

ساختمان داده ها

درخت جستجوی دودویی (Binary Search Trees)

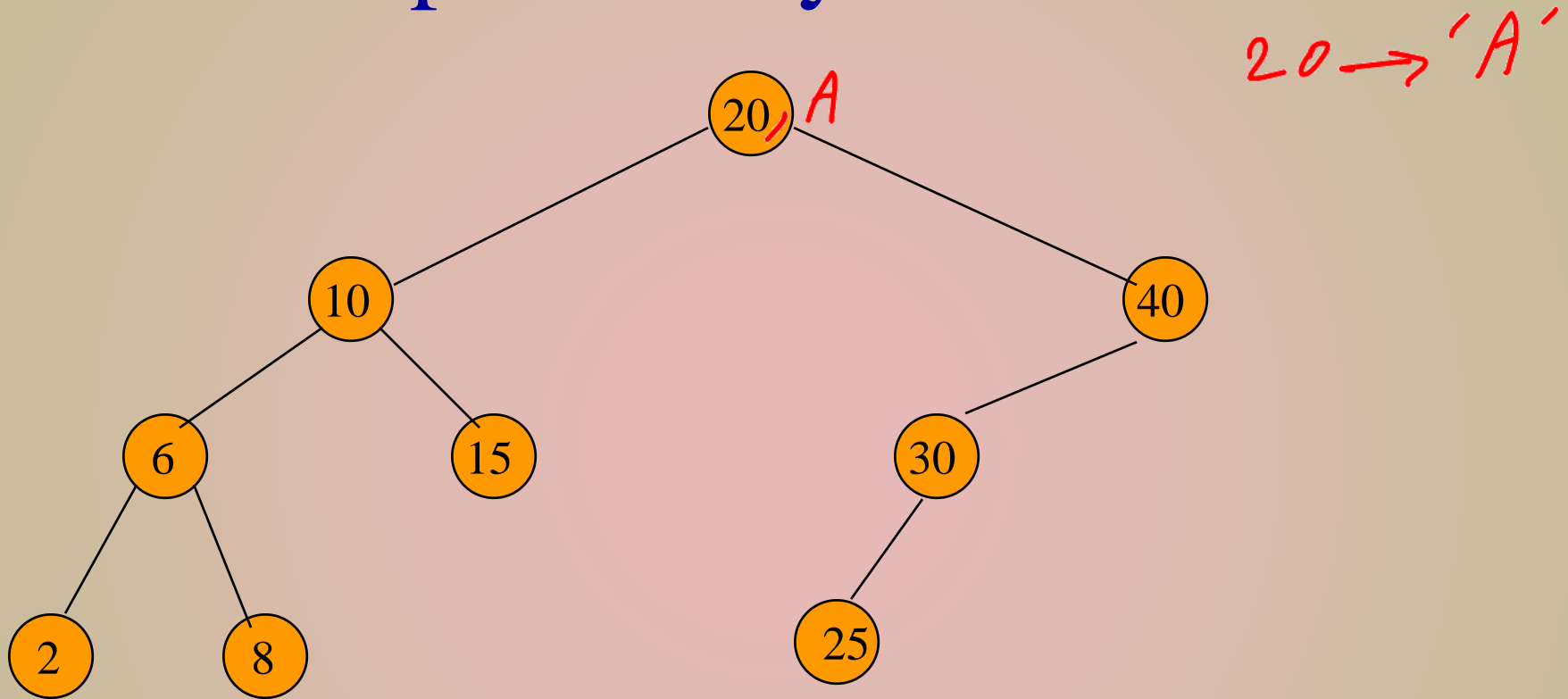
مدرس: غیاثی شیرازی
دانشگاه فردوسی مشهد

Definition Of Binary Search Tree

key \longrightarrow value

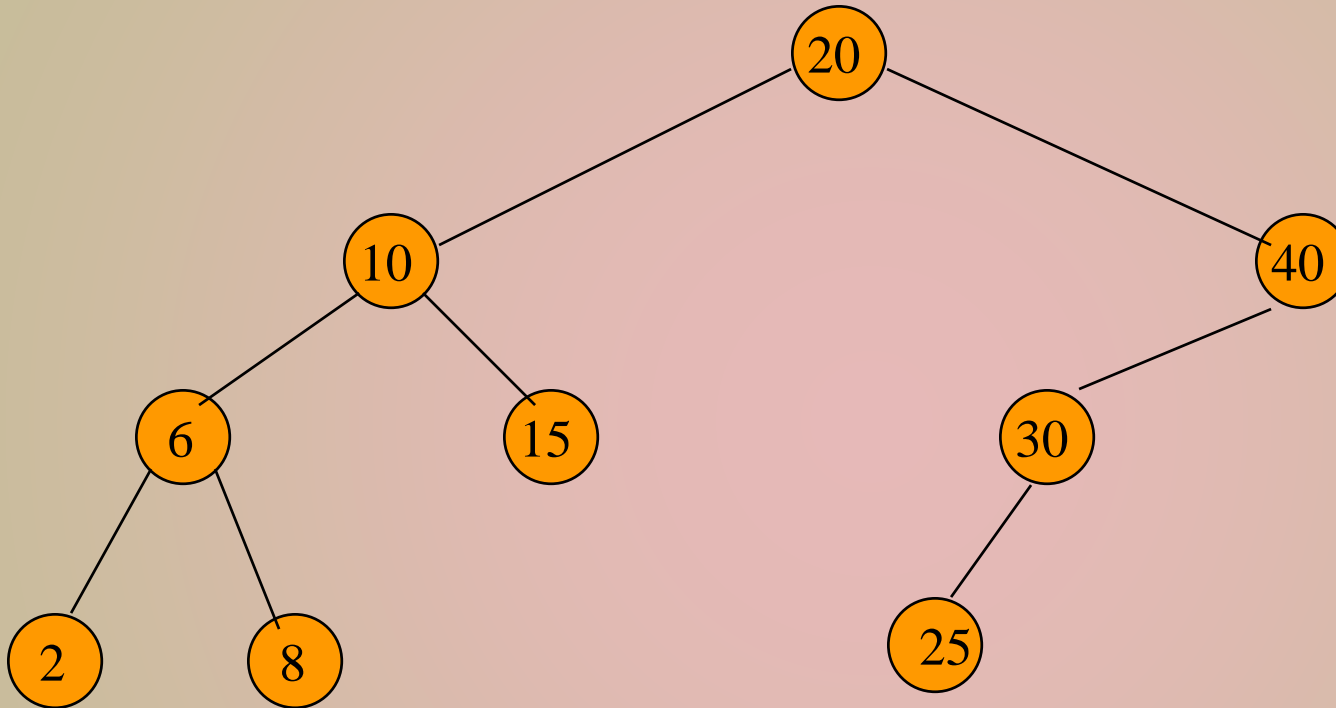
- A binary tree.
- Each node has a (key, value) pair.
- For every node **x**, all keys in the left subtree of **x** are smaller than that in **x**.
- For every node **x**, all keys in the right subtree of **x** are greater than that in **x**.

Example Binary Search Tree



Only keys are shown.

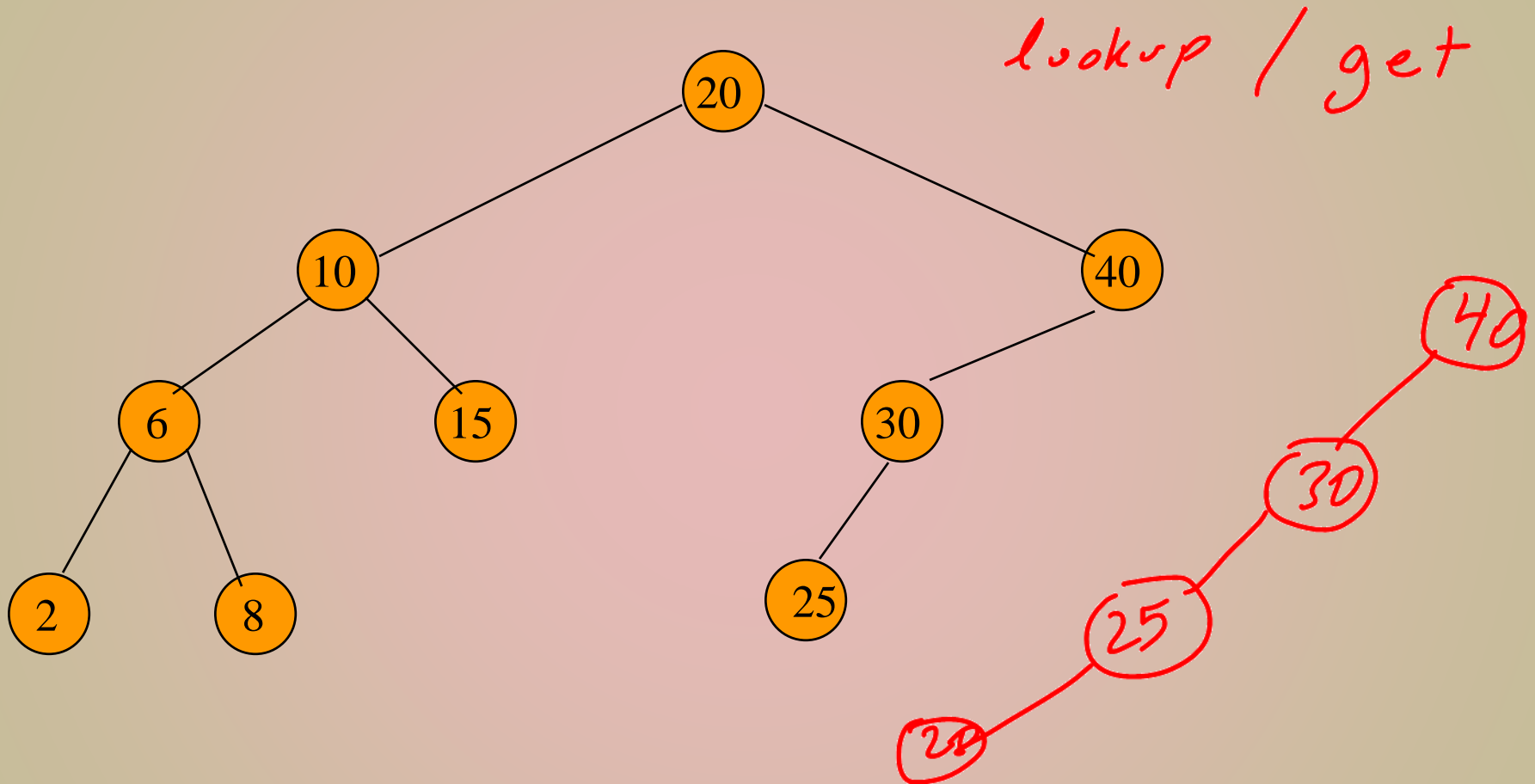
The Operation Ascend()



2, 6, 8, 10, 15, 20, 25, 30, 40

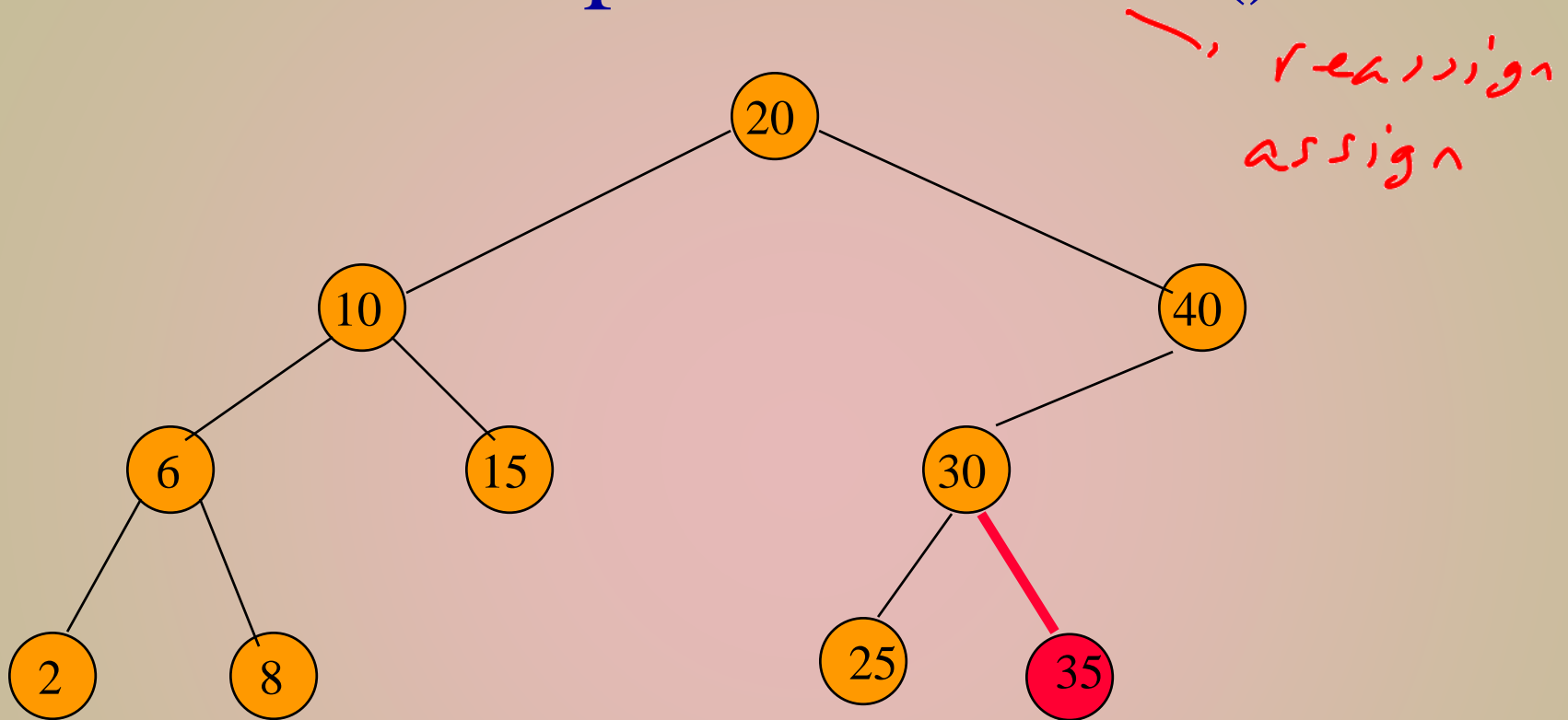
Do an inorder traversal. $O(n)$ time.

The Operation Get()



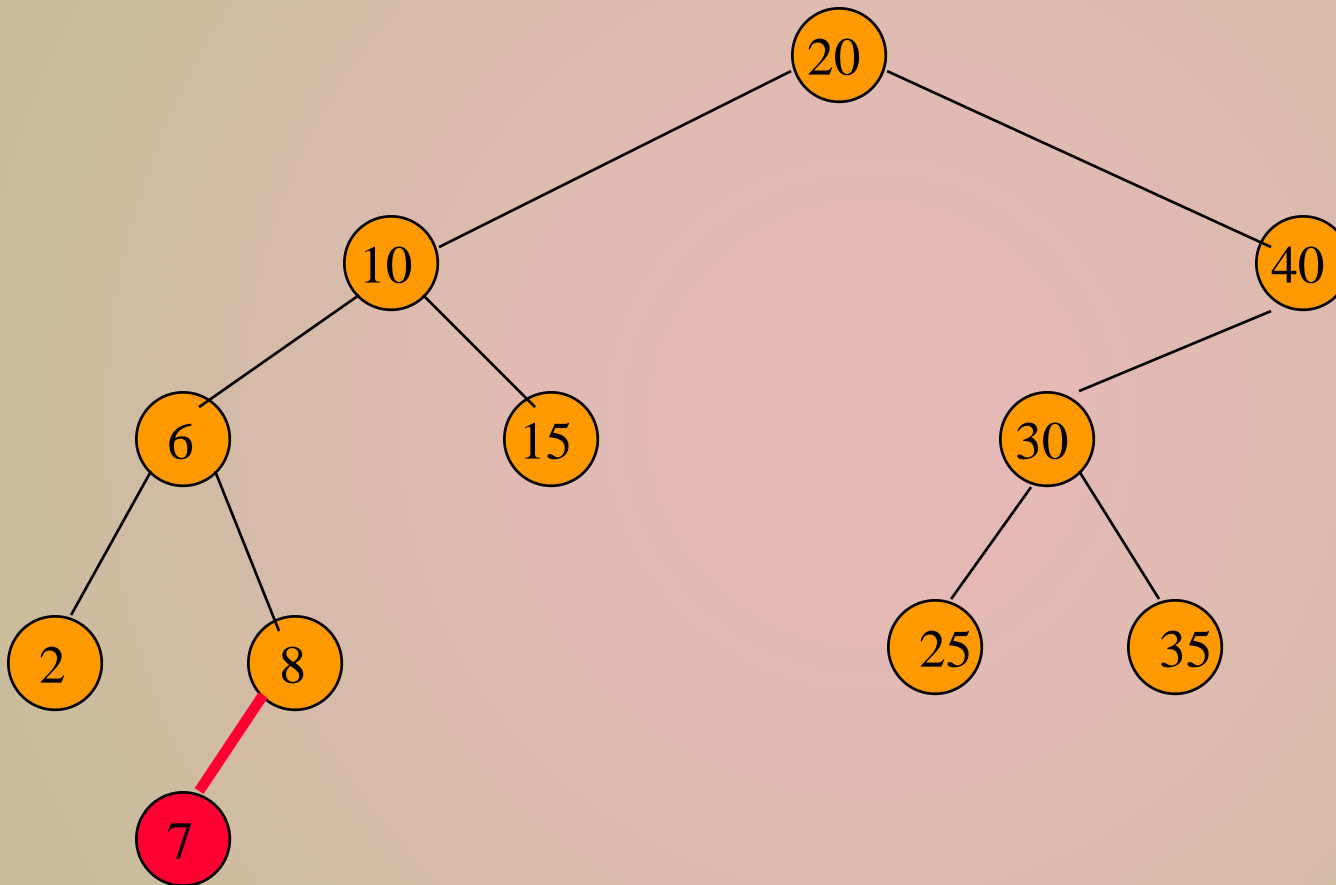
Complexity is $\Theta(\text{height}) = \Theta(n)$, where n is number of nodes/elements.

The Operation Insert()



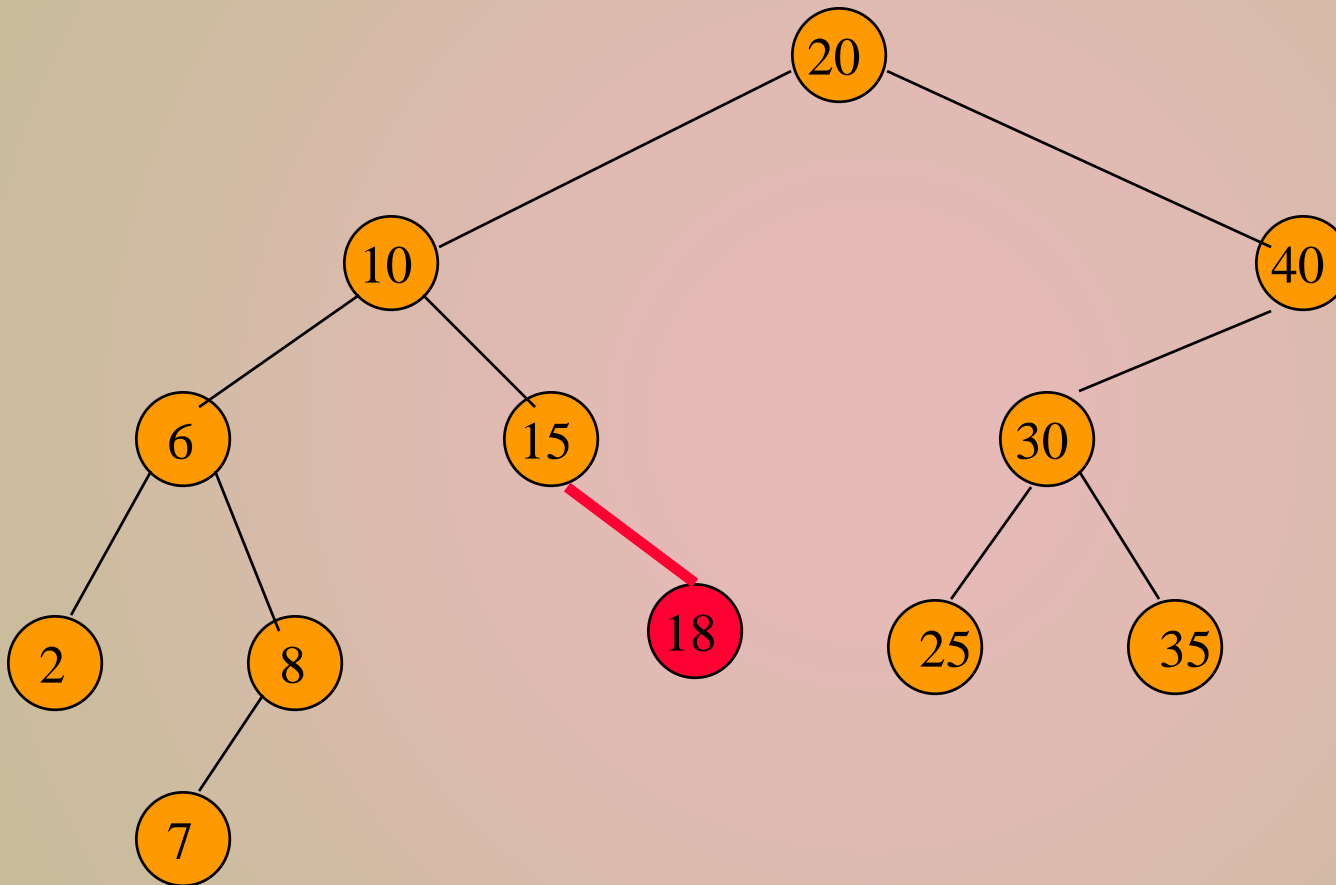
Insert a pair whose key is **35**.

The Operation Insert()



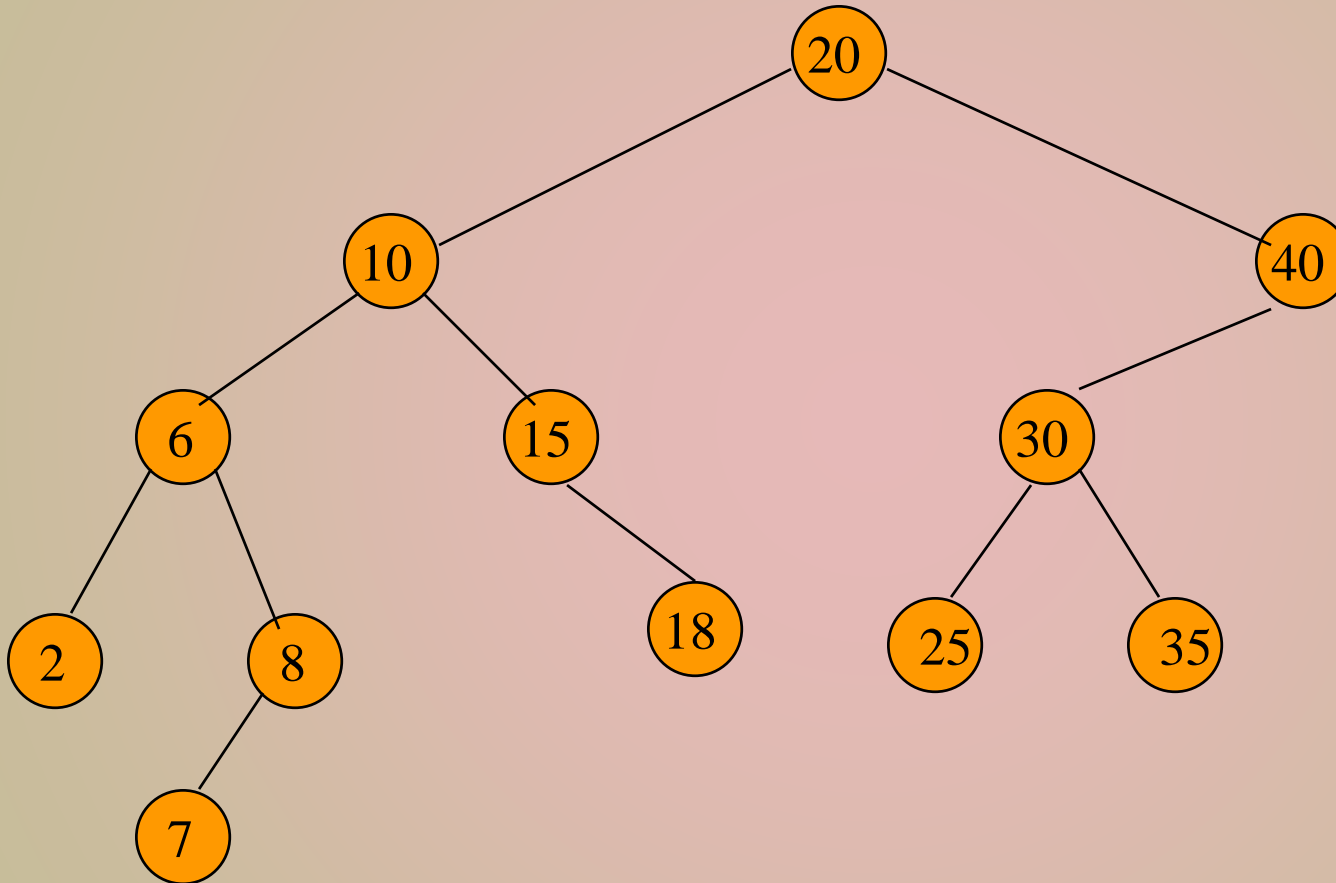
Insert a pair whose key is **7**.

The Operation Insert()



Insert a pair whose key is 18.

The Operation Insert()



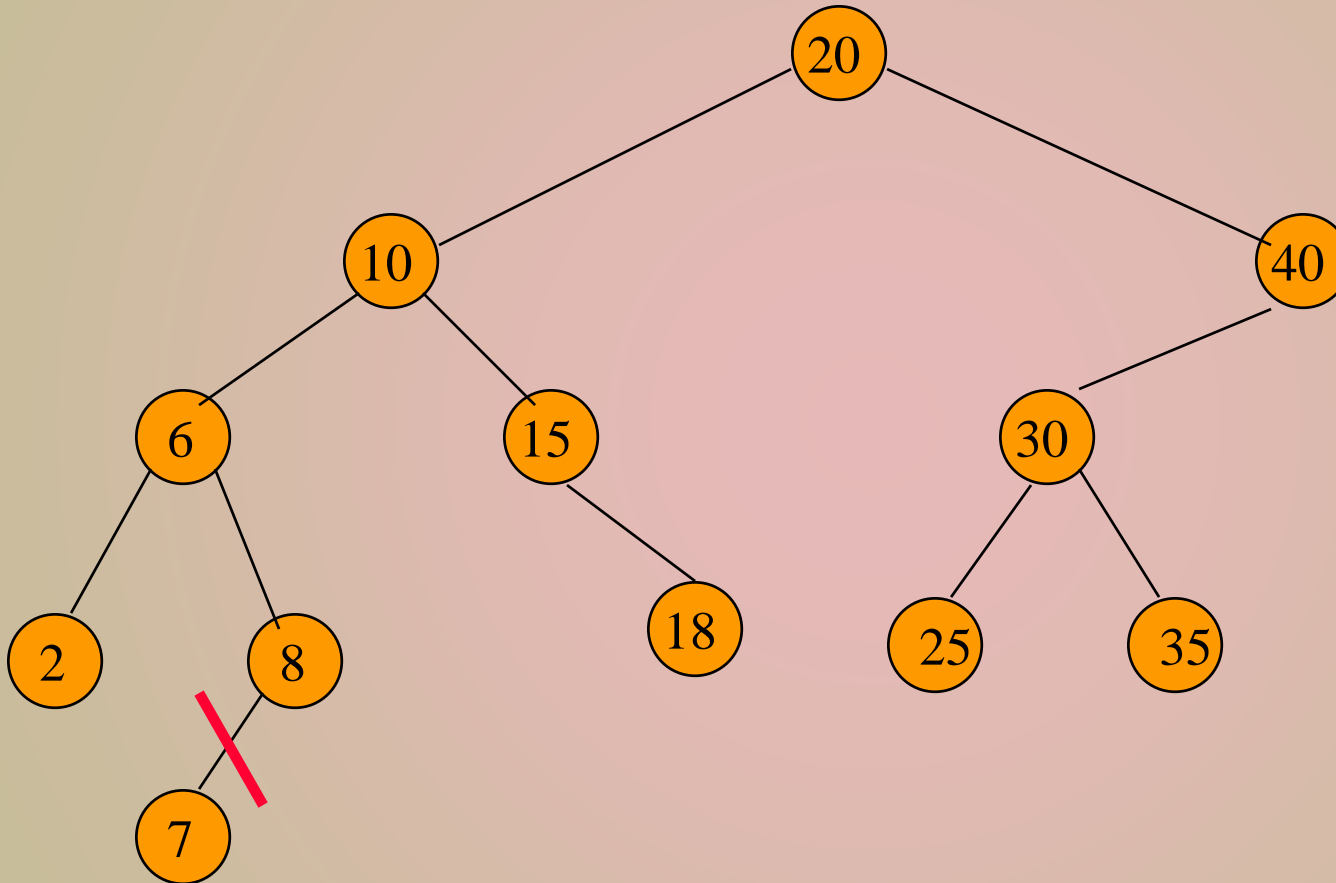
Complexity of `Insert()` is $O(\text{height})$.

The Operation Delete()

Four cases:

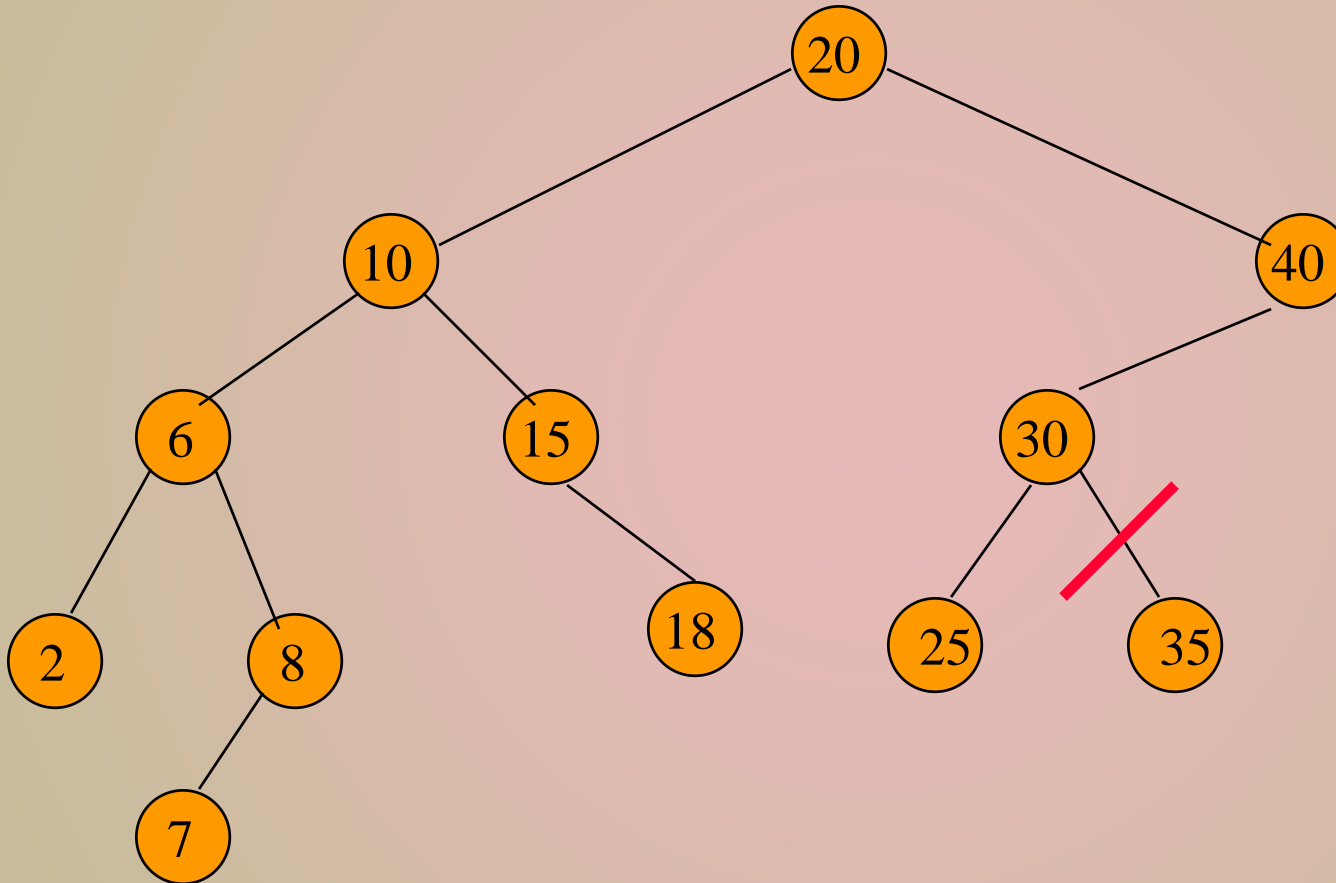
- No element with delete key.
- Element is in a leaf.
- Element is in a degree 1 node.
- Element is in a degree 2 node.

Delete From A Leaf



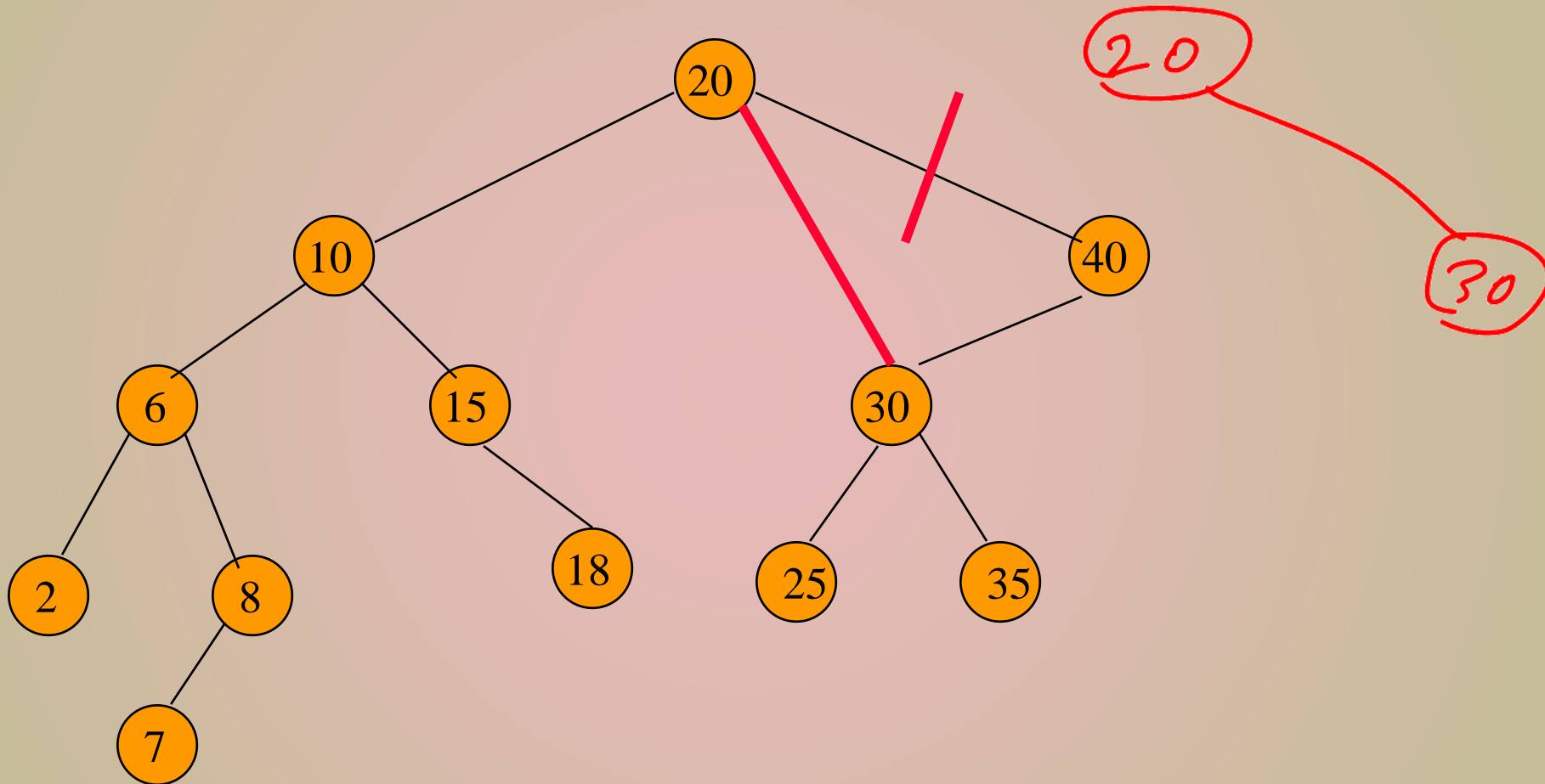
Delete a leaf element. key = 7

Delete From A Leaf (contd.)



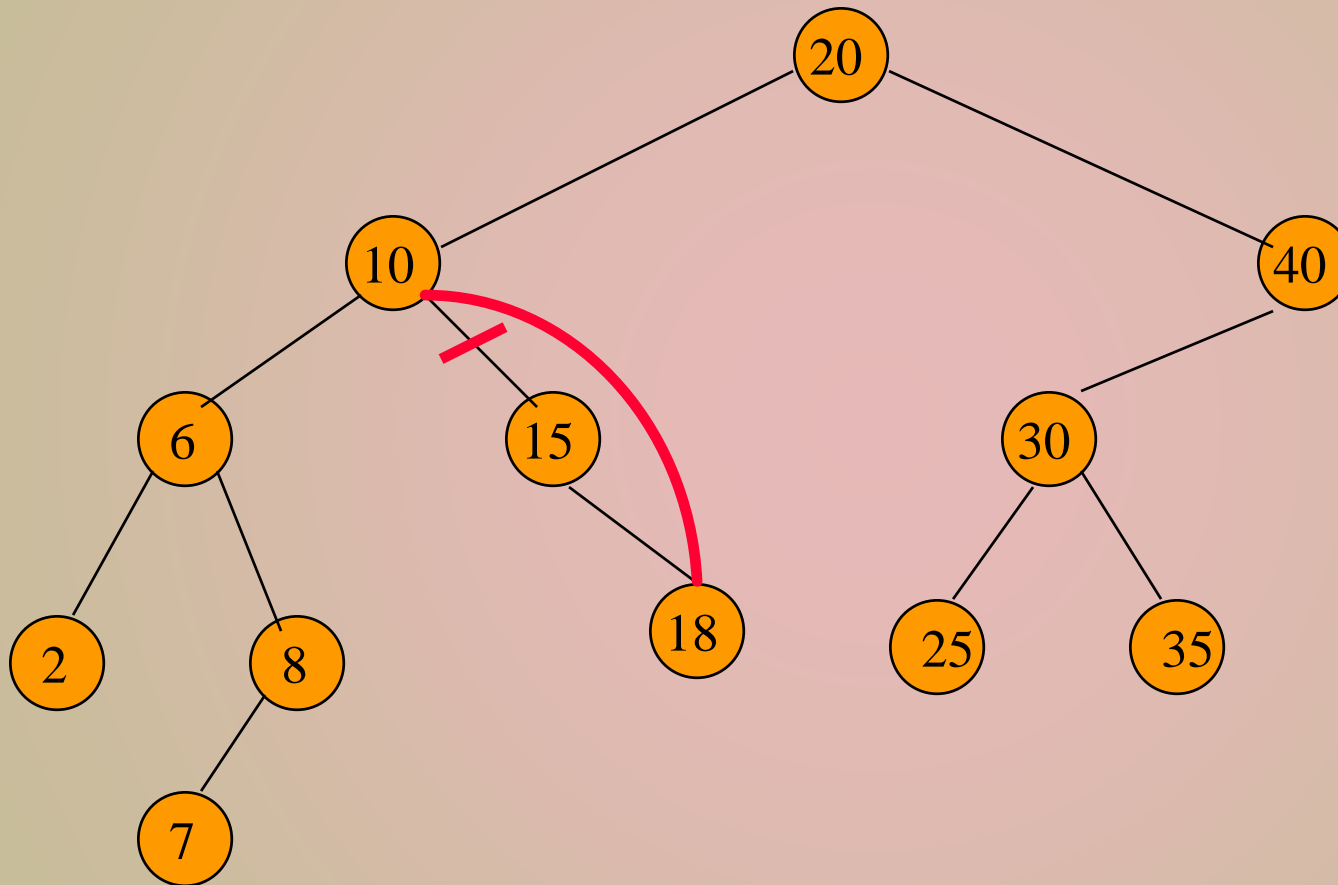
Delete a leaf element. key = 35

Delete From A Degree 1 Node



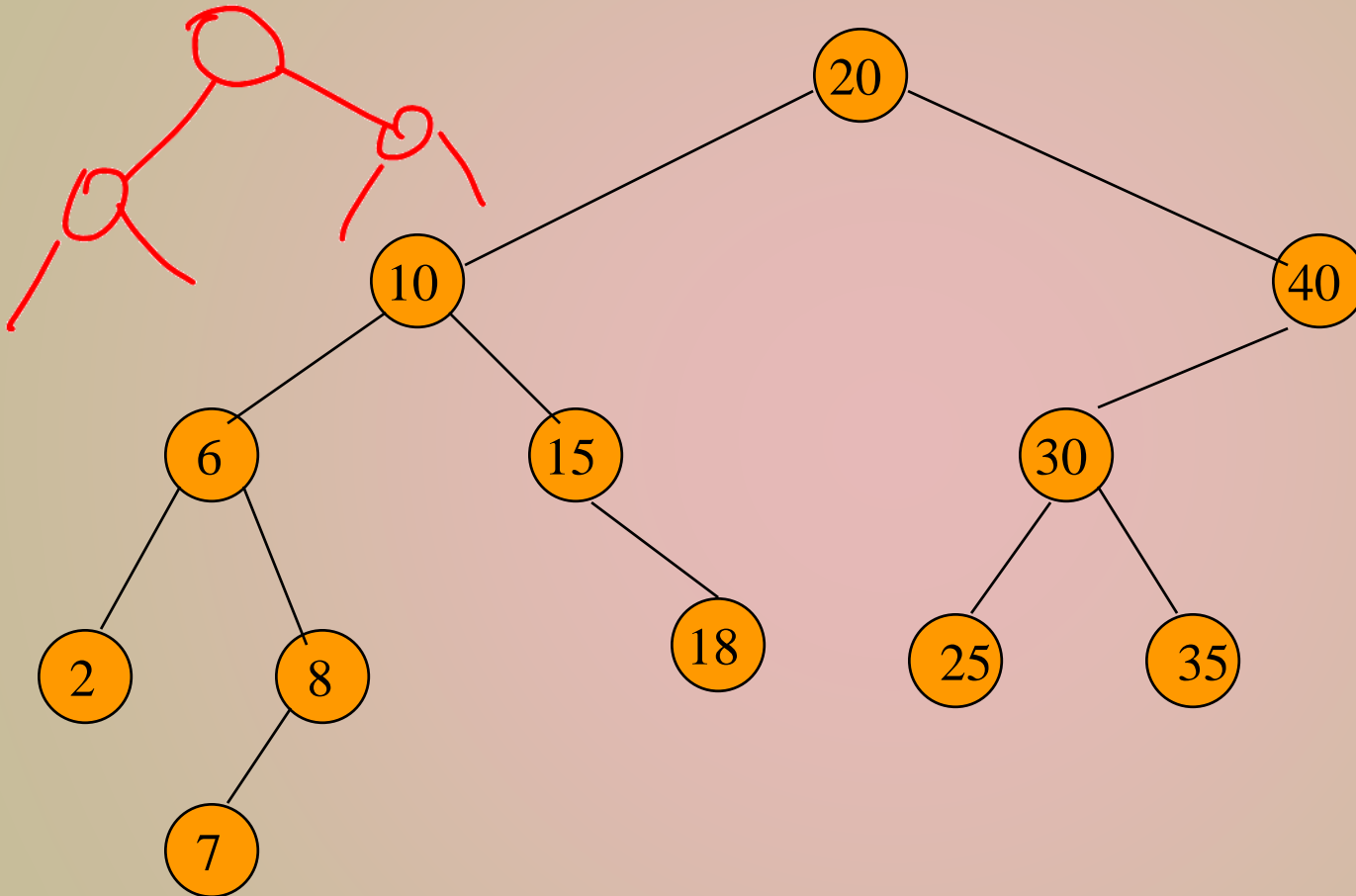
Delete from a degree 1 node. key = 40

Delete From A Degree 1 Node (contd.)



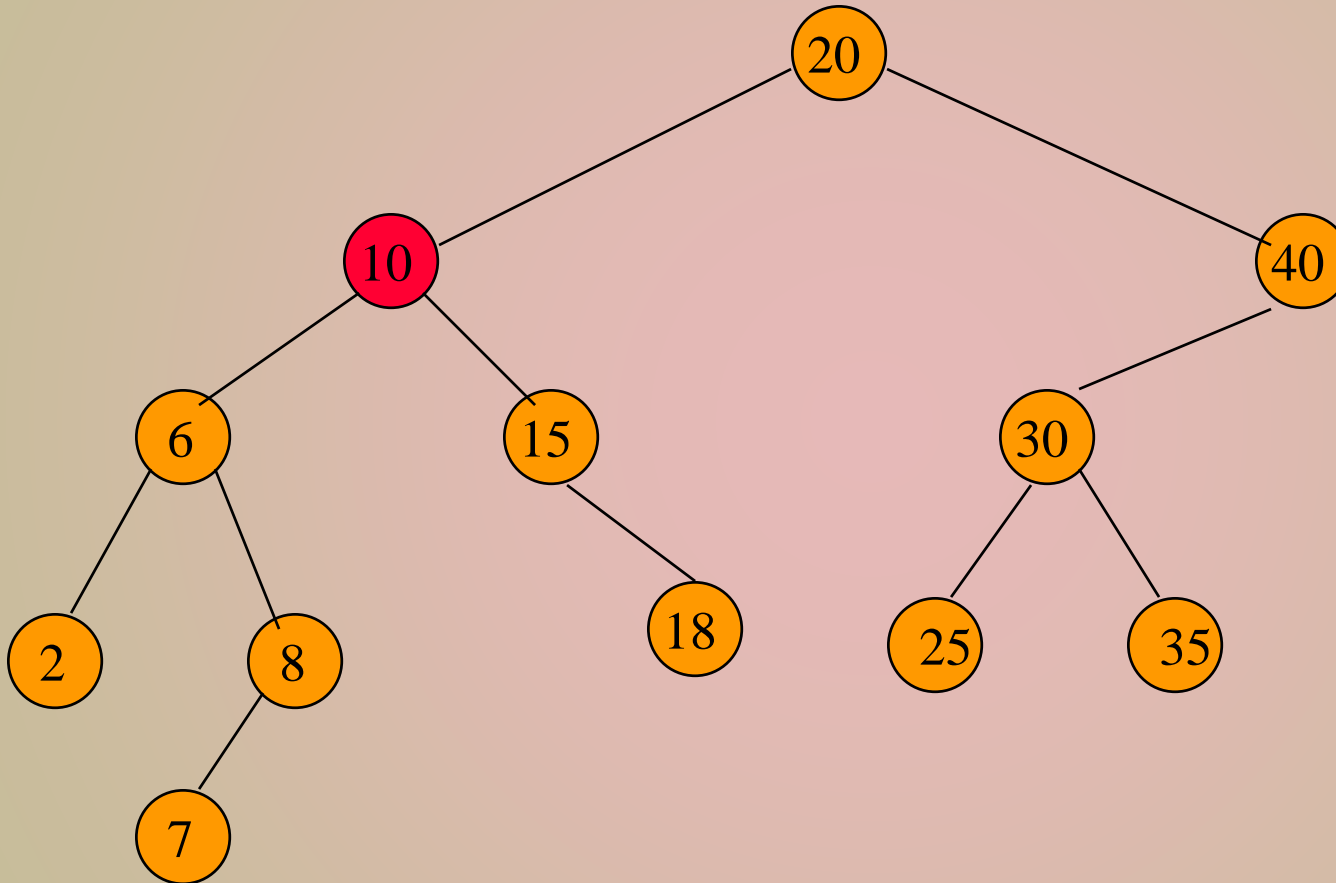
Delete from a degree **1** node. key = **15**

Delete From A Degree 2 Node



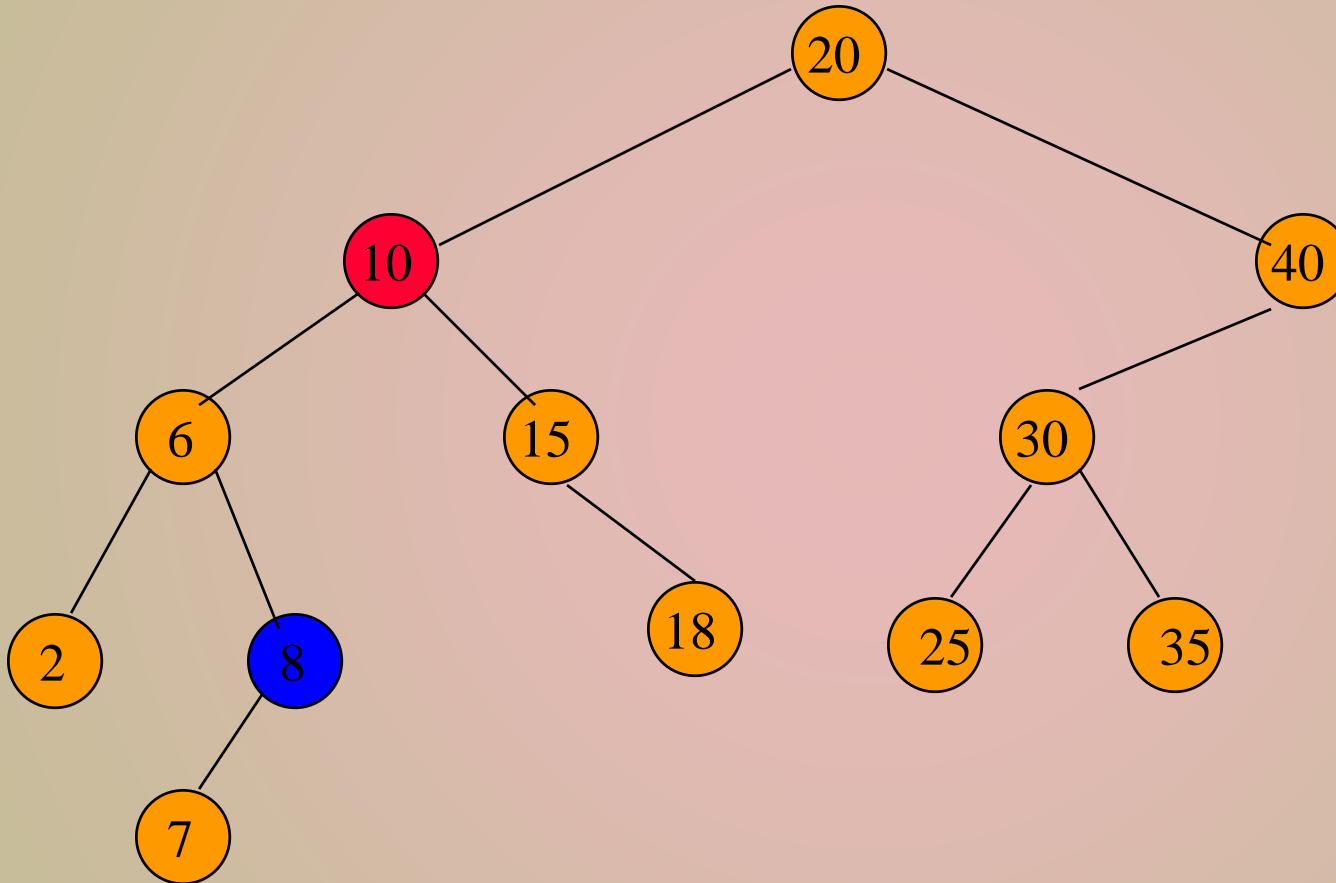
Delete from a degree 2 node. key = 10

Delete From A Degree 2 Node



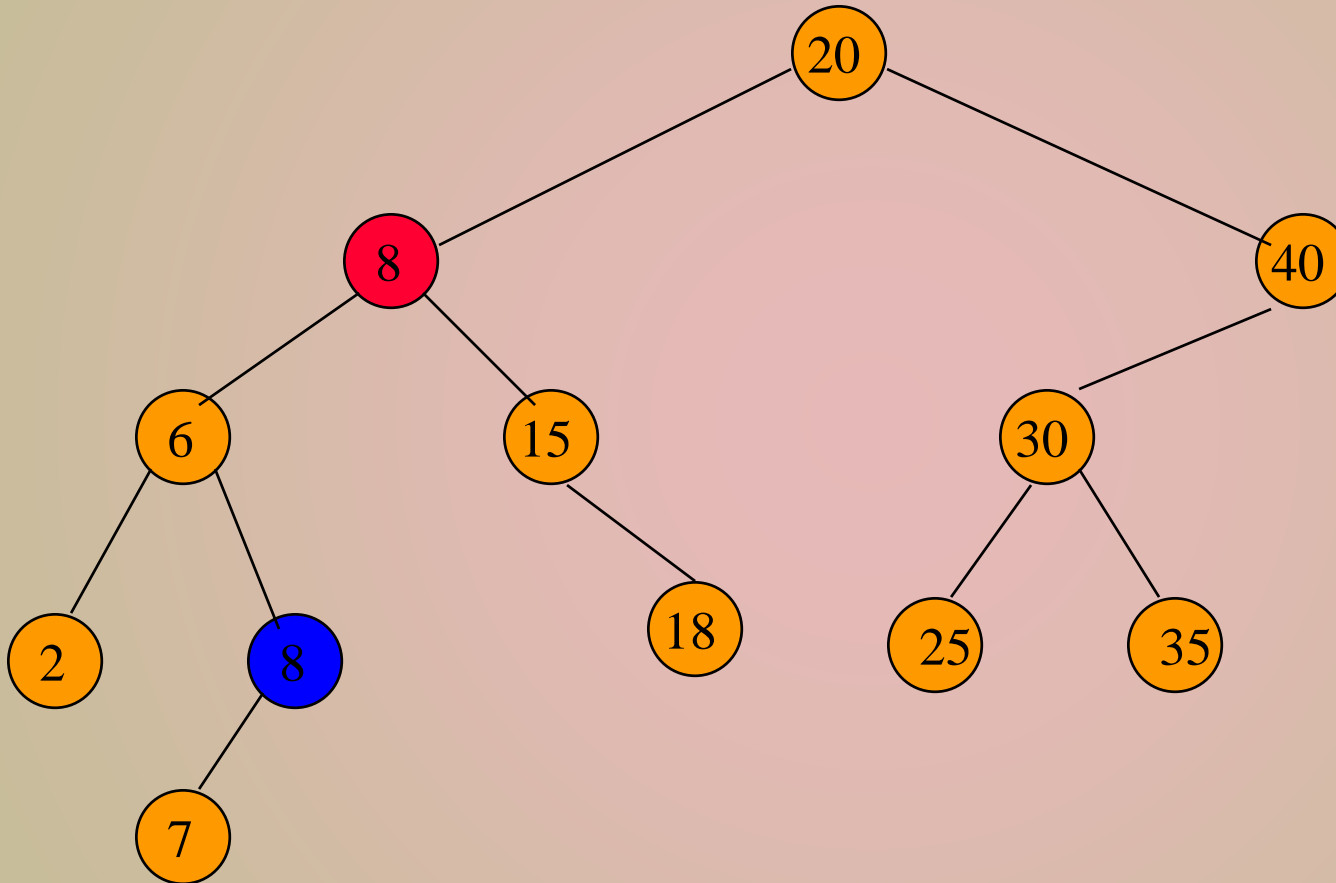
Replace with largest key in left subtree (or smallest in right subtree).

Delete From A Degree 2 Node



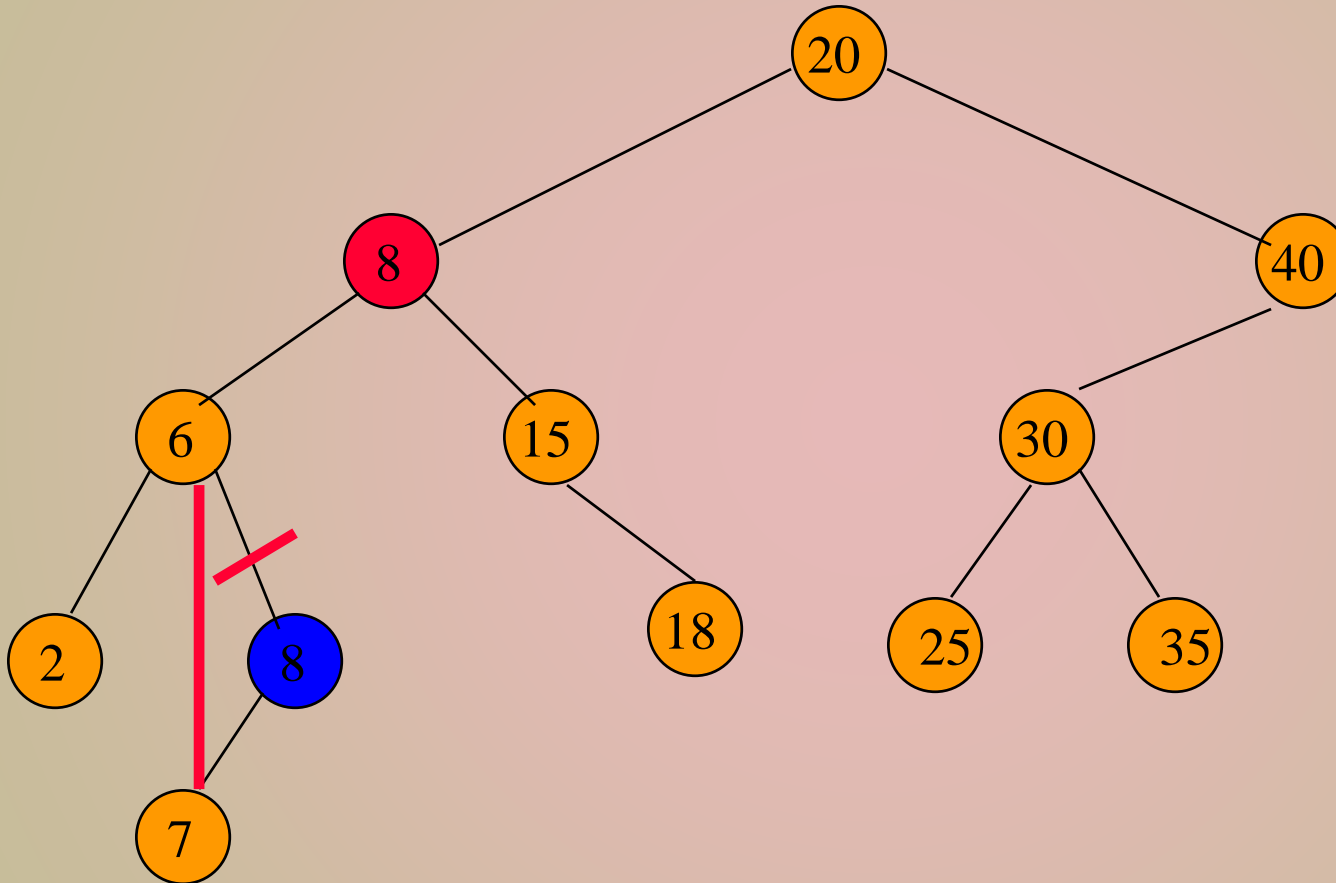
Replace with largest key in left subtree (or smallest in right subtree).

Delete From A Degree 2 Node



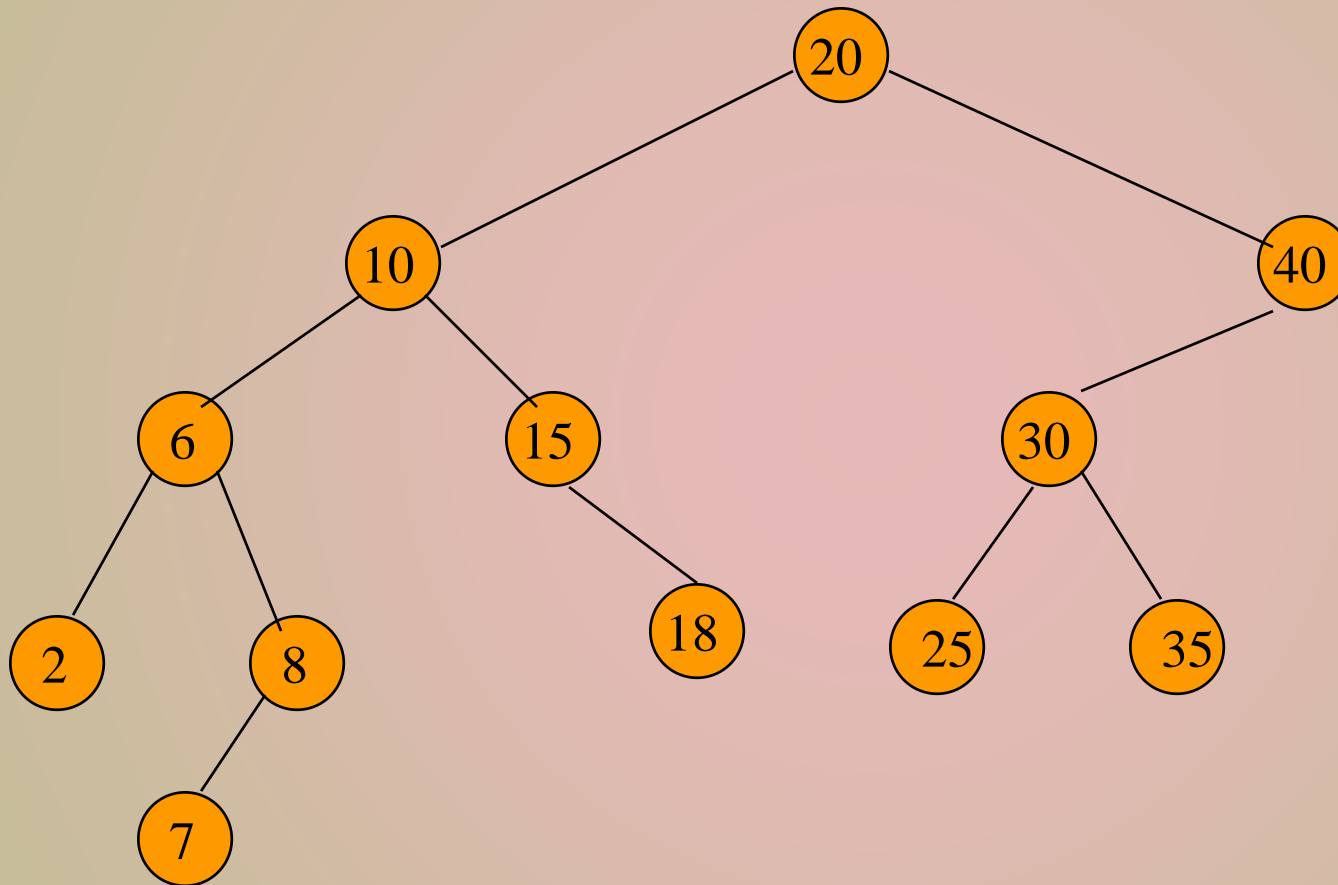
Replace with largest key in left subtree (or smallest in right subtree).

Delete From A Degree 2 Node



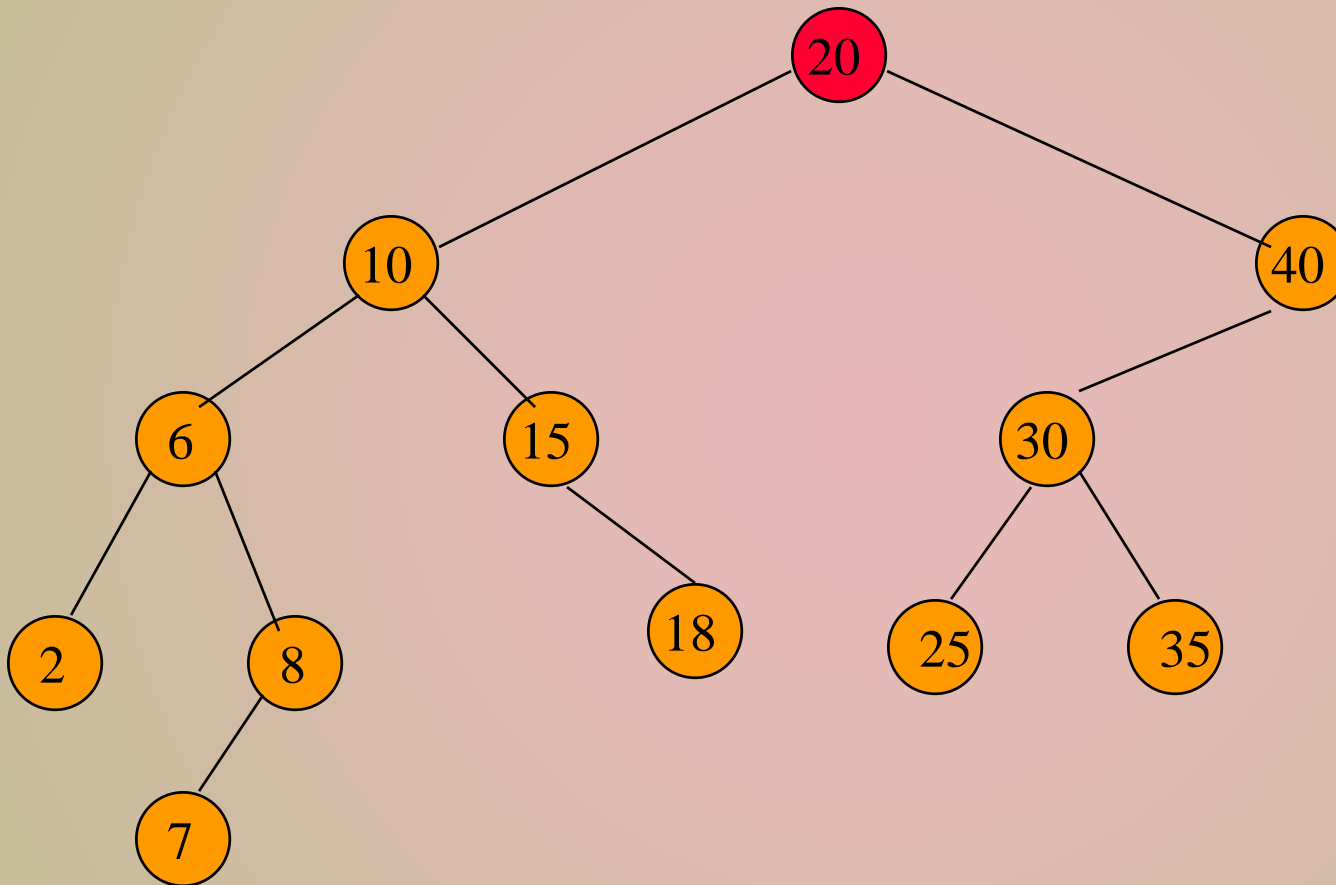
Largest key must be in a leaf or degree **1** node.

Another Delete From A Degree 2 Node



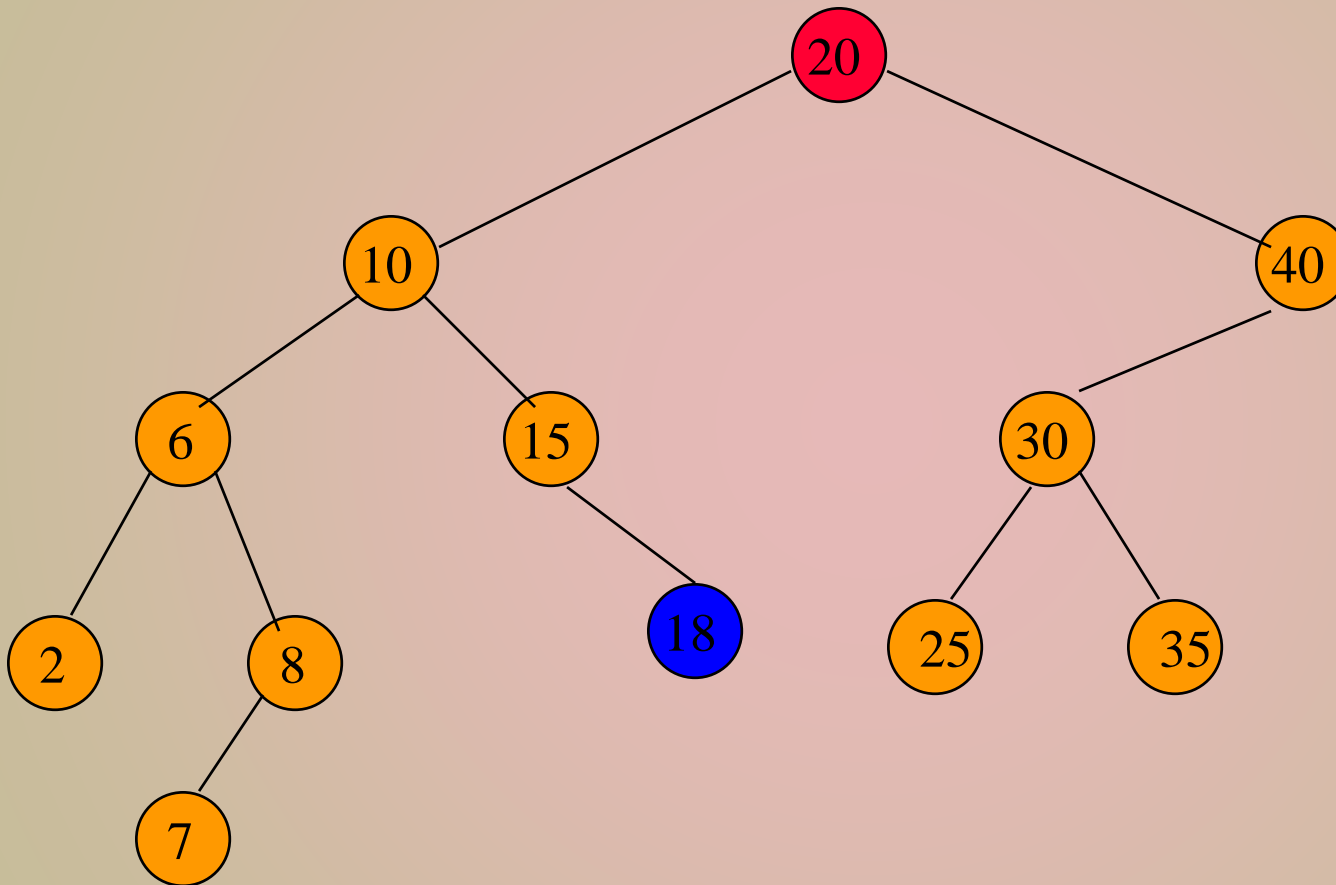
Delete from a degree 2 node. key = 20

Delete From A Degree 2 Node



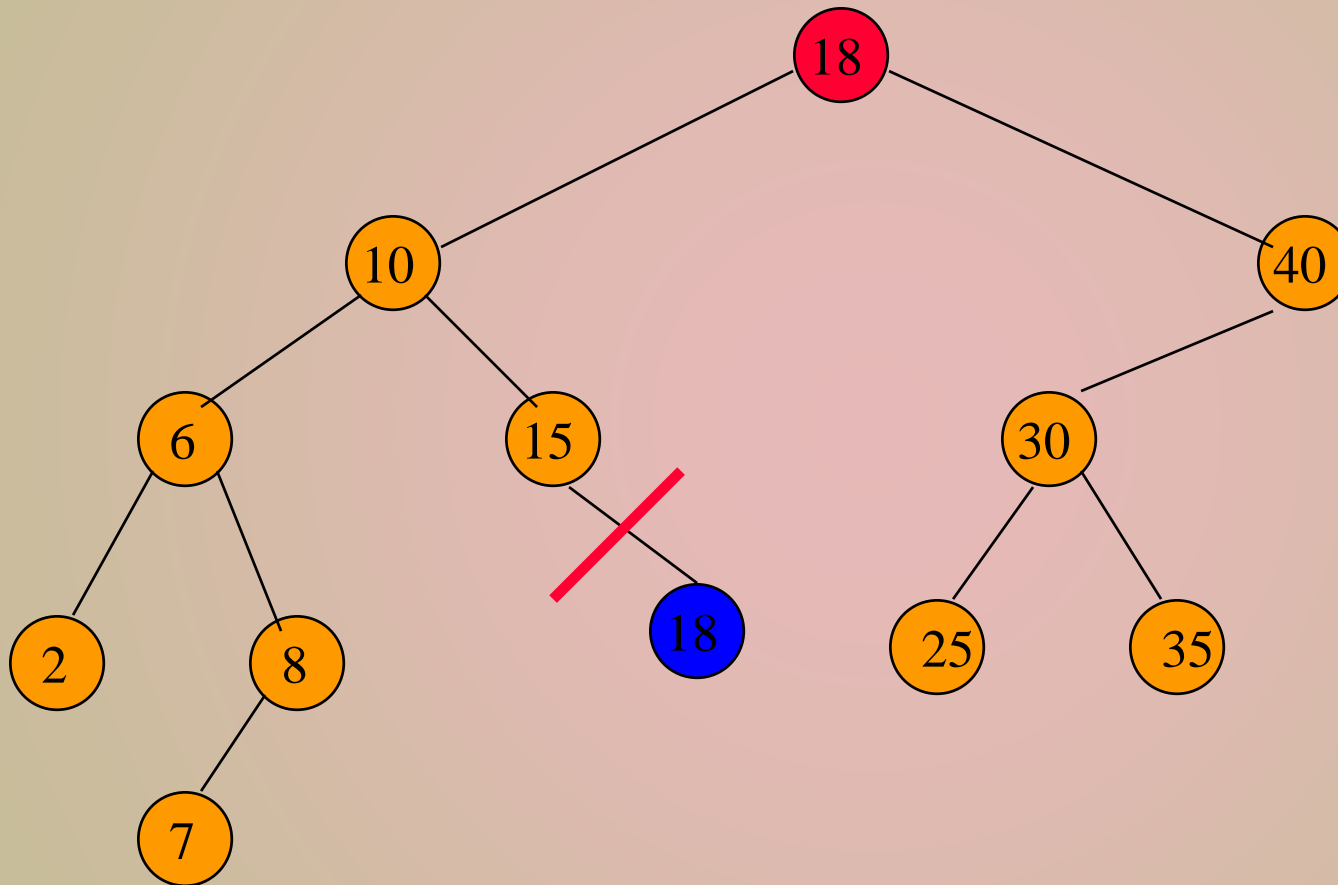
Replace with largest in left subtree.

Delete From A Degree 2 Node



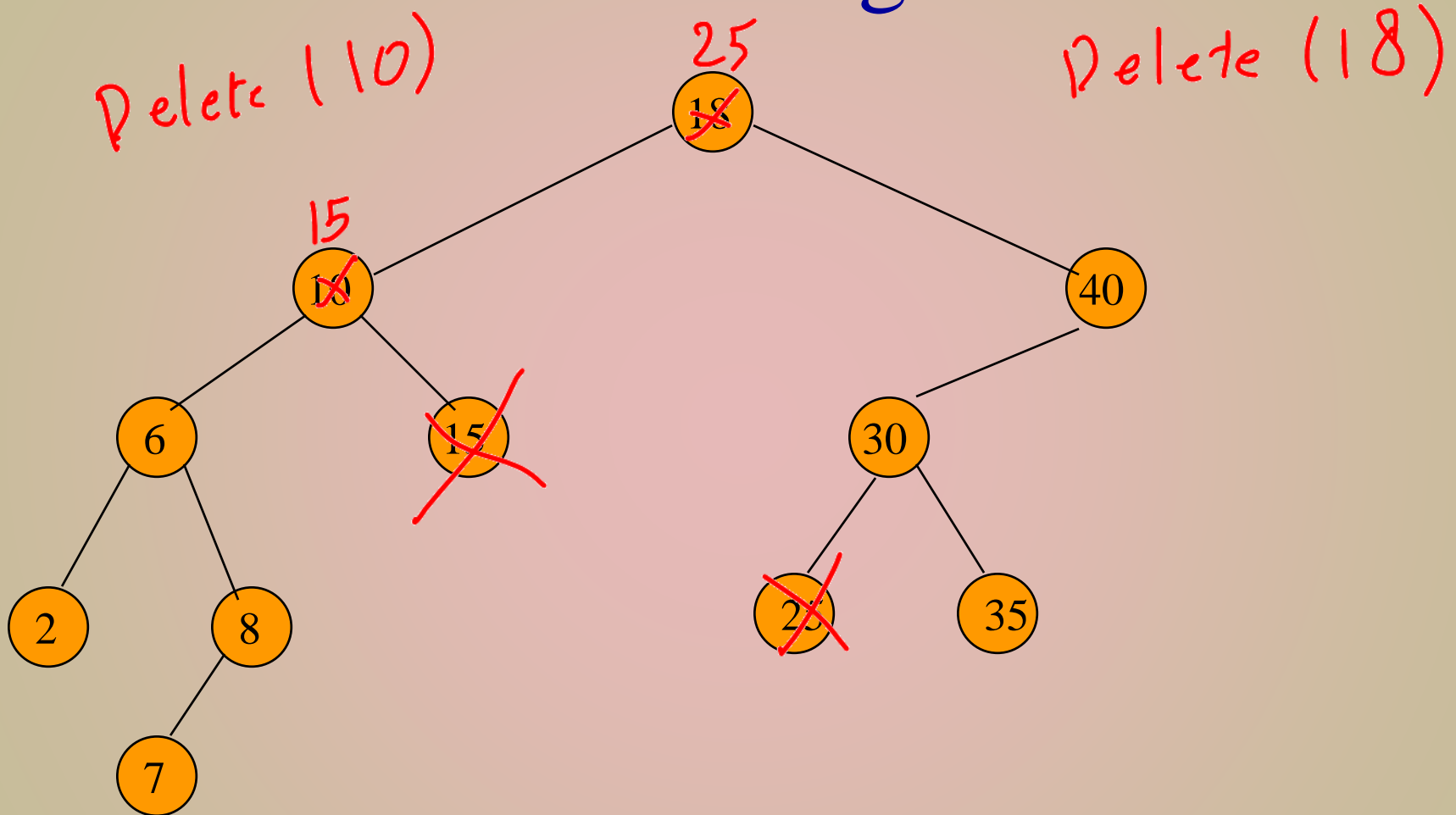
Replace with largest in left subtree.

Delete From A Degree 2 Node



Replace with largest in left subtree.

Delete From A Degree 2 Node



Complexity is $O(\text{height})$.