Due 23:59 Oct 19 (Sunday). There are 100 points in this assignment.
Submit your answers **(must be typed)** in pdf file to CourSys
`https://coursys.sfu.ca/2025fa-cmpt-705-x1/`.
Submissions received after 23:59 will get penalty of reducing points: 20 and 50 points deductions for submissions received at $[00:00, 00:10]$ and $(00:10, 00:30]$ of Oct 20, respectively; no points will be given to submissions after $00:30$ of Oct 20.

1. (Chapter 7 Problems 1 and 2 of the text book) 15 points

   (a) List all minimum $s-t$ cuts in the flow network pictured in Figure 1 (i). The capacity of each edge appears as a label next to the edge.

   (b) What is the minimum capacity of an $s-t$ cut in the flow network in Figure 1 (ii)? Again, the capacity of each edge appears as a label next to the edge.

   Figure 1 (iii) shows a flow network on which an $s-t$ flow has been computed. The capacity of each edge appears as a label next to the edge, and the numbers in boxes give the amount of flow sent on each edge.

   (c) What is the value of this flow? Is this a maximum (s,t) flow in this graph?

   (d) Find a minimum $s-t$ cut in the flow network pictured in Figure 1 (iii), and give the capacity of the minimum cut.

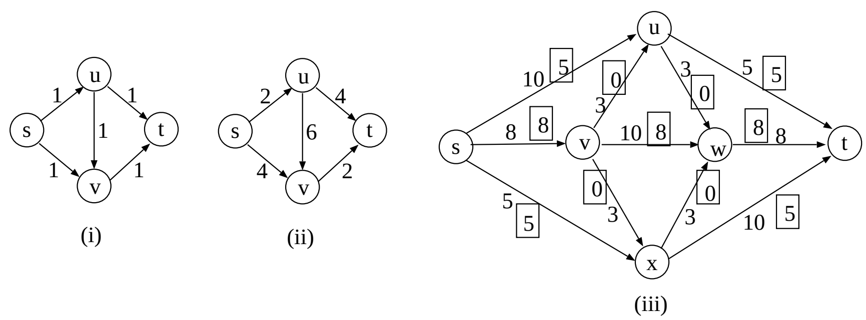


Figure 1: Figures for questions (a), (b), (c) and (d).

2. 10 points

   Figure 2 gives a flow network G and a function $f : E(G) \rightarrow R^+$. The capacity of each arc appears as a label next to the arc, the value assigned to each arc by f is in the box next to the arc.

   (a) Is f a flow on G? If yes, why? and give the residual graph G_f w.r.t. f. If no, why?

   (b) Implement Ford-Fulkerson Algorithm for the maximum flow problem and show the the max-flow found by your implementation for G. You do not need to submit your program codes.

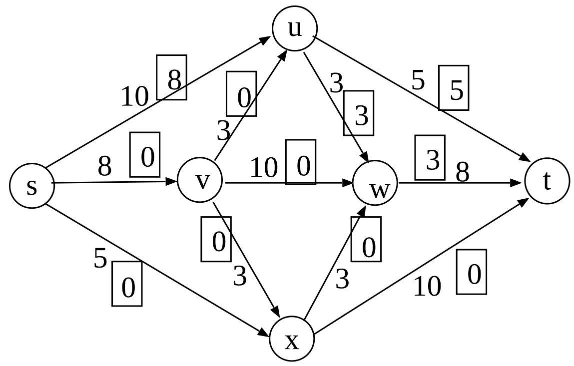


Figure 2: Figure for question 2.

3. (Chapter 7 Problem 7 of the text book) 15 points

   Consider the problem to connect a set of mobile clients to a set of base stations, each client needs to be connected to exactly one station. The position of each client (station) is specified by its (x, y) coordinates in the plane. There is a range parameter r, a client can only be connected to a station that is within distance r. There is also a load parameter L, no more than L clients can be connected to any single station. Design a polynomial time algorithm for the following problem and analyze your algorithm: Given the positions of a set of n clients and a set of k base stations, as well as, the range and the load parameters, decide whether every client can be connected simultaneously to a base station, subject to the range and load conditions above.

4. (Chapter 7 Problem 8 of the text book) 15 points

   The blood of human is divided into four types, A, B, AB, O. In a blood transfusion, a patient of type A can receive only types A and O, one of type B can receive only types B and O, one of type O can receive only type O, and one of type AB can receive any of the four types.
   (a) Let S_O, S_A, S_B, S_{AB} be the units of blood supply for the four types respectively. Let d_O, d_A, d_B, d_{AB} be the units of blood demand for the four types respectively. Give an algorithm which, given blood supplies and demands, evaluates if the supplies suffice to the demands in polynomial time in the total units of supplies, and analyze your algorithm.
   (b) Assume the blood supplies are: $S_O = 50$, $S_A = 36$, $S_B = 11$ and $S_{AB} = 8$. Assume the blood demands are $d_O = 45$, $d_A = 42$, $d_B = 8$ and $d_{AB} = 3$. Are the blood supplies enough to satisfy the demands? Prove your answer.

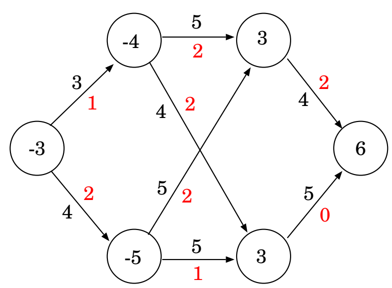
5. (Chapter 7 Problem 14 of the text book) 15 points

   An Escape Problem is defined as follows: given a digraph $G(V, E)$ (e.g., a road network), a subset $X \subset V$ of nodes (designated as populated nodes), and a subset $S \subset V$ of nodes (designated as safe nodes) with $X \cap S = \emptyset$, find evacuation routes from populated nodes to safe nodes in case of an emergency. A set of evacuation routes is defined as a set of paths in G so that (i) each node in X is the tail of one path, (ii) the last node on each path lies in S, and (iii) the paths do not share any arcs. Such a set of paths gives a way for the occupants of the populated nodes to "escape" to S, without overly congesting any arc in G.

Give an algorithm which, given G, X, and S, decides in polynomial time whether such a set of evacuation routes exists, and analyze your algorithm.

6. 15 points

   Figure 3 gives a circulation with demands and lower bounds instance H. (a) Reduce H to a circulation with demands instance H'. (b) Reduce the circulation with demands instance H' to a flow network G. To find a max-flow in G, assume that an argument-path with the largest bottleneck capacity f in G is found and a flow of value f is obtained. (c) Give the residual graph G_f w.r.t. f. (d) Find a solution for H based on a max-flow in G.



A ciculation with demands and lower bounds instance, black numbers are arc capacity, red numbers are lower bound.

Figure 3: Figure for question 6.

7. (Chapter 7 Problem 31 of the text book) 15 points

   There are n boxes $1,..,n$, each box i has size $s(i)$ specified by (x_i, y_i, z_i), $x_i, y_i, z_i > 0$. We say $s(i) < s(j)$ if $x_i < x_j, y_i < y_j$ and $z_i < z_j$. A box i can be put inside box j if $s(i) < s(j)$. For any two boxes i and i' with $s(i) < s(j)$ and $s(i') < s(j)$, both i and i' can not be put inside j if i is not put inside i' or i' is not put inside i. But for a sequence of boxes $i_1, i_2, .., i_k$ with $s(i_1) < s(i_2) < \cdots < s(i_k)$, i_1 can be put inside i_2, then i_2 put inside i_3,.., and finally i_{k-1} put inside i_k. In this case, all boxes $i_1, .., i_{k-1}$ are inside box i_k and only i_k is visible. The nesting arrangement for a set of n boxes to put one box inside another such that the number of visible boxes is minimized. Give a polynomial time algorithm which, given a set of n boxes $1,..,n$ and $s(1),..,s(n)$, solves the nesting arrangement problem, and analyze your algorithm.