tau} = \beta E_t[u'(y_{t+1})\frac{Q_{t+1}}{P_{t+1}}]\frac{A_t}{\bar{e}_t}$ *rather than (10). Compared to the no-tax equilibrium, effort levels are always strictly lower and welfare is strictly higher.*

The logic of the proposition and the argument in the proof goes through, when imposing the linear tax in some other manner, distributing the proceeds lump-sum to the group of taxed agents.[10] Generally, effort provision is imposing a wasteful externality on all other effort-providing agents in the economy, since the additional quantity of Bitcoins is not affected: it, therefore, should not surprise that welfare can be improved per taxing that activity. Note that this conclusion is likely to change in a setting where the total hash rate provided by the network is relevant for the security of the mining network and thus the cryptocurrency as a whole.

7. Multiple private currencies and free entry

So far we have analyzed competition between two fiat currencies without addressing entry of new currencies. This section offers a sketch on how to extend our analysis to that situation. A full development is left to future research.

While the supply of any particular cryptocurrency may evolve deterministically over time and be bounded, the total supply of all cryptocurrencies may be stochastic and unbounded due to entry. One particular property of monetary equilibria is that they are self-eliminating backwards in time. As argued in Kareken and Wallace (1981), if tomorrow's price of a currency is zero, its price today is zero as well. For a new fiat currency to enter (or a new coin offering to succeed), the opposite effect must be true. The fundamental pricing Eq. (1) will hold for any cryptocurrency with a nonzero price at date t. That equation expresses in particular the logic, that agents accept a currency at some date t at a strictly positive price only because they believe that the currency will have a positive price at the following date $t+1$ with some nonzero probability. Put differently, if today sufficiently many agents believe that tomorrow sufficiently many agents believe that the currency has value, then its price today is positive indeed. In that sense, monetary equilibria are self-fulfilling, see also Sockin and Xiong (2018). Suppose that the sale of a new currency or coin in $t+1$ takes (mining) effort in date t. A miner mines the first coin only if her joint belief about the measure of agents who will accept the currency tomorrow as well as the price of the currency justifies her effort level today.

The equilibrium condition (1) requires sellers to be marginally indifferent in accepting all positively priced currencies. Assume that the quantity of any cryptocurrency cannot decrease. Let Q_t^k denote the price of cryptocurrency k at date t. Let n_t the random quantity of distinct cryptocurrencies available for purchase at date t. Denote by $B^k_{t_0^k} \geq 0$ the initial stock of cryptocurrency k, where $t_0^k \geq 0$ is the time of the initial coin offering of currency k. Let B_t^k the stock of that cryptocurrency at date t where $B_t^k = 0$ for $t < t_0^k$ before the ICO. Then, the real price bound of Corrolary 2 generalizes to $\sum_{k=1}^{n_t} \frac{Q_t^k}{P_t} B^k_{t_0^k} \leq \bar{y}$, since we must have $\sum_{k=1}^{n_t} \frac{Q_t^k}{P_t} B_t^k \leq y_t$. The result on Bitcoin disappearance may no longer obtain. If over time new currencies enter faster than established currencies devalue, the total purchasing power of all cryptocurrencies may not vanish and can even increase.

8. Conclusions

This paper analyzes the evolution of cryptocurrency prices and the consequences for monetary policy in a model, in which a cryptocurrency such as Bitcoin coexists and competes with a traditional fiat money (Dollar) for usage. A central bank targets a stochastic inflation level for the Dollar via appropriate monetary injections, while Bitcoin production is decentralized via proof-of-work and Bitcoin supply may only increase over time. Both monies can be used for transactions. We derive a "fundamental pricing equation" for Bitcoin prices when both currencies are simultaneously in use. It implies that Bitcoin prices form a martingale in a special case. Due to currency competition, the block rewards are not a tax on Bitcoin holders but are financed by Dollar taxes imposed by the central bank. We provide a "speculative price bound" when Bitcoins are held back in transactions in the hope of a Bitcoin price appreciation. We provide conditions, under which no speculation in Bitcoins arises. Price volatility therefore does not invalidate the medium-of-exchange function of Bitcoin. We further provide a general method for constructing equilibria, thereby demonstrating their existence. Specific examples show that the Bitcoin price might appreciate or depreciate in expectation, or a mix thereof. We study the implications for monetary policy under a "conventional" as well as an "unconventional" scenario. In the conventional scenario, the Bitcoin price

[10] The results might get altered, if the tax schedule is nonlinear, as one could now introduce multiple equilibria. They would also be altered, if, say, the tax receipts collected from green agents are redistributed to red agents. Of course, equilibrium-altering redistributive taxes do not require the taxation of Bitcoin production.

evolves exogenously, thereby driving the Dollar injections needed by the central bank to achieve its inflation target. In the unconventional scenario, we suppose that the inflation target is achieved for a range of monetary injections, which then, however, influence the price of Bitcoins. We discuss the taxation of Bitcoin production.

For our analysis, one should think of Bitcoin as a representative of the family of intrinsically worthless, storable, non-dividend paying objects, which are used as a medium of exchange but whose price process is not manipulated or stabilized by a third institution such as a central bank. We abstract from several features which distinguish Bitcoin from for instance Visa, Paypal, and cash such as censorship resistance, transparency, and speed of trading. These characteristics may be important for future research when thinking about Bitcoin speculation.

Acknowledgment

Linda's work was conducted in the framework of the ECODEC laboratory of excellence, bearing the reference ANR-11-LABX-0047.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jmoneco.2019.07.002.

References

Bewley, T., 1977. The permanent income hypothesis: a theoretical formulation. J. Econ. Theory 16 (2), 252–292.
Cong, L. W., He, Z., Li, J., 2018. Decentralized mining in centralized pools. draft.
Fernández-Villaverde, J., Sanches, D., 2016. Can currency competition work? draft.
Garratt, R., Wallace, N., 2017. Bitcoin 1, bitcoin 2,...: an experiment in privately issued outside monies. draft.
Hayek, F., 1976. The Denationalization of Money. Institute of Economic Affairs.
Huberman, G., Leshno, J. D., Moallemi, C. C., 2017. Monopoly without a monopolist: an economic analysis of the bitcoin payment system. draft.
Kareken, J., Wallace, N., 1981. On the indeterminacy of equilibrium exchange rates. Q. J. Econ. 96 (2), 207–222.
Lagos, R., Wright, R., 2005. A unified framework for monetary theory and policy analysis. J. Polit. Econ. 113 (3), 463–484.
Ljungvist, L., Sargent, T.J., 2018. Recursive Macroeconomic Theory, Fourth ed. MIT Press.
Manuelli, R.E., Peck, J., 1990. Exchange rate volatility in an equilibrium asset pricing model. Int. Econ. Rev. 559–574.
Prat, J., Walter, B., 2018. An equilibrium model of the market for bitcoin mining. draft.
Schilling, L., Uhlig, H., 2019. Currency substitution under transaction costs. In: AEA Papers and Proceedings, pp. 83–87.
Sockin, M., Xiong, W., 2018. A model of cryptocurrencies. draft.
Townsend, R., 1980. Models of money with spatially separated agents. In: Kareken, J., Wallace, N. (Eds.), Models of Monetary Economies. Federal Reserve Bank of Minneapolis, pp. 265–303..
Zhu, Y., Hendry, S., 2018. A framework for analyzing monetary policy in an economy with e-money. draft.