

COT 5405 Programming Assignment 3

Chapter 4 (Greedy Algorithms), unsolved problem 13

Algorithm Design by Kleinberg and Tardos, 1st Edition

Problem statement

13. A small business—say, a photocopying service with a single large machine—faces the following scheduling problem. Each morning they get a set of jobs from customers. They want to do the jobs on their single machine in an order that keeps their customers happiest. Customer i 's job will take t_i time to complete. Given a schedule (i.e., an ordering of the jobs), let C_i denote the finishing time of job i . For example, if job j is the first to be done, we would have $C_j = t_j$; and if job j is done right after job i , we would have $C_j = C_i + t_j$. Each customer i also has a given weight w_i that represents his or her importance to the business. The happiness of customer i is expected to be dependent on the finishing time of i 's job. So the company decides that they want to order the jobs to minimize the weighted sum of the completion times, $\sum_{i=1}^n w_i C_i$.

Design an efficient algorithm to solve this problem. That is, you are given a set of n jobs with a processing time t_i and a weight w_i for each job. You want to order the jobs so as to minimize the weighted sum of the completion times, $\sum_{i=1}^n w_i C_i$.

Example. Suppose there are two jobs: the first takes time $t_1 = 1$ and has weight $w_1 = 10$, while the second job takes time $t_2 = 3$ and has weight

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$w_2 = 2$. Then doing job 1 first would yield a weighted completion time of $10 \cdot 1 + 2 \cdot 4 = 18$, while doing the second job first would yield the larger weighted completion time of $10 \cdot 4 + 2 \cdot 3 = 46$.

Brute Force solution

- Data: 10 test cases, each with k jobs (with random t_k and w_k) for test case k .
- Implementation:
 - Input: k jobs.
 - Generate $k!$ possible sequences.
 - Calculate cost for each sequence.
 - Return sequence with minimum cost.

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Terminal - kartik@level2: ~/Dropbox/academic/algo/assignments/assignment_3
File Edit View Terminal Tabs Help
kartik@level2:~/Dropbox/academic/algo/assignments/assignment_3$ python3 job_sequencing.py
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ITERATION: 1
JOBS: [{'job_no': 0, 'time': 18, 'weight': 73}]
BF SEQUENCE: (0,) COST: 1314 TIME: 4.5299530029296875e-06
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ITERATION: 2
JOBS: [{'job_no': 0, 'time': 98, 'weight': 9}, {'job_no': 1, 'time': 33, 'weight': 16}]
BF SEQUENCE: (1, 0) COST: 1707 TIME: 3.5762786865234375e-06
-----
ITERATION: 3
JOBS: [{'job_no': 0, 'time': 64, 'weight': 98}, {'job_no': 1, 'time': 58, 'weight': 61}, {'job_no': 2, 'time': 84, 'weight': 49}]
BF SEQUENCE: (0, 1, 2) COST: 23808 TIME: 5.4836273193359375e-06
-----
ITERATION: 4
JOBS: [{'job_no': 0, 'time': 27, 'weight': 13}, {'job_no': 1, 'time': 63, 'weight': 4}, {'job_no': 2, 'time': 50, 'weight': 56}, {'job_no': 3, 'time': 78, 'weight': 98}]
BF SEQUENCE: (3, 2, 0, 1) COST: 17699 TIME: 1.7404556274414062e-05
-----
ITERATION: 5
JOBS: [{'job_no': 0, 'time': 99, 'weight': 1}, {'job_no': 1, 'time': 90, 'weight': 58}, {'job_no': 2, 'time': 35, 'weight': 93}, {'job_no': 3, 'time': 30, 'weight': 76}, {'job_no': 4, 'time': 14, 'weight': 41}]
BF SEQUENCE: (4, 2, 3, 1, 0) COST: 21205 TIME: 9.107589721679688e-05
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ITERATION: 6
JOBS: [{'job_no': 0, 'time': 4, 'weight': 3}, {'job_no': 1, 'time': 4, 'weight': 84}, {'job_no': 2, 'time': 70, 'weight': 2}, {'job_no': 3, 'time': 49, 'weight': 88}, {'job_no': 4, 'time': 28, 'weight': 55}, {'job_no': 5, 'time': 93, 'weight': 4}]
BF SEQUENCE: (1, 4, 3, 0, 5, 2) COST: 10687 TIME: 0.000591278076171875
-----
ITERATION: 7
JOBS: [{'job_no': 0, 'time': 68, 'weight': 29}, {'job_no': 1, 'time': 98, 'weight': 57}, {'job_no': 2, 'time': 64, 'weight': 71}, {'job_no': 3, 'time': 30, 'weight': 45}, {'job_no': 4, 'time': 30, 'weight': 87}, {'job_no': 5, 'time': 29, 'weight': 98}, {'job_no': 6, 'time': 59, 'weight': 38}]
BF SEQUENCE: (5, 4, 3, 2, 6, 1, 0) COST: 59531 TIME: 0.005026102066040039
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ITERATION: 8
JOBS: [{'job_no': 0, 'time': 3, 'weight': 54}, {'job_no': 1, 'time': 72, 'weight': 83}, {'job_no': 2, 'time': 13, 'weight': 24}, {'job_no': 3, 'time': 81, 'weight': 93}, {'job_no': 4, 'time': 38, 'weight': 16}, {'job_no': 5, 'time': 96, 'weight': 43}, {'job_no': 6, 'time': 93, 'weight': 92}, {'job_no': 7, 'time': 65, 'weight': 55}]
BF SEQUENCE: (0, 2, 1, 3, 6, 7, 5, 4) COST: 91221 TIME: 0.04486680030822754
-----
ITERATION: 9
JOBS: [{'job_no': 0, 'time': 65, 'weight': 86}, {'job_no': 1, 'time': 25, 'weight': 39}, {'job_no': 2, 'time': 37, 'weight': 76}, {'job_no': 3, 'time': 64, 'weight': 65}, {'job_no': 4, 'time': 51, 'weight': 76}, {'job_no': 5, 'time': 5, 'weight': 62}, {'job_no': 6, 'time': 32, 'weight': 96}, {'job_no': 7, 'time': 52, 'weight': 54}, {'job_no': 8, 'time': 86, 'weight': 23}]
BF SEQUENCE: (5, 6, 2, 1, 4, 0, 7, 3, 8) COST: 88761 TIME: 0.4334287643432617
-----
ITERATION: 10
JOBS: [{'job_no': 0, 'time': 47, 'weight': 71}, {'job_no': 1, 'time': 90, 'weight': 100}, {'job_no': 2, 'time': 87, 'weight': 95}, {'job_no': 3, 'time': 48, 'weight': 12}, {'job_no': 4, 'time': 57, 'weight': 85}, {'job_no': 5, 'time': 66, 'weight': 14}, {'job_no': 6, 'time': 100, 'weight': 21}, {'job_no': 7, 'time': 67, 'weight': 51}, {'job_no': 8, 'time': 48, 'weight': 63}, {'job_no': 9, 'time': 94, 'weight': 4}]
BF SEQUENCE: (0, 4, 8, 1, 2, 7, 3, 5, 6, 9) COST: 125498 TIME: 4.881145238876343
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Greedy Solution Pseudocode

- Objective: Minimize $\sum w_i C_i$ where C_i is the finish time of job i .
- Greedy thinking: Short jobs (small t_i values) with high priority (large w_i values) are executed first.
- Pseudo code:
 - For every job in set, store $r_i = t_i / w_i$.
 - Sort the list r keeping its corresponding job number i in track.
 - The obtained sequence is the optimal sequence that returns the least cost possible.
- Code: https://github.com/kjain-ucf/misc/algorithms_assignment_3/job_sequencing.py

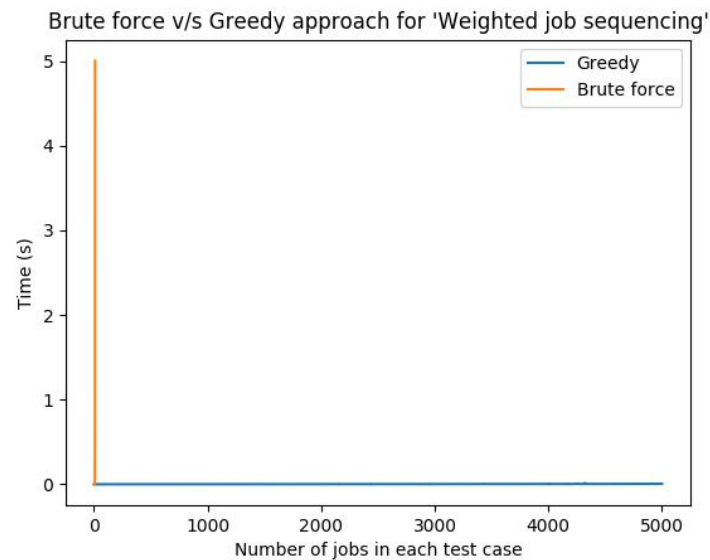
Validation (10 tests)

- Note: Used a seed to generate the same random job sets to replicate results. Comment line number 9 to get different results for every run.
- Note: Same sequences are obtained with both the approaches as expected

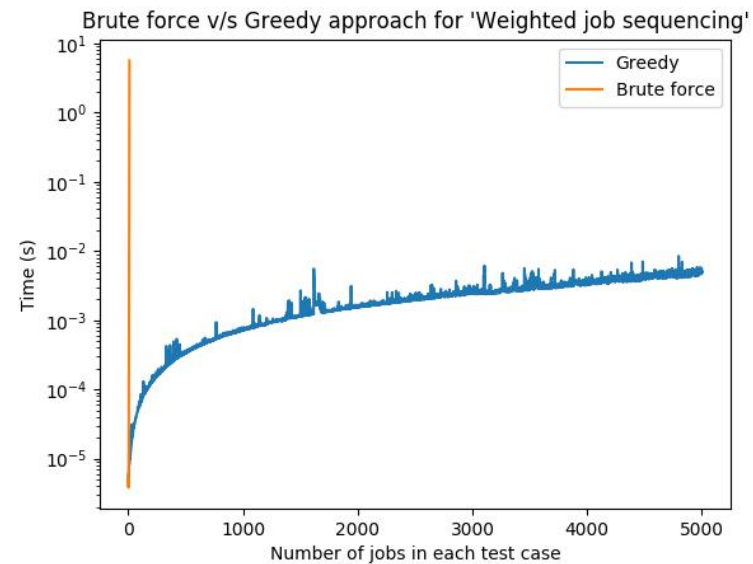
```
Terminal - kartik@level2: ~/Dropbox/academic/algo/assignments/assignment_3
File Edit View Terminal Tabs Help
kartik@level2:~/Dropbox/academic/algo/assignments/assignment_3$ python3 job_sequencing.py
-----
ITERATION: 1
GREEDY SEQUENCE: [0] COST: 1314 TIME: 1.5735626220703125e-05
BF SEQUENCE: (0,) COST: 1314 TIME: 1.4543533325195312e-05
-----
ITERATION: 2
GREEDY SEQUENCE: [1, 0] COST: 1707 TIME: 4.5299530029296875e-06
BF SEQUENCE: (1, 0) COST: 1707 TIME: 3.814697265625e-06
-----
ITERATION: 3
GREEDY SEQUENCE: [0, 1, 2] COST: 23808 TIME: 4.0531158447265625e-06
BF SEQUENCE: (0, 1, 2) COST: 23808 TIME: 5.7220458984375e-06
-----
ITERATION: 4
GREEDY SEQUENCE: [3, 2, 0, 1] COST: 17699 TIME: 4.291534423828125e-06
BF SEQUENCE: (3, 2, 0, 1) COST: 17699 TIME: 1.7404556274414062e-05
-----
ITERATION: 5
GREEDY SEQUENCE: [4, 2, 3, 1, 0] COST: 21205 TIME: 5.245208740234375e-06
BF SEQUENCE: (4, 2, 3, 1, 0) COST: 21205 TIME: 9.799003601074219e-05
-----
ITERATION: 6
GREEDY SEQUENCE: [1, 4, 3, 0, 5, 2] COST: 10687 TIME: 5.7220458984375e-06
BF SEQUENCE: (1, 4, 3, 0, 5, 2) COST: 10687 TIME: 0.0005960464477539062
-----
ITERATION: 7
GREEDY SEQUENCE: [5, 4, 3, 2, 6, 1, 0] COST: 59531 TIME: 5.7220458984375e-06
BF SEQUENCE: (5, 4, 3, 2, 6, 1, 0) COST: 59531 TIME: 0.00508427619934082
-----
ITERATION: 8
GREEDY SEQUENCE: [0, 2, 1, 3, 6, 7, 5, 4] COST: 91221 TIME: 7.867813110351562e-06
BF SEQUENCE: (0, 2, 1, 3, 6, 7, 5, 4) COST: 91221 TIME: 0.04427027702331543
-----
ITERATION: 9
GREEDY SEQUENCE: [5, 6, 2, 1, 4, 0, 7, 3, 8] COST: 88761 TIME: 1.0967254638671875e-05
BF SEQUENCE: (5, 6, 2, 1, 4, 0, 7, 3, 8) COST: 88761 TIME: 0.4296393394470215
-----
ITERATION: 10
GREEDY SEQUENCE: [0, 4, 8, 1, 2, 7, 3, 5, 6, 9] COST: 125498 TIME: 1.33514404296875e-05
BF SEQUENCE: (0, 4, 8, 1, 2, 7, 3, 5, 6, 9) COST: 125498 TIME: 4.767407417297363
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Runtime graphs

X linear scale, Y linear scale



X linear scale, Y log scale



X log scale, Y log scale

