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Quantile regression based probabilistic forecasting of renewable energy generation and building electrical load: A state of the art review

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A R T I C L E I N F O

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A B S T R A C T

With the increasing penetration of renewable energy in smart grids and the increasing building electrical load, their accurate forecasting is essential for system design, control and associated optimizations. To date, probabilistic forecasting methods have attracted increasing attentions as they can assess various uncertainty impacts. Among them, quantile regression based probabilistic forecasting methods are more popular and experience fast developments. However, there is little review that systematically covers their similarities and differences in the aspects of mechanism, feature and effectiveness in applications. This paper, therefore, provides a comprehensive review of quantile regression-related methods for renewable energy generation and building electrical load. Firstly, according to their principles/mechanisms, existing quantile regression based probabilistic forecasting methods are classified into two major categories, namely statistic-based methods and machine learning-based methods. Meanwhile, their respective strengths and limitations are comparatively analyzed and summarized. Next, their practical applications and effectiveness are systematically reviewed. On the basis of the above review part, a discussion focusing on the current research gaps and potential research opportunities is presented regarding quantile regression future developments. The timely review can help improve researchers' understanding and facilitate further improvements of the quantile regression based probabilistic forecasting methods.

Nomenclature

ANN	Artificial Neural Network
CRPS	Continuous Ranked Probability Score
CWC	Coverage Width-based Criterion
ELM	Extreme Learning Machine
GAQR	General Additive Quantile Regression
GBDT	Gradient Boosted Decision Tree
IS	Interval Sharpness

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KDE	Kernel Density Estimation
LQR	Local Quantile Regression
LSSVM	Least Square Support Vector Machine
NWP	Numerical Weather Prediction
PICP	Prediction Interval Coverage Probability
PINAW	Prediction Interval Normalized Average Width
QR	Quantile Regression
QRA	Quantile Regression Averaging
QRDL	Quantile Regression Deep Learning
QRF	Quantile Regression Forest
QRNN	Quantile Regression Neural Network
SVQR	Support Vector Quantile Regression
MSE	mean square error

1. Introduction

With the development of information technology, the traditional power grid has evolved towards a smart grid [1,2], which integrates generators and consumers to supply electricity efficiency and economically [3,4]. For energy resources, renewable energy is gradually replacing fossil fuels, which brings uncontrollable uncertainty to the power grid, and for consumers, the peak demand for electricity may lead to insufficient or unstable voltage. Therefore, this integration presents new issues for the efficient operation of the smart grid, and accurate renewable energy, as well as electricity load forecasting, are effective measures to solve these problems. With the growing amount of data from energy systems, there is a need for utilities to quantify the uncertainty in future generation and demand, especially for wind power, solar power, and electricity demand [5,6].

1.1. Related reviews

There are a variety of papers that have reviewed renewable energy and electricity load forecasting in recent years. Jung and Broadwater [7] presented an overview of research on wind power forecasting and discussed some promising methods for improving the performance of the forecasting models, while Zhang et al. [8] focused on the application of probabilistic wind power generation forecasting, and the evaluation metrics of probabilistic forecasting are summarized in this work. For solar power forecasting, Wang et al. [9] provided taxonomy research of deterministic solar power forecasting, which included regression methods and optimization methods, and the research on probabilistic solar power forecasting is surveyed by van der Meer and Munkhammar [10]. Electricity load forecasting is mainly applied in the area of buildings, and numerous related papers have been published [11–13], most of the literature focus on deterministic load forecasting methods, which are mature technologies recently.

It can be concluded from the above literature that the methods for renewable energy and electricity load forecasting can be divided into deterministic forecasting and probabilistic forecasting. However, as renewable energy data changes intermittently and randomly in reality, deterministic forecasting methods cannot fully describe the uncertainty of renewable energy and load by providing a single-valued expectation series, while probabilistic forecasting methods can provide more information about uncertainty for decision-makers in the energy market. Therefore, probabilistic forecasting methods have become more and more appealing for researchers in recent years, and probabilistic forecasting can play an increasingly greater role in the energy market by providing reliable and sufficient future information.

1.2. Motivation of the review

Although the above-mentioned review papers on probabilistic forecasting have provided a comprehensive overview of probabilistic forecasting methods from their perspective, their focus remains on the general summarization of the different types of probabilistic forecasting methods, while the detailed and specific review of probabilistic forecasting methods is still deficient. Since quantile regression (QR), which estimate conditional quantiles of the response variable by minimizing the quantile loss, has received growing interest in the field of probabilistic prediction [10,14], it is imperative to provide a specific review that traces the development of QR-based probabilistic forecasting methods and helps researchers further understand them.

Based on the above background, the aims of this review paper can be summed as follows:

- First, we aim to provide a reasonable classification of QR-based probabilistic forecasting methods. As a variety of novel methods based on QR have been proposed in recent years, reasonable classification can help researchers better understand the mechanisms.
- Second, we want to summarize the applications of QR-based probabilistic forecasting methods in different fields and find distinctions as well as common ground among these applications, which may provide new perspectives for improving the prediction performance of existing models.
- Finally, we focus on identifying the shortcomings of existing studies and trying to bridge these gaps by summing up several potential research directions.

1.3. Structure of the review

The remainder of this paper is organized as follows. Section 2 provides a general introduction to QR-based probabilistic forecasting methods and research methodology. Different categories of QR-based probabilistic forecasting methods and corresponding applications are reviewed from Section 3 to Section 4 respectively. The analysis and potential research directions of QR-based probabilistic forecasting methods are discussed in detail in Section 5. Finally, concluding remarks are presented in Section 6.

2. Definition and methodology

2.1. Basic definition of quantile regression

This section presents the basic principles of QR-based probabilistic forecasting method and provides an overview of evaluation metrics for probabilistic forecasting performance. As in QR, the quantiles are described by simple polynomials, which are not suitable for capturing the complex and uncertain behavior of renewable energy and electricity consumption [15]. Therefore, to overcome the shortcoming, various approaches have been adopted to improve the forecasting performance.

In general, the most intuitive manner of probabilistic prediction is to determine the specified probability distributions, such as beta distributions [16,17] and Gaussian distributions [18,19]. However, in many cases, the distribution of actual values is not consistent with the assumed distribution, which may lead to inaccurate predictions. QR can estimate different quantiles of actual values directly, and thus the assumptions of the distribution are not required. It was first introduced by Koenker and Bassett [20] and has been widely used in the area of renewable energy and electricity load prediction. An illustration of the QR method is shown in Fig. 1. The basic idea of QR is to approximate the conditional quantiles by a parametric function ξ,

$$q_\theta(\mathbf{x}) = \xi(x_i, \theta, b) \tag{1}$$

here ω(θ) and b are the regression coefficients.

The estimation of regression coefficients ω(θ) and b is realized by minimizing the quantile loss (pinball loss)

$$\sum \rho_\theta(y - q_\theta(\mathbf{x})) \tag{2}$$

where the quantile loss function ρ_θ is defined by:

$$\rho_\theta = \begin{cases} \theta\ldots.., y - q_\theta(\mathbf{x}) \geq 0 \\ \theta - 1, y - q_\theta(\mathbf{x}) < 0 \end{cases} \tag{3}$$

2.2. Methodology

To choose the related articles to review, we have conducted a systematic analysis of the relevant application research of quantile prediction in wind energy, solar energy, and building energy consumption. We have used the Web of Science core collection database for this purpose, as it is the world's largest comprehensive academic information resource. In this study, the current state of the existing literature on quantile prediction in wind energy, solar energy, and building electrical load published in recent years (2016–2022) has been analyzed. The formulated keywords are presented as follows:

TS= (("quantile regression" or "quantile regression prediction*") AND ("load" or "building*" or "solar*" or " wind*"))

The search results included 357 articles that met the search criteria. To identify a manageable subset of papers, an initial filtering of the results was performed: the publication had to be in the Citation Topics Meso of Power system & Electric vehicles, AI & Machine learning, and Statistical methods, which resulted in 216 records. While manually categorizing these papers, it was noted that 119 studies were not directly relevant to the aim of this review, although they met the search criteria. Therefore, 97 papers remained for detailed analysis.

Fig. 2 shows the distribution of tiles for these papers. It is evident that for the research objects, the research on electric load forecasting accounts for a relatively high proportion, while the research on solar energy forecasting accounts for a relatively low proportion. For the research methods, most of the research targets probability density forecasting, and only a small proportion is

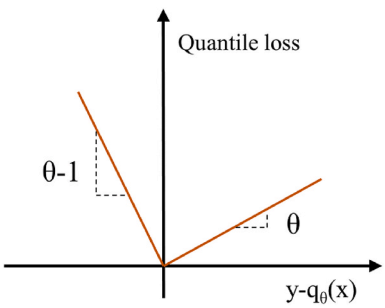

Fig. 1. Illustration of the quantile loss function.

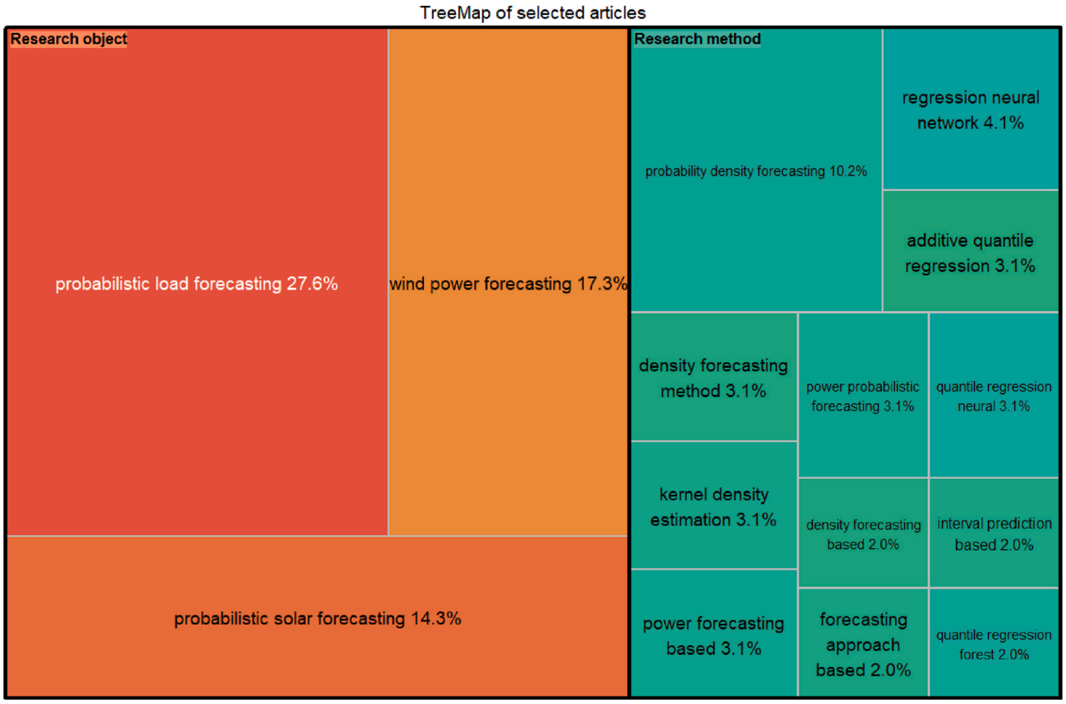

Fig. 2. Tree Map of selected articles with respect to the title of papers.

interval forecasting, which indicates that the QR method will not be directly applied to probability forecasting but further improved to probability density forecasting by, for example, kernel function methods.

3. Overview of quantile regression methods

3.1. Quantile regression methods

In general, existing QR-based probabilistic forecasting methods can be grouped into two main categories, namely statistics-based methods and machine learning-based methods. Fig. 3 shows the classification of QR-based probabilistic forecasting methods. The classification in Fig. 3 mainly depends on the different mechanisms for estimating quantiles. Statistics-based methods leverage the assumption of typical statistical approach linear regression to estimate the quantile, while machine learning-based methods take advantage of machine learning methods in solving nonlinear problems, which has more obvious improvement on prediction performance compared with statistical methods.

3.1.1. Generalized additive quantile regression

Although the assumption of linear regression is simple and intuitive, the relationship between input variables and output variables does not always conform to this assumption in practical applications. Therefore, to generalize the definition of the relationship mentioned above, the generalized additive model was developed by Hastie and Tibshirani [21], which regarded the output variables Y as a sum of smooth functions of input variables X. Smooth function is utilized to describe the nonlinear relationship between X and Y, and provides high flexibility for additive models. Inspired by this method, Gaillard et al. [22] proposed a methodology that performs QR using a generalized additive model, namely generalized additive quantile regression (GAQR).

Fig. 4 illustrates the basic idea of GAQR. The assumption of linear regression in the QR method is converted to additivity assumption by a generalized additive model, and thus the θ-th quantile of y_t is expressed as:

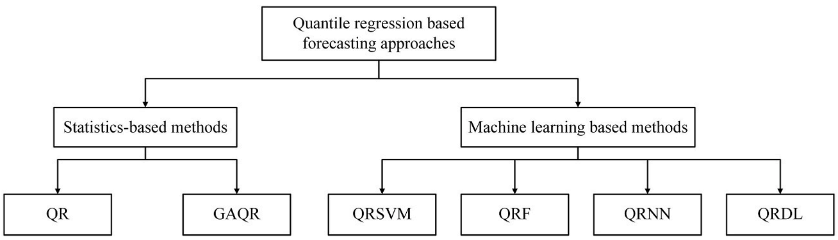

Fig. 3. Classification of QR-based probabilistic forecasting methods.

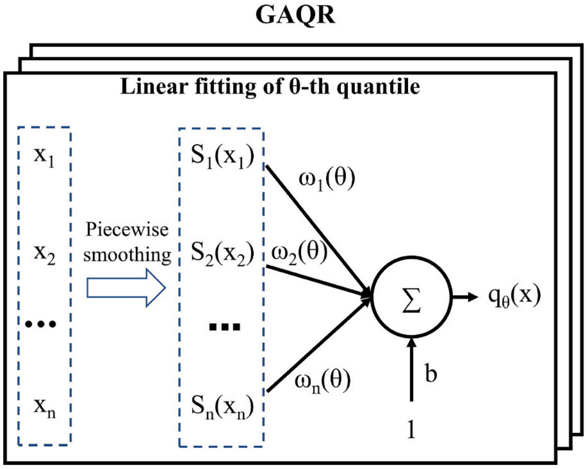

Fig. 4. Flowchart of generalized additive quantile regression.

$$q_y(\theta|\mathbf{x_t}) = s(\mathbf{x_t}) = \sum_{i=1}^{n} s_{i,\theta}(x_{t,i}) \tag{4}$$

where $\mathbf{x_t} = \{x_{t,1}, x_{t,2}, \cdots x_{t,m}\}$ $(1 \leq t \leq n)$ and $s_{i,\theta}(x_{t,i})$ is the smooth function of the input variable $x_{t,i}$ $(1 \leq i \leq n)$. In general, commonly used smooth functions include P-splines, B-splines, and cubic splines.

GAQR allows for more flexibility than linear models, and more interpretability than nonlinear black box models. However, they can suffer from a low performance compared to full complexity models when interactions truly exist in input variables [5], therefore, the main difficulty in GAQR lies in how to find the appropriate form of nonlinear function [23].

3.1.2. Quantile regression neural network

Quantile regression neural network (QRNN) is essentially an extension of a neural network [24]. Typical QRNN is also composed of three layers: input layer, hidden layer, and output layer, however, the training goal of QRNN is to minimize the quantile loss between the targets and the outputs.

Fig. 5 illustrates the basic idea of the QR neural network method. The main restriction of applying the pinball loss function to machine learning models is that the loss function is not differentiable everywhere, which prevents the training of models from using gradient descent, and as a result, the weights of input variables cannot be optimized.

A possible way to address this issue is to find a smooth approximation function, which can be trained by the gradient descent method. Zheng [25] proposed a smooth function to replace the pinball loss function and made the training process possible in QR by gradient descent. Hatalis et al. [26] extended Zheng's smooth approximation for neural networks, and showed superior performance

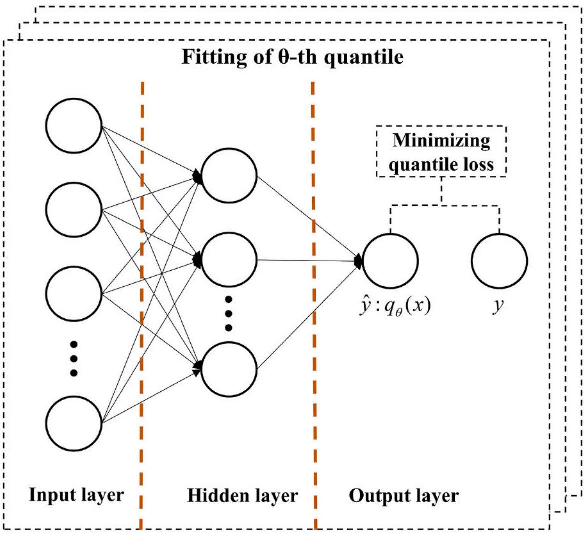

Fig. 5. Illustration of quantile regression neural network method.

compared to other models. Furthermore, the most commonly used approximation function is the Huber norm [27], which is differentiable everywhere, and it is introduced by Wang et al. [28] to guide the training process of the long short-term memory (LSTM) network.

The Huber norm is essentially a combination of the L1 norm and L2 norm, which is calculated as:

$$H(e_t) = \begin{cases} \frac{1}{2\varepsilon}(e_t)^2, |e_t| \le \varepsilon \\ |e_t| - \frac{\varepsilon}{2}, |e_t| > \varepsilon \end{cases} \quad (5)$$

where ε is the threshold for the L1 and L2 norm. Although the non-differentiable problem of the pinball loss function when the error is 0 can be solved by the Huber norm, it is tricky to optimize the threshold magnitude of the Huber norm [29].

QRNN has the advantage of being able to store the input information over the entire network, thus having high fault tolerance, and it is capable of parallel processing [30]. On the contrary, the disadvantage of QRNN is very memory-consuming due to its naive training scheme [31]. In addition, the traditional QRNN suffers from the problem of over-fitting and over-training [32].

3.1.3. Support vector quantile regression

Support vector quantile regression (SVQR) is derived from support vector machine (SVR) [33], the basic idea of SVQR is to find a linear regression function that could predict the result with an acceptable deviation from the actual target.

Fig. 6 illustrates a typical SVQR model, and the width of the margins is a hyper-parameter that is usually pre-defined. SVQR approach is first proposed by Takeuchi and Furuhashi [34] to address the quantile crossing problem.

SVQR, which accounts for a relatively small percentage of the reviewed literature, has a slight advantage over QRNN and QRF in terms of predictive performance, but is far more computationally expensive than other models [35]. Recently, efforts have been paid to selecting appropriate kernel functions to improve its prediction performance [23].

3.1.4. Quantile regression forest

Quantile regression forest (QRF) is a QR method derived from the random forest [36]. For each final leaf of each tree, one does not compute the mean of the forecasting results, but instead their empirical cumulative distribution function. Once the forest is built, one determines a new vector of predictors and its associated leaf in each tree by following the splitting according to the predictors' values. An illustration of the quantile regression forest method is shown in Fig. 7.

The tree structure of QRF is as easy to understand as RF, which enhances the interpretability of the prediction results [37]. Meanwhile, QRF can output the prediction results of different quantiles simultaneously, and avoid the existence of overfitting, which improves the stability of the model prediction. However, since the principle of QRF is similar to RF, that is, the prediction results are primarily based on expectations [38], its prediction performance would have a certain gap compared with other machine learning-based methods.

3.1.5. Quantile regression deep learning

Recently, deep learnings (DL) have achieved promising performance in many fields [39,40], which prompted lots of explorations

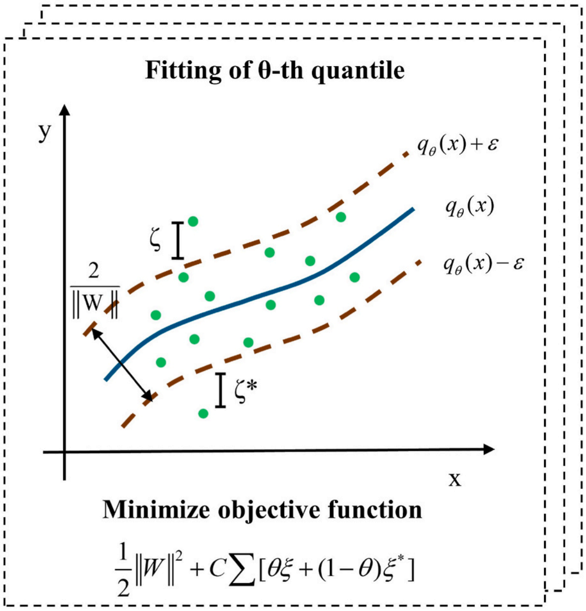

Fig. 6. Illustration of support vector quantile regression method.

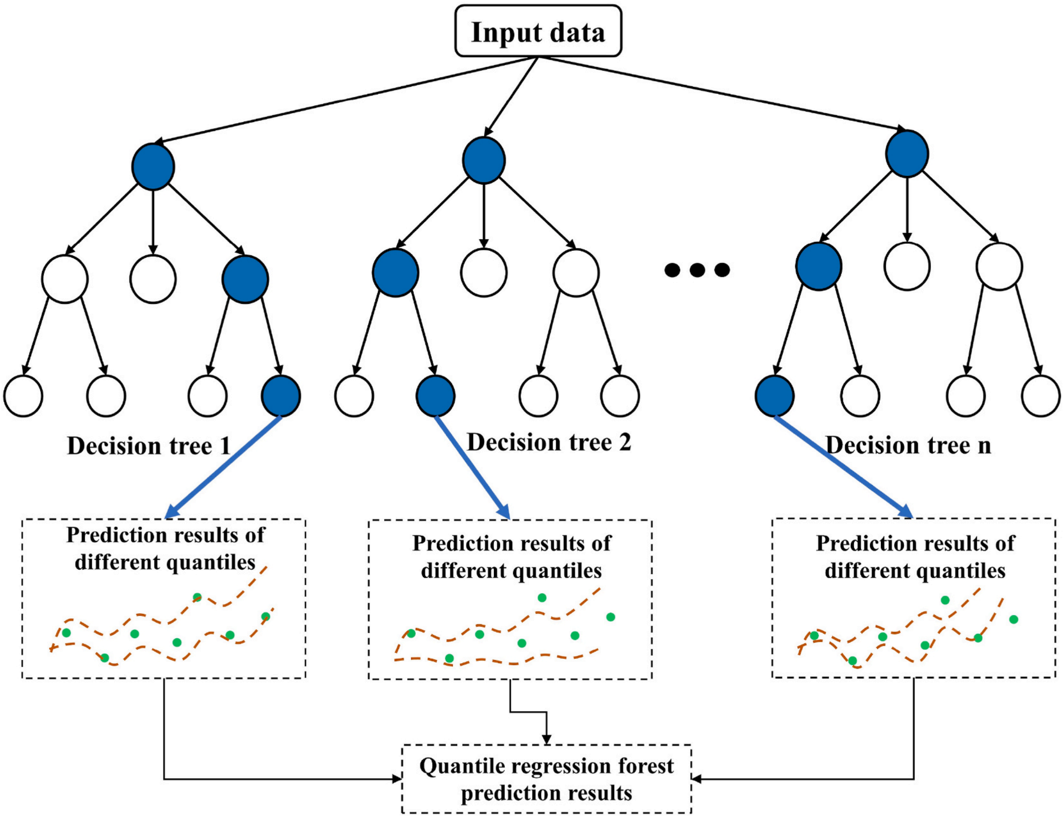

Fig. 7. Illustration of quantile regression forest method.

on DL in probabilistic load forecasting. Among the most popular DL models integrated with QR are QR-LSTM, QR-deep neural network (QR-DNN), and QR-Convolutional neural network (QR-CNN). As the complexity and diversity of DL, these methods are not elaborated on in this paper, the reader can refer to reference [28,41–43] for details.

The main advantage of QRDL is that its improvement in prediction performance is much greater than that of other models, due to its remarkable nonlinear approximation capability [44,45]. However, the disadvantage of QRDL is that its complex structure and the nature of a large number of variables can lead to expensive computational costs and clumsy models [46].

With the description of the advantages and disadvantages of different QR methods above, the main features of these methods are summarized in Table 1.

3.2. Evaluation metrics for forecast results

General metrics can be utilized to evaluate the performance of interval results as well as probability density results, and a brief description of general metrics is described as follows:

Prediction interval coverage probability (PICP) evaluates whether the actual value is within the calculated prediction interval limits. However, it becomes clear that it is not that the higher value of the PICP, the better, as a fairly wide prediction interval may lead

Table 1
Comparison among quantile regression methods.

Methods	Complexity level	Computational cost	Accuracy
GAQR	★	★	★
QRNN	★★★	★★'	★★★
SVQR	★★★	★★★★★	★★★★
QRF	★★	★★★	★★
QRDL	★★★★★	★★★	★★★★★

Note.
★: Very easy to achieve, very low computational cost, and very low accuracy.
★★: Easy to achieve, low computational cost, and low accuracy.
★★★: Moderate method complexity, moderate computational cost, and acceptable accuracy.
★★★★: High method complexity, high computational cost, and high accuracy.
★★★★★: Very high method complexity, very high computational cost, and very high accuracy.

to meaningless results. Therefore, a narrow prediction interval, as well as a high PICP, should be the main purpose of the forecast. While prediction interval normalized averaged width (PINAW) can measure the width of the prediction interval. These two metrics are defined below:

$$PICP = \frac{1}{N}\sum_{i=1}^{N}\alpha_i \tag{6}$$

$$PINAW = \frac{1}{NE}\sum_{i=1}^{N}(U_i - L_i) \tag{7}$$

where $\alpha_i = 1$ if the actual value lies within the prediction interval, and $\alpha_i = 0$ otherwise. L_i and U_i represent the lower and the upper boundaries of the prediction interval respectively. E is the difference between the maximum and the minimum actual values, and its main role is to normalize the result.

Furthermore, there is an extended version of PINAW, namely interval sharpness, which is a more comprehensive index to assess the width of the prediction interval, by rewarding the narrow PI and penalizing the wide one [47], and it is defined by

$$IS = \frac{1}{N}\sum_{i=1}^{N}\begin{cases} -2\delta_i - 4(L_i - y_i)., y_i < L_i \\ -2\delta_i ...., L_i \le y_i \le U_i \\ -2\delta_i - 4(y_i - U_i)., y_i > U_i \end{cases} \tag{8}$$

where δ_i is the width of the prediction interval at time step i, and the higher value of IS means better forecasting performance.

Another type of general metric is skill scores, which can comprehensively rank probabilistic models [48]. The pinball loss is a typical metric, and it is also the target function to be minimized in QR, the expression of its formula can be found in Eq. (3). Winkler score is proposed by Winkler [49] to assess the prediction interval, and it is very similar to Eq. (8), as both of them focus on the penalty of observations lie outside the constructed interval. For a central $(1-\alpha) \times 100\%$ prediction interval, the Winkler score is defined as:

$$Winkler\ score = \begin{cases} \delta_i + \frac{2}{\alpha}(L_i - y_i)., y_i < L_i \\ \delta_i ...., L_i \le y_i \le U_i \\ \delta_i + \frac{2}{\alpha}(y_i - U_i)., y_i > U_i \end{cases} \tag{9}$$

4. Practical applications of quantile regression methods

4.1. Applications of statistical based quantile regression methods

As described in section 2.1, for the basic QR method, the form of independent quantiles is in keeping with linear regression. However, due to the defects of basic QR in dealing with nonlinear problems, it is simplistic to model the energy prediction by using basic QR in practice. To alleviate the restriction of QR applications, QR methods are usually combined with other advanced approaches to obtain accurate prediction results in early cases.

4.1.1. Applications of linear quantile regression methods

For photovoltaic power-related forecasting, researchers mainly focus on the feature extraction of input variables [50]. Lauret et al. [48] explored the impact of different QR inputs on the predicted results, and the results demonstrated that numerical weather prediction data in the input variables positively affect the performance of the intra-day probabilistic solar forecasts, and the performance improvement on continuous ranked probability score (CRPS) can be up to nearly 12% compared to models that do not take into account weather variables. A similar study on the input variables of QR was proposed by Alonso-Suarez et al. [51]. In this work, the instability of the cloud regime and the Earth's surface reflectance are considered as the input variables, which is rarely mentioned in other literature, and results showed that these two variables improve the forecast performance in all four selected sites. Besides, the effect of aggregation of customers and an increasing share of photovoltaic power in the net load on probabilistic forecasts performance are also investigated by van der Meer et al. [52].

For probabilistic load forecasting, hybrid methods of QR are often adopted [53,54], Guo et al. [55] first combined deep neural networks with QR for short-term electricity load forecasting, and the proposed model performs better than other benchmark models in point prediction, but they did not compare the performance between the proposed model and benchmark models in probabilistic prediction. A similar method to leverage the point prediction results was proposed by Wang et al. [56], which combined historical load data with prediction results to establish a probabilistic residual forecasting model. David et al. [57] proposed a time series method combining the point forecasting model and QR model. This work conducted a comprehensive analysis of 20 different models for intraday probabilistic forecasting, and these models derive from three different prediction models and seven QR-based models by permutation and combination. Based on the results of the case study, the authors noted that linear models in QR, weighted QR and gradient boosting decision trees are the most efficient models for generating probabilistic forecasts without the use of exogenous variables.

Recently, hybrid methods of QR also attract attention in the area of wind power forecasting [58,59]. Huang et al. utilized SVR with QR [60]. The key point of the point prediction results-based methods is to establish a high-accuracy point prediction model, as it can be

Table 2
Publications on probabilistic forecasting using statistic-based quantile regression methods.

Ref	Temporal granularity	Forecast horizon	Method	Input variables	Remarks
[48]	1 h	1 h–6 h	QR	Global horizontal irradiance; Weather forecasts	Highlight: Combine ground telemetry and weather forecasts to improve the quality of the intra-day probabilistic forecasts.
Performance: CRPS 66.2–91.6.					
[50]	15 min	72 h	QR	Solar power; Weather forecasts	Highlight: Combine the Gradient-Boosted Regression Trees method with quantile regression for probabilistic forecasts.
Performance: CRPS, but no conclusive quantification.					
[51]	10 min	10 min-3 h	QR	Global horizontal irradiance;	Highlight: Quantify the impact of adding local short-term solar irradiance variability and spatially averaged satellite albedo to solar probabilistic forecast.
Performance: PICP, PINAW, CRPS, but no conclusive quantification.					
[52]	30 min	30 min	QR	PV power	Highlight: Investigated the effect of the aggregation of time series and an increasing share of photovoltaic (PV) power on prediction intervals in the local electricity distribution grid.
Performance: PICP, PINAW, CRPS, but no conclusive quantification.					
[60]	15 min	3 h	QR	Wind speed; Wind direction	Highlight: Combine EHS-based SVR and EHS-based QR approach for wind power forecasting.
Performance: Coverage ratio of confidence intervals 82.3785%.					
[53]	Monthly	Monthly	QR	Electrical load; Natural gas load; Natural gas price et al.	Highlight: Utilize the predicted results of different models to establish a quantile regression model.
Performance: No performance metric was applied to assess the prediction intervals.					
[54]	1h	168h	QR	Wind power	Highlight: Explores the impact of historical load and wind power factors on short-term power load via LASSO-QR.
Performance: PICP from 0.9879 to 0.9892; PINAW from 0.17% to 0.18%.					
[55]	Daily	Daily	QR	Electricity consumption data; Weather data; Calendar data	Highlight: Combine deep learning methods with quantile regression.
Performance: No performance metric was applied to assess					

(continued on next page)

Table 2 (*continued*)

Ref	Temporal granularity	Forecast horizon	Method	Input variables	Remarks
the prediction intervals.					
[57]	1 h	1 h–6 h	QR	Global horizontal irradiance	Highlight: Use past solar irradiance data as inputs, and compare the performance of different point prediction results-based methods.
Performance: CRPS, but no conclusive quantification.					
[61]	10 min	10 min–30 min	QR	Wind power	Highlight: Use quantile regression averaging and variational mode decomposition-based hybrid models for wind power forecasting.
Performance: PICP from 0.9 to 0.91; PINAW from 18.86% to 41.06%.					
[58]	1 h	24 h	QR	Wind power; Temperature; Air pressure; Wind speed; Wind direction	Highlight: Use a two-stage attention mechanism for stable wind power forecasting.
Performance: Performance: CRPS improvement of 45.7%, Pinball loss improvement of 47.44% compared to benchmark models.					
[62]	1h	24h	QR	Wind power; Meteorological forecasts	Highlight: Propose a new concept of distributed approaches for wind power forecasting.
Performance: Quantile score from 3.206 to 4.621.					
[66]	5min	1h–24h	QR	Wind power; Meteorological forecasts	Highlight: Quantify the relationship between forecastability and wind series characteristics by QR.
Performance: forecastability, but no conclusive quantification.					
[59]	10min	10min–30min	QR	Wind speed	Highlight: Combine the data denoising technique, statistical models, shallow neural networks, deep neural networks, and QR, using MMOTA optimization algorithms for combination forecasts.
Performance: PICP from 88% to 94%; PINAW from 0.0247 to 0.0467.					
[64]	1h	24h	QR	Electricity consumption data; Temperature data	Highlight: Present CPQR-based method to automatically identify building energy use patterns.
Performance: No performance metric was applied to assess the prediction intervals.					

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Table 2 (*continued*)

Ref	Temporal granularity	Forecast horizon	Method	Input variables	Remarks
[65]	1h	1h–12h	QR	Solar radiation; Meteorological forecasts	Highlight: Develop a robust multi-step solar irradiance prediction model by using Transformer deep learning and the QR method.
Performance: No conclusive quantification.					
[67]	/	/	QR	Energy consumption; Building information	Highlight: Utilize quantile regression to differentiate and understand the main drivers of energy consumption throughout the conditional distribution.
Performance: No performance metric was applied to assess the prediction intervals.					
[84]	15min	1h	QR	Electricity data; Weather data; Calendar data	Highlight: Identify factors that have a significant influence on demand and residual demand by QR.
Performance: No performance metric was applied to assess the prediction intervals.					
[85]	15 min-1 h	18 h–36 h	GAQR	Meteorological forecasts; forecasted wind power	Highlight: To extend existing point forecast systems to probabilistic forecast systems by GAQR.
Performance: Inter quartile range (the difference between 75% and 25% quantiles), but no conclusive quantification.					
[86]	15min	15min-6h	GAQR	PV power	Highlight: Exploit off-site information to improve forecasting of photovoltaic production.
Performance: Reliability, Sharpness, but no conclusive quantification.					
[87]	Daily	Daily	GAQR	Solar radiation	Highlight: Develop copula-base nonlinear quantile regression for daily diffuse radiation estimation.
Performance: The training time of the CNQR models are far less than that of SVM.					
[88]	5min	Intra-hour.			
Intra-day.					
Day ahead	GAQR	Ground-based measurements; Sky-camera images; Satellite-imagery features; Numerical weather data.	Highlight: Provide a series of benchmarks for probabilistic solar forecasting on a standardized dataset.		
Performance: Quantile loss from 12 to 16.3.					
[89]	1h	1h	GAQR	PV power	Highlight: Address Bayesian bootstrap in real-time probabilistic PV power forecasting.
Performance: ACE from 0.97% to 2.02%;					

(*continued on next page*)

Table 2 (*continued*)

Ref	Temporal granularity	Forecast horizon	Method	Input variables	Remarks
PINAW from 7.135 to 11.637.					
[90]	4s	1min	GAQR	Solar irradiance	Highlight: Generate sub-minute solar irradiance forecasts for high-temporal-resolution stochastic simulation.
Performance: PICP 0.933; PINAW 482.7; Quantile loss 27.3.					

concluded from the literature that better performance of point prediction results leads to better performance of probabilistic prediction results. Zhang et al. [61] adopted variational mode decomposition and several base point prediction models to obtain different deterministic predictions, and use the QR method to generate probabilistic interval prediction results. In this work, three base models LSSVM, ESN, and ELM are established for subseries respectively, and thus generate the final predictions by summing up predictions of all the subseries.

Since the main advantage of QR is its simplicity of form and interpretability, more and more researchers are applying it to the characterization of energy properties [62–65]. Feng et al. [66] developed a forecastability quantification method by characterizing the relationship between forecastability and wind series entropy using QR, they found that wind characteristics can provide valuable information to wind farm developers and power system operators. Roth and Rajagopal [67] used QR to benchmark building energy use, this method can help users understand the main drivers of energy consumption throughout the conditional distribution. Moreover, the efficiency scores associated with the energy use of benchmarked buildings also provided information on the energy-saving potential for building managers.

4.1.2. Applications of generalized additive quantile regression methods

Due to the stronger nonlinear fitting ability of GAQR compared to QR, therefore, QR has been used as a benchmark in many studies to demonstrate the advantage of GAQR prediction performance. Pritchard [68] combined GAQR with the persistence method, and the results showed that the proposed model was a fairly successful forecasting tool for wind power. Extension methods for GAQR mainly focus on feature engineering [69,70] and model structure optimization [71–73]. Haque et al. [74] proposed a hybrid deterministic model including wavelet transform, fuzzy ARTMAP network, firefly optimization, and support vector machine, and it was a fairly complex model for point prediction. The forecast results obtained from the proposed hybrid model were then used by the QR method to get probabilistic wind power forecasting results. Despite the outperformance of the proposed model in point prediction, the probabilistic forecasting performance of this model was not significantly better than the simple BPNN model. Similarly, wavelet transform and deep belief networks are combined to make deterministic wind power forecasting in Ref. [47], and three commonly used prediction models are chosen to fully validate the effectiveness of the proposed algorithm. Compared with the benchmark models, the both deterministic and probabilistic performance of the proposed model is optimal in terms of various seasons and prediction horizons. Besides, owing to the flexibility of GAQR, it has been widely utilized as the benchmark method in probabilistic wind power forecasting [29,75,76].

Gaillard et al. [22] proposed the mixed method of QR and generalized additive model for probabilistic electric load forecasting, and the performance of the proposed method is as good as the model proposed by the team Tololo, which ranked first in the Global Energy Forecasting Competition 2014. Sigauke et al. [77] improved the forecasting performance of GAQR by a variable selection method, and results show that GAQR with the variables selection method can give the most accurate forecasts. In the reviewed literature, there are also other approaches to improve the forecasting performance of the GAQR methods, such as data preprocessing [78], kernel density estimation [79,80], Gaussian process [81], and residual distribution estimation [82]. The deep learning method was also adopted for probabilistic load forecasting, Wang et al. [83] combined wavelet transform-based deep convolutional neural network method with GAQR to generate high-accuracy probabilistic forecasting results, and they found that more accurate deterministic forecasting performance generally means less uncertainty on the probabilistic forecasting results.

Table 2 provides an overview of the statistic-based quantile regression methods reviewed in this section.

4.2. Applications of machine learning-based quantile regression methods

All the aforementioned machine learning-based quantile regression methods are widely used in wind energy, solar energy, as well as load forecasting applications, which range from common machine learning models to cutting-edge deep learning models, and the extensive research results provide researchers with a wealth of reference information and different research perspectives.

4.2.1. Applications of quantile regression neural network

As stated in Section 3.1.2, in the training process of machine learning models, it is impossible to use pinball loss instead of mean square error as the objective function directly. Therefore, researchers mainly focus on the approximation method of the pinball loss function in recent years. He and Li [91] adopted the quantile regression neural network for probabilistic wind power forecasting. However, the QRNN proposed by Taylor is not perfect as stated in this reference [92], as the objective function of QRNN is not differentiable everywhere, which may lead to sub-optimal model parameters [93]. In general, the optimization process of the objective

Table 3
Publications on probabilistic forecasting using machine learning-based quantile regression methods.

Ref	Temporal granularity	Forecast horizon	Method	Input variables	Remarks
[91]	1 h	1 h	QRNN	Wind power	Highlight: Develop quantile regression neural network and Epanechnikov kernel function using Unbiased cross-validation for wind power probability density prediction. Performance: PICP from 0.8393 to 0.9048; PINAW from 16.94% to 21.34%.
[94]	10min	30min	QRNN	Wind power	Highlight: Develop a highly efficient and reliable method for wind power forecasting. Performance: ACD from 0.1132 to 0.1490; AW from −0.22% to 0.4%.
[95]	1h	1h	QRF	Wind power	Highlight: Combine instance-based transfer learning and GBDT to increase the performance of wind power forecasting. Performance: Quantile score from 0.0132 to 0.039.
[96]	10 min	10min-3h	QRNN	Wind power	Highlight: Take advantage of ELM and quantile regression for wind power forecasting. Performance: Reliability and skill score of sharpness, but no conclusive quantification.
[97]	10min	10min	QRNN	Wind speed	Highlight: Develop a new quantile regression model CQR-ORELM for improving model robustness and predictive capability. Performance: MAE from 0.0132 to 0.0385.
[100]	15–30 min	Weekly	QRNN	Electrical load data	Highlight: Propose quantile regression neural network using triangle kernel function for probability density load forecasting. Performance: PICP 98.8%–99.8%, PINAW 9.6%–16.3%.
[102]	1h	24–168h	SVQR	Wind power/Solar power	Highlight: Present FIG-SVQR model to forecast time series of wind. Performance: PICP from 0.9405 to 0.9812; PINAW from 19.87% to 23.58%.
[105]	10 min	66 h and 72 h	QRF	Photovoltaic data; Weather forecasts	Highlight: Compare the performance of QR and QRF for forecasting photovoltaic production. Performance: CRPS, but no conclusive quantification.
[108]	1 h	24 h	QRF	Load data; Temperature	Highlight: Propose multi-variable quantile regression to conduct multi-variable prediction for categorized loads. Performance: Three probability-based assessments were used.
[106]	1 h	24 h	QRF	Load data	Highlight: Solve the reliability issue of direct PI construction by proposing an alternative quantile determination method. Performance: Quantile loss 2.82; Winkler score 25.14%–49.46%.
[109]	1 h	24 h	QRCN	Wind speed; Solar irradiance; Humidity; PV power	Highlight: Propose a two-stage training strategy to combine convolutional neural networks and quantile regression. Performance: PICP, PINAW, CRPS, but no conclusive quantification.
[113]	1h	6h–18h	QRNN	Global radiation	Highlight: Compare the performance of different QR based methods for probabilistic global radiation. Performance: CRPS, but no conclusive quantification.
[114]	1 h	24 h	QRF	Photovoltaic data; Weather forecasts; Calendar data	Highlight: Compare the performance of different combination strategies for probabilistic models. Performance: PICP, PINAW, CRPS, but no conclusive quantification.
[115]	1min	1h	QRF	Meteorological variables; Solar power	Highlight: Develop improved threshold-based methods for performance assessment of PV modules based on the QRF. Performance: PICP, PINAW, but no conclusive quantification.
[111]	15min	3 days	QRDL	PV power; Meteorological forecasts	Highlight: Propose QRCNN for the probabilistic forecast of the power output of the clustered PV plants in a region. Performance: ACE 0.035; PINAW 0.047; Quantile loss 1.71.
[116]	15 min	15 min	QRF, QRNN, SVQR	Wind speed; Wind direction	Highlight: Combine 13 different models by Meta-learning strategy for highly precise wind power forecasting. Performance: CRPS 0.3337 and 0.9890.
[42]	30min	30min-3h	QRDL	Wind speed	Highlight: integrates quantile regression loss function into IndRNN network to yield the prediction intervals. Performance: PICP from 94.89% to 95.34%; PINAW from 0.1163 to 0.1853.
[117]	1h	1h	QRNN	Wind speed	Highlight: Implement EMD to reduce the noise of QR based wind speed forecasting results. Performance: PICP 93%; PINAW 18.3%.
[118]	15min	30min-1h	QRNN	Wind power	Highlight: Develop an adaptive bilevel programming model to generate nonparametric PIs of wind power. Performance: ECP from 94.11% to 95.42%; AW from 0.2346 to 0.2821.
[119]	15min	15min	QRNN	Wind speed	Highlight: Develop a new comprehensive index to optimize QRELM for wind power forecasting. Performance: PICP 89.17%; ACE -0.83%.

(continued on next page)

Table 3 (*continued*)

Ref	Temporal granularity	Forecast horizon	Method	Input variables	Remarks
[120]	1h	1h	SVQR	Wind speed	Highlight: Develop a two-step probabilistic wind forecasting method based on pinball loss optimization. Performance: Quantile loss from 2.886 to 4.891.
[112]	15min	15min	QRDL	Wind speed	Highlight: Propose QRMGM to reduce the training time without significantly decreasing the prediction accuracy. Performance: CP from 0.952 to 0.958; MWP from 0.365 to 0.57; MC from 0.383 to 0.599.
[121]	1h–2h	1h–2h	QRNN	Wind speed	Highlight: Develop a novel hybrid model composed of QRNN with MCEEMDAN and GOA algorithm to obtain precise wind speed forecasting results. Performance: Sharpness from 1.4665 to 3.2316.

function can be regarded as a linear programming problem, but the methods of minimizing the pinball loss function are not described in detail in many cases [94,95]. In addition, the efficient model of neural networks, ELM, is also applied to wind energy prediction by integrating with QR [96,97].

Fernandez-Jimenez et al. [98] firstly introduced QRNN to probabilistic photovoltaic power forecasting. The input variables as well as the number of neurons in the hidden layer and the weight decay regularization factor are optimized by the GA method to improve prediction accuracy. Cheng et al. [99] proposed a more complex QRNN method for photovoltaic power generation probabilistic prediction. In this work, the weight corresponding to different weather variables changes with predicted data, therefore, the adaptive weighting algorithm, KNN algorithm, and neural network quantile regression are combined to realize the probability prediction of photovoltaic power generation.

He et al. [100] proposed a short-term probability density forecasting method, which utilized the QRNN and triangle kernel function, and the probabilistic forecasting results of this model showed superiority in comparison with that of the radial basis function quantile regression method. To improve the performance of medium-term probabilistic load forecasting, Gan et al. [101] incorporated the uncertainty of hourly temperature and load variation into the prediction model, and the result turns out that the proposed method can improve predictive performance by nearly 20% compared with QR and the multi-layer perceptron model. As QRNN is not suitable for large datasets due to the unaffordable computational cost, Zhang et al. [32] a more computationally efficient QRNN method, which incorporates batch training and noise layers in deep learning areas to reduce the training cost.

4.2.2. Applications of support vector quantile regression

The support vector quantile regression (SVQR) approach is first proposed by Takeuchi and Furuhashi [34] to address the quantile crossing problem. At present, SVQR is mainly applied in the field of economics, while the applications in the field of energy are mainly contributed by He et al. [102] for wind power and solar power forecasting, and showed better probabilistic prediction performance compared to other state-of-the-art models. Furthermore, the authors [103] combined three different kinds of kernel functions with SVQR, and select the best one for the prediction model, it can be concluded that the kernel function can improve the prediction performance of SVQR. Due to the excessive time cost of SVQR in the model training process, it has not received much attention in these applications.

4.2.3. Applications of quantile regression forest

Researchers have applied QRF with a preference for its stability in prediction problems. Ehsan et al. [104] compared the performance of QRF and BART when predicting the maximum wind speed at 10 m using selected convective weather variables. Results showed that in terms of systematic and random error metrics, no significant differences were exhibited between the two models, while in terms of ensemble verification statistics, significantly better performances were reported for the ensemble realizations generated using the QRF model than the BART model. Zamo et al. [105] utilized QR and QRF methods to forecast daily photovoltaic power, and the input variables are chosen from the ensemble numerical weather prediction (NWP) system, PEARP. The rank histogram method is regarded as the post-processing technique for forecasting models, however, there is no significant improvement to the reliability of forecasting models. Zhang et al. [106] adopted QRF for probabilistic load forecasting, and introduced a recursive feature elimination method to eliminate irrelevant features of input variables, results found that the introduced method can further improve the reliability of QRF by incorporating an extra validation process to determine the best quantile pair.

Since the QRF method can output predictions in different quartiles simultaneously, researchers have taken advantage of this to improve the efficiency of the model [107]. Xing et al. [108] proposed a hybrid framework combining QRF with MQR for improving the performance of multiple loads forecasting at the same time. The proposed method showed advantages in assessing the uncertain information of future demand load. Similarly,

4.2.4. Applications of quantile regression deep learning

The superiority of QRDL in terms of prediction accuracy has been widely demonstrated [44]. Huang and Wei [109] proposed a hybrid quantile regression method QCNN for daily-ahead probabilistic forecasting of photovoltaic power. In their work, the deep learning method CNN was first utilized to extract the features of PV power influence factors, and then the extracted features were used as the inputs of the QR method. It was worth noting that the training strategies of CNN and QR in the QCNN framework are different and independent due to the non-differentiable loss functions of the QR, thus the proposed two-stage training strategy could fully realize the potential of CNN in forecasting. Results showed that the proposed method achieved a PICP between 84.1% and 90.1%, and PINAW between 15.9% and 30.6% depending on different weather conditions, which are more accurate and reliable compared with the other benchmark methods. Zhang et al. [110] investigated the advantages of applying DNN with skip connections in electric load forecasting, it is found that skip connections bring improvements in the performance of NNs, and the improvements are more pronounced as the number of network layers increases.

Nevertheless, the increasing complexity of the quantile regression deep learning methods is a problem that cannot be ignored researchers have made efforts on this issue [42,111]. Zhang et al. [112] proposed a hybrid model combining Quantile Regression and Minimal Gated Memory Network for short-term wind speed forecasting. In their study, Minimal Gated Memory Network is utilized to reduce the training time as the design of the MGM simplifies the structure of the LSTM and does not significantly reduce the prediction accuracy. Lu et al. [41] developed a sparse-group Lasso-quantile regression deep neural network model for electricity load forecasting, they introduced a standard regularized penalty term in the loss function to avoid network redundancy, and the results showed that the training time of the proposed model is only a quarter of that of QR method.

Table 3 provides an overview of the machine learning-based quantile regression methods reviewed in this section.

5. Summary and future works

In the previous sections, the theory and form of QR-based probabilistic forecasting methods as well as their applications have been comprehensively summarized. It can be concluded that QR-based probabilistic forecasting methods have been widely studied by researchers in the past two decades, and plenty of high-performance methods have emerged. Based on this background, the analysis of the methods and potential opportunities for future works are discussed and summarized as follows.

In recent years, statistical-based QR methods are mainly applied to energy pattern identification and control optimization, different from prediction problems, these two aspects of research focus on the reliability and interpretability of the methods without high requirements on the performance of prediction. Therefore, existing studies have paid much attention to statistical-based QR methods because they can explicitly represent the relationship between predicted values and input variables and are simple to adopt. While in the field of predictive modeling, machine learning-based QR methods have been a hot research topic, and various feature extraction and parameter tuning methods have been developed to improve the accuracy of the models. In particular, cutting-edge deep learning methods have also been applied to this field with remarkable results.

However, with the continuous improvement of machine learning-based QR methods, the complexity of the models also increases, leading to an increased computational burden, so researchers also start to contribute to the field of model computational efficiency improvement, and they mainly focus on the model simplification for deep learning, which brings down the computational cost to a similar level to general machine learning algorithms while maintaining the high performance of deep learning.

The primary research of the reviewed literature has focused on machine learning-based QR methods, while wind power forecasting-related research accounts for nearly 50% of the reviewed studies. The reason for this is that previous research mainly focuses on improving the QR method, and wind power forecasting is the main research object due to the requirements of placing bids in energy markets [122]. Besides, with the development of smart power grids, load forecasting has attracted more and more attention in recent years. Therefore, state-of-the-art methods, such as QRDL, account for a high percentage of the load forecasting application [108, 123–125], which indicates that load forecasting is the main area with potential for future applications of QR methods.

Furthermore, the performance evaluation metrics for probabilistic forecasting in the reviewed papers have not been unified yet, and the proportion of each metric in different applications is demonstrated in Fig. 8. It is shown that the metrics that researchers prefer to use in each application are different, while PICP and PINAW are relatively widely used in these applications. In addition to the diversity of metrics, other factors lead to the disunion of evaluation metrics. For example, the Winkler score, quantile score, and CRPS are regarded as the metrics to evaluate the sharpness of probabilistic forecasting results in Ref. [126], while in Ref. [10], these metrics are considered comprehensive measurements that take both reliability and sharpness into consideration, and the detailed definition of reliability and sharpness refers to the study of Gneiting et al. [127]. More controversially, Nowotarski and Weron [126] concluded that CWC should be avoided in the evaluation of probabilistic forecasts, and this conclusion derived from the reliability and usability discussion of CWC by Ref. [128] and Ref. [129]. However, CWC is still adopted by researchers for load forecasting evaluation [130]. Therefore, the utilization of CWC in probabilistic forecasting is worth further study.

Based on the above analysis of QR-based probabilistic forecasting in various applications, it has become apparent that future works should focus on fulfilling the research gaps in the previous sections, and the potential opportunities for further research on QR-based methods mainly include:

(1) *Reduce the computational cost of QR-based methods.* With the continuous improvement of QR methods, the computational cost of models is getting higher and higher. In the modeling process, model training and parameter tuning are the two most time-consuming parts, so the simplification of complex models and improvement of parameter tuning methods are possible research hotspots in future work.
(2) *Increase model interpretability.* Model interpretability has been initially studied in the field of point prediction, while it has not been discussed in the literature in the context of QR methods. Model interpretability is important for the application of the model, for example, readers need to know the magnitude of the effect of different input variables on the quantile prediction results, which can improve the credibility of the model prediction. Therefore, how to improve the interpretability of the model will be one of the main research hotspots in the near future.

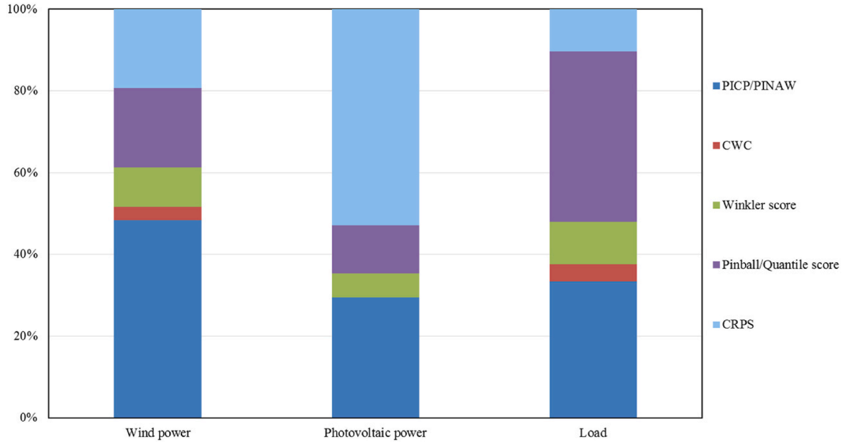

Fig. 8. The proportion of various evaluation metrics for different applications.

(3) *Pay more attention to the comparison of state-of-the-art methods.* Although researchers are constantly exploring new approaches to improve the performance of QR-based methods, the comparison among these up-to-date methods has not been fully studied. In particular, the comparison between different machine learning-based methods is worthwhile for researchers, as these methods are widely used in probabilistic forecasting.

(4) *Unified evaluation metrics.* Reliability and sharpness are the two complementary properties when evaluating a probabilistic forecast, however, most of the metrics mentioned above focus on the sharpness property of the forecasting results, and coming up with a metric that considers both properties is a key problem that needs to be solved urgently. It is worth noting that although CWC meets the above conditions, it is still a controversial metric, and there may be cases where it is not applicable.

6. Conclusions

In this paper, the QR-based forecasting methods are summarized and classified according to their principle. On this basis, the two main types of applications that implement QR-based forecasting methods are also described in detail in this work. It can be concluded that QR-based methods are suitable for probabilistic forecasting due to their ease of use, and the increasing number of related publications indicates that QR-based methods are attracting more and more attention from researchers.

A significant contribution of this review is the comparison of the advantages and disadvantages of the two different types of methods. Statistic-based methods rely heavily on the assumption of the relationship between input and output variables, proper selection of the relationship may lead to the satisfying performance of probabilistic forecasting results, but it may become knotty when dealing with the complex nonlinear problem. Machine learning-based methods can generate higher performance of probabilistic forecasting results due to the powerful fitting ability of machine learning methods and no assumptions made about the distribution of errors, however, the models are always unexplainable and cannot extrapolate beyond the range of the training data. Besides providing a reference and guidance for the selection of the most suitable probability prediction method of renewable energy and power load, this paper also summarizes several research gaps and future potential opportunities for improving the forecasting performance of QR-based methods.

This comprehensive review can help researchers to identify the commonly used QR-based probabilistic forecasting methods and provide them with a comprehensive overview of the applications. Besides, decision-makers and companies in the energy market can learn about recent advances in forecasting techniques to develop more efficient market strategies from this review.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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