Afleveringsopgave 1

INTRODUKTION TIL STATISTIK OG SANDSYNLIGHEDSTEORI

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4.9.2023

Chapter 1 - Exercise 14

In this exercise we let A and B be two events such that:

$$P(A) = 0.4, P(B) = 0.7, P(A \cup B) = 0.9$$

In each of the following exercises we have to find the probability of the statements.

a

$$P(A \cap B)$$

From example 1.10 exercise e, we have proven that:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Here we can now put in the known values for $P(A \cup B), P(A) and P(B)$.

$$0.9 = 0.4 + 0.7 - P(A \cap B)$$

We can now calculate and solve this equation:

$$0.9 = 0.11 - P(A \cap B)$$

$$-0.2 = -P(A \cap B)$$

$$P(A \cap B) = 0.2$$

Therefore we have now found that $P(A \cap B)$ = 0.2

b

$$P(A^c \cap B)$$

Since that $P(A^c \cap B) = P(B - A)$ we can use example d from 1.10, which gives us the following equation:

$$P(B - A) = P(B) - P(B \cap A)$$

We now insert the known values and solve the equation:

$$P(B - A) = 0.7 - 0.2$$

$$P(B - A) = 0.5$$

Since $B - A = A^c \cap B$ we have now shown that $P(A^c \cap B) = 0.5$

c

$$P(A-B)$$

We can here use exercise D again:

$$P(A-B) = P(A) - P(A \cap B)$$

I will now insert the values:

$$P(A-B) = 0.4 - 0.2$$

$$P(A-B) = 0.2$$

Therfore P(A-B) = 0.2

d

$$P(A^c - B)$$

Again here we first use exercise D from 1.10

$$P(A^c - B) = P(A^c) - P(A^c \cap B)$$

From exercise A in 1.10, we also know that:

$$A^c = 1 - A$$

We now insert our values:

$$A^c = 1 - 0.4$$

$$A^c = 0.6$$

We can now insert our values in the one from exercise D:

$$P(A^c - B) = 0.6 - 0.5$$

Therefore we get that:

$$P(A^c - B) = 0.1$$

e

$$P(A^c \cup B)$$

Here we can use exercise E from example 1.10:

$$P(A^c \cup B) = P(A^c) + P(B) - P(A^c \cap B)$$

We can now insert the known values:

$$P(A^c \cup B) = 0.6 + 0.7 - 0.5$$

Therefore we get that

$$P(A^c \cup B) = 0.8$$

f

$$P(A \cap (B \cup A^c))$$

By using theorem 1.2, distributive law, from the book we can transform this expression to:

$$P((A \cap B) \cup (A \cap A^c))$$

Since $A \cap A^c$ results in the empty set we get:

$$P((A \cap B) \cup \emptyset)$$

Since the union with the empty set results in the same set we get:

$$P(A \cap B)$$

And from earlier exercise we know that this is 0.2. Therefore:

$$P(A \cap (B \cup A^c)) = 0.2$$