# QEA Project 3: Helmholtz Resonators (Bottle Lab)

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November 8, 2025

## 1 Introduction

This project investigates the behavior of a Helmholtz resonator, a system in which a cavity of air connected to the outside environment through a narrow neck oscillates at a characteristic resonant frequency when disturbed. A familiar example is the tone produced when blowing across the opening of a bottle.

In this experiment, we explored how the resonant frequency of a bottle changes as its internal air volume varies. The air volume was controlled by adding increasing amounts of water to the bottle. For each water level, the bottle was excited by blowing gently across the opening, and the resulting sound was recorded using MATLAB, which extracted the dominant frequency from the signal using an FFT. These measured frequencies were then compared to the theoretical predictions from the simplified Helmholtz resonator model, derived via a lumped-parameter approximation and linearization.

The primary goals of this project were:

1. To measure the Helmholtz resonance frequency as a function of internal cavity volume.
2. To derive the approximate resonant frequency expression using Newton's second law, the adiabatic gas relation, and linearization.
3. To compare the measured frequencies to theoretical predictions and evaluate the accuracy and limitations of the Helmholtz model.

## 2 Experimental Procedure

A single bottle served as the resonant cavity for this experiment. The inner diameter and length of the bottle neck were measured using a ruler, while the masses of the empty bottle and a completely water-filled bottle were recorded using a digital food scale. These measurements were used to estimate both the total internal bottle volume and the amount of air present during each trial.

A provided MATLAB script was used to record acoustic signals via the laptop's built-in microphone and determine the dominant frequency using a Fast Fourier Transform (FFT). To excite resonance, air was blown steadily across the bottle opening for approximately five seconds. For each trial, the combined mass of the bottle and water, as well as the dominant resonant frequency, were recorded. A total of 30 measurements were collected, ranging from an empty bottle to one nearly filled with water.

### 2.1 Data Collection

The results from all 30 trials are listed in Table 1. Each entry specifies the measured water volume inside the bottle and the corresponding resonant frequency.

| Trial | Water Volume (mL) | Frequency (rad/sec) |
|---|---|---|
| 1 | 200 | 1192.5486 |
| 2 | 232 | 1255.3804 |
| 3 | 249 | 1289.3096 |
| 4 | 260 | 1319.4689 |
| 5 | 281 | 1359.6813 |
| 6 | 291 | 1389.8406 |
| 7 | 305 | 1437.5928 |
| 8 | 322 | 1500.4247 |
| 9 | 337 | 1535.6105 |
| 10 | 363 | 1647.4512 |
| 11 | 371 | 1678.8671 |
| 12 | 377 | 1749.2388 |
| 13 | 393 | 1755.5220 |
| 14 | 401 | 1822.1237 |
| 15 | 421 | 1940.2476 |
| 16 | 436 | 2055.8582 |
| 17 | 442 | 2109.8936 |
| 18 | 449 | 2167.6989 |
| 19 | 457 | 2241.8405 |
| 20 | 465 | 2343.6281 |
| 21 | 476 | 2470.5485 |
| 22 | 486 | 2636.4246 |
| 23 | 493 | 2715.5927 |
| 24 | 500 | 2909.1148 |
| 25 | 515 | 3347.6811 |
| 26 | 525 | 4051.3979 |
| 27 | 535 | 4424.6191 |
| 28 | 543 | 5203.7341 |
| 29 | 552 | 6302.0349 |
| 30 | 560 | 7458.1410 |

Table 1: Measured resonant frequency as a function of water volume.

## 2.2 Table of Values

To relate measured resonance frequencies to the geometry of the bottle, the total air volume for each trial was calculated from the mass measurements. The mass of water added to the bottle was found using

$$m_{\text{water}} = m_{\text{total}} - m_{\text{empty}},$$

and converted to volume using the density of water:

$$V_{\text{water}} = \frac{m_{\text{water}}}{\rho_{\text{water}}}.$$

The remaining air volume within the bottle was then determined by

$$V_{\text{air}} = V_{\text{bottle}} - V_{\text{water}}.$$

These computed volumes, together with the recorded resonance frequencies, were used to compare the experimental results to theoretical predictions based on the Helmholtz resonance model. Table 2 summarizes the relevant physical and derived parameters used in the analysis.

| **Quantity** | **Value** |
|---|---|
| Density of water | $1000\,\mathrm{Kg/m^3}$ |
| Density of air | $1.204,\mathrm{Kg/m^3}$ |
| Heat capacity ratio of air | 7/5 |
| Ambient pressure | 1 atm |
| Neck diameter | 19.03 mm |
| Neck length | 79.1 mm |
| Empty bottle mass | 200 g |
| Filled bottle mass | 580 g |
| Cross-sectional area of bottle neck | $284.42\,\mathrm{mm^2}$ |
| Neck volume | $22498\,\mathrm{mm^3}$ |
| Neck air mass | 2.7087 g |
| Bottle volume | 380 mL |
| Empty cavity volume | 325 mL |

Table 2: Measured and computed physical system parameters.

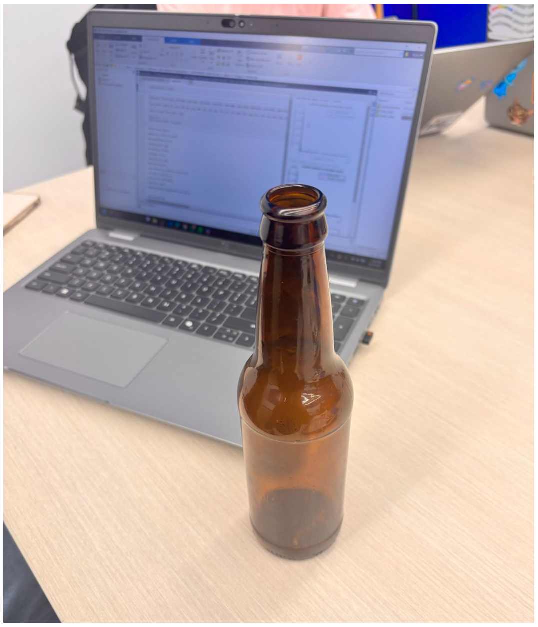

Figure 1: Bottle used for the Helmholtz resonator experiment.

# 3 Mathematical Model and Derivation Summary

## 3.1 Derivation Summary

The bottle acts as a Helmholtz resonator, where the air in the neck behaves as a mass and the cavity air provides a restoring spring. The mass of air in the neck is determined by its density, cross-sectional area, and length.

$$m = \rho L A$$

A displacement of the neck air moves a proportional volume in the cavity. The total cavity volume is the sum of the static volume and this displaced volume.

$$V = V_0 + A\Delta x$$

Assuming adiabatic compression of the cavity air, the pressure varies with volume according to:

$$PV^\gamma = P_0 V_0^\gamma \quad \Rightarrow \quad P \approx P_0 \left(1 + \frac{A}{V_0}\Delta x\right)^{-\gamma}$$

Applying Newton's second law to the neck air gives a nonlinear equation of motion. For small displacements, this can be linearized into a simple harmonic oscillator:

$$m\,\Delta\ddot{x} + k\,\Delta x = 0, \quad k = \frac{\gamma A^2 P_0}{V_0}$$

The natural frequency of this system is:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{\gamma A P_0}{\rho L V_0}}$$

This shows that the resonant frequency increases as the cavity volume decreases, allowing us to predict the pitch of sound produced when blowing across the bottle opening.

# 4 Results

## 4.1 Data Analysis Discussion

The data collected consisted of the measured angular frequencies of the acoustic resonance of the bottle for 30 different water volumes. Each measurement corresponds to a specific total air volume within the bottle, determined from the mass and density of the added water. MATLAB was used to compute the total air volume and evaluate the theoretical predictions of the Helmholtz resonator model.

### 4.1.1 Computation of System Parameters:

All system parameters were derived from physical measurements of the bottles geometry and mass, combined with known material properties (density and pressure). The following equations were used to compute the intermediate quantities required for the theoretical analysis:

Cross sectional area of neck:

$$A = \pi \left(\frac{d}{2}\right)^2 \tag{1}$$

Neck volume:

$$V_{\text{neck}} = A\,L \tag{2}$$

Mass of air in the neck:

$$m_{\text{air,neck}} = \rho_{\text{air}}\,A\,L \tag{3}$$

Mass of water added to the bottle:

$$m_{\text{water}} = m_{\text{total}} - m_{\text{empty}} \tag{4}$$

Volume of Water inside the bottle:

$$V_{\text{water}} = \frac{m_{\text{water}}}{\rho_{\text{water}}} \tag{5}$$

The total volume of the bottle (including the neck) was calculated by filling the entire bottle with water and then measuring its content in milliliters.

Remaining air volume (cavity) inside the bottle:

$$V_{\text{air}} = V_{\text{bottle}} - V_{\text{water}} \tag{6}$$

Using the computed parameters, the total air volume for each trial was calculated and plotted against the measured angular frequency values obtained from the FFT.

The theoretical frequencies were calculated using the Helmholtz relation:

$$\omega_{\text{theoretical}} = \sqrt{\frac{\gamma A P_0}{\rho_{\text{air}} L V_{\text{air}}}},$$

where:
$\gamma$ is the heat capacity ratio of air,
$A$ is the neck cross-sectional area,
$P_0$ is the ambient pressure,
$\rho_{\text{air}}$ is the density of air,
$L$ is the neck length, and
$V_{\text{air}}$ is the cavity volume.

This equation predicts that the frequency varies inversely with the square root of the total air volume. As the air volume decreases, the resonant frequency is expected to increase sharply.

## 4.2 Plot

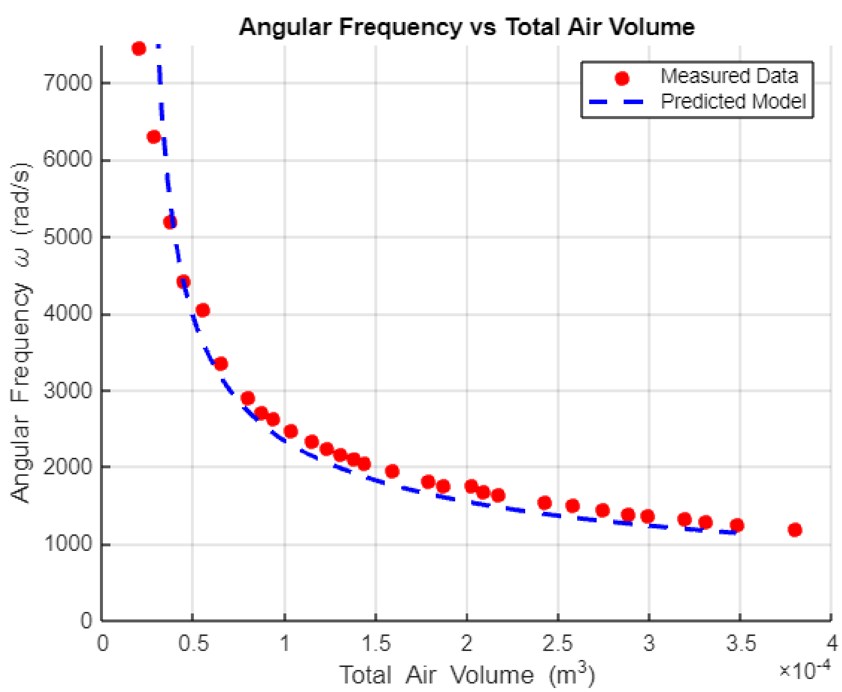


Figure 2: Measured (red markers) and predicted (blue dashed line) angular frequencies as a function of total air volume. The plot shows the expected inverse-square-root dependence between air volume and angular frequency.

The experimental data show a strong inverse-square-root relationship between air volume and resonance frequency. For large air volumes, the measured frequencies align almost perfectly with the theoretical predictions. This confirms that the Helmholtz model accurately captures the resonance behavior under conditions of small-amplitude oscillations and laminar airflow.

At smaller air volumes, the measured frequencies are consistently slightly higher than the predicted curve. This deviation is clearly visible in the left portion of the plot and can be attributed to the breakdown of ideal model assumptions as the air column moves more rapidly and nonlinearly at higher frequencies.

### 4.2.1 Model Assumptions:

This model relies in 3 primary assumptions:

1. Uniform cavity pressure: The air in the bottle cavity is assumed to compress and expand uniformly, with negligible pressure gradients.
2. Small oscillations: The motion of the air plug in the neck is assumed to be small relative to the cavity volume.
3. Ideal geometry: The neck and cavity are treated as perfect cylinders with smooth and constant cross-sectional areas. In reality, the bottle neck is slightly tapered and irregular, which alters the effective air mass and stiffness of the system.

### 4.2.2 Sources of Error and Deviation

1. Turbulent and nonlinear flow: At smaller air volumes, airflow becomes partially turbulent, violating the small-amplitude and laminar-flow assumptions and producing higher measured frequencies
2. Non-cylindrical neck geometry: The neck's tapered and irregular shape alters the effective length and area, affecting both the mass and stiffness terms in the model.
3. Measurement uncertainty: Small errors in estimating air volume, water mass, or the dominant FFT frequency can contribute to small discrepancies.

# 5 Conclusion

This experiment demonstrated the resonant behavior of a bottle acting as a Helmholtz resonator. By varying the internal air volume through the addition of water and measuring the resulting resonant frequencies, we observed a clear inverse–square–root trend: as cavity volume decreased, the resonant frequency increased. These measurements showed strong agreement with theoretical predictions obtained from a simplified model derived by applying Newton's second law, assuming adiabatic compression, and linearizing the system to yield an equivalent mass–spring form.

The Helmholtz model provided accurate predictions for moderate to large air volumes, while systematic deviations emerged at very small volumes, where assumptions such as uniform cavity pressure and small oscillations begin to break down. Possible sources of error include geometric irregularities in the bottle neck, turbulence at the opening, and measurement uncertainty. Overall, this project verified that the dominant parameters governing Helmholtz resonance determine bottle pitch in a predictable manner, demonstrating the usefulness and limitations of the idealized model.