Due 23:59 Nov 16 (Sunday). There are 100 points in this assignment.
Submit your answers **(must be typed)** in pdf file to CourSys
`https://coursys.sfu.ca/2025fa-cmpt-705-x1/`.
Submissions received after 23:59 will get penalty of reducing points: 20 and 50 points deductions for submissions received at $[00:00, 00:10]$ and $(00:10, 00:30]$ of Nov 17, respectively; no points will be given to submissions after $00:30$ of Nov 17.

1. (Chapter 11 Problem 1 of the text book) 15 points

   There are n containers $1, .., n$ of weights $w_1, .., w_n$ and some trucks, each truck can hold at most K units of weight (w_i and K, $w_i \leq K$, are positive integers). Multiple containers can be put on a truck subject to the weight restriction K. The problem is to use a minimum number of trucks to carry all containers. A greedy algorithm for this problem is as follows: start with an empty truck and put containers $1, .., j$ on the truck to have this truck loaded (i.e., $\sum_{1 \leq i \leq j} w_i \leq K$ and $\sum_{1 \leq i \leq j+1} w_i > K$); then put containers $j+1, j+2, ...$ on a new empty truck to have this truck loaded; continue this process until all containers are carried.

   (a) Give an example of a set of containers and a value K to show that the greedy algorithm above does not give an optimal solution.

   (b) Prove the greedy algorithm is a 2-approximation algorithm.

2. (Chapter 11 Problem 2 of the text book) 10 points

   Given a set S of strings, for any two strings p and q in S, a distance $d(p, q) \geq 0$ is defined. Given a similarity threshold value $\Delta \geq 0$, two strings p and q are called similar if $d(p, q) \leq \Delta$. A subset R of S is called a representative set of S if for any string $p \in S$, there is a string $q \in R$ with $d(p, q) \leq \Delta$. The minimum representative set problem is to find a representative set of minimum size.

   Give a polynomial time $O(\log n)$-approximate algorithm for the problem and analyze the algorithm.

3. (Chapter 11 Problem 5 of the text book) 15 points

   (a) Consider a load balancing problem instance of 10 machines and n jobs $S = \{1, .., n\}$ with $1 \leq t_i \leq 50$ for every i and $\sum_{i=1}^{n} t_i \geq 3000$. Prove that the greedy algorithm discussed in class finds a solution of makespan T with $T \leq (1.2)(\sum_{1 \leq i \leq n} t_i)/10$ for this instance.

   (b) Implement the greedy algorithm to find a solution for the load balancing problem instance in (a) and compare the solution with the lower bound $(\sum_{1 \leq i \leq n} t_i)/10$ on the makespan of the instance (each t_i can be generated randomly). Report your results for T, $(\sum_{i=1}^{n} t_i)/10$ and $T/((\sum_{i=1}^{n} t_i)/10)$ with $n = 100$.

4. (Chapter 11 Problem 7 of text book) 15 points

   Given a set of customers $\{1, 2, .., n\}$, each customer has a value v_i and is shown to one of the advertisements (ads) $A_1, .., A_m$. A selection of ads to customers is to assign one ad to each customer. For a set C_j of customers assigned ad A_j, the total value of C_j is $v(C_j) = \sum_{i \in C_j} v_i$. The spread of the selection is $\min_{1 \le j \le m}\{v(C_j)\}$. The problem of finding the maximum spread is NP-hard. Give a (1/2)-approximation algorithm for finding the maximum spread for any input instance with $v_i \le (\sum_{k=1}^{n} v_k)/(2m)$ for $1 \le i \le n$, and analyze the algorithm.

5. (Chapter 11 Problem 8 of text book) 15 points

   For every instance of the load balancing problem discussed in class, there exists an order of the jobs so that when Greedy-Balance processes the jobs in this order, it produces an assignment of jobs to machines with the minimum possible makespan.

6. (Chapter 10 Problem 2 of text book) 15 points

   Given a 3-SAT instance Φ of n variables, a truth assignment $f : \{x_1, .., x_n\} \to \{0, 1\}$ and an integer $d \ge 0$, the procedure Explore(Φ, f, d) below answers whether there is a truth assignment $f' : \{x_1, .., x_n\} \to \{0, 1\}$ such that $d(f, f') = |\{i | f(x_i) \ne f'(x_i)\}| \le d$ and f' satisfies Φ.

   Explore(Φ, f, d)
       if f satisfies Φ then return YES
       else if $d = 0$ then return NO
           else
               let $C = (l_1 \vee l_2 \vee l_3)$ be a clause of Φ that is not satisfied by f;
               let f_i ($i = 1, 2, 3$) be the truth assignment obtained from
                   f by inverting the assigned value to l_i;
               if Explore($\Phi, f_1, d-1$)=YES then return YES;
               if Explore($\Phi, f_2, d-1$)=YES then return YES;
               if Explore($\Phi, f_3, d-1$)=YES then return YES;
       return NO

   (a) Prove Explore(Φ, f, d) returns YES iff there is a satisfying assignment f' with $d(f, f') \le d$. Analyze the running time of Explore(Φ, f, d) as a function of n and d.

   (b) Using Explore(Φ, f, d) as a subroutine, give an algorithm which decides whether a 3-SAT instance is satisfiable or not in $O(n^{O(1)}(\sqrt{3})^n)$ time.

7. (Chapter 10 Problem 5 of the text book) 15 points

   Given a node weighted graph G (each node v of G is assigned a positive weight $w(v)$, a minimum weight dominating set D of G is a subset of $V(G)$ such that for every node u of G, either $u \in D$ or u is adjacent to a node $v \in D$ and $\sum_{v \in D} w(v)$ is minimized. Give a polynomial time dynamic programming algorithm (optimal solution structrue, Bellman equation, pseudo code, and running time) to find a minimum weight dominating set D and the weight of D in a tree.