

# Homework 2

STA 307

Malcolm, Vejay, Rohan, Tyler

February 22, 2024

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from ISLP import load_data
from sklearn.datasets import load_iris
from sklearn.preprocessing import StandardScaler
from sklearn.decomposition import PCA

AUTO = load_data("Auto")
```

## Question 1

```
selected_columns = ['mpg', 'cylinders', 'displacement', 'weight', 'acceleration', ]
X = AUTO[selected_columns]
y = AUTO['origin'] # Origin of car (1. American, 2. European, 3. Japanese)
print(X.head)

print("American cars: {}".format((y == 1).sum())) # American
print("European cars: {}".format((y == 2).sum())) # European
print("Japanese cars: {}".format((y == 3).sum())) # Japanese
```

```
<bound method NDFrame.head of          mpg  cylinders  displacement  weight  acceleration
0      18.0           8        307.0     3504         12.0
1      15.0           8        350.0     3693         11.5
2      18.0           8        318.0     3436         11.0
3      16.0           8        304.0     3433         12.0
4      17.0           8        302.0     3449         10.5
..      ...           ...           ...           ...           ...
387    27.0           4        140.0     2790         15.6
388    44.0           4         97.0     2130         24.6
389    32.0           4        135.0     2295         11.6
390    28.0           4        120.0     2625         18.6
391    31.0           4        119.0     2720         19.4
```

```
[392 rows x 5 columns]>
American cars: 245
European cars: 68
```

Japanese cars: 79

## Question 2

```
print("Standard deviation")
print(X.std())
print("\n")
print("Mean")
print(X.mean())
```

Standard deviation

```
mpg          7.805007
cylinders     1.705783
displacement 104.644004
weight       849.402560
acceleration  2.758864
dtype: float64
```

Mean

```
mpg          23.445918
cylinders     5.471939
displacement 194.411990
weight       2977.584184
acceleration  15.541327
dtype: float64
```

## Question 3

a

```
print("Original data size:", X.shape)
# Implement PCA with 2 principal components
pca = PCA(n_components=2)

# Fit the PCA model to the data and transform the data
X_reduced = pca.fit_transform(X)
print("Reduced data size:", X_reduced.shape)

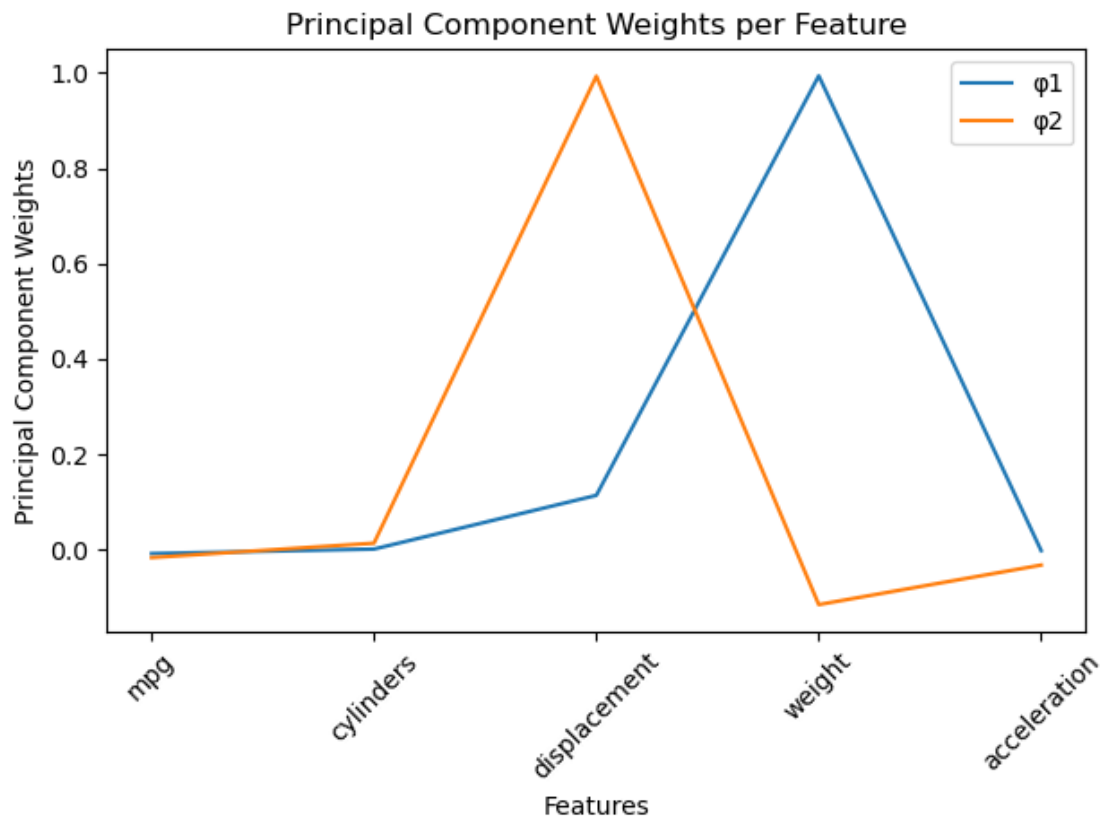
# principal components
components = pca.components_
x = np.arange(components.shape[1]) # 6

# Plot the first and second principal components
plt.plot(x, components[0], label='1')
plt.plot(x, components[1], label='2')

# Define feature names as tick labels
names = ['mpg', 'cylinders', 'displacement', 'weight', 'acceleration']
plt.xticks(ticks=x, labels=selected_columns, rotation=45)
```

```
# Labeling the axes and the legend
plt.xlabel('Features')
plt.ylabel('Principal Component Weights')
plt.title('Principal Component Weights per Feature')
plt.legend()

# Show plot
plt.tight_layout()
plt.savefig("plot.png", bbox_inches='tight')
plt.close()
```



b

It seems pca is capturing the data with larger scales this can lead to misleading conclusions and the importance of features

## Question 4

```
# Manually standardize the data to have mean 0 and standard deviation 1
means = np.mean(X, axis=0)
stds = np.std(X, axis=0)
Z = (X - means) / stds

# Verify that all features in Z have a mean of approximately zero and a standard deviation of one
means_Z = np.mean(Z, axis=0)
```

```
stds_Z = np.std(Z, axis=0)

print(means_Z, stds_Z)
```

```
mpg          1.450087e-16
cylinders    -1.087565e-16
displacement -7.250436e-17
weight       -1.812609e-17
acceleration  4.350262e-16
dtype: float64 mpg          1.0
cylinders     1.0
displacement  1.0
weight        1.0
acceleration  1.0
dtype: float64
```

## Question 5

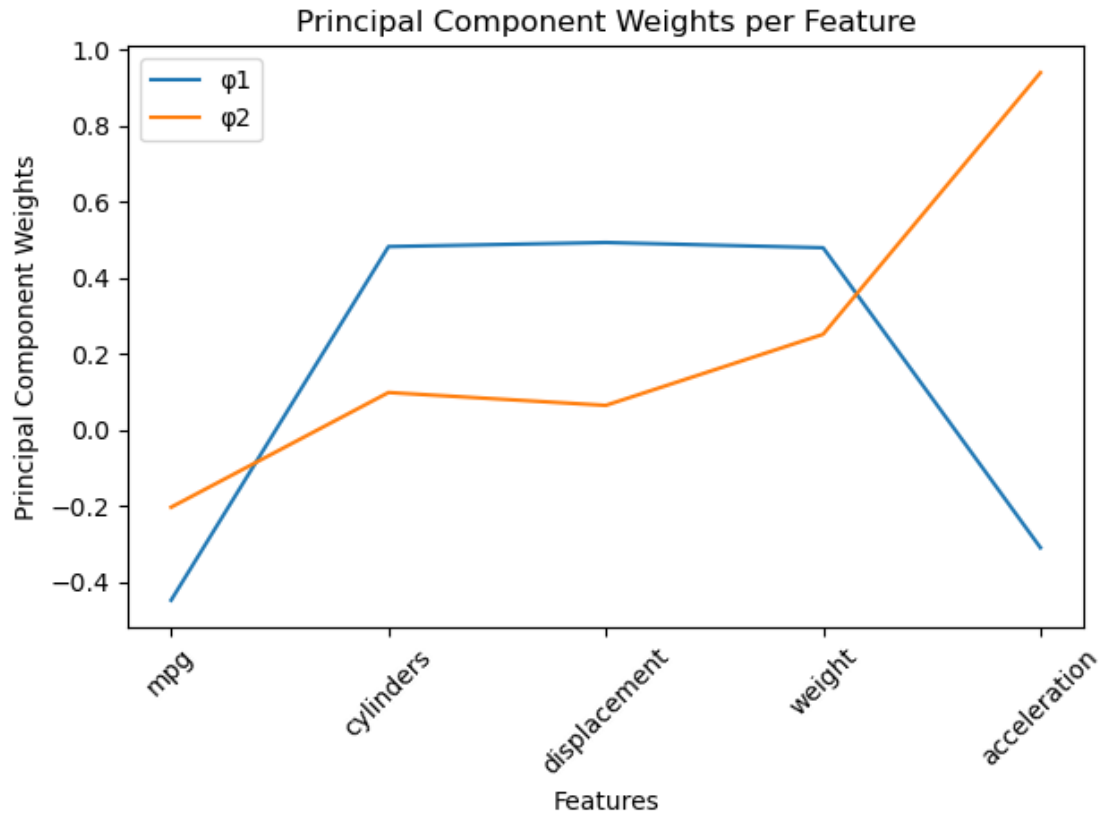
a

```
pca = PCA(n_components=2)
Z_reduced = pca.fit_transform(Z)
print("Reduced data size:", Z_reduced.shape)
components = pca.components_
plt.plot(x, components[0], label='1')
plt.plot(x, components[1], label='2')

plt.xticks(ticks=x, labels=selected_columns, rotation=45)

plt.xlabel('Features')
plt.ylabel('Principal Component Weights')
plt.title('Principal Component Weights per Feature')
plt.legend()

# Show plot
plt.tight_layout()
plt.savefig("plot-standard.png", bbox_inches='tight')
plt.close()
# plt.show()
```



**b**

It seems that the three most important features are 'cylinders', 'displacement', and 'weight', as the first principal component ( $\phi_1$ ) weighs these more heavily, with 'displacement' appearing to have the highest contribution to variance by a small margin.

**c**

```
dot_product_mock = np.dot(components[0], components[1])

# Calculate the magnitude (norm) of each principal component to check if it's equal to one
magnitude_phi1_mock = np.linalg.norm(components[0])
magnitude_phi2_mock = np.linalg.norm(components[1])
print("Dot product", dot_product_mock)
print("Magnitude phi 1", magnitude_phi1)
print("Magnitude phi 2", magnitude_phi2)
```

Dot product -1.1102230246251565e-16

Magnitude phi 1 1.0000000000000002

Magnitude phi 2 1.0000000000000002

## Question 6