QEA Project 1: Fidget Spinner

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1 Introduction

Fidget spinners provide a simple test case for studying rotational dynamics. In this lab, we analyzed the decay of angular velocity of a freely moving fidget spinner. A video recording was processed in MATLAB to extract angular velocity data, which was then compared against mathematical models. While the theoretical model includes quadratic drag, viscous damping, and Coulomb friction, our results showed that quadratic drag was negligible. A linear first-order ODE with viscous and Coulomb friction provided the best fit to the data. This allowed us to simulate the spinner's motion and evaluate how accurately the simplified model matched experiment.

2 System Model

The rotational dynamics are described by:

$$I\alpha = \sum \tau \tag{1}$$

where I is the moment of inertia, $\alpha = \dot{\omega}$ is angular acceleration, and $\sum \tau$ is the total torque. The general form of the torque includes:

- Quadratic drag: $-d_1\omega^2$
- Viscous drag: $-d_2\omega$
- Coulomb friction: $-d_3$

In normalized form:

$$\dot{\omega}(t) = a\omega^2 + b\omega + c \tag{2}$$

However, regression on experimental data showed that $a \approx 0$. Thus, the best-fit model reduced to a linear first-order ODE:

$$\dot{\omega}(t) = b\omega + c \tag{3}$$

This simplified model still captures the system's dissipative behavior but with fewer terms.

3 Experimental Procedure

The experiment was conducted using a standard three-arm fidget spinner placed flat on a whiteboard surface. The background was chosen to be uniform and static in order to minimize visual noise when processing the video. A phone camera (30 frames per second) was positioned directly above the spinner, oriented so that the field of view contained the spinner and minimal background. The camera was kept at a fixed constant height to avoid disturbances. Lighting was kept consistent to prevent shadows from interfering with the pixel intensity measurements.The spinner was spun manually by hand until it reached a steady rotational speed.

Recording began only after the spinner was already in motion and continued until several seconds after the spinner had come to rest, ensuring that the complete decay of angular velocity was captured. Once the recording was completed, the video was transferred to a computer for analysis. A region of interest covering a portion of one spinner arm was selected to track brightness oscillations. The periodic change in pixel intensity was later used to extract the angular velocity of the spinner. All processing steps were performed in MATLAB, using provided FFT-based functions to estimate angular velocity and acceleration.

4 Data Analysis

The raw FFT output initially provided an apparent frequency three times greater than the true angular velocity, a discrepancy caused by the three-arm structure of the spinner. Dividing the measured frequency by the number of arms yielded the correct $\omega(t)$. To estimate angular acceleration, finite differences were computed between successive angular velocity values, producing $(\omega, \dot{\omega})$ data pairs suitable for regression analysis.

A quadratic model of the form $\dot{\omega} = a\omega^2 + b\omega + c$ was first tested, but the fitted value of a was negligible, indicating that quadratic drag did not significantly contribute to the dynamics. Consequently, a reduced linear model $\dot{\omega} = b\omega + c$ was adopted, and this form provided an excellent fit to the data.

Using the fitted parameters and the initial angular velocity, we simulated the system in MATLAB with the ODE45 solver. The simulated trajectory aligned very closely with the measured data, confirming the accuracy of the linear model. Finally, this model was used to predict how changes in spinner density would affect the decay of angular velocity. As expected, higher density (and thus higher moment of inertia) led to longer spin times, consistent with the theoretical scaling of rotational inertia.

5 Lab Deliverables

5.1 Picture of Fidget Spinner.

Figure 1: Fidget Spinner Used for the Lab

5.2 Sketch of Experimental Setup for Recording the Video.

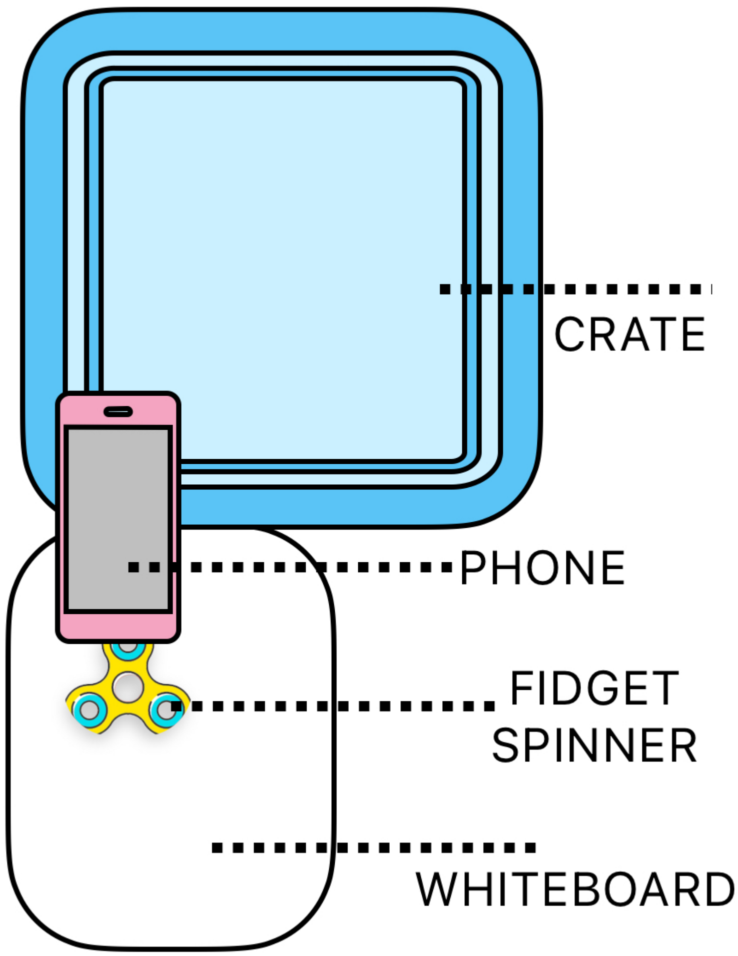

Figure 2: Top View of the Lab Setup

5.3 Written Description of Experimental Procedure.

Please refer to Section 3: Experimental Procedure.

5.4 Units of the Angular Velocity and Angular Acceleration.

Angular velocity is measured in units of radians per second ($rad * s^-1$), and angular acceleration is radians per second squared ($rad * s^-2$).

5.5 Physical Quantities Values Table.

Quantity Description	Symbol	Value	Units
Estimated quadratic drag constant	a	0	–
Estimated viscous damping constant	b	-0.0265	Nms/rad
Estimated Coulomb friction constant	c	-0.2948	Nm^2/C^2
Video frame rate	F_s	30	fps
Estimated initial angular velocity	$\omega(t=0)$	68.82	rad/sec

Table 1: Estimated Physical Quantities

5.6 Plot of Angular Acceleration as a Function of Angular Velocity.

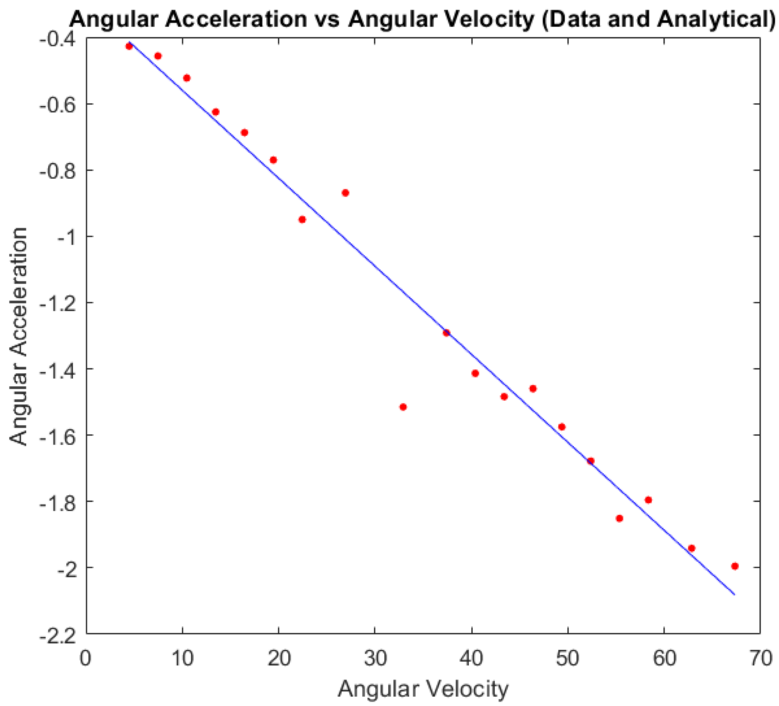

Figure 3: Angular Acceleration vs Angular Velocity

5.7 Does the quadratic fit seem to capture the dynamics of the fidget spinner? Why or why not?

A quadratic fit does not capture the dynamics of the fidget spinner. The quadratic drag constant was so close to zero that it was better to ignore it completely. We decided on using a linear fit model, which is a much closer fit.

5.8 What about our model makes it first order? Is the model linear or nonlinear? Is the system homogeneous or forced?

Our model is first-order because the highest order or derivative is one. The model that best fits our data is a first-order one. This is a forced system, as the term for Coulomb friction does not include any variables.

5.9 Plot that compares the measured values of omega(t) with the solution simulated using ODE45 (with the same initial condition).

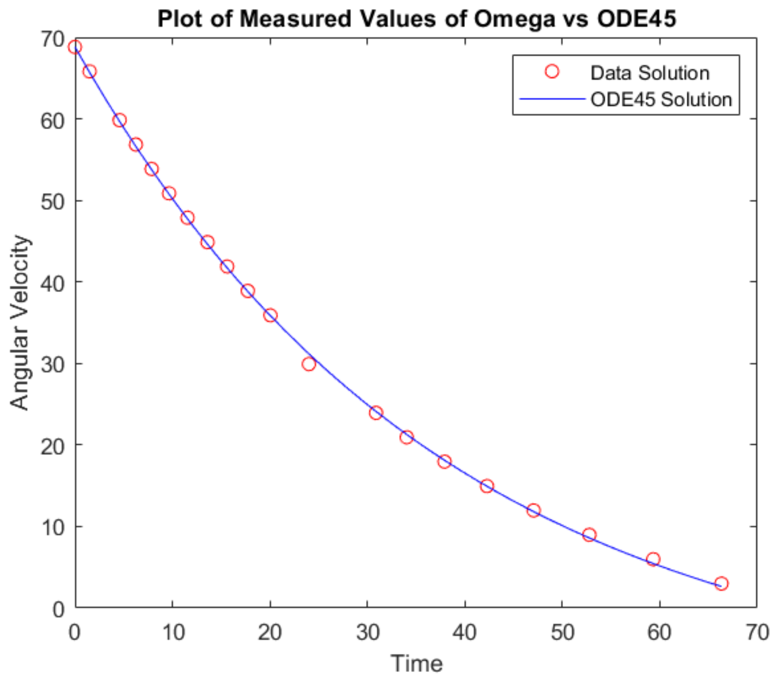

Figure 4: Data vs ODE45

5.10 Based on a comparison between the simulated solution with the measured data, does the model seem accurate?

The model seems very accurate when compared to the data. All data points are directly on the ODE45 solution.

5.11 Quiver plot that visualizes the ODE using the parameter values of a, b, and c that were measured.

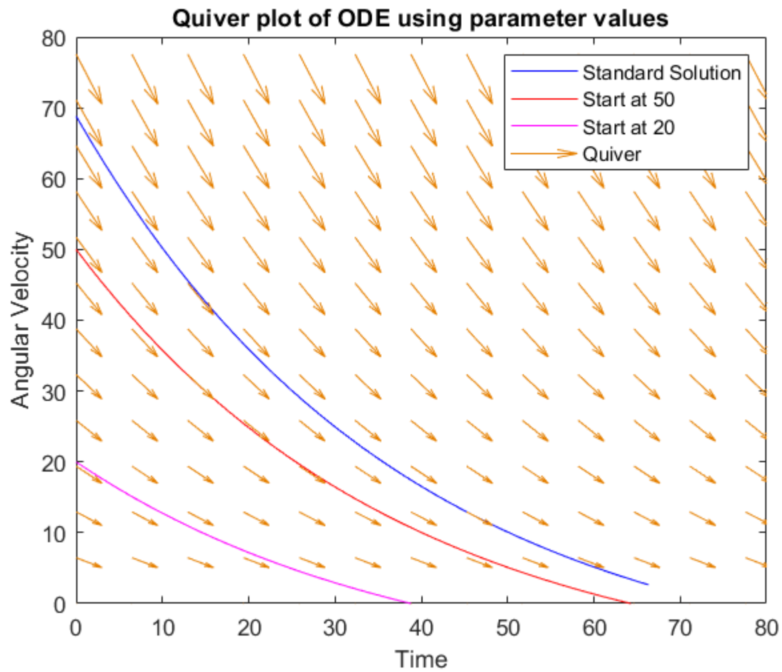

Figure 5: Quiver Plot of Three Different Solutions

5.12 How are the angular velocity of the fidget spinner and the measured frequency of the video signal related to one another?

The measured frequency of the video is how many times an arm crosses the area in one second. All three arms cross every rotation, so 1/3 of this value is the number of rotations in 1 second. This is exactly what the angular velocity is. $omega = f/3$

5.13 Why is the measured video frequency divided by the number of wings to calculate angular velocity?

Angular velocity is measured in radians per second, based on how many rotations it does in a second. If every time a spoke passes is one third of a rotation, so if we count every spoke as a rotation we will get three times the number of rotations. So, we need to divide by the number of spokes.

5.14 What is the stroboscopic effect?

The stroboscopic effect is an effect that causes rotating objects in film (or vision) to appear to stop or even move backwards. This is due to the way apertures (cameras or eyes) capture motion. Film is just a combination of pictures taken very close to each other and played back. If an object is rotating so fast it completes exactly one rotation per frame, then it appears to be still as its in the same spot every time the camera takes a picture. Then, if the object completes close to one rotation between frames, it will appear just behind where it was in the previous frame, and thus appear to move backwards. Because the fidget spinner is symmetrical about 120 degree slices, it appears the same every third of a rotation. This is clearly observed in our video.

5.15 Why does the fidget spinner stop in finite time instead of decaying exponentially like an RC circuit, and how does Coulomb friction influence this behavior?

The previously-examined first order systems don't come to rest technically, as they continue to approach the resting point but never actually reach it. In practice, they get so close that we can just assume the values are the same. This is a fundamental property of exponential decay, because any non-zero number cannot be raised to a power and return 0. So, the Ae^{-at} term will never reach 0. The fidget spinner, however does actually reach rest. Because of this, the system will not fit an exponential curve.

5.16 How does density affect the decay?

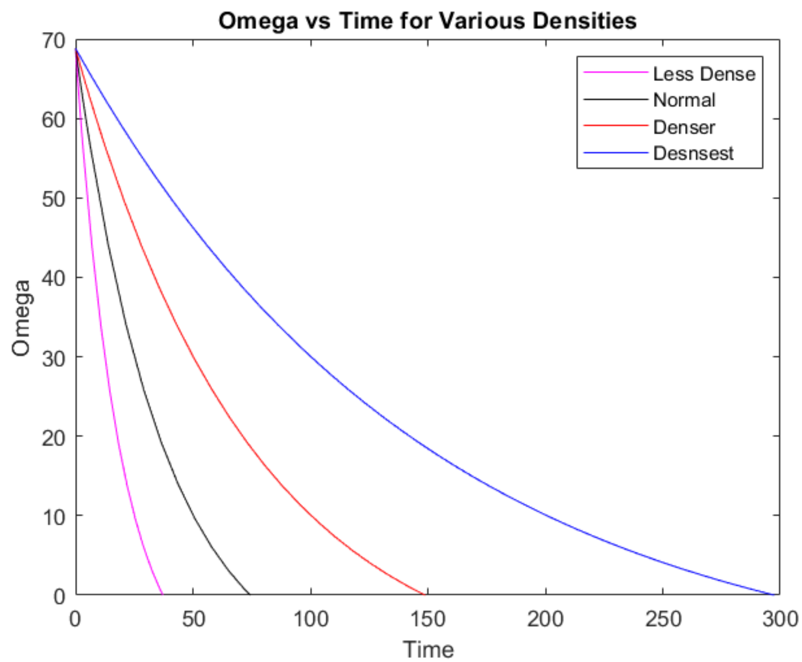

Figure 6: Omega vs Time for Different Densities

Increasing the density of the fidget spinner slows the decay of angular velocity over time. A denser spinner has a higher moment of inertia, which resists changes in rotational speed. As a result, it takes longer to come to rest. This effect is visible in simulations and plots, where spinners made of denser materials show slower decay curves compared to less dense ones.

5.17 How does the plot of $t_{\text{stop}}/t^*_{\text{stop}}$ vs. $\rho_{\text{new}}/\rho_{\text{old}}$ behave? What trend do you notice?

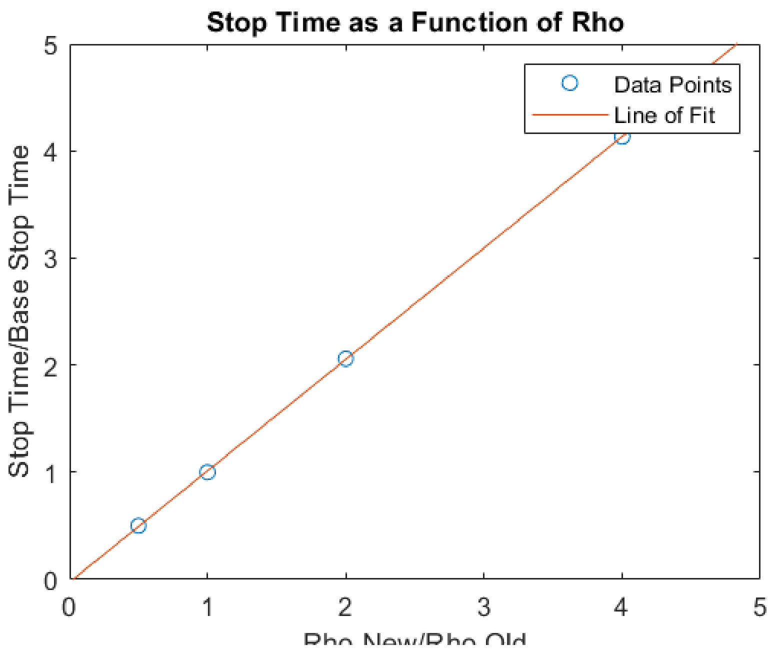

Figure 7: Stop Time as a Function of Density

Scaling appears to be a clear linear relationship, and very nearly a 1 to 1. The linear equation that describes the relationship of the above plot is $t = 1.0406p - 0.0273$, and the line of fit it generates is very close to the data. Obviously this isn't perfectly accurate, as spinners with very low density would accordingly stop spinning before they started, but it is a useful model. In fact, based on the likely errors within the data and the proximity to a 1 to 1 relationship, we can assume it is so.

5.18 How does scaling time by $\rho_{\text{new}}/\rho_{\text{old}}$ lead to a new form of the ODE? How is this related to the stop time plot?

$$\frac{\rho_{new}}{\rho_{old}} \dot{\omega}(t) = a\omega(t)^2 + b\omega(t) + c$$

$$\dot{\omega}(t) = \frac{d\omega}{dt} = \frac{d\omega}{dt} \cdot \frac{d\tilde{t}}{d\tilde{t}} = \frac{d\omega}{d\tilde{t}} \cdot \frac{d\tilde{t}}{dt}$$

$$\tilde{t} = t \Big/ \frac{\rho_{new}}{\rho_{old}}, \qquad \frac{d\tilde{t}}{dt} = \frac{\rho_{old}}{\rho_{new}}$$

$$\frac{d\omega}{d\tilde{t}} \cdot \frac{\rho_{old}}{\rho_{new}}$$

$$\left[\frac{d\omega}{d\tilde{t}} \cdot \frac{\cancel{\rho_{old}}}{\cancel{\rho_{new}}}\right] \cdot \frac{\cancel{\rho_{new}}}{\cancel{\rho_{old}}} = a\omega(t)^2 + b\omega(t) + c$$

$$\therefore \frac{d\omega}{d\tilde{t}} = a\omega(t)^2 + b\omega(t) + c$$

Figure 8: Transforming time by $\rho_{\text{new}}/\rho_{\text{old}}$ yields an equivalent ODE form.

5.19 Using the trend from the plot, estimate the stop time for diamond and gold spinners based on their densities. Can you do this without re-simulating?

Yes, we can plug it into the fit line formula found above. Here we assume it is a 1 to 1 relationship between the ratios. Using $t_{stop}/71.856 = 3.5/1.2$ for diamond we get a stop time of 209.58 seconds, and a time of 1156.88 seconds for gold.

5.20 Is increasing the material density an effective way to increase the spin time?

Yes, it is a very effective way. The gold spinner will spin for over 16 times as long as the plastic.

6 Conclusion & Further Work

This lab demonstrated how empirical data can guide and simplify theoretical modeling. While the initial torque balance included quadratic drag, our analysis showed that viscous and Coulomb friction dominated the spinner's deceleration. This justified modeling the system with a linear first-order ODE, which produced simulations that closely matched experimental angular velocity data.

A key insight was that the spinner came to rest in finite time—unlike exponential decay systems such as RC circuits. This behavior reflects the influence of constant Coulomb friction, which prevents the system from approaching zero asymptotically and instead forces a complete stop. Video analysis also revealed a stroboscopic effect, where the spinner appeared to pause or reverse direction due to aliasing. Correcting for this using FFT filtering was essential for extracting accurate angular velocity measurements.

Overall, the lab highlighted how nonlinear dynamics, system identification, and signal processing can work together to build predictive models of physical systems. Future improvements could include higher frame-rate recordings or testing spinners with different geometries to explore how design choices affect drag and spin duration.