

Economics – Optional (2023, B-I)

Economics - I

Advanced Microeconomics

Elasticity of Demand

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Elasticity of Demand - Meaning & Significance

- ◆ Law of demand depicts only the direction of change in quantity demanded of a commodity in response to a change in its price.
- ◆ This does not tell us by how much or to what extent the quantity demanded of a good will change in response to a change in its price. Answer to this question is provided by the concept of price elasticity of demand.
- ◆ As demand for a good is determined by its price, income of the people, prices of related goods, etc., the concept of elasticity of demand therefore refers to the degree of responsiveness of quantity demanded of a good to a change in its price, consumers' income and prices of related goods.

Price Elasticity of Demand

$$\epsilon_{Q,P} = \frac{\text{percentage change in quantity}}{\text{percentage change in price}}$$

$$\epsilon_{Q,P} = \frac{\Delta Q}{\Delta P} \frac{P}{Q}$$

- ◆ When changes in prices are significant, suggested to use the mid-point formula.

Elasticity of Demand - Meaning & Significance

Table : Interpretation of different values of Price Elasticity of Demand

	Value of $\epsilon_{Q,P}$	Classification	Meaning
1	0	Perfectly inelastic demand	Quantity demanded is completely insensitive to price.
2	Between 0 and -1	Inelastic demand	Quantity demanded is relatively insensitive to price.
3	-1	Unitary elastic demand	Percentage increase in quantity demanded is equal to percentage decrease in price.
4	Between -1 and - ∞	Elastic demand	Quantity demanded is relatively sensitive to price.
5	- ∞	Perfectly elastic demand	Any increase in price results in quantity demanded decreasing to zero, and any decrease in price results in quantity demanded increasing to infinity.

Note: Values are negative because $\epsilon_{Q,P}$ is negative, but to avoid confusion, we can use absolute values of $\epsilon_{Q,P}$

Elasticity of Demand - Meaning & Significance

Elasticities along Specific Demand Curves

Linear Demand Curves

$$Q = a - bP$$

The inverse demand curve for the linear demand curve is given by

$$P = \frac{a}{b} - \frac{1}{b} Q$$

The term a/b is called the **choke price**. This is the price at which the quantity demanded falls to 0. It may be verified that quantity demanded falls to 0 at the choke price by substituting $P = a/b$ into the equation of the demand curve:

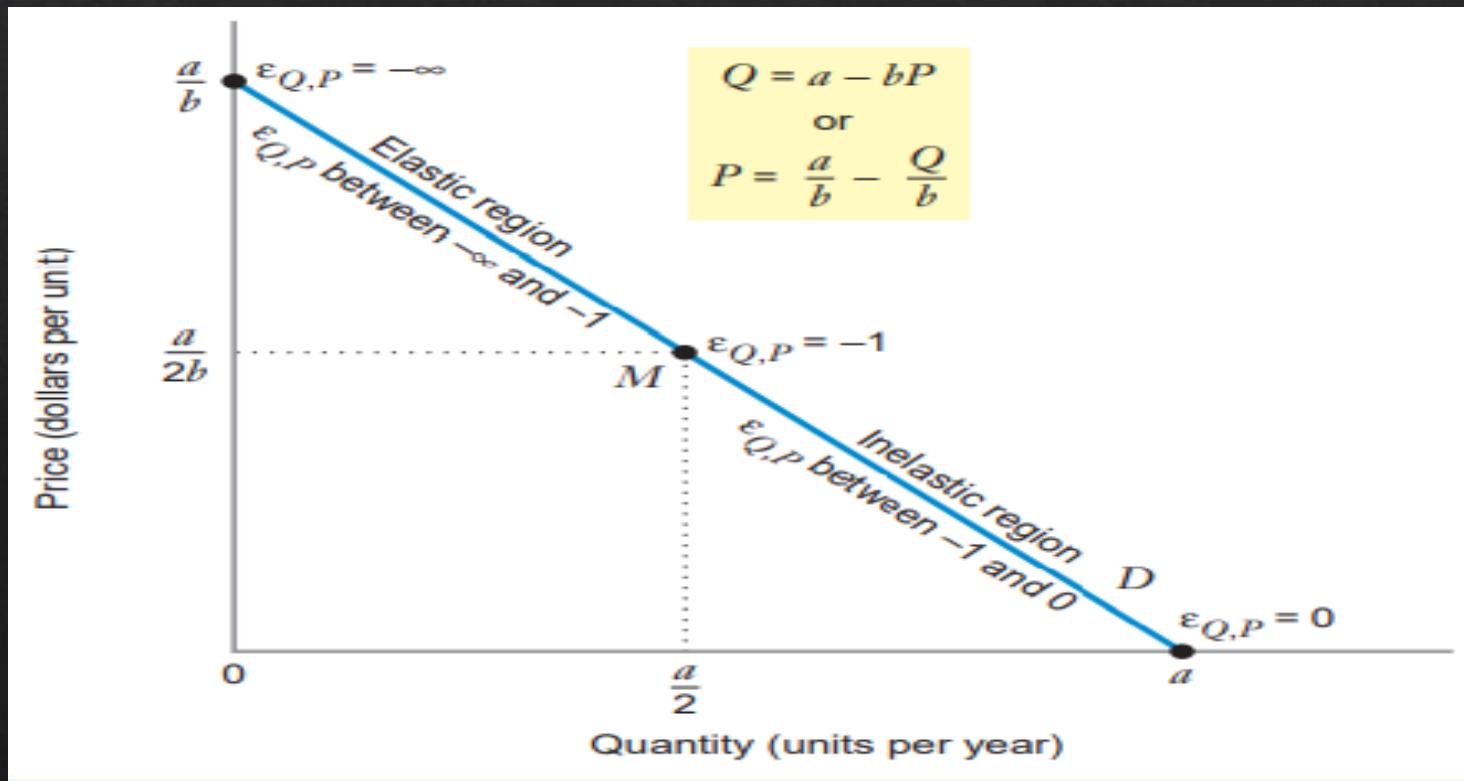
$$\begin{aligned} Q &= a - b\left(\frac{a}{b}\right) \\ &= a - a \\ &= 0 \end{aligned}$$

price elasticity of demand for the linear demand curve

$$\epsilon_{Q,P} = \frac{\Delta Q}{\Delta P} \frac{P}{Q} = -b \frac{P}{Q}$$

Elasticities along Specific Demand Curves

- On a linear demand curve, the price elasticity of demand varies as we move along the curve. Between the choke price a/b (where $Q = 0$) and a price of $a/2b$ at the midpoint M of the demand curve, the price elasticity of demand is between $-\infty$ and -1 . This is known as the elastic region of the demand curve. For prices between $a/2b$ and 0 , the price elasticity of demand is between -1 and 0 . This is the inelastic region of the demand curve.



Elasticities along Specific Demand Curves

- Constant Elasticity Demand Curves

$$Q = aP^{-b}$$

- where a and b are positive constants. For the constant elasticity demand curve, the price elasticity is always equal to the exponent $-b$
- Proof:

$$\frac{dQ}{dP} = -baP^{-(b+1)}$$

Forming the expression for the point elasticity of demand, we have

$$\begin{aligned}\epsilon_{Q, P} &= \frac{dQ}{dP} \frac{P}{Q} \\ &= -baP^{-(b+1)} \times \frac{P}{aP^{-b}} \text{ (substituting in the expression for } Q\text{)} \\ &= -b \text{ (after canceling terms)}\end{aligned}$$

This shows that the price elasticity of demand for the constant elasticity demand curve is simply the exponent in the equation of the demand curve, $-b$. (For more on the use of derivatives, see the Mathematical Appendix at the end of the book.)

Elasticities along Specific Demand Curves

- ◆ Measurement of Price Elasticity of Demand at a Point of Demand Curve
- ◆ The Measure of price elasticity of demand is:

$$e_p = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

- ◆ $\Delta Q/\Delta P$ = reciprocal of the slope of the demand curve, thus

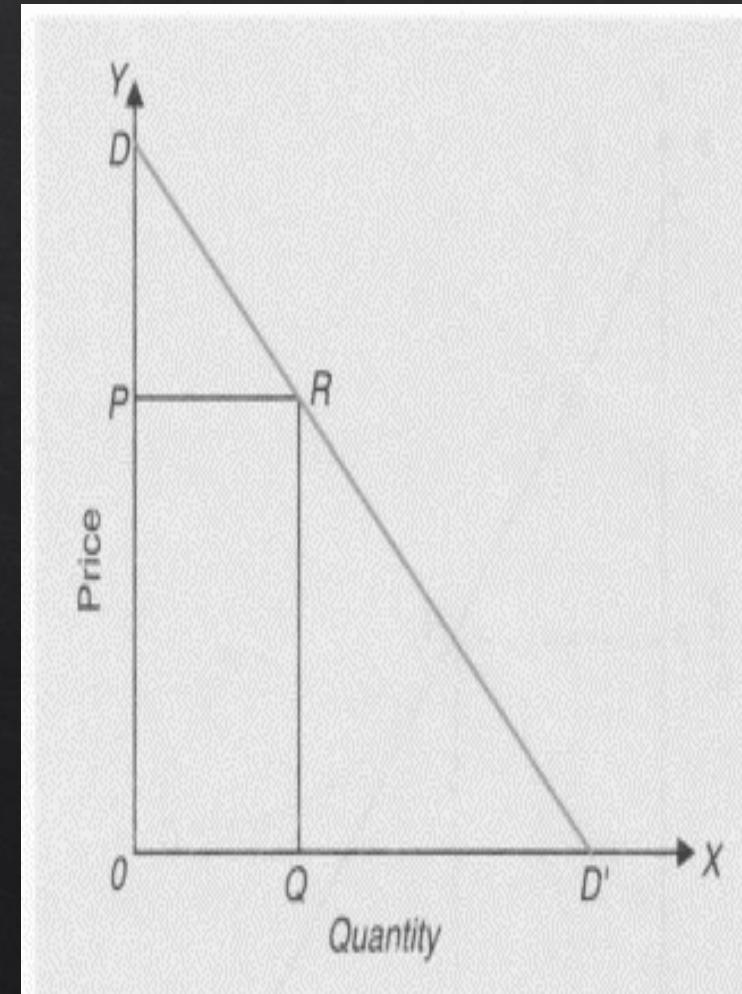
$$e_p = \frac{1}{\text{slope}} \cdot \frac{P}{Q}$$

- ◆ In figure, at point R, original $P = OP$ and original $Q = OQ$ and the slope of DD' is :

$$\frac{\Delta P}{\Delta Q} = \frac{PD}{PR}$$

Substituting these values in the above formula we have

$$\begin{aligned} e_p &= \frac{1}{\frac{PD}{PR}} \times \frac{OP}{OQ} \\ &= \frac{PR}{PD} \times \frac{OP}{OQ} \end{aligned}$$



Elasticities along Specific Demand Curves

- ◊ In the figure, PR = OQ, therefore,

$$e_p = \frac{OP}{PD}$$

- ◊ Measuring price elasticity by taking the ratio of these distances on the vertical axis, that is OP/PD is called vertical axis formula.
- ◊ If two aright-angled triangles are similar, ratio of their sides will also be similar. Therefore,

$$e_p = \frac{OP}{PD} = \frac{RD'}{RD}$$

- ◊ RD' is the lower segment of the demand curve DD' at point R and RD is its upper segment. Therefore,

$$e_p = \frac{RD'}{RD} = \frac{\text{lower segment}}{\text{upper segment}}$$

- ◊ Measuring price elasticity at a point on the demand curve by measuring the ratio of the distances of lower segment and upper segment is a popular method of measuring point price elasticity on a demand curve.

Elasticities along Specific Demand Curves

- ◆ If the demand curve is not a straight line , and is a non-linear curve, we have to draw a tangent TT' at the given point R on the demand curve DD' and then measure price elasticity by finding the ratio of the lower to upper segment of the tangent drawn at any point of the non-linear demand curve.

Comparing Price Elasticity on the Two Demand Curves with Different Slopes

Given that price elasticity at a point on the demand curve is equal to lower segment/upper segment.

$$e_p \text{ on } DA \text{ at point E} = EA/ED$$

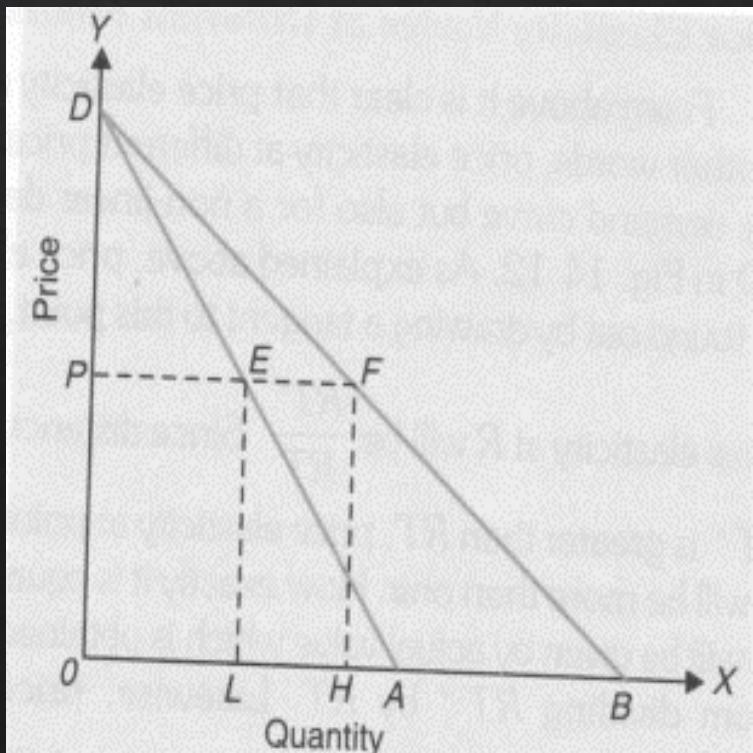
$$e_p \text{ on } DB \text{ at point F} = FB/FD$$

Now, take triangle ODA which is a right-angled triangle in which PE is parallel of OA , it follows $EA/ED = OP/PD$. Thus, e_p on $DA = OP/PD$

Now in ΔODB , PF is parallel to OB , therefore $FB/FD = OP/PD$

It is clear, e_p on points E and F on two demand curves = OP/PD

Thus, elasticity of demand at points E and F are equal though the slopes of these two demand curves are different



Elasticities along Specific Demand Curves

Price Elasticity of Demand and Total Revenue

Price change Elasticity greater than one elasticity less than one Elasticity equal to one

$$(e > 1)$$

$$(e < 1)$$

$$(e_p = 1)$$

Price falls

TE increases

TE decreases

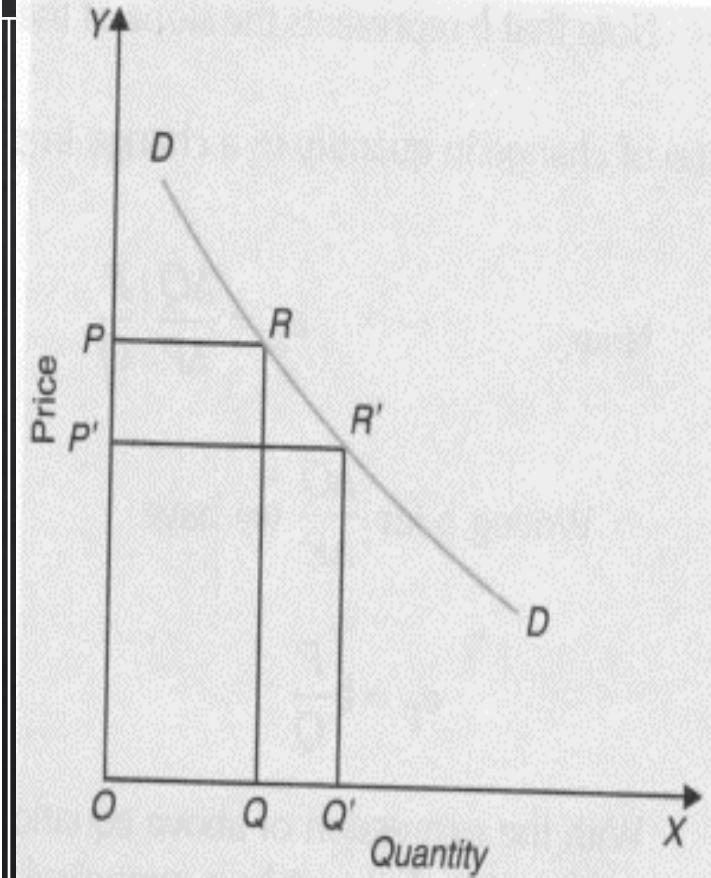
No change in TE

Price rises

TE decreases

TE increases

No change in TE



Determinants of Elasticity of Demand

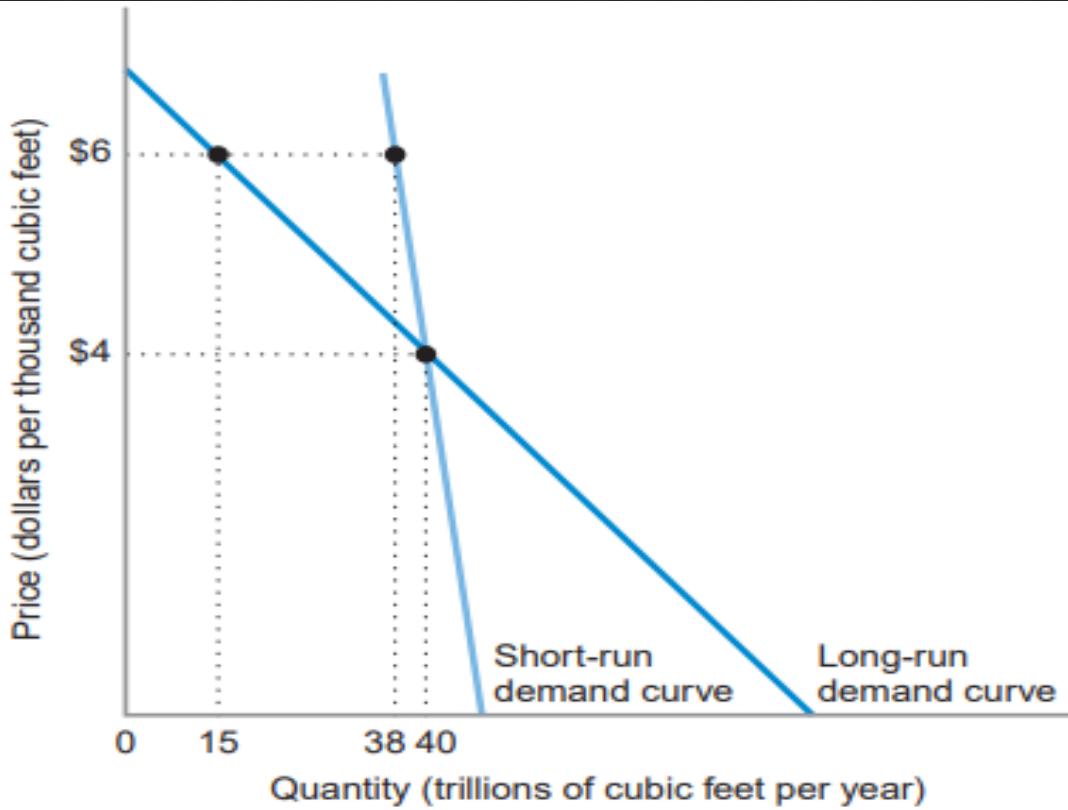
- ❖ *The Availability of Substitutes.* Demand tends to be more price elastic when there are good substitutes for a product .
- ❖ *The Proportion of Consumer's Income Spent.* Demand tends to be more price elastic when a consumer's expenditure on the product is large .
- ❖ *The Number of Uses of a Commodity.* The greater the number of uses to which a commodity can be put, the greater will be its price elasticity of demand. To illustrate, milk has several uses. If its price rises to a very high level, it will be used only for essential purposes such as feeding the children and sick persons.
- ❖ *Complementarity Between Goods.* Households are generally less sensitive to the changes in prices of goods that are complementary with each other or which are jointly used as compared to those goods which have independent demand or used alone. For example, for the running of automobiles, besides petrol, lubricating oil is also used. Now, if price of lubricating oil goes up, it will mean a very small increase in the total cost of running the automobile, since the use of oil is much less as compared to other things such as petrol. Thus, the demand for lubricating oil tends to be inelastic.
- ❖ *Time and Elasticity.* The element of time also influences the elasticity of demand for a commodity. Demand tends to be more elastic if the time involved is long

Greater Elasticity in the Long Run Than in the Short Run

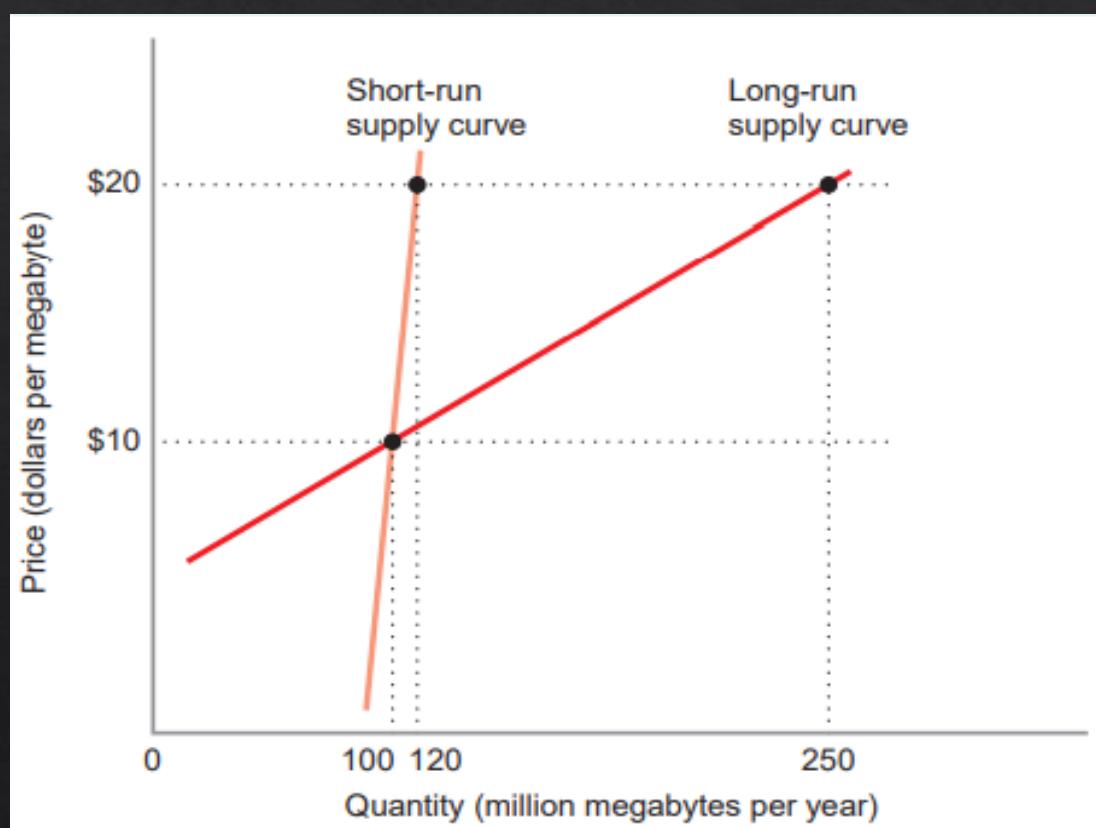
- ❖ Thus, it is useful to distinguish between the long-run demand curve for a product--the demand curve that pertains to the period of time in which consumers can fully adjust their purchase decisions to changes in price--and the short-run demand curve—the demand curve that pertains to the period of time in which consumers cannot fully adjust their purchasing decisions to changes in price. We would expect that for products, such as natural gas, for which consumption is tied to physical assets whose stocks change slowly, long-run demand would be more price elastic than short-run demand. **The long-run demand curve is “flatter” than the short-run demand curve.**
- ❖ Similarly, firms sometimes cannot fully adjust their supply decisions in response to changes in price. For example, in the short run, a producer of semiconductors might not be able to increase its supply of chips in response to an increase in price by very much because it faces a capacity constraint--a facility can only produce so many chips, even if extra workers are hired. However, if the price increase is expected to be permanent, then the firm can expand the capacity of its existing facilities or build new ones. The increase in the quantity supplied as a result of the price increase will thus be greater in the long run than in the short run.

Greater Elasticity in the Long Run Than in the Short Run

Short-Run and Long-Run Demand Curves for Natural Gas



Short-Run and Long-Run Supply Curves for Semiconductors



Cross Elasticity of Demand

- Very often demand for two goods are so related to each other that when price of any of them changes, the demand for the other goods also changes. Therefore, the degree of responsiveness of change in the demand for one good in response to change in price of another good represents the cross elasticity of demand of one goods for the other

$$\text{Coefficient of cross elasticity of demand of } X \text{ for } Y = \frac{\text{Proportionate change in the quantity demanded of } X}{\text{Proportionate change in the price of good } Y}$$

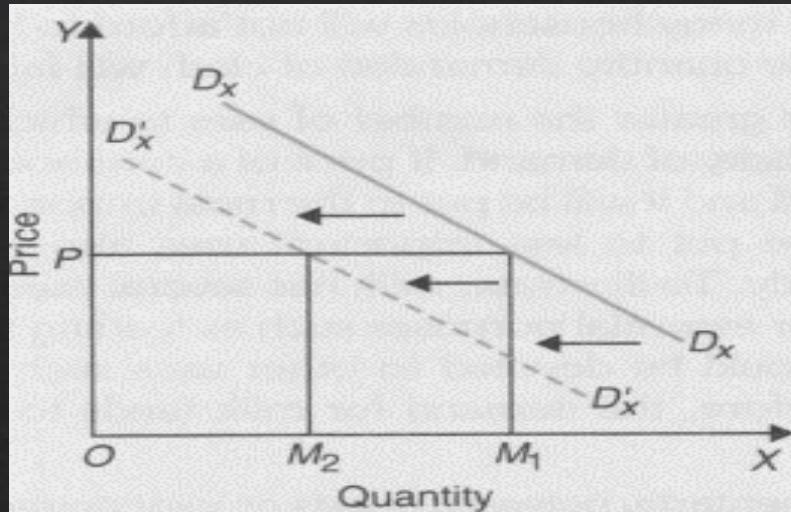
$$\text{or, } e_c = \frac{\frac{\Delta q_x}{q_x}}{\frac{\Delta p_y}{p_y}} = \frac{\Delta q_x}{q_x} \times \frac{p_y}{\Delta p_y}$$

$$= \frac{\Delta q_x}{q_x} \times \frac{p_y}{\Delta p_y}$$

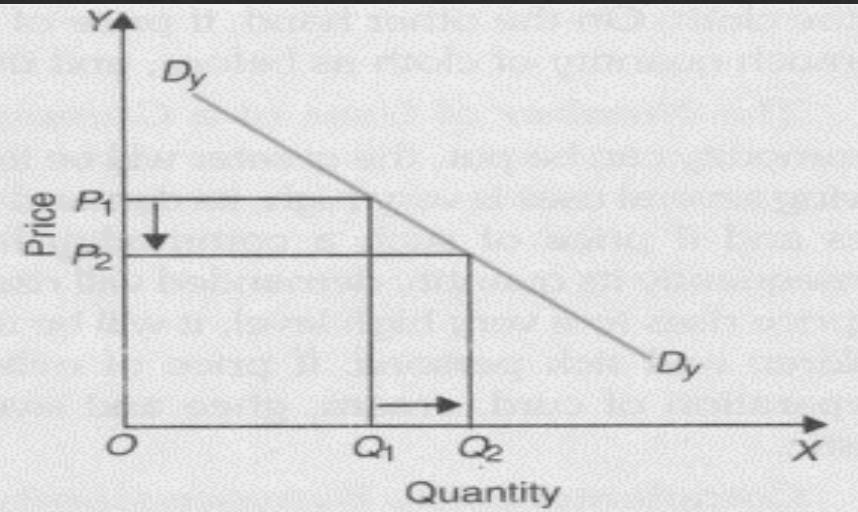
$$= \frac{\Delta q_x}{\Delta p_y} \times \frac{p_y}{q_x}$$

Cross Elasticity of Demand

- Demand Relations Between Two Substitutes Goods



(a) Demand for Goods X



(b) Demand for Goods Y

- Cross Elasticity of Demand: Substitutes and Complements

- Substitutes:** When two goods are substitutes of each other, then as a result of the rise in price of one good, the quantity demanded of the other good increases. Therefore, the cross elasticity of demand between the two substitute goods is positive, that is, in response to the rise in price of one good, the demand for the other good rises. Substitute goods are also known as competing goods.

Cross Elasticity of Demand

- ❖ **Complements:** When the two goods are complementary with each other such as bread and butter, tea and milk, etc., the rise in price of one good brings about the decrease in demand for the other. Therefore, the cross elasticity of demand between the two complementary goods is negative. Thus, according to the classification based on the concept of cross elasticity of demand, goods X and Y are substitutes or complements according as the cross elasticity of demand is positive or negative.

Importance of Cross Elasticity of Demand for Business Decision Making

- ❖ The concept of cross elasticity of demand is of great importance in managerial decision making for formulating proper price strategy. Multiproduct firms often use this concept to measure the effect of change in price of one product on the demand for other products. For example, Maruti Udyog Ltd. produces Maruti Suzuki Dzire, Maruti Suzuki Brezza and Maruti Suzuki Baleno.
- ❖ These products are good substitutes of each other and therefore cross elasticity of demand between them is very high. If Maruti Udyog decides to lower the price of Maruti Dzire, it will significantly affect the demand for Maruti Brezza and Baleno. So it will formulate a proper price strategy fixing appropriate price for its various products.

Cross Elasticity of Demand

- ❖ **Second,** the concept of cross elasticity is used in defining the boundaries of an industry and in measuring interrelationship between industries. An industry is defined as a group of firms producing similar products (that is, products with a high positive cross elasticity of demand). For example, cross elasticity of demand between Maruti Brezza, Hyundai Kreta, Ford Ecosport is positive and quite high. They therefore belong to the same industry (i.e., automobiles).
- ❖ Further, the concept of cross elasticity of demand is extremely used in deciding cases relating to Anti-trust laws and monopolistic practices used by firms. It so happens that in order to reduce competition that one dominant firm producing a product with a high cross elasticity of demand with the products of other firms tries to take over them and thereby establish a monopoly or different firms try to merge with each other to form a cartel to enjoy monopolistic profits. These actions are held illegal by Anti-trust or anti-monopoly laws.

Income Elasticity of Demand

- Income elasticity of demand shows the degree of responsiveness of quantity demanded of a good to a small change in income of consumers.

$$\text{Income Elasticity}(\epsilon_{yd}) = \frac{\text{Proportionate Change in purchases of a good}}{\text{Proportionate change in income}}$$

- Let M stand for an initial income, ΔM for a small change in income, Q for the initial quantity purchased demand, ΔQ for a change in quantity purchased as a result of a change in income and e_I for income elasticity of demand. Then

$$\begin{aligned} e_I &= \frac{\frac{\Delta Q}{Q}}{\frac{\Delta M}{M}} = \frac{\Delta Q}{Q} \times \frac{M}{\Delta M} \\ &= \frac{\Delta Q}{\Delta M} \cdot \frac{M}{Q} \end{aligned}$$

Income Elasticity of Demand

Income Elasticity, Normal Goods and Inferior Goods

- ❖ Value of zero income elasticity of demand is of great significance. Zero income elasticity of demand for a good implies that a given increase in income does not at all lead to any increase in quantity demanded of the good or increase in expenditure on it.
- ❖ When $\varepsilon_{yd} > 0$, then an increase in income leads to the increase in quantity demanded of the goods. This happens in case of normal goods. On the other side there are all those goods which $\varepsilon_{yd} < 0$, (that is, negative) and in such cases increase in income leads to the fall in quantity demanded of the goods. Goods having negative income elasticity are known as inferior goods.
- ❖ Zero-income elasticity is a significant value, for it represents a dividing line distinguishing normal goods from inferior goods.

Income Elasticity, Luxuries and Necessities

- ❖ $\varepsilon_{yd} = 1$, represents a useful dividing line. If the $\varepsilon_{yd} > 1 \rightarrow$ luxury goods.
- ❖ If, $\varepsilon_{yd} < 1$, \rightarrow reflects necessary goods).

Income Elasticity of Demand

Income Elasticity Defined in Terms of Expenditure

As defined above,

$$e_i = \frac{\Delta Q}{Q} \div \frac{\Delta M}{M} = \frac{\Delta Q}{Q} \times \frac{M}{\Delta M}$$

Multiplying the numerator and denominator by P , we get

$$e_i = \frac{\Delta Q \cdot P}{Q \cdot P} \times \frac{M}{\Delta M}$$

Now, as explained above, $Q \cdot P$ is the expenditure made on the good and $\Delta Q \cdot P$ is the change in expenditure made as a result of change in income. Let E stand for the expenditure made on the good. Then the above equation will become:

$$e_i = \frac{\Delta E}{E} \times \frac{M}{\Delta M}$$

or $e_i = \frac{\Delta E}{\Delta M} \cdot \frac{M}{E}$

Thus, income elasticity = $\frac{\text{Change in expenditure on a good}}{\text{Change in income}} \times \frac{\text{Income}}{\text{Expenditure on a good}}$

Income Elasticity of Demand

- ◆ Another important value of income elasticity is the reciprocal of proportion of consumer's income spent on a good, that is $1/K_x$ where K_x stands for the proportion of consumer's income spent on a good X.
- ◆ The value of $1/K_x$ for the income elasticity of demand is significant because when income elasticity for a good equal $1/K_x$, then the whole of the increase in consumer's income will be spent on the increase in quantity purchased of the good X.
- ◆ Proof: We know that ξ_{yd} is

$$\frac{M}{E} \cdot \frac{\Delta E}{\Delta M}$$

- ◆ Suppose ΔM is spent on the good X, then ΔE on the good X = to ΔM , price of good X remaining the same. Substituting ΔM for ΔE in the above measure of income elasticity, we get

$$\begin{aligned} e_i &= \frac{M}{E} \cdot \frac{\Delta M}{\Delta M} = \frac{M}{E} = \frac{1}{E/M} \\ &= \frac{1}{K_x} \text{ (where } K_x = E/M = \text{proportion of income spent on good X).} \end{aligned}$$

Income Elasticity of Demand

Measuring Income Elasticity at a Point on an Engel Curve

- An Engel curve shows the relationship between quantity demanded of a good and level of consumer's income. Since with the increase in income, normally more quantity of the good is demanded, Engel curve slopes upward (i.e., it has a positive slope).

Measuring Income Elasticity at Point R on the Engel Curve

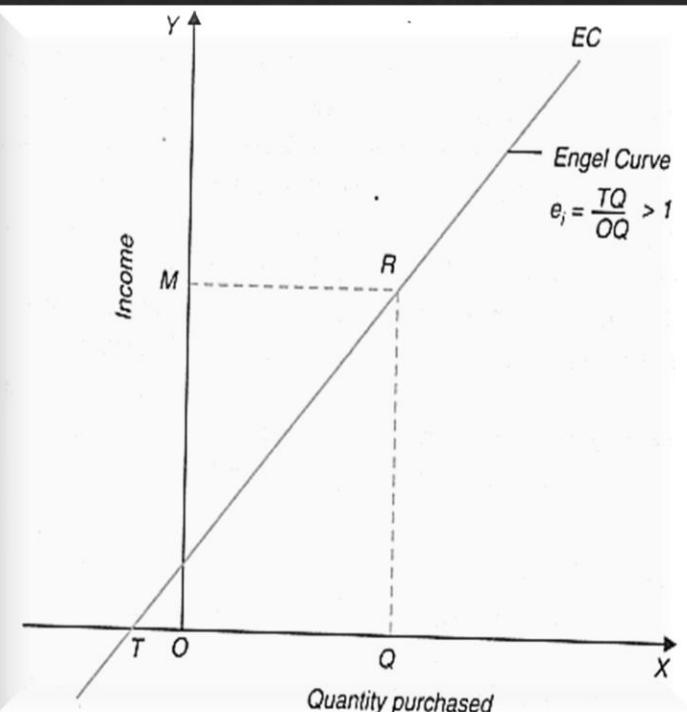
At point R, the $OQ = Q_d$ at OM level of income. Let us extend the Engel curve EC downward so that it meets the X-axis at point T.

$\Delta Q / \Delta M$ is the reciprocal of slope of EC, thus, re-writing

$$e_i = \frac{1}{\Delta M / \Delta Q} \cdot \frac{M}{Q}$$

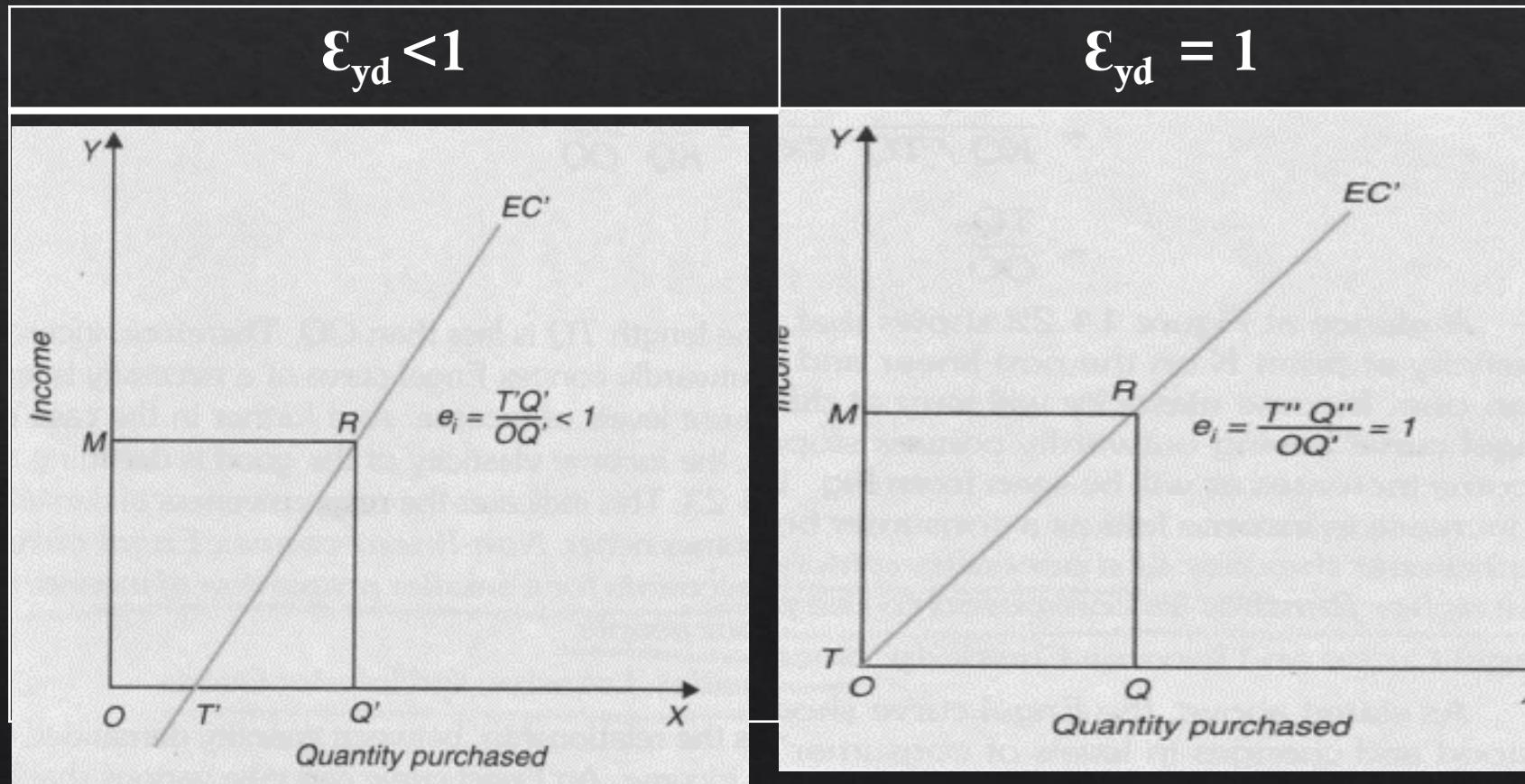
At R, Slope of EC = $\Delta Q / \Delta M = RQ / TQ$, and $M = RQ$ and $Q = OQ$
Then, we get

$$\begin{aligned} e_i &= \frac{1}{\Delta M / \Delta Q} \cdot \frac{M}{Q} = \frac{1}{RQ / TQ} \cdot \frac{RQ}{OQ} \\ &= \frac{TQ}{RQ} \cdot \frac{RQ}{OQ} = \frac{TQ}{OQ} \end{aligned}$$



Income Elasticity of Demand

- Since $TQ > OQ$, ϵ_{yd} at point R > 1 . It may therefore be concluded that if Engel curve on being extended downward meets X-axis to the left of the point of origin' income elasticity will be greater than one.



Income Elasticity of Demand

Income Elasticity and Proportion of Income Spent

- ❖ There is a useful relationship between income elasticity for a good on the one hand and proportion of income spent on it. The relationship between the two is described in the following three propositions:
 - If proportion of income spent on the good remains the same as income increases, then income elasticity for the good is equal to one.

$$\frac{E + \Delta E}{M + \Delta M} = \frac{E}{M}$$

or, $M(E + \Delta E) = E(M + \Delta M)$

$$EM + M \cdot \Delta E = EM + E \Delta M$$

Since EM occurs on both sides of the equation, it will cancel out. Therefore, $M \Delta E = E \Delta M$

or $\frac{\Delta E}{\Delta M} \cdot \frac{M}{E} = 1$

Income Elasticity of Demand

- › If the proportion of income spent on the good rises as income increases, the income elasticity is greater than unity.

As stated above, E/M is the proportion of income spent on a good, E stands for expenditure made on the good and M for consumer's income. Suppose income increases by ΔM and as a result the income spent on the good rises by ΔE , the new proportion of income spent on the good will equal to

$$\frac{E + \Delta E}{M + \Delta M}.$$

$$\frac{E + \Delta E}{M + \Delta M} > \frac{E}{M}$$

or

$$\begin{aligned}M(E + \Delta E) &> E(M + \Delta M) \\EM + M \cdot \Delta E &> EM + E \cdot \Delta M \\M \cdot \Delta E &> E \cdot \Delta M\end{aligned}$$

$$\frac{\Delta E}{\Delta M} \cdot \frac{M}{E} > 1$$

Hence, $e_t > 1$.

- › If the proportion of income spent on the good decreases as income increases, income elasticity is less than one.

Income Elasticity of Demand

Sum of Income Elasticities, Budget Constraint and Expenditure

- ❖ Increase in income can be spent on a number of goods and services demanded by a consumer.
- ❖ Relationship = sum of change in the proportion of expenditure spent on all goods and services in the consumer's basket must be unity
- ❖ Suppose that consumer has an income equal to M and further that all his income is spent on two goods X and Y . Let P_x and P_y represent the prices of two goods X and Y respectively. Let income increases by ΔM and as a result quantity purchased of two goods increase by ΔQ_x and ΔQ_y . Then, budget constraint can be written as

$$\Delta M = P_x \cdot \Delta Q_x + P_y \cdot \Delta Q_y$$

Dividing both sides of the budget equation by ΔM we have

$$\frac{\Delta M}{\Delta M} = \frac{P_x \cdot \Delta Q_x}{\Delta M} + \frac{P_y \cdot \Delta Q_y}{\Delta M}$$

or,

$$\frac{P_x \cdot \Delta Q_x}{\Delta M} + \frac{P_y \cdot \Delta Q_y}{\Delta M} = 1$$

Income Elasticity of Demand

- ❖ The above equation implies that the
 - sum of the proportion of increase in income spent on good X = $P_x \cdot \Delta Q_x / \Delta M$ and the proportion of increase in income spent on good Y = $P_y \cdot \Delta Q_y / \Delta M$ must be = 1
- ❖ Multiplying the first term on the left side of equation by Q_x/Q_x and M/M and multiplying the second term by Q_y/Q_y and M/M , we have

$$\frac{P_x \cdot Q_x}{M} \cdot \frac{\Delta Q_x}{\Delta M} \cdot \frac{M}{Q_x} + \frac{P_y \cdot Q_y}{M} \cdot \frac{\Delta Q_y}{\Delta M} \cdot \frac{M}{Q_y}$$

- ❖ Using K_x to denote proportion of income spent on good X, K_y for the proportion of income spent on good Y, ϵ_{xi} for income elasticity of demand for good X, and ϵ_{yi} , for income elasticity of demand for good Y, then from the above equations, we have:

$$K_x \epsilon_{xi} + K_y \epsilon_{yi} = 1$$

- ❖ The above equation, called **Engel's Aggregation**, reveals that the weighted sum of the income elasticities of demand for all goods (weights being the proportion of income spent on goods) must equal unity

Income Elasticity of Demand

- ❖ This for example, means that when income increases by 10 per cent, the budget constraint requires that purchases or consumption of goods as a whole must also increase by 10 per cent. If there is a 10 per cent increase in income, if the consumption of some goods increases by less than 10 per cent, the consumption of others must increase by more than 10 per cent so that the increase in income must somehow be spent on the goods.
- ❖ That is, the increase in consumption of goods whose income elasticity of demand is less than one must be offset by increase in consumption of others for which income elasticity of demand is greater than one. Thus, on the average, consumption of goods must increase by the same proportion as income.
- ❖ This equation implies that given the two goods on which a consumer spends his income, if we know the proportion of income spent of one good and the income elasticity for that good, we can calculate the income elasticity of demand for the other good.

Income Elasticity of Demand

Importance of Income Elasticity for Business Firms

- ❖ First, the firms producing products which have a high-income elasticity have great potential for growth in an expanding economy. For example, if for a firm's product income elasticity of demand is greater than one, it means that it will gain more than proportionately to the increase in national income. Thus, firms which are producing products having high income elasticity are more interested in forecasting the level of aggregate economic activity (i.e., level of national income) because the demand for their products greatly depends on the level of overall economic activity. Therefore, the demand for luxuries fluctuates very much during different phases of business cycles. During boom periods, demand for luxuries increases very much and declines sharply during recessionary periods.
- ❖ On the other hand, the demand for products with low-income elasticity will not be greatly affected by the fluctuations in aggregate economic activity. During booms the demand for these products will not increase much and during recessions it will not decrease sharply. It is generally necessities for which demand is not much income elastic. Of course, to share the benefits of increasing national income firms currently producing products with low-income elasticity would try to enter the industries demand for whose products is highly income elastic as this would ensure better growth opportunities.

Income Elasticity of Demand

- ❖ The knowledge of income elasticity of demand also plays a significant role in designing marketing strategies of the firms. If income of people is an important determinant of demand for a product, the firms producing products with high income elasticity of demand will be located in those areas or set up their sales outlets in those cities or regions where incomes are increasing rapidly. Besides, the firms will direct their advertising campaigns and other sales promotion activities to those segments of people whose income is high and also increasing rapidly. This is to ensure higher growth of sales of their products.
- ❖ The concept of income elasticity of demand shows clearly why farmers' income do not rise equal to that of urban people engaged in manufacturing industries. Income elasticity of demand for agricultural products such as food grains is less than one. This implies that it is difficult for the farmers' income from agriculture to increase in proportion to the expanding national income. Thus, farmers' income cannot keep pace with the urban people who derive their income from industries producing goods with high income elasticity of demand.