Homework 2

STA 307

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```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from ISLP import load_data
from sklearn.datasets import load_iris
from sklearn.preprocessing import StandardScaler
from sklearn.decomposition import PCA

AUTO = load_data("Auto")
```

Question 1

```
selected_coloumns = ['mpg','cylinders', 'displacement',  'weight', 'acceleration', ]
X = AUTO[selected_coloumns]
y = AUTO['origin'] # Origin of car (1. American, 2. European, 3. Japanese)
print(X.head)

print("American cars: {0}".format((y == 1).sum())) # American
print("European cars: {0}".format((y == 2).sum())) # European
print("Japanese cars: {0}".format((y == 3).sum())) # Japanese
```

```
<bound method NDFrame.head of       mpg  cylinders  displacement  weight  acceleration
0    18.0          8         307.0    3504          12.0
1    15.0          8         350.0    3693          11.5
2    18.0          8         318.0    3436          11.0
3    16.0          8         304.0    3433          12.0
4    17.0          8         302.0    3449          10.5
..    ...        ...           ...     ...           ...
387  27.0          4         140.0    2790          15.6
388  44.0          4          97.0    2130          24.6
389  32.0          4         135.0    2295          11.6
390  28.0          4         120.0    2625          18.6
391  31.0          4         119.0    2720          19.4

[392 rows x 5 columns]>
American cars: 245
European cars: 68
```

```
Japanese cars: 79
```

Question 2

```
print("Standard deviation")
print(X.std())
print("\n")
print("Mean")
print(X.mean())
```

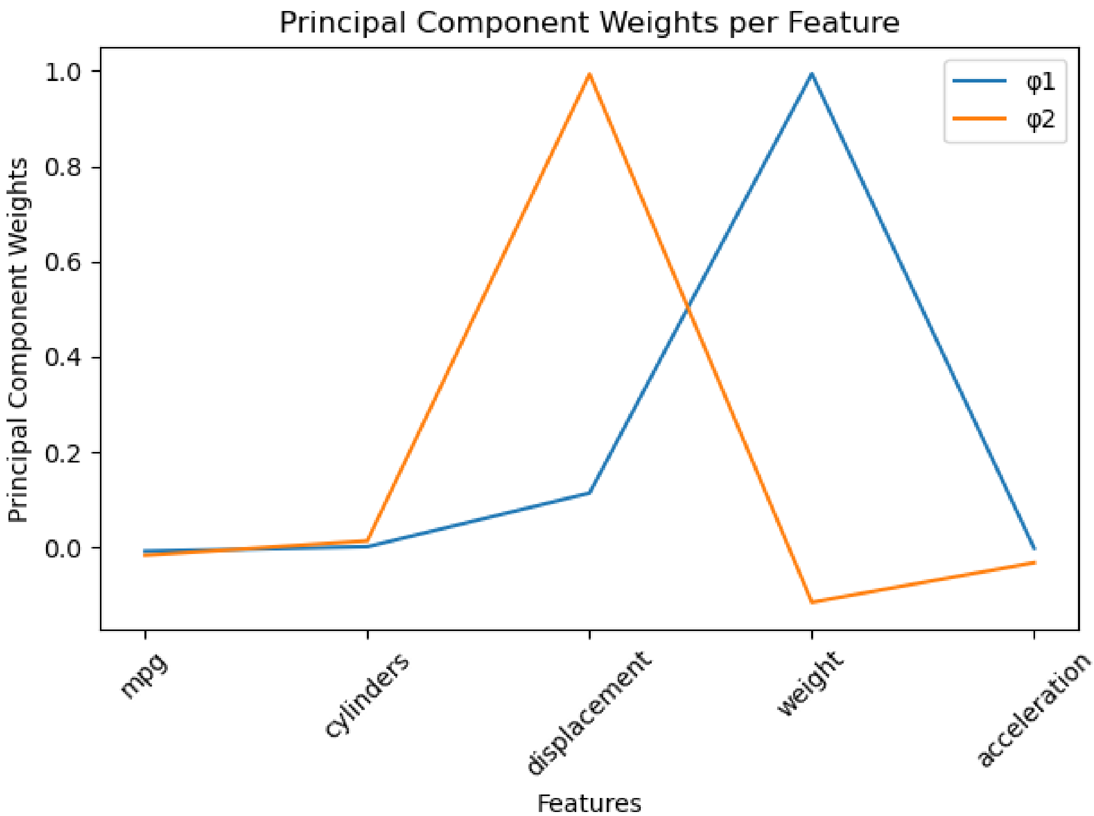
```
Standard deviation
mpg               7.805007
cylinders         1.705783
displacement    104.644004
weight          849.402560
acceleration      2.758864
dtype: float64


Mean
mpg               23.445918
cylinders          5.471939
displacement     194.411990
weight          2977.584184
acceleration      15.541327
dtype: float64
```

Qustion 3

a

```

print("Original data size:",X.shape)
# Implement PCA with 2 principal components
pca = PCA(n_components=2)

# Fit the PCA model to the data and transform the data
X_reduced = pca.fit_transform(X)
print("Reduced data size:", X_reduced.shape)

#principal components
components = pca.components_
x = np.arange(components.shape[1]) # 6

# Plot the first and second principal components
plt.plot(x, components[0], label='1')
plt.plot(x, components[1], label='2')

# Define feature names as tick labels
names = ['mpg', 'cylinders', 'displacement', 'weight', 'acceleration']
plt.xticks(ticks=x, labels=selected_coloumns, rotation=45)
```

```
# Labeling the axes and the legend
plt.xlabel('Features')
plt.ylabel('Principal Component Weights')
plt.title('Principal Component Weights per Feature')
plt.legend()

# Show plot
plt.tight_layout()
plt.savefig("plot.png", bbox_inches='tight')
plt.close()
```



b

It seems pca is caputring the data with larger scales this can lead to misleading conclusions about the importance of features

Question 4

```
means = np.mean(X, axis=0)
stds = np.std(X, axis=0)
Z = (X - means) / stds

means_Z = np.mean(Z, axis=0)
stds_Z = np.std(Z, axis=0)
```

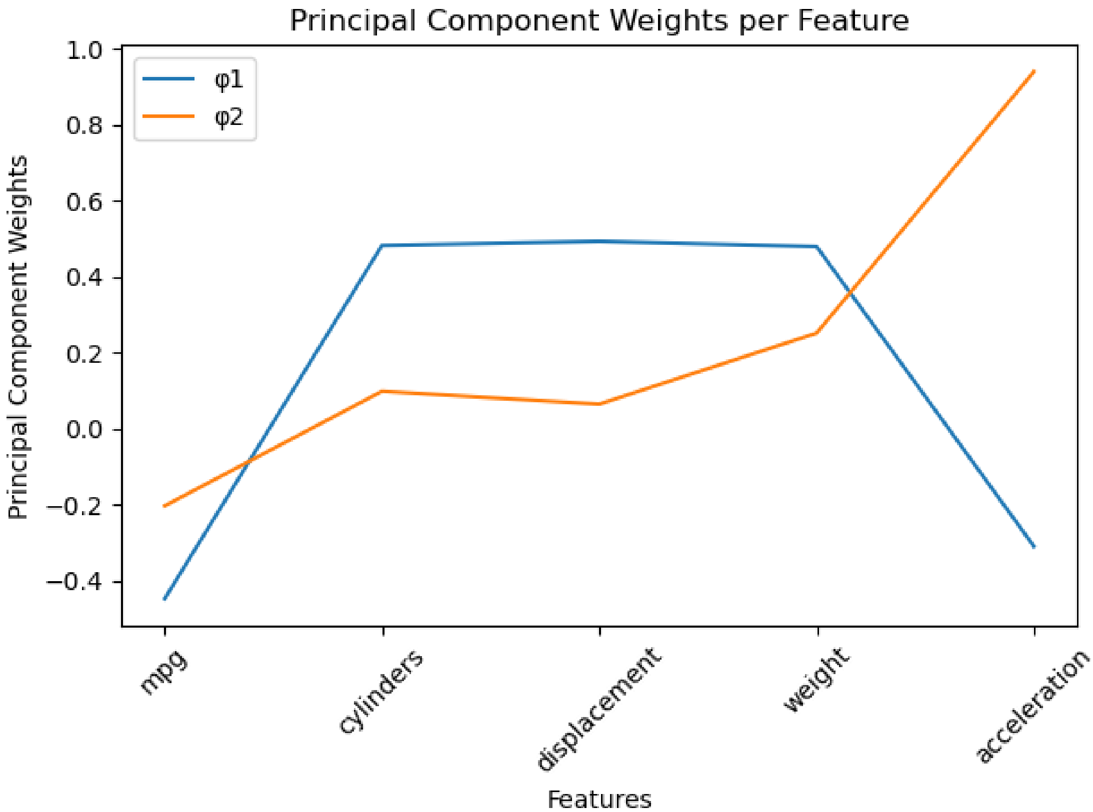
```
print(means_Z, stds_Z)
```

```
mpg              1.450087e-16
cylinders       -1.087565e-16
displacement    -7.250436e-17
weight          -1.812609e-17
acceleration     4.350262e-16
dtype: float64 mpg             1.0
cylinders        1.0
displacement     1.0
weight           1.0
acceleration     1.0
dtype: float64
```

Question 5

a

```
pca = PCA(n_components=2)
Z_reduced = pca.fit_transform(Z)
print("Reduced data size:", Z_reduced.shape)
components = pca.components_
plt.plot(x, components[0], label='1')
plt.plot(x, components[1], label='2')

plt.xticks(ticks=x, labels=selected_coloumns, rotation=45)

plt.xlabel('Features')
plt.ylabel('Principal Component Weights')
plt.title('Principal Component Weights per Feature')
plt.legend()

# Show plot
plt.tight_layout()
plt.savefig("plot-standard.png", bbox_inches='tight')
plt.close()
# plt.show()
```



b

It seems that the three most important features are 'cylinders', 'displacement', and 'weight', as the first principal component ($\phi 1$) weighs these more heavily, with 'displacement' appearing to have the highest contribution to variance by a small margin.

c

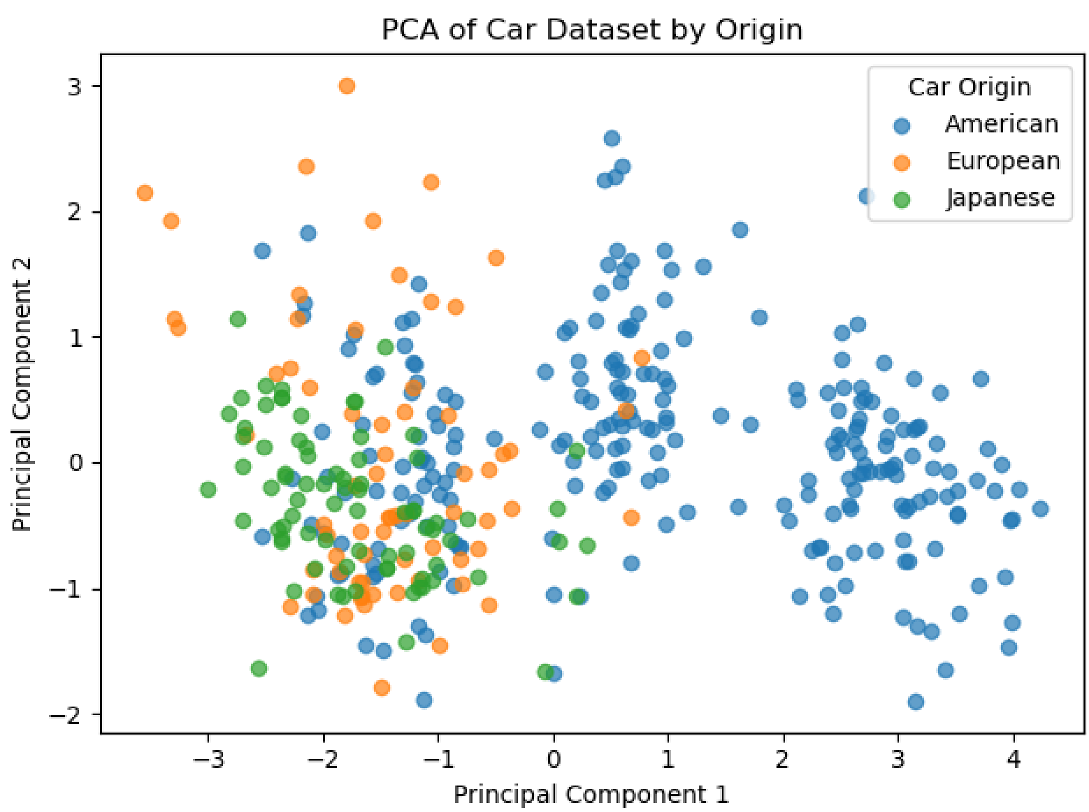
```
dot_product_mock = np.dot(components[0], components[1])

# Calculate the magnitude (norm) of each principal component to check if it's equal to one
magnitude_phi1_mock = np.linalg.norm(components[0])
magnitude_phi2_mock = np.linalg.norm(components[1])
print("Dot product", dot_product_mock)
print("Magnitude phi 1",magnitude_phi1)
print("Magnitude phi 2", magnitude_phi2)
```

```
Dot product -1.1102230246251565e-16
```

Question 6

a

```
# Simulating dimensionality-reduced data
# Plotting
origins = ["American", "European", "Japanese"]
# plt.figure(figsize=(8, 6))
for origin in [1, 2, 3]:
    subset = Z_reduced[origin == y]
    plt.scatter(subset[:, 0], subset[:, 1], label=origins[origin - 1], alpha=0.7)

plt.xlabel('Principal Component 1')
plt.ylabel('Principal Component 2')
plt.title('PCA of Car Dataset by Origin')
plt.legend(title='Car Origin')
plt.tight_layout()
plt.savefig("plot-scatter.png")
plt.close()
```



b

c

Question 7

a

b

c