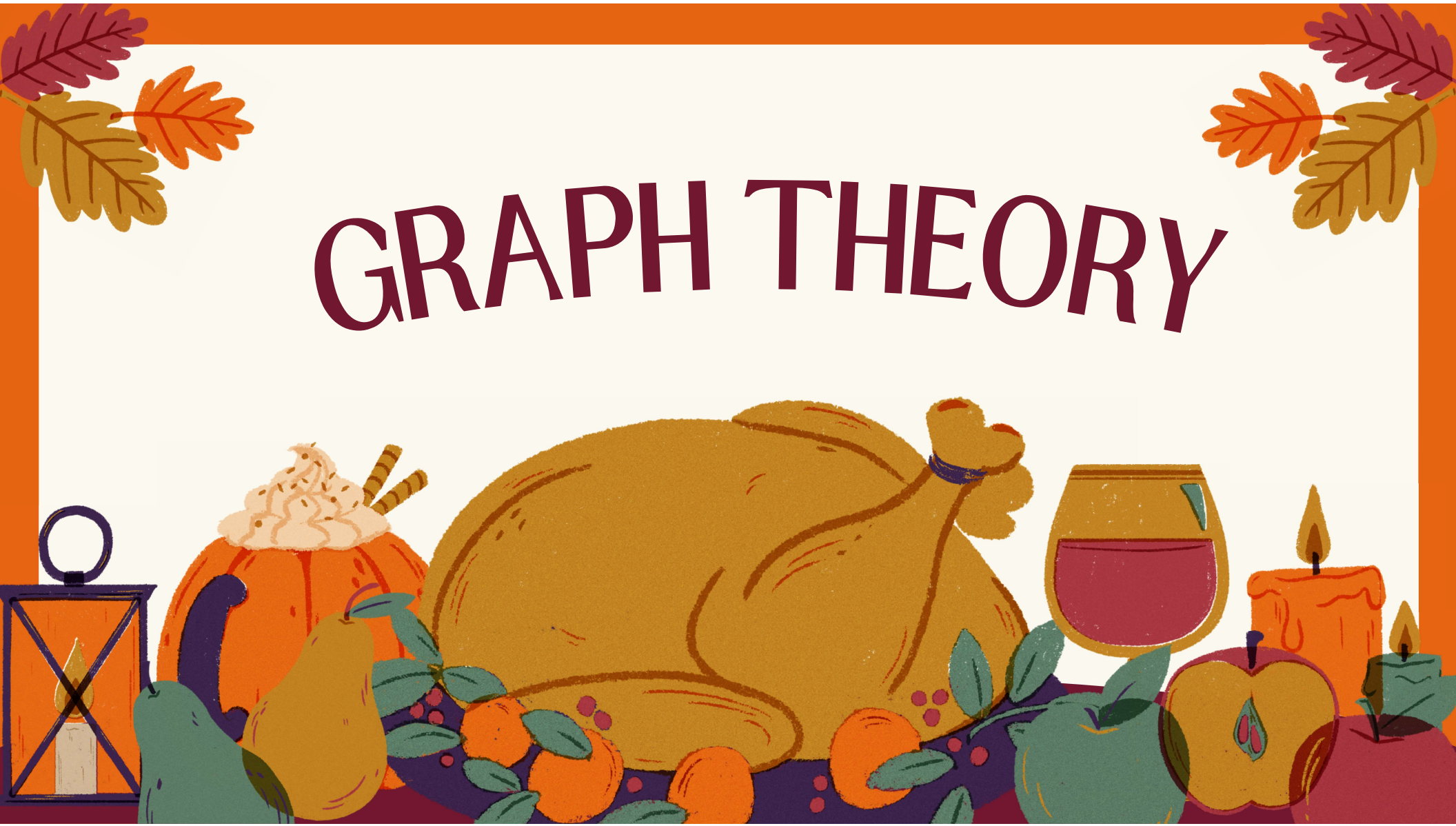
GRAPH THEORY



Graph Theory

Graph theory is a fascinating field of mathematics that deals with graphs—mathematical structures used to model pairwise relations between objects.

Graph theory provides a powerful framework for understanding relationships, analyzing networks, and solving practical problems across various domains.

Directed vs Undirected Graphs

In an undirected graph, edges do not have a specific direction.
In a directed graph, edges have a specific direction.

Undirected Graphs	Directed Graphs
• Edges are bidirectional. • Symmetric relationships. • Simpler to understand and manipulate. • Requires less memory.	• Edges have direction. • Asymmetric relationships. • Efficient for modeling processes or dependencies. • Suitable for efficient traversal in a specified direction.

The Problem

- Königsberg (now Kaliningrad, Russia) was a city situated on both sides of the Pregel River. It included two large islands, Kneiphof and Lomse, connected by seven bridges.

- The challenge was to find a walk through the city that would cross each bridge once and only once.

- Solutions involving reaching an island or mainland bank without using a bridge were explicitly unacceptable.

Topological Sort Requirement

- Euler realized that the choice of route within each landmass was irrelevant. The critical aspect was the sequence of bridges crossed.
- He abstracted the problem, focusing solely on the list of landmasses and the bridges connecting them.
- In modern terms, he replaced each landmass with an abstract vertex (or node) and each bridge with an abstract connection (an “edge”).
- The resulting mathematical structure is a graph.

Graph Representation

- Euler observed that whenever one enters a vertex (landmass) via a bridge, they also leave it via a bridge (except at the endpoints of the walk).

- If every bridge had been traversed exactly once, the number of bridges touching each landmass must be even (half toward the landmass, half away from it).

- However, all four landmasses in the original problem were touched by an odd number of bridges (one by 5 bridges, and each of the other three by 3).

Negative Resolution

- Euler proved that the problem had no solution. The difficulty lay in developing a suitable technique of analysis and rigorous mathematical tests.

- His work established the foundations of graph theory and demonstrated the power of abstracting real-world problems into mathematical structures.

Thank you for coming!