1. Fundamental Matrix Estimation from Point Correspondences:

(a) Implement the linear least-squares eight-point algorithm and report the returned fundamental matrix.

給定兩個座標係(u, v, 1)和(u', v', 1)上的點，計算點集合所構成的矩陣 Z 並求解 ZF=0。為了要讓 least square error 最小取出 Z.T*Z 最小的 eigenvector 即為 fundermental matrix。

為了保證 fundermental matrix rank2 的特性，我用 SVD 分解 fundermental matrix 並把第三個 egienvalue 設為零，最後再乘上原本的 U 跟 V。

```
f in (a):
[[ 5.63087200e-06  2.77622828e-05 -1.07623595e-02]
 [-2.74976583e-05  6.74748522e-06  1.22519240e-02]
 [ 6.42650411e-03 -1.52182033e-02  9.99730547e-01]]
```

(b) Implement the normalized eight-point algorithm and report the returned fundamental matrix.

大概步驟跟 a 相同，但點會先減掉每個點集合的平均、rescale 距離後才計算 eight-point algorithm。在確定 rank2 的時候也會乘上 normalize 中計算出來的 scaling factor 使 F 符合 2-norm=1 的限制。

```
f in (b):
[[-1.33351218e-05  3.20320025e-04  2.08672226e-02]
 [ 2.22048310e-04  2.83731330e-05 -7.52241693e-01]
 [-1.37802168e-02  6.19621780e-01 -2.22656232e-01]]
```

(c) Plot the epipolar lines and calculate average distance between feature points and epipolar lines.

Average distance

For (a): linear least-squares version

```
----Average distance between points and lines in (a)----
Fig.1: 9.701438832240644
Fig.2: 14.568227199632618
```
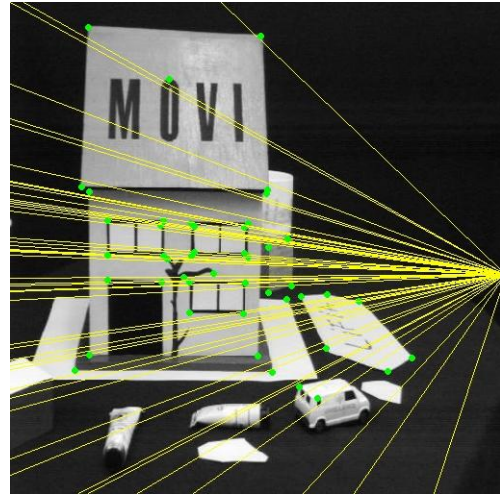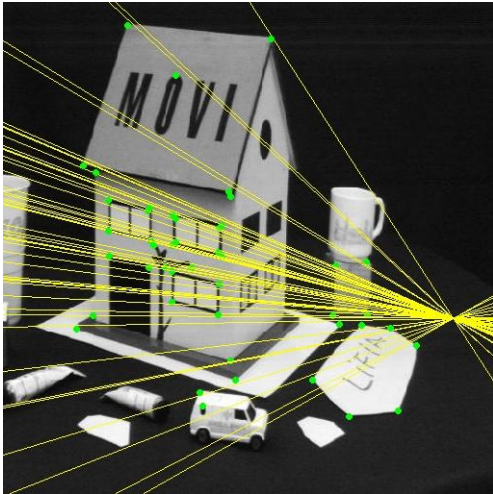
For (b): normalized version

```
----Average distance between points and lines in (b)----
Fig.1: 0.8894960616888047
Fig.2: 0.8917172367781785
```
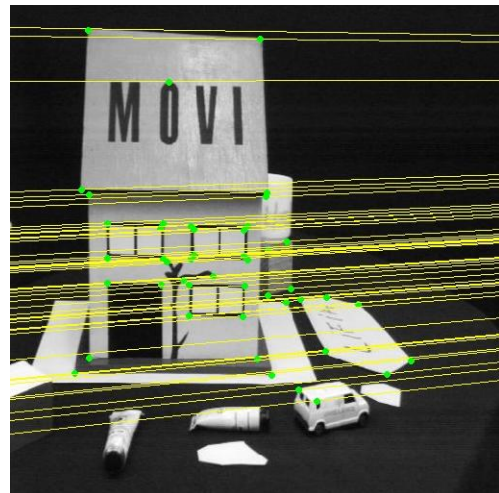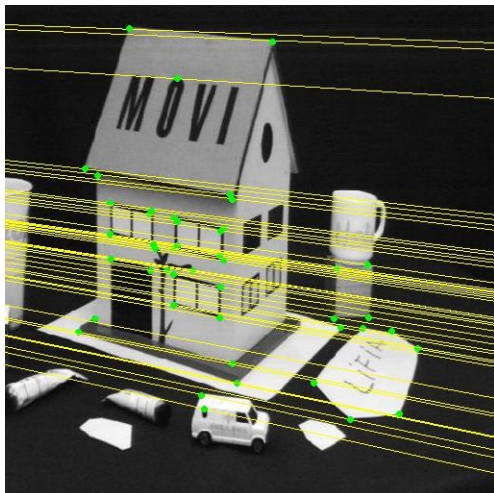
給定線 $ax + by + c = 0$ 與點 (x_0, y_0)

點與線的距離為 $\dfrac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$

For (a): linear least-squares version: fig1. (left) and fig2. (right)

For (b): linear least-squares version: fig1. (left) and fig2. (right)

2. Homography transform:

(a) Implement a function that estimates the homography matrix H that maps a set of interest points to a new set of interest points Describe your implementation.
轉換前後的點以 homogeneous coordinate 表示之後，跟第一題一樣求出方程組矩陣 Z 並用 SVD 計算出對應最小的 eigenvector，在 scaling 的部分我一樣將 H 的 norm 設為一。

(b) Specify a set of point correspondences for the source image of the Delta building and the target one. Compute the 3X3 homography matrix. The rectification is to make the new image plane parallel to the front building as best as possible. Please
找到對應台達的四個點以及映射後的對應點(我設原影像大小的四個角)，因為是 backward 回去所以要將映射後的對應點當 source 原本台達的四個點當 target 找 homography matrix，scaling 的 constraint 一樣是將 H 的 norm 設為 1。在 bilinear interpolation 時將 RGB 3 個 channels 分開做 interpolation。

Source image with selected region

Homography matrix

```
H:
 [[-6.92569676e-04   3.36418178e-04  -6.33526425e-01]
  [ 4.70644115e-05  -6.49362296e-06  -7.73718634e-01]
  [ 5.64029490e-08   5.57857637e-07  -1.77458396e-03]]
```

Rectified image