Energy-Throughput Tradeoff with Optimal Sensing Order in Cognitive Radio Networks

M. Santhoshkumar and K. Premkumar
Department of Electronics and Communication Engineering
Indian Institute of Information Technology,
Design and Manufacturing, Kancheepuram
Chennai 600 127, INDIA
Email: kpk@iiitdm.ac.in

***Abstract*—We consider the problem of optimal sensing in a cognitive radio network which has N primary users (PUs) and one secondary user (SU). Our objective is to maximize the throughput that the SU gets for a given budget on the sensing energy. We consider a time-slotted system and the activity of each PU follows a two state (idle and busy) discrete time Markov chain. At the beginning of each time-slot, the SU senses channels in an optimal sensing order (i.e., in the descending order of posterior probability of a channel being idle) until it finds an idle channel. When the SU finds an idle channel, it stops sensing and transmits its packet in the remaining part of the time-slot. At the end of each time-slot, SU updates the posterior probability of N channels. SU spends finite energy to sense and declare a decision (idle or busy) on each channel. Therefore, sensing energy of the SU drains linearly as the number of channels sensed increases. In this work, we study the energy-throughput tradeoff of SU with optimal sensing order. In particular, we find the number of channels that should be sensed in each time-slot such that we achieve maximum throughput with a bound on sensing energy. Our numerical results show the optimum number of channels M, to be sensed in each time-slot for perfect and imperfect sensing.**

***Index Terms*—cognitive radios, sensing, throughput.**

I. Introduction

The ever increasing demand for high data rate motivates us to study the use of under-utilized and unused spectrum. This leads to the study of dynamic spectrum allocation, or in general, Cognitive Radios (CRs). A CR allows an unlicensed, also called, Secondary User (SU) to access the licensed band which is not currently occupied by a (licensed, or) Primary User (PU) thereby improving the spectrum utilisation. At the same time, SUs should not provide any interference to any of the PUs. To know the spectrum availability and to avoid the interference, an SU has to sense the spectrum. Based on sensing outcome, SU decides whether or not to attempt for its transmission. Hence, spectrum sensing plays a significant role in cognitive radio.

To sense a channel, SU has to spend an amount of energy. In general, SUs are energy constrained devices, and thus, the energy needs to be used efficiently. In case of multiple channels, SU has to find an idle channel for the transmission. An SU senses channels sequentially until it finds an idle channel. The sensing process drains its energy reserves. In contrast, sensing too many channels, ensures an idle channel, which increases its throughput. This leads to the study of energy–throughput tradeoff. In particular, we ask the following question: *what is the optimal number of channels that the SU has to sense so that it achieves a maximum throughput.*

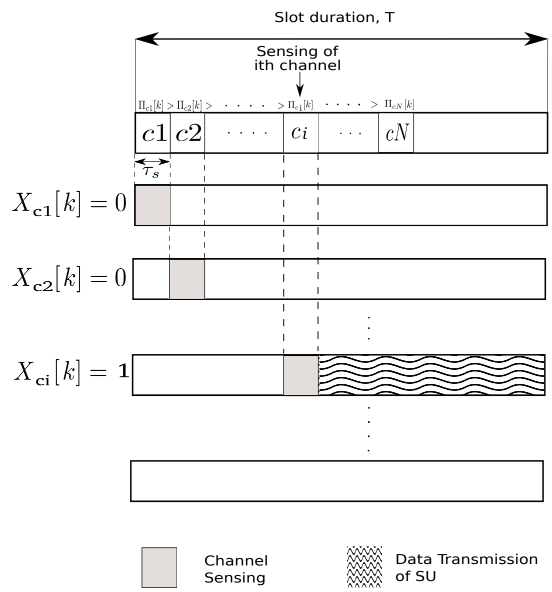

Fig. 1. An illustration of sensing process. In each time-slot k, the SU sequentially senses channels in the order of decreasing $\Pi_i[k]s$. Sensing for each channel requires a time-span of τ_s time units. The sensing procedure is done until $X_{A[k]}[k] = 1$ for some channel $A[k]$, or the number of channels sensed is M whichever is smaller.

A. Previous Work

In [1] – [4] the authors study various sensing-throughput tradeoffs, and obtain optimum sensing time. In [5] and [6], the authors pose energy efficient opportunistic spectrum access problem as a partially observable Markov decision process (POMDP), and obtain an optimum sensing policy. In [7], authors consider the channel sensing order (see Figure 1), in which, the channels are sensed sequentially based on descending order of posterior probability, and the posterior probability is updated based on all the observations until current time-slot for every channel. However, the authors do not consider the sensing energy of SU in the sensing procedure.

None of the work so far considers energy-throughput tradeoff for the case of multiple channels with optimum sensing

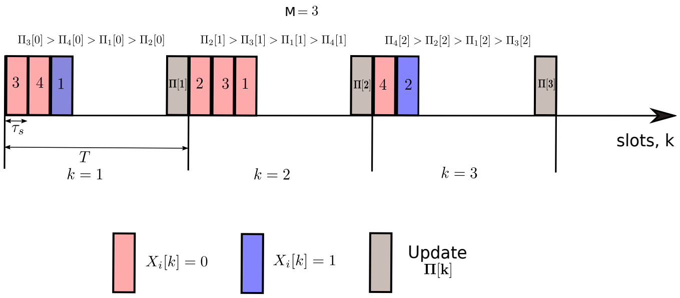

Fig. 2. An illustration of sequential sensing. We consider 4 channels and look at the evolution across time-slots $k = 1, 2, 3$ (for $M = 3$). At the beginning of each time-slot k, the SU has $\Pi_i[k-1]$s that are computed using the observations made until time-slot $k-1$. At the beginning of time-slot 1, we observe that $\Pi_3[0] > \Pi_4[0] > \Pi_1[0] > \Pi_2[0]$. At $k = 1$, channel 1 is observed to be idle, and is used by the SU for transmitting its packet. At the beginning of time-slot $k = 2$, the a posteriori probabilities are in the following order: $\Pi_2[1] > \Pi_3[1] > \Pi_1[1] > \Pi_4[1]$. Since we chose $M = 3$, sensing has to be stopped within first 3 sensing even if the SU observes that all channels are busy. At the beginning of $k = 3$, the SU observes a channel to be idle after having observed 2 channels, and so, the sensing procedure is stopped after sensing channel 2. Thus, the SU sees an opportunity for transmitting in channel 2 during the remaining portion of time-slot 3. Note that the ordering on $\Pi_i[k-1]$s from maximum to minimum is optimum for the case $p_{i,11} > p_{i,01}$ (see Eqn. (4)).

order. In this work, we describe a tradeoff where we achieve optimum throughput with a budget on sensing energy, i.e., we find the optimum number of channels to be sensed in each time-slot. Our objective is to find an optimum number of channels to sense M, such that the throughput is maximum with a constraint on average sensing energy.

B. *Organization of the paper*

The rest of the paper is organized as follows. In Section II, we explain the model of our network, and the activity of the PUs. We also describe the sensing model in Section II. In Section III, we formulate the optimal sensing problem, which aims to maximize the throughput for a given average sensing energy. Numerical Results are discussed in Sections IV. Finally, we conclude in Section V.

II. System Model

We consider a Cognitive Radio Network (CRN) that consists of a set of N PUs and one SU. Each PU has access to a unique channel, and the set of channels (or, the set of PUs) is denoted by $\mathcal{C} = \{1, 2, 3, \cdots, N\}$.

We consider a slotted-time channel, in which time-slots are indexed by $k \in \mathbb{Z}^+$, where $\mathbb{Z}^+ = \{1, 2, 3, \cdots\}$. We assume that all packets of PUs are of same length, and that the length of a time-slot is the same as that of a packet of a PU, which is considered to be T (i.e., time-slots and packets of a PU are of length T time units). Packets of SU shall be of different lengths, as this depends on how much time the SU gets in each time-slot for its transmission.

At the beginning of time-slot k, each PU $i \in \mathcal{C}$ either has a packet to transmit (indicated by 0), or does not have any packets to transmit (indicated by 1). Let $\Theta_i[k] \in \{0, 1\}$ represent the *busy/idle* state of PU i during time-slot k, where busy is indicated by 0 (meaning that the PU has a packet to transmit), and idle is indicated by 1 (meaning that the PU does not have any packets to transmit). We assume that the *busy/idle* state of each PU i during each time-slot k follows a two state discrete-time Markov chain (DTMC), the transition probability matrix (TPM) of which is given by

$$\boldsymbol{P}_i = \begin{bmatrix} p_{i,00} & p_{i,01} \\ p_{i,10} & p_{i,11} \end{bmatrix}. \tag{1}$$

In this problem, we assume that the TPM $\boldsymbol{P}_i$ of each PU i is known to the SU. Let $\boldsymbol{\Theta}[k]$ denote the vector of states of all PUs during time-slot k, i.e., $\boldsymbol{\Theta}[k] = [\Theta_1[k], \Theta_2[k], \cdots, \Theta_N[k]] \in \{0, 1\}^N$.

A. *Sensing Model*

At the beginning of each time-slot k, the SU makes an observation about the state of a number of PUs in a sequential order. If at the beginning of time-slot k, the SU chooses to sense channel i, it obtains an observation (which is the inference obtained from its measurement) which is denoted by $X_i[k]$. In perfect sensing, the observation is the same as that of the actual state of the channel, i.e., $X_i[k] = \Theta_i[k]$. In imperfect sensing, the observation may not reflect the actual state of the channel due errors in measurement and inference. The model that we follow for observation is described as follows.

$$\begin{aligned} \mathbb{P}\{X_i[k] = 1 \mid \Theta_i[k] = 0\} &= \alpha, \qquad (2) \\ &= 1 - \mathbb{P}\{X_i[k] = 0 \mid \Theta_i[k] = 0\}, \\ \mathbb{P}\{X_i[k] = 1 \mid \Theta_i[k] = 1\} &= \beta, \qquad (3) \\ &= 1 - \mathbb{P}\{X_i[k] = 0 \mid \Theta_i[k] = 1\}. \end{aligned}$$

Note that α and β depend on the quality of measured samples, signal–to–noise ratio of samples, and the detection/inference procedure.

III. Problem Formulation

At the beginning of each time-slot k, the SU senses and obtains an observation of the channel that is most-likely to be idle. If the observation happens to be 0 (meaning that the sensed channel is busy), the SU then obtains an observation of another channel that is next most-likely to be idle. The SU keeps sensing until it sees an idle channel. In order to restrict the time spent for sensing, we keep the number of sensing attempts to M, i.e., if the SU finds all channels to be busy in its first M sensing attempts, it will give up sensing, and no transmission of packets of SU takes place during time-slot k; otherwise, SU transmits its packet during the remaining portion of time-slot k (see Figure 2).

After having obtained an observation of channel i, the SU computes the a posteriori probability that PU i is idle given all observation until time-slot k, which is denoted by $\Pi_i[k]$. Thus,

$$\begin{aligned} \Pi_i[k] &= \mathbb{P}\{\Theta_i[k] = 1 \mid X_i[1], X_i[2], \cdots, X_i[k]\} \\ &= \mathbb{P}\{\Theta_i[k] = 1 \mid X_i[1:k]\}, \end{aligned}$$

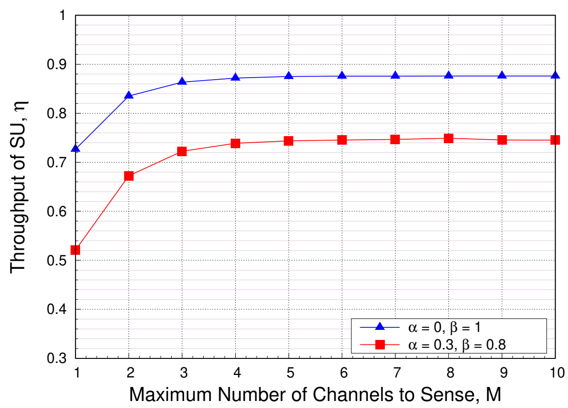

Fig. 3. Throughput of SU, η is plotted as a function of M for $p_{00} = 0.75, p_{11} = 0.85, N = 10$, and $K = 10^5$ slots. $p_{00} = 0.75, p_{11} = 0.85, N = 10$, and $K = 10^5$ slots. The plot has both perfect sensing $\alpha = 0, \beta = 1$ and imperfect sensing $\alpha = 0.3, \beta = 0.8$.

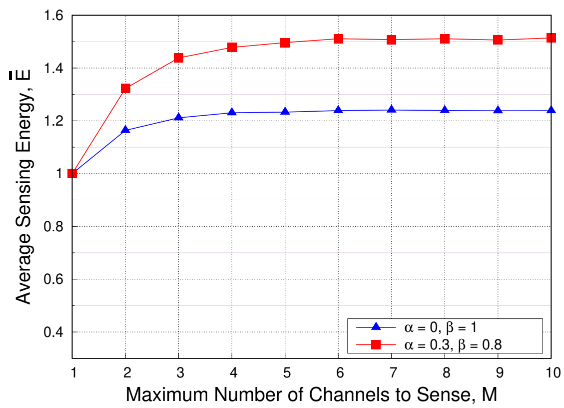

Fig. 4. Average Sensing Energy, $\overline{E}$ is plotted as a function of M for $p_{00} = 0.75, p_{11} = 0.85, N = 10$, and $K = 10^5$ slots. The plot has both perfect sensing $\alpha = 0, \beta = 1$ and imperfect sensing $\alpha = 0.3, \beta = 0.8$.

where $X_i[1:k] := [X_i[1], X_i[2], \cdots, X_i[k]]$ is the vector of observations of channel i from time-slot 1 to time-slot k. Note that $X_i[1:k] \in \{0, 1, \phi\}^k$, where ϕ denotes no observation.

Define $\mathbf{\Pi}[k]$ as the vector of posterior probabilities as $\mathbf{\Pi}[k] := [\Pi_1[k], \Pi_2[k], \cdots, \Pi_N[k]] \in [0,1]^N$. However, at the beginning of time-slot k, the SU does not have observation $X_i[k]$ of any channel $i \in \mathcal{C}$. Hence, the SU computes the following prior probabilities

$$\begin{aligned}\Lambda_i[k] &= \mathbb{P}\{\Theta_i[k] = 1 \mid X_i[1:k-1]\} \\ &= (1 - \Pi_i[k-1])\, p_{i,01} + \Pi_i[k-1] p_{i,11} \\ &= p_{i,01} + (p_{i,11} - p_{i,01})\, \Pi_i[k-1]. \end{aligned} \tag{4}$$

The $\Lambda_i[k]$s are sorted in the decreasing order, and let

$$\Lambda_{c1}[k] \;\geqslant\; \Lambda_{c2}[k] \;\geqslant\; \Lambda_{c2}[k] \;\geqslant\; \Lambda_{cN}[k].$$

From Eqn. (4), it is clear that for the case $p_{i,11} - p_{i,01} > 0$, sorting of channels based on the descending order of $\Lambda_i[k]$s is the same as the sorting based on the descending order of $\Pi_i[k-1]$s, and for the case $p_{i,11} - p_{i,01} < 0$, sorting of channels for sensing is the same as that of sorting based on the ascending order of $\Pi_i[k-1]$s.

The SU starts observing channels sequentially in the following order” $c1$ followed by $c2$, $c3$, and so on, until it senses an idle channel, or it senses a maximum of M channels. Let $N[k]$ be the number of channels sensed by the SU during time-slot k. Note that $N[k] \leqslant M$. For sensing a channel, the SU takes τ_s time units. We assume that the sensing time is the same for all channels. Therefore, to sense $N[k]$ channels, it requires $\tau_s N[k]$ time units. If the SU finds an idle channel at the end of sensing $N[k]$ channels, the SU uses the remaining time $T - \tau_s N[k]$ for its transmission as shown in Figure 2. The maximum sensing capability M is chosen such that the SU sees an appreciable residual time for transmitting its packets.

Recall that $N[k]$ is the number of sensing attempts by the SU during time-slot k, and let $A[k]$ be the channel that is sensed by the SU in its $N[k]$th sensing attempt. Also, let $C[k] \in \{0, 1\}$ be the resultant of sensing of the channel $A[k]$, where $C[k] = 0$ means that all channels are sensed as busy, and $C[k] = 1$ indicates that the last sensed channel, $A[k]$ is idle. Note that $C[k] = X_{A[k]}[k]$.

Therefore, the length of time available for the SU during time-slot k, which is denoted by $S[k]$ is given by

$$S[k] = \begin{cases} T - \tau_s N[k], & \text{if } C[k] = 1 \text{ and } \Theta_{A[k]}[k] = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

In the case of imperfect sensing, even if the SU observes that the channel $A[k]$ is idle, it is possible that the actual state of channel $A[k]$ is busy (i.e., $\Theta_{A[k]}[k] = 0$), in which case the SU derives no useful throughput during time-slot k.

We define throughput of the SU, denoted by η, as the long-term average fraction of time-slot that SU gets. Thus,

$$\begin{aligned}\eta &:= \lim_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K} \frac{S[k]}{T}, \\ &= \lim_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K} \left[\frac{T - \tau_s N[k]}{T} \mathbf{1}_{\{C[k]=1, \Theta_{A[k]}[k]=1\}} \right]. \end{aligned} \tag{6}$$

We define average sensing energy, denoted by $\overline{E}$, as the average energy expended by the SU for sensing in each time-slot. $\overline{E}$ is given by

$$\begin{aligned}\overline{E} &:= \lim_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K} E_1 N[k], \\ &= E_1 \cdot \lim_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K} N[k], \end{aligned} \tag{7}$$

where E_1 is the energy spent for sensing a channel once.

Our objective is to maximize the throughput of SU for a given budget on the average sensing energy. Hence, we formulate the problem by maximizing the throughput subject to a constraint on average sensing energy, which is given by

$$\begin{aligned} &\max \quad \eta \\ &\text{s.t.} \quad \overline{E} \leqslant e, \end{aligned} \tag{8}$$

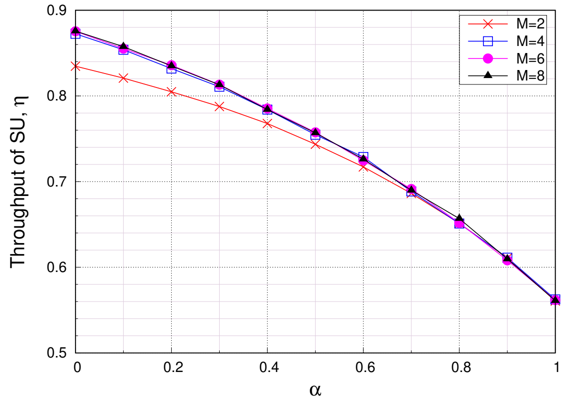

Fig. 5. Throughput of SU, η is plotted as a function of α for various values of M, and with $N = 10, \beta = 1.0, p_{00} = 0.75, p_{11} = 0.85$, and $K = 10^5$ slots.

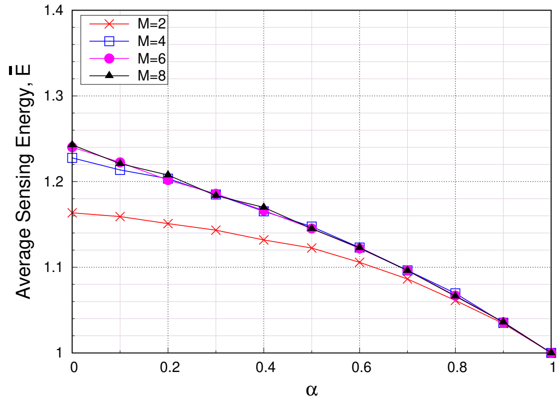

Fig. 6. Average Sensing Energy, $\overline{E}$ is plotted as a function of α for various values of M, and with $N = 10, \beta = 1.0, p_{00} = 0.75, p_{11} = 0.85$, and $K = 10^5$ slots.

where e is the sensing energy budget of the SU.

IV. Results and Discussion

Figure 3 shows the throughput η as a function of the maximum number of channels to sense M. As M increases, there is a higher probability that the SU finds an idle channel, and hence, η increases with M. Also, in the case of imperfect sensing, there is a loss in throughput due to miss detection (which happens with probability $1-\beta$). Hence, the throughput is less in the case of imperfect sensing than in the case of perfect sensing.

Figure 4 shows the average sensing energy $\overline{E}$ for different values of M. As M increases, the number of attempts to find an idle channel by the SU increases, and hence, $\overline{E}$ increases with M. Also, in the case of imperfect sensing, because of miss detection (which happens with probability $1 - \beta$), the number of sensing attempts increases. Hence, the average sensing energy required is more in the case of imperfect sensing than in the case of perfect sensing.

Figures 3 and 4 illustrate the tradeoff between throughput, η and average sensing energy, $\overline{E}$. As the average sensing energy $\overline{E}$ increases, the throughput η also increases. We note that both the throughput, η and the average sensing energy, $\overline{E}$ saturates at $M = 5$. This is due to the following reason: even though the SU can potentially sense up to M channels, it finds an idle channel, on the average, in few attempts.

Figure 5 shows the throughput η versus α. We note that the throughput decreases as α increases. As α increases, the SU stops sensing earlier, and might catch a channel which is actually busy, resulting in a loss of throughput. We also observe that the throughput increases as M increases (which is already explained before). But, for $M > 4$, the increment in throughput is only marginal. Thus, we can design a system with $M = 4$.

Figure 6 shows the variation of average sensing energy $\overline{E}$ with α. We observe that the sensing energy decreases as α increases. This is also due to stopping the sensing procedure earlier than the case of $\alpha = 0$. Hence, the average sensing energy decreases. Also, we observe that the average sensing energy increases with M (the phenomenon which we explained before). Also, we observe that for $M > 4$, the increment in average sensing energy is only marginal.

V. Conclusions

In this paper, we study the average sensing energy–throughput tradeoff in a Cognitive Radio Network having N PUs and one SU. We consider a sequential channel sensing policy based on an optimal sensing order. The number of sensing attempts is limited to M. We obtain the throughput and the average sensing energy, for various scenarios: with a range of M, for both perfect and imperfect sensing. Our results show that there exists an optimum M for the number of sensing attempts for throughput to be maximum.

In future, we focus on an analytical treatment of the tradeoff problem, and solve for the optimal sensing policy.

References

[1] A. Ghasemi and E. S. Sousa, "Optimization of spectrum sensing for opportunistic spectrum access in cognitive radio networks," in *Consumer Communications and Networking Conference*, Jan. 2007.

[2] Y. C. Liang, Y. Zeng, E. C. Y. Peh, and A. T. Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 4, pp. 4–17, Apr. 2008.

[3] Y. Pei, Y. Liang, K. C. Teh, and K. H. Li, "Sensing-throughput tradeoff for cognitive radio networks: A multiple-channel scenario," in *2009 IEEE 20th International Symposium on Personal, Indoor and Mobile Radio Communications*, Sep. 2009, pp. 1257–1261.

[4] Z. Quan, S. Cui, H. V. Poor, and A. H. Sayed, "Optimal multiband joint detection for spectrum sensing in cognitive radio networks," *IEEE Transactions on Signal Processing*, vol. 57, no. 3, pp. 1128–1140, March 2009.

[5] A. T. Hoang, Y. Liang, D. T. C. Wong, Y. Zeng, and R. Zhang, "Opportunistic spectrum access for energy-constrained cognitive radios," *IEEE Transactions on Wireless Communications*, vol. 8, no. 3, pp. 1206–1211, March 2009.

[6] Y. Pei, Y. Liang, K. C. Teh, and K. H. Li, "Energy-efficient design of sequential channel sensing in cognitive radio networks: Optimal sensing strategy, power allocation, and sensing order," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 8, pp. 1648–1659, Sep. 2011.

[7] H. T. Cheng and W. Zhuang, "Simple channel sensing order in cognitive radio networks," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 4, pp. 676–688, April 2011.