Due 23:59 Nov 2 (Sunday). There are 100 points in this assignment.
Submit your answers **(must be typed)** in pdf file to CourSys
`https://coursys.sfu.ca/2025fa-cmpt-705-x1/`.
Submissions received after 23:59 will get penalty of reducing points: 20 and 50 points deductions for submissions received at $[00:00, 00:10]$ and $(00:10, 00:30]$ of Nov 3, respectively; no points will be given to submissions after $00:30$ of Nov 3.

1. (Chapter 8 Problem 1 of text book) 10 points

   For each of the two questions below, decide whether the answer is (i) "Yes," (ii) "No," or (iii) "Unknown, because it would resolve the question of whether $P = NP$." Give a brief explanation of your answer. (a) The decision version of the Interval Scheduling Problem from Chapter 4 is defined as follows: Given a collection of intervals on a timeline, and a bound k, does the collection contain a subset of nonoverlapping intervals of size at least k? Question: Is it the case that Interval-Scheduling$\leq_P$Vertex Cover? (b) Question: Is it the case that Independent-Set$\leq_P$Interval-Scheduling?

2. (Chapter 8 Problem 2 of the text book) 15 points

   Let A be an $n \times m$ array with each element $A[i, j] \geq 0$. The diverse subset problem is defined as follows: Given array A and integer $0 < k \leq n$, whether there is a subset of at least k rows of A such that for every column j, $1 \leq j \leq m$, at most one row i in the subset with $A[i, j] > 0$. Prove the diverse subset problem is NP-complete.

3. (Chapter 8 Problem 5 of the text book) 15 points

   Let $A = \{a_1, .., a_n\}$ and $B_1, .., B_m$ be a collection of subsets of A, a subset H of A is a hitting set for $B_1, .., B_m$ if $H \cap B_i \neq \emptyset$ for every $1 \leq i \leq m$. The Hitting Set problem is that given A and $B_1, .., B_m$, and integer $k > 0$, whether there is a hitting set of size k or not. Prove the hitting set problem is NP-complete.

4. (Chapter 8 Problem 13 of the text book) 15 points

   The winner determination for combinatorial auction problem is defined as: Gievn $I = \{i_1, .., i_n\}$ of n items, m bids $(S_1, x_1), .., (S_m, x_m)$, where $S_i \subseteq I$ and $x_i \geq 0$ for $1 \leq i \leq m$, and a bound B, whether there is a subset $A \subseteq \{(S_1, x_1), .., (S_m, x_m)\}$ such that $\sum_{(S_i, x_i) \in A} x_i \geq B$ and for any $(S_i, x_i), (S_j, x_j)$ with $i \neq j$, $S_i \cap S_j = \emptyset$. Prove the problem is NP-complete.

5. (Chapter 8 Problem 20 of the text book) 15 points

   Let $p_1, .., p_n$ be n points. A distance $d(p_i, p_j)$ between p_i and p_j is denoted for every pair of points p_i and p_j with $d(p_i, p_i) = 0$, $d(p_i, p_j) > 0$ for $i \neq j$ and $d(p_i, p_j) = d(p_j, p_i)$. For a subset C of $\{p_1, .., p_n\}$, the diameter of C is defined as $d(C) = \max_{p_i, p_j \in C} d(p_i, p_j)$. Prove the following low-diameter clustering problem is NP-complete: Given a bound B and k, whether $p_1, .., p_n$ can be partitioned into k clusters $C_1, .., C_k$ such that $d(C_i) \leq B$ for every i.

6. (Chapter 8 Problem 21 of the text book) 15 points

   The fully compatible configuration (FCC) problem is defined as follows: An instance of FCC consists of disjoint sets of $A_1, .., A_k$, each A_i is a set of options, and a set P of incompatible pairs (x, y), where $x \in A_i$ and $y \in A_j$ with some $1 \leq i \neq j \leq k$. The FCC problem is to decide whether there is fully configuration which is a selection of one element from each A_i ($1 \leq i \leq k$) so that no pair of selected elements is in P. Prove the problem is NP-complete.

7. (Chapter 9 Problem 1 of text book) 15 points

   A monotone 3-SAT instance $\Phi(x_1, .., x_n)$ is a Boolean formula of the form $C_1 \wedge .. \wedge C_k$, each C_i is a disjunction of 3 nonnegative variables. The monotone QSAT problem is the decision problem whether

   $$\exists x_1 \forall x_2 ... \exists x_{n-2} \forall x_{n-1} \exists x_n \Phi(x_1, .., x_n)$$

   is satisfiable, $\Phi(x_1, .., x_n)$ is a monotone 3-SAT instance. Give an algorithm which solves the monotone QSAT problem in polynomial time in n.