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GATE

GRADUATE APTITUDE TEST IN ENGINEERING

TOPIC-WISE PREVIOUS YEARS' SOLVED QUESTION PAPERS

27 YEARS

ELECTRONICS AND COMMUNICATION ENGINEERING

2020

HIGHLIGHTS

- Includes around 27 years' GATE questions arranged chapter-wise
- Detailed solutions for better understanding
- Includes the latest GATE solved question papers with detailed analysis
- Comprehensively revised and updated

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27 YEARS

ELECTRONICS AND COMMUNICATION ENGINEERING 2020

Pearson

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First Impression

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Dedication

This book is dedicated to almighty GOD, my parents, wife Hemani, daughter Aarchishya, and son Ruchirangad. It was not possible for me to take up this task without their support.

Contents

Preface

Graduate Aptitude Test in Engineering (GATE) is an all India examination that primarily tests the comprehensive understanding of the candidate in various undergraduate subjects in Engineering/Technology/Architecture and postgraduate level subjects in Science. Owing to multifaceted opportunities open to any good performer, the number of aspirants appearing for the GATE examination is increasing significantly every year. Apart from giving the aspirant a chance to pursue an M.Tech. from institutions such as the IITs /NITs, a good GATE score can be highly instrumental in landing the candidate a plush public sector job, as many PSUs are recruiting graduate engineers on the basis of their performance in GATE. The GATE examination pattern has undergone several changes over the years—sometimes apparent and sometimes subtle. It is bound to continue to do so with changing technological environment.

We are pleased to present before the readers ***GATE Topic-wise Previous Years' Solved Question Papers*** **for** ***Electronics and Communication Engineering*** which is a one-stop solution for GATE aspirants to crack the GATE exam. The book includes around 27 years GATE questions segregated topic-wise along with exam analysis which is provided at the beginning of every unit. This in turn will help the aspirants to get an idea about the pattern and weightage of questions asked in the examination. The salient features of the book are given below.

Salient Features

- Includes around 27 years GATE questions arranged topic-wise.
- Gate 2019 paper with topic-wise Analysis.
- Detailed solutions for better understanding.
- Topic-wise detailed analysis of previous year questions provided for each section.

I acknowledge the help provided in compilation and by giving technical inputs by Pearson Editorial Team and many others.

Despite of our best efforts, some errors may have inadvertently crept into the book. Constructive comments and suggestions to further improve the book are welcome and shall be acknowledged gratefully.

Rajiv Kapoor

Reviewers

We would like to thank the below mentioned reviewers for their continuous feedback and suggestions which has helped in shaping this book.

Jyotsna Singh	Associate Professor, Netaji Subhas Institute of Technology, Dwarka, New Delhi
S. K. Mydhili	Assistant Professor (Senior Grade), SVS College of Engineering, Coimbatore, Tamil Nadu
Arun Khosla	Associate Professor, National Institute of Technology, Jalandhar, Punjab
Manav Bhatnagar	Associate Professor, Indian Institute of Technology, New Delhi
B.V.R. Reddy	Professor and Dean, USET, Guru Gobind Singh Indraprastha University, Dwarka, Delhi

About The Author

Rajiv Kapoor, Ph.D. in Electronics and Communication Engineering from Panjab University (Punjab Engineering College), Chandigarh and has worked in industries of repute and also worked in Engineering institutions of repute and having a total experience of 23 years in teaching Engineering students. He is presently working as professor in Electronics and Communication Engineering Department at Delhi Technological University (Formerly Delhi College of Engineering). He published more than 100 papers in the various Journals/Conferences of repute and has filed 6 patents also. He has undertaken 21 R&D projects and is not only a strong believer but a practitioner for '**MAKE in INDIA**'. He has an endeavour to design new products for Industries and work for society with in-depth knowledge of the subject, he has honed his skills to foresee the obstacles of students in their quest to prepare them for these examinations (GATE, NET, etc.) and successfully weeds them out.

Syllabus: Electronics and Communication Engineering

Networks, Signals and Systems

Network Solution Methods: Nodal and mesh analysis; Network theorems: superposition, Thevenin and Norton's, maximum power transfer; Wye-Delta transformation; Steady state sinusoidal analysis using phasors; Time domain analysis of simple linear circuits; Solution of network equations using Laplace transform; Frequency domain analysis of RLC circuits; Linear 2-port network parameters: driving point and transfer functions; State equations for networks.
Continuous-time Signals: Fourier series and Fourier transform representations, sampling theorem and applications; Discrete-time signals: discrete-time Fourier transform (DTFT), DFT, FFT, Z-transform, interpolation of discrete-time signals; LTI systems: definition and properties, causality, stability, impulse response, convolution, poles and zeros, parallel and cascade structure, frequency response, group delay, phase delay, digital filter design techniques.

Control Systems

Basic control system components; Feedback principle; Transfer function; Block diagram representation; Signal flow graph; Transient and steady-state analysis of LTI systems; Frequency response; Routh-Hurwitz and Nyquist stability criteria; Bode and root-locus plots; Lag, lead and lag-lead compensation; State variable model and solution of state equation of LTI systems.

Electronic Devices

Energy bands in intrinsic and extrinsic silicon; Carrier transport: diffusion current, drift current, mobility and resistivity; Generation and recombination of carriers; Poisson and continuity equations; P-N junction, Zener diode, BJT, MOS capacitor, MOSFET, LED, photo diode and solar cell; Integrated circuit fabrication process: oxidation, diffusion, ion implantation, photolithography and twin-tub CMOS process.

Analog Circuits

Small signal equivalent circuits of diodes, BJTs and MOSFETs; Simple diode circuits: clipping, clamping and rectifiers; Single-stage BJT and MOSFET amplifiers: biasing, bias stability, mid-frequency small signal analysis and frequency response; BJT and MOSFET amplifiers: multi-stage, differential, feedback, power and operational; Simple op-amp circuits; Active filters; Sinusoidal oscillators: criterion for oscillation, single-transistor and opamp configurations; Function generators, wave-shaping circuits and 555 timers; Voltage reference circuits; Power supplies: ripple removal and regulation.

Digital Circuits

Number systems; Combinatorial circuits: Boolean algebra, minimization of functions using Boolean identities and Karnaugh map, logic gates and their static CMOS implementations, arithmetic circuits, code converters, multiplexers, decoders and PLAs; Sequential circuits: latches and flip-flops, counters, shift-registers and finite state machines; Data converters: sample and hold circuits, ADCs and DACs; Semiconductor memories: ROM, SRAM, DRAM; 8-bit microprocessor (8085): architecture, programming, memory and I/O interfacing.

Communications

Random processes: Autocorrelation and power spectral density, properties of white noise, filtering of random signals through LTI systems; Analog communications: amplitude modulation and demodulation, angle modulation and demodulation, spectra of AM and FM, superheterodyne receivers, circuits for analog communications; Information theory: entropy, mutual information and channel capacity theorem; Digital communications: PCM, DPCM, digital modulation schemes, amplitude, phase and frequency shift keying (ASK, PSK, FSK), QAM, MAP and ML decoding, matched filter receiver, calculation of bandwidth, SNR and BER for digital modulation; Fundamentals of error correction, Hamming codes; Timing and frequency synchronization, inter-symbol interference and its mitigation; Basics of TDMA, FDMA and CDMA.

Electromagnetics

Electrostatics; Maxwell's equations: differential and integral forms and their interpretation, boundary conditions, wave equation, Poynting vector; Plane waves and properties: reflection and refraction, polarization, phase and group velocity, propagation through various media, skin depth; Transmission lines: equations, characteristic impedance, impedance matching, impedance transformation, S-parameters, Smith chart; Waveguides: modes, boundary conditions, cut-off frequencies, dispersion relations; Antennas: antenna types, radiation pattern, gain and directivity, return loss, antenna arrays; Basics of radar; Light propagation in optical fibers.

Important Tips for GATE Preparation

The followings are some important tips which would be helpful for students to prepare for GATE examination.

1. Go through the pattern (using previous years' GATE paper) and syllabus of the exam and start preparing accordingly.
2. Preparation time for GATE depends on many factors, such as, individual's aptitude, attitude, fundamentals, concentration level etc., Generally rigorous preparation for four to six months is considered good but it may vary from student to student.
3. Make a list of books which cover complete syllabus, contains solved previous year questions and mock tests for practice based on latest GATE pattern. Purchase these books and start your preparation.
4. Make a list of topics which needs to be studied and make priority list for study of every topic based upon the marks for which that particular topic is asked in GATE examination. Find out the topics which fetch more marks and give more importance to those topics. Make a timetable for study of topics and follow the timetable strictly.
5. An effective way to brush up your knowledge about technical topics is group study with your friends. During group study you can explore new techniques and procedures.
6. While preparing any subject highlight important points (key definitions, equations, derivations, theorems and laws) which can be revised during last minute preparation.
7. Pay equal attention to both theory and numerical problems. Solve questions (numerical) based on latest exam pattern as much as possible, keeping weightage of that topic in mind. Whatever topics you decide to study, make sure that you know everything about it.
8. Try to use short-cut methods to solve problems instead of traditional lengthy and time consuming methods.
9. Go through previous years' papers (say last ten years), to check your knowledge and note the distribution of different topics. Also analyze the topics in which you are weak and concentrate more on those topics. Always try to solve papers in given time, to obtain an idea how many questions you are able to solve in the given time limit.
10. Finish the detail study of topics one and a half month before your exam. During last month revise all the topics once again and clear leftover doubts.

UNIT I

NETWORK THEORY

EXAM ANALYSIS

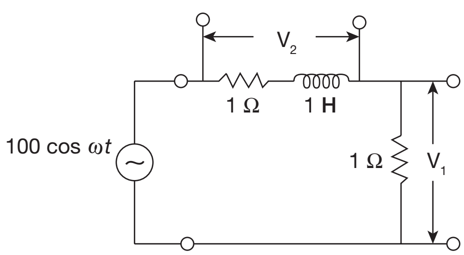
Exam Year	92	93	94	95	96	97	98	99	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15			16			17		18	19
																								Set 1	Set 2	Set 3	Set 1	Set 2	Set 3	Set 1	Set 2		
1 Marks Ques.	-	2	4	7	3	4	6	2	2	4	2	4	5	5	6	2	2	3	3	3	4	3	2	3	3	1	1	4	1	1	2	2	1
2 Marks Ques.	5	4	1	-	1	2	-	3	2	4	2	7	5	6	-	4	7	4	3	3	4	6	4	3	4	4	3	4	3	2	4	4	3
3 Marks Ques.	-	-	-	-	1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5 Marks Ques.	-	3	4	2	3	2	1	2	4	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Total Marks	**10**	**25**	**26**	**17**	**23**	**21**	**11**	**18**	**26**	**27**	**21**	**18**	**15**	**17**	**6**	**10**	**16**	**11**	**9**	**9**	**12**	**15**	**10**	**9**	**11**	**9**	**7**	**12**	**7**	**5**	**10**	**10**	**7**
Basics of Network Analysis	1	2	-	2	-	4	2	1	2	3	2	3	5	2	-	-	-	2	1	1	3	5	-		5			2		3		-	-
Sinusoidal Steady State	1	2	1	4	2	-	1	-	-	1	-	2	5	3	-	1	-	1	2	1	-	1	-		2			3		3		1	1
Network Theorems	-	2	1	1	-	-	1	2	1	1	1	1	-	2	-	2	1	1	-	2	2	2	-		4			1		1		-	1
Transient Analysis	1	-	1	-	1	1	-	-	-	-	1	2	2	1	2	1	4	2	1	1	1	-	-		3			2		-		1	-
Two Port Networks	1	-	1	-	-	1	1	1	-	2	-	1	2	2	2	-	2	-	2	1	2	-	-		2			3		-		1	1
Graph Theory and State Equations		1	-	-	-	1	-	1	1	-		-	1	1	-	-	-	1	-	-	-	-	-		1			2		1		1	1
Network Functions	-	-	1	-	-	-	-	-	1	-	-	1	-	1	2	2	1	1	-	-	-	1	-		-			2		1		-	-

Chapter 1

Basics of Network Analysis

One-mark Questions

1. In the circuit shown in the figure, the positive angular frequency ω (in radians per second) at which the magnitude of the phase difference between the voltages V_1 and V_2 equals $\frac{\pi}{4}$ radians, is ____________. **[2017]**



Solution: From the given data

Let $Z_1 = 1\,\Omega = 1\angle 0°\,\Omega$

$Z_2 = 1 + j\omega = \sqrt{1+\omega^2}\angle\theta_2\,\Omega$

Where $\theta_2 = \tan^{-1}\omega$

$V_1 = Z_1 . i(t)$

$V_2 = Z_2 . i(t)$

Let $i(t) = I_m \angle\theta$ Amp.

$V_1 = I_m \angle\theta \times 1$

$V_2 = I_m \angle\theta \times \sqrt{1+\omega^2}\angle\theta_2$

$V_2 = I_m \sqrt{1+\omega^2}\angle\theta + \theta_2$

Given $\theta + \theta_2 - \theta = \frac{\pi}{4}$

$\therefore \theta_2 = \frac{\pi}{4}$

$\text{Tan}^{-1}\left(\frac{\omega}{1}\right) = \theta_2$

$\therefore \omega = 1 \text{ rad/sec}$

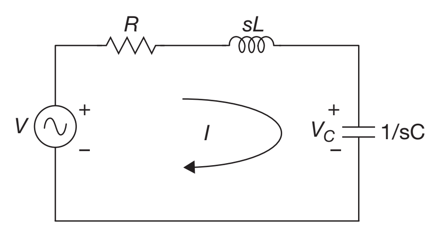
Hence, the correct answer is (0.9 to 1.1).

2. A connection is made consisting of resistance A in series with a parallel combination of resistances B and C. Three resistors of value 10 Ω, 5 Ω, 2 Ω are provided. Consider all possible permutations of the given resistors in to the positions A, B, C, and identify the configurations with maximum possible overall resistance, and also the ones with minimum possible overall resistance. The ratio of maximum to minimum values of the resistances (up to second decimal place) is __________. **[2017]**

3. The damping ratio of a series RLC circuit can be expressed as **[2015]**

(A) $\frac{R^2C}{2L}$ (B) $\frac{2L}{R^2C}$

(C) $\frac{R}{2}\sqrt{\frac{C}{L}}$ (D) $\frac{2}{R}\sqrt{\frac{L}{C}}$



Solution: $I = \dfrac{V}{R + sL + \dfrac{1}{sC}}$

$V_C = \dfrac{I}{sC}$

$\therefore \quad V_C = \left\{\dfrac{V}{R + sL + \dfrac{1}{sC}}\right\} \times \dfrac{1}{sC}$

$\dfrac{V_C}{V} = H(s) = \dfrac{1}{s^2LC + sRC + 1}$

$H(s) = \dfrac{1/LC}{s^2 + s\left(\dfrac{R}{L}\right) + 1/LC}$

From the above T/F compare to 2nd order system

$$\omega_n = \omega_o = \frac{1}{\sqrt{LC}}$$

$$2\zeta\omega_n = \frac{R}{L}$$

$$\zeta = \frac{R}{2L} \times \sqrt{LC} = \frac{R}{2}\sqrt{\frac{C}{L}} = \frac{1}{2Q}$$

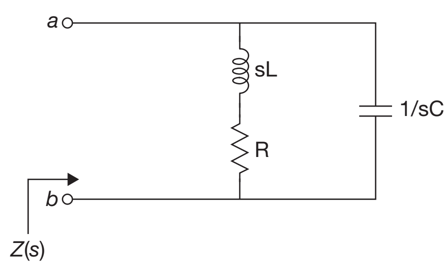
Hence, the correct option is (C).

4. An LC tank circuit consists of an ideal capacitor C connected in parallel with a coil of inductance L having an internal resistance R. The resonant frequency of the tank circuit is **[2015]**

(A) $\frac{1}{2\pi\sqrt{LC}}$ (B) $\frac{1}{2\pi\sqrt{LC}}\sqrt{1-R^2\frac{C}{L}}$

(C) $\frac{1}{2\pi\sqrt{LC}}\sqrt{1-\frac{1}{R^2C}}$ (D) $\frac{1}{2\pi\sqrt{LC}}\left(1-R^2\frac{C}{L}\right)$

Solution: From the given data the equivalent tank circuit is



$$Z(s) = \{R + sL\} \parallel \frac{1}{sC}$$

$$= \frac{(R+sL)\frac{1}{sC}}{R+sL+\frac{1}{sC}}$$

$$Z(s) = \frac{R+sL}{s^2 LC + RCs + 1}$$

$$Z(j\omega) = \frac{R+j\omega L}{-\omega^2 LC + j\omega RC + 1} = \frac{R+j\omega L}{(1-\omega^2 LC) + j\omega RC}$$

$$Z(j\omega) = \frac{(R+j\omega L)\{(1-\omega^2 LC) - j\omega RC\}}{(1-\omega^2 LC)^2 + (\omega RC)^2}$$

at resonance

$$I_{mag}\{Z(j\omega)\} = 0$$

$$I_{mag}(z\} = 0$$

$$\therefore \quad -j\,\omega R^2C + j\,\omega L\,(1 - \omega^2 LC) = 0$$

At resonance $\omega = \omega_0$

$$R^2C = L\,[1 - \omega_0^2\,LC\,\}$$

$$\frac{R^2C}{L} = 1 - \omega_0^2\,LC$$

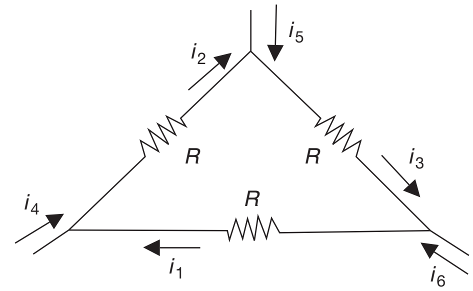
$$\omega_o^2 LC = 1 - \frac{R^2C}{L}$$

$$\omega_o = \frac{1}{\sqrt{LC}}\cdot\sqrt{1-\frac{R^2C}{L}}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}}\cdot\sqrt{1-\frac{R^2C}{L}}$$

Hence, the correct option is (B).

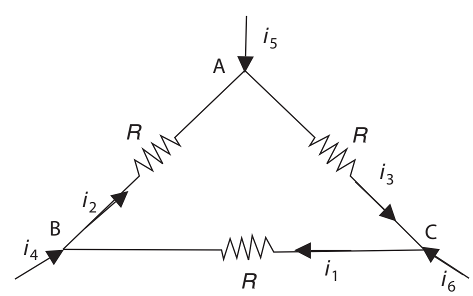
5. Consider the configuration shown in the figure which is a portion of a larger electrical network



For $R = 1\ \Omega$ and currents $i_1 = 2$ A, $i_4 = -1$ A, $i_5 = -4$ A, which one of the following is TRUE?

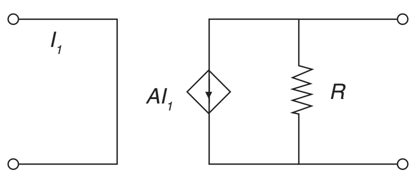
(a) $i_6 = 5$ A
(b) $i_3 = -4$ A
(c) Data is sufficient to conclude that the supposed currents are impossible
(d) Data is insufficient to identify the currents i_2, i_3 and i_6 **[2014]**

Solution: (a)



At A, $i_5 = i_3 - i_2$
At B, $-i_4 = i_1 - i_2$
At C, $i_6 = i_1 - i_3$
$i_6 + (i_2 + i_5) - i_1 = 0$
$i_6 + i_1 + i_4 + i_5 - i_1 = 0$
$i_6 + (2 - 1 - 4) - 2 = 0$
$i_6 = 5$A
Hence, the correct option is (a).

6. The circuit shown in the figure represents a



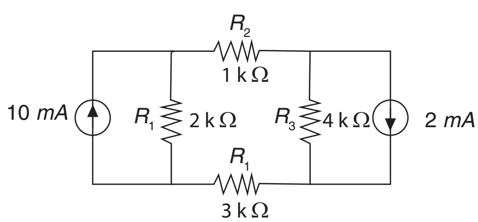
(a) voltage controlled voltage source
(b) voltage controlled current source
(c) current controlled current source
(d) current controlled voltage source **[2014]**

Solution: (c)

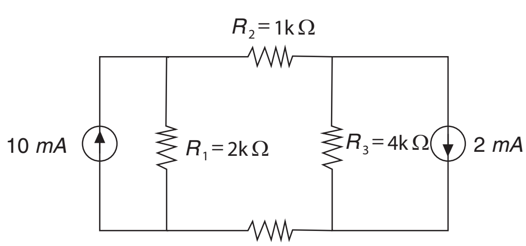
Output current = function (input current) = CCCS

Hence, the correct option is (c).

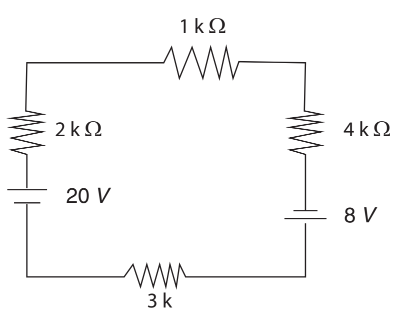
7. The magnitude of current (in mA) through the resistor R_2 in the figure shown is______ **[2014]**



Solution: 2.8 mA



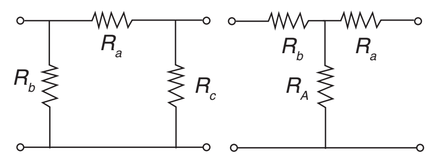
KVL:



$20 = 2I + I + 4I - 8 + 3I$

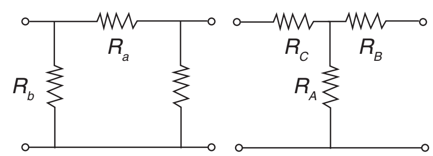
$I = 2.8$ mA

8. Consider a delta connection of resistors and its equivalent star connection as shown below. If all elements of the delta connection are scaled by a factor k, $k > 0$, the elements of the corresponding star equivalent will be scaled by a factor of



(a) k^2 (b) k
(c) $1/k$ (d) $\sqrt{k}$ **[2013]**

Solution: (b)



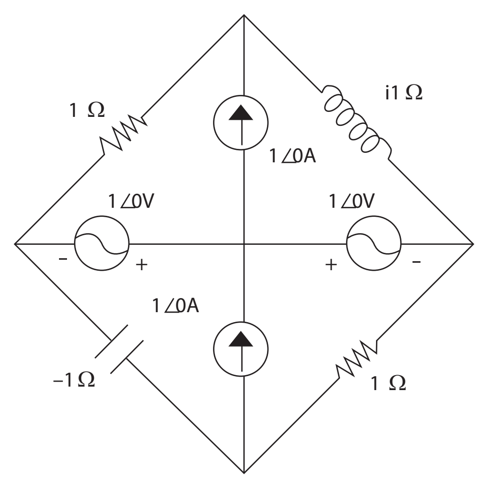
Appling star –delta conversion for the given circuit

$$R'_A = \frac{R_b R_c}{R_b + R_c + R_a} = \frac{K^2(R_b R_c)}{K[R_b + R_c + R_a]} = KR_A$$

$$R_A = KR_A$$

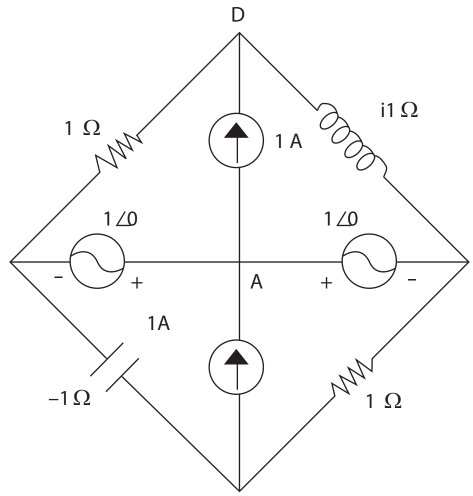
Hence, the correct option is (b).

9. In the circuit shown below, the current through the inductor is



(a) $\frac{2}{1+j}$ A (b) $\frac{-1}{1+j}$ A
(c) $\frac{1}{1+j}$ A (d) 0A **[2012]**

Solution: (c)



Applying KCL at node A, there will be no current in voltage sources.

Applying KVL in upper loop,

$$1+i+(i+1)j1+1\angle 0^\circ-1\angle 0^\circ=0$$

$$i=\frac{-j}{1+j}$$

$\therefore$ Current through inductor $=\dfrac{1-j}{j+1}=\dfrac{1}{1+j}$

Hence, the correct option is (c).

10. The average power delivered to an impedance $(4-j3)\Omega$ by a current 5 cos (100 πt + 100) A is

(a) 44.2 W (b) 50 W
(c) 62.5 W (d) 125 W **[2012]**

Solution: (b)

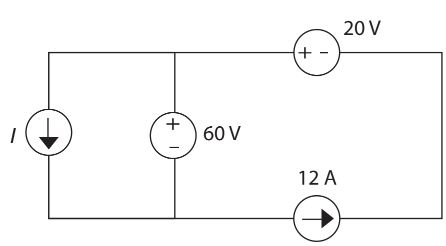
Average power is equivalent to RMS power

$$I_{rms}=\frac{I}{\sqrt{2}}=\frac{5}{\sqrt{2}}$$

$$P=I_{rms}^2R=\frac{25}{2}\times 4=50\text{W}$$

Hence, the correct option is (b).

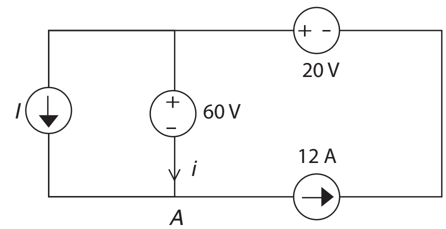
11. In the interconnection of ideal sources shown in the figure, it is known that the 60 V source is absorbing power.



Which of the following can be the value of the current source I?

(a) 10 A (b) 13 A
(c) 15 A (d) 18 A **[2009]**

Solution: (a)



Since 60 V is absorbing power, it means the current is entering positive terminal. Applying KCL at A,

$I+i=12$

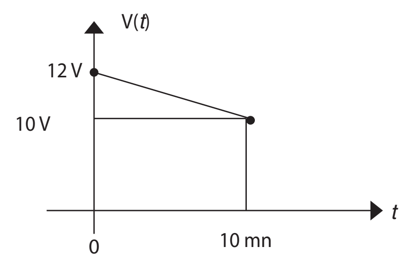
$I=12-i$ value of $i<12$

So, out of given options, only (a) can be the answer.

Hence, the correct option is (a).

12. A fully charged mobile phone with a 12 V battery is good for a 10 minute talk-time. Assume that, during the talk-time the battery delivers a constant current of 2 A and its voltage drops linearly from 12 V to 10 V as shown in the figure. How much energy does the battery deliver during this talk-time?

(a) 220 J (b) 12 kJ
(c) 13.2 kJ (d) 14.4 J **[2009]**



Solution: (c)

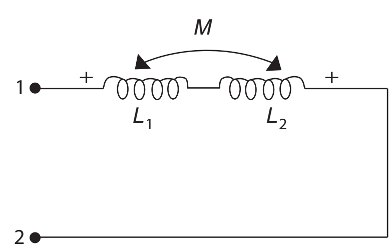
Given current $I = 2$**A**

$$E=VIT=I(VT)=\frac{2\times(12+10)}{2}\times 10\times 60$$

$E = 13.2$ kJ.

Hence, the correct option is (c).

13. The equivalent inductance measured between the terminals 1 and 2 for the circuit shown in the figure is



(a) $L_1 + L_2 + M$ (b) $L_1 + L_2 - M$
(c) $L_1 + L_2 + 2M$ (d) $L_1 + L_2 - 2M$

[2004]

Solution: (d)

Appling KVL for the given circuit, equation can be given as

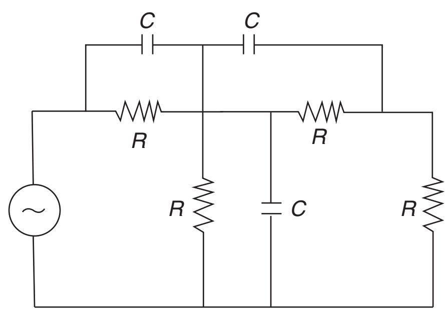
$$V = -M\frac{dI}{dt} + L_1\frac{dI}{dt} + -M\frac{dI}{dt} + L_2\frac{dI}{dt}$$

$$= (L_1 + L_2 - 2M)\frac{dI}{dt}$$

So, equivalent inductance = $L_{equ} = L_1 + L_2 - 2M$.

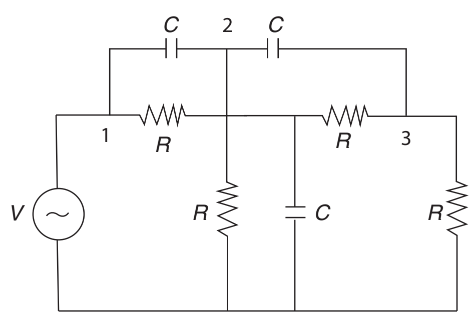
Hence, the correct option is (d).

14. The minimum number of equations required to analyze the circuit shown in the figure is



(a) 3 (b) 4
(c) 6 (d) 7 **[2003]**

Solution: (a)

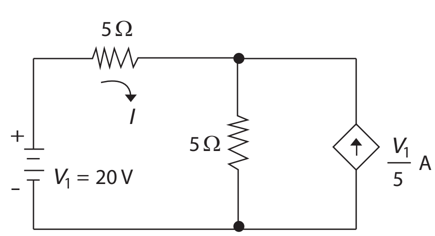


The node voltages are numbered as 1, 2, 3

Number of equation = Number of nodes = 3.

Hence, the correct option is (a).

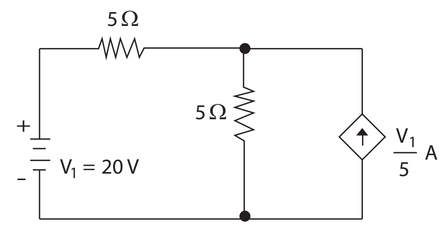
15. The dependent current source shown in the figure



(a) delivers 80 W (b) absorbs 80 W
(c) delivers 40 W (d) absorbs 40 W

[2002]

Solution: (a)



Applying KCL at node A,

$$\frac{V_1 - V_A}{5} + \frac{V_1}{5} = \frac{V_A}{5}$$

$\therefore V_1 = V_A$

Power delivered by current source (as current is leaving the positive terminal) = $V_A \times I$.

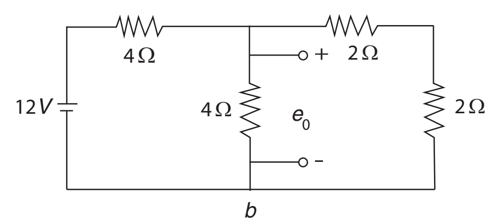
$$= V_1 \times \frac{V_1}{5}$$

$$= \frac{V_1^2}{5}$$

$$= \frac{\overset{4}{\cancel{20}} \times 20}{\cancel{5}} = 80\text{W}$$

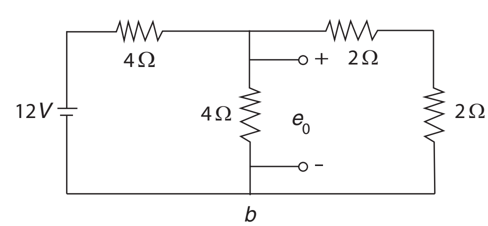
Hence, the correct option is (a).

16. The voltage e_0 in the figure is



(a) 2 V (b) $\frac{4}{3}$ V
(c) 4 V (d) 8 V **[2001]**

Solution: (c)



Equivalent resistance for the circuit = 4 + 4||4 = 4 + 2 = 6Ω.

Total current $= \dfrac{12}{6} = 2\text{A}.$

$\therefore\ e_0 = 2 \times \dfrac{4}{4+4} \times 4 = \dfrac{2 \times 16}{8} = 4\text{V}.$

Hence, the correct option is (c).

17. If each branch of a Delta circuit has impedance $\sqrt{3}$ Z, then each branch of the equivalent Wye circuit has impedance

(a) $\dfrac{Z}{\sqrt{3}}$ (b) 3 Z

(c) $3\sqrt{3}Z$ (d) $\dfrac{Z}{3}$ **[2001]**

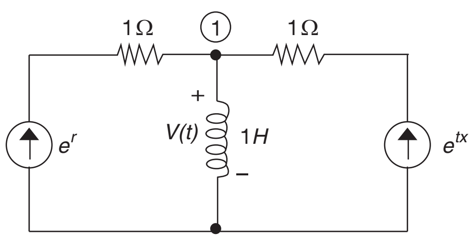
Solution: (a)

$Z_\Delta = 3Z_Y$

$\sqrt{3}Z_\Delta = 3Z_Y \Rightarrow Z_Y = \dfrac{Z_\Delta}{\sqrt{3}}$

Hence, the correct option is (a).

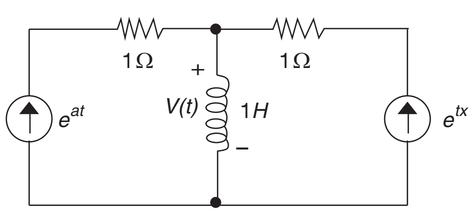
18. In the circuit of the figure, the voltage $V(t)$ is



(a) $e^{at} - e^{bt}$ (b) $e^{at} + e^{bt}$

(c) $ae^{at} - be^{bt}$ (d) $ae^{at} + be^{bt}$ **[2000]**

Solution: (d)



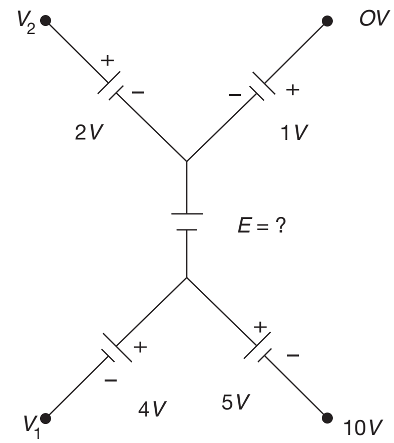
The voltage across inductor is given by differentiation of current across it. So, the voltage across 1H inductor is given by

$$V(t) = L\frac{d}{dt}(i_1 + i_2)$$

$$V(t) = \frac{di_1}{dt} + \frac{di_2}{dt} = ae^{at} + be^{bt}$$

Hence, the correct option is (d).

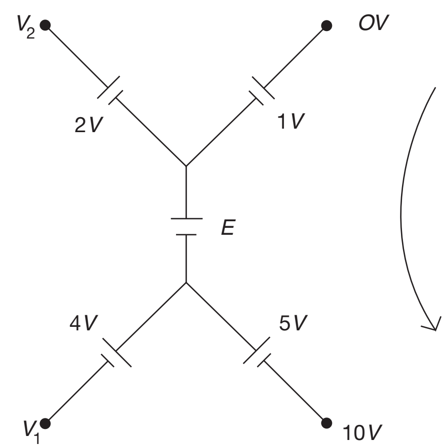
19. In the circuit of the figure, the value of the voltage source E is



(a) –16 V (b) 4 V

(c) –6 V (d) 16 V **[2000]**

Solution: (a)



Applying KVL in the given arrow direction and solving

$0 = 1 + E + 5 + 10,$

$E = -16$ V.

Hence, the correct option is (a).

20. The nodal method of circuit analysis is based on

(a) KVL and Ohm's law

(b) KCL and Ohm's law

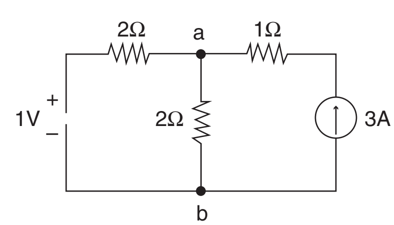
(c) KCL and KVL

(d) KCL, KVL and Ohm's law **[1998]**

Solution: (b)

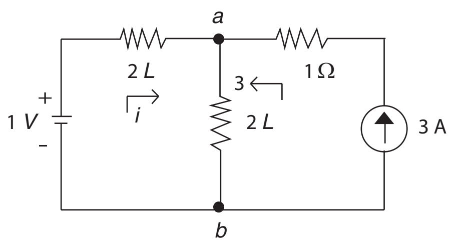
Nodal analysis is based on KCL and ohm's law.

21. The voltage across the terminals a and b in the figure is



(a) 0.5 V (b) 3.0 V
(c) 3.5 V (d) 4.0 V **[1998]**

Solution: (c)

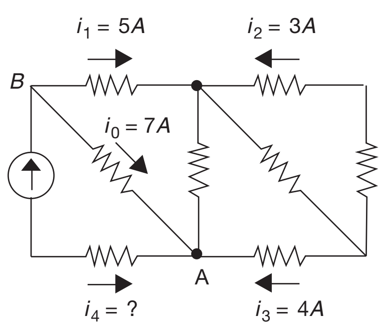


Applying KVL at first mess, $1 = 2i + 2(i + 3)$

$$1 = 2i + 2i + 6$$
$$-5 = 4i$$
$$i = \frac{-5}{4}$$
$$V_{ab} = 2i + 6 = 2 \times \frac{-5+6}{4} = -2.5 + 6$$
$$V_{ab} = 3.5 \text{ V.}$$

Hence, the correct option is (c).

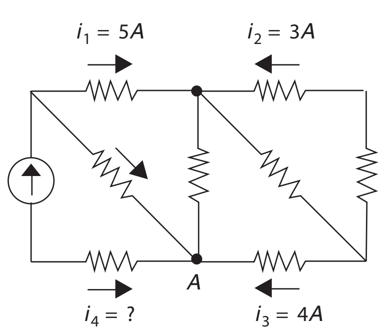
22. The current i_4 in the circuit of the figure is equal to____



(a) 12 A (b) −12 A
(c) 4 A (d) None of these

[1997]

Solution: (b)

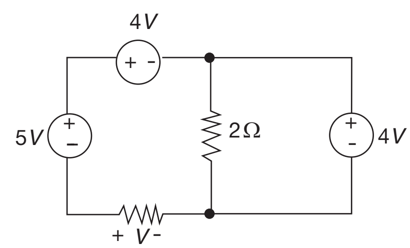


Appling KCL at node B, sum of all currents will be zero. So,

$$i_4 + i_1 + i_0 = 0$$
$$i_4 = -i_1 - i_0 = -5 - 7$$
$$i_4 = -12\text{A}$$

Hence, the correct option is (b).

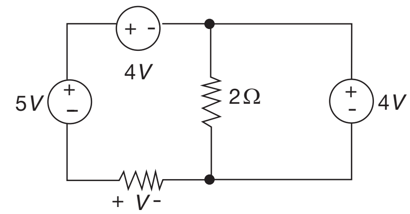
23. The voltage V in the figure is equal to____



(a) 3 V (b) –3 V
(c) 5 V (d) None of these

[1997]

Solution: (a)



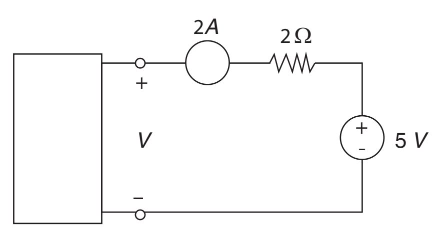
Applying KVL in outer mesh,

$5 = 4 + 4 - V$.

So, $V = 3V$.

Hence, the correct option is (a).

24. The voltage V in the figure is always equal to____



(a) 9 V (b) 5 V
(c) 1 V (d) None of these

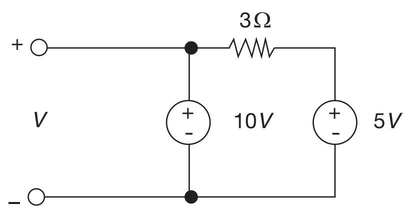
[1997]

Solution: (d)

Since the voltage across current source is unknown, we cannot apply KVL to find V.

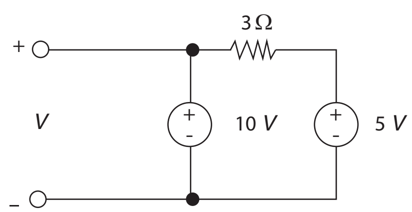
Hence, the correct option is (d).

25. The voltage V in the figure is **[1997]**



(a) 10 V (b) 15 V
(c) 5 V (d) None of the these

Solution: (a)

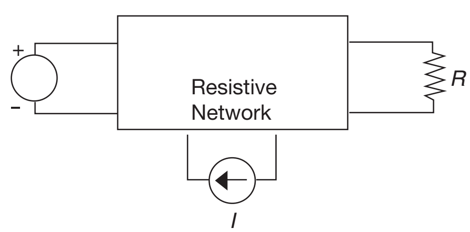


Voltage in parallel connection remains equal. So,

$V = 10$ V.

Hence, the correct option is (a).

26. A DC circuit shown in the figure given below has a voltage source V, a current source I and several resistors. A particular resistor R dissipates a power of 4 W when V alone is active. The same resistor R dissipates a power of 9 W when I alone is active. The power dissipated by R when both sources are active will be



(a) 1 W (b) 5 W
(c) 5 W (d) 25 W **[1995]**

Solution: (d)

Superposition theorem cannot be applied for nonlinear parameters (power).

When V_1 is applied, power $P_1 = 4$ W.

When I is applied, $P_2 = 9$W.

With both sources, $P = \left(\sqrt{P_1} + \sqrt{P_2}\right)^2$

$P = (2 + 3)^2 = 25$ W.

Hence, the correct option is (d).

27. Two 2H inductance coils are connected in series and are also magnetically coupled to each other, the coefficient of coupling being 0.1. The total inductance of the combination can be

(a) 0.4 H (b) 3.2 H
(c) 4.0 H (d) 4.4 H **[1995]**

Solution: (d)

Given inductance of first and second coils as $L_1 = 2$H, $L_2 = 2$H and coupling coefficient as $K = 0.1$, let mutual inductance between these coils be given by M. Then equivalent inductance is given as

$L_{eq} = L_1 + L_2 + 2M = 2 + 2 + 2K\sqrt{L_1 L_2}$

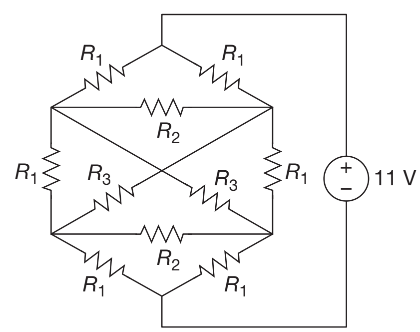
where $M = K\sqrt{L_1 L_2} = 4 + 0.1\sqrt{2 \times 2}$

$= 4.4$ H.

Hence, the correct option is (d).

Two-marks Questions

1. Consider the network shown below with $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$ and $R_2 = 3\ \Omega$.. The network is connected to a constant voltage source of 11 V.



The magnitude of current (in amperes, accurate to two decimal places) through the source is ________. **[2018]**

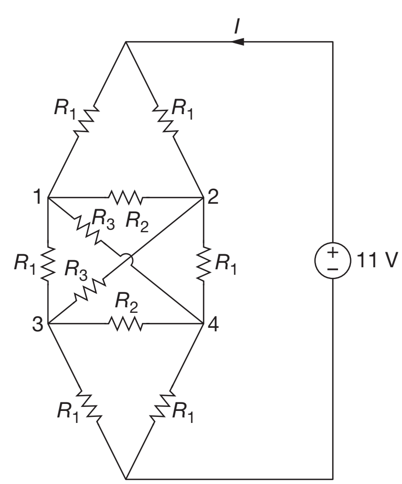
Solution:

Resistance $R_1 = 1\Omega$

Resistance $R_2 = 2\Omega$

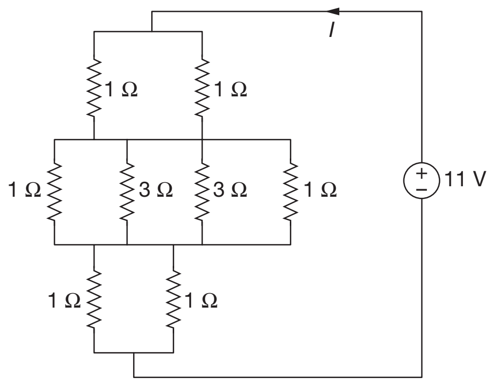
Resistance $R_3 = 3\Omega$

Current $I = ?$



We know that above network is symmetrical, so redrawing the given circuit.

$$V_1 = V_2 \text{ and } V_3 = V_4$$



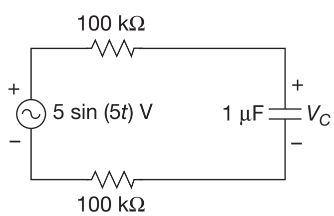
$$R_{eq} = \{(1\|1) + \{1.5\|0.5\} + 0.5\}$$

$$= 0.5 + 0.5 + 0.375 = \frac{11}{8}\Omega$$

$$I = \frac{11}{11/8} = 8 \text{ Amp}$$

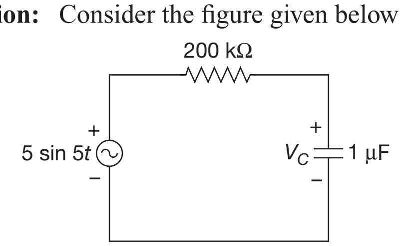
Hence, the correct answer is 7.9 to 8.1.

2. For the circuit given in the figure, the voltage V_c (in volts) across the capacitor is **[2017]**



(A) $1.25\sqrt{2}\ \sin(5t - 0.25\pi)$
(B) $1.25\sqrt{2}\ \sin 5t - 0.125\pi$
(C) $2.5\sqrt{2}\ \sin(5t - 0.25\pi)$
(D) $2.5\sqrt{2}\ \sin(5t - 0.125\pi)$

Solution: Consider the figure given below



$$V_c(t) = ?$$

We know that:

$$H(S) = \frac{V_c(S)}{V_{in}(S)}$$

$$H(S) = \frac{1}{1 + j\omega RC}$$

$$H(j\omega) = \frac{1}{1 + j5 \times 200 \times 10^3 \times 1 \times 10^{-6}}$$

$$= \frac{1}{1 + j1}$$

$$H = \frac{1}{\sqrt{2}} \angle -45°$$

$$V_c(t) = \frac{5}{\sqrt{2}} \sin(5t - 45°) \text{ V}$$

$$V_c(t) = 2.5\sqrt{2}\ \sin(5t - 45°) \text{ V}$$

Hence, the correct option is (C)

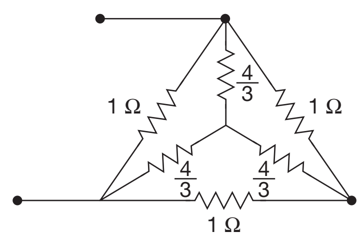
3. In the given circuit, each resistor has a value equal to 1 Ω.

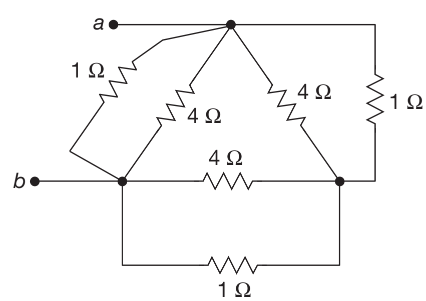
What is the equivalent resistance across the terminals a and b? **[2016]**

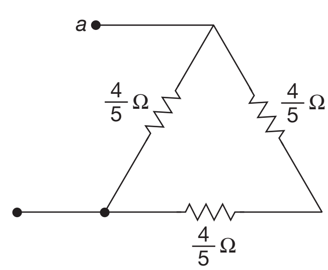
(A) $\frac{1}{6}\Omega$ (B) $\frac{1}{3}\Omega$

(C) $\frac{9}{20}\Omega$ (D) $\frac{8}{15}\Omega$

Solution: Apply $\Delta \leftrightarrow Y$ conversion to the given network







$$\therefore \qquad R_{ab} = \left(\frac{4}{5}\right) // \left(\frac{8}{5}\right)$$

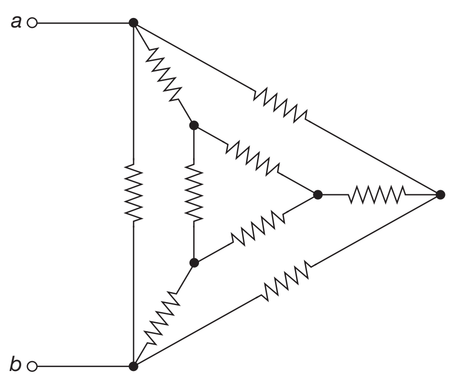
$$R_{ab} = \frac{\frac{4}{5} \times \frac{8}{5}}{\frac{4}{5} + \frac{8}{5}}$$

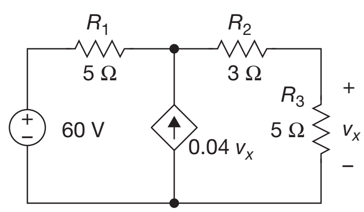
$$R_{ab} = \frac{8 \times 4}{25} \times \frac{5}{12}$$

$$= \frac{8}{15}\Omega$$

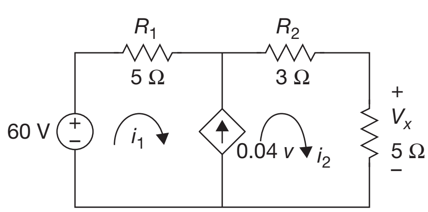
Hence, the correct option is (D).

4. In the circuit shown in the figure, the magnitude of the current (in amperes) through R_2 is ____. **[2016]**





Solution: Consider the labelled figure given below



Mesh equation

$$-60 + i_1(5\Omega) + i_2(3\Omega) + i_2(5\Omega) = 0$$

$$5i_1 + 8i_2 = 60$$

$$V_x = i_2(5\Omega) \qquad \text{(i)}$$

Now we have,

$$i_1 - i_2 = -0.04V_x$$

$$i_1 = \frac{v_x}{5} - 0.04V_x$$

$$i_1 = 0.16V_x$$

$$i_2 = 0.2V_x$$

$$5(0.16V_x) + 8(0.2V_x) = 60$$

$$V_x\,[0.8 + 1.6] = 60$$

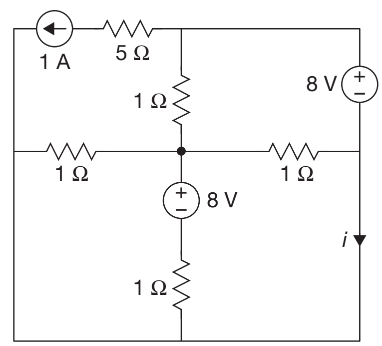
$$V_x = 25 \text{ volts}$$

$$i_2 = \frac{25}{5} = 5A$$

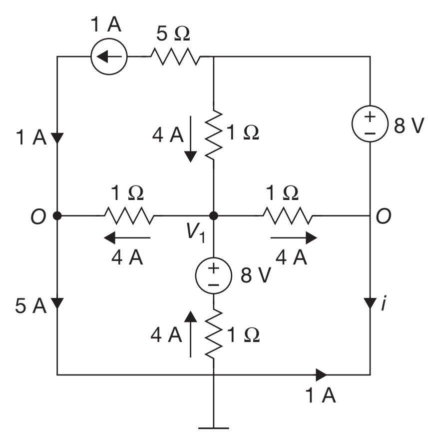
$$i_{R_2} = 5A$$

Hence, the correct Answer is (5 A).

5. In the figure shown, the current I (in ampere) is ___ . **[2016]**



Solution:



Apply nodal analysis at node V_1

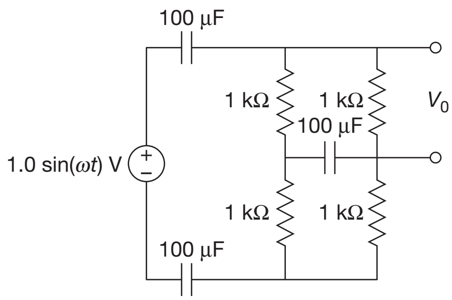
$$\frac{V_1 - 0}{1} + \frac{V_1 - 8}{1} + \frac{V_1 - 0}{1} + \frac{V - 8}{1} = 0$$

$$4V_1 = 16 \Rightarrow V_1 = 4 \text{ volts}$$

$\therefore \quad I = -1$ Amp

Hence, the correct Answer is (−1 amp).

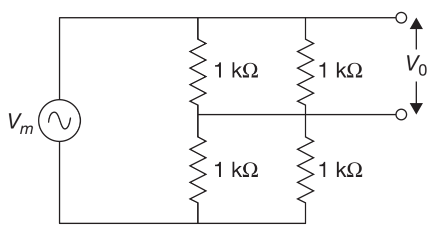
6. At very high frequencies, the peak output voltage V_0 (in Volts) is _______. **[2015]**



Solution: At high frequencies the given equivalent network.

Shown in below

$Z_c \to$ short circuit

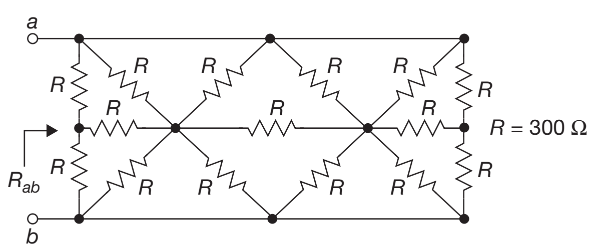


$$V_o = \frac{V_m \times 1k}{2k} = \frac{V_m}{2}$$

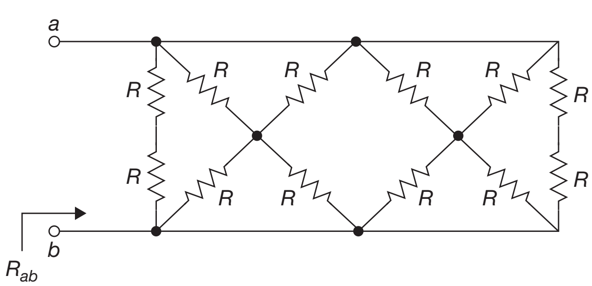
$\therefore$ The peak o/p voltage $= \frac{1}{2} = 0.5$ volts

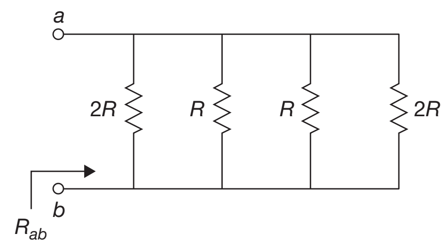
Hence, the correct Answer is (0.49 to 0.51).

7. In the network shown in the figure, all resistor are identical with R = 300 Ω. The resistance R_{ab} (in Ω) of the network is ______. **[2015]**



Solution: The given network is in balanced mode, so redraw equivalent network.





$\therefore \quad R_{ab} = (2R||2R)||(R||R) = (R||R||R)$

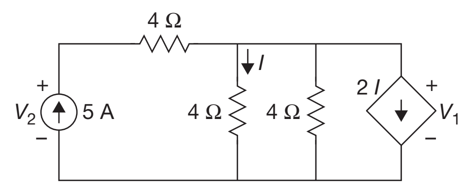
$= R/3$

But give R = 300 Ω

$\therefore \quad R_{ab} = 100\ \Omega$

Hence, the correct Answer is (99.5 to 100.5).

8. In the given circuit, the values of V_1 and V_2 respectively are **[2015]**



(A) 5 V, 25 V (B) 10 V, 30 V
(C) 15 V, 35 V (D) 0 V, 20 V

Solution: Apply nodal analysis to the given network.

$$-5 + \frac{V_1}{4} + \frac{V_1}{4} + 2I = 0$$

$$\frac{V_1}{2} + 2I = 5$$

But, $$I = \frac{V_1}{4}$$

$$\frac{V_1}{2} + \frac{V_1}{2} = 5$$

$\Rightarrow \quad V_1 = 5$ volts

Apply KVL in 1st loop

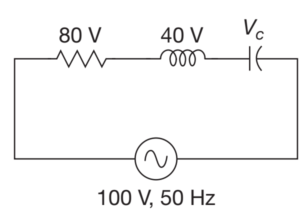
$$V_2 - 4 \times 5 - V_1 = 0$$

$$V_2 = 20 + 5$$

$$= 25 \text{ Volts}$$

Hence, the correct option is (A).

9. The voltage (V_C) across the capacitor (in Volts) in the network shown is ______. **[2015]**



Solution: We know

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} \quad \text{(or)} \quad \sqrt{V_R^2 + (V_C - V_L)^2}$$

$$100 = \sqrt{(80)^2 + (V_L - V_C)^2}$$

$$(100)^2 = 80^2 + (V_L - V_C)^2$$

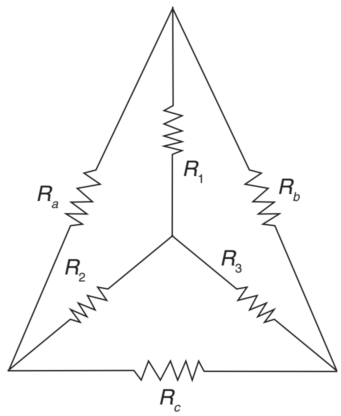
$$(V_C - V_L)^2 = 3600$$

$$V_C - V_L = 60$$

$$V_C = 100 \text{ V}$$

Hence, the correct Answer is (100).

10. A Y-network has resistances of 10 Ω each in two of its arms, while the third arm has a resistance of 11 Ω. In the equivalent A-network, the lowest value (in Ω) among the three resistances is______. **[2014]**



$$R_a = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_b = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

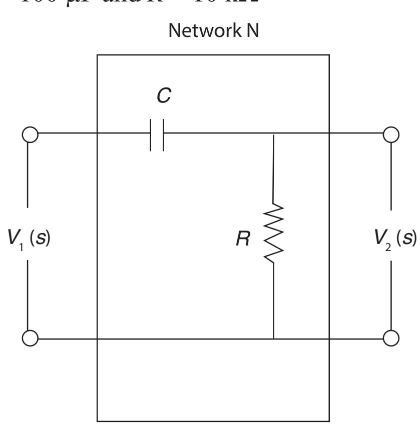
$$R_c = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

Here $R_1 = 11\Omega$, $R_2 = 10\Omega$, $R_3 = 10\Omega$

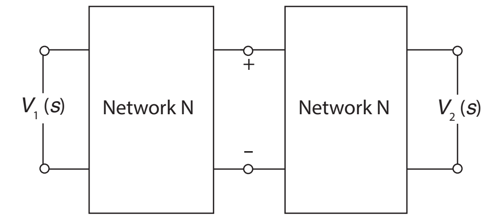
$\therefore R_a = R_b = 32\ \Omega$, $R_c = 29.09\ \Omega \rightarrow$ lowest value

11. Consider the building block called 'Network N' shown in the figure.

Let $C = 100\ \mu F$ and $R = 10\ k\Omega$



Two such blocks are connected in cascade, as shown in the figure.



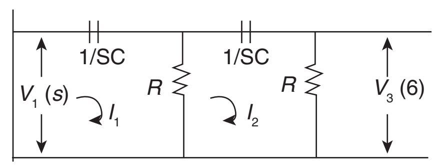
The transfer function $\frac{V_3(s)}{V_1(s)}$ of the cascaded network is

(a) $\frac{s}{1+s}$ (b) $\frac{s^2}{1+3s+s^2}$

(c) $\left(\frac{s}{1+s}\right)^2$ (d) $\left(\frac{s}{2+s}\right)^2$

[2014]

Solution: (b)



Applying KVL, $\left(R + \frac{1}{SC}\right) I_1(s) - RI_2(s) = V_1(s)$

$$\left(R + \frac{1}{SC}\right) I_2(s) - RI_2(s) = 0$$

$$V_3(s) = I_2(s)R$$

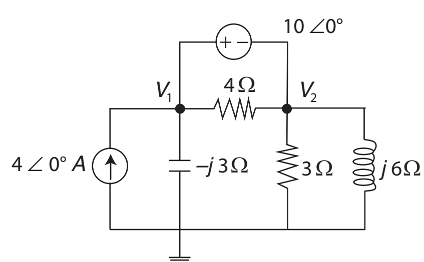
$$\therefore \frac{V_3(s)}{V_1(s)} = \frac{s^2R^2C^2}{s^2R^2C^2 + 1 + 3RCs}$$

$$R = 10\ k\Omega,\ C = 100\ \mu F,\ RC = 1$$

$$\frac{V_3(s)}{V_1(s)} = \frac{s^2}{s^2 + 3s + 1}$$

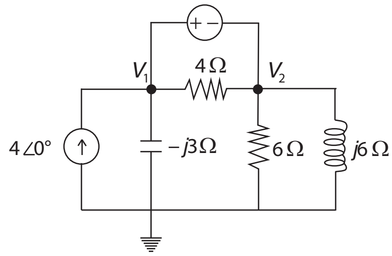
Hence, the correct option is (b).

12. In the circuit shown in the figure, the value of node voltage V_2 is



(a) $22 + j2$ V (b) $2 + j22$ V
(c) $22 - j2$ V (d) $2 - j22$ V **[2014]**

Solution: (d)



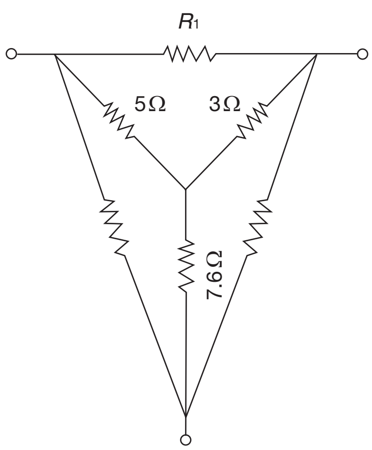
$$V_1 - V_2 = 10\angle 0°;\ \frac{V_1}{-3j} + \frac{V_2}{6j} + \frac{V_2}{6} = 4\angle 0°$$

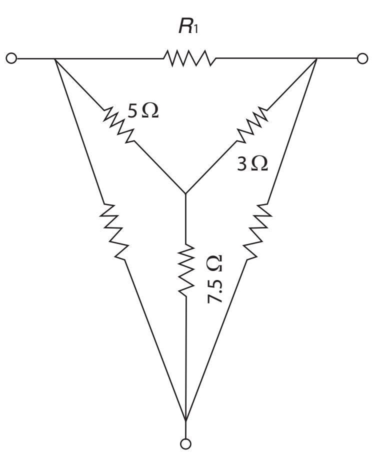
$$\therefore V_2 = \frac{20 + j24}{(-1 + j)} = 22.09\angle -84.8$$

$$V_2 = 2 - 22j$$

Hence, the correct option is (d).

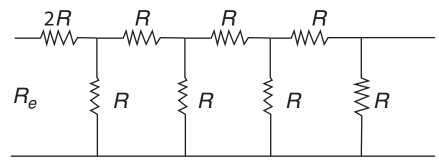
13. For the Y-network shown in the figure, the value of R_1 (in Ω) in the equivalent Δ-network is_____. **[2014]**





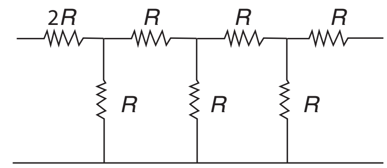
$$R_1 + \frac{5 + 3 + 5 \times 3}{7.5} = 10\Omega$$

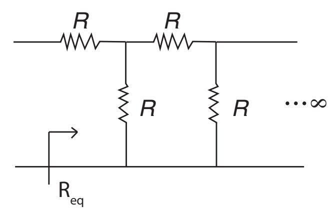
14. The equivalent resistance in the infinite ladder network shown in the figure is R_e

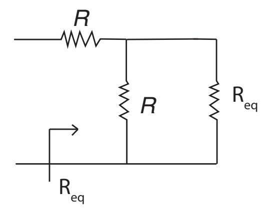


The value R_e/R is_______. **[2014]**

Solution:



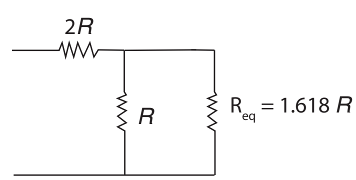




$$R_{eq} = \frac{R + R.R_{eq}}{R + R_{eq}}$$

$$R_{eq} = \left(1 + \frac{\sqrt{5}}{2}\right)R$$

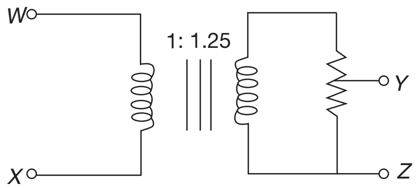
$$R_{eq} = 1.618\ R$$



$$R_{in} = 2R + \frac{R \times 1.618\ \text{R}}{R + 1.618\ \text{R}}$$

$$R_{in} = 2.618\ \text{R} \rightarrow \frac{R_{in}}{R} = 2.618$$

15. The following arrangement consists of an ideal transformer and an attenuator which attenuates by a factor of 0.8. An ac voltage $V_{WX1} = 100$ V is applied across WX to get an open circuit voltage V_{YZ1} across YZ. Next, an ac voltage V_{YZ2} = 100 V is applied across YZ to get an open circuit voltage V_{WX2} across WX. Then, V_{YZ1}/V_{WX1}, V_{WX1}/V_{YZ2} are, respectively,



(a) 125/100 and 80/100
(b) 100/100 and 80/100
(c) 100/100 and 100/100
(d) 80/100 and 80/100 **[2013]**

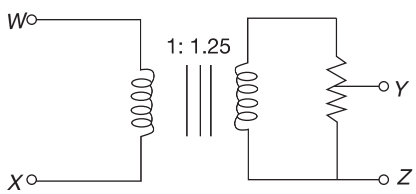
Solution: (b)

$$V_{YZ_1} = 100 \times 1.25 \times 0.8 = 100 \text{ V}$$

When 100 V is applied at YZ, this will appear across secondary winding.

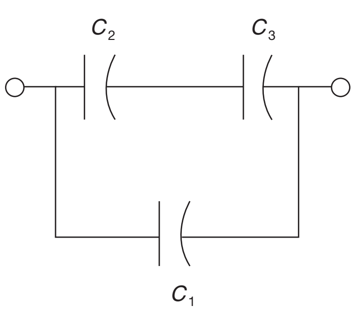
V_{wx2} = 100/1.25 = 80V

$$\frac{V_{YZ_1}}{V_{WX_1}} = \frac{100}{100}, \frac{V_{WX_2}}{V_{YZ_2}} = \frac{80}{100}$$



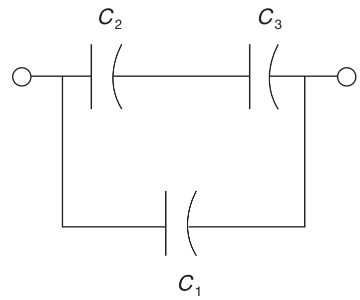
Hence, the correct option is (b).

16. Three capacitors C_1, C_2 and C_3 whose values are 10 μF, 5 μF, and 2 μF, respectively, have breakdown voltages of 10 V, 5 V and 2 V, respectively. For the interconnection shown below, the maximum safe voltage in volts that can be applied across the combination, and the corresponding total charge in μC stored in the effective capacitance across the terminals, are, respectively



(a) 2.8 and 36 (b) 7 and 119
(c) 2.8 and 32 (d) 7 and 80
[2013]

Solution: (c)



$Q_1 = C_1V_1 = 100\ \mu C$
$Q_2 = C_2V_2 = 25\ \mu C$
$Q_3 = C_3V_3 = 4\ \mu C$

In series, charge remains same so, the charge on C_2 and C_3 must be 4 μC.

$$C_{23} = \frac{C_2C_3}{C_2 + C_3} = \frac{10}{7}\mu F$$

$$V_{23} = \frac{\theta}{C_{23}} = \frac{4\mu C}{\frac{10}{7}\mu C} = \frac{28}{10} = 2.8 \text{ V}$$

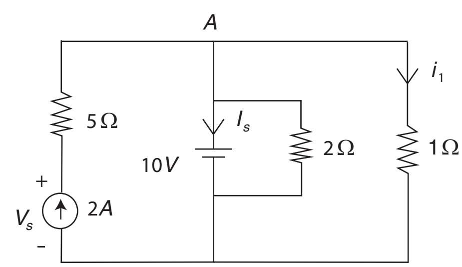
In parallel, voltage will remain the same

$V_1 = V_{23} = 2.8\ V$
$Q_1 = C_1V_1 = 28\ \mu C$
Total charge = $Q_1 + Q_{23}$ = 32 μC

Hence, the correct option is (c).

Common Data For Questions 8 and 9

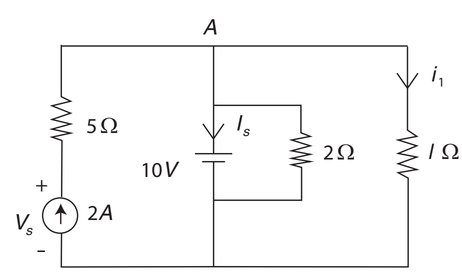
Consider the following figure:



17. The current I_s in Amps in the voltage source, and voltage V_s in volts across the current source respectively, are

(a) 13, –20 (b) 8, –10
(c) –8, 20 (d) –13, 20 **[2013]**

Solution: (d)



In parallel, voltage remains same, so the voltage across 1Ω remains 10 V.

$$i_1 = \frac{10}{1} = 10\text{ A}$$

Voltage across 2Ω = 10 V

$$\therefore i_2 = \frac{10}{2} = 5\text{ A}$$

Applying KCL at A, $-2 + I_f + i_2 + i_1 = 0$

$I_S = -13$ A

$V_A = 10\text{ V} = V_S = V_A + 10 = 20\text{V}$

Hence, the correct option is (d).

18. The current in the 1 Ω resistor in Amps is

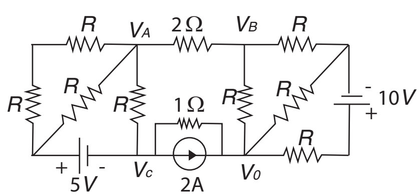
(a) 2 (b) 3.33
(c) 10 (d) 12 **[2013]**

Solution: (c)

Current in 1 Ω resistor is $= I_1 = \frac{10}{1} = 10$ Amp

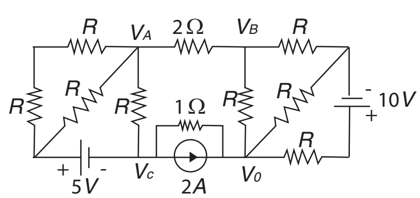
Hence, the correct option is (c).

19. If $V_A - V_B = 6\ V$, then $V_c - V_D$ is

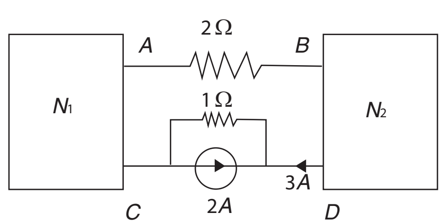


(a) –5 V (b) 2 V
(c) 3 V (d) 6 V **[2012]**

Solution: (a)



Since $V_A - V_B$ is known, we can replace the left and right half as shown.

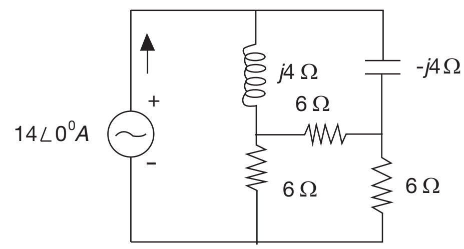


Total current = $5A$ through 1Ω in anti-clock wise direction

$\therefore V_{CD} = -5 \times 1 = -5\text{V}$

Hence, the correct option is (a).

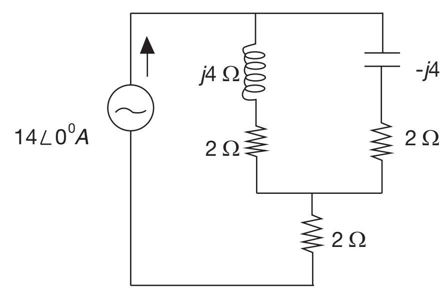
20. In the circuit shown below, the current I is equal to



(a) 1.4 ∠0°A (b) 2.0 ∠0°A
(c) 2.8 ∠0°A (d) 3.2 ∠0°A **[2011]**

Solution: (b)

Applying delta to star conversion,



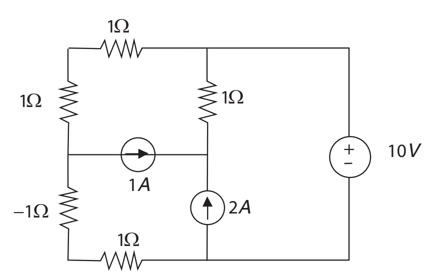
$$Z_{eq} = (2 + 4j) \parallel (2 - 4j) + 2$$

$$Z_{eq} = 7\Omega$$

$$I = \frac{V}{Z_{eq}} = \frac{14\angle 0^0}{7} = 2\angle 0°\text{A}$$

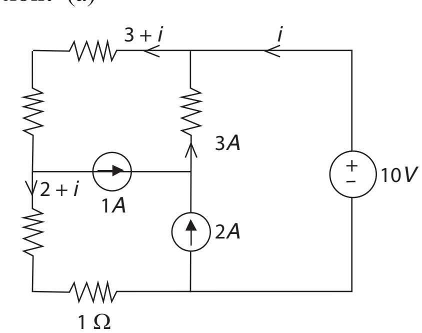
Hence, the correct option is (b).

11. In the circuit shown, the power supplied by the voltage source is



(a) 0 W (b) 5 W
(c) 10 W (d) 100 W **[2010]**

Solution: (a)



Applying KVL in outer loop,

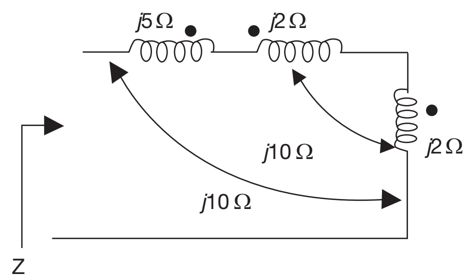
$(3 + i) \times + (2 + i) \times 2 = 10$

$i = 0$

$\therefore$ Power supplied by voltage source $= VI = 0$

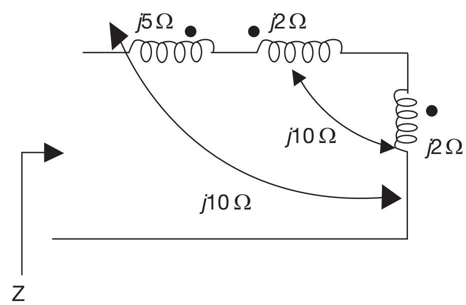
Hence, the correct option is (a).

22. Impedance Z as shown in the given figure is



(a) $j29\ \Omega$ (b) $j9\ \Omega$
(c) $j19\ \Omega$ (d) $j39\ \Omega$ **[2005]**

Solution: (b)



Impedance of the circuit will be given by

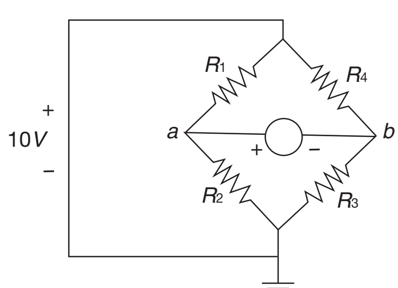
$Z = X_{L_1} + X_{L_2} + X_{L_3} + 2M_{23} - 2M_{13}$

$Z = j5 + j2 + j2 + 2 \times 10j - 2 \times 10j$

$Z = 9\,j\,\Omega$

Hence, the correct option is (b).

23. If $R_1 = R_2 = R_4 = R$ and $R_3 = 1.1\ R$ in the bridge circuit shown in the figure, then the reading in the ideal voltmeter connected between a and b is

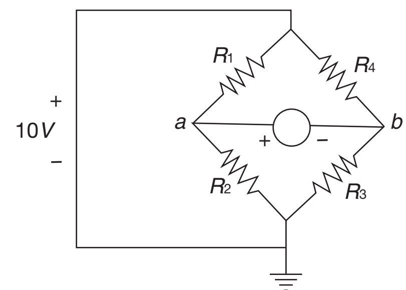


(a) $0.238\ V$ (b) $0.138\ V$
(c) $-0.238\ V$ (d) $1\ V$ **[2005]**

Solution: (c)

For given circuit given parameters are

$R_1 = R_2 = R_4 = R$ and $R_3 = 1.1R$



$$V_a = \frac{R_2}{R_1 + R_2} \times 10 = 5\,V$$

$$V_b = \frac{R_5}{R_3 + R_5} \times 10 = \frac{1.1}{2.1} \times 10 = 5.238\,V$$

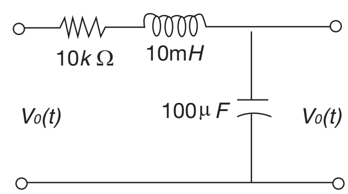
So, voltage across terminal ab is

$V_{ab} = V_a - V_b = -0.238$ V.

Hence, the correct option is (c).

24. For the circuit shown in the figure, the initial conditions are zero. Its transfer function

$$H(s) = \frac{V_c(s)}{V_i(s)} \text{ is}$$



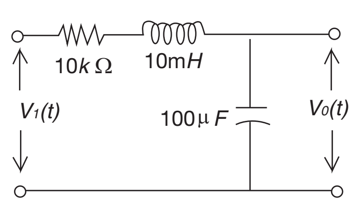
(a) $\dfrac{1}{s^2 + 10^6 s + 10^6}$ (b) $\dfrac{10^6}{s^2 + 10^3 s + 10^6}$

(c) $\dfrac{10^3}{s^2 + 10^3 s + 10^6}$ (d) $\dfrac{10^6}{s^2 + 10^6 s + 10^6}$

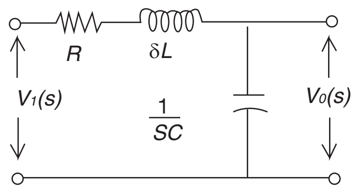
[2004]

Solution: (d)

Given circuit



Writing in s-admin,



Appling KVL in given circuit

$$V_i(s) = I(s)\left\langle R + sL + \frac{1}{sc}\right\rangle$$

$$V_i(s) = \frac{I(s)}{sc}(s^2LC + sCR + 1)$$

$$V_0(s) = \frac{I(s)}{SC}$$

$$\therefore \frac{V_0(s)}{V_i(s)} = \frac{1}{s^2LC + sCR + 1} = \frac{\frac{1}{LC}}{s^2 + \frac{L}{R}s + \frac{1}{LC}}$$

$$\frac{V_0(s)}{V_i(s)} = \frac{10^6}{s^2 + 10^6 s + 10^6}$$

Hence, the correct option is (d).

25. The transfer function $H(s) = \frac{V_0(s)}{V_i(s)}$ of an R–L–C circuit is given by $H(s) = \frac{10^6}{s^2 + 20s + 10^6}$.

The Quality factor (Q-factor) of this circuit is

(a) 25 (b) 50
(c) 100 (d) 5000 **[2004]**

Solution: (b)

Transfer function is given as,

$$H(s) = \frac{10^6}{s^2 + 20s + 10^6}$$

Comparing parameters with standard equation

$$\frac{W_0^2}{s^2 + \frac{f_0}{Q}s + w_0^2}$$

$$\frac{f_0}{Q} = 20 \Rightarrow Q = \frac{f_0}{20}$$

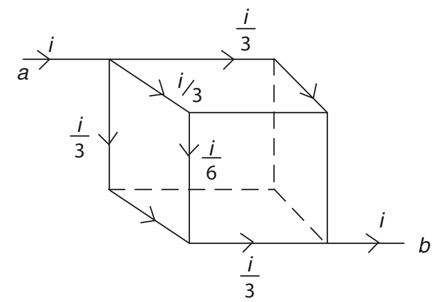
So $f_0 = 10^3 \Rightarrow Q = \frac{10^3}{20} = 50$

Hence, the correct option is (b).

26. Twelve 1 Ω resistances are used as edges to form a cube. The resistance between two diagonally opposite corners of the cube is

(a) $\frac{5}{6}\,\Omega$ (b) 1 Ω
(c) $\frac{6}{5}\,\Omega$ (d) $\frac{3}{2}\,\Omega$ **[2003]**

Solution:(a)



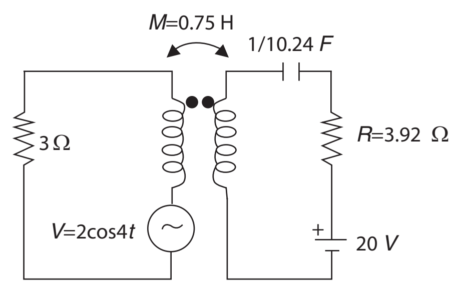
Equivalent resistant can be calculated as ratio of voltage to current across terminal

$$V_{ab} = \frac{i}{3}\times 1 + \frac{i}{6}\times 1 + \frac{i}{3}\times 1$$

$$R_{eq} = \frac{V_{ab}}{i} = \frac{5}{6}\,\Omega$$

Hence, the correct option is (a).

27. The current flowing through the resistance R in the circuit in the figure has the form P cos 4t, where P is



(a) $(0.18 + j\,0.72)$
(b) $(0.46 + j\,1.90)$
(c) $-(0\ 18 + j\,1.90)$
(d) $(0.23 - 0.35\,j)$ cos 4t **[2003]**

Solution: (d)

Here the value of inductance is not given we can ignore it

$$I_1 = \frac{2\cos 4t}{3} = 0.67\cos 4t \text{ i.e.} \omega = 4$$

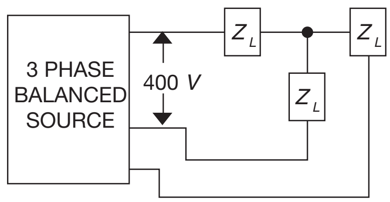
$$I_2 = \frac{-j\omega M}{R - [j/\omega C]} = \frac{-j4\times 0.75 I_1}{3.92 - 2.56j}$$

$$= \frac{-j3\times 0.67 Cos4t}{3.92 - 2.56j}\times\frac{3.92 + 2.56j}{3.92 + 2.5j}$$

$$= (0.23 - 0.35j) Cos4t$$

Hence, the correct option is (d).

28. If the three-phase balanced source in the figure delivers 1500 W at a leading power factor 0.844. then the value of Z_L (in ohm) is approximately



(a) 90 ∠32.44° (b) 80 ∠32.44°
(C) 80 ∠–32.44° (d) 90∠–32.44°

[2002]

Solution: (d)

Power delivered = $3V_P I_P COS\theta$

$$3V_P I_P \cos\theta = 1500$$

$$3\left(\frac{V_L}{\sqrt{3}}\right)\left(\frac{V_L}{\sqrt{3}Z_L}\right)\cos\theta = 1500$$

$$Z_L = \frac{V_L^2 \cos\theta}{1500} = \frac{400^2 \times 0.844}{1500}$$

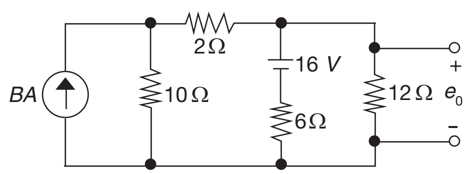
$Z_L = 90\Omega$

$\theta = \cos^{-1}(0.84) = 32.44$

As power factor is leading, it implies load is capacitive $= \theta = -32.44^o$

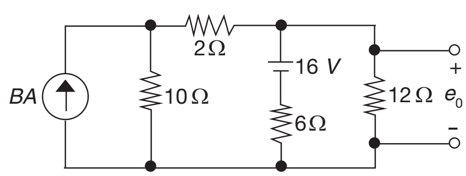
Hence, the correct option is (d).

29. The voltage e_0 in the figure is

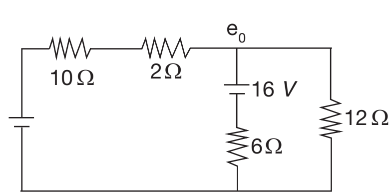


(a) 48 V (b) 24 V
(c) 36 V (d) 28 V **[2001]**

Solution: (d)



Converting current source of 8A to equivalent voltage source,



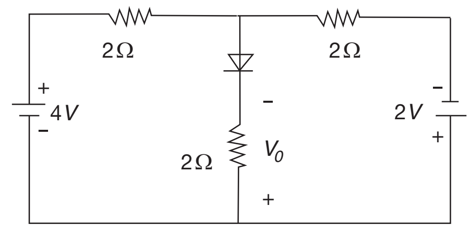
Appling KCL to calculate e_0

$$\frac{e_0 - 80}{12} + \frac{e_0}{12} + \frac{e_0 - 16}{6} = 0$$

$e_0 = 28\ V$

Hence, the correct option is (d).

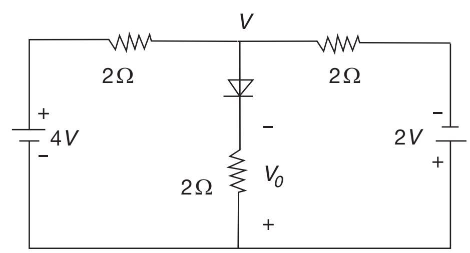
30. For the circuit in the figure, the voltage V_0 is



(a) 2 V (b) 1 V
(c) −1 V (d) None of these

[2000]

Solution: (d)



Since diode is Forward Bias, so it will be replaced by short circuit wire, and now applying KCL for voltage V.

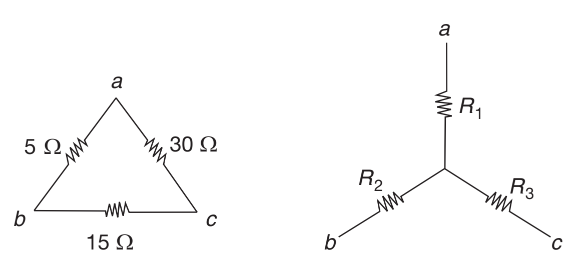
$$\frac{V-4}{2} + \frac{V}{2} + \frac{V+2}{2} = 0$$

$$V = \frac{2}{3}$$

$$V_0 = V = \frac{-2}{3}$$

Hence, the correct option is (d).

31. ADelta-connected network with its Wye-equivalent is shown in the figure. The resistances R_1, R_2 and R_3 (in ohms) are respectively



(a) 1.5, 3 and 9 (b) 3.9 and 1 5
(c) 9, 3 and 1.5 (d) 3, 1.5 and 9

[1999]

Solution: (d)

For delta to star conversion, the equivalent resistant in star network is given by

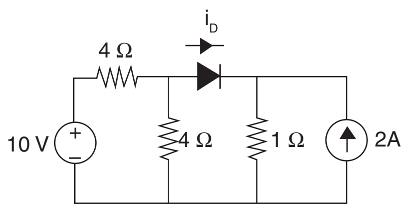
$$R_1 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{150}{20+30} = 3\Omega$$

$$R_2 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{-75}{50} = 1.5\Omega$$

$$R_3 = \frac{R_a R_c}{R_a + R_b + R_c} = \frac{\overset{3}{\cancel{15}} \times 3\cancel{0}}{\underset{1}{\cancel{5}}\,\cancel{0}} = 9\Omega$$

Hence, the correct option is (d).

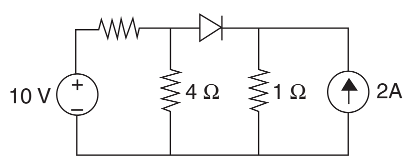
32. In the circuit shown in the figure the current i_D through the ideal diode (zero cut in voltage and zero forward resistance) equals

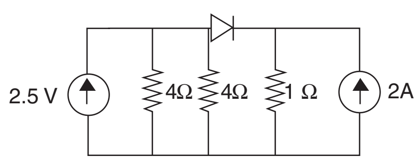


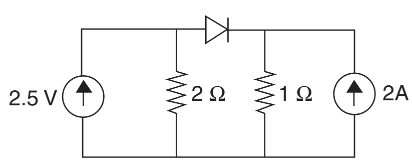
(a) 0 A (b) 4 A
(c) 1 A (d) None of these

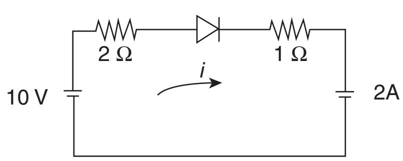
[1997]

Solution: (c)









From the figure,

$$i = \frac{5-2}{3} = 1\text{A}$$

Hence, the correct option is (c).

33. A network contains linear resistors and ideal voltage sources. If values of all the resistors are doubled, then the voltage across each resistor is

(a) halved (b) doubled
(c) increased by four times (d) not changed

t

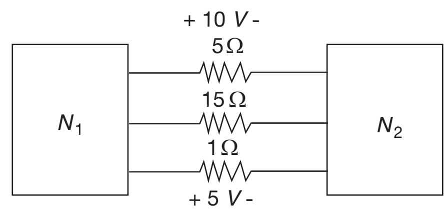
[1993]

Solution: (d)

When the value of all resistors is doubled then the current in corresponding resistors is halved, hence voltage remains unchanged.

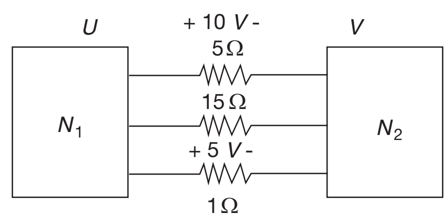
Hence, the correct option is (d).

34. The two electrical sub networks N_1 and N_2 are connected through three resistors as shown in the figure. The voltages across 5 ohm resistor and 1 ohm resistor are given to be 10 V and 5 V, respectively. Then voltage across 15 ohm resistor is



(a) −105 V (b) + 105 V
(c) −15 V (d) + 15V **[1993]**

Solution: (a)



When two networks are connected then net incoming current is equal to net outgoing current

Current through 5Ω, $i_5 = \frac{10}{5} = 2\text{A}$

Current through 1Ω, $i_1 = \frac{5}{1} = 5\text{A}$

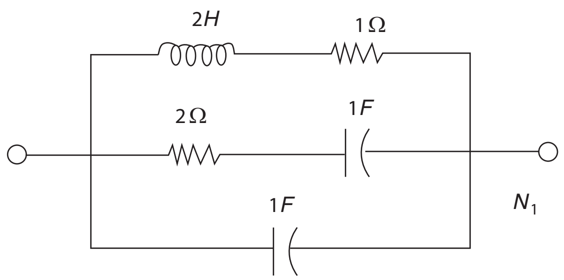
So, current through 15Ω resistor,

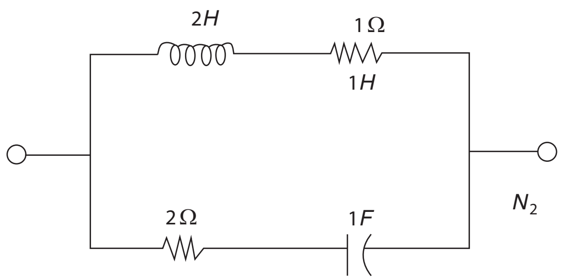
$$i_{15} = -(i_1 + i_5) = -7\text{A}$$

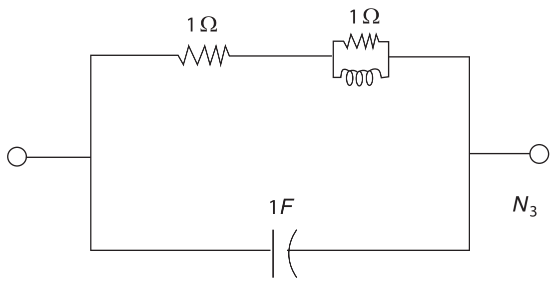
$$\therefore V_{15} = 15(-7) = -105 \text{ V}$$

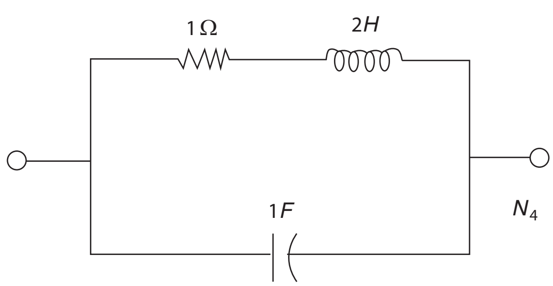
Hence, the correct option is (a).

35. Of the networks N_1, N_2, N_3 and N_4 of figure, the networks having identical driving point function are







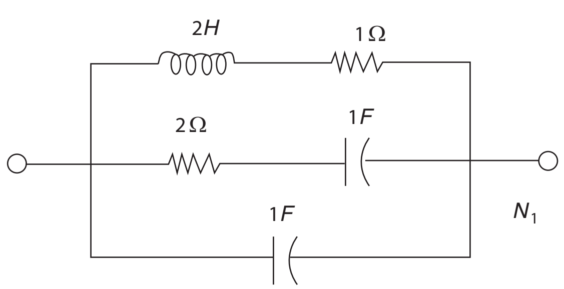


(a) N_1 and N_2 (b) N_2 and N_4
(c) N_1 and N_3 (d) N_1 and N_4

[1992]

Solution: (c)

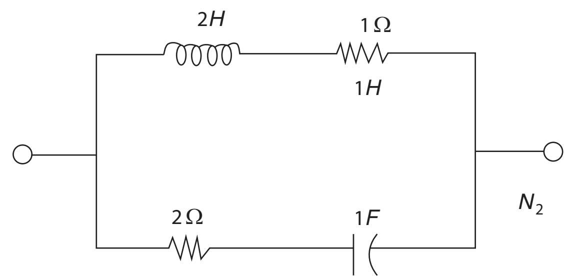
To calculate identical driving point function we need to calculate driving point admittance for each network (for parallel branches admittance can be easily calculated)



For network 1

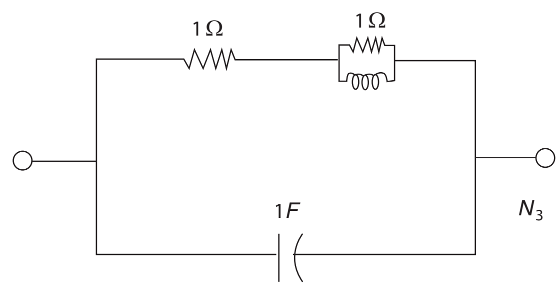
$$y_1(b) = s + \frac{1}{2s+1} + \frac{1}{\frac{1}{s}+2}$$

$$y_1(b) = \frac{2s^2+2s+1}{2s+1}$$



For network 2

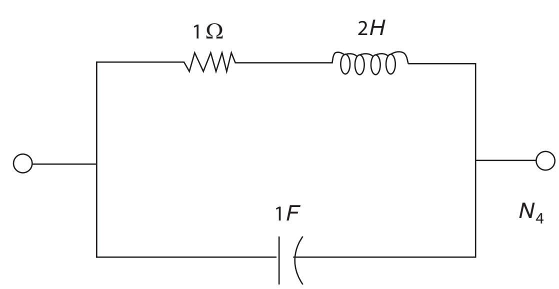
$$y_2(b) = \frac{1}{2s+1} + \frac{1}{2+\frac{1}{s}} = \frac{1+s}{2s+1}$$



For network 3

$$y_3(b) = s + \frac{1}{1+\frac{1}{1+\frac{1}{s}}} = s + \frac{1+s}{s+1+s}$$

$$y_3(b) = \frac{2s^2+2s+1}{2s+1}$$

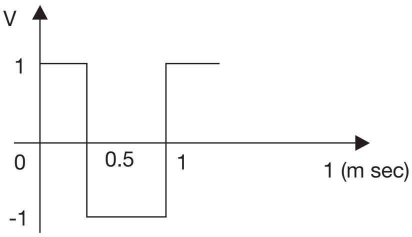


For network 4

$$y_4(b) = s + \frac{1}{2s+1} = \frac{2s^2+s+1}{2s+1}$$

Hence, the correct option is (c).

36. A square waveform as shown in figure is applied across 1 mH ideal inductor. The current through the inductor is a.........wave of..........peak amplitude. **[1987]**



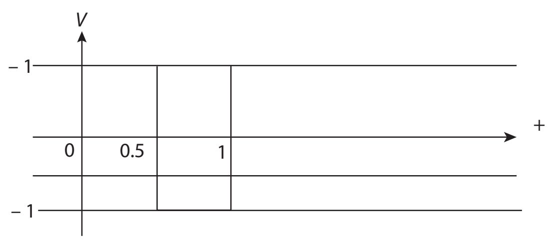
Solution: triangular and 2

Given $L = 1$ mH

Voltage across inductor is given as, $V = L\dfrac{di}{dt}$

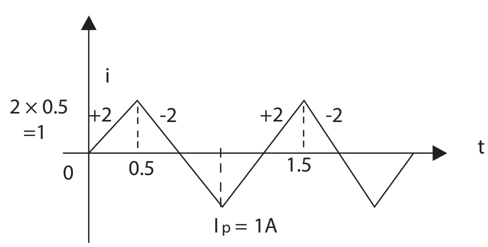
$$\frac{di}{dt} = \frac{V}{L}$$

$$i = \frac{1}{L}\int_{-\infty}^{t} Vdt = \frac{1}{L}\int_{0}^{t} Vdt$$



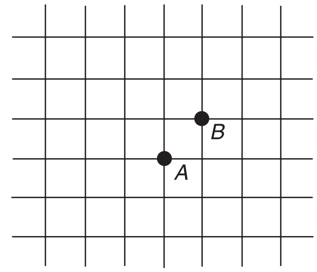
The current through inductor is integration applied voltage across the inductor. As integration of step voltage is triangular wave, so current through inductor is a triangular wave and slope of triangular wave = step change in square wave

$$\text{slope} = \frac{1-0}{0.5-0} = 2$$

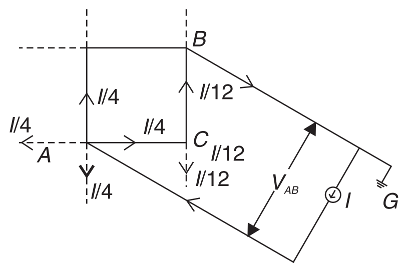


FIVE-MARKS QUESTIONS

1. An infinite grid is built up by connecting in the manner indicated in figure, where each branch represents one ohm resistor. Calculate the effective resistance between the nodes A and B.



Solution:



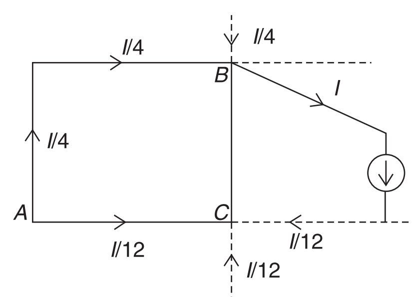
$$R_{AB} = \frac{V_{AB}}{I}$$

Connect current source IA between A and G. I enters terminal A, goes through the infinite grid and comes back to through ground.

$$V_{1AB} = V_{AC} + V_{CB} = \left(\frac{I}{4}\times 1\right) + \left(\frac{I}{12}\times 1\right) = \frac{I}{3}$$

Now,

Connect a current source, I between B and G. I enters the infinite grid through G and leaves B.



$$V_{2AB} = V_{AC} + V_{CB} = \left(\frac{I}{12}\times 1\right) + \left(\frac{I}{4}\times 1\right) = \frac{I}{3}$$

$$\therefore V_{AB} = V_{1AB} + V_{2AB} = \frac{I}{3} + \frac{I}{3} = \frac{2I}{3}$$

$$\therefore R_{AB} = \frac{V_{AB}}{I} = \frac{2}{3}\,\Omega$$

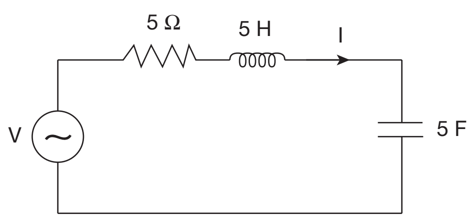
Chapter 2

Sinusoidal Steady State

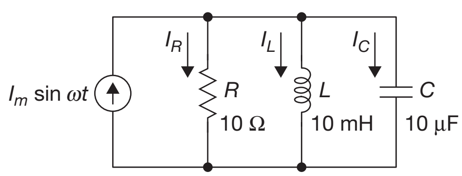
One-mark Questions

1. In the circuit shown, V is a sinusoidal voltage source. The current I is in phase with voltage V. The ratio

$\dfrac{\text{amplitude of voltage across the capacitor}}{\text{amplitude of voltage across the resistor}}$ is______. **[2017]**



2. The figure shows an RLC circuit with a sinusoidal current source.



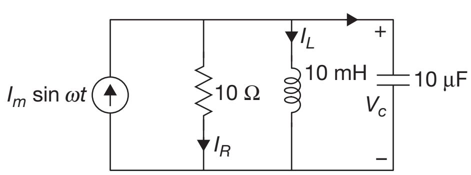
At resonance, the ratio $|I_L|/|I_R|$ i.e., the ratio of the magnitudes of the inductor current phasor and the resistor current phasor, is ______. **[2016]**

Solution: Resistance $R = 10$ ohms

Inductance $L = 10$ mH

Capacitance $C = 10$ microfarad (μF)

We know that voltage across all the elements remain constant



Using the resonance condition when R, L, C are connected in parallel

$\omega = \dfrac{1}{\sqrt{(RLC)}}$, we get the value from which the ratio of I_L/I_R can be evaluated.

Current across the inductor is

$$I_L = \frac{V_c}{j\omega 10\times 10^{-3}} \Rightarrow |I_L| = \frac{100V_c}{\omega}$$

Current across resistance will be

$$I_R = \tfrac{V_c}{10} \Rightarrow |I_R| = \frac{V_c}{10}$$

At resonance,

$$\omega = \frac{1}{\sqrt{10\times 10\times 10^{-9}}} = \frac{1}{\sqrt{10^{-7}}} = \sqrt{10^7}$$

The required ratio of current across inductor and resistance is

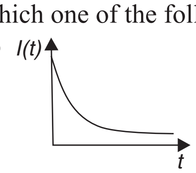
$$\frac{|I_L|}{|I_R|} = \frac{100V/\sqrt{10^7}}{V/10} = \frac{1000}{\sqrt{10^7}} = \frac{1}{\sqrt{10}} = 0.316$$

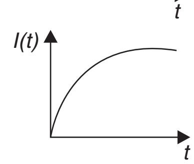
Hence, the correct Answer is (0.316).

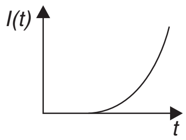
3. A series RC circuit is connected to a DC voltage source at time $t = 0$. The relation between the source voltage V_s, the resistance R, the capacitance C, and the current $i(t)$ is given below:

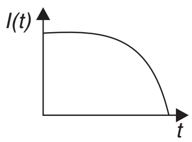
$$V_s = Ri(t) + \frac{1}{C}\int_0^t i(t)\,dt$$

Which one of the following represents the current $i(t)$?

(a) 

(b) 

(c) 

(d) 

[2014]

Solution: (a)

$$V(s) = RI(s) + \frac{I(s)}{SC}$$

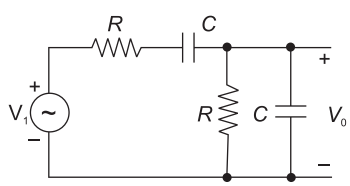
$$I(s) = \frac{V(s)}{R + \frac{1}{SC}}; \text{V(s)} = 1/5$$

$$I(s) = \frac{1}{R\left(s + \frac{1}{RC}\right)}$$

$$i(t) = \frac{1}{R}e^{-t/RC}$$

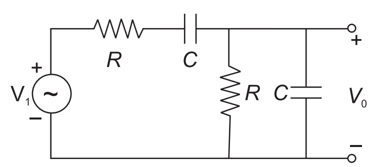
Hence, the correct option is (a).

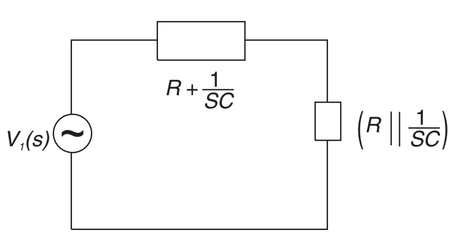
4. The circuit shown below is driven by a sinusoidal input $v_i = V_p \cos(t/\text{RC})$. The steady output v_0 is



(a) $(V_p/3)\cos(t/\text{RC})$ (b) $(V_p/3)\sin(t/\text{RC})$
(c) $(V_p/2)\cos(t/\text{RC})$ (d) $(V_p/2)\sin(t/\text{RC})$ **[2011]**

Solution: (a)





$$V_1(s) = \left(R + \frac{1}{SC}\right)I(s) + \frac{R.\frac{1}{SC}}{R + \frac{1}{SC}}I(s)$$

$$V_1(s) = \left(\frac{1 + SCR}{SC}\right)I(s) + \frac{R}{1 + SCR}I(s)$$

$$V_i = V_p \cos\left(\frac{t}{RC}\right), W = \frac{1}{RC}$$

$$V_i(s) = \left(\frac{1 + j\omega CR}{j\omega C}\right)I(s) + \left(\frac{R}{1 + j\omega CR}\right)I(s)$$

$$V_i(s) = \left[\frac{(1+j)R}{j} + \frac{R}{1+j}\right]I(s)$$

$$I(s) = \frac{V(s)}{3R}(1+j)$$

$$V_0(s) = \left(R \,||\, \frac{1}{SC}\right)I(s)$$

$$V_0(s) = \frac{\frac{R}{SC}}{R + \frac{1}{SC}}I(s)$$

$$V_0(s) = \frac{V_1(s)}{3}$$

$$V_0(t) = \frac{V_i(t)}{3} = \frac{V_p}{3}\cos\left(\frac{t}{RC}\right)$$

$W = 1/RC$

Hence, the correct option is (a).

5. For a parallel RLC circuit which one of the following statements is NOT correct?

(a) The bandwidth of the circuit decreases if R is increased
(b) The bandwidth of the circuit remains same if L is increased
(c) At resonance, input impedance is a real quantity.
(d) At resonance, the magnitude of input impedance attains its minimum value **[2010]**

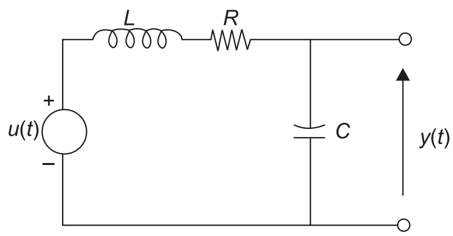
Solution: (d)

$$\text{B.W.} = \frac{1}{RC} \text{ for parallel } R\text{-}L\text{-}C$$

At resonance, impedance is maximum in parallel *R-L-C*.

Hence, the correct option is (d).

6. The condition on R, L and C such that the step response $y(t)$ in the figure has no oscillations is



(a) $R \geq \frac{1}{2}\sqrt{\frac{L}{C}}$ (b) $R \geq \sqrt{\frac{L}{C}}$

(c) $R \geq 2\sqrt{\frac{L}{C}}$ (d) $R = \sqrt{\frac{1}{LC}}$

[2005]

Solution: (c)

$$Z = \frac{R}{2}\sqrt{\frac{C}{L}} \geq 1$$

$$\therefore R > 2\sqrt{\frac{L}{C}}$$

Hence, the correct option is (c).

7. In a series, RLC circuit, R = 2 kΩ, L = 1 H, and $C = \frac{1}{400}\,\mu\text{F}$

The resonant frequency is

(a) 2×10^4 Hz (b) $\frac{1}{\pi} \times 10^4$ Hz

(c) 10^4 Hz (d) $2\pi \times 10^4$ Hz

[2005]

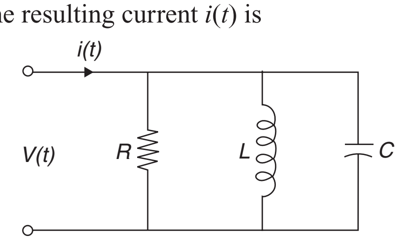
Solution: (b)

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{10^4}{TC}\text{Hz}$$

Hence, the correct option is (b).

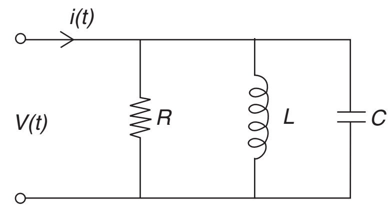
8. The circuit shown in the figure, with

$R = \frac{1}{3}\Omega,\ L = \frac{1}{4}\text{H},\ C = 3\text{ F}$ has input voltage $V(t)$ = sin $2t$. The resulting current $i(t)$ is



(a) 5 sin $(2t + 53.1°)$
(b) 5 sin $(2t – 53.1°)$
(c) 25 sin $(2t + 53.1°)$
(d) 25 sin $(2t – 53.1°)$ **[2004]**

Solution: (a)



$i(t) = V(t)\,.\,Y$

$$i(t) = V(t)\left[\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right]$$

$$i(t) = \sin 2t\left[3 + \frac{4}{2} + j \times 2 \times 3\right]$$

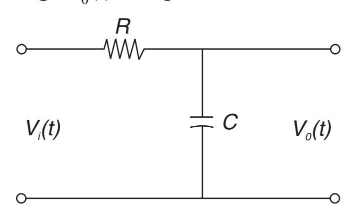
$$i(t) = \sin 2t[3 - 2i + 6j]$$

$$i(t) = \sin 2t[3 + 4j]$$

$$i(t) = 5\sin 2t\angle \tan^{-1}\frac{4}{3} = 5\sin(2t + 53.1°)$$

Hence, the correct option is (a).

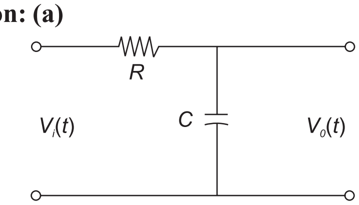
9. For the circuit shown in the figure, the time constant RC = 1 ms. The input voltage is $v_i(t) = \sqrt{2}\ \sin 10^3 t$. The output voltage $v_0(t)$ is equal to



(a) $\sin(10^3 t – 45°)$ (b) $\sin(10^3 t + 45°)$
(c) $\sin(10^3 t – 53°)$ (d) $\sin(10^3 t + 53°)$

[2004]

Solution: (a)



$$V_0(t) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} - V_i(t)$$

$$V_0(t) = \frac{1}{1 + j\omega CR}\sqrt{2}\sin 10^3 t$$

$V_0(t) = \sin(10^3\ t – 45^0)$

Hence, the correct option is (a).

10. A series RLC circuit has a resonance frequency of 1 kHz and a quality factor $Q = 100$. If each of R, L and C is doubled from its original value, the new Q of the circuit is

(a) 25 (b) 50
(c) 100 (d) 200 **[2003]**

Solution: (b)

$f_0 = 1$ kHz, $\vartheta = 100$

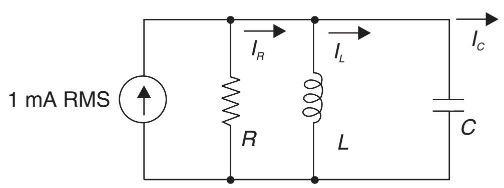
$\theta = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$ for series RLC

$$\theta_2 = \frac{1}{2R}\sqrt{\frac{2L}{2C}} = \frac{1}{2R}\sqrt{\frac{L}{C}} = \frac{\theta_1}{2}$$

$\theta_2 = 50$

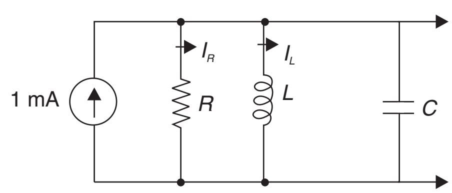
Hence, the correct option is (b).

11. The parallel RLC circuit shown in figure is in resonance, In this circuit



(a) $|I_P| < 1$ mA (b) $|I_R + I_L| > 1$ mA
(c) $|I_R + I_C| > 1$ mA (d) $|I_L + I_C| > 1$ mA **[1998]**

Solution: (b)



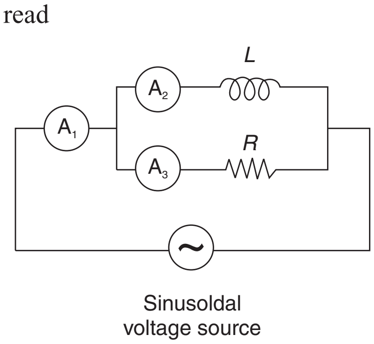
$I - I_R = 1$ mA

$$|I_R - J_L| = \sqrt{J_R^2 + I_L^2} > 1 \text{ mA}$$

$|I_R + I_L| > 1$ mA

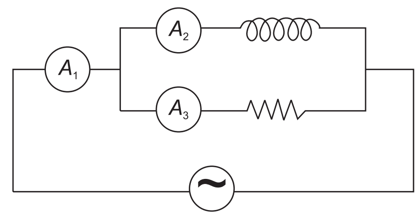
Hence, the correct option is (b).

12. In the below given figure, A_1, A_2 and A_3 are ideal ammeters. If A_2 and A_3 read 3 A and 4 A, respectively, then A_1 should read



(a) 1 A
(b) 5 A
(c) 7 A
(d) None of the above **[1996]**

Solution: (b)

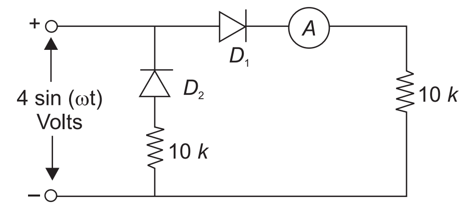


$A_1^2 - A_2^2 + A_3^2$

$A_1^2 = 3^2 + 4^2 = 25 \Rightarrow A_1 = 5\text{A}$

Hence, the correct option is (b).

13. In the circuit of the figure, assume that the diodes are ideal and the metre is an average indicating ammeter. The ammeter will read



(a) $0.4\sqrt{2}$ mA (b) 0.4 mA

(c) $\dfrac{0.8}{\pi}$ mA (d) $\dfrac{0.4}{\pi}$ mA **[1996]**

Solution: (d)

When V_{in} is (+) ve:- D_1 will conduct for (+) ve half cycle.

$$I_{avg} = \frac{V_m}{TC} \times \frac{1}{R} = \frac{4}{\pi} \times \frac{1}{10 \times 10^3}$$

$$I_{avg} = \frac{0.4}{\pi} \text{mA}$$

Hence, the correct option is (d).

14. A DC voltage source is connected across a series R-L-C circuit. Under steady-state conditions, the applied DC voltage drops entirely across the

(a) R only
(b) L only
(c) C only
(d) R and L combination **[1995]**

Solution: (c)

For DC supply, inductor behaves as short circuit and capacitor behaves as an open circuit. So, under steady-state conditions, the applied DC voltage drops entirely across capacitor only.

Hence, the correct option is (c).

15. Consider a DC voltage source connected to a series R-C circuit. When the steady-state reaches, the ratio of the energy stored in the capacitor to the total energy supplied by the voltage source is equal to

(a) 0.362 (b) 0.500
(c) 0.632 (d) 1.000 **[1995]**

Solution: (b)

$$W_S = CV_S^2 \text{ and } W_C = \frac{1}{2}CV_S^2$$

$$\frac{W_C}{W_S} = 0.5$$

Hence, the correct option is (b).

16. The current, $i(t)$, through a 10–Ω resistor in series with an inductance, is given by

$i(t) = 3 + 4\sin(100t + 45°) + 4\sin(300t + 60°)$ amperes.

The RMS value of the current and the power dissipated in the circuit are:

(a) $\sqrt{4}$ A, 410 W, respectively
(b) $\sqrt{35}$ A, 350 W, respectively
(c) 5 A, 250 W, respectively
(d) 11 A, 1210 W, respectively **[1995]**

Solution: (c)

$i(t) = 3 + 4\sin(100t + 45°) + 4\sin(300t + 60°)$

$$I_{rms} = \sqrt{3^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^x} = 5\text{A}$$

$$P = I_{rms}^2 R = (5)^2 \times 10 = 250\text{W}$$

Hence, the correct option is (c).

17. A series R-L-C circuit has a Q of 100 and an impedance of $(100 + j0)$ Ω at its resonant angular frequency of 10^7 radians/sec. The values of R and L are:

R = ________ohms. L = ______henries. **[1995]**

Solution:

$Z = R + j(X_L - X_C) = 100 + j0$

$$\therefore R = 100\Omega,\ Q = \frac{wL}{R} \text{ and } L = \frac{QR}{\omega} = \frac{100\times100}{10^7}$$

$L = 1$ mH

18. A series LCR circuit consisting of $R = 10$ D, $IX_L I = 20$ Ω and $IX_C I = 20$ Ω is connected across an A.C. supply of 200 V rms. The rms voltage across the capacitor is

(a) 200 ∠–90° V (b) 200 ∠ + 90° V
(c) 400 Z + 90° V (d) 400 Z – 90° V

[1994]

Solution: (d)

$\therefore X_L = X_C \Rightarrow$ circuit is at resonance

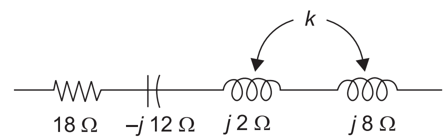
$$I = \frac{V}{R} = \frac{200}{10} = 20\text{A}$$

$V_C = I(-jX_C) = -j400$

$V_C = 400\angle -90°$

Hence, the correct option is (d).

19. In the series circuit shown in figure, for series resonance, the value of the coupling coefficient k will be



(a) 0.25 (b) 0 5
(c) 0.999 (d) 10 **[1993]**

Solution: (a)

At resonance net capacitive impedance = net inductive impedance.

So, $|X_L| = |X_C| = |J12|$

$X_L = J12$

Let k denote the coupling coefficients then

$$X_L = X_{L1} + X_{L2} + 2k\sqrt{X_{L1}X_{L2}}$$

$$J2 + J8 + 2k\sqrt{J2J8} = J12$$

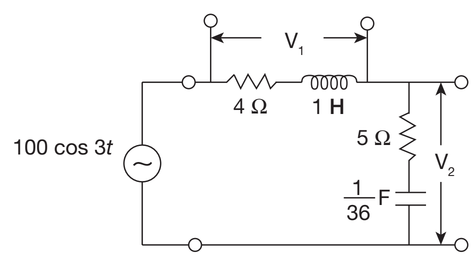
$$= 2kj4 = j2$$

$$k = 0.25$$

Hence, the correct option is (a).

Two-marks Questions

1. The Figure shows an RLC circuit exited by the sinusoidal voltage 100 cos $(3t)$ Volts, where t is in seconds. The ratio $\frac{\text{amplitude of } V_2}{\text{amplitude of } V_1}$ is ________. **[2017]**



Solution: From the given data, $\omega = 3$ rad/sec

$V_{in} = 100\ \angle 0°$ V

$Z_1 = 4 + j\times3\times1 = (4 + j3)\Omega = 5\ \angle 36.86°$

$$Z_2 = 5 - j\times\frac{36}{3} = (5 - j12)\Omega = 13\ \angle -67.38°\ \Omega$$

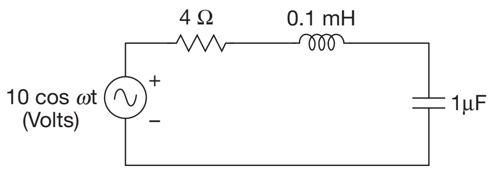
$$V_2 = \frac{Z_2}{Z_1 + Z_2}\times V_{in}$$

$$V_1 = \frac{Z_1}{Z_1 + Z_2}\times V_{in}$$

$$\frac{|V_2|}{|V_1|} = \frac{Z_2}{Z_1}$$
$$= \frac{13}{5}$$
$$= 2.6$$

Hence, the correct answer is (2.55 to 2.65).

2. In the circuit shown, at resonance, the amplitude of the sinusoidal voltage (in Volts) across the capacitor is _______. **[2015]**



Solution: We know for a RLC series circuit

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

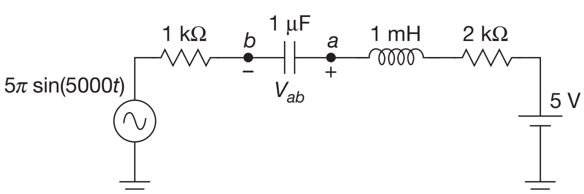
$$Q = \frac{1}{4}\sqrt{\frac{10^{-4}}{10^{-6}}} = \frac{10}{4} = 2.5$$

$$V_C = QV\angle{-90°}$$

$$|V_C| = 2.5 \times 10 = 25 \text{ V}$$

Hence, the correct Answer is (24 to 26).

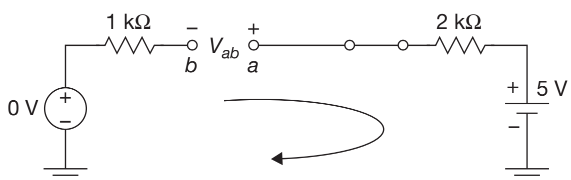
3. In the circuit shown, the average value of the voltage V_{ab} (in Volts) in steady state condition is ________. **[2015]**



Solution: At steady state redraw the given circuit

C – Open circuit

L – Shot circuit



$$0 + V_{ab} + 0 - 5 \text{ V} = 0$$

$$V_{ab} = 5 \text{ V}$$

Hence, the correct Answer is (4.9 to 5.1).

4. A 230 V rms source supplies power to two loads connected in parallel. The first load draws 10 kW at 0.8 leading power factor and the second one draws 10 kVA at 0 8 lagging power factor. The complex power delivered by the source is

(a) $(18 + j\,1.5)$ kVA (b) $(18 - j\,1.5)$ kVA
(c) $(20 + j\,1.5)$ kVA (d) $(20 - j\,1\,5)$ kVA

[2014]

Solution: (b)

$V_{\text{rms}} = 230$ V, Z $\Rightarrow$ 10 kw at 0.8 leading

$Z_2 \Rightarrow 10\,\text{KVA}$ at 0.8 lagging power factor

$$P_1 = V_1 I_1 \cos\phi$$

$$I_1 = \frac{P_1}{V_1 \cos\phi} = 54.34\angle 36.86$$

$$P = VI_2$$

$$I_2 = 43.47\angle{-36.86°}$$

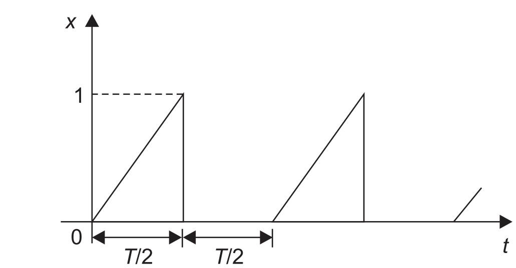
$$\vec{I} = \vec{I}_1 + \vec{I}_2 = 78.25 + j6.25$$

$$P = VI^* = 230\,(78.25 - j6.25)$$

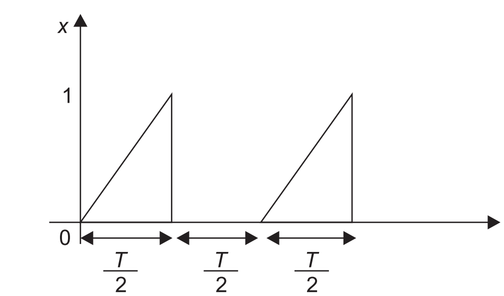
$$P = (18 - j1.5) \text{ KVA}$$

Hence, the correct option is (b).

5. A periodic variable x is shown in the figure as a function of time. The root-mean-square (rms) value of x is________. **[2014]**



Solution: 0.408



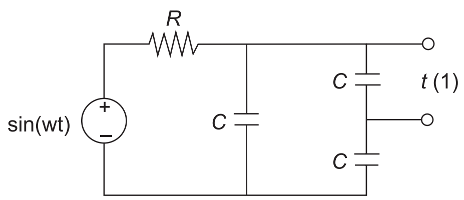
$$\text{RMS Value} = \sqrt{\frac{1}{T}\int_0^T \eta^2(t)dt}$$

$$= \sqrt{\frac{1}{T}\int_0^{T/2}\left(\frac{2t}{T}\right)^2 dt}$$

$$= \sqrt{\frac{1}{T}\int_0^{T/2} \frac{4}{T^2} t^2 dt} = 0.408$$

6. The steady-state output of the circuit shown in the figure is given by

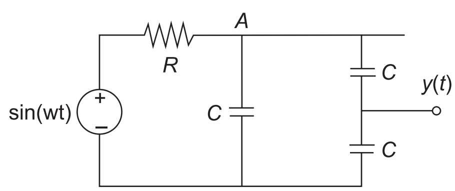
$y(t) = A(\omega)\sin(\omega t + \phi(\omega))$. If the amplitude $A(\omega)| = 0.25$, then the frequency ω is



(a) $\frac{1}{\sqrt{3}RC}$ (b) $\frac{2}{\sqrt{3}RC}$

(c) $\frac{1}{RC}$ (d) $\frac{2}{RC}$ **[2014]**

Solution: (b)



Applying KCL at A,

$$\frac{V_A - \sin wt}{R} + \frac{V_A}{\frac{1}{j\omega C}} + \frac{V_A}{\frac{2}{j\omega C}} = 0$$

$$V_A = \frac{2}{2 + 3RCj\omega}$$

$$Y = \frac{V_A}{2} = \frac{1}{2 + 3RCj\omega}$$

$$|A(\omega)| = \frac{1}{4}$$

$$\frac{1}{4} = \frac{1}{\sqrt{4 + (qw^2R^2C^2)}}$$

$$w = \frac{2}{\sqrt{3}RC}$$

Hence, the correct option is (b).

7. Two magnetically uncoupled inductive coils have Q factors q_1 and q_2 at the chosen operating frequency. Their respective resistance is R_1 and R_2. When connected in series, their effective Q factor at the same operating frequency is

(a) $q_1 + q_2$

(b) $(1/q_1) + (1/q_2)$

(c) $(q_1R_1 + q_2R_2)/(R_1 + R_2)$

(d) $(q_1R_2 + q_2R_1)/(R_1 + R_2)$ **[2013]**

Solution: (c)

$$\theta_1 = \frac{\omega L_1}{R_1}, \quad \theta_2 = \frac{\omega L_2}{R_2}$$

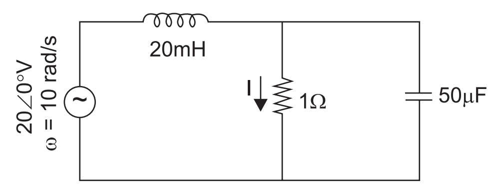
$$\omega L_1 = q_1 R_1 \text{ and } \omega L_2 = q_2 R_2$$

$$\therefore \text{Q.R} = \omega L_1 + \omega L_2 = q_1R_1 + q_2R_2$$

$$q = \frac{q_1R_1 + q_2R_2}{R_1 + R_2}$$

Hence, the correct option is (c).

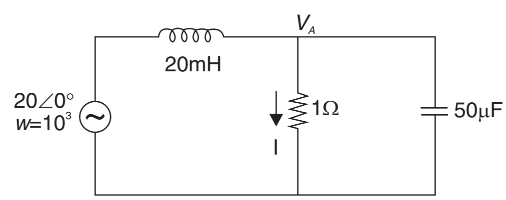
8. The current I in the circuit shown is



(a) $-j1$ A (b) $-j1$ A

(c) 0 A (d) 20 A **[2010]**

Solution: (a)



$$\frac{V_A - 20\angle 0°}{j\omega L} + \frac{V_A}{1} + \frac{V_A}{\frac{1}{j\omega C}} = 0$$

$$V_A\left[\frac{-j}{20} + 1 + \frac{j}{20}\right] = -j$$

$$V_A = -j = 1\angle -90°$$

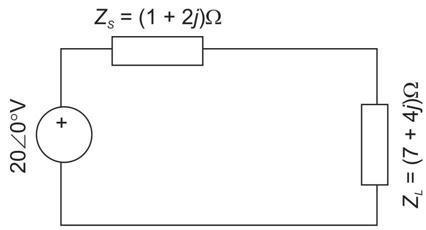
$$I = \frac{V_A}{1} = 1\angle -90° \text{ A}$$

Hence, the correct option is (b).

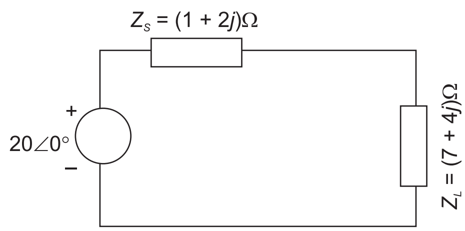
9. An AC source of RMS voltage 20 V with internal impedance $Z_s = (1 + 2j)\Omega$ feeds a load of impedance $Z_L = (7 + 4j)\Omega$ in the figure below. The reactive power consumed by the load is

(a) 8 VAR (b) 16 VAR

(c) 28 VAR (d) 32 VAR **[2009]**



Solution: (b)

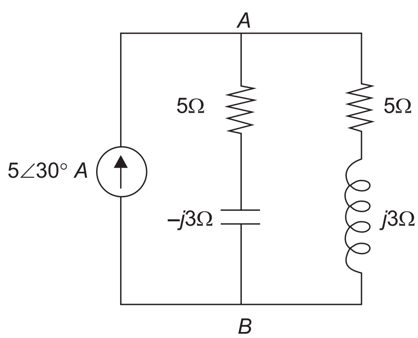


$$I = \frac{20\angle 0°}{(1+2j)+(7+4j)} = \frac{10}{4+3j} = 2\angle -368i$$

Reactive power = $I^2X_L = 4 \times 4 = 16$ VAR

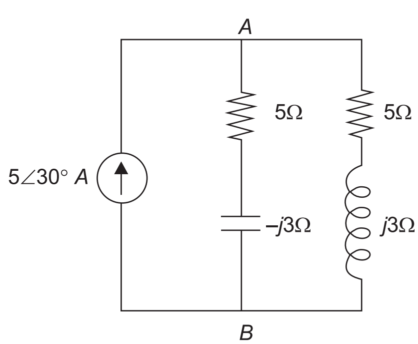
Hence, the correct option is (b).

10. In the AC network shown in the figure, the phasor voltage V_{AB} (in volts) is



(a) 0 (b) $5\angle 30°$
(c) $12.5\angle 30°$ (d) $17\angle 30°$ **[2007]**

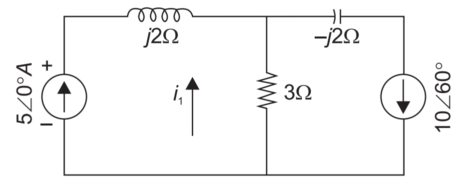
Solution: (d)



$$V_{AB} = 5\angle 30° \times \frac{(5+3j)}{(5+3j+5-3j)} = 17\angle 30°$$

Hence, the correct option is (d).

11. For the circuit shown in the figure, the instantaneous current $i_1(t)$ is



(a) $\dfrac{10\sqrt{3}}{2}\angle 90°$ A

(b) $\dfrac{10\sqrt{3}}{2}\angle -90°$ A

(c) $5\angle 60°$ A

(d) $5\angle -60°$ A **[2005]**

Solution: (a)

$$5\angle 0° + i_1 = 10\angle 60°$$

$$i_1 = 10\angle 60° - 5\angle 0°$$

$$i_1 = \frac{10\sqrt{3}}{2} < 90°\ \text{A}$$

Hence, the correct option is (a).

12. Consider the following statements S_1 and S_2

S_1: At the resonant frequency the impedance of a series *R-L-C* circuit is zero.

S_2: In a parallel G-L-C circuit, increasing the conductance G results in increase in its Q factor.

Which one of the following is correct?

(a) S_1 is FALSE and S_2 is TRUE
(b) Both S_1 and S_2 are TRUE
(c) S_1 is TRUE and S_2 is FALSE
(d) Both S_1 and S_2 are FALSE **[2004]**

Solution: (d)

Both the statements are false.

Correct S_1: At resonant frequency, impedance is minimum for series *R-L-C*

Correct S_2: In parallel G-L-C, increasing G decreases ϑ factor.

Hence, the correct option is (d).

13. An input voltage $v(t) = 10\sqrt{5}$ cos (t + 10°) $+10\sqrt{5}$ (t + 10°) V is applied to a series combination of resistance $R = 1\ \Omega$ and an inductance $L = 1$ H. The resulting steady-state current $i(t)$ in ampere is

(a) $10\cos(t + 55°) + 10\cos(2t + 10° + \tan^{-1}2)$

(b) $10\cos(t + 55°) + 10\,J - 10\sqrt{\dfrac{3}{2}}\cos(2t + 55)$

(c) $10\cos(t - 35°) + 10\cos(2t + 10° - \tan^{-1}2)$

(d) $10\cos(t - 35) + 10\sqrt{\dfrac{3}{2}}\cos(2t - 35°)$

[2003]

Solution: (c)

$$v(t) = 10\sqrt{5}\cos(t+10°) + 10\sqrt{5}\cos(2t+10°)$$

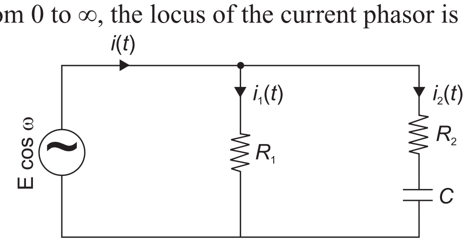
$R = 1\Omega,\ L = 1\text{H}$

$$i(t) = \frac{V(t)}{R + j\omega L} = \frac{V_1(t)}{R + j\omega L} + \frac{V_2(t)}{R + j\omega_2 L}$$

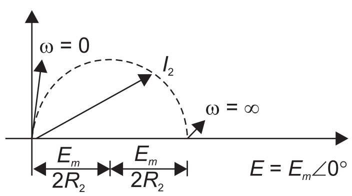
$$i(t) = \frac{10\sqrt{5}\cos(t+10°)}{\sqrt{2}\angle 45°} + \frac{10\sqrt{5}\cos(2t+10°)}{\sqrt{5}\angle \tan^{-1} 2}$$

Hence, the correct option is (c).

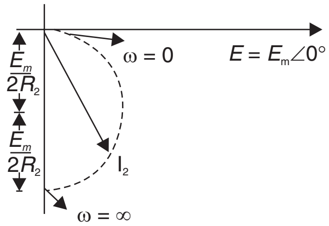
14. When the angular frequency ω in the figure is varied from 0 to ∞, the locus of the current phasor is given by



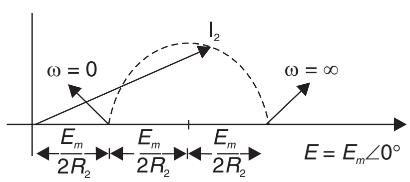
(a)



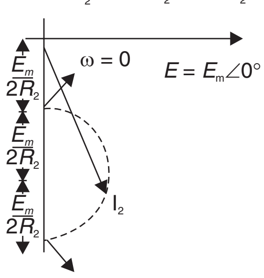
(b)



(c)

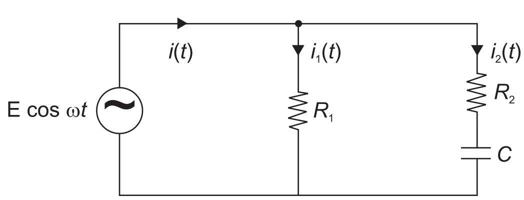


(d)



[2001]

Solution: (a)



$$i_2(t) = \frac{E_m \cos w_t}{R_2 + \dfrac{1}{j\omega C}}$$

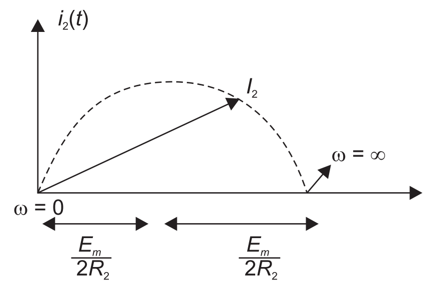
$$i_2(t) = E_m\angle 0° - \frac{j\omega C}{1 + j\omega C R_2}$$

$$i_2(t) = \frac{E_m\angle 0° \times \angle 90° \times \omega C}{\sqrt{1 + W^2C^2R^2}\angle \tan^{-1}\omega C R_2}$$

$$i_2(t) = \frac{E_m \omega C}{\sqrt{1 + W^2C^2 + R^2}}\angle 90° - \tan^{-1}\omega C R_2$$

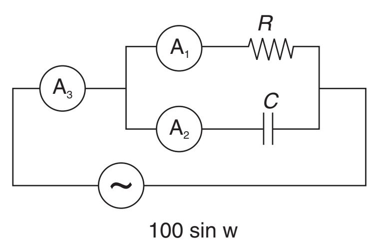
When $w = 0$ $i_2(t) = 0, \angle 90°$

When $w = \infty \Rightarrow i_2(t) = \dfrac{E_m}{R_2}\angle 0°$



Hence, the correct option is (a).

15. In the given figure, A_1, A_2 and A_3 are ideal ammeters. If A_1 reads 5A, A_2 reads 12 A, then A_3 should read



(a) 7 A (b) 12 A
(c) 13 A (d) 17 A **[1993]**

Solution: (c)

Given $I_{A1} = I_R = 5\text{A}$

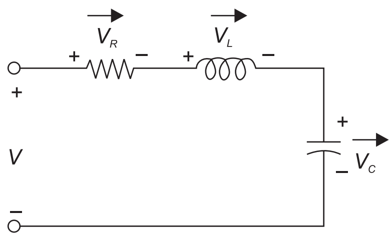
$I_{A2} = I_C = 12\text{A}$

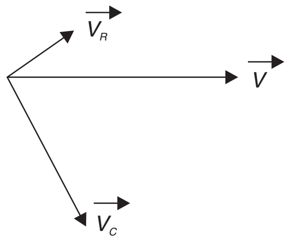
Resultant of current through resistor and capacitor will be

$$I_{A3} = \sqrt{I_R^2 + I_C^2} = 13\text{A}$$

Hence, the correct option is (c).

16. For the series *R-L-C* circuit in the below given figure 1, the partial fissure diagram at a certain frequency is a shown in figure 2. The operating frequency of the circuit is

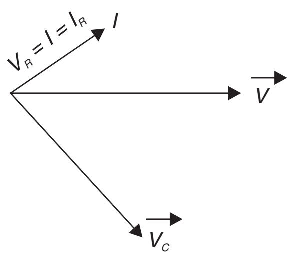




(a) equal to the resonance frequency
(b) less than the resonance frequency
(c) greater than the resonance frequency
(d) not zero **[1992]**

Solution: (b)

Since the current is leading voltage, it behaves as a capacitive circuit.



So, voltage across capacitor will be greater than voltage across inductor

$V_C > V_L$

$IX_C > IX_L$

$X_C > X_L$

$$\frac{1}{\omega C} > \omega L$$

$$\omega^2 < \frac{1}{LC} \Rightarrow \omega < w_r$$

Hence, the correct option is (b).

17. In a series RLC high Q circuit, the current peaks at a frequency

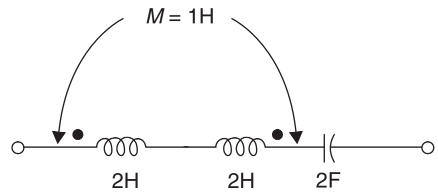
(a) equal to the resonant frequency
(b) greater than the resonant frequency
(c) less than the resonant frequency
(d) none of the above is true **[1991]**

Solution: (a)

In series RLC circuit at resonance frequency, impedance is minimum. So, the current is maximum.

Hence, the correct option is (a).

18. The resonant frequency of the series circuit shown in figure is

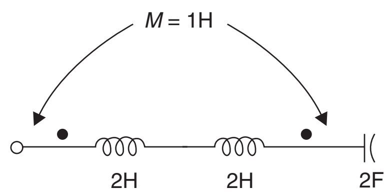


(a) $\dfrac{1}{4\pi\sqrt{3}}$ Hz (b) $\dfrac{1}{4\pi}$ Hz

(c) $\dfrac{1}{2\pi\sqrt{10}}$ Hz (d) $\dfrac{1}{4\pi\sqrt{2}}$ Hz

[1990]

Solution: (b)



Equivalent inductance will be given as

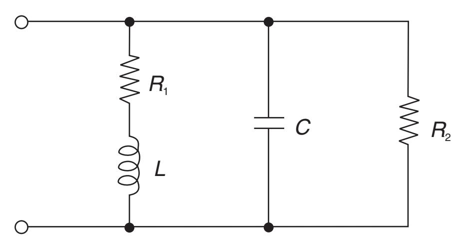
$L_{eq} = L_1 + L_2 - 2M$

$L_{eq} = 2 + 2 - 2 = 2H$

$$j = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{2\times 2}} = \frac{1}{4\pi} Hz$$

Hence, the correct option is (b).

19. The half-power bandwidth of the resonant circuit of figure can be increased by:



(a) increasing R_1 (b) decreasing R_1
(c) increasing R_2 (d) decreasing R_2

[1989]

Solution: (a) and (d)

Selectivity $\propto$ Q

$$Q = \frac{f}{B.W}$$

$$Q = \frac{f}{B.W} \text{ or } B.W \propto \frac{1}{Q}$$

$$B.W \propto \frac{1}{\text{selectivity}}$$

If $R_1 \to 0$ and $R_1 \to \infty$

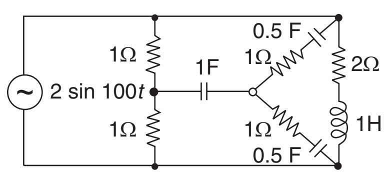
then circuit will have only L and C element and has selectivity.

HPBW can be reduced by decreasing selectivity and vice versa.

So, by increasing series resistance R_1 and decreasing parallel resistance R_2 the half power BW can be increased.

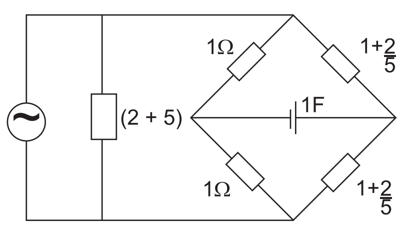
Hence, the correct option is (a) and (b).

20. The value of current through the 1 Farad capacitor of figure is



(a) zero (b) one
(c) two (d) three **[1987]**

Solution: (a)



$$1\left(1+\frac{2}{5}\right)=1\left(1+\frac{2}{5}\right)$$

Circuit given is the same as a bridge circuit, as product of opposite arms impedance is equal. So, current through the diagonal element ($1F$) = 0.

Hence, the correct option is (a).

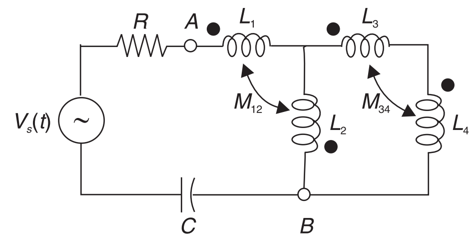
Five-marks Questions

1. For network in figure, R = 1 kΩ, L_1 = 2H, L_2 = 5H, L_3 = 1H, L_4 = 4H and C = 0.2μF. The mutual inductances are M_{12} = 3 H and M_{34} = 2H. Determine

(a) the equivalent inductance for the combination of L_3 and L_4.

(b) the equivalent inductance across the points A and B in the network.

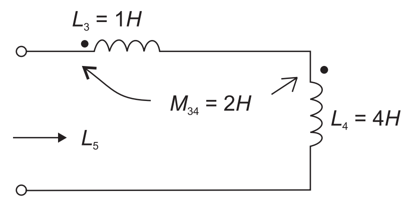
(c) the resonant frequency of the network.



[2002]

Solution:

(a) Let L_5 be the equivalent inductance for the combination of L_3 and L_4 as shown in below figure.

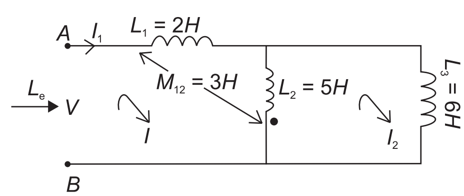


The coupling is series aiding

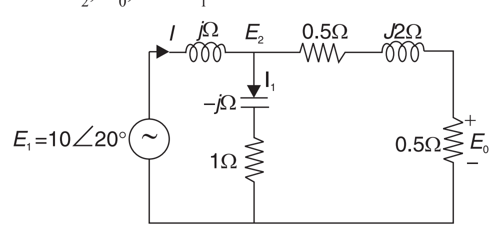
$L_5 = L_3 + L_4 + 2M34$

$= 1 + 4 + 4 = 9H$

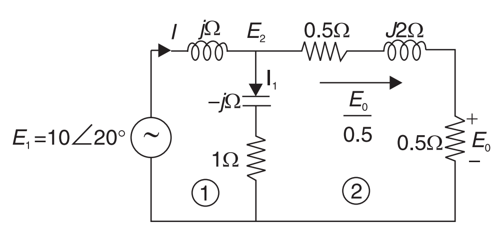
(b) The circuit for calculating the equivalent inductance L_e across the points A and B is shown in below figure.



2. For the circuit shown in figure determine the phasorsE_2, E_0, land 1_1.



Solution:



For loop 2,

$$\frac{E_0}{0.5}(0.5+j2)+E_0=I_1(1-j)$$

$\Rightarrow E_0(2 + j4) = I_1(1 - j)$ (1)

KCL at node E_2

$$\Rightarrow I = I_1 + \frac{E_0}{0.5} \quad (2)$$

$E_2 = I_1(1 - j)$ (3)

For loop 1,

$$E_1 = jI + E_2 = 10\underline{|20°} \quad (4)$$

$\Rightarrow JI + I_1(1 - J) = 10e_{j20°}$ (5)

From equation (2) and (5),

$$(1-j)I_1 + jI_1 + \frac{jE_0}{0.5} = 10e^{j20°} \quad (6)$$

$\Rightarrow I_1 + 2JE_0 = 10e^{J20°}$

$\Rightarrow I_1 = 10e^{j20°} - 2jE_0$ (7)

From equation (1) and (7),

$\Rightarrow E_0(2 + J4) = (10e^{j20°} - 2JE_0$

$\Rightarrow E_0(4 + J6) = (1 - J)10e^{j20°}$

$$\Rightarrow E_0 = \frac{5(1-j)}{2+j3}e^{j20°}$$

$$= 1.96e^{-j81.3°}V \quad (8)$$

From equation (1),

$E_0(2 + j4) = (1 - j1)I_1$

$$\Rightarrow I_1 = \frac{2+j4}{1-j1}E_0 = 6.2e^{j27.13°}A \quad (9)$$

From equation (3) and (9),

$E_2 = (1 - j)I_1$

$= (1 - j)6.2e^{j27.13°}$

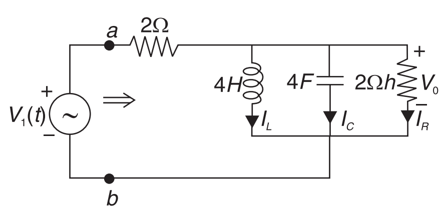
$= 8.77e^{-j17.9°}$

From equation (2) and (8)

$$\Rightarrow I = I_1 + \frac{E_0}{0.5} = I_1 + 2E_0$$

$$= 6.2e^{-j9.75°}A$$

3. For the circuit in figure which is in steady state,



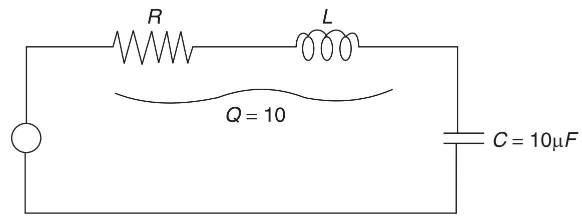
(a) Find the frequency ω_0 at which the magnitude of the impedance across terminals a, b reaches maximum.

(b) Find the impedance across a, b at the frequency ω^0.

(c) If $V_i(t) = V\sin((\omega_0 t)$, find $i_L(t)$, $i_c(t)$, $i_R(t)$. **[2000]**

4. A coil with a quality factor (Q) of 10 is put in series with a capacitor C_1 of 10 μF, and the combination is found to draw maximum current When sinusoidal voltage of frequency 50 Hz is applied. A second capacitor C_2 is now connected in parallel with the capacitor. What should be the capacitance of C_2 for the combined circuit to act purely as a resistance for a sinusoidal excitation at a frequency of 100 Hz? Calculate the rms current drawn by the combined circuit at 100 Hz if the applied voltage is 100 V(rms). **[1999]**

Solution:



$$f_r = 50 \text{ Hz} = \frac{1}{2\pi\sqrt{LC_1}}$$

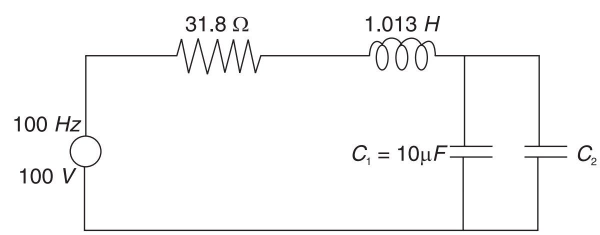
$$\Rightarrow L = \frac{1}{4\pi^2 C_1(50)^2} = \frac{1}{4\pi^2 \times 10 \times 10^{-6} \times 2500}$$

$$= 1.013 \text{ H.}$$

$$Q = \frac{\omega_0 L}{R} = 10$$

$$\Rightarrow R = \frac{\omega_0 L}{10} = \frac{2\pi \times 50 \times 1.013}{10} = 31.8\Omega$$

Now,



$$f_r = \frac{1}{2\pi\sqrt{LC_{eq}}}$$

$$\Rightarrow C_{eq} = \frac{1}{4\pi^2 L f_r^2} = \frac{1}{4\pi^2(1.013)(100)^2}$$

$\Rightarrow C_{eq} = 2.5$ μF $= C_1 + C_2$

It is not possible

It means C_2 is in series with C_1.

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 2.5$$

$$\Rightarrow \frac{C_2 \times 10}{C_2 + 10} = 2.5$$

$$\Rightarrow 10C_2 = 2.5C_2 + 25$$

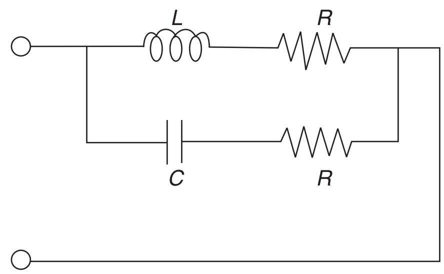
$$\Rightarrow 7.5\ C_2 = 25$$

$$\Rightarrow C_2 = \frac{25}{7.5} = 3.33\ \mu F$$

$$\therefore I_{rms} = \frac{V_{rms}}{R} = \frac{100}{31.8} = 3.14A \text{ Ans}$$

5. Determine the frequency of resonance and the resonant impedance of the parallel circuit shown in figure. What happens when $L = CR^2$?

$L = CR^2$?



[1998]

Solution:

$$y_L = \frac{1}{R + j\omega L} = \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

$$= \frac{R}{R^2 + \omega^2 + L^2} - j\frac{\omega L}{R^2 + \omega^2 L^2}$$

$$y_C = \frac{1}{R + \frac{1}{j\omega C}} = \frac{j\omega C}{1 + j\omega RC}$$

$$= \frac{j\omega C(1 - j\omega RC)}{1 + \omega^2 R^2 C^2}$$

$$\frac{\omega^2 RC^2}{1 + \omega^2 R^2 C^2} + j\frac{\omega C}{1 + \omega^2 R^2 C^2}$$

$$\Rightarrow y = y_L + y_C$$

For resonance, $I_m(y) = 0$

$$\Rightarrow \frac{-\omega L}{R^2 + \omega^2 L^2} + \frac{\omega C}{1 + \omega^2 R^2 C^2} = 0$$

$$\Rightarrow L(1 + \omega_0^2 R^2 C^2) = C(R^2 + \omega_0^2 L^2)$$

$$\Rightarrow L + \omega_0^2 R^2 C^2 L = R^2 C + \omega_0^2 L^2 C$$

$$\omega_0^2 LC\,(R^2C - L) = R^2 C - L$$

$$\Rightarrow \omega_0^2 = \frac{1}{LC}$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$y = \frac{R}{R^2 + \omega_0^2 L^2} + \frac{\omega_0^2 RC^2}{1 + \omega_0^2 R^2 C^2}$$

Put $\omega_0^2 = \dfrac{1}{LC}$

$$\therefore y = \frac{R}{R^2 + \frac{L}{C}} + \frac{\frac{RC}{L}}{1 + \frac{R^2 C}{L}}$$

$$\Rightarrow y = \frac{RC}{R^2C + L} + \frac{RC}{R^2C + L} = \frac{2RC}{R^2C + L}$$

$$\Rightarrow Z = \frac{1}{y} = \frac{R^2C + L}{2RC}$$

If, $L = R^2C$

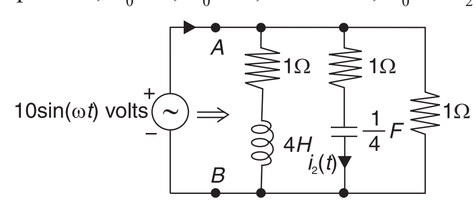
$$\therefore Z = \frac{R^2C + R^2C}{2RC} = R$$

So, at $L = R^2C$, the circuit resonant at infinite number of frequencies.

6. In the circuit of figure all currents and voltages are sinusoids of frequency ω rad/sec.

(a) Find the impedance to the right of (A, B) at $\omega = \infty$ rad/sec and $\omega = \infty$ rad/sec.

(b) If $\omega = \omega_0$ rad/sec and $i_1(t) = I\sin(\omega_0 t)A_1$ where I is positive, $\omega_0 \neq 0$, $\omega_0 \neq \infty$, then find I, ω_0 and $i_2(t)$.



[1997]

Solution:

(a) At $\omega = 0$

$X_L = \omega L = 0$ short circuit

$$X_C = \frac{1}{\omega c} = \infty \Rightarrow \text{open circuit}$$

$\Rightarrow R_{AB} = 1||1 = 0.5\Omega$

At $\omega \to \infty$

$X_L = \omega L \to \infty, \Rightarrow$ open circuit

$$X_C = \frac{1}{\omega C} = 0 \Rightarrow \text{short circuit}$$

$\Rightarrow R_{AB} = 1||1 = 0.5\ \Omega$

(b) $\omega = \omega_0$ = resonant frequency

$$y_{AB} = \frac{1}{1+4s} + \frac{1}{1+\frac{4}{s}} + 1$$

$$y_{AB} = \frac{8s^2+19s+8}{4s^2+17s+4}$$

$$= \frac{-8\omega_0^2+8+j19\omega_0}{-4\omega_0^2+4+j17\omega_0}$$

y_{AB} is purely real if $\omega_0 = 1$

i.e., $y_{AB} = \dfrac{19}{17} \Rightarrow Z_{AB} = \dfrac{17}{19}\Omega$

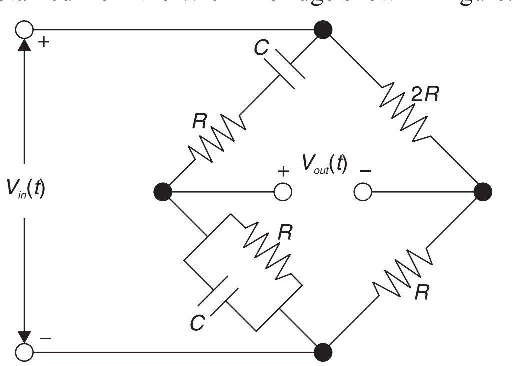
$$i_1(t) = \frac{V_1(t)}{Z_{AB}} = \frac{190}{17}\sin\omega t = I\sin\omega t$$

$$\Rightarrow I = \frac{190}{17}A$$

$$i_1(t) = \frac{V(t)}{1+\frac{4}{j\omega_0}} = \frac{V(t).j\omega_0}{4+j\omega_0} = \frac{10\sin\omega t.i}{4+i}$$

$$= \frac{10}{\sqrt{17}}\sin(\omega t + 76°)A$$

7. Calculate the frequency at which zero – transmission is obtained from the Wien – bridge shown in figure.



[1994]

Solution: $Z_1 = R + \dfrac{1}{Cs}$, $Z_2 = 2R$

$$Z_3 = \frac{R\times\frac{1}{Cs}}{R+\frac{1}{Cs}} = \frac{R}{RCs+1},\ Z_4 = R$$

For balance condition,

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$$

$$\Rightarrow \frac{\frac{RCs+1}{Cs}}{\frac{R}{RCs+1}} = \frac{2R}{R}$$

$\Rightarrow (RCs + 1)^2 = 2RCs$

$\Rightarrow 1 + R^2C^2s^2 = 0$

$$\Rightarrow s^2 = \frac{-1}{R^2C^2}$$

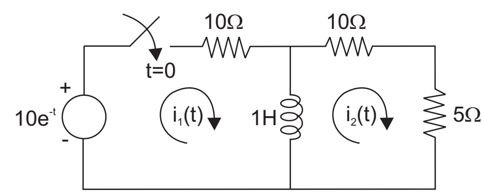
$$\Rightarrow (j\omega)^2 = \frac{-1}{R^2C^2}$$

$$\Rightarrow -\omega^2 = \frac{-1}{R^2C^2}$$

$$\Rightarrow \omega = \frac{1}{RC}$$

or $f = \dfrac{1}{2\pi RC}$

8. Write down the mesh equations of the following network in terms of $i_1(t)$ and $i_2(t)$. Derive the differential equation for $i_1(t)$ from these and Sove it.



[1994]

Solution: For loop 1,

$$10i_1(t) + \frac{di_1(t)}{dt} - \frac{di_2(t)}{dt} = 10e^{-t} \quad (1)$$

For loop 2,

$$15i_2(t) + \frac{di_2(t)}{dt} - \frac{di_1(t)}{dt} = 0 \quad (2)$$

Adding both equations,

$10i_1(t) + 15i_2(t) = 10e^{-t}$

$$\Rightarrow i_2(t) = \frac{10e^{-t}}{15} - \frac{10}{15}i_1(t) \quad (3)$$

Substituting $i_2(t)$ in equation (1), we get

$$10i_i(t) + \frac{di_1(t)}{dt} - \frac{10}{15}\frac{de^{-t}}{dt} + \frac{10}{15}\frac{di_1(t)}{dt} = 10e^{-t}$$

$$\Rightarrow \frac{5}{3}\frac{di_1(t)}{dt} + 10i_1(t) = \frac{10}{15}\frac{de^{-t}}{dt} + 10e^{-t}$$

$$\Rightarrow \frac{di_1(t)}{dt} + 6i_1(t) = \frac{1}{25}\left[\frac{dV_i(t)}{dt} + 15V_1(t)\right]$$

And, $V_i(t) = 10e^{-t}.u(t)$

Take the laplace transform,

$$\Rightarrow (s+6)I_1(s) = \frac{1}{25}(s+15).\frac{10}{s+1}$$

$$\Rightarrow I_1(s) = \frac{2(s+15)}{(5)(s+6)(s+1)}$$

$$= \left(\frac{2.8}{s+1} - \frac{1.8}{s+6}\right)0.4$$

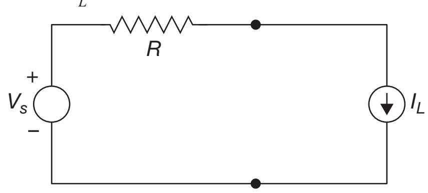
$$= (1.12\ e^{-t} - 0.72\ e^{-6t})\ v(t)$$

Chapter 3

Network Theorems

One-mark Questions

1. In the circuit shown below, V_s is a constant voltage source and I_L is a constant current load.

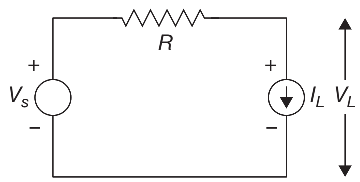


The value of I_L that maximizes the power absorbed by the constant current load is **[2016]**

(A) $\frac{V_{ref}}{2R}$ (B) $\frac{V_s}{2R}$

(C) $\frac{V_s}{R}$ (D) ∞

Solution:



Load voltage can be expressed as

$$V_L = V_S - I_L R$$

Load power is $P_L = V_L \cdot I_L$

$$P_L = (V_S - I_L R)\, I_L$$

To get I_L that maximize the power observed by the load

$$\frac{dP_L}{dI_L} = 0$$

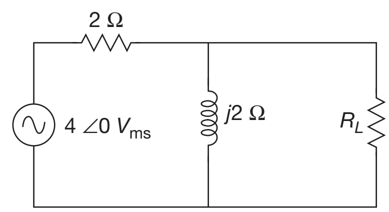
$$V_S - 2I_L R = 0$$

$$2I_L R = V_S$$

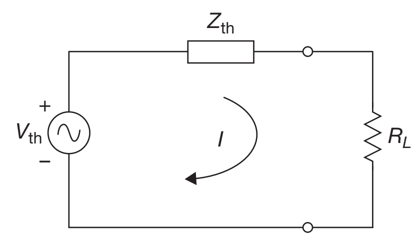
$$I_L = \frac{V_s}{2R}$$

Hence, the correct option is (B).

2. In the given circuit, the maximum power (in Watts) that can be transferred to the load R_L is _______. **[2015]**



Solution: Find the Thevenin equivalent circuit at the terminal of R_L



$$Z_{th} = (2 \,||\, j_2) = (1 + j_1)\ \Omega = \sqrt{2}\angle 45°\,\Omega$$

$$R_L = |Z_{th}| = \sqrt{2}\ \Omega$$

$$V_{th_{rms}} = \frac{4\times j_2}{2+j_2} = 2 + 2i = 2\sqrt{2}\ \angle 45°\ \text{V}$$

$$I_{rms} = \frac{V_{th}}{Z_{th}+R_{th}} = \frac{2\sqrt{2}\angle 45°}{1+j1+1.414} = \frac{2\sqrt{2}\angle 45°}{2.414+j1}$$

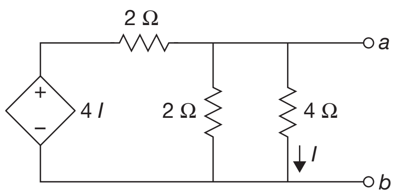
$$I_{rms} = \frac{2\sqrt{2}\angle 45°}{2.613\angle 22.5} = 1.082\ \angle 22.5°\ \text{Amp}$$

$$P_{max} = \frac{1}{2}|I|^2.R_L = |I_{rms}|^2\ .R_L$$

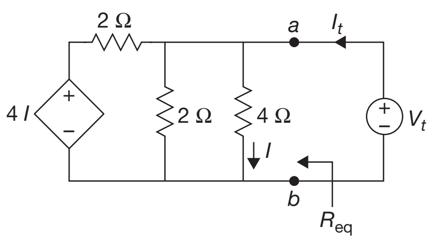
$$= 1.655\ \text{watts}$$

Hence, the correct Answer is (1.6 to 1.7).

3. In the circuit shown, the Norton equivalent resistance (in Ω) across terminals $a - b$ is _____.



Solution: The given network consists only dependent source, connect one test source and find the equivalent resistance across the test source.



$$R_{eq} = R_N = \frac{V_t}{I_t}$$

Applying Nodal analysis

$$\frac{V_t - 4I}{2} + \frac{V_t}{2} + I = I_t$$

$$V_t - 4I + V_t + 2I = 2I_t$$

$$\Rightarrow \qquad 2V_t - 2I = 2I_t$$

$$\text{but} \qquad I = \frac{V_t}{4}$$

$$\Rightarrow \qquad 2V_t - \frac{V_t}{2} = 2I_t$$

$$3V_t = 4I_t$$

$$\frac{V_t}{I_t} = \frac{4}{3}$$

$$R_N = 1.33\ \Omega$$

Hence, the correct Answer is (1.3 to 1.35).

4. Norton's theorem states that a complex network connected to a load can be replaced with an equivalent impedance

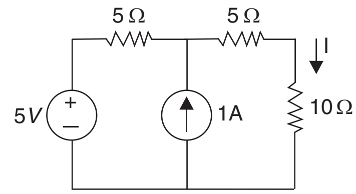
(a) in series with a current source
(b) in parallel with a voltage source
(c) in series with a voltage source
(d) in parallel with a current source **[2014]**

Solution: (d)

In parallel with current source

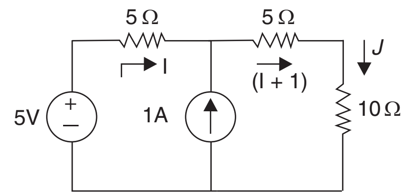
Hence, the correct option is (d)

5. In the figure shown, the value of the current I (in Amperes) is__________.



[2014]

Solution: 0.5 A



Applying KVL,

$5 = 5I + 5(I + 1) + 10(I + 1)$

$5 = 20I + 5 + 10$

$|I| = 0.5\text{A}$

6. A source $v_s(t) = \omega\cos 100\,\pi t$ has an internal impedance of $(4 + j3)\Omega$. If a purely resistive load connected to this source has to extract the maximum power out of the source, its value in Ω should be

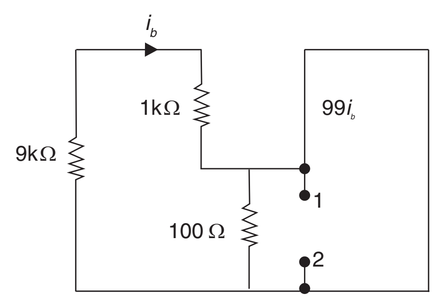
(a) 3 (b) 4
(c) 5 (d) 7 **[2013]**

Solution: (c)

$$R_L = \sqrt{R_S^2 + X_S^2} = 5\Omega$$

Hence, the correct option is (c)

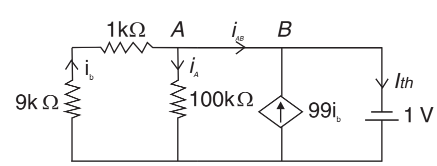
7. The impedance looking into nodes 1 and 2 in the given circuit is



(a) 50 Ω (b) 100 Ω
(c) 5 kΩ (d) 10.1 kΩ

[2012]

Solution: (a)



Applying KCL, $i_{AB} + 99_{ib} = I_{th}$

$$i_b = i_A + i_{AB}$$

$\therefore$ $100ib - i_A = I_{th}$

Applying KVL in outer loop,

$10 \times 10^3 ib = 1$

$ib = 10^{-4}$ A

$i_A = -100 i_b$

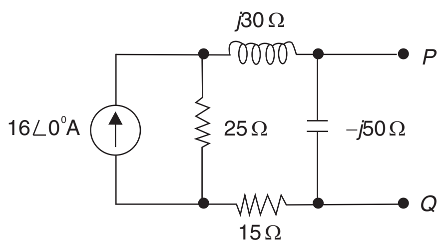
$\therefore$ $100 i_b + 100 i_b = I_{th}$

$I_{th} = 200 i_b = 0.02$

$$Z_{in} = \frac{1}{I_{th}} = 50\,\Omega$$

Hence, the correct option is (a)

8. In the circuit shown below, the Norton equivalent current in amperes with respect to the terminals P and Q is

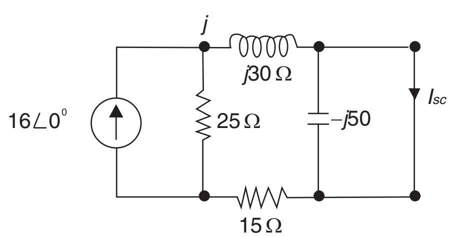


$$I_{sc} = \frac{25}{15 + 30j + 25} \quad 16\angle 0^{O} = 8\angle \quad 36.86$$

(a) $6.4 - j4.8$ (b) $6.56 - j7.87$
(c) $10 + j0$ (d) $16 + j0$ **[2011]**

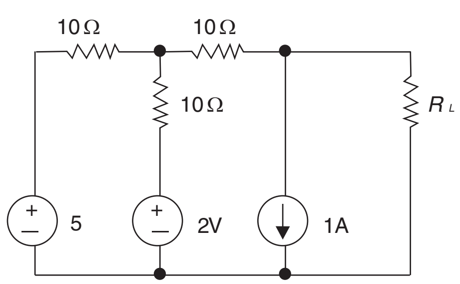
Solution: (a)

In order to find the Norton equivalent, we have to find I_{SC}.



Hence, the correct option is (a)

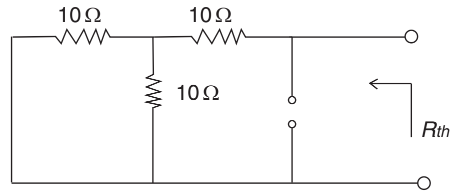
9. In the circuit shown below, the value of R_L such that the power transferred to R_L is maximum is



(a) 5 Ω (b) 10 Ω
(c) 15 Ω (d) 20 Ω **[2011]**

Solution: (c)

For MPT, $R_L = R_{th}$



$R_{th} = 10 + 10 \parallel 10 = 15\Omega$

Hence, the correct option is (c)

10. An independent voltage source in series with an impedance $Z_s = R_s + jX_s$ delivers a maximum average power to a load impedance Z_L, when

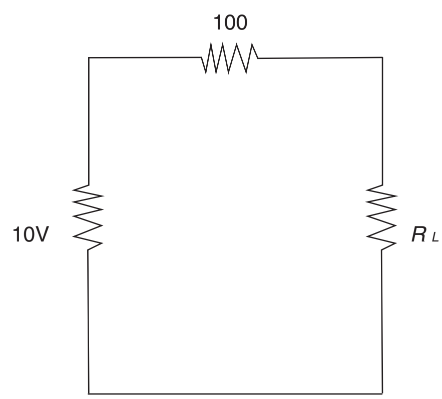
(a) $Z_L = R_s + jX_s$
(b) $Z_L = R_s$
(c) $Z_L = jX_s$
(d) $Z_L = \text{R} - jX_s$ **[2007]**

Solution: (d)

For MPT, $Z_L = Z_S^* = R_S - jX_S$

Hence, the correct option is (d)

11. The maximum power that can be transferred to the load resistor R_L from the voltage source in the figure is



(a) 1 W (b) 10 W
(c) 0.25 W (d) 0.5 W **[2005]**

Solution: (c)

$$= \frac{V_{th}}{4R_L} = \frac{V_{th}^2}{4R_S} = \frac{100}{4 \times 100} = 0.25\,\text{W}$$

Hence, the correct option is (c)

12. A source of angular frequency 1 rad/sec has a source impedance consisting of 1 Ω. resistance in series with 1 H inductance. The load that will obtain the maximum power transfer is

(a) 1 Ω resistance
(b) 1 Ω resistance in parallel with 1 H inductance

(c) 1 Ω resistance in series with 1 F capacitor
(d) 1 Ω resistance in parallel with 1 F capacitor **[2003]**

Solution: (c)

$$Z_1 = Z_S^* = 1 - j(1\Omega \,||\, 1F)$$

Hence, the correct option is (c)

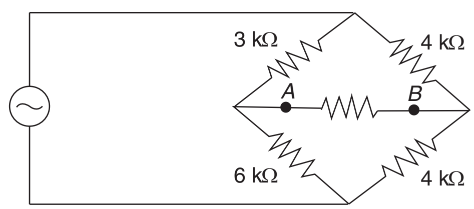
13. Superposition theorem is NOT applicable to networks containing
(a) non–linear elements
(b) dependent voltage sources
(c) dependent current sources
(d) transformers **[1998]**

Solution: (a)

Super position works on principle of linearity, so is applicable to linear network only.

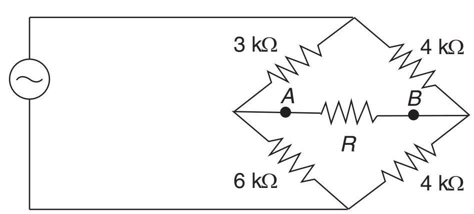
Hence, the correct option is (a)

14. The value of the resistance R, connected across the terminals, A and B, (see the figure), which will absorb the maximum power, is



(a) 4.00 kΩ (b) 4 11 kΩ
(c) 8.00 kΩ (d) 9 00 kΩ **[1995]**

Solution: (a)



$R_{AB} = 3||6 + 4||4 = 4$ kΩ

Hence, the correct option is (a)

15. A generator of internal impedance, Z_G, delivers maximum power to a load impedance, Z_L, only if Z_L = ... **[1994]**

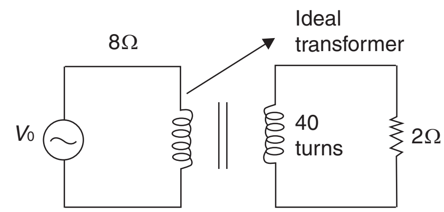
Solution: $R_G - jX_G$

$$Z_L = Z_G^*$$

$$Z_G = R_G + JX_G$$

$$so \quad Z_L = R_G - JX_G$$

16. If the secondary winding of the ideal transformer shown in the circuit of figure has 40 turns, the number of turns in the primary winding for maximum power transfer to the 2 Ω resistor will by



(a) 20 (b) 40
(c) 80 (d) 160 **[1993]**

Solution: (c)

$$\left(\frac{n_2}{n_1}\right)^2 = \frac{\|Z_L\|}{|Z_S|}$$

$$\left(\frac{n_2}{n_1}\right)^2 = \frac{2}{8} + \frac{1}{4}$$

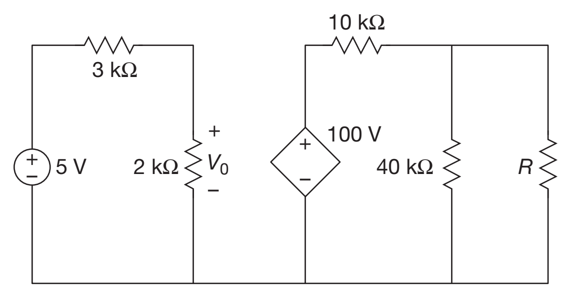
$$\frac{n_2}{n_1} = \frac{1}{2}$$

$n_1 = 2n_2 = 2 \times 40 = 80.$

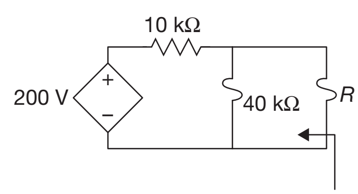
Hence, the correct option is (c)

Two-marks Questions

1. In the circuit shown in the figure, the maximum power (in watt) delivered to the resistor R is _____. **[2016]**



Solution:



Maximum power transfer theorem states that a network will transfer maximum power to load if load impedance will be equal to its internal impedance, i.e., Rth. To find Rth disconnect the load and deactivate all independent sources and find the equivalent impedance across the open circuit.

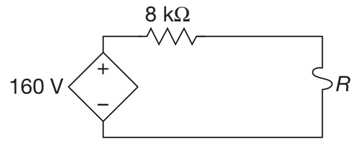
$$R_{th} = \frac{V_{OC}}{I_{SC}}$$

$$V_{OC} = 200x\frac{40}{50}$$

$$= 160 \text{ V}$$

$$I_{SC} = 20 \text{ mA}$$

$$R_{th} = \frac{160}{20} = 8k\Omega$$



$$p = \left(\frac{160}{8+R}\right)^2 x\,R$$

For maxima or minima

$$\frac{dP}{dR} = 0$$

$$\left[\frac{(8+R)^2 - 2R(8+R)}{(8+R)^2}\right] = 0$$

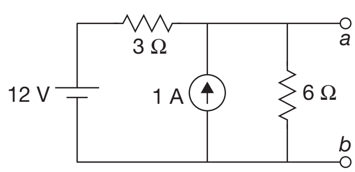
$$R = 8\,\Omega$$

Hence, at load impedance of 8 ohm network will transfer maximum power

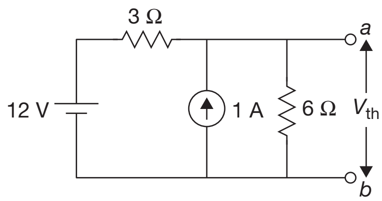
$$p_{max} = \frac{(160)^2}{4x8} = \frac{160\times 160}{4\times 8} = \frac{800}{1000} = 0.8 \text{ W}$$

Hence, the correct Answer is (0.8).

2. For the circuit shown in the figure, the Thevenin equivalent voltage (in Volts) across terminals a–b is ________. **[2015]**



Solution:



Apply Nodal analysis:

$$\frac{V_{th}-12}{3} - 1 + \frac{V_{th}}{6} = 0$$

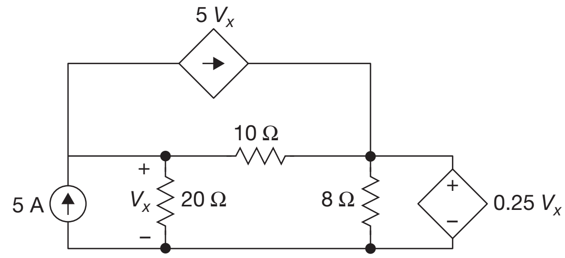
$$2(V_{th} - 12) - 6 + V_{th} = 0$$

$$3V_{th} - 24 - 6 = 0$$

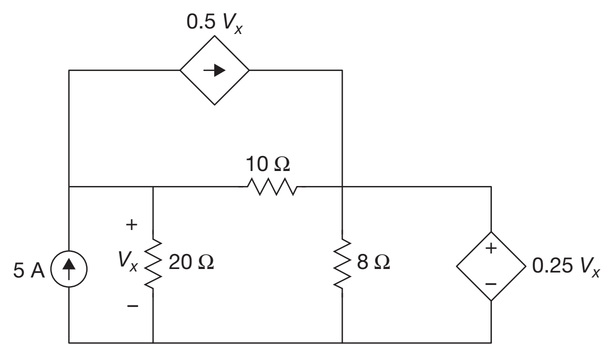
$$V_{th} = 10 \text{ volts}$$

Hence, the correct Answer is (10).

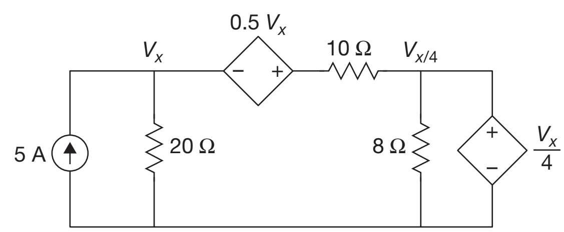
3. In the circuit shown, the voltage V_x (in Volts) is ________. **[2015]**



Solution:



Redraw the given network



Applying Nodal analysis.

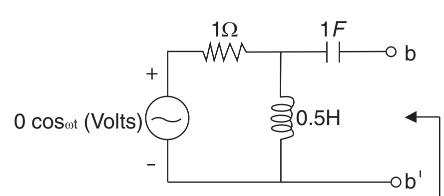
$$-5 + \frac{V_x}{20} + \frac{V_x + 5V_x - 0.25V_x}{10} = 0$$

$$100 = V_x + 2\{5.75\ V_x\}$$

$$V_x = 8 \text{ V}$$

Hence, the correct Answer is (7.95 to 8.05).

4. In the circuit shown in the figure, the angular frequency ω (in rad/s), at which the Norton equivalent impedance as seen from terminals b-b' is purely resistive, is______.



[2014]

Solution:

$Z_{th} = ?$

$$Z_{th} = \frac{1 \times j0.05\omega}{1 + j0.5\omega} + \frac{1}{j\omega}$$

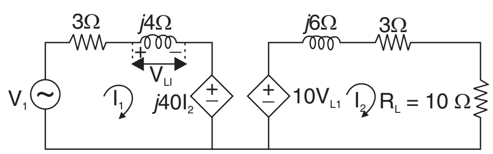
$$Z_{th} = \frac{2 - \omega^2 + j\omega}{2j\omega - \omega^2}$$

$$Z_{th} = -\left(\frac{2\omega^2 + \omega^4 + 2\omega^2}{\omega^2 + 2\omega^2}\right) + j\left(\frac{\omega^3 - 4\omega}{\omega^4 + 2\omega^2}\right)$$

Put $J_m(Z_{th}) = 0$

$\therefore$ ω = 2 rad/s

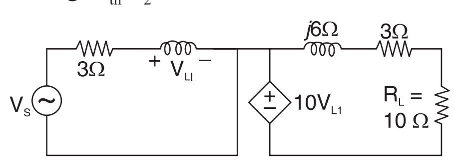
5. In the circuit shown below, if the source voltage $V_s = 100\angle 53.13°$V then the Thevenin's equivalent voltage in volts as seen by the load resistance R_L is



(a) $100\angle 90°$ (b) $800\angle 0°$
(c) $800\angle 90°$ (d) $100\angle 90°$ **[2013]**

Solution: (c)

For finding, V_{th}, $I_2 = 0$

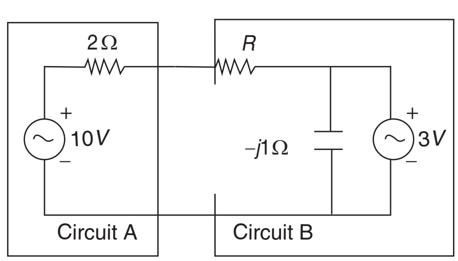


$$V_{L1} = \frac{j4}{j4 + 3} \times V_s = 80\angle 90°$$

$$V_{th} = 10V_{L1} + (3 + 6j)I_2 = 800\angle 90°$$

Hence, the correct option is (c)

6. Assuming both the voltage sources are in phase, the value of R for which maximum power is transferred from circuit A to circuit B is



(a) 0.8 Ω (b) 1.4 Ω
(c) 2 Ω (d) 2.8 Ω **[2012]**

Solution: (a)

Current through $R = \frac{7}{2+R}$

Current through capacitor $= \frac{3}{j}$

Current through 3V source $= \left(\frac{7}{2+R}\right)\frac{-3}{j}$

$$= \frac{7}{(2+R)} + 3j$$

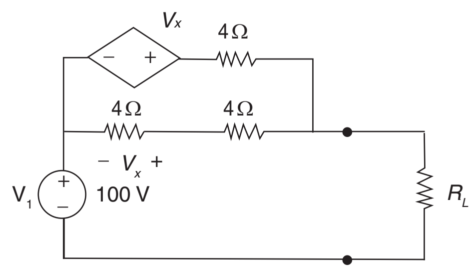
Power delivered from circuit B to A

$$P = \left(\frac{7}{2+R}\right)^2 R + \left(\frac{7}{2+R} - 3j\right)3$$

$$\frac{\partial P}{\partial R} = 0 \Rightarrow R = 0.8\Omega$$

Hence, the correct option is (a)

7. In the circuit shown, what value of R_L maximizes the power delivered to R_L?

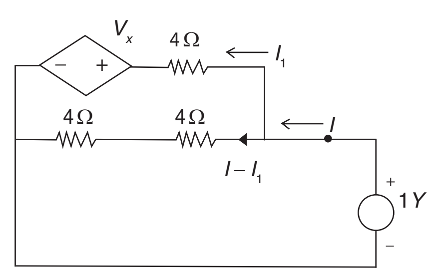


(a) 2.4 Ω (b) $\frac{8}{3}$ Ω
(c) 4 Ω (d) 6 Ω **[2009]**

Solution: (c)

For maximum power transfer, $R_L = R_{eq}$

Applying a source of 1V, we have



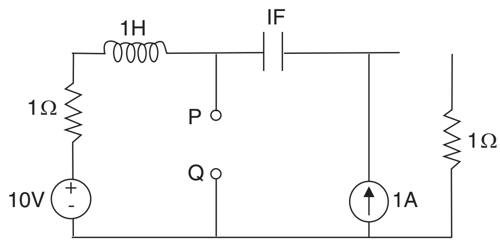
$1 = 4I1 + Vx$

$Vx = 4(I - I_1)$

$$\therefore R_{eq} = \frac{1}{I} = 4 - 2$$

Hence, the correct option is (c)

8. The Thevenin equivalent impedance Z_{th} between the nodes P and Q in the following circuit is



(a) 1 (b) $1+s+\frac{1}{s}$

(c) $2+s+\frac{1}{s}$ (d) $\frac{s^2+s+1}{s^2+2s+1}$

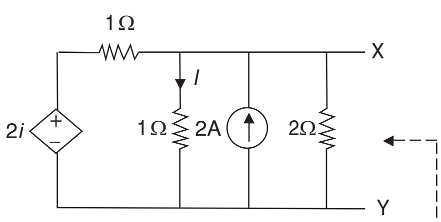
[2008]

Solution: (a)

$$Z_{eq} = (s+1) \,||\, \left(1+\frac{1}{s}\right) = 1$$

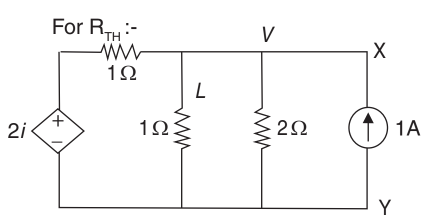
Hence, the correct option is (a)

9. For the circuit shown in the figure, the Thevenin voltage and resistance looking into X–Y are



(a) 4/3 V, 2 Ω (b) 4 V, 2/3 Ω
(c) 4/3 V, 2/3 Ω (d) 4 V, 2 Ω **[2007]**

Solution: (d)



$$1+2i-V = V+\frac{V}{2}$$

$1 + 2i = 2.5$ V

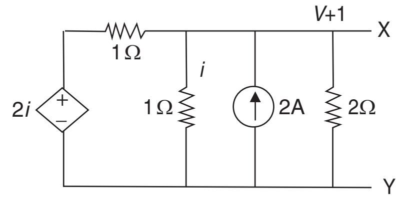
i = $V \Rightarrow 1 + 20 = 2.5$ V

$0.5V = 1$

$V = 2$

$$R_{th} = \frac{V}{1} = 2\Omega$$

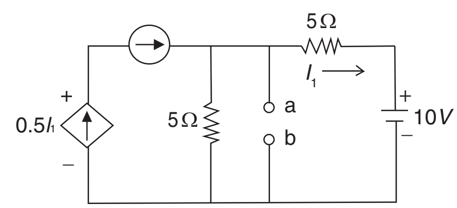
For Vth:–



$$2 = \frac{V_{th}}{2} + \frac{V_{th}}{1} + \frac{V_{th}-2i}{1},\ i = \text{Vth}$$

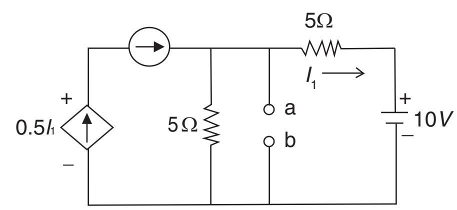
Hence, the correct option is (d)

10. For the circuit shown in the figure, Thevenin's voltage and Thevenin's equivalent resistance at terminals a–b are



(a) 5 V and 2 Ω (b) 7.5 V and 2.5 Ω
(c) 4 V and Ω (d) 3 V and 2.5 Ω **[2005]**

Solution: (b)

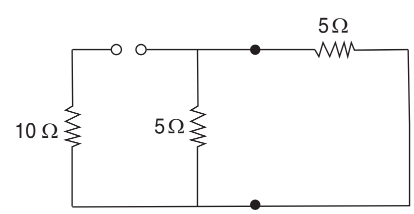


$$\frac{V_{ab}}{5} + \frac{V_{ab}-10}{5} = 1$$

$V_{ab} = 7.5\ V$

$$\frac{0.5J_1}{J_1} = \frac{5}{R} \Rightarrow R = 10\Omega$$

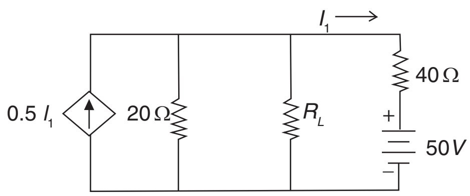
R_{th}:–



$R_{th} = 5||5 = 2.5\Omega$

Hence, the correct option is (b)

11. In the network of the figure, the maximum power is delivered to R_L if its value is

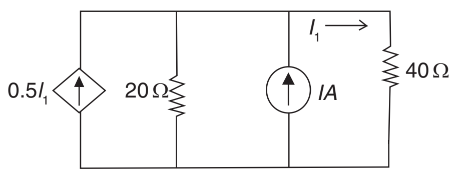


(a) 16 Ω (b) $\frac{40}{3}\Omega$

(c) 60 Ω (d) 20 Ω **[2002]**

Solution: (a)

Short–circuit the independent voltage source and apply 1A sauce in place of R_L



$$0.5J_1 + 1 = I_1 + \frac{V}{20}$$

$$0.5J_1 + \frac{V}{20} = 1$$

$$0.5\frac{V}{40} + \frac{V}{20} = 1$$

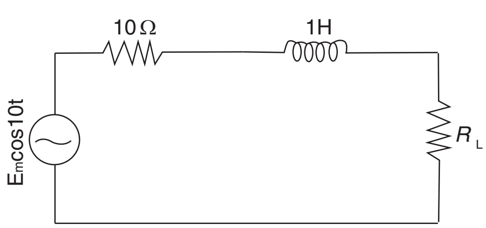
$$V + 4V = 80$$

$$V = 16\ V$$

$$R_{th} = \frac{V}{1} = 16\Omega$$

Hence, the correct option is (a)

12. In the figure, the value of the load resistor R which maximizes the power delivered to it is



(a) 14.14 Ω (b) 10 Ω

(c) 200 Ω (d) 28.28 Ω **[2001]**

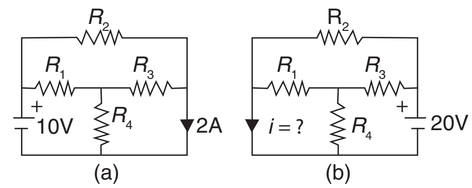
Solution: (a)

$$R_L = \sqrt{R^2 + X_L^2} \text{ for MPT}$$

$$R_L = \sqrt{100 + 100} = 14.14\Omega$$

Hence, the correct option is (a)

13. Use the data of the figure (a). The current I in the circuit of figure (b) will be _____.



(a) –2 A (b) 2 A

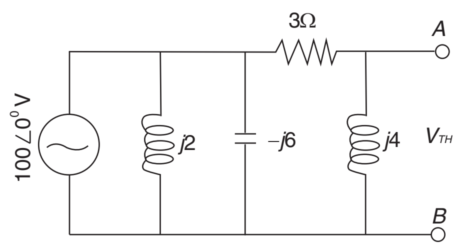
(c) –4 A (d) + 4 A **[2000]**

Solution: (c)

Since the N/W is reciprocal and linear, doubling voltage will double current

$i = -4$A.

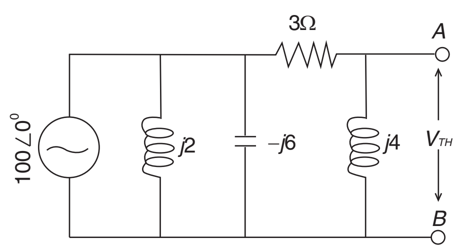
14. The Thevenin equivalent voltage V_{TH} appearing between the terminals A and B of the network shown in the figure is given by



(a) $j16(3 - j4)$ (b) $j16(3 + j4)$

(c) $16(3 + j4)$ (d) $16(3 - j4)$ **[1999]**

Solution: (a)



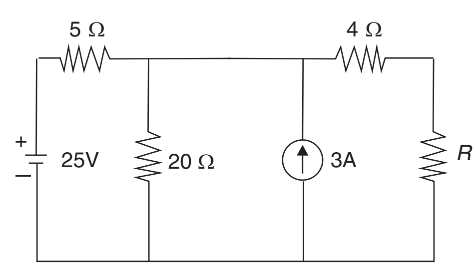
$$V = 100\angle 0°$$

$$V_{th} = 100\angle 0° \times \frac{4j}{3 + 4j}$$

Vth = $16j$ (3 – 4j)

Hence, the correct option is (a)

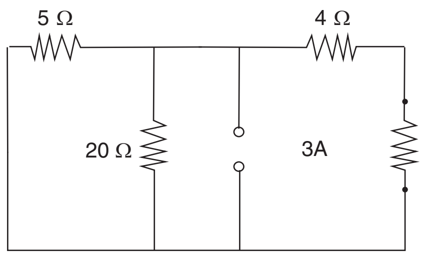
15. The value of R (in ohms) required for maximum power transfer in the network shown in the figure is



(a) 2 (b) 4
(c) 8 (d) 16 **[1999]**

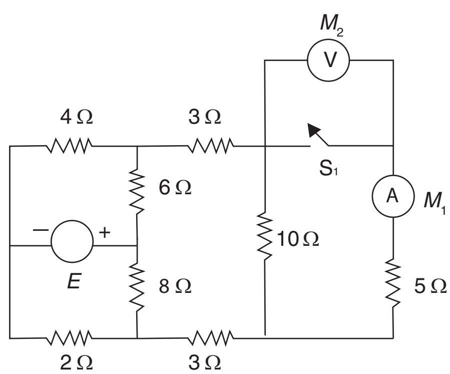
Solution: (c)

$R_{th} = 4 + (5 \parallel 20) = 8\Omega$



Hence, the correct option is (c)

16. In the circuit of the figure given below, when switch S_1 is closed, the ideal ammeter M_1 reads 5A. What will the ideal voltmeter M_2 read when S_1 is kept open ? (The value of E is not specified.) **[1993]**



Solution: 50 V

Across switch S_1, $I_{sc} = 5$ amp

We have to find V_{th}

$V_{th} = I_{SC} R_{th}$

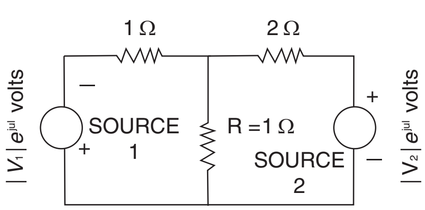
$I_{SC} = 5\text{A}$

$R_{th} = [(4 \parallel 6 + 2 \parallel 8) + 3 + 3] \parallel 10 + 5$

$R_{th} = 10\Omega$

$V_{th} = 10 \times 5 = 50$ V.

17. In the circuit of figure, the power dissipated in the resistor R is '1' W when only source '1' is present and '2' is replaced by a short. The power dissipated in the same resistor R is 4 W when only source '2' is present and '1' is replaced by a short. When both the sources '1' and '2' are present, the power dissipated in R will be :.



(a) 1 W (b) 3 W
(c) 3 W (d) 5 W **[1989]**

Solution: (a)

Given

$P_1 = 1$ W

$P_2 = 1$ W

Since the polarity of both the sources is different

$$P = (\sqrt{P_1} - \sqrt{P_2})^2$$

$$P = (\sqrt{1} - \sqrt{4})^2 = (1-2)^2$$

$$P = 1\text{ W}$$

Hence, the correct option is (a)

18. A load, $Z_L = R_L + jX_L$, is to be matched, using an ideal transformer, to a generator of internal impedance, $Z_s = R_s + jX_s$. The turns ratio of the transformer required is

(a) $\sqrt{|Z_L/Z_S|}$ (b) $\sqrt{|Z_L/R_S|}$

(c) $\sqrt{|Z_L/R_S|}$ (d) $\sqrt{|Z_L/R_S|}$ **[1989]**

Solution: (a)

$$\frac{Z_L}{Z_S} = \left(\frac{n_2}{n_1}\right)^2 \Rightarrow \frac{n_2}{n_1} = \sqrt{\frac{Z_L}{Z_S}}$$

Hence, the correct option is (a)

19. If an impedance Z_L is connected across a voltage source V with source impedance Z_s, then for maximum power transfer the load impedance must be equal to

(a) source impedance Z_s
(b) complex conjugate of Z_s
(c) real part of Z_s
(d) imaginary part of Z_s **[1988]**

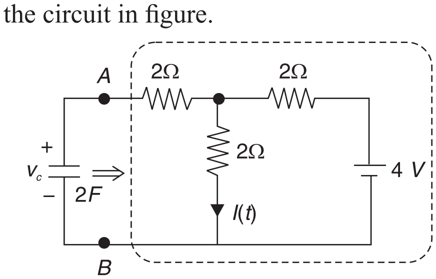
Solution: (b)

For maximum power transfer $Z_L = Z_S{}^*$

Hence, the correct option is (b)

Five-marks Questions

1. For the circuit in figure.



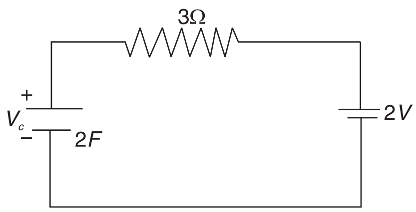
(a) Find the Thevenin equivalent of the sub circuit faced by the capacitor across the terminals A, B.
(b) Find $V_c(t)$, $t > 0$, given $v_c(0) = 0$.
(c) Find $i(t)$, $t > 0$. **[2000]**

Solution: (a)

$$V_{AB} = V_{th} = \frac{2}{2+2} \times 4 = 2V \text{ i.e. Thevenin voltage}$$

And, $R_{th} = 2\|2 + 2 = 1 + 2 = 3\Omega$ i.e. Thevenin Resistance

(b)



$V_C(\sigma) = V_C(0^+) = OV$

$V_c(\infty) = 2V$

$\tau = R_{th}C = 3 \times 2 = 6$ sec. (Time constant)

$$V_c(t) = V_\infty + (1_i - V_\infty)\, e^{-t/\infty}$$
$$= 2 + (0 - 2)e^{-t/b}$$
$$= 2(1 - e^{-t/6})\text{V Ans.}$$

(c) $$V_{0^+} = \frac{2\,\|\,2}{2\,\|\,2+2} \times 4 = \frac{1}{1+2} \times 4 = \frac{4}{3}V$$

$$i_{0^+} = \frac{V_{0^+}}{2} = \frac{2}{3}A$$

$$i_\infty = \frac{4}{4} = 1A$$

$\tau = RC = 6$ sec

$$i(t) = i_\infty + (i_{0+} - i_\infty)e^{-t/\tau}$$

$$= 1 + \left(\frac{2}{3} - 1\right)e^{-t/b}$$

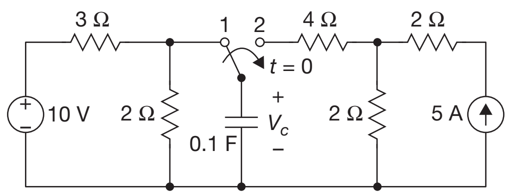
$$= 1 - \frac{1}{3}e^{-t/6}\,Aa$$

Chapter 4

Transient Analysis

One-mark Questions

1. The switch has been in position 1 for a long time and abruptly changes to position 2 at $t = 0$.



If time t is in seconds, the capacitor voltage V_c (in volts) for $t > 0$ is given by **[2016]**

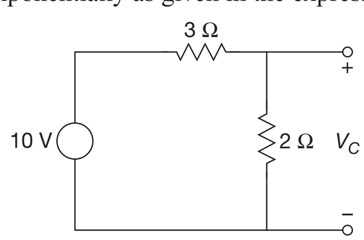
(A) $4(1 - \exp(-t/0.5))$

(B) $10 - 6\exp(-t/0.5)$

(C) $4(1 - \exp(-t/0.6))$

(D) $10 - 6\exp(-t/0.6)$

Solution: As we know that capacitor gets S.C. when AC supply is used and remain O.C when the supply is in form of DC. Here the voltage appearing across C will not die out immediately after switching, instead it will decay exponentially as given in the expression below.



For time $t < 0$:

$$V_C = 10 \times 2/5 = 4\text{V}$$

$$V_c(0^-) = 4\text{ V}$$

$$\underline{\text{For } t > 0:}$$

$$\text{at } t = 0^+:$$

$$V_c(0^+) = 4\text{V}$$

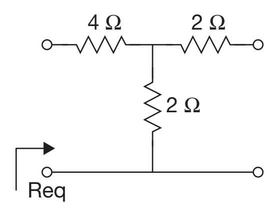
For time $t \to \infty$

$$V_c = 5 \times 2 = 10\text{ V}$$

$$V_c(t) = V_f + (V_i - V_f)\, e^{-t/T}$$

$$= 10 + (4 - 10)e^{-t/T}$$

Now consider the figure given below



$$\text{Req} = 4 + 2 = 6\ \Omega$$

Time constant

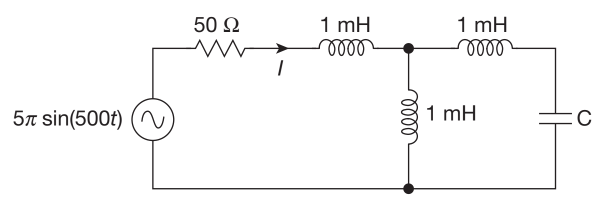
$$T = RC = 6 \times 0.1 = 0.6 \text{ sec}$$

The voltage across capacitor can be expressed as

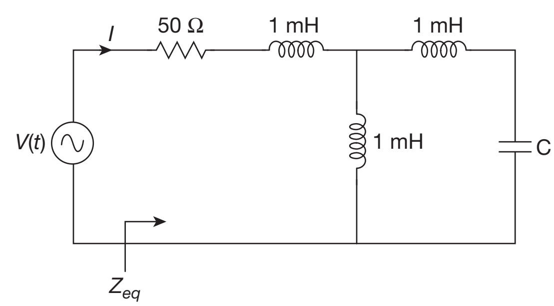
$$V_c(t) = 10^{-6} \cdot e^{(-t/0.6)}$$

Hence, the correct option is (D).

2. In the circuit shown, the current I flowing through the 50 Ω resistor will be zero if the value of capacitor C (in μF) is _______. **[2015]**



Solution: Given network is



Find the equivalent impedance in S-domain

$$Z(s) = (R + SL) + \{SL \parallel (SL + {}^{1}\!/_{SC})\}$$

Let $Z(s) = Z_1 + Z_2$

$$Z_2 = \frac{SL\left\{SL + \frac{1}{SC}\right\}}{SL + SL + \frac{1}{SC}} = \frac{SL\left\{S^2 LC + 1\right\}}{2S^2 LC + 1}$$

But, given $I = 0$

So $Z_2(s) = \infty$ or open circuit

$\therefore \quad 2S^2 LC + 1 = 0$

Sub $S = j\omega$ and L value

$$\omega = 5000$$

$$-2\,\omega^2 LC + 1 = 0$$

$$LC = \frac{1}{2\omega^2}$$

$$C = \frac{1}{2L\omega^2}$$

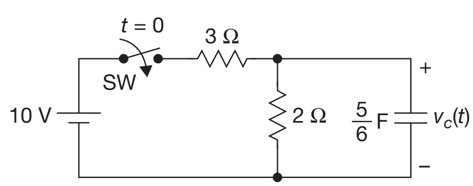
$$C = \frac{1}{2\times1\times10^{-3}\times25\times10^{6}}$$

$$C = 0.02 \times 10^{-3}\ \text{F}$$

$$C = 20\ \mu\text{F}$$

Hence, the correct Answer is (20).

3. In the circuit shown, switch SW is closed at $t = 0$. Assuming zero initial conditions, the value of $V_c(t)$ (in Volts) at t = 1 sec is ______ **[2015]**



Solution: Given all initial conditions are zero

For $t < 0$, switch was opened

So $V_C(0^-) = 0$ V

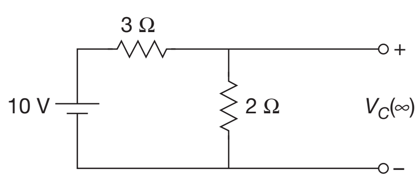
For $t > 0$; switch was closed

at $\qquad t = 0^+$

$$V_C(0^+) = V_C(0^-) = 0\ \text{V}$$

If $t \to \infty$

circuit is in steady state



$$V_C(\infty) = \frac{2\times10}{2+3} = 4V$$

$$\Rightarrow \quad V_c(t) = V_c(\infty) + \{V_c(0^-) - V_C(\infty)\}.\,e^{-t/\tau}$$

$$\tau = R_{eq}.C$$

$$R_{eq} = (3||2) = 1.2\ \Omega$$

$$C = \frac{5}{6}F$$

$$RC = \frac{5}{6}\times\frac{6}{5}$$

$$\tau = 1\ \text{sec}$$

$$V_c(t) = 4 + \{0 - 4\}.e^{-t/1}$$

$$V_c(t) = 4\ \{1 - e^{-t}\}\ \text{volts}$$

at t = 1 sec

$$V_c(1) = 4\ \{1 - e^{-1}\} = 2.528\ \text{V}$$

Hence, the correct Answer is (2.48 to 2.58).

4. For maximum power transfer between two cascaded sections of an electrical network, the relationship between the output impedance Z_1 of the first section to the input impedance Z_2 of the second section is

(a) $Z_1 = Z_1$ (b) $Z_2 = -Z_1$
(c) $Z_2 = Z_1^*$ (d) $Z_2 = -Z_1^*$

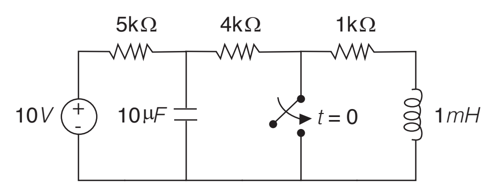
[2014]

Solution: (c)

$Z_2 = Z_1^*$ for MPT

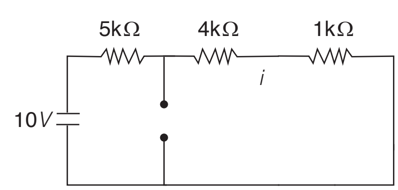
Hence, the correct option is (c).

5. In the figure shown, the ideal switch has been open for a long time. If it is closed at $t = 0$, then the magnitude of the current (in mA) through the 4 kΩ resistor at $t = 0+$ is__________. **[2014]**



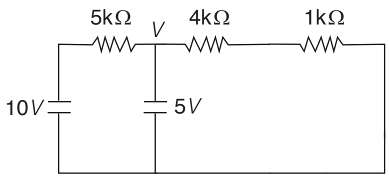
Solution: 1.25 mA

At $t = 0^-$



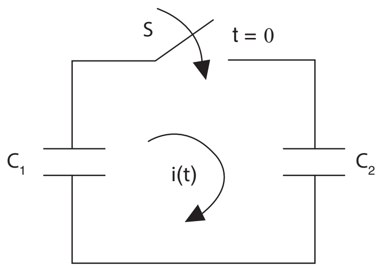
$$i = \frac{10\ \text{V}}{10\ \text{k}\Omega} = 1\ \text{mA}$$

At $t = 0$ switch gets closed



$$I = \frac{5}{4} = 1.25 \text{ mA}$$

6. In the following figure, C_1 and C_2 are ideal capacitors. C_1 had been charged to 12V before the ideal switch S is closed at $t = 0$. The current $i(t)$ for all t is



(a) zero
(b) a step function
(c) an exponentially decaying function
(d) an impulse function **[2012]**

Solution: (d)

$R = 0 \Rightarrow T = 0 \Rightarrow$ capacitor allows sudden change of voltage through it when impulse response is found i.e. excitation = $\delta(t)$

Hence, the correct option is (d).

7. In the following circuit, the switch S is closed at $t = 0$. The rate of change of current $\frac{di}{dt}(0^+)$ is given by

(a) 0 (b) $\frac{R_s I_s}{L}$

(c) $\frac{(R+R_s)I_s}{L}$ (d) ∞ **[2008]**

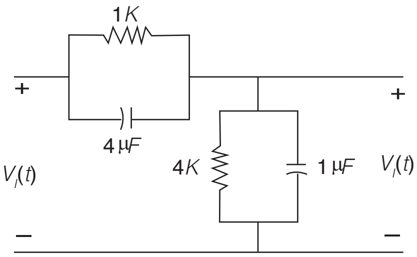
Solution:(b)

$$V_L = L\frac{di}{dt}(0^+)$$

$$\frac{di}{dt}(0^+) = \frac{V_L}{I} = \frac{I_S R_S}{L}$$

Hence, the correct option is (b).

8. In the figure shown below, assume that all the capacitors are initially uncharged. If $v_1(t) = 10\,u(t)$ volts, $v_0(t)$ is given by



(a) $8e^{-1/0.004}$ volts
(b) $8(1 - e^{-t/0.004})$ volts
(c) $8\,u(t)$ volts
(d) 8 volts **[2006]**

Solution: (c)

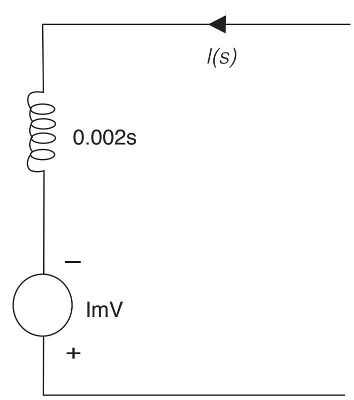
As $R_1C_1 = R_2C_2$

$$V_0 = \frac{Z_2}{Z_1+Z_2}V_1(t) = 0.8\times 10\,u(t)$$

$$V_0 = \delta u(-1)$$

Hence, the correct option is (c).

9. A 2 mH inductor with some initial current can be represented as shown below where s is the Laplace Transform variable. The value of initial current is



(a) 0.5 A (b) 2.0 A
(c) 1.0 A (d) 0.0 A **[2006]**

Solution: (a)

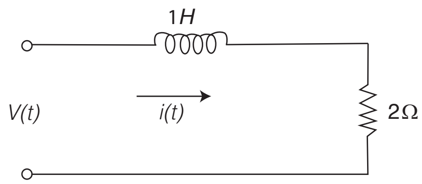
$$V = L\frac{di}{dt}$$

$V(s) = sLI(s) - Li(0^+)$

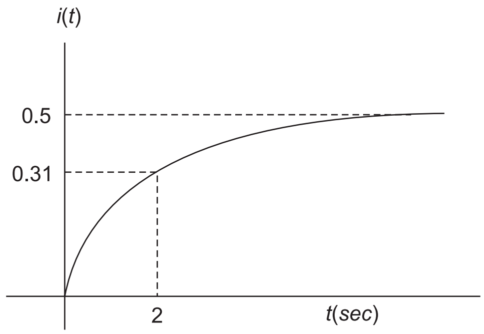
$-Li(0^+) = -1 \text{ mV} \Rightarrow i(0^+) = 0.5 \text{ A}$

Hence, the correct option is (a).

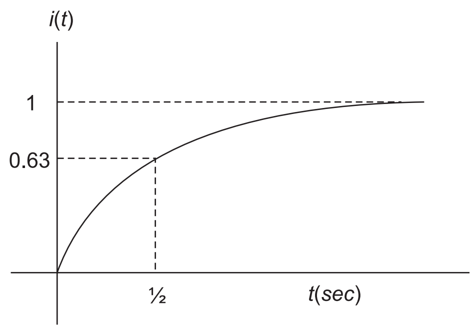
10. For the R-L circuit shown in the figure, the input voltage $v_1(t) = u(t)$. The current $i(t)$ is



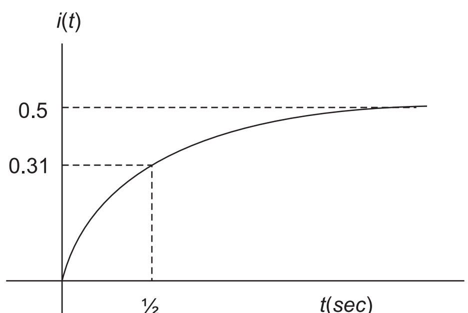
(a)



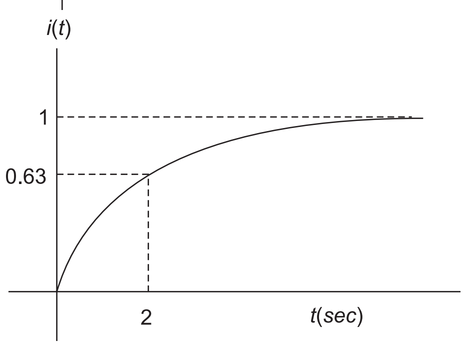
(b)



(c)



(d)



[2004]

Solution: (c)

$$V_1(t) = U(t) \rightarrow V_1(s) = \frac{1}{s}$$

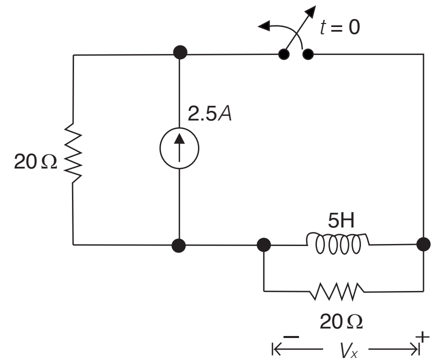
$$I(s) = \frac{1}{s(2+s)} = \left[\frac{1}{s} - \frac{1}{s+2}\right] \times \frac{1}{2}$$

$$i(t) = \frac{1}{2}\left\langle 1 - e^{-2t}\right\rangle$$

At $t = 0$, $i(t) = .5$

Hence, the correct option is (c).

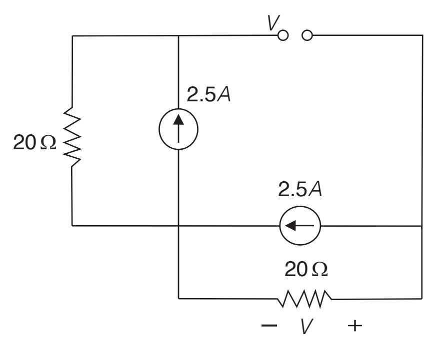
11. In the figure, the switch was closed for a long time before opening at $t = 0$ The voltage V_x at $t = 0+$ is



(a) 25 V (b) 50 V
(c) –50 V (d) 0 V **[2002]**

Solution: (c)

Steady state, $i_L(0^-) = 2.5\text{A}$



$\therefore V = -2.5 \times 20 = -50\text{V}$

The circuit is given. Assume that the switch S is in position 1 for a long time and thrown to position 2 at $t = 0$.

Hence, the correct option is (c).

12. A ramp voltage, $v(t) = 100\ t$ volts, is applied to a RC differentiating circuit with $R = 5\ \text{k}\Omega$ and $C = 4\mu\text{F}$. The maximum output voltage is

(a) 0.2 volt (b) 2.0 volts
(c) 10.0 volts (d) 50.0 volts

[1994]

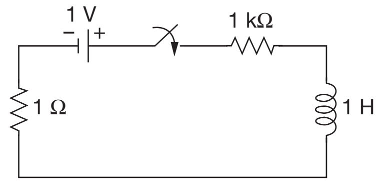
Solution: (b)

$$V = RC\frac{dv}{dt} = 2\text{V}$$

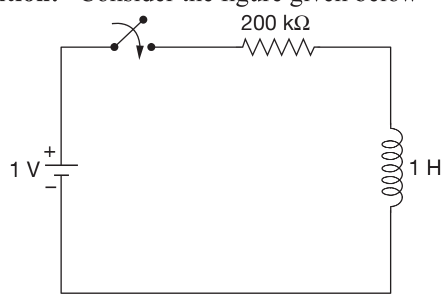
Hence, the correct option is (b).

Two-marks Questions

1. For the circuit given in the figure, the magnitude of the loop current (in amperes, correct to three decimal places) 0.5 second after closing the switch is ________. **[2018]**



Solution: Consider the figure given below



As per circuit diagram given above we have

$$I_L(o^-) = I_L(o^+) = 0$$

For time $t > 0$:

As $t \to \infty$

$$I_L(\infty) = \frac{1}{2}\text{ A}$$

$$\tau = \frac{L}{R} = \frac{1}{2} = 0.5\text{ sec}$$

We know that:

$$i(t) = I_O(1 - e^{-t/\tau})\text{Amp}$$

$$= 0.5\,(1 - e^{-t/0.5})$$

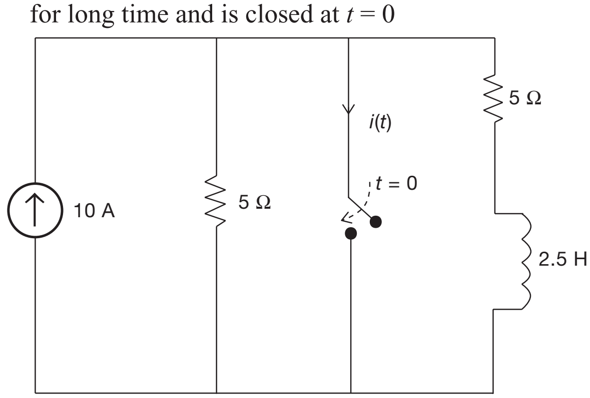
At $t = 0.5$ sec

$$I_L(0.5) = 0.5\,(1 - e^{-1})$$

$$= 0.316$$

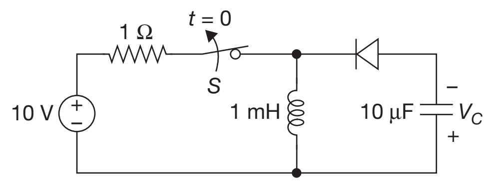
Hence, the correct answer is 0.284 to 0.348.

2. The switch in the circuit, shown in the figure, was open for long time and is closed at $t = 0$



The current $i(t)$ (in ampere) at $t = 0.5$ seconds is ________. **[2017]**

3. The switch S in the circuit shown has been closed for a long time. It is opened at time $t = 0$ and remains open after that. Assume that the diode has zero reverse current and zero forward voltage drop.



The steady state magnitude of the capacitor voltage V_c (in volts) is ______. **[2016]**

Solution: Inductance $L = 1$ mH,

Voltage $Vc = 10 \times 10^{-6}$ F,

Resistance $R = 1$ ohm

Energy from inductor transfers to capacitor

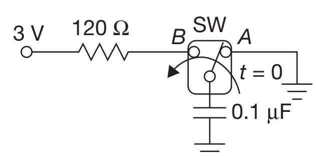
$$\frac{1}{2}LI^2 = \frac{1}{2}CV^2$$

$$\frac{1}{2}\times 10^{-3}\times 100 = \frac{1}{2}\times 10\times 10^{-6}\times V_c^2$$

$$V_c = \sqrt{10^4} = 100\text{ volts}$$

Hence, the correct Answer is (100 volts).

4. In the circuit shown, the switch SW is thrown from position A to position B at time $t = 0$. The energy (in μJ) taken from the 3 V source to charge the 0.1 μF capacitor from 0 V to 3 V is **[2015]**



(A) 0.3 (B) 0.45
(C) 0.9 (D) 3

Solution: For $t < 0$:

at $t = 0^-$, the switch connected to position A

So, $V_C(0^-) = 0$ V.

For $t > 0$:

at $t = 0^+$. The switch was connected to position B.

$$V_C(0^+) = V_C(0^-) = 0\text{ V}$$

If $t \to \infty$,. $V_C(\infty) = 3$ V

$$\tau = Rc = 120 \times 0.1 \times 10^{-6} = 12\ \mu\text{ sec}$$

$$V_C(t) = V_o\{1 - e^{-t/\tau}\} = 3.\{1 - e^{-t/\tau}\}\text{ volts}$$

$$i_C(t) = C.\frac{dV_c(t)}{dt} = 3\times 0.1\times 10^{-6}\left\{\frac{d}{dt}\right\}\left\{1-e^{-t/\tau}\right\}$$

$$i_c(t) = \frac{0.3\times 10^{-6}}{\tau}\left\{e^{-t/\tau}\right\}$$

$$i_c(t) = \frac{0.3}{12}.e^{-t/\tau}$$

We know Instantaneous power of source = $V(t)\cdot i(t)$

$$P(t) = 3.\left\{\frac{0.3}{12}\cdot e^{-t/\tau}\right\}$$

$$= \frac{0.3}{4}\cdot e^{-t/\tau}$$

$$E = \int_0^\infty p.dt = \int_0^\infty \frac{0.3}{4}\cdot e^{-t/\tau}$$

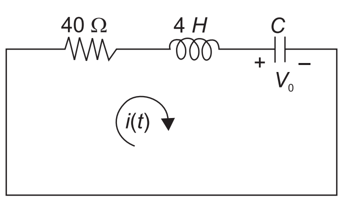
$$= \frac{0.3}{4}\{\tau\} = \frac{0.3}{4}\times 12\,\mu J$$

$$E = 0.9\ \mu\text{J}$$

Hint: Total energy of the source = CV^2

Hence, the correct option is (C).

5. In the circuit shown in the figure, the value of capacitor C (in mF) needed to have critically damped response $i(t)$ is______.



[2014]

Solution: 10 mF

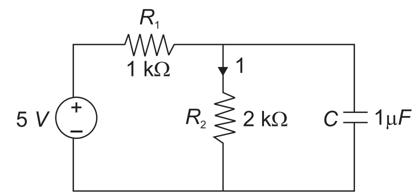
For series R-L-C, $\theta = \frac{1}{R}\sqrt{\frac{L}{C}}$

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\xi = 1 \Rightarrow \left(\frac{2}{R}\right)^2 = \frac{C}{L}$$

$$C = \frac{4}{R^2}L = 10\text{ mF}$$

6. In the figure shown, the capacitor is initially uncharged. Which one of the following expressions describes the current $I(t)$ (in mA) for $t > 0$?



(a) $I(t) = \frac{5}{3}(1-e^{-t/\tau}),\ \tau = \frac{2}{3}\text{ m sec}$

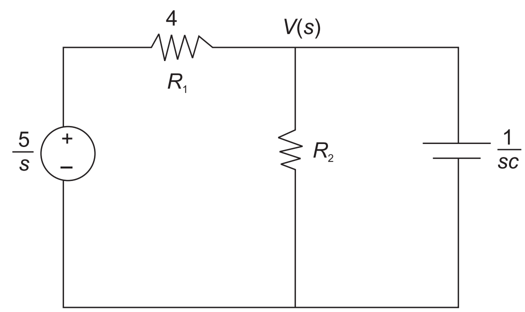
(b) $I(t) = \frac{5}{2}(1-e^{-t/\tau}),\ \tau = \frac{2}{3}\text{ m sec}$

(c) $I(t) = \frac{5}{3}(1-e^{-t/\tau}),\ \tau = 3\text{ m sec}$

(d) $I(t) = \frac{5}{2}(1-e^{-t/\tau}),\ \tau = 3\text{ m sec}$

[2014]

Solution: (a)



Applying KCL, $\frac{V(s)-\frac{5}{6}}{R_1} + \frac{V(s)}{R_2} + \frac{V(s)}{SC} = 0$

$R_1 = 1\ \text{k}\Omega$, $R_2 = 2\ \text{k}\Omega$, $C = 1\ \mu\text{F}$

$$V(s)\left[1+\frac{1}{2}+s\right] = \frac{5}{6}$$

$$V(s) = \frac{5}{s\left(s+\frac{3}{2}\right)}$$

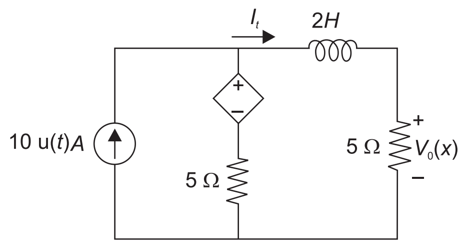
$$V(s) = \frac{10}{3s} + \frac{10}{3\left(3+\frac{3}{2}\right)}$$

$$V(t) = \frac{10}{3}\left[1-e^{-\frac{2}{3}t}\right]$$

$$i(t) = \frac{V(t)}{R_2} = \frac{5}{3}\left[1-e^{-\frac{3}{2}t}\right]\text{mA}$$

Hence, the correct option is (a).

7. In the circuit shown in the figure, the value of $v_0(t)$ (in volts) for $t \to \infty$ is____.



[2014]

Solution: 31.25 V

At $t \to \infty$ $L \to S.C$ $C \to O.C.$

$V_x = 5i_x$

$$\text{Also, } 10 = \frac{V_x - Z_{ix}}{5} + i_x$$

$$10 = \frac{5i_x - 2i_x}{5} + i_x$$

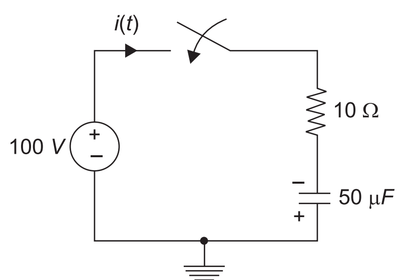
$$10 = \frac{3i_x}{5} + i_x$$

$10 = 1.6i_x$

$$i_x = \frac{100}{16}$$

$$V_x = 5i_x = \frac{500}{16} = 31.25\text{V}$$

8. In the circuit shown below, the initial charge on the capacitor is 2.5 mC, with the voltage polarity as indicated. The switch is closed at time $t = 0$. The current $i(t)$ at a time t after the switch is closed is



(a) $i(t) = 15\exp(-2 \times 10^3 t)$ A
(b) $i(t) = 5\exp(-2 \times 10^3 t)$ A
(c) $i(t) = 10\exp(-2 \times 10^3 t)$ A
(d) $i(t) = -5\exp(-2 \times 10^3 t)$ A **[2011]**

Solution: (a)

$Q_0 = 2.5$ mC, $C = 50$ μF

$$V = \frac{Q}{C} = 50\text{V}$$

$$V(\infty) = 100 \text{ V}$$

$$V(0^+) = -50 \text{ V}$$

$$V(t) = V(\infty) + \left[V(0) - V(\infty)\right]e^{-t/T}$$

$T = RC = 5 \times 10^{-4}$

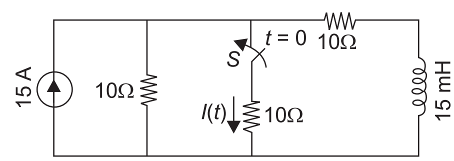
$V(t) = 100 - 150e^{-t/T}$

$$i(t) = C\frac{dv}{dt} = \frac{+150}{t} \times C \times e^{-t/\tau}$$

$$i(t) = 15e^{-2\times10^3 t}\text{A}$$

Hence, the correct option is (a).

9. In the circuit shown, the switch S is open for a long time and is closed at $t = 0$ The current $i(t)$ for $t \geq 0+$ is



(a) $i(t) = 0.5 - 0\ 125e^{-1000t}$ A
(b) $i(t) = 1.5 - 0.125\ e^{-1000t}$ A
(c) $i(t) = 0.5 - 0.5e^{-1000t}$ A
(d) $i(t) = 0.375\ e^{-1000t}$ A **[2010]**

Solution: (a)

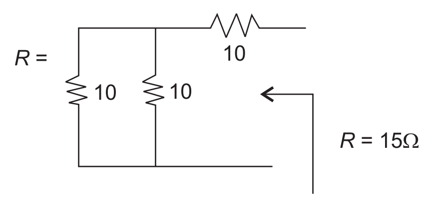
When circuit is in steady state then

$$i_L = \frac{0.75\text{A}}{2}\ (t = 0) = 0.375 \text{ A}$$

At $t = \infty$ $i_L = 0.5$A $(t = \infty)$

$$i_L(t) = i(\infty) + \left[i(0) - i(\infty)\right]e^{\frac{-tR}{L}}$$

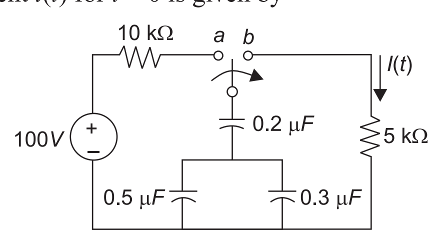
$$i_L(t) = 0.5 - 0.125e^{-\frac{tR}{L}}$$



$\therefore\ i(t) = 0.5 - 0.125\ e^{-1000t}$

Hence, the correct option is (a).

10. The switch in the circuit shown was on position a for a long time, and is moved to position b at time $t = 0$. The current $i(t)$ for $t > 0$ is given by

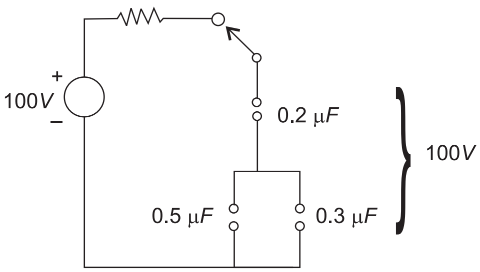


(a) $0.2e^{-125t}u(t)$ mA (b) $20e^{-1250t}u(t)$ mA
(c) $0\ 2e^{-1250t}, u(t)$ mA (d) $20e^{-1000t}u(t)$ mA

[2009]

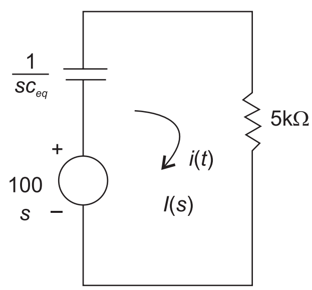
Solution: (b)

In position a:-



$C_{eq} = (0.5 + 0.3) \parallel 0.2 = 0.16\ \mu F$

$t > 0$



$$I(s) = \frac{100}{s} \times \frac{1}{\left(\frac{1}{Sc_{eq}} + 5\right)}; C_{eq} = 16\ \mu F$$

$$i(t) = \frac{100}{5} \times e^{-t \times 1250}$$

$$i(t) = 20e^{-1250t}$$

11. The time domain behaviour of an RL circuit is represented by

$$L\frac{di}{dt} + Ri = V_0(1 + Be^{-Rt/L}\ \sin t)u(t)$$

For an initial current of $i(0) = \frac{V_0}{R}$ the steady-state value of the current is given by

(a) $i(t) \to \frac{V_0}{R}$

(b) $i(t) \to \frac{2V_0}{R}$

(c) $i(t) \to \frac{V_0}{R}(1 - B)$

(d) $i(t) \to \frac{V_0}{R}(1 + B)$

[2009]

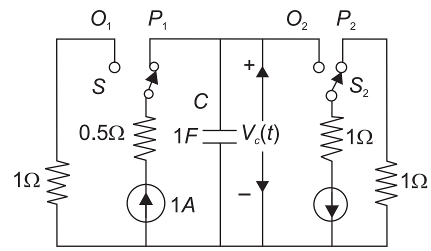
Solution: (a)

Hence, the correct option is (a)

12. The circuit shown in the figure is used to charge the capacitor C alternately from two current sources as indicated. The switches S_1 and S_2 are mechanically coupled and connected as follows:

For $2nT \le t < (2n + 1)T$, $(n = 0, 1, 2...)$ S_1 to P_1 and S_2 to P_2.

For $(2n + nT \le t < (2n + 2)T$, $(n = 0,1,2....)$ S_1 to Q_1 and S_2 to Q_2



Assume that the capacitor has zero initial charge. Given that $u(t)$ is a unit step function, the voltage $V_c(t)$ across the capacitor is given by

(a) $\sum_{n=0}^{\infty}(-1)^n tu(t - nT)$

(b) $u(t) + 2\sum_{n=1}^{\infty}(-1)^n u(t - nT)$

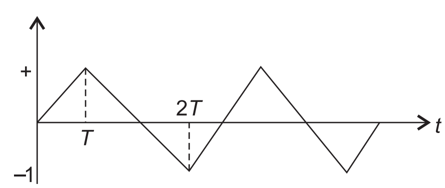
(c) $tu(t) + 2\sum_{n=1}^{\infty}(-1)^n (t - nT)u(t - nT)$

(d) $\sum_{n=0}^{\infty}\left[0.5 - e^{-(t-2nT)} + 0.5e^{-(t-2nT-T)}\right]$

[2008]

Solution: (c)

The waveform of voltage $V_C(t)$ is as shown:



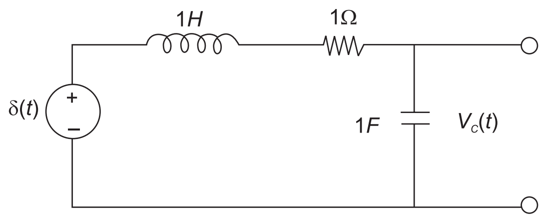
$V_C(t) = tu(t) - 2(t - T)\ u(t - T) + 2(t - 2T)\ u(t - 2T)$

$$V_C(t) = tu(t) + 2\sum_{n=1}^{\infty}(-1)^n (t - nT)u(t - nT)$$

Hence, the correct option is (c)

Common Data for Questions 9 and 10

The following series RLC circuit with zero initial conditions is excited by a unit impulse function $\delta(t)$.



13. For $t > 0$, the output voltage $V_c(t)$ is

(a) $\frac{2}{\sqrt{3}}\left(e^{-\frac{1}{2}t} - e^{-\frac{\sqrt{3}}{2}t}\right)$

(b) $\frac{2}{\sqrt{3}} t e^{-\frac{1}{2}t}$

(c) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right)$

(d) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right)$

[2008]

Solution (d)

$$V_C(s) = \frac{1}{s} \times \frac{1}{\frac{1}{s} + s + 1}$$

$$1 \times \frac{1/s}{\frac{s^2+s+1}{s}} = \frac{1}{(s^2+s+1)}$$

$$V_C(s) = \frac{1}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$V_C(t) = \frac{2}{\sqrt{3}} e^{-1/2} \sin\left(\frac{\sqrt{3}t}{2}\right)$$

Hence, the correct option is (d).

14. For $t > 0$, the voltage across the resistor is

(a) $\frac{1}{\sqrt{3}}\left(e^{-\frac{\sqrt{3}}{2}} - e^{-\frac{1}{2}t}\right)$

(b) $e^{-\frac{1}{2}t}\left[\cos\left(\frac{\sqrt{3}t}{2}\right) - \frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}t}{2}\right)\right]$

(c) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}t}{2}\right)$

(d) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{3}t}{2}\right)$

[2008]

Solution: (b)

$$V_R(s) = \frac{1}{\left(s+1+\frac{1}{s}\right)} = \frac{s}{s^2+s+1}$$

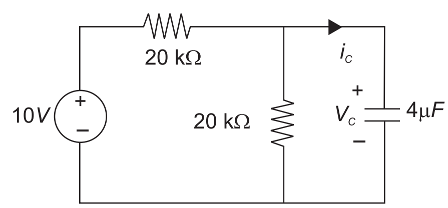
$$V_R(s) = \frac{s+\frac{1}{2}-\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$V_R(s) = \frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)^2} - \frac{\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$V_R(t) = e^{-t/2}\cos\left(\frac{\sqrt{3}t}{2}\right) = \frac{1}{\sqrt{3}} e^{-t/2} \sin\left(\frac{\sqrt{3}t}{2}\right)$$

Hence, the correct option is (b).

15. In the circuit shown, V_c is 0 volts at $t = 0$ sec. For $t > 0$, the capacitor current $i_c(t)$, where t is in seconds, is given by



(a) 0.50 exp (–25 t) mA
(b) 0.25 exp (–25 t) mA
(c) 0.50 exp (–12.5 t)mA
(d) 0.25 exp (–6.25 t) mA **[2007]**

Solution: (a)

We know that $i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/PC}$

At infinity capacitor will behave as output and $i(\infty) = 0$

For finding i(0), S.C. the capacitor.

$$\therefore i_C(0) = \frac{10}{20} = 0.5 \text{ mA}$$

$$T = R_{eq}C(20 \| 20)\text{k}\Omega \times 4 \,..\text{F} = 40 \text{ ms}$$

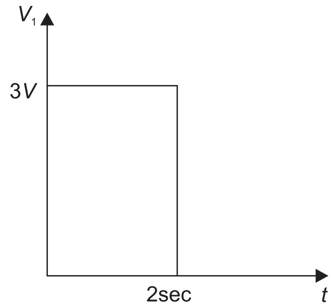
$$\therefore i_C(t) = 0.5e^{-25t}\text{mA}$$

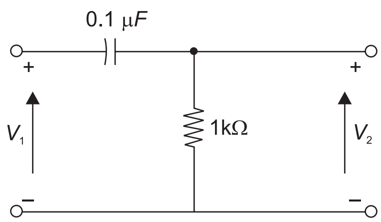
Hence, the correct option is (a).

16. A square pulse of 3 volts amplitude is applied to C-R circuit shown in the figure. The capacitor is initially uncharged. The output voltage V_2 at time $t = 2$ sec is

(a) 3 V (b) –3 V
(c) 4 V (d) –4V

Solution: (b)





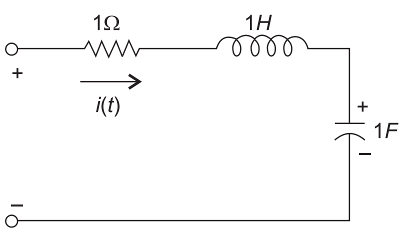
[2005]

As $RC = 100\ \mu s \Rightarrow$ steady state will be reached in approx. 500 μs and at 2s, it will be in state.

$\therefore V_C = 3V,\ V_2 = -V_C = -3\text{V}$

Hence, the correct option is (b).

17. The circuit shown in the figure has initial current $I_L(0) = 1$ A through the inductor and an initial voltage $v_c(0^-) = -1$ V across the capacitor. For input $v(t) = u(t)$, the Laplace transform of the current $i(t)$ for $t > 0$ is



(a) $\dfrac{s}{s^2+s+1}$ (b) $\dfrac{s+2}{s^2+s+1}$

(c) $\dfrac{s-2}{s^2+s+1}$ (d) $\dfrac{s-2}{s^2+s+1}$ **[2004]**

Solution: (b)

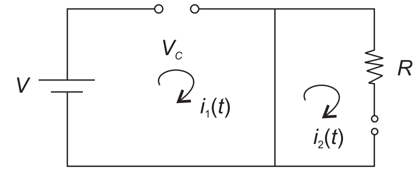
$$V(t) = Ri(t) + L\frac{di}{dt}(t) + \frac{1}{C}\int_0^\infty i(t)dt$$

$$V(s) = RI(s) + sLI(s) - LI(0^{-1}) + \frac{I(1)}{SC} + \frac{V_C(0^+)}{s}$$

$$\therefore I(s) = \frac{s+2}{s^2+s+1}$$

Hence, the correct option is (a).

18. At $t = 0+$, the current i_1 is

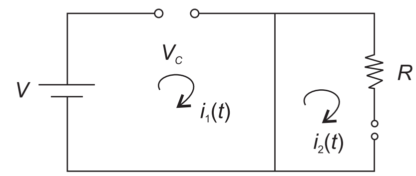


(a) $\dfrac{-V}{2R}$ (b) $\dfrac{-V}{R}$

(c) $\dfrac{-V}{4R}$ (d) zero **[2003]**

Solution: (a)

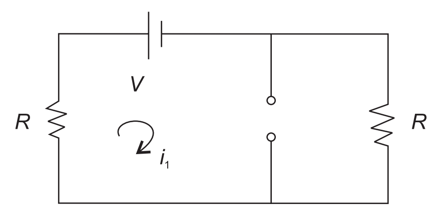
In steady state,



$i_1(t) = i_2(t) = 0$

$V_C(0^-) = V$

At $t = 0^+$



$$\therefore 4 = \frac{-V}{2R}$$

Hence, the correct option is (a).

19. $I_1(s)$ and $I_2(s)$ are the Laplace transforms of $i_1(t)$ and $i_2(t)$, respectively. The equations for the loop currents $I_1(s)$ and $I_2(s)$ for the circuit shown in the figure, after the switch is brought from position 1 to position 2 at $t = 0$, are

(a) $$\begin{bmatrix} R + Ls + \dfrac{1}{Cs} & -Ls \\ -Ls & R + \dfrac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{V}{s} \\ 0 \end{bmatrix}$$

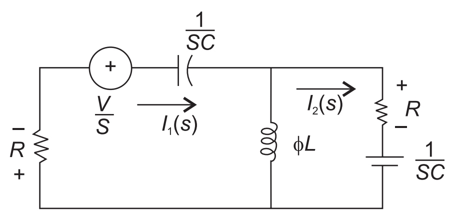
(b) $$\begin{bmatrix} R + Ls + \dfrac{1}{Cs} & -Ls \\ -Ls & R + \dfrac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} -\dfrac{V}{s} \\ 0 \end{bmatrix}$$

(c) $$\begin{bmatrix} R + Ls + \dfrac{1}{Cs} & -Ls \\ -Ls & R + Ls + \dfrac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} -\dfrac{V}{s} \\ 0 \end{bmatrix}$$

(d) $$\begin{bmatrix} R+Ls+\dfrac{1}{Cs} & -Ls \\ -Ls & R+Ls+\dfrac{1}{Cs} \end{bmatrix}\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}=\begin{bmatrix} \dfrac{V}{s} \\ 0 \end{bmatrix}$$

[2003]

Solution: (c)



KVL in loop 1:-

$$I_1(b)R+\frac{V}{s}+I_1(b).\frac{1}{SC}+\left[J_1(s)-I_2(s)\right]OL=0$$

KVL in loop 2:-

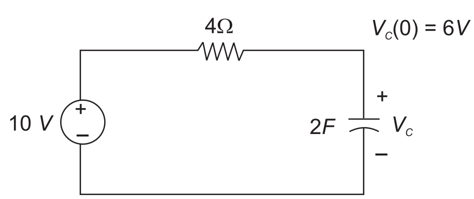
$$\left[I_2(s)-I_1(s)sL+I_2(s)R+I_2(s).\frac{1}{SC}=0\right](2)$$

$\therefore$ From (1) and (2)

$$\begin{bmatrix} R+sL+\dfrac{1}{SC} & -sL \\ -sL & R+sL+\dfrac{1}{SC} \end{bmatrix}\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}=\begin{bmatrix} \dfrac{-V}{s} \\ 0 \end{bmatrix}$$

Hence, the correct option is (c).

20. In the circuit of the figure the energy absorbed by the 4 Ω resistor in the time interval



(a) 36 Joules
(b) 16 Joules
(c) 256 Joules
(d) None of the above **[1997]**

Solution: (b)

$V_C(\infty)=10\text{V}$

$V_C(0)=\sigma\text{V}$

$V_R = 10 - 6 = 4\text{V}$

$I_R = 1\text{A}$

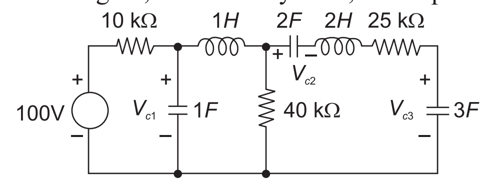
$i(t)=i(\infty)_t+[i(0)-i(\infty)]e^{-t/PC}$

$i(t)_\infty = e^{t/\delta}$

$$E=\int_0^\infty i^2Rdt=16\text{J}$$

Hence, the correct option is (b).

21. The voltage V_{C1}, V_{C2} and V_{C3} across the capacitors in the circuit in figure, under steady state, are respectively.

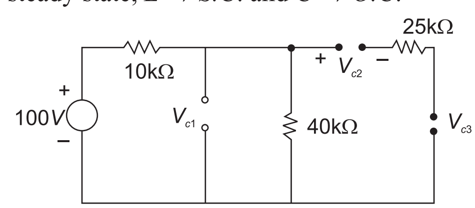


(a) 80 V, 32 V, 48 V (b) 80 V, 48 V, 32 V
(c) 20 V, 8 V, 12 V (d) 20 V, 12 V, 8

[1996]

Solution: (b)

At steady state, $L \to S.C.$ and $C \to O.C.$



$$V_{C_1}=100\times\frac{40}{50}=80\text{V}$$

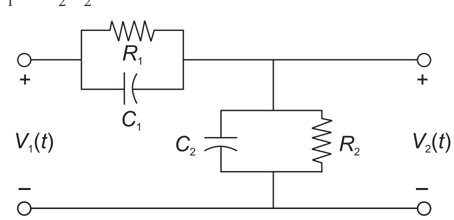
$$V_{C_2}=80\times\frac{3}{2+3}=48\text{V}$$

$$V_{C_3}=80-48=32\text{V}$$

Hence, the correct option is (b).

22. For the compensated attenuator of figure, the impulse response under the condition

$R_1C_1=R_2C_2$ is



(a) $\dfrac{R_2}{R_1+R_2}[1-e^{\frac{1}{R_1C_1}}]u(t)$

(b) $V_2(t)=\dfrac{R_2}{R_1+R_2}\delta(t)$

(c) $\dfrac{R_2}{R_1+R_2}u(t)$

(d) $\dfrac{R_2}{R_1+R_2}e^{\frac{1}{R_1C_1u(t)}}$

[1992]

Solution: (b)

$$\frac{V_2(b)}{V_1(b)} = \frac{Z_2(s)}{Z_2(s)+Z_1(s)}$$

$$Z_2(s) = \frac{R_2}{SC_2R_2+1}$$

$$Z_1(b) = \frac{R_1}{SC_1R_1+1}$$

$$R_1C_1 = R_2C_2$$

$$\therefore \frac{V_2(s)}{V_1(s)} = \frac{R_2}{R_1+R_2}$$

$$V_2(t) = \frac{R_2}{R_1+R_2}\delta(t)$$

Hence, the correct option is (b).

23. A 10 Ω resistor, a 1 H inductor and 1 μF capacitor are connected in parallel. The combination is driven by a unit step current. Under the steady-state condition, the source current flows through:

(a) the resistor
(b) the inductor
(c) the capacitor only
(d) all the three elements **[1989]**

Solution: (b)

In steady state, current flows through the inductor

Hence, the correct option is (b).

24. If the Laplace transform of the voltage across a capacitor of value of $\frac{1}{2}$ F is

$$V_C(s) = \frac{s+1}{s^3+s^2+s+1}$$

the value of the current through the capacitor at $t = 0+$ is

(a) 0 A (b) 2 A
(c) (1/2) A (d) 1 A **[1989]**

Solution: (c)

$$I = \frac{Cdv}{dt} \Rightarrow I(s) = SCV(s)$$

$$\lim_{t\to 0^+} i(t) = \lim_{s\to 0} SI(s)$$

$$= \lim_{s\to\infty} \frac{s}{2}\times\frac{(s+1)}{(s^3+s^2+s+1)}$$

$$= \lim_{s\to\infty} \frac{s}{2}\frac{s(s+1)}{2(s^2+1)(s+1)} = \frac{s}{2(s^2+1)}$$

$$= \frac{1}{2}\text{A}$$

Hence, the correct option is (c).

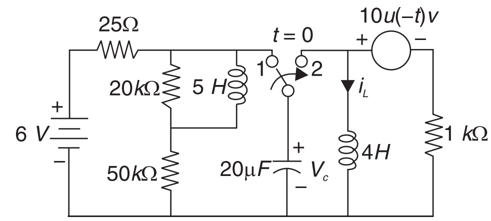
Five-marks Questions

1. The switch in figure, has been in position 1 for a long time and is then moved to position 2at = 0.

(a) Determine $V_c(0^+)$ and $I_L(0^+)$

(b) Determine $\frac{dv_c(t)}{dt}$ at $t = 0^+$

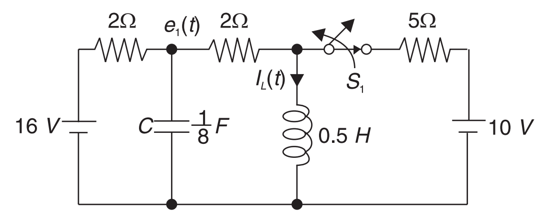
(c) Determine $V_c(t)$ for $t > 0$.



[2002]

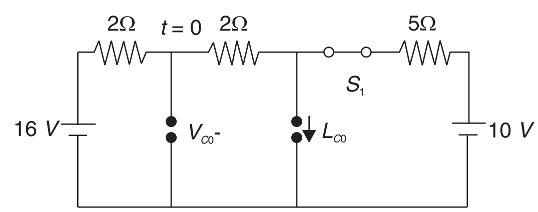
2. The circuit shown in figure is operating steady–state with switch S_1 closed. The switch S_1 is opened at $t = 0$.

(a) Find i_L (0^+)
(b) Find e_1 (O^+).
(c) Using nodal equations and Laplace transform approach, find an expression for the voltage across the capacitor for all $t > 0$.



[2001]

Solution: at $t = 0^-$



for 16V source →

$L'_L(0^-) = 16/4 = 4$Amp

$$i_{0^-} = i_{0^+} = \frac{10}{5} = 2A$$

For 10V source →

$i''_L(0^-)10/5 = 2$Amp

$V''_C(0^-) = 0V$

So

$i_L(0^-) = 1'_L(0^-) = 1''_L(0^-)$

$= 4 + 2$

$i_L(0^-) = 6$ Amp.

$V_C(0^-) = V_C'(60–) + V_C''(0^-)$

$= 8 + 0$

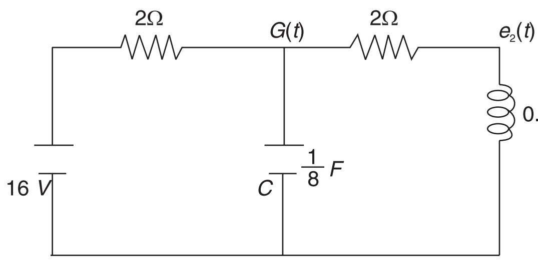
$= 8V$

$V_C(0^-) = 8\text{V}$

(a) $i_L(0^-) = i_L(0^-) = 6$ Amp

(b) $e_1(0^-) = e_1(0^-) = 8V$

(c) at $t = 0^+ \rightarrow$



Apply Nodal Analysis at node $e_1(t)$

$$\frac{e_1(t)-16u(t)}{2}+\frac{1}{8}\frac{d}{dt}e_1(t)+\frac{e_1(t)-e_2(t)}{2}=0 \quad (1)$$

Apply nodal analysis at node $e_2(t)$

$$\frac{e_2(t)-c_1(t)}{2}+\frac{1}{0.5}\int e_2(t)dt=0 \quad (2)$$

From equation (i)

$$\frac{de_1(t)}{dt}+8e_1(t)-4e_2(t)-64u(t)=0 \quad (3)$$

taking the laplace transform on both side

$e_y(0^-) = 8V$

$$-8+SE_1(S)+8E_1(S)-4E_2(S)-\frac{64}{S}=0$$

$$(S+8)E_1(S)-4E_2(S)=\frac{64}{S}+8$$

$$=\frac{8S+64}{S} \quad (1)$$

taking laplace transform of equation (2)

$i_L(0^-) = 6$ Amp

$$0.5\,S\,E_2(S)-0.5E_1(S)+2\frac{E_2(S)}{S}+\frac{6}{S}=0$$

$$0.5\,S\,E_1(S) + (0.5\,S + 2)\,E_2(S) = -6 \quad (4)$$

by Solving equation (4) and (5)

$$E_1(S)=\frac{8S^2+48S+256}{S(S^2+8S+32)}$$

$$=\frac{8(S^2+8S+32)}{S(S^2+8S+32)}-\frac{16S}{S(S^2+8S+32)}$$

$$=\frac{8}{5}-\frac{16}{S^2+8S+32}$$

$$=\frac{8}{S}-\frac{4\times 4}{(S+4)^2+4^2}$$

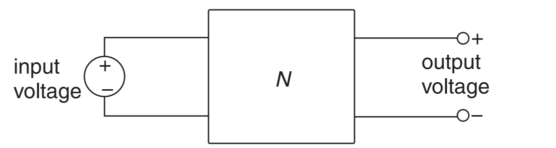
taking inverse laplace transform

$e_1(t) = 84(t) - 4e^{-4t}\sin u(t)\,u(t)$

$e_1(t) = y(2 - e^{-4t}\sin u(t))u(t).V$

3. The network N in figure consists only of two elements: a resistor of 1 Q and an inductor of L Henry. A 5 V source is connected at the input $t = 0$. The inductor current is zero at $t = 0$. The output voltage is found to be $5e^{-3t}$ V, for $t > 0$.

(a) Find the voltage transfer function of the network.
(b) Find L, and draw the configuration of the network.
(c) Find the impulse response of the network.



[2000]

Solution:

(a) $$V_0(s)=\frac{5}{s+3}$$

$$V_i(s)=\frac{5}{3}$$

$$H(s)=\frac{V_0(s)}{V_i(s)}=\frac{s}{s+3}$$

(b) For series $R - L$ n/w

$$H(S)=\frac{LS}{R+LS}=\frac{LS}{1+LS}=\frac{S}{S+1/L}$$

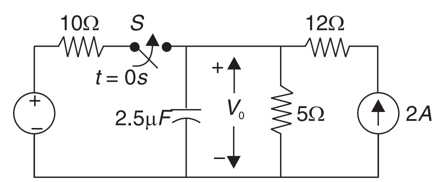
$$\frac{1}{L}=3$$

$$L=\frac{1}{3}H$$

(c) $$h(t)=L^{-1}\left(\frac{s}{s+3}\right)=L^{-1}\left(1-\frac{3}{s+3}\right)$$

$h(t) = \delta(t) - 3e^{-3t}$

4. In the circuit of figure, the switch 'S' has remained open for a long time. The switch closes instantaneously at t = 0.



(a) Find V_0 for $t \leq 0$ and as $t \rightarrow \infty$.
(b) Write an expression for V_0 as a function of time for $0 < t < \infty$.
(c) Evaluate V_0 at $t = 25$ μsee. **[1999]**

Solution:

(a) At steady state capacitor behave as open circuit

$V_{0-} = 2 \times 5 = 10V$

(b) $V_{0t} = \mathrm{V}_{0-} = 10V$

$$V_{\infty}^{1} = \left(2 \times \frac{10}{10+5}\right) \times 5 = \frac{20}{3}V$$

V_{∞} due to $25V$

$$V_{\infty}^{11} = 25 \times \frac{5}{5+10} = \frac{125}{15} = \frac{25}{3}$$

$$V_{\infty} = V^{1}_{\infty} + V^{11}_{\infty}$$

$$= \frac{20}{3} + \frac{25}{3} = 15V$$

$$V_C(t) = V_{\infty}(V_0 - V_{\infty})e^{-t/\tau}$$

$$= 15 + (10 - 15)e^{-t/t}$$

$$= 15 - 5e^{-t/t}$$

$$\tau = \text{Req}\ C$$

$$\tau = \frac{10}{3} \times 2.5 \times 10^{-6}$$

$$= \frac{25}{3} \times 10^{-6} = 8.33 \times 10^{-6}$$

$$V_C(t) = 15 - 5e^{-t/8}33 \times 10^{-6}$$

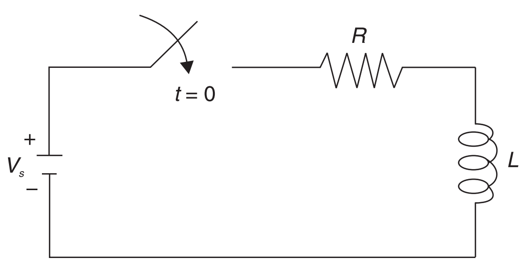
$$= 15 - 5e^{-1.2} \times 10^{5t} V$$

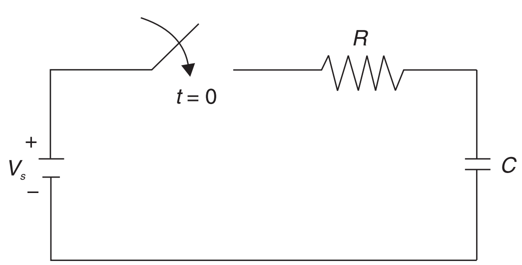
(c) $V_0(25\mu\text{sec}) = 15 – 5e^{-1.2 \times 10^5 \times 25 \times 10^{-6}}$

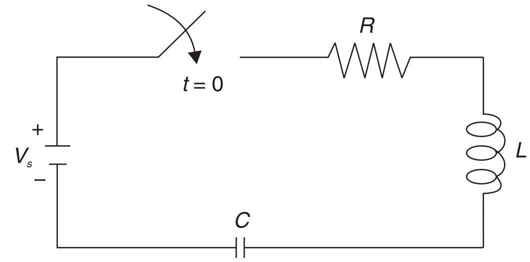
$V_0 = 15 - 5\ (0.05)$

$= 14.75\ V$

5. Match each of the items A, B and C on the left, with an appropriate item on the right. In toe circuit shown in Figure (a) to (c), assuming initial voltages across capacitors and currents through the inductors to be zero at the time of switching ($t = 0$), then at any time $t > 0$,







(1) Current increases monotonically with time.

(2) Current decreases monotonically with time.

(3) Current remains constant at V/R

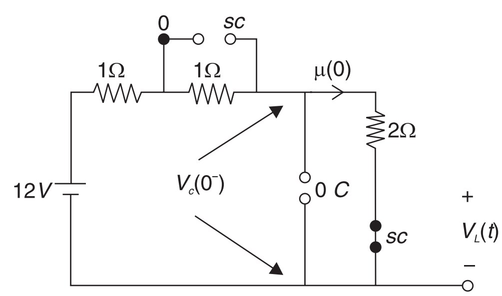
(4) Current first increases, then decreases.

(5) No current can ever flow. **[1996]**

6. The circuit shown in figure below, is initially in its steady–state. The switch is opened at $t = 0$.

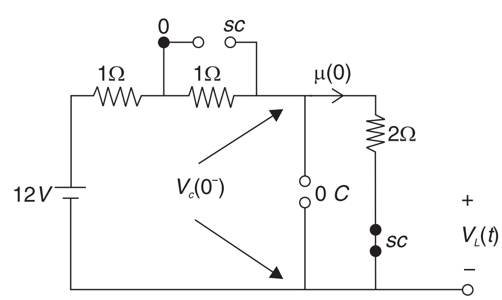
(i) Determine the initial voltage, $V_c(0)$, across the capacitor, and the initial current, $i_L(0^-)$, through the inductor.

(ii) Calculate the voltage, $v_L(t)$, across the inductors for $t > 0$.



[1994]

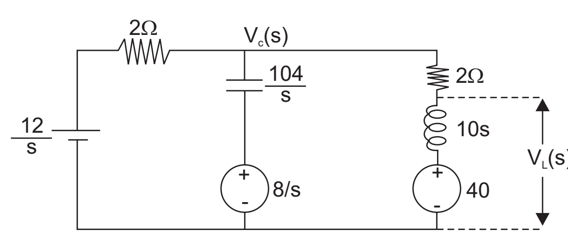
Solution: (i) At steady state



$$i_L(0^-) = \frac{12}{1+2} = 4A$$

$V_C(0) = 2 \times i_L(0^-) = BA$

(ii) for $t > 0$, drawing the equivalent circuit in s–domain



Using KCL,

$$\frac{V_C(S)-12/S}{2}+\frac{V_C(S)-8/S}{10^4/S}+\frac{V_C+40}{10S+2}=0$$

On Solving,

$$V_C(S)=\frac{80s^2+20\times10^4 s+12\times10^4}{10s^3+5\times10^4 s^2+2\times10^4 s} \quad (1)$$

$$\therefore V_L(S)=\left(\frac{V_C(S)+40}{10S+2}\right)10S-40$$

On putting the value of $V_C(S)$ from equation (i)

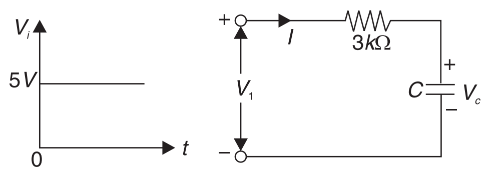
We get,

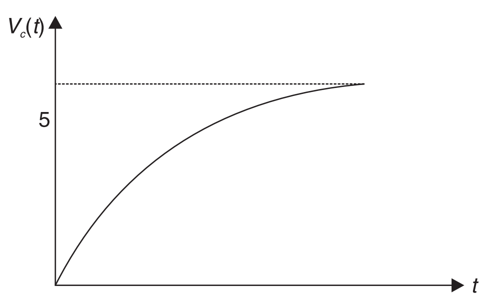
$$V_L(S)\cong\frac{4}{s+5000}-\frac{4}{s+0.4}$$

Taking laplace transform

$V_L(t) = (ye^{-500t} - 4e^{-0.4t})V$

7. In the following circuit the capacitance varies as C = KQ, where K is a constant equal to 0.5 Farads/Coulomb and Q, the charge on the capacitor in Coulombs. Determine the current through the circuit and sketch the voltage waveform across the capacitor (V_c) for a step input V_i as shown in figure.





[1993]

Solution:

$$i_{0^+}=\frac{5}{3k}=1.67\text{ mA}$$

$i_\infty = 0$

$\tau = RC$

$C = 0.5\vartheta$

$\tau = 0.5R\vartheta$

$i(t) = i_\infty + (i_1 - i_\infty)e^{-t/\tau}$

$= 0 + (1.67 - 0)e^{-t/0.5R\vartheta}$

$= 1.67\, e^{-2t/R\vartheta}$

$V_{0-} = V_{0+} = OV$

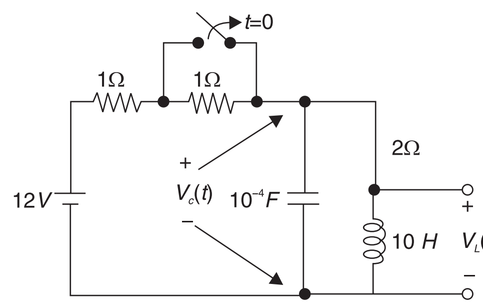
$V_\infty = 5V$

$T = RC$

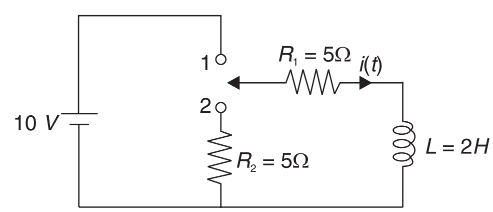
$V_i(t) = V_\infty + (V_{0+} - V_\infty)e^{-t/\tau}$

$= (5 + 10 - 5)e^{-t/RC}$

$= 5(1 - e^{-t/RC})$



8. The network shown in figure is initially under steady–state condition with the switch in position 1. The switch is moved from position 1 to position 2 at $t = 0$. Calculate the current (i) through R_1 after switching.



[1991]

Solution:

$$i_{0^-}=i_{0^+}=\frac{10}{5}=2A$$

$i_\infty = 0$

$R_{eq} = R_1 + R_2 = 5 + 5 = 10\Omega$

$L = 2H$

$$\tau=\frac{L}{R}=\frac{2}{10}=\frac{1}{5}$$

$i(t) = i_\infty + (i_{0+} - i_\infty)e^{-t/\tau}$

$= 0 + (2 - 0)\, e^{-t/15}$

$= 2e^{-5t}; + \geq 0$

or $i(t) = 2.e^{-5t}u(t)$

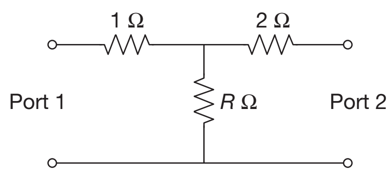
Chapter 5

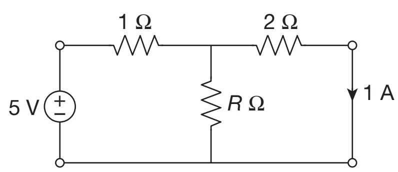
Two Port Networks

One-mark Questions

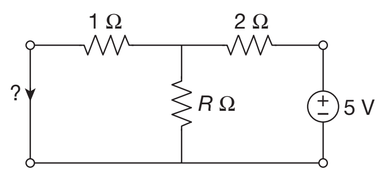
1. Consider the two-port resistive network shown in the figure. When an excitation of 5 V is applied across Port 1, and Port 2 is shorted, the current through the short circuit at Port 2 is measured to be 1 A (see(a) in the figure).

Now, if an excitation of 5 V is applied across port 2, and Port 1 is shorted (see (b) in the figure), what is the current through the short circuit at Port 1? **[2019]**





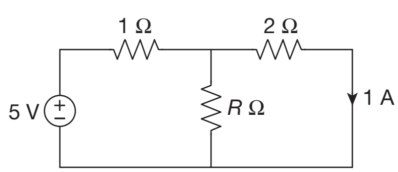
(a)

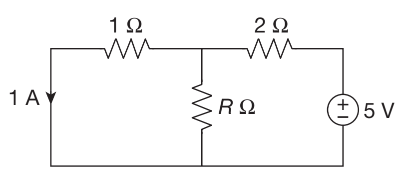


(b)

(A) 0.5 A (B) 2.5 A
(C) 1 A (D) 2 A

Solution:





According to reciprocity theorem

$$\frac{\text{Response}}{\text{Excitation}} = \text{Constant}$$

$$\therefore \frac{V}{I} = \text{contant}$$

$$\therefore \frac{V_1}{I_1} = \frac{V_2}{I_2}$$

$$\frac{5}{I_2} = \frac{5}{1}$$

$$\therefore I_2 = 1 \text{ Amp}$$

Hence, the correct option is (C)

2. Consider the sequence $x[n] = a^n u[n] + b^n u[n]$, where $u[n]$ denote the unit step sequence and $0 < |a| < |b| < 1$. The region of convergence (ROC) of the z transform of $x[n]$ is **[2016]**

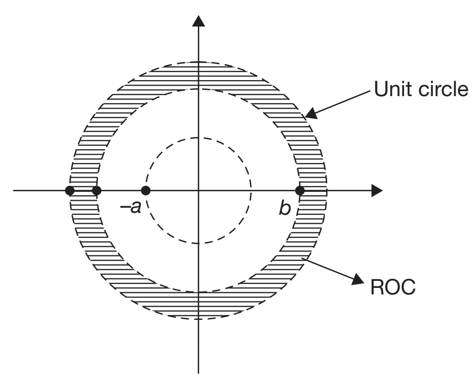
(A) $|z| > |a|$ (B) $|z| > |b|$
(C) $|z| < |a|$ (D) $|a| < |z| < |b|$

Solution: Given $x[n] = a^n \cdot u(n) + b^n \cdot u(n)$

It is a right-sided signal.

So ROC is $|z| > a$ and $|z| > b$

But given $0 < |a| < |b| < 1$



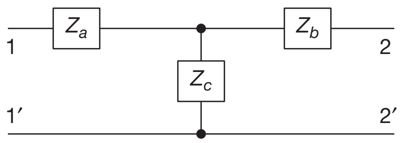
$\therefore$ ROC $|z| > |b|$

Hence, the correct option is (B).

3. The z-parameter matrix for the two-port network shown is

$$\begin{bmatrix} 2j\omega & j\omega \\ j\omega & 3+2j\omega \end{bmatrix}$$

Where the entries are in Ω. Suppose $z_b(j\omega) = R_b + j\omega$.



Then the value of R_b (in Ω) equals ______. **[2016]**

Solution: $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_a + z_c & z_c \\ z_c & z_b + z_c \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

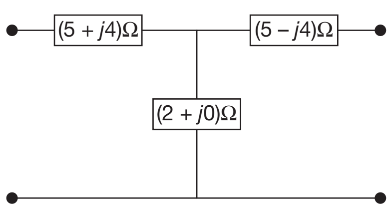
$$Z_c = j\omega$$
$$Z_b + Z_c = 3 + 2j\omega$$
$$R_b + j\omega + j\omega = 3 + 2j\omega$$
$$R_b = 3\,\Omega$$

As we know that the values of z parameters for t model can be given as

$$Z_{11} = Z_a + Z_b$$
$$Z_{12} = Z_{21} = Z_b$$
$$Z_{22} = Z_c + Z_b$$

Hence, the correct Answer is (3 Ω).

4. The ABCD parameters of the following 2-port network are **[2015]**



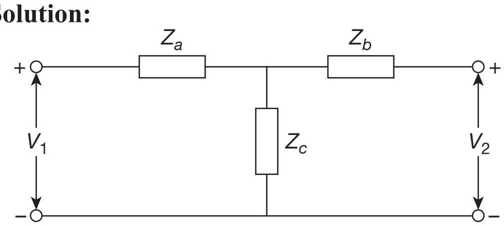
(A) $\begin{bmatrix} 3.5+j2 & 20.5 \\ 20.5 & 3.5-j2 \end{bmatrix}$

(B) $\begin{bmatrix} 3.5+j2 & 30.5 \\ 0.5 & 3.5-j2 \end{bmatrix}$

(C) $\begin{bmatrix} 10 & 2+j0 \\ 2+j0 & 10 \end{bmatrix}$

(D) $\begin{bmatrix} 7+j4 & 0.5 \\ 30.5 & 7-j4 \end{bmatrix}$

Solution:



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Let $\quad I_2 = 0$

$$C = \frac{I_1}{V_2} \qquad [\because I_2 = 0]$$

$$\therefore \quad C = \frac{1}{Z_{21}}$$

We know for a T-network

$$Z_{21} = Z_C = 2\,\Omega$$
$$C = \frac{1}{2} = 0.5\Omega$$

From the given options Choice B is correct

Hence, the correct option is (B).

5. If the scattering matrix [SI] of a two port network is

$$S = \begin{bmatrix} 0.2\angle 0^\circ & 0.9\angle 90^\circ \\ 0.9\angle 90^\circ & 0.1\angle 90^\circ \end{bmatrix}$$

then the network is

(a) lossless and reciprocal
(b) lossless but not reciprocal
(c) not lossless but reciprocal
(d) neither lossless nor reciprocal **[2010]**

Solution: (c)

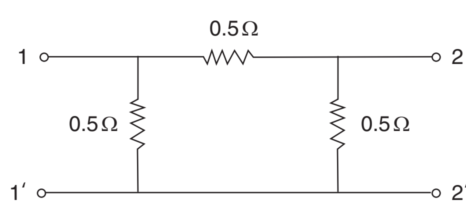
For reciprocal network

$S_{12} = S_{21}$

but $|S_{11}|^2 + |S_{12}|^2 \neq 1 \Rightarrow$ not loss less

Hence, the correct option is (c)

6. For the two port network shown below, the short-circuit admittance parameter matrix is

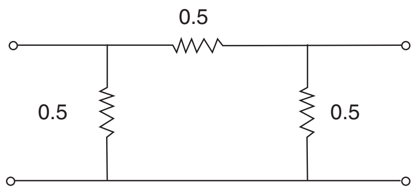


(a) $\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} S$ (b) $\begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix} S$

(c) $\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} S$ (d) $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} S$

[2010]

Solution: (a)



Y parameters will be written as:

$$y_a = y_b = y_c = \frac{1}{.5} = 2$$
$$y_{11} = y_{22} = 2 + 2 = 4$$
$$y_{21} = y_{21} = -2$$

Hence, the correct option is (a)

7. A two-port network is represented by ABCD parameters given by

$$\begin{bmatrix} V_1 \\ I_t \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

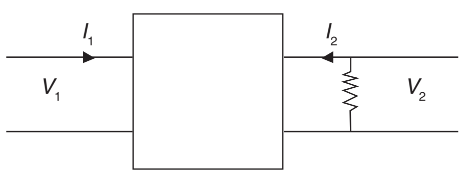
If port-2 is terminated by R_L, the input impedance seen at port-1 is given by

(a) $\dfrac{A + BR_L}{C + DR_L}$ (b) $\dfrac{AR_L + C}{BR_L + D}$

(c) $\dfrac{DR_L + A}{BR_L + C}$ (d) $\dfrac{B + AR_L}{D + CR_L}$

[2006]

Solution: (d)



ABCD parameters will be written by

$V_1 = AV_2 - BI_2$

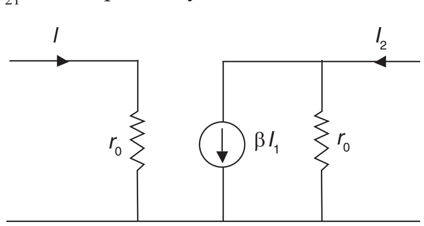
$I_1 = CV_2 - DI_2$ and $V_2 = -I_2R_L$

$$\frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2} = \frac{-AI_2R_L - BI_2}{-CI_2R_L - DI_2}$$

$$J/P \text{ imp} = \frac{AR_L + B}{CR_L + D}$$

Hence, the correct option is (d)

8. In the two-port network shown in the figure below, Z_{12}, and Z_{21} are, respectively



(a) re and βr_0 (b) 0 and $-\beta r_0$

(c) 0 and βr_0 (d) re and $-\beta r_0$

[2006]

Solution: (b)

When $I_1 = 0 \Rightarrow V_1 = 0$

$$\therefore \frac{V_1}{I_2} = 0 = Z_{12}$$

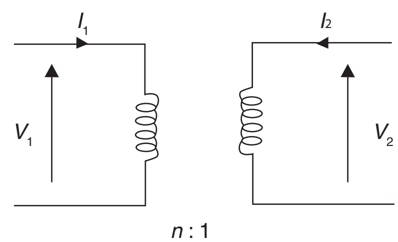
When $I_2 = 0 \Rightarrow V_2 = -\beta I_1 r_0$

$$\frac{V_2}{I_1} = -\beta x_0 = Z_{21}$$

Hence, the correct option is (b)

9. The ABCD parameters of an ideal n:1 transformer shown in the figure are $\begin{bmatrix} n & 0 \\ 0 & X \end{bmatrix}$

The value of X will be



(a) n (b) $\dfrac{1}{n}$

(c) n^2 (d) $\dfrac{1}{n^2}$ **[2005]**

Solution: (b)

For transformer, $\dfrac{I_2}{J_1} = \dfrac{V_1}{V_2} = \dfrac{n}{1}$

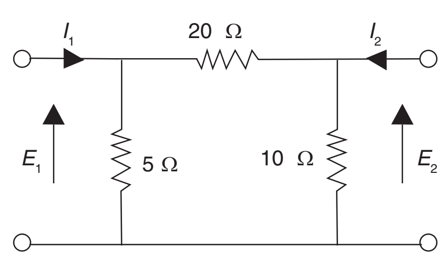
$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = n, \; D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{V_2}{V_1} = \frac{1}{n}$$

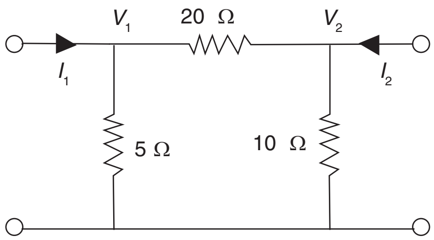
Hence, the correct option is (b)

10. The admittance parameter Y_{12} in the two-port network in figure is



(a) −0.2 mho (b) 0.1 mho
(c) −0.05 mho (d) 0.05 mho **[2001]**

Solution: (c)



$$I_1 = \frac{V_1}{5} + \frac{(V_1 - V_2)}{20}$$

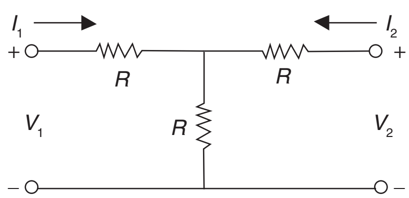
$$I_1 = V_1\left[\frac{1}{5} + \frac{1}{20}\right]\frac{-V_2}{20}$$

$$I_1 = V_1\left[\frac{1}{4}\right]\frac{-V_2}{20}$$

Hence, the correct option is (c)

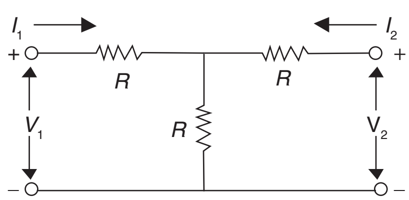
$$I_1 = Y_{11}V_1 + Y_{12}V_2 \Rightarrow Y_{12} = \frac{-1}{20} = -0.05 \text{ mho}$$

11. A two-port network is shown in the figure. The parameter h^{21} for this network can be given by



(a) −1/2 (b) +1/2
(c) −3/2 (d) +3/2 **[1999]**

Solution: (a)



$$V_1 = 2RI_1 + RI_2$$
$$V_2 = RI_1 + 2RI_2$$
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

Put $V_2 = 0$

$$\therefore\ 0 = RI_1 + 2RJ_2 \Rightarrow \frac{I_2}{I_1} = \frac{-1}{2}$$

Hence, the correct option is (a)

12. The short-circuit admittance matrix of a two-port network is

$$\begin{bmatrix} 0 & -1/2 \\ 1/2 & 0 \end{bmatrix}$$

The two-port network is
(a) non-reciprocal and passive
(b) non-reciprocal and active
(c) reciprocal and passive
(d) reciprocal and active **[1998]**

Solution:(b)

$y^{12} \neq y^{21} \Rightarrow$ N/W is non-reciprocal and active

Hence, the correct option is (b)

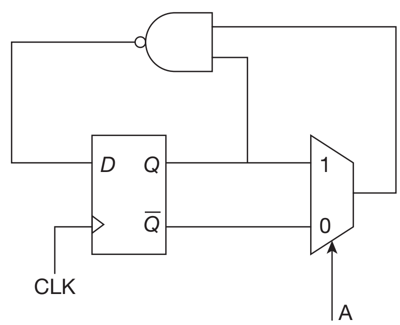
13. The condition that a two-port network is reciprocal can be expressed in terms of its ABCD parameters as______ **[1994]**

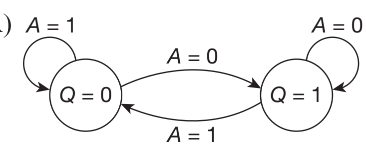
Solution: AD – BC = 1

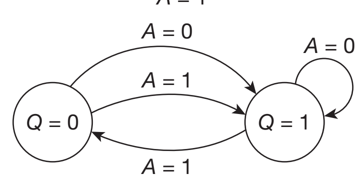
The condition is $AD - BC = 1$

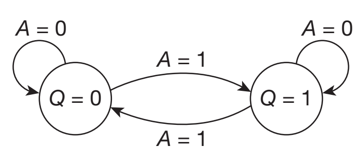
Two-marks Questions

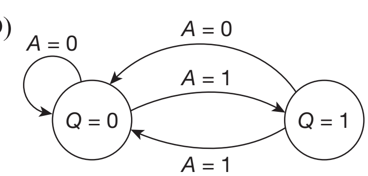
1. The state transition diagram for the circuit shown is **[2019]**



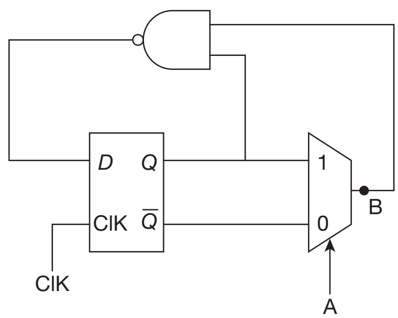
(A)


(B)


(C)


(D)


Solution:



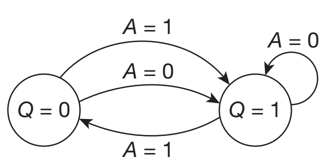
$$B = \overline{A}\,\overline{Q} + AQ$$

$$D = \overline{B\cdot Q}$$

$$= \overline{\overline{A}\,\overline{Q} + AQ\cdot Q}$$

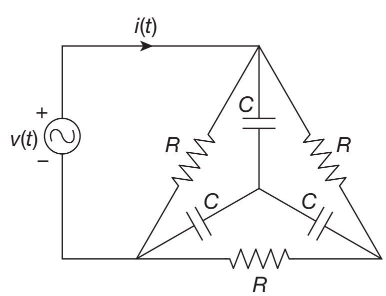
$$= A\oplus Q + \overline{Q}$$

Present state	i/P	Next state
Q		Q^+
0	0	1
0	1	1
1	0	1
1	1	0



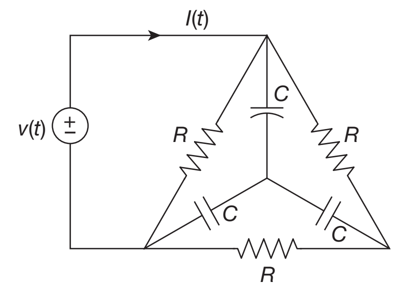
Hence, the correct option is (B).

2. In the circuit shown, if $v(t) = 2\sin(1000\,t)$ volts, $R = 1$ kΩ and $C = 1$ μF, then the steady-state current $i(t)$, in milliamperes (mA), is **[2019]**

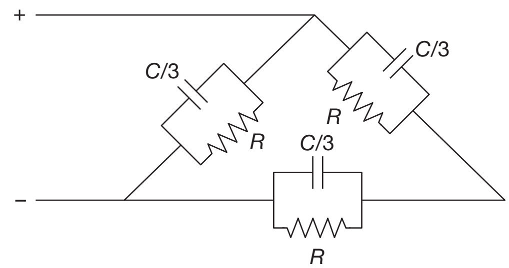


(A) $3\sin(1000\,t) + \cos(1000\,t)$
(B) $\sin(1000\,t) + \cos(1000\,t)$
(C) $\sin(1000\,t) + 3\cos(1000\,t)$
(D) $2\sin(1000\,t) + 2\cos(1000\,t)$

Solution:

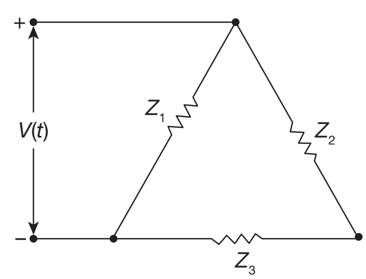


$V(t) = 2\sin(1000t)$
$R = 1$ K
$C = 1\mu$F
Using Y to Δ conversion for the viner circuit.



$$Z = \frac{R\times\dfrac{1}{jWC/3}}{R+\dfrac{1}{jWC/3}} = \frac{R}{1+jWR\dfrac{C}{3}}$$

$$= 900 - 300i$$



$$Z_T = \frac{2}{3}Z_1 = \frac{2}{3}[900 - 300i]$$

$$= \frac{2\times 10^3}{3+j} = 600 - 200i$$

$$I(t) = \frac{V(t)}{Z_t} = \frac{2\sin(1000t)}{600-200i}$$

$$= [(3+j)\sin(100t)]$$

$$i(t) = 3\sin 100t + j\sin 1000t$$

$$= 3\sin 1000t + 1 < 90\sin 1000t$$

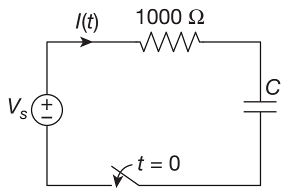
$$= 3\sin 1000t + \sin 1000t + 90$$

$$i(t) = 3\sin 1000t + \cos 1000t$$

Hence, the correct optiojn is (A).

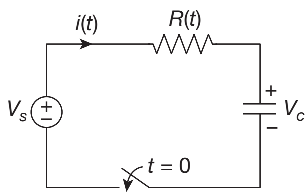
3. The RC circuit shown below has a variable resistance $R(t)$ given by the following expression:

$$R(t) = R_0\left(1 - \frac{t}{T}\right) \text{ for } 0 \le t \le T$$



Where $R_0 = 1\ \Omega$ and $C = 1$ F. We are also given that $T = 3R_0C$ and the source voltage is $V_S = 1$ V. If the current at time $t = 0$ is 1 A, then the current $I(t)$, in amperes, at time $t = T/2$ is ______ (rounded off to 2 decimal places). **[2019]**

Solution:



$$R(t) = R_0\left(1 - \frac{t}{T}\right) \quad 0 \le t \le T$$

$$R_0 = 1\Omega$$

$$C = 1F$$

$$T = 3R_0C$$

$$V_S = 1V$$

$$R(t) = 1\cdot\left(1 - \frac{t}{3}\right)$$

$$R(t)\cdot i(t) = \frac{1}{C}\int i(t)\,dt = V_S$$

$$\left[1 - \frac{t}{3}\right]\cdot P(t) + \int i(t)\,dt = 1.$$

On differentiation w.r. to t

$$\left(1 - \frac{t}{3}\right)\frac{di(t)}{dt} - \frac{i(t)}{3} = -i(t)$$

$$(3-t)\frac{di(t)}{dt} + 2i = 0$$

$$\frac{di}{i} = \frac{-2}{(3-t)}dt$$

Integrating path sides we get

$$\ln i = 2\ln(3-t) + (nCC).$$

$$i(t) = C(3-t)^2$$

$$i(o) = 1A$$

$$l = C(3)^2$$

$$C = \frac{1}{9}$$

$$i(t) = \frac{1}{9}(3-t)^2\ A$$

$$\text{for } t = \frac{T}{2}$$

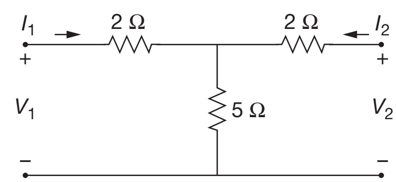
$$i(1.5) = \frac{1}{9}[3-1.5]^2$$

$$= \frac{1}{9}\times\left(\frac{3}{2}\right)^2 = \frac{1}{4} = 0.25$$

Hence, the correct answer is 0.25.

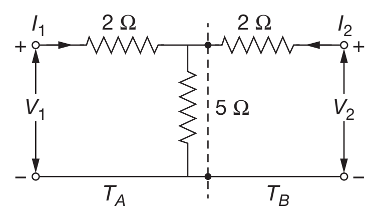
4. The $ABCD$ matrix for a two-port network is defined by:

$$\begin{bmatrix} v_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$



The parameter B for the given two – port network (in ohms, correct to two decimal places) is ______. **[2018]**

Solution: Consider the figure given below



From the above figure

$$T_A = \begin{bmatrix} 1 + \frac{Z_A}{Z_B} & Z_A \\ \frac{1}{Z_B} & 1 \end{bmatrix}$$

$$T_A = \begin{bmatrix} 1.4 & 2 \\ 0.2 & 1 \end{bmatrix}$$

$$T_B = \begin{bmatrix} 2 & 2 \\ 0 & 1 \end{bmatrix}$$

$$T = [T_A][T_B]$$

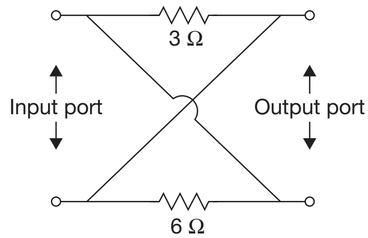
$$T = \begin{bmatrix} 2.8 & 4.8 \\ 0.4 & 1.4 \end{bmatrix}$$

From the above matrix T

$$B = 4.8$$

Hence, the correct answer is 4.3 to 5.3.

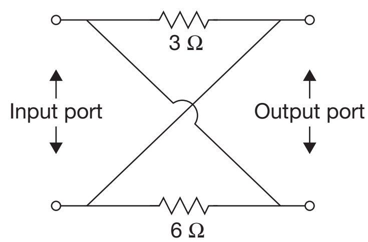
5. The z–parameter matrix $\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$ for the two port network shown is **[2016]**



(A) $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 9 & -3 \\ 6 & 9 \end{bmatrix}$ (D) $\begin{bmatrix} 9 & 3 \\ 6 & 9 \end{bmatrix}$

Solution:

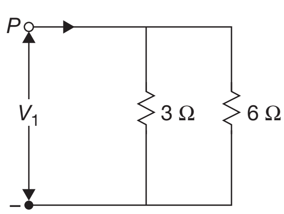


As we know that the equations for Z parameters are

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Let, $I_2 = 0$



$$Z_{11} = \frac{V_1}{\text{I}_1} \quad \text{and} \quad Z_{21} = \frac{V_2}{\text{I}_1}$$

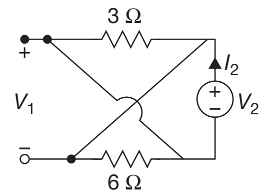
$$\frac{V_1}{I_1} = 2\ \Omega$$

$$Z_{21} = \frac{V_2}{I_1} = -\frac{V_1}{I_1} = -2\ \Omega$$

$$\frac{V_2}{I_1} = Z_{21} = -2\ \Omega$$

Let $I_1 = 0$

$$Z_{12} = \frac{I_2}{V_1} \quad \text{and} \quad Z_{21} = \frac{V_2}{I_1}$$



$$Z_{22} = (3 \parallel 6)$$

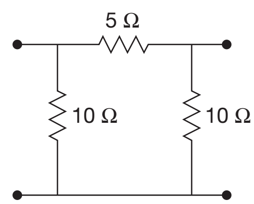
$$= 2\ \Omega$$

$$Z_{12} = \frac{V_1}{I_2} = -\frac{V_2}{I_2}$$

$$= -2\ \Omega$$

Hence, the correct option is (A).

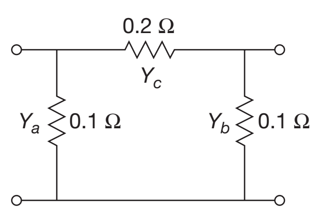
6. The 2-port admittance matrix of the circuit shown is given by **[2015]**



(A) $\begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}$ (B) $\begin{bmatrix} 15 & 5 \\ 5 & 15 \end{bmatrix}$

(C) $\begin{bmatrix} 3.33 & 5 \\ 5 & 3.33 \end{bmatrix}$ (D) $\begin{bmatrix} 0.3 & 0.4 \\ 0.4 & 0.3 \end{bmatrix}$

Solution:



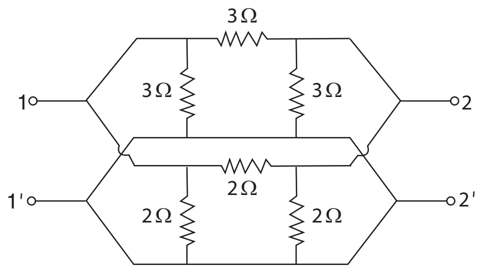
We know for π-network

$$[Y] = \begin{bmatrix} Y_a + Y_c & Y_c \\ Y_c & Y_b + Y_c \end{bmatrix}$$

$$= \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{bmatrix} \Omega$$

Hence, the correct option is (A).

7. In the h-parameter model of the two-port network given in the figure shown, the value of h_{22} (in S) is_____ **[2014]**



Solution:

For parallel combination, equivalent admittance is given by $[Y] = [Y_1] + [Y_2]$

$$[y_1] = \begin{bmatrix} \frac{1}{3}+\frac{1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{1}{3}+\frac{1}{3} \end{bmatrix}$$

$$[y_2] = \begin{bmatrix} \frac{1}{2}+\frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{2}+\frac{1}{2} \end{bmatrix}$$

$$\therefore \quad [y] = \begin{bmatrix} \frac{5}{3} & \frac{-5}{6} \\ \frac{-5}{6} & \frac{5}{3} \end{bmatrix}$$

$$I_1 = \frac{5}{3}V_1 - \frac{5}{6}V_2$$

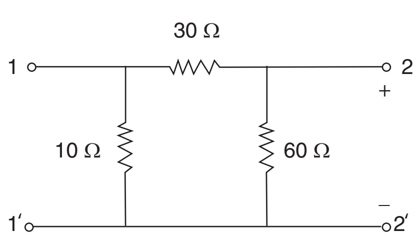
$$I_2 = \frac{-5}{6}V_1 + \frac{5}{3}V_2$$

$$h_{22} = \left.\frac{I_1}{V_2}\right|_{I_1=0}$$

When $I_1 = 0 \Rightarrow V_1 = \frac{V_2}{2}$

$$h_{22} = \frac{I_2}{V_2} = \frac{15}{12} = 1.2s$$

8. For the two-port network shown in the figure, the impedance (Z) matrix (in Ω) is



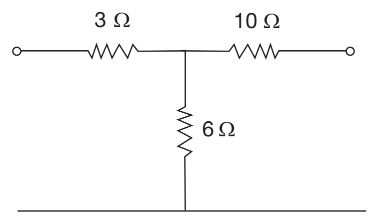
(a) $\begin{bmatrix} 6 & 24 \\ 42 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} 9 & 8 \\ 8 & 24 \end{bmatrix}$

(c) $\begin{bmatrix} 9 & 6 \\ 6 & 24 \end{bmatrix}$ (d) $\begin{bmatrix} 42 & 6 \\ 6 & 60 \end{bmatrix}$

[2014]

Solution: (c)

Applying Y to Z conversion



$$[Z] = \begin{bmatrix} 9\Omega & 6\Omega \\ 6\Omega & 24\Omega \end{bmatrix}.$$

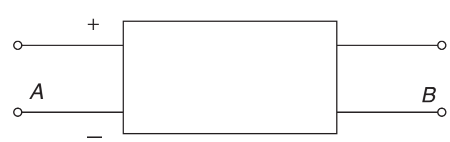
Hence, the correct option is (c)

Common Data for Questions 3 and 4

With 10 V dc connected at port A in the linear nonreciprocal two-port network shown below, the following were observed.

(i) 1 Ω connected at port B draws a current of 3 A.

(ii) 2.5 Ω connected at port B draws a current of 2 A.



9. For the same network, with 6 V dc connected at port A, 1 Ω connected at port B draws 7/3 A. If 8 V dc is connected to port A, the open circuit voltage at port B is

(a) 6 V (b) 7 V

(c) 8 V (d) 9 V **[2012]**

Solution: (b)

Given

$V_1 = 10\text{V}, V_2 = 3\text{V}, I_2 = -3\text{A}$

$V_1 = AV_2 - BI_2$

$10 = 3A + 3B$ (1)

$V_2 = 5V, I_2 = -2A$

$10 = 5A + 2B$ (2)

$\therefore A = \frac{10}{9}, B = \frac{20}{9}$ from (1) and (2)

Given:- $V_1 = \delta V, (V_2)_{oc} = ?$

$I_2 = 0$

$\therefore V_1 = AV_2$

$$(V_2)_{oc} = \frac{V_1}{A} = \frac{80}{10/9} = 7.2\text{V}$$

Hence, the correct option is (b)

10. With 10 V dc connected at port A, the current drawn by 7 Ω connected at port B is

(a) 3/7 A (b) 5/7 A
(c) 1 A (d) 9/7 A **[2012]**

Solution: (c)

Using ABCD parameters for given circuits we can write

$V_1 = AV_2 - BI_2$ and $V_2 = -7I_2$

$\therefore\ 10 = -7I_2A - BI_2$

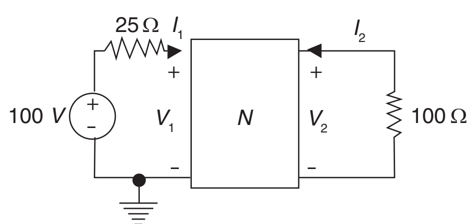
$$10 = \frac{-70}{9} I_2 \frac{-20}{9} I_2$$

$I_2 = -1\text{A}$

Hence, the correct option is (c)

11. In the circuit shown below, the network N is described by the following Y matrix:

$$Y = \begin{bmatrix} 0.1S & -0.01S \\ 0.01S & 0.1S \end{bmatrix}. \text{ The voltage gain } \frac{V_2}{V_1} \text{ is}$$



(a) 1/90 (b) −1/90
(c) −1/99 (d) −1/11 **[2011]**

Solution: (d)

Y parameters can be written as

$I_2 = Y_{21}V_1 + Y_{22}V_2$

$$V_2 = -I_2R_L = -100I_2 \Rightarrow I_2 = \frac{-V_2}{100}$$

$$\therefore\ \frac{-V_2}{100} = 0.01V_1 + 0.1V_2$$

$V_1/V_2 = -1/11$

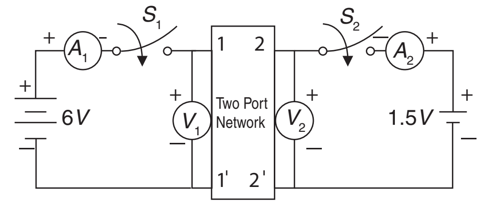
Hence, the correct option is (d)

Linked Answer Questions 6 to 7.

A two-port network shown below is excited by external dc sources. The voltages and currents are measured with voltmeters V_1 V_2 and ammeters A_1 A_2 (all assumed to be ideal) as indicated. Under following switch conditions, the readings obtained are:

(i) S-Open, S_2-closed $A_1 = 0\ A$, $V_1 = 4.5\ V$, $V_2 = 1.5V$, $A_2 = 1\ A$

(ii) S_1-Closed. S_2-Open $A_1 = 4\ A$, $V_1 = 6\ V$, $V_2 = 6\ V$, $A_2 = 0\ A$



12. The Z-parameter matrix for this network is

(a) $\begin{bmatrix} 1.5 & 1.5 \\ 4.5 & 1.5 \end{bmatrix}$ (b) $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 4.5 \end{bmatrix}$

(c) $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 1.5 \end{bmatrix}$ (d) $\begin{bmatrix} 4.5 & 1.5 \\ 1.5 & 4.5 \end{bmatrix}$

[2008]

Solution: (c)

When I = 0

$V_1 = 4.5$ V

$V_2 = 1.5$ V

$J_2 = 1$ A

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 4.5$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 1.5$$

When $I_2 = 0$ then $I_1 = 4$A

$V_1 = 6V$, $V_2 = 6$V

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{6}{4} = 1.5$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{6}{4} = 1.5$$

$$[Z] = \begin{bmatrix} 1.s & 4.s \\ 1.s & 1.s \end{bmatrix}$$

Hence, the correct option is (c)

13. The h-parameter matrix for this network is

(a) $\begin{bmatrix} -3 & 3 \\ -1 & 0.67 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & -1 \\ 3 & 0.67 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 3 \\ 1 & 0.67 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 1 \\ -3 & -0.67 \end{bmatrix}$

[2008]

Solution: (a)

$$h_{11} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}} = -3$$

$$h_{21} = \frac{-Z_{21}}{Z_{22}} = -1$$

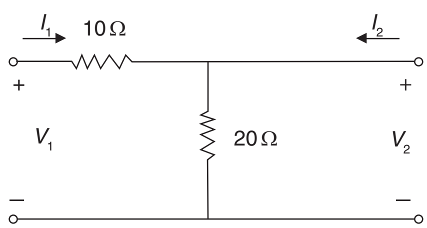
$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = 3$$

$$h_{22} \left.\frac{I_2}{V_2}\right|_{I_1=0} = 0.67$$

$$[h] = \begin{bmatrix} -3 & 3 \\ -1 & 0.67 \end{bmatrix}$$

Hence, the correct option is (a)

14. The h parameters of the circuit shown in the figure are

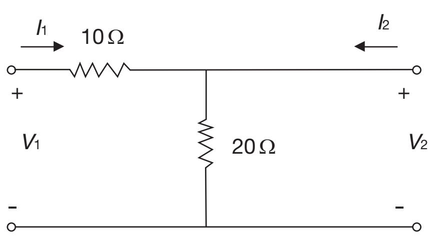


(a) $\begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.3 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & -1 \\ -1 & 0.05 \end{bmatrix}$

(c) $\begin{bmatrix} 30 & 20 \\ 20 & 20 \end{bmatrix}$ (d) $\begin{bmatrix} 10 & 1 \\ -1 & 0.05 \end{bmatrix}$

[2005]

Solution: (d)



$V_1 = 30I_1 + 20I_2$

$V_2 = 20I_1 + 20I_2$

$I_2 = -20I_1 + V_2$ and $I_2 = h_{21}I_1 + h_{22}V_2$

$$\therefore h_{21} = \frac{-20}{20} = -1,\ h_{22} = \frac{1}{20}$$

$$V_1 = 30I_1 + \frac{20(V_2 - 20I_1)}{20}$$

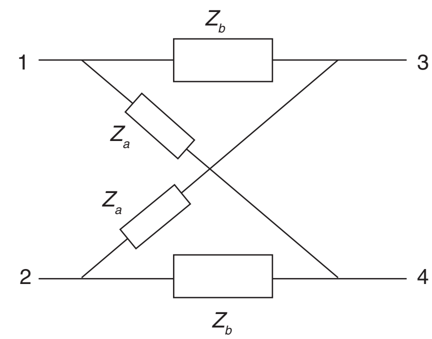
$V_1 = 10I_1 + V_2$

$\therefore h_{11} = 10, h_{12} = 1$

Hence, the correct option is (d)

15. For the lattice circuit shown in the figure, $Z_a = J2\Omega$ and $Z_b = 2Q$. The values of the open circuit impedance parameters

$Z\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$ are



(a) $\begin{bmatrix} 1-j & 1+j \\ 1+j & 1+j \end{bmatrix}$

(b) $\begin{bmatrix} 1-j & 1+j \\ -1+j & 1-j \end{bmatrix}$

(c) $\begin{bmatrix} 1+j & 1+j \\ 1-j & 1-j \end{bmatrix}$

(d) $\begin{bmatrix} 1+j & -1+j \\ -1-j & 1+j \end{bmatrix}$ **[2004]**

Solution: (d)

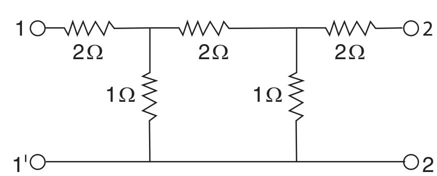
$$\text{For lattice N/W } [Z] = \begin{bmatrix} \frac{Za+Zb}{2} & \frac{Za-Zb}{2} \\ \frac{Za-Zb}{2} & \frac{Za+Zb}{2} \end{bmatrix}$$

$Za = 2j, Zb = 2\Omega$

$$[Z] = \begin{bmatrix} 1+j & -1+j \\ -1+j & 1+j \end{bmatrix}$$

Hence, the correct option is (d)

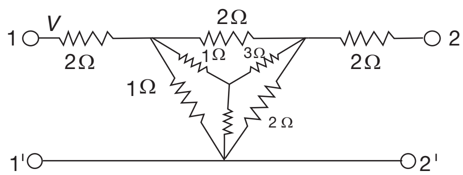
16. The impedance parameters Z_{11} and Z_{12} of the two-port network in the figure are

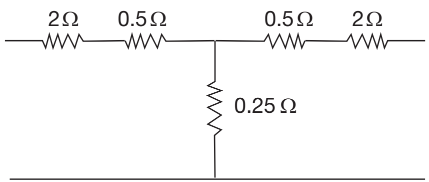


(a) $Z_{11} = 2.75\ \Omega$ and $Z_{12} = 0.25\Omega$
(b) $Z_{11} = 3\Omega$ and $Z_{12} = 0.5\ \Omega$
(c) $Z_{11} = 3Q$ and $Z_{12} = 0.25\ \Omega$
(d) $Z_{11} = 2.25\ \Omega$ and $Z_{12} = 0.5\ \Omega$ **[2003]**

Solution: (a)

Using Δ-Y conversion



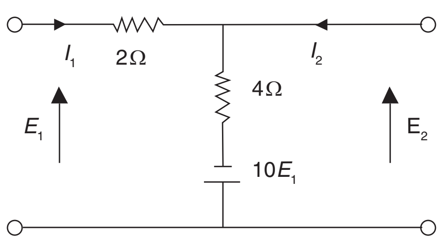


$$\therefore [Z] = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix}$$

$$[Z] = \begin{bmatrix} 2.75 & 0.25 \\ 0.25 & 2.75 \end{bmatrix}$$

Hence, the correct option is (a)

17. The Z parameters Z_{11} and Z_{21} for the two-port network in the figure are



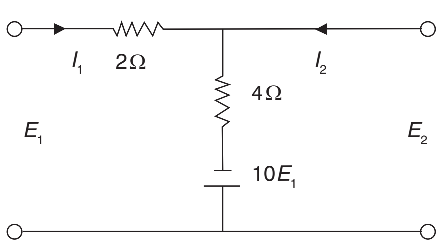
(a) $Z_{11} = \frac{-6}{11}\Omega;\ Z_{21} = \frac{16}{11}\Omega$

(b) $Z_{11} = \frac{6}{11}\Omega;\ Z_{21} = \frac{4}{11}\Omega$

(c) $Z_{11} = \frac{6}{11}\Omega;\ Z_{21} = \frac{-16}{11}\Omega$

(d) $Z_{11} = \frac{4}{11}\Omega;\ Z_{21} = \frac{4}{11}\Omega$ **[2001]**

Solution: (c)



$$E_1 = 6I_1 + 4I_2 - 10E_1 \Rightarrow 11E_1 = 6I_1 + 4I_2$$

$$E_1 = \frac{6}{11}J_1 + \frac{4}{11}I_2 \Rightarrow Z_{11} = \frac{6}{11}\Omega$$

$$E_2 = 4J_1 + 4I_2 - 10E_1$$

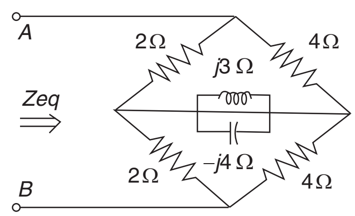
$$E_2 = 4I_1 + 4I_2 - 10\left\langle \frac{6}{11}J_1 + \frac{4}{11} \right\rangle J_2$$

$$E_2 = 4I_1 + 4I_2 - \frac{60}{11}J_1 - \frac{40}{11}J_2$$

$$E_2 = \frac{-16}{11}J_1 + \frac{4I_2}{11} \Rightarrow Z_{21} = \frac{-16}{11}\Omega$$

Hence, the correct option is (c)

18. In the circuit of the figure given below, the equivalent impedance seen across terminals A, B is



(a) (16/3) Ω
(b) (8/3) Ω
(c) (8/3 + 12 j) Ω
(d) None of the above **[1997]**

Solution: (b)

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4} \Rightarrow \text{Bridge is balanced}$$

$$\therefore\ Z_{eq} = (2 \parallel 4) + (2 \parallel 4) = \frac{8}{3}\Omega$$

Hence, the correct option is (b)

19. For a two-port network to be reciprocal,

(a) $z_{11} = z_{22}$ (b) $y_{21} = y_{12}$
(c) $h_{21} = -h_{12}$ (d) AD – BC = 0 **[1992]**

Solution: (b) and (c)

$$\left.\begin{aligned} y_{11} &= y_{12} \\ h_{21} &= -h_{12} \end{aligned}\right\} \text{ for reciprocal N/W}$$

Hence, the correct option is (b) and (c)

20. Two two-port networks are connected in cascade. The combination is to be represented as a single two-port network. The parameters of the network are obtained by multiplying the individual

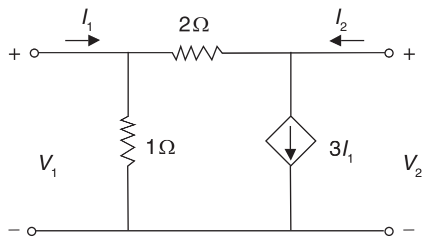
(a) z-parameter matrices
(b) h-parameter matrices
(c) y-parameter matrices
(d) ABCD parameter matrices **[1991]**

Solution: (d)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

Hence, the correct option is (d)

21. The open circuit impedance matrix of the two port network shown in figure is

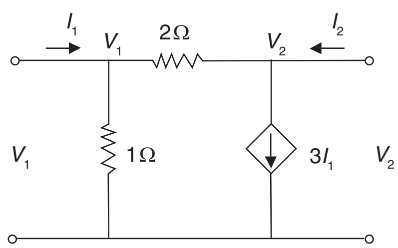


(a) $\begin{bmatrix} -2 & 1 \\ -8 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & -8 \\ -8 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$

[1990]

Solution: (a)



$$I_1 = \frac{V_1}{1} + \frac{V_1 - V_2}{2} = \frac{3V_1}{2} - \frac{V_2}{2}$$

$$I_2 = 3I_1 + \frac{V_2 - V_1}{2}$$

$$I_2 - 3I_1 + \frac{V_1}{2} = \frac{V_2}{2}$$

$$V_2 = 2I_2 - 6I_1 + V_1 \quad (1)$$

$$V_2 = 2I_2 - 6J_1 + \left(J_1 + \frac{V_2}{2}\right) \times \frac{2}{3}$$

$$V_2 = 2I_2 - CI_1 + \frac{2}{3}I_1 + \frac{V_2}{3}$$

$$\frac{2V_2}{3} = 2I_2 - \frac{16}{3}I_1$$

$V_2 = 3I_2 - 8I_1$

$Z_{22} = 3\Omega$, $Z_{21} = -8\Omega$

Also, by substituting V_2 in equation (1),

$V_1 = -2I_1 + I_2$

$Z_{11} = -2\ \Omega$, $Z_{12} = 1\ \Omega$

Hence, the correct option is (a).

22. For the transfer function of a physical two-port network:

(a) all the zeros must lie only in the left hall of the s-plane

(b) the poles may lie anywhere in the s-plane

(c) the poles lying on the imaginary axis must be simple

(d) a pole may lie at origin **[1989]**

Solution: (c) and (d)

The poles lying on imaginary axis must be simple and a pole may lie at origin.

Hence, the correct option is (c) and (d).

23. The condition AD – BC – 1 for a two-port network implies that the network is a:

(a) reciprocal network

(b) lumped element network

(c) lossless network

(d) unilateral element network **[1989]**

Solution: (a)

For reciprocal N/W, $AD - BC = 1$

Hence, the correct option is (a).

24. Two two-port networks are connected in parallel. The combination is to be represented as a single two-port network. The parameters of this network are obtained by addition of the individual

(a) z parameters

(b) h parameters

(c) y parameters

(d) ABCD parameters **[1988]**

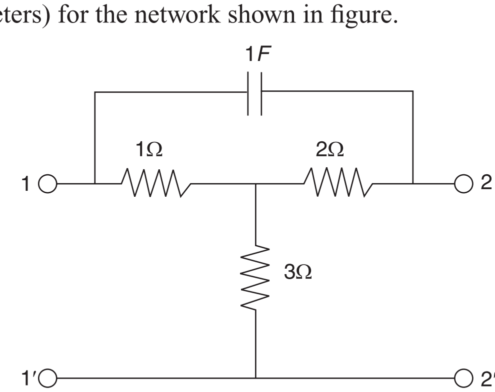
Solution: (c)

For parallel connection, $[Y] = [Y]_A + [Y]_B$

Hence, the correct option is (c).

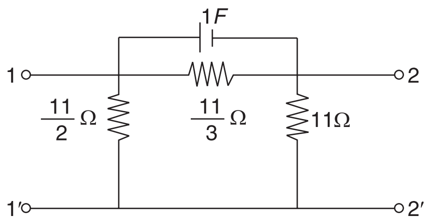
Five-marks Questions

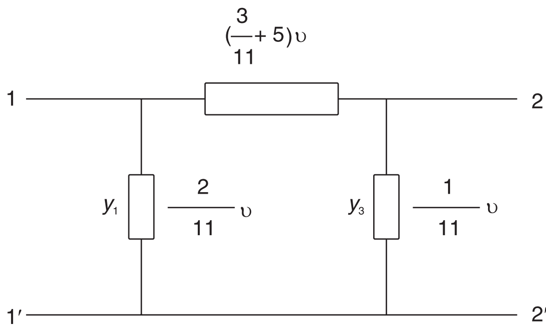
1. Find the Y–parameters (short circuit admittance parameters) for the network shown in figure.

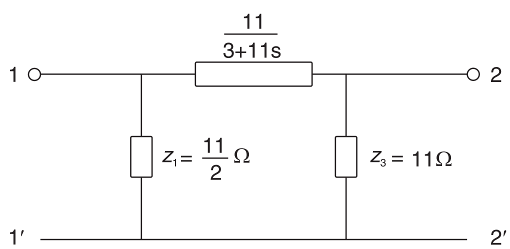


[1993]

Solution: Using star–delta conversion







$V_2 = 0 \rightarrow$

$$y_{11} = \frac{I_1}{V_1} = \frac{1}{2i} = \frac{5}{11} + s\ (y_{11} \neq y_1 + y_2)$$

$$y_{21} = \frac{I_2}{V_1} = -V_2 = -\left(\frac{3}{11} + s\right)$$

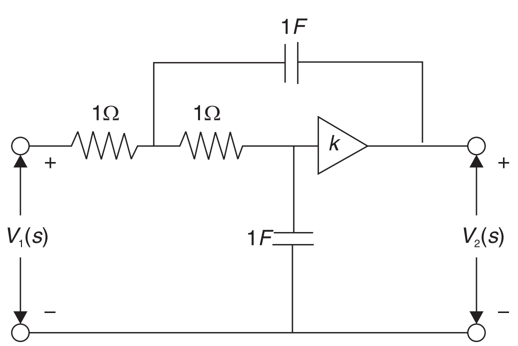
$V_1 = 0 \rightarrow$

$$y_{12} = -y_2 = -\left(\frac{3}{11} + s\right)$$

$$y_{22} = y_2 + y_3 = \frac{4}{11} + s$$

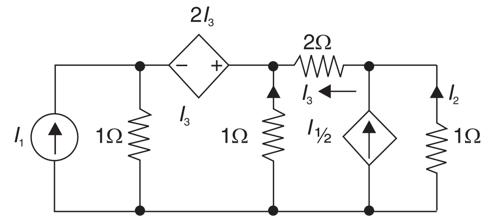
2. Assuming that the amplifier shown in figure below, is a voltage–controlled voltage source, show that the voltage transfer function of the network is given by

$$\mathrm{T(s)} = \frac{\mathrm{V_2(s)}}{\mathrm{V_1(s)}} = \frac{\mathrm{K}}{\mathrm{s^2 + (3 - K)s + 1}}$$



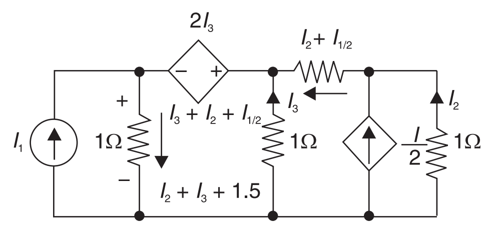
[1994]

3. Find the current–transfer–ratio, I_2/I_2 for the network shown in below figure. Also mark all branch currents.



[1995]

Solution:



Apply mesh analysis in both loop(11)

$(I_3 + I_2 + 1.5I_1) \times 1 + (I_3) \times 1 + 2I_3 = 0$

$1.5I_1 + I_2 + 4I_3 = 0$

Apply mesh analysis

$$(I_2)\times 1 + \left(I_2 + \frac{I_1}{2}\right)\times 2 - (I_3 \times 1) = 0$$

$I_1 + 3I_2 = I_3$

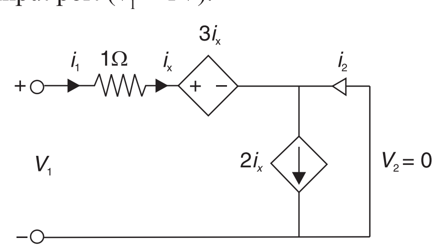
Substitute I_3 in equation (i)

$1.5I_1 + 2I_2 + 4(I_1 + 3I_2) = 0$

$1.5I_1 + I_2 + 4I_1 + 12I_2 = 0$

$$\frac{I_2}{I_1} = \frac{-11}{26}$$

4. For the 2–port network shown in figure determine the *h*–parameters. Using these parameters, calculate the output (port '2') voltage v_2, when the output port is terminated in a 3| resistance and a 1 V(DC) is applied at the input port (v_1 = 1V).

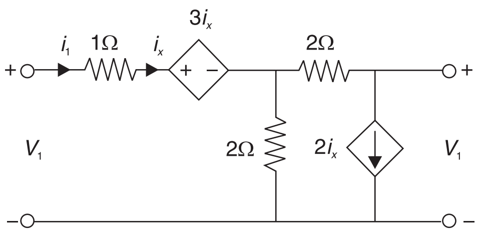


[1995]

Solution: $I_y = I_x$

$I_Y = I_X$

$V_2 = 0 \rightarrow$



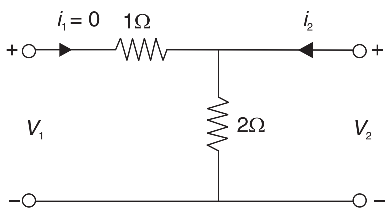
$$h_{11} = \frac{V_1}{I_1} = \frac{1\times i_1 + 3i_1' + 0}{i_l} = y$$

$$h_{21} = \frac{I_2}{I_1} = \frac{I_1}{I_2} = 1$$

i'_1=0

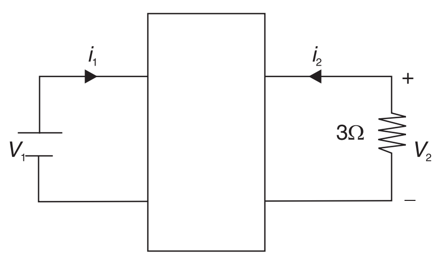
$lx = 0$

$2ix = 0$



$V_1 = 4I_1 + V_2$

$I_2 = I_1 + 05V_2$



$V_1 = 1V$

$V_2 = -3L_2$

$l_1 = l_1 - 1.5\ I_2$

$2.5\ l_2 = l_1$

$$I_1 = 2.5\left(-\frac{V_2}{3}\right) = \frac{-2.5V_2}{3}$$

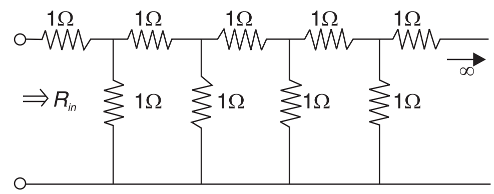
$$V_1 = 4I_1 + V_2$$

$$1 = 4\left(\frac{-25V_2}{3}\right) + V_2 = \frac{-10V_2}{3} + V_2$$

$$1 = \frac{-10V_2 + 3V_2}{3} = \frac{-7V_2}{3}$$

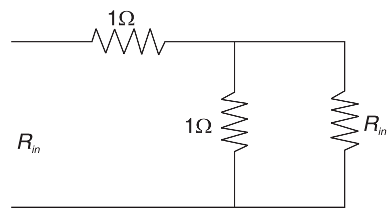
$$V_2 = \frac{-3}{7}V$$

5. Find the input resistance R_{in} of the infinite section resistive network shown in figure.



[1996]

Solution:



$$R_{in} = 1 + \frac{1\times R_{in}}{1 + R_{in}} = \frac{1 + R_{in} + R_{in}}{1 + R_{in}}$$

$R^2_{in} + R_{in} = 1 + 2R_{in}$

$R^2_{in} - R_{in} - 1 = 0$

$$R_{in} = \frac{+1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

R_{in} cannot be negative

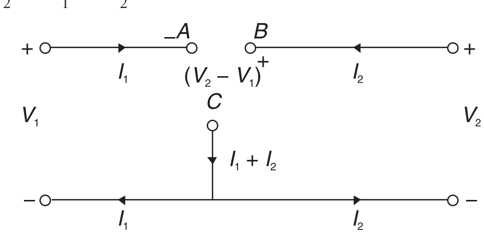
$$R_{in} = \frac{1 + \sqrt{5}}{2}\Omega$$

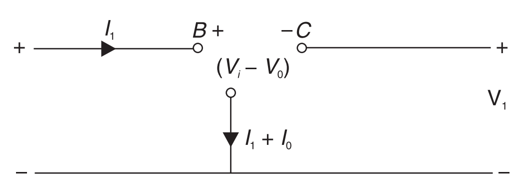
6. The open circuit impedance matrix Z_{oc} of a three–terminal two–port network with A as the input terminal, B as the output terminal, and C as the common terminal, is given as $[Z_{oc}] = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$

Write down the short circuit admittance matrix Y_{SC} of the network viewed as a two–port network, but now taking B as the input terminal, C as the output terminal and A as the common terminal. **[1996]**

Solution: $V_1 = 2I_1 + 5I_2$

$V_2 = 3I_1 + 7I_2$





$I_i = y_{11}V_i + V_{12}V_b$

$I_0 = y_{21}V_1 + y_{22}V_0$

$V_1 = -V_b$

$V_2 = -V_i - V_b$

$I_1 = -(I_i + I_0)$

$I_2 = I_i$

$-V_0 = -2(I_1 + I_0) + 5I_i$

$-V_0 = 3I_i + 2I_0$

$V_i - V_0 = -3(I_i + I_0) + 7I_i$

$V_i = V_0 + 4I_i - 3I_0$

$V_i = I_i - I_0$

$$[2] = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$$

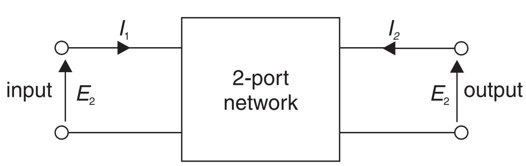
$$[y] = [2]^{-1} = \begin{bmatrix} -2 & -1 \\ -3 & -1 \end{bmatrix}$$

7. The admittance parameters of a 2–port network shown in figure are given by $Y_{11} = 2$ mho, $Y_{12} = -0.5$ mho, $Y_{21} = 4.8$ mho, $Y_{22} = 1$ mho. The output port is terminated with a load admittance $Y_L = 0.2$ mho. Find E_2 for each of the following conditions:

(a) $E_1 = 10\angle 0°$V

(b) $I_1 = 10\angle 0°$A

(c) A source $10\angle 0°$ V in series a 0.25Ω resistor is connected to the input port.

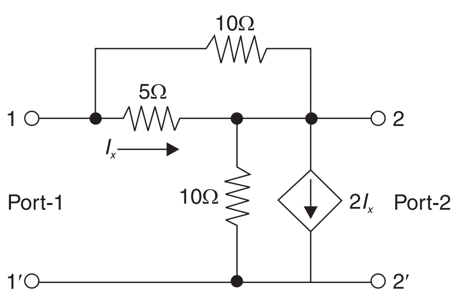


[2001]

8. Consider the network in figure 1.

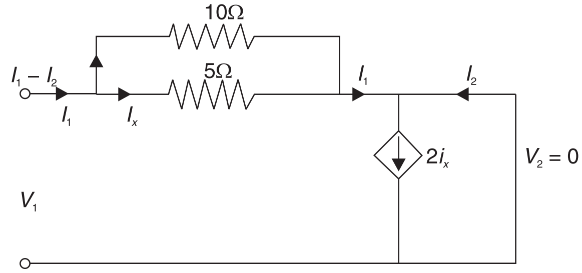
(a) Find its short–circuit admittance parameters.

(b) Find the open–circuit impedance Z_{22}.



[2002]

Solution: When $V_2 = 0$



from the fig,

$2I_x = I_1 + I_2$

$V_1 = (5||10)I_1 = \dfrac{10}{3}I_1$

$\Rightarrow y_{11} = \dfrac{I_1}{V_1} = \dfrac{3}{10} = 0.3℧$

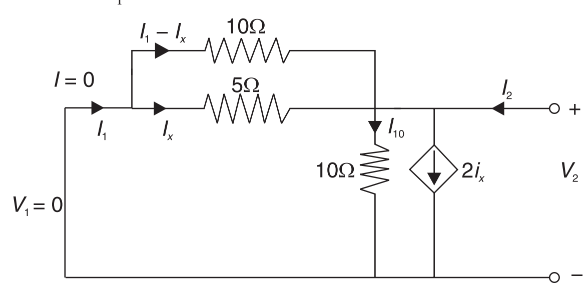
Also, $5I_x = 10(I_1 - I_x) = 10I_1 - I_0I_x$

$\Rightarrow 3I_x = 2I_1$

$\Rightarrow I_x = \dfrac{2}{3}I_1$

$$\therefore y_{21} = \frac{I_2}{V_1} = \frac{2I_x - I_1}{\frac{10}{3}I_1} = \frac{\frac{4}{3}I_1 - I_1}{\frac{10}{3}I_1} = \frac{1}{10} = 0.1℧$$

When $V_1 = 0$,



$$V_2 = \frac{-10}{3}I_1$$

$$\Rightarrow y_{12} = \frac{I_1}{V_2} = -0.3℧$$

$$I_x = \frac{2}{3}I_1$$

$$\Rightarrow I_1 + I_2 = \frac{V_2}{10} + 2I_x$$

$$= \frac{V_2}{10} + \frac{4}{3}I_1$$

$$\Rightarrow I_2 = \frac{V_2}{10} + \frac{1}{3} I_1$$

$$= \frac{V_2}{10} + \frac{1}{3}\left(\frac{-3}{10}\right) V_2$$

$$\Rightarrow I_2 = 0$$

$$\Rightarrow y_{22} = \frac{I_2}{V_2} = 0$$

$$\therefore [y] = \begin{bmatrix} 0.3 & -0.3 \\ 0.1 & 0 \end{bmatrix} \text{Ans.}$$

(b) $Z_{22} = \frac{y_{11}}{|y|} = \frac{0.3}{0.03} = 10\Omega$ Ans.

Chapter 6

Graph Theory and State Equations

One-mark Questions

1. The network is described by the model is

$$\dot{x}_1 = 2x_1 - x_2 + 3u$$

$$\dot{x}_2 = -4x_2 - u$$

$$y = 3x_1 - 2x_2$$

The transfer function $H(s)\left(=\frac{Y(s)}{U(s)}\right)$ is **[2015]**

(A) $\frac{11s+35}{(s-2)(s+4)}$ (B) $\frac{11s-35}{(s-2)(s+4)}$

(C) $\frac{11s+38}{(s-2)(s+4)}$ (D) $\frac{11s-38}{(s-2)(s+4)}$

Solution: $A = \begin{bmatrix} 2 & -1 \\ 0 & -4 \end{bmatrix}$

$$B = \begin{bmatrix} 3 \\ -1 \end{bmatrix}; C = \begin{bmatrix} 3 & -2 \end{bmatrix}$$

$D = 0$, we know

$$H(s) = C.\,(SI - A)^{-1}.\,B + D$$

$$(SI - A) = \begin{bmatrix} S-2 & 1 \\ 0 & S+4 \end{bmatrix}$$

$$(SI - A)^{-1} = \frac{1}{|SI - A|}.\text{adj}(SI - A)$$

$$|SI - A| = (S - 2)\,(S + 4)$$

$$(SI - A)^{-1} = \frac{1}{|SI - A|}\begin{bmatrix} S+4 & -1 \\ 0 & S-2 \end{bmatrix}$$

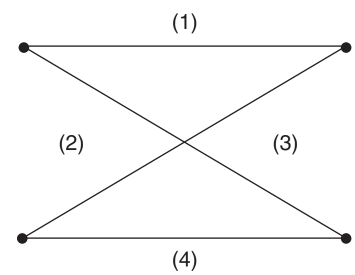
$$\therefore\; H(s) = \frac{1}{(S-2)(S+4)}.\begin{bmatrix} 3 & -2 \end{bmatrix}\begin{bmatrix} S+4 & \overset{\rightarrow}{-1} \\ 0 & S-2 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$H(s) = \frac{\begin{bmatrix} 3 & -2 \end{bmatrix}\begin{bmatrix} 3S+13 \\ -S+2 \end{bmatrix}}{(S-2)(S+4)} = \frac{9S+39+2S-4}{(S-2)(S+4)}$$

$$\therefore\; H(s) = \frac{11S+35}{(S-2)(S+4)}$$

Hence, the correct option is (A).

2. In the following graph, the number of trees (P) and the number of cut–sets (Q) are



(a) $P = 2, Q = 2$ (b) $P = 2, Q = 6$

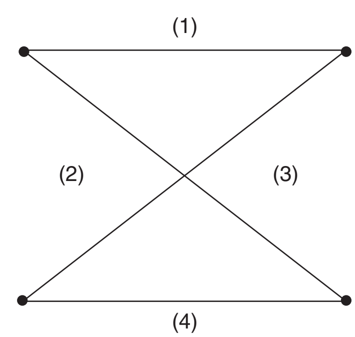
(c) $P = 4, Q = 6$ (d) $P = 4, Q = 10$

[2008]

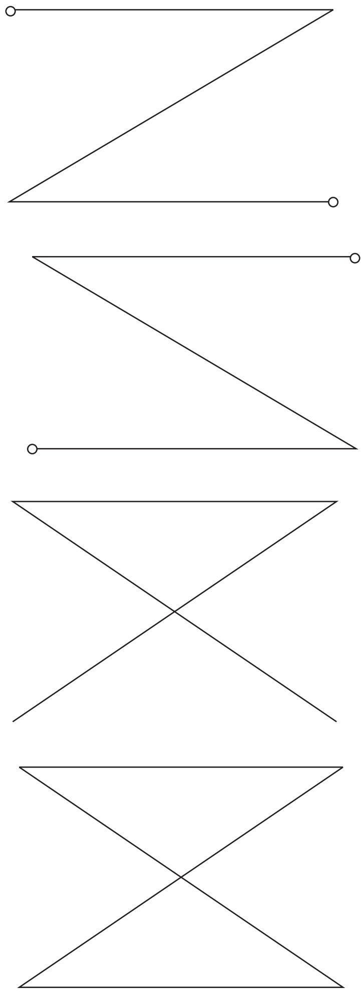
Solution: (c)

Hence, the correct option is (c)

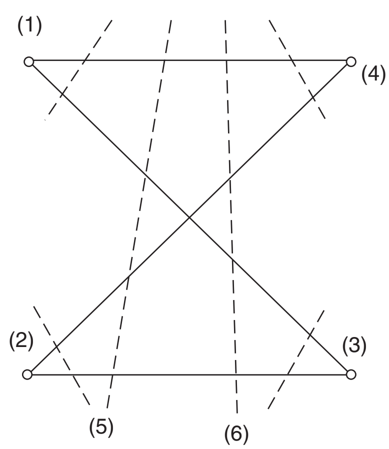
For given graph, number of tree can be calculated as



Number of trees = 4

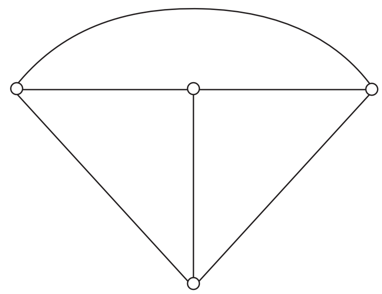


And cut set will be drown as



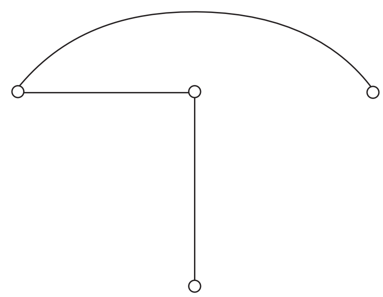
Total cut–sets = 6

3. Consider the network graph shown in the figure. Which one of the following is NOT a ‘tree’ of this graph?

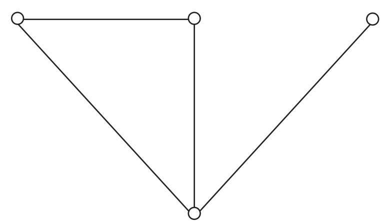


[2004]

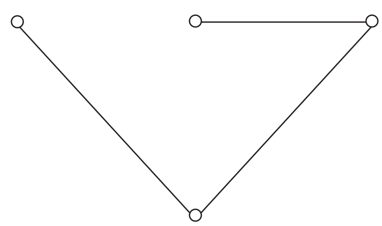
(a)



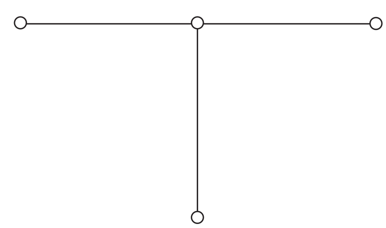
(b)



(c)



(d)

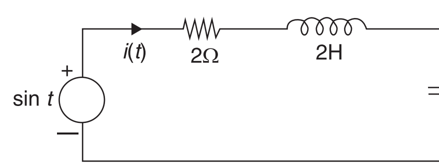


Solution: (b)

The tree doesn’t form closed loop.

Hence, the correct option is (b)

4. The differential equation for the current $i(t)$ in the circuit of the figure is



(a) $2\frac{d^2i}{dt^2}+2\frac{di}{dt}+i(t)=\sin t$

(b) $\frac{d^2i}{dt^2}+2\frac{di}{dt}+2(t)=\cos t$

(c) $2\frac{d^2i}{dt^2}+2\frac{di}{dt}+i(t)=\cos t$

(d) $\frac{d^2i}{dt^2}+2\frac{di}{dt}+2(t)=\sin t$ **[2003]**

Solution: (c)

Appling KVL for given loop

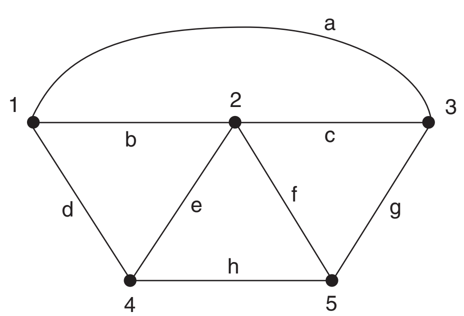
$$\sin t = 2i+\frac{2di}{dt}+\int idt$$

Differentiation:–

$$\cos t=\frac{2di(t)}{dt}+\frac{2d^2i(t)}{dt^2}+i(t)$$

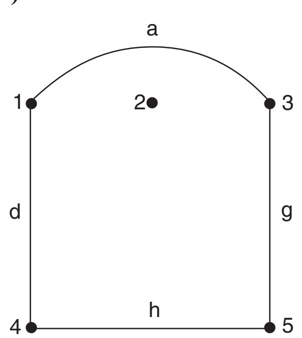
Hence, the correct option is (c)

5. Identify which of the following is NOT a tree of the graph shown in the figure



(a) begh (b) defg
(c) adhg (d) aegh **[1999]**

Solution: (c)



(c) does not form closed loop. So adhg is not a tree.

Hence, the correct option is (c)

6. A network has seven nodes and five independent loops. The number of branches in the network is
(a) 12 (b) 12
(c) 11 (d) 10 **[1998]**

Solution: (c)

Number of independent loops or mess is given by

$e = b-(n-1)$

$b = e+(n-1)=5+(7-1)=5+6$

$b = 11$

Hence, the correct option is (c)

7. The number of independent loops for a network with n nodes and b branches is
(a) $n-1$
(b) $b-n$
(c) $b-n+1$
(d) independent of the number of nodes **[1996]**

Solution: (c)

The number of independent loops for a network with n nodes and b branches is $b-(n-1)=b-n+1$.

Hence, the correct option is (c)

Two-marks Questions

1. Relative to a given fixed tree of a network,
(a) Link currents form an independent set
(b) Branch currents form an independent set
(c) Link voltages form an independent set
(d) Branch voltages form an independent set **[1992]**

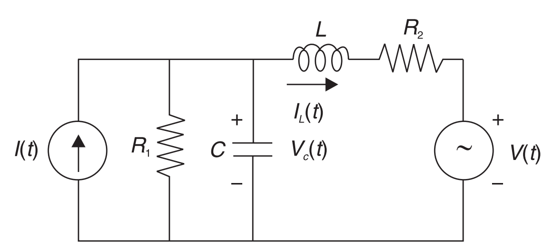
Solution:(a)

Link current forms an independent set.

Hence, the correct option is (a)

Five-marks Questions

1. Refer to the circuit shown in figure



Choosing the voltage $v_c(t)$ across capacitor, and the current $i_L(t)$ through the inductor as state variables, i.e.

$$[X(t)]=\begin{bmatrix}V_c & (t)\\ i_L & (t)\end{bmatrix}$$

Write the state equation in the from

$\frac{d}{dt}[x(t)]=[A][\times(t)+[B][u(t)]$ and find [A], [B] and [u(t)]. **[1996]**

Solution: Apply KCL,

$$\frac{CdV_C(t)}{dt}+\frac{V_C}{R}+i_L(t)=I(t)$$

$$\Rightarrow \frac{dV_C(t)}{dt}+\frac{1}{R_1C}V_C+\frac{i_L(t)}{C}=\frac{I(t)}{C}$$

$$\Rightarrow \frac{dV_C(t)}{dt}=\frac{-1}{R_1C}V_C-\frac{1}{C}i_L(t)+\frac{1}{C}I(t) \quad (1)$$

Apply KVL in right side loop,

$$L\frac{di_L}{dt}+i_LR_2+V(t)-V_L=0$$

$$\Rightarrow \frac{di_L}{dt}=\frac{1}{L}V_L-\frac{R_2}{L}i_L-\frac{1}{L}V(t) \quad (2)$$

From equation (1) and (2)

$$\begin{bmatrix} V_C \\ i_L \end{bmatrix}=\begin{bmatrix} \frac{-1}{R_1C} & \frac{-1}{C} \\ \frac{1}{L} & \frac{-R_2}{C} \end{bmatrix}\begin{bmatrix} V_C \\ i_L \end{bmatrix}+\begin{bmatrix} \frac{1}{C} & 0 \\ 0 & \frac{-1}{L} \end{bmatrix}\begin{bmatrix} I(t) \\ V(t) \end{bmatrix}$$

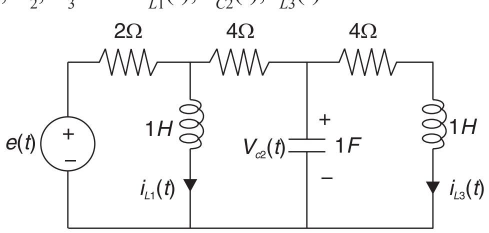
$$\Rightarrow \frac{d}{dt}\begin{bmatrix} V_C(t) \\ i_L(t) \end{bmatrix}=\begin{bmatrix} \frac{-1}{R_1C} & \frac{-1}{C} \\ \frac{1}{L} & \frac{-R_2}{L} \end{bmatrix}\begin{bmatrix} V_C(t) \\ i_L(t) \end{bmatrix}+\begin{bmatrix} \frac{1}{C} & 0 \\ 0 & \frac{-1}{L} \end{bmatrix}\begin{bmatrix} I(t) \\ V(t) \end{bmatrix}$$

$$\Rightarrow A=\begin{bmatrix} \frac{-1}{R_1C} & \frac{-1}{C} \\ \frac{1}{L} & \frac{-R_2}{L} \end{bmatrix}$$

$$B=\begin{bmatrix} \frac{1}{C} & 0 \\ 0 & \frac{-1}{L} \end{bmatrix}$$

$$u(t)=\begin{bmatrix} I(t) \\ V(t) \end{bmatrix}$$

2. For the circuit shown in figure choose state variables X_1, X_2, X_3 to be $i_{L1}(t), v_{C2}(t), i_{L3}(t)$

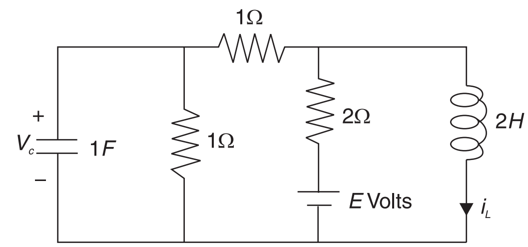


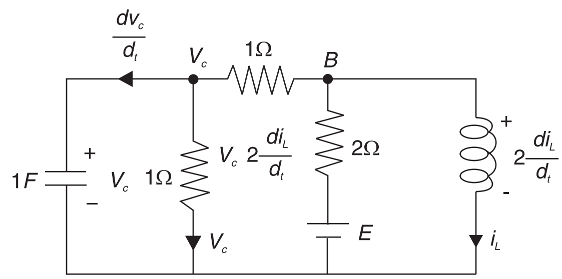
(a) Write the state equations.

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}=A\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}+B[e(t)]$$

(b) If $e(t)=0,\ t\geq 0,\ i_{L1}(0)=0,\ v_{o2}(0)=0,_2(0)=0,$ $i_{La}(0)=1A$, then what would the total energy dissipated in the resistor in the interval $(0,\infty)$ be? **[1997]**

3. For the circuit in figure, write the state equation using v_c and i_L as state variable.





Solution:

Apply KCL at node V_C

$$\frac{dV_C}{dt}+V_C+\left(V_C-\frac{di_L}{dt}\right)=0$$

$$\Rightarrow \frac{dV_C}{dt}=-2V_C+\frac{2di_L}{dt} \quad (1)$$

Apply KCL at node B

$$V_C-\frac{2di_L}{dt}=\frac{di_L}{dt}-\frac{E}{2}+i_L$$

$$\Rightarrow \frac{3di_L}{dt}=V_C-i_L+\frac{E}{2}$$

$$\Rightarrow \frac{di_L}{dt}=\frac{1}{3}V_C-\frac{1}{3}i_L+\frac{1}{6}E \quad (2)$$

From equation (1) and (2),

$$\frac{dV_C}{dt}=-2V_C+2\left(\frac{1}{3}V_C-\frac{1}{3}i_L+\frac{E}{6}\right)$$

$$=\frac{-4}{3}V_C-\frac{2}{3}i_L+\frac{1}{3}E \quad (3)$$

From eq (2) and (3), we get

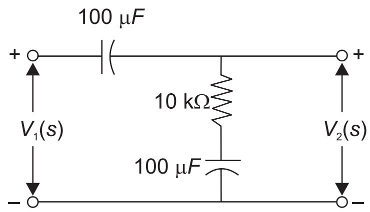
$$\begin{bmatrix} V_C \\ i_L \end{bmatrix}=\begin{bmatrix} -\frac{4}{3} & -\frac{2}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix}\begin{bmatrix} V_C \\ i_L \end{bmatrix}+\begin{bmatrix} 1/3 \\ 1/6 \end{bmatrix}[E]$$

Chapter 7

Network Functions

One-mark Questions

1. The transfer function $\frac{V_2(s)}{V_1(s)}$ of the circuit shown below is



(a) $\frac{0.5s+1}{s+1}$ (b) $\frac{3s+6}{s+2}$

(c) $\frac{s+2}{s+1}$ (d) $\frac{s+1}{s+2}$ **[2013]**

Solution: (d)

From a given circuit, transfer function can be given as

$$\frac{V_2(s)}{V_1(s)} = \frac{R+1/SC}{R+2/SC}$$

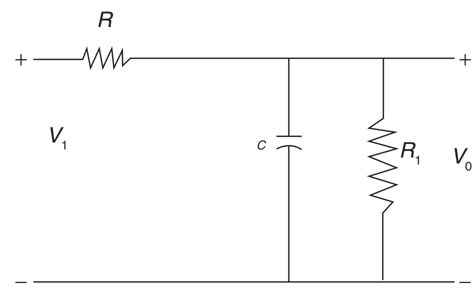
$$\Rightarrow \frac{RSC+1}{RSC+2}$$

$$RC = \frac{10\times 10^3 \times 100}{10^6} = 1\,\text{sec}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{s+1}{s+2}$$

Hence, the correct option is (d)

2. If the transfer function of the following network is $\frac{V_0(s)}{V_1(s)} = \frac{1}{2+\text{sCR}}$.



The value of the load resistance R_L is

(a) $R/4$ (b) $R/2$

(c) R (d) $2R$ **[2009]**

Solution: (c)

From the given circuit, transfer function can be given as

$$\frac{V_0(a)}{V_i(s)} = \frac{R_L \,||\, 1/sC}{R + R_L \,||\, \frac{1}{sC}}$$

$$\Rightarrow \frac{1}{1+\frac{R}{R_L \,||\, \frac{1}{sC}}}$$

$$\Rightarrow \frac{1}{1+\frac{R(SCR+1)}{R_L}}$$

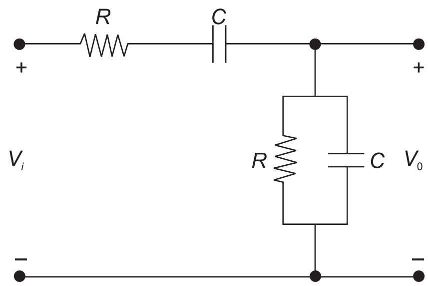
Comparing with $\frac{1}{2+sCL}$

We get $\frac{R}{R_L} = 1$

$R_L = R$

Hence, the correct option is (c)

3. The RC circuit shown in the figure is

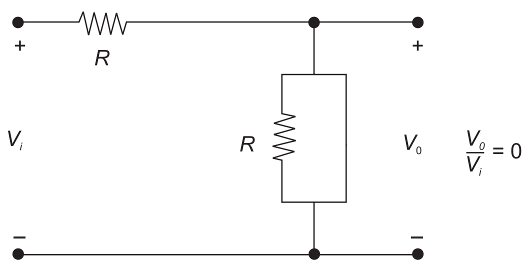


(a) a low-pass filter
(b) a high-pass filter
(c) a band-pass filter
(d) a band reject filter **[2007]**

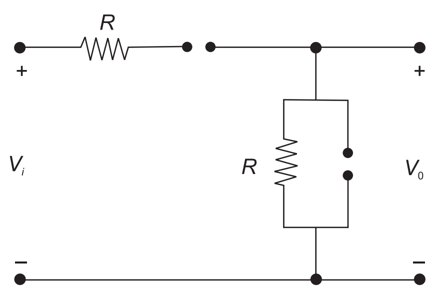
Solution: (c)

When $\omega \to \infty$, capacitor $\to$ short circuited.

Circuit looks like

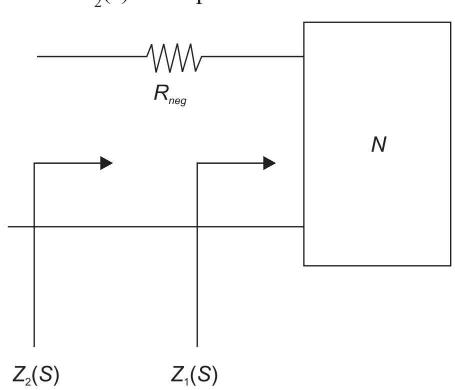


At $\omega \to \infty$, Capacitor $\to$ open circuited.



Hence, the correct option is (c)

4. A negative resistance R_{neq} is connected to a passive network N having driving point impedance as shown below. For Z_2(s) to be positive real



(a) $|R_{neg}| \leq ReZ_1(j\omega), \forall\omega$
(b) $|R_{neg}| \leq |Z_1(j\omega)|, \forall\omega$
(c) $|R_{neg}| \leq |ImZ_1(j\omega)|, \forall\omega$
(d) $|R_{neg}| \leq |\leq\angle Z_1(j\omega), \forall\omega$ **[2006]**

Solution:(a)

For $Z_2(s)$ to be positive real, $R_e|Z(s)| \geq |R_{neg}|$

$\Rightarrow |R_{neg}| \leq R_e\,|Z_1(j\omega)|$ for all ω.

Hence, the correct option is (a)

5. The first and the last critical frequencies (singularities) of a driving point impedance function of a passive network having two kinds of elements are a pole and a zero, respectively. The above property will be satisfied by

(a) RL network only
(b) RC network only
(c) LC network only
(d) RC as well as RL networks **[2006]**

Solution: (b)

RC impedance junction has
(i) 1st critical frequency due to pole
(ii) Last critical frequency due to zero

Hence, the correct option is (b)

6. The first and the last critical frequency of an RC-driving point impedance function must respectively be

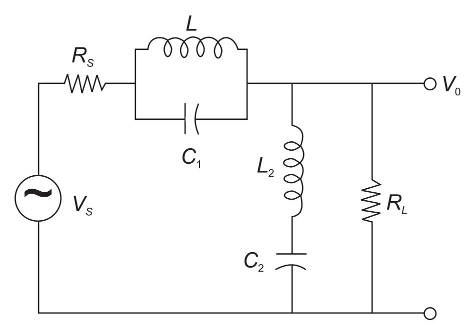
(a) a zero and a pole
(b) a zero and a zero
(c) a pole and a pole
(d) a pole and a zero **[2005]**

Solution: (d)

For stability, poles and zero interface on real axis. Since its *RC,* first pole should come and zero at last.

Hence, the correct option is (d)

7. The circuit of the figure represents a



(a) low-pass filter
(b) high-pass filter
(c) band-pass filter
(d) band-reject filter **[2000]**

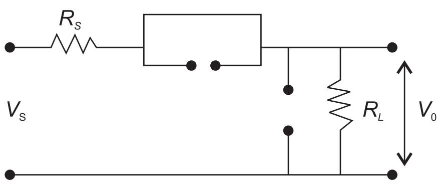
Solution: (d)

Analyzing the circuit for $\omega = 0$

$\Delta\omega = \infty;$

At $\omega = 0$

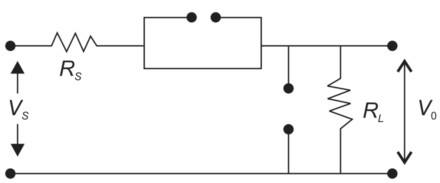
$\omega L = 0 \rightarrow$ inductor (SC)



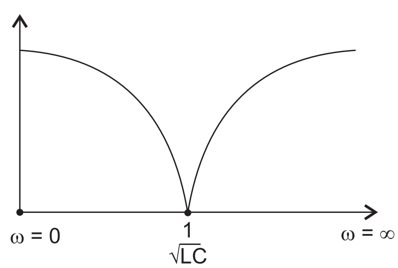
$$\text{Capacitor} = \frac{1}{\omega C} = \infty \text{ (OC)}$$

$$\frac{V_0}{V_S} = \frac{R_L}{R_L + R_S} \quad \text{(finite value)}$$

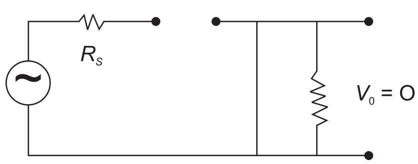
At $\omega = \infty$



$$\frac{V_0}{V_S} = \frac{R_L}{R_L + R_S} \quad \text{(finite value)}$$



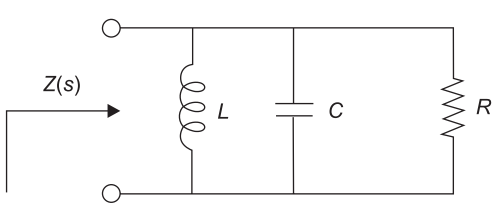
At $\omega = \dfrac{1}{\sqrt{LC}}$



Hence, the correct option is (d)

Two-marks Questions

1. The driving point impedance of the following network



is given by $Z(s) = \dfrac{0.2s}{s^2 + 0.1s + 2}$. The component values are

(a) $L = 5\text{H}, R = 0.5\ \Omega, C = 0.1\ \text{F}$
(b) $L = 0.1\ \text{H}, R = 0.5\ \Omega, C = 5\ \text{F}$
(c) $L = 5\text{H}, R = 2\ \Omega, C = 0.1\ \text{F}$
(d) $L = 0.1\ \text{H}, R = 2\ \Omega, C = 5\ \text{F}$ **[2008]**

Solution: (d)

Equivalent admittance for given RLC series circuit can be given as

$$y(s) = \frac{1}{R} + \frac{1}{sL} + SC \qquad (1)$$

$$= \frac{1}{R} + \frac{s^2 LC}{sL}$$

$$y(s) = \frac{1}{2(s)} = \frac{s^2 + 0.1s + 2}{0.2s}$$

$$= \frac{s^2}{0.2s} + \frac{0.1s}{0.2s} + \frac{2}{0.2s}$$

$$= 5s + \frac{1}{2} + \frac{10}{s} \qquad (2)$$

by comparing (1) and (2)

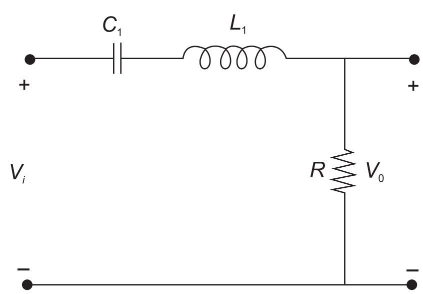
$R = 2\Omega$

$C = 5\text{F}$

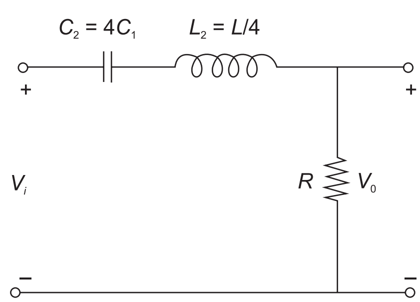
$L = 0.1\text{H}$

Hence, the correct option is (d)

2. Two series resonant filters are as shown in the figure. Let the 3-dB bandwidth of Filter 1 be B_1 and that of Filter 2 be B_2. The value of $B_1 B_2$ is



Filter 1



Filter 2

(a) 4 (b) 1
(c) $\dfrac{1}{2}$ (d) $\dfrac{1}{4}$ **[2007]**

Solution: (d)

Bandwidth of series RLC circuit $= \dfrac{R}{L}$

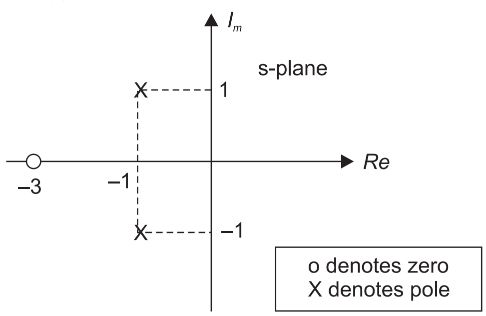
$$B_1 = \frac{R}{L_1};\ B_2 = \frac{R}{L_2}$$

$$B_2 = \frac{4R}{L_1}$$

$$\frac{B_1}{B_2} = \frac{1}{4}$$

Hence, the correct option is (d)

3. The driving-point impedance $Z(s)$ of a network has the pole-zero locations as shown in the figure. If $Z(0) = 3$, then $Z(s)$ is



(a) $\dfrac{3(s+3)}{s^2+2s+3}$ (b) $\dfrac{2(s+3)}{s^2+2s+2}$

(c) $\dfrac{3(s-3)}{s^2-2s-2}$ (d) $\dfrac{2(s-3)}{s^2-2s-3}$

[2003]

Solution: (b)

From pole –zero diagram Impedance function can be written as,

$$Z(s) = \frac{K(S-Z)}{(S-P_1)(S-P_2)}$$

$$= \frac{K(S+3)}{(S+1+j)(S+1-j)}$$

$$Z(S) = \frac{K(S+3)}{(S+1)^2 - J^2}$$

$$= \frac{K(S+3)}{(S+1)^2+1}$$

$$Z(0)\Big|_{\omega=0} = 0$$

$$\Rightarrow \frac{3K}{2} = 3 \Rightarrow k = 2$$

$$\therefore\ Z(S) = \frac{2(S+3)}{S^2+2S+2}$$

Hence, the correct option is (b)

4. Indicate True/False and give reason for the following question.

$Z(s) = \dfrac{5}{s^2+4}$ represents the input impedance of a network. **[1994]**

Solution: FALSE.

For $Z(s)$ to represent the input impedance of a passive network, the numerator and denominator degrees should not differ by more than 1.

5. The necessary and sufficient condition for a rational function of s, $T(s)$, to be a driving point impedance of an RC network is that all poles and zeros should be

(a) simple and lie on the negative real axis of the s-plane

(b) complex and lie in the left half of the s-plane

(c) complex and lie in the right half of the s-plane

(d) Simple and lie on the positive real axis of the s-plane **[1991]**

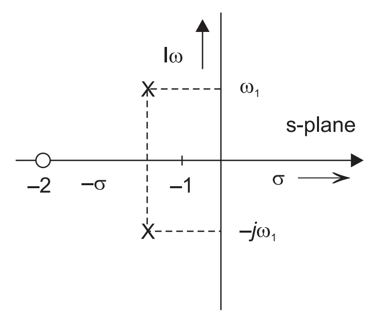
Solution: (a)

Simple and lie on the negative real axis of the s-plane.

The poles and zeros of the Z_{RC}(s) should be simple and alternate on the negative real axis of the s-plane.

Hence, the correct option is (a)

6. A driving point admittance function has pole and zero locations as shown below. The range of s for which the function can be realized using passive elements is



(a) σ < – 1 (b.) σ > 1

(c) σ < 1 (d) σ > – 1 **[1988]**

Solution: (b)

Function can be realized when

$\sigma - 1 > 0$

$\sigma > 1$

Hence, the correct option is (b)

UNIT II

SIGNALS AND SYSTEMS

EXAM ANALYSIS

Exam Year	92	93	94	95	96	97	98	99	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14–1	14–2	14–3	14–4	15			16			17		18	19
																											Set 1	Set 2	Set 3	Set 1	Set 2	Set 3	Set 1	Set 2		
1 Mark Questions	-	-	5	6	3	2	11	3	3	4	3	4	3	6	3	1	2	3	2	3	2	7	4	3	3	4	5	3	3	4	3	3	3	3	3	4
2 Marks Questions	3	4	-	1	1	2	-	2	3	1	5	3	6	6	3	3	8	5	3	4	3	3	4	4	4	4	1	3	4	3	2	6	3	6	6	3
5 Marks Questions	-	2	1	2	4	1	1	-	3	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Total Marks	**6**	**18**	**10**	**18**	**25**	**11**	**16**	**7**	**24**	**11**	**13**	**10**	**15**	**18**	**9**	**7**	**18**	**13**	**8**	**11**	**8**	**13**	**12**	**9**	**11**	**12**	**7**	**9**	**11**	**10**	**7**	**15**	**9**	**15**	**15**	**10**
Chapter wise marks distribution																																				
Basics of Signals and Systems	-	-	-	-	-	-	-	-	1	2	-	2	2	2	3	-	3	-	-	-	2	3	1	3	4	6		2			4		1		1	1
Divide and Conquer	-	-	-	1	-	-	1	-	2	2	1	-	3	1	-	-	3	-	-	1	2	2	3	2	1	1		4			2		7		-	-
Greedy Method	2	2	1	1	1	-	2	2	2	-	1	1	-	1	-	-	-	1	-	1	-	-	-	-	-	-		2			1		2		1	-
Dynamic Programming	2	2	1	2	2	3	4	1	1	-	2	5	5	6	1	2	6	3	-	-	2	1	-	3	-	2		-			4		1		-	-
P and NP Concepts	-	4	2	3	2	3	1	1	3	1	2	1	-	-	2	1	-	2	2	5	1	4	2	-	-	-		2			1		-		-	1
Optimal Binary Search Tree	2	-	-	-	-	-	1	3	-	1	2	1	3	1	1	2	4	3	5	2	1	-	3	3	5	-		4			3		1		-	1
Miscellaneous Topics	-	-	-	-	-	-	-	-	-	-	-	-	-	7	-	2	2	2	1	2	-	2	2	-	-	2		1			1		1		1	3
Miscellaneous Topics	-	-	1	1	-	-	2	-	-	-	-	-	2	-	2	-	-	2	-	-	-	1	1	-	1	1		2			2		1		1	1

Chapter 1

Basics of Signals and Systems

ONE-MARK QUESTIONS

1. Consider the random process

$$X(t) = U + Vt$$

Where U is a zero-mean Gaussian random variable and V is a random variable uniformly distributed between 0 and 2. Assume that U and Y are statistically independent. The mean value of the random process at $t = 2$ is ______. **[2017]**

Solution: $x(t) = U + Vt$

At, $\quad t = 2s, x(t) = x(2) = U + 2V$

$$E[x(t)] = E[U + 2V]$$

$$= E[U] + 2E[V]$$

$$E[U] = 0$$

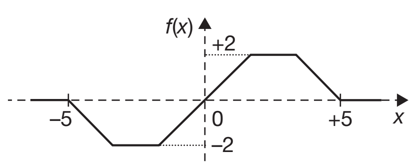
$$E[V] = \int_{-\infty}^{\infty} f_v(V)dv$$

$$= \int_0^2 \frac{1}{2} dv = 1$$

$$\therefore \; E[x(t)] = 0 + 2(1) = 2$$

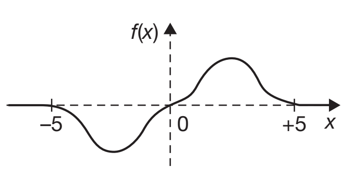
Hence, the correct answer is (1.9 to 2.1).

2. Consider the plot of $f(x)$ versus x as shown below

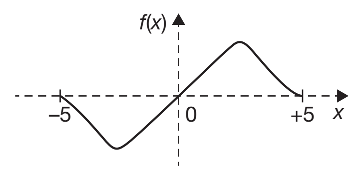


Suppose $F(x) = \int_{-5}^{x} f(y)\,dy$. Which one of the following is a graph of $F(x)$? **[2016]**

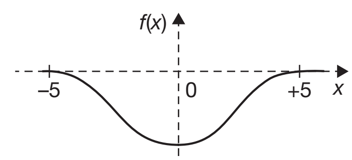
(A)



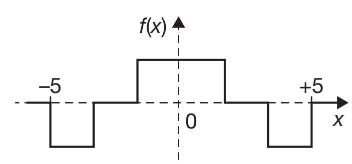
(B)



(C)



(D)



Solution: $F(x)$ is an implicit function here and from the given figure, $f(x)$ is linear so the result will be in quadratic form. Thus option (c) is correct with –ve peak.

Hence, the correct option is (C).

3. Which one of the following is an Eigen function of the class of all continuous time, linear, time invariant systems ($u(t)$ denotes the unit step function)? **[2016]**

(A) $e^{j\omega_0 t}u(t)$ (B) $\cos(\omega_0 t)$

(C) $e^{j\omega_0 t}$ (D) $\sin(\omega_0 t)$

Solution: $e^{j\omega o t}$ is an Eigen function and also continuous, time invariant and linear out of all given option. So option (C)

Hence, the correct option is (C).

4. The energy of the signal $x(t) = \dfrac{\sin(4\pi t)}{4\pi t}$ is ______. **[2016]**

Solution: The given signal is

$$x(t) = \frac{\sin(4\pi t)}{4\pi t} \text{ is}$$

According to Parseval's identity, Energy content of $x(t)$ is

$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\times(\omega)|^2 \, d\omega$$

For $\frac{\sin at}{\pi t} \xleftrightarrow{F.T} P_a(\omega) = \begin{cases} 1. & |\omega| < a \\ 0, & |\omega| > a \end{cases}$

For $\frac{1}{4}\frac{\sin 4\pi T}{\pi t} \xleftrightarrow{F.T} P_a(\omega) = \begin{cases} \frac{1}{4} & |\omega| < 4\pi \\ 0, & |\omega| > 4\pi \end{cases}$

So

$$\text{Energy of } x(t) = \frac{1}{2\pi} \int_{-4\pi}^{+4\pi} \left(\frac{1}{4}\right)^2 d\omega = \frac{1}{2\pi} \times \frac{1}{16}(8\pi)$$

$$= \frac{1}{4} = 0.25$$

Hence, the correct Answer is (0.25).

5. If the signal $x(t) = \frac{\sin(t)}{\pi t} * \frac{\sin(t)}{\pi t}$ with * denoting the convolution operation, then $x(t)$ is equal to **[2016]**

(A) $\frac{\sin(t)}{\pi t}$ (B) $\frac{\sin(2t)}{2\pi t}$

(C) $\frac{2\sin(t)}{\pi t}$ (D) $\left(\frac{\sin(t)}{\pi t}\right)^2$

Solution: We know that convolution of two sinc pulses is again sinc pulse

The given signal is

$$x(t) = \frac{\sin(t)}{\pi t} * \frac{\sin(t)}{\pi t}$$

Let, $x_1(t) = \frac{\sin(t)}{\pi t}$

$$x(t) = x_1(t) * x_2(t)$$

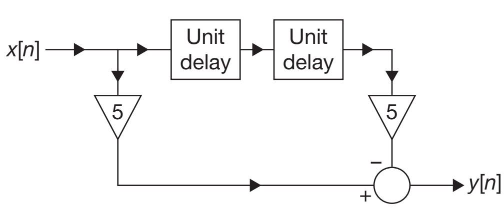
$$X(\omega) = X_1(\omega).\ X_2(\omega) = X_1(\omega)$$

$\therefore$ $X(\omega) = X_1(\omega)$

$$x(t) = x_1(t) = \frac{\sin(t)}{\pi t}$$

Hence, the correct option is (A).

6. The direct form structure of an FIR (finite impulse response) filter is shown in the figure



The filter can be used to approximate a **[2016]**

(A) low-pass filter (B) high-pass filter
(C) band-pass filter (D) band-stop filter

Solution: $x[n] \rightarrow$ unit delay $\rightarrow x[n-1]$

from the figure

$$y[n] = 5x[n] - 5x[n-2]$$

$$y(z) = 5x(z) - 5z^{-2} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = 5 - 5z^{-2}$$

Putting $z = e^{j\omega}$

$$H(e^{j\omega}) = 5(1 - e^{-j2\omega})$$

As frequency vary from $-\infty$ to $+\infty$, ω varies from $-\pi$ to $+\pi$

At low frequency $\omega = -\pi$

$$H\left(e^{j(-\pi)}\right) = 5\ (1-1) = 0$$

At high frequency $2 = \pi$

$$H\left(e^{j(\pi)}\right) = 5\ (1-1) = 0$$

At intermediate frequency $\omega = \frac{\pi}{2}$

$$H\left(e^{j\frac{\pi}{2}}\right) = 5\left(1 - e^{-j2\times\frac{\pi}{2}}\right) = 5\ (1-(-1)) = 10$$

So it is band pass filter

Hence, the correct option is (C).

7. Input $x(t)$ and output $y(t)$ of an LTI system are related by the differential equation $y''(t) - y'(t) - 6y(t) = x(t)$. If the system is neither causal nor stable, the impulse response $h(t)$ of the system is **[2015]**

(A) $\frac{1}{5}e^{3t}u(-t) + \frac{1}{5}e^{-2t}u(-t)$

(B) $-\frac{1}{5}e^{3t}u(-t) + \frac{1}{5}e^{-2t}u(-t)$

(C) $\frac{1}{5}e^{3t}u(-t) - \frac{1}{5}e^{-2t}u(t)$

(D) $-\frac{1}{5}e^{3t}u(-t) - \frac{1}{5}e - u(t)$

Solution: $y''(t) - y(t) - 6y(t) = x(t)$

By taking laplace transforms on both the sides

$$S^2Y(S) - Y(S) - 6Y(S) = X(S)$$

$$Y(S)\ [S^2 - S - 6] = X(S)$$

$$H(S) = \frac{Y(S)}{X(S)} = \frac{1}{S^2 - S - 6} = \frac{1}{(S+2)(S-3)}$$

$$H(S) = \frac{1}{5(S-3)} - \frac{1}{5(S+2)}$$

$\because$ System is not causal so

$$H(t) = -\frac{1}{5}e^{+3t}u(-t) + \frac{1}{5}e^{-2t}u(-t)$$

Hence, the correct option is (B).

8. A discrete time signal $x[n] = \sin(\pi^2 n)$, n being an integer, is **[2014]**

(a) periodic with period π
(b) periodic with period π^2
(c) periodic with period $\pi/2$
(d) not periodic

Solution: (d)

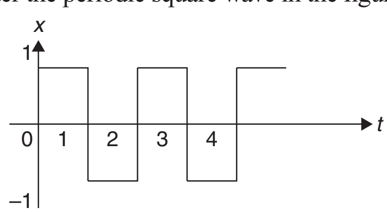
$x(n) = \sin(x^2 n)$

$$N = \frac{2\pi}{\omega} \times k$$

$$\therefore N = \frac{2\pi}{\pi^2} \times k = \frac{2k}{\pi}$$

where k is min integer so that N is a natural number.
No integer is possible.
Hence, the correct option is (d).

9. Consider the periodic square wave in the figure shown



The ratio of the power in the seventh harmonic to the power in the fifth harmonic for this waveform is closest in value to ____ **[2014]**

Solution: 0.51

For wave form $x(t)$

$$x(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a\, k\delta(\omega - k\omega)$$

$$\Rightarrow a_k = \frac{1}{2\pi}\int_0^2 x(t)e^{-jk\omega_0 t}dt$$

$$\Rightarrow a_k = \frac{1}{2\pi}\left[\int_0^1 1\cdot e^{-jk\omega_0 t}dt + \int_1^2 -1\cdot e^{-j\omega_0 kt}\,dt\right]$$

$$= \frac{1}{2\pi}\left[\frac{e^{-jk\omega_0 t}}{-j^{k\omega_0}}\right]_0^1 + \frac{1}{2\pi}\left[\frac{e^{-jk\omega_0 t}}{j^{k\omega_0}}\right]_1^2$$

$$= \frac{1}{2\pi j^{k\omega_0}}\left[1 - 2e^{-jk\omega_0} + 2e^{-2jk\omega_0}\right]$$

$$\frac{1}{2\pi j^{k\omega_0}}\left[1 - 2e^{-jk\bar{n}/2} + 2e^{-2jk\pi/2}\right] \therefore \omega_0 = \frac{\pi}{2}$$

$$\text{Now } \frac{\text{The power of the } 7^{\text{th}} \text{ harmonic}}{\text{power of the } 5^{\text{th}} \text{ harmonic}} = \left(\frac{a_7}{a_5}\right)^2$$

$$a_7 = \frac{1}{n^2 j7}[1-2j-2] = \frac{1}{n^2 j^7}[-1-2j]$$

$$= (2\pi\, a_7)^2 = 5$$

$$a_5 = (2\pi a_5)^2 = 7$$

$$\therefore \left(\frac{a_7}{a_5}\right)^2 = \left(\frac{5}{7}\right)^2 = \frac{25}{49} = 0.51$$

10. For a periodic signal $v(t) = 30\sin 100t + 10\cos 300t + 6\sin(500t + \pi/4)$, the fundamental frequency in rad/s is **[2013]**

(a) 100 (b) 300
(c) 500 (d) 1500

Solution: (a)

For $30\sin 100\,t \rightarrow \omega_1 = 100$.
For $10\cos 300\,t \rightarrow \omega_2 = 300$.
For $6\sin 500\,t \rightarrow \omega_3 = 500$
Fundamental frequency = HCF of (100, 300, 500) = 100
Hence, the correct option is (a).

11. The input and output of a continuous time system are, respectively, denoted by $x(t)$ and $y(t)$. which of the following descriptions corresponds to a causal system? **[2008]**

(a) $y(t) = x(t-2) + x(t+4)$
(b) $y(t) = (t-4) \times (t+1)$
(c) $y(t) = (t+4) \times (t-1)$
(d) $y(t) = (t+5) \times (t+5)$

Solution: (c)

For a causal system, o/p should depend only on present value or past values of I/P.
Hence, the correct option is (c).

12. The Dirac delta function $\delta(t)$ is defined as **[2006]**

(a) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(b) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$

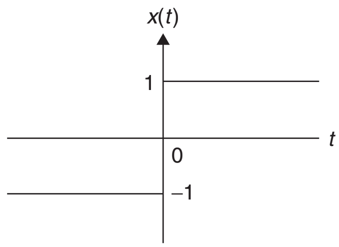
(c) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

(d) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

Solution: (d)

$\delta(t)$ is an impulse fx^n with infinite amplitude at $t = 0$ and zero amplitude for other values and also the area under an impulse function is equal to 1.
Hence, the correct option is (d).

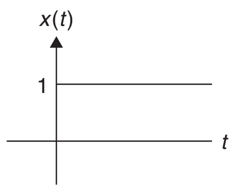
13. The function $x(t)$ is shown in the figure. Even and odd parts of a unit step function $u(t)$ are respectively. **[2005]**



(a) $\frac{1}{2},\frac{1}{2}x(t)$ (b) $\frac{-1}{2},\frac{1}{2}x(t)$

(c) $\frac{1}{2},\frac{-1}{2}x(t)$ (d) $\frac{-1}{2},\frac{-1}{2}x(t)$,

Solution: (a)

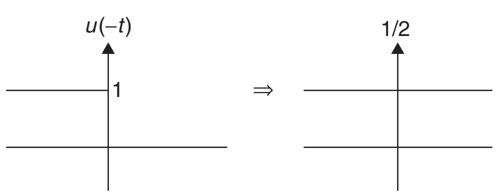


Unit step function

For even function, even part is given as,

Now, $x_e(t)=\frac{x(t)+x(-t)}{2}$ for a function $u(t)$

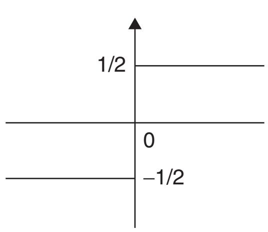
$$x_e(t)=\frac{u(t)+u(-t)}{2}=\frac{1}{2}$$



$$\therefore x_e(t)=\frac{1}{2}$$

For odd function,

now $x_0(t)=\frac{x_0(t)-x_0(-t)}{2}=\frac{u(t)-u(-t)}{2}=\frac{1}{2}x(t)$



$$\Rightarrow \frac{1}{2}x(t)$$

$$\therefore x_o(t)=\frac{1}{2}x(t)$$

Hence, the correct option is (a).

14. The power in the signal

$s(t)=8(\cos)\left(20\pi t-\frac{\pi}{2}\right)+4\sin(15\pi t)$ is **[2005]**

(a) 40 (b) 41

(c) 42 (d) 82

Solution: (a)

For a sinusoidal signal $Am\cos w_m t$ or $Am\sin w_m t$

$$\text{Power}=\frac{Am}{2}$$

$$\therefore \text{Power}=\frac{8^2}{2}+\frac{4^2}{2}=32+8=40$$

Hence, the correct option is (a).

15. Let $\delta(t)$ denote the delta function. The value of the integral $\int_{-\infty}^{\infty}\delta(t)\cos\left(\frac{3t}{2}\right)dt$ is **[2001]**

(a) 1 (b) –1

(c) 0 (d) $\frac{\pi}{2}$

Solution: (a)

As for function $f(t)$

$$\int_{-\infty}^{\infty} f(t)\delta(t)\,dt=f(o)$$

So, $\int_{-\infty}^{\infty}\delta(t)\cos\left(\frac{3t}{2}\right)dt=\cos\frac{3\times 0}{2}=1$

Hence, the correct option is (a).

16. If a signal $f(t)$ has energy E, the energy of the signal $f(2t)$ is equal to **[2001]**

(a) E (b) $\frac{E}{2}$

(c) $2E$ (d) $4E$

Solution: (b)

$$E=\int_{-\infty}^{\infty} f^2(t)\,dt$$

$$E^1=\int_{\infty}^{\infty} f^2(2t)\,dt$$

$$=\int_{-\infty}^{\infty} f^2(u)\frac{du}{2}=\frac{E}{2}$$

Let $2t=u$

Then $2\,dt=du$

$\therefore E^1=E/2$

Hence, the correct option is (b).

17. A system with an input $x(t)$ and output $y(t)$ is described by the relation: $y(t) = T \times (t)$. This system is________. **[2000]**

(a) linear and time invariant
(b) linear and time varying
(c) non-linear and time invariant
(d) non-linear and time varying

Solution: (b)

$y(t) = tx(t)$
Let $x(t) = x_1(t)$; then $y(t) = t\, x_1(t)$.
Similarly for $x(t) = x_2(t) \therefore y_2(t) = tx_2(t)$
$\therefore y_1(t) + y_2(t) = t[x_1(t) + x_2(t)]$ (1)
Now let $x(t) = x_1(t) + x_2(t)$
then $y(t) = t[x_1(t) + x_2(t)]$ (2)
As equations (1) and (2) are equal, so it follows superposition principle.
Now let $x'(t) \to ax(t)$
Then $y'(t) = \text{at}\, x(t)$
Also $a\, y(t) = \text{at}\, x(t)$
Hence, homogeneity is also satisfied, so, linear system
Also $y(t - t_o) = (t - t_o)\, x(t - t_o)$
So, the system is a time varying system.
Hence, the correct option is (b).

Two-mark Questions

1. Consider the signal $f(t) = 1 + 2\cos(\pi t) + 3\sin\left(\frac{2\pi}{3}t\right) + 4\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)$, where t is in seconds. Its fundamental time period, in seconds, is _____. **[2019]**

Solution:

$$f(t) = 1 + \underbrace{2\cos\Pi t}_{T_1} + \underbrace{3\sin\left(\frac{2\Pi t}{3}\right)}_{T_2} + \underbrace{4\cos\left(\frac{\Pi t}{2} + \frac{\Pi}{4}\right)}_{T_3}$$

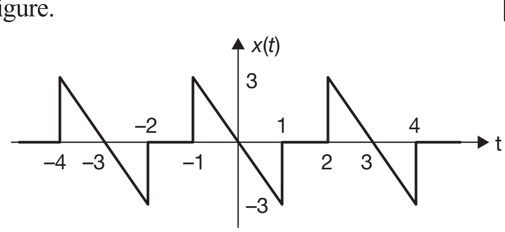
$$T_1 = \frac{2\cancel{\Pi}}{\cancel{\Pi}} = 2;\ T_2 = \frac{2\cancel{\Pi}}{\frac{2\cancel{\Pi}}{3}} = 3;\ T_3 = \frac{2\cancel{\Pi}}{\frac{\cancel{\Pi}}{2}} = 4$$

$f(t)$

$$T = LCM(T_1, T_2, T_3)$$
$$= LCM(2, 3, 4)$$
$$= 12$$

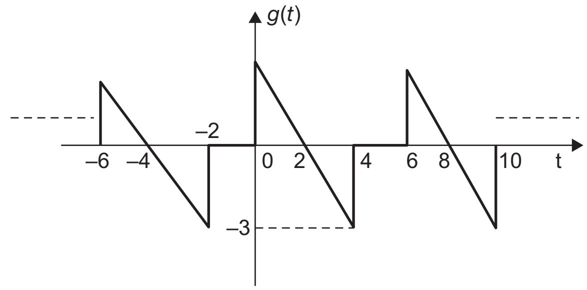
Hence, the correct answer is 12.

2. The waveform of a periodic signal $x(t)$ is shown in the figure. **[2015]**



A signal $g(t)$ is defined by $g(t) = x\left(\frac{t-1}{2}\right)$. The average power of $g(t)$ is _____

Solution: $g(t) =$



$$\text{Now average power} = \underset{T\to\infty}{Lim}\frac{1}{T}\int_{-\infty}^{+\infty}|g(t)|^2\,dt$$

$$\text{Now} \qquad g(t)_T = \begin{cases} -\frac{3}{2}x + 3, & 0 \le t \le 4 \\ 0 & 4 \le t \le 6 \end{cases}$$

Period $T = 6$

$$\text{So}\quad P_{av} = \frac{1}{6}\int_0^4\left(-\frac{3}{2}x + 3\right)dx$$

$$= \frac{1}{6}\int_0^4\left[\frac{9}{4}x^2 + 9 - 9x\right]$$

$$= \frac{1}{6}\left[\frac{9}{4}.\frac{x^3}{3} + 9x - \frac{9x^2}{2}\right]_0^4$$

$$= \frac{1}{6}\left[\frac{9}{4}.\frac{4\times4\times4}{3} + 9\times4 - \frac{9}{2}\times4\times4\right]$$

$$= \frac{1}{6}[48 + 36 - 24\times3]$$

$= 14 - 12 = 2$

Hence, the correct Answer is (2).

3. Let $h(t)$ denote the impulse response of a causal system with transfer function $\frac{1}{s+1}$.

Consider the following three statements
S_1: The system is stable
S_2: $\frac{n(t+1)}{n(t)}$ is independent of t for $t > 0$
S_3: A non-causal system with the same transfer function is stable.
For the above system **[2014]**

(a) only S_1 and S_2 are true.
(b) only S_2 and S_3 are true.
(c) only S_1 and S_3 are true
(d) S_1, S_2 and S_3 are true.

Solution: (a)

$$T(s)=\frac{1}{s+1}$$

$$h(t)=L^{-1}[T(s)]=e^{-1}$$

That is, a decaying exponential, so the system is stable.

$$\frac{h(t+1)}{h(t)}=\frac{e^{-(t+1)}}{e^{-t}}=e^{-1}$$

e^{-1} is independent of Z.
A non-causal system for the same transfer function will not include $j\omega$ as σ will be less than 1. So, system will not be stable.
Hence, the correct option is (a).

4. Let $x(t)$ be a wide sense stationary (*WSS*) random with power spectral density $s_x(f)$. If $y(t)$ is the process defined as $y(t) = x(2t - 1)$, the power spectral density $S_y(f)$ is_____ **[2014]**

(a) $s_y(f)=\frac{1}{2}s_x\left(\frac{f}{2}\right)e^{-j\pi f}$

(b) $s_y(f)=\frac{1}{2}s_x\left(\frac{f}{2}\right)e^{-j\pi f/2}$

(c) $s_y(f)=\frac{1}{2}s_x\left(\frac{f}{2}\right)$

(d) $s_y(f)=\frac{1}{2}s_x\left(\frac{f}{2}\right)e^{j2\pi f}$

Solution: (c)
In case of PSD, no effect of shifting takes place.
For $x(t)\xrightarrow{PSD}s_x(f)$

$$x(2t)\xrightarrow{PSD}\frac{1}{2}s_x(f/2)$$

$$x(2t-1)\rightarrow\frac{1}{2}s_x(f/2)$$

$$s_y(t)\rightarrow\frac{1}{2}s_x(f/2)$$

Hence, the correct option is (c).

5. The unilateral Laplace transform of $f(t)$ is $\frac{1}{s^2+s+1}$. Which one of the following is the unilateral Laplace transform of $g(t)=t\cdot f(t)$? **[2014]**

(a) $\frac{-s}{(s^2+s+1)^2}$ (b) $\frac{-(2s+1)}{(s^2+s+1)^2}$

(c) $\frac{s}{(s^2+s+1)^2}$ (d) $\frac{2s+1}{(s^2+s+1)^2}$

Solution: (d)

$$f(3)=\frac{1}{s^2+s+1}$$

For $f(t)\leftrightarrow F(s)$

$$t.f(t)\leftrightarrow\frac{-d\,F(s)}{ds}$$

$$-\frac{d}{ds}\frac{1}{(s^2+s+1)}=\frac{-[-(2s+1)]}{(s^2+s+1)^2}$$

$$=\frac{2s+1}{(s^2+s+1)^2}$$

Hence, the correct option is (d).

6. A stable linear time-invariant (LTI) system has a transfer function $H(s)=\frac{1}{s^2+s-6}$. To make this system causal it needs to be cascaded with another LTI system having a transfer function H_1(s). A correct choice for $H_1(s)$ among the following option is. **[2014]**
(a) $s = 2$ pole should be added
(b) $s = 2$ zero should be added
(c) $s = 3$ pole should be added
(d) $s = 3$ zero should be added

Solution: (b)

$$H(s)=\frac{1}{s^2+s-6}=\frac{1}{(s+3)(s-2)}$$

for the system to be causal. All poles must lie on the left half plane. So, for making the system causal $s = 2$, zero should be added.
Hence, the correct option is (b).

7. A causal LTI system has zero initial conditions and impulse response $h(t)$. Its input $y(t)$ and output $x(t)$ are related through the linear constant coefficient differential equation.

$$\frac{d^2y(t)}{dt^2}+\frac{\alpha\,dy(t)}{dt}+\alpha^2y(t)=x(t)$$

Let another signal $g(t)$ be defined as

$$g(t)=\alpha^2\int_0^t h(\tau)\,d\tau+\frac{dh(t)}{dt}+ah(t)$$

If $G(s)$ is the Laplace transform of $g(t)$, then the number of poles of $G(S)$ is______. **[2014]**

Solution: 1

$$\frac{d^2y(t)}{dt^2}+\frac{\alpha\,dy(t)}{dt}+\alpha^2y(t)=x(t)$$

$$(s^2+\alpha s+\alpha^2)\,y(s)=x(s)$$

$$H(s)=\frac{y(s)}{x(s)}=\frac{1}{s^2+\alpha s+\alpha^2}$$

Now $g(t)=\alpha^2\int_0^t h(\tau)\,dt+\frac{dn(\tau)}{dt}+h(t)$

$$G(s)=\left(\frac{\alpha^2}{s}+s+\alpha\right)H(s)$$

$$=\frac{\alpha^2+\cancel{s}^2+\alpha s}{s}\times\frac{1}{s^2+\alpha\cancel{s}+\alpha^2}=\frac{1}{s}\text{ sw}$$

Number of poles at $G(s) = 1$.

8. The impulse response of a continuous time system is given by $h(t)=\delta(t-1)+\delta(t-3)$. The value of the step response at $t = 2$ is **[2013]**

(a) 0 (b) 1
(c) 2 (d) 3

Solution: (b)

step response $= \int$ Impulse response, dt

$\therefore$ step response $= u\ (t{-}1) + u\ (t{-}3)$

At $t = 2$ step response $= u\ (1) + u(-1)$

As $u(t) = 0$ for $t < 0$ $\therefore u\ (-1) = 0$

Hence step response = 1.

Hence, the correct option is (b).

9. The input $x(t)$ and output $y(t)$ of a system are related as $y(t)=\int^{t} x(\tau)\cos(\tau)d\tau$. The system is **[2012]**

(a) time invariant and stable
(b) stable and not time invariant
(c) time invariant and not stable
(d) not time invariant and not stable

Solution: (d)

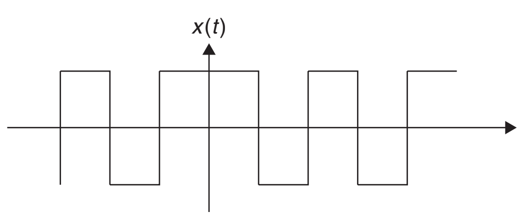
$$y^1(t)_{x=(t-t_0)}=\int_{-\infty}^{t}x(t-t_0)\cos(3(t-t_0))\,dt$$

$$y^1(t)=\int^{t}x(t-t_0)\cos(3(t-t_0))\,dt$$

$$y(t-t_0)=\int_{-\infty}^{t-t_0}x(t)\cos(3t)\,dt$$

$\therefore$ System is time variant

Also for an I/p like as shown



$x(t)\cos\omega_0 t$ will be +ve and hence $y(t)$ will be infinite, and so an unstable system.

Hence, the correct option is (d).

10. Let $x(t)$ be the input and $y(t)$ be the output of a continuous time system. Match the system properties P_1, P_2, and P_3 with system relations R_1, R_2, R_3, R_4. **[2008]**

Properties

P_1 = linear but not time invariant
P_2 = Time invariant but not linear
P_3 = linear and time invariant

Relations

$$R_1 = y\ (t) = t^2x(t)$$

$$R_2 = y(t)=t\,|x(t)|$$

$$R_3 = y(t)=|x(t)|$$

$$R_4 = y\ (t) = x\ (t-s)$$

(a) $(P_1, R_1), (P_2, R_3), (P_3, R_4)$
(b) $(P_1, R_2), (P_2, R_3), (P_3, R_4)$
(c) $(P_1, R_3), (P_2, R_1), (P_3, R_2)$
(d) $(P_1, R_1), (P_2, R_2), (P_3, R_3)$

Solution: (a)

$R_1 = y(t) = t^2x(t)$

$a_1y_1\ (t) + a_2y_2\ (t) = t^2\ [a_1x_1\ (t) + a_2x_2\ (t)]$

$\therefore$ Linear system

$y(t-t_0) = (t-t_0)^2x(t-t_0)$

Not time invariant

$$R_2 : y(t) = t\ |\ x(t)\ |$$

This system is not linear and also time variant

$$R_3 : y(t) = |x(t)|$$

This system is not linear as if a −ve then $ay(t)$ can be −ve but $|ax\ (t)| = +$ve

$$y\ (t-t_0) = |x\ (t-t_0)$$

$\therefore$ No delay, invariant system

$$R_4 = y(t) = x(t-5)$$

$$a_1y_1(t) + a_2y_2(t) = a_1x_1(t-5) + a_2x_2(t-5)$$

So, linear system

Also $y(t-t_0) = x(t-t_0-5)$

So, time-invariant system.

Hence, the correct option is (a).

11. A system with input $x[n]$ and output $y[n]$ is given as $y[n]=\left(\sin\frac{5}{6}\pi n\right)\times(n)\ x[n]$. The system is **[2006]**

(a) linear, stable and invertible
(b) non-linear, stable and non-invertible
(c) linear, stable and non-invertible
(d) linear, unstable and invertible

Solution: (c)

For linearity

$$x(n)=x_1(n)\Rightarrow y_1(n)=\left(\sin\frac{5}{6}\pi n\right)x_1(n)$$

For $x(n)=x_2(n)\Rightarrow y_2(n)=\left(\sin\frac{5}{6}\pi n\right)x_2(n)$

If

$$x(n)=x_1(n)+x_2(n)\Rightarrow y(n)=\left(\sin\frac{5}{6}\pi n\right)(x_1(n)+x_2(n))$$

$$\therefore y(n) = y_1(n) + y_2(n)$$

Also $ay(n) = F[ax(n)]$
$\therefore$ Linear system
Also for a bounded I/P we get bounded o/p
So, stable system
Now if $n = 0$, $y(n) = 0$
$n = 2$, $y(n) = 0$
If for different values of $\frac{I}{P}$, $y(n)$ is same then the system is non-invertible.
Hence, the correct option is (c).

12. Consider the sequence $x[n] = [-4 - 5j \; 1 + 2j \; 4]$.
The conjugate anti-symmetric part of the sequence is **[2004]**

(a) $[-4-2\cdot5j \;\; 2j \;\; 4-2\cdot5j]$
(b) $[-j+1 \; j2 \; -5]$
(c) $[-j5 \; j2 \; 0]$
(d) $[-4 \; 1 \; 4]$

Solution: (a)
For a conjugate anti-symmetric part

$$x_{\text{CAS}}(n) = \frac{x(n) - x^*(-n)}{2}$$

$$\Rightarrow x(n) = [-4-5j \quad 1+j^2 \quad 4]$$

$$\Rightarrow x^*(-n) = [4 \quad 1-j^2 \quad -4+5j]$$

$$\Rightarrow \frac{x(n) - x^*(-n)}{2} = \left[-4 - \frac{5}{2}j \quad 2j \quad 4 - \frac{5}{2}j\right]$$

$$\Rightarrow x_{\text{CAS}}(n) = [-4-2\cdot5j \;\; 2j \;\; 4-2\cdot5j]$$

Hence, the correct option is (a).

13. Let P be linearity, Q be time invariance, R be causality and S be stability. A discrete time system has the input–output relationship.

$$y(n) = \begin{cases} x(n) & n \geq 1 \\ o & n = o \\ x(n+1) & n \leq -1 \end{cases}$$

where $x(n)$ is the input and $y(n)$ is the output. The above system has the properties **[2003]**

(a) P, S but not Q, R
(b) P, Q, S, but not R
(c) P, Q, R, S
(d) Q, R, S but not P

Solution: (a)

As the $\frac{I}{P}$ signal is varying at different time instances, so it is a time variant system.
Also $y(n) = x(n + 1)$, so it depends on future values, hence a non-casual system. For a bounded $\frac{I}{P}$ this system provides bound of P. So, it is stable
Also, this satisfies homogeneity and super position theorem and so it is linear.
Hence, the correct option is (a).

14. An excitation is applied to a system at $t = T$ and its response is zero for $-\infty < t < T$. Such a system is a **[1991]**

(a) non-causal system
(b) unstable system
(c) causal system
(d) unstable system

Solution: (c)
For a system whose o/p is zero at the instant as $t < T$ when the $\frac{I}{P}$ is applied on T is termed as causal system.
Hence, the correct option is (c).

Five-marks Question

1. A system having a unit impulse response $h(n) = u(n)$ is excited by a signal $x(n) = \alpha^n u(n)$. Determine the output $y(n)$ **[1996]**

Solution:

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} h(k) \, . \, x(n-k)$$

given, $x(n) = \alpha^n u(n)$
$h(n) = u(n)$

So, $\quad y(n) = \sum_{k=0}^{n} \alpha^{n-k}$ for $n \geq 0$,

$$= \alpha^n \sum_{k=0}^{n} \alpha^{-k} = \frac{\alpha^{n+1} - 1}{\alpha - 1} \quad n \geq 0$$

$$= 0 \; n < 0$$

Chapter 2

LTI Systems Continues and Discrete

One-mark Questions

1. Let the input be u and the output be y of a system, and the other parameters are real constants. Identify which among the following systems is not a linear system: **[2018]**

   (A) $\frac{d^3y}{dt^3}+\frac{d^2y}{dt^2}+a_2\frac{dy}{dt}+a_3y=b_3u+b_2\frac{du}{dt}+b_1\frac{d^2u}{dt^2}$

   (with initial rest conditions)

   (B) $y(t)=\int_0^t e^{a(t-r)}\,\beta u(\tau)d\tau$

   (C) $y=au+b,\ b\neq 0$

   (D) $y=au$

   Solution:

   Hence, the correct option is (C)

2. A discrete-time all-pass system has two of its poles at $0.25\angle 0°$ and $2\angle 30°$. Which one of the following statements about the system is TRUE? **[2018]**

   (A) It has two more poles at $0.5\angle 30°$ and $4\angle 0°$.

   (B) It is stable only when the impulse response is two-sided.

   (C) It has constant phase response over all frequencies.

   (D) It has constant phase response over the entire z-plane.

   Solution:

   Hence, the correct option is (B)

3. Consider a single input and single output discrete-time system with $x[n]$ as input and $y[n]$ as output, where the two are related as: **[2017]**

$$y[n]=\begin{cases} n\left|x[n]\right|, & \text{for } 0\le n\le 10 \\ x[n]-x[n-1], & \end{cases}$$

Which one of the following statements is true about the system?

(A) It is causal and stable

(B) It is casual but not stable

(C) It is not casual but stable

(D) It is neither casual nor stable

Solution:

$$y\{n\}=\begin{cases} nx\{n\}; & \text{for } 0\le n\le 10 \\ x\{n\}-x\{n-1\}; & \text{otherwise}\end{cases}$$

The output in both the case is depending on either the present or past values of input, so it is causal and in both the cases, bounded input will lead to bounded output, Hence, stable.

4. Consider the following statements about the linear dependence of the real valued functions $y_1 = 1$, $y_2 = x$ and $y_3 = x^2$ over the field of real numbers.

   I. y_1, y_2, and y_3 are linearly independent on $-1\le x\le 0$

   II. y_1, y_2, and y_3 are linearly dependent on $0\le x\le 1$

   III. y_1, y_2, and y_3 are linearly independent on $0\le x\le 1$

   IV. y_1, y_2, and y_3 are linearly dependent on $-1\le x\le 0$

   Which one among the following is correct? **[2017]**

   (A) Both I and II are true

   (B) Both I and III are true

   (C) Both II and IV are true

   (D) Both III and IV are true

   Solution: Given $y_1 = 1$, $y_2 = x$, and $y_3 = x^2$

   For $-1\le x\le 0$ or $0\le x\le 1$, the linear combination of y_1, y_2, and y_3, $ay_1+by_2+cy_3=0$ only when $a=b=c=0$

   $\therefore$ y_1, y_2, and y_3 are linearly independent on $-1\le x\le 0$ as well as on $0\le x\le$

   $\therefore$ Both I and III are true Choice (B)

   Hence, the correct option is (B).

5. Consider the following statements for continuous-time linear time invariant (LTI) systems. **[2017]**

I. There is no bounded input bounded output (BIBO) stable system with a pole in the right half of the complex plane.

II. There is no causal and BIBO stable system with a pole in the right half of the complex plane.

Which one among the following is correct?

(A) Both I and II are true
(B) Both I and II are not true
(C) Only I is true
(D) Only II is true

Solution: Only I is true a causal system may be stable may not be stable [e.g., $e^t u(t)$ is causal but unstable due to RHP.]

Hence, the correct option is (D).

6. An LTI system with unit sample response $h[n] = 5\delta[n] - 7\delta[n-1] + 7\delta[n-3] - 5\delta[n-4]$ is a **[2017]**

(A) Low-pass filter (B) High-pass filter
(C) Band-pass filter (D) Band-stop filter

7. The input $x(t)$ and the output $y(t)$ of a continuous-time system are related as $y(t) = \int_{t-T}^{t} x(u)du$. The system is **[2017]**

(A) Linear and time-variant
(B) Linear and time-invariant
(C) Non-linear and time-variant
(D) Non-linear and time-invariant

8. Two sequences $x_1[n]$ and $x_2[n]$ have the same energy. Suppose $x_1[n] = \alpha 0.5^n u[n]$, where α is a positive real number and $u[n]$ is the unit step sequence. Assume **[2015]**

$$x_2[n] = \begin{cases} \sqrt{1.5} & \text{for } n = 0,.1 \\ 0 & \text{otherwise} \end{cases}$$

Then the value of α is ________

Solution: Energy of $x_2[n] = \sum_{\infty} \left[x[n]\right]^2$

$$= \sum_{n=0}^{1} 1.5 = 1.5 + 1.5$$

$$= 3$$

Now Energy of $x_1[n]$ = Energy of $x_2[n]$

So $$3 = \sum_{n=-\infty}^{+\infty} \alpha(0.5)^n u[n]$$

$$= \sum_{n=0}^{\infty} \alpha(0.5)^n$$

$$3 = \frac{\alpha}{1-0.5} = \frac{\alpha}{0.5}$$

$$\alpha = 3 \times 0.5 = 1.5$$

Hence, the correct Answer is (1.49 to 1.51).

9. A continuous, linear time-invariant filter has an impulse response $h(t)$ described by

$$h(t) = \begin{cases} 3 & \text{for } 0 \le t \le 3 \\ 0 & \text{otherwise} \end{cases}$$

When a constant input of value 5 is applied to this filter, the steady-state output is ____________. **[2014]**

Solution: 45

$$h(t) = \begin{cases} 3 & \text{for } 0 \le t \le 3 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = 5$$

$$H(s) = \int_{-\infty}^{\infty} h(t)e^{-st}dt = \int_{0}^{3} 3e^{-st}dt = \left.\frac{3e^{-st}}{-s}\right|_0^3$$

$$= 3\left[\frac{1-e^{-3s}}{s}\right]$$

$$x(s) = \frac{5}{s}$$

$$y(s) = x(s)H(s) = 3\left[\frac{1}{s} - \frac{e^{-3s}}{s}\right].\frac{5}{s}$$

For steady-state output

$$\underset{t\to\infty}{Lt}\, y(t) = \underset{s\to 0}{Lt}\, Sy(s) = \underset{s\to 0}{Lt}\, 3\cancel{s}\left[\frac{1-e^{-3s}}{s}\right].\frac{5}{\cancel{s}}$$

$$\Rightarrow \underset{s\to 0}{Lt}\, \frac{5\times 3}{s}[1-e^{-3s}] = \underset{s\to 0}{Lt}[1+e^{-3s}]3\times 3\times 5 = 45$$

10. The input $-3e^{-2t}u(t)$, where $u(t)$ is the unit step function, is applied to a system with transfer function $\frac{s-2}{s+3}$. If the initial value of the output is −2, then the value of the output of steady state is _________ **[2014]**

Solution: 0

$$x(t) = -3e^{-2t}u(t)$$

$$H(s) = \frac{s-2}{s+3}$$

$$x(s) = \frac{-3}{s+2}$$

$$\underset{x\to\infty}{Lt}\, y(t) = \underset{y\to 0}{Lt}\, Sy(s) = \underset{y\to 0}{Lt}\, S\left(\frac{s-2}{s+3}\right)\left(\frac{-3}{s+2}\right) = 0$$

11. The sequence $x[n] = 0.5^n u[n]$, where $u[n]$ is the unit step sequence, is convolved with itself to obtain $y[n.]$ Then $\sum_{n=-\infty}^{+\infty} y[n]$ is ____________ **[2014]**

Solution: 4

Given $x(n) = 0.5^n u[n]$

$$y(n) = x(n) \otimes h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$$y(n) = \sum_{k=-\infty}^{\infty} 0.5^k \mu(k).0.5^{n-k} u(n-k)$$

$y(n) = (0.5)^n (n+1) \mu(n)$

Let $A = \sum_{n=0}^{\infty} a^n (n+1)$ also at $z = 0.5$

$$\therefore \quad A = \sum_{n=0}^{\infty} a^n (n+1)$$

$\therefore$ Solving above equation

$$A(1-z) = \frac{1}{1-z}$$

$$A = \frac{1}{(1-z)^2}$$

$$A = \frac{.1}{(1-0.5)^2} = \frac{1}{(0.5)^2} = 4$$

$$= x(0)\,h(4) + x(1)\,h(3) + x(2)\,h(2)$$
$$+ x(3)\,h(1) + x(4)\,h(0)$$
$$= 0 + 3 + 4 + 3 + 0$$
$$= 10$$

12. Two systems with impulse response $h_1(t)$ and $h_2(t)$ are connected in cascade. Then the overall impulse response of the cascaded system is given by **[2013]**

(a) product of $h_1(t)$ and $h_2(t)$
(b) sum of $h_1(t)$ and $h_2(t)$
(c) convolution of $h_1(t)$ and $h_2(t)$
(d) subtraction of $h_1(t)$ from $h_1(t)$

Solution: (c)

For the two systems connected in cascade, output is given as convolution of transfer fx^n of two systems.
Hence, the correct option is (c).

13. Which one of the following statements is NOT TRUE for a continuous time causal and stable LTI system? **[2013]**

(a) All the poles of the system must lie on the left side of the jw axis
(b) Zeros of the system can lie anywhere in the s-plane
(c) All the poles must lie with $|s| = 1$
(d) All the roots of the characteristics equation must be located on the left side of the jw axis.

Solution: (c)

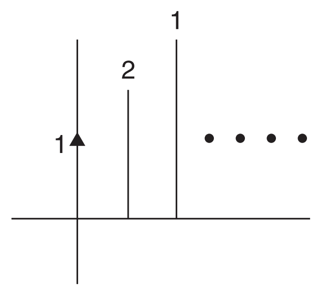
For a causal and stable system, all poles must lie within $|s| = 1$.
Hence, the correct option is (c).

14. A system is defined by its impulse response $h(n) = 2^n u(n-2)$, the system is **[2011]**

(a) stable and causal
(b) causal but not stable
(c) stable but not causal
(d) unstable and non-causal

Solution: (b)

$h(n) = 2^n u(n-2)$
This system depends only on present or past inputs, so, causal system.
For $2^n u(n-2)$



So, it is an increasing function and hence an unstable system.
Hence, the correct option is (b).

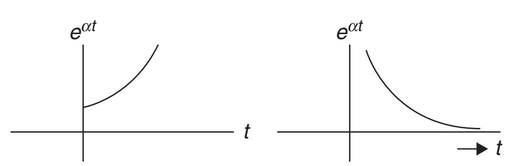
15. The impulse response $h(t)$ of a linear time-invariant continuous time system is described by $h(t) = \exp(dt)$ ut + $\exp(bt)$ $u(-t)$, where $u(t)$ denotes the unit step function and α and β are real constants. This system is stable if **[2008]**

(a) α is positive and β is positive
(b) α is negative and β is negative
(c) α is positive and β is negative
(d) α is negative and β is positive

Solution: (d)

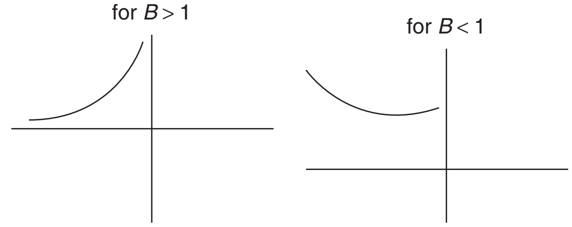
$$h(t)e^{\alpha t}\mu(t) + e^{\beta t}\mu(-t)$$

Here for $\alpha < 1$



$\therefore \alpha$ should be –ve

for $B > 1$ for $B < 1$

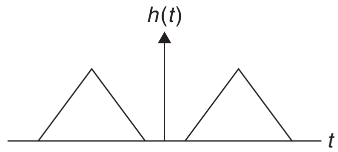


$\therefore$ β should be greater than 0, i.e. +ve

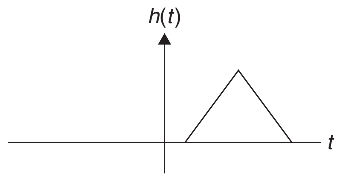
Hence, the correct option is (d).

16. Which of the following can be impulse response of a causal system?

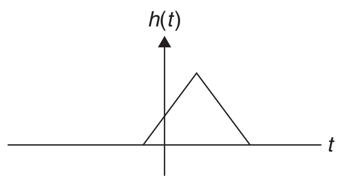
(a)



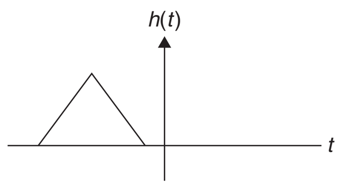
(b)



(c)



(d) **[2005]**



Solution: (b)

For a causal system $h(t) = 0$ for $t < 0$.

Hence, the correct option is (b).

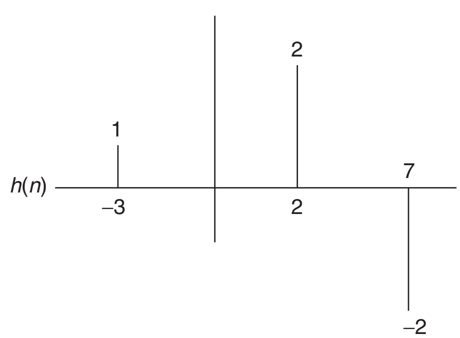
17. The impulse response $h[n]$ of a linear time-invariant system is given by

$h[n] = u[n + 3] + u[n - 2] - 2u[n - 7]$,

where $u[n]$ is the unit step sequence. The above system is **[2004]**

(a) stable but not causal
(b) stable and causal
(c) causal but unstable
(d) unstable and not causal

Solution: (a)



$\therefore$ As the output amplitude is finite for finite input, system is stable. But as $u(n + 3)$ term is present so not causal.

Hence, the correct option is (a).

18. Convolution of $x(t + 5)$ with impulse function $\delta(t - 7)$ is equal to **[2002]**

(a) $x(t - 12)$ (b) $x(t + 12)$
(c) $x(t - 2)$ (d) $x(t + 2)$

Solution: (c)

$$x(t+5)\otimes\delta(t-7) = x(t-2)$$

$$[f(t)\otimes\delta(t-t_0) = f(t-t_0)]$$

Hence, the correct option is (c).

19. The unit impulse response of a linear time-invariant system is the unit step function $u(t)$. For $t > 0$, the response of the system to an excitation $e^{-at}u(t)$, $a > 0$ will be **[1998]**

(a) ae^{-at} (b) $\left(\frac{1}{a}\right)\left(1-e^{-at}\right)$
(c) $a(1 - e^{-at})$ (d) $1 - e^{-at}$

Solution: (b)

$$y(t) = u(t)\otimes e^{-at}u(t)$$

$$= \int_{-\infty}^{\infty} u(t)e^{-at}u(t-)dt$$

$$= \int_0^t e^{-at}dt = \left.\frac{e^{-at}}{-a}\right|_0^{t} = \frac{1}{a}\left[1-e^{-at}\right]$$

Hence, the correct option is (b).

20. Let $h(t)$ be the impulse response of a linear time in variant system. Then the response of the system for any input $u(t)$ is **[1995]**

(a) $\int_0^t h(\tau)u(t-\tau)d\tau$

(b) $\frac{d}{dt}\int_0^t h(\tau)u(t-\tau)d\tau$

(c) $\int_0^t \left[\int_0^t h(\tau)u(t-\tau)d\tau\right]dt$

(d) $\int_0^t h^2(\tau)u(t-\tau)d\tau$

Solution: (a)

$$y(t) = x(t)\otimes h(t)$$

$$= u(t)\otimes h(t)$$

$$= \int_{-\infty}^{\infty} h(\tau)u(t-\tau)dt$$

As $u(t) = 0$ for $t < 0$

$$= \int_0^t h(t)u(t-\tau)dt$$

Hence, the correct option is (a).

Two-marks Questions

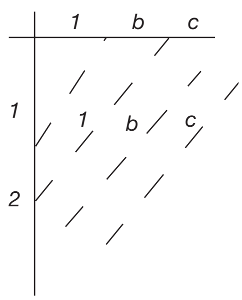
1. Two discrete-time signals $x[n]$ and $h[n]$ are both non-zero only for $n = 0, 1, 2$ and are zero otherwise. It is given that $x[0] = 1, x[1] = 2, x[2] = 1, h[0] = 1$.

Let $y[n]$ be the linear convolution of $x[n]$ and $h[n]$. Given that $y[1] = 3$ and $y[2] = 4$, the value of the expression $(10y[3] + y[4])$ is ________. **[2017]**

Solution: Given $x(n) = \{1, 2, 1\}$
$\uparrow$

$h(n) = \{1, b, c\}$
$\uparrow$

using cross table method



$y(0) = 1$; given $y(1) = 3$

$\Rightarrow 2 + b = 3 \ \Rightarrow b = 1$

given $y(2) = 4$

$\Rightarrow 2b + c + 1 = 4$

$\Rightarrow c = 4 - 2 - 1$

$\Rightarrow c = 1$

$y(3) = b + 2c = 3$

$y(4) = c = 1$

$10y(3) + y(4) = 10 \times 3 + 1 = 31.$

Hence, the correct answer is (31 to 31).

2. Let $h[n]$ be the impulse response of discrete-time linear time invariant (LTI) filter. The impulse response is given by $h(0) = \frac{1}{3}$; $h[1] = \frac{1}{3}$; $h[2] = \frac{1}{3}$; and $h[n] = 0$ for n < 0 and n > 2.

Let $H(\omega)$ be the discrete-time fourier transform (DTFT) of $h[n]$, where ω is normalized angular frequency in radians. Given that $H(\omega_0) = 0$ and $0 < \omega_0 < \pi$, the value of ω_0 (in radians) is equal to ________. **[2017]**

Solution: Given

$$h[0] = \frac{1}{3};\ h[1] = \frac{1}{3};\ h[2] = \frac{1}{3}$$

$$H(\omega) = \sum_{n-\infty}^{\infty} h(n) e^{-j\omega n}$$

$$H(\omega) = \frac{1}{3} + \frac{1}{3}e^{-j\omega} + \frac{1}{3}e^{-j2\omega}$$

if $\omega = \frac{2\pi}{3}$; $H(\omega) = 0$

So, $\omega = 2.0943$ rad/sec

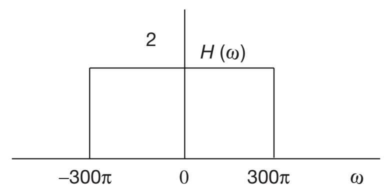
Hence, the correct answer is (2.05 to 2.15).

3. A continuous time signal $x(t) = 4\cos(200\pi t) + 8\cos(400\pi t)$, where t is in seconds, is the input to a linear time invariant (LTI) filter with the impulse response

$$h(t) = \begin{cases} \dfrac{2\sin(300\pi t)}{\pi t}, & t \neq 0 \\ 600, t = 0 \end{cases}$$

Let $y(t)$ be the output of this filter. The maximum value of $|y(t)|$ is ____________. **[2017]**

Solution: The Frequency response of the filter can be given as



In the input $x(t) = 4\cos(200\pi t) + 8\cos(400\pi t)$ only the first component is passed through the system.

Output of the system is

$y(t) = 8\cos(200\pi t)$

$|y(t)| = 8$

Hence, the correct answer is (7.9 to 8.1).

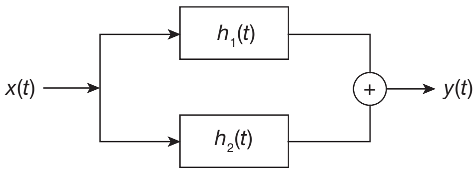
4. Consider an LTI system with magnitude response

$$|H(f)| = \begin{cases} 1 - \dfrac{|f|}{20} & |f| \leq 20 \\ 0, & |f| > 20 \end{cases}$$ and phase response arg $\{H(f)\} = -2f$

If the input to the system is $x(t) = 8\cos\left(20\pi t + \frac{\pi}{4}\right) + 16\sin\left(40\pi t + \frac{\pi}{8}\right) + 24\cos\left(80\pi t + \frac{\pi}{16}\right)$ then the average power of the output signal $y(t)$ is ___________. **[2017]**

5. The transfer function of a causal LTI system is $H(s) = 1/s$. If the input to the system is $x(t) = [\sin(t)/\pi t]u(t)$, where $u(t)$ is a unit step function, the system output y(t) as $t \to \infty$ is ___________. **[2017]**

6. Consider the parallel combination of two LTI systems shown in the figure



The impulse responses of the systems are

$$h_1(t) = 2\delta(t+2) - 3\delta(t+1)$$
$$h_2(t) = \delta(t-2)$$

If the input $x(t)$ is a unit step signal, then the energy of $y(t)$ is ________. **[2017]**

7. A signal $2\cos\left(\frac{2\pi}{3}t\right) - \cos(\pi t)$ is the input to an LTI system with the transfer function $H(s) = e^s + e^{-s}$.

If C_k denotes the kth coefficient in the exponential Fourier series of the output signal, then C_3 is equal to **[2016]**

(A) 0 (B) 1
(C) 2 (D) 3

Solution: By writing in Fourier form ($s = j\omega$)

$$H(\omega) = e^{j\omega} + e^{-j\omega} = 2\cos\omega$$

$$\text{Input } x(t) = 2\cos\left(\frac{2\pi}{3}t\right) - \cos(\pi t)$$

When input is componential input $x(t) = Ae^{j\omega t}$

Output $y(t) = H(\omega)Ae^{j\omega t}$

$$x(t) = e^{j\frac{2\pi}{3}t} + e^{-j\frac{2\pi}{3}t} - \frac{1}{2}\left[e^{j\pi t} + e^{-j\pi t}\right]$$

Fundamental frequency $(\omega_0) = \dfrac{\pi}{3}$

$$x(t) = e^{j2\omega_0 t} + e^{-j2\omega_0 t} - \frac{1}{2}e^{j3\omega_0 t} - \frac{1}{2}e^{-j3\omega_0 t}\; y(t)$$

$$= H(\omega)\cdot e^{j\omega t}$$

$$= H(2\omega_0)\, e^{j2\omega_0 t} + H(-2\omega_0)\, e^{-j2\omega_0}$$

$$-\frac{1}{2}H(3\omega_0)e^{j3\omega_0 t} - \frac{1}{2}H(-3\omega_0)e^{-j3\omega_0 t}$$

$$H(2\omega_0) = 2\cos(2\omega_0) = 2\cos\left(\frac{2\pi}{3}\right) = -1$$

$$H(-2\omega_0) = 1$$

$$H(3\omega_0) = 2\cos(3\omega_0) = 2\cos\left(3\times\frac{\pi}{3}\right) = -2$$

$$H(-3\omega_0) = 2$$

$$y(t) = -e^{j2\omega_0 t} - -e^{-j2\omega_0 t} + -e^{j3\omega_0 t} - -e^{-j3\omega_0 t}$$

for $k = 3$, $C_3 = 1$

Hence, the correct option is (B).

8. A second order linear time invariant system is described by the following state equations

$$\frac{d}{dt}x_1(t) + 2x_1(t) = 3u(t)$$

$$\frac{d}{dt}x_2(t) + x_2(t) = u(t)$$

Where $x_1(t)$ and $x_2(t)$ are the two state variables and $u(t)$ denotes the input. If the output $c(t) = x_1(t)$, then the system is **[2016]**

(A) controllable but not observable
(B) observable but not controllable
(C) both controllable and observable
(D) neither controllable nor observable

Solution: $\dfrac{d}{dt}x_1(t) + 2x_1(t) = 3u(t)$

$$\frac{d}{dt}x_2(t) + 2x_2(t) = u(t)$$

Output $c(t) = x_1(t)$

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}-2 & 0\\ 0 & 1\end{bmatrix}x(t) + \begin{bmatrix}3\\ 1\end{bmatrix}u(t)$$

$$c(t) = [1 \quad 0]x(t)$$

test for controllability:

$$T_c = [B \quad AB]$$

$$AB = \begin{bmatrix}-2 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}3\\ 1\end{bmatrix} = \begin{bmatrix}-6\\ 1\end{bmatrix}$$

$$T_c = \begin{bmatrix}3 & -6\\ 1 & 1\end{bmatrix}$$

$$IT_cI \neq 0$$

∴ It is controllable

Test for observability:

$$T_0 = [C^T\, A^T C^T]$$

$$A^T C^T = \begin{bmatrix}-2 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \begin{bmatrix}-2\\ 0\end{bmatrix}$$

$$T_0 = \begin{bmatrix}1 & -2\\ 0 & 0\end{bmatrix}$$

$$IT_0I = 0$$

∴ so it is not observable

Hence, the correct option is (A).

9. The impulse response of an LTI system can be obtained by **[2015]**

(A) Differentiating the unit ramp response.
(B) Differentiating the unit step response.
(C) Integrating the unit ramp response.
(D) Integrating the unit step response.

Solution: The impulse response of an LTI system can be obtained by differentiating the unit step response.

Hence, the correct option is (B).

10. The result of the convolution $x(-t) * \delta(-t - t_0)$ is **[2015]**

(A) $x(t + t_0)$ (B) $x(t - t_0)$
(C) $x(-t + t_0)$ (D) $x(-t - t_0)$

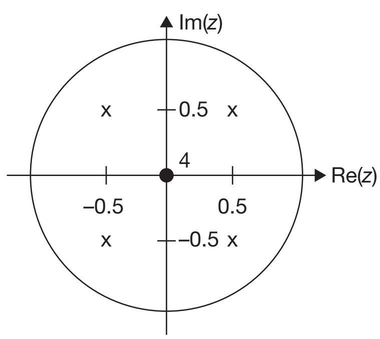
Solution: The result of convolution $x(-t) * \delta(-t - t_0)$ is

Convolution with delta function gives the same function with shifting

So $x(-t) * \delta(-t - t_0) = x(-t - t_0)$

Hence, the correct option is (D).

11. The pole-zero diagram of a causal and stable discrete-time system is shown in the figure. The zero at the origin has multiplicity 4. The impulse response of the system is $h[n]$. If $h[0] = 1$, we can conclude **[2015]**



(A) $h[n]$ is real for all n
(B) $h[n]$ is purely imaginary for all n
(C) $h[n]$ is real for only even n
(D) $h[n]$ is purely imaginary for only odd n

Solution: According to pole zero pattern

$$H(z) = \frac{Z^4}{\left[(Z+0.5)^2 + .(0.5)^2\right]\left[(Z-0.5)^2 + (0.5)^2\right]}$$

$$= \frac{Z^4}{\left(Z^2 + 0.5 - Z\right)\left(Z^2 + 0.5 + Z\right)} = \frac{Z^4}{Z^4 + 0.25}$$

$H(z) = 1 - 0.25\, Z^{-4}$ ….

$h[n] = [1, 0, 0, 0, -0.25….]$

Now

$h[0] = 1$

So $h[n]$ is real for all n

Hence, the correct option is (A).

12. For a function $g(t)$, it is given that $\int_{-\infty}^{\infty} g(t) = e^{jwt}dt = we^{-2w^2}$ for any real value w. If $y(t) = \int_{-\infty}^{t} g(t)dt$, then $\int_{-\infty}^{\infty} y(t)dt$ is ________ **[2014]**

(a) 0 (b) $-j$
(c) $\frac{-j}{2}$ (d) $\frac{j}{2}$

Solution: (b)

$$\int_{-\infty}^{\infty} g(t)e^{-jwt}dt = we^{-2w^2}$$

$$G(w) = we^{-2w^2}$$

$$y(t) = \int_{-\infty}^{t} g(t)dt,$$

i.e., $f\left[\int g(t)\right] = \frac{we^{-2w^2}}{jw}$

$$y(t) \xleftrightarrow{F} \frac{we^{-2w^2}}{jw}$$

$$\int_{-\infty}^{\infty} y(t)e^{-jwt}dt = \frac{\cancel{w}e^{-2w^2}}{j\cancel{w}}$$

for $w = 0$

$$\int y(t)dt = \frac{1}{j} = -j$$

Hence, the correct option is (b).

13. Consider a discrete-time signal

$$x[n] = \begin{cases} n & \text{for } 0 \le n \le 10 \\ 0 & \text{otherwise} \end{cases}$$

If $y[n]$ is the convolution of $x[n]$ with itself, the value of $y[n]$ is __________ **[2014]**

Solution: 10

$$x[n] = \begin{cases} n & \text{for } 0 \le n \le 10 \\ 0 & \text{otherwise} \end{cases}$$

$h(n) = x(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \Rightarrow y(n) = \sum_{k=0}^{n} x(k)h(n-k)$$

14. Let $y[n]$ denote the convolution of $h[n]$ and $g[n]$, where $h[n] = (1/2)^n u[n]$ and $g[n]$ is a causal sequence. If $y[0] = 1$ and $y[1] = ½$, then $g[1]$ equals **[2012]**

(a) 0 (b) ½
(c) 1 (d) 3/2

Solution: (a)

$$h(x)=\left(\frac{1}{2}\right)^n u(n)\ g(n)=x(n)u(n)$$

$y[0] = 1$ $y[1]$ $1/2$ $g[1] = ?$

$$y[n]=g(n)\otimes h(n)=\sum_{k=-\infty}^{\infty} x(k)u(k)\left(\frac{1}{2}\right)^{n-k} u(n-k)$$

$$y(n)=\sum_{k=0}^{n} x(k)(1/2)^{n-k}$$

$$y(0)=x(0)\left(\frac{1}{2}\right)=x(0)=1$$

$$y(1)=\sum_{k=0}^{1} x(k)\left(\frac{1}{2}\right)^{1-k}=x(0)\left(\frac{1}{2}\right)+x(1)\left(\frac{1}{2}\right)$$

$$\frac{1}{2}=\frac{1}{2}+x(1)\therefore \quad x(1)=0$$

$x(1)=g(1)=0$

Hence, the correct option is (a).

15. A discrete time linear shift-invariant system has an impulse response $h[n]$ with $h[0]=1$, $h[1]=-1$, $h[2]=2$, and zero otherwise $x[n]$ with $x[0]=x[2]=1$ and zero otherwise. The number of non-zero samples in the output sequence $y[n]$ and the value of $y[2]$ are, respectively, **[2008]**

(a) 5, 2 (b) 6, 2
(c) 6, 1 (d) 5, 3

Solution: (d)

$$h(n)=\left\{\begin{matrix}1,-1,2\\ \uparrow\end{matrix}\right\}$$

$$x(n)=\left\{\begin{matrix}1,0,1\\ \uparrow\end{matrix}\right\}$$

$$y(n)=x(n)\otimes h(n)$$

$$y(n)=\left\{\begin{matrix}1,-1,3,-1,2\\ \uparrow\end{matrix}\right\}$$

$\therefore$ Number of non-zero samples = 5

$y[2] = 3$.

Hence, the correct option is (d).

16. The impulse response $h[n]$ of a linear time-invariant system is given as

$$h[n]=\begin{cases}-2\sqrt{2} & n=1,-1\\ 4\sqrt{2} & n=2-2\\ 0, & \text{otherwise}\end{cases}$$

If the input to the above system is the sequence $e^{j\pi n/4}$, then the output is **[2004]**

(a) $4\sqrt{2}e^{j\pi n/4}$ (b) $4\sqrt{2}e^{-j\pi n/4}$
(c) $4e^{j\pi n/4}$ (d) $-4e^{j\pi n/4}$

Solution: (d)

$y(n)=H(e^{jw}).e^{j\pi n/4}$ [As per the concept of eigen vectors]

$$H(e^{jw})=\sum_{n=-\infty}^{\infty} h(n)e^{-jwn}$$

$$=-2\sqrt{2}e^{jw}-2\sqrt{2}e^{jw}+4\sqrt{2}e^{-2jw}+4\sqrt{2}e^{2jw}$$

$$=-2\sqrt{2}\times 2\cos w+8\sqrt{2}\cos 2w$$

$$=-4\sqrt{2}\cos w+8\sqrt{2}\cos 2w$$

$$w=\frac{\pi}{4}=H(e^{jw})=-4\sqrt{2}.\frac{1}{\sqrt{2}}=-4$$

$$\therefore \quad y(n)=-4e^{j\pi n/4}$$

Hence, the correct option is (d).

17. The impulse response functions of four linear systems S_1, S_2, S_3, S_4 are given, respectively, by $h_1(t) = 1$ $h_2(t) = u(t)$

$$h_3(t)=\frac{u(t)}{t+1}\ h_4(t)=e^{-3t}u(t)$$

where $u(t)$ is the unit step function. Which of these systems is time invariant, causal and stable? **[2001]**

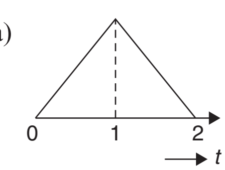
(a) S_1 (b) S_2
(c) S_3 (d) S_4

Solution: (d)

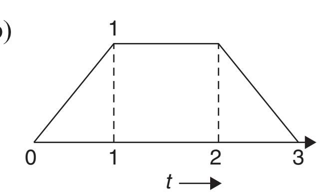
$h_1(t)\rightarrow$ non-causal
$h_2(t)\rightarrow$ time-variant
$h_3(t)\rightarrow$ time-variant
$h_4(t)\rightarrow$ all conditions satisfied
Hence, the correct option is (d).

18. Let $u(t)$ be the step function. which of the waveforms in the figure corresponds to the convolution of $u(t) - u(t - 1)$ with $u(t) - u(t - 2)$? **[2000]**

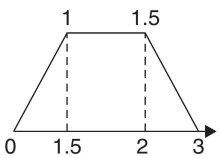
(a)



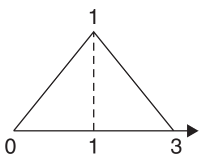
(b)



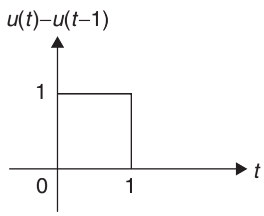
(c)

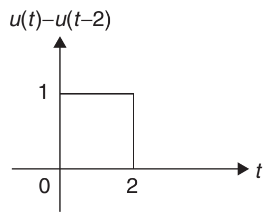


(d)

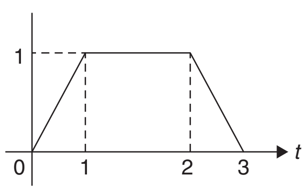


Solution: (b)



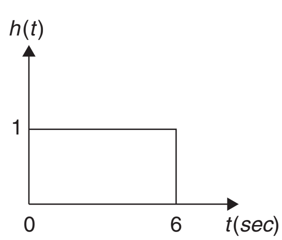


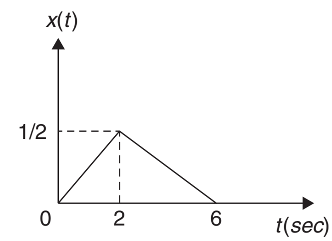
When two rectangular waveforms of different widths are convolved, then the result is a trapezoidal waveform with the slope equal to the product of amplitudes of both signals given and the width of the constant amplitude starts from upper hand limit of width signal.



Hence, the correct option is (b).

19. The impulse response and the excitation function of a linear time-invariant causal system are shown in Figures (a) and (b), respectively. The output of the system at $t = 2$ sec is equal to





[1990]

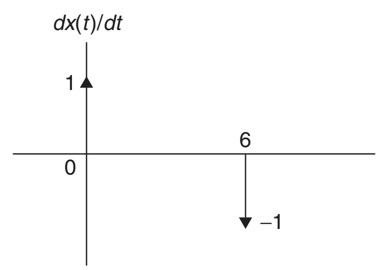
(a) 0 (b) $\frac{1}{2}$

(c) $\frac{3}{2}$ (d) 1

Solution: (b)

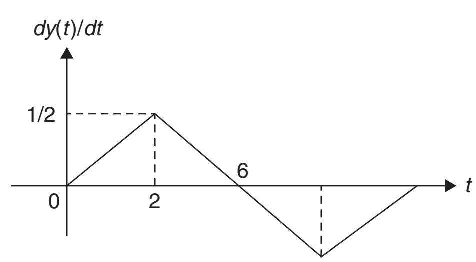
$$y(t) = h(t) \otimes x(t)$$

$$\frac{dy(t)}{dt} = h(t) \otimes \frac{dx(t)}{dt}$$



$$\Rightarrow \quad \delta(t) - \delta(t-6)$$

$$\therefore \quad \frac{dy}{dt}(t) = x(t) - x(t-6)$$



$$y(t) = \int \frac{dy(t)}{dt} = dt = \int_0^{\frac{1}{2}} \frac{1}{4} dt = \frac{1}{2}$$

Hence, the correct option is (b).

Chapter 3

Fourier Series

One-mark Questions

1. Let $x(t)$ be a periodic function with period $T = 10$. The Fourier series coefficients for this series are denoted by a_k, that is

$$x(t) = \sum_{K=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

The same function $x(t)$ can also be considered as a periodic function with period $T^1 = 40$. Let b_k be the Fourier series coefficients when period is taken as T_1. If $\sum_{K=-\infty}^{\infty} |a_k| = 16$, then $\sum_{K=-\infty}^{\infty} |a \backslash b_k|$ is equal to **[2018]**

(A) 256 (B) 64
(C) 16 (D) 4

Solution: We know that change in time period does not change the fourier series coefficients

$$\therefore \qquad \sum_{k=-\infty}^{\infty} |b_k| = \sum_{k=-\infty}^{\infty} |a_k| = 16$$

Hence, the correct option is (C)

2. A periodic signal $x(t)$ has a trigonometric Fourier series expansion

$$x(t) = a_o + \sum_{n-1}^{\infty}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

If $x(t) = -x(-t) = -x(t - \pi/\omega_0)$, we can conclude that **[2017]**

(A) a_n are zero for all n and b_n are zero for n even
(B) a_n are zero for all n and b_n are zero for n odd
(C) a_n are zero for n even and b_n are zero for n odd
(D) a_n are zero for all n odd and b_n are zero for n even

Solution: Given $x(t) = -x(-t)$ means the signal is odd

$x(t) = -x\left(t - \dfrac{\pi}{\omega_0}\right)$, which says that the signal is half wave symmetric.

Condition for half wave symmetry is $x(t) = -x\left(t \pm \dfrac{T}{2}\right)$

So that, signal is odd and half wave symmetric. In the resulting fourier series expansion, a_0 and a_n will exist only for odd harmonics (due to half wave symmetry)

Hence, the correct option is (A).

3. Let $\tilde{x}[n] = 1 + \cos\left(\dfrac{\pi n}{8}\right)$ be a periodic signal with period 16. Its DFS coefficients are defined by $a_k = \dfrac{1}{16}\sum_{n=0}^{15}\tilde{x}[n]\exp\left(-j\dfrac{\pi}{8}kn\right)$ for all k. the value of the coefficient a_{31} is ________. **[2015]**

Solution: $X[n] = 1 + \cos\left(\dfrac{\pi n}{8}\right)$ $\qquad \because N = 16$

$$= 1 + \frac{e^{j\frac{2\pi}{16}} + e^{-j\frac{2\pi n}{16}}}{2}$$

$$= 1 + \frac{1}{2}e^{j\frac{2\pi n}{16}} + \frac{1}{2}e^{-j\frac{2\pi n}{16}}$$

$$a_0 = 1,\ a_1 = \frac{1}{2},\ a_{-1} = \frac{1}{2}$$

So $\qquad a_{0+16} = a_{16} = 1$

$$a_{-1+16} = a_{15} = \frac{1}{2}$$

$$a_{31} = a_{16+15} = a_{15} = \frac{1}{2}$$

Hence, the correct Answer is (0.48 to 0.52).

4. The trigonometric Fourier series of an even function does not have the **[2011]**

(a) *dc* term
(b) cosine term
(c) sine terms
(d) odd harmonic terms

Solution: (c)

For an even function $b_n = 0 \therefore$ no sine terms are present.
Hence, the correct option is (c).

5. The Fourier series of a real periodic function has only

P. Cosine terms if it is even

Q. Sine terms if it is even
R. Cosine terms if it is odd
S. Sine terms if it is odd
Which of the above statements are correct? **[2009]**

(a) *P* and *S*
(b) *P* and *R*
(c) *Q* and *S*
(d) *Q* and *R*

Solution: (a)
For an even fx^n, $b_n = 0$, so sine terms are omitted and only cosine terms are present.
For an odd fx^n, $a_0 = 0$, $a_n = 0$; so only sine terms are present.
Hence, the correct option is (a).

6. Choose the function $f(t)$, $-\infty < t < \infty$, for which a Fourier series cannot be defined, **[2005]**
(a) $3 \sin(25t)$
(b) $4 \cos(20t + 3) + 2 \sin(710t)$
(c) $\exp(-|t|) \sin 25t$
(d) 1

Solution: (c)
Fourier series expression of a signal $x(t)$ must be periodic
- $f(t) = 3 \sin(25t)$
  It is periodic signal.
- $f(t) = 4 \cos(20t + 3) + 2\sin(710t)$

$$T_1 = \frac{2\pi}{20}; \; T_2 = \frac{2\pi}{710} \cdot \frac{T_1}{T_2} = \frac{71}{2} \Rightarrow \text{rational}$$

∴ Periodic
- $f(t) = e^{-|t|} \sin(25t)$

This signal is not periodic due to modulus fx^n. So, Fourier series representation is not possible.
Hence, the correct option is (c).

7. The Fourier series expression of a real periodic signal with fundamental frequency f_0 is given by

$gp(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_0 t}$. It is given that $C_3 = 3 + j5$. Then C_{-3} is **[2003]**

(a) $5 + j3$
(b) $-3 - j5$
(c) $-5 + j3$
(d) $3 - j5$

Solution: (d)
For a Fourier series
$C_n = C_{-n}{}^*$
$C_{-3} = 3 - 5j$
Hence, the correct option is (d).

8. Which of the following cannot be the Fourier series expression of a periodic signal? **[2002]**
(a) $x(t) = 2 \cos t + 3 \cos t$
(b) $x(t) = 2 \cos\pi t + 7 \cos t$
(c) $x(t) = \cos t + 0.5$
(d) $x(t) = 2 \cos 1.5\pi t + \sin 3.5\pi t$

Solution: (b)
For Fourier series expression, $x(t)$, should be periodic for $x(t) = 2 \cos t + 3 \cos 3t$

$$T_1 = 2\pi T_2 = \frac{2\pi}{3} \;\therefore\; \frac{T_1}{T_2} = 3 = \text{rational}$$

∴ Periodic for $x(t) = 2 \cos\pi t + 7 \cos t$

$$T_1 = \frac{2\pi}{\pi} = 2; \; T_2 = 2\pi \; \frac{T_1}{T_2} = \frac{1}{\pi} \Rightarrow \text{Not periodic}$$

As not $x(t)$ is not periodic. So, Fourier series expression is not possible.
Hence, the correct option is (b).

9. The trigonometric Fourier series of a periodic time function can have only **[1998]**
(a) cosine terms
(b) sine terms
(c) cosine and sine terms
(d) *dc* and cosine terms

Solution: (c)
Trigonometric Fourier series consists of cosine and sine terms.
Hence, the correct option is (c).

10. A periodic signal $x(t)$ of period T_0 is given by $x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & T_1 < |t| < \frac{T_0}{2} \end{cases}$. The component of $x(t)$ is **[1998]**

(a) $\frac{T_1}{T_0}$ (b) $\frac{T_1}{(2T_0)}$

(c) $\frac{2T_1}{T_0}$ (d) $\frac{T_0}{T_1}$

Solution: (c)
For *dc* component

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt$$

$$= \frac{1}{10}\left[\int_{-T_1}^{T_1} 1.dt\right] = \frac{2T_1}{T_0}$$

$$a_0 = \frac{2T_1}{T_0}$$

Hence, the correct option is (c).

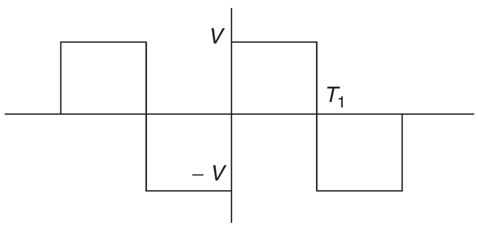
11. The trigonometric Fourier series of an even function of time does not have **[1996]**
(a) the *dc* term
(b) cosine terms
(c) sine terms
(d) odd harmonic terms

Solution: (c)
For even periodic $fx^n b_n = 0$
$\therefore$ Sine terms are not present.
Hence, the correct option is (c).

12. The RMS value of a rectangular wave of period T, having a value of $+V$ for a duration $T_1(< 7)$ and $-V$ for the duration $T - T_1 = T_2$ equals: **[1995]**

(a) V (b) $\frac{T_1 - T_2}{T}V$
(c) $\frac{V}{\sqrt{2}}$ (d) $\frac{T_1}{T_2}V$

Solution: (a)



$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2 dt} = \frac{1}{T}\left[\int_0^T V^2 dt + \int_{T_1}^T V^2 dt\right]$$

$$= \sqrt{\frac{1}{T}\left[V^2 T_1 + V^2[T - T_1]\right]} = V$$

Hence, the correct option is (a).

13. The Fourier series of an odd periodic function contains only **[1994]**

(a) odd harmonics (b) even harmonics
(c) cosine terms (d) sine terms

Solution: (b)
For odd harmonic fx^n
$a_0 = a_n = 0$
$\therefore$ Only sine terms are present.
Hence, the correct option is (b).

Two-marks Questions

1. Let $x(t)$ be a continuous time periodic signal with fundamental period $T = 1$ seconds. Let $\{a_K\}$ be the complex fourier series coefficients of $x(t)$, where k is integer valued. Consider the following statements about $x(3t)$:

I. The complex fourier series coefficients of $x(3t)$ are $\{a_K\}$ where k is integer valued

II. The complex fourier series coefficients of $x(3t)$ are $\{3a_K\}$ where k is Integer valued

III. The fundamental angular frequency of $x(3t)$ is 6π rad/s

For the three statements above which one of the following is correct? **[2017]**

(A) Only II and III are true
(B) Only I and III are true
(C) Only III is true
(D) Only I is true

Solution: Fundamental period $T = 1$ s. $\omega_0 = 2\pi$ time scaling property of complex fourier series says that

$x(t) \leftrightarrow a_k$

$x(at) \leftrightarrow a_k$

But $T \to \frac{T}{\alpha}$ and $\omega_0 \to \alpha(\omega_0)$

$x(t) \longleftrightarrow a_k$

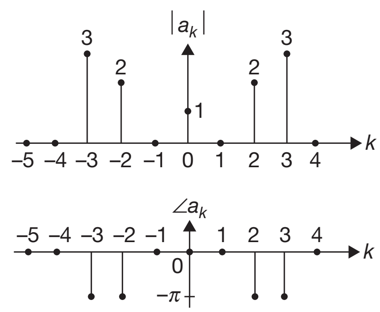
$x(3t) \longleftrightarrow a_k$

$\omega_0 \longleftrightarrow 3(2\pi) = 6\pi$

I and III are true.

Hence, the correct option is (B).

2. The magnitude and phase of the complex Fourier series coefficient a_k of a periodic signal $x(t)$ are shown in the figure. Choose the correct statement from the four choices given. Notation C is the set of complex numbers, R is the set of purely real numbers, and P is the set of purely imaginary numbers. **[2015]**



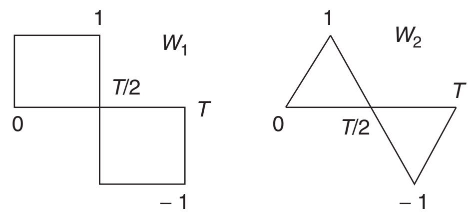
(A) $x(t) \in R$
(B) $x(t) \in P$
(C) $x(t) \in (C - R)$
(D) the information given is not sufficient to draw any conclusion about $x(t)$

Solution: Magnitude spectrum of a_k is even so corresponding signal $x(t)$ in time domain is real and even.

$$x(t) \in R$$

Hence, the correct option is (A).

3. One period (0, 7) each of two periodic waveforms w_1 and w_2 are shown in the figure. The magnitudes of the n^{th} Fourier series coefficients of w_1 and w_2, for $n \geq 1$, n odd, are respectively proportional to:



[2000]

(a) $|n|^{-3}$ and $|n|^{-2}$
(b) $|n|^{-2}$ and $|n|^{-3}$
(c) $|n|^{-1}$ and $|n|^{-2}$
(d) $|n|^{-4}$ and $|n|^{-2}$

Solution: (c)
In Fourier series representation

$$C_n \propto \frac{1}{n^1} \text{ for } fx^n \text{ consisting of unit steps}$$

$$C_n \propto \frac{1}{n^2} \text{ for } fx^n \text{ consisting of ramps.}$$

Hence, the correct option is (c).

4. The Fourier series representation of an impulse train denoted by $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$ is given by **[1999]**

(a) $\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \exp\frac{-j2\pi nt}{T_0}$

(b) $\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \exp\frac{-j\pi nt}{T_0}$

(c) $\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \exp\frac{j\pi nt}{T_0}$

(d) $\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \exp\frac{-j2\pi nt}{T_0}$

Solution: (d)

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

for Fourier series

$$f(t) = \sum_{n=-\infty}^{\infty} C_k e^{jkw_0 t}$$

for an impulse function $C_k = \frac{1}{T_0}$

$$\therefore \; f(t) = \frac{1}{T_0}\sum_{n=-\infty}^{\infty} e^{jkw_0 t}$$

$$w_0 = \frac{2\pi}{T_0} \text{ and } k \to n$$

$$f(t) = \frac{1}{T_0}\sum_{n=-\infty}^{\infty} e^{h\frac{n2\pi}{10}t}$$

Hence, the correct option is (d).

5. Fourier series of the periodic function (period 2π) defined by

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x & 0 < x < n \end{cases} \text{ is}$$

$$\frac{\pi}{4} + \sum_{1}^{\infty}\left[\frac{1}{\pi n^2}[\cos(n\pi) - 1]\cos(nx) - \frac{1}{n}\cos(n\pi)\sin(nx)\right].$$

By putting $x = \pi$ in the above equation, one can deduce that the sum of the series

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots \text{ is}$$ **[1993]**

Solution: $\frac{\pi^2}{8}$

At $x = \pi$

$$f(\pi) = \frac{\pi}{4} + \sum_{1} \frac{1}{\pi n^2}[\cos n\pi - 1]\cos n\pi$$
$$- \frac{1}{n}\cos(n\pi)\sin(n\pi)$$
$$= \frac{\pi}{4} + \sum_{1} \frac{1}{\pi n^2}\left[(-1)^{2n} - (-1)^n\right] - 0$$
$$= \frac{\pi}{4} + \sum_{1} .\frac{1}{\pi n^2}\left[1 - (-1)^n\right]$$
$$= \frac{\pi}{4} + \frac{2}{\pi} + \frac{2}{9\pi} + \frac{2}{25\pi} + \cdots$$
$$= \frac{\pi}{4} + \frac{2}{\pi}\left[1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right]$$

As the $f(x)$ has a period of 2π, so, the $f(x)$ converges to middle value at $\frac{2\pi}{2}$ where π is the point of discontinuity at which value changes from 0 to 1

$$\therefore \; \frac{\pi}{2} = \frac{\pi}{4} + \frac{2}{\pi}\left[1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right]$$

$$\left[\frac{\frac{\pi}{2} - \frac{\pi}{4}}{\frac{2}{\pi}}\right] = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

$$\therefore \; 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$

6. Which of the following signals is/are periodic? **[1992]**
(a) $s(t) = \cos 2t + \cos 3t + \cos 5t$
(b) $s(f) = \exp(j8\pi t)$
(c) $s(f) = \exp(-7t)\sin 10\pi t$
(d) $s(t) = \cos 2t \cos 4t$

Solution: (a), (b) and (d)

For a periodic signal, ratio of time period must be a rational number

$s(t) = \cos 2t + \cos 3t + \cos 5t$

$$T_1 = \frac{2\pi}{2} = \pi;\ T_2 = \frac{2\pi}{3}\ T_3 = \frac{2\pi}{5}$$

$$\frac{T_1}{T_2} = \frac{\pi}{2\pi} \times 3 = \frac{3}{2} \rightarrow \text{rational number}$$

$$\frac{T_2}{T_3} = \frac{2\pi}{3} \times \frac{5}{2\pi} = \frac{5}{3} \rightarrow \text{rational number}$$

$$\frac{T_1}{T_3} = \frac{5\pi}{2\pi} = \frac{5}{3} \rightarrow \text{ rational number}$$

- $s(t) = e^{j8\pi t}$ so, it is periodic

  $e^{jw_0 t}$ is always a period signal

- $s(t) = e^{-7t} \sin 10\pi t$

  Here $T_1 = \dfrac{2\pi}{7}\ T_2 = \dfrac{2\pi}{10\pi}$

  $$\frac{T_1}{T_2} = \frac{10\pi}{7} \neq \text{Rational Number}$$

  So, not periodic.

- $s(t) = \cos 2t \cos 4t = \dfrac{1}{2}[\cos 6t + \cos 2t]$

$$T_1 = \frac{2\pi}{6}\ T_2 = \frac{2\pi}{2}$$

$$\frac{T_1}{T_2} = \frac{1}{3} \Rightarrow \text{Rational No, so periodic}$$

Hence, the correct options are (a), (b) and (d).

7. A half-wave rectified sinusoidal waveform has a peak voltage of 10V. Its average value and the peak value of the fundamental component are, respectively, given by **[1987]**

(a) $\dfrac{20}{\pi}$ V, $\dfrac{10}{\sqrt{2}}$ V (b) $\dfrac{10}{\pi}$ V, $\dfrac{10}{\sqrt{2}}$ V

(c) $\dfrac{10}{\pi}$ V, 5 V (d) $\dfrac{20}{\pi}$ V, 5 V

Solution: (c)

For a half wave rectified sinusoid

$$V_{av} = \frac{V_m}{\pi} = \frac{10}{\pi}\ \text{V}$$

and fundamental component, $\dfrac{V_m}{2} = \dfrac{10}{2} = 5$ V

Hence, the correct option is (c).

Chapter 4

Fourier Transforms

One-mark Questions

1. A real valued signal $x(t)$ limited to the frequency band $|f|'' \frac{W}{2}$ passed through a linear time-invariant system whose frequency response is

$$H(f) = \begin{cases} e^{-J4\pi f} & |f| \le \frac{W}{2} \\ 0 & |f| > \frac{W}{2} \end{cases}$$

The output of system is **[2014]**

(a) $x(t+4)$ (b) $x(t-4)$

(c) $x(t+2)$ (d) $x(t-2)$

Solution: (d)

$$\text{Given } H(f) \neq \begin{cases} e^{-J4\pi f} & |f| \le \frac{W}{2} \\ 0 & |f| > \frac{W}{2} \end{cases}$$

$$H(f) = e^{-J4\pi f} \Rightarrow H(\omega) = e^{-J2\omega}$$

$$\Rightarrow h(t) = \delta(t-2)$$

As we know that

$$y(t) = x(t) * h(t) = x(t) * \delta(t-2) = x(t-2)$$

so $y(t) = x(t-2)$

Hence, the correct option is (d).

2. A Fourier transform pair is given by

$$\left(\frac{2}{3}\right)^n (n+3) \xleftrightarrow{F\cdot T} \frac{A \cdot e^{-J6\pi f}}{1-\left(\frac{2}{3}\right)e^{-J'2\pi f}}$$

where $u[n]$ denotes the unit step sequence, and the value of A is ______ **[2014]**

Solution: 3.3375

$$\text{Let } x[n] = \left(\frac{2}{3}\right)^n u(n+3)$$

Discrete Fourier transform is given as

$$x(e^{J'\omega}) = \sum_{n=-}[n]e^{-J\omega n} = \sum_{n=-3}\left(\frac{2}{3}\right)^n e^{-J\omega n}$$

Put $n+3=m$

$$\text{So } x\left(e^{J'\omega}\right) = \sum_{m=0}\left(\frac{2}{3}\right)^{m-3} e^{-J\omega(m-3)}$$

$$= \left(\frac{3}{2}\right)^3 \sum_{m=0}\left(\frac{2}{3}e^{-J\omega}\right)^m e^{J\omega 3}$$

Applying ∞ G.P summation

$$= \frac{3.375e^{J'2\pi f\,3}}{1-\frac{2}{3}e^{-J2\pi f}}$$

So, $A = 3.3375$.

3. An FIR system is described by the system function $H(z) = 1+\frac{7}{2}z^{-1}+\frac{3}{2}z^{-2}$. The system is **[2014]**

(a) maximum phase

(b) minimum phase

(c) mixed phase

(d) zero phase

Solution: (c)

Poles of system $H(z)$ are lying inside and outside of the unit circle. So, system will be the mixed phase system. Hence, the correct option is (c).

4. Let $g(t) = e^{-\pi t^2}$ and $h(t)$ is filter matched to $g(t)$. If $g(t)$ is applied as input to $h(t)$, then the Fourier transform of the output is **[2013]**

(a) $e^{-\pi f^2}$

(b) $e^{-\frac{\pi f^2}{2}}$

(c) $e^{-\pi|f|}$

(d) $e^{-2\pi f^2}$

Solution: (d)

$g(t)$ is a Gaussian pulse whose Fourier transform is also Gaussian pulse.

$$g(t) = e^{-\pi t^2} \xleftrightarrow{F\cdot T} e^{-\pi f^2} = G(\omega)$$

o/p $Y(\omega) = G(\omega)\cdot G(\omega) = e^{-\pi f^2}\cdot e^{-\pi f^2} = e^{-2\pi f^2}$

Hence, the correct option is (d).

5. A function is given by $f(t) = \sin^2 t + \cos 2t$. Which of the following is true? **[2009]**

(a) f has frequency component of 0 and $\frac{1}{2\pi}H_3$

(b) f has frequency component of 0 and $\frac{1}{\pi}H_3$

(c) f has frequency component of $\frac{1}{2\pi}$ and $\frac{1}{\pi}H_3$

(d) f has frequency component of 0, $\frac{1}{2\pi}$ and $\frac{1}{\pi}H_3$

Solution: (b)

Given $f(t) = \frac{1}{2}(1-\cos 2t) + \cos 2t$

Frequency components are $f_1 = 0$

$$f_2 = \frac{\omega 2}{2\pi} = \frac{2}{2\pi} = \frac{1}{\pi}H_3$$

Hence, the correct option is (b).

6. Let $x(t) \leftrightarrow x(j\omega)$ be Fourier transform pair. The Fourier transform of signal $x(5t–3)$ in terms of $x(j\omega)$ is given as **[2006]**

(a) $\frac{1}{5}e^{\frac{j3\omega}{5}}X\left(\frac{J\omega}{5}\right)$ (b) $\frac{1}{5}e^{-\frac{j3\omega}{5}}X\left(\frac{j\omega}{5}\right)$

(c) $\frac{1}{5}e^{-j3\omega}X\left(\frac{J\omega}{5}\right)$ (d) $\frac{1}{5}e^{j3\omega}X\left(\frac{J\omega}{5}\right)$

Solution: (a)

Using scaling and shifting property

$x(t) \leftrightarrow x(j\omega)$

$$X\left[5\left(t-\frac{3}{5}\right)\right] = \frac{1}{5}\times\left(\frac{J\omega}{5}\right)e^{\frac{j3\omega}{5}}$$

Hence, the correct option is (a).

7. Let $x(t)$ be the input to a linear time-invariant system. The required o/p is 4 $x(t–2)$. The transfer function of the system should be. **[2003]**

(a) $4e^{j4\pi f}$ (b) $2e^{-j8\pi f}$

(c) $4e^{-j4\pi f}$ (d) $2e^{j8\pi f}$

Solution: (c)

$$y(t) = 4x(t-2)$$

Taking Laplace transform: $y(s) = 4e^{-2\delta}x(s)$

$$\Rightarrow \frac{y(s)}{x(s)} = 4\cdot e^{-2\delta}$$

but $s = j2\pi f$

$\Rightarrow$ So, $H(f) = 4e^{-j4\pi f}$

Hence, the correct option is (c).

8. The Fourier transform of $F\{e^{-t}u(t)\}$ is equal to $\frac{t}{1+j2\pi f}$. Therefore, $F\left\{\frac{1}{t+j2\pi t}\right\}$ is **[2002]**

(a) $e^{f}u(f)$ (b) $e^{-f}u(f)$

(c) $e^{f}u(-f)$ (d) $e^{-f}u(-f)$

Solution: (d)

Given $f(t) = e^{-t}u(t) \xleftrightarrow{F\cdot T} \frac{t}{1+2\pi f\cdot j}$

From duality property,

$$\frac{t}{1+2\pi j} \xleftrightarrow{F\cdot T} f(t) = e^{-f}u(-f)$$

Hence, the correct option is (d).

9. A linear phase channel with phase delay T_p and group delay T_g must have **[2002]**

(a) $T_p = T_g =$ constant

(b) $T_p \propto f$ and $T_g \propto f$

(c) $T_p =$ constant and $T_g \propto f$

(d) $T_p \propto f$ and $T_p =$ constant, (f denotes frequency)

Solution: (a)

For a distortionless system, phase is given by $\theta(w) = -wt_0$

then phase delay $T_p = \frac{-\theta(w)}{w} = t_0$

and group delay $T_g = \frac{d\theta(w)}{dw} = t_0$

So, $T_p = T_g =$ constant.

Hence, the correct option is (a).

10. The Fourier transform of signal $x(t) = e^{-Bt^2}$ is of the following form where A and B are constants: **[2000]**

(a) $Ae^{-B|f|}$ (b) $A\cdot e^{-Bf^2}$

(c) $A + B|f^2|$ (d) $A\cdot e^{-Bf}$

Solution: (b)

Given $f(t) = e^{-Bt^2}$ (1)

Let $f(t) \xleftrightarrow{F\cdot T} F(\omega)$

Differentiating equation (1) both side.

$$\frac{df(t)}{dt} = -2at\,e^{-at^2} \xleftrightarrow{F\cdot T} (J\infty)F(\omega)$$

$$\Rightarrow t\cdot e^{-at^2} \xleftrightarrow{FT} \left(\frac{-J\omega}{2a}\right)F(\omega) \quad (2)$$

Now multiplication by t in time-domain, $\leftrightarrow$ different into domain.

So,

$$t\cdot e^{-at^2} \longleftrightarrow J\cdot\frac{dF(\omega)}{d\omega} \quad (3)$$

From equations (2) and (3)

$$\Rightarrow \cancel{J}\cdot\frac{dF(\omega)}{d\omega} = \left(\frac{-\cancel{J}\omega}{2a}\right)F(\omega)$$

$$\Rightarrow \frac{dF(\omega)}{F(\omega)} = \left(\frac{-1}{2a}\right)\omega d\omega \Rightarrow$$

Integrating on both sides

$$\log F(\omega) = \frac{-1}{2a}\cdot\frac{\omega^2}{2} = \frac{\omega^2}{4a}$$

Or $F(\omega) = e$

put $a = 3$, and $\omega = 2\pi f$

$$\Rightarrow F(f) = e^{\frac{-\pi f^2}{3}} = \text{Gaussion pulse}$$

Hence, the correct option is (b).

11. A signal $x(t)$ has a Fourier transform $x(\omega)$. If $x(t)$ is a real and odd function of t, then $x(\omega)$ is **[1999]**

(a) a real and even function of ω
(b) an imaginary and odd function of ω
(c) an imaginary and even function of ω
(d) a real and odd function of ω

Solution: (b)
Example of real and odd function of t is
$x(t) = A \sin \omega_0 t$
then Fourier transform of $x(t)$

$x(\omega) = Aj\pi\left[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)\right] \Rightarrow$ Imaginary and odd function ω

Hence, the correct option is (b).

12. The Fourier transform of voltage signal $x(t)$ is $x(f)$. The unit of $|x(f)|$ is **[1998]**

(a) Volt (b) Volt-sec
(c) Volt/sec (d) Volt^{2-}

Solution: (b)
Fourier transform of signal $x(t)$ is given as

$$x(f)\int_{-} x(t)\cdot e^{-j2\pi ft}dt$$

if $x(t) \rightarrow$ volt,

then $x(f) \rightarrow$ Volt-sec $\rightarrow \dfrac{Volt}{(1/Sec)} \rightarrow \dfrac{Volt}{Hertz}$

Hence, the correct option is (b).

13. The Fourier transform of function $x(t)$ is $X(t)$. The Fourier transform of $\dfrac{dx(t)}{dt}$ will be **[1998]**

(a) $\dfrac{dx(f)}{dt}$ (b) $j2\pi f x(f)$

(c) $x(f)$ (d) $\dfrac{x(f)}{jf}$

Solution: (b)

$$\frac{dx(t)}{dt} \longrightarrow (j2\pi f)x(f).$$

$$x(t) \xleftrightarrow[\text{Transform}]{\text{Fourier}} x(f)$$

Hence, the correct option is (b).

14. The amplitude spectrum of a Gaussian pulse is **[1998]**

(a) uniform
(b) a Sine function
(c) Gaussian
(d) an impulse function

Solution: (c)
Normalized Gaussian pulse is defined as $x(t) = e^{-\pi t^2}$
Fourier transfer will be

$x(t) = e^{-\pi t^2} \xleftrightarrow{F \cdot T} e^{-\pi f^2} = X(t) \Rightarrow$ Gaussian pulse

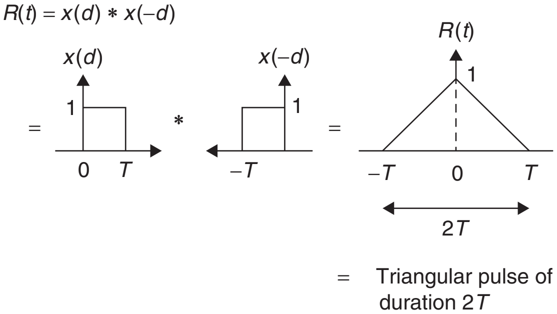
Hence, the correct option is (c).

15. The ACF of a rectangular pulse of duration T is: **[1998]**

(a) a rectangular pulse of duration T
(b) a rectangular pulse of duration $2T$
(c) a triangular pulse of duration T
(d) a triangular pulse of duration $2T$

Solution: (d)
ACF for signal $x(t)$ is given by.

$R(t) = x(d) * x(-d)$



Hence, the correct option is (d).

16. The function $f(t)$ has Fourier transform $g(\omega)$. The Fourier transform $g(t)$ **[1997]**

$$g(t) = \int_{-} g(t)e^{-J\omega t}dt, \quad \text{is}$$

(a) $\dfrac{1}{2\pi}f(\infty)$ (b) $\dfrac{1}{2\pi}f(-\omega)$

(c) $2\pi f(-\omega)$ (d) name of these

Solution: (c)
Using duality property of Fourier transform

$$f(t) \xleftrightarrow{F \cdot T} g(\omega).$$

Then Fourier transform of $g(t)$

$$g(t) \xleftrightarrow{F \cdot T} 2\pi f(-\omega)$$

Hence, the correct option is (c).

17. The Fourier transform of a real valued time signal has **[1996]**

(a) odd symmetry
(b) even symmetry

(c) conjugate symmetry
(d) No symmetry.

Solution: (d)

Fourier transform of signal $x(t)$ is given by

$$X(i\omega) = \int_{-} x(t)\cdot e^{-J\omega t}dt\ .$$

Taking conjugate on both sides,

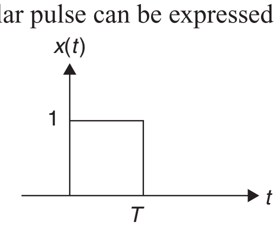
$$\Rightarrow X^*(j\omega) = \left[\int_{-} x(t)\cdot e^{-J\omega t}dt\right]^* = \int_{-} x^*(t)\cdot e^{+J\omega t} = X(-J\omega)$$

$\Rightarrow$ So $x*(j\omega) = x(-j\omega) \Rightarrow$ Conjugate symmetric.
$\therefore$ $[x^*(t) = x(t)$ for real valued signal.
Hence, the correct option is (d).

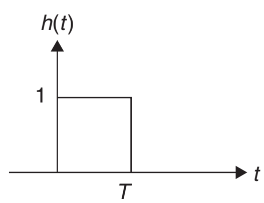
18. A rectangular pulse of duration T is applied to a filter method to this input. The output of filter is a **[1996]**
(a) rectangular pulse of duration T.
(b) rectangular pulse of duration $2T$.
(c) triangular pulse
(d) sine function

Solution: (c)

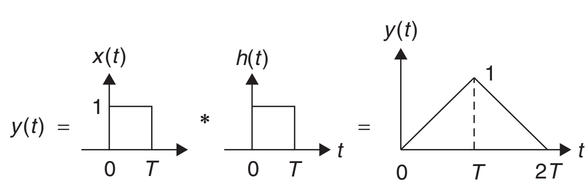
A rectangular pulse can be expressed as



For matched filter o/p for i/p $x(t)$ is
$h(t) = x(T - t)$
$h(t)$ can be represented as



then time duration output will be $y(t) = x(t) * h(t)$



= Triangular pulse of duration $2T$

Hence, the correct option is (c).

19. The 3-dB bandwidth of a typical second-order system with the Transfer function **[1994]**

$$\frac{e(s)}{R(s)} = \frac{wn^2}{\delta^2 + 2\xi wn + wn^2},\ is\ given\ by$$

(a) $\omega n\sqrt{1-2\xi^2}$
(b) $\omega n\sqrt{(1-\xi^2)+\sqrt{\xi^4-\xi^2+1}}$
(c) $\omega n\sqrt{(1-2\xi^2)+\sqrt{4\xi^4-4\xi^2+2}}$
(d) $\omega n\sqrt{(1-2\xi^2)-\sqrt{4\xi^4-4\xi^2+2}}$

Solution: (d)

Given transfer function

$$H(s) = \frac{c(s)}{R(s)} = \frac{\omega n^2}{s^2 + 2\xi\omega n s + \omega n^2} \qquad (1)$$

Put $s = J'\omega$, and divide numerator and denominator by ωn^2

$$H(J\omega) = \frac{t}{\left(1 - \frac{\omega^2}{\omega n^2}\right) + J'\left(\frac{2\xi\omega n}{\omega n}\right)} \qquad (2)$$

Let $x = \frac{\infty}{\infty n}$ from ξg^n (2)

$$H(j\omega) = \frac{t}{(1-x^2) + j(2\xi x)} \qquad (3)$$

at 3 dB frequency $|H(j\omega_c)| = 1/\sqrt{2}$, from ξg^n (3)

$$\Rightarrow \frac{1}{\sqrt{(1-x^2)+(2\xi x^2)}} = \frac{1}{\sqrt{2}} \qquad (4)$$

Solving equation (4)

$$x = \sqrt{(1-2\xi^2) \pm (4\xi^2 - 4\xi^2 + 2)} \qquad (5)$$

as $x = \frac{\omega c}{\omega n}$

So from equation (5)

$$\omega c = {}^{\omega n}\sqrt{(1-2\xi^2) \pm \sqrt{(4\xi^2 - 4\xi^2 + 2)}}$$

As ω_C can't be negative,

So $\omega_c = {}^{\omega_n}\sqrt{(1-2\xi^2) + \sqrt{4\xi^4 - 4\xi^2 + 2}}$

Hence, the correct option is (d).

Two-marks Questions

1. Consider the signal $x[n] = 6\delta[n+2] + 3\delta[n+1] + 8\delta[n] + 7\delta[n-1] + 4\delta[n-2]$.
If $x(e^{j\omega})$is the discrete-time Fourier transform of $x[n]$, then $\frac{1}{\pi}\int_{-\pi}^{\pi} x\left(e^{j\omega}\right)\sin^2(2\omega)\,d\omega$ is equal to ____. **[2016]**

Solution: $x[n] = 6\,\delta\,[n+2] + 3\,\delta\,[n+1] + 8\,\delta\,[n] + 7\,\delta\,[n-1] + 4\,\delta\,[n-2]$

$x[n] = \{6, 3, 8, 7, 4\}$ its DTFT π is $X(e^{j\omega})$

Now

$$\frac{1}{\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\sin^2(2\omega)\,d\omega$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\left(\frac{1-\cos 4\omega}{2}\right)d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)d\omega - \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)\cos 4\omega\, d\omega$$

$$= x[0] - \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)\times\left(\frac{e^{j4\omega}+e^{-j4\omega}}{2}\right)d\omega$$

$$= 8 - \frac{1}{4\pi}\left[\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)d\omega + \int_{-\pi}^{\pi} X\left(e^{j\omega}\right)e^{-j4\omega}d\omega\right]$$

$$= 8 - \frac{1}{\pi}\left[x[4]+x(-4)\right] = 8 - \frac{1}{2}[0+0] = 8$$

Hence, the correct Answer is (8).

2. The value of the integral $\int_{-} \sin c^2(5t)$ is **[2014]**

Solution: 0.2

Let $f(t) = \sin c(5t) = Sa(5\pi t)$

$(\because \sin c(t) = Sa(\pi t))$

Fourier transform of Sa (t) is given as

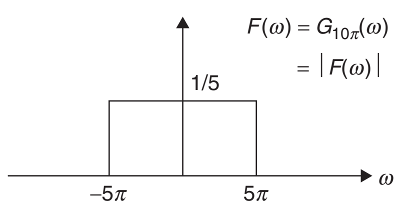
$$TSa\left(t\cdot\frac{T}{2}\right) \xleftrightarrow{F\cdot T} 2\pi G_T(\omega)$$

$$\frac{T}{2} = 5\pi \Rightarrow T = 10\pi$$

$$\Rightarrow 10\pi Sa(5\pi t) \longleftrightarrow \frac{2\cancel{\pi}}{5} G_{10\pi}(\omega)$$

$$\Rightarrow ESD_f = |F(\omega)|^2$$

$$E_f = \int_{-} f^2(t)dt = \frac{1}{2\cancel{\pi}}\int_{-} ESD_f\, d\omega$$



$$\Rightarrow \int_{-} \sin c^2(5t)dt = \frac{1}{2\cancel{\pi}}\int_{-5\pi}^{5\pi}\left(\frac{1}{5}\right)^2 d\omega = \frac{1}{2\pi}\times\frac{1}{25}\times 10\pi = \frac{1}{5} = 0.2$$

3. The Fourier transform of a signal $h(t)$ is **[2012]**
$H(J'\omega) = (2\cos\omega)(\sin 2\omega)/\omega$ the value of $h(o)$ is

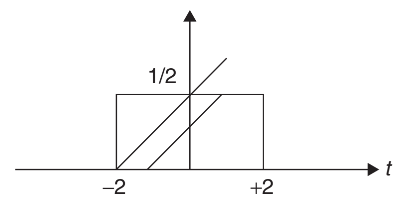
(a) 1/4
(b) 1/2
(c) 1
(d) 2

Solution: (a)

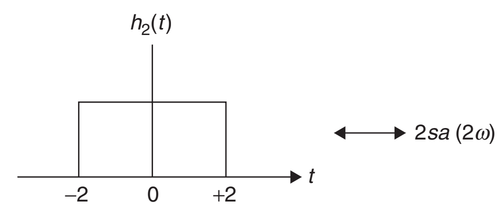
$$H(J\omega) = 2\cos\omega\left(\frac{2\sin 2\omega}{2\omega}\right) = H_1(J\omega)\cdot H_2(J'\omega)$$

Given
where
$\Rightarrow H_1\,(j\omega) = 2\cos\omega =$

$$H_2(j\omega) = \frac{2\sin 2\omega}{2\omega} = 2\delta a(2\omega)$$



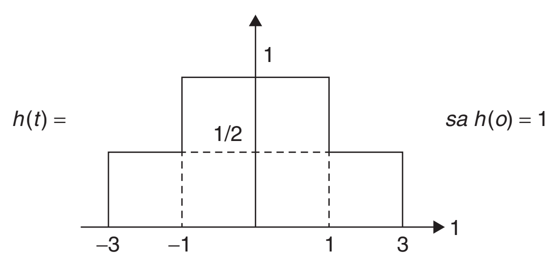
$\Rightarrow$



$$\Rightarrow H_1(J\omega) = 2\cos\omega = e^{J\omega 1} + e^{J\omega(-1)}$$

So, $H(J\omega) = e^{J\omega}H_2(J\omega) + e^{-J\omega}H_2(J\omega)$

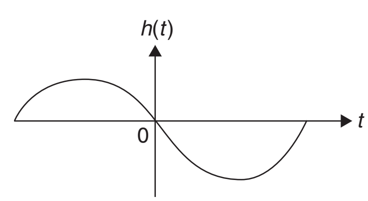
Or $h(t) = h_2(t+1) + h_2(t-1)$



Hence, the correct option is (a).

4. Consider a system whose i/p and o/p y are related by the equation

$y(t) = \int_{-} x(t-\tau)h(2\tau)dt$, and $h(t)$ is shown in graph

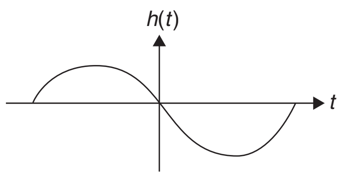


Which of the following your properties are possessed by the system? BIBO: Bounded *i/p* fixes *o/p*
Causal: the system is causal
LP: The system is low pass
LTI: the system is linear and time-invariant **[2009]**
(a) Causal, LP
(b) BIBO, LTI
(c) BIBO, Causal, LTI
(d) LP, LTI

Solution: (b)

$$y(t) = \int_{-} x(t-\tau)\, h(2\tau) d\tau$$

given



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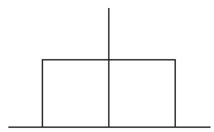


The system is LTI, BIBO, but not a low pass filter.
Hence, the correct option is (b).

5. The output of the system to sinusoid input $x(t)$ = 2cos (2t) for all time t is **[2008]**
(a) 0
(b) $2^{-0.2}\cos(2t - 0.125\pi)$
(c) $2^{-0.25}\cos(2t - 0.125\pi)$
(d) $2^{-0.5}\cos(2t - 0.25\pi)$

Solution: (d)
Given input $x(t) = 2\cos 2t$

then $x(\omega) = 2\pi[\delta(\omega-2)+\delta(\omega+2)]$

$$\Rightarrow H(\omega) = \frac{1}{1+J\omega}$$

o/p =

$$y(\omega) = H(\omega)\cdot x(\omega) = \frac{1}{2+j\omega}\left[(2\pi)[\delta(\omega-2)+\delta(\omega+2)]\right]$$

$$= \frac{\pi}{2}\left[\delta(\omega-2)+\delta(\omega+2)\right] - j\frac{\pi}{2}\left[\delta(\omega-2)-\delta(\omega+2)\right]$$

$$= \frac{\cos 2t}{2} + \frac{\sin 2t}{2} = \frac{\sqrt{2}}{2}\left[\frac{1}{\sqrt{2}}\cos 2t + \frac{1}{\sqrt{2}}\sin 2t\right]$$

$$= 2^{-0.5}\cdot\cos(2t - 0.25\pi)$$

Hence, the correct option is (d).

6. The frequency response $H(\omega)$ of this system in terms of angular frequency ω, is given by $H(\omega)$ = **[2008]**

(a) $\dfrac{1}{1+2J\omega}$ (b) $\dfrac{\sin(\omega)}{\omega}$

(c) $\dfrac{1}{1+j\omega}$ (d) $\dfrac{j\omega}{2+j\omega}$

Solution: (a)
Given $h(t) = e^{-2t}u(t)$

$$H(\omega) = \int_{-} h(t)e^{-J\omega t}dt = \int_0 e^{-(2tJ\omega)^t}dt = \frac{-1}{(2tJ\omega)}\left[e^{-(2tJ\omega)^t}\right]_0^{\infty}$$

$$= \frac{1}{1+2J\omega}$$

Hence, the correct option is (a).

7. The signal $x(t)$ is described by

$x(t)\begin{cases}1 & \text{for } -1 \le t \le 1\\ 0 & \text{otherwise}\end{cases}$ Two of the angular frequencies where Fourier transform became zero are **[2008]**
(a) π, 2π
(b) 0.5π, 1.5π
(c) 0, π,
(d) 2π, 2.5π

Solution: (a) and (c)
Fourier transform

$$x(\omega)\int_{-} x(t)e^{-J\omega t}dt = \int_{-1}^{1} e^{-J\omega t}dt = \frac{1}{J\infty}\left(e^{J\omega} - e^{-J\omega}\right)$$

$\Rightarrow x(\omega) = 0$
So $e^{J\omega} - e^{-J\omega} = 0$
$\Rightarrow e^{2J\omega} = \phi \Rightarrow \omega = \pi, 2\pi$ or $0.\pi$
Hence, the correct options are (a) and (c).

8. The 3-*dB* bandwidth of the LOW-pass signal, $e^{-t}u(t)$, where $u(t)$ is the unit step function, is given by **[2007]**

(a) $\dfrac{1}{2\pi}H_z$

(b) $\dfrac{1}{2\pi}\sqrt{\sqrt{2}-1}\ Hz$

(c) ∞
(d) 1 H_z

Solution: (a)

Laplace transform of $e^{-t}u(t) \xleftrightarrow{L\cdot T} \dfrac{1}{(s+1)}$

$\therefore$ Magnitude at 3*dB* frequency $= \dfrac{t}{\sqrt{2}}$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{1}{s+1} = \frac{1}{\sqrt{1+\omega^2}} \Rightarrow \omega = 1 \text{ real or } f = \frac{1}{2\pi}Hz$$

Hence, the correct option is (a).

9. The output $y(t)$ of a linear time-invariant system is related to its input $x(t)$ by the equation $y(t) = 0.5\,x(t - t_d + T) + x(t - t_d) + 0.5x(t - t_d + T)$. The filter transfer function $H(\omega)$ of such system is given by **[2005]**
(a) $(1 + \cos\omega T)\, e^{-J\omega td}$
(b) $(1 + 0.5\cos\omega T)\, e^{-J\omega td}$
(c) $(1 - \cos\omega T)\, e^{-J\omega td}$
(d) $(1 - 0.5\cos\omega T)\, e^{-J\omega td}$

Solution: (a)

Given $y(t) = 0.5\ x\ (t - t_d + T\ ) + 0.5x\ (t - t_d + T\ ) + x(t - t_d)$

taking Fourier transform

$$y(\omega) = \left[0.5e^{j\omega(-t_d+T)} + 0.5e^{j\omega(-t_d+T)} + e^{-j\omega t_d}\right]x(\omega)$$

$$\Rightarrow H(\omega) = \frac{y(\omega)}{x(\omega)} = e^{-j\omega t_d}\left[0.5e^{j\omega t} + 0.5e^{-j\omega t} + 1\right]$$

$$H(\omega) = e^{-j\omega t_d}\left[\cos\omega T + 1\right]$$

Hence, the correct option is (a).

10. Match the following and choose the correct combination. **[2005]**

Group-1

E. Continuous and aperiodic signal
F. Continuous and periodic signal
G. Discrete and aperiodic signal
H. Discrete and periodic signal

Group-2

1. Fourier representation is continuous and periodic
2. Fourier representation is discrete and periodic
3. Fourier representation is continuous and periodic
4. Fourier representation is discrete and periodic

(a) E – 3, F – 2, G – 4, H – 1
(b) E – 1, F – 3, G – 2, H – 4
(c) E – 1, F – 2, G – 3, H – 4
(d) E – 2, F – 1, G – 4, H – 3

Solution: (c)

Continuous and aperiodic signal has continuous and periodic representation.

Continuous and periodic signal has discrete and periodic representation.

Hence, the correct option is (c).

11. For a signal $x(t)$, the Fourier transform is $x(f)$, then the inverse Fourier Transform of $x(3f + 2)$ is given by **[2005]**

(a) $\frac{1}{2}\times\left(\frac{t}{2}\right)e^{j3\pi t}$ (b) $\frac{1}{3}\times\left(\frac{t}{3}\right)e^{-j4\pi\frac{1}{3}}$

(c) $3\times(3t)e^{-j4\pi t}$ (d) $x(3t + 2)$

Solution: (b)

Scaling and translation property for Fourier transform pair are given as.

$$\Rightarrow x(at) \xleftrightarrow{F\cdot T} \frac{1}{|a|}\times\left(\frac{f}{a}\right)$$

$$\Rightarrow x(t - t_0) \xleftrightarrow{F\cdot T} x(f)e^{-J2\pi f t_0}$$

$$\times\left[3\left(f + \frac{2}{3}\right)\right] = \frac{1}{3}\times\left(\frac{t}{3}\right)\cdot e^{-J4\pi\frac{t}{3}}$$

Hence, the correct option is (b).

12. The Fourier transform of a conjugate symmetric function is always: **[2004]**

(a) Imaginary
(b) Conjugate anti-symmetric
(c) Real
(d) Conjugate symmetric

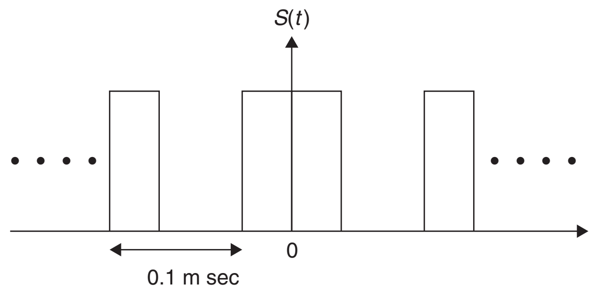
Solution: (c)

Fourier transform of a symmetric function is a purely real *fn*.

Hence, the correct option is (c).

13. A rectangular pulse train $s(t)$ as shown in the figure is convolved with the signal $\cos^2(4\pi \times 10^3 t)$.

Convolved signal will be a



[2004]

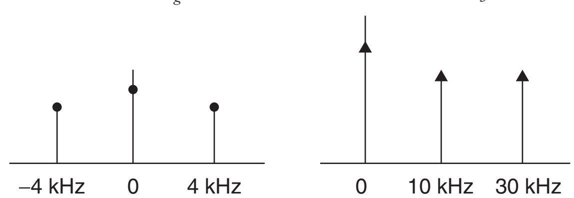
(a) D.C. (b) 12 kH_3 sinusoid
(c) 9 kH_3 sinusoid (d) 14 kH_3 sinusoid

Solution: (a)

Time period for pulse train $T_o = 0.1$ msec $= 10^{-4}$ sec

$$f_0 = \frac{1}{T_0} = 10kH_3$$

Fundamental f_g in $\cos^2(4\pi \times 10^3 t)$ is $= 4kH_3$

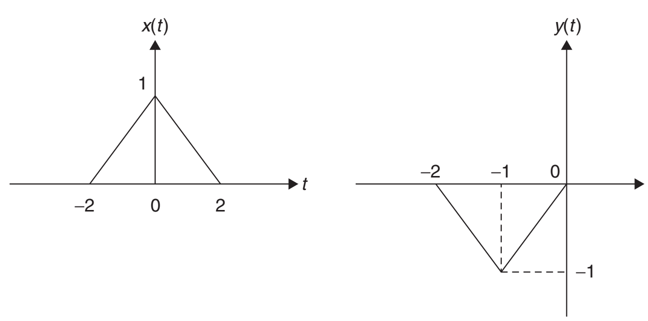


So only at '0' we get output after convolution. (only odd harmonics are present)

↑ Constant in time domain.

Hence, the correct option is (a).

14. Let $x(t)$ and $y(t)$ (with Fourier transforms $x(f)$ and $y(f)$ respectively) be related as shown in the figure **[2004]**



(a) $\frac{-1}{2}x\left(\frac{f}{2}\right)e^{-J2\pi f}$ (b) $\frac{-1}{2}x\left(\frac{f}{2}\right)e^{j2\pi f}$

(c) $x\left(\frac{f}{2}\right)e^{j2\pi f}$ (d) $-x\left(\frac{f}{2}\right)\cdot e^{-j2\pi f}$

Solution: (b)

From the plot, $y(t)$ can be represented as $y(t) = -x(2(t)+1)$

Using property: $x(t-t_0) \longleftrightarrow x(f)e^{-J2\pi f t_0}$ $: t_0 = -1$

$$x(at) \longleftrightarrow \frac{1}{|a|} \times \left(\frac{f}{a}\right) \qquad a = -2$$

So, $y(f) = \frac{-1}{2} \times \left(\frac{f}{2}\right)e^{J2\pi f}.$

Hence, the correct option is (b).

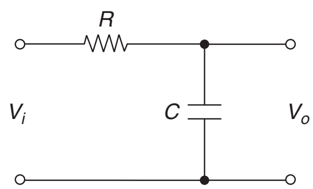
15. Let $H(f)$ denote the frequency response of the RC-LPF. Let f_1 be the highest frequency such that $o \le |f| \le f_1$: $\frac{|H(f_1)|}{H(o)} \ge 0.95$ then f_1 (in H_3) is **[2003]**

(a) 327.8 (b) 163.9

(c) 52.2 (d) 104.4

Solution: (c)

Rc-LPF can be given as



Transfer function will be given by.

$$\frac{vo}{vi} = \frac{\frac{1}{j2\pi f \cdot c}}{R + \frac{t}{J \cdot 2\pi f \cdot c}} = \frac{t}{1 + j2\pi f \cdot RC} \qquad (1)$$

$$H(f) = \frac{1}{1 + j2\pi f RC}, \quad H(o) = 1$$

$\Rightarrow$ cutoff filter is cancelled at point where magnitude is 5% of maximum value.

$$\text{So, } \frac{|H(f_1)|}{|H(o)|} = \frac{1}{\sqrt{1 + 4\pi^2 f^2 R^2 C^2}} \ge 0.95. \qquad (2)$$

Put a value of R and C, and find $f_1 \Rightarrow f_{1\max} = 52.2$Hz.

Hence, the correct option is (c).

16. Let $t_g(f)$ be the group delay function of the given RC-LPF and $f_2 = 100\text{H}_3$. Then $t_g(f_2)$(ms) **[2003]**

(a) 0.717 (b) 7.17

(c) 71.7 (d) 4.505

Solution: (c)

Given, $H(\omega) = \frac{1}{1 + \omega RC}$

and $\theta w = -\tan^{-1}(RC\omega)$

from equation (1) 4.19

put a value of R, C and w to calculate t_g

$$t_g = \frac{d\theta(w)}{dw} = \frac{RC}{1 + R^2C^2w^2},$$

put $R = 10^3$, $C = 1 \times 10^{-6} f = 100H_3$

So $t_g = 0.717$ ms.

Hence, the correct option is (c).

17. If Fourier transform of deterministic signal $g(t)$ is $G(f)$, then

(1) the Fourier transform of $g(t-2)$ is

(2) the Fourier transform of $g(t/2)$ is

(a) $G(f) \cdot e^{-J4\pi f}$ (b) $G(2f)$

(c) $2G(2f)$ (d) $G(f-2)$

Match each of the items (1) and (2) on the left with the most appropriate item a, b, c, or d on the right. **[1997]**

Solution: (1 – a, 2 – c)

Given $g(t) \xleftrightarrow{F \cdot T} G(f)$

then $g(t-2) \xleftrightarrow{F \cdot T} e^{-j2\pi 2f} G(f) = G(f) \cdot e^{-j4\pi f}$

and $g\left(\frac{t}{2}\right) \xleftrightarrow{F \cdot T} \frac{1}{(½)} G\left(\frac{f}{½}\right) = 2G(2f)$

18. Match each of items A, B and C with an appropriate item from 1, 2, 3, 4 and 5. **[1995]**

List-I

(A) Fourier transform of a Gaussian function

(B) Convolution of rectangular pulse with itself

(C) Current through an inductor for a step input voltage.

List-II

(1) Gaussian function

(2) Rectangular pulse

(3) Triangular pulse

(4) Ramp function

(5) Zero

Solution: (A – 1, B – 3, C – 4)

(A) Gaussian pulse is given as $f(t) = e^{-\pi t 2}$

So, $e^{-\pi t^2} \xleftrightarrow{F \cdot T} e^{-\pi f^2}$

(B) Conversion of a rectangular pulse with a rectangular pulse is a triangular pulse.

(C) Current through an inductor is given by

$$i_L = \frac{1}{L}\int V dt$$

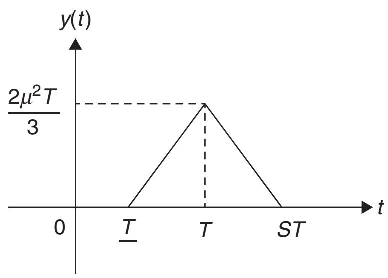
For step input $V = u(t)$

So, $i_L = \frac{1}{L}\int u(t)dt = \frac{1}{L}r(t)$

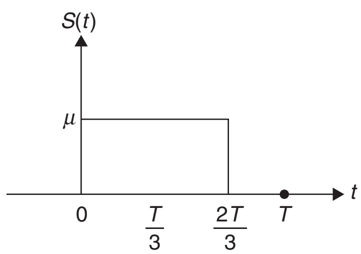
19. Sketch the waveform (with properly marked axis) at the output of a match filter matched for a signal $S(t)$, of duration T, given by

$$S(t) = \begin{cases} A & for\ 0 \le t \le \frac{2T}{3} \\ 0 & for\ \frac{2T}{3} \le t \le T \end{cases}$$ **[1993]**

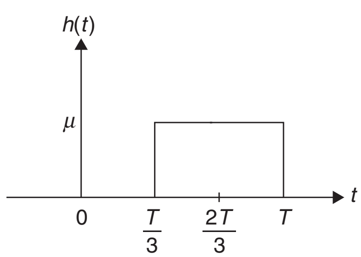
Solution:



Signal $S(t)$ is given by



Impulse-response of a matched filter is given by $h(t) = S(T–t)$



The output of the matched filter $y(t)$

$y(t) = S(t)*h(t)$

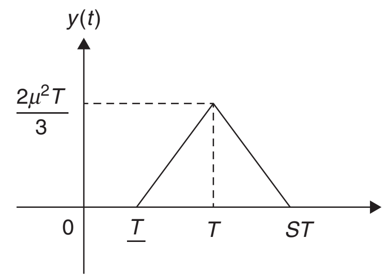
$$=> \frac{dy(t)}{dt} = s(t)*\frac{dh(t)}{dt} = \left\{u(t) - u\left(t - \frac{2T}{3}\right)\right\} * \left\{S\left(t - \frac{T}{3}\right) - \delta(t - T)\right\}$$

$$= u\left(t - \frac{T}{3}\right) - u(t - T) - u(t - T) + u\left(t - \frac{5T}{3}\right)$$

$$= u\left(t - \frac{T}{3}\right) - 2u(t - T) - u\left(t - \frac{5T}{3}\right)$$

$$=> y(t) = \int_{-}^{t}\left(\frac{u}{t} - \frac{T}{3}\right) - 2u(t - T) + u\left(t - \frac{5T}{3}\right)dt$$

$$= r\left(t - \frac{T}{3}\right) - 2r(t - T) + r\left(t - \frac{5T}{3}\right)$$



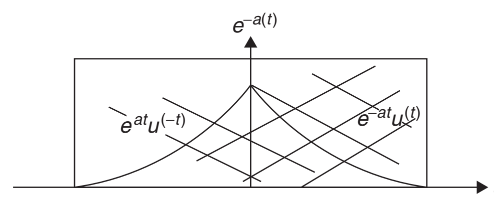
20. If $G(f)$ represents the Fourier transform of a Signal $g(t)$, which is real and odd symmetric in time, then **[1992]**

(a) $G(f)$ is complex
(b) $G(f)$ is imaginary
(c) $G(f)$ is real
(d) $G(f)$ is real and non-negative

Solution: (b)

We take a real and odd symmetric signal as exempla, and verify its properties.

Let $f(t) = -e^{at}\, u(-t) + e^{-at}\, u(t)$



real and odd symmetric.

taking Fourier transform

$$F(\omega) = \int_{-} f(t)\cdot e^{-J\omega t}dt = -\int_{-}^{0} e^{at}\cdot e^{-J\omega t}dt + \int_{0} e^{-at}\cdot e^{-J\omega t}dt$$

$$= \frac{1}{\cancel{1}+aJ} = -\frac{1}{a - J\omega} + \frac{1}{a + J\omega} = \frac{2J\omega}{a^2 + \omega^2}$$

So, $F(\omega) = \frac{2J\omega}{a^2 + \omega^2} \Rightarrow$ poorly imaginary and odd.

Hence, the correct option is (d).

21. The magnitude and phase function for a distortionless filter should, respectively, be **[1990]**

	(Magnitude)	**(Phase)**
(a)	Linear	Constant
(b)	Constant	Constant
(c)	Constant	Linear
(d)	Linear	Linear

Solution: (c)

For distortionless filter

$$f(t) \to \boxed{h(t)\, or\ h(\omega)} \to A\cdot f(t - t_0)$$

If Fourier transform of function $f(t) \to F(\omega)$,

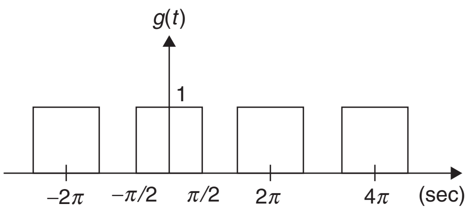
then Fourier transform of $A\cdot f(t - t_0) \to A\, F(\omega)\cdot e^{-J\omega t0}$

$$So,\quad H(\omega) = \frac{y(\omega)}{x(\omega)} = A\cdot e^{-J\omega t_0}$$

Taking Inverse Fourier transform of $H(\omega)$
$h(t) = A\,\delta(t - t_0)$.
Taking magnitude and phase of $H(\omega)$
$|H(\omega) = A|$ and $\angle H(\omega) = -\omega\, t_0$
as $H(\omega) = A \Rightarrow$ constant (Magnitude Response)
and $\angle H(\omega) = -\omega\, t_0 \Rightarrow$ Linear (Phase Response)
Hence, the correct option is (c).

Five-marks Questions

1. A periodic signal $g(t)$ is shown in figure. Determine the PSD of $g(t)$. **[2001]**



Solution:
$g(t)$ is a periodic signal with, $T_0 = 2\pi$

$$f_0 = \frac{1}{T_0} = \frac{1}{2\pi}$$

$$\omega_0 = 2\pi f_0 = \frac{1}{2\pi} \times 2\pi = 1 \text{ rad/sec}$$

Express $g(t)$ as a Fourier series,

$$g(t) = \sum_{n=-\infty}^{\infty} e_m e^{j2\pi n f_0 t}$$

$$C_n = \frac{1}{T_0}\int_0^{T_0} e^{-j2n\pi f_0 t}\,dt$$

C_n can be calculated as,

$$C_n = \frac{1}{2}\frac{\sin\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)} = \frac{1}{2} s_a\left(\frac{n\pi}{2}\right)$$

$$C_0 = \frac{1}{2}$$

$C_n = 0 \quad n = \pm 2, \pm 4, \pm 6 \ldots\ldots\ldots$

$$C_n = \frac{1}{\pi n} \quad n = \pm 1, \pm 5, \pm 9$$

$$C_n = \frac{-1}{n\pi} \quad n = \pm 3, \pm 7, \pm 11 \ldots\ldots.$$

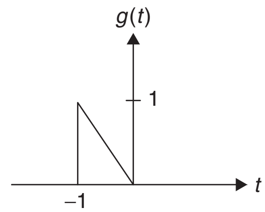
Let, PSD of $g(t)$

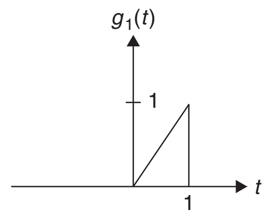
$$S_g(f) = \sum_{n=-\infty}^{\infty} |C_n|^2 = \sum_{n=-\infty}^{\infty}\left(0.5 sa\left(\frac{n\pi}{2}\right)\right)^2$$

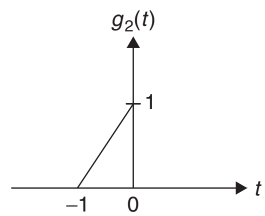
For $n = \pm 1, \pm 3, \pm 5 \ldots\ldots\ldots$

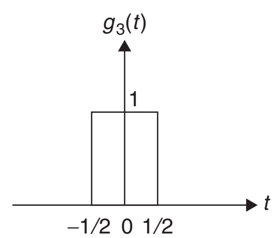
2. The Fourier transform $G(\omega)$ of the signal $g(t)$ in figure 1 is given as:

$G(\omega) = \frac{1}{\omega^2}(e^{j\omega} - j\omega e^{j\omega} - 1)$. Using the information, and the time-shifting and time-scaling properties, determine the Fourier transform of signals in figures (2), (3) and (4). **[2000]**









Solution:

If $g(t) \xleftrightarrow{F.T} G(\omega)$

then, $g(at) \xleftrightarrow{F.T} \frac{1}{|a|} G\left(\frac{\omega}{a}\right)$

$$g(-t) \xrightarrow{F.T.} G(-\omega)$$

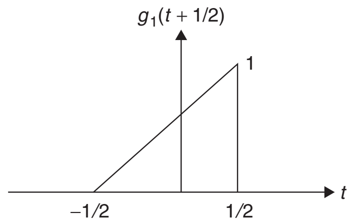
$$g(t - t_0) \xrightarrow{F.T.} e^{-j\omega t_0} G(\omega)$$

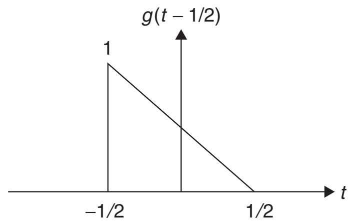
Using above properties, $G_1(\omega) = G(-\omega)$

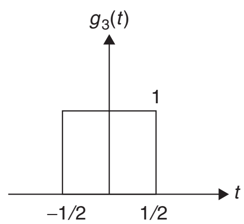
$$G_1(\omega) = \frac{1}{\omega^2}\left[e^{-j\omega} + j\omega e^{-j\omega} - 1\right]$$

and $g_2(t) = g(-t - 1) = g(-(t + 1))$

$$G_2(\omega) = e^{j\omega} G_1(\omega) = \frac{1}{\omega^2}[1 + j\omega - e^{j\omega}]$$







$$g_3(t) = g_1\left(t+\frac{1}{2}\right) + g\left(t-\frac{1}{2}\right)$$

$$G_3(\omega) = e^{j\omega/2}.\frac{1}{\omega^2}\left[e^{-j\omega} + j\omega e^{-j\omega} - 1\right]$$

$$+e^{-j\omega/2}\frac{1}{\omega^2}[e^{j\omega} - j\omega^{t\omega} - 1]$$

Solving

$$= \frac{1}{j\omega}[e^{0.5j\omega} - e^{-0.5j\omega}]$$

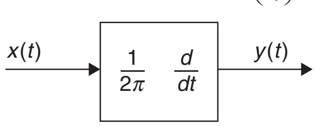
$$= \frac{2}{\omega}\sin\left(\frac{\omega}{2}\right) = s_a\left(\frac{\omega}{2}\right)$$

3. A deterministic signal $x(t) = \cos 2\pi t$ is passed through a differentiator as shown in figure.

(a) Determine the autocorrelation $R_{xx}(\pi)$ and the power spectral density $S_{xx}(f)$.

(b) Find the output power spectral density $s_{yy}(f)$.

(c) Evaluate $R_{xy}(0)$ and $R_{xy}\left(\frac{1}{4}\right)$. **[2000]**



Solution:

Given signal is periodic with $T_0 = 1$ sec

(a) and $f_0 = 1$ Hz ACF

$$R_{xx}(\tau) = \frac{1}{T}\int^{\tau_0} x(t)\cdot d(t-\tau)dt$$

$$= \int_0^1 \cos(2\pi t)\cos 2\pi(t-\tau)dt$$

Using, 2 cos$A$ cos$B$ = cos(A – B) + cos(A + B)

$$R_{xx}(\tau) = \frac{1}{2}\cos(2\pi\tau) = \frac{1}{2}\cos(2\pi\tau)\int_0^1 dt + \frac{1}{2}\int_0^1 \cos(4\pi t - 2\pi\tau)dt$$

$$= \frac{1}{2}\cos(2\pi\tau) + \frac{1}{4\pi}[\sin(4\pi - 2\pi\tau) - \sin(-2\pi\tau)]$$

$$= \frac{1}{2}\cos(2\pi\tau) + \frac{1}{4\pi}[\sin(4\pi)\cos(2\pi\tau)$$

$$-\cos(4\pi)\sin(2\pi\tau) + \sin(2\pi\tau)]$$

Solving

$$= \frac{1}{2}\ \cos 2\pi\tau$$

$$R_{xx}(\tau) \xleftrightarrow{F.T} S_{xx}(f) = \frac{1}{4}[f(f-1) + f(f+1)]$$

(b) $y(t) = \dfrac{1}{2\pi}\dfrac{d}{dt}x(f) = \dfrac{1}{2\pi}\dfrac{d}{dt}\cos 2\pi t = -\sin 2\pi t$

$$y(t) = \cos\left[2\pi\left(t+\frac{1}{4}\right)\right] = x\left(t+\frac{1}{4}\right)$$

As time shift has no effect on ACF and PSD

So,

$$R_{yy}(\tau) = R_{xx}(\tau) = \frac{1}{2}\cos(2\pi t)$$

$$S_{yy}(f) = S_{xx}(f) = \frac{1}{4}[f(f-1) + f(f+1)]$$

(c) $R_{xy}(\tau) = \dfrac{1}{T_0}\displaystyle\int_0^{T_0} x(t)\cdot y(t-\tau)dt$

$$= \frac{1}{T_0}\int_0^{T_0}\cos(2\pi t).[-\sin\{2\pi(t-\tau)]dt$$

$$= \frac{-1}{T_0}\left[\int_0^{T_0}\sin(u\pi t - 2\pi t)dt - \int_0^{T_0}\sin 2\pi, dt\right]$$

$$R_{xy}(\tau) = \frac{1}{2}\sin(2\pi\tau)\int_0^{T_0} dt$$

$$= \frac{1}{2}\sin(2\pi t)$$

$$R_{xy}(0) = 0$$

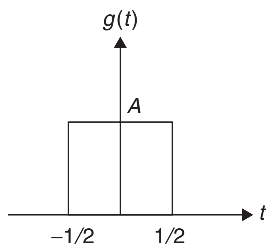
$$R_{xy}\left(\frac{1}{4}\right) = \frac{1}{2}\sin\left(\frac{\pi}{2}\right) = \frac{1}{2}$$

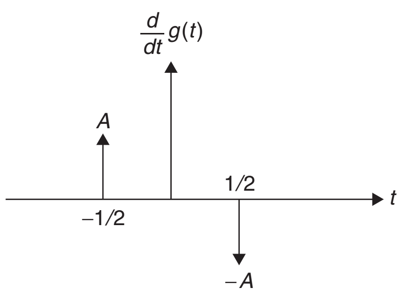
4. Consider a rectangular pulse $g(t)$ existing between $t = -\frac{T}{2}$ and $\frac{T}{2}$. Find and sketch the pulse obtained by convolving $g(t)$ with itself. The Fourier transform of $g(t)$ is a sine function. Write down the Fourier transform of the pulse obtained by the above convolution. **[1998]**

Solution:

Given that, $g(t) = A \qquad t \le \left|\frac{T}{2}\right|$

$\qquad\qquad = 0 \qquad$ otherwise.

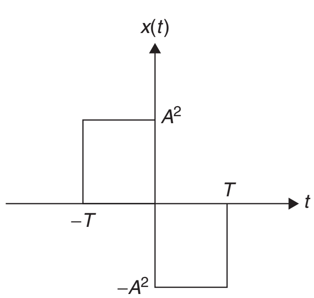




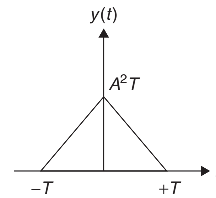
Let, $y_1(t) = g(t) * \frac{d}{dt} g(t)$ then, $y(t) = \int_t y_1(t)dt$

$$y_1(t) = g(t) * Af\left(t + \frac{T}{2}\right) - g(f) * Af\left(t - \frac{T}{2}\right)$$

$$\Rightarrow \; y_1(t) = Ag\left(t + \frac{T}{2}\right) - Ag\left(t - \frac{T}{2}\right)$$



$$y(t) = \int y_1(t)dt$$



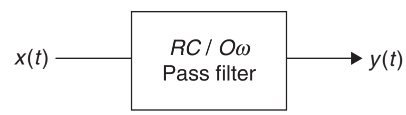
$$g(t) \xrightarrow{F.T} G(f) = AT \sin c\,(fT)$$

$$y(y) = G(f)\,.\,G(f) = A^2T^2 \sin c^2\,(fT)$$

$$x(t) = \cos 2\pi$$

5. A signal $3\sin(\pi f_0 t) + \cos(3\pi f_0 t)$ is applied to an RC low pass filter of 3 dB cut-off frequency f_0. Determine and plot the output power spectrum and also calculate the total input and output normalized power. **[1996]**

Solution:



Given, $x(t) = 3\sin(\pi f_0 t) + \cos(3\pi f_0 t)$

$$x(f) = 3\sin\left(2\pi\frac{f_0}{2}t\right) + \cos\left(2\pi\frac{3f_0}{2}t\right)$$

the frequency response of TC low pass filter.

$$H(f) = \frac{1}{1 + j2\pi RC}$$

3 dB cut off frequency, $f_0 = \dfrac{1}{2\pi RC}$

$$\therefore\; H(f) = \frac{1}{1 + J\left(\dfrac{f}{f_0}\right)}, \quad LH(f) = -\tan^{-1}\left(\frac{f}{f_0}\right)$$

$$\theta_1 = -\tan^{-1}\left(\frac{1}{2}\right) \text{ at } f = \frac{f_0}{2}$$

$$\theta_2 = -\tan^{-1}\left(\frac{3}{2}\right) \text{ at } f = \frac{3f_0}{2}$$

$$|H(f)| = \frac{1}{\sqrt{1 + \left(\dfrac{f}{f_b}\right)^2}},$$

$$|H(f_1)| = \frac{1}{\sqrt{1 + \left(\dfrac{1}{2}\right)^2}} = \frac{2}{5} \text{ at } f = \frac{f_0}{2}$$

$$|H(f_2)| = \frac{1}{\sqrt{1+\left(\frac{3}{2}\right)^2}} = \frac{2}{\sqrt{13}} \text{ at } f_0 = \frac{3f_0}{2}$$

So output $y(f)$

$$y(f) = 3\times\frac{2}{\sqrt{5}}\sin(\pi f_0 f + \theta_1) + 5\frac{2}{\sqrt{13}}\cos(3\pi f_0 tP\theta_2)$$

the input periodic signal consists of 1st harmonic of $f_0/2$ having power $= \left(\frac{3}{\sqrt{2}}\right)^2 = 4.5$ W

Power of 3rd harmonic $= \left(\frac{5}{\sqrt{2}}\right)^2 = 12.5$

the input power spectral is given by,

$$f_i(t) = \frac{9}{2}\left[f\left(f-\frac{f_0}{2}\right) + f\left(f+\frac{f_0}{2}\right)\right] + \frac{25}{4}f\left(f-\frac{3f_0}{2}\right) + \frac{25}{4}f\left(f+\frac{3f_0}{2}\right)$$

So, total input power = 4.5 + 12.5 = 17 W

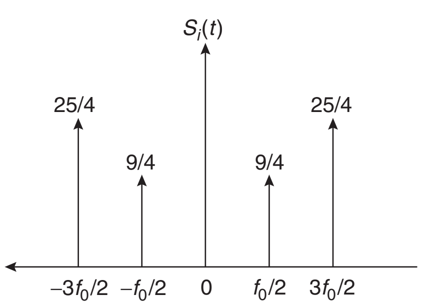
Output power $= \left(\frac{6}{\sqrt{5}\sqrt{2}}\right)^2 = \frac{18}{5}$ W at $\frac{f_0}{2}$ and

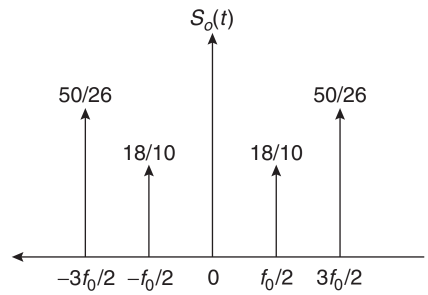
$= \left(\frac{10}{\sqrt{3}\sqrt{2}}\right)^2 = \frac{50}{13}$ W at $\frac{3f_0}{2}$

Total output power $= \frac{18}{5} + \frac{50}{13} = \frac{484}{65}$ W

The output power spectrum is given by,

$$S_0(t) = \frac{18}{10}\left[f\left(f-\frac{f_0}{2}\right) + f\left(f+\frac{f_0}{2}\right) + \frac{50}{26}\left[f\left(f-\frac{3f_0}{2}\right) + f\left(f-\frac{3f_0}{2}\right)\right]\right]$$





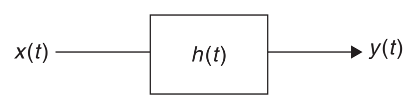
6. An input signal A exp$(-\alpha t)$ with $\alpha > 0$ is applied to a causal filter, the impulse response of which is A exp$(-\alpha t)$. Determine the filter output sketch it as a function of time and label the important points. **[1996]**

Solution:

Input $x(t) = Ae^{-\tau}u(t) \xrightarrow{F.T.} x(f) = \dfrac{A}{\alpha + j2\pi f}$

Impulse response

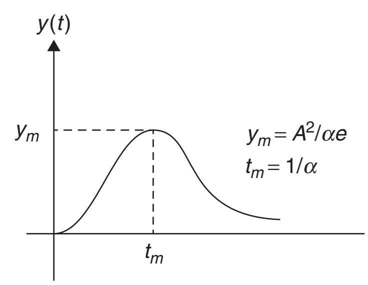
$$h(f) = Ae^{-\alpha t}u(t) \xrightarrow{F.T.} H(f) = \frac{1}{\alpha + j2\pi f}$$



$$y(t) = x(t) * h(t) \to x(s)\,.\,H(s)$$

$$y(f) = \frac{A}{(\alpha + j2\pi f)} \times \frac{A}{(\alpha + j2\pi f)} = \frac{A^2}{(\alpha + j2\pi f)^2}$$

$$y(t) = A^2 te^{-\alpha t}u(t) \quad f.e^{-\alpha t}u(t) \to \frac{1}{(\alpha + j2\pi f)^2}$$



$$\frac{d}{dt}y(t) = 0 \text{ at } t = tm$$

Solving, $t_m = \dfrac{1}{\alpha}$

$$y_m = y(tm) = A^2 tm\, e^{-\alpha tm} = \frac{A^2 e^{-1}}{\alpha} = \frac{A^2}{\alpha e}$$

7. A signal $v(t) = [1 + m(t)\cos(\omega_c t)$ is detected using a square law detector, having the characteristic $v_0 = v^2$. If the Fourier transform of $m(t)$ is constant, M_0, extending from $-f_m$ to $+f_m$, sketch the Fourier transform of $v_0(t)$ in the frequency range $-f_m < f < f_m$. **[1995]**

Solution:

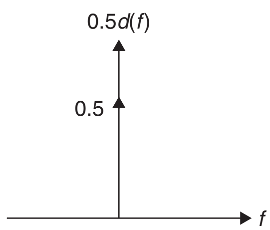
$V_0(t) = V(t)^2 = (1 + m(t))^2 \cos^2(\omega_c t)$

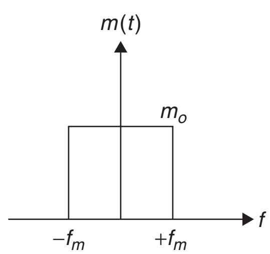
$$= [1+m^2(t)+2m(t)]\left\{\frac{1+2\cos 2\,\omega_c t}{2}\right\}$$

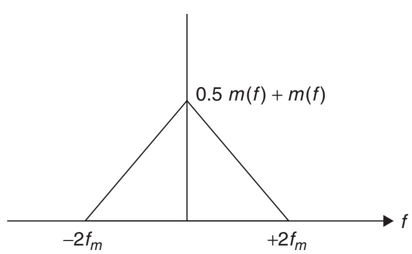
$= [0.5 + m(t) + 0.5\ m^2(t)] + [0.5 + m(t) + (0.5)m^2(t)] \cos 2\pi f_c$

Let, $V_1(t) = 0.5 + m(t) + 0.5\ \mathrm{m}^2(t)$

$V_1(f) = 0.5f(f) + \mathrm{m}(f) + 0.5\ ]\ m(f) * m(f)$







$V_2(t) = V_1(t) \cos 2\omega_c t$

$$V_2(f) = \frac{1}{2}[V_1(f-2f_c)+v_1(f+2f_c)]$$

Because of frequency shift by $2f_c$, the spectrum for $y_2(t) \neq 0$ in the range of $-2f_c - 2f_m$ to $-2f_c + 2f_m$ and $2f_c - 2f_m$ to $2f_c + 2f_m$.

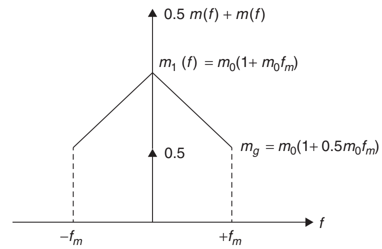
and $y_2(t) = 0$ for $-f_n < f < +f_m$ provided $f_c \geq 1.5\ c$

$$V_0(f) = V_1(f) + V_2(f)$$

$$V_0(f) = V_1(f) + V_2(f)$$

$$V_0(f) = V_1(f),\ \text{for} -f_m < y < +f_m.$$

So the spectrum of $V_0(t)$ for $f_m < f < f_m$ is the addition of three spectrum



8. A signal, $f(t) = e^{-at}\,u(t)$, where $u(t)$ is the unit step function, is applied to the input of a low-pass filter having $|H(\omega)| = \dfrac{b}{\sqrt{\omega^2+b^2}}$.

Calculate the value of the ratio, $\dfrac{a}{b}$ for which 50% of the input signal energy is transferred to the output. **[1994]**

Solution:

$$e^{-at}u(t) \xrightarrow{F.T} F(\omega) = \frac{1}{a+j\omega}$$

Input energy spectrum density, $(ESD)_x = |F(\omega)|^2$

$$= \frac{1}{a^2+\omega^2}$$

Input signal energy $E_x = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} |F(\omega)^2\, d\omega = \frac{1}{2a}$

Output spectrum density

$$|y(\omega)|^2 = |H(\omega)|^2 |F(\omega)|^2 = \frac{b^2}{(\omega^2+b^2)} \times \frac{1}{(a^2+\omega^2)}$$

Multiplication in frequency domain is equivalent to convolution in time domain.

So, $R_y(\tau) = \dfrac{b}{2}e^{-b|\tau|}\,\dfrac{1}{2a}e^{-a|\tau|}$

at $t = 0$,

$$R_y(0) = E_y = \frac{b}{4a}\left[\int_{-\infty}^{0} e^{b^t} e^{at}\, d\tau + \int_{0}^{\infty} e^{-b\tau}.e^{-a\tau}.d\tau\right]$$

$$= \frac{b}{2a(a+b)} = \frac{1}{2}Ex.$$

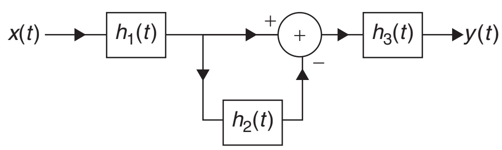
(or) $\dfrac{b}{2a(a+b)} = \dfrac{1}{4a}$

(or) $b = a$, so $a/b = 1$

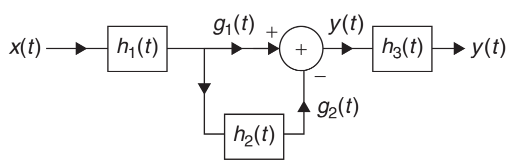
9. Consider the following interconnection of the three LTI systems (Figure), $h_1(t)$, $h_2(t)$ and $h_3(t)$ are the impulse responses of these three LTI systems with $H_1(\omega)$, $H_2(\omega)$ and $H_3(\omega)$ as their respective Fourier transforms. Given that

$$h_1(t) = \frac{d}{dt}\left[\frac{\text{Sin}(\omega_0 t)}{2\pi t}\right],\ h_2(\omega) = \exp\left[\frac{-j2\pi\omega}{\omega_0}\right]$$

$h_3(t) = u(t)$ and $x(t) = \sin 2\omega_0 t + \cos\left(\omega_0 \dfrac{t}{2}\right)$ Find the output $y(t)$. **[1993]**



Solution:

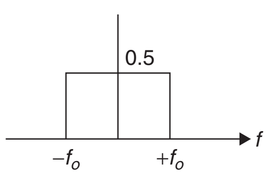


Given that,

$$h_1(t) = \frac{d}{dt}\left(\frac{\sin(\omega_0 t)}{2\pi t}\right) = \frac{d}{dt}\left[\frac{f_0 \sin 2\pi f_0 t}{2\pi f_0 t}\right]$$

$$= \frac{d}{dt}[f_0 \sin e(2\pi f_0 t)]$$

$$f_0 \sin c(2\pi f_0 t) \xleftrightarrow{F.T}$$



$$h_1(t) = \frac{d}{dt}[(f_0 \sin c(2\pi f_0 t)] \xrightarrow{F.T}$$

$$H_1(t) = j2\pi f \times 0.5 = j\pi f \qquad |f| \geq f_0$$
$$= 0 \qquad |f| > f_0$$

In the input, $x(t)$ there are two terms with frequencies $2f_0$ and $f_0/2$, the term with frequency $2f_0$ will not pass through the block with impulse response. $H_1(t)$ or transfer function $H(t)$ as its cut-off frequency is f_0.

$$\cos\left(\frac{\omega_0 t}{2}\right) \to \frac{1}{2}\left[f\left(f - \frac{f_0}{2}\right) + f\left(f + \frac{f_0}{2}\right)\right]$$

$$g_1(t) \xrightarrow{F.T} G_1(f) = H_1(f)\left[\frac{1}{2} f\left(f - \frac{f_0}{2}\right) + \frac{1}{2} f\left(f + \frac{f_0}{2}\right)\right]$$

$$G_1(t) = \frac{j\pi f_0}{4} f\left(f - \frac{f_0}{2}\right) - \frac{j\pi f_0}{4} f\left(f + \frac{f_0}{2}\right)$$

$$\Rightarrow\ = \frac{j\pi f_0}{4}\left[f\left(f - \frac{f_0}{2}\right) - f\left(f + \frac{f_0}{2}\right)\right]$$

$V_1(t) = 10 \cos 2000\ \pi t + 4 \sin 200\ \pi t$

$V_1(t)^2 = 100 \cos^2 2000\ \pi t + 16 \sin^2 200\ \pi t$
$\quad + 80 \cos (2000\ \pi t) . \sin(200\ \pi t)$

$$= 100\left(\frac{\cos 4000\pi t + 1}{2}\right) + 16\left(\frac{1 - \cos 400\pi t}{2}\right)$$
$$+ 80\left(\frac{\sin 2200\pi t - \sin 1800\pi t}{2}\right)$$

So $V_2(t)$ has frequencies (in Hz)
= 0, 200, 900, 1100 and 2000
$V_1(t)$ has = 100, 1000 Hz,
B.W. of B.P.F. is from 800 to 1200 Hz,
So only term 900 Hz, 1000 Hz and 1100 Hz will pass to the output $V_3(t)$

$V_3(t) = 10 \cos (2000\ \pi t) + 4 \sin (2200\ \pi t)$
$\quad - 4 \sin (1800\ \pi t)$
$V_3(t) = 10 \cos (2000\ \pi t) + 8 \cos (2000\ \pi t)$
$\quad \sin (200\ \pi t)$
$\quad = 2 \cos 2000\ \pi t\ [5 + 4 \sin (200\ \pi t)]$

$$G_2(f) = G_1(f) . H_2(f),\ H_1(t) = \exp\left[-j2\not{\ } \frac{f}{f_h}\right]$$

$$G_g(f) = \frac{j\pi f_0}{4}\left[f\left(f - \frac{f_0}{2}\right)e^{-j\pi} - f\left(f + \frac{f_0}{2}\right)e^{j\pi}\right]$$

$$= \frac{j\pi f_0}{4}\left[-f\left(f - \frac{f_0}{2}\right) + f\left(f + \frac{f_0}{2}\right)\right]$$

$$y_1(f) = G_1(t) - G_2(t) = \frac{j\pi f_0}{2}\left[f\left(f - \frac{f_0}{2}\right) - f\left(f + \frac{f_0}{2}\right)\right]$$

$$y_1(t) = \frac{e^{j\pi f_0}}{2}\left[e^{+2\pi j \frac{f_0}{2} t} - e^{-j2\pi \frac{f_0}{2} t}\right]$$

$$= -\frac{\omega_0}{2} \sin\left(\frac{\omega_0 t}{2}\right)$$

given from block 3

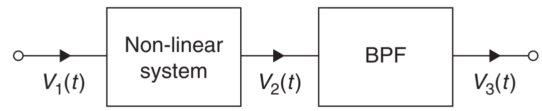
$$h_3(t) = u(t) = \int_{-\infty}^{t} f(t)dt$$

$$y(f) = \int y_1(t)dt = -\frac{\omega_0}{2}\int \sin\left(\frac{\omega_0}{2}f\right)dt$$

$$= \cos\left(\frac{\omega_0}{2}\right)f$$

10. Obtain an expression for the signal $v_3(t)$ in figure for $v_1(t) = 100\cos(2000\pi t) + 4\sin(200\pi t)$.

Assume that $v_2(t) = v_1(t) + 0.1v^2 1(t)$ and that the BPF is an idea! unity gain filter with pass band from 800 Hz to 1200 Hz. **[1993]**



Chapter 5

Laplace Transforms

One-mark Questions

1. Let $Y(s)$ be the unit-step response of a causal system having a transfer function $G(s) = \dfrac{3-s}{(s+1)(s+3)}$ that is $Y(s) = \dfrac{G(s)}{s}$. The forced response of the system is **[2019]**

(A) $u(t)$
(B) $2u(t)$
(C) $u(t) - 2e^{-t} + e^{-3t}u(t)$
(D) $2u(t) - 2e^{-t}u(t) + e^{-3t}u(t)$

Solution: $Y(S) = \dfrac{3-S}{S(S+1)(S+3)}$

Applying partial fraction method

$$Y(S) = \frac{A}{S} + \frac{B}{S+1} + \frac{C}{S+3}$$

$$A = 1; B = -2; C = 1.$$

$$Y(S) = \frac{1}{S} + \frac{-2}{S+1} = \frac{1}{S+3}$$

On taking inverse laplace transform

$$Y(t) = u(t) - 2e^{-t}u(t) + e^{-3t}u(t)$$

Forced response : $u(t)$

Hence, the correct option is (A).

2. Which one of the following is a property of the solutions to the Laplace equation: $\nabla^2 f = 0$? **[2016]**

(A) The solutions have neither maxima nor minima anywhere except at the boundaries.
(B) The solutions are not separable in the coordinates.
(C) The solutions are not continuous.
(D) The solutions are not dependent on the boundary conditions.

Solution: It is known that laplace equation has maxima and minima at boundaries only and at all remaining points there are no such conditions.

Hence, the correct option is (A).

3. Let $x(t) = \alpha\, s(t) + s(-t)$ with $s(t) = \beta e^{-4t}u(t)$, where $u(t)$ is unit step function. If the bilateral Laplace transform of $x(t)$ is **[2015]**

$$X(s) = \frac{16}{s^2 - 16} \quad -4 < \text{Re}\{s\} < 4;$$

then the value of β is ______.

Solution: $x(t) = \alpha s(t) + s(-t)$

$$s(t) = \beta e^{-4t}u(t)$$

$$x(t) = \alpha\beta e^{-4t}u(t) + \beta e^{+4t}u(-t)$$

$$X(s) = \frac{\alpha\beta}{s+4} + \frac{-\beta}{s-4}$$

$$= \frac{\alpha\beta}{s+4} - \frac{\beta}{s-4}$$

$$= \frac{\alpha\beta(s-4) - \beta(s+4)}{s^2-16}$$

$$X(s) = \frac{\alpha\beta s - \alpha\beta.4 - \beta s - \beta.4}{s^2 - 16} = \frac{16}{s^2-16}$$

It is given

So $\quad \alpha\beta s - \beta s = 0$

$\beta s(\alpha - 1) = 0$

∴ $\quad \alpha = 1$

and $\quad -\alpha\,\beta.4 - \beta.4 = 16$

$-\beta.4 - \beta.4 = 16$

$-8\beta = 16$

$\beta = -2$

Hence, the correct Answer is (–2).

4. The impulse response of a system is $h(t) = tu(t)$. For an input $u(t-1)$, the output is **[2013]**

(a) $\dfrac{t^2}{2}u(t)$ (b) $\dfrac{t(t-1)}{2}u(t-1)$

(c) $\dfrac{(t-1)^2}{2}u(t-1)$ (d) $\dfrac{t^2-1}{2}u(t-1)$

Solution: (c)
Given $h(t) = t\,u(t)$
Taking LT on both sides

$$X(S) = \frac{e^{-S}}{S} \qquad (1)$$

Input $x(t) = u(t-1)$
Taking LT on both sides

$$X(S) = \frac{e^{-S}}{S} \qquad (2)$$

$$\therefore H(S) = \frac{Y(S)}{X(S)}$$

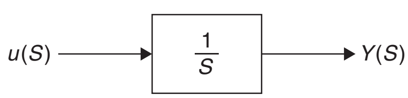
$$\therefore Y(S) = H(S)\cdot X(S)$$

Equation value of $H(S)$ & $X(S)$ from (1) and (2) taking inverse Laplace transform

$$y(t) = \frac{(t-1)^2}{2}u(t-1)$$

Hence, the correct option is (c).

5. Assuming zero initial condition, the response $y(t)$ of the system given below to a unit step input $u(t)$ is **[2013]**



(a) $u(t)$ (b) $t\,u(t)$

(c) $\frac{t^2}{2}u(t)$ (d) $e^{-t}u(t)$

Solution: (b)

$$H(S) = \frac{Y(S)}{u(S)} = \frac{1}{S}$$

$$x(t) = u(t) \quad \{\text{unit step input}\}$$

Taking Laplace transform on both sides

$$X(S) = \frac{1}{S}$$

$$Y(S) = H(S)\cdot X(S)$$
$$= \frac{1}{S}\times\frac{1}{S} = \frac{1}{S^2}$$

Taking inverse Laplace transform

$$y(t) = t\,u(t)$$

Hence, the correct option is (b).

6. The unilateral Laplace transform of $f(t)$ is $\frac{1}{S^2+S+1}$.
The unilateral Laplace transform of $tf(t)$ is **[2012]**

(a) $\frac{S}{(S^2+S+1)^2}$ (b) $\frac{-(2S+1)}{(S^2+S+1)^2}$

(c) $\frac{-S}{(S^2+S+1)^2}$ (d) $\frac{(2S+1)}{(S^2+S+1)^2}$

Solution: (d)

$$L[f(t)] = \frac{1}{S^2+S+1} = f(S)$$

$$\therefore L[t\,f(t)] = (-1)^1\frac{d}{dS}f(S)$$
$$= -1\frac{d}{dS}\left(\frac{1}{S^2+S+1}\right)$$
$$= -\left[\frac{-1}{(S^2+S+1)^2}\right](2S+1) = \frac{(2S+1)}{(S^2+S+1)^2}$$

Hence, the correct option is (d).

7. If the unit step response of a network is $(1-e^{\alpha t})$, then its unit impulse response is **[2011]**

(a) $\alpha e^{-\alpha t}$ (b) $\alpha^{-1}e^{-\alpha t}$

(c) $(1-\alpha^{-1})e^{-\alpha t}$ (d) $(1-\alpha)e^{-\alpha t}$

Solution: (a)
Given unit step response $S(t)$ $(1-e^{\alpha t})$
For a given response, unit impulse response is

$$h(t) = \frac{ds(t)}{dt}$$
$$= \frac{d(1-e^{\alpha t})}{dt} = \alpha e^{-\alpha t}$$

Hence, the correct option is (a).

8. If the Laplace transform of a signal $y(t)$ is
$Y(S) = \frac{1}{S(S-1)}$, then final value is____. **[2007]**

(a) -1 (b) 0

(c) 1 (d) unbounded

Solution: (d)
Here final value theorem will not be applicable as all of system are not lining left half of s-plane,
$\therefore$ $S = 1$ is right S-plane pole.
$\therefore$ unbounded.
Hence, the correct option is (d).

9. The Laplace transform of $i(t)$ is given by

$I(S) = \frac{2}{S(S+1)}$, As $t \to \infty$, the value of $i(t)$ tends to

[2003]

(a) 0 (b) 1
(c) 2 (d) ∞

Solution: (c)
Using final value theorem

$$\lim_{t\to\infty} i(t) = \lim_{S\to 0} S \cdot I(S)$$

$$= \lim_{S\to 0} S \cdot \frac{2}{S(S+1)} = 2$$

Hence, the correct option is (c).

10. The transfer function of a system is given by $H(s) = \dfrac{1}{S^2(S-2)}$. The Impulse response of the system is **[2001]**

(a) $(t^2 * e^{-2t})\, u(t)$ (b) $(t * e^{2t})\, u(t)$
(c) $(t - e^{-2t})\, u(t)$ (d) $(t - e^{-2t})\, u(t)$

Solution: (b)
Taking inverse Laplace transfer to calculate impulse response

$$h(t) = L^{-1}[H(S)] = L^{-1}\left[\frac{1}{S^2(S-2)}\right] = L^{-1}\left[\frac{1}{S^2} \times \frac{1}{S-2}\right]$$

$$= (t * e^{2t})u(t)$$

Hence, the correct option is (b).

11. Given that

$$L(f(t)) = \frac{S+2}{S^2+1}, \quad L\left[g(t)\right] = \frac{S^2+1}{(S+3)(S+2)}$$

$h(t) = \int_0^t f(t)g(t-T)dT$. The $L\,[h(t)]$ is **[2000]**

(a) $\dfrac{S^2+1}{S+3}$

(b) $\dfrac{1}{S+3}$

(c) $\dfrac{S^2+1}{(S+3)(S+2)} + \dfrac{(S+2)}{S^2+1}$

(d) None of these

Solution: (b)
h (*t*) shows convolution of *f* (*t*) and *g* (*t*) and convolution in time domain equivalent to multiplication in s-domain

$$\therefore L[h(t)] = L[f(t)] \times L[g(t)] = \frac{1}{S+3}$$

Hence, the correct option is (b).

12. If $L(f(t)) = F(s)$, then $L(f(t-T))$ is equal to **[1999]**

(a) $e^{ST} F(S)$ (b) $e^{-ST} F(S)$

(c) $\dfrac{F(s)}{1+e^{ST}}$ (d) $\dfrac{F(s)}{1-e^{-ST}}$

Solution: (b)

Let,
$$f(t) \xrightarrow{L\cdot T} F(s)$$
$$f(t-T) \xrightarrow{L\cdot T} F(s)e^{-ST}$$

Time shifting properly of Laplace transforms.
Hence, the correct option is (b).

13. If $L(f(t)) = \dfrac{\pi}{s^2+\omega^2}$, then the value of $\lim_{t\to\infty} f(t)$ **[1998]**

(a) cannot be determined
(b) is zero
(c) is unity
(d) is infinite.

Solution: (a)

$$F(s) = \frac{\omega}{S^2+\omega^2} \xleftrightarrow{I\cdot L\cdot T} f(t) = \sin\omega t$$

$\Rightarrow \lim_{t\to\infty} f(t) = \sin\omega t$ = lie between -1 and $+1$ cant determined.
Hence, the correct option is (a).

14. The Laplace transform of $e^{\alpha t} \cdot \cos(\alpha t)$ is equal to **[1997]**

(a) $\dfrac{(s-\alpha)}{(s-\alpha)^2+\alpha^2}$ (b) $\dfrac{(s+\alpha)}{(s+\alpha)^2+\alpha^2}$

(c) $\dfrac{1}{(s-\alpha)^2}$ (d) None of these

Solution: (a)
Laplace transform of $\cos\alpha t u(t)$ is given as

$$\cos(\alpha t)u(t) \xleftrightarrow{L\cdot T} \frac{S}{S^2+\alpha^2}$$

Multiplying by $e^{\alpha t}$ will result shifting in *frequency* domain

$$e^{\alpha t}\cos(\alpha t)u(t) \xleftrightarrow{L\cdot T} \frac{(s-\alpha)}{(s-\alpha)^2+\alpha^2}$$

Hence, the correct option is (a).

15. If $L(f(t)) = \dfrac{2(s+1)}{s^2+2s+5}$ then $f(0^t)$ and $f(\infty)$ are given by **[1995]**

(a) 0, 2 respectively
(b) 2, 0 respectively
(c) 0, 1 respectively
(d) 2/5, 0 respectively.

Solution: (b)

$$f(o^t) \neq \lim_{s\to\infty} \frac{S2(s+1)}{s^2+2s+1}$$
$$= 2$$

$$f(\infty) = \lim_{s\to 0} SF(s) = \lim_{s\to 0} \frac{S2(s+1)}{S^2+2s+5}$$

$t(\infty) = 0.$

Hence, the correct option is (b).

16. The transfer function of a linear system is the **[1995]**

(a) ratio of the output $v_0(t)$, and input $v_i(t)$

(b) ratio of the derivatives of the output and *i/p*

(c) ratio of the Laplace transform of the output and that of the *i/p* with odd initial conditions zero

(d) None of these

Solution: (c)

Transfer function given $H(s) = \dfrac{y(s)}{x(s)}$ = ratio of Laplace transform of output to the input.

Hence, the correct option is (c).

17. The final values theorem is used to find the **[1995]**

(a) Steady state value of the system output

(b) Initial value of the system output

(c) transient behaviour of the system output

(d) none of these

Solution: (a)

Final value theorem used to find the final value or steady state value of system o/p. $F(\infty) = \lim_{s\to\infty} SF(s)$

Hence, the correct option is (a).

18. The Laplace transform of unit ramp function starting at $t = a$ is **[1994]**

(a) $\dfrac{1}{(s+a)^2}$ (b) $\dfrac{e^{-as}}{(s+a)^2}$

(c) $\dfrac{e^{-as}}{s^2}$ (d) $\dfrac{4}{s^2}$

Solution: (c)

Let ramp function be $r(t)$

$$r(t) \xleftrightarrow{L\cdot T} \frac{1}{s^2}$$

$$r(t-a) \xleftrightarrow{L\cdot T} e^{-as} \times \frac{1}{s^2} = \frac{e^{-as}}{s^2}$$

Hence, the correct option is (c).

19. Indicate whether the following statement is true/False.

Give reason for your answer. If $G(s)$ is a stable transfer function, the $F(s)\dfrac{1}{G(s)}$ is always a stable transfer function. **[1994]**

Solution: FALSE

If $G(s)$ is stable, all poles must lie in left half of S-plane and there is no restriction on its zeros, which can lie also on right half of S plane. The Inverse function $F(s)\dfrac{1}{G(s)}$ may or may not be stable. The zeros of $G(s)$ lie in the right half of s plane, hence given statement is not True.

So $F(s)\dfrac{1}{G(s)}$ is not always a stable function.

Two-marks Questions

1. A sequence $x[n]$ is specified as

$$\begin{bmatrix} x[n] \\ x[n-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ for } n \geq 2$$

The initial conditions are $x[0] = 1$, $x[1] = 1$, and $x[n] = 0$ for $n < 0$. The value of $x[12]$ is ______. **[2016]**

Solution: Given $\begin{bmatrix} x[n] \\ x[n-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, for $n \geq 2$

where $x[0] = 1$, $x[1] = 1$.

For $n = 2$, we have

$$\begin{bmatrix} x[2] \\ x[1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^2 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} x[2] \\ x[1] \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

For $n = 3$, we have

$$\begin{bmatrix} x[3] \\ x[2] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^3 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} x[3] \\ x[2] \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Similarly, for $n = 4$, we have $\begin{bmatrix} x[4] \\ x[3] \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

$\therefore$ $x[2] = 2$; $x[3] = 3$ and $x[4] = 5$.

From these values, we can observe that

$$x[n] = x[n-1] + x[n-2];$$

$$\therefore \quad x[5] = x[4) + x[3] = 8$$

$$x[6] = x[5) + x[4] = 13$$

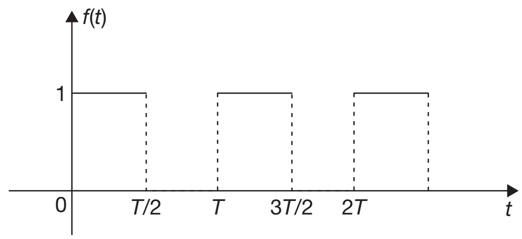
$$x[7] = x[6) + x[5] = 21$$

$$\vdots$$

$$x[12] = x[11] + x[10] = 233.$$

Hence, the correct Answer is (233).

2. The Laplace transform of the causal periodic square wave of period T shown in the figure below is **[2016]**



(A) $F(s) = \dfrac{1}{1+e^{-sT/2}}$

(B) $F(s) = \dfrac{1}{s\left(1+e^{\frac{-sT}{2}}\right)}$

(C) $F(s) = \dfrac{1}{s\left(1-e^{-sT}\right)}$

(D) $F(s) = \dfrac{1}{1-e^{-sT}}$

Solution: Given periodic signal

$$f(t) = \begin{cases} 1 & 0 < t \le T/2 \\ 0 & T/2 < t \le T \end{cases}$$

Laplace transform for periodic signal is

$$F(s) = \frac{1}{1-e^{-sT}}\int_0^T f(t)e^{-st}dt$$

$$= \frac{1}{1-e^{-ST}}\int_0^{T/2} 1\cdot e^{-st}dt$$

$$= \frac{1}{1-e^{-ST}}\left[\frac{e^{-st}}{-s}\right]_0^{T/2}$$

$$= \frac{1}{1-e^{-ST}}\cdot\frac{1}{S}\cdot\left(e^{-sT/2}-1\right)$$

$$= \frac{\left(1-e^{-sT/2}\right)}{s\left(1-e^{-ST}\right)} = \frac{\left(1-e^{-sT/2}\right)}{s\left(1-e^{-ST/2}\right)\left(1+e^{-ST/2}\right)}$$

$$= \frac{1}{s\left(1+e^{-ST/2}\right)}$$

Hence, the correct option is (B).

3. Consider the function $g(t) = e^{-t}\sin(2\pi t)\,u(t)$ where $u(t)$ is the unit step function. The area under $g(t)$ is ________. **[2015]**

Solution: $g(t) = e^{-t}\sin(2\pi t)\,u(t)$

By the property.

$$G(s) = \frac{2\pi}{(S+1)^2+(2\pi)^2}$$

$$G(s) = \int_{-\infty}^{+\infty} g(t)e^{-st}dt$$

So $$G(0) = \int_{-\infty}^{+\infty} g(t)dt = \text{area under } g(t)$$

$$= \frac{2\pi}{1+4\pi^2} = \frac{6.28}{40.4384} = 0.155$$

Hence, the correct Answer is (0.14 to 0.16).

4. Let the signal $f(t) = 0$ outside the interval $[T_1, T_2]$, where T_1 and T_2 are finite. Furthermore, $|f(t)| < \infty$. The region of convergence (RoC) of the signal's bilateral Laplace transform $F(s)$ is **[2015]**

(A) a parallel strip containing the j Ω axis.
(B) a parallel strip not containing the j Ω axis.
(C) the entire s-plane.
(D) a half plane containing the j Ω axis.

Solution: For a finite duration time signal, ROC of Laplace transform is entire S-plane.

Hence, the correct option is (C).

5. A system is described by the following differential equation, where $u(t)$ is the input to the system and $y(t)$ is the output of the system

$$y'(t)+5y(t) = u(t)$$

When $y(o) = 1$ and $u(t)$ is a unit step function, $y(t)$ is **[2014]**

(a) $0.2 + 0.8\,e^{-5t}$
(b) $0.2 - 0.2\,e^{-5t}$
(c) $0.8 - 0.2\,e^{-5t}$
(d) $0.8 - 0.8\,e^{-5t}$

Solution:(a)
Given $y'(t)+5y(t) = u(t)$
taking Laplace transform

$$SY(s) - y(o) + 5Y(s) = \frac{1}{S} \qquad (y(0)=1)$$

$$(S+5)Y(S) - 1 = \frac{1}{S}$$

$$\Rightarrow (S+5)Y(S) - 1 = \frac{1}{S}+1$$

$$Y(S) = \frac{(1+S)}{S}\times\frac{1}{(S+5)} = \frac{(S+1)}{S(S+5)}$$

Taking partial fraction

$$\frac{A}{S}+\frac{B}{(S+5)} = \frac{(S+1)}{S(S+5)}$$

Solving $A(S+5)+BS = S+1$
and taking $S = 0$
$5A = 1 \therefore A = 0.2$
Taking $S = -5$

$-5B = -4 \therefore B = \frac{-4}{-5} = 0.8$

Now $Y(S) = \frac{0.2}{5} + \frac{0.8}{S+5}$

$\therefore$ Taking inverse LT,
we get answer $y(t) = 0.2 + 0.8\ e^{-5t}$
Hence, the correct option is (a).

6. A system is described by the differential equation $\frac{d^2y}{dt^2} + \frac{5dy}{dt} + 6y(t) = x(t)$. Let $x(t)$ be a rectangular pulse given by

$$x(t) = \begin{cases} 1 & 0 < t < 2 \\ 0 & otherwise \end{cases}$$

Assuming that $y(o) = 0$ and $\frac{dy}{dt} = 0$ at $t = 0$,

Laplace transform of $y(t)$ is **[2013]**

(a) $\frac{e^{-2S}}{S(S+2)(S+3)}$

(b) $\frac{1-e^{-2S}}{S(S+2)(S+3)}$

(c) $\frac{e^{-2S}}{(S+2)(S+3)}$

(d) $\frac{1-e^{-2S}}{(S+2)(S+3)}$

Solution: (b)

$$x(t) = \begin{cases} 1 & 0 < t < 2 \\ 0 & otherwise \end{cases}$$

$$\therefore x(t) = u(t) - u(t-2)$$

Taking Laplace transform

$$X(S) = \frac{1}{S} - \frac{e^{-2S}}{S} = \frac{1-e^{-2S}}{S}$$

given

$$\frac{d^2y}{dt^2} + \frac{5dy}{dt} + 6y(t) = x(t)$$

Apply Laplace transform $(y(o) = 0;\ \frac{dy}{dt} = 0$

$at \quad t = 0)$

$$\Rightarrow S^2Y(S) + 5SY(S) + 6(S) = X(S)$$

$$\Rightarrow Y(S)\left[S^2 + 5S + 6\right] = \frac{1-e^{-2S}}{S}$$

$$\Rightarrow Y(S) = \frac{1-e^{-2S}}{S(S+2)(S+3)}$$

Hence, the correct option is (b).

7. An input $x(t) = \exp(-2t)\ u(t) + \delta(t-6)$ is applied to an LTI system with impulse response $h(t) = u(t)$. The output is **[2011]**

(a) $[1 - \exp(-2t)]\ u(t) + u(t+6)$
(b) $[1 - \exp(-2t)]\ u(t) + u(t-6)$
(c) $0.5\ [1 - \exp(-2t)]\ u(t) + u(t+6)$
(d) $0.5\ [1 - \exp(-2t)]\ u(t) + u(t-6)$

Solution: (d)

$$x(t) = e^{-2t}u(t) + S(t-6)$$

Taking Laplace transform

$$X(S) = \frac{1}{S+2} + e^{-6S}$$

$$h(t) = u(t)$$

$$\therefore H(S) = \frac{1}{S}$$

$\therefore$ output

$$Y(S) = X(3)\cdot H(S)$$

$$Y(S) = \left\{\frac{1}{S+2} + e^{-6S}\right\}\frac{1}{S}$$

$$Y(S) = \frac{1}{S(S+2)} + \frac{1}{S}e^{-6S}$$

$$Y(S) = \frac{1}{2}\left(\frac{1}{S} - \frac{1}{S+2}\right) + \frac{1}{S}e^{-6S}$$

Taking Inverse Laplace Transform

$$y(t) = 0.5(1-e^{-2t})u(t) + 4(t-6)$$

Hence, the correct option is (d).

8. If $F(S) = L[f(t)] = \frac{2(S+1)}{S^2+4S+7}$ then the initial and final values of $f(t)$ are respectively **[2011]**

(a) 0, 2 (b) 2, 0
(c) 0, 2/7 (d) 2/7, 0

Solution: (b)

$$F(S) = L[f(t)] = \frac{2(S+1)}{S^2+4S+7}$$

Initial value of $f(t)$

$$\underset{t\to 0}{Lt}\ f(t) = \underset{t\to 0}{Lt}\ S\ F(S) = \frac{S\cdot 2(S+1)}{S^2+4S+7}$$

Taking S common

$$\underset{t\to 0}{Lt}\ f(t) = \underset{S\to\infty}{Lt}\ \frac{S^2\cdot 2\left(1+\frac{1}{S}\right)}{S^2\left(1+\frac{4}{S}+\frac{7}{S^2}\right)}$$

$$= \frac{2(1+0)}{(1+0+0)} = 2$$

For final value

$$\underset{t\to 0}{Lt}\, f(t) = \underset{t\to 0}{Lt}\, S \cdot F(S)$$

$$= \frac{S \cdot 2(S+1)}{S^2 4S + 7} = 0$$

Hence, the correct option is (b).

9. A continuous time LTI system is described by

$$\frac{d^2y(t)}{dt^2} + \frac{4dy(t)}{dt} + 3y(t) = \frac{2dx(t)}{dt} + 4x(t).$$

Assuming zero initial condition, the response $y(t)$ of the above system for the input $x(t) = e^{-2t}u(t)$ is given by **[2010]**

(a) $(e^t - e^{3t})u(t)$

(b) $(e^{-t} - e^{-3t})u(t)$

(c) $(e^{-t} + e^{-3t})u(t)$

(d) $(e^t + e^{3t})u(t)$

Solution: (b)

Given equation

$$\frac{d^2y(t)}{dt^2} + \frac{4dy(t)}{dt} + 3y(t) = \frac{2dx(t)}{dt} + 4x(t)$$

Considering zero initial condition, Laplace transform on both side

$$S^2Y(S) + 4SY(S) + 3Y(S) = 2SX(S) + 4X(3)$$

$$\Rightarrow (S^2 + 4S + 3)Y(S) = (2S+4)X(S)$$

$$\frac{Y(S)}{X(s)} = \frac{2S+4}{S^2+4S+3} = \frac{2(S+2)}{(S+1)(S+3)}$$

Now

$$x(t) = e^{-2t}u(t)$$

$$X(S) = \frac{1}{S+2}$$

Sub-value of $X(S)$ in the equation

$$Y(S) = \frac{2(S+2)}{(S+1)(S+3)(S+2)} = \frac{2}{(S+1)(s+3)}$$

$$= \frac{1}{S+1} - \frac{1}{S+3}$$

Taking inverse Laplace transform

$$y(t) = \left(e^{-1} - e^{-3t}\right)u(t)$$

Hence, the correct option is (b).

10. Given that $F(S)$ is one-sided Laplace transform of $f(t)$, the Laplace transform of

$\int_0^t f(\tau)d\tau$ is **[2009]**

(a) $S\,F(S) - f(o)$ (b) $\frac{1}{S}F(S)$

(c) $\int_0^S f(\tau)d'$ (d) $\frac{1}{S}[F(S) - f(o)]$

Solution: (b)

Given $f(t) \xleftrightarrow{L\cdot T} F(s)$,

then $\int_0^t f(\tau)d\tau = \frac{F(S)}{S} + \frac{f^{-1}(o^+)}{S} = \frac{F(S)}{S}.$

$f^{-1}(o^+)$ shows initial conditions, which are zero.

Hence, the correct option is (b).

11. Consider the function of $f(t)$ having Laplace transform

$$F(s) = \frac{\omega_0}{S^2 + \omega_0^2} \qquad \text{Re}\{S\} > 0$$

The final value of $f(t)$ would be______. **[2006]**

(a) 0 (b) 1

(c) $-1 \le f(\infty) \le 1$ (d) ∞

Solution: (c)

Given $F(s) = \dfrac{\omega_0}{S^2 + \omega_0^2}$

$$f(t) = L^{-1}(F|S) = L^{-1}\left\{\frac{\omega_0}{S^2 + \omega_0^2}\right\} = \sin\omega_0 t$$

So $-1 \le f(t) \le 1$

Hence, the correct option is (c).

12. The Laplace transform of a continuous time signal $x(t)$ is $X(S) = \dfrac{5-S}{S^2-S-2}$. If Fourier transform of signal exists, then $x(t)$ is **[2002]**

(a) $e^{2t}u(t) - 2e^{-t}u(t)$

(b) $-e^{2t}u(-t) + 2e^{-t}u(t)$

(c) $-e^{2t}u(t) - 2e^{-t}u(t)$

(d) $e^{2t}u(-t) - 2e^{-t}u(t)$

Solution: (d)

Given

$$X(S) = \frac{5-S}{S^2-S-2} = \frac{5-S}{(S+1)(S-2)} = \frac{-2}{(S+1)} + \frac{1}{(S-2)}$$

taking inverse Laplace transform.

$$x(t) = -2e^{-t}u(t) + e^{2t}u(-t)$$

ROC will include $\sigma = 0$ line

Hence, the correct option is (d).

13. A linear time invariant system has an impulse response e^{2t}, for $t > 0$, if initial condition is 0 and input is e^{3t}, the output for $t > 0$ is **[2000]**

(a) $e^{3t} - e^{2t}$

(b) e^{5t}

(c) $e^{3t} + e^{2t}$
(d) None of the above

Solution: (a)
Given

$$h(t) = e^{2t} \xleftrightarrow{L\cdot T} \frac{1}{S-2} = H(s)$$

$$X(t) = e^{3t} \xleftrightarrow{L\cdot T} \frac{1}{S-3} = X(s) \quad o/p$$

$$\Rightarrow Y(S) = X(S)\cdot H(S) = \frac{1}{(S-2)(S-3)} = \frac{-1}{S-2} + \frac{1}{S-3}$$

Inverse Laplace transform
$o/p\, Y(t) = e^{3t} - e^{2t}$
Hence, the correct option is (a).

14. Match each of items 1, 2 on the left with most appropriate item A, B, C, D on right.
In case of linear time-invariant system **[1997]**
(1) poles in the right half plane implies
(2) Impulse response zero for $t \le 0$ implies
(a) Exponential delay o/p.
(b) System is causal.
(c) No stored energy in system.
(d) System is unstable.

Solution: (1 – D, 2 – B)
(i) Poles at right half of s-plane $\Rightarrow$ system unstable
(ii) $h(t) = 0$, for $t \le 0 \Rightarrow$ causal system.

15. The inverse Laplace transform of the function $\dfrac{(s+5)}{(s+1)(s+3)}$ is **[1996]**

(a) $2e^{-t} - e^{-3t}$ (b) $2e^{-t} + e^{-3t}$
(c) $e^{-t} - 2e^{-3t}$ (d) $e^{-t} + 2e^{-3t}$

Solution: (a)
Given

$$F(s) = \frac{s+5}{(s+1)(s+3)} = \frac{2}{(s+1)} - \frac{1}{(s+3)}$$

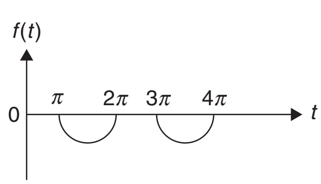
Taking inverse Laplace transform,

$$f(t) = 2e^{-t} - e^{-3t}$$

Hence, the correct option is (a).

16. The Laplace transform of the periodic function $f(t)$ described by curve below i.e. **[1993]**

$$f(t)\begin{cases} \sin t & \text{if } (2n-1)\pi \le t \le 2n\pi \quad (n = 1,2,3\ldots) \\ 0 & \text{otherwise} \end{cases}$$



Solution: $F(s) = \dfrac{-e^{-\pi s}}{(s^2+1)(1-e^{-\pi s})}$

Curve $f(t)$ can be generated from sine curve or cosine curve.
Let
$f_1(t) = -[\sin(t-)u(t-\pi) + \sin(t-2\pi)\cdot u(t-2\pi)]$
The given signal $f(t)$ can be represented as

$$f_1(t)\begin{cases} f(t) & 0 < t \le 2\pi \\ 0 & \text{otherwise} \end{cases}$$

Also $\sin t\, u(t) \longrightarrow \dfrac{1}{s^2+1}$, and using shifting property

$$x(t-t_0) \longrightarrow e^{-t_0 s}\cdot x(s)$$

Then

$$F_1(s) = -\left[\frac{e^{-\pi s}}{s^2+1} + \frac{e^{-2\pi s}}{s^2+1}\right] = -e^{-\pi s}\left[\frac{1+e^{-\pi s}}{s^2+1}\right]$$

$$\Rightarrow F(s) = \frac{F_1(s)}{1-e^{T_0 s}} = \frac{F_1(s)}{1-e^{-2\pi s}} = e^{-\pi s}\left(\frac{1+e^{-\pi s}}{s^2+1}\right)\frac{1}{(1-e^{-2\pi} s)}$$

$$F(s) = \frac{-e^{-\pi s}}{(s^2+1)(1-e^{-\pi s})}$$

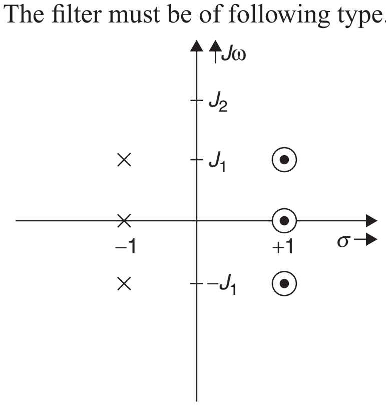
17. If $F(s) = L(f(t)) = \dfrac{k}{(s+1)(s^2+4)}$, then $\lim f(t)$ is given by **[1993]**
(a) $k/4$
(b) zero
(c) ∞
(d) undefined

Solution: (d)

Given $F(s) = \dfrac{k}{(s+1)(s^2+4)}$, poles $= -1, \pm 2J$

Since poles are not at left half of s-plane so the time value theorem cannot be applied.
So $\lim_{t\to\infty} f(t) \to$ indeterminate
Hence, the correct option is (d).

18. The pole-zero pattern of a certain filter is shown in the figure. The filter must be of following type. **[1991]**



(a) low-pass
(b) high-pass
(c) all pass
(d) band-pass

Solution: (c)

The pole zero pattern shown is symmetrical about imaginary axis. Such property is shown by all pass filter.
Hence, the correct option is (c).

19. The voltage across an impedance in a network is $v(s) = z(s)\, I\,(s)$, where $v(s)$, $z(s)$ and $I\,(s)$ are the Laplace transform of the corresponding time functions $v(t)$, $z(t)$ and $i(t)$. The voltage $v(t)$ is **[1991]**

(a) $v(t) = z(t) - i(t)$

(b) $v(t) = \int_0^t i(t)\cdot z(t-T)dT$

(c) $v(t) = \int_0^t i(t)\cdot z(t+T)dT$

(d) $v(t) = z(t) + i(t)$

Solution: (c)

Multiplication of two functions in frequency domain is equivalent to convolution in time domain

So $v(t) = \int_0^t i(\tau)\cdot z(t-\tau)d\tau$

Hence, the correct option is (c).

20. The response of an initially relaxed linear constant parameter to a unit impulse applied at $t = 0$ is $4e^{-2t}u(t)$. The response of this network to a unit step function will be. **[1990]**

(a) $2[1-e^{-2t}]\, u(t)$
(b) $4[e^{-t}-e^{-2t}]\, u(t)$
(c) $\sin 2t$
(d) $[1-4e^{-4t}]\, u(t)$

Solution: (a)

Given $h(t) = 4e^{-2t} \xrightarrow{1\cdot T} H(s)\dfrac{4}{(s+2)}$

Given that $x(d) \longrightarrow u(t) \xrightarrow{1\cdot T} x(s) = \dfrac{1}{s}$

$$\Rightarrow H(s) = \frac{Y(s)}{X(s)} \Rightarrow Y(s) = X(s)\cdot H(s) = \frac{1}{s}\cdot\frac{4}{(s+2)}$$

$$Y(s) = 2\left[\frac{1}{s} - \frac{1}{(s+2)}\right]$$

Taking Inverse Laplace transform on both side,

$$y(t) = 2[1-e^{-2t}]u(t).$$

Hence, the correct option is (a).

21. Specify the filter type if its voltage transfer function $H(s)$ is given by

$$H(s) = \frac{K(s^2+\omega_0^2)}{S^2+(\omega_0/Q)s+\omega_0^2}$$ **[1988]**

(a) all pass filter (b) low pass filter
(c) band pass filter (d) notch filter

Solution: (d)

Given that $H(s) = \dfrac{K(s^2+\omega_0^2)}{S^2+\left(\dfrac{\omega_0}{Q}\right)s+\omega_0^2}$

put $H(\infty) = K$, $H(\infty) \neq 0$

$$H(\infty) = \frac{K(1+0)}{1+0+0} = K, \qquad H(\infty) \neq 0$$

So, the given transfer function is a notch filter.
Hence, the correct option is (d).

22. The Laplace transform of function $f(t)\, u(t)$, where $f(t)$ is periodic with period T, is $A(s)$ times the Laplace transform of its first period, then **[1988]**

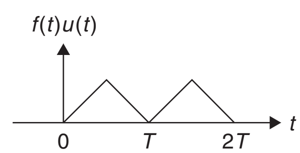
(a) $A(s) = S$ (b) $A(s) = \dfrac{1}{1-e^{-Ts}}$

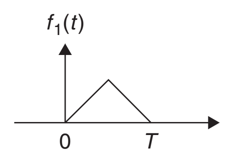
(c) $A(s) = \dfrac{1}{1+e^{+Ts}}$ (d) $A(s) = e^{Ts}$

Solution: (b)

Given function represents causal period signal
$f(t)\, u(t) = 0$: $t > 0$
period of $f(t) = T$ for $t > 0$

Let $f_1(t) = f(t)u(t)$
$= 0$
$0 \le t \le T$ otherwise,





$$f(t)u(t) = \sum_{n=0}^{\infty} f_1(t-nT)$$

Let $e^{-\pi f^2}$

$$f_1(t-nT) \xrightarrow{1\cdot T} e^{-nTs}F_1(s)$$

$$f(t)u(t) \xrightarrow{1\cdot T} F(s) = \sum_{n=0}^{\infty} e^{-nTs}F_1(s) = \frac{F_1(s)}{1-e^{-Ts}}$$

$f(t)u(t) \longrightarrow \dfrac{1}{1-e^{-sT}} \times$ transform of first period of $f(t)$

$u(t)$

$$A(S) = \frac{1}{1-e^{Ts}}$$

Hence, the correct option is (b).

23. Laplace transform of the function $tu(t)$ and $\sin tu(t)$ is, respectively: **[1987]**

(a) $\frac{1}{S^2}, \frac{S}{S^2+1}$ (b) $\frac{1}{S}, \frac{1}{S^2+1}$

(c) $\frac{1}{S^2}, \frac{1}{S^2+1}$ (d) $S, \frac{S}{S^2+1}$

Solution: (c)

Laplace transform of function $f(t)$ is given as

$$F|S| = \int_{-\infty}^{\infty} f(t)e^{-st}dt$$

$$y(t) = tu(t)$$

$$F(S) = \int_{-\infty}^{\infty} tu(t)\cdot e^{-st}dt = \int_{0}^{\infty} t\cdot e^{-st}dt$$

$$\Rightarrow F(S) = \left[\frac{t\cdot e^{-st}}{(-S)}\right]_0^{\infty} - \left[\int_0^{\infty} 1\cdot\frac{e^{-st}}{(-S)}dt\right] = [0-0] - \left[\frac{e^{-st}}{S^2}\right]_0^{\infty}$$

$$\Rightarrow F(S) = \frac{-1}{S^2}[0-1] = \frac{1}{S^2}$$

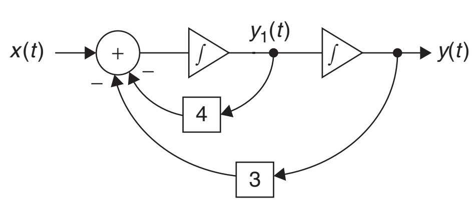
When $f(t) = \sin at\cdot u(t)$

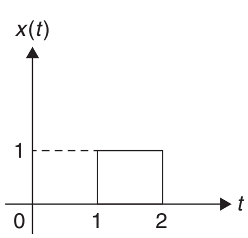
similarly for $\sin t\, u(t)$ the transform is $\frac{1}{S^2+1}$

Hence, the correct option is (c).

FIVE-MARKS QUESTIONS

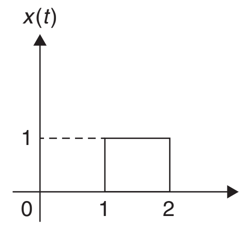
1. For the linear, time-invariant system whose block diagram is shown in figure (a), with input $x(t)$ and output $y(t)$
   (a) Find the transfer function.
   (b) Find the step response of the system [i.e. find $y(t)$ when $x(t)$ is a unity step function and the initial conditions are zero].
   (c) Find $y(t)$, if $x(t)$ is as shown in figure (b), and the initial conditions are zero. **[2000]**





Solution:

(a)



$$y_1(s) = \frac{x(s) - 4y_1(s) - 3y(s)}{s}$$

$$y(s) = \frac{y_1(s)}{2}, \Rightarrow y_1(s) = sy(s)$$

$$\Rightarrow \frac{x(s) - 4sy(s) - 3y(s)}{s}$$

$$= \left[\frac{1}{3} - \frac{1}{2}e.e^{-t} + \frac{1}{6}e^3e^{-3t}\right]u(t-1)$$

$$-\left[\frac{1}{3} - \frac{1}{2}e^2e^{-t} + \frac{1}{6}e^6e^{-3t}\right]u(t-2)$$

Transform function,

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2+4s+3} = \frac{1}{(s+1)(s+3)}$$

(b) $x(t) = u(t)$,

$$x(s) = \frac{1}{s},$$

$$H(s) = \frac{Y(s)}{X(s)} = \xrightarrow{\alpha} \Rightarrow y(s) = h(s)\,.\,x(s)$$

$$\Rightarrow Y(s) = \frac{1}{s(s+1)(s+3)} = \frac{1}{3s} - \frac{1}{2(s+2)} + \frac{1}{6(s+3)}$$

$$y(t) = \left[\frac{1}{3} - \frac{1}{2}d^{-t} + \frac{1}{6}e^{-35}\right]v(t)$$

(c) $x(t) = u(t-1) - u(t-2)$

For LTI systems,

$u(t) \to y(t)$

$u(t-1) \to x(t-1)$

$u(t-2) \to y(t-2)$

So, $x(t) = u(t-1) - u(t-2) \to y(t-1) - y(t-2)$

$$= \left[\frac{1}{3} - \frac{1}{2}e^{-(t-1)} + \frac{1}{6}e^{-3(t-1)}\right]u(t-1)$$

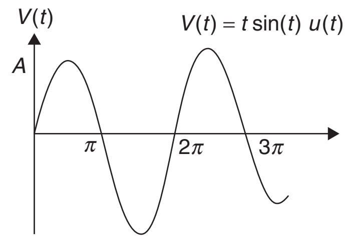
2. A sinusoidal signal, $v(t) = A\sin(t)$, is applied to an ideal full-wave rectifier. Show that the Laplace transform of the output can be written in the form,

$$V_0(s) = \frac{A}{s^2+1}\cot h(\alpha s)$$

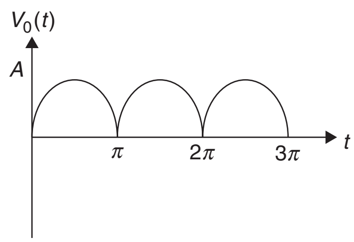
where α is a constant. Determine the value of α.

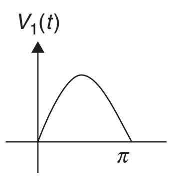
[1995]

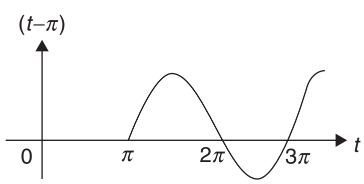
Solution:



$$V(t) \xrightarrow{L.T.} V(s) = \frac{A}{s^2+1}$$







$$V_1(t) = V(t) + V(t-\pi)$$

$$V(t-\pi) \xrightarrow{L.T} \frac{A}{s^2+1}e^{-\pi s}$$

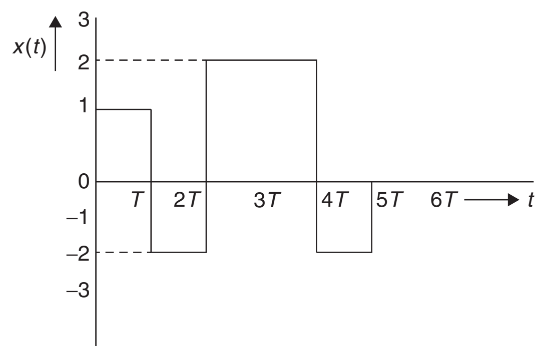
$$V_1(t) \xrightarrow{L.T.} V_1(s) = V(s) + \frac{A}{s^2+1}e^{-\pi s}$$

$$V_1(s) = \frac{A}{(s^2+1)} + \frac{A}{(s^2+1)}e^{-\frac{\pi}{s}}$$

$$= \frac{A}{(s^2+1)} \times \frac{e^{\frac{-\pi s}{2}}\left[e^{\frac{\pi s}{2}} + e^{\frac{-\pi s}{2}}\right]}{e^{\frac{-\pi s}{2}}\left[e^{\frac{\pi s}{2}} - e^{\frac{-\pi s}{2}}\right]}$$

$$V_0(s) = \frac{A}{s^2+1}\cot h\left(\frac{\pi s}{2}\right), \text{ So, } \alpha = \frac{\pi}{2}$$

3. Find the Laplace transform of the waveform $x(t)$ shown in figure. **[1991]**



Solution:

$x(t)$ is expressed as,

$$x(t) = u(t) - 3u(t-T) + 4u(t-2T) - 4u(t-4T) + 2$$

$$\text{So, } x(s) = \frac{1}{s} - \frac{3e^{-Ts}}{s} + \frac{4e^{-2Ts}}{s} - \frac{e^{4e^{-4/s}}}{s} + \frac{2e^{-sTs}}{s}$$

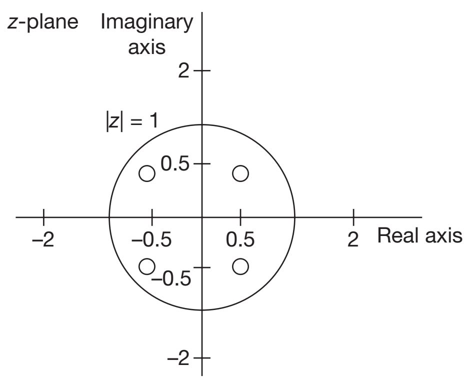
$$= \frac{1}{s}\left[1 - 3e^{-Ts} + 4e^{-2Ts} - 4e^{-4Ts} + 2e^{-sTs}\right]$$

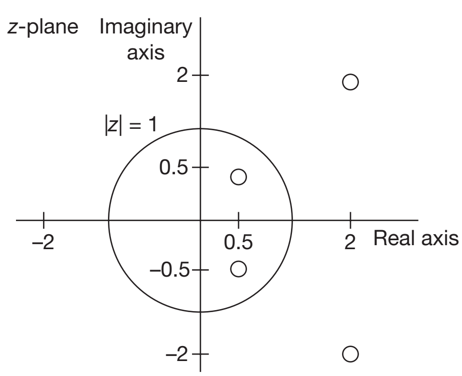
Chapter 6

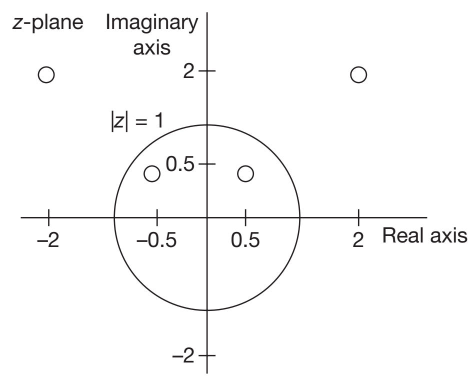
Z-Transform

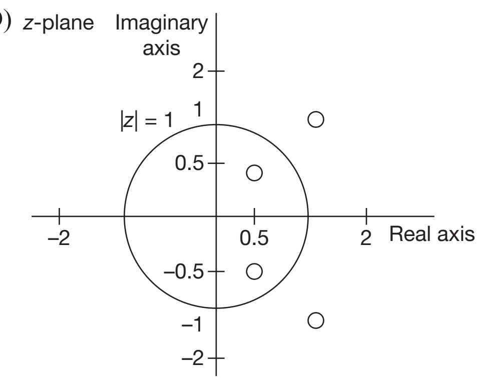
One-mark Questions

1. Let $H(z)$ be the z-transform of a real-valued discrete-time signal $h(n)$. If $p(z) = H(z)\,H\left(\frac{1}{z}\right)$ has a zero at $z = \frac{1}{2} + \frac{1}{2}j$ and $p(z)$ has a total of four zeros, which one of the following plots represents all the zeros correctly? **[2019]**

(A) 

(B) 

(C) 

(D) 

Solution: $P(z) = H(z)\cdot H\left(\frac{1}{z}\right)$

Zero at $z = \frac{1}{z} + \frac{1}{2}j$

if z is zero, $\bar{z}$ will also be zero

$$\bar{z} = \overline{\left(\frac{1}{2} + \frac{1}{2}j\right)} = \frac{1}{2} - \frac{1}{2}j$$

also $\frac{1}{z}$ & $\frac{1}{\bar{z}}$ will also be zero

$$\frac{1}{z}=\frac{1}{\frac{1}{2}+\frac{1}{2}j}\times\frac{\frac{1}{2}-\frac{1}{2}j}{\frac{1}{2}-\frac{1}{2}j}=1-j$$

$$\frac{1}{\bar{z}}=1+j$$

Hence, the correct option is (D).

2. The residues of a function $f(z)=\frac{1}{(z-4)(z+1)^3}$ are **[2017]**

(A) $\frac{-1}{27}$ and $\frac{-1}{125}$ (B) $\frac{1}{125}$ and $\frac{-1}{125}$

(C) $\frac{-1}{27}$ and $\frac{1}{5}$ (D) $\frac{1}{125}$ and $\frac{-1}{5}$

Solution: Given $f(z)=\frac{1}{(z-4)(z+1)^3}$

$z=4$ and $z=-1$ are the singularities of $f(z)$

$z=4$ is a simple pole of $f(z)$

$$\therefore \operatorname{Res}_{z=4}[f(z)]=\underset{z\to 4}{Lt}[(z-4)f(z)]$$

$$=\underset{z\to 4}{Lt}\left[(z-4).\frac{1}{(z-4)(z+1)^3}\right]$$

$$=\underset{z\to 4}{Lt}\left[\frac{1}{(z+1)^3}\right]$$

$$=\frac{1}{(4+1)^3}$$

$$\therefore \operatorname{Res}_{z=4}[f(z)]=\frac{1}{125}$$

$z=-1$ is a pole of order 3 for $f(z)$

$$\therefore \operatorname{Res}_{z=-1}[f(z)]=\frac{1}{(3-1)!}\left(\underset{z\Rightarrow -1}{Lt}\left[\frac{d^{3-1}}{dz^{3-1}}((z+1)^3 f(z))\right]\right)$$

$$=\frac{1}{2}\left(\underset{z\to -1}{Lt}\left[\frac{d^2}{dz^2}\left(\frac{1}{z-4}\right)\right]\right)$$

$$=\frac{1}{2}\left(\underset{z\to -1}{Lt}\left[\frac{2}{(z-4)^3}\right]\right)$$

$$=\frac{1}{2}\left(\frac{2}{(-1-4)^3}\right)$$

$$\therefore \operatorname{Res}_{z=-1}[f(z)]=\frac{-1}{125}$$

Hence, the correct option is (B).

3. A continuous time function $x(t)$ is periodic with period T. The function is sampled uniformly with a sampling period T_s. In which one of the following cases is the sampled signal periodic? **[2016]**

(A) $T=\sqrt{2}\,T_s$ (B) $T=1.2\,T_s$

(C) Always (D) Never

Solution: As we know that in Nyquist theorem,

$$Fs \geq 2Fm$$

or $$T_m \geq 2Ts$$

or $$T \geq kTs$$

Hence, the correct option is (B).

4. For $f(z)=\frac{s(z)}{z^2}$ the residue of the pole at $z=0$ is ____. **[2016]**

Solution: Given $f(Z)=\frac{\sin Z}{Z^2}$

$Z=0$ is a pole of order 2 for $f(Z)$;

the residue of $f(Z)$ at $Z=a$ is given by,

$$\frac{1}{(n-1)!}\underset{Z\to a}{Lt}\left[\frac{d^{n-1}}{dZ^{n-1}}(Z-a)^n f(Z)\right]$$

Here, $Z=a=0$ and $n=2$;

$\therefore$ Res $[f(Z)]=Z=0$

$$=\frac{1}{(2-1)!}\underset{Z\to 0}{Lt}\left[\frac{d}{dZ}\left[(Z-0)^2\frac{\sin Z}{Z^2}\right]\right]$$

$$=\underset{Z\to 0}{Lt}\left[\frac{d}{dZ}(\sin Z)\right]$$

$$=\underset{Z\to 0}{Lt}\ \cos Z$$

$$=1$$

Hence, the correct Answer is (1).

5. A discrete-time signal $x[n]=\delta[n-3]+2\delta[n-5]$ has z–transform $X(z)$. If $Y(z)=X(-z)$ is the z–transform of another signal $y[n]$, then **[2016]**

(A) $y[n]=x[n]$ (B) $y[n]=x[-n]$

(C) $y[n]=-x[n]$ (D) $y[n]=-x[-n]$

Solution: We know the z-transform $a^n\cdot x(n)\leftrightarrow X(Z/a)$

$$(-1)^n\cdot x(n)\leftrightarrow X(-Z)$$

given $x(n)=\delta[n-3]+2\delta[n-5]$

and

$$Y[Z] = X[-Z]$$

$$\therefore \quad y[n] = (-1)^n \cdot x[n]$$

$$y[n] = (-1)^n [\delta[n-3] + 2\delta[n-5]]$$

$$\therefore \quad y[n] = -\delta[n-3] - 2\delta[n-5]$$

or $\quad y[n] = -(\delta[n-3] + 2\delta[n-5]$

$$= -x[n]$$

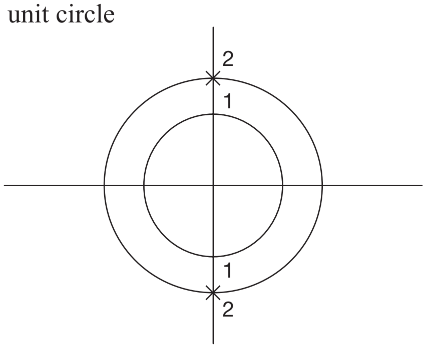
$$= -x[n].$$

Hence, the correct option is (C).

6. Suppose $x[n]$ is an absolutely summable discrete-time signal. Its z-transform is a rational function with two poles and two zeros. The poles are at $z = \pm 2j$. Which one of the following statements is TRUE for the signal $x[n]$? **[2015]**

(A) It is a finite duration signal.
(B) It is a causal signal
(C) It is a non-causal signal
(D) It is a periodic signal.

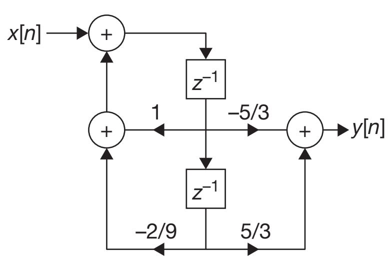
Solution: $x[n]$ is absolute summable so, ROC must include unit circle



and poles are at 2. So ROC is inside the circle so $x[n]$ is non-causal signal.

Hence, the correct option is (C).

7. A realization of a stable discrete time system is shown in the figure. If the system is excited by a unit step sequence input $x[n]$, the response $y[n]$ is **[2015]**



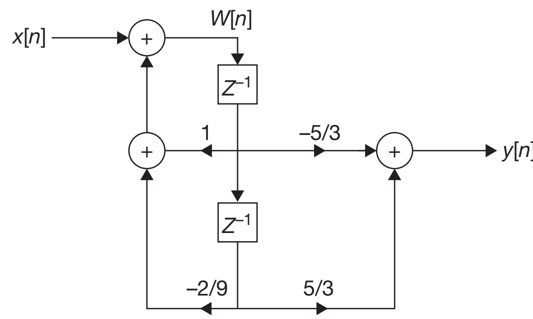
(A) $4\left[-\frac{1}{3}\right]^n u[n] - 5\left[-\frac{2}{3}\right]^n u[n]$

(B) $5\left[-\frac{2}{3}\right]^n u[n] - 3\left[-\frac{1}{3}\right]^n u[n]$

(C) $5\left[\frac{1}{3}\right]^n u[n] - 5\left[\frac{2}{3}\right]^n u[n]$

(D) $5\left[\frac{2}{3}\right]^n u[n] - 5\left[\frac{1}{3}\right]^n u[n]$

Solution:



Let $W[n] = x[n] + W[n-1] - \frac{2}{9} W[n-2]$

By taking Z transform

$$W(z) = X(z) + Z^{-1} W(z) - \frac{2}{9} W(z) \cdot Z^{-2}$$

$$W(z)\left(1 - Z^{-1} + \frac{2}{9} Z^{-2}\right) = X(z)$$

$$\frac{W(z)}{X(z)} = \frac{1}{\left(1 - Z^{-1} + \frac{2}{9} Z^{-2}\right)} = \frac{1}{\left(1 - \frac{1}{3} z^{-1}\right)\left(1 - \frac{2}{3} z^{-1}\right)}$$

Now $\quad y[n] = -\frac{5}{3} W[n-1] + \frac{5}{3} W[n-2]$

$$Y(z) = \left(-\frac{5}{3} Z^{-1} + \frac{5}{3} Z^{-2}\right) W(z)$$

$$Y(z) = \frac{+\frac{5}{3} Z^{-1}\left(Z^{-1} - 1\right) X(z)}{\left(1 - \frac{1}{3} Z^{-1}\right)\left(1 - \frac{2}{3} Z^{-1}\right)}$$

Now unit step response of $X(z) = \frac{1}{\left(1 - z^{-1}\right)}$

$$Y(z) = \frac{-\frac{5}{3} Z^{-1}}{\left(1 - \frac{1}{3} Z^{-1}\right)\left(1 - \frac{2}{3} Z^{-1}\right)}$$

$$= \frac{A}{\left(1 - \frac{1}{3} Z^{-1}\right)} + \frac{B}{\left(1 - \frac{2}{3} Z^{-1}\right)}$$

$$A = 5$$

$$B = -5$$

So $Y[z] = \dfrac{5}{1-\frac{1}{3}z^{-1}} - \dfrac{5}{1-\frac{2}{3}z^{-1}}$

$$y[n] = 5\left(\frac{1}{3}\right)^n u[n] - 5\left(\frac{2}{3}\right)^n u[n]$$

Hence, the correct option is (C).

8. For an all-pass system $H(z) = \dfrac{(z^{-1}-b)}{(1-az^{-1})}$, where $|H(e^{-j\omega})| = 1$, for all ω. If $Re(a) \neq 0$, $Im(a) \neq 0$, then b equals **[2014]**

(a) a
(b) a^*
(c) $1/a^*$
(d) $1/a$

Solution: (b)

For a given system, $H(z) = \dfrac{(z^{-1}-b)}{(1-az^{-1})}$

The poles lie at z = 0 and the zero lie at z = 1/b. a is complex in nature so

$a = |a| \cdot e^{i<a}$

for an all pass system, $|He^{J\omega}| = 1$

⇒ for all ω, if the pole lies at d then the zero must be at 1/a

$$\therefore \text{ zero}\left(\frac{1}{b}\right) = \frac{1}{|a|e^{JLa}} - \frac{1}{a} \quad or \quad b = a*$$

Hence, the correct option is (b).

9. C is a closed path in the z-plane given by $|z|$ = 3. The value of the integral

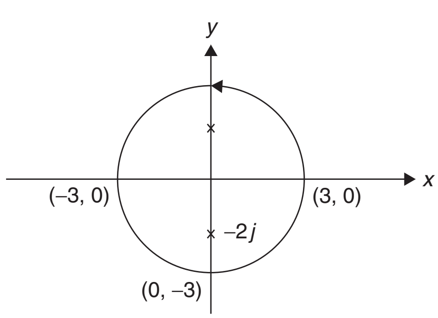
$$\oint_c \left(\frac{z^2 - z + 4j}{z + 2j}\right) dz \text{ is}$$ **[2014]**

(a) $-4\pi(1+j2)$ (b) $4\pi(3-j2)$

(c) $-4\pi(3+j2)$ (d) $4\pi(1-j2)$

Solution: (c)

Location of poles can be shown as:



So $\int_c \dfrac{z^2 - z + 4j}{z + 2j}$, poles $= z = -2j$

Poles are inside of $|z|$=3 using Cauchy integral formula

$$\int \frac{z^2 - z + 4j}{z + 2j} = 2\pi i\left[\lim_{zx \to 2j} z^2 - z + 4j\right] = -\pi[2j + 3]$$

Hence, the correct option is (c).

10. Let $x[n] = x[-n]$. Let $X(z)$ be the z-transform of $x[n]$. If $0.5 + j0.25$ is zero of $X(z)$, which one of the following must also be a zero of $X(z)$. **[2014]**

(a) $0.5 - j0.25$ (b) $\dfrac{1}{(0.5 + j0.25)}$

(c) $\dfrac{1}{(0.5 - j0.25)}$ (d) $2 + j4$

Solution: (b)

Given $x(n) = x(-n)$,

Using z-transform property then

$x(n) \rightarrow x[z]$

$x(-n) \rightarrow x[z^{-1}]$

Now, let $(0.5 + j\, 0.25)$ is a zero of $x(z)$ for $x(n)$

then $x(-n) \rightarrow x(z^{-1}) = \dfrac{1}{0.5 + j0.25}$

zero of $x(z)$.

Hence, the correct option is (b).

11. If $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$, then the region of convergence (ROC) in its Z-transform in the Z-plane will be **[2012]**

(a) $\dfrac{1}{3} < |z| < 3$ (b) $\dfrac{1}{3} < |z| < \dfrac{1}{2}$

(c) $\dfrac{1}{2} < |z| < 3$ (d) $\dfrac{1}{3} < |z|$

Solution: (c)

Given signal $x(n) = \left(\dfrac{1}{3}\right)^{|n|} - \left(\dfrac{1}{3}\right)^n u(n)$

can be written as

$$x(n) = \left(\frac{1}{3}\right)^n 4(n) + \left(\frac{1}{3}\right)^{-n} u(-n-1) - \left(\frac{1}{2}\right)^n u(n)$$

Using z-transform property:

$$\left(\frac{1}{3}\right)^n u(n) \longleftrightarrow \frac{1}{1-\frac{1}{2}z^{-1}} \qquad Roc \quad |z| > \frac{1}{3}$$

$$\left(\frac{1}{3}\right)^n u(-n-1) \longleftrightarrow \frac{-1}{1-3z^{-1}} \qquad Roc \quad |z| < 3$$

$$\left(\frac{1}{2}\right)^n u(n) \longleftrightarrow \frac{1}{1-\frac{1}{2}z^{-1}} \qquad Roc \quad |z| > \frac{1}{2}$$

Overall ROC will be intersection of these ROC i.e.

$$\frac{1}{2} < |z| < 3$$

Hence, the correct option is (c).

12. Consider the z-transform $X(z) = 5z^2 + 4z^{-1} + 3$; $0 < |z|$ ° . The inverse z-transform $x[n]$ is **[2010]**

(a) $5\delta[n+2] + 3\delta[n] + 4\delta[n-1]$
(b) $5\delta[n-2] + 3\delta[n] + 4\delta[n+1]$
(c) $5u[n+2] + 3u[n] + 4u[n-1]$
(d) $5u[n-2] + 3u[n] + 4u[n+1]$

Solution: (a)

$$\delta[n+n_0] \xleftrightarrow{z} z^{n_0} \times (z) = 5z^2 + 4z^{-1} + 3; \quad 0 < |z| < \infty$$

$$\therefore x(n) = 5\delta(n+2) + 4\delta(n-1) + 3\delta(n)$$

Hence, the correct option is (a).

13. Two discrete time systems with impulse response $h_1[n] = \delta[n-1]$ and $h_2[n] = \delta[n-2]$ are connected in cascade. The overall impulse response of the cascaded system is **[2010]**

(a) $\delta[n-1] + \delta[n-2]$
(b) $\delta[n-4]$
(c) $\delta[n-3]$
(d) $\delta[n-1]\ \delta[n-2]$

Solution: (c)

$$h_1[n] = \delta[n-1] \xleftrightarrow{z} n_1(z) = z^{-1}$$
$$h_2[n] = \delta[n-1] \xleftrightarrow{z} n_2(z) = z^{-2}$$

Overall impulse response in Z-domain

$$n(z) = n_1(z)n_2(z)$$
$$= z^{-1}z^{-2}$$
$$= z^{-3}$$

Overall impulse response in discrete-time domain

$$h[n] = \delta(n-3)$$

Hence, the correct option is (c).

14. The ROC of z-transform of the discrete time sequence **[2009]**

$$x(n) = \left(\frac{1}{3}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(-n-1) \text{ is}$$

(a) $\frac{1}{3} < |z| < \frac{1}{2}$ (b) $|z| > \frac{1}{2}$

(c) $|z| < \frac{1}{3}$ (d) $2 < |z| < 3$

Solution: (a)

$$x(n) = \left(\frac{1}{3}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(-n-1)$$

$\left(\frac{1}{3}\right)^n u(n)$ is right sided signal, so ROC will be $|z|>1/3$.

$\left(\frac{1}{3}\right)^n u(n)$ is left sided signal, so ROC will be $|z|<1/2$

$\Rightarrow$ ROC of the function will be:

$$\frac{1}{3} < |z| < \frac{1}{2}$$

Hence, the correct option is (a).

15. If the region of convergence of $x_1[n] + x_2[n]$ is $\frac{1}{3} < |z| < \frac{2}{3}$, then the region of convergence of $x_1[n] - x_2[n]$ includes **[2006]**

(a) $\frac{1}{3} < |z| < 3$ (b) $\frac{2}{3} < |z| < 3$

(c) $\frac{3}{2} < |z| < 3$ (d) $\frac{1}{3} < |z| < \frac{2}{3}$

Solution: (d)

$$x(n) = x_1(n) + x_2(n)$$

ROC is $\frac{1}{2} < |z| < \frac{1}{3}$

Now $x_1(n) = x_1(n) - x_2(n)$

$\rightarrow$ROC will remain the same as the intersection will not change due to $-x$ sign

$\rightarrow$ROC is $\frac{1}{2} < |z| < \frac{1}{3}$

ROC of $x_1(n) + x_2(n)$ and $x_1(n) - x_2(n)$ is the same.
Hence, the correct option is (d).

16. The region of convergence (ROC) of z-transform of the sequence

$\left(\frac{5}{6}\right)^n u(n) - \left(\frac{6}{5}\right)^n u(-n-1)$ must be **[2005]**

(a) $|z| < \frac{5}{6}$ (b) $|z| > \frac{5}{6}$

(c) $\frac{5}{6} < |z| < \frac{5}{6}$ (d) $\frac{6}{5} < |z| <$

Solution: (c)
Given sequence is

$$x(n) = \left(\frac{5}{6}\right)^n u(n) - \left(\frac{6}{5}\right)^n u(-n-1)$$
$$x(n) = x_1(n) + x_2(n)$$

ROC of $x(z)$ = [ROC of $x_1(z)$] $\cap$ [ROC of $x_2(z)$]

$$\left(\frac{5}{6}\right)^n u(n) \longleftrightarrow \frac{1}{1 - \frac{5}{6}z^{-1}},$$

$$\text{Roc } |z| > \therefore L[h(t)] = L[f(t)] \times L[g(t)] = \frac{1}{S+3}$$

$$-\left(\frac{6}{5}\right)^n u(-n-1) \longleftrightarrow \frac{1}{1-\left(\frac{6}{5}\right)z^{-1}}, \text{ Roc } |z| < \frac{6}{5}$$

ROC of $x(z)$ will be $\frac{5}{6} < |z| < \frac{6}{5}$

Hence, the correct option is (c).

17. The z-transform of a system is **[2004]**

$$H(z) = \frac{z}{z-0.2}$$

If the ROC is $|z| < 0.2$, then the impulse response of the system is

(a) $(0.2)^n u[n]$ (b) $(0.2)^n u[-n-1]$
(c) $-(0.2)^n u[n]$ (d) $-(0.2)^n u[-n-1]$

Solution: (d)

$$H(z) = \frac{z}{z-0.2} \quad ,|z| < 0.2$$

$$H(z) = \frac{1}{1-0.2z^{-1}} \longleftrightarrow \frac{1}{1-r} \cdot r < 1$$

$0.2\, Z^{-1} < 1 \Rightarrow |z| > 0.2$
But given ROC is $|z| < 0.2$
Hence, the correct option is (d).

18. A sequence $x(n)$ with the z-transform $X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$ is applied as an input to a linear, time-invariant system with the impulse response $h(n) = 2\delta(n-3)$ where

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$ **[2003]**

The output at $n = 4$ is

(a) -6 (b) Zero
(c) 2 (d) -4

Solution: (b)

$$x(z) = z^4 + z^{+2} - 2z + 2 - 3z^{-4}$$
$$h(n) = 2\delta(n-3)$$
$$h(z) = 2z^{-3}$$
$$y(z) = h(z) \cdot x(z)$$
$$y(z) = 2z^{-3}(z^4 + z^2 - 2z + 2 - 3z^{-4})$$
$$y(z) = 2z + 2z^{-1} - 4z^{-2} + 4z^{-3} - 6z^{-7}$$
$$y(n) = 2[\delta(n+1) + \delta(n-1) - 2\delta(n-2) + 2\delta(n-3) - 3\delta(n-7)]$$

At $n = 4$ $y(4) = 0$

$$\boxed{h(n) = -(0.2)^n u(-n-1)}$$

Hence, the correct option is (b).

19. The region of convergence of the z-transform of a unit step function is **[2001]**

(a) $|z| > 1$
(b) $|z| < 1$
(c) (Real part of z) > 0
(d) (Real part of z) < 0

Solution: (a)

$$h(n) = u(n)$$
$$H(z) = \sum_{n=0}^{\infty} 1 - z^{-n}$$

For RVC

$$\sum_{n=0}^{\infty} z^{-n} < \infty$$
$$1 + z^{-1} + z^{-2} + \ldots < \infty$$
$$|z^{-1} < 1| \rightarrow |z| > |1|$$

Hence, the correct option is (a).

20. The z-transform $F(z)$ of the function $f(nT) = a^{nT}$ is______. **[1999]**

(a) $\frac{z}{z-a^T}$ (b) $\frac{z}{z+a^T}$
(c) $\frac{z}{z-a^{-T}}$ (d) $\frac{z}{z+a^{-T}}$

Solution : (a)

$$f(nT) = a^{nT}$$
$$z[f(nT)] = \sum_{n=0}^{\infty} f(nT)z^{-n} = \sum_{n=0}^{\infty} a^{nT} z^{-n}$$
$$= \sum_{n=0}^{\infty} (a^T z^{-1})^n = \frac{1}{1-a^T z^{-1}}$$
$$= \frac{z}{z-a^T}$$

Hence, the correct option is (a).

21. The z-transform of the time function **[1998]**

$$\sum_{k=0}^{f} d(n-k) \text{ is}$$

(a) $\frac{z}{z-a^T}$ (b) $\frac{z}{z+a^T}$
(c) $\frac{z}{z-a^{-T}}$ (d) $\frac{z}{z+a^{-T}}$

Solution: (*)
Given Signal

$$x(n) = \sum_{k=0}^{\infty} S(n-k) = S(n) + S(n-1) + S(n-2) + \ldots$$
$$x(z) = 1 + z^{-1} + z^{-2} + \ldots$$
$$= \frac{1}{1-z^{-1}} = \frac{z}{z-1}$$

Two-mark Questions

1. A continuous time filter with transfer function $H(s) = \frac{2s+6}{s^2+6s+8}$ is converted to a discrete time filter with transfer function $G(z) = \frac{2z^2-0.5032z}{z^2-0.5032z+k}$ so that the impulse response of the continuous time filter, sampled at 2 Hz, is identical at the sampling instants to the impulse response of the discrete time filter. The value of k is ______. **[2016]**

Solution: To find the Inverse Laplace of transfer function $H(s)$ we need to calculate the values using partial fraction formula as given below:

$$H(s) = \frac{2s+6}{s^2+6s+8}$$

$$H(s) = \frac{A}{(s+2)} + \frac{B}{s+4}$$

$$A = H(s)\cdot(s+2) \text{ at } s = -2$$

$$A = \frac{2}{2} = 1$$

$$B = H(s)\,(s+4) \text{ at } s = -4$$

$$B = \frac{-2}{-2} = 1$$

Now we have

$$H(s) = \frac{1}{s+2} + \frac{1}{s+4}$$

$$h(t) = ILT\{H(S)\} = (e^{-2t} + e^{-4t})u(t)$$

We know that, $T_S = \frac{1}{f_s} = \frac{1}{2}$

Substituting $t = n\cdot T_S$ we get

$$h(nT_S) = (e^{-2nT_s} + e^{-4nT_s})\cdot u(nT_S)$$

$$= e^{-n}\cdot u(n) + e^{-2n}\cdot u(n)$$

$$H(Z) = ZT\{h(nT_S)\}$$

$$H(Z) = \frac{Z}{Z-e^{-1}} + \frac{Z}{Z-e^{-2}}$$

$$= \frac{Z}{Z-0.367} + \frac{Z}{Z-0.135}$$

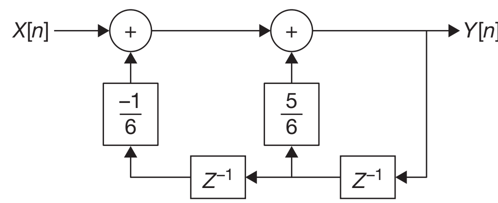
$$= \frac{Z^2-0.135Z+Z^2-0.367Z}{Z^2-0.5032Z+0.049}$$

$$= \frac{2Z^2-0.5032Z}{Z^2-0.5032Z+0.049}$$

$\therefore \quad K = 0.049$

Hence, the correct Answer is (0.049).

2. For the discrete-time system shown in the figure, the poles of the system transfer function are located at **[2015]**



(A) 2, 3 (B) $\frac{1}{2}, 3$

(C) $\frac{1}{2}, \frac{1}{3}$ (D) $2, \frac{1}{2}$

Solution: $Y[n] = \frac{5}{6}y[n-1] - \frac{1}{6}y[n-2] + x[n]$

By taking z transform on both sides

$$Y(z)\left[1 - \frac{5}{6}Z^{-1} + \frac{1}{6}Z^{-2}\right] = X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1}{\left(1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}\right)} \Rightarrow = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}$$

So poles are at $Z = \frac{1}{2}, \frac{1}{3}$

Hence, the correct option is (C).

3. Two causal discrete-time signals $x[n]$ and $y[n]$ are related as $y[n] = \sum_{m=0}^{n} x[m]$. If the z-transform of $y[n]$ is $\frac{2}{z(z-1)^2}$, the value of $x[2]$ is ______. **[2015]**

Solution: As we know the accumulation property

If $$y[n] = \sum_{m=0}^{n} x[m]$$

then $$Y(z) = \frac{X(z)}{(1-z^{-1})}$$

$$\Rightarrow \quad \frac{2}{Z(Z-1)^2} = \frac{X(z)}{(1-Z^{-1})} = \frac{Z\,X(z)}{(Z-1)}$$

So $$X(z) = \frac{2Z^{-3}}{(1-Z^{-1})}$$

$$x[n] = 2\,u[n-3]$$

So at $n = 2$

$$x[2] = 0$$

Hence, the correct Answer is (0).

4. Let $X[n] = \left(-\frac{1}{9}\right)^n u(n) - \left(-\frac{1}{3}\right)^n u(-n-1)$. The Region of Convergence (ROC) of the z-transform of $x[n]$ **[2014]**

(a) is $|z| > \frac{1}{9}$ (b) is $|z| > \frac{1}{3}$

(c) is $\frac{1}{3} > |z| > \frac{1}{9}$ (d) does not exit

Solution: (c)

Given

$$x(n) = \underbrace{\left(\frac{-1}{9}\right)^n u(n)}_{x_1(n)} - \underbrace{\left(\frac{-1}{3}\right)^n u(-n-1)}_{x_2(n)}$$

Here $x_1(n)$ is right sided signal, so

$$x_1(n) = \frac{1}{1-\left(\frac{-1}{9}\right)z^{-1}} \qquad Roc \quad |z| \geq \left|\frac{1}{9}\right| \quad or \quad |z| > \frac{1}{9}$$

and $x_2(n) = \dfrac{1}{1-\left(\frac{-1}{3}\right)z^{-1}} \qquad Roc \quad |z| \leq \dfrac{1}{3}$

Roc of overall system will be $\frac{1}{9} < |z| < \frac{1}{3}$

Hence, the correct option is (c).

5. The input-output relationship of a casual stable LTI system is given as $y[n] = \alpha y[n-1] + \beta x[n]$

If the impulse response $h[n]$ of this system satisfies the condition $\sum_{n=0}^{\infty} h[n] = 2$, the relationship between α and β is **[2014]**

(a) $\alpha = 1 - \beta/2$ (b) $\alpha = 1 + \beta/2$

(c) $\alpha = 2\beta$ (d) $\alpha = -2\beta$

Solution: (a)

Given relationship is

$$x(n) = \alpha z(n-1) + \beta x(n)$$

Taking z-transform will give

$$y(z) = \alpha z^{-1} y(z) + \beta x(z)$$

Or, $\dfrac{y(z)}{x(z)} = H(z) = \dfrac{\beta}{1-\alpha z^{-1}}$

Taking inverse z-transform

$$n(n) = \beta \cdot \alpha^n u(n)$$

also, given $\sum_{n=0}^{\infty} h(n) = 2 \Rightarrow \sum_{n=0}^{\infty} \beta \cdot \alpha^n u(n) = 2$

so $\dfrac{\beta}{1-\alpha} = 2 \Rightarrow \beta = 2 - 2\alpha \quad OR \quad \alpha = 1 - \dfrac{\beta}{2}$

Hence, the correct option is (a).

6. Let $H_1(z) = (1 - pz^{-1})^{-1}$, $H_2(z) = (1 - qz^{-1})^{-1}$, $H(z) = H_1(z) + rH_2(z)$. The quantities p, q, r are real numbers. Consider $p = \frac{1}{2}$, $q = \frac{1}{4}$, $|r| < 1$. If the zero of $H(z)$ lies on the unit circle, then $r =$ ______. **[2014]**

Solution: 0.5

Given $H_1(z) = \dfrac{1}{1-pz^{-1}}$

and $H_2(z) = \dfrac{1}{1-gz^{-1}}$

also given $p = ½$ and $g = -1/4$

So $H(z) = H_1(z) + rH_2(z) = \dfrac{1}{1-pz^{-1}} + \dfrac{1}{1-gz^{-1}}$

Solving

$$H(z) = \frac{\left(1+\frac{1}{4}z^{-1}\right) + r\left(1-\frac{1}{2}z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}$$

Zero of

$$H(z) \Rightarrow (1+r) - (g+pr)z^{-1} = 0$$

or,

$$z = \frac{g+pr}{1+r}$$

Since the zero of $H(z)$ lies on the unit circle; therefore $|z| = 1$ or $z = \pm 1$, taking $z = 1$ we get

$$z = \frac{g+pr}{1+r} = 1.$$

Solving the equation with $p = 1/2$ and $g = -1/4$ we will get $r = 2.5$,

For $|r| < 1$ or $-1 < r < 1$

Taking $z = -1$, in equation (iii) we will get $r = 0.5$.

7. The z-transform of the sequence $x[n]$ is given by $X(z) = \dfrac{1}{\left(1-2z^{-1}\right)^2}$, with the region of convergence $|z|$ 2. Then, $x[2]$ is ________. **[2014]**

Solution: 12

Given $x(z) = \dfrac{1}{(1-2z^{-1})^2}, \qquad |z| > 2 \qquad (1)$

$$\Rightarrow x(z) = (1-2z^{-1})^{-2}$$

Expanding equation (1) using binomial expression

$$x(z) = 1 + 4z^{-1} + 12z^{-2} + \ldots$$

Taking inverse z-transform

$$x(n) = \{1, 4, 12, \ldots$$

or $x[2] = 12$

8. Two systems $H_1(z)$ and $H_2(z)$ are connected in cascade as shown below. The overall output $y(n)$ is the same as the input $x(n)$ with a one unit delay. The transfer function of the second system $H_2(z)$ is **[2011]**

$$x(n) \rightarrow \boxed{H_1(z) = \frac{(1-0.4z^{-1})}{(1-0.6z^{-1})}} \rightarrow \boxed{H_2(z)} \rightarrow y(n)$$

(a) $\dfrac{(1-0.6z^{-1})}{z^{-1}(1-0.4z^{-1})}$ (b) $\dfrac{z^{-1}(1-0.6z^{-1})}{(1-0.4z^{-1})}$

(c) $\dfrac{z^{-1}(1-0.4z^{-1})}{(1-0.6z^{-1})}$ (d) $\dfrac{(1-0.4z^{-1})}{z^{-1}(1-0.6z^{-1})}$

Solution: (b)

$$y[n] = x[n-1]$$

Taking Z-transform both sides

$$y(z) = z^{-1} \times (z)$$

$$\frac{y(z)}{x(z)} = z^{-1}$$

For cascaded system

$$n(z) - n_1(z) \cdot n_2(z)$$

$$z^{-1} = \frac{(1-0.4z^{-1})}{(1-0.6z^{-1})} n_2(z) \quad \therefore n_2(z) = \frac{(1-0.6z^{-1})}{(1-0.4z^{-1})}$$

Hence, the correct option is (b).

9. The transfer function of a discrete time LTI system is given by **[2010]**

$$H(z) = \frac{2 - \frac{3}{4}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

Consider the following statements:

S_1: The system is stable and causal for

ROC: $|z| > \dfrac{1}{2}$

S_2: The system is stable but not causal for

ROC: $|z| < \dfrac{1}{4}$

S_3: The system is neither stable nor causal

for ROC: $\dfrac{1}{4} < |z| < \dfrac{1}{2}$

Which one of the following statements is valid?

(a) Both S_1 and S_2 are true
(b) Both S_2 and S_3 true
(c) Both S_1 and S_3 are true
(d) S_1, S_2 and S_3 are all true

Solution: (c)

1. A discrete time LII system is causal if the ROC of its system function is the exterior of a circle, including infinity.
2. A discrete-time LTI system is stable if the ROC of its system function includes the unit circle, $|z| = 1$

$$H(z) \frac{\left(1 - \frac{1}{4}z^{-1}\right) + \left(1 - \frac{1}{2}z^{-1}\right)}{\left(1 - \frac{1}{4}z^{-1}\right) + \left(1 - \frac{1}{2}z^{-1}\right)}$$

$$\Rightarrow H(z) = \frac{1}{1 - \frac{1}{4}z^{-1}} + \frac{1}{1 - \frac{1}{2}z^{-1}}$$

For ROC: $|z| > \dfrac{1}{2}$, the system is stable and causal

For ROC: $|z| < \dfrac{1}{4}$, ROC does not include unit circle. So system is not stable.

For ROC: $\dfrac{1}{4}$, ROC does not include unit circle. So system in not stable

Also ROC is not the exterior of $|z| = \dfrac{1}{2}$. So it is not causal.

Hence, the correct option is (c).

10. A system with transfer function $H(z)$ has impulse $h(n)$ defined as $h(2) = 1$, $h(3) = -1$ and $h(k) = 0$ otherwise. Consider the following statements **[2009]**

S_1: $H(z)$ is a low-pass filter

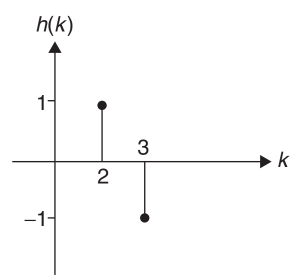
S_2: $H(z)$ is an FIR filter.

Which of the following is correct?

(a) Only S_2 is true
(b) Both S_1 and S_2 are false
(c) Both S_1 and S_2 are true, and S_2 is a reason for S_1
(d) Both S_1 and S_2 are true, but S_2 is not a reason for S_1

Solution: (a)

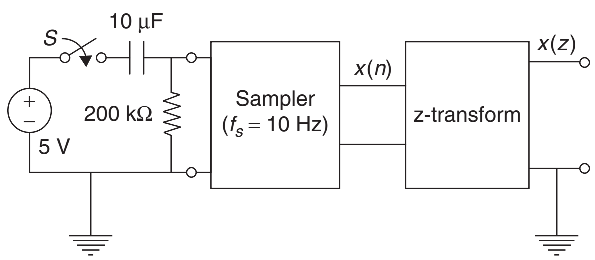
$h(2) = 1$
$h(3) = -1$
$h(k) =$ otherwise



$\Rightarrow$It is an FIR filter and not a low pass filter.
Hence, the correct option is (a).

Statement for Linked Answer Questions 8 and 9.

In the following network, the switch is closed at $t = 0^-$ and the sampling starts from $t = 0$. The sampling frequency is 10 Hz.



11. The expression and the region of convergence of the z-transform of the sampled signal are **[2008]**

(a) $\dfrac{5z}{z-e^5}, |z| < e^{-5}$

(b) $\dfrac{5z}{z-e^{-0.05}}, |z| < e^{-0.05}$

(c) $\dfrac{5z}{z-e^{-0.05}}, |z| > e^{-0.05}$

(d) $\dfrac{5z}{z-e^{-5}}, |z| > e^{-5}$

Solution: (c)

$$X(z) = \sum_{n=0}^{\infty} 5e^{-0.05n} z^{-n}$$

$$= 5\sum_{n=0}^{\infty}\left(e^{-0.05}z^{-1}\right)^{-n}$$

For $e^{-0.05}z^{-1} < 1$

$\Rightarrow |z| > e^{-0.05}$

$\Rightarrow$

$x(z) = \dfrac{5z}{z-e^{-0.05}}$

Pole should be inside unit circle

$$\sqrt{\frac{\alpha}{2}} < 1$$

$$\boxed{|\alpha| < 2}$$

Hence, the correct option is (c).

12. The sample $x(n)(n = 0,1,2, ...)$ is given by **[2008]**

(a) $5(1-e^{-0.05n})$

(b) $5e^{-0.05n}$

(c) $5(1-e^{-5n})$

(d) $5e^{-5n}$

Solution: (b)

Using the voltage divider rule

$$V_R(S) = \left(\frac{200\times10^3}{200\times10^3 + \dfrac{1}{10\times10^{-6}S}}\right)\frac{5}{S}$$

$$= \frac{10}{2S+1} = 5e^{-0.5t}$$

$\therefore$ samples are: $x(n) = 5e^{-0.05n}$

Hence, the correct option is (b).

13. The z-transform $X[z]$ of sequence $x[n]$ is given by $X[z] = \dfrac{0.5}{1-2z^{-1}}$. It is given that the region of convergence of $X[z]$ includes the unit circle. The value of $x[0]$ is **[2007]**

(a) –0.5 (b) 0

(c) 0.25 (d) 0.5

Solution: (0)

$$x(z) = \frac{0.5}{1-2z^{-1}}$$

ROC includes unit circle left handed system

$$x(n) = -(0.5)2^{-n}u(-n-1)$$
$$x(o) = 0$$

14. A causal LTI system is described by the difference equation

$2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]$. **[2004]**

The system is stable only if

(a) $|\alpha| = 2, |\beta| < 2$

(b) $|\alpha| > 2, |\beta| > 2$

(c) $|\alpha| < 2, \infty$ any value of β

(d) $|\beta| < 2$, any value of α

Solution: (c)

$$2y(n) = \alpha y[n-2] - 2x[n] + \beta x[n-1]$$

Taking z-transform

$$2y(z) = \alpha y(z)z^{-2} - 2x(z) + \beta x(z)z^{-1}$$
$$y(z)\left[2-\alpha z^{-2}\right] = z(z)[\beta z^{-1} - 2]$$
$$\frac{y(z)}{x(z)} = \frac{\beta z^{-1}-2}{2-\alpha z^{-2}} = T\cdot F$$

For stable B can have only value

$$2-\alpha z^{-2} = 0 \;\Rightarrow\; z = \sqrt{\frac{\alpha}{2}}$$

Hence, the correct option is (c).

15. If the impulse response of a discrete-time system is $h[n] = -5^n u[-n-1]$, then the system function $H(z)$ is equal to **[2002]**

(a) $\dfrac{-z}{z-5}$ and the system is stable

(b) $\dfrac{z}{z-5}$ and the system is stable

(c) $\dfrac{-z}{z-5}$ and the system is unstable

(d) $\dfrac{z}{z-5}$ and the system is unstable

Solution: (b)

$$h(n) = -5^{nu}[-n-1]$$

$$u(n) \longleftrightarrow \frac{z}{z-1}$$

$$u(n-1) \longleftrightarrow (z^{-1}) \times \frac{z}{z-1}$$

$$u(-n-1) \longleftrightarrow \frac{1}{1-z}$$

$$5^n u(-n-1) \longleftrightarrow \frac{1}{5} \times \frac{5z}{5-z}$$

$$\boxed{-5^n u(-n-1) \longleftrightarrow \frac{z}{-}}$$

The system is stable because ROC includes unit circle. Hence, the correct option is (b).

16. The z-transform of a signal is given by **[1999]**

$$C(z) = \frac{1}{4}\frac{z^{-1}\left(1-z^{-4}\right)}{\left(1-z^{-1}\right)^2}. \text{ Its final value is}$$

(a) 1/4 (b) zero
(c) 1.0 (d) infinity

Solution: (c)

$$C(z) = \frac{1}{4}\frac{z^{-1}(1-z^{-4})}{(1-z^{-1})^2}$$

Final value theorem

$$\lim_{n\to\infty} x(n) = \lim_{z\to 1}(1-z^{-1})x(z)$$

$$= \lim_{z\to 1} \cancel{(1-z^{-1})}\,\frac{1}{4}\frac{z^{-1}(1-z^{-4})}{(1-z^{-1})^{\cancel{2}}}$$

$$= \lim_{z\to 1}\frac{1}{4}\frac{z^{-1}(1-z^{-4})}{(1-z^{-1})} \to \left(\frac{0}{0}\right)$$

For apply L-hospital rule

$$\lim_{n\to\infty} x(n) = \lim_{z\to 1}\frac{1}{4}\frac{1}{z}\frac{\left(1-\frac{1}{z^4}\right)}{\left(1-\frac{1}{z}\right)} = \lim_{z\to 1}\frac{1}{4}\frac{\left(z^4-1\right)}{z^4\left(z-1\right)}$$

$$\lim_{n\to\infty} x(n) = \lim_{z\to 1}\frac{1}{4}\frac{4z^3}{5z^4-4z^3}$$

$$= \frac{1}{4}\times 4 = 1$$

Final value = 1
Hence, the correct option is (c).

17. A linear discrete-time system has the characteristics equation $z^3 - 0.81z = 0$. The system **[1992]**

(a) is stable
(b) is marginally stable
(c) is unstable
(d) stability cannot be assessed from the given information

Solution: (a)
A system is stable when all the poles lie inside the unit circle in z-plane

$$z^3 - 0.81z = 0 \Rightarrow z(z^2 - 0.81) = 0$$

$$z(z-0.9)(z+0.9) = 0$$

$$z = 0, -0.9, 0.9$$

All the poles are inside unit circle hence stable.
Hence, the correct option is (a).

18. The z-transform of the following real exponential sequence **[1990]**
$X(nT) = a^n,\ nT \ge 0$
$= 0,\ nT < 0,\ a > 0$

(a) $\dfrac{1}{1-z^{-1}}; |z| > 1$ (b) $\dfrac{1}{1-az^{-1}}; |z| > a$

(c) 1 for all z (d) $\dfrac{1}{1-\mathrm{a}z^{-1}}; |\mathrm{z}| < a$

Solution: (b)

$$x(nT) = a^n, \quad nT \ge 0$$
$$= 0, \quad nT < 0, \quad a > 0$$

$$x(nT) = a^n u(n), \quad a > 0$$

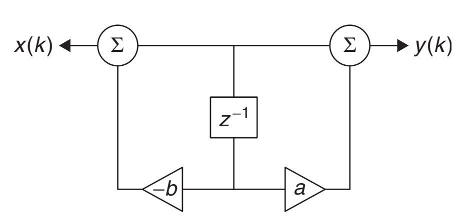
$$x(n) = a^n u(m), \quad a > 0$$

$$a^n u(m) = \frac{z}{z-a} \cdot |z| > |a|$$

$$x(z) = \frac{1}{1-az^{-1}}; \quad |z| > a$$

Hence, the correct option is (b).

19. Consider the system shown in the figure below. The transfer function $Y(z)/X(z)$ of the system is

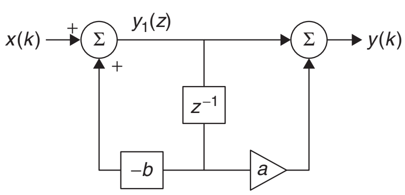


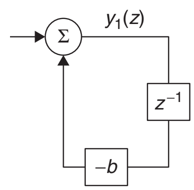
[1988]

(a) $\dfrac{1+az^{-1}}{1+bz^{-1}}$ (b) $\dfrac{1+bz^{-1}}{1+az^{-1}}$

(c) $\dfrac{1+az^{-1}}{1-bz^{-1}}$ (d) $\dfrac{1-bz^{-1}}{1+az^{-1}}$

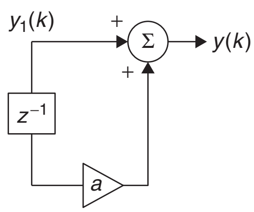
Solution: (a)





$$\frac{y_1(z)}{x(z)}=\frac{1}{(1+bz^{-1})} \tag{1}$$

Now, $\dfrac{y(z)}{y_1(z)}$



$$y(z)=y_1(z)+az^{-1}y_1(z)$$

$$\frac{y(z)}{y_1(z)}=1+az^{-1} \tag{2}$$

Now

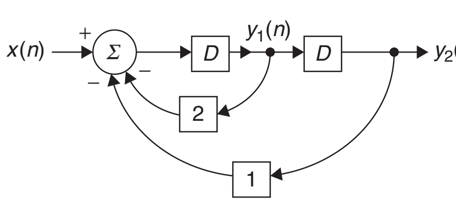
$$\frac{y(z)}{x(z)}=\frac{y(z)}{y_1(z)}\times\frac{y_1(z)}{x(z)}=(1+az^{-1})\times\frac{1}{(1+bz^{-1})}$$

$$\text{T.F}=\frac{y(z)}{x(z)}=\frac{1+az^{-1}}{1+bz^{-1}}$$

Hence, the correct option is (a).

Five-marks Questions

1. In figure, a linear time invariant discrete system is shown. Blocks labelled D represent unit delay elements. For $n < 0$, you may assume that $x(n)$, $y_1(n)$, $y_2(n)$ are all zero.



(a) Find the expression for $y_1(n)$ and $y_2(n)$ in terms of $x(n)$.

(b) Find the transfer function $Y_2(z)/X(z)$ in the z- domain.

(c) If $x(n) = 1$ at $n = 0 = 0$ otherwise

Find $y_2(n)$. **[1997]**

Solution:

Given, $x(n) = 1 \;\; n = 0$

$\qquad\quad = 0 \;\; n \neq 0$

So, $\quad x(n) = f(n)$

$$f(n)\xleftrightarrow{z.T}1$$

$$\text{So, } y_2(z)=H(z).x(z)=H(z).1.=H(z)=\frac{1}{(z+1)^2}$$

$$u(n)\xleftrightarrow{Z.T}\frac{z}{z-1}$$

$$nu(n)\xleftrightarrow{Z.T}-\frac{2d}{dz}\left(\frac{z}{2-1}\right)=\frac{z}{(z-1)^2}$$

$$(-1)^n nu(m)\xleftrightarrow{Z.T}\frac{-z}{(z+1)^2}$$

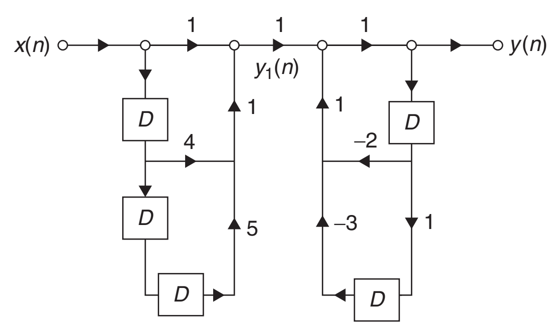
$$(-1)^{n-1}(n-1)u(n-1)\rightarrow\frac{-1}{(2+1)^2}$$

$$y_2(n) = (-1)^n\,(n-1)u(n-1)$$

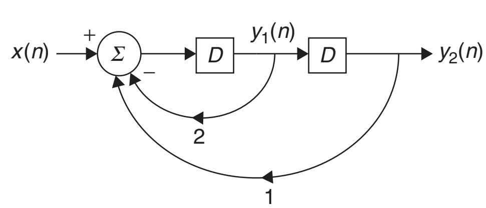
$$y. = (-1)^n\,(n-1)\;\;\forall n \geq 1$$

$$= 0 \;\; n \leq 0$$

2. In the linear time-invariant system shown in figure, blocks labelled D represent unit delay elements. Find the expression for $y(n)$, and also the transfer function $\dfrac{Y(z)}{X(z)}$ in the z- domain **[1996]**



Solution:



(a) $y_1(n) = x(n-1) - 2y_1\,(n-1) - y_1\,(n-2)$

$\quad y_2(n) = y_1(n-1) = x(n-2) - 2y_1\,(n-2) - y_2\,(n-1)$

$\quad y_3(n) = y(n-2) - 2y_2\,(n-1) - y_2\,(n-2)$

(b) $y_2(n) + 2y_2(n-1) + y_2(n-2) = x(n-2)$

taking Z-transform of both sides.

$y_2(z)\,[1 + 2z^{-1} + z^{-2}] = z^{-2} \times (2)$

So, $H(z) = \dfrac{y_2(2)}{x(2)} = \dfrac{z^{-2}}{1+2z^{-1}+z^{-2}} = \dfrac{1}{(z+1)^2}$

3. The output of a system is given in difference equation form as
$y(k) = ay(k-1) + x(k)$
where $x(k)$ is the input. If $x(k) = 0$ for $k \neq 0$,
$x(0) = 1$, and $y(0) = 0$, find $y(k)$ for all k.
Determine the range of 'a' for which $y(k)$ is bounded. **[1988]**

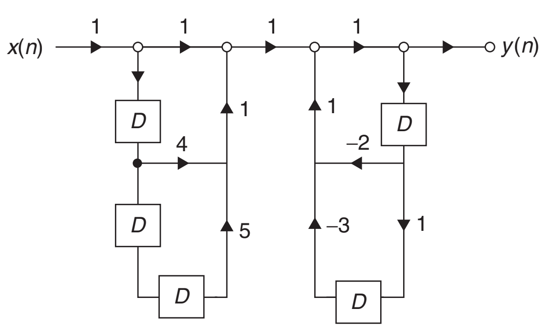
Solution:
Given, $x(k) = 0\ k \neq 0$ so $x(k)$ is a
$x(k) = 1\ k = 0$,
discrete, impulse, $x(k) = f(k)$
given that, $y(0) = 0$
$y(k) = ay(k-1) + x(k)$
$y(k) = ay(k-1) + f(k)$
So,
$k = 1 \rightarrow y(1) = ay(0) + f(1) = 0$
$k = 2 \rightarrow y(2) = ay(1) + f(2) - 0$
$k = 0 \rightarrow y(0) = ay(-1) + f(0) - ay(-1) + 1 = 0$

$$y(1) = \frac{-1}{a} = -a^{-1}$$

$$k = -1 = y(-1) = a^{-1}$$

$$= ay(-2) + f(-1) = ay(-2)$$

$y(-2) = -a^2$



$y_1(n) = y(n) + 4x(n-1) + 5x(n-3)$
$y(n) = y_1(n) - 2y(n-1) - 3y(n-2)$
$y(n + 2y)(n-1) \neq 3y(n-2) = y_1(n)$
$= x(n) - 4x(n-1) + 5x(x-3)$
taking Z-transform on both sides

$$y(z)[1+2z^{-1}+3z^{-2}] = x(z)[1+4z^{-1}+5z^{-3}]$$

So, $H(z) = \dfrac{y(z)}{x(z)} = \dfrac{3^3z^3+4z^2+5}{2(z^2+2z+3)}$

$y(-2) = -a^{-2}$
$y(k) = -a^{-k}$ for $k \leq -1$
$y(k) = 0$ for $k > -1$

$$y(k) = \left\{\ldots, -\frac{1}{a^4}, -\frac{1}{a^3}, -\frac{1}{a^2}, -\frac{1}{a}, \underset{\uparrow}{0}, 0, 0, \ldots\right\}$$

for $a = 1$

$$y(k) = \left\{\ldots\ldots, -1, -1, -1, \underset{\uparrow}{0}, 0, 0\ldots\right\}$$

for $a > 1$, let $a = 2$,

$$y(k) = \left\{\ldots, \frac{1}{8}, -\frac{1}{4}, -\frac{1}{2}, \underset{\uparrow}{0}, 0, 0\ldots\right\}$$

for $a < 1$, let $a = \dfrac{1}{2}$

$$y(k) = \left\{\ldots., -8, -4, -2, \underset{\uparrow}{0}, 0, 0\ldots.\right\}$$

So for $y(k)$ to be bounded
$|a| \geq 1$

Chapter 7

DTFT and DFT

One-mark Questions

1. Consider two real sequences with time-origin marked by the bold value,

$$x_1[n] = \{1, 2, 3, 0\}, x_2[n] = \{1, 3, 2, 1\}$$

Let $X_1(k)$ and $X_2(k)$ be 4-points DFTs of $x_1[n]$ and $x_2[n]$, respectively.

Another sequence $x_3[n]$ is derived by taking 4-point inverse DFT of $X_3(k) = X_1(k)\, X_2(k)$.

The value of $x_3[2]$ is ________. **[2015]**

Solution: By applying the property

Multiplication in frequency domain = Convolution in time domain.

$$x_1[n] \otimes x_2[n] = X_1(k) \cdot X_2(k)$$

	1	2	3	0
1	1	2	3	0
3	3	6	9	0
2	2	4	6	0
1	1	2	3	0

$$x_3[n] = \{ 1, 5, 11, 14, 8, 3, 0\}$$

$$x_3[2] = 11$$

Hence, the correct Answer is (10.9 to 11.1).

2. For an N-point FFT algorithm with $N = 2^m$, which one of the following statement is TRUE? **[2010]**

(a) It is not possible to construct a signal flow graph with both input and output in normal order.

(b) The number of butterflies in the m^{th} state is N/m.

(c) In-place computation requires storage of only $2N$ node data.

(d) Computation of a butterfly requires only one complex multiplication.

Solution: (d)

For an N point FFT algorithm with $N = 2^m$, computation of a butterfly requires only one complex multiplication and two complex additions.

Hence, the correct option is (d).

3. Let $x(n) = \left(\frac{1}{2}\right)^n u(n)$, $y(n) = x^2(n)$ and $Y\left(e^{j\omega}\right)$ be the Fourier transform of $y(n)$. Then $Y\left(e^{j0}\right)$ is **[2005]**

(a) $\frac{1}{4}$ (b) 2

(c) 4 (d) $\frac{4}{3}$

Solution: (d)

Given signal $x(n) = \left(\frac{1}{2}\right)^n u(n)$

and $x(n) = x^2(n) = \left(\frac{1}{2}\right)^{2n} (u(n))^2$

$$\Rightarrow y(n) = \left(\frac{1}{4}\right)^n u(n)$$

Taking z-transform

$$y(z)\frac{1}{1\frac{1}{4}z^{-1}}$$

Put $z = e^{j\omega} \Rightarrow y(e^{j\omega}) = \dfrac{1}{1\frac{1}{4}e^{-J\omega}}$

At $w = 0$, $y(e^{Jo}) = \dfrac{1}{1-\frac{1}{4}} = \frac{4}{3} Ans$

Hence, the correct option is (d).

Two-marks Questions

1. It is desired to find a three-tap causal filter which gives zero signal as an output to an input of the form

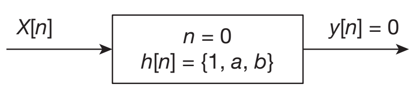
$$x[n] = c_1 \exp\left(-\frac{j\pi n}{2}\right) + c_2 \exp\left(\frac{j\pi n}{2}\right)$$

Where c_1 and c_2 are arbitrary real numbers. The desired three-tap filter is given by and

$h[0] = 1, h[1] = a, h[2] = b$

$h[n] = 0$ for $n < 0$ or $n > 2$.

What are the values of the filter taps a and b if the output is $y[n] = 0$ for all n, when $x[n]$ is as given above? **[2019]**



(A) $a = 0, b = -1$
(B) $a = 1, b = 1$
(C) $a = -1, b = 1$
(D) $a = 0, b = 1$

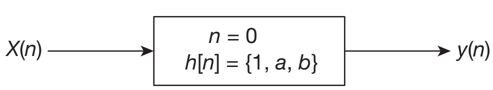
Solution:

$$X[n] = C_1 e^{-\left(\frac{J\Pi n}{2}\right)} + C_2 e^{j\Pi n/2}$$

$n[0] = 1$

$n[1] = a$

$n[2] = b$



Applying fourrier transform

$$H\left(e^{jw}\right) = 1 + ae^{-jw} + be^{-j2w}$$

For input $X[n] = C_1 e^{-j\frac{\Pi}{2}n} + C_2 e^{j\frac{\Pi}{2}n}$

output $y[n] = 0$ if

$$\left|H\left(W = -\frac{\Pi}{2}\right)\right| = 0$$

$$1 + ae^{-j\frac{\Pi}{2}} + be^{-i\Pi} = 0$$

$$|1 - aj - b| = 0$$

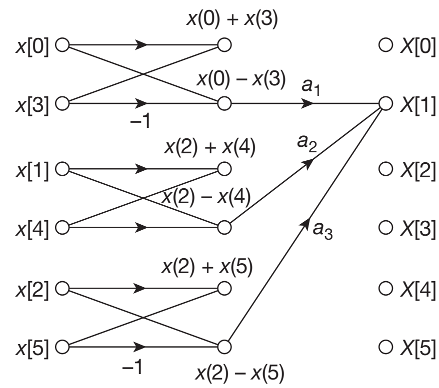
$$|1 - b - aj| = 0$$

$$1 - b = 0 \quad a = 0$$

$$b = 1 \quad a = 0$$

Hence, the correct option is (D).

2. Consider a six-point decimation-in-time. Fast Fourier Transform (FFT) alogorithm, for which the signal-flow graph corresponding to $X[1]$ is shown in the figure. Let $W_6 = \exp\left(-\frac{j2}{6}\right)$ In the figure, what should be the value of the coefficients a_1, a_2, a_3 in terms of W_6 so the $X[1]$ is obtained correctly? **[2019]**



(A) $a_1 = 1, a_2 = W_6^2, a_3 = W_6$

(B) $a_1 = -1, a_2 = W_6^2, a_3 = W_6$

(C) $a_1 = -1, a_2 = W_6, a_3 = W_6^2$

(D) $a_1 = 1, a_2 = W_6, a_3 = W_6^2$

Solution: Six point $DIT - DFT$

$$X(K) = \sum_{n=0}^{5} X[n] W_6^{Kn}$$

$$X(1) = X(0) + X(1)W_6^1 + X(2)W_6^2 + X(3)W_6^3 + X(4)W_6^4 + X(5)W_6^5$$

$$W_n^{K+\frac{N}{2}} = -W_n^K \qquad [\text{property twiddle factor}]$$

$$\therefore W_6^3 = -W_6^O$$

$$W_6^4 = -W_6^1$$

$$W_6^5 = -W_6^2$$

From the graph we have

$$X(1) = [X(0) - X(3)]a_1 + [X(1) - X(4)]a_2 + [X(2) - X(5)]a_3$$

on companion we get

$a_1 = 1$

$a_2 = W_6^1$

$a_3 = W_6^2$

Hence, the correct option is (D).

3. Let $h[n]$ be a length-7 discrete-time finite impulse response filter, given by

$h[0] = 4 \quad h[1] = 3, \quad h[2] = 2 \quad h[3] = 1$

$h[-1] = -3, \quad h[-2] = -2, \quad h[-3] = -1$

and $h[n]$ is zero for $|n| \geq 4$. A length-3 finite impulse response approximation $g[n]$ of $h[n]$ has to be obtained such that

$$E(h, g) = \int_{-\pi}^{\pi} \left|H\left(e^{j\omega}\right) - G\left(e^{j\omega}\right)\right|^2 d\omega$$

is minimized, where $H(e^{j\omega})$ and $G(e^{j\omega})$ are the discrete-time. Fourier transforms of $h[n]$ and $g[n]$, respectively. For the filter that minimizes. $E(h, g)$, the value of $10g[-1] + g[1]$. Rounded off to 2 decimal places, is ______ **[2019]**

Solution: $h[n] = [-1, -2, -3, 4, 3, 2, 1]$

To minimize $E(h, g)$ make a rectangular window of $h[n] = g[n]$

On applying parseval's theorem for *DTFT*.

We get,

$= 10\, g[-1] + g[1]$

$= 10(-3) + 3 = 27$

Hence, the correct answer is 27.

4. Let $X[k] = k + 1, 0 \leq K \leq 7$ be 8-point DFT of a sequence $x[n]$, where $X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi nk/N}$.

The value (correct to two decimal places) of $\sum_{n=0}^{3} x[2n]$ is _______. **[2018]**

Solution: $X(K) = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$$\sum_{n=0}^{3} x[2n] = x[0] + x[2] + x[4] + x[6]$$

$$= 4.5 - 0.5 - 0.5j - 0.5 - 0.5 + 0.5j$$

$$= 4.5 - 1.5 = 3$$

Hence, the correct answer is 2.9 to 3.1.

5. The Discrete Fourier Transform (DFT) of the 4 point sequence

$x[n] = \{x[0], x[2], x[3]\} = \{3, 2, 3, 4\}$ is

$x[k] = \{x[0], x[1], x[2], x[3]\} = \{12, 2j, 0, -2j\}$

If $x_1[k]$ is the DFT of the 12 point sequence $x_1[n] = \{3, 0, 0, 2, 0, 0, 3, 0, 0, 4, 0, 0\}$ the value of $\left|\dfrac{x_1[8]}{x_1[11]}\right|$

is ____ **[2016]**

Solution:

$$x[n] = \{3, 2, 3, 4\}$$

$$x_1[n] = \{3, 0, 0, 2, 0, 0, 3, 0, 0, 4, 0, 0\}$$

$$x_1[n] = X[n/3]$$

Using the following identities

$$x[n] \xleftrightarrow{DFT} X[k]$$

$$x_1[n] = x\left[\frac{n}{s}\right] \xleftrightarrow{DFT} X(3k) = X_1[k]$$

$$X_1[k] = [12, -2j, 0, 2j, 12, -2j, 0, 2j, 12, -2j, 0, 2j]$$

$$\frac{X_1[8]}{X_1[11]} = \left(\frac{12}{2j}\right) = 6.$$

6. A continuous time speech signal $x_a(t)$ is sampled at a rate of 8 kHz and the samples are subsequently grouped in blocks, each of size N. The DFT of each block is to be computed in real time using the radix – 2 decimation in frequency FFT algorithm. If the processor performs all operations sequentially, and takes 20 μs for computing each complex multiplication (including multiplications by 1 and –1) and the time required for addition/subtraction is negligible, then the maximum value of N is **[2016]**

Solution: Frequency $F_m = 8$KHz

Size of block $= N$

All the operations are performed sequentially and $t_{comp} = 20$ μs

Each block is computed using radix 2 DIF FFT algorithm, therefore

$$N = 2^{12} = 4096$$

Hence, the correct Answer is (4096).

7. Consider a discrete time periodic signal $x[n] = \sin\left(\dfrac{\pi n}{5}\right)$. Let a_k be the complex Fourier series coefficients of $x[n]$. The coefficients $\{a_k\}$ are non-zero when $k = Bm \pm 1$, where m is any integer. The value of B is ______. **[2014]**

Solution: 10

Discrete time Fourier series

$$x(n)\sum_{k=0}^{n-1} ake^{jk\left(\frac{2\pi}{n}\right)n}$$

$$= a_{-1}e^{-J\frac{2\pi}{n}} + a_0 + a_1 e^{j\frac{2\pi}{n}n} \quad (1)$$

$$x(n) = \frac{1}{2j}e^{-\frac{J}{5}n} + \frac{1}{2j}e^{J\frac{\pi}{5}n}$$

Also $x(n) = \sin\left(\dfrac{\pi n}{5}\right)$

Where $N = 10$, $\left[\dfrac{2\pi}{N} = \dfrac{\pi}{5} \Rightarrow N = 10\right]$

By comparing, $a_1 = \frac{1}{2J}$ and $a_9 = a-1 = \frac{-1}{2j}$

as, the TFS Coeff. a_k is also periodic with $N = 10$

$a_1 = a_{11} = a_{21} \ldots$

$a_1 = a_9 = a_{19} \ldots$

Given that a_k is non-zero for $k = Bm \pm 1$

$$Bm \pm 1 = 1 \quad 9 \ldots$$
$$-1 \quad 11 \ldots$$

For $m = 0$ for $m = 1$

To satisfy this condition, B must be '10' where 'm' is any integer.

8. The DFT of a vector $[a\ b\ c\ d\ ]$ is the vector $[\alpha \beta \gamma \delta]$. Consider the product.

$$\begin{bmatrix} p & q & r & s \end{bmatrix} = \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{bmatrix}$$

The DFT of the vector $[\, p\ q\ r\ s]$ is a scaled versions of **[2013]**

(a) $\left[\alpha^2\ \beta^2\ \gamma^2\ \delta^2\right]$

(b) $\left[\sqrt{\alpha}\ \sqrt{\beta}\sqrt{\gamma}\ \sqrt{\delta}\right]$

(c) $\left[\alpha+\beta\ \ \beta+\delta\ \ \delta+\gamma\ \ \gamma+\alpha\right]$

(d) $\left[\alpha\ \ \beta\ \ \ddagger\ \ \delta\right]$

Solution: (a)

Given DFT of vector $[a, b, c, d\,]$ is $[\alpha, \beta, \gamma, \delta]$

i.e.

$$\begin{bmatrix} \lambda \\ \gamma \\ \beta \\ \delta \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} a & +b & +c & +d \\ a & -jb & -c & +jd \\ a & -b & +c & -d \\ a & +jb & -c & -jd \end{bmatrix}$$

Given

$$\begin{bmatrix} p & q & r & s \end{bmatrix} = \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{bmatrix} \qquad (1)$$

$$\left[a^2 + bd + c^2bd \quad ab + ab + cd + cd \quad 2ac + b^2 + d^2 \quad 2ad + 2bc\right] \qquad (2)$$

DFT of $pqrs$ is given as:

$$\begin{bmatrix} p & q & r & s \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= [(p+q+r+s) \quad (p - jq - pr + s)$$
$$(p-q+r-s)\, p + jq - r - js]$$
$$= \left[\lambda^2 \quad \beta^2 \quad \gamma^2 \quad \delta^2\right]$$

From (1) and (2)

$$p+q+r+s = (a^2 + c^2 + 2bd) + (2ab + 2cd)$$
$$+ (b^2 + d^2 + 2ac) + (2ad + 2bc)$$
$$= (a+b+c+d)^2 = a^2$$

Hence, the correct option is (a).

9. The first six points of the 8-point DFT of a real valued sequence are 5, 1 $-j3$, 0, 3 $-j4$, and 3 + $j4$. The last two points of the DFT are respectively. **[2011]**

(a) 0, 1 $-j3$ (b) 0, 1 + $j3$

(c) 1 + $j3$, 5 (d) 1 $-j3$, 5

Solution: (b)

Given $x(n) \rightarrow$ real

$x(k) \rightarrow$ conjugate symmetric

$$\Rightarrow x[k] = x^*[n-k]$$
$$n = 8$$
$$\Rightarrow x[k] = x^*[8 \cdot k]$$
$$x[6] = x^*[8-6] = x^*[2] = 0$$
$$x[7] = x^*[8-7] = x^*[1] = 1 + j3$$

Hence, the correct option is (b).

10. The four-point discrete Fourier Transform (DFT) of a discrete time sequence {1, 0, 2, 3} is **[2009]**

(a) [0, –2 + 2j, 2, –2 – 2j]

(b) [2, 2 + 2j, 6, 2 – 2j]

(c) [6, 1 – 3j, 2, 1 + 3j]

(d) [6, –1 + 3j, 0, –1 – 3j]

Solution: (d)

q Point DFT of sequence {1, 0, 2, 3} is given by

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1+2+3 \\ 1-2+3j \\ 1+2-3 \\ 1-2-3j \end{bmatrix} = \begin{bmatrix} 6 \\ -1+3j \\ 0 \\ -1-3j \end{bmatrix}$$

Hence, the correct option is (d).

11. $\{x(n)\}$ is a real-valued periodic sequence with a period N. $x(n)$ and $X(k)$ form N-point Discrete Fourier Transform (DFT) pairs. The DFT $Y(k)$ of the sequence **[2008]**

$$y(n) = \frac{1}{N}\sum_{r=0}^{N-1} x(r)\,x(n+r) \text{ is}$$

(a) $|X(K)|^2$

(b) $\frac{1}{N}\sum_{r=0}^{N-1} X\ (r)X^{*}(k+r)$

(c) $\frac{1}{N}\sum_{r=0}^{N-1} X\ (r)X\ (k+r)$

(d) 0

Solution: (a)

DFT of $y(n) = |x(k)|^2$

Hence, the correct option is (a).

12. A five-point sequence $x[n]$ is given as $x[-3] = 1$, $x[-2] = 1$, $x[-1] = 0$, $x[0] = 5$, $x[1] = 1$.
Let $X\left(e^{j\omega}\right)$ denote the discrete-time Fourier transform of $x[n]$. The value of $\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)d\omega$ is **[2007]**

(a) 5 (b) $10\ \pi$

(c) $16\ \pi$ (d) $5 + \varphi\ 10\ \pi$

Solution: (b)

Given $X(e^{j\omega}) = e^{3j\omega} + e^{2j\omega} + 0 + 5 + e^{-j\omega}$

$$\because \int_{-\pi}^{\pi} e^{aj\omega}d\omega = 0 \quad if \quad a \neq 0$$

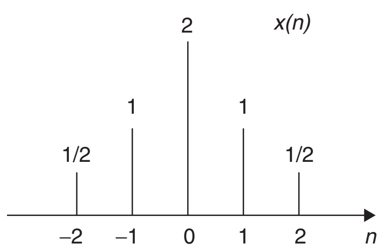
$$\Rightarrow \int_{-\pi}^{\pi} X(e^{j\omega})d\omega = 0\left|\frac{e^{3j\omega}}{3j} + \frac{e^{2j\omega}}{2j} + 5\omega + \frac{e^{-j\omega}}{-j}\right|_{-\pi}^{1}$$

$$= 5\pi + 5\pi = 10\pi$$

Hence, the correct option is (b).

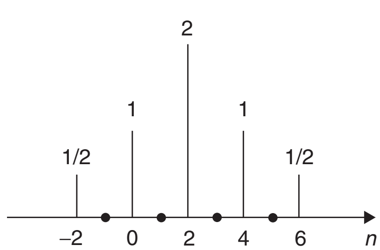
Statement of Linked answer Question 7 and 8

A sequence $x(n)$ has non-zero values as shown in the figure

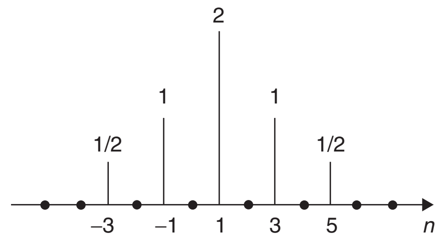


13. The sequence $y(n) = \left\{x\left(\frac{n}{2}-1\right)\right.$ for n even will be = 0, for n odd **[2005]**

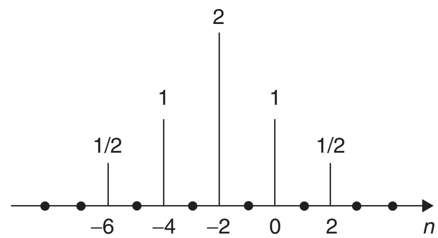
(a)



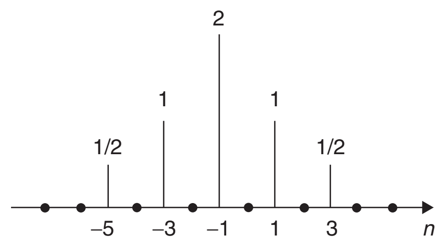
(b)



(c)



(d)



Solution: (a)

Given $y(n) = x\left(\frac{n}{2}-1\right)$ for n even

$=0$ n odd

So, $n = 0$, $y(0) = x(-1) = 1$

$n = 2$, $y(2) = x(0) = 2$

$n = 4$, $y(4) = x(1) = 1$

$n = 6$, $y(6) = x(2) = ½$

Hence, the correct option is (a).

14. The Fourier transform of $y(2n)$ will be **[2005]**

(a) $e^{-j2\omega}[\cos 4\omega + 2\cos 2\omega + 2]$

(b) $[\cos 2\omega + 2\cos \omega + 2]$

(c) $e^{-j\omega}[\cos 2\omega + 2\cos\omega + 2]$

(d) $e^{-\frac{j\omega}{2}}[\cos 2\omega + 2\cos \omega + 2]$

Solution: (c)

Given $y(2n) = x(n-1)$

From graph $f(n) = y(2n)$

$$= \frac{1}{2}f(n+1) + f(n) + 2f(n-1)$$

$$+ f(n-2) + \frac{1}{2}f(n-3)$$

Taking 2-transform

$$F(z) = \frac{1}{2}z + 1 + 2z^{-1} + z^{-2} + \frac{1}{2}z^{-3}$$

Put $z = e^{j\omega}$

$$F(e^{j\omega}) = \frac{1}{2}e^{j\omega} + 1 + 2e^{j\omega} + e^{-2j\omega} + \frac{1}{2}e^{-3j\omega}$$

$$= e^{-j\omega}\left[\frac{e^{2j\omega} + e^{-2j\omega}}{2} + e^{j\omega} + e^{-j\omega} + 2\right]$$

$f(x) = e^{-J\omega}\,[\cos 2\omega + 2\cos\omega + 2]$

Hence, the correct option is (c).

15. A signal $x(n) = \sin(\omega_0 n + f)$ is the input to a linear time-invariant system having a frequency response $H\left(e^{j\omega}\right)$. If the output of the system is $Ax(n - n_0)$, then the most general form of $\angle H(e^{j\omega})$ will be **[2005]**

(a) $-n_0\omega_0 + \beta$ for any arbitrary real β

(b) $-n_0\omega_0 + 2\pi k$ for any arbitrary integer k.

(c) $n_0\omega_0 + 2\omega k$ for any arbitrary integer k.

(d) $-n_0\omega_0\phi$

Solution: (b)

$y(n) = A\times(n - n_o) = A\sin[\omega_o(n - n_o) + f]$

Given, $$-\frac{d\theta(\omega)}{d\omega} = n_o (= t_g)$$

$\Rightarrow \theta(\omega) = n_o \int d\omega = -n_o\omega_o + k$

So, to avoid phase change k should be an integral multiple of 2π

$\therefore \theta(\omega) = -n_o\omega_o + 2\pi k$.

Hence, the correct option is (b).

Chapter 8

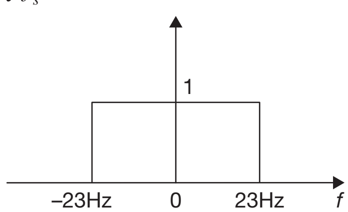
Sampling

ONE-MARK QUESTIONS

1. A continuous time sinusoid of frequency 33 Hz is multiplied with a periodic dirac impulse train of frequency 46 Hz. The resulting signal is passed through an ideal analog low pass filter with a cutoff frequency of 23 Hz. The fundamental frequency (in Hz) of the output is _____. **[2016]**

Solution: Frequency $f_m = 33$ Hz

Frequency $f_s = 46$ Hz



$f_{o/p} = n.fs \pm f_m$.

$n = 1, 2, \ldots\ldots$ etc

(f(o/p) = 46 + 33, 46 – 33, 92 – 33, 92 + 33 … etc. Hence, we get frequency of o/p is 13 Hz, 79 Hz….. etc. But LPF has cut-off $f_c = 23$ Hz so it allows up to 23 Hz only. Hence only 13 Hz will pass through LPF)

$$f_{o/p} = \pm 33 Hz, 46 \pm 33 \text{ Hz.....etc.}$$

$$f_{o/p} = 13 \text{ Hz}, 33 \text{ Hz}, 79 \text{ Hz .....etc.}$$

But $f_c = 23$ Hz so it allows up to 23 Hz only.

Hence, the correct Answer is (13).

2. Consider the signal $x(t) = \cos(6\pi t) + \sin(8\pi t)$, where t is in seconds. The Nyquist sampling rate (in samples/second) for the signal $y(t) = x(2t + 5)$ is **[2016]**

(A) 8 (B) 12
(C) 16 (D) 32

Solution: We know that

$$x(t) = \cos(6\pi t) + \sin(8\pi t)$$

and $$y(t) = x(2t + 5)$$

As we know that Nyquist sampling rate = $2 f_m$ where f_m is the maximum frequency of the input signal and for the given signal $x(t)$, $f_m = 8$ Hz.

Apply time multiplication and shifting property. It is a periodic signal so shifting does not affect the overall time period.

Let, $y(t) = x(a\,t + b)$

$a = 2$. [$a > 1$, so time compressor]

($\therefore$ $y(t) = x(2t)$ No effect of time shift)

$$f \,\alpha\, \frac{1}{T}$$

$\therefore$ $x(t)$ Nyquist sampling rate = $2f_m$ samples/second.

$y(t)$ Nyquist sampling rate = $2 \times f_{sx(t)} = 2 \times 8 = 16$ samples/sec

Hence, the correct option is (C).

3. An analog baseband signal, bandlimited to 100 Hz, is sampled at the Nyquist rate. The samples are quantized in to four message symbols that occur independently with probabilities $P_1 = P_4 = 0.125$ and $P_2 = P_3$. The information rate (bits/sec) of the message source is _____ . **[2016]**

Solution: Frequency $f_m = 100$Hz

Probability $P_1 = P_4 = \frac{1}{8} = 0.125$

Probability $P_2 = P_3 = \frac{3}{8}$

Number of quantization levels (L) = 4

And we know

$$L = 2^n$$

So, $$4 = 2^n$$

$$\Rightarrow \quad 2^2 = 2^n$$

$$\Rightarrow \quad n = 2$$

Entropy can be calculated as

$$H = -\{P_1 \log_2 P_1 + P_2 . \log_2 P_2 + P_3 . \log_2 P_3 + P_4 . \log_2 P_4\}$$

$$= \left\{2 \times \frac{1}{8}.\log_2 8 + 2 \times \frac{3}{8}.\log_2 {}^{8}\!/\!_{3}\right\}$$

$$= \left\{\frac{1}{4} \times 3 + \frac{3}{4} \times 1.414\right\}$$

= 1.81 bits/symbols

Information rate can be calculated as

$$R = n \times f_m \times H$$
$$= 2 \times 100 \times 1.81$$
$$= \quad 200 \times 1.81 = 362.0 \text{ bits/sec}$$

Hence, the correct Answer is (362.0 bits/sec).

4. Consider a continuous-time signal defined as

$$x(t) = \left(\frac{\sin(\pi t/2)}{(\pi t/2)}\right) * \sum_{n=-\infty}^{\infty} \delta(t - 10n)$$

Where '*' denotes the convolution operation and t is in seconds. The Nyquist sampling rate (in samples/sec) for $x(t)$ is _______. **[2015]**

Solution: $x(t) = \dfrac{\sin(\pi 2)}{(\pi t/2)} \otimes \displaystyle\sum_{n=-\infty}^{+\infty} \delta(t - 10n)$

As we know that convolution in time domain is equal to multiplication in frequency domain

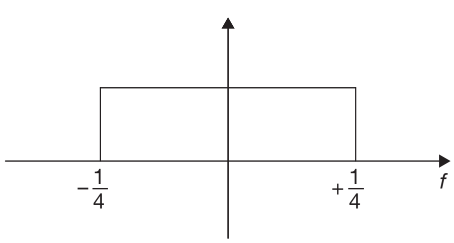
So $\displaystyle\sum_{n=-\infty}^{+\infty} \delta(t-10n) \overset{F.T}{\leftrightarrow} \frac{1}{10} \sum_{n=-\infty}^{+\infty} \delta(f - kf_s)$

$$= \frac{1}{10} \sum_{n=-\infty}^{+\infty} \delta(f - kf_s)$$

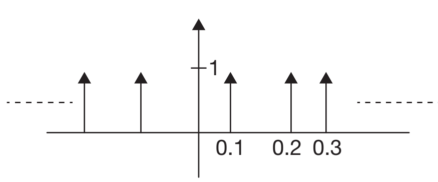
and $F_s = \dfrac{1}{T_s} = 0.1 \dfrac{1}{10} \displaystyle\sum_{n=-\infty}^{+\infty} \delta(f - 0.1k)$

So $\displaystyle\sum_{n=-\infty}^{t_\infty} \delta(t-10n) \xleftrightarrow{F.T} \frac{1}{10} \sum_{n=-\infty}^{+\infty} \delta(f - 0.1k)$

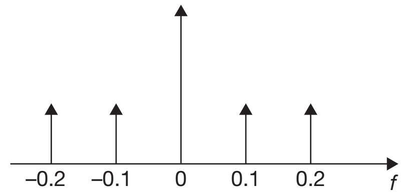
$$\frac{\sin(\pi t/z)}{(\pi t/z)} \xleftrightarrow{F.T}$$



$$\sum_{n=-\infty}^{+\infty} \delta(t-10n) \xleftrightarrow{F.T}$$



After multiplication



So Nyquist sampling rate = 2 × 0.2

= 0.4 samples/sec

Hence, the correct Answer is (0.39 to 0.41).

5. A sinusoidal signal of 2 kHz frequency is applied to a delta modulator. The sampling rate and step-size Δ of the delta modulator are 20,000 samples per second and 0.1 V, respectively. To prevent slope overload, the maximum amplitude of the sinusoidal signal (in Volts) is **[2015]**

(A) $\dfrac{1}{2\pi}$ (B) $\dfrac{1}{\pi}$

(C) $\dfrac{2}{\pi}$ (D) π

Solution: As we know that in delta modulator to prevent slope overload

$$\frac{\Delta}{T_S} \geq \frac{sm(t)}{dt}$$

$$\frac{\Delta}{T_S} \geq A\omega_m$$

Where, Δ = step size

T_s = sampling period

A = Amplitude of message signal.

ω_m = frequency of message signal

$$0.1 \times 20{,}000 = A \times 2\pi \times 2 \times 10^3$$

$$\therefore \qquad A = \frac{1}{2\pi}$$

Hence, the correct option is (A).

6. The signal $\cos\left[10\pi t + \dfrac{\pi}{4}\right]$ is ideally sampled at a sampling frequency of 15 Hz. The sampled signal is passed through a filter with impulse response $\left(\dfrac{\sin(\pi t)}{\pi t}\right)\cos\left(40\pi t - \dfrac{\pi}{2}\right)$. The filter output is **[2015]**

(A) $\dfrac{15}{2}\cos\left(40\pi t - \dfrac{\pi}{4}\right)$

(B) $\dfrac{15}{2}\left(\dfrac{\sin(\pi t)}{\pi t}\right)\cos\left(10\pi t + \dfrac{\pi}{4}\right)$

(C) $\frac{15}{2}\cos\left(10\pi t-\frac{\pi}{4}\right)$

(D) $\frac{15}{2}\left(\frac{\sin(\pi t)}{\pi t}\right)\cos\left(40\pi t-\frac{\pi}{2}\right)$

Solution: Let $x(t) = \cos\left(10\pi t+\frac{\pi}{4}\right)$

By ignoring phase

$$x'(t) = \cos(10\pi t)$$

$$X'(f) = \frac{1}{2}[\delta(f-5) + \delta(f+5)]$$

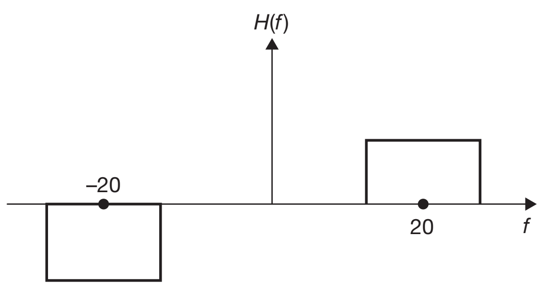
Impulse response

$$h(t) = \left(\frac{\sin \pi t}{\pi t}\right)\cos\left(40\pi t-\frac{\pi}{2}\right)$$

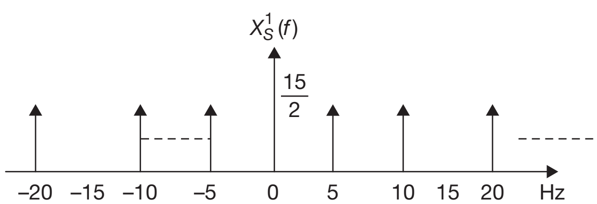
$$= \sin c\,\pi\, t \sin 40\,\pi\, \mathrm{t}$$

$$H(f) = \text{rect} \otimes \frac{1}{2j}[\delta(f-20) - \delta(f+20)]$$

So



After sampling of $X'(f)$ with 15 Hz the signal will repeat after 15 Hz.



And the amplitude will depend upon that how much amplitude impulse train is used to sample the $x(t)$.

So $Y(f) = X_s'(f)H(f)$

By seeing the spectrum only one delta function will be present at 20 kHz.

So $\frac{1}{2j}(\delta[f-20] - \delta(f+20))$

$$y(t) = \frac{15}{2}\sin(40\pi t)$$

$$= \frac{15}{2}\cos\left(40\pi t-\frac{\pi}{2}\right)$$

By adding phase shift

$$= \frac{15}{2}\cos\left(40\pi t-\frac{\pi}{2}+\frac{\pi}{4}\right)$$

$$= \frac{15}{2}\cos\left(40\pi t-\frac{\pi}{4}\right)$$

Hence, the correct option is (A).

7. Consider two real valued signals, $x(t)$ band-limited to [–500 Hz, 500 Hz] and $y(t)$ band-limited to [–1 Hz, 1 Hz]. For $z(t) = x(t)\,y(t)$. The Nyquist sampling frequency (in kHz) is ______. **[2014]**

Solution: 3000

Using multiplication property,

$$X_1(t)\cdot X_2(t) = X_1(\omega) * X_2(\omega)$$

So highest f_2 contained by convolve signal $z(t) = 1500H_3$

$\therefore$ Nyguist rate = 2 × 1500 = $3000H_3$.

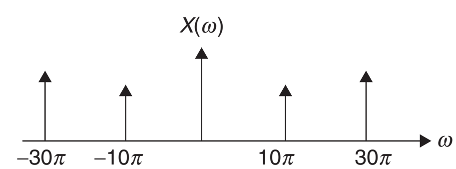
8. Let $x(t) = \cos(10\pi t) + \cos(30\pi t)$ be sampled at 20 Hz and reconstructed using an ideal low-pass filter with cut-off frequency of 20 Hz. The frequency/frequencies present in the reconstructed signal is/are **[2014]**

(a) 5 Hz and 15 Hz only
(b) 10 Hz and 15 Hz only
(c) 5 Hz, 10 Hz and 15 Hz only
(d) 5 Hz only

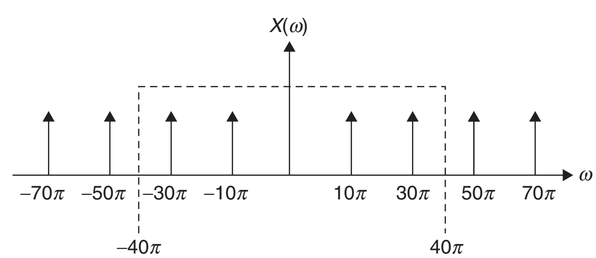
Solution: (a)

Given $x(t) = \cos 10\pi t + \cos 30\pi t$

Given $f_S = 20H_3, \Rightarrow \omega_S = 40\pi\, {}^{rad}/_{\sec}$



$$y(\omega) = \sum_{k=-} X(\omega - k\omega_S)$$



Applying LPF we will get 10π and 30π at o/p or $5H_3$, $15kH_3$

Hence, the correct option is (a).

9. For a given sample-and-hold circuit, if the value of the hold capacitor is increased, then **[2014]**

(a) drop rate decreases and acquisition time decreases
(b) drop rate decreases and acquisition time increases

(c) drop rate increases and acquisition time decreases
(d) drop rate increases and acquisition time increases

Solution: (b)
We know

$$Q = CV = it \qquad (1)$$

$$\text{So } t = \frac{CV}{i} \qquad (2)$$

From (2) $t \lambda C \Rightarrow$ as capacitance will increase the acquisition time will increase for capacitor drop rate is given as $\frac{dv}{dt}$

$$i = c\frac{dv}{dt} \quad \text{or} \quad \frac{dx}{dt} \times \frac{i}{c}$$

$$\Rightarrow \frac{dv}{dt} = \frac{i}{c}$$

As capacitor value increases, the voltage drop rate decreases.
Hence, the correct option is (b).

10. A band-limited signal with a maximum frequency of 5 KHz is to be sampled. According to the sampling theorem, the sampling frequency which is not valid is **[2013]**

(a) 5 kHz (b) 12 kHz
(c) 15 kHz (d) 20 kHz

Solution: (a)
Given

$f_S \min = 2f_m$

$(f_S)\min = 2 \times 5 = 10kH_3.$

So $f_S \geq 10kH_3$
Hence, the correct option is (a).

11. The transfer function of a zero-order-hold system is **[1998]**

(a) $\left(\frac{1}{s}\right)(1+e^{-sT})$ (b) $\left(\frac{1}{s}\right)(1-e^{-sT})$
(c) $1-(1/s)e^{-sT}$ (d) $1+(1/s)e^{-sT}$

Solution: (b)
The impulse response $h(t)$ of zero-order hold system is $h(t) = U(t) - U(t-T)$

$$\text{So, } H(s) = \frac{1}{s} - \frac{1}{s}e^{-ST} = \frac{(1-e^{-ST})}{S}$$

Hence, the correct option is (b).

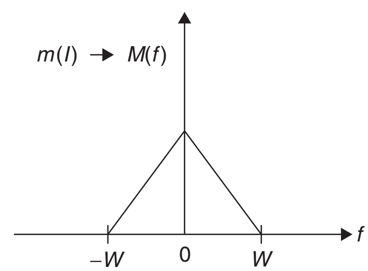
12. Flat top sampling of low pass signals **[1998]**
(a) gives rise to aperture effect
(b) implies oversampling
(c) leads to aliasing
(d) introduce delay distortion

Solution: (a)
Flat-top sampling of low pass signals gives rise to aperture effect.
Hence, the correct option is (a).

13. A 1.0 kHz signal is flat-top sampled at the rate of 1800 samples/sec and the samples are applied to an ideal rectangular LPF with cut-off frequency of 1100 Hz. Then the output of the filter contains **[1995]**
(a) only 800 Hz component
(b) 800 Hz and 900 Hz components
(c) 800 Hz and 1000 Hz components
(d) 800 Hz, 900 Hz and 1000 Hz components

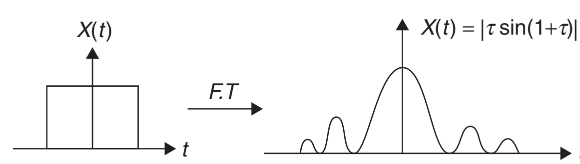
Solution: (c)
Given f_s = 1800 Samples/sec
f_m = 1000 Hz
The Spectrum of Sampled signal would have $nf_s \pm f_m$
So, 1000, 1800 ± 1000 Hz, 3600 ± 1000 Hz…
So, 1000, 800, 2800, 2600, 4600 Hz…
The cutoff frequency of LPF is 1100 Hz. So the o/p filter will contain 800 Hz and 1000 Hz components.
Hence, the correct option is (c).

14. Increased pulse-width in that flat-top sampling leads to **[1994]**



(a) attention of high frequencies in reproduction
(b) attention of low frequencies in reproduction
(c) greater aliasing errors in reproduction
(d) no harmful effects in reproduction

Solution: (a)



As pulse width τ is increased, the width $1/T$ of the first lobe of spectrum is decreased.

$$Y(S) = \frac{S^2+1}{S^2+2S+1} \times \frac{e^S}{S^2+1} = \frac{e^S}{S^2+2S+1} = \frac{e^S}{(S+1)^2}$$

Take inverse Laplace Transform

$$y(t) = (t+1)e^{-(t+1)}$$

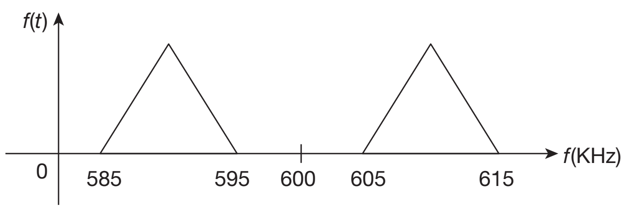
$$y(\ ) = \lim_{s\to 0} SY(S) = \lim_{s\to 0} \frac{S \cdot e^S}{(S+1)^2} = 0$$

So at steady state, Y remains zero for all sampling $f_2 \cdot \omega_s$.
Hence, increased pulse-width in flat-top sampling leads to attenuation of high frequencies in reproduction.
Hence, the correct option is (a).

Two-marks Questions

1. A voice signal $m(t)$ is in the frequency range 5 kHz to 15 kHz. The signal is amplitude-modulated to generate as AM signal $f(t) = A(1 + m(t))\cos 2\pi f_c t$, where f_c = 600 kHz. The AM signal $f(t)$ is to be digitized and archived. This is done by first sampling $f(t)$ at 1.2 times the Nyquist frequency, and then quantizing each sample using a 256-level quantizer. Finally, each quantized sample is binary coded using K bits, where K is the minimum number of bits required for the encoding. The rate, in Megabits per second (rounded off to decimal places). Of the resulting stream of coded bits is ______ Mbps. **[2019]**

Solution:



$$f_N = \frac{2f_H}{K}$$

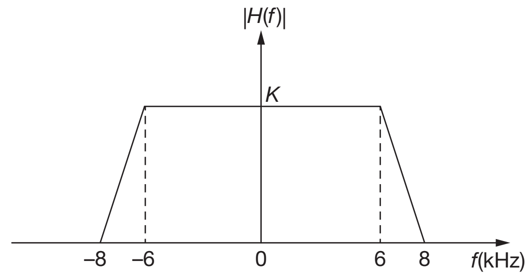
$$K = \left(\frac{f_H}{BW}\right) = \frac{615}{30} = 20$$

$$f_N = \frac{2\times 615}{20} = 61.5 \text{ KHz}$$

$$f_S = 1.2 f_N = 1.2\times 61.5 \text{ KHz} = 73.8 \text{ KHz}$$

$$P_B = nfs = 8\times 73.8 = 590.4 \text{ kbps}$$

2. A band limited low-pass signal $x(t)$ of bandwidth 5 kHz is sampled at a sampling rate f_s. The signal $x(t)$ is reconstructed using the reconstruction filter $H(f)$ whose magnitude response is shown below:



The minimum sampling rate f_s (in kHz) for perfect reconstruction of $x(t)$ is ________. **[2018]**

Solution: Minimum sampling rate f_s should be

$$f_s \geq 5 + 8 \Rightarrow f_s \geq 13$$

So, $f_s(\text{min})$ = 13 kHz
Hence, the correct answer is 13.

3. The signal $x(t) = \sin(14000\pi t)$, where t is in seconds, is sampled at a rate of 9000 samples per second. The sampled signal is the input to an ideal lowpass filter with frequency response $H(f)$ as follows :

$$H(f) = \begin{cases} 1, & |f| \leq 12 \text{ kHz} \\ 0, & |f| > 12 \text{ kHz} \end{cases}$$

What is the number of sinusoids in the output and their frequencies in kHz? **[2017]**
(A) Number = 1, frequency = 7
(B) Number = 3, frequencies = 2,7,11
(C) Number = 2, frequencies = 2,7
(D) Number =2, frequencies = 7, 11

4. A continuous time filter with transfer function $H(s) = \dfrac{2s+6}{s^2+6s+8}$ is converted to a discrete time filter with transfer function $G(z) = \dfrac{2z^2 - 0.5032z}{z^2 - 0.5032z + k}$ so that the impulse response of the continuous time filter, sampled at 2 Hz, is identical at the sampling instants to the impulse response of the discrete time filter. The value of k is _____. **[2016]**

Solution: To find the Inverse Laplace of transfer function $H(s)$ we need to calculate the values using partial fraction formula as given below:

$$H(s) = \frac{2s+6}{s^2 + 6s + 8}$$

$$H(s) = \frac{A}{(s+2)} + \frac{B}{s+4}$$

$$A = H(s)\cdot(s+2) \text{ at } s = -2$$

$$A = \frac{2}{2} = 1$$

$$B = H(s)\,(s+4) \text{ at } s = -4$$

$$B = \frac{-2}{-2} = 1$$

Now we have

$$H(s) = \frac{1}{s+2} + \frac{1}{s+4}$$

$$h(t) = ILT\,\{H(S)\} = (e^{-2t} + e^{-4t})u(t)$$

We know that, $T_S = \dfrac{1}{f_s} = \dfrac{1}{2}$

Substituting $t = n \cdot T_S$ we get

$$h(nT_S) = (e^{-2nT_s} + e^{-4nT_s}) \cdot u(nT_S)$$

$$= e^{-n} \cdot u(n) + e^{-2n} \cdot u(n)$$

$$H(Z) = ZT\{h(nT_S)\}$$

$$H(Z) = \frac{Z}{Z - e^{-1}} + \frac{Z}{Z - e^{-2}}$$

$$= \frac{Z}{Z - 0.367} + \frac{Z}{Z - 0.135}$$

$$= \frac{Z^2 - 0.135Z + Z^2 - 0.367Z}{Z^2 - 0.5032Z + 0.049}$$

$$= \frac{2Z^2 - 0.5032Z}{Z^2 - 0.5032Z + 0.049}$$

$\therefore \quad K = 0.049$

Hence, the correct Answer is (0.049).

5. An LTI system having transfer function $\dfrac{s^2+1}{s^2+2s+1}$ and input $x(t) = \sin(t+1)$ are in steady state. The output is sampled at a rate ω_s rad/s to obtain the final output $\{y(k)\}$. Which of the following is true? **[2009]**

(a) y is zero for all sampling frequencies ω_s
(b) y is non-zero for all sampling frequencies ω_s
(c) y is non-zero for $\omega_s > 2$ but zero for $\omega_s < 2$
(d) y is zero for $\omega_s > 2$ but non-zero for $\omega_s < 2$

Solution: (a)

$$X(s) \rightarrow \boxed{H(s) = \frac{s^2+1}{s^2+2s+1}} \rightarrow X(s)$$

$x(t) = \sin(t+1) \rightarrow \omega = 1$

Laplace transform

$$X(s) = \frac{\omega}{s^2+\omega^2}e^s = \frac{1}{1^2+s^2}e^s = \frac{e^s}{s^2+1}$$

$\Rightarrow y(s) = H(s)^*(s)$

Hence, the correct option is (a).

6. A signal $m(t)$ with bandwidth 500 Hz is first multiplied by a signal $g(t)$ where

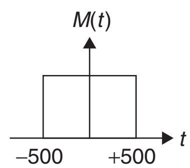
$$g(t) = \sum_{k=-} (-1)^k \delta\,(t - 0.5\times10^{-4}k)$$

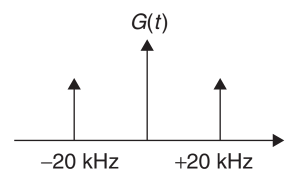
The resulting signal is then passed through an ideal low pass filter with bandwidth 1 kHz. The output of low pass fitter would be **[2006]**

(a) $\delta(t)$ (b) $m(t)$
(c) 0 (d) $m(t)\,\delta(t)$

Solution: (b)

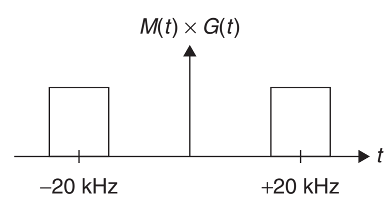
Given





$m(t)\,g(t) \leftrightarrow M(t)\,G(t)$

So,



low-pass filter $fe = 1$ kHz, so $o/p = 0$

Hence, the correct option is (b).

7. A 1 kHz sinusoidal signal is ideally sampled at 1500 samples/sec and the sampled signal is passed through an ideal low-pass filter with cut-off frequency 800 Hz. The output signal has the frequency **[2004]**

(a) zero Hz (b) 0.75 kHz
(c) 0.5 kHz (d) 0.25 kHz

Solution: (c)

Given $fs = 1.5$ kHz,
$fm = 1$ kHz
available frequency components $= nfs \pm fm$
So, 1 kHz, 2.5 kHz, 0.5 kHz.
$\because$ LPF has $fe = 0.8$ kHz.
So only 0.5 kHz will be at o/p.
Hence, the correct option is (c).

8. The transfer function of a zero-order hold is **[1988]**

(a) $\dfrac{1-\exp(-Ts)}{s}$ (b) $1/s$
(c) 1 (d) $1/[-\exp(-Ts)]$

Solution: (a)

Given Impulse response of system is

$$h(t) = u(t) - u(t-T),\ T \rightarrow \text{sampling period.}$$

Taking Laplace transform.

$$H(S) = \frac{1}{S} - \frac{1}{S}e^{-TS} = \frac{1-e^{-TS}}{S}$$

Hence, the correct option is (a).

9. A signal containing only two frequency components (3 kHz and 6 kHz) is sampled at the rate of 8 kHz, and then passed through a low pass filter with a cut-off frequency of 8 kHz. The filter output **[1988]**

(a) is an undistorted version of the original signal
(b) contains only 3 kHz component
(c) contains the 3 kHz component and a spurious component of 2 kHz
(d) contains both the components of the original signal and two spurious components of 2 kHz and 5 kHz

Solution: (d)
fs = 9000 samples/sec.

$$fm_1 = 3kH_3, \quad fm_2 = 6kH_3$$

Then spectrum of sampled signal would have $nfs \pm fm$
So, $3kH_3$, 8 ± 3, $16 \pm 3\ldots = 3kH_3$, $5kH_3$, $11kH_3$, …
$6kH_3$, 8 ± 6, $16 \pm 6\ldots = 6kH_3$, $2kH_3$, $14kH_3$, …
Cut of frequency of low pass filter = $8kH_3$,
So filter output would have,
$3kH_3$, $6kH_3$, $2kH_3$ and $5kH_3$
Hence, the correct option is (d).

UNIT III

CONTROL SYSTEMS

EXAM ANALYSIS

Exam Year	92	93	94	95	96	97	98	99	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14-1	14-2	14-3	14-4	15			16			17		18	19
																											Set 1	Set 2	Set 3	Set 1	Set 2	Set 3	Set 1	Set 2		
1 Marks Ques.	-	-	5	8	-	-	9	4	1	4	4	2	2	3	2	1	2	2	3	3	1	1	2	2	1	2	3	2	3	2	2	2	2	3	2	1
2 Marks Ques.	4	2	3	-	-	2	1	3	1	3	4	6	9	7	9	9	7	4	3	3	4	4	3	3	3	3	3	3	3	3	3	2	2	2	3	3
5 Marks Ques.	-	-	1	-	3	2	3	3	2	3	3	-	-	-	-	-	-	-	-	-	9	9	8	8	7	8	-	-	-	-	-	-	-	-	-	-
Total Marks	**8**	**2**	**16**	**8**	**15**	**14**	**26**	**25**	**13**	**25**	**27**	**14**	**20**	**17**	**20**	**19**	**16**	**10**	**9**	**9**	-	-	-	-	-	-	**9**	**8**	**9**	**8**	**8**	**6**	**6**	**7**	**8**	**7**
Chapter wise marks distribution																																				
Basics	-	-	1	1	-	-	1	-	-	3	-	-	2	1	7	2	3	-	1	-	1	-	-	-	-	-		2			1		-		1	-
Block Diagram and SFG	-	-	-	1	-	2	-	-	-	1	-	2	2	-	-	-	-	-	1	4	-	2	-	1	1	-		3			2		1		-	1
Compensators and Controllers	2	-	-	-	-	-	-	1	-	-	-	1	-	2	2	4	2	1	2	-	4	-	-	-	-	-		2			-		2		-	-
Time Response Analysis	-	-	5	4	-	-	5	3	-	1	3	2	2	4	-	2	5	2	-	1	-	2	2	1	2	3		2			3		1		1	-
Stability Analysis	-	2	2	-	-	-	2	2	3	2	7	2	5	4	7	1	4	-	-	-	2	-	1	-	-	2		1			3		2		1	1
Root Locus	2	-	1	-	-	-	-	2	-	1	1	1	1	2	-	2	-	2	-	1	-	-	-	-	2	-		3			1		-		-	-
Frequency Analysis	2	-	2	2	-	-	2	-	-	1	-	3	4	3	2	2	-	4	1	1	-	1	3	2	-	1		4			4		2		1	1
State Space Analysis	2	-	-	-	-	2	-	3	-	-	-	2	4	1	2	6	2	1	4	2	2	4	2	4	2	2		1			-		1		1	1

Chapter 1

Basics

One-mark Questions

1. The output of a standard second-order system for a unit step input is given as

$y(t)=1-\frac{2}{\sqrt{3}}e^{-t}\cos\left(\sqrt{3}t-\frac{\pi}{6}\right)$. The transfer function of the system is **[2015]**

(A) $\frac{2}{(s+2)(s+\sqrt{3})}$ (B) $\frac{1}{s^2+2s+1}$

(C) $\frac{3}{s^2+2s+3}$ (D) $\frac{4}{s^2+2s+4}$

Solution: Given

$$y(t) = 1-\frac{2}{\sqrt{3}}.e^{-t}.\cos\left(\sqrt{3}t-\pi/6\right)$$

we know

$$c(t) = 1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}.\sin(\omega_d t+\theta)$$

$$\omega_d = \sqrt{3}\text{ rad/sec}$$

$$\zeta = \cos\theta = \cos(-\pi/6) = 0.866$$

$$\omega_d = \omega_n.\sqrt{1-\zeta^2}$$

$$\omega_d = \sqrt{\omega_n^2-(\zeta\omega_n)^2}$$

$$3 = \omega_n^2-1$$

$$\omega_n^2 = 4$$

$\Rightarrow$ $\omega_n = 2$ rad/sec

$\therefore$ $$T(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$\therefore$ $$T(s) = \frac{4}{s^2+2s+4}$$

Hence, the correct option is (D).

2. A system with transfer function

$$G(s) = \frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$$

is excited by sin (ωt). The steady-state output of the system is zero at

(a) $\omega = 1$ rad/s (b) $\omega = 2$ rad/s

(c) $\omega = 3$ rad/s (d) $\omega = 4$ rad/s

[2012]

Solution: (c)

Given transfer function

$$H(s) = \frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$$

Input $x(s) = \sin wt$

Taking Laplace transform

$$x(s) = \frac{w}{s^2+w^2}$$

$\therefore$ Output $C(s) = X(s).H(s)$

$$C(s) = \frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}.\frac{w}{s^2+w^2}$$

$$\therefore \underset{t\to\infty}{Lt}\ c(t) = \underset{s\to 0}{Lt}\ s\,c(s)$$

$$= \underset{s\to 0}{Lt}\ s.\frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}.\frac{w}{s^2+w^2}$$

If $s^2+w^2 = s^2+9$ for $\underset{t\to\infty}{Lt}\ c(t)=0$

$w^2 = 9$

$w = 3$ rad/sec, steady-state output will be zero.

Hence, the correct option is (c)

3. A system with the transfer function $\frac{Y(s)}{X(s)} = \frac{s}{s+p}$ has an output $y(t)=\cos\left(2t-\frac{\pi}{3}\right)$ for the input signal $x(t)=p\cos\left(2t-\frac{\pi}{3}\right)$. Then, the system parameter p is

(a) $\sqrt{3}$ (b) $\dfrac{2}{\sqrt{3}}$

(c) 1 (d) $\dfrac{\sqrt{3}}{2}$ **[2010]**

Solution: (b)

$$\frac{y(s)}{x(s)} = \frac{s}{s+p}$$

Phase difference between input and output

$$\phi = \frac{-\pi}{3} - \left(-\frac{\pi}{2}\right) = \frac{\pi}{6} = 30^\circ \text{ and } w = 2 \text{ rad/s.}$$

From the transfer function

$$\phi = 90^\circ - \tan^{-1}\frac{w}{p}$$

$$\therefore 90^\circ - \tan^{-1}\frac{2}{p} = 30^\circ$$

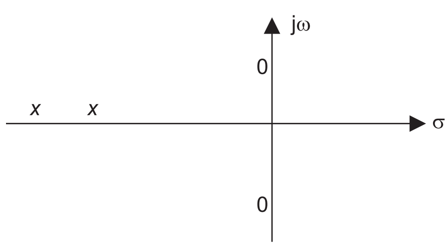
$$\Rightarrow \tan^{-1}\frac{2}{p} = 60^\circ$$

$$\Rightarrow \frac{2}{p} = \tan 60^\circ = \sqrt{3}$$

$$p = \frac{2}{\sqrt{3}}$$

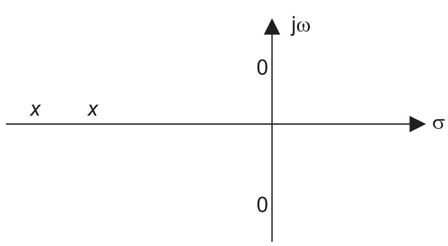
Hence, the correct option is (b)

4. The pole-zero plot given below corresponds to a



(a) Low-pass filter (b) High-pass filter
(c) Band-pass filter (d) Notch filter **[2008]**

Solution: (d)

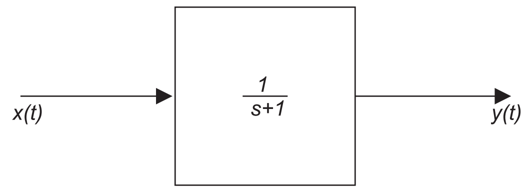


$$\therefore G(s) = \frac{s^2 + as + b}{s^2 + ps + q}$$

which is the transfer function of notch filter.

Hence, the correct option is (d)

5. In the system shown below, $x(t) = (\sin t)u(t)$. In steady-state, the response $y(t)$ will be

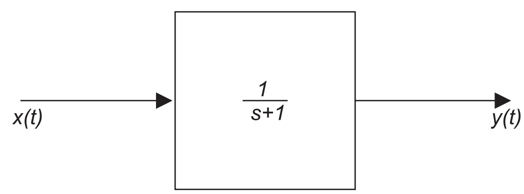


(a) $\dfrac{1}{\sqrt{2}}\sin\left(t - \dfrac{\pi}{4}\right)$ (b) $\dfrac{1}{\sqrt{2}}\sin\left(t + \dfrac{\pi}{4}\right)$

(c) $\dfrac{1}{\sqrt{2}}e^{-t}\sin t$ (d) $\sin t - \cos t$

[2006]

Solution: (a)



gives $x(t) = (\sin t)\, u(t)$.

$\therefore$ In steady-stake

$y(t) = n(t) + h(t)$

$\therefore y(s) = x(s).\, H(s)$

$$H(jw) = \frac{1}{s+1} = \frac{1}{\sqrt{2}}\angle -45^\circ$$

$$= \frac{1}{\sqrt{2}}\angle -\frac{\pi}{4}$$

$x(t) = \sin(t)\, u(t)$

$$y(t) = \frac{1}{\sqrt{2}}\sin\left(t - \frac{\pi}{4}\right)$$

Hence, the correct option is (a)

6. Despite the presence of negative feedback, control systems still have problems of instability because the

(a) components used have nonlinearities
(b) dynamic equations of the systems are not known exactly
(c) mathematical analysis involves approximations
(d) system has large negative phase angle at high frequencies **[2005]**

Solution: (a)

Hence, the correct option is (a)

7. The transfer function of a tachometer is of the form

(a) Ks (b) $\frac{K}{s}$

(c) $\frac{K}{(s+1)}$ (d) $\frac{K}{s(s+1)}$ **[1998]**

Solution: (a)

The tachometer input is assumed to be shaft position. Thus, it differentiates shaft position to angular rate and multiplies it by the gain, So, transfer function of a tachometer is of Ks.

Hence, the correct option is (a)

8. The transfer function of a linear system is the

(a) ratio of the output, $v_0(t)$ and input $v_i(t)$.
(b) ratio of the derivatives of the output and the input.
(c) ratio of the Laplace transform of the output and that of the input with all initial conditions zeros.
(d) none of these **[1995]**

Solution: (c)

Given a linear system, the transfer function, $G(s)$, of the system is the ratio of the transform of the output to the transform of the input with all initial conditions zero.

Hence, the correct option is (c)

9. Tachometer feedback in a d.c. position control system enhances stability. State True or False. **[1994]**

Solution: True

Tachometers are electromechanical devices that convert mechanical energy into electrical energy. It is a derivative feedback. So, it adds zero at origin. Hence, it improves the damping characteristics of the system.

Two-marks Questions

1. Negative feedback in a closed-loop control system does not **[2015]**

(A) reduce the overall gain
(B) reduce bandwidth
(C) improve disturbance rejection
(D) reduce sensitivity to parameter variation

Solution: Negative feedback effects in closed loop system:

(i) Reduces the overall gain
(ii) Bandwidth increases
(iii) Reduces sensitivity to parameter variation
(iv) Reduces noise effect
(v) Improves disturbance rejection ratio

Hence, the correct option is (B).

2. A linear, time-invariant, causal continuous time system has a rational transfer function with simple poles at $s = -2$ and $s = -4$, and one simple zero at $s = -1$. A unit step $u(t)$ is applied at the input of the system. At steady state, the output has constant value of 1. The impulse response of this system is

(a) $[\exp(-2t) + \exp(-4t)]\, u(t)$
(b) $[-4\exp(-2t) + 12\exp(-4t) - \exp(-t)]\, u(t)$
(c) $[-4\exp(-2t) + 12\exp(-4t)]\, u(t)$
(d) $[-0.5\exp(-2t) + 1.5\exp(-4t)]\, u(t)$ **[2008]**

Solution: (c)

given transfer function

$$G(s) = \frac{k(s+1)}{(s+2)(s+4)}$$

Unit step input, $R(s) = \frac{1}{s}$

Output $C(s) = R(s)\,.\,G(s)$

$\therefore \underset{s\to 0}{Lt}\; s\,c(s) = 1$

$$\Rightarrow \underset{s\to 0}{Lt}\; \frac{s.k(s+1)}{s(s+2)(s+4)} = 1$$

$$\Rightarrow \frac{k}{8} = 1$$

$\therefore k = 8$

$$G(s) = \frac{8(s+1)}{(s+2)(s+4)}$$

Applying partial fraction

$$\frac{8(s+1)}{(s+2)(s+4)} = \frac{A}{s+2} + \frac{B}{s+4}$$

$8(s+1) = A(s+4) + B(s+2)$

Let $S = -4$

$-24 = -2B \quad \therefore B = 12$

Let $S = -2$

$\Rightarrow -8 = 2A \quad \therefore A = -4$

$$\therefore G(s) = \frac{-4}{s+2} + \frac{12}{s+4}$$

$g(t) = (-4e^{-2t} + 12e^{-4t})\, u(t)$

which is the impulse response of the system.

Hence, the correct option is (c)

3. The frequency response of a linear time-invariant system is given by

$$H(f) = \frac{5}{1 + j10\pi f}.$$

The step response of the system is

(a) $5(1 - e^{-5t})\, u(t)$ (b) $5(1 - e^{-t/5})\, u(t)$

(c) $\frac{1}{5}\left(1 - e^{-5t}\right)u(t)$ (d) $\frac{1}{(s+5)(s+1)}$

[2007]

Solution: (b)

Frequency response

$$H(f)=\frac{5}{1+j10\pi f}$$

$$H(s)=\frac{5}{1+5s}=\frac{5}{5\left(\frac{1}{5}+s\right)}=\frac{1}{\left(s+\frac{1}{5}\right)}$$

Step response $G(s) = R(s).\ H(s)$ $\quad \therefore R(s)=\frac{1}{s}$

$$C(s)=\frac{1}{s}\times\frac{1}{\left(s+\frac{1}{5}\right)}=\frac{A}{s}+\frac{B}{\left(s+\frac{1}{s}\right)}$$

$$\Rightarrow 1=A\left(s+\frac{1}{5}\right)+BS$$

Let s = 0

$$1=\frac{1}{5}A \quad \therefore A = 5$$

Let $S=-\frac{1}{5}$

$$1=-\frac{1}{5}B \qquad \therefore B = -5$$

$$C(s)=\frac{5}{s}-\frac{5}{\left(s+\frac{1}{5}\right)}$$

$\therefore c(t) = 5[1 - e^{-t/5}]\ u(t)$

Hence, the correct option is (b)

4. Consider two transfer functions

$$G_1(s)=\frac{1}{s^2+as+b} \text{ and } G_2(s)=\frac{s}{s^2+as+b}$$

The 3-dB bandwidths of their frequency responses are, respectively

(a) $\sqrt{a^2-4b},\sqrt{a^2+4b}$

(b) $\sqrt{a^2+4b},\sqrt{a^2-4b}$

(c) $\sqrt{a^2-4b},\sqrt{a^2-4b}$

(d) $\sqrt{a^2+4b},\sqrt{a^2+4b}$ **[2006]**

Solution: (c)

For the given transfer function.

$$G_1(s)=\frac{1}{s^2+as+b}$$

Taking determinant of the characteristics equation

D of $G_1=\sqrt{a^2-4b}$

$\therefore$ 3-dB bandwidth $=\sqrt{a^2-4b}$

$$G_2(s)=\frac{s}{s^2+as+b}$$

$\therefore$ Determinant of the characteristics equation

$$\text{D of } G_2=\sqrt{a^2-4b}$$

$\therefore$ 3 dB-Bandwidth of $G_2=\sqrt{a^2-4b}$

Hence, the correct option is (c)

5. The unit step response of a system starting from rest is given by

$c(t) = 1 - e^{-2t}$ for $t \geq 0$

The transfer function of the system is

(a) $\frac{1}{1+2s}$ (b) $\frac{2}{2+s}$

(c) $\frac{1}{2+s}$ (d) $\frac{2s}{1+2s}$ **[2006]**

Solution: (b)

$c(t) = 1 - e^{-2t}$ – (unit step response)

$$\therefore\ c(s)=\frac{1}{s}-\frac{1}{s+2}=\frac{s+2-3}{s(s+2)}=\frac{2}{s(s+2)}$$

$$\therefore R(s)=\frac{1}{s}\therefore\ H(s)=\frac{C(s)}{R(s)}=\frac{2}{2+S}$$

Hence, the correct option is (b)

6. The unit impulse response of a system is $h(t) = e^{-t}$, $t \geq 0$.

For this system, the steady-state value of the output for unit step input is equal to

(a) -1 (b) 0

(c) 1 (d) ∞ **[2006]**

Solution: (c)

Unit impulse response of a system

$h(t) = e^{-t}$, $t \geq 0$

$$\therefore H(s)=\frac{1}{s+1}$$

$$R(s)=\frac{1}{s}$$

$\therefore$ Output $C(s)=H(s).R(s)=\frac{1}{(s+1)}.\frac{1}{s}$

Solving by partial fraction

$$\Rightarrow \frac{.1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$1 = A\ (S + 1) + BS$

Let $S = 0$

$1 = A \qquad \therefore A = 1$

Let $S = -1$

$1 = -B \qquad \therefore B = -1$

Output $c(s) = \frac{1}{s} - \frac{1}{s+1}$

$= (1 - e^{-t})\ u(t)$

When $t = \infty$ at steady state

Output = 1

Hence, the correct option is (c)

7. A system described by the following differential equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = x(t)$ is initially at rest. For input $x(t) = 2u(t)$, the output $y(t)$ is

(a) $(1 - 2\ e^{-t} + e^{-2t})\ u(t)$
(b) $(1 + 2e^{-t} - 2e^{-2t})\ u(t)$
(c) $(0.5 + e^{-t} + 1.5e^{-2t})\ u(t)$
(d) $(0.5 + 2e^{-t} + 2e^{-2t})\ u(t)$ **[2004]**

Solution: (a)

Differential equations: $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = x(t)$

Taking Laplace transform on both sides.

$\Rightarrow s^2y(s) + 3sy(s) + 2y(s) = x(s)$ (i)

Input given: $x(t) = 24(t)$

$x(s) = \frac{2}{5}$ (ii)

Sub value of $x(s)$ in equation (i)

$\therefore (s^2 + 3s + 2)y(s) = \frac{2}{s}$

$y(s) = \frac{2}{s(s+2)(s+1)}$ (iii)

Using partial fraction

$$\frac{2}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+1}$$

$\Rightarrow 2 = A(s + 2)(s + 1) + B(s)(s + 1) + C(s)(s + 2)$

Let $S = 0$

$2 = A(2) \quad \therefore A = 1$

Let $S = -1$

$2 = C(-1)\ (+1)$

$2 = -C \quad \therefore C = -2$

Let $S = -2$

$2 = B(-2)(-1)$

$\therefore 2 = 2B \quad \therefore B = 1$

$\therefore y(s) = \frac{1}{s} + \frac{1}{s+2} - \frac{2}{s+1}$

Taking inverse Laplace transform

$y(t) = [1 + e^{-2t} - 2e^{-t}]\ u\ (t)$.

Hence, the correct option is (a)

8. An electrical system and its signal-flow graph representations are shown in figures (a) and (b), respectively. The values of G_2 and H, respectively, are

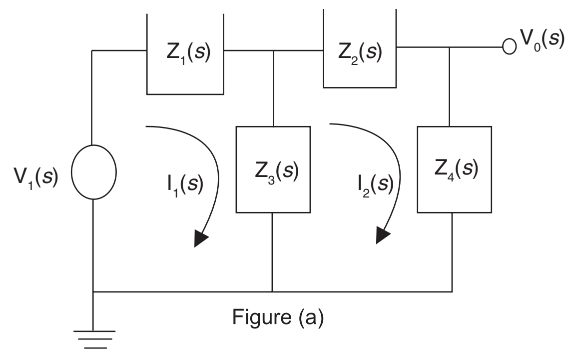


Figure (a)

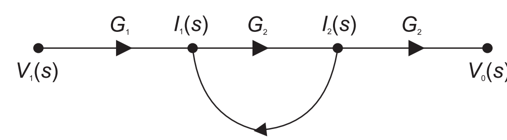


Figure (b)

(a) $\frac{Z_3(s)}{Z_2(s)+Z_3(s)+Z_4(s)}, \frac{-Z_3(s)}{Z_1(s)+Z_3(s)}$

(b) $\frac{-Z_3(s)}{Z_2(s)-Z_3(s)+Z_4(s)}, \frac{-Z_3(s)}{Z_1(s)+Z_3(s)}$

(c) $\frac{Z_3(s)}{Z_2(s)+Z_3(s)+Z_4(s)}, \frac{Z_3(s)}{Z_1(s)+Z_3(s)}$

(d) $\frac{-Z_3(s)}{Z_2(s)-Z_3(s)+Z_4(s)}, \frac{Z_3(s)}{Z_1(s)+Z_3(s)}$ **[2001]**

Solution: (c)

Applying KVL in both loop,

for 1st loop

$Vi(s) = I_1(s)\ Z_1(s) + [I_1(s) - I_2(s)]Z_3(s)$

$\Rightarrow V_i(s) = I_1(s)\ [Z_1(s) + Z_3(s)] - I_2(s)\ Z_3(s)$

$$\Rightarrow \frac{V_1(s)}{Z_1(s)+Z_3(s)} = \frac{I_1(s) - I_2(s)Z_3(s)}{????}$$

$$I_1(s) = \frac{V_i(s)}{Z_1(s)+Z_3(s)} + \frac{I_2(s)+Z_3(s)}{Z_1(s)+Z_3(s)} \qquad \text{(i)}$$

In 2nd loop

$[I_2(s) - I_1(s)]Z_3(s) + I_2(s)Z_2(s) + I_2(s)\,Z_4(s) = 0$

$\Rightarrow I_2(s)Z_3(s) - I_1(s)Z_3(s) + I_2(s)Z_2(s) + I_2(s)Z_4(s) = 0$

$$\Rightarrow \therefore G_2 = \frac{I_2(s)}{I_1(s)} = \frac{Z_3(s)}{Z_2(s)+Z_3(s)+Z_4(s)} \quad \text{(ii)}$$

From signal flow graph.

$I_1(s) = V_iG_1(s) + I_2(s)H(s)$ (iii)

Comparing equation (i) and (iii)

$$G_1(s) = \frac{.1}{Z_1(s)+Z_3(s)}; \quad H(s) = \frac{Z_3(s)}{Z_1(s)+Z_3(s)}$$

Hence, the correct option is (c)

9. The open-loop DC gain of a unity negative feedback system with closed-loop transfer function

$\frac{s+4}{s^2+7s+13}$ is

(a) $\frac{4}{13}$ (b) $\frac{4}{9}$

(c) 4 (d) 13 **[2001]**

Solution: (b)

Closed loop transfer function $= \frac{s+4}{s^2+7s+13}$

$$\Rightarrow \frac{G(s)}{1+G(s)H(s)} = \frac{s+4}{s^2+7s+13}$$

For a unity feedback $H(s) = 1$

$$\frac{G(s)}{1+G(s)} = \frac{s+4}{s^2+7s+13}$$

$$\therefore \frac{1+G(s)}{G(s)} = \frac{s^2+7s+43}{s+4}$$

$$\Rightarrow \frac{1}{G(s)}+1 = \frac{s^2+7s+13}{s+4}$$

$$\Rightarrow \frac{1}{G(s)} = \frac{s^2+7s+13}{s+4} - 1$$

$$\Rightarrow \frac{1}{G(s)} = \frac{s^2+7s+13-s-41}{s+4}$$

$$\Rightarrow \frac{1}{G(s)} = \frac{s^2+6s+9}{s+4}$$

$$\therefore G(s) = \frac{s+4}{s^2+6s+9}$$

For D.C. S = 0

$$\therefore G(s) = \frac{4}{9}$$

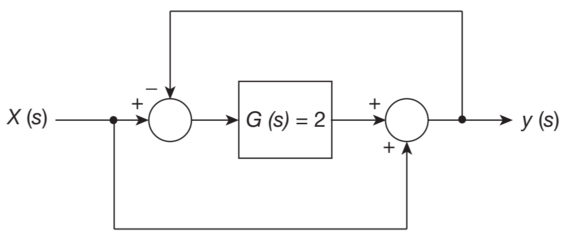
Hence, the correct option is (b)

Chapter 2

Block Diagram and SFG

One-mark Questions

1. For the system shown in the figure $Y(s)/X(s) =$ ________ **[2017]**



Solution: Apply Mason's gain formula between $Y(s)$ and $X(s)$

Number of Forward Paths = 2

Forward path gain, $P_1 = 2$, $P_2 = 1$

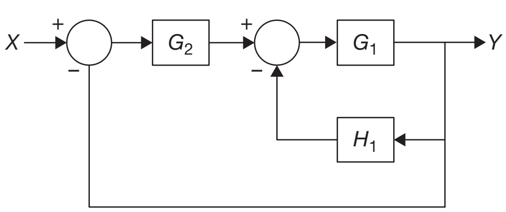
Number of Loops = 1

Loop Gain, $L_1 = -2$

$$\therefore \frac{Y(s)}{X(s)} = \frac{2+1}{1-(-2)} = \frac{3}{3} = 1$$

Hence, the correct answer is (1).

2. The block diagram of a feedback control system is shown in the figure. The overall closed loop gain G of the system is **[2016]**



(A) $G = \dfrac{G_1G_2}{1+G_1H_1}$

(B) $G = \dfrac{G_1G_2}{1+G_1G_2+G_1H_1}$

(C) $G = \dfrac{G_1G_2}{1+G_1G_2H_1}$

(D) $G = \dfrac{G_1G_2}{1+G_1G_2+G_1G_2H_1}$

Solution: Since the block diagram and its transfer function is given, therefore, Mason formula can be easily applied to evaluate the closed loop gain.

No. of forward paths = 1

$\therefore \quad P_1 = G_2\, G_1$

Individual loop gains

$L_1 = -G_1\, H_1$

$L_2 = -G_2\, G_1$

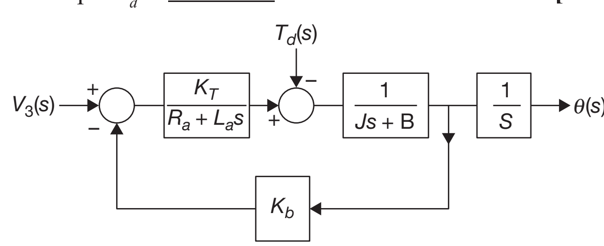
Applying the mason's formula we get

$$\therefore \quad \frac{Y}{X} = \frac{P_1\Delta_1}{\Delta}$$

$$\frac{Y}{X} = \frac{G_1G_2}{1-\left(-G_1H_1 - G_1G_2\right)} = \frac{G_1G_2}{1+G_1G_2+G_1H_1}$$

Hence, the correct option is (B).

3. The position control of a DC servo-motor is given in the figure. The values of the parameters are $K_r = 1$ N-m/A, $R_a = 1\ \Omega$, $L_a = 0.1$ H, $J = 5$ kg-m^2, $B = 1$ N-m/ (rad/sec) and $K_b = 1$ V/(rad/sec). The steady-state position response (in radians) due to unit impulse disturbance torque T_d is ________. **[2015]**



Solution: Let $H(s)=\dfrac{\theta(s)}{T_d(s)}$

From the given Block diagram

Forward path gain $=\dfrac{-1}{s(Js+B)}$

$$\Delta_1 = 1$$

and Individual loop gain $L_1=\dfrac{-K_b.K_T}{(Js+B)(R_a+L_a s)}$

$$H(s)=\frac{P_1\Delta_1}{\Delta}$$

$$\Delta=\frac{1+K_b.K_T}{(Js+B)(R_a+L_a s)}=\frac{(Js+B)(R_a+L_a s)+K_b\,K_T}{(Js+B)(R_a+L_a s)}$$

$$\Rightarrow H(s)=\frac{-1/s(Js+B)}{\Delta}$$

$$H(s)=\frac{-1}{S(JS+B)}\times\frac{(JS+B)(R_a+L_a s)}{(Js+B)(R_a+L_a s)+K_b.K_T}$$

Impulse response $H(s)$

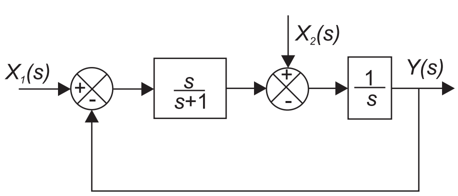
$$h(\infty)=\underset{S\to 0}{Lt}\,S.H(s)=\frac{-(B)(R_a)}{(B)(R_a)+K_b.K_T}$$

Given $K_T = 1$ N-m/A, $R_a = 1\ \Omega$, $L_a = 0.1$ M, $J = 5$ kg-m^2, $B = 1$ N-m/ (rad/sec) and $K_B = 1$ V

$$h(\infty)=\frac{-1\times 1}{1\times 1+1\times 1}=-0.5$$

Hence, the correct Answer is (–0.51 to –0.49).

4. For the following system,



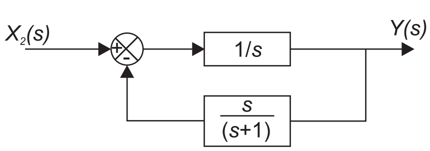
when $X_1(s) = 0$, the transfer function is $\dfrac{Y(s)}{X_2(s)}$

(a) $\dfrac{s+1}{2}$ (b) $\dfrac{1}{s+1}$

(c) $\dfrac{s+2}{s(s+1)}$ (d) $\dfrac{s+1}{s(s+2)}$

[2014]

Solution: (d)



Transfer function

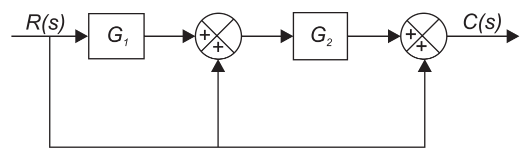
$$z(s)=\frac{y(s)}{x_2(s)}$$

$$=\frac{G(s)}{1+G(s)H(s)}$$

$$\therefore\ z(s)=\frac{\frac{1}{s}}{1+\frac{1}{s}\times\frac{s}{s+1}}=\frac{(s+1)}{s(s+2)}$$

Hence, the correct option is (d)

5. Consider the following block diagram in the figure.



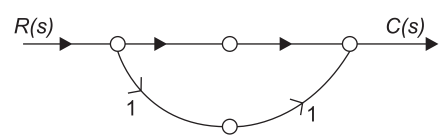
The transfer function $\dfrac{C(s)}{R(s)}$ is

(a) $\dfrac{G_1G_2}{1+G_1+G_2}$ (b) $G_1\,G_2+G_1+1$

(c) $G_1\,G_2+G_2+1$ (d) $\dfrac{G_1}{1+G_1+G_2}$

[2014]

Solution: (c)



Signal flow graph:

Forward path:

$P_1 = G_1G_2$

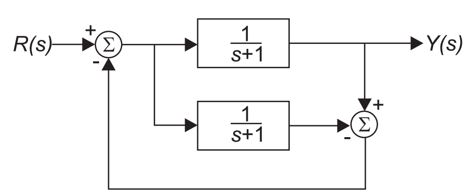
$P_2 = G_2.1$

$P_3 = 1.1 = 1$

So, transfer function $\dfrac{C(s)}{R(s)}=G_1G_2+G_2+1$

Hence, the correct option is (c)

6. The transfer function $Y(s)/R(s)$ of the system shown is

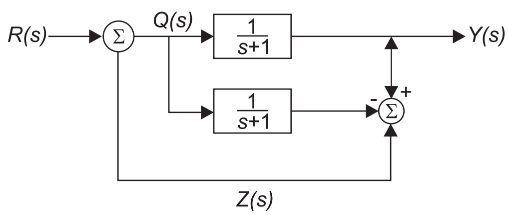


(a) 0 (b) $\dfrac{1}{s+1}$

(c) $\dfrac{2}{s+1}$ (d) $\dfrac{2}{s+3}$

[2010]

Solution:(b)



$$Z(s) = Q(s)\left[\frac{1}{s+1} - \frac{1}{s+1}\right]$$

$$= 0$$

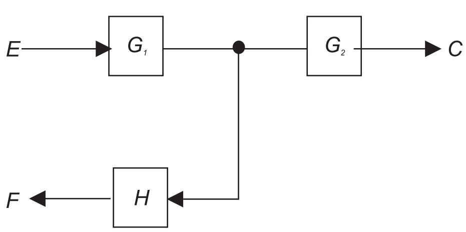
So $Q(s) = R(s) - 0 = R(s)$

$$y(s) = \frac{Q(s)}{s+1} = \frac{R(s)}{s+1}$$

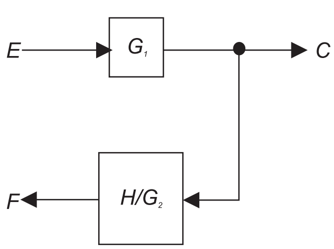
$$\frac{Y(s)}{R(s)} = \frac{1}{s+1}$$

Hence, the correct option is (b)

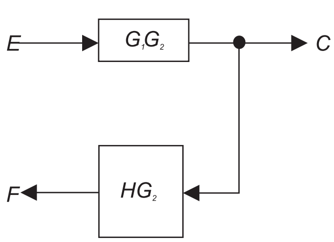
7. The equivalent of the block diagram in the figure is given as



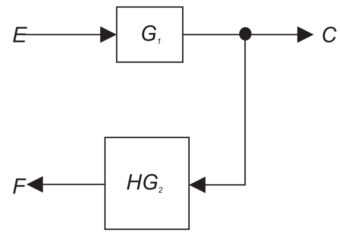
(a)



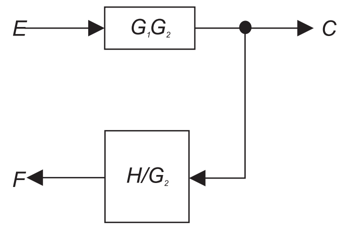
(b)



(c)

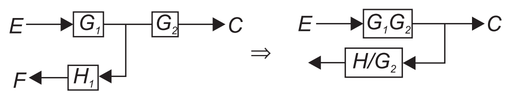


(d)



[2001]

Solution: (d)



Hence, the correct option is (d)

8. Signal flow graph is used to find

(a) Stability of the system

(b) Controllability of the system

(c) Transfer function of the system

(d) Poles of the system.

[1995]

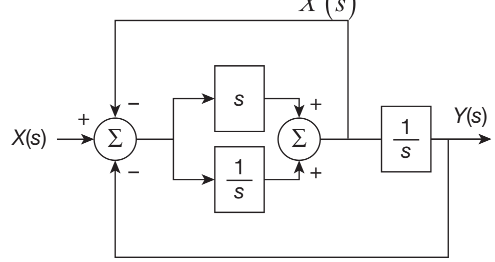
Solution: (c)

Transfer function of the system

Hence, the correct option is (c)

TWO-MARKS QUESTIONS

1. The block diagram of a system is illustrated in the figure shown, where $X(s)$ is the input and $Y(s)$ is the output. The transfer function $H(s) = \dfrac{Y(s)}{X(s)}$ is **[2019]**



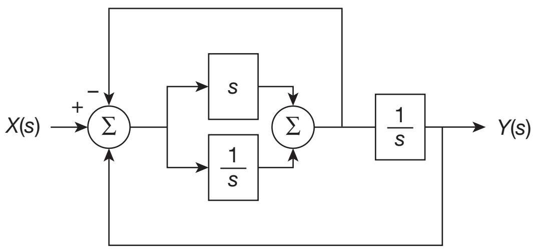
(A) $H(s) = \dfrac{s^2+1}{2s^2+1}$

(B) $H(s) = \dfrac{s^2+1}{s^3+2s^2+s+1}$

(C) $H(s) = \dfrac{s+1}{2s^2+s+1}$

(D) $H(s) = \dfrac{s^2+1}{s^3+s^2+s+1}$

Solution:



Using Mason's gain formula.

$P_1 = 1 \qquad \Delta_1 = 1$

$P_2 = \dfrac{1}{S^2} \qquad \Delta_2 = 1$

$$\frac{Y(S)}{X(S)} = \frac{P_1\Delta_1 + P_z\Delta_2}{\Delta}$$

$$= \frac{1.1 + 1\cdot\frac{1}{S^2}}{1 - \left\{-1 - \frac{1}{S^2} - S - \frac{1}{S}\right\}}$$

$$= \frac{\frac{S^2+1}{S^2}}{2 + \frac{1}{S^2} + S + \frac{1}{S}}$$

On sum plefilation

$$= \frac{S^2+1}{2S^2+S^3+S+1}$$

Hence, the correct option is (B)

2. The forward path transfer function and the feedback path transfer function of a single loop negative feedback control system are given as

$$G(s) = \frac{K(s+2)}{s^2+2s+2} \quad \text{and} \quad H(s) = 1,$$

respectively. If the variable parameter K is real positive, then the location of the breakaway point on the root locus diagram of the system is **[2016]**

Solution: From the given data

$$G(s)H(s) = \frac{k(s+2)}{s^2+2s+2}$$

$$1 + G(s)H(s) = 0$$

$$K = -\frac{(s^2+2s+2)}{s+2}$$

Break away point occurs only at the point of where multiple poles are present. Break away point exist at

$$\frac{dk}{ds} = 0$$

$$(s+2)(2s+2) - (S_2 + 2s + 2)\cdot 1 = 0$$

$$s_2 + 4s + 2 = 0$$

$$s_1 = -0.585,\ s_2 = -3.414$$

Sub S_1 and S_2 values in characteristic equation

Find the valid point ($k > 0$)

At $\quad s = -3.414$

$$1 + \frac{k\{-1.414\}}{6.8273} = 0$$

$$k = 4.828$$

$\therefore \quad k > 0$ at $s = -3.414$

Break point occurs at $s = -3.414$.

Hence, the correct Answer is (–3.414).

3. By performing cascading and/or summing/differencing operations using transfer function blocks $G_1(s)$ and $G_2(s)$, one cannot realize a transfer function of the form **[2015]**

(A) $G_1(s)\,G_2(s)$

(B) $\dfrac{G_1(s)}{G_2(s)}$

(C) $G_1(s)\left[\dfrac{1}{G_1(s)} + G_2(s)\right]$

(D) $G_1(s)\left[\dfrac{1}{G_1(s)} - G_2(s)\right]$

Solution: By performing cascading/summing/differencing operations using T/F blocks $G_1(s)$ and $G_2(s)$, we cannot realize transfer function of the form $\dfrac{G_1(s)}{G_2(s)}$

Hence, the correct option is (B).

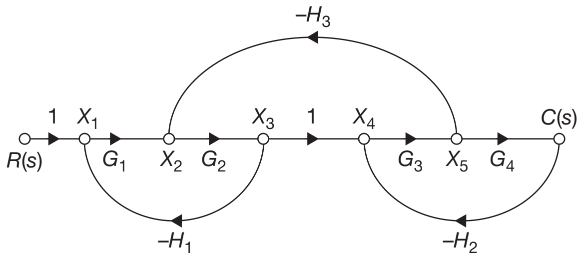
4. For the signal flow graph shown in the figure, the value of $\dfrac{C(s)}{R(s)}$ is **[2015]**

(A) $\dfrac{G_1G_2G_3G_4}{1 - G_1G_2H_1 - G_3G_4H_2 - G_2G_3H_3 + G_1G_2G_3G_4H_1H_2}$

(B) $\dfrac{G_1G_2G_3G_4}{1 + G_1G_2H_1 + G_3G_4H_2 + G_2G_3H_3 + G_1G_2G_3G_4H_1H_2}$

(C) $\dfrac{1}{1+G_1G_2H_1+G_3G_4H_2+G_2G_3H_3+G_1G_2G_3G_4H_1H_2}$

(D) $\dfrac{1}{1\ G_1G_2H_1\ G_3G_4H_2\ G_2G_3H_3+G_1G_2G_3G_4H_1H_2}$



Solution: From the SFG

no. of forward paths = 1

forward path gain $P_1 = G_1\ G_2\ G_3\ G_4$

$\Delta_1 = 1 - \{\text{gain of non-touching loop gains}\}$

$= 1 - 0 = 1$

Individual loop gains:-

$L_1 = -G_1\ G_2\ H_1$

$L_2 = -G_3\ G_4\ H_2$

$L_3 = -G_2\ G_3\ H_3$

Two non-touching loop gains:-

$L_1\ L_2 = G_1\ G_2\ G_3\ G_4\ H_1\ H_2$

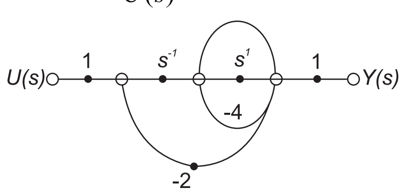
$\Delta = 1 - \{L_1 + L_2 + L_3\} + L_1\ L_2$

$\Delta = 1 + \{G_1G_2H_1\} + G_3G_4H_2 + G_2\ G_3\ H_3 + G_1\ G_2\ G_3\ G_4\ H_1H_2$

$$\frac{C(s)}{R(s)} = \frac{P_1\ \Delta_1}{\Delta}$$

Hence, the correct option is (B).

5. The signal flow graph for a system is given below. The transfer function $\dfrac{Y(s)}{U(s)}$ for this system is



(a) $\dfrac{s+1}{5s^2+6s+2}$ (b) $\dfrac{s+1}{s^2+6s+2}$

(c) $\dfrac{s+1}{s^2+4s+2}$ (d) $\dfrac{s+1}{5s^2+6s+2}$

[2013]

Solution: (a)

Using Mason's gain formula

$$\frac{C}{R} = \frac{P_K\Delta_K}{\Delta}$$

$$\Delta = 1 - [-2s^{-1} - 2s^{-2} - 4 - 4s^{-1}]$$

$$\Delta = 1 + \frac{2}{s} + \frac{2}{s^2} + 4 + \frac{4}{s} = \frac{5s^2+6s+2}{s^2}$$

As $P_1 = s^{-2} = \dfrac{1}{s^2}$

$P_2 = \dfrac{1}{s^2}$

$\Delta_1 = 1$ $\quad\therefore\ \Delta_2 = 2$

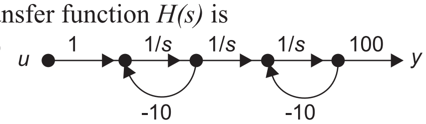
$$\frac{y(s)}{u(s)} = \frac{\Sigma P_K\Delta_K}{\Delta} = \frac{\frac{1}{s^2}\times 1 + \frac{1}{s}\times 1}{\frac{5s^2+6s+2}{s^2}}$$

$$\Rightarrow \frac{y(s)}{u(s)} = \frac{s+1}{5s^2+6s+2}$$

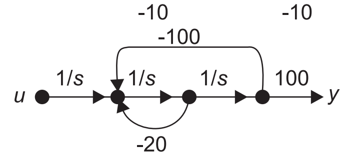
Hence, the correct option is (a)

6. The signal flow graph that DOES NOT model the plant transfer function *H(s)* is

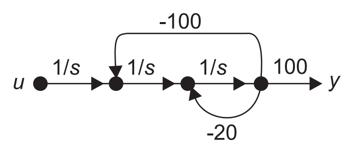
(a)



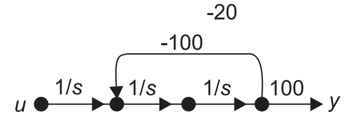
(b)



(c)



(d)



[2011]

Solution: (d)

Which is not equal to *H(s)*

$$\frac{Y(s)}{U(s)} = \frac{100}{\frac{100}{}} = \frac{100}{s(s\quad 100)}$$

Hence, the correct option is (d)

7. The gain margin of the system under closed loop unity negative feedback is $G(s)H(s) = \dfrac{100}{s(s+10)^2}$

(a) 0 dB (b) 20 dB
(c) 26 dB (d) 46 dB

[2011]

Solution: (c)

given $G(s)H(s) = \dfrac{100}{s(s+10)^2}$

For phase cross-over frequency,

$-180 = -90° - 2\tan^{-1}(w/10)$

$w = 10$ rad/sec

Put $s = jw$

$$G(jw)H(jw) = \frac{100}{jw(jw+100)^2}$$

$$1G(jw)H(jw)\big|_{w=10}$$

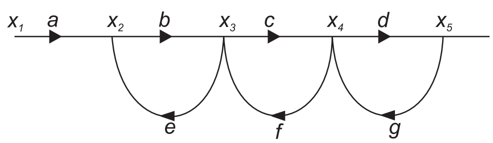
$$= \frac{100}{w(w^2+100)} = \frac{100}{10(200)} = \frac{1}{20}$$

$$\text{G.M.} = \frac{1}{|G(jw)H(jw)|} = 20$$

G.M. in dB = 20 log20 = 26 dB

Hence, the correct option is (c)

8. Consider the signal flow graph shown in the figure. The gain x_5/x_1 is



(a) $\dfrac{1-(be+cf+dg)}{abc}$

(b) $\dfrac{bedg}{1-(be+cf+dg)}$

(c) $\dfrac{abcd}{1-(be+cf+dg)+bedg}$

(d) $\dfrac{1-(be+cf+dg)+bedg}{abcd}$ **[2004]**

Solution: (c)

$P_1 = abcd$, $\Delta = 1$

Loop again: $L_1 =$ be, $L_2 = cf$, $L_3 = dg$

Non-touching loops are L_1 and $L_3 = bedg$

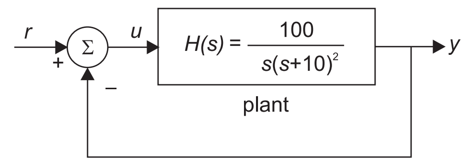
$$\therefore \frac{x_5}{x_1} = \frac{P_1\Delta}{1-(\text{loop again})}$$

$$\therefore \frac{x_5}{x_1} = \frac{abcd}{1-(be+cf+dg)+bedg}$$

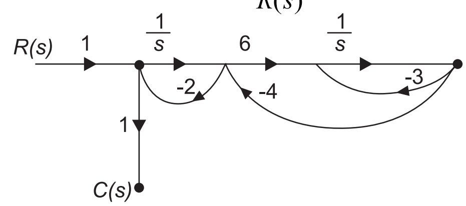
Hence, the correct option is (c)

Common Data for Question 5 and 6

The input-output transfer function of a plant $H(s) = \dfrac{100}{s(s+10)^2}$. The plant is placed in a unity negative feedback configuration as shown in the figure below.



9. The signal flow graph of a system is shown in the figure. The transfer function $\dfrac{C(s)}{R(s)}$ of the system is



(a) $\dfrac{6}{s^2+29s+6}$

(b) $\therefore G(s) = \dfrac{s(s+27)}{s^2+29s+6}$

(c) $\dfrac{6s}{s^2+29s+6}$

(d) $\dfrac{s(s+2)}{s^2+29s+6}$ **[2003]**

Solution: (d)

$$G(s) = \frac{P\Delta_1}{1-(\text{loop gain})+\text{pair of non}-\text{touching loops}}$$

$$\therefore \Delta = +\frac{3}{s}+\frac{24}{s} = \frac{5\quad 27}{s}$$

$P_1 = 1$

L_1 and L_3 are non-touching

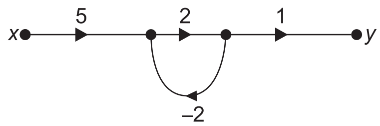
$$\therefore G(s) = \frac{\dfrac{(s+27)}{s}}{1-\left(-\dfrac{3}{s}-\dfrac{24}{s}-\dfrac{2}{s}\right)+\dfrac{-2}{s}\times\dfrac{-3}{s}}$$

$$= \frac{\dfrac{s+27}{s}}{1+\dfrac{29}{s}+\dfrac{6}{s^2}}$$

$$\therefore G(s) = \frac{s(s+27)}{s^2+29s+6}$$

Hence, the correct option is (d)

10. In the signal flow graph of the below figure y/x equals



(a) 3
(b) 5/2
(c) 2
(d) None of the above [1997]

Solution: (c)

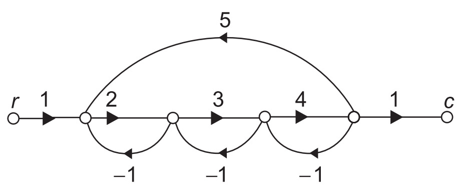
Using Masson's gain formula

$$\frac{C}{R} = \frac{P_K \Delta_K}{\Delta}$$

$$\Rightarrow \frac{C}{R} = \frac{5\times2\times1}{1-(-4)} = \frac{10}{1+4} = \frac{10}{5} = 2$$

Hence, the correct option is (c)

11. .In the signal flow graph of the gain c/r will be



(a) 11/9 (b) 22/15
(c) 24/23 (d) 44/23

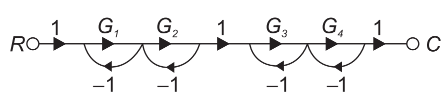
[1991]

Solution: (d)

Using Masson's gain formulae

Hence, the correct option is (d)

12. The C/R for the signal flow graph in the below given figure is



(a) $\dfrac{G_1G_2G_3G_4}{(1+G_1G_2)(1+G_3G_4)}$

(b) $\dfrac{G_1G_2G_3G_4}{(1+G_1+G_2+G_1G_2)(1+G_3+G_4+G_3G_4)}$

(c) $\dfrac{G_1G_2G_3G_4}{(1+G_1+G_2)(1+G_3+G_4)}$

(d) $\dfrac{G_1G_2G_3G_4}{(1+G_1+G_2+G_3+G_4)}$

[1989]

Solution: (c)

Using Masson's gain formulae

$$\frac{C}{R} = \frac{P_K \Delta_K}{\Delta}$$

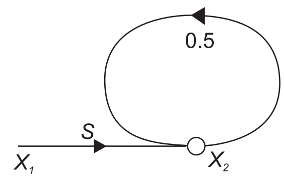
$$\frac{C}{R} =$$

$$\frac{G_1G_2G_3G_4}{1-[-G_1-G_2-G_3-G_4]+[G_1G_3+G_1G_4+G_2G_3+G_2G_4]}$$

$$\frac{C}{R} = \frac{G_1G_2G_3G_4}{(1+G_1+G_2)(1+G_3+G_4)}$$

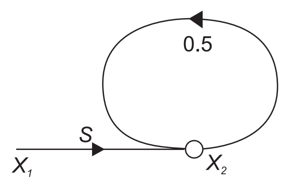
Hence, the correct option is (c)

13. In the signal flow graph shown in the below figure, $X_2 = TX_1$ where T is equal to



(a) 2.5 (b) 5
(c) 5.5 (d) 10 [1987]

Solution: (d)



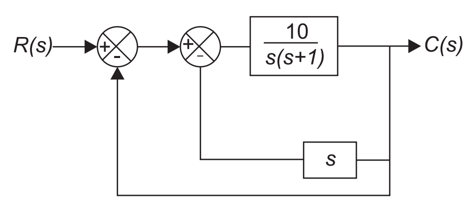
$X_2 = TX_1$

$0 = 1 - 05 = 0.5$

$$\therefore \frac{x_2}{x_1} = \frac{5}{\Delta} = \frac{5}{0.5} = 10$$

Hence, the correct option is (d)

14. For the system shown in figure the transfer function $\dfrac{C(s)}{R(s)}$ is equal to:



(a) $\dfrac{10}{s^2+s+10}$ (b) $\dfrac{10}{s^2+11s+10}$

(c) $\dfrac{10}{s^2+9s+10}$ (d) $\dfrac{10}{s^2+2s+10}$

[1987]

Solution: (b)

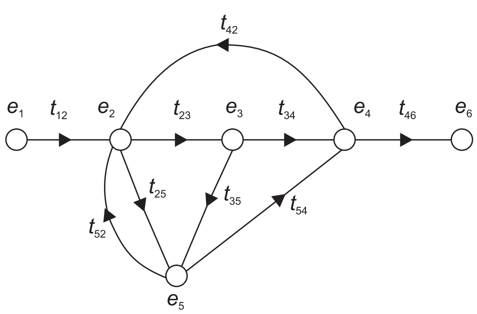
$$\frac{C(s)}{R(s)}=\frac{\dfrac{\frac{10}{s(s+1)}}{1+\frac{10}{s(s+1)}\times s}}{\dfrac{1+\frac{10}{s(s+1)}}{1+\frac{10}{s(s+1)}\times s}\times 1}$$

$$\therefore\ \frac{C(s)}{R(s)}=\frac{\dfrac{10}{s(s+1)+10s}}{1+\dfrac{10}{s(s+1)+10s}}=\frac{10}{s^2+11s+10}$$

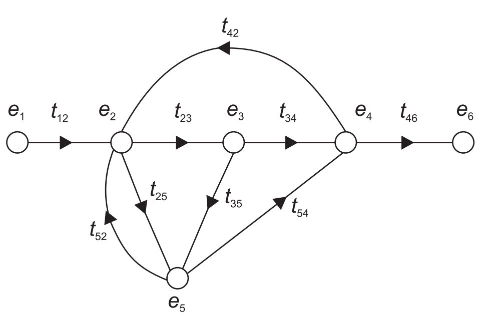
Hence, the correct option is (b)

FIVE-MARKS QUESTIONS

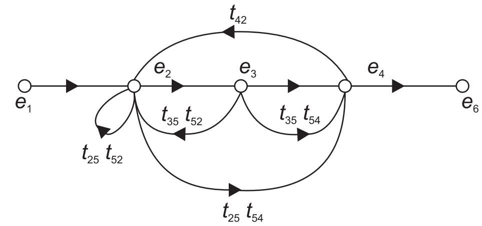
1. Reduce the signal flow graph shown in figure below, to obtain another graph which does not contain the node e_5. (Also, remove any self-loop from the resulting graph)



Solution:



Remaining node R_5



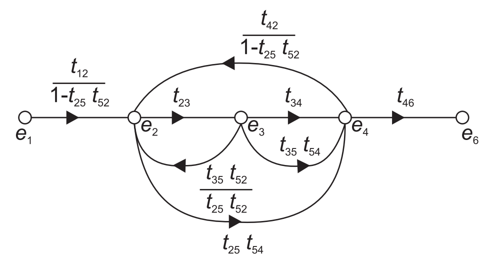
$\therefore$ node equation at l_2

$l_2 = l_1t_{12} + l_3t_{35}t_{52} + l_2t_{25}t_{52} + l_4t_{42}$

$\Rightarrow l_2(1 - t_{25}t_{52}) = e_1t_{12} + e_3t_{35}t_{52} + e_4t_{42}$

$$\Rightarrow e_2 = e_1\frac{t_{12}}{1-t25^552}+e_3\frac{t_{35}t_{52}}{1-t_{25}t_{52}}+e_4\frac{t_{42}}{1-t_{25}t_{52}}$$

Reduced signal flow graph.



2. Draw a signal flow graph for the following set of algebraic equations:

$y_2 = ay_1 - gy_3$

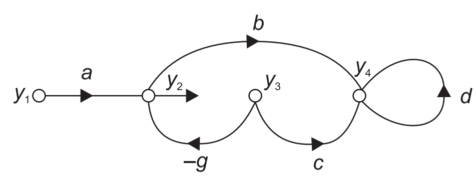
$y_3 = ey_2 + cy_4$

$y_4 = by_2 - dy_4$

Hence, find the gains $\dfrac{y_2}{y_1}$ and $\dfrac{y_2}{y_1}$ **[1998]**

Solution:

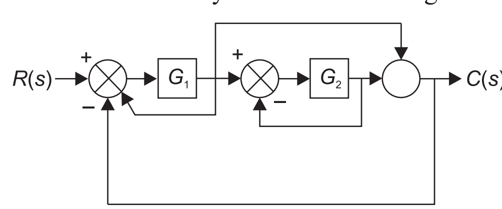
$\therefore$ signal flow graph;



$$\therefore\ \frac{y_2}{y_1}=\frac{a(1+d)}{1+d+eg+bcg+\text{deg}}$$

$$\therefore\ \frac{y_3}{y_1}=\frac{ae(1+d)+abc}{1+d+eg+bcg+deg}$$

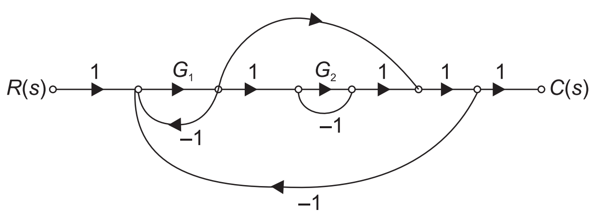
3. A feedback control system is shown in figure



(a) Draw the signal-flow graph that represents the system.
(b) Find the total number of loops in the graph and determine the loop-gains of all the loops.
(c) Find the number of all possible combinations of non-touching loops taken two at a time.
(d) Determine the transfer function of the system using the signal-flow graph. **[2001]**

Solution: (a)

signal flow graph representing the system



(b) Total number of loops in the graph is 4

i.e., $L_1 = -G_1$

$L_2 = -G_2$

$L_3 = -G_1$

$L_4 = -G_1G_2$

Touching loops, are

$L_1L_2 = G1\mathrm{G}_2$

$L_2L_3 = G_1\mathrm{G}_2$

(d) By mason's gain formulae

$$\frac{C(s)}{R(s)} = \frac{P_K\Delta_K}{\Delta}$$

$$\text{T.F.} = \frac{G_1G_2 + G_1(1+G_2)}{1-(G_1 - G_2 - G_1 - G_1G_2) + (G_1G_2 + G_1G_2)}$$

$$\frac{C(s)}{R(s)} = \frac{G_1(1+2G_2)}{1+2G_1+G_2+3G_1G_2}$$

Hence, the correct option is (a)

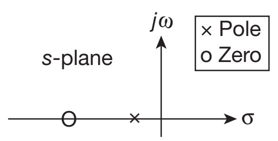
Chapter 3

Compensators and Controllers

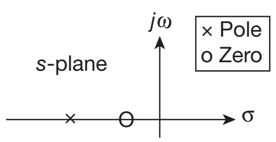
ONE-MARK QUESTIONS

1. Which of the following can be the pole-zero configuration of a phase-lag controller (lag compensator)? **[2017]**

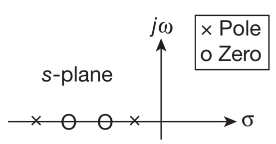
(A)



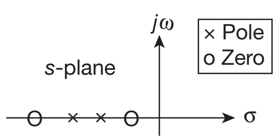
(B)



(C)



(D)



Solution: It is evident that from pole zero configuration Choice (A) satisfies phase lag compensator.

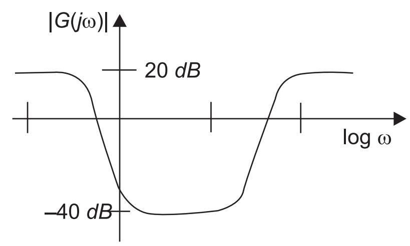
2. Which of the following statement is incorrect? **[2017]**

(A) Leading compensator is used to reduce the settling time.
(B) Lag compensator is used to reduce the steady state error.
(C) Lead compensator may increase the order of a system
(D) Lag compensator always stabilizes an unstable system.

Solution: By adding lag compensation, one pole is added which is nearer to origin will degrade the stability of a system

Hence, the correct option is (D).

3. The magnitude plot of a rational transfer function $G(s)$ with real coefficients is shown below. Which of the following compensators has such a magnitude plot?



(a) Lead compensator
(b) Lag compensator
(c) PID compensator
(d) Lead-lag compensator **[2009]**

Solution: (d)

The given plot is for a Lead log compensator

Hence, the correct option is (d)

4. A PD controller is used to compensate a system. Compared to the uncompensated system, the compensated system has

(a) a higher type number
(b) reduced damping
(c) higher noise amplification
(d) larger transient overshoot **[2003]**

Solution: (c)

When bandwidth increases then signal-to-noise ratio decreases and system become more prone to noise.

Hence, the correct option is (c)

5. The transfer function of a phase lead controller is $\frac{1+3Ts}{1+Ts}$. The maximum value of phase provided by this controller is

(a) 90° (b) 60°
(c) 45° (d) 30° **[1998]**

Solution: (d)

For lead compensator

$$\text{T.F.} = \frac{a\,(1+T's)}{1+a\,T's} = \frac{1+3Ts}{1+Ts}$$

So $T' = 3T$

$$a\,T' = T$$

$$a\,3T' = T$$

$$a = \frac{1}{3}$$

$$\phi_{\max} = \sin^{-1}\left(\frac{1-a}{1+a}\right) = \sin^{-1}\left(\frac{1-\frac{1}{3}}{1+\frac{1}{3}}\right)$$

$$= \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

Hence, the correct option is (d)

Two-marks Questions

Common Data for Questions 1 and 2:

The transfer function of a compensator is given as

$$G_c(s) = \frac{s+a}{s+b}$$

1. G_c (s) is a lead compensator if

(a) $a = 1$, $b = 2$
(b) $a = 3$, $b = 2$
(c) $a = -3$, $b = -1$
(d) $a = 3$, $b = 1$ **[2012]**

Solution: (a)

given $G_c(s) = \left(\frac{s+a}{s+b}\right)$

phase $P = \tan^{-1}(w/a) - \tan^{-1}(w/b)$

for load comparators phase must be positive

$$\therefore \frac{w}{a} > \frac{w}{b} \Rightarrow a < b$$

$\therefore a = 1, b = 2$

Hence, the correct option is (a)

2. The phase of the above lead compensator is maximum at

(a) $\sqrt{2}$ rad/s (b) $\sqrt{3}$ rad/s
(c) $\sqrt{6}$ rad/s (d) $1\sqrt{3}$ rad/s **[2012]**

Solution: (a)

For phase to be maximum

$$\frac{\partial p}{\partial w} = 0$$

$$\frac{\frac{1}{a}}{1+\frac{w^2}{a^2}} - \frac{\frac{1}{b}}{1+\frac{w^2}{b^2}} = 0$$

$$\Rightarrow \frac{1}{1+w^2} - \frac{2}{w^2+4} = 0$$

$w^2 + 4 - 2 - 2w^2 = 0$

$w^2 = 2$

$w = \sqrt{2}$ rad/sec

Hence, the correct option is (a)

3. A unity negative feedback closed loop system has a plant with the transfer function

$G(s) = \frac{1}{s^2+2s+2}$ and a controller $G_c(s)$ in the feed forward path. For a unit step input, the transfer function of the controller that gives minimum steady-state error is

(a) $G_c(s) = \frac{s+1}{s+2}$,

(b) $G_c(s) = \frac{s+2}{s+1}$,

(c) $G_c(s) = \frac{(s+1)(s+4)}{(s+2)(s+3)}$,

(d) $G_c(s) = 1 + \frac{2}{5} + 3s$,

[2010]

Solution: (d)

Steady-state error

$$e_{ss} = \underset{s\to 0}{Lt}\, \frac{S\,R(s)}{1+G(s)G_c(s)}$$

$r(t) = G\,(t)$

$$R(s) = \frac{1}{s}$$

$$\therefore e_{ss} = \underset{s\to 0}{Lt}\, \frac{s.\frac{1}{s}}{1+G(s)G_c(s)} = \underset{s\to 0}{Lt}\, \frac{1}{1+G(s)G_c(s)}$$

From the options given

For $G_c(s) = \frac{s+1}{s+2}$, $e_{ss} = \frac{2}{3}$

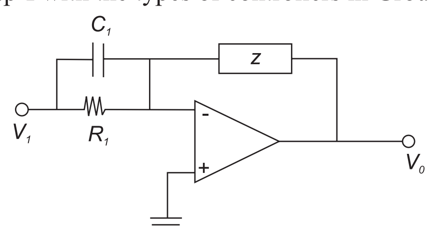
For $G_c(s) = \frac{s+2}{s+1}$, $e_{ss} = \frac{1}{3}$

For $G_c(s) = \frac{(s+1)(s+4)}{(s+2)(s+3)}$, $e_{ss} = \frac{3}{5}$

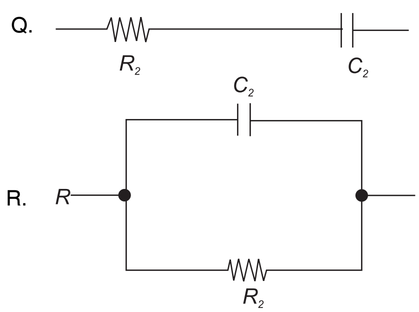
For $G_c(s) = 1+\frac{2}{s}+3s$, $e_{ss} = 0$

Hence, the correct option is (d)

4. Group I gives two possible choices for the impedance Z in the diagram. The circuit elements in Z satisfy the condition $R_2C_2 > R_1C_1$. The transfer function $\frac{V_0}{V_i}$ represents a kind of controller. Match the impedances in Group-I with the types of controllers in Group-II.



Group-I



Group-II

1. PID controller
2. Lead compensator
3. Lag compensator

(a) $Q-1, R-2$ (b) $Q-1, R-3$
(c) $Q-2, R-3$ (d) $Q-3, R-2$

[2008]

Solution :(b)

$$\frac{V_i(R_1C_1s+1)1}{1} = \frac{-V_0}{2} \qquad \therefore \text{ given } R_2C_2 > R_1\,C_1$$

$$\Rightarrow \frac{V_0}{V_i} = \frac{-z(R_1C_1s+1)}{R_1}$$

For Q, $z = \frac{R_2C_2s+1}{C_2s+1}$

$$\therefore \frac{V_0}{V_i} = \frac{(R_2C_2s+1)}{(C_2s+1)}\cdot\frac{(R_1C_1s+1)}{R_1}$$

∴ Controller in PID controller

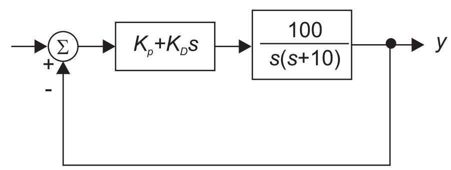
For R

$$\therefore Z = \frac{R_2}{R_2C_2s+1}$$

$$\therefore \frac{V_0}{V_i} = -\frac{R_2}{(R_2C_2s+1)}\cdot\frac{(R_1C_1s+1)}{R_1}$$

∴ Controller is log compensator

5. A control system with a PD controller is shown in the figure. If the velocity error constant $K_v = 1000$ and the damping ratio $\varsigma = 0.5$, then the values of K_p and K_D are



(a) $K_p = 100, K_D = 0.09$
(b) $K_p = 100, K_D = 0.9$
(c) $K_p = 10, K_D = 0.09$
(d) $K_p = 10, K_D = 0\ 9$ **[2007]**

Solution: (b)

$$\frac{Y(s)}{R(s)} = \frac{Gs}{1+Gs\,H(s)} = \frac{(k_p+k_Ds)}{1+(k_p+k_Ds).\frac{100}{s(s+10)}}$$ characteristics equation $1 + G(s)\,H(s)$

$$K_v = \underset{s\to 0}{Lt}\; s\,G(s)\,H(s)$$

given $K_v = 1000$

$$\Rightarrow 1000 = \underset{s\to 0}{Lt}\; s\times\frac{(k_p+k_Ds)100}{s(s+10)}$$

$$\therefore 100\cancel{0} = \frac{k_p\times 1\cancel{00}}{1\cancel{0}}$$

$\therefore k_p = 100$

Now characteristics equation $(1 + G(s)\,H(s)) = 0$

$$1+\frac{(k_p+k_Ds)100}{s(s+10)} = 0$$

Put $k_p = 100$

$= s^2 + 10s + 10000 + 100k_Ds = 0$

$= s^2 + (10 + 100k_D)s + 10^4 = 0$ (i)

Standard 2nd order equation

$$s^2 + 2\xi w_n s + w_n^2 = 0 \text{ (ii)}$$

Comparing equation (i) and (ii)

So, $w_n = 100$; $2\,\xi w_n = 10 + 1000k_D$

$\xi = 0.5$; $2 \times 0.5 \times 100 = 2\xi w_n$

$100 = 10 + 100\,k_D$

$$k_D = \frac{90}{100} = 0.9$$

6. The open-loop transfer function of a plant is given as $G(s) = \dfrac{1}{s^2 - 1}$. If the plant is operated in a unity feedback configuration, then the lead compensator that can stabilize this control system is

(a) $\dfrac{10(s-1)}{s+2}$ (b) $\dfrac{10(s+4)}{s+2}$

(c) $\dfrac{10(s+2)}{s+10}$ (d) $\dfrac{10(s+2)}{s+10}$

[2007]

Solution: (a)

$$\text{given} = G(s) = \frac{1}{s^2 - 1} = \frac{1}{(s+1)(s-1)}$$

load compensator $C(s)$ should first stabilize the plant, i.e., remove $\dfrac{1}{(s-1)}$ term

$$\therefore G(s)\,C(s) = \frac{1}{(s+1)(s-1)} \times \frac{10(s-1)}{s+2}$$

7. The transfer function of a phase-lead compensator is given by

$$G_e(s) = \frac{1+3Ts}{1+Ts} \; where\; T > 0$$

The maximum phase-shift provided by such a compensator is

(a) $\pi/2$ (b) $\pi/3$

(c) $\pi/4$ (d) $\pi/6$ **[2006]**

Solution: (d)

Phase-lead compensator

$$G_e(s) = \frac{1+3Ts}{1+Ts} \text{ where } T > 0$$

Max. phase shift

$$\phi_m = \angle G_e(s)$$

$$\phi = \tan^{-1} 3wT - \tan^{-1} wT$$

For maximum phase shift

$$\frac{d\phi}{dt} = 0$$

$$\therefore \frac{3T}{1+(3T+w)^2} = \frac{T}{1+(Tw)^2}$$

$\Rightarrow 3[1+(Tw)2] = 1 + (3Tw)^2$

$\Rightarrow 3 + 3T^2w^2 = 1 + 9T^2w^2$

$\Rightarrow 6T^2w^2 = 2$

$$\Rightarrow T^2w^2 = \frac{1}{3}$$

$$\Rightarrow Tw = \frac{1}{\sqrt{3}}$$

$$\phi_{\max} = \tan^{-1} 3 \times \frac{1}{\sqrt{3}} - \tan^{-1}\frac{1}{\sqrt{3}}$$

$$= \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

8. A double integrator plant, $G(s) = \dfrac{K}{s^2}$, H(s) = 1 is to be compensated to achieve the damping ratio $\varsigma = 0.5$, and an undamped natural frequency, $\omega_0 = 5$ rad/s. Which one of the following compensator $G_e(s)$ will be suitable?

(a) $\dfrac{s+3}{s+9.9}$ (b) $\dfrac{s+9.9}{s+3}$

(c) $\dfrac{s-6}{s+8.33}$ (d) $\dfrac{s+6}{s}$ **[2005]**

Solution: (a)

$\xi = 0.5$

$\cos^{-1} 0.5 = 60°$

$\theta = 60°$

$$\angle G(s) = \left.\frac{k}{s^2}\right|_{s=-2.5+4.33j}$$

$$= -2\tan^{-1}\frac{4.33}{-2.5};\, 120°$$

Putting s = −2.5 + 4.33j

$$\frac{k(s+3)}{s^2(s+9.9)} = \frac{0.5 + j4.33}{7.4 + j4.33} = 53°$$

9. A process with open-loop model

$G(s) = \dfrac{Ke^{-sT_D}}{\tau S + 1}$ is controlled by a PID controller. For this process.

(a) the integral mode improves transient performance
(b) the integral mode improves steady-state performance
(c) the derivative mode improves transient performance
(d) the derivative mode improves steady-state performance. **[1992]**

Solution: (b)

Integral mode improves steady-state performance.

10. The transfer function of a simple RC network functioning as a controller is:

$$G_c(s) = \frac{s+z_1}{s+p_1}$$

The condition for the RC network to act as a phase lead controller is:

(a) $p_1 < z_1$ (b) $p_1 = 0$
(c) $p_1 = z_1$ (d) $p_1 > z_1$ **[1990]**

Solution: (d)

$$G(s) = \frac{s+z_1}{s+p_1}$$

$$\therefore \theta = \tan^{-1}\left(\frac{w}{z_1}\right) - \tan^{-1}\left(\frac{w}{p_1}\right)$$

For phase lead controller

$\theta > 0$

$$\tan^{-1}\left(\frac{w}{z_1}\right) - \tan^{-1}\left(\frac{w}{p_1}\right) > 0$$

$$\tan^{-1}\left(\frac{w}{z_1}\right) \Rightarrow \tan^{-1}\left(\frac{w}{p_1}\right)$$

$\Rightarrow p_1 > z_1$

Two-marks Questions

1. The transfer function of a first – order controller is given as

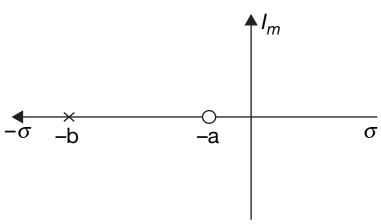
$$G_c(s) = \frac{K(s+a)}{s+b}$$

where K, a and b are positive real numbers. The condition for this controller to act as a phase lead compensator is **[2015]**

(A) $a < b$ (B) $a > b$
(C) $K < ab$ (D) $K > ab$

Solution: Given $G_c(s) = \dfrac{K(s+a)}{(s+b)}$

Given it is a phase lead controller, so zero is dominating



$\therefore$ $a < b$

Hence, the correct option is (A).

2. A lead compensator network includes a parallel contribution of R and C in the feed-forward path. If the transfer function of the compensator is $G_c(s) = \dfrac{s+2}{s+4}$, the value of RC is ________ **[2015]**

Solution: Given

$$G_C(s) = \frac{s+2}{s+4}$$

$$G(s) = \frac{0.5\{1+0.5s\}}{(1+0.25s)} = K.\left\{\frac{1+s\tau}{1+\beta\tau s}\right\}$$

$\tau = 0.5$ sec

Hence, the correct Answer is (0.5).

Five-marks Questions

1. A unity feedback system has the plant transfer function

$$G_P(s) = \frac{1}{(s+1)(2s+1)}$$

(a) Determine the frequency at which the plant has a phase lag of 90°.

(b) An integral controller with transfer function $G_c(s) = \dfrac{k}{s}$ is placed in the forward path of the feedback system.

Find the value of k such that the compensated system has an open loop gain margin of 2.5.

(c) Determine the steady state errors of the compensated system to unit-step and unit-ramp inputs. **[2002]**

Solution:

(a) $\therefore\ \theta = -90°$

$$\theta = -\tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{2\omega}{1}\right)$$

$$\Rightarrow -90 = -\tan^{-1}(\omega) - \tan^{-1}(2\omega)$$

$$\Rightarrow \tan^{-1}(\omega) + \tan^{-1}(2\omega) = 90$$

$$\left(\frac{\omega+2\omega}{1-2\omega}\right) = \tan 10 = \infty$$

$$\Rightarrow 1 - 2\omega^2 = 0$$

$$\omega = \frac{1}{\sqrt{2}} \text{ rad/sec}$$

(b) given $G(s) = G_P(s).\ G_C(s)$

$$G(s) = \frac{1}{(s+1)(2s+1)} \times \frac{k}{s} = \frac{k}{s(s+1)(2s+1)}$$

$$\therefore \omega_{PC} = -90° - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$$

$$= -180°$$

$$\omega_{PC} = \frac{1}{\sqrt{2}} \text{ rad/sec}$$

$$|G(j\omega)|_{\omega PC} = \frac{k}{\left|\left(j\frac{1}{\sqrt{2}}\right)\left(i\frac{1}{\sqrt{2}}+1\right)\left(2\frac{1}{\sqrt{2}}+1\right)\right|}$$

gain margin $G(Gm) = 2.5$

$$\therefore \frac{1}{(G(j\omega))_{\omega PC}} = Gm$$

$$\therefore (G(j\omega))_{\omega PC} = \frac{1}{Gm} = \frac{1}{2.5} = 0.4$$

$$\therefore \frac{k}{\left(\frac{1}{\sqrt{2}}\right)\left|\left(\sqrt{\frac{1}{2}}\right)^2+1\right|\left(\sqrt{(\sqrt{2})^2+1}\right)} = 0.4$$

$$R = 0.4\frac{1}{\sqrt{2}}\times(\sqrt{3})(\sqrt{3})$$

$$k = 0.4\times\frac{3}{2} = 0.6$$

(c) $k_p = \lim_{s\to 0} G(s)H(s)$

$$= \underset{s\to 0}{Lt}\ \frac{k}{s(s+1)(2s+1)} = \infty$$

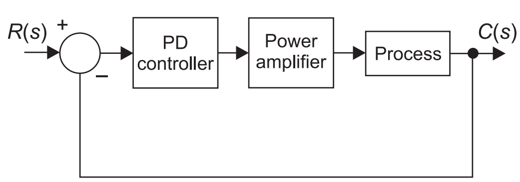
$$e_{ss} = \frac{1}{1+k_p} = 0$$

$$k_v = \underset{s\to 0}{Lt}\ S(G(s)H(s))$$

$$= \underset{s\to 0}{Lt}\ \frac{k}{(s+1)(2s+1)} = k = 0.6$$

$$e_{ss} = \frac{1}{k_v} = \frac{1}{0.6} = \frac{10}{6} = 1.667$$

2. For the feedback control system shown in the figure, the process transfer function is $G_P(s) = \frac{1}{s(s+1)}$, and the amplification factor of the power amplifier is K > 0. the design specifications required for the system, time constant is 1 sec and a damping ratio of 0.707.



(a) Find the desired locations of the closed loop poles.
(b) Write down the required characteristic equation for the system. Hence determine the PD controller transfer function $G_p(s)$ when K = 1.
(c) Sketch the root-locus for the system. **[2001]**

Solution:

Transfer function of PD controller $(G_1) = K_p + S_{kD}$

Transfer function of power amplifier $(G_2) = k$

Transfer function of process

$$(G_3) = \frac{1}{s(s+1)}$$

$$\therefore G(s) = G_1G_2G_3 = \frac{k(k_p+s_{kD})}{s(s+1)}$$

Overall transfer function $= \frac{G(s)}{1+G(s)}$

$$\therefore \frac{C(s)}{R(s)} = \frac{\frac{k(k_p+s_{kD})}{s(s+1)}}{\frac{1+k(k_p+s_{kD})}{s(s+1)}} = \frac{k(k_p+s_{kD})}{s^2+s+(k_p+s_{kD})k}$$

$$\frac{C(s)}{R(s)} = \frac{+k_Ds}{s^2+s(1+kk_D)+kk_P}$$

$\therefore$ As *tw* characteristics equation

$$s^2+2\xi\omega_n s+\omega_n^2 = 0$$

$$\therefore \text{given } \tau = \frac{1}{\xi\cos n} = 1$$

$$\omega_n = \frac{1}{\xi}$$

given $\xi = 0.707$

$\therefore$ Poles

s_1 and $s_2 = \ s = \xi\omega_n\sqrt{1-\xi^2}$

$$s = -1 \pm j1.41\sqrt{1-(0.707)^2}$$

$s = -1 \pm j1.41 \times 0.07$

$s = -1 \pm j$

$\therefore s_1 = -1 + j$

$s_2 = -1 - j$

(b) $$\frac{C(s)}{R(s)} = \frac{k(k_p+k_Ds)}{s^2+s(1+kk_D)+kk_P}$$

Characteristics equation

$= s^2 + s(1 + k_0k) + kk_p$

$2\xi\omega_n = 1+kk_D$

$\therefore 1 + kk_D = 2 \times 1 - 2$

$0 \Rightarrow kk_D = 1$

given $k = 1$

$1.k_p = 2$

$k_p = 2$

$1.k_D = 1$

$k_D = 1$

$k_P + k_D s = 2 + s \times 1 = s + 2$

(c) $G(s) = \dfrac{k(k_D s + k_p)}{s(s+1)}$

$= \dfrac{k(s+2)}{s(s+1)}$

$\therefore$ Number of pole(p)= 2

And number of Zero(z) = 1

$\therefore p - z = 2 - 1 = 1$

$\therefore \theta = \dfrac{180°}{1} = 180°$

Breakaway point

$1 + \dfrac{k(s+2)}{s(s+1)} = 0$

$\therefore \dfrac{dk}{ds} = \dfrac{(s+1)(2s-1)-(s^2-5)(1)}{(s+2)^2}$

As $\dfrac{dk}{ds} = 0$

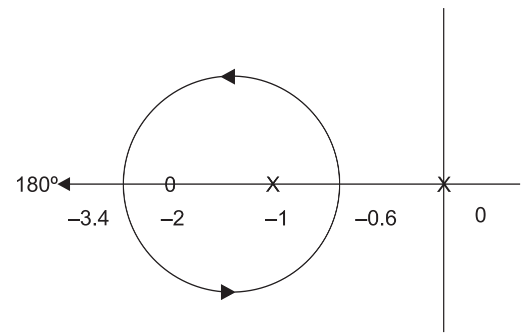
$\therefore -2s^2 - s - 4s - 2 + s^2 + 5 = 0$

$\therefore s^2 + 4s + 2 = 0$

$\therefore s = \dfrac{-4 \pm \sqrt{16-8}}{2}$

$s = -2 \pm \sqrt{2}$

$s = -0.6, -3.4$



Chapter 4

Time Response Analysis

One-mark Questions

1. Consider $P(s) = s^3 + a_2s^2 + a_1s + a_0$ with all real coefficients. It is known that its derivative $p^I(s)$ has no real roots. The number of real roots of $p(s)$ is **[2018]**

(A) 0 (B) 1
(C) 2 (D) 3

Solution: As per problem

$$p(s) = S^3 + a_2s^2 + a_1s + a_0$$

derivative of above will be $p^I(s) = 0$ has no real roots, so it is imaginary complex conjugates. (2nd order system)

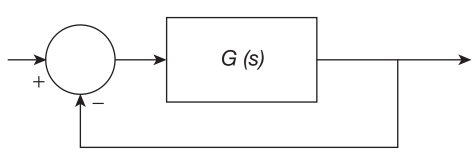
It is 3rd order system so remaining one root must be a real root.

Hence, the correct option is (B)

2. The open loop transfer function

$$G(s) = \frac{(s+1)}{s^p(s+2)(s+3)}$$

where p is an integer, is connected in unity feedback configuration as shown in the figure.



Given that the steady-state error is 0 for unit step input and is 6 for unit ramp input, the value of the parameter p is __________. **[2017]**

Solution:

$$e_{ss} = \frac{1}{1+k_p}\text{(For unit step input)}$$

$$K_p = 1 + \underset{s\to 0}{G(S)H(s)} = \infty(\text{For } e_{ss} \text{ to be 'o'})$$

$$k_p = \underset{s\to o}{\text{Lt}} \frac{(S+1)}{s^P(s+2)(S+3)} = \infty(\text{'}P\text{' must be } \geq 1)$$

$$e_{ss} = \frac{1}{k_v} = 6 \text{ (for unit reamp input)}$$

$$k_v = \underset{s\to 0}{Lt}\; SG(S)H(S) = \frac{1}{6}$$

$$k_v = \underset{s\to 0}{\text{Lt}}\; S.\frac{(S+1)}{S^P(S+2)(S+3)} = \frac{1}{6} \to \quad \text{for } P = 1$$

$\boxed{\therefore P = 1}$

Hence, the correct answer is (0.99 to 1.01).

3. Match the inferences X, Y and Z about a system, to the corresponding properties of the elements of first column in Routh's table of the system characteristic equation **[2016]**

X: The system is stable	P: When all elements are positive
Y: The system is unstable	Q: When any one element is zero
Z: The test breaks down	R: When there is a change in sign of coefficients

(A) X → P, Y → Q, Z → R
(B) X → Q, Y → P, Z → R
(C) X → R, Y → Q, Z → P
(D) X → P, Y → R, Z → Q

Solution: From the RH criterion:

X → P
Y → R
Z → Q

Change in sign shows that system is unstable. The system is stable when all the elements of Routh array are +ve. When any row of Routh array becomes zero, then it indicates that breakdown has occurred.

Hence, the correct option is (D).

4. The response of the system $G(s) = \dfrac{s-2}{(s+1)(s+3)}$ to the unit step input $u(t)$ is $y(t)$. The value of $\dfrac{dy}{dt}$ at $t = 0^+$ is ________. **[2016]**

Solution: We are given

$$Y(s) = \frac{s-2}{s(s+1)(s+3)}$$

Applying the concept of partial function we get

$$\frac{s-2}{s(s+1)(s+3)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$$

After solving above partial fraction, we get

$$A = -1/3,\ B = 3/2,\ C = -5/6$$

$$Y(s) = \frac{\frac{-1}{3}}{s} + \frac{\frac{3}{2}}{s+1} - \frac{\frac{5}{6}}{s+3}$$

$$y(t) = \frac{-1}{3} + \frac{3}{2}e^{-t} - \frac{5}{6}e^{-3t}$$

$$\left.\frac{dy(t)}{dt}\right| = \frac{-3}{2}e^{-t} + \frac{5}{2}e^{-3t}$$

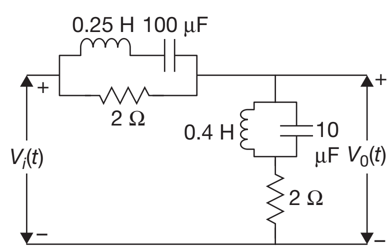
$$\Rightarrow \quad \frac{-3}{2} + \frac{5}{2} = \frac{2}{2} = 1$$

Hence, the correct Answer is (1).

5. In the RLC circuit shown in the figure, the input voltage is given by

$$V_i(t) = 2\cos(200t) + 4\sin(500t).$$

The output voltage $V_0(t)$ is **[2016]**



(A) $\cos(200t) + 2\sin(500t)$
(B) $2\cos(200t) + 4\sin(500t)$
(C) $\sin(200t) + 2\cos(500t)$
(D) $2\sin(200t) + 4\cos(500t)$

Solution: The input voltage is

$$V_{in} = 2\cos(200t) + 4\sin(500t)$$

From the above relation,

$$\omega_1 \cdot t = 200t \quad \Rightarrow \quad \omega_1 = 200 \text{ rad/s}$$

And

$$\omega_2 \cdot t = 500t \quad \Rightarrow \quad \omega_2 = 500 \text{ rad/sec}$$

Also we have,

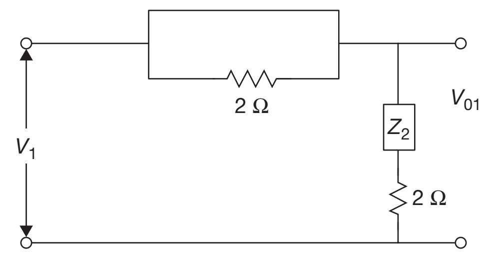
$$V_{in} = V_1 + V_2$$

If $\quad V_{in} = V_1 : \omega_1 = 200$ rad/sec

$\quad V_{in} = V_2 : \omega_2 = 500$ rad/sec

Case (i):

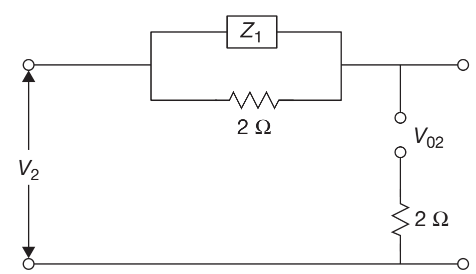
$$V_{in} = 2\cos(200t)$$



$$\Rightarrow \quad V_{01} = V_1 = 2\cos(200t)$$

Case (ii):

$$V_{in} = V_2 = 4\sin(500t)$$



$$V_{02} = V_2$$

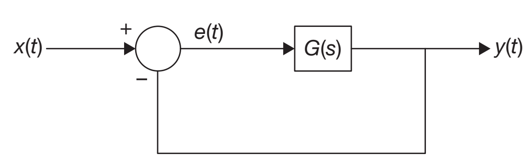
$$\therefore \quad V_0 = V_{01} + V_{02}.$$

Hence, the correct option is (B).

6. For the unity feedback conrol system shown in the figure, the open loop transfer function $G(s)$ is given as

$$G(s) = \frac{2}{s(s+1)}.$$

The steady state error e_{ss} due to a unit step input is **[2016]**



(A) 0 (B) 0.5
(C) 1.0 (D) ∞

Solution: Open loop transfer function is

$$G(s) = \frac{2}{s(s+1)}$$

For unity feedback control $H(s) = 1$

As we know that

$$\therefore \quad e_{ss} = \underset{s\to 0}{lt} \frac{S.R(s)}{1+G(s)H(s)}$$

Given $R(s) = \frac{1}{s}$ (for unit step input)

$$\therefore \quad e_{ss} = \underset{s\to 0}{lt} \frac{1}{1+\frac{2}{s(s+1)}} = \frac{1}{\infty} = 0$$

Hence, the correct option is (A).

7. The characteristics equation of an LTI system is given by $F(s) = s^5 + 2s^4 + 3s^3 + 6s^2 - 4s - 8 = 0$. The number of roots that lie strictly in the left half s-plane is ________. **[2015]**

Solution: Given characteristic equation

$$F(s) = s^5 + 2s^4 + 3s^3 + 6s^2 - 4s - 8 = 0$$

Applying RH criterion

s^5	1	3	−4
s^4	2	6	8
s^3	0(2)	0(3)	
s^2	$\frac{12-6}{2}$	−8	
s^1	$\frac{9+16}{3}$		
s^0	−8		

s^3: row having all zeros

So $\quad A(s) = 0$

$$2s^4 + 6s^2 - 8 = 0$$

$$s^4 + 3s^2 - 8 = 0$$

$$s^4 + 3s^2 - 8 = 0$$

$$\frac{dA(s)}{ds} = 0$$

$$\Rightarrow \quad 4s^3 + 6s = 0$$

$$w_n^2 = 0$$

$$2s^2 + 3 = 0$$

and roots of A.E is Let $s^2 = y$

$$2y^2 + 6y - 8 = 0$$

$$y^2 + 3y - 4 = 0$$

$y = 1, -4$

$s^2 = 1$ and $s^2 = -4$

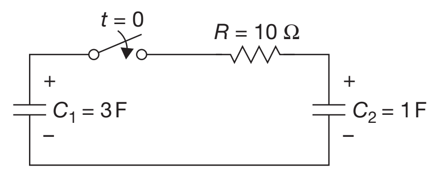
$s = \pm 1$ and $s = \pm 2$j and

one sign change 1 root in RHS of S-plane and 2 roots are on imaginary axis

$\therefore$ so only two poles exist in LHS of s-plane

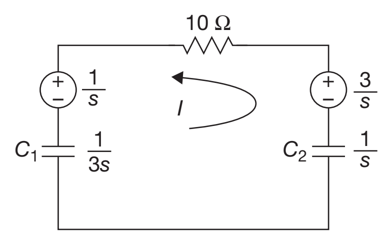
Hence, the correct Answer is (2).

8. In the circuit shown, the initial voltages across the capacitors C_1 and C_2 are 1 V and 3 V, respectively. The switch is closed at time $t = 0$. The total energy dissipated (in Joules) in the resistor R until steady state is reached, is ______ **[2015]**



Solution: $V_{c_1}(0) = 1$ V and $V_{C_2}(0) = 3$ V

Redraw the given network in S-domain



$$I(s) = \frac{\frac{3-1}{S}}{10+\frac{1}{s}+\frac{1}{3s}} = \frac{2}{\left(10s+\frac{4}{3}\right)}$$

$$i(t) = \text{ILT}\{I(s)\}$$

$$= \frac{1}{5}.e^{-4/30t}\text{; for } t \geq 0$$

$$\text{Energy dissipated} = \int_0^\infty i^2(t).R.dt$$

$$= \int_0^\infty \frac{1}{25}.e^{-4/15t}.10.dt = 1.5 \text{ J}$$

Hence, the correct Answer is (1.4 to 1.6).

9. The natural frequency of an undamped second-order system is 40 rad/s. If the system is damped with a damping ratio 0.3, the damped natural frequency in rad/s is______. **[2014]**

Solution: 38.16 rad/sec

given $w_n = 40$ rad/sec

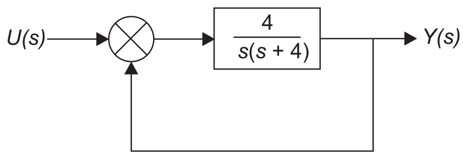
$\zeta = 0.3$

$$w_d = w_n\sqrt{1-\xi^2}$$

$$= 40\sqrt{1-0.3^2} = 40\sqrt{1-0.09}$$

$$= 40\sqrt{0.91} = 38.16 \text{ rad/sec}$$

10. For the second order closed-loop system shown in the figure, the natural frequency (in rad/s) is



(a) 16 (b) 4
(c) 2 (d) 1 **[2014]**

Solution: (c)

Closed transfer function of the given system

$$\frac{C(s)}{R(s)} = \frac{\dfrac{4}{s(s+4)}}{1+\dfrac{4}{s(s+4)}} = \frac{4}{s^2+4s+4} \qquad \text{(i)}$$

Standard transfer function

$$\frac{kw_n^2}{s^2+2\xi w_n + w_n^2} \qquad \text{(ii)}$$

Comparing equation (i) and (ii)

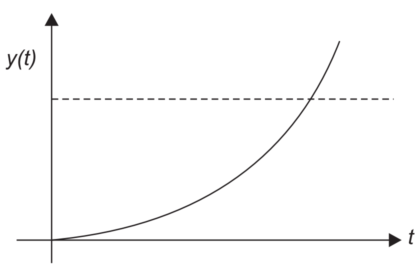
$w_n^2 = 4$

$w_n = 2$ rad/sec

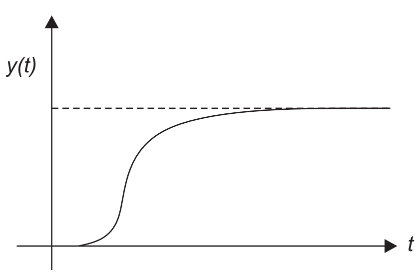
Hence, the correct option is (c).

11. The differential equation $100\dfrac{d^2y}{dt^2} - 20\dfrac{dy}{dt} + y\ .\ x(t)$ describes a system with an input *x(t)* and an output *y(t)*. The system which is initially relaxed is excited by a unit step input. The output *y(t)* can be represented by the waveform

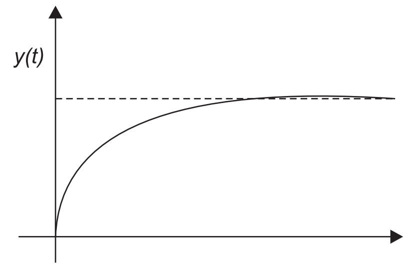
(a)



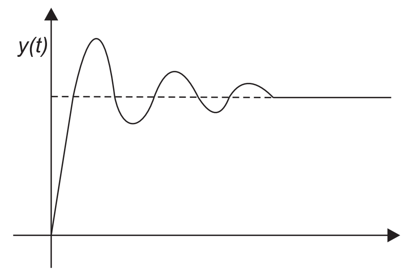
(b)



(c)



(d)



[2011]

Solution: (a)

Given differential equation

$$100\frac{d^2y}{dt^2} - 20\frac{dy}{dt} + y = x(t)$$

Take Laplace transform on both sides

$$100s^2 y(s) - 20s\, y(s) + y(s) = \frac{x(s)}{s}$$

$$\Rightarrow \frac{y(s)}{x(s)} = \frac{\dfrac{1}{s}}{(100s^2 - 20s + 1)}$$

$$\Rightarrow \frac{y(s)}{x(s)} = \frac{1}{s(10s-1)(10s-1)} = \frac{1}{s(10s-1)^2}$$

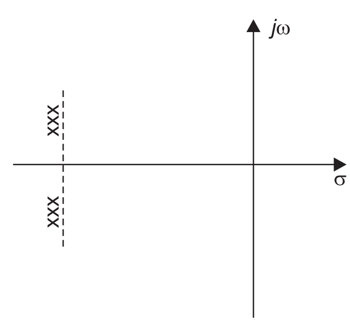
$$\therefore \text{Poles are} = 0, \ \frac{1}{10}, \frac{1}{10}$$

Systems are unstable, poles lie on right-hand side

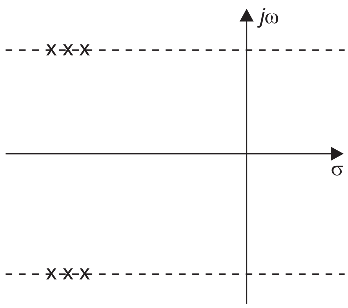
Hence, the correct option is (a).

12. Step responses of a set of three second-order under-damped systems all have the same percentage over-shoot. Which of the following diagrams represents the poles of three systems?

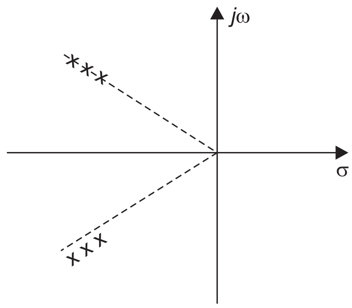
(a)



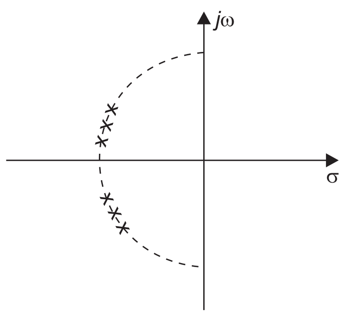
(b)



(c)



(d)



[2008]

Solution: (c)

Maximum overshoot depends on and

$$\therefore m_p = e^{-\xi\pi\sqrt{1-\xi^2}}$$

where $\xi = \cos^{-1}\theta$

$\therefore \theta$ is the angle made by pole from negative real axis.
For m_p to be same, θ is made same.

Hence, the correct option is (c).

13. Consider a system with the transfer function $G(s) = \dfrac{s+6}{Ks^2+s+6}$. Its damping ratio will be 0.5 when the value of K is

(a) 2/6 (b) 3
(c) 1/6 (d) 6 **[2002]**

Solution: (c)

$$G(s) = \frac{s+6}{k\,s^2+s+6} = \frac{s+6}{k\left(s^2+\dfrac{s}{k}+\dfrac{6}{k}\right)}$$

Characteristics equation

$s^2 + 2\zeta w_n + w_n^{\,2}$ given $\zeta = 0.5$

on comparing

$$w_n^2 = \frac{6}{k} \quad \therefore w_n = \sqrt{\frac{6}{k}}$$

$$2\xi w_n = \frac{1}{k}$$

$$\Rightarrow 2 \times 0.5 \times \sqrt{\frac{6}{k}} = \frac{1}{k}$$

$$\Rightarrow \frac{6}{k} = \frac{1}{k^2}$$

$$\Rightarrow k = \frac{1}{6}$$

Hence, the correct option is (c).

14. If the characteristic equation of a closed-loop system is $s^2 + 2s + 2 - 0$, then the system is

(a) overdamped
(b) critically damped
(c) underdamped
(d) undamped **[2001]**

Solution: (c)

$s^2 + 2s + 2 = 0$

$\therefore\ s^2 + 2\zeta w_n + w_n^2 = 0$

$2\zeta w_n = 2 \qquad \xi = \dfrac{1}{w_n}$

$w_n^2 = 2$

$\zeta < 1$ (underdamped)

Hence, the correct option is (c).

15. For a second-or der system with the closed-loop transfer function

$T(s) = \dfrac{9}{s^2+4s+9}$ the settling time for 2 per cent band, in seconds, is

(a) 1.5 (b) 2.0
(c) 3.0 (d) 4.0 **[1999]**

Solution: (b)

$$T(s) = \frac{9}{s^2+4s+9}$$

$$\therefore w_n^2 = 9 \therefore h$$

$$w_n = \mathbf{3}$$

$$\mathbf{2}\zeta w_n = \mathbf{4}$$

$$\therefore \zeta w_n = \mathbf{2}$$

$$\therefore t_s = \frac{4}{\xi w_n} = \frac{4}{2} = 2$$

Hence, the correct option is (b).

16. If $F(s)=\dfrac{\omega}{s^2+\omega^2}$, then the value of $\lim_{\to\infty} f(t)$,
{where $F(s)$ is the $L[f(t)]$}
(a) cannot be determined (b) is zero
(c) is unity (d) is infinite

[1998]

Solution: (a)

$$F(s)=\frac{2}{s^2+w^2}$$

$s^2 = w^2 = 0$

$\therefore s = \pm jw$

$\therefore$ poles are pure imaginary, hence final value of function cannot be determined.

Hence, the correct option is (a).

17. Consider a feedback control system with loop transfer function

$$G(s)H(s)=\frac{K(1+0.5s)}{s(1+s)(1+2s)}$$

The type of the closed loop system is
(a) zero (b) one
(c) two (d) three

[1998]

Solution: (b)

$$G(s).H(s)=\frac{k(1+0.5s)}{s(1+s)(s+2s)}$$

$\therefore$ Type of the system is determined by number of poles at origin of open loop transfer function.

In this it is only 1.

Hence, the correct option is (b).

18. Consider a unity feedback control system with open-loop transfer unction $G(s)=\dfrac{K}{s(s+1)}$ the steady-state error of the system due to a unit step input is
(a) zero (b) K
(c) $1/K$ (d) infinite **[1998]**

Solution: (a)

$$G(s)=\frac{K}{s(s+1)}$$

Unity feedback $H(s) = 1$

$$k_p = \underset{s\to 0}{Lt}\ G(s)\ H(s)$$

$$= \underset{s\to 0}{Lt}\ \frac{k}{s(s+1)} = \infty$$

$$e_{ss} = \frac{A}{1+k_p} = \frac{A}{1+\infty} = 0$$

Hence, the correct option is (a).

19. For a second order system, damping ratio, (ζ), is $0 < \zeta < 1$, then the roots of the characteristic polynomial are
(a) real but not equal (b) real and equal
(c) complex conjugates. (d) imaginary

[1995]

Solution :(c)

Damping ratio ζ is $0<\zeta<1$

For under damped system, roots or poles are $s=-\xi w_n \pm jw_n\sqrt{1-\xi^2}$

Hence, the correct option is (c).

20. If $L[f(t)] = \dfrac{2(s+1)}{s^2+2s+5}$ then $f(0+)$ and $f(\infty)$ are given by
(a) 0, 2 respectively
(b) 2, 0 respectively
(c) 0, 1 respectively
(d) 2/5, 0 respectively.
[Note: 'L' stands for 'Laplace Transform of']

[1995]

Solution: (b)

$$L[f(t)]=\frac{2(s+1)}{s^2+2s+5}=F(s)$$

For $f_{(0^+)} = \underset{s\to\infty}{Lt}\ s\,F(s) = \underset{s\to\infty}{Lt}\ \dfrac{2s^2+2s}{s^2+2s+5}$

$$\underset{s\to\infty}{Lt}\ \frac{s^2\left(2+\frac{2}{s}\right)}{\left(1+\frac{2}{s}+\frac{5}{s^2}\right)}=2$$

For $f_{(\infty)^*} = \underset{s\to 0}{Lt}\ s\,F(s) = \underset{s\to 0}{Lt}\ \dfrac{s(2s+2)}{s^2+2s+5}=0$

Hence, the correct option is (b).

21. The step error coefficient of a system $\dfrac{1}{(s+6)(s+1)}$ with unity feedback is
(a) 1/6 (b) ∞
(c) 0 (d) 1

[1995]

Solution: (a)

$$G(s)=\frac{1}{(s+b)(s+1)}$$

Unity feedback $H(s) = 1$

$$\underset{\to}{Lt}\ G(s)\ H(s)$$

Hence, the correct option is (a).

22. The final value theorem is used to find the
(a) steady-state value of the system output
(b) initial value of the system output
(c) transient behaviour of the system output
(d) none of these. **[1995]**

Solution: (a)

The final value theorem is used to find the steady-state value of the system.

23. The poles of a continuous time oscillator are…… **[1994]**

Solution: Pure imaginary.

The poles of a continuous time oscillator are pure imaginary.

Two-marks Questions

1. For a unity feedback control system with the forward path transfer function $G(s) = \dfrac{K}{s(s+2)}$.

The peak resonant magnitude M_r of the closed-loop frequency response is 2. The corresponding value of the gain K (Correct to two decimal places is ________. **[2018]**

Solution: The transfer function is given as

$$G(s) = \frac{K}{s(s+2)}$$

$$\text{CLTF} = \left(\frac{K}{s^2+2s+K}\right);\ \xi = \frac{2}{2\sqrt{K}} = \frac{1}{\sqrt{K}}$$

peak resonant magnitude can be calculated using

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 2$$

$$\frac{1}{2\left(\frac{1}{\sqrt{K}}\right)\sqrt{1-\frac{1}{K}}} = 2$$

$$1 = 4\left(\frac{1}{\sqrt{K}}\right)\sqrt{\frac{K-1}{K}}$$

Simplifying the above expression we get

$$K = 4\sqrt{K-1}$$

$$K^2 = 16\,(K-1)$$

$$K^2 - 16K + 16 = 0$$

$$K = 14.9282\ \xi\ K = 1.071$$

$$K = 14.9282 \text{ is valid.}$$

Hence, the correct answer is 14 to 17.

2. Which one of the following options correctly describes the locations of the roots of the equation $s^4 + s^2 + 1 = 0$ on the complex plane? **[2017]**

(A) Four left half plan (LHP) roots
(B) One right half plane (RHP) root, one LHP root and two roots on the imaginary axis.
(C) Two RHP roots and two LHP roots.
(D) All four roots are on the imaginary axis.

Solution:

S^4	1	1	1
S^3_2	0	0	0
S^2	1/2	1	0
S^1	–6	0	0
S^0	1	0	0

No. of sign changes = 2

No. RHP = 2

No of symmetric with respect of origin = 4

$\therefore$ No of LHP = 2

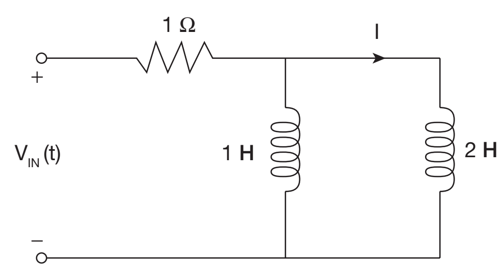
No of imaginary axis Poles = 0

Hence, the correct option is (C).

3. In the circuit shown, the voltage V_{IN} (t) is described by:

$$V_{IN}(t)\begin{cases} 0, & \text{for } t < 0 \\ 15 \text{ volts}, & \text{for} \le 0 \end{cases}$$

where t is in seconds. The time (in seconds) at which the current I in the circuit will reach the value 2 Amperes is ___________. **[2017]**



Solution: From the given data

For $t < 0$

$V_{in}(t) = 0$ V, so $I(0^-) = 0$ amp

For $t > 0$:-

At $t = 0^+$, $I(0^+) = I(0^-) = 0$A

As $t \longrightarrow \infty$

$$i(\infty) = 15 \text{ A.}$$

$$L_{eq} = (2 \parallel 1) = \frac{2}{3}\text{H}$$

$$R = 1\,\Omega$$

$$\therefore \quad \tau = \frac{L}{R} = \frac{2}{3}\text{sec}$$

$$\therefore i_s(t) = i(\infty) + \{i(0) - i(\infty)\}.e^{-t/\tau}$$

$$i(t) = 15[1 - e^{-1.5t}]$$

but $I(t) = \dfrac{1 \times \text{is}(t)}{1+2} = 5.[1 - e^{-1.5t}]\,\text{Amg}$

given $i(t) = 2$ A Then $t = ?$

$2 = 5\,[I - e^{-1.5t}]$

$e^{-1.5\,t} = 0.6$

$t = 0.34$ sec.

Hence, the correct answer is (0.3 to 0.4).

4. A unity feedback control system is characterized by the open-loop transfer function $G(S) = \dfrac{2(s+1)}{s^3 + ks^2 + 2s + 1}$

The value of k for which the system oscillates at 2 rad/s is ____________. **[2017]**

Solution: Given, open-loop transfer function

$$G(S) = \frac{2(s+1)}{s^3 + Ks^2 + 2s + 1}$$

$\therefore$ Characteristic equation is $1 + G(S) = 0$

$$1 + \frac{2(S+1)}{S^3 + KS^2 + 2S + 1} = 0$$

$$\Rightarrow S^3 + KS^2 + 4S + 4 = 0$$

Form Routh Table

S^3	1	4
S^2	K	3
S^1	$\dfrac{4K-3}{K}$	0
S^0	3	0

For the system to oscillate at $\omega_{mar} = 2$ rad/s, all S^1 row of elements are to be zero.

$$\therefore \frac{4K_{mar} - 3}{K_{mar}} = 0$$

$$\Rightarrow K_{mar} = \frac{3}{4} = 0.75$$

Hence, the correct answer is (0.75).

5. A network consisting of a finite number of linear resistor (R), inductor (L) and capacitor (C) elements, connected all in series or all in parallel, is excited with a source of the form. **[2016]**

$$\sum_{k=1}^{3} a_k \cos(k\omega_0 t)$$

where $a_k \neq 0,\ \omega_0 \neq 0$

The source has nonzero impedance. Which one of the following is a possible form of the output measured across a resistor in the network?

(A) $\sum_{k=1}^{3} b_k \cos(k\omega_o t + \phi_k)$ where $b_k \neq a_k,\ \forall\, k$

(B) $\sum_{k=1}^{4} b_k \cos(k\omega_o t + \phi_k)$ where $b_k \neq 0,\ \forall\, k$

(C) $\sum_{k=1}^{3} a_k \cos(k\omega_o t + \phi_k)$

(D) $\sum_{k=1}^{2} a_k \cos(k\omega_o t + \phi_k)$

Solution: If R, L, C are in series, and unit values $R = 1$, $L = 1$, $C = 1$ then the transfer function of the system

$$H(s) = R + sL + \frac{1}{Cs} = 1 + s + \frac{1}{s}$$

$$= \frac{s + s^2 + 1}{s} = \frac{s^2 + s + 1}{s}$$

At $\omega = k\omega_o$, $H(j\omega)$ will be in the form of $M < \phi$

$$y(t) = H(j\omega)\, x(t)$$

$$= \sum_{k=1}^{3} a_k M \cdot \cos(k\omega_o t + \phi)$$

$$= \sum_{k=1}^{3} b_k M.\cos(k\omega_o t + \phi)$$

Where $a_k \neq b_k$.

Hence, the correct option is (A).

6. A first order low pass filter of time constant T is excited with different input signals (with zero initial conditions up to $t = 0$), Match the excitation signals X, Y, Z with the corresponding time responses for $t \geq 0$: **[2016]**

X: Impulse P: $1 - e^{-\frac{t}{T}}$

Y: Unit step Q: $t - T\left(1 - e^{\frac{-t}{T}}\right)$

Z: Ramp R: $e^{\frac{-t}{T}}$

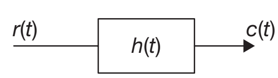
(A) X $\rightarrow$ R, Y $\rightarrow$ Q, Z $\rightarrow$ P

(B) X $\rightarrow$ Q, Y $\rightarrow$ P, Z $\rightarrow$ R

(C) X $\rightarrow$ R, Y $\rightarrow$ P, Z $\rightarrow$ Q

(D) X $\rightarrow$ P, Y $\rightarrow$ R, Z $\rightarrow$ Q

Solution:



$$c(t) = r(t) \cdot h(t)$$

$$C(s) = R(s) \cdot H(s)$$

$$X \rightarrow R,\ Y \rightarrow P,\ Z \rightarrow Q$$

Hence, the correct option is (C).

7. The open-loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K}{s(s+2)}.$$

For the peak overshoot of the closed loop system to a unit step input to be 10% the value of K is _______. **[2016]**

Solution: Transfer function is given as

$$G(S) = \frac{K}{S(S+2)}$$

Given $\quad \text{Mp\%} = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \times 100$

$$\frac{-\xi\pi}{\sqrt{1-\xi^2}} = ln\ (0.1)$$

The characteristic eqn of this system is $1 + G(s)H(S) = 0$

i.e. $s^2 + 2s + k = 0$

comparing it with standard equation

$$s^2 + 2\xi\omega_n s + \omega_n{}^2 = 0$$

we get, the natural freq and damping ratio as given below

$$\xi = 0.5911 = 0.6$$

$$2\xi\ \omega_n = 2$$

$$2 \times 0.6 \times \omega_n = 2$$

$$\omega_n = 1.66$$

$$K = \omega_n^2 = K = 1.66^2 = 2.77.$$

Hence, the correct Answer is (2.77).

8. The transfer function of a linear time invariant system is given by

$$H(s) = 2s^4 - 5s^3 + 5s - 2$$

The number of zeros in the right half of the s-plane is _______. **[2016]**

Solution: Transfer function is

$$H(s) = 2s^4 - 5s^3 + 5s - 2$$

Apply RH criteria

S^4	2	0	-2
S^3	-5	5	
S^2	2	-2	
S^1	0(4)	0	
S^0	-2		

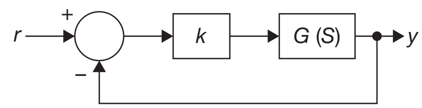
3 – sign changes hence 3 roots on right half of s – plane.

From the Routh array, we observed that one row is completely zero, which shows that break down has occurred and there are 3 sign changes in the 1st column indicating the instability.

Hence, the correct Answer is (3).

9. In the feedback system shown below $G(s) = \frac{1}{(s^2 + 2s)}$.

The step response of the closed loop system should have minimum settling time and have no overshoot.



The required value of gain k to achieve this is ___ **[2016]**

Solution: $\frac{Y(s)}{R(s)} = \frac{k}{s^2 + 2s + k}$

The characteristics equation can be given as

$$S^2 + 2s + K$$

Comparing this equation with standard equation,

we get,

$$S^2 + 2\zeta\omega_n s + \omega_n{}^2 = 0.$$

Minimum settling time and no over shoot occurs at critical case.

$$2\zeta\omega_n = 2$$

$$\zeta = 1$$

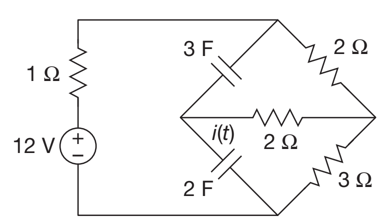
$$\omega_n = 1 \text{ rad/sec}$$

$$k = \omega_n^2 = 1$$

$$k = 1$$

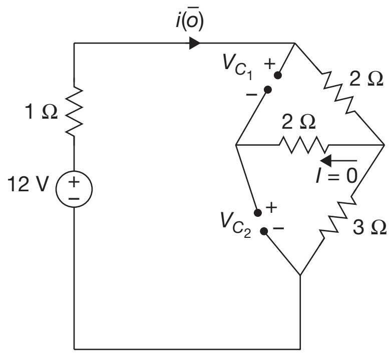
Hence, the correct Answer is (1).

10. Assume that the circuit in the figure has reached the steady state before time $t = 0$ when the 3 Ω resistor suddenly burns out, resulting in an open circuit. The current $i(t)$ (in ampere) at $t = 0$ is _____. **[2016]**



Solution: From the given data, the circuit is in steady state before $t = 0$.

At $t = 0^-$, circuit is in steady state so in steady state capacitors open circuit.



$$i(0^-) = \frac{12}{6} = 2A$$

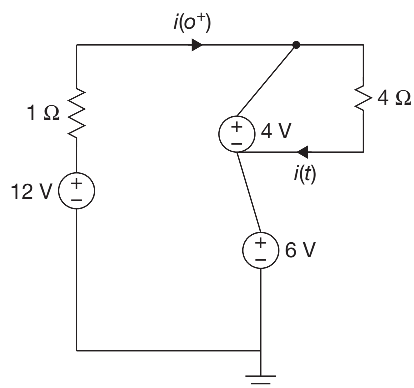
$$\therefore \qquad Vc_1(0^-) = 2 \times 2 = 4\text{ V}$$

$$Vc_2(0^-) = 3 \times 2 = 6\text{ V}$$

But at $t = 0^+$; $Vc\,(0^-) = Vc(0^+)$

for $t > 0$:

Redraw the given circuit



$$i(t) = \frac{10-6}{4}\text{ amp}$$

$$= 1\text{ amp}$$

Hence, the correct option is (1 amp).

11. The first two rows in the Routh table for the characteristic equation of a certain closed loop control system are given as: **[2016]**

S^3	1	(2K + 3)
S^2	2K	4

The range of k for which the system is stable is

(A) $-2.0 < K < 0.5$ (B) $0 < k < 0.5$

(C) $0 < k < \infty$ (D) $0.5 < K < \infty$

Solution: The range of k can be easily obtained using routh array and is used as under

Applying RH criterion to given table

S_3	1	$2k + 3$
S_2	$2k$	4
S_1	$\dfrac{2k(2k+3)-4}{2k}$	0
S_0	4	

The system will only be stable when all the values of column 1 are positive, i.e., >0

We know the first $k > 0$ and

$$2k(2k+3) - 4 \geq 0$$

$$4k^2 + 6k - 4 \geq 0$$

But for marginal stable system

$$4k^2 + 6k - 4 = 0$$

or
$$2k^2 + 3k - 2 = 0$$

$$2k^2 + 4k - k - 2 = 0$$

$$2k\,\{k+2\} - 1\,\{k+2\} = 0$$

$$k + 2 = 0$$

and

$$2k - 1 = 0$$

$$k = 1/2$$

and

$$k = -2$$

But k always positive so for stable system $k > 0$ and $k > 0.5$

$$\therefore \qquad 0.5 < k < \infty$$

Hence, the correct option is (D).

12. A unity negative feedback system has an open-loop transfer function $G(s) = \dfrac{K}{s(s+10)}$. The gain K for the system to have a damping ratio of 0.25 is________. **[2015]**

Solution: Given OLTF $G(s) = \dfrac{K}{s(s+10)}$

Characteristic equation $s^2 + 10s + K = 0$

Given
$$\zeta = 0.25$$

$$2\,\zeta\omega_n = 10$$

$$\omega_n = \sqrt{K}$$

$$0.5 \cdot \omega_n = 10$$

$$\omega_n = 20$$

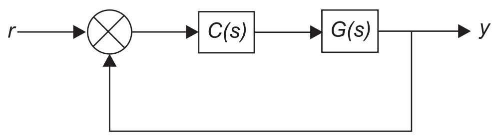
$$\sqrt{K} = 20$$

$$K = 400$$

Hence, the correct Answer is (400).

13. For the following feedback system

$G(s) = \dfrac{1}{(s+1)(s+2)}$. The 2%-settling time of the step response is required to be less than 2 seconds.



Which one of the following compensators $C(s)$ achieve this?

(a) $3\left(\dfrac{1}{s+5}\right)$ (b) $5\left(\dfrac{0.03}{s}+1\right)$

(c) $2(s+4)$ (d) $4\left(\dfrac{s+8}{s+3}\right)$

[2014]

Solution: (c)

$$G(s) = \frac{1}{(s+1)(s+2)}$$

$$H(s) = 1$$

$$\therefore T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{1}{1+(s+1)(s+2)}$$

$$T(s) = \frac{1}{s^2+3s+3} \quad \text{(i)}$$

Standard transfer function

$$\frac{kw_n^2}{s^2+2\xi w_n s + w_n^2} \quad \text{(ii)}$$

Comparing equation (1) with (ii)

$2\zeta w_n = 3$

$\xi w_n = \dfrac{3}{2} = 1.5$

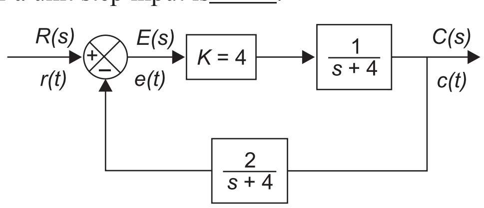
Settling time $t_s = \dfrac{4}{\xi w_n} = \dfrac{4}{1.5} = 2.67$

which is greater than 2 sec to make settling time (t_s) less than 2 sec, PD controller can be used

$\therefore$ comparing all option, option (c) is correct.

Hence, the correct option is (c).

14. The steady-state error of the system shown in the figure for a unit step input is______. **[2014]**



Solution : (0.49 to 0.52)

$$e_{ss} = \underset{s\to 0}{Lt}\ sE(s)$$

$$e_{ss} = \underset{s\to 0}{Lt} \frac{s.R(S)}{1+G(s)H(s)}$$

Here $G(s) = \dfrac{k}{s+2} = \dfrac{4}{s+2}$; $H(s) = \dfrac{2}{s+4}$;

$$R(s) = \frac{1}{s}$$

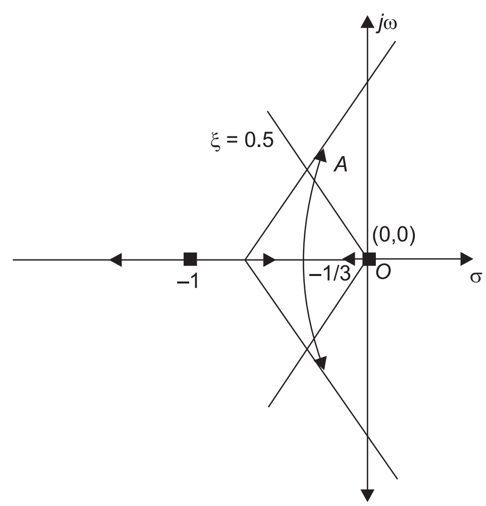
$$e_{ss} = \underset{s\to 0}{Lt} \frac{s.\frac{1}{s}}{1+\frac{4}{s+2}\times\frac{2}{s+4}}$$

$$= \frac{1}{1+\frac{\cancel{4}}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{4}}} = \frac{1}{1+1} = \frac{1}{2}$$

$$\therefore e_{ss} = 0.5$$

Hence, the correct option is (c).

15. The characteristic equation of a unity negative feedback system is $1 + KG(s) = 0$. The open loop transfer function $G(s)$ has one pole at 0 and two poles at -1. The root locus of the system for varying K is shown in the figure.



The constant damping ratio line, for $\zeta = 0.5$, intersects the root locus at point A. The distance from the origin to point A is given as 0 5. The value of K at point A is_______. **[2014]**

Solution: (0.31 to 0.41)

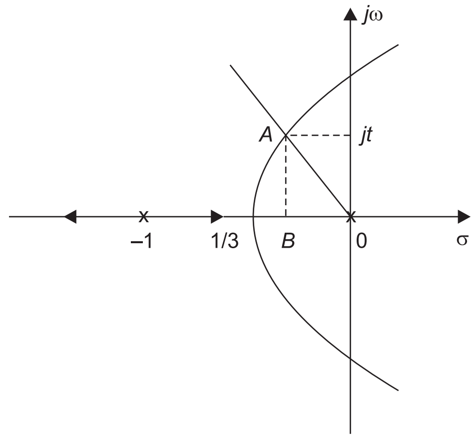
According to the question

$$G(s)H(s) = \frac{k}{s(s+1)^2} \quad \text{(i)}$$

given $\zeta = 0.5$

As $\zeta = \cos\theta$

$\therefore = \cos^{-1}\zeta = \cos^{-1}0.5 = 60°$



Let the co-ordinate of point A be $(-a + jb)$.

Distance from origin to point A is 0.5

$\therefore OA = 0.5$

$$\therefore \cos 60° = \frac{OB}{OA}$$

$\therefore OB = 0.25$

Similarly

$$\sin 60° = \frac{BA}{OA}$$

$\therefore BA = jb = 0.433j$

Co-ordinates of $A = -0.25 + 0.43j$

$\therefore$ As $|G(s)\,H(s)| = 1$ (ii)

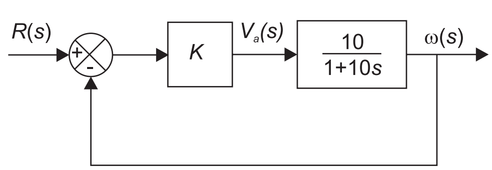
$s = -0.25 + 0.43j$

subvalues of s in equation (ii)

$$\therefore \left|\frac{k}{s(s+1)^2}\right| = 1$$

$\therefore\ k = 0.37$

16. The open-loop transfer function of a dc motor is given as $\dfrac{\omega(s)}{V_a(a)} = \dfrac{10}{1+10s}$. When connected in feedback as shown below, the approximate value of K_a that will reduce the time constant of the closed loop system by one hundred times as compared to that of the open-loop system is



(a) 1 (b) 5

(c) 10 (d) 100 **[2013]**

Solution: (c)

$$\frac{w(s)}{V_0(s)} = \frac{10}{1+10s}$$

$1 + Ts = 1 + 10s$

$\therefore T_s = 10\text{s}$

$\therefore$ Time constant of open loop system = 10

Time constant of closed loop $T' = \dfrac{T}{100}$

$$\frac{10}{100} = \frac{1}{10}$$

$$\frac{W(s)}{R(s)} = \frac{ka \times \dfrac{10}{1+10s}}{1 + ka\dfrac{10}{1+10s}}$$

$$\Rightarrow \frac{W(s)}{R(s)} = \frac{10ka}{1+10s+10ka} = \frac{10ka}{10s+(1+10ka)}$$

$$\Rightarrow \frac{W(s)}{R(s)} = \frac{10ka}{(1+10ka)\left(1+\dfrac{10s}{(1+10ka)}\right)}$$

$$T' = \frac{10}{1+10ka} = \frac{1}{10}$$

$= 1 + 10ka = 100$

$10ka = 99$

$$ka = \frac{99}{10} = 9.9 \cong 10$$

Hence, the correct option is (c).

17. The unit step response of an under-damped second order system has steady-state value of –2. Which one of the following transfer functions has these properties?

(a) $\dfrac{-2.24}{s^2+2.59s+1.12}$

(b) $\dfrac{-3.82}{s^2+1.91s+1.91}$

(c) $\dfrac{-2.24}{s^2-2.59s+1.12}$

(d) $\dfrac{-3.82}{s^2-1.91s+1.91}$ **[2009]**

Solution: (b)

Given steady-state value = –2

$\therefore$ transfer function should be under damped for these properties.

From the given options

solve for all transfer function to find ζ (damping ratio

$$\frac{-3.82}{s^2+1.918+1.91}$$

$$\therefore w_n^2 = 1.91 \quad \therefore w_n \cong 1.4$$

$$2\zeta w_n = 1.91$$

$$\xi = \frac{1.91}{2.8} < 1$$

Hence, the correct option is (b).

18. Group I lists a set of four transfer functions. Group II gives a list of possible step responses $y(t)$. Match the step responses with the corresponding transfer functions.

Group-I

$$P = \frac{25}{s^2+25}$$

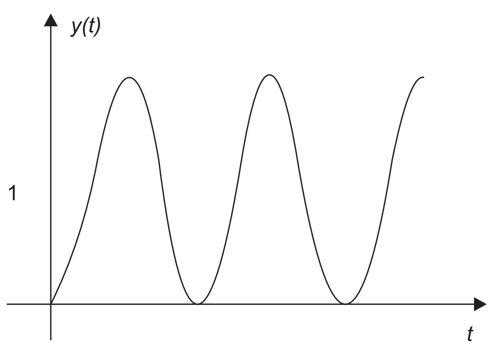
$$Q = \frac{36}{s^2+20s+36}$$

$$R = \frac{36}{s^2+12s+36}$$

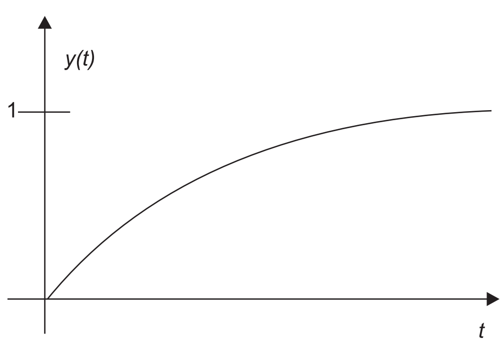
$$S = \frac{36}{s^2+7s+49}$$

Group-II

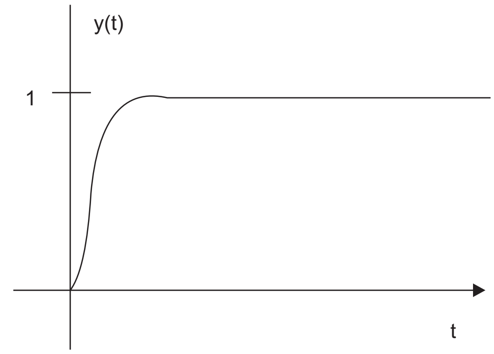
(1)



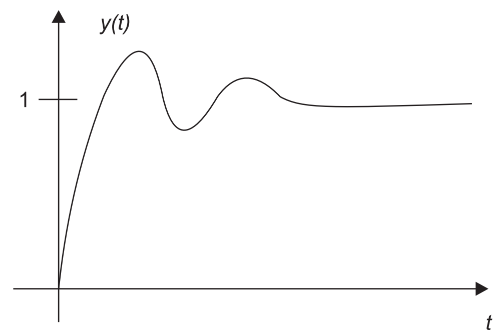
(2)



(3)



(4)



(a) P–3, Q–1, R–4, S–2
(b) P–3, Q–2, R–4, S–1
(c) P–2, Q–1, R–4, S–3
(d) P–3, Q–4, R–1, S–2 **[2008]**

Solution: (d) $G(s) = \dfrac{w_n^2}{s^2+2\xi w_n s + w_n^2}$

So $p = \dfrac{25}{s^2+25}$

$\therefore 2\,\zeta w_n = 0 \quad \therefore$ P is Undamped

$\therefore \zeta = 0$

For $Q = \dfrac{36}{s^2+20s+36}$

$$\therefore w_n^2 = 36 \quad \therefore w_n = 6$$

$$2\zeta\, w_n = 20$$

$$\xi = \frac{20}{2\times 6} = 1.67$$

$\therefore Q$ is over damped

For $R = \dfrac{36}{s^2+12s+36}$

$$\therefore w_n^2 = 36 \quad w_n = 6$$

$$2\zeta w_n = 12$$

$$\xi = \frac{12}{2\times 6} = 1$$

$\therefore R$ is critically damped

For $s = \dfrac{49}{s^2+7s+49}$

$\therefore w_n^2 = 49 \quad w_n = 7$

$\xi = \dfrac{7}{2\times 7} = 0.5$

$\therefore s$ is under damped.

Hence, the correct option is (d).

19. The magnitude of frequency response of an under damped second order system is 5 at 0 rad/sec and peaks to $\dfrac{10}{\sqrt{3}}$ at $5\sqrt{2}$ rad/sec. The transfer function of the system is

(a) $\dfrac{500}{s^2+10s+100}$ (b) $\dfrac{375}{s^2+5s+75}$

(c) $\dfrac{720}{s^2+12s+144}$ (d) $\dfrac{1125}{s^2+25s+225}$

[2008]

Solution (a):

Only (a) satisfies the given conditions.

20. The transfer function of a plant is $T(s) = \dfrac{5}{(s+5)(s^2+s+1)}$. The second-order approximation of $T(s)$ using dominant pole concept is

(a) $\dfrac{1}{(s+5)(s+1)}$ (b) $\dfrac{5}{(s+5)(s+1)}$

(c) $\dfrac{5}{s^2+s+1}$ (d) $\dfrac{1}{s^2+s+1}$ **[2007]**

Solution: (d)

$$T(s) = \frac{5}{(s+5)(s^2+s+1)}$$

In dominant pole concept, time constant factor has to be eliminated.

$$\therefore T(s) = \frac{5}{5\left(1+\frac{s}{5}\right)(s^2+s+1)}$$

$$= \frac{1}{s^2+s+1}$$

Hence, the correct option is (d).

21. In the derivation of expression for peak per cent overshoot, $M_p = \exp\left(\dfrac{-\pi\xi}{\sqrt{1-\xi^2}}\right)\times 100$ which one of the following conditions is NOT required?

(a) System is linear and time invariant
(b) The system transfer function has a pair of complex conjugate poles and no zeroes
(c) There is no transportation delay in the system
(d) The system has zero initial conditions **[2005]**

Solution: (c)

There is no transportation delay in the system.

22. A ramp input applied to a unity feedback system results in 5% steady-state error. The type number and zero frequency gain of the system are, respectively,

(a) 1 and 20 (b) 0 and 20
(c) 0 and 1/20 (d) 1 and 1/20

[2005]

Solution: (a)

$E(s) = \dfrac{R(s)}{1+G(s)}$ where $R(s) = \dfrac{1}{s^2}$

$$\therefore e_{ss} = \underset{s\to 0}{Lt}\ s.E(s)$$

$$= \underset{s\to 0}{Lt}\ s.\frac{R(s)}{1+G(s)}$$

$$= \underset{s\to 0}{Lt}\ \frac{1}{s(1+G(s))} \text{ given (finite)}$$

$$\therefore k_v = \underset{s\to 0}{Lt}\ s\,G(s)$$

$$e_{ss} = \underset{s\to 0}{Lt}\ \frac{1}{s\,G(s)} = 5\% = \frac{1}{20}$$

$\therefore k = 20$; k_v is finite for type 1 system

Hence, the correct option is (a).

23. A causal system having the transfer function $H(s) = \dfrac{1}{s+2}$ is excited with 10 $u(t)$. The time at which the output reaches 99% of its steady state value is

(a) 2.7 sec (b) 2.5 sec
(c) 2,3 sec (d) 2.1 sec **[2004]**

Solution: (c)

$$H(s) = \frac{1}{s+2}$$

$r(k) = 10u(t)$

$$\therefore R(s) = \frac{10}{s}$$

$$\therefore \text{Output } C(s) = H(s).R(s) = \frac{1}{s+2}.\frac{10}{s}$$

Using partial fraction

$$\frac{10}{s(s+2)} = \frac{A}{s+2} + \frac{B}{s}$$

$10 = AS + B(s + 2)$

$\Rightarrow$ Let $s = 0$

$10 = 2B \therefore\ B = 5$

Let $s = -2$

$10 = -2A \therefore\ A = -5$

$$\therefore C(s) = \frac{-5}{s+2} + \frac{5}{s}$$

$\therefore\ C(t) = 5(1 - e^{-2t})$

At $t = 0$ (steady-state value)

$C(t) = t(1 - e^{0}) = 5$

To reach 99% of steady-state value

$0.99 \times 5 = 5[1 - e^{-2t}]$

$\Rightarrow 1 - e^{-2t} = 0.99$

$\Rightarrow e^{-2t} = 0.1$

$\Rightarrow -2t = \ln 0.1$

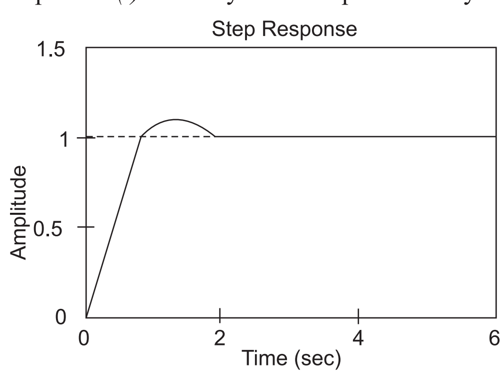
$\Rightarrow -2t = -4.6$

$\Rightarrow t = 2.3$ sec

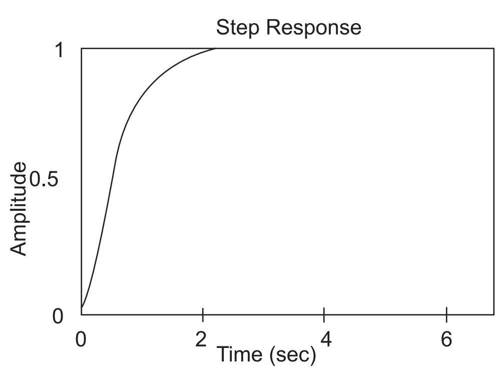
Hence, the correct option is (c).

24. A second-order system has the transfer function $\frac{C(s)}{R(s)} = \frac{4}{s^2+4s+4}$. With *r(t)* as the unit-step function, the response *c(t)* of the system is represented by

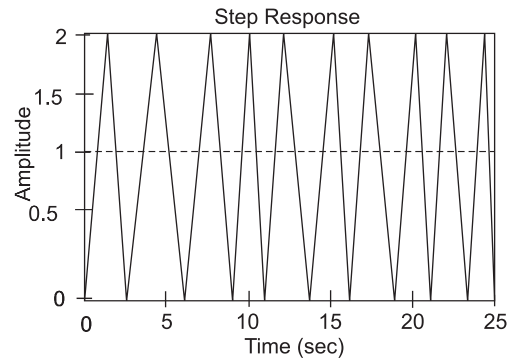
(a)



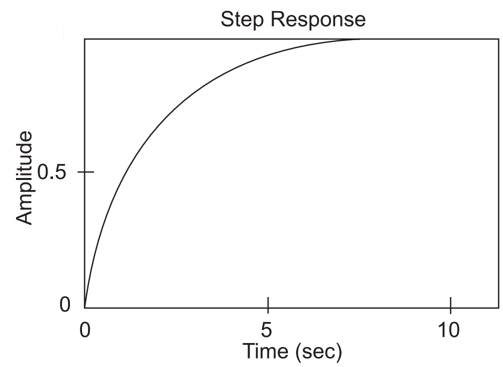
(b)



(c)



(d)



[2003]

Solution: (b)

given $\frac{C(s)}{R(s)} = \frac{4}{s^2+4s+4}$

$\therefore w_n^2 = 4 \qquad \therefore\ w_n = 2$

$2\zeta w_n = 4$

$\therefore \zeta = 1$ (critical damping)

Settling time $= t_s = \frac{4}{\xi w_n} = \frac{4}{1\times 2} = 2\,\text{sec}$

Hence, the correct option is (b).

25. The transfer function of a system is $G(s) = \frac{100}{(s+1)(s+100)}$. For a unit-step input to the system the approximate settling time for 2% criterion is

(a) 100 sec (b) 4 sec

(c) 1 sec (d) 0.01 sec **[2002]**

Solution: (b)

$$G(s) = \frac{100}{(s+1)(s+100)}$$

Taking dominant pole consideration, $s = -100$ pole is not taken. $G(s) = \frac{100}{s+1}$, it is a 1st order system

$$\therefore t_s = 4t = 4 \times 1 = 4\,\text{sec}$$

Hence, the correct option is (b).

26. If the closed-loop transfer function *T(s)* of a unity negative feedback system is given by

$$[SI - A]^{-1} = \frac{1}{(s+1)(s+1)}\begin{bmatrix} s+1 & 0 \\ 1 & s+1 \end{bmatrix} =$$

$$= \begin{bmatrix} \dfrac{1}{(s+1)} & 0 \\ \dfrac{1}{(s+1)^2} & \dfrac{1}{(s+1)} \end{bmatrix}$$

then the steady-state error for a unit ramp input is

(a) $\phi(t) = e^{At} = L^{-1}\{(SI - A)\}$

(b) $e^{at} = \begin{bmatrix} e^{-t} & 0 \\ te^{-t} & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$

$u;\ y = [1 \quad 1 \quad 1]\begin{bmatrix} x_1 \\ x_2 \\ xs_3 \end{bmatrix}$ **[1999]**

Solution: (d)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix} 4$$

$$\therefore A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

For type 2, Ramp input

$$k_v = \underset{s \to 0}{Lt}\ s\,G(s)$$

$$k_v = \infty$$

$$e_{ss} = \frac{1}{k_v} = 0$$

Hence, the correct option is (d).

27. The unit impulse response of a linear time invariant system is the unit step function *u(t)*. For $t > 0$, the response of the system to an excitation e^{-at} *u(t),* a > 0 will be

(a) *ae* (b) $(1/a)(1\ e^{-at})$

(c) $a(1\ e^{-at})$ (d) $1\ e^{-at}$ **[1998]**

Solution: (b)

$$e^{at}u(t)$$

$$R(s) = \frac{1}{s+a} \qquad \therefore H(s) = \frac{1}{s}$$

$$\therefore\ C(s) = R(s)\,.H(s)$$

$$C(s) = \frac{1}{(s+a)}.\frac{1}{s}$$

Solving by partial fraction

$$\frac{1}{s(s+a)} = \frac{A}{s+a} + \frac{B}{s}$$

$\Rightarrow 1 = AS + B(s + a)$ Let $s = 0$

$\therefore 2B = 1 \qquad \therefore B = \dfrac{1}{a}$

Let $s = -a$

$-aA = 1 \qquad \therefore A = \dfrac{-1}{a}$

$$\therefore C(s) = \frac{1}{a}\left[\frac{1}{s} - \frac{1}{s+a}\right] \quad \therefore C(t) = \frac{1}{a}[1 - e^{-at}]$$

Hence, the correct option is (b).

28. The response of an LCR circuit to a step input is

(a) Overdamped
(b) Critically damped
(c) Oscillatory

If the transfer function has

(1) poles on the negative real axis
(2) poles on the imaginary axis
(3) multiple poles on the positive real axis
(4) poles on the positive real axis
(5) Multiple poles on the negative real axis.

[1994]

Solution: (a) overdamped → 1 (poles on the negative real axis)

$$\left|\frac{k}{s(s+1)}\right| = 1$$

(b) critically damped → 5 (multiple poles on the negative real axis.

$\zeta = \mathbf{1}$

(c) oscillatory → 2 (poles on the imaginary axis)

$$\xi = 0$$

$$s = \pm jw_n$$

Hence, the correct option is (a).

29. Match the following codes with List-I with List-II:

List-I

(a) Very low response at very high frequencies
(b) Overshoot
(c) Synchro-control transformer output

List-II

(1) Low pass systems
(2) Velocity damping
(3) Natural frequency
(4) Phase-sensitive modulation
(5) Damping ratio. **[1994]**

Solution:

(a) very low response at very high frequencies → low pass system.
(b) over shoot → damping ratio
(c) synchro-control transformer output → phase-sensitive modulation.

Solution :(c)

Damping ratio ζ is $0 < \zeta < 1$

For under damped system, roots or poles are

Hence, the correct option is (a).

30. A second-order system has a transfer function given by $\dot{x}_1(t) + 2x_1(t) = u(t)$

If the system, initially at rest, is subjected to a unit step input at $t = 0$, the second peak in the response will occur at

(a) πsec (b) $\pi/3$sec
(c) $2\pi/3$ sec (d) $\pi/2$ sec. **[1991]**

Solution: (a)

$$G(s) = \frac{25}{s^2 + 8s + 25}$$

$$\therefore w_n^2 = 25$$

$$w_n = \pm 5$$

$$2\zeta w_n = 8$$

$$\zeta = 8 = 0.8$$

$$w_d = w_n\sqrt{1-\xi^2} = 5\sqrt{1-(0.8)^2} = 3$$

For 2nd peak $n = 3$

$$t_p = \frac{n\pi}{w_d} = \frac{3\pi}{3} = \pi \text{ sec}$$

Hence, the correct option is (a).

31. A unity-feedback control system has the open-loop transfer function $a_1 = 0, a_2 \neq 0, a_3 \neq 0$,. If the input to the system is a unit ramp, the steady-state error will be

(a) 0 (b) 0.5
(c) 2 (d) Infinity **[1991]**

Solution: (a)

$$G(s) = \frac{4(1+2s)}{s^2(s+2)}$$

Unit ramp i.e. $H(s) = 1$

$$k_v = \underset{s\to 0}{Lt}\, sG(s)\,H(s)$$

$$= \underset{s\to 0}{Lt} \frac{s\,4(1+2s)}{s^2(s+2)}$$

$$= \underset{s\to 0}{Lt} \frac{4(1+2s)}{s(s+2)} = \infty$$

$$e_{ss} = \frac{A}{kV} = \frac{A}{\infty} = 0$$

Hence, the correct option is (a).

32. The steady-state error of a stable 'type 0' unity feedback system for a unit step function is

(a) 0 (b) $\frac{1}{1+K_P}$
(c) ∞ (d) $\frac{1}{K_P}$

[1990]

Solution: (b)

The steady-state error of a stable 'type 0' unity feedback system for a unit step function is $\frac{1}{1+k_p}$.

Hence, the correct option is (b).

33. A critically damped, continuous-time, second order system, when sampled, will have (in Z domain)

(a) a simple pole
(b) double pole on real axis
(c) double pole on imaginary axis
(d) a pair of complex conjugate poles. **[1988]**

Solution: (b)

2nd order system when sampled

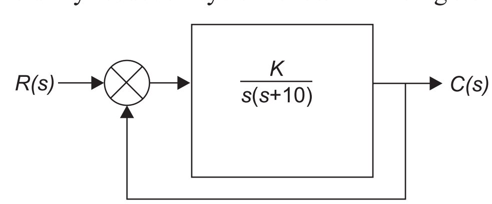
$\zeta = 1$

$\therefore s = -\zeta w_n, -\zeta w_n$

$\therefore s = -w_n, -w_n$

Hence, the correct option is (b).

34. The unity feedback system shown in the figure has:



(a) Zero steady-state position error
(b) Zero steady-state velocity error
(c) Steady-state position error $\frac{K}{10}$ units
(d) Steady-state velocity error $\frac{K}{10}$ units **[1987]**

Solution: (a)

given $G(s) = \dfrac{k}{s(s+10)}$

$H(s) = 1$

$\therefore k_p = \underset{s\to 0}{Lt}\ G(s)\,H(s)$

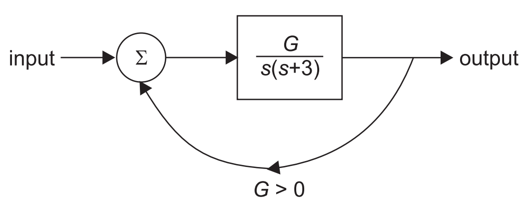
$$= \underset{s\to 0}{Lt}\ \frac{k}{s(s+10)} = \infty$$

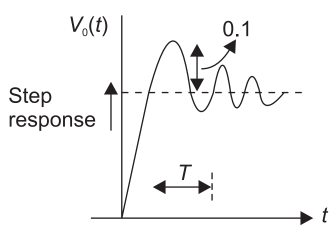
$$\therefore e_{ss} = \frac{A}{1+k_p} = \frac{A}{1+\infty} = 0$$

Hence, the correct option is (a).

Five-marks Questions

1. The block diagram of a feedback system is shown in the figure.





(a) Find the closed loop transfer function.
(b) Find the minimum value of G for which the step response of the system would exhibit an overshoot, as shown in figure.
(c) For G equal to twice this minimum value, find the time period T indicated in the figure. T = 1.96 sec. **[2000]**

Solution:

(a) Close loop transfer function

$$H(s) = \frac{\dfrac{G}{s(s+3)}}{1+\dfrac{G}{s(s+3)}} = \frac{G}{s^2+3s+G} \qquad (1)$$

(b) from figure

$$\text{overshoot} = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = 0.1$$

$$\Rightarrow -\pi\xi = \sqrt{1-\xi^2}\,\ln(0.1)$$

$$\Rightarrow (\pi\xi)^2 = (1-\xi^2)(2.3)^2$$

$$\Rightarrow \xi^2 = \frac{(2.3)^2}{\pi^2+(2.3)^2} = 0.349$$

$$\Rightarrow \xi = \sqrt{0.349} = 0.5907$$

Now comparing (1) with second order characteristics equation

$$\Rightarrow s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + 3s + G$$

$$\Rightarrow \omega_n = \sqrt{G}$$

$$\Rightarrow 2\xi\omega_n = 3,$$

$$\Rightarrow 2\times 0.5907\times\sqrt{G} = 3$$

$$G = 6.45$$

(c) New value of

$G \approx 13$

$\omega_n = \sqrt{12.96} = 3.6$

$2\xi\omega_n = 3$ (from equation (1))

$$\xi = \frac{3}{2\omega_n} = \frac{3}{2\times 3.6} = 0.42$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\omega_d = 3.6\sqrt{1-(0.42)^2}$$

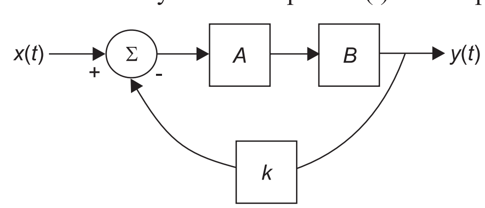
$\omega_d = 3.27$ rad/sec

$$\omega_d = \frac{2\pi}{T} = 3.27$$

$$\text{Time period(T)} = \frac{2\pi}{3.27} = 1.92\,\text{sec}$$

2. For figure shows the block diagram representation of a control system. The system in block A has in impulse response $h_A(t) = e^{-t}u(t)$. The system in block B has an impulse response.

$h_B(t) = e^{-2t}u(t)$. The block 'k' is an amplifier by a factor k. For the over all system the input is $x(t)$ and output $y(t)$.



(a) Find the transfer function $\frac{y(S)}{x(S)}$ when $k = 1$.
(b) Find the impulse response when $k = 0$.
(c) Find the values of k for which the system becomes unstable. **[1997]**

Solution: Impulse response $h_A(t) = e^{-t}u(t)$ of Block A

$$H_A(s) = \frac{1}{s+1}$$

Impulse response of Block B ($h_B(t) = e^{-2}t(ut)$)

$$H_B(s) = \frac{1}{s+2}$$

(a) given $k = 1$

$$\text{T.F.} = \frac{Y(s)}{X(s)} = \frac{G(s)}{1+G(s).H(s)}$$

where $G(s) = H_A(s) \,.\, H_B(s)$

$$= \frac{1}{s+1} \times \frac{1}{s+2} = \frac{1}{(s+1)(s+2)}$$

$H(s) = 1$

$$\therefore \frac{Y(s)}{X(s)} = \frac{\frac{1}{(s+1)(s+2)}}{1+\frac{1}{(s+1)}\cdot\frac{1}{(s+2)}.1}$$

$$= \frac{1}{(s+1)(s+2)+1} = \frac{1}{s^2+2s+s+2+1}$$

$$\therefore \frac{Y(s)}{X(s)} = \frac{1}{s^2+3s+3}$$

(b) when $K = 0$

Impulse response $H(s)$

$$\therefore \; H(s) = \frac{1}{(s+1)(s+2)}$$

take partial fraction

$$\frac{1}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

Let $S = -1$

$1 = A \qquad \therefore A = 1$

Let $S = -2$

$1 = -B \qquad \therefore B = -1$

$$\therefore H(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$h(t) = e^{-t}u(t) - e^{2t}u(t)$

$= (e^{-t} - e^{-2t})u(t)$

(c) For system to table

$1 + GH = 0$

$$\therefore 1 + \frac{1}{(s+1)(s+2)}.k = 0$$

$s^2 + 3s + 2 + k = 0$

using Rough Hurwitz criterion

s^2	1	$2 + k$
s^1	3	0
s^0	$2 + k$	

for stable system

$2 + k > 0$

$k > -2$

for unstable system

$k < -2$

Chapter 5

Stability Analysis

One-mark Questions

1. The Nyquist stability criterion and the Routh criterion both are powerful analysis tools for determining the stability of feedback controllers. Identify which of the following statements is FALSE: **[2018]**

(A) Both the criteria provide information relative to the stable gain range of the system

(B) The general shape of the Nyquist plot is readily obtained from the Bode magnitude plot for all minimum-phase systems.

(C) The Routh criterion is not applicable in the condition of transport lag, which can be readily handled by the Nyquist criterion.

(D) The closed-loop frequency response for a unity feedback system cannot be obtained from the Nyquist plot.

Solution: The closed-loop frequency response for a unity feedback system cannot be obtained from the Nyquist plot.

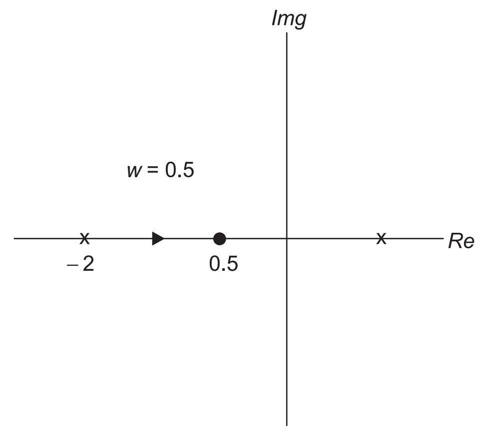
Hence, the correct option is (D)

2. The forward path transfer function of a unity negative feedback system is given by $G(s) = \dfrac{K}{(s+2)(s-1)}$

The value of K which will place both the poles of the closed-loop system at the same location is ____. **[2014]**

Solution:

Given that, $G(s) = \dfrac{k}{(s+2)(s-1)}$



Using Root locus method, the break point can be obtained as

$\Rightarrow 1 + G(s) = 0$

$$\Rightarrow 1 + \frac{k}{(s+2)(s-1)} = 0$$

or $k = -(s + 2)(s - 1)$

$$\frac{dk}{ds} = -2s - 1 = 0 \text{ or } s = -0.5$$

To have both the poles at the same direction

$|G(s)|_{s = 0.5} = 1$

$k = 2.25$.

3. If the closed-loop transfer function of a control system is given as $T(s) = \dfrac{s-5}{(s+2)(s+3)}$, then it is

(a) an unstable system
(b) an uncontrollable system
(c) a minimum phase system
(d) a non-minimum phase system **[2007]**

Solution: (d)

Due to the location of zero at right half, the system is a non-minimum phase system.

Hence, the correct option is (d)

4. The open-loop transfer function of a unity-gain feedback control system is given by $G(s) = \dfrac{K}{(s+1)(s+2)}$

The gain margin of the system in dB is given by

(a) 0 (b) 1
(c) 20 (d) ∞ **[2006]**

Solution: (d)

For 2nd order system G.M. = ∞

Hence, the correct option is (d)

5. The gain margin for the system with open-loop transfer function $G(s)H(s) = \dfrac{2(1+s)}{s^2}$ is

(a) ∞ (b) 0
(c) 1 (d) –∞ **[2004]**

Solution: (d)

$$\angle G(s)H(s) = -180 + \tan^{-1}(w)$$

for, $w_\phi = -180 + \tan^{-1} w = -180$

$w = 0$

So = $|G(s)H(s)| = \dfrac{2\sqrt{1+w^2}}{w^2} = \infty.$

$$\text{G.M.} = \frac{1}{\infty} = 0 \text{ in db}$$

Hence, the correct option is (d)

6. The phase margin of a system with the open-loop transfer function $G(s)H(s) = \dfrac{(1-s)}{(1+s)(2+s)}$ is

(a) 0 (b) 63.4°
(c) 90° (d) ∞ **[2002]**

Solution: (d)

w_g is where $|G(s)\ H(s)| = 1$

So, $\left|\dfrac{1-s}{(1+s)(2+s)}\right| = \dfrac{\sqrt{1+w^2}}{\sqrt{1+w^2}\sqrt{4+w^2}} = 1$

$\Rightarrow \sqrt{4+w^2} = 1$

$w^2 = -3$ imaginary, so no gain cross over fg or P.M. = ∞

Hence, the correct option is (d)

7. An amplifier with resistive negative feedback has two left half plane poles in its open-loop transfer function. The amplifier

(a) will always be unstable at high frequency
(b) will be stable for all frequency
(c) may be unstable, depending on the feedback factor
(d) will oscillate at low frequency **[2000]**

Solution: (c)

For a Resistive Network feedback factor is always greater than unity. So, overall gain will decrease a feedback may lead unstability F NOT Properly applied

Hence, the correct option is (c)

8. The gain margin (in dB) of a system having the loop transfer function $G(s)H(s) = \dfrac{\sqrt{2}}{s(s+1)}$ is

(a) 0 (b) 3
(c) 6 (d) ∞ **[1999]**

Solution: (d)

The given system is a second-order system so its gain margin is infinity.

Hence, the correct option is (d)

9. The phase margin (in degrees) of a system having the loop transfer function $G_2(s)H(s) = \dfrac{s}{s^2 + as + b}$ is

(a) 45° (b) –30°
(c) 60° (d) 30° **[1999]**

Solution: (d)

The gain cross over frequency is defined (w_g), where gain is 1,

So, $\left|\dfrac{2\sqrt{3}}{jw(1+jw)}\right| = 1$ or $\dfrac{2\sqrt{3}}{w\sqrt{1+w^2}} = 1$

Solving,

$$\angle G(jw)H(jw) = -90° - \tan^{-1} w = -90 - \tan^{-1}\sqrt{3}$$

∴ PM = 180° – 150 = +30°.

Hence, the correct option is (d)

10. The number of roots of

$s^3 + 5s^2 + 7s + 3 = 0$ in the left half of the s-plane is

(a) zero (b) one
(c) two (d) three **[1998]**

Solution: (d)

Using *R-H* criteria

s^3	1	7
s^2	5	3
S	6.4	0
s^0	3	

There is no sign change in 1st column, so there is no roots that lie in RHS of s-plane, or all three roots die in left half of s-plane.

Hence, the correct option is (d)

11. The open loop transfer function of a unity feedback open loop system is $\dfrac{2s^2+6s+5}{(s+1)^2(s+2)}$.

The characteristic equation of the closed loop system is

(a) $2s^2 + 6s + 5 = 0$
(b) $(s + 1)^2(s + 2) = 0$
(c) $2s^2 + 6s + 5 + (s + 1)^2(s + 2) = 0$
(d) $2s^2 + 6s + 5 - (s + 1)^2(s + 2) = 0$ **[1998]**

Solution: (c)

Given from

$2s^2 + 6s + 5 + (s + 1)^2(s + 2) = 0$,

characteristics equation is given by

$1 + GH = 0$.

So, $1+\dfrac{2s^2+6s+5}{(s+1)^2(s+2)}=0$

or, $2s^2 + 6s + 5 + (s + 1)^2(s + 2) = 0$.

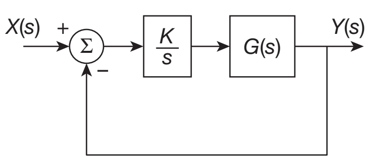
Hence, the correct option is (c)

Two-marks Questions

1. Consider a unity feedback system. as in the figure shown, with an integral compensator $\dfrac{K}{s}$ and open-loop transfer function

$$G(s) = \frac{1}{s^2 + 3s + 2}$$

where $K > 0$. The positive value of K for which there are exactly two poles of the unity feedback system on the $j\omega$ axis is equal to ______ (rounded off to two decimal places) **[2019]**



Solution: Charecterstic equation of the given function is

$$S^3 + 3S^2 + 2S + K = 0$$

Applying Routh-Herrvitz critereas

S^3	1	2
S^2	3	K
S^1	$\dfrac{6-K}{3}$	0
S^0	K	

$6 - K = 0$

$K = 6$

2. A linear time invariant (LTI) system with the transfer function $G(s)=\dfrac{K(s^2+2s+2)}{s^2-3s+2}$ is connected in unity feedback configuration as shown in the figure. **[2017]**

For the closed loop system shown, the root locus for $0 < K < \infty$ intersects the imaginary axis for $K = 1.5$. The closed loop system is stable for

(A) $K > 1.5$ (B) $1 < K < 1.5$
(C) $0 < K < 1$ (D) no positive value of K

Solution:

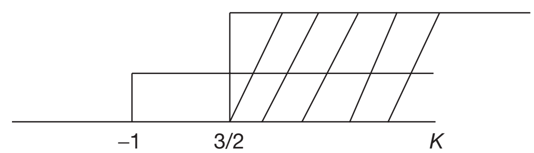
$C.E = S^2 - 3S + 2 + K(S^2 + 2S + 2) = 0$

$C.E = S^2(1+k) + S(2k-3) + 2 + 2k = 0$

$(1+k) > 0 \Rightarrow k > -1$

$(2k-3) > 0 \Rightarrow k > \dfrac{3}{2}$

$(2+2k) > 0 \Rightarrow k > -1$



$\therefore k > \dfrac{3}{2}$, i.e., $k > 1.$:

Hence, the correct option is (A).

3. A plant transfer function is given as $G(s) = \left[K_P + \dfrac{K_I}{s}\right]\dfrac{I}{s(s+2)}$. When the plant operates in a unity feedback configuration, the condition for the stability of the closed loop system is **[2015]**

(A) $K_P > \dfrac{K_I}{2} > 0$ (B) $2K_I > K_P > 0$
(C) $2K_I < K_P$ (D) $2K_I > K_P$

Solution: $G(s) = \left(K_P + \dfrac{K_I}{s}\right)\left(\dfrac{1}{s(s+2)}\right)$.

$$G(s) = \frac{(K_I + K_P s)}{s^2(s+2)}$$

Characteristic equation is

$$S^3 + 2S^2 + K_p S + K_1 = 0.$$

Apply RH criterion:

S^3	1	K_P
S^2	2	K_I
S^1	$\dfrac{2K_p - K_I}{}$	0
S^0	K_I^2	

It is stable only when $K_I > 0$ and $2K_P - K_I > 0$

$$2K_P > K_I$$

$$K_P > \frac{K_I}{2} > 0$$

Hence, the correct option is (A).

4. Consider a transfer function

$$G_p(s) = \frac{ps^2 + 3ps - 2}{s^2 + (3+p)s + (2-p)}$$ with a positive real parameter. The maximum value of p until which G_p remains stable is ______. **[2014]**

Solution: Given transfer function

$$G_p(s) = \frac{ps^2 + 3ps + 2}{s^2 + (3+p)s + (2-p)}$$

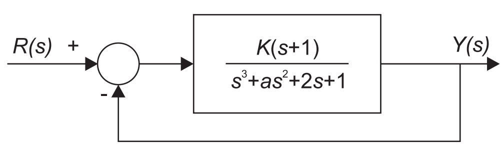
the characteristics equation is $= s^2 + (3 + p)s + (2 - p)$

Using *RH* criteria.

s^2	1	$(2 - p)$
S	$(3 + p)$	0
s^0	$(2 - p)$	0

For system to be stable, $2 - p \geq 0$ or $P_{max} = 2$.

5. The feedback system shown below oscillates at 2 rad/s when



(a) $K = 2$ and $a = 0.75$

(b) $K = 3$ and $a = 0.75$

(c) $K = 4$ and $a = 0$ 5

(d) $K = 2$ and $a = 0.5$ **[2012]**

Solution: (a)

Characteristics equation

$$1 + G(1)H(s) = 0,\ 1 + \frac{k(s+1)}{s^3 + as^2 + 2s + 1} = 0$$

$s^3 + as^2 + (2 + k)s + (1 + k) = 0$

RH table is

s^3	1	$(k + 2)$
s^2	A	$(k + 1)$
s^1	$\frac{a(2+k)-(k+1)}{9} = 0$	
s°	$(k + 1)$	

for oscillation, $\frac{a(2+k)-(k+1)}{9} = 0 \Rightarrow a = \left(\frac{k+1}{k+2}\right)$

Now, $as^2 + (k + 1) = 0$

$-aw^2 + (k + 1) = 0$

given $w = 2$ rad/sec

so put value of a, $k = -1, 2$

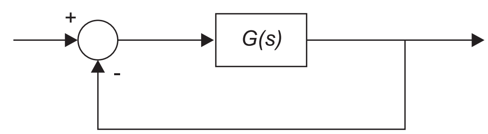
for $k = -1$, $a = 0$ and system will not oscillate for this value.

So $k = 2,\ a = \frac{k+1}{k+2} = 0.75$

Hence, the correct option is (a)

6. A certain system has transfer function

$G(s) = \frac{s+8}{s^2 + \alpha - 4}$, where α is a parameter. Consider the standard negative unity feedback configuration as shown below.



Which of the following statements is true?

(a) The closed loop system is never stable for any value of α

(b) For some positive values of α, the closed loop system is stable, but not for all positive values

(c) For all positive values of α, the closed loop system is stable

(d) The closed loop system is stable for all values of α, both positive and negative **[2008]**

Solution: (c)

Closed loop gain is

$$\frac{G(s)}{1+G(s)} = \frac{s+8}{s^2 + \alpha s + 4 + s + 8}$$

Characteristics equation, $g(s) = s^2 + (\alpha + 1)s + 4$

So, close loop system will be stable only for $\alpha > -1$. Therefore, for all +ve value of α system is stable.

Hence, the correct option is (c)

7. The number of open right half plane poles of $G(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3}$ is

(a) 0 (b) 1

(c) 2 (d) 3 **[2008]**

Solution: (c)

Characteristic equation is

$g(s) = s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$

Put $s = \frac{1}{z}$, $g'(z) = 3z^5 + 5z^4 + 6z^3 + 3z^2 + 2z + 1$

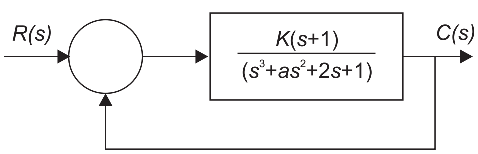
Routh array,

z^5	3	6	2
z^4	5	3	1
z^3	$\frac{21}{5}$	$\frac{7}{5}$	
z^2	$\frac{4}{3}$	1	
z^1	$\frac{-7}{4}$		
z^0	1		

Since there are two sign changes in routh array, so two poles are lying at right half of s-plane

Hence, the correct option is (c)

8. The positive values of 'K' and 'a' so that the system shown in the figure below oscillates at a frequency of 2 rad/sec respectively are



(a) 1,0.75 (b) 2,0.75
(c) 1,1 (d) 2,2 **[2006]**

Solution: (b)

$$1+G(s)H(s)=1+\frac{k(s+1)}{s^3+as^2+2s+1}=0$$

$$\frac{s^3+as^2+2s+1+ks+k}{s^3+as^2+2s+1}=0$$

$s_3 + as_2 + (2 + k)s + 1 + 1 = 0$

s^3	1	$2+k$	
s^2	a	$k+1$	
S	$\frac{a(2+k)-(1+k)}{a}$		0
s^0	$(k+1)$		

for oscillations $\frac{a(2+k)-(k+1)}{a}=0$

$$\Rightarrow 9=\frac{(k+1)}{(k+2)}$$

$\Rightarrow as^2 + k + 1 = 0 \Rightarrow s = jw,\ s^2 = -w^2 = -4$

$\Rightarrow -4a + k + 1 = 0$

$$\Rightarrow\ a=\frac{k+1}{4}\Rightarrow\frac{k+1}{4}=\frac{k+1}{k+2}\quad\Rightarrow k=2$$

$a = 0.75.$

Hence, the correct option is (b)

Common Data for Questions 6 and 7

Consider a unity-gain feedback control system whose open-loop transfer function is $G(s)=\frac{as+1}{s^2}$

9. The value of `a‘ so that the system has a phase-margin equal to$\pi/4$ is approximately equal to

(a) 2.40 (b) 1.40
(c) 0.84 (d) 0.74 **[2006]**

Solution: (c)

$$\text{P.M.} = \frac{\pi}{4}$$

$$\Rightarrow 180 + \tan^{-1}(aw) - 180^\circ = \frac{\pi}{4}$$

$\Rightarrow aw = 1$

for gain cross over frequency, $|G(s)| = 1$

$$\sqrt{\frac{1+a^2w^2}{w^2}}=1\ \Rightarrow w^2=\sqrt{2}\ \Rightarrow w=2^{\frac{1}{4}}$$

$$a=\frac{1}{2^{\frac{1}{4}}}=0.84$$

Hence, the correct option is (c)

10. With the value of `a‘ set for phase-margin of $\pi/4$, the value of unit-impulse response of the open-loop system at $t = 1$ second is equal to

(a) 3.40 (b) 2.40
(c) 1.84 (d) 1.74 **[2006]**

Solution: (c)

$$G(s)=\frac{0.84s+1}{s^2}$$

$H(s) = 1,\ R(s) = 1 \Rightarrow C(s) = G(s).\ R(s)$

$$\Rightarrow C(s)=\frac{0.84s+1}{s^2}$$

taking inverse Laplace transform

$$c(t)=L^{-1}\left[\frac{1+0.84s}{s^2}\right]=(t+0.84)u(t)$$

at, $t = 1$,

$c(t) = 1 + 0.84 = 1.84$

Hence, the correct option is (c)

11. The gain and phase crossover frequencies in rad/sec are, respectively

(a) 0.632 and 1.26 (b) 0.632 and 0.485
(c) 0.485 and 0.632 (d) 1.26 and 0.632

[2005]

Solution: (d)

Gain cross over frequency defined where gain is 1

$|G(s)| = 1$

$$\Rightarrow \frac{3}{w(w^2+r)^{\frac{1}{2}}} = 1 \Rightarrow w_{gc} = 1.26$$

phase cross over frequency where $\angle GH = 180$

$$\Rightarrow w_{pc} = 0.682\left(\frac{w\phi}{2} = 2\cos 2w\phi\right)$$

Hence, the correct option is (d)

12. Based on the above results the gain and phase margins of the system will be

(a) −7.09 and 87.5°
(b) 7.09 and 87.5°
(c) 7.09 dB and −87.5°
(d) −7.09 dB and −87.5° **[2005]**

Solution: (d)

$$\text{G.M. at } w\phi = \frac{3}{0.632(0.632^2+4)^{\frac{1}{2}}}$$

$a = 2.26$

$$\text{G.M.} = 20\log\frac{1}{a}$$

$$20\log\frac{1}{2.26} = -7.09$$

as G.M. is –ve so system is unstable, PM is also negative.

P.M. = − 87.5°

Hence, the correct option is (d)

13. The open-loop transfer function of a unity feedback system is $G(s) = \dfrac{k}{s(s^2+s+2)(s+3)}$

The range of K for which the system is stable is

(a) $\frac{21}{4} > K > 0$ (b) $13 > K > 0$
(c) $\frac{21}{4} < K < \infty$ (d) $-6 < K < \infty$

[2004]

Solution: (a)

$$G(s) = \frac{k}{s(s^2+s+2)(s+3)},\ H(s) = 1$$

$$1 + G(s)H(s) = 1 + \frac{k}{s(s^3+3s^2+s^2+3s+2s+6)}$$

$$= \frac{s^4+4s^3+5s^2+6s+k}{s(s^3+4s^2+5s+6)}$$

$1 + G(s)H(s) = 0 \Rightarrow s^4 + 4s^3 + 5s^2 + 6s + k = 0$

RH – table,

s^4	1	5	k
s^3	4	6	
s^2		k	
s	$\dfrac{\frac{7}{2}\times 6-4k}{\frac{7}{2}}$		0
s^0	K		

for system to be stable, $k > 0$

$$\Rightarrow \frac{7}{2}\times 6 - 4k > 0$$

$$\Rightarrow k < \frac{21}{4}$$

So, $0 < k < \dfrac{21}{4}$

Hence, the correct option is (a)

14. For the polynomial $P(s) = s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15$, the number of roots which lie in the right half of the s-plane is

(a) 4 (b) 2
(c) 3 (d) 1 **[2004]**

Solution: (b)

$P(s) = s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15$

s^5	1	2	3
s^4	1	2	15
s^3	$0(E)$	-12	0
s^2	$\dfrac{2E+12}{E}$	15	0
S	$\dfrac{-12(2E+12)}{E}-15E$		
s^0	15		

Let E is a small +ve number. Then coefficient, s^2,

$$\frac{2E+12}{E} \rightarrow +\text{ve}$$

$$\frac{-12(2E+12)}{E} \rightarrow -\text{ve}$$

∴ So, two sign changes from s^2 to s and s to $s°$

∴ Two roots on RHS of s plane.

Hence, the correct option is (b)

Common Data for Questions 12 and 13.

The open loop transfer function of a unity feedback system is given by $G(s) = \dfrac{3e^{=2s}}{s(s+2)}$

15. The gain margin and the phase margin of a feedback system with $G(s)H(s) = \dfrac{s}{(s+100)^3}$ are

(a) 0 dB, 0° (b) ∞, ∞
(c) ∞, 0° (d) 88.5 dB, ∞

[2003]

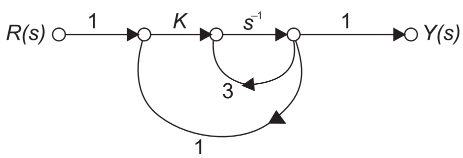
Solution: (b)

$$G(s)H(s) = \frac{s}{(s+100)^3}$$

Gain margin and phase margin of system cannot be determined.

Hence, the correct option is (b)

16. The system shown in the figure remains stable when



(a) $K < -1$ (b) $-1 < K < 1$
(c) $1 < K < 3$ (d) $K < -3$ **[2002]**

Solution: (d)

$$\text{Given } \frac{Y(s)}{R(s)} = \frac{\frac{k}{s}}{1-\left(\frac{g}{s}+\frac{k}{s}\right)} = \frac{k}{s-(3+k)}$$

for system to be stable.

$3 + k < 0$

$k < -3$

Hence, the correct option is (d)

17. The characteristic polynomial of a system is $q(s) = 2s^5 + s^4 + 4s^3 + 2s^2 + 2s + 1$. The system is

(a) stable
(b) marginally stable
(c) unstable
(d) oscillatory **[2002]**

Solution: (c)

RH – table,

s^5	2(1)	4(2)	2(1)
s^4	1	2	1
s^3	0	0	0
s^2			
S			
s^0			

In the third row all coefficients are zero. So, different rows

$$\frac{d}{ds}(s^4 + 2s^2 + 1) = 0 \quad s = \pm j, s = \pm \mathrm{j}$$

$$\frac{d}{ds}(s^2 + 1) = 0 \Rightarrow s = 0.$$

Double root on imaginary axis, so system is unstable.

Hence, the correct option is (c)

18. The system with the open loop transfer function $G(s)H(s) = \dfrac{1}{s(s^2+s+1)}$ has a gain margin of

(a) – 6 dB (b) 0 dB
(c) 3.5 dB (d) 6 dB **[2002]**

Solution: (b)

Given, $G(s)H(s) = \dfrac{1}{s(s^2+s+1)}$

w when $\angle G(s)H(s) = -180$

$$\Rightarrow -180 = -90 - \tan^{-1}\frac{w}{1+w^2} \Rightarrow w_{pc}^2 = 1 \text{ rad/sec.}$$

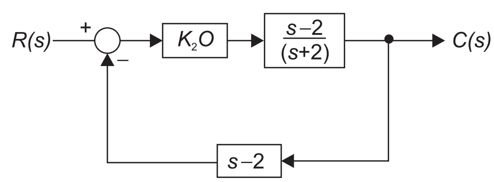
$$\text{So, } |G(s)H(s)| = \frac{1}{\sqrt{\left|(1-w^2)\right|^2 + |jw|^2}}$$

at $w_{pc} = 1$ rad/sec.

$\therefore$ G.M. $= -20 \log 1 = 0$

Hence, the correct option is (b)

19. The feedback control system in the figure is stable.



(a) for all $K \geq 0$ (b) only is $K \geq 0$
(c) only if $0 \leq K < 1$ (d) only if $0 \leq K \leq 1$

[2001]

Solution: (c)

Given transfer function

$$T.F = \frac{G_1G_2}{1+G_1G_2H} = \frac{\frac{k(s-2)}{(s+2)^2}}{1+\frac{k(s-2)(s-2)}{(s+2)^2}}$$

$$= \frac{k(s-2)}{(s+2)^2 + k(s-2)^2}$$

Characteristic equation is $= s^2 + 4 + 4s + ks^2 - 4ks + 4k$

or $(1 + k)s^2 + 4(4 - 4k)s + 4k + 4 = 0$

RH – table.

s^2 $(1 + k)$ $(4k + 4)$

$S \quad k(4-4k) \quad 0$

$S \quad 4(k+1)$

for system to be stable.

$k(4-4k) > 0 \Rightarrow k < 1$ and $k \geq 0$.

also, $(k+1) > 0 \Rightarrow k > -1$

so $0 \leq k < 1$

Hence, the correct option is (c)

20. A system described by the transfer function $H(s) = \dfrac{1}{s^3 + \alpha s^2 + Ks + 3}$ is stable.

The constraints on α and K are

(a) $\alpha > 0, \alpha K < 3$ (b) $\alpha > 0, \alpha K > 3$
(c) $\alpha < 0, \alpha K > 3$ (d) $\alpha < 0, \alpha K < 3$

[2000]

Solution: (b)

R-H table is

	1	k
	3	
	$\dfrac{\propto k - 3}{\propto}$	

for system to be stable.

$\propto > 0, \dfrac{\propto k - 3}{\propto} > 0$ or $\propto k > 3$.

Hence, the correct option is (b)

21. If $G(s)$ is a stable transfer function, then $F(s) = \dfrac{1}{G(s)}$ is always a stable transfer function.

State True or False. **[1994]**

Solution: False

$$\text{T.F. of } G(s) = \frac{(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)}$$

$$\text{T.F. of } G(s) = \frac{(s+p_1)(s+p_2)}{(s+z_1)(s+z_2)}$$

for stability of $G(s)$, poles of $G(s)$ should be at left half s plane, so far stable $G(s)$ $F(s)$ have zeros at left half of s plane. So, $F(s)$ need not to be stable.

22. If $s^3 + 3s^2 + 4s + A = 0$, then all the roots of this equation are in the left half plane provided that

(a) $A > 12$ (b) $-3 < A < 4$
(c) $0 < A < 12$ (d) $5 < A < 1s2$ **[1993]**

Solution: (c)

$\rightarrow s^3 + 3s^2 + 4s + A = 0$

Using *RH*

s^3	1	4
s^2	3	A
S	$\dfrac{12-A}{3}$	0
s^0	A	

for stability $\dfrac{12-A}{3} > 0 \Rightarrow 0 < A < 12$

Hence, the correct option is (c)

23. An electromechanical closed-loop control system has the following characteristic equation: $s^3 + 6K\,s^2 + (K+2)s + 8 = 0$, where K is the forward gain of the system. The condition for closed loop stability is:

(a) $K = 0.528$ (b) $K = 2$
(c) $K = 0$ (d) $K = -2.258$

[1990]

Solution: (a)

Given $s^3 + 6ks^2 + (k+2)s + 8 = 0$

Apply RH,

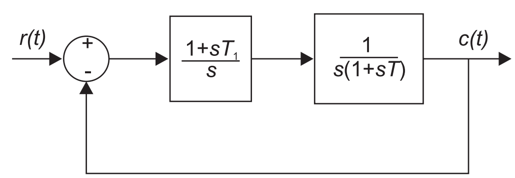
s^3	1	$k+2$
s^2	$6k$	8
S	$\dfrac{6k^2+12k-8}{6k}$	
s°	8	

For stable system,

$\Rightarrow k = -2.528, +0.528$ as $k > 0$, so $k \Rightarrow 0.528$

Hence, the correct option is (a)

24. In order to stabilize the system shown in the below given figure. Ti should satisfy:



(a) $T_i = -T$ (b) $T_i = T$
(c) $T_i < T$ (d) $Ti > -T$. **[1989]**

Solution: (d)

$$G(s) = \frac{1+sTi}{s} \times \frac{1}{s(1+ST)}$$

$H(s) = 1$

$$\therefore T(s) = \frac{G(s)}{1+G(s)H(s)}$$

Characteristics equation

$1 + G(s)\,H(s) = 0$

$$1 + \frac{(1+sTi)}{s} \times \frac{1}{s(1+sT)} \times 1 = 0$$

$= s^2(1+sT) + (1+sTi) = 0$

$= s^3T + s + sTi + 1 = 0$

Using R-H criterion

s^3	T	Ti
s^2	1	1
s^1	$Ti - T$	0
s^0	Ti	0

For stability 1st column should be positive

$\therefore Ti - T > 0$

$Ti > T$

Hence, the correct option is (d)

25. Consider a characteristic equation given by

$s^4 + 3s^3 + 5s^2 + 6s + K + 10$

The condition for stability is

(a) $K > 5$ (b) $-10 < K$
(c) $K > -4$ (d) $-10 < K < -4$

[1988]

Solution: (d)

Characteristics equation

$s^4 + 3s^3 + 5s^2 + 6s + k + 10$

using $R - H$ criterion

s^4	1	5	$k + 10$
s^3	3	6	0
s^2	3	$k + 10$	0
s^1	$\dfrac{-12-3k}{3}$		
s^0	$k + 10$		

For stable system, all coefficients in 1st column are positive

$$\therefore \frac{-12-3k}{3} > 0$$

$-12 - 3k > 0$

$3k < -12$

$k < -4$

and

$k + 10 > 0$

$k > -10$

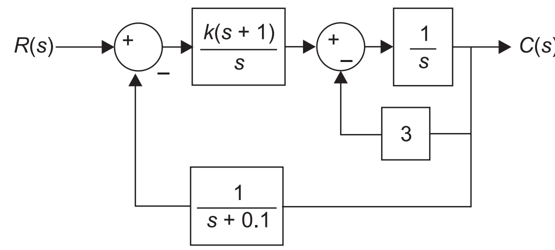
so the value of k lies between

$-10 < k < -4$.

Hence, the correct option is (d)

Five-marks Questions

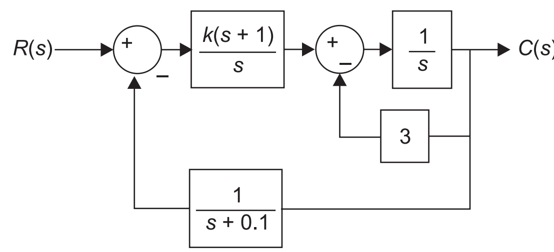
1. Consider the feedback control system shown in figure.



(a) Find the transfer function of the system and its characteristic equation.
(b) Use the Routh-Hurwitz criterion to determine the range of K for which the system is stable.

[2001]

Solution:



$$\therefore G_1(s) = \frac{k(s+1)}{s}$$

$$G_s(s) = \frac{1/s}{1+\frac{1}{s}\times 3} = \frac{1}{(s+3)}$$

$$\therefore G(s) = G_1(s) \times G_2(s)$$

$$= \frac{k(s+1)}{s} \times \frac{1}{(s+3)}$$

$$\therefore \text{T.F.} \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

where $H(s) = \dfrac{1}{(s+0.1)}$

$$\therefore \frac{C(s)}{R(s)} = \frac{\frac{k(s+1)}{s} \times \frac{1}{(s+3)}}{1+\frac{1}{(s+0.1)} \times \frac{k(s+1)}{s} \times \frac{1}{(s+3)}}$$

$$= \frac{k(s+1)(s+0.1)}{s(s+0.1)(s+3)+k(s+1)}$$

$$\frac{C(s)}{R(s)} = \frac{k(s+1)(s+0.1)}{s^3+3.1s^2+s(k+0.3)+k}$$

$\therefore$ Characteristic equation

$$s^3 + 3.1s^2 + s(k + 0.3) + k = 0$$

Using Routh-Hurwitz criterion

s^3	1	$k + 0.3$
s^2	3.1	k
s^1	$\frac{2.1k+0.93}{3.1}$	0
0		
s^0	k	

For stable system'

$k > 0$ and

$$\frac{2.1k+0.93}{3.1} > 0$$

$\therefore$ $2.1\,k + 0.93 > 0$

$k > -0.44$

$\therefore$ For stability $k > 0$

2. The loop transfer function of a feedback control system is given by

$G(s)H(s) = \frac{k'(s+1)}{s(1+Ts)(1+2s)}$, $k > 0$. Using Routh-Hurwitz criterion, determine the region of K–T plane in which the closed-loop system is stable. **[1999]**

Solution: $G(s)H(s) = \frac{k(s+1)}{s(1+7s)(1+2s)}$

Characteristics equation

$1 + G(s)\,H(s) = 0$

$$1+\frac{k(s+1)}{s(1+7s)(1+2s)}$$

$\Rightarrow s(1 + 7s)(1 + 2s) + k(s + 1) = 0$

$\Rightarrow s(1 + 2s + 7s + 27s^2) + ks + k = 0$

$\Rightarrow 27s^3 + 2s^2 + 7s^2 + s + ks + k = 0$

$\Rightarrow 27s^3 + s^2(2 + 7) + s(1 + k) = 0$

s^3	αT	$1 + k$
s^2	$2 + 7$	K
s^1	$(1+k)-\frac{27k}{2+7}$	0
0		
s^0	K	

For a stable system

$k > 0$, $\quad 2T > 0$

$T > 0$

$2 + T > 0 \Rightarrow T > -2$

and

$$(k+1) = \frac{2kt}{2+7} > 0$$

$$k+1 > \frac{2kt}{2+7}$$

$k > 0$ and $T > 0$

$(k + 1)(2 + 7) > 2\,kT$

$\Rightarrow 2k + 2 + Tk + T > 2\,kT$

$\Rightarrow 2k + 7 + 2 > kT$

$\Rightarrow 2(k + 1) > T(k - 1)$

$$\Rightarrow \frac{2(1+k)}{(k-1)} > T$$

$\therefore$ Region in which closed-loop system is stable $= T < \frac{2(1+k)}{(k-1)}$

3. The characteristic equation of a feedback control system is $s^4 + 20s^3 + 15s^2 + 2s + K = 0$

(i) Determine the range of K for the system to be stable.

(ii) Can the system be marginally stable? If so, find the required value of K and the frequency of sustained oscillations. **[1998]**

Solution: For

$$s^4 + 20s^2 + 15s^2 + 2 + k = 0$$

Using Routh-Hurwitz criterion

s^4	1	15	K
s^3	20	2	0
s^2	$\frac{298}{20}$	0	K
k	0		
s^1	$2-\frac{20k\times 20}{298}$	0	0
0	0		
s^0	K		

For stable system

$k > 0$

and $\frac{2-20k\times 20}{298} > 0$

$$\Rightarrow 2 > \frac{400k}{298}$$

$\therefore$ $k < 1.49$

Range of k is $0 < k < 1.49$

(ii) For oscillation

For marginal stable value of $k = 1.49$

$$\frac{298}{20}s^2 + k = 0 \text{ (from } s^2\text{)}$$

$19.9s^2 + 1.49 = 0$

$10s^2 + 1 = 0$

$$s^2 = \frac{-1}{10}$$

$$s \pm j\frac{1}{\sqrt{10}}$$

$$+j\omega_n = \pm j\frac{1}{\sqrt{10}}$$

$$\omega_n = \frac{1}{\sqrt{10}} \text{ rad/sec}$$

4. A system having an open loop transfer function $G(s) = \dfrac{k(s+3)}{s(s^2+2s+2)}$ is used in control system with unity negative feedback. Using the Routh-Hurwitz criterion, find the range of values of k for which the feedback system is stable. **[1996]**

Solution: $G(s) = \dfrac{k(s+3)}{s(s^2+2s+2)}$ $H(s)$ **= 1**

$$\text{T.F.} = \frac{G(s)}{1+G(s)H(s)}$$

Characteristic equation

$\therefore 1 + G(s)\, H(s) = 0$

$$\Rightarrow 1 + \frac{k(s+3)}{s^3+2s^2+2s} = 0$$

$H(s) = 1$

$$\therefore \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$\frac{\frac{G}{s(s+3)}}{1+\frac{G}{s(s+3)}} = \frac{G}{s(s+3)+G}$$

$$\therefore \frac{C(s)}{R(s)} = \frac{G}{s^2+3s+G} \qquad \text{(i)}$$

(b) For overshoot

$$m_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100 = 10$$

$\therefore \xi = 0.6$

$\omega_n^2 = G$

$\omega_n = \sqrt{G}$

$\therefore 2\xi\omega n = 3$ (from eq(i))

$$\therefore \omega_n = \frac{3}{2\times 0.6}$$

$\therefore \sqrt{G} = 2.5$

$G = 6.25$

Using Routh-Hurwitz condition

s^3	1	$2+k$
s^2	2	$3k$
s^1	$\dfrac{4+2k-3k}{2}$	0
s^0	3	

For system to the stable

$3k > 0$

$k > 0$

$$\frac{4+2k-3k}{2} > 0$$

$4 - k > 0$

$k < 4$

$\therefore$ Range of stability

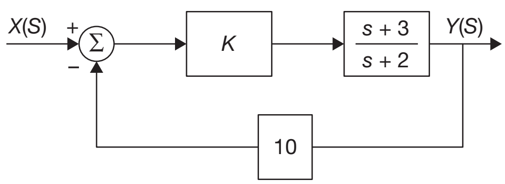
$0 < k < 4$

Chapter 6

Root Locus

One-mark Questions

1. For the system shown in the figure, $s = -2.75$ lies on the root locus if K is ________. **[2015]**



Solution: From the given block diagram

$$T(s) = \frac{K(s+3)/(s+2)}{1+\dfrac{10K(S+3)}{(s+2)}} = \frac{K(s+3)}{(s+2)+10K(s+3)}$$

$$T(s) = \frac{K(s+3)}{(10K+1)s+(2+30K)}$$

But given $S = -2.75$ is on Root locus so it must satisfy the magnitude condition.

$$|T(s)| = 1 \text{ at } S = -2.75$$

$$\frac{K(0.25)}{(10K+1)(-2.75)+2+30K} = 1$$

$$K(0.25) = -27.5K - 2.75 + 2 + 30K$$

$$-2.5K = -0.75$$

$$K = 0.3$$

Hence, the correct Answer is (0.29 to 0.31).

2. A unity negative feedback system has the open-loop transfer function $G(s) = \dfrac{K}{s(s+1)(s+3)}$. The value of the gain $K(>0)$ at which the root locus crosses the imaginary axis is ________. **[2015]**

Solution: Given OLTF $G(s) = \dfrac{K}{s(s+1)(s+3)}$

Characteristic equation is $1 + G(s)H(s) = 0$

$$s(s^2 + 4s + 3) + K = 0$$

$$s^3 + 4s^2 + 3s + K = 0$$

Apply RH criterion,

S^3	1	3
S^2	4	K
S^1	$\dfrac{12-k}{4}$	0
S^0	K	

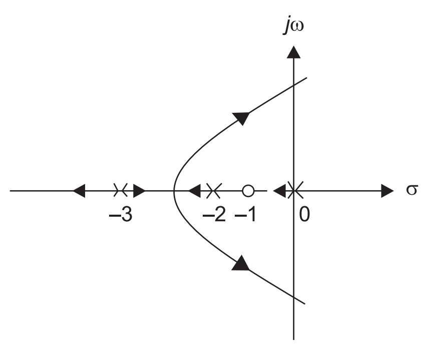
Given $K > 0$.

It is crossing imaginary axis let S^1 row all zeros.

$$\therefore \quad \frac{12-k}{4} = 0 \quad \Rightarrow \quad K = 12$$

Hence, the correct Answer is (12).

3. The root locus plot for a system is given below. The open loop transfer function corresponding to this plot is given by



(a) $G(s)H(s) = K\dfrac{s(s+1)}{(s+2)(s+3)}$

(b) $G(s)H(s) = K\dfrac{(s+1)}{s(s+2)(s+3)^2}$

(c) $G(s)H(s) = K\dfrac{1}{s(s-2)(s+2)(s+3)}$

(d) $G(s)H(s) = K\dfrac{(s+1)}{s(s+2)(s+3)}$ **[2011]**

Solution: (b)

From the plot we can observe that one pole terminates at zero at −1 and three poles terminate to ∞. It means there are four poles and 1 zero. There are two poles at −3 on both sides. It means there are two poles at –3.

Hence, the correct option is (b)

4. Given $G(s)H(s) = \dfrac{K}{s(s+1)(s+3)}$, the point of intersection of the asymptotes of the root loci with the real axis is

(a) –4 (b) 1.33
(c) −1.33 (d) 4 **[2004]**

Solution : (c)

$$\text{Centroide } = \frac{\Sigma P - \Sigma Z}{P - Z} = \frac{-1-3}{3} = -1.33$$

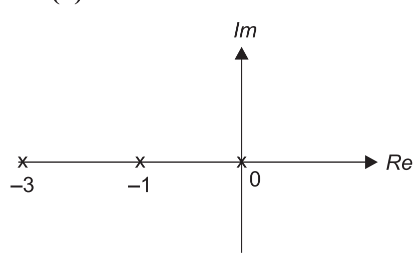
Hence, the correct option is (c)

5. Which of the following points is NOT on the root locus of a system with the open-looptransfer function

$$G(s)H(s) = \frac{K}{s(s+1)(s+3)}$$

(a) $s = -j\sqrt{3}$ (b) $s = -1.5$
(c) $s = -3$ (d) $s = -\infty$ **[2002]**

Solution: (b)

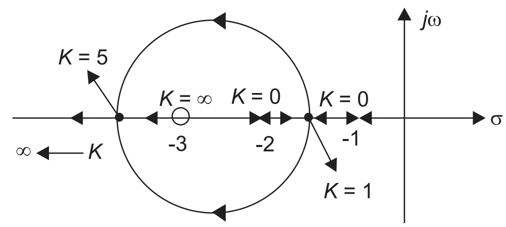


Root locus lies where number of poles and zeros to right half of the pole is odd.

$\therefore$ $s = -1.5$ does not lie on root locus.

Hence, the correct option is (b)

6. The root-locus diagram for a closed-loop feedback system is shown in the figure. The system is overdamped



(a) only if $0 \le K \le 1$
(b) only if $1 < K < 5$
(c) only if $K > 5$
(d) if $0 \le K < 1$ or $K > 5$ **[2001]**

Solution: (d)

For overdamping roots of characteristics equation should lie on –ve axis and be unequal.

Hence, the correct option is (d)

7. If the open-loop transfer function is a ratio of a numerator polynomial of degree 'm' and a denominator polynomial of degree 'n,' then the integer (n-m) represents the number of

(a) breakaway points (b) unstable poles
(c) separate root loci (d) asymptotes
[1994]

Solution: (d)

Difference between poles and zeros will give number of asymptotes.

Hence, the correct option is (d)

Two-marks Questions

1. The open loop transfer function of a unity feedback control system is

$$G(s) = \frac{K}{s^2+5s+5}.$$

The value of K at the breakaway point of the feedback control system's root locus plot is **[2016]**

Solution: Now we have transfer function as

$$G(s) = \frac{K}{s^2+5s+5}.$$

The characteristics equation for open loop TF is

$$1 + G(S)H(S) = 0$$

$$S^2 + 5S + 5 + K = 0$$

$$K = -S^2 - 5S - 5$$

To find break away point differentiate the above equation w.r.t s, that is

$$\frac{dK}{ds} = 0$$

$$\Rightarrow \quad -2S - 5 = 0$$

$$S = -2.5$$

$$K = -(-2.5)^2 - 5(-2.5) - 5$$

$$= 1.25$$

Break away point is that point in root locus where two complex poles are present causing the break away

Hence TF has break away point at $S = -2.5$ at $k = 1.25$

Hence, the correct Answer is (1.25).

2. An open-loop transfer function of a plant in a unity feedback configuration is given as $(s) = \dfrac{K(s+4)}{(s+8)(s^2-9)}$.

The value of the gain $K(>0)$ for which -1 + j_2 lies on the root locus is ______ **[2015]**

Solution: $G(s) = \dfrac{K(S+4)}{(s+8)(s^2-9)}$

$$G(s) = \frac{K(s+4)}{(s+8)(s+3)(s-3)}$$

Given $S_1 = -1 + j_2$ lies on root locus so

$$|G(s)| = 1 \quad \text{at} \quad S = S_1$$

$$G(S_1) = \frac{K\{3+j2\}}{(7+j2)(2+j2)(-4+j2)}$$

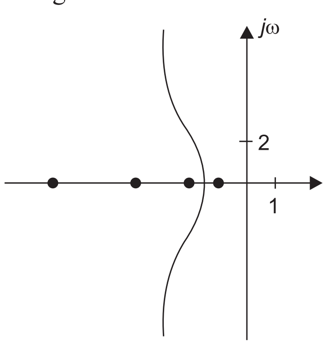
$$|G(j\omega)| = 1$$

$$\frac{K\{\sqrt{13}\}}{(\sqrt{51}).\sqrt{8}.\sqrt{20}} = 1$$

$$K = 25.05$$

Hence, the correct Answer is (25 to 26).

3. In the root locus plot shown in the figure, the pole/zero marks and the arrows have been removed. Which one of the following transfer functions has this root locus?



(a) $\dfrac{s+1}{(s+2)(s+4)(s+7)}$

(b) $\dfrac{s+4}{(s+1)(s+2)(s+7)}$

(c) $\dfrac{s+7}{(s+1)(s+2)(s+4)}$

(d) $\dfrac{(s+1)(s+2)}{(s+7)(s+4)}$ **[2014]**

Solution: (b)

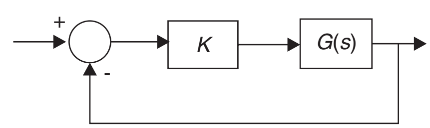
Since the root locus always emerges from the break-away points, i.e., when two poles are at a part of same root locus lie. So, $\sigma = -1$ and -2 surely be poles.

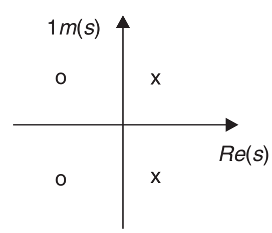
We also know root locus terminates at zero. So $s = -4$ is zero and $s = -7$ is a pole.

$$\therefore \; T(s) = \frac{(s+4)}{(s+1)(s+2)(s+7)}$$

Hence, the correct option is (b)

4. The feedback configuration and the pole-zero locations of $G(s) = \dfrac{s^2 - 2s + 2}{s^2 + 2s + 2}$ are shown below. The root locus for Negative values of K, i.e. for $-\infty < K < 0$, has break-away/break-in points and angle of departure at pole P (with respect to the positive real axis) equal to





(a) $\pm\sqrt{2}$ and 0°
(b) $\pm\sqrt{2}$ and 45°
(c) $\pm\sqrt{3}$ and 0°
(d) $\pm\sqrt{3}$ and 45° **[2009]**

Solution : **(b)**

$$1 + G(s)\,H(s) = 0$$

$$1 + \frac{k(s^2 - 2s + 2)}{s^2 + 2s + 2} = 0 \Rightarrow k = \frac{s^2 + 2s + 2}{s^2 - 2s + 2}$$

$$\frac{dk}{ds} = 0,$$

So, $(s^2 - 2s + 2)(2s + 2) - (s^2 + 2s + 2)(s - 1) = 0$

$\Rightarrow s = \pm\sqrt{2}$

Angle of departure is

$\phi_D = 180° + \phi$

Where $\phi = \Sigma\theta_z - \Sigma\phi_p - 135$

$\phi_D = 180 - 135 = 45°$

Hence, the correct option is (b)

5. A unity feedback control system has an open-loop transfer function $G(s) = \dfrac{K}{s(s^2 + 7s + 12)}$.

The gain K for which $s = -1 + j1$ will lie on the root locus of this system is

(a) 4 (b) 5.5
(c) 6.5 (d) 10 **[2007]**

Solution: (d)

Transfer function $= \dfrac{G(s)}{1 + G(s)H(s)}$; $H(s) = 1$

for the point $s = -1 + j1$ to lie on root locus $1 + G(s) = 0$

$$\Rightarrow 1 + \frac{k}{s(s^2 + 7s + 12)} = 0$$

$\Rightarrow [s^2 + 7s + (k)] + (12) = 0$

Putting $s = -1 + j$,

$\Rightarrow (-1 + j)(1 - 2j - 1 - 7 + 7j + 12) + k = 0$

$K = +10.$

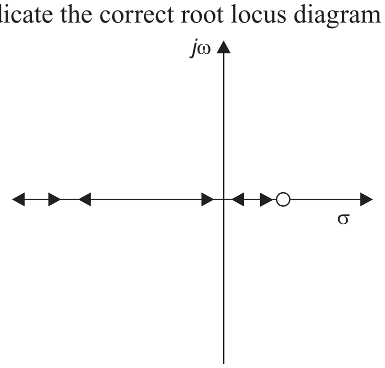
Hence, the correct option is (d)

6. A unity feedback system is given as

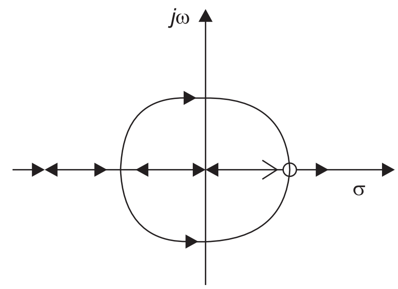
$$G(s) = \frac{K(1 - s)}{s(s + 3)}$$

Indicate the correct root locus diagram **[2005]**

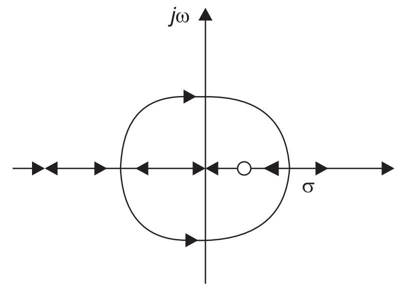
(a)



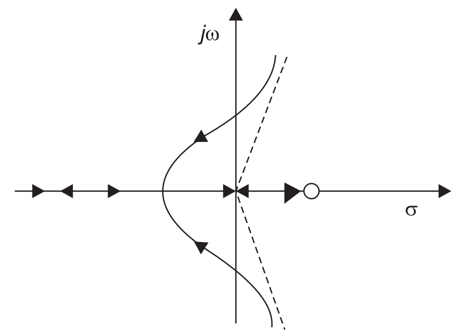
(b)



(c)



(d)



Solution: (c)

$1 + G(s)H(s) = 0$, $k = \dfrac{s^2 + 3s}{1 - s}$

For break away and break in point $\dfrac{dk}{ds} = 0$,

$$\frac{dk}{ds} = (1 - s)(2s + 3) + s^2 + 3s = 0$$

$\Rightarrow s = 3, -1$

So, -1 will be break away point and 3 is break in point.

Hence, the correct option is (c)

7. The root locus of the system $G(s)H(s) = \dfrac{K}{s(s + 2)(s + 3)}$ has the break-away point located at

(a) $(-0.5, 0)$ (b) $(-2.548, 0)$
(c) $(-4, 0)$ (d) $(-0.784, 0)$

[2003]

Solution: (d)

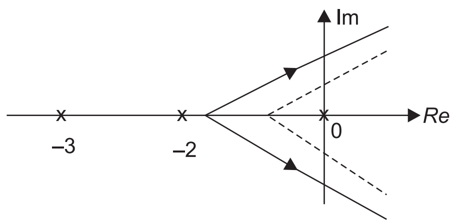
$1 + G(s)H(s) = 0$

$$1 + \frac{k}{s(s + 2)(s + 3)} = 0 \Rightarrow k = -1\,(s^3 + 5s^2 + 6s)$$

$$\frac{dk}{ds} = -(3s^2 + 10s + 6)$$

for break away point. $\frac{dk}{ds} = 0$

or $3s^2 + 10s + 6 = 0 \Rightarrow s = -0.784, -2.55$



$$\text{Centroid } = \frac{-2-3}{-3} = \frac{-5}{3} = -1.66$$

$$\text{angle of asymptotes } = (2k+1)\frac{\pi}{p-z} = (2k+1)\frac{\pi}{3}$$

$$= \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

So break away point is $(-0.784, 0)$ as -2.55 does not lie on R.L.

Hence, the correct option is (d)

8. Consider the points $s_1 = -3 + j4$ and $s_2 = -3 - j2$ in the s-plane. Then, for a system with the open-loop transfer function

$$G(s)H(s) = \frac{K}{(s+1)^4}$$

(a) s_1 is on the root locus, but not s_2
(b) s_2 is on the root locus, but not s_1
(c) both s_1 and s_2 are on the root locus
(d) neither s_1 nor s_2 is on the root locus **[1999]**

Solution: (b)

$s_1 = -3 + 4j$

$$G(s)H(s) = \frac{k}{(-3+4j+1)^4} = \frac{k}{(-2+4j)^4}$$

$\angle G(s)H(s) = -4\tan^{-1}(-2) = -4.66° \neq$ odd multiple of π

So, s_1 does not lie on root locus

$s_2 = -3 - j2$

$$G(s)H(s) = \frac{k}{(-3-j2+1)^4} = \frac{k}{(-2-2j)^4}$$

$\angle G(s)H(s) = -4[\tan^{-1}(1)] = -\pi =$ odd multiple of π

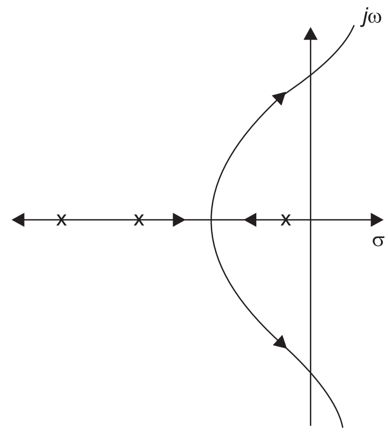
So, s_2 lies on root locus.

Hence, the correct option is (b)

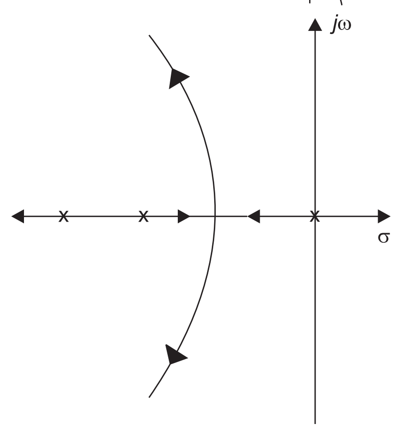
9. Given a unity feedback system with open-loop transfer function.

$G(s) = \frac{k}{(s+0.1)(s+0.2)}$, The root locus plot of the system is of the form. **[1992]**

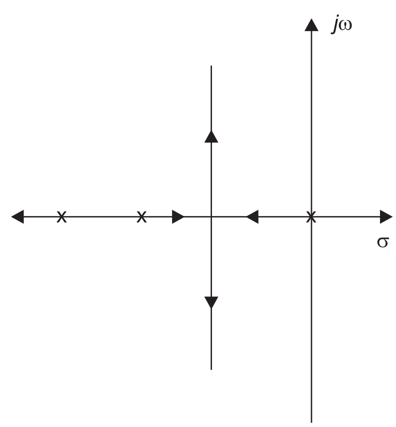
(a)



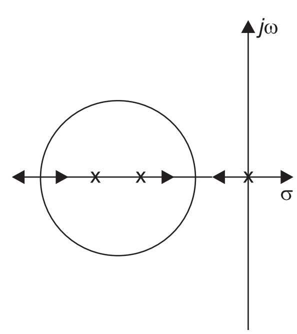
(b)



(c)



(d)



Solution: (a)

$P = 3, Z = 0, P - Z = 3.0 = 3$

$$\theta = \frac{(2q+1)180°}{p-z} \Rightarrow \theta = 60°, 180°, 300°$$

So problem (a) will be satisfied

Hence, the correct option is (a)

10. The characteristic equation of a feedback control system is given by $s^3 + 5s^2 + (K + 6)s\ K = 0$, where $K > 0$ is a scalar variable parameter. In the root-locus diagram of the system, the asymptotes of the root-loci for large values of K meet at a point in the s-plane, whose coordinates are

(a) (–3,0) (b) (–2, 0)
(c) (–1,0) (d) (2,0) **[1991]**

Solution : (b)

Open loop transfer function = $G(s)H(s)$

$$= \frac{k(s+1)}{s^3+5s^2+6s}$$

$$\Rightarrow G(s)H(s) = \frac{k(s+1)}{s(s+2)(s+3)}$$

poles = 0, −2, −3, zeros = −1

$$\text{centroid} = \frac{(-2-3)-(-1)}{3-1} = -2 = (-2,0)$$

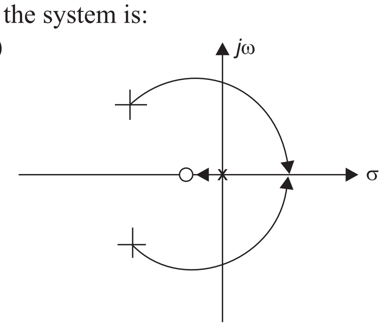
Hence, the correct option is (b)

11. The transfer function of a closed loop system is

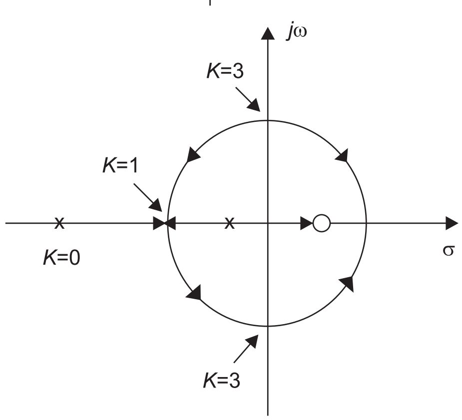
$$T(s) = \frac{K}{s^2+(3-K)(s+1)}$$

Where K is these forward path gain, the root locus plot of the system is: **[1990]**

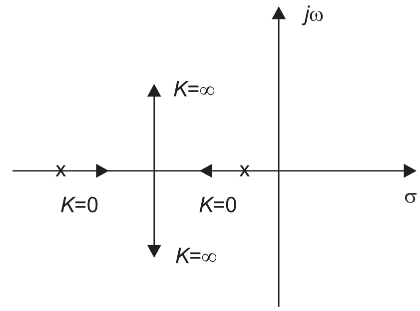
(a)



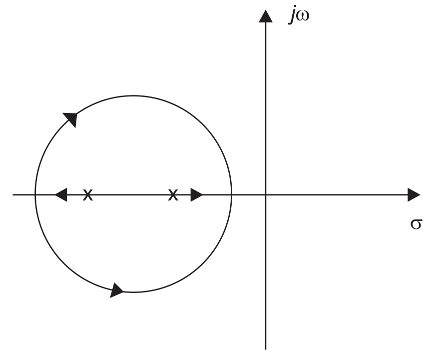
(b)



(c)



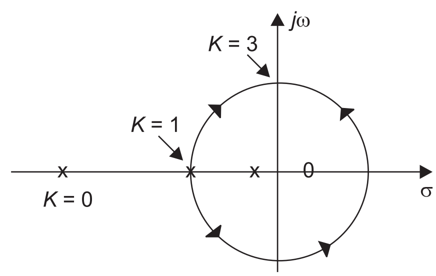
(d)



Solution : (b)

$$\text{Given T.F} = \frac{k}{s^2+(3-k)(s+1)}$$

root locus plot will be



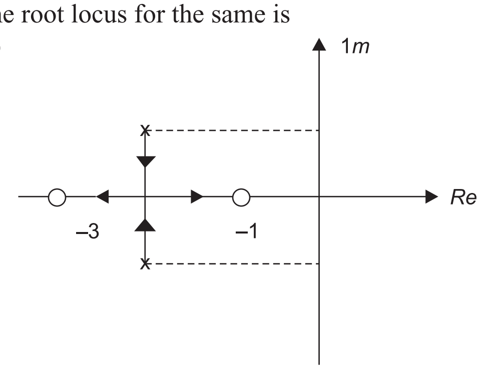
Hence, the correct option is (b)

12. The OLTF of a feedback system is $G(s)\ H(s)$

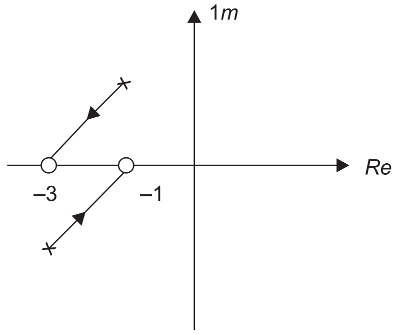
$$= \frac{K(s+1)(s+3)}{s^2+4s+8}.$$

The root locus for the same is **[1989]**

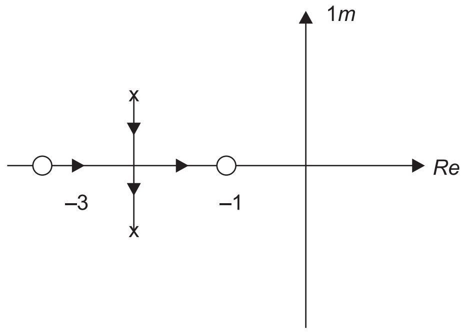
(a)



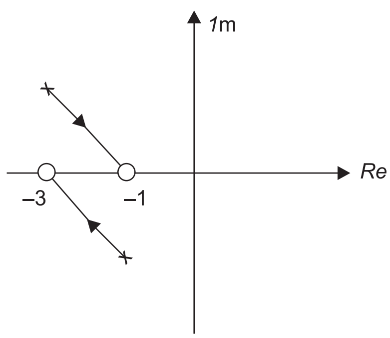
(b)



(c)



(d)



Solution: (a)

Answer (b) and (d) are wrong as root locus is symmetrical about real axis, and (c) is wrong as root locus direction is from poles to zero.

Hence, the correct option is (a)

13. Consider a closed-loop system shown in figure (a) below. The root locus for it is shown in figure (b). The closed loop transfer function for the system is

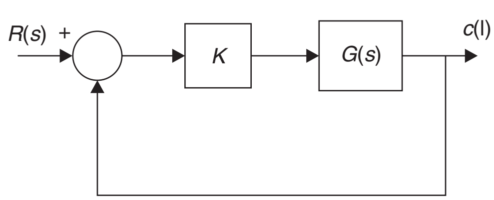


Fig. (a)

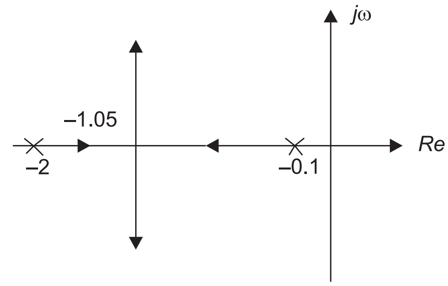


Fig. (b)

(a) $\dfrac{\quad}{1+(0.5s+1)(10s+1)}$

(b) $\dfrac{K}{(s+2)(s+0.1)}$

(c) $\dfrac{K}{1+K(0.5s+1)(10s+1)}$

(d) $\dfrac{K}{K+0.2(0.5s+1)(10s+1)}$ **[1988]**

Solution: (d)

$$G(s)=\frac{k}{(s+0.1)(s+2)}$$

$$\text{T.F}=\frac{K.G(s)}{1+K.G(s)}$$

$$T.F.=\frac{\frac{k}{(s+0.1)(s+2)}}{1+\frac{k}{(s+0.1)(s+2)}}$$

$$T.F.=\frac{k}{(s+0.1)(s+2)+K}$$

$$T.F.=\frac{k}{(s+0.1)(s+2)+K}$$

$$T.F.=\frac{k}{k+0.2(1+10s)(1+0.5s)}$$

$$T.F.=\frac{k}{k+(s+0.1)\ (s+2)}$$

$$=\frac{k}{k+(0.2)\ (1+10s)(1+0.5s)}$$

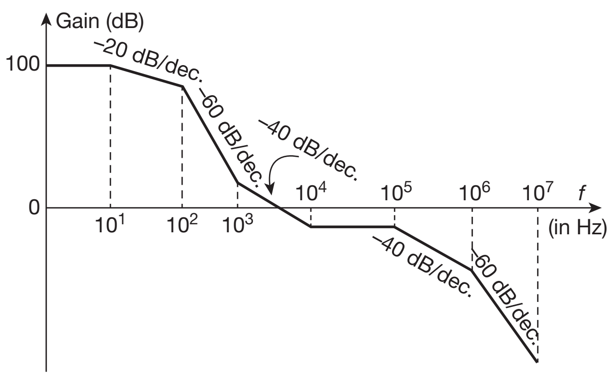
Hence, the correct option is (d)

Chapter 7

Frequency Analysis

One-mark Questions

1. For an LTI system, the Bode plot for its gain is as illustrated in the figure shown. The number of system poles N_p and the number of system zeros N_z in the frequency range 1 Hz $\le f \le 10^7$ Hz is **[2019]**



(A) $N_p = 6, N_z = 3$ (B) $N_p = 7, N_z = 4$
(C) $N_p = 5, N_z = 2$ (D) $N_p = 4, N_z = 2$

Solution: Addition of zero changes slope by +20 dB/decade

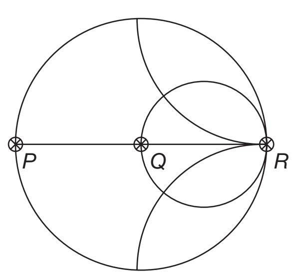
Addition of pole changes slope by –20 dB/decase

$N_p = 6$

$N_z = 3$

Hence, the correct option is (A)

2. The points P, Q and R shown on the smith chart (normalized impedance chart) in the following figure represent: **[2018]**



(A) P : Open Circuit Q : Short Circuit, R : Matched Load
(B) P : Open Circuit Q : Matched Load, R : Short Circuit
(C) P : Short Circuit Q : Matched Load, R : Open Circuit
(D) P : Short Circuit Q : Open Circuit , R : Matched Load

Solution: We know that he constant resistance circle, $r = 0$ and the constant reactance circle, $x = 0$ pass through the point 'P'. therefore

$$z = r + jx = 0$$

Thus point 'P' represents short circuit.

At the point 'R' $r = \infty$ and $x = \infty$ which gives $z = \infty$. Therefore point 'R' represents open circuit.

The constant resistance circle $r = 1$ passes through the point Q and $x = 0$.

$z = r$

$$\frac{Z_L}{Z_0} = 1$$

$Z_L = Z_0$ (Load impedance matched to the line characteristic impedance)

Hence, point 'Q' represents matched load.

Hence, the correct option is (C)

3. A closed loop control system is stable if the Nyquist plot of the corresponding open loop transfer function **[2016]**

(A) Encircles the s-plane point $(-1 + j0)$ in the counterclockwise direction as many times as the number of right half s-plane poles.
(B) Encircles the s-plane point $(0 - j1)$ in the clockwise direction as many times as the number of right half s-plane poles.
(C) Encircles the s-plane point $(-1 + j0)$ in the counterclockwise direction as many times as the number of left half s-plane poles.

(D) Encircles the s-plane point $(-1 + j0)$ in the counterclockwise direction as many times as the number of right half s-plane zeros.

Solution: In Nyquist plot, for an open loop transfer function, condition required is N – P = Z where N is the number of encirclement of pt. (–1, 0). P = no of poles of open loop system at the right side and Z is no of poles of close loop system at the right side.

Hence, the correct option is (A).

4. The number and direction of encirclements around the point $-1 + j0$ in the complex plane by the Nyquist plot of $G(s) = \dfrac{1-s}{4+2s}$ is **[2016]**

(A) zero
(B) one, anti clockwise
(C) one, clockwise
(D) two, clockwise

Solution: We are given that

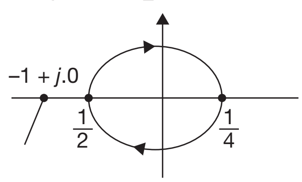
$$G(s) = \frac{1-s}{4+2s}$$

The magnitude and phase can be evaluated as

$$M = \frac{\sqrt{1+\omega^2}}{\sqrt{16+4\omega^2}},\ \phi = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

$$\omega = 0 \qquad \omega = \alpha$$

$$\frac{1}{4}\angle 0 \qquad \frac{1}{2}\angle -180°$$



Since, (–1,0) lies on the left and is not encircled therefor the system is stable and no of encirclement is 0.

Hence, the correct option is (A).

5. The polar plot of the transfer function $G(s) = \dfrac{10(s+1)}{s+10}$ for $0 \le \omega < \infty$ will be in the **[2015]**

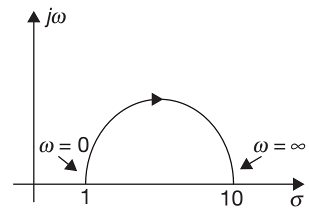
(A) first quadrant (B) second quadrant
(C) third quadrant (D) fourth quadrant

Solution: Given $G_c(s) = \dfrac{10(s+1)}{(s+10)}$

Zero is dominating, so it is a lead network.

$\Phi = \angle G_c(j\omega) = \tan^{-1}\left\{\dfrac{\omega}{1}\right\} - \tan^{-1}\left\{\dfrac{\omega}{10}\right\}$ any value of $0 \le \omega \le \infty$.

$\Phi \rightarrow$ positive value so, it is in 1st quadrant.



Hence, the correct option is (A).

6. The transfer function of a mass-spring-damper system is given by

$$G(s) = \frac{1}{Ms^2 + Bs + K}$$

The frequency response data for the system are given in the following table.

ω in rad/s	|G(jω)| in dB	Arg (G(jω)) in deg
0.01	–18.5	–0.2
0.1	–18.5	–1.3
0.2	–18.4	–2.6
1	–16	–16.9
2	–11.4	–89.4
3	–21.5	–151
5	–32.8	–167
10	–45.3	–174.5

The unit step response of the system approaches a steady state value of ________. **[2015]**

Solution: $G(s) = \dfrac{1}{MS^2 + BS + K}$

for Unit step response $R(s) = \dfrac{1}{s}$

$$C(s) = R(s)\cdot G(s) = \frac{1}{s\left(Ms^2 + Bs + K\right)}$$

Steady state value (or) F.V

$$= \underset{S\to 0}{Lt}\ s.C(s) = 1/K$$

Find the 'K' value from the given table

$$G(j\omega) = \frac{1}{-M\omega^2 + jB\omega + K}$$

$$|G(j\omega)| = \frac{1}{\sqrt{\left(K-\omega^2 M\right)^2 + (B\omega)^2}}$$

If ω = 1

20 log $|G(j\omega)|$ = –16

$|G(j\omega)| = 0.158$

$$(6.329)^2 = (K - M)^2 + B^2$$

$$40 = (K - M)^2 + B^2 \qquad \text{(i)}$$

$$-\tan^{-1}\left\{\frac{B\omega}{K-\omega^2 M}\right\} = -16.9°$$

$$\frac{B}{K-M} = 0.30$$

$$B^2 = 0.0923\,(K - M)^2$$

$$40 = (K - M)^2 + 0.09\,23(K - M)^2$$

$$40 = (1.0923)(K - M)^2$$

$$K - M = 6.05$$

$$\frac{B\omega}{K-\omega^2 M} = \tan\Phi$$

at

$$\omega = 2$$

$$\frac{2B}{K-4M} = \tan(89.4) = 95.489$$

and

$$|G(j\omega)|_{\omega=2} = 0.269$$

$$13.8 = (K - 4M)^2 + 4B^2 \qquad \text{(ii)}$$

$$13.8 = 9119(K - 4M)^2$$

$$K - 4M = 0.038$$

$$4(K - M = 6.05)$$

From the above equations, we obtain K = 8.054 and M = 2.004

But

$$C(\infty) = 1/K$$

$$C(\infty) = 1/8.054$$

$$C(\infty) = 0.124$$

2nd alternate method:

From the above analysis, we know

$$C(\infty) = \frac{1}{K}$$

From the given data at ω = 0.01 rad/sec ≈ 0

$$|G(j\omega)| \text{ in dB} = -18.5$$

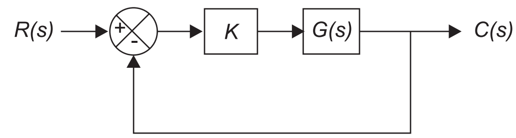
$$\therefore \quad 20\log_{10}\left|\frac{1}{K}\right| = -18.5$$

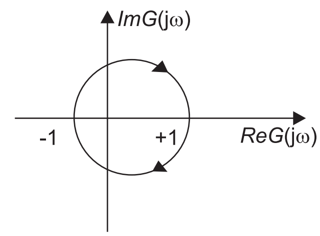
$$K = 10^{0.925} = 8.4139$$

$$\therefore \quad C(\infty) = \frac{1}{8.4B9} = 0.1188$$

Hence, the correct Answer is (0.10 to 0.13).

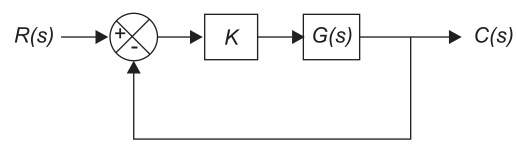
7. Consider the feedback system shown in the figure. The Nyquist plot of $G(s)$ is also shown. Which one of the following conclusions is correct?

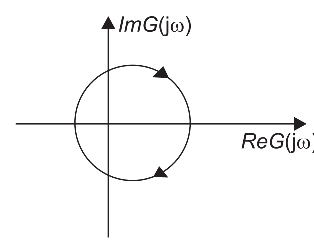




(a) $G(s)$ is an all-pass filter
(b) $G(s)$ is a strictly proper transfer function
(c) $G(s)$ is a stable and minimum-phase transfer function
(d) The closed loop system is unstable for sufficiently large and positive K **[2014]**

Solution: (d)





From the Nyquist plot, it can be seen that it does not encircle the critical point (−1, 0)

$N = P - Z, N = 0$

The closed loop-system is unstable for large and positive k.

Hence, the correct option is (d).

8. In a Bode magnitude plot, which one of the following slopes would be exhibited at high frequencies by a 4th order all-pole system?

(a) −80 dB/decade (b) −40 dB/decade
(c) +40 dB/decade (d) +80 dB/decade

[2014]

Solution: (a)

A 4th order all pole system means that the system must be having no zero or s-terms in numerator and s^4 terms in denominator or 4 pole i.e.

$$H(s) \propto \frac{1}{s^4}$$

One pole exhibits −20 dB/dec slope; so four pole exhibits a slope of −80 dB/decade.

$$PM = 180 + \angle GH\Big|_{w=w_{gc}}$$

from the given transfer function

$$\angle G(jw) = -\tan^{-1}\left(\frac{w_{gc}}{0.1}\right) - \tan^{-1}\left(\frac{w_{gc}}{1}\right) - \tan^{-1}\left(\frac{w_{gc}}{10}\right)$$

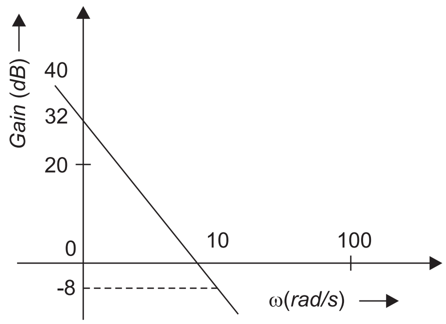
or $\angle G(jw) = -84.289° - 45° - 5.711° = -135°$

$\therefore$ from equations (ii) and (iii), we get phase margins

$PM = 180 - 135 = 45°$.

Hence, the correct option is (a).

9. The Bode plot of a transfer function $G(s)$ is shown in the figure below:



The gain ($20\log|G(s)|$) is 32 dB and −8 dB at 1 rad/s and 10rad/s, respectively. The phase is negative for all ω. Then $G(s)$ is

(a) $\dfrac{39.8}{s}$ (b) $\dfrac{39.8}{s^2}$

(c) $\dfrac{32}{s}$ (d) $\dfrac{32}{s^2}$ **[2013]**

Solution : (b)

10 rad/s to 1 rad/s is 1 decade 32 – (−8) = 40 dB

So, the slope is 40 dB/decade. It means there are two poles at origin, which means either option (b) or option (d) is correct, put $w = 1$ rad/sec in both.

$$20\log\left(\frac{39.8}{(1)^2}\right) = 32\,\text{dB}$$

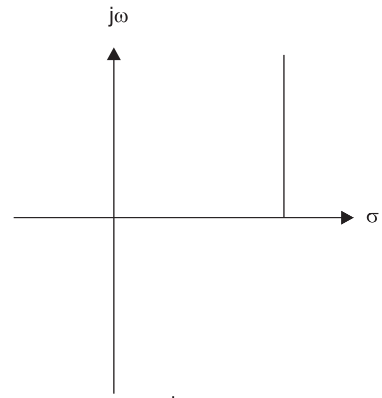
$$20\log\left(\frac{32}{12}\right) = 30.1\,\text{dB}$$

So option (b) is correct.

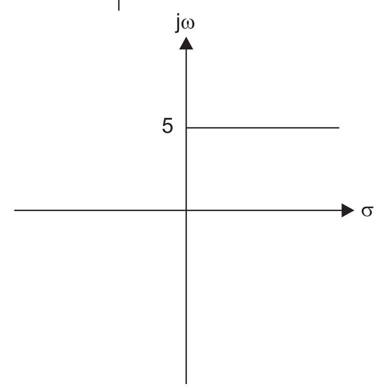
Hence, the correct option is (b).

10. For the transfer function $G(j\omega) = 5 + j\omega$, the corresponding Nyquist plot for positive frequency has the form **[2011]**

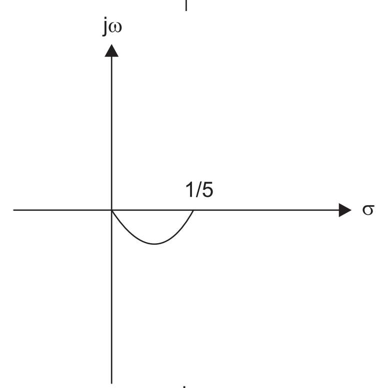
(a)



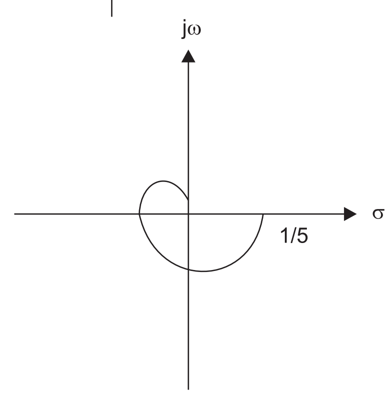
(b)



(c)



(d)



Solution: (a)

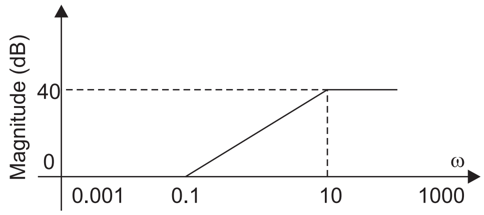
$G(jw) = 5 + jw$

$|G(jw)| = \sqrt{25 + w^2}$

at, $w = 0$ $|G(\phi)| = \sqrt{25} = 5$

Hence, the correct option is (a).

11. For the asymptotic Bode magnitude plot shown below, the system transfer function can be



(a) $\dfrac{10s+1}{0.1s+1}$ (b) $\dfrac{100s+1}{0.1s+1}$

(c) $\dfrac{100s}{10s+1}$ (d) $\dfrac{0.1s+1}{10s+1}$ **[2010]**

Solution: (a)

$$\text{T.F., } G(s)H(s) = \frac{k\left(1+\dfrac{s}{0.1}\right)}{\left(1+\dfrac{s}{10}\right)}$$

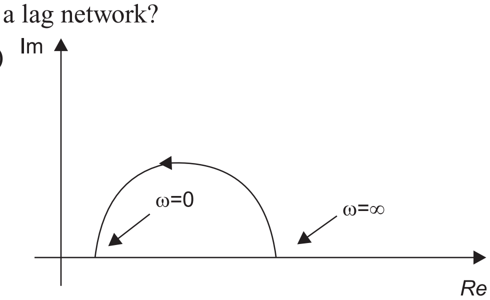
Here, 20 log $k = 0$, $k = 1$

So $G(s)H(s) = \dfrac{10s+1}{0.1s+1}$

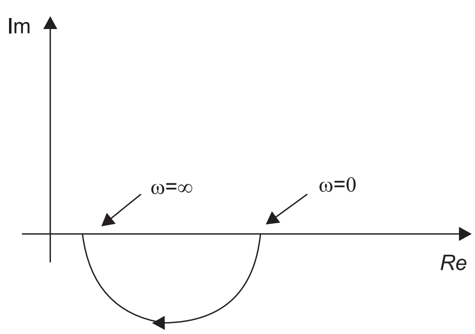
Hence, the correct option is (a).

12. Which one of the following polar diagrams correspond to a lag network? **[2005]**

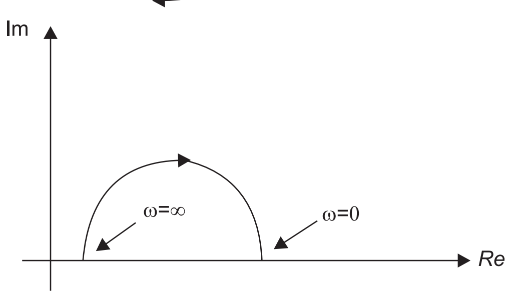
(a)



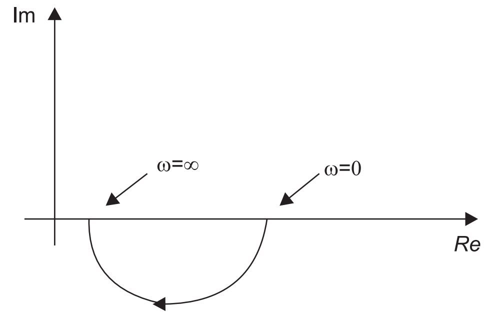
(b)



(c)



(d)



Solution: (d)

Let $\dfrac{1}{s+1}$ be a lag network.

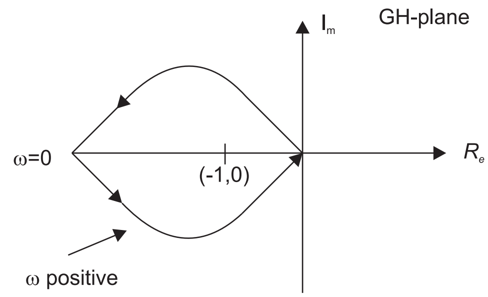
At $w = 0$, Mag $= \infty$ $\angle = \tan^{-1} 0 = 0$

At $w = \infty$ Mag $= 0$, $\angle = -\tan^{-1}(\infty) = -90°$

If in the direction of w increasing phase shift is decreasing system is lag network.

Hence, the correct option is (d).

13. In the figure, the Nyquist plot of the open-loop transfer function $G(s)H(s)$ of a system is shown. If $G(s)H(s)$ has one right-hand pole, the closed-loop system is



(a) always stable
(b) unstable with one closed-loop right hand pole
(c) unstable with two closed-loop right hand poles
(d) unstable with three closed-loop right hand poles **[2003]**

Solution: (a)

The encirclement of critical point (−1,0) in ACW direction is once.

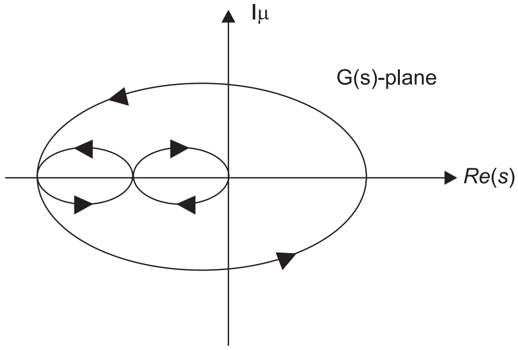
$\therefore N = 1, P = 1$ (given)

$Z = P - N = 0$

No zero is RH of s-plane. So system is stable.

Hence, the correct option is (a).

14. The Nyquist plot for the open loop transfer function $G(s)$ of a unity negative feedback system is shown in the figure, if $G(s)$ has no pole in the right-half of s-plane, the number of roots of the system characteristic equation in the right-half of s-plane is



(a) 0 (b) 1
(c) 2 (d) 3 **[2001]**

Solution :(a)

$N = 0$, (1 encirclement in CW direction and other in ACW).

$P = 0$ (no pole in right half)

$N = P - Z$

$Z = P - N = 0$

So, no roots on RH of s-plane.

Hence, the correct option is (a).

15. In the Bode-plot of a unity feedback control system, the value of phase of $G(j\omega)$ at the gain cross over frequency is $-125°$. The phase margin of the system is

(a) $-125°$ (b) $-55°$
(c) $55°$ (d) $125°$ **[1998]**

Solution: (c)

$\phi = -125°$

P. M. $= 180 + \phi \;= 180° - 125 = 55°$

Hence, the correct option is (c).

16. The Nyquist plot of a loop transfer function $G(j\omega)$ $H(j\omega)$ of a system encloses the (1 –, j0) point. The gain margin of the system is

(a) less than zero (b) zero
(c) greater than zero (d) infinity **[1998]**

Solution: (a)

A system is unstable when Nyquist plot of $G(jw)$ $H(jw)$ encloses the point $(-1 + j0)$ and gain margin of unstable system is always –ve. Therefore, the gain margin of the given system is less than zero.

Hence, the correct option is (a).

17. Non-minium phase transfer function is defined as the transfer function

(a) which has zeros in the right-half s-plane
(b) which has zeros only in the left-half s-plane
(c) which has poles in the right-half s-plane
(d) which has poles in the left-half s-plane **[1995]**

Solution: (a)

The non-minimum phase transfer function is defined as the transfer function which has zeros in the right half of s-plane.

Hence, the correct option is (a).

18. The 3–dB bandwidth of a typical second-order system with the transfer function $\dfrac{C(s)}{R(s)} = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$ is given by

(a) $\omega_n\sqrt{1-2\xi^2}$

(b) $\omega_n\sqrt{(1-\xi^2)+\sqrt{\xi^4-\xi^2+1}}$

(c) $\omega_n\sqrt{(1-2\xi^2)+\sqrt{4\xi^4-4\xi^2+2}}$

(d) $\omega_n\sqrt{(1-2\xi^2)-\sqrt{4\xi^4-4\xi^2+2}}$ **[1994]**

Solution: (c)

For example, $w_n\sqrt{(1-2\xi^2)+\sqrt{4\xi^4-4\xi^2+2}}$

Hence, the correct option is (c).

19. The open loop frequency response of a system at two particular frequencies is given by : 1.2 $\angle -180°$ and 1.0 $\angle -190°$. The closed loop unity feedback control is then______ **[1994]**

Solution: at 180° → $|GH| = 1.2$

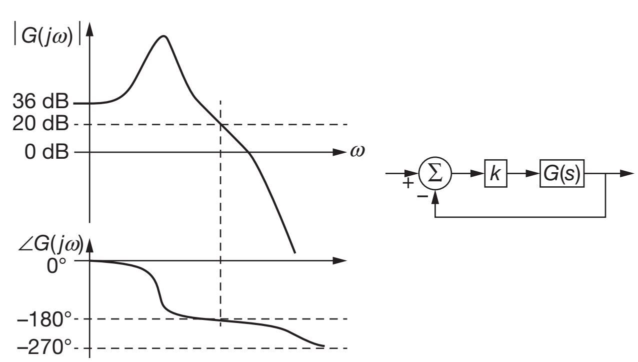
$$\text{G.M} = 20\log\left(\frac{1}{|GH|}\right) = 20\log\left(\frac{1}{1.2}\right) = -1.6 \text{ dB}$$

at $|GH| = 1 \rightarrow \phi = -190°$

P.M. $= 180° + \phi = 180 - 190 = -10°$, since both GM and PM are negative so system is unstable.

Two-marks Questions

1. The figure below shows the Bode magnitude and plots of a stable transfer function $G(s) = \dfrac{n_0}{s^3 + d_2 s^2 + d_1 s + d_0}$ **[2018]**



Consider the negative unity feedback configuration with gain k in the feed forward path.

The closed loop is stable for $K < K_0$. The maximum value of K_0 is ________.

Solution: Open loop transfer function OLTF = $KG(s)$

We know that for the system to be stable 20 log $\left|\frac{1}{KG(s)}\right|_{w=w_{pc}} > 0$ dB

$$\{-20\log K - 20\log|G(jw)|\} > 0 \text{ dB}$$

$$|-20\log K - 20| > 0 \text{ dB}$$

$$|-20\log K - 20| > 0 \text{ dB}$$

$$20 \log K < -20$$

$$\log K < -1$$

$$K < 0.1$$

Hence, the correct answer is 0.1.

2. The Nyquist plot of the transfer function

$$G(s) = \frac{K}{(s^2+2s+2)(s+2)}$$

does not encircle the point $(-1 + j0)$ for $K = 10$ but does encircle the point $(-1 + j0)$ for $K = 100$. Then the closed loop system (having unity gain feedback) is **[2017]**

(A) Stable for $K = 10$ and stable for $K = 100$
(B) Stable for $K = 10$ and unstable for $K = 100$
(C) Unstable for $K = 10$ and stable for $K = 100$
(D) Unstable for $K = 10$ and unstable for $K = 100$

Solution: The above problem can be solved by R_H criteria.

$C.E = S^3 + 2S^2 + 2S^2 + 4S + 2S + 4 + K = 0$

$C.E = S^3 + 4S^2 + 6S + 4 + K = 0$

For the system to be stable

$24 > (4 + k)$

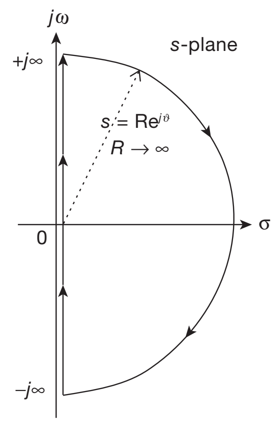
$(4 + k) < 24$

$0 < K < 20$

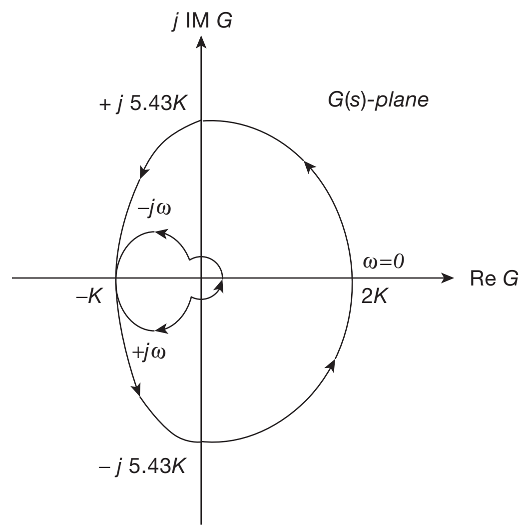
Hence, the correct option is (B).

3. A unity feedback control system is characterized by the open-loop transfer function $G(s) = \frac{10K(s+2)}{s^3+3s^2+10}$

The Nyquist path and the corresponding Nyquist plot of G(s) are shown in the figure below.



Nyquist Path for *G*(*s*)



Nyquist Plot of *G*(*s*)

If $0<K<1$, then the number of poles of the closed-loop transfer that lie in the right-half of the s-plane is **[2017]**

(A) 0 (B) 1
(C) 2 (D) 3

Solution: Given open-loop transfer function

$$G(S) = \frac{10k(S+2)}{S^3+3S^2+10}$$

Characteristice equation is $1 + G(S) = 0$

$$1+\frac{10k(S+2)}{S^3+3S^2+10} = 0$$

$$\Rightarrow S^3+3S^2+10KS+(20K+10) = 0$$

Form Routh Table

S^3	1	$10K$
S^2	3	$2K+10$
S^1	$\frac{10k-10}{3}$	0
S^0	$20k+10$	0

$$\frac{10K-10}{3} > 0 \Rightarrow k > 1$$

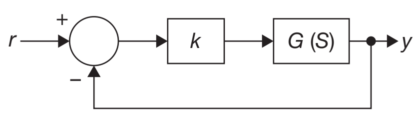
$$20K+10>0 \quad \Rightarrow K > -\tfrac{1}{2}$$

$\therefore$ If $K > 1 \rightarrow$ The closed-loop system is stable If $0 < K < 1 \rightarrow$ The closed-loop system is unstable. The number of sign changes in the first column of routh table are = 2.

$\therefore$ The number of poles of the closed-loop transfer function that lie in the Right-half of S-plane are = 2

Hence, the correct option is (C).

4. In the feedback system below $G(s) = \dfrac{1}{(s+1)(s+2)(s+3)}$. **[2016]**



The positive value of k for which the gain margin of the loop is exactly 0 dB and the phase margin of the loop is exactly zero degree is ______.

Solution: Gain margin = 0 dB

Phase = –180°

On equating the gain to 0 we get

$$20\log\left[\frac{\sqrt{\omega^2+1}\sqrt{\omega^2+4}\sqrt{\omega^2+9}}{k}\right]=0$$

$$(\omega^2+1)(\omega^2+4)(\omega^2+9)=k^2$$

$$\tan^{-1}(\omega)-\tan^{-1}\left(\frac{\omega}{2}\right)-\tan^{-1}\left(\frac{\omega}{3}\right)=-180°$$

$$\tan^{-1}\left[\frac{\omega+\frac{\omega}{2}}{1-\frac{\omega^2}{2}}\right]+\tan^{-1}\left(\frac{\omega}{3}\right)=180°$$

$$\Rightarrow \quad \omega=\sqrt{11}$$

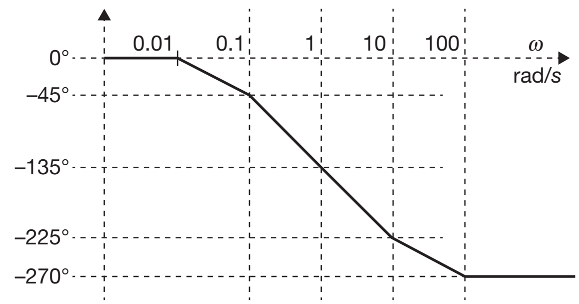
$$k=\sqrt{12\times15\times20}=60$$

Hence, the correct Answer is (60).

5. The asymptotic Bode phase plot of

$$G(s)=\frac{k}{(s+0.1)(s+10)(s+p_1)},$$

with k and p_1 both positive is shown below



The value of p_1 is ______. **[2016]**

Solution: As we know that phase of the system is

$$\phi=-\tan^{-1}\left(\frac{\omega}{0.1}\right)-\tan^{-1}\left(\frac{\omega}{10}\right)-\tan^{-1}\left(\frac{\omega}{p_1}\right)$$

Phase margin = 180 + phase of a system

At ω = 1, phase of a system – 135°

$$P_m = 180 - 135° = 45°$$

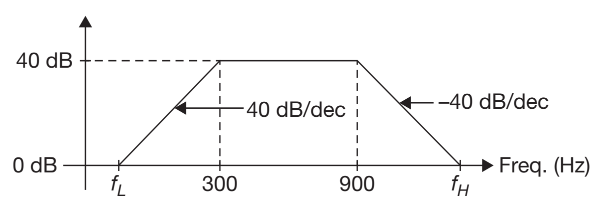
At ω = 1,

$$-\tan^{-1}\left(\frac{1}{0.1}\right)-\tan^{-1}\left(\frac{1}{10}\right)-\tan^{-1}\left(\frac{1}{p_1}\right)=45°$$

$$\tan^{-1}\left(\frac{1}{p_1}\right)=-135° \Rightarrow p_1=1$$

Hence, the correct Answer is (1).

6. Consider the Bode plot shown in the figure. Assume that all the poles and zeros are real-valued.



The value of $f_H - f_L$ (in Hz) is _______. **[2015]**

Solution: From the given Bode plot

$$G(j\omega)=\frac{K(s+2\pi f_L)^2(s+2\pi f_H)^2}{(s+2\pi\times300)^2(s+2\pi\times900)^2}$$

From the Bode plot

$$40=\frac{40-0}{\log(300)-\log f_L}$$

$$\log_{10}\left(\frac{300}{f_L}\right)=1 \Rightarrow f_L=\frac{300}{10}=30\,Hz$$

and

$$-40=\frac{40-0}{\log(900)-\log f_H}$$

$$-1=\frac{1}{\log(900/f_H)}$$

$$(f_H/900) = 10$$

$$f_H = 9000 \text{ Hz}$$

$$f_H - f_L = 8970 \text{ Hz}$$

Hence, the correct Answer is (8970).

7. The phase margin (in degrees) of the system $G(s) = \dfrac{10}{s(s+10)}$ is ______ **[2015]**

Solution: Given $G(s) = \dfrac{10}{s(s+10)}$

$$\text{P.M} = 180° + \Phi$$

$$\Phi = \Rightarrow G(j\omega) \text{ at } \omega = \omega_{gc}$$

at $\quad \omega = \omega_g c\ |G(j\omega)| = 1$

$$\frac{10}{\omega\sqrt{\omega^2+100}} = 1$$

$$100 = \omega^2(\omega^2 + 100)$$

$$\omega^4 + 100\,\omega^2 - 100 = 0$$

Let $\quad \omega^2 = x$

$$X^2 + 100X - 100 = 0$$

$$X_1 = 0.99,\ X_2 = -100$$

$$\omega^2 = 0.99 \qquad \omega \rightarrow +\text{ve only}$$

$$\omega_{gc} = 0.9948 \text{ rad/sec}$$

$$\Phi = -90° - \tan^{-1}\left\{\frac{\omega}{10}\right\}$$

$$\Phi = -90° - 5.682$$

$$\Phi = -95.682$$

$$\text{PM} = 180° + \Phi$$

$$\text{P.M} = 84.3178°$$

Hence, the correct Answer is (84 to 84.5).

8. The phase margin in degrees of

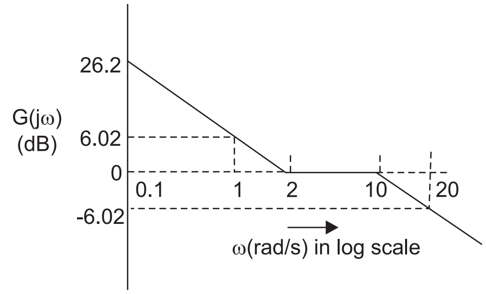
$G(s) = \dfrac{10}{(s+0.1)+(s+1)(s+10)}$ calculated using the asymptotic Bode plot is_____. **[2014]**

Solution : $\dfrac{10}{0.1\times10\left(\dfrac{s}{0.1}+1\right)\left(\dfrac{s}{1}+1\right)\left(\dfrac{s}{10}+1\right)}$

$$= \frac{10}{\left(\dfrac{s}{0.1}+1\right)\left(\dfrac{s}{1}+1\right)\left(\dfrac{s}{10}+1\right)}$$

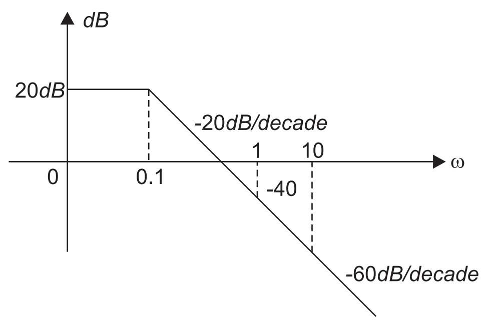
Amount of shift = 20 log k = 20 log 10 = 20 dB

9. The Bode asymptotic magnitude plot of a minimum phase system is shown in the figure.



If the system is connected in a unity negative feedback configuration, the steady–state error of the closed loop system, to a unit ramp input, is________. **[2014]**

Solution :



$$-20\,\text{dB} = \frac{20-0}{\log 0.1 - \log w_{gc}}$$

where $26.02\big|_{w=0.1} = 20\log k - 20\log 0$

or, $k = 1.99 \simeq 2$

so, $G(s) = \dfrac{20(s+2)}{2s(s+10)}$

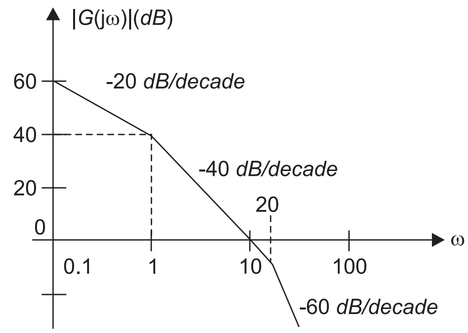
Steady-state error for ramp input is $e_{ss} = \dfrac{A}{ka}$
$\Rightarrow ka = \lim\limits_{s\to0} sG(s)$

From equation (iii)

$$ka = \lim_{s\to0} s \times \frac{10(s+2)}{s(s+10)} = 2$$

$$\therefore e_{ss} = \frac{1}{2} = 0.5$$

10. The asymptotic Bode plot of a transfer function is as shown in the figure. The transfer function $G(s)$ corresponding to this Bode plot is



(a) $\dfrac{1}{(s+1)(s+20)}$

(b) $\dfrac{1}{s(s+1)(s+20)}$

(c) $\dfrac{100}{s(s+1)(s+20)}$

(d) $\dfrac{100}{s(s+1)(1+0.05s)}$ **[2007]**

Solution: (c)

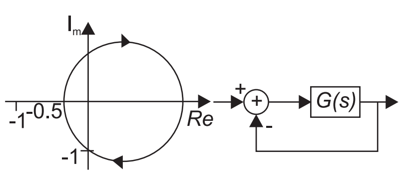
$$G(s)=\frac{k}{s(s+1)(s+20)}=\frac{\frac{k}{20}}{s(1+s)\left(1+\frac{s}{20}\right)}$$

Bode plot is in (1 + *ST*) form.

$= -20 \log w + 20 \log k = |60\text{ dB}|_{w=0.1}$

Solving $\Rightarrow k = 100$

$$\text{So } G(s)=\frac{100}{s(s+1)(1+0.05s)}=\frac{100}{s(s+1)(s+20)}$$



Hence, the correct option is (c).

Common Data for Question 11 and 12

The Nyquist plot of a stable transfer function $G(s)$ is shown in the figure. We are interested in the stability of the closed loop system in the feedback configuration shown

11. Which of the following statement is true?
(a) $G(s)$ is an all-pass filter
(b) $G(s)$ has a zero in the right-half plane
(c) $G(s)$ is the impedance of a passive network
(d) $G(s)$ is marginally stable **[2009]**

Solution: (b)

From the plot, $G(s)$ has a zero in the right half of plane.

Hence, the correct option is (b).

12. The gain and phase margins of $G(s)$ for closed loop stability are
(a) 6 dB and 180° (b) 3 dB and 180°
(c) 6 dB and 90° (d) 3 dB and 90°
[2009]

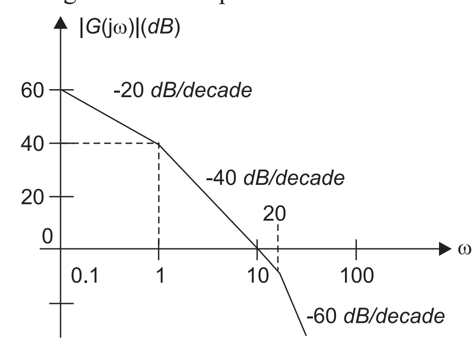
Solution: (c)

$$\text{Gain margin } = 20\log\frac{1}{x} = 20\log\frac{1}{0.5}$$
$$= 20 \log 2 = 6 \text{ dB}$$

and phase margin = 90°

Hence, the correct option is (c).

13. The asymptotic Bode plot of a transfer function is as shown in the figure. The transfer function $G(s)$ corresponding to this Bode plot is



(a) $\dfrac{1}{(s+1)(s+20)}$

(b) $\dfrac{1}{s(s+1)(s+20)}$

(c) $\dfrac{100}{s(s+1)(s+20)}$

(d) $\dfrac{100}{s(s+1)(1+0.05s)}$ **[2007]**

Solution: (c)

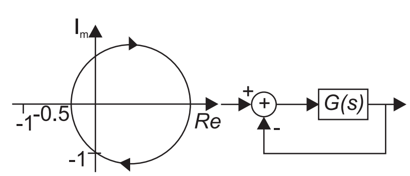
$$G(s)=\frac{k}{s(s+1)(s+20)}=\frac{\frac{k}{20}}{s(1+s)\left(1+\frac{s}{20}\right)}$$

Bode plot is in (1 + ST) form.

$= -20 \log w + 20 \log k = |60\text{ dB}|_{w=0.1}$

Solving $\Rightarrow k = 100$

$$\text{So } G(s)=\frac{100}{s(s+1)(1+0.05s)}=\frac{100}{s(s+1)(s+20)}$$



Hence, the correct option is (c).

14. The Nyquist plot of $G(j\omega)\, H(j\omega)$ for a closed loop control system, passed through $(-1, j0)$ point in the GH plane. The gain margin of the system in dB is equal to

(a) infinite (b) greater than zero
(c) less than zero (d) zero **[2006]**

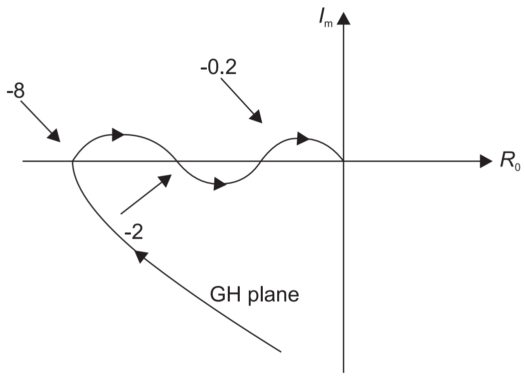
Solution: (d)

$$\text{G.M.} = 20 \log \frac{1}{a} \text{ dB}$$

$a = 1$, G.M. = 0

Hence, the correct option is (d).

15. The polar diagram of a conditionally stable system for open loop gain $K = 1$ is shown in the figure. The open loop transfer function of the system is known to be stable. The closed loop system is stable for



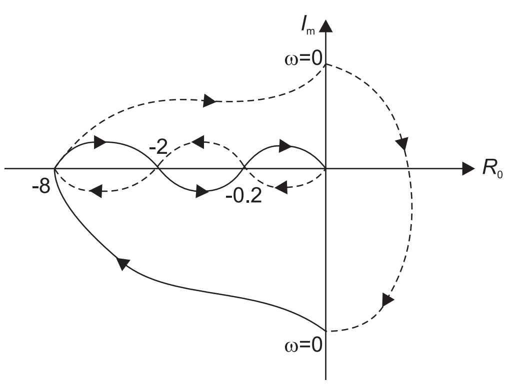
(a) $K < 5$ and $\frac{1}{2} < K \frac{1}{8}$

(b) $K < \frac{1}{8}$ and $\frac{1}{2} < K < 5$

(c) $K < \frac{1}{8}$ and $5 < K$

(d) $K < \frac{1}{8}$ and **[2005]**

Solution: (b)



system is stable in region −0.2 to −2 and on the left side of −8 as number of encirclement there is zero.

$0.2\, k < 1 \quad k < 5$

$2k > 1 \quad k > 0.5$

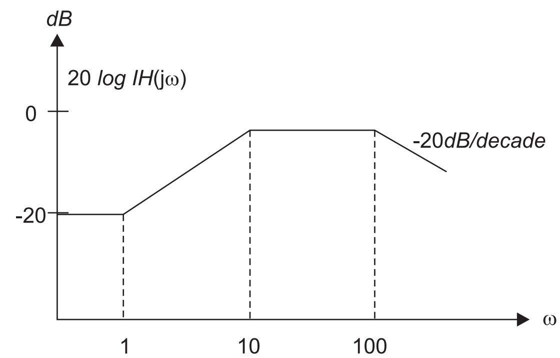
$0.5 < k < 5$

$1 > 8k$

$k < \frac{1}{8}$ (negative sign only shows that it is on negative axis)

Hence, the correct option is (b).

16. Consider the Bode magnitude plot shown in the figure. The transfer function $H(s)$ is



(a) $\dfrac{(s+10)}{(s+10)(s+100)}$

(b) $\dfrac{10(s+1)}{(s+10)(s+100)}$

(c) $\dfrac{10^2(s+1)}{(s+10)(s+100)}$

(d) $\dfrac{10^3(s+100)}{(s+1)(s+10)}$ **[2004]**

Solution : (c)

$20 \log k = -20$ dB

$\Rightarrow k = 10^{-1} = 0.1$

Zero at $w = 1$ and poles at $w = 10, 100$

$$H(s) = \frac{k(s+1)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)} = \frac{100(s+1)}{(s+10)(s+100)}$$

Hence, the correct option is (c).

17. A system has poles at 0 01 Hz, 1 Hz and 80 Hz; zeros at 5 Hz, 100 Hz and 200 Hz. The approximate phase of the system response at 20 Hz is

(a) −90° (b) 0°
(c) 90° (d) −180° **[2004]**

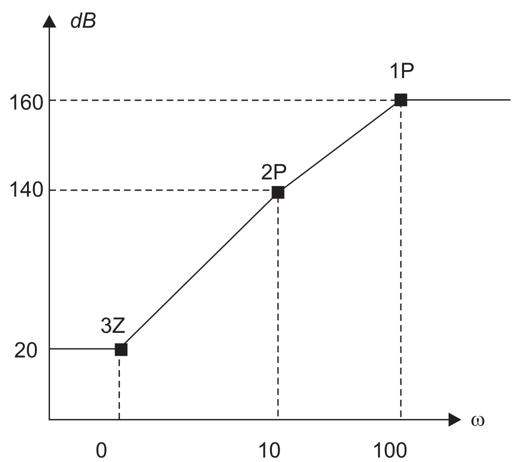
Solution: (a)

Pole at 0.01 and 1 Hz gives −180 phase. Zero at 5Hz gives +90 phase.

$\therefore$ at 20 Hz $-90°$ phase is provided.

Hence, the correct option is (a).

18. The approximate Bode magnitude plot of a minimum-phase system is shown in the figure. The transfer function of the system is



(a) $10^8 \dfrac{(s+0.1)^3}{(s+10)^2(s+100)}$

(b) $10^7 \dfrac{(s+0.1)^3}{(s+10)(s+100)}$

(c) $10^8 \dfrac{(s+0.1)^2}{(s+10)^2(s+100)}$

(d) $10^9 \dfrac{(s+0.1)^3}{(s+10)(s+100)^2}$ **[2003]**

Solution: (a)

$w = 0.1$ to 10, $+120$ dB change

$\therefore$ 3 zeros at 0.1

$w = 10$ to 100, -40 dB

[i.e., +60 to +20] change

so two poles at $w = 100$

$$\Rightarrow T(s) = \frac{k(s+0.1)^3}{(s+10)^2(s+100)}$$

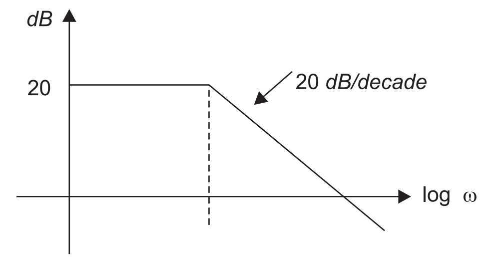
$$\Rightarrow 20\log\left(\frac{k}{w}\right)\Bigg|_{w=10} = 140$$

$$\Rightarrow \frac{k}{w}\Bigg|_{w=10} = 10^7$$

$\therefore k = 10^8$

Hence, the correct option is (a).

19. Bode plot of a stable system is shown in figure. The transfer function of the system is



[1992]

Solution :

Corner $fq = 1$ rad/sec

$$\frac{1}{T} = 1 \Rightarrow T = 1$$

$y = mx + c$

$\Rightarrow 20 = 0 + c \Rightarrow c = 20$

$c = 20 \log k = 20 \quad k = 10$

$$\text{T.F} = \frac{k}{1+TS} = \frac{10}{1+S}$$

20. The open-loop transfer function of a feedback control system is $G(s)\cdot H(s) = \dfrac{1}{(s+1)^3}$

The gain margin of the system is

(a) 2 (b) 4

(c) 8 (d) 16 **[1991]**

Solution: (c)

$w_{pc} \Rightarrow -3\tan^{-1}(w_{pc}) = -180°$

$w_{pc} = \sqrt{3}$ rad/sec

$$|G.H| = x = \frac{1}{\sqrt{1+w_{pc}^2}} = \frac{1}{\left(\sqrt{1+\left(\sqrt{3}\right)^2}\right)^3} = \frac{1}{8}$$

$$\text{G.M.} = \frac{1}{x} = 8.$$

Hence, the correct option is (c).

21. From the Nicholas chart one can determine the following quantities pertaining to a closed loop system:

(a) Magnitude and phase

(b) Band width

(c) Only magnitude

(d) Only phase **[1989]**

Solution: (a)

Nicholas chart is magnitude versus phase pilot.

Hence, the correct option is (a).

22. A system has fourteen poles and two zeros. Its high frequency asymptote in its magnitude plot has a slope of:

(a) −40 dB/decade (b) −240 dB/decade
(c) −280 dB/decade (d) −320 dB/decade

[1987]

Solution: (b)

Given $P = 14$, $Z = 2$, $(P - Z) = 14 - 2 = 12$

Slope $= -20(P - Z) = -20(12) = -240$ dB/dec.

Hence, the correct option is (b).

23. The polar plot of $G(s) = \dfrac{10}{s(s+1)^2}$ intercepts real axis at $\omega = \omega_0$. Then, the real part and ω_0 are, respectively, given by

(a) −2.5, 1 (b) −5, 0.5
(c) −5, 1 (d) −5, 2 **[1987]**

Solution : (c)

$$G(s) = \frac{10}{s(s+1)^2},$$

$$\angle G = -180 = -90° - 2\tan^{-1}(w)$$

$2\tan^{-1}(w) = 45° \Rightarrow w = 1$

$w_{pc} = 1$ rad/sec

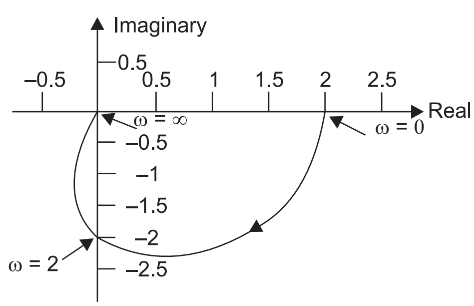
$$|G|_{w=w_{pc}} = \frac{10}{w(1+w^2)} = \frac{10}{1(1+1)} = 5$$

At $w = w$ the polar plot crosses the negative real axis at −5.

Hence, the correct option is (a).

Five-marks Questions

1. The Nyquist plot of an all-pole second order open-loop system is shown in the figure is obtain the transfer function of the system.



Solution: System in type 0, order 2

$$\therefore G(s)H(s) = \frac{k}{(s+p_1)(s+p_2)}$$

$P_1P_2 =$ Poles

$$\therefore G(j0)H(j0) = \frac{k}{p_1p_2} = 2$$

$$k = 2\,p_1p_2$$

$$\Rightarrow G(j\omega)H(j\omega) = \frac{k}{(j\omega+p_1)(j\omega+p_2)}$$

$$= \frac{k}{-\omega^2 + p_1p_2 + j\omega(p_1+p_2)}$$

∴ Considering imaginary axis.

(Real part) = Re $[G(j\omega)\,H(j\omega)] = 0$

$\therefore -\omega^2 + p_1p_2 = 0$

$p_1p_2 = \omega^2$

$\omega = 2$

$p_1p_2 = 2^2 = 4$

Sub value of p^1p^2 in equation (i)

$k = 2p_1p_2 = 2 \times 4 = 8$

∴ Sub value of ω, k and p_1p_2 in equation (ii)

$$\therefore G(j\omega)H(j\omega) = \frac{k}{-\omega^2 + p_1p_2 + j\omega(p_1+p_2)}$$

$$|G(j_2)\,H(j_2)| = \frac{8}{2(p_1+p_2)} = 2$$

thus

$\therefore p_1 + p_2 = 2$ (iii)

∴ To find p_1 and p_2 values

$\therefore (p_1 - p_2)^2 = R_e(p_1, p_2)^2 - p_1p_2$

$\Rightarrow (p_1p_2)^2 = 2^2 - 4 \times 4$

$\Rightarrow (p_1 - p_2)^2 = 4 - 16 = -12$

$\Rightarrow p_1 - p_2 = j2\sqrt{3}$ (iv)

Adding equation (iii) and (iv)

$p_1 + p_2 = 2$

$p_1 - p_2 = j2\sqrt{3}$

$2p_1 = 2 + j2\sqrt{3}$

$= 1 + j\sqrt{3}$

Sub value of p_1 in equation (iii)

$p_1 + p_2 = 2$

$\Rightarrow 2p_1 = 2 + j2\sqrt{3}$

$p_2 = 1 - j\sqrt{3}$

$$\therefore G(s)H(s) = \frac{8}{[s+1+j\sqrt{3}][(s+1-j\sqrt{3}]}$$

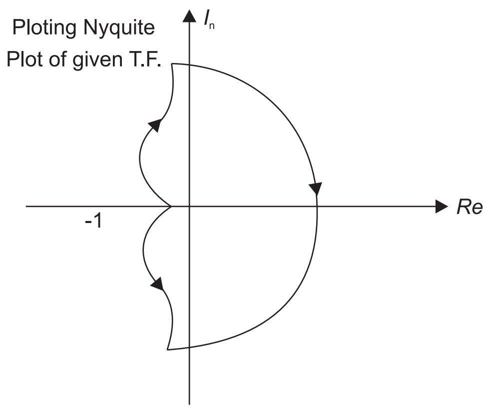
$$G(s)H(s) = \frac{8}{(s+1)^2 - (j\sqrt{3})^2} = \frac{8}{s^2+1+2s+3}$$

$$= \frac{8}{s^2+2s+4}$$

2. Consider a feedback system with the open-loop transfer function. Given by $G(s)H(s) = \frac{K}{s(2s+1)}$. Examine the stability of the closed-loop system using Nyquist stability.

[1999]

Solution: given $G(s)H(s) = \dfrac{k}{s(2s+1)}$



$S_1 \rightarrow$ polar plot

$S_2 \rightarrow S = \lim\limits_{x\to\infty} \mathrm{Re}^{j\theta}$

$\therefore R = \infty$

$$\theta = \frac{+\pi}{2} \text{ to } \frac{\pi}{2}$$

$S_3 \rightarrow$ Inverse Polar plot

$S_4 \rightarrow S = \lim\limits_{R\to 0} \mathrm{Re}^{j\theta}$

$R = 0$

$$\therefore \theta = \frac{-\pi}{2} \text{ to } \frac{+\pi}{2}$$

$N = 0$

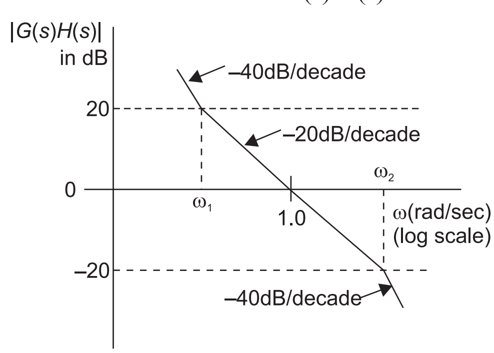
$P = 0$

$N = P - Z$

$Z = 0$

given system is stable.

3. The asymptotic Bode plot of the minimum phase open-loop transfer function G(s) H(s) is as shown in figure. Obtain the transfer function G(s) H(s)



Solution: given

(i) Initially

Slope = −40 dB bidecade

$\therefore$ No. of poles at origin $\frac{1}{s^2} = 2$

(ii) $\omega_1 \rightarrow 1^{st}$ corner frequency

Change on slope at ω_1 = +20 dB decade

$\therefore$ zero at $\frac{1}{\omega_i} = 1$

(iii) $V_2 \rightarrow 2^{nd}$ corner frequency

Change in slope at ω_2 = 20 dB/decade

So, poles at $\frac{1}{\omega_2} = 1$

Equation between ω_1 and ω_2

$y = -20 \log \omega + k$

$0 = -20 \log(1) + k$

$k = 0$

at ω_1

$20 = -20 \log(\omega_1) + k$

$\Rightarrow 20 = -20 \log(\omega_1) + 0$

$\omega_1 = 0.1$

at ω_2

$-20 = -20 \log(\omega_2) + k$

$-20 = -20 \log(\omega_2) + 0$

$\therefore \omega_2 = 10$

On initial line

$20 = -40 \log(\omega_1) + k_1$

$20 = -40 \log(0.1) + k_1$

$k_1 = -20$

$k_1 = 20 \log(c_2)$

$\Rightarrow -20 = 20 \log(k_2)$

$c_2 = 0.1$

$$\text{T.F.}G(s)H(s) = \frac{c_2\left(1+\frac{2}{\omega_1}\right)}{s^2\left(1+\frac{s}{\omega_2}\right)}$$

$$= \frac{0.1\left(1+\frac{s}{0.1}\right)}{s^2\left(1+\frac{s}{10}\right)}$$

$$\text{T.F.} = \frac{10(s+0.1)}{s^2(s+10)}$$

4. The loop transfer function of a single loop control system is given by

$G(s)H(s) = \dfrac{100}{s(1+0.01s)} e\text{-}sT.$ Using Nyquist criterion, find the condition for the closed loop system to be stable **[1998]**

Solution:

$$|G(s)H(s)| = \left|100/\omega\sqrt{1+(0.01\omega)^2}\right|_{\omega=wge} = 1$$

$$\Rightarrow \omega\sqrt{1+(0.01\omega)^2} = 100$$

$$\Rightarrow \omega^2(1+(0.01\omega)^2) = 10000$$

$$\Rightarrow \omega^2(!+0.0001\omega^2) = 10000$$

$\Rightarrow 0.0001\omega^4 + \omega^2 - 10000 = 0$

let $\omega^2 = y$

$0.0001\ y_2 + y - 10000 = 0$

$$\therefore y = \frac{-1 \pm \sqrt{1+4}}{0.0002}$$

$$y = \frac{-1 \pm \sqrt{5}}{0.0002}$$

$y = 6180$

$\therefore$ As $y = \omega^2$

$\omega^2 = 6180$

$\omega_{gc} = 70.6$ rad/sec

$$\phi = \omega_{gc}T - \frac{\pi}{2} - \tan^{-1}(\omega_{gc}0.01)$$

$$\phi = -78.67 - 90° - \tan^{-1}(78.6 \times 0.01)$$

$$\phi = -78.67 - 90° - 38.17°$$

$$= -78.67 - 128.17°$$

$$\text{p.m.} = 180 + \phi$$

$$= 180 + (-78.67 - 128.17°)$$

If pm is +ve, system becomes stable.

$P_m > 0$

$180° - 128.17° - 78.67 > 0$

$$\Rightarrow -78.67\left(\frac{180}{\pi}\right) > -51.83$$

$T > 0.0115$

Chapter 8

State Space Analysis

One-mark Questions

1. Consider the state space realization $\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -9 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 45 \end{bmatrix}u(t)$, with the initial condition $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, where $u(t)$ denotes the unit step function. The Value of $\lim\limits_{x\to\infty}\left|\sqrt{x_1^2(t)+x_2^2(t)}\right|$ is ________. **[2017]**

Solution: From given state space realization,

$$\dot{x}_1(t) = 0$$

$$\dot{x}_2(t) = -9x_2(t) + 45u(t) \qquad (1)$$

Apply Laplace transfrom to equation (1)

$$SX_1(s) - x_1(0) = 0 \Rightarrow X_1(s) = 0 \qquad (2)$$

$$SX_2(s) - x_2(0) = -9x_2(s) + 45\left(\frac{1}{5}\right)$$

$$(S+9)X_2(S) = \frac{45}{S}$$

$$X_2(S) = \frac{45}{S(S+9)} = \frac{A}{S} + \frac{B}{S+9} = \frac{5}{S} - \frac{5}{S+9}$$

Apply inverse LT to equations (2) and (3)

$$x_1(t) = 0;$$

$$x_2(t) = 5u(t) - 5e^{-9t}u(t)$$

$$\lim_{t\to\infty}\sqrt{x_1^2(t)+x_2^2(t)}$$

$$\lim_{x\to\infty}|x_2(t)| = 5$$

Hence, the correct answer is (5).

2. The state variable representation of a system is given as

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}x;\ x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$y = [0 \quad 1]x$$

The response $y(t)$ is **[2015]**

(A) $\sin(t)$ (B) $1 - e^t$

(C) $1 - \cos(t)$ (D) 0

Solution: From the given state equation

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix};\ x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$y = [0 \quad 1]x$$

$$\therefore \qquad y = x_2$$

$$x(t) = \Phi(t) \cdot x(0)$$

$$\Phi(t) = L^{-1}\{(SI - A)^{-1}\}$$

$$\begin{bmatrix} s & -1 \\ 0 & s+1 \end{bmatrix} = (SI - A)$$

$$\Phi = \frac{1}{s(s+1)}\begin{bmatrix} s+1 & 1 \\ 0 & s \end{bmatrix}$$

$$\Phi = \begin{bmatrix} 1/s & 1/s(s+1) \\ 0 & 1/(s+1) \end{bmatrix}$$

$$X(s) = \begin{bmatrix} 1/S & 1/s(s+1) \\ 0 & 1/(s+1) \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$X(s) = \begin{bmatrix} 1/s \\ 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$x(t) = 1 \cdot x_1(t) + 0 \cdot x_2(t)$$

$$\therefore \qquad y = 0$$

Hence, the correct option is (D).

3. Consider the system $\frac{dx}{dt} = Ax + Bu$ with $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} p \\ q \end{bmatrix}$ where p and q are arbitrary real numbers.

   Which of the following statements about the controllability of the system is true?

   (a) The system is completely state controllable for any nonzero values of p and q
   (b) Only $p = 0$ and $q = 0$ result in controllability
   (c) The system is uncontrollable for all values of p and q
   (d) We cannot conclude about controllability from the given data **[2009]**

   Solution: (c)

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};\ B = \begin{bmatrix} p \\ q \end{bmatrix}$$

   Controllability, $QC = [BAB \ldots A^{n-1}B] \neq 0$

$$A.B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$$

   So $Q_c = \begin{bmatrix} p & p \\ q & q \end{bmatrix} = 0 \Rightarrow$ uncontrollable, for all p and q

   Hence, the correct option is (c)

4. A linear system is equivalent represented by two sets of state equations. $X = AX + BU$ and $W = CW + DU$.

   The eigen values of the representations are also computed as [λ] and [μ]. Which one of the following statements is true?

   (a) [λ] = [μ] and $X = W$
   (b) [λ] = [μ] and $X \neq W$
   (c) [λ] ≠ [μ] and $X = W$
   (d) [λ] ≠ [μ] and $X \neq W$ **[2005]**

   Solution: (a,b)

   Eigen value of A = [λ]

   Eigen value of W = [μ]

   The eigen value of a system are always unique.

   So [λ] = [μ]

   But a system can be represented by different state models having different sets of state variables

   $x = w$

   $x \neq aw$

   Both are possible conditions.

5. The transfer function $Y(s)/U(s)$ of a system described by the state equations $\dot{x}(t) = -2x)(t) + 2u(t)$ and $y(t) =$ 0 $5x(t)$ is

   (a) $0.5/(s - 2)$ (b) $1/(s - 2)$
   (c) $0.5/(s + 2)$ (d) $1/(s + 2)$ **[2002]**

   Solution: (d)

   $\dot{x}(t) = 2x(t) + 2u(t)$ (1)

   $y(t) = 0.5\ x(t)$ (2)

   taking Laplace transform of (1)

   $s \times (s) = -2 \times (s) + 2u(s)$

$$\Rightarrow x(s) = \frac{2u(s)}{(s+2)}$$

   Laplace transform of (1)

   $y(s) = 0.5 \times (s)$

$$y(s) = \frac{0.5 \times 2u(s)}{(s+2)}$$

$$\therefore \frac{y(s)}{u(s)} = \frac{1}{(s+2)}$$

   Hence, the correct option is (d)

6. The system mode described by the state equations

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}x + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u,\ Y = \begin{bmatrix} 1 & 1 \end{bmatrix}x\ is$$

   (a) controllable and observable
   (b) controllable, but not observable
   (c) observable, but not controllable
   (d) neither controllable nor observable **[1999]**

   Solution: (a)

   $Q0 = [BABA^2B\ -\ A^{n-1}B]$

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ C = [1 \quad 1]$$

$$AB = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\therefore Q_0 = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}_{2\times 2} \neq 0$$

   ∴ Order = 2, rank = 2

   Controllable

$$Q_0 = [C^T \ A^T C^T \ C^T A^{T^2} \ldots]$$

$$C^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ A^T = \begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix}$$

$$\therefore Q_0 = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix} \neq 0$$

   rank 2 observable.

   Hence, the correct option is (a)

Two-marks Questions

1. Let the state-space representation of an LTI system be $\dot{X}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t) + du(t)$ where A, B, C are matrices, d is a scalar. $u(t)$ is the input to the system and $y(t)$ is its output. Let $B = [0\ 0\ 1]^T$ and $d = 0$. Which one of the following options for A and C will ensure that the transfer function of this LTI system is $H(s) = \dfrac{1}{s^2 + 3s^2 + 2s + 1}$? **[2019]**

(A) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix}$ and $C = [0\ 0\ 1]$

(B) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$ and $C = [0\ 0\ 1]$

(C) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$ and $C = [1\ 0\ 0]$

(D) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix}$ and $C = [1\ 0\ 0]$

Solution: $\dot{X}(t) = Ax(t) + Bu(t)$

$$Y(t) = Cx(t) + Du(t)$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad d = 0$$

$$H(s) = \frac{1}{S^3 + 3S^2 + 2S + 1}$$

From the standard four

$$\frac{Y(S)}{U(S)} = \frac{b_0}{a_3S^3 + a_2S^2 + a_4S + a_0}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix};\ B = \begin{bmatrix} 0 \\ 0 \\ b_0 \end{bmatrix};\ C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

On complaring we get

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

Hence, the correct option is (C).

2. The state equation and the output equation of a control system are given below:

$$X = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} X + \begin{bmatrix} 2 \\ 0 \end{bmatrix} U,$$

$$Y = [1.5\ 0.625]\,x.$$

The transfer function representation of the system is: **[2018]**

(A) $\dfrac{3s+5}{s^2+4s+6}$ (B) $\dfrac{3s-1.875}{s^2+4s+6}$

(C) $\dfrac{4s+1.5}{s^2+4s+6}$ (D) $\dfrac{6s+5}{s^2+4s+6}$

Solution: T.F $= C(SI - A)^{-1} \cdot B + D$

$$(SI - A)^{-1} = \begin{bmatrix} S+4 & 1.5 \\ -4 & S \end{bmatrix}^{-1}$$

$$= \frac{1}{S(S+4)+6} \begin{Bmatrix} S & -1.5 \\ 4 & S+4 \end{Bmatrix}$$

$$= \begin{bmatrix} \dfrac{S}{S^2+4S+6} & \dfrac{-1.5}{S^2+4S+6} \\ \dfrac{4}{S^2+4S+6} & \dfrac{S+4}{S^2+4S+6} \end{bmatrix} = [1.5\ 0.625]$$

$$\begin{bmatrix} \dfrac{S}{S^2+4S+6} & \dfrac{-1.5}{S^2+4S+6} \\ \dfrac{4}{S^2+4S+6} & \dfrac{S+4}{S^2+4S+6} \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$= [1.5\ 0.625] \begin{bmatrix} \dfrac{2S}{S^2+4S+6} \\ \dfrac{8}{S^2+4S+6} \end{bmatrix}$$

$$= \frac{3S+5}{S^2+4S+6}$$

Hence, the correct option is (A)

3. A second-order LTI system is described by the following state equations,

$$\frac{d}{dt}x_1(t) - x_2(t) = 0$$

$$\frac{d}{dt}x_2(t) + 2x_1(t) + 3x_2(t) = r(t)$$

where $x_1(t)$ and $x_2(t)$ are the two state variables and r(t) denotes the input. The output $c(t) = x_1(t)$. The system is **[2017]**

(A) undamped (oscillatory)
(B) underdamped
(C) critically damped
(D) overdamped

Solution: From given state equation

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = -2x_1(t) - 3x_2(t) + 1r(t)$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\overline{X}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

Characterstic equation is $|S\overline{I} - \overline{A}| = \overline{0}$

$$\left| \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix} \right| = 0$$

Charactersitic equation is : $S^2 + 3S + 2 = 0$

Roots of characterstics equation are, $S = -1, -2$

∴ The system is overdamped, ∵Poles are (–)ve real and unequal.

Hence, the correct option is (D).

4. Consider the state space model of a system, as given below.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$$

$$u;\ y = [1 \quad 1 \quad 1]\begin{bmatrix} x_1 \\ x_2 \\ xs_3 \end{bmatrix}$$

The system is
(a) controllable and observable
(b) uncontrollable and observable
(c) uncontrollable and unobservable
(d) controllable and unobservable **[2014]**

Solution: (b)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix} 4$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

Controllability

$$Q_0 = \left[B : AB : A^2B\right] = \begin{bmatrix} 0 & 4 & -8 \\ 4 & -4 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$|Q_0| = 0$

∴ uncontrollable.

Observability:

$$Q_0 = [C^T : C^TC^T : C^{2T}C^T] = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 4 \end{bmatrix}$$

$|Q_0| = 1$ ∴ observable

Hence, the correct option is (b)

5. An unforced linear time invariant (LTI) system is represented by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

If the initial conditions are $x_1(0) = 1$ and $x_2(0) = -1$, the solution of the state equation is
(a) $x_1(t) = -1, x_2(t) = 2$
(b) $x_1(t) = -e^{-t}, x_2(t) = 2e^{-t}$
(c) $x_1(t) = -e^{-t}, x_2(t) = -2e^{-t}$
(d) $x_1(t) = -e^{-t}, x_2(t) = -2e^{-t}$ **[2014]**

Solution: (c)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{x}_1 = -x_1 - 2x_2 \quad (1)$$

$$\dot{x}_2 = -2x_2 \quad (2)$$

Applying Laplace transform in equations (1) and (2)

$\Rightarrow sx_1 = x_1(0) - x_1(s)$ $\{x_1(0) = 1\}$

So, $x_1(s) = \dfrac{1}{s+1}$

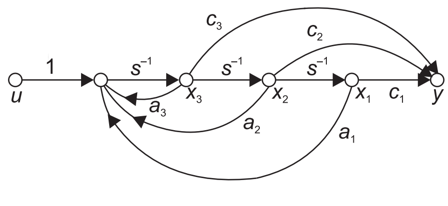
or $e^{-t} = x_1(t)$

$\Rightarrow sx_2(s) = x_2(0) = -2x_2(s)$ $\{x_2(0) = -1\}$

$\Rightarrow x_2(s) = \dfrac{1}{s+2}$ $x_2(t) = e^{-2t}$

Hence, the correct option is (c)

6. Consider the state space system expressed by the signal flow diagram shown in the figure.



The corresponding system is
(a) always controllable
(b) always observable
(c) always stable
(d) always unstable **[2014]**

Solution: (a)

The stage equation from the state diagram.

$\dot{x}_1 = 0$

$\dot{x}_2 = 0$

$\dot{x}_3 = a_3x_3 + a_2x_2 + a_1x_1 + 4$

also $y = c_1x_1 + c_2x_2 + c_3x_3$

Thus, state matrix.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_1 & a_2 & a_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$y = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

System is controllable if the rank of controllability matrix Q_c is equal to the rank of state matrix A. However, if Q_c is a square matrix the condition is $|Q_c| \neq 0$

$Q_c = [B \ : \ AB \ : \ A^2B]$

$$Q_c = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & a_3 & a_3^2 \end{bmatrix}$$

rank of Q_c = rank if $A = 1$ controllable system.

Observability: The system is said to be observable if the rank of observability matrix Q_0 is equal to rank of state matrix A, for square Q_0 the observability condition is $|Q_0| \neq 0$.

$Q_0 = [C^T : A^TC^T : (A^T)^2C^T]$

$$\therefore Q_0 = \begin{bmatrix} c_1 & a_1c_3 & a_1a_3c_3 \\ c_2 & c_2c_3 & a_2a_3c_3 \\ c_3 & a_3c_3 & a_3a_3c_3 \end{bmatrix}$$

rank of $Q_0 \neq$ Rank of A, also $|Q_0| = 0$ the system is not observable.

Hence, the correct option is (a)

7. The state equation of a second-order linear system is given by

$\dot{x}(t) = Ax(t), x(0) = x_0$

For $x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$ and for $x_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$,

$$x(t) = \begin{bmatrix} e^{-t} - e^{-2t} \\ -e^{-t} + 2e^{-2t} \end{bmatrix}.$$

When $x_0 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $x(t)$ is

(a) $\begin{bmatrix} -8e^{-t} + 11e^{-2t} \\ 8e^{-t} - 22e^{-2t} \end{bmatrix}$ (b) $\begin{bmatrix} 11e^{-t} - 8e^{-2t} \\ -11e^{t} + 16e^{-2t} \end{bmatrix}$

(c) $\begin{bmatrix} 3e^{-t} - 5e^{-2t} \\ -3e^{-t} + 10e^{-2t} \end{bmatrix}$ (d) $\begin{bmatrix} -5e^{-t} + 6e^{-2t} \\ -5e^{-t} + 6e^{-2t} \end{bmatrix}$

[2014]

Solution: (b)

Given $\dot{x}(t) = Ax(t)$, $x(0) = x0$

Taking Laplace transform

$sx(s) - x(0) = Ax(s)$

$[SI - A]\, x(s) = x(0)$

$x(s) = [SI - A]^{-1}x(0)$ (1)

for, $y_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$

for $y_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $x(t) = \begin{bmatrix} e^{-t} - e^{-2t} \\ -e^{-t} + 2e^{-2t} \end{bmatrix}$

using linearity property in equation (1)

$k_1x_1(t) = L^{-1}[SI - A]^{-1}x_1(0)\, k_1$

$k_2x_2(t) = L^{-1}[SI - A]^{-1}x_2(0)\, k_2$

Using Linearity Property

$k_1x_1(t) + k_2x_2(t) = L^{-1}[SI - A]^{-1}$ (2)

$[k_1x_1(0) + k_2x_2\,(0)]$

again $x_3(s) = [SI - A]^{-1}x_3(0)$

So, $k_1\begin{bmatrix} 1 \\ -1 \end{bmatrix} + k_2\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} k_1 0 + 0k_2 \\ -k_1 + k_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \Rightarrow \begin{matrix} k_1 = 3 \\ k_2 = 8 \end{matrix}$$

From (2) $x(t) = k_1x_1(t) + k_2x_2(t)$

$$= 3\begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} + 8\begin{bmatrix} e^{-t} - e^{-2t} \\ -e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$= \begin{bmatrix} 11e^{-t} - 8e^{-2t} \\ -11e^{-t} + 16e^{-2t} \end{bmatrix}$$

Hence, the correct option is (b)

8. The state transition matrix $\varphi(t)$ of a system $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is

(a) $\begin{bmatrix} t & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ 1 & t \end{bmatrix}$ (d) $\begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$ **[2014]**

Solution: (d)

given, $\begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ [SI - A] = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}$$

$[SI - A] = s_2$

$$\phi(t) = L^{-1}[SI - A]^{-1} = L^{-1}\left[\frac{1}{s^2}\begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix}\right] = \begin{bmatrix} 1 & + \\ 0 & 1 \end{bmatrix}$$

9. The state-variable equations of the system in the figure above are

(a) $\dot{X} = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$

$y = \begin{bmatrix} 1 & -1 \end{bmatrix} X + u$

(b) $\dot{X} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$

$y = \begin{bmatrix} -1 & -1 \end{bmatrix} X + u$

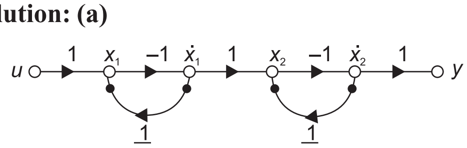
(c) $\dot{X} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$

$y = \begin{bmatrix} -1 & -1 \end{bmatrix} X - u$

(d) $\dot{X} = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$ **[2013]**

$y = \begin{bmatrix} 1 & -1 \end{bmatrix} X - u$

Solution: (a)



So, $\dot{x}_1 - x_1 - 4$

$\dot{x}_2 = -(x_2 + \dot{x}_1) = -(x_2 - x_1 - 4)$

$\dot{x}_2 = x_1 - x_2 + 4$

$y = \dot{x}_2 = x_1 - x_2 + 4$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} 4$$

or $\dot{x} = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} -1 \\ 1 \end{bmatrix} 4$

Hence, the correct option is (a)

10. The state transition matrix e^{At} of the system shown in figure above is

(a) $\begin{bmatrix} e^{-t} & 0 \\ te^{-t} & e^{-t} \end{bmatrix}$ (b) $\begin{bmatrix} e^{-t} & 0 \\ -te^{-t} & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{-t} & 0 \\ e^{-t} & e^{-t} \end{bmatrix}$ (d) $\begin{bmatrix} e^{-t} & -te^{-t} \\ 0 & e^{-t} \end{bmatrix}$

[2013]

Solution: (a)

$$A = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$$

$$[SI - A]^{-1} = \frac{1}{(s+1)(s+1)} \begin{bmatrix} s+1 & 0 \\ 1 & s+1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{1}{(s+1)} & 0 \\ \frac{1}{(s+1)^2} & \frac{1}{(s+1)} \end{bmatrix}$$

$$\phi(t) = e^{At} = L^{-1}\left\{(SI - A)^{-1}\right\}$$

$$e^{At} = \begin{bmatrix} e^{-t} & 0 \\ te^{-t} & e^{-t} \end{bmatrix}$$

Hence, the correct option is (a)

11. The state variable description of an LTI system is given by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

where y is the output and u is the input. The system is controllable for

(a) $a_1 \neq 0, a_2 = 0, a_3 \neq 0$

(b) $a_1 = 0, a_2 \neq 0, a_3 \neq 0$

(c) $a_1 = 0, a_2 \neq 0, a_3 = 0$

(d) $a_1 \neq 0, a_2 \neq 0, a_3 = 0$ **[2012]**

Solution: (d)

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & a_1 & 0\\ 0 & 0 & a_2\\ a_3 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}u$$

$$y=\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

$$QC = [BABA^2B] \qquad (1)$$

$$AB=\begin{bmatrix}0 & a_1 & 0\\ 0 & 0 & a_2\\ a_3 & 0 & 0\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}0\\a_2\\0\end{bmatrix}$$

$$AAB = A^2B=\begin{bmatrix}0 & a_1 & 0\\ 0 & 0 & a_2\\ a_3 & 0 & 0\end{bmatrix}\begin{bmatrix}0\\a_2\\0\end{bmatrix}=\begin{bmatrix}a_1a_2\\0\\0\end{bmatrix}$$

$$Q_c=\begin{bmatrix}0 & 0 & a_1a_2\\ 0 & a_2 & 0\\ 1 & 0 & 0\end{bmatrix}$$

for controllable system $|Q_0| \neq 0$

$(0 - a_1a_a^2) \neq 0$

or

or $a_1 \neq 0$

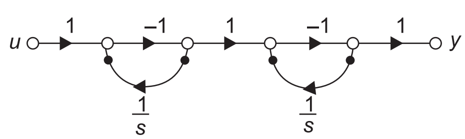
$a_2 \neq 0$

Hence, the correct option is (d)

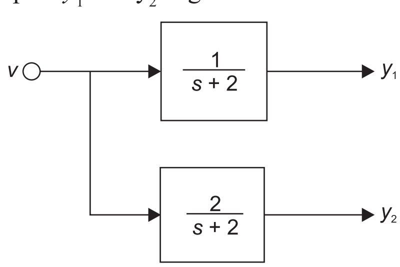
Statement for Linked Answer Questions 12 and 13:

The state diagram of a system is shown below described by the state-variable equations.

$\dot{X} = AX + Bu; \qquad y = CX + Du$



12. The block diagram of a system with one input u and two outputs y_1 and y_2 is given below.



A state space model of the above system in terms of the state vector x and the output vector $y = \begin{bmatrix}y_1 & y_2\end{bmatrix}^T$ is

(a) $\dot{x} = [2]x + [1]u; \qquad y = \begin{bmatrix}1 & 2\end{bmatrix}x$

(b) $\dot{x} = [-2]x + [1]u; \qquad y = \begin{bmatrix}1\\2\end{bmatrix}x$

(c) $\dot{x} = \begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix}x + \begin{bmatrix}1\\1\end{bmatrix}u; \qquad y = \begin{bmatrix}1 & 2\end{bmatrix}x$

(d) $\dot{x} = \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}x + \begin{bmatrix}1\\2\end{bmatrix}u; \qquad y = \begin{bmatrix}1\\2\end{bmatrix}x$ **[2011]**

Solution: (b)

$$\frac{y_1(s)}{u(s)}=\frac{1}{s+2};\ \frac{y_2(s)}{u(s)}=\frac{2}{(s+2)}$$

$$\frac{y_1(s)}{x_1(s)}\cdot\frac{x_1(s)}{u(s)}=\frac{1}{s+2}$$

$$\frac{y_2(s)}{y_2(s)}\frac{y_2(s)}{u(s)}=\frac{2}{(s+2)}$$

$$\Rightarrow \frac{x_1(s)}{u(s)}=\frac{1}{s+2} \text{ and } \frac{x_1(s)}{x_1(s)}=1$$

$$\frac{y_2(s)}{u(s)}=\frac{1}{s+2} \text{ and } \frac{y_2(s)}{u(s)}=2$$

$sx_1 + 2x_1(s) = u(s)$

and $y_1(s) = x_1(s)$

$sx_2(s) + 2x_2(s) = u(s)$

$y_2(s) = 2x_2(s$

$\dot{x}_1(t)+2x_1(t)=u(t))$

$y_2(t) = 2x_2(t)$

So $y=\begin{bmatrix}y_1 & y_2\end{bmatrix}^T=\begin{bmatrix}1 & 2\end{bmatrix}^T=\begin{bmatrix}1\\2\end{bmatrix}$

$\dot{x}_2(t)+2x_2(t)=u(t)$

$\dot{x}_1(t)+2x_2(t)=u(t)$

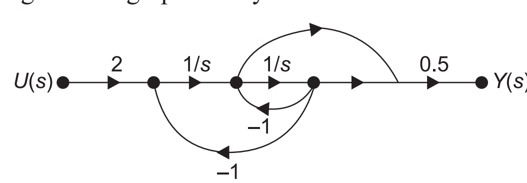
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-2 & 0\\ -2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}u(t)$$

or $\dot{x} = [-2]x + [1]^4$

Hence, the correct option is (b)

Common Data for Questions 13 and 14:

The signal flow graph of a system is shown below:



13. The state variable representation of the system can be

(a) $\dot{x} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$

$y = \begin{bmatrix} 0 & 0.5 \end{bmatrix} x$

(b) $\dot{x} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$

$y = \begin{bmatrix} 0 & 0.5 \end{bmatrix} x$

(c) $\dot{x} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$

$y = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} x$

(d) $\dot{x} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$ **[2010]**

$y = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} x$

Solution: (b)

$$x = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$

$y = [0 \quad 0.5]x$

Hence, the correct option is (b)

14. The transfer function of the system is

(a) $\dfrac{s+1}{s^2+1}$ (b) $\dfrac{s-1}{s^2+1}$

(c) $\dfrac{s+1}{s^2+s+1}$ (d) $\dfrac{s-1}{s^2+s+1}$

[2010]

Solution: (c)

forward path gain

$$P_1 = 2\left(\frac{1}{s}\right)\left(\frac{1}{s}\right)(0.5) = \frac{1}{s^2}$$

$$P_2 = 2\left(\frac{1}{s}\right)(1)(0.5) = \frac{1}{s}$$

$\Delta_1 = 1$

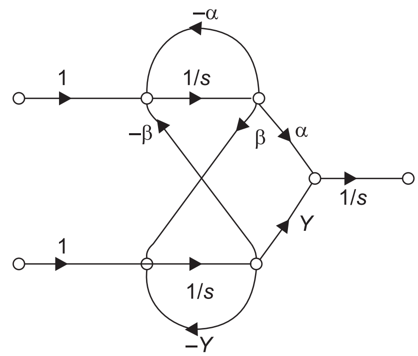
$\Delta_2 = 1$

$$\Delta = 1 - \left\{-\frac{1}{s} - \frac{1}{s^2}\right\} = 1 + \frac{1}{s} + \frac{1}{s^2}$$

$$\text{T.F} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{s+1}{s^2+s+1}$$

Hence, the correct option is (c)

15. A signal flow graph of a system is given below.



The set of equations that correspond to this signal flow graph is

(a) $\dfrac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} \beta & -\gamma & 0 \\ \gamma & \alpha & 0 \\ -\alpha & -\beta & 0 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

(b) $\dfrac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 0 & \alpha & \gamma \\ 0 & -\alpha & -\gamma \\ 0 & \beta & -\beta \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

(c) $\dfrac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} -\alpha & \beta & 0 \\ -\beta & -\gamma & 0 \\ \alpha & \gamma & 0 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

(d) $\dfrac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} -\gamma & 0 & \beta \\ \gamma & 0 & \alpha \\ -\beta & 0 & -\alpha \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

[2008]

Solution: (d)

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} -\gamma & 0 & \beta \\ \gamma & 0 & \alpha \\ -\beta & 0 & -\alpha \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Hence, the correct option is (d)

Common Data for Questions 13 and 14:

Consider a linear system whose state space representation is $\dot{x}(t) = Ax(t)$. If the initial state vector of the system is $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then the system response is $x(t) = \begin{bmatrix} e^{-2t} \\ -2e^{-2t} \end{bmatrix}$. If the initial state vector of the system changes to $x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. then the system response becomes

$$x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$$

16. The eigen-value and eigen-vector pairs (λ_i, v_i) for the system are

(a) $\left(-1,\begin{bmatrix}1\\1\end{bmatrix}\right)$ and $\left(-2,\begin{bmatrix}1\\2\end{bmatrix}\right)$

(b) $\left(-2,\begin{bmatrix}1\\-1\end{bmatrix}\right)$ and $\left(-1,\begin{bmatrix}1\\-2\end{bmatrix}\right)$

(c) $\left(-1,\begin{bmatrix}1\\-1\end{bmatrix}\right)$ and $\left(-2,\begin{bmatrix}1\\-2\end{bmatrix}\right)$

(d) $\left(-2,\begin{bmatrix}1\\-1\end{bmatrix}\right)$ and $\left(1,\begin{bmatrix}1\\-2\end{bmatrix}\right)$ **[2007]**

Solution: (a)

Sum of the eigen value = trace of the principle diagonal matrix.

Sum = −3, only (a) satisfies.

Hence, the correct option is (a)

17. The system matrix A is

(a) $\begin{bmatrix}0 & 1\\-1 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 1\\-1 & -2\end{bmatrix}$

(c) $\begin{bmatrix}1 & 1\\-1 & -2\end{bmatrix}$ (d) $\begin{bmatrix}0 & 1\\-2 & -3\end{bmatrix}$

[2007]

Solution: (d)

Multiplication of eigen values = determinant of matrix

$\therefore$ *Det* = 2 satisfies by last option.

Hence, the correct option is (d)

18. The state space representation of a separately excited DC servo motor dynamics is given as

$$\begin{bmatrix}\dfrac{d\omega}{dt}\\ \dfrac{di_a}{dt}\end{bmatrix} = \begin{bmatrix}-1 & 1\\-1 & -10\end{bmatrix}\begin{bmatrix}\omega\\ i_a\end{bmatrix} + \begin{bmatrix}0\\10\end{bmatrix}u$$

where ω is the speed of the motor, i_a is the armature current and u is the armature voltage. The transfer function $\dfrac{\omega(s)}{U(s)}$ of the motor is

(a) $\dfrac{10}{s^2+11s+11}$ (b) $\dfrac{1}{s^2+11s+11}$

(c) $\dfrac{10s+10}{s^2+11s+11}$ (d) $\dfrac{1}{s^2+s+1}$ **[2007]**

Solution: (a)

$$\begin{bmatrix}\dfrac{dw}{dt}\\ \dfrac{di_a}{dt}\end{bmatrix} = \begin{bmatrix}-1 & 1\\-1 & -10\end{bmatrix}\begin{bmatrix}w\\ i_a\end{bmatrix} + \begin{bmatrix}0\\10\end{bmatrix}4$$

$$\Rightarrow \frac{dw}{dt} = -w + i_a \quad (1)$$

$$\frac{di_a}{dt} = -w - 10i_a + 10u \quad (2)$$

taking Laplace transform (1) and (2)

$sw(s) = -w(s) + Ia(s)$

$\Rightarrow (s + 1)\, w(s) = Ia(s)$ (3)

$\Rightarrow sI_a(s) = -w(s) - 10I_a(s) + 10\, u(s)$

$\Rightarrow [s^2 + 11s + 11]w(s) = 10\, u(s)$

$$\frac{w(s)}{u(s)} = \frac{10}{(s^2+11s+11)}$$

Hence, the correct option is (a)

19. A linear system is described by the following state equation

$$\dot{X}(t) = AX(t) + BU(t), \quad A\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$$

The state-transition matrix of the system is

(a) $\begin{bmatrix}\cot t & \sin t\\ -\sin t & \cos t\end{bmatrix}$

(b) $\begin{bmatrix}-\cos t & \sin t\\ -\sin t & -\cos t\end{bmatrix}$

(c) $\begin{bmatrix}-\cos t & -\sin t\\ -\sin t & \cos t\end{bmatrix}$

(d) $\begin{bmatrix}\cos t & -\sin t\\ \cos t & \sin t\end{bmatrix}$ **[2006]**

Solution: (a)

$$\phi(t) = L^{-1}[SI - A]^{-1}$$

$$= L^{-1}\left[\begin{bmatrix}s & 0\\0 & s\end{bmatrix} - \begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}\right]^{-1} = L^{-1}\begin{bmatrix}s & -1\\1 & s\end{bmatrix}^{-1}$$

$$= L^{-1}\begin{bmatrix}\dfrac{s}{s^2+1} & \dfrac{1}{s^2+1}\\ \dfrac{-1}{s^2+1} & \dfrac{s}{s^2+1}\end{bmatrix}[D = s^2+1] = \begin{bmatrix}\cos t & \sin t\\ -\sin t & \cos t\end{bmatrix}$$

Hence, the correct option is (a)

20. The state variable equations of a system are:

1. $\dot{x}_1 = -3x_1 - x_2 + u$
2. $\dot{x}_2 = 2x_1 \qquad y = x_1 + u$

The system is

(a) controllable but not observable
(b) observable but not controllable
(c) neither controllable nor observable
(d) controllable and observable **[2004]**

Solution: (d)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$QC = [BAB], \quad AB = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

$$Q_C = \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix} \neq 0$$

$\therefore$ Controllable,

$$Q_0 = \begin{bmatrix} C^T & A^T C^T \end{bmatrix} : C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, A^T = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$$

$$\therefore Q_0 = \begin{bmatrix} 1 & -3 \\ 0 & -1 \end{bmatrix} \neq 0$$

Observable.

Hence, the correct option is (d)

21. Given $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the state transition matrix eAt is given by

(a) $\begin{bmatrix} 0 & e^{-t} \\ e^{-t} & 0 \end{bmatrix}$ (b) $\begin{bmatrix} e^{t} & 0 \\ 0 & e^{t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-t} \end{bmatrix}$ (d) $\begin{bmatrix} 0 & e^{t} \\ e^{t} & 0 \end{bmatrix}$ **[2004]**

Solution: (b)

$$[SI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} s-1 & 0 \\ 0 & s-1 \end{bmatrix}$$

$$e^{At} = [SI - A]^{-1} = \begin{bmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{1}{s-1} \end{bmatrix} = \begin{bmatrix} e^{t} & 0 \\ 0 & e^{t} \end{bmatrix}$$

Hence, the correct option is (b)

22. The zero-input response of a system given by the state-space equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ and } \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} is$$

(a) $\begin{bmatrix} te^{t} \\ t \end{bmatrix}$ (b) $\begin{bmatrix} e^{t} \\ t \end{bmatrix}$

(c) $\begin{bmatrix} e^{t} \\ te^{t} \end{bmatrix}$ (d) $\begin{bmatrix} t \\ te^{t} \end{bmatrix}$ **[2003]**

Solution: (c)

$$(SI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$$

$$(SI - A)^{-1} = \frac{\begin{bmatrix} (s-1) & 0 \\ 1 & (s-1) \end{bmatrix}}{(s-1)^2} = \begin{bmatrix} \frac{1}{(s-1)} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{(s-1)} \end{bmatrix}$$

$$L^{-1}[SI \wp A]^{-1} \quad e^{At} \quad \begin{bmatrix} e^{t} & 0 \\ \cdot\ t & t \end{bmatrix}$$

$$\Rightarrow x(t) = e^{At}[x(0)] = \begin{bmatrix} e^{t} & 0 \\ te^{t} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e^{t} \\ te^{t} \end{bmatrix}$$

Hence, the correct option is (c)

23. For the system described by the state equation

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

If the control signal is given by $u = [-0.5\ -3\ -5]\ x + v$, then the eigen values of the closed-loop system will be

(a) 0, –1,–2 (b) 0, –1, –3
(c) –1, –1,–2 (d) 0, –1, –1 **[1999]**

Solution: (a)

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [0.5 - 3 - 5]x + v$$

$$\therefore \dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} x + v$$

Char. equation

$= \lambda^3 + 3\lambda^2 + 2\lambda + 0 = 0$

$\lambda = 0, -1, -2.$

Hence, the correct option is (a)

24. A certain linear time invariant system has the state and the output equations given below:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$y=[1 \quad 1]\begin{bmatrix}x_1\\ x_2\end{bmatrix} is$$

$x_1(0) = 1$, $x_2(0) = -1$, $u(0) = 0$ then $\left.\dfrac{dy}{dt}\right|_{t=0}$ is

(a) 1
(b) –1
(c) 0
(d) None of the above **[1997]**

Solution: (a)

$$\dot{x}_1 = x_1 - x_2$$

$$\dot{x}_1(0) = x_1(0) = x_2(0) = 1-(-1) = 2$$

$$\dot{x}_2 = x_2$$

$$\dot{x}_2(0) = x_2(0) = -1$$

$$y = x_1 + x_2$$

$$\frac{dy}{dt} = \dot{x}_1 + \dot{x}_2,\ \left.\frac{dy}{dt}\right|_{t=0} = \dot{x}_1(0)+\dot{x}_2(0) = 2-1 = 1$$

Hence, the correct option is (a)

25. A linear time-invariant system is described by the state variable model

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$Y=[1 \quad 2]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

(a) The system is completely controllable.
(b) The system is not completely controllable.
(c) The system is completely observable.
(d) The system is not completely observable. **[1992]**

Solution: (b, c)

$$A=\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix},\ B=\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$AB=\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}=\begin{bmatrix}0\\ -2\end{bmatrix}$$

$$Q_c=\begin{bmatrix}B & AB\end{bmatrix}=\begin{bmatrix}0 & 0\\ 1 & -2\end{bmatrix}\Rightarrow$$

$|Q_c| = 0 - 0 = 0$

So, system is not controllable.

$$A^T=\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}\ C=[1 \quad 2],\ C^T=\begin{bmatrix}1\\ 2\end{bmatrix}$$

$$\Rightarrow A^TC^T=\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}\begin{bmatrix}1\\ 2\end{bmatrix}=\begin{bmatrix}-1\\ -4\end{bmatrix}$$

$$Q_0=\begin{bmatrix}C^T & A^TC^T\end{bmatrix}=\begin{bmatrix}1 & -1\\ 2 & -4\end{bmatrix}\Rightarrow$$

$$|Q_0| = -4-(-2) = -2 \neq 0$$

So, the given system is observable

26. A liner second-order single-input continuous-time system is described by following set of differential equations

$\dot{x}_1(t) = -2x(t) + 4x_2(t)$;

$\dot{x}_2(t) = 2x_1(t) - x_2(t) + u(t)$, Where $x_1(t)$ and $x_2(t)$ are the state variables and $u(t)$ is the control variable. The system is

(a) controllable and stable
(b) controllable but unstable
(c) uncontrollable and unstable
(d) uncontrollable but stable. **[1991]**

Solution: (b)

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-2 & 4\\ +2 & -1\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u(t)$$

$$Q_c=\begin{bmatrix}B & AB\end{bmatrix}$$

$$B=\begin{bmatrix}0\\ 1\end{bmatrix};\ AB=\begin{bmatrix}-2 & 4\\ +2 & -1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}=\begin{bmatrix}4\\ -1\end{bmatrix}$$

$$Q_0=\begin{bmatrix}0 & 4\\ 1 & -1\end{bmatrix}$$

$$|Q_0| = 0-4 = 4 \neq 0$$

Hence, the given system is controllable characteristics equation

$[SI - A] = 0$

$$\begin{bmatrix}(s+2) & -4\\ -2 & s+1\end{bmatrix}=0$$

$(s + 2)(s + 1) - 8 = 0 \quad s = 4.37, 1.37$

Since, pole lies at right half of s plane so system is unstable.

Hence, the correct option is (b)

27. Given the following state-space description of a system

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-2 & 0\\ 0 & -4\end{bmatrix}s+\begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$y=[0 \quad 1]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Find the state-transition matrix. **[1988]**

Solution: $A = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix}$

$$SI - A = \begin{bmatrix} s+2 & 0 \\ 0 & s+4 \end{bmatrix}$$

$$[SI-A]^{-1} = \frac{1}{(s+2)(s+4)} \qquad =|$$

$$\begin{bmatrix} s+4 & 0 \\ 0 & s+2 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+2} & 0 \\ 0 & \frac{1}{s+4} \end{bmatrix}$$

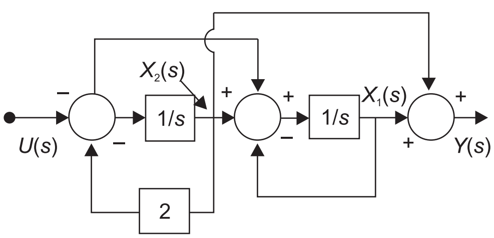
State transition matrix

$$\phi(t) = e^{At} = L^{-1}\{[SI-A]^{-1}\}$$

$$\phi(t) = e^{At} = \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-4t} \end{bmatrix}$$

Five-marks Questions

1. The block diagram of a linear time invariant system is given in the figure is



(a) Write down the state variable equations for the system in matrix form assuming the state vector to be $[x_1(t)x_2(t)]^T$.

(b) Find out state transition matrix.

(c) Determine $y(t)$, $t \geq 0$, when the initial values of the state at time $t = 0$ are $x_1(0) = 1$,and $x_2(0) = .1$ **[2002]**

Solution: From the figure given

$$x_2'(t) = -2x_2(t) - u(t)$$

$$x_1'(t) = -x_1(t) + x_2(t) + u(t)$$

$$y(t) = x_1(t) + x_1(t)$$

State equation

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix}4$$

$$\therefore\ x_1(t) = 2^{e^{-t}} - e^{-2t}$$

$$x_2(t) = e^{-2t}$$

$$y(t) = x_1(t) + x_2(t) = 2e^{-t} - e^{-2t} + e^{-2t}$$

$$y(t) = 2^{e^{-t}}$$

2. A certain linear, time-invariant system has the state and output representation shown below:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(a) Find the eigen values (natural frequencies) of the system.

(b) If $u(t) = \delta(t)$ and $x_1(0^+) = x_2(0^+) = 0$, find $x_1(t)$, $x_2(t)$ and $y(t)$, for $t > 0$.

(c) When the input is zero, choose initial conditions $x_1(0_+)$ and $x_2(0_+)$ such that $y(t) = Ae{-}^{2t}$ for $t > 0$. **[2000]**

Solution: $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}4$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(a) $|S - A| = 0$

$$\begin{bmatrix} s+2 & -1 \\ 0 & s+3 \end{bmatrix} = 0$$

$$\Rightarrow (s+2)(s+3) - 0 = 0$$

$$\Rightarrow (s+2)(s+3) = 0$$

$$\therefore s = -2, -3$$

(b) State transition matrix

$$[S-A] = \begin{bmatrix} s+2 & -1 \\ 0 & s+3 \end{bmatrix}$$

$$\Rightarrow [S-A]^{-1} = \frac{1}{(s+2)(s+3)}\begin{bmatrix} s+3 & 1 \\ 0 & s+2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+2} & \frac{1}{(s+2)(s+3)} \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

$$\phi(t) = e^{AT} = L^{-1}\left[(s-A)^{-1}\right]$$

$$e^{AT} = \begin{bmatrix} e^{-2x}. & e^{-2t} - e^{-3t} \\ 0 & e^{-3t} \end{bmatrix}$$

$$\therefore \text{As } x(t) = \left[L^{-1}\{S-A\}^{-1}\right]x(0) + L^{-1}\left[(S-A)^{-1} f_s.4(s)\right]$$

$$\Rightarrow [S-A]^{-1}.B = \begin{bmatrix} \frac{1}{(s+2)} & \frac{1}{(s+2)(s+3)} \\ 0 & \frac{1}{(s+3)} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1}{s+2} \\ 0 \end{bmatrix}$$

$$L^{-1}\{(S-A)^{-1}.B\} = [e^{-2t}]$$

$$x(t) = l^{At}x(0) + \begin{bmatrix} e^{-2t} \\ 0 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} e^{-2t} \\ 0 \end{bmatrix} - \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

So, $x_1(t) = e^{-2t}$

$x_2(t) = 0$

$$y(t) = [11]\begin{bmatrix} e^{-2t} \\ 0 \end{bmatrix} = e^{-2t}$$

(c) $$x(t) = \begin{bmatrix} e^{-2t} & e^{-2t} - e^{-3t} \\ 0 & e^{-3t} \end{bmatrix}\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}$$

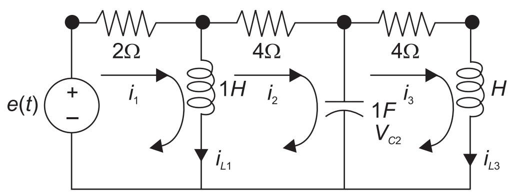
$$y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = x_1(t) + x_2(t)$$

$y(t) = e^{-2t}\, x_1(0) + (e^{-2t} - e^{-3t})\, x_2(0) + e^{-3t}x_2(0)$

$y(t) = Ae^{-2t}$

So, $x_1(0) = A$

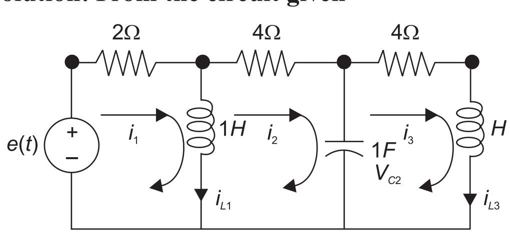
$x_2(0) = 0$

3. 

For the circuit shown in the figure choose state variables as x_1, x_2, x_3 to be $i_{L1}(t)$, $v_{c2}(t)$, $i_{L3}(t)$. Write the state equations

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \mathrm{A}\begin{bmatrix} x_1 \\ x_1 \\ x_1 \end{bmatrix} + \mathrm{B}[e(t)]$$ **[1997]**

Solution: From the circuit given



Apply KVL in Loop 1

$$\frac{di_L}{dt} = e(t) - 2i_1 = e(t) - 2(i_{L_1} - i_2)$$

$$\frac{di_{L_1}}{dt} = e(t) = 2i_{L_1} - 2i_2 \quad \text{(i)}$$

$$\therefore i_2 = \frac{\dfrac{di_{L_1}}{dt} - V_{C_2}}{4} \quad \text{(ii)}$$

So sub eq (ii) in (i)

$$\frac{di_{L_1}}{dt} = e(t) - 2i_{L_1} - 2\left(\frac{di_{L_1} - V_{C_2}}{4}\right)$$

$$\therefore \frac{di_{L_1}}{dt} = e(t) - 2i_{L_1} - \frac{1}{2}\frac{di_{L_1}}{dt} - \frac{V_{C_2}}{2}$$

$$\Rightarrow \frac{3}{2}\frac{di_{L_1}}{dt} = e(t) - 2i_{L_1} + \frac{1}{2}V_{c_2}$$

$$\Rightarrow \frac{di_{L_1}}{dt} = -\frac{4}{3}i_{L_1} + \frac{1}{3}V_{C_2} + \frac{2}{3}e(t) \quad \text{(iii)}$$

In 3rd loop

$$\Rightarrow \frac{dV_{C_2}}{dt} = C_2 - i_{L_3}$$

Sub value of i_2 in above equation

$$\frac{dv_{C_2}}{dt} = \frac{\dfrac{di_{L_1}}{dt} - V_{C_2}}{4} - i_{L_3}$$

$$\Rightarrow \frac{dV_{C_2}}{dt} = \frac{1}{4}\left[-\frac{4}{3}i_{L_1} + \frac{1}{3}V_{C_2} + \frac{2}{3}l(p)\right] - \frac{1}{4}V_{C_2} - i_{L_3}$$

$$\Rightarrow \frac{dV_{C_2}}{dt} = -\frac{1}{3}i_{L_1} - \frac{1}{6}V_{C_2} - i_{L_3} + \frac{1}{6}l(t) \quad \text{(iv)}$$

$$\Rightarrow \frac{di_{L_3}}{dt} = V_{C_2} - 4i_{L_3} \quad \text{(v)}$$

4. Obtain a state space representation in diagonal form for the following system **[1996]**

$$\frac{d^3y}{dt^3} + 6\frac{d^2}{dt^2} + 11\frac{dy}{dt} + 6y = 6u(t)$$

Solution: The given system is

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y = 6u(t)$$

$$\Rightarrow s^3y(s) + 6s^2y(s) + 11sy(s) + 6y(s) = 6u(s)$$

$$\Rightarrow y(s)\,[s^3 + 6s^2 + 11s + 6] = 6u(s)$$

$$\text{T.F.} = \frac{y(s)}{u(s)} = \frac{6}{s^3 + 6s^2 + 11s + 6}$$

$$\Rightarrow \frac{6}{(s+1)(s+2)(s+3)}$$

Taking partial Fraction.

$$\Rightarrow \frac{6}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$\Rightarrow 6 = A(s + 2)\,(s + 3) + B(s + 1)\,(s + 3) + C(s + 1)\,(s + 2)$

$\Rightarrow \mu t\; s = -1$

$6 = A(1)(2) \quad \therefore A = 3$

let $s = -2$

$6 = B(-1)(1) \quad \therefore B = -6$

let $s = -3$

$6 = C(-2)(-1), \qquad \therefore C = 3$

$$\therefore \text{T.F.} = \frac{y(s)}{u(s)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$\frac{y(s)}{u(s)} = \frac{3}{s+1} - \frac{6}{s+2} + \frac{3}{s+3}$$

$x(t) = x_1(t) + x_2(s) + x_3(s)$

$y(t) = x_1(t) + x_2(t) + x_3(t)$

$$x_1(s) = \frac{3}{s+1}u(s)$$

take inverse laplace transform

$x_1 = 3\,u(t) - x_1'\,(t)$ (i)

$$x_2(s) = \frac{-6}{s+2}u(s)$$

Taking inverse laplace transform

$x_2' - 2x_2 = 6u(t)$ (ii)

$$x_3(s) = \frac{3}{s+3}4(s)$$

Taking inverse laplace transform

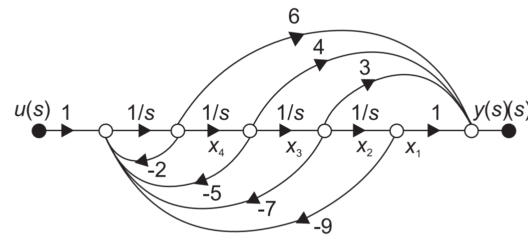
$x_3' = -3x_3 + 3u(t)$ (iii)

from equation (i), (ii) and (iii)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$$

$$y(s) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

5. From the signal flow graph shown in figure obtain the state space model with x_1, x_2, x_3 and x_4 as state variables and write the transfer function directly from the state space model.



Solution: Consider the system given

$x_1' = x_2$

$x_2' = x_3$

$x_4' = x_4$

$\dot{x}_4 = G(s) - 2\dot{x}4 - 5x_3 - 7x_2 - 9x_1$

$\therefore\; y = x_1 + 3x_2 + 4x_3 + 6xy$

$\therefore$ State equation representation as given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -9 & -7 & -5 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} [0]$$

$$[y] = \begin{bmatrix} 1 & 3 & 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\text{T.F.} = \frac{y(s)}{x(s)} = \frac{6s^2 + 4s^2 + 3s + 1}{s^4 + 2s^3 + 5s^2 + 7s + 9}$$

UNIT IV

ELECTRONIC DEVICES AND CIRCUITS

EXAM ANALYSIS

Exam Year	92	93	94	95	96	97	98	99	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14-1	14-2	14-3	14-4	15			16			17		18	19
																											Set 1	Set 2	Set 3	Set 1	Set 2	Set 3	Set 1	Set 2		
1 Marks Ques.	-	-	4	11	-	1	2	1	-	2	1	5	3	3	4	2	4	2	2	3	1	3	3	3	3	3	2	4	2	2	1	0	3	3	4	1
2 Marks Ques.	3	1	-	2	1	4	1	1	-	-	-	5	7	3	4	3	4	1	4	3	3	-	3	6	4	4	1	4	1	4	3	1	4	3	3	4
5 Marks Ques.	-	-	-	-	-	-	-	-	-	1	-	-	-	-	-	-	-	-	-	-							-	-	-	-	-	-	-	-	-	-
Total Marks	**6**	**2**	**4**	**15**	**2**	**9**	**4**	**3**	**0**	**2**	**1**	**15**	**17**	**9**	**12**	**8**	**12**	**4**	**10**	**9**	**7**	**3**	**9**	**15**	**11**	**11**	**4**	**12**	**4**	**10**	**7**	**2**	**11**	**9**	**10**	**9**
Chapter wise marks distribution																																				
Basics		2	-	1	4	-	7	-	-	-		-	5	7	4	6	1	4	2	5		-	1	1	3	2		8			-		-		-	-
PN Junction	-	2	-	3	-	-	4	-	-	-	1	7	4	3	5	7	3	-	2	2	1	1	2	2	1	-		5			8		9		4	5
BJT		4	-	3	8	2	2	-	3	-		-	3	6	2	1	-	5	2	3		6	1	3	4	4		-			3		4		-	-

Chapter 1

Basic

One-mark Questions

1. A bar of Gallium Arsenide (GaAs) is doped with Silicon such that the Silicon atoms occupy Gallium and Arsenic sites in the GaAs crystal. Which one of the following statements is true? **[2017]**

(A) Silicon atoms act as *p*-type dopants in Arsenic sites and *n*-type dopants in Gallium sites

(B) Silicon atoms act as *n*-type dopants in Arsenic sites and *p*-type dopants in Gallium sites

(C) Silicon atoms act as *p*-type dopants in Arsenic as well as Gallium sites

(D) Silicon atoms act as *n*-type dopants in Arsenic as well as Gallium sites

Solution: Si is a IVth group element, so it acts like *p*-type dopant in Vth group sites *p*, As ....etc.) and it is acts like a *n*-type dopant like in IIIrd Group sites (B, Al, Ga,… etc).

Hence, the correct option is (A).

2. An n^+–n Silicon device is fabricated with uniform and non-degenerate donor doping concentrations of $N_{D_1} = 1 \times 10^{18}$ cm^{-3} and $N_{D_2} = 1 \times 10^{15}$ cm^{-3} corresponding to the n^+ and n regions, respectively. At the operational temperature T, assume complete impurity ionization, $kT/q = 25$ mV, and intrinsic carrier concentration to be $n_i = 1 \times 10^{10}$ cm^{-3}. What is the magnitude of the built-in potential of this device? **[2017]**

(A) 0.748 V (B) 0.460 V

(C) 0.288 V (D) 0.173 V

Solution: Form the given data

$N_{D_1} = 1 \times 10^{18}$ atoms/cm^3

$N_{D_2} = 1 \times 10^{15}$ atoms/cm^3

$V_T = 25$ mv

$V_0 = ?$

$$N_A = \frac{n_i^2}{N_{D_1}}$$

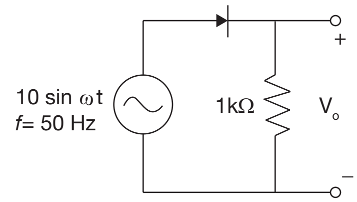
$$= \frac{1\times10^{20}}{1\times10^{15}} = 1\times10^{5} \text{ atoms/cm}^3$$

$$V_0 = V_T\, \ell n \left[\frac{N_A . N_D}{n_i^2}\right]$$

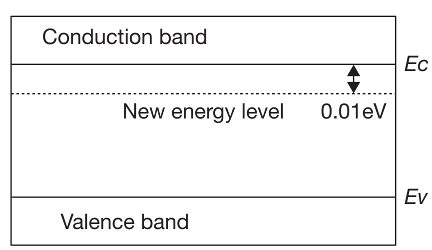
$$V_0 = 25 \times 10^{-3}\ \ell n\ \frac{1\times10^{18}\times10^{5}}{1\times10^{20}} = 0.173 \text{ V}$$

Hence, the correct option is (D).

3. The output V_0 of the diode circuit shown in the figure is connected to an aveaging DC voltmeter. The reading on the DC voltmeter in Volts, neglecting the voltage drop across the diode, is __________. **[2017]**



4. A small percentage of impurity is added to an intrinsic semiconductor at 300 K. Which one of the following statements is true for the energy band diagram shown in the following figure? **[2016]**



(A) Intrinsic semiconductor doped with pentavalent atoms to form *n*-type semiconductor.

(B) Intrinsic semiconductor doped with trivalent atoms to form *n*-type semiconductor.

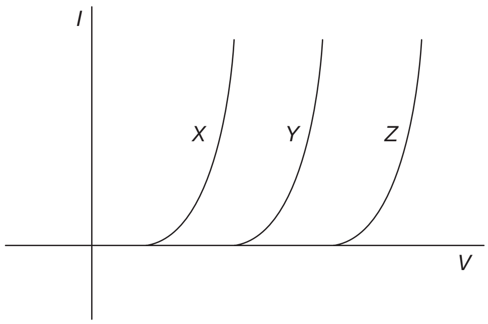
(C) Intrinsic semiconductor doped with pentavalent atoms to form *p*-type semiconductor.

(D) Intrinsic semiconductor doped with trivalent atoms to form *p*-type semiconductor.

Solution: Here Fermi energy level is lying just below the conduction band which shows that it is *n*-type semiconductor doped, i.e., an intrinsic semiconductor is doped with pentavalent impurity to form n-type.

Hence, the correct option is (A).

5. The I–V characteristics of three types of diodes at the room temperature, made of semiconductors X, Y and Z are shown in the figure. Assume that the diodes are uniformly doped and identical in all respects except their materials. If E_{gx}, E_{gy} and E_{gz} are the band gaps of X, Y and Z respectively, then **[2016]**



(A) $E_{gx} > E_{gy} > E_{gz}$
(B) $E_{gx} = E_{gy} = E_{gz}$
(C) $E_{gx} < E_{gy} < E_{gz}$
(D) no relationship among these band gaps exists

Solution: The energy band gap of a semiconductor is directly proportional to the cut in voltage. More the cut in voltage, more will be the energy band gap of the semiconductor, therefore the relation of E_g of three semiconductors X, Y, Z is

$$E_{gz} > E_{gy} > E_{gx}.$$

Hence, the correct option is (C).

6. For a silicon diode with long P and N regions, the acceptor and donor impurity concentrations are 1×10^{17} cm^{-3} and 1×10^{15} cm^{-3}, respectively. The lifetimes of electrons in P region and holes in N region are both 100 μs. The electron and hole diffusion coefficients are 49 cm^2/s and 36 cm^2/s, respectively. Assume $kT/q = 26$ mV, the intrinsic carrier concentration is 1×10^{10} cm^{-3}, and $q = 1.6 \times 10^{-19}$ C. When a forward voltage of 208 mV is applied across the diode, the hole current density (in nA//cm^2) injected from P region to N region is ________. **[2015]**

Solution: From the given data

$N_A = 1 \times 10^{17}$ cm^{-3} and $N_D = 1 \times 10^{15}$ cm^{-3}

$\tau_P = \tau_n = 100$ μs

$D_n = 49$ cm^2/s and $D_P = 36$ cm^2/s

$V_T = 26$ mV

$n_i = 1 \times 10^{10}$ cm^{-3}

$q = 1.6 \times 10^{-19}$ C

forward voltage

$$V = 208 \text{ mV}$$

$$I_h = ?$$

$$I = I_0\left(e^{V/nV_T} - 1\right),$$

But,

$$I = I_h$$

Consider $n = 1$ for small currents

$$I_o \approx \frac{q.D_P.P_n}{L_P}.A$$

$$P_{no} = \frac{n_i^2}{N_D} = \frac{10^{20}}{1\times10^{15}} = 10^5$$

$$L_P = \sqrt{D_P.\tau_P} = \sqrt{36\times10^{-4}}$$

$$L_P = 6 \times 10^{-2} \text{ cm}$$

Hole current density $= \dfrac{I_h}{A}$

$$J_h = \frac{q.p_{no}.D_P}{L_P}.\left[e^{V_D/V_T} - 1\right]$$

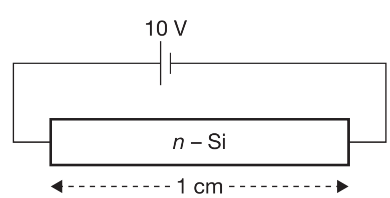
$$J_h = \frac{1.6\times10^{-19}\times10^5\times36}{6\times10^{-2}}\left\{e^{208/26} - 1\right\}$$

$$J_h = 9.6 \times 10^{-12} \times 2980$$

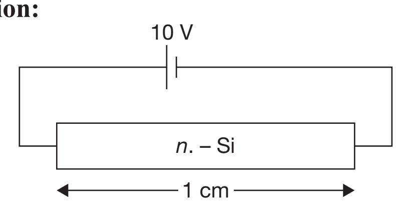
$$J_h = 28.608 \text{ nA/cm}^2$$

Hence, the correct Answer is (28 to 30).

7. A dc voltage of 10 V is applied across an n-type silicon bar having a rectangular cross-section and a length of 1 cm as shown in figure. The donor doping concentration N_D and the mobility of electrons μ_n are 10^{16} cm^{-3} and 1000 cm^{-2}V^{-1}s^{-1}, respectively. The average time (in μs) taken by the electrons to move from one end of the bar to other end is ______. **[2015]**



Solution:



$N_D = 10^{16}$ cm^{-3} and $\mu_n = 1000$ cm^2/V-sec

$$V_d = \mu E = \frac{\mu V}{L} = \frac{1000 \times 10}{1} = 10^4 \text{ cm/sec}$$

We know

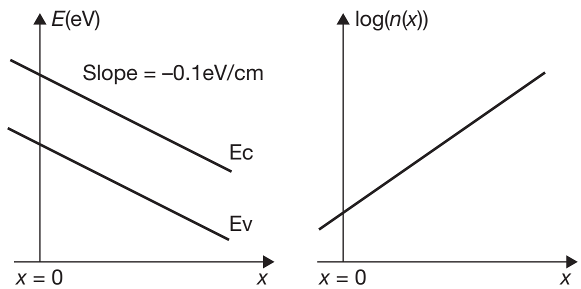
$$\text{drift velocity} = \frac{\text{distance}}{\text{time}} = \frac{L}{\tau}$$

$$\therefore \quad \tau = \frac{L}{V_d} = \frac{1\text{ cm}}{10^4 \text{ cm/sec}} = 10^{-4} \text{ sec}$$

$$\therefore \quad \tau = 100\ \mu \text{ sec}$$

Hence, the correct Answer is (95 to 105).

8. The energy band diagram and the electron density profile $n(x)$ in a semiconductor are shown in the figures. Assume that $n(x) = 10^{15} e^{(q\alpha x/kT)}$ cm^{-3}, with $\alpha = 0.1$ V/cm and x expressed in cm. Given $\frac{kT}{q} = 0.026$ V, D_n = 36 cm^2 s^{-1}, and $\frac{D}{\mu} = \frac{kT}{q}$. The electron current density (in A/cm^2) at $x = 0$ is **[2015]**



(A) -4.4×10^{-2} (B) -2.2×10^{-2}
(C) 0 (D) 2.2×10^{-2}

Solution: From the given data electron density profile

$$n(x) = 10^{15} . e^{\left(\frac{q \propto x}{KT}\right)} \text{ cm}^{-3}$$

$$\alpha = 0.1 \text{ V/cm}$$

$$\frac{KT}{q} = V_T = 0.026 \text{ V} = 26 \text{ mV}$$

$$D_n = 36 \text{ cm}^2\text{/sec}$$

We know

$$J_n = q . D_n . \frac{dn}{dx} = \text{diffusion current density}$$

$$\therefore \quad j_n(x) = q . D_n . \frac{d}{dx}(10^{15} \times e^{(\frac{q\alpha x}{KT})})$$

$$\Rightarrow \quad J_n(x) = 10^{15} \times 1.6 \times 10^{-19} \times 36 \times \frac{q \propto}{KT} . e^{q \propto x/KT}$$

at $X = 0$

$$J_n(0) = 10^{15} \times 1.6 \times 10^{-19} \times 36 \times \frac{0.1}{0.026} \times 1$$

$$\therefore \quad J_n(0)_{(diff)} = 2.2 \times 10^{-2} \text{ A/cm}^2$$

and

$$J_{n(\text{drift})} = n(x) . q . \mu_n . E$$

$$\frac{D}{\mu} = V_T$$

$$\mu_n = \frac{D_n}{V_T} = \frac{36}{0.026} = 1384.61 \text{ cm}^2\text{/V-S}$$

$$J_n(0)_{(\text{drift})} = 10^{15} \times 1.6 \times 10^{-19} \times 1384.5 \times E_x$$

$$E_x = \frac{-KT}{q} . \frac{1}{n(x)} . \frac{dn(x)}{dx} = -\alpha$$

$$= -0.1 \text{ V/cm}$$

$$\therefore \quad J_{n \text{ drift}} = -2.2 \times 10^{-12} \text{ A/cm}^2$$

$$J = J_{n(\text{drift})} + J_{n(\text{driff})}$$

$$= -2.2 \times 10^{-12} + 2.2 \times 10^{-12}$$

$$= 0$$

Hence, the correct option is (C).

9. A silicon bar is doped with donor impurities $N_D = 2.25 \times 10^{15}$ atoms cm^{-3}. Given that the intrinsic carrier concentration of silicon at $T = 300$ K is $n_I = 1.5 \times 10^{10}$ cm^{-3} and assuming complete impurity ionization, the equilibrium electron and hole concentrations are

(a) $n_0 = 1.5 \times 10^{16}$ cm^{-3}, $p_0 = 1.5 \times 10^5$ cm^{-3}
(b) $n_0 = 1.5 \times 10^{10}$ cm^{-3}, $p_0 = 1.5 \times 101^5$ cm^{-3}
(c) $n_0 = 225 \times 10^{15}$ cm^{-3}, $p_0 = 1.5 \times 10^{10}$cm^{-3}
(d) $n_0 = 2.25 \times 10^{15}$ cm^{-3}, $p_0 = 1 \times 10^5$ cm^{-3} **[2014]**

Solution: (d)

$n \approx N_D = 2.25 \times 10^{15}$ cm^3

$$p = \frac{n_i^2}{n} = 10^5 \text{ cm}^{-3}$$

Hence, the correct option is (d)

10. A thin p-type silicon sample is uniformly illuminated with light which generates excess carriers. The recombination rate is directly proportional to

(a) the minority carrier mobility
(b) the minority carrier recombination lifetime
(c) the majority carrier concentration
(d) the excess minority carrier concentration

[2014]

Solution: (d)

Hence, the correct option is (d)

11. At $T = 300$ K, the hole mobility of a semiconductor μ_p = 500 cm^2/V^{-s} and $\frac{kT}{q} = 26$ mV . The hole diffusion constant D_p in cm²/s is______. **[2014]**

Solution: 13 cm²/s.

By Einstein relation, $D_p = \mu_P VT = 500 \times 26 \times 10^{-3} = 13$ cm²/s.

12. At $T = 300$ K, the band gap and the intrinsic carrier concentration of GaAs are 1.42 eV and 10^6 cm^{-3}, respectively. In order to generate electron hole pairs in GaAs, which one of the wavelength (λ_C) ranges of incident radiation is most suitable? (Given that: Plank's constant is 6.62×10^{-34} J-s, velocity of light is 3×10^{10} cm/s and charge of electron is1.6×10^{-19} C.)

(a) 0.42 μm < λ_C < 0.87 μm
(b) 0.87 μm < λ_C < 1.42 μm
(c) 1.42 μm < λ_C < 1.62 μm
(d) 1.62 μm < λ_C < 6.62 μm **[2014]**

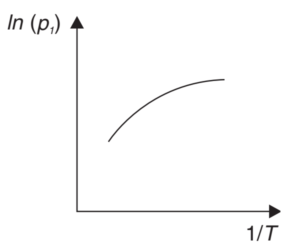
Solution: (a)

$$E_g(eV) \le \frac{1.24}{\lambda(\mu m)}$$

$$\lambda_c \le \frac{1.24}{E_g} = \frac{1.24}{1.42} \quad \lambda \le 0.87\mu m$$

Hence, the correct option is (a)

13. In the figure, *ln* (ρ_i) is plotted as a function of *1/T*, where ρ_i is the intrinsic resistivity of silicon, T is the temperature, and the plot is almost linear.



The slope of the line can be used to estimate

(a) band gap energy of silicon (E_g)
(b) sum of electron and hole mobility in silicon $(\mu_n + \mu_p)$
(c) reciprocal of the sum of electron and hole mobility in silicon $(\mu_n + \mu_p)^{-1}$
(d) intrinsic carrier concentration of silicon (n_i) **[2014]**

Solution: (a)

Hence, the correct option is (a)

14. The cut-off wavelength (in μm) of light that can be used for intrinsic excitation of a semiconductor material of band gap $Eg = 1.1$ eV is________. **[2014]**

Solution: 1.12

$$\lambda_{(\mu m)} \le \frac{1.24}{E_g(eV)}$$

$$\lambda \le \frac{1.24}{1.1} \quad \lambda \le 1.12\mu m$$

$\therefore \lambda_{max} = 1.12$ μm.

15. In IC technology, dry oxidation (using dry oxygen) as compared to wet oxidation (using stem or water vapour) produces

(a) superior quality oxide with a higher growth rate
(b) inferior quality oxide with a higher growth rate
(c) inferior quality oxide with a lower growth rate
(d) superior quality oxide with a lower growth rate **[2013]**

Solution: (d)

The answer is superior quality oxide with a lower growth rate.

Hence, the correct option is (d)

16. Drift current in semiconductors depends upon

(a) only the electric field
(b) only the carrier concentration gradient
(c) both the electric field and the carrier concentration
(d) both the electric field and the carrier concentration gradient **[2011]**

Solution: (c)

Drift current = $J.A. = Nq\,\mu_n E$

$\therefore I \propto E, I \propto N.$

Hence, the correct option is (c).

17. Thin gate oxide in a CMOS process is preferably grown using

(a) wet oxidation (b) dry oxidation
(c) epitaxial deposition (d) ion implantation **[2010]**

Solution: (b)

Dry oxidation provides better isolation from impurities.

Hence, the correct option is (b).

18. In an n-type silicon crystal at room temperature, which of the following can have a concentration of 4×10^{19} cm^{-3}?

(a) Silicon atoms
(b) Holes
(c) Dopant atoms
(d) Valence electrons **[2009]**

Solution: (c)

For a highly doped silicon, ratio is 1:10^3

$\therefore$ Dopant atoms = $1/10^3 \times 5 \times 10^{22} = 5 \times 10^{19}$

$\approx 4 \times 10^{19}$ cm^{-3}.

Hence, the correct option is (c).

19. The ratio of the mobility to the diffusion coefficient in a semiconductor has the unit

(a) V^{-1} (b) cm V^{-1}
(c) V cm^{-1} (d) V s **[2009]**

Solution: (a)

According to the Einstein theorem,

$$\frac{D^n}{\mu^n} = V_T \Rightarrow \frac{\mu^n}{D^n} = 1/V_T$$

$\therefore \frac{\mu}{D}$ has the unit of V^{-1}.

Hence, the correct option is (a).

20. Which of the following is true?

(a) A silicon wafer heavily doped with boron is a p^+ substrate.

(b) A silicon wafer lightly doped with boron is a p^+ substrate.

(c) A silicon wafer heavily doped with arsenic is a p^+ substrate.

(d) A silicon wafer lightly doped with arsenic is a p^+ substrate. **[2008]**

Solution: (a)

Boron is a *p*-type material and arsenic is an *n*-type material.

Hence, the correct option is (a).

21. A silicon wafer has 100 nm of oxide on it and is inserted in a furnace at a temperature above 1000° C for further oxidation in dry oxygen. The oxidation rate

(a) is independent of current oxide thickness and temperature.

(b) is independent of current oxide thickness but depends on temperature.

(c) slows down as the oxide grows.

(d) is zero as the existing oxide prevents further oxidation. **[2008]**

Solution: (d)

The oxide layer of 100 nm acts as an insulator for further oxidation.

Hence, the correct option is (d).

22. The electron and hole concentrations in an intrinsic semiconductor are n_i per cm^3 at 300 K. Now, if acceptor impurities are introduced with a concentration of N_A per cm^3 (where $N_A >> n_i$), the electron concentration per cm^3 at 300 K will be

(a) n_i (b) $n^i + N^A$

(c) $N_A - n_i$ (d) $\frac{n_i^2}{N_A}$ **[2007]**

Solution: (d)

$$n = \frac{n_i^2}{N_A}$$

Hence, the correct option is (d).

23. The concentration of minority carriers in an extrinsic semiconductor under equilibrium is

(a) directly proportional to the doping concentration.

(b) inversely proportional to the doping concentration.

(c) directly proportional to the intrinsic concentration.

(d) inversely proportional to the intrinsic concentration. **[2006]**

Solution: (b)

$$\text{Minority concentration} = \frac{n_i^2}{\text{Majority concentration}}$$

$\therefore$ Minority concentration is inversely proportional to majority concentration.

Hence, the correct option is (b).

24. Under low-level injection assumption, the injected minority carrier current for an extrinsic semiconductor is essentially the

(a) diffusion current (b) drift current

(c) recombination current (d) induced current **[2006]**

Solution: (a)

The current is by diffusion as there is concentration gradient due to low-level injection.

Hence, the correct option is (a).

25. The band gap of silicon at room temperature is

(a) 1.3 eV (b) 0.7 eV

(c) 1.1 eV (d) 1.4 eV **[2005]**

Solution: (c)

At 300 k, band gap of silicon is 1.1 eV.

Hence, the correct option is (c).

26. The primary reason for the widespread use of silicon in semiconductor device technology is

(a) abundance of silicon on the surface of the Earth.

(b) larger band gap of silicon in comparison to germanium.

(c) favourable properties of silicon-dioxide (SiO_2).

(d) lower melting point. **[2005]**

Solution: (a)

Silica is abundant on the Earth. This is one of the major reasons for using Si.

Hence, the correct option is (a).

27. The impurity commonly used for realizing the base region of a silicon *n-p-n* transistor is

(a) Gallium (b) Indium

(c) Boron (d) Phosphorus **[2004]**

Solution: (c)

Boron is used in *n-p-n* transistor.

Hence, the correct option is (c).

28. *N*-type silicon is obtained by doping silicon with

(a) Germanium (b) Aluminium

(c) Boron (d) Phosphorus **[2003]**

Solution: (d)

Phosphorus has excess electrons and thus *n*-type semiconductor is formed.

Hence, the correct option is (d).

29. The band gap of silicon at 300 K is
(a) 1.36 eV (b) 1.10 eV
(c) 0.80 eV (d) 0.67 eV **[2003]**

Solution: (b)

At 300 *k*, band gap of silicon is 1.1eV.

Hence, the correct option is (b).

30. The intrinsic carrier concentration of silicon sample at 300 K is 1.5×10^{16} rn^{-3}. If after doping, the number of majority carriers is $5 \times 10^{20}/m^3$, the minority carrier density is
(a) 4.50×10^{11} m^{-3} (b) 3.33×10^4 m^{-3}
(c) 5.00×10^{20} m^{-3} (d) $3.00 \times 10 \times^{-5}$ m^{-3}
[2003]

Solution: (a)

$n_i = 1.5 \times 10^{16}$ m^{-3}

Let majority carrier be represented by N.

$N = 5 \times 10^{20}$ m^{-3}

$$\therefore \text{ Minority carrier density } = \frac{n_i^2}{N} = \frac{2.25\times10^{32}}{5\times10^{20}}$$

$= 4.5 \times 10^{11}$m^{-3}

Hence, the correct option is (a).

31. The units of q/kT are
(a) V (b) V–^{1}
(c) J (d) J/K **[1997]**

Solution: (b)

$q/kT = 1/V_T$ (V_T= thermal voltage)

q/kT has the units V^{-1}.

Hence, the correct option is (b).

32. The drift velocity of electrons in silicon:
(a) is proportional to the electric field for all values of electric field
(b) is independent of the electric field.
(c) increases at low values of electric field and decreases at high values of electric field exhibiting negative differential resistance.
(d) increases linearly with electric field at low values of electric field and gradually saturates at higher values of electric field. **[1995]**

Solution: (d)

Drift velocity increases linearly with electric field at low values, sub-linearly and at medium values and gets saturated at high values.

Hence, the correct option is (d).

33. The probability that an electron in a metal occupies the Fermi level at any temperature (> K) is:
(a) 0 (b) 1
(c) 0.5 (d) 1.0 **[1995]**

Solution: (c)

The asked probability is 0.5.

Hence, the correct option is (c).

34. In a *p*-type Si simple, the hole concentration is 2.25×10^{15} cm^{-3}. The intrinsic carrier concentration is 1.5×10^{-10} cm^{-3}, and the electron concentration is
(a) Zero (b) 10^{10} cm^{-3}
(c) 10^5 cm^{-3} (d) 1.5×10^{25} cm^{-3}
[1995]

Solution: (c)

$n_i^2 = n*p$ [as per mass action law]

$$\therefore n = \frac{n_i^2}{P} = \frac{2.25\times10^{20}}{2.25\times10^{15}} = 10^5 \text{ atoms cm}^{-3}$$

35. A small concentration of minority carries is injected into a homogeneous semiconductor crystal at one point. An electric field of 10 V/cm is applied across the crystal and this moves the minority carriers a distance of 1 cm in 20 μsec. The mobility (in cm^2 V-sec) will be
(a) 1,000 (b) 2,000
(c) 5,000 (d) 500,000 **[1995]**

Solution: (c)

$E = 10$ V/cm; $d = 1$ cm $t = 20$ μ sec.

$$V^d = \frac{d}{t} = \frac{1}{20\times10^{-6}} = \frac{10^6}{20}\text{cm/sec}.$$

$$\mu = \frac{V^d}{E} = \frac{10^6}{20\times10} = \frac{10^5}{20} = 5000 \text{ cm}^2\text{/V-sec}$$

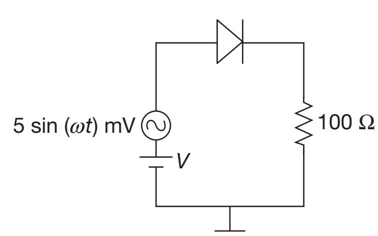
36. A *p*-type silicon sample has a higher conductivity compared to an *n*-type silicon sample having the same dopant concentration. Is it True or False? **[1994]**

Solution: False

No, it is false because *n*-type sample will be having a large number of elections whose mobility is high as compared to holes.

TWO-MARKS QUESTION

1. A dc current of 26 μA flows through the circuit shown. The diode in the circuit is forward biased and it has an ideality factor of one. At the quiescent point, the diode has a junction capacitance of 0.5 nF. Its neutral region resistances can be neglected. Assume that the room temperature thermal equivalent voltage is 26 mV.



For $\omega = 2 \times 10^6$ rad/s, the amplitude of the small-signal component of diode current (in μA, correct to one decimal place) is _______. **[2018]**

Solution:

Voltage $V_T = 26$ mV

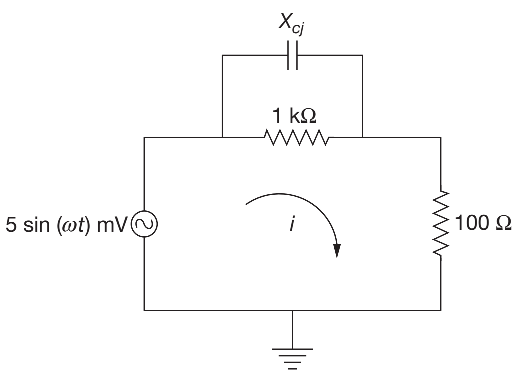
Current $I_D = 2.6$ μA

Resistance

$$r_d = \frac{V_T}{I_D} = \frac{26 \text{ mA}}{26 \text{ μA}} = 1 \text{ K}\Omega$$

Impedance

$$X_{cj} = \frac{1}{\omega_{cj}} = 1 \text{ k}\Omega$$



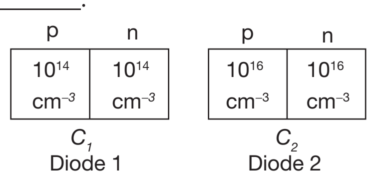
$$Z = \left(r_d \,||\, \frac{1}{j\,\omega_{cj}} \right) + 100\ \Omega = 600 - j500\ \Omega$$

The amplitude of the small signal component of diode is

$$\frac{5}{100\sqrt{61}} = 6.4 \text{ μA}$$

Hence, the correct answer is 6.2 to 6.6.

2. As shown, two silicon (Si) abrupt p-n junction diodes are fabricated with uniform donor doping concentrations of $N_{D_1} = 10^{14}$ cm^{-3} and $N_{D_2} = 10^{16}$ cm^{-3} in the n-regions of the diodes, and uniform acceptor doping concentrations of $N_{A_1} = 10^{14}$ cm^{-3} and $N_{A_2} = 10^{16}$ cm^{-3} in the p-regions of the diodes, respectively. Assuming that the reverse bias voltage is >> built-in potentials of the diodes, the ratio C_2/C_1 of their reverse bias capacitances for the same applied reverse bias, is ____________. **[2017]**



Solution: We know s $C = \frac{\varepsilon A}{W}$

$$W = \sqrt{\frac{2\varepsilon_s}{q}\left[\frac{1}{N_A} + \frac{1}{N_D}\right] V_j}$$

$$\therefore C_T \alpha \frac{1}{W}$$

$$\frac{C_{T_1}}{C_{T_2}} = \frac{W_2}{W_1}$$

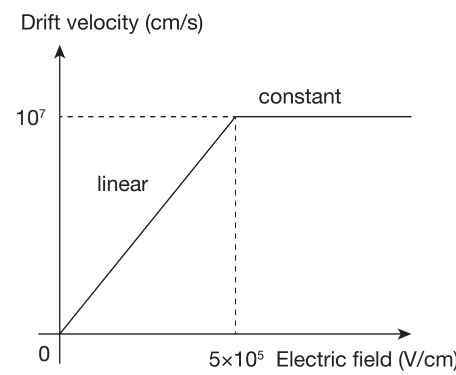
$$\frac{C_{T_1}}{C_{T_2}} = \frac{\sqrt{\frac{1}{10^{16}} + \frac{1}{10^{16}}}}{\sqrt{\frac{1}{10^{14}} + \frac{1}{10^{14}}}}$$

$$= \sqrt{\frac{10^{14}}{10^{16}}} = \frac{1}{10}$$

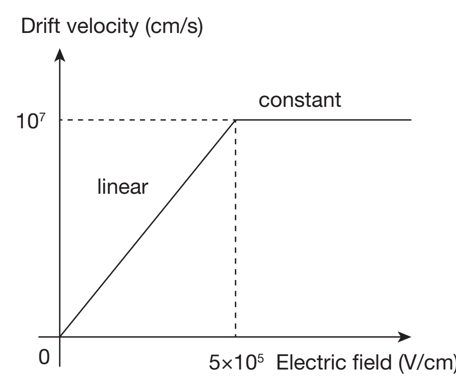
$$\therefore C_{T_2} = 10.C_{T_1}$$

Hence, the correct answer is (10).

3. The dependence of drift velocity of electrons on electric field in a semiconductor is shown below. The semiconductor has a uniform electron concentration of $n = 1 \times 10^{16}$ cm^{-3} and electronic charge $q = 1.6 \times 10^{-19}$C. If a bias of 5 V is applied across a 1 μm region of this semiconductor, the resulting current density in this region, in kA/cm^2, is________. **[2017]**



Solution:



Form the given data

$V = 5$ Volts, $L = 1$ μm

$$E = \frac{V}{L} = 5\times10^6 \text{ V/m.} = 5\times10^4 \text{ V/cm}$$

We Know

$$V_d = \mu E$$

$$J = n.q.V_d$$

from the given data

$$\mu = m = \frac{10^7 - 0}{5\times10^5 - 0} = 20$$

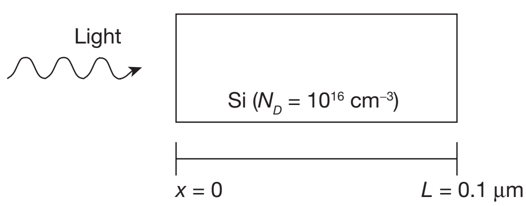
$$\therefore J = 1\times10^{16} \times 1.6\times10^{-19} \times 20 \times 5\times10^{14}$$

$$= 0.032 \times 5\times10^4 = 1.6\times10^3 \text{ A/cm}^2$$

$$J = 1.6 \text{ KA/cm}^2$$

Hence, the correct answer is (1.5 to 1.7).

4. As shown, a uniformly doped silicon (Si) bar for length $L = 0.1$ µm with a donor concentration $N_D = 10^{16}$ cm^{-3}, is illuminated at $x = 0$ such that electron and hole pair are generated at the rate of $G_L = G_{L0}\left(1 - \frac{x}{L}\right), 0 \le 0X \le L,$ where $G_{L0} = 10^{17}$ cm^{-1}s^{-1}. Hole lifetime is 10^{-4} s, electronic charge $q = 1.6 \times 10^{-19}$ C, hole diffusion coefficient $D_p = 100$ cm²/s and low level injection condition prevails. Assuming a linearly decaying steady state excess hole concentration that goes to 0 at $x = L$, the magnitude of the diffusion current density at $x = L/2$, in A/cm², is________. **[2017]**



Solution: From the given data

$$N_D = 10^{16} \text{ cm}^{-3}$$

$$L = 0.1\ \mu\text{m} = 1\times10^{-5} \text{ cm}$$

$$G_L = G_{LO}\left(1 - \frac{x}{L}\right), 0 \le x \le L$$

$$G_{LO} = 10^{17} \text{ cm}^{-3}\text{s}^{-1}$$

$$\tau_p = 10^{-4}\text{ s}$$

$q = 1.6 \times 10^{-19}$c and $D_p = 100$ cm²/sec

$$J_p = ?$$

$$P = P_0 + \Delta P$$

$$G_L = \frac{\Delta P}{\tau_p}$$

$$\Delta P = G_L.\tau_p$$

$$P = P_{no} + G_{Lo}.\tau_P\left(1 - \frac{x}{L}\right)$$

$$J_{p.diff} = -q.D_p.\frac{\partial p}{\partial x}$$

$$J_{p.diff} = -q.D_p.\frac{\partial}{\partial x}\left\{p_{no} + G_{Lo}.\tau_P\left(1 - \frac{x}{L}\right)\right\}$$

$$J_{p.diff} = -q.D_p.G_{L0}.\tau_p\frac{\partial}{\partial x}\left\{\left(1 - \frac{x}{L}\right)\right\}$$

$$\therefore J_{P.diff} = \frac{q.D_p.\tau_p.GL_0}{L}$$

$$J_{P.diff} = \frac{1.6\times10^{-19}\times100\times10^{-4}\times10^{17}}{0.1\times10^{-4}}$$

$$J_{P.diff} = \frac{1.6\times10^{-3}}{10^{-4}} = 16 \text{ A/cm}^2$$

Hence, the correct answer is (15.9 to 16.1).

5. A MOS capacitor is fabricated on p-type Si (Silicon) where the metal work function is 4.1 eV and electron affinity of Si is 4,4 eV. $E_c - E_F = 0.9$ eV, where E_c and E_F are the conduction band minimum and the Fermi energy levels of Si, respectively. Oxide $\in_r = 3.9$. $\in_o = 8.85 \times 10^{-14}$ F/cm, oxide thickness $t_{ox} = 0.1$ µm and electronic charge $q = 1.6 \times 10^{-19}$ C. If the measured flat band voltage of this capacitor is −1 V, then the magnitude of the fixed charge at the oxide-semiconductor interface, in nC/cm², is _________. **[2017]**

6. An electron (q_1) is moving in free space with velocity 10^5 m/s towards a stationary electron (q_2) far away. The closest distance that this moving electron gets to the stationary electron before the repulsive force diverts its path is________ × 10^{-8} m.

[Given, mass of electron $m = 9.11 \times 10^{-31}$ kg, charge of electron $e = -1.6 \times 10^{-19}$ C, and permittivity $\varepsilon_0 = (1/36\pi)\times10^{-9}$ F/m] **[2017]**

Solution:

$$q_1 \xrightarrow{V=10^5 \text{ m/s}} \longrightarrow r \longleftarrow$$

• moving • stationary

$$\frac{1}{2}\text{mv}^2 = \left(\frac{q_1q_2}{4\pi\varepsilon_0 r}\right) \Rightarrow \text{Force Per Distance}$$

$$r = \frac{2\times-1.6\times10^{-19}\times-1.6\times10^{-19}}{4\pi\times\frac{10^{-9}}{36\pi}\times9.11\times10^{-31}\times(10^5)^2}$$

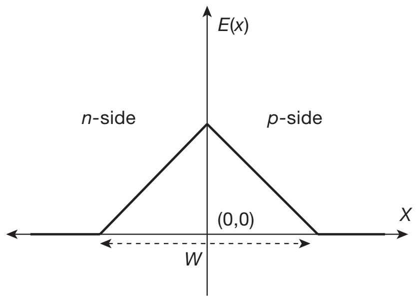
$$\Rightarrow \quad r = 5.058 \times 10^{-8}\text{ m}$$

Hence, the correct answer is (4.55 to 5.55).

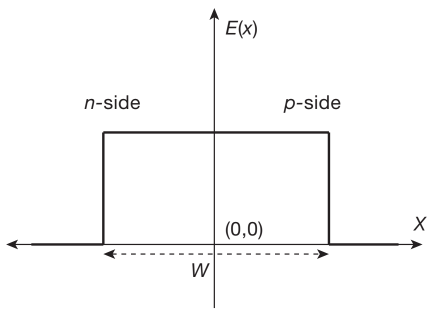
7. An abrupt pn junction (located at $x = 0$) is uniformly doped on both p and n sides. The width of the depletion region is W and the electric field variation in the

x-direction is $E(x)$. Which of the following figures represents the electric field profile near the pn junction? **[2017]**

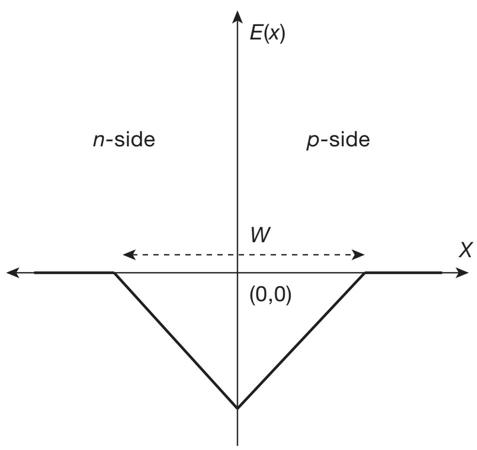
(A)



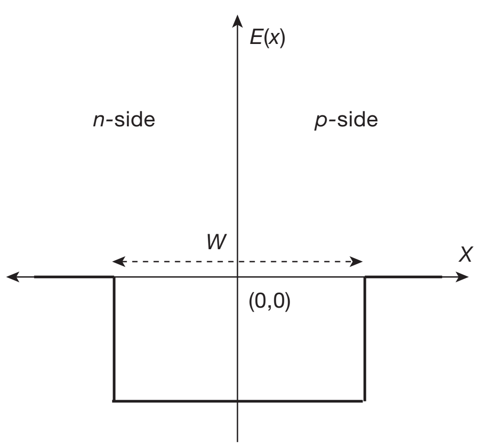
(B)



(C)



(D)



8. Consider a silicon p–n junction with a uniform acceptor doping concentration of 10^{17} cm^{-3} on the p side and a uniform donor doping concentration of 10^{16} cm^{-3} on the n – side. No external voltage is applied to the diode Given: $kT/q = 26$ mV, $n_i = 1.5 \times 10^{10}$ cm^{-3}, $\varepsilon_{si} = 12\varepsilon_0$, $\varepsilon_0 = 8.85 \times 10^{-14}$F/cm, an and $q = 1.6 \times 10^{-19}$ C. The charge per unit junction area (nC cm^{-2}) in the depletion region on the p–side is ______. **[2016]**

Solution: Number density of acceptor atoms $N_A = 10^{17}$ cm^{-3}

Charge $q = 1.6\ 10^{-19}$ C

Number density of donor atoms $N_D = 10^{16}$ atoms cm^{-3}

Number density of intrinsic charge carriers $n_i = 1.5 \times 10^{10}$ cm^{-3}

Voltage $V_T = 26$ mV

$$\varepsilon_{si} = 12\varepsilon_o$$

$$W = \sqrt{\frac{2\varepsilon_s}{q}\left[\frac{1}{N_A}+\frac{1}{N_D}\right]\cdot V_j}$$

$$C_T = q \cdot N_A \cdot A \cdot \left|\frac{dw}{dv}\right|$$

$$Q_p = -q \cdot N_A \,.\, \mathrm{W_p}\, \mathrm{A}$$

$$V_j = V_T \ell_n \left[\frac{N_A \cdot N_D}{n_i^2}\right] V.$$

$$\frac{Q_p}{A} = -q \cdot N_A \cdot w_p$$

$$V_j = 26\times10^{-3}\times\ln\left[\frac{10^{17}\times10^{16}}{2.25\times10^{20}}\right]$$

$$= 26\times10^{-3}\times\ln\left[4.44\times10^{12}\right]$$

$$= 0.757 \text{ V}$$

$$W = \sqrt{\frac{2\times12\times8.85\times10^{-14}}{1.6\times10^{-19}}\left[\frac{1}{10^{17}}+\frac{1}{10^{16}}\right]\times 0.757}$$

$$W = \sqrt{1.1054\times10^{-9}}$$

$$W = 3.324\ \times10^{-5} \text{ cm}$$

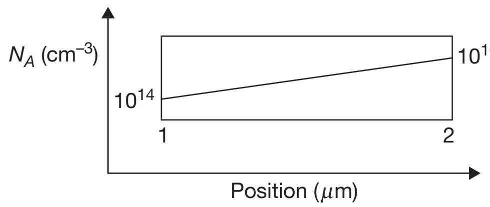
$$W_p = \frac{N_D}{N_A+N_D}\cdot W = 3.022 \times 10^{-6} \text{ cm.}$$

$$\frac{Q}{A} = -1.6\times10^{-19}\times10^{17}\times3.022\times10^{-6}$$

$$= -48.36 \text{ nC/cm}^2.$$

Hence, the correct Answer is (–48).

9. The figure shows the doping distribution in a p type semiconductor in log scale.



The magnitude of the electric field (in kV/cm) in the semiconductor due to non uniform doping is ______. **[2016]**

Solution: Now using the relation

$$q \cdot D_P \cdot \frac{\partial P}{\partial X} = q \cdot \mu_P \cdot \text{PE}$$

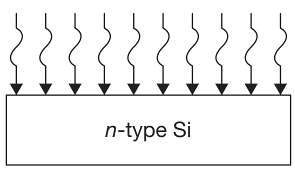
$$P = N_A$$

$$E = \frac{V_T}{N_A} \cdot \frac{\partial P}{\partial X}$$

$$\Rightarrow \quad \varepsilon = V_T \cdot \frac{d}{dx} \ln \{N_A(X)\}$$

Hence, the correct Answer is (1010 to 1.25).

10. Consider a silicon sample at $T = 300$ K, with a uniform donor density $N_d = 5 \times 10^{16}$ cm^{-3}, illuminated uniformly such that the optical generation rate is $G_{opt} = 1.5 \times 10^{20}$ cm^{-3}s^{-1} throughout the sample. The incident radiation is turned off at $t = 0$. Assume low level injuection to be valid and ignore surface effects. The carrier lifetimes are $\tau_{po} = 0.1$ µs and $\tau_{no} = 0.5$ µs.



The hole concentration at $t = 0$ and the hole concentration at $t = 0.3$ µs, respectively, are **[2016]**

(A) 1.5×10^{13} cm^{-3} and 7.47×10^{11} cm^{-3}
(B) 1.5×10^{13} cm^{-3} and 8.23×10^{11} cm^{-3}
(C) 7.5×10^{13} cm^{-34} and 3.73×10^{11} cm^{-3}
(D) 7.5×10^{13} cm^{-3} and 4.12×10^{11} cm^{-3}

Solution: Donor density $N_d = 5 \times 10^{16}$ cm^{-3}
Optical generation rate $G_{opt} = 1.5 \times 10^{20}$ cm^{-3} s^{-1}
Carrier lifetimes $\tau_{po} = 0.1$ µsec
Carrier lifetimes $\tau_{no} = 0.5$ µs
Now using the relation

$$Na = \frac{n_i^2}{N_d} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{16}}$$

$$= 4.5 \times 10^3 \text{ atoms/cm}^3$$

$$\text{Generation rate} = \frac{\delta\rho}{\tau_{\rho o}}$$

$$\Delta p = G_{opt} \times \tau_{po} = 1.5 \times 10^{20} \times 0.1\ \mu$$

$$\Delta p = 1.5 \times 10^{13} \text{ cm}^{-3}$$

$$P_t = p_0 \cdot \exp(-t/\tau_p)$$

$$= 1.5 \times 10^{13} \cdot \exp\left(\frac{-0.3 \times 10^{-6}}{0.1 \times 10^{-6}}\right)$$

$$= 7.47 \times 10^{11} \text{ cm}^{-3}$$

Hence, the correct option is (A).

11. A voltage V_G is applied across a MOS capacitor with metal gate and p-type silicon substrate at $T = 300k$. The inversion carrier density (in number of carriers per unit area) for $V_G = 0.8$ V is 2×10^{11} cm^{-2}. For $V_G = 1.3$ V, the inversion carrier density is 4×10^{11} cm^{-2}. What is the value of the inversion carrier density for $V_G = 1.8$ V? **[2016]**

(A) 4.5×10^{11} cm^{-2}
(B) 6.0×10^{11} cm^{-2}
(C) 7.2×10^{11} cm^{-2}
(D) 8.4×10^{11} cm^{-2}

Solution: The following relation can be given for computing the value of inversion carrier density.

$$Q_{inv} = K(V_{GS} - V_T),\ V_{GS} > V_t$$

$$|Q_{inv}| = q \cdot Ni$$

$$N_i \rightarrow \text{Inversion carrier density}$$

from the given data

(i) $K(0.8 - V_T)) = q \times 2 \times 10^{11}$
(ii) $K(1.3 - V_T) = q \times 2 \times 10^{11}$

From (i) and (ii)

$$\frac{1}{2} = \frac{0.8 - V_T}{1.3 - V_T}$$

$$V_T = 0.3 \text{ V}$$

and

$$K = 4.q \times 10^{11}$$

$$1.6 \times 10^{-19} \times N_i = 4 \times 10^{11} \times 1.6 \times 10^{-19} \times 1.5$$

$$N_i = 6 \times 10^{11} \text{ cm}^{-2}$$

Hence, the correct option is (B).

12. Consider the avalanche breakdown in a silicon P + N junction. The n–region is uniformly doped with a donor density N_D. Assume that breakdown occurs when the magnitude of the electric field at any point in the device becomes equal to the critical field E_{crit}. Assume E_{crit} to be independent of N_D. If the built in voltage of the P + N junction is much smaller than the breakdown voltage V_{BR}, The relationship between V_{BR} and N_D is given by, **[2016]**

(A) $V_{BR} \times \sqrt{N_D}$ = constant

(B) $N_D \times \sqrt{V_{BR}}$ = constant

(C) $N_D \times V_{BR}$ = constant

(D) $\dfrac{N_D}{V_{BR}}$ = constant

Solution: $W = \sqrt{\dfrac{2\varepsilon}{q}\left(\dfrac{1}{N_A}+\dfrac{1}{N_D}\right)Vo}$

From above relation we conclude that

$$W \propto \text{doping}$$

and

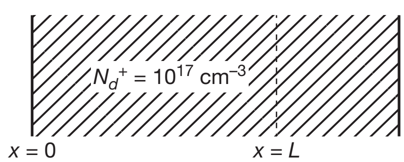
$$W \propto \text{breakdown voltage}$$

$$\text{Break down voltage} \propto \frac{1}{\text{doping}}$$

$$V_{BR} \times N_D = \text{constant}$$

Hence, the correct option is (C).

13. Consider a region of silicon devoid of electrons and holes, with an ionized donor density of $N_a^+ = 10^{17}\ \text{cm}^{-3}$. The electric field at $x = 0$ is 0 V/cm and the electric field at $x = L$ is 50 kV/cm in the positive x-direction. Assume that the electric field is zero in the y and z directions.



Given q = 1.6 × 10^{-19} coulomb, $\in_0$ = 8.85 × 10^{-14} F/cm, $\in_r$ = 11.7 for silicon, the value of L in nm is ____. **[2016]**

Solution: Number density $N_d = 10^{17}\text{cm}^{-3}$

For silicon $\in_r = 11.7$

Permittivity of free space $\in_0 = 8.85 \times 10^{-14}$ F/cm

Now we know relation between field and potential is

$$E = \frac{V}{x}$$

Also $\quad E = 0 \quad \text{at} \quad x = 0$

$$E = 5\ 0\text{KV/cm} \quad \text{at} \quad x = L$$

$$\frac{V}{L} = 50\ \text{KV/cm}$$

$$L = \frac{V}{50\ \text{KV}}$$

$$W = \sqrt{\frac{2\varepsilon}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)Vo}$$

and (but it is devoid of electrons therefore)

$$E = \frac{q.Nd.W}{\in_s}$$

$$W = L = \frac{E \in_S}{q.N_d}$$

$$= \frac{50\times10^3\times11.7\times8.85\times10^{-14}}{1.6\times10^{-19}\times10^{17}}$$

$$= 32.35\ \text{nm.}$$

Hence, the correct Answer is (32.35 nm).

14. The I–V characteristics of the zener diodes D_1 and D_2 are shown in figure I. These diodes are used in the circuit given in figure II. If the supply voltge is varied from 0 to 100 V, then the breakdown occurs in **[2016]**

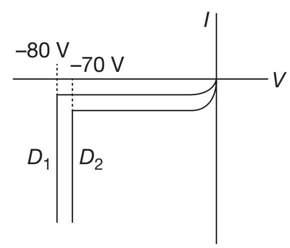


Figure I

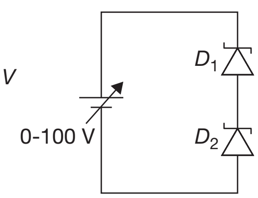


Figure II

(A) D_1 only
(B) D_2 only
(C) both D_1 and D_2
(D) none of D_1 and D_2

Solution: Voltage V_{z1} = 80 V

Voltage V_{z2} = 70 V

$V_{in} \rightarrow$ 0 to 100 V only

When we will vary the $V_{in} > 80$ V

$D_1 \rightarrow$ break down.

When 0 < V_{in} < 70, none of the Zener diode reaches breakdown but when 70 < V_{in} < 80, only D_1 and D_2 will conduct but D_2 will not break down. When V_{in} > 80 the diode D_1 reaches breakdown. Hence, option (A) is correct because the max voltage across it will remain 80 V which will not decrease to cause the breakdown of D_2.

Hence, the correct option is (A).

15. Consider a long channel NMOS transistor with source and body connected together. Assume that the electron mobility is independent of V_{GS} and V_{DS}. Given,

$$g_m = 0.5\ \mu\text{A/V}$$

for $\quad V_{DS} = 50$ mV $\quad$ and $\quad V_{GS} = 2$ V,

$$g_d = 8\ \mu\text{A/V}$$

for $\quad V_{GS} = 2$ V $\quad$ and $\quad V_{DS} = 0$ V, Where,

$$g_m = \frac{\partial I_D}{\partial V_{GS}} \quad \text{and} \quad g_d = \frac{\partial I_D}{\partial V_{DS}}.$$

The threshold voltage (in volts) of the transistor is ______. **[2016]**

Solution: We are given that

$$g_m = 0.5\ \mu A/V$$

for $V_{DS} = 50$ mV and $V_{GS} = 2$ V

$$g_d = 8\ \mu A/V$$

for $V_{GS} = 2V$ and $V_{DS} = 0V)$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} \quad \text{and} \quad g_d = \frac{\partial I_D}{\partial V_{DS}}$$

(for NMOS operating in active region)

$$I_D = K^1\left[(V_{GS} - V_T)\cdot V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

$$g_d = \frac{\partial I_D}{\partial V_{DS}} = K^1\{(V_{GS} - V_T)\cdot 1 - V_{DS}\}$$

Substituting the given values, we get

$$8 \times 10^{-6} = K^1\ \{(2 - V_T) - 0\}$$

$$K^1 = \frac{8\times 10^{-6}}{2 - V_T} \qquad \text{(i)}$$

Also we have,

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = K^1\{(1-0)\cdot V_{DS} - 0\} = K^1 \cdot V_{DS}$$

$$K^1\ V_{DS} = 0.5 \times 10^{-6}$$

$$K^1 = \frac{0.5\times 10^{-6}}{50\times 10^{-3}} \qquad \text{(ii)}$$

From Eq. (i) and Eq. (ii) we get

$$\frac{8\times 10^{-6}}{2 - V_t} = \frac{0.5\times 10^{-6}}{50\times 10^{-3}} \quad 2 - V_T = \frac{8\times 50\times 10^{-3}}{0.5}$$

$$V_T = 2 - 0.8 = 1.2 \text{ Volts}$$

Hence, the correct Answer is (1.2 Volts).

16. An n-type silicon sample is uniformly illuminated with light which generates 10^{20} electron-hole pairs per cm^3 per second. The minority carrier lifetime in the sample is 1 µs. In the steady state, the hole concentration in the sample is approximately 10^x, where x is an integer. The value of x is ______. **[2015]**

Solution: From the given data n-type Si sample

e-h pair generation = 10^{20} pairs/cm^{-3}/s = Δg

$$\tau_p = 1\ \mu s$$

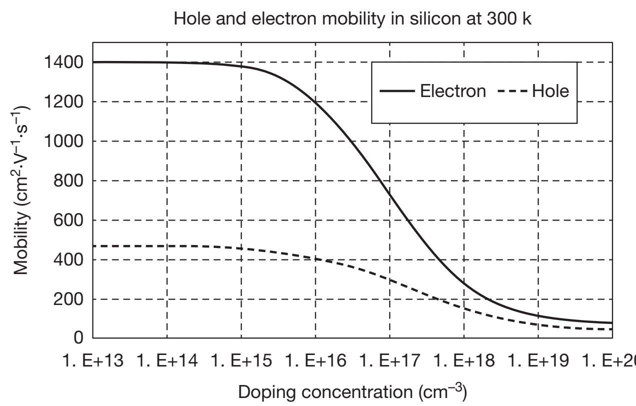
$$\Rightarrow \quad \Delta g = \frac{\Delta p}{\tau_p}$$

$$\therefore \quad \Delta_p = \Delta_g \cdot \tau_p = 10^{20} \times 10^{-6} = 10^{14} = 10^x$$

$$\therefore \quad x = 14$$

Hence, the correct Answer is (14).

17. A piece of silicon is doped uniformly with phosphorous with a doping concentration of 10^{16}/cm^3. The expected value of mobility versus doping concentration for silicon assuming full dopant ionization is shown below. The charge of an electron is 1.6×10^{-19} C. The conductivity (in S cm^{-1}) of the silicon sample at 300 K is ______. **[2015]**



Solution: From the given data

$$N_D = 10^{16}\ \text{cm}^{-3}$$

$$q = 1.6 \times 10^{-19}\ \text{C}$$

from the graph at $N_D = 10^{16}$ cm^{-3}

$$\mu = 1200\ \text{cm}^2/\text{V-sec}$$

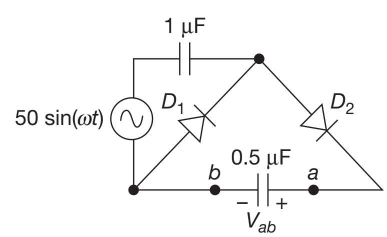
$$\therefore \quad \sigma = N_D \cdot q \cdot \mu_n$$

$$= 10^{16} \times 1.6 \times 10^{-19} \times 1200$$

$$\sigma = 1.92\ \text{S/cm}$$

Hence, the correct Answer is (1.8 to 2.0).

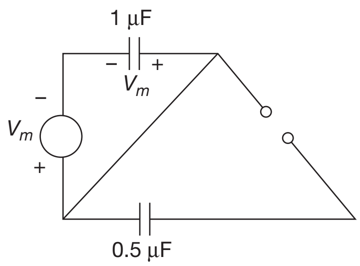
18. In the circuit shown, assume that diodes D_1 and D_2 are ideal. In the steady state condition, the average voltage V_{ab} (in Volts) across the 0.5 µF capacitor is ______. **[2015]**



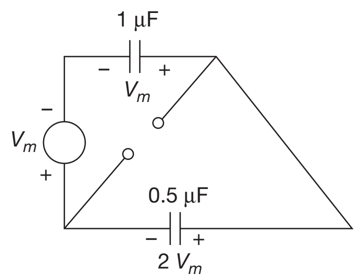
Solution: Given circuit is voltage doubler.

For first half cycle

D_1 is on and D_2 is off



For next half cycle.



So $\quad V_{ab} = 2 \times 50$

$= 100$ V

19. A silicon sample is uniformly doped with donor type impurities with a concentration of 10^{16}/cm^3. The electron and holemobility in the sample are 1200 cm^2/V-s and 400 cm^2/V-s respectively. Assume complete ionization of impurities. The charge of an electron is 1.6×10^{-19} C. The resistivity of the sample (in Ω-cm) is ______. **[2015]**

Solution: From the given data

$$N_D = 10^{16} \text{ cm}^{-3}$$

$$\mu_n = 1200 \text{ cm}^2/\text{V-s}$$

$$\mu_p = 400 \text{ cm}^2/\text{V-s}$$

$$q = 1.6 \times 10^{-19} \text{ C}$$

$$\rho = ?$$

$$\rho = 1/\sigma$$

$$\sigma = (n \cdot \mu_n + p\, \mu_p)q$$

But $\quad n >> p$ (n-type)

$$\sigma_n \approx n\, \mu_n \cdot q$$

$$n = N_D$$

$$\sigma_n \approx N_D \cdot \mu_n \cdot q$$

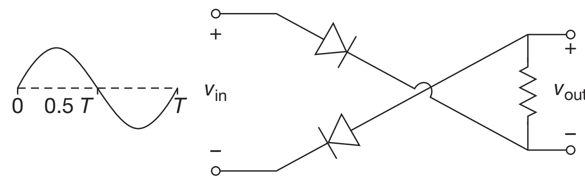
$$\sigma_n = 10^{16} \times 1200 \times 1.6 \times 10^{-19}$$

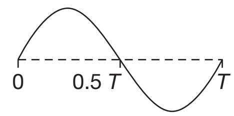
$$= 1.92\ \mho/\text{cm}$$

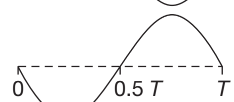
$$\therefore \quad \rho = \frac{1}{1.92} = 0.52\ \Omega\text{-cm}$$

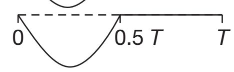
Hence, the correct Answer is (0.50 to 0.54).

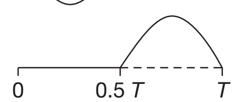
20. For the current with ideal diodes shown in the figure, the shape of the output (v_{out}) for the given sine wave input (v_{in}) will be **[2015]**



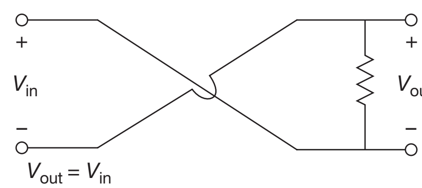
(A) 

(B) 

(C) 

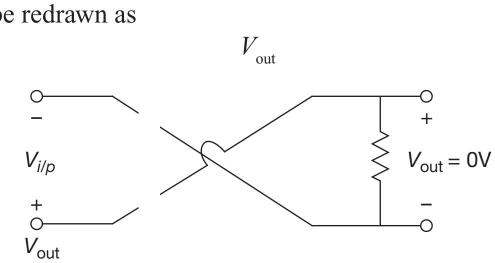
(D) 

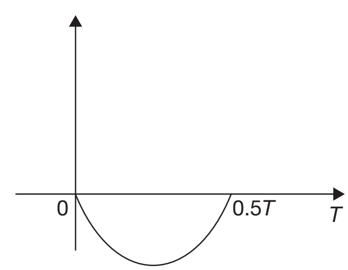
Solution: Where positive input cycle is applied then circuit can be redrawn as



$$V_{\text{out}} = -V_{\text{in}}$$

When negative input cycle is applied then circuit can be redrawn as





Hence, the correct option is (C).

21. The doping concentrations on the p-side and n-side of a silicon diode are 1×10^{16} cm^3 and 1×10^{17}cm^{-3}, respectively. A forward bias of 0.3 V is applied to the diode. At T = 300 K, the intrinsic carrier concentration of silicon $n_i = 1.5 \times 10^{10}$cm^{-3}

and $\frac{kT}{q} = 26\ mV$. The electron concentration at the edge of the depletion region on the p-side is

(a) 2.3×109 cm^{-3} (b) 1×10^{16} cm
(c) 1×10^{17} cm (d) 2.15×10^{6}cm^{-3} **[2014]**

Solution: (a)

$$n^{po} e \frac{\mathrm{V}}{kt}$$

where $n_{p0} = \frac{n_i^2}{NA}$ in p-type

$$\therefore n_{p0} = \frac{2.25 \times 10^{20}}{10^{16}} = 2.25 \times 10^4$$

$$n_p = 2.25 \times 10^4 e^{\frac{0.3}{26 \times 10^{-3}}}$$

$n_p = 2.306 \times 10^9$ cm^{-3}

Hence, the correct option is (a).

22. Assume electronic charge $q = 1.6 \times 10^{-19}$C, $kT/q = 25$ mV and electron mobility $\mu n = 1000$ cm^2/V-s. If the concentration gradient of electrons injected into a p-type silicon sample is 1×10^{21} cm^4, the magnitude of electron diffusion current density (in A/cm^2) is **[2014]**

Solution: (4000)

$$J = qDn \frac{dn}{dx}$$

$$\frac{Dn}{\mu n} = V_T \Rightarrow D_n = V_T \mu n$$

$$J = q\mu_n V_T \frac{dn}{dx} = 1.6 \times 10^{-19} \times 1000 \times 25 \times 10^{-3} \times 10^{21}$$

$J = 4000$ A/cm^2

23. When a silicon diode having a doping concentration of $N_A = 9 \times 10^{16}$ cm^{-3} on p-side and $N_D = 1 \times 10^{16}$ cm^{-3} on n-side is reverse biased, the total depletion width is found to be 3 µm. Given that the permittivity of silicon is 1.04×10^{-12} F/cm, the depletion width on the p-side and the maximum electric field in the depletion region, respectively, are

(a) 2.7 µm and 2.3×10^5 V/cm
(b) 0.3 µm and 4.15 × 105 V/cm
(c) 0.3 µm and 0.42 × 105 V/cm
(d) 2.1 µm and 0.42 × 105 V/cm **[2014]**

Solution: (b)

$W_P N_A = W_N N_D$

Let $W_N = x.\ \mu m$

then $W_P = (3 - x)$ µm

$$(3-x) \times 9 \times \cancel{10^{16}} = x \times \cancel{10^{16}}$$

$$27 - 9x = x \quad x = 2.7\ \mu am$$

$$\therefore \begin{array}{l} W_N = 2.7\ \mu\text{m} \\ W_P = 0.3\ \mu\text{m} \end{array}$$

$$E_{g\,max} = \frac{q}{\varepsilon} W_N N_D$$

$$= \frac{1.6 \times 10^{-19}}{1.04 \times 10^{-12}} \times 2.7 \times 10^{-4} \times 10^{16} = 4.15 \times 10^5 \text{ V/cm}$$

24. Consider a silicon sample doped with $N_D = 1 \times 10^{15}$ cm^{-3} donor atoms. Assume that the intrinsic carrier concentration $n_i = 1.5 \times 10^{10}$ cm^{-3}. If the sample is additionally doped with $N_A = 1 \times 10^{18}$ cm^{-3} acceptor atoms, the approximate number of electrons cm^{-3} in the sample, at $T = 300$ K, will be_____.

[2014]

Solution: 225.22

$N_D = 10^{15}$ cm^{-3}

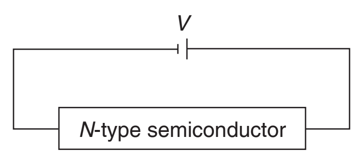
$N_A = 10^{18}$ cm^{-3}

$\therefore$ Net holes $= (10^{18} - 10^{15})$ cm^{-3}

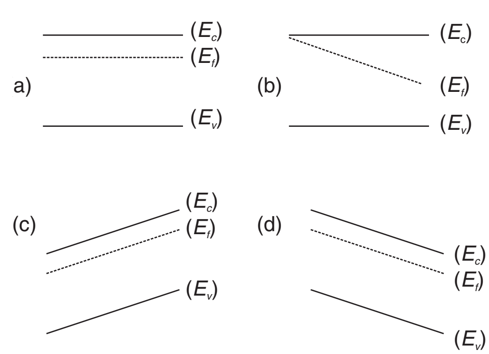
$= 999 \times 10^5$ cm^{-3}

$$\text{Now } n = \frac{n_i^2}{p} = \frac{2.25 \times 10^{20}}{999 \times 10^{15}} = 225.22 \text{ cm}^{-3}$$

25. An N-type semiconductor having uniform doping is biased as shown in the figure



If E_c is the lowest energy level of the conduction band, E_v is the highest energy level of the valance band and E_F is the Fermi level, which one of the following represents the energy band diagram for the biased N-type semiconductor?



[2014]

Solution: (c)

n-type SC E_f lies near to conduction band.

Hence, the correct option is (c)

26. The magnitude of the electric field at $x = 0.5$ μm is
(a) 1 kV/cm (b) 5 kV/cm
(c) 10 kV/cm (d) 26 kV/cm **[2010]**

Solution: (c)

$E = V/x$

At $x = 0.5$ μm $V = 0.5$ V

$$\therefore E = \frac{0.5}{0.5 \times 10^{-4}} = 10 \text{ kV/cm}$$

Hence, the correct option is (c)

27. The magnitude of the electron drift current density at $x = 0.5$ μm is
(a) 2.16×10^4 A/cm^2 (b) 1.08×10^4 A/cm^2
(c) 4.32×10^3 A/cm^2 (d) 6.48×10^2 A/cm^2
[2010]

Solution: (a)

Election drift current = $J.A$

Let A = 1.

$$J = \sigma E = N_D q \mu_n E$$

$J = 10^{16} \times 1.6 \times 10^{-19} \times 1350 \times 10^4 = 2.16 \times 10^4$ A/cm^2

Hence, the correct option is (a)

28. Silicon is doped with boron to a concentration of 4×10^{17} atoms cm^{-3}. Assume the intrinsic carrier concentration of silicon to be 1.5×10^{10} cm^{-3} and the value of kT/q to be 25 mV at 300 K. Compared to undoped silicon, the Fermi level of doped silicon:
(a) goes down by 0.13 eV
(b) goes up by 0.13 eV
(c) goes down by 0.427 eV
(d) goes up by 0.427 eV **[2008]**

Solution: (c)

$N_A = P = 4 \times 10^{17}$ atoms cm^{-3}, $n_i = 1.5 \times 10^{10}$ cm^{-3}

$$\frac{kT}{q} = 25 \text{ mV at } 300 \text{ K}$$

In n- type,

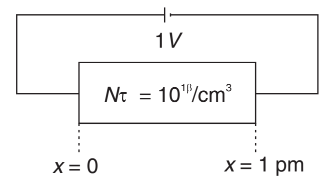
$$\text{Shift } = \frac{KT}{q} \log^e \left(\frac{NA}{n_i} \right) = 25 \times 10^{-3} \log^e \left(\frac{4 \times 10^{17}}{1.5 \times 10^{10}} \right)$$

= 0.427 eV.

Hence, the correct option is (c)

Common Data Questions 9 and 10

The silicon sample with unit cross-sectional areas shown below is in thermal equilibrium. The following information is given: $T = 300$ K, electronic charge = 1.6×10^{-19} C, thermal voltage = 26 mV and electron mobility = 1350 cm^2/V-s.



29. The majority carriers in an n-type semiconductor have an average drift velocity v in a direction perpendicular to a uniform magnetic field B. The electric field E induced due to the Hall effect acts in the direction
(a) $v \times B$ (b) $B \times v$
(c) along v (d) opposite to v **[2006]**

Solution: (b)

$E = B \times v$ according to the Hall Effect.

Hence, the correct option is (b)

30. A heavily doped n-typed semiconductor has the following data:

Hole-electron mobility ratio: 0.4

Doping concentration: 4.2×10^8 atoms/m^3

Intrinsic concentration: 1.5×10^4 atoms/m^3

The ratio of conductance of the n-type semiconductor to that of the intrinsic semiconductor of same material and at the same temperature is given by
(a) 0.00005 (b) 2,000
(c) 10,000 (d) 20,000 **[2006]**

Solution: (d)

$$\frac{\mu_p}{\mu_n} = 0.4 \quad n = 4.2 \times 10^8 \text{ atoms/cm}^3$$

ni = 1.5×10^4 atoms cm^{-3}

$$\frac{\sigma_n}{\sigma_i} = \frac{nq\mu_n}{n_i q(\mu_n p \mu_p)} = \frac{n \cancel{\mu_n}}{n_i \cancel{\mu_n} \left(1 + \frac{\mu_p}{\mu_n}\right)}$$

$$\frac{\sigma_n}{\sigma_i} = \frac{4.2 \times 10^8}{1.5 \times 10^4 (1 + 0.4)} = 20,000$$

Hence, the correct option is (d)

31. A silicon sample A is doped with 10^{18} atoms/ cm^3 of boron. Another sample B of identical dimensions is doped with 10^{18} atoms cm^{-3} of phosphorus. The ratio of electron to hole mobility is 3. The ratio of conductivity of the sample A to B is
(a) 3 (b) 1/3
(b) 2/3 (d) 3/2 **[2005]**

Solution: (b)

For n type σ = $nq\mu n$ sample $A \rightarrow p$-type

For p type σ = $p_q \mu_p$ sample $B \rightarrow n$-type

Now $\dfrac{\sigma_n}{\sigma_p} = \dfrac{n\mu n}{p\mu p}$

$$\frac{\sigma_B}{\sigma_A} = \frac{\sigma_n}{\sigma_p} = 3/1$$

$$\frac{\sigma_A}{\sigma_B} = 1/3$$

Hence, the correct option is (b)

32. The reistivity of a uniformly doped n-type silicon sample is 0.5 Ω cm. If the electron mobility (μ_n) is 1250 cm^2/V-sec and the charge of an electron is 1.6×10^{-19} Coulomb, the donor impurity concentration (N_D) in the sample is

(a) 2×10^{16} cm^{-3} (b) 1×10^{16} cm^{-3}
(c) 2.5×10^{15} cm^{-3} (d) 2×10^{15} cm^{-3}

[2004]

Solution: (c)

ρ = 0.5 Ω-cm, μn = 1250 cm^2/v-sec, q = 1.6×10^{-19}c

$$\sigma = 0.5\Omega - \text{cm}, \quad \Rightarrow \frac{1}{\rho} = \frac{1}{N_D q \mu_n}$$

$$N_D = \frac{\rho}{q\mu n} = \frac{0.5}{1.6\times 10^{-19}\times 1250} = 2.5\times 10^{15}/\text{cm}^3$$

Hence, the correct option is (c)

33. The longest wavelength that can be absorbed by silicon which has the band gap of 1.12 eV is 1.1 μm. If the longest wavelength that can be absorbed by another material is 0.87 μm, then the band gap of this material is

(a) 1.416 eV (b) 0.886 eV
(c) 0.854 eV (d) 0.706 eV **[2004]**

Solution: (a)

$$\therefore E_a = \frac{1.24}{\lambda}$$

where E_a is in eV and λ in μm.

$\therefore E_a \propto 1/\lambda$

x = 1.416 eV.

Hence, the correct option is (a)

34. The neutral base width of a bipolar transistor, biased in the active region, is 0.5 μm. The maximum electron concentration and the diffusion constant in the base are 10^{14} cm^{-3} and D_n = 25 cm^2/sec, respectively. Assuming negligible recombination in the base, the collector current density is (the electron charge is 1.6×10^{-19} coulomb)

(a) 800 A/cm^2 (b) 8 A/cm^2
(c) 200 A/cm^2 (d) 2 A/cm^2 **[2004]**

Solution: (b)

x = 0.5 μm, $n = 10^{14}$ cm^{-3}, D_n = 25cm^2/sec.

$$J = qD_n\frac{dn}{dx} = 1.6\times 10^{-19}\times 25\times \frac{10^{14}}{6.5\times 10^{-4}}$$

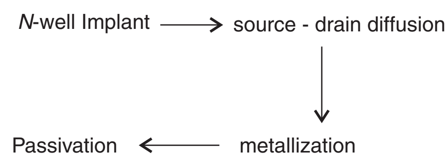
= 8 A/cm^2

Hence, the correct option is (b)

35. If P is Passivation, Q is n-well implant, R is metallization and S is source/drain diffusion, then the order in which they are carried out in a standard n-well CMOS fabrication process is

(a) P-Q-R-S (b) Q-S-R-P
(c) R-P-S-Q (d) S-R-Q-P **[2003]**

Solution: (b)



Hence, the correct option is (b)

36. The intrinsic carrier density at 300 K is 1.5×10^{10} cm^{-3} in silicon for n-type silicon doped to 2.25×10^{15} atoms cm^{-3}, the equilibrium electron and hole densities are

(a) $n = 1.5 \times 10^{15}$, $p = 1.5 \times 10^{10}$ cm^{-3}
(b) $n = 1.5 \times 10^{10}$, $p = 2.25 \times 10^{15}$ cm^{-3}
(c) $n = 2.25 \times 10^{15}$, $p = 1.0 \times 10^{5}$ cm^{-3}
(d) $n = 1.5 \times 10^{10}$, $p = 1.5 \times 10^{10}$ cm^{-3} **[1997]**

Solution: (c)

$n_i = 1.5 \times 10^{10}/$ cm^3. $N_D = 2.25 \times 10^{15}$ atoms/cm^3

$n \approx N_D$

$\therefore n_i$ = 2.25 × 1015/ cm

$$P = \frac{n_i^2}{n} = \frac{2.25\times 10^{20}}{2.25\times 10^{15}} = 10^5/\text{cm}^3$$

Hence, the correct option is (c)

37. The electron concentration in a sample of uniformly doped n-type silicon at 300 K varies linearly from 10^{17} cm^{-3} at $x = 0$ to 6×10^{16} cm^{-3} at $x = 2$ μ_{rn}. Assume a situation that electrons are supplied to keep this concentration gradient constant with time. If electronic charge is 1.6×10^{-9}, Coulomb and the diffusion constant D_n = 35cm^2/s, the current density in the silicon, if no electric field is present, is

(a) zero (b) 120 A/cm^2
(c) +1120 A/cm^2 (d) –1120 A/cm^2

[1997]

Solution: (d)

n_1 = 10^{17}/ cm3 $x_1 = 0$

n_2 = 6×10^{16}/ cm3 x_2 = 2μm

$q \quad = 1.6 \times 10^{-19}$c

$D_n \quad = 35$ cm^2/ sec

$$J = qD_n \frac{dn}{dx} = 1.6\times10^{-19} \times 35 \times \left[\frac{6\times10^{16} - 10^{17}}{2\times10^{-4}}\right]$$

$J = -1120$ A / cm^2

Hence, the correct option is (d)

38. An n-type silicon bar 0.1 cm long and 100 μrn^2 in cross-sectional area has a majority carrier concentration of 5 × 10^{20}/m^3 and the carrier mobility is 0.13 m^2/V-s at 300 K. If the charge of an electron is 1.6 × 10^{-19} coulomb, then the resistance of the bar is

(a) 10^6 Ω (b) 10^4 Ω

c) 10^{-1} Ω (d) 10^{-1} Ω **[1997]**

Solution: (a)

$l = 0.1$ cm, $A = 100$ μm^2, $n = 5 \times 10^{20}$/m^3, $\mu = 0.13$ m^2/ vs

$$l = \frac{0.1}{100} = 10^{-3}\text{m} \quad A = 100\times10^{-12}\text{ m}$$

$$R = \rho l / A$$

$$\rho = \frac{1}{\sigma} = \frac{1}{nq\mu} = \frac{1}{5\times10^{20}\times1.6\times10^{-19}\times0.13}$$

$$\rho = 5/52\ \Omega\text{-m}$$

Now $R = \dfrac{5}{52} \times \dfrac{10^{-3}}{10^{-10}} \cong 10^6\,\Omega$

Hence, the correct option is (a)

39. A semiconductor is irradiated with light such that carriers are uniformly generated throughout its volume. The semiconductor is n-type with $N_D = 10^{19}$ cm^{-3}. If the excess electron concentration in the steady state is Δn = 10^{15} cm^{-3} and if τ_p = 10 μsec (minority carries life time), the generation rate due to irradiation

(a) is 10^{20} $e - h$ pairs/cm^3/s

(b) is 10^{24} $e - h$ pairs/cm^3/s

(c) is 10^{10} $e - h$ pairs/cm^3/s

(d) cannot be determined, as the given data is insufficient. **[1992]**

Solution: (a)

Generation rate in an-type semiconductor $= \dfrac{\Delta p}{Tp}$

$\Delta p = \Delta n = 10^{15}$/cm^3

So, generation rate $= \dfrac{10^5}{10\times10^{-6}} = 10^{20}\, e - h$ pairs/cm^3.

Hence, the correct option is (a)

40. A silicon sample is uniformly doped with 10^{16} phosphorus atoms cm^{-3} and 2 × 10^{16} boron atoms cm^{-3}. If all the dopants are fully ionized, the material is

(a) n-type with carrier concentration of 10^{16} cm^{-3}

(b) p-type with carrier concentration of 10^{16} cm^{-3}

(c) p-type with carrier of 2 × 10^{16} cm^{-3}

(d) n-type with a earner concentration of 2 × 10^{16} cm^{-3} **[1991]**

Solution: (b)

As $n < p$ (or) $N_A > N_D$, ∴ semiconductor is p-type with carrier concentration of 10^{16} cm^{-3}.

Hence, the correct option is (b)

41. Under high electric fields, in a semiconductor with increasing electric field,

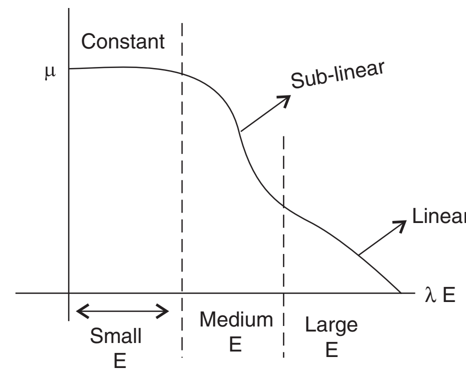
(a) the mobility of charge carriers decreases

(b) the mobility of the carries increases

(c) the velocity of the charge carriers saturates

(d) the velocity of the charge carriers increases **[1990]**

Solution: (a) and (c)



From figure at high electric field, mobility of the carrier decreases and drift velocity saturated.

Hence, the correct option is (a) and (c)

42. Due to illumination by light, the electron and hole concentrations in a heavily doped N-type semiconductor increase by Δn and Δp, respectively. If n_1 is the intrinsic concentration then,

(a) $\Delta n < \Delta p$ (b) $\Delta n > \Delta p$

(c) $\Delta n = \Delta p$ (d) $\Delta n \times \Delta p = \eta^2$

[1989]

Solution: (c)

The light results in breaking of covalent bonds. Thus, $\Delta n = \Delta p$.

Hence, the correct option is (c)

43. The concentration of ionized acceptors and donors in a semiconductor are N_A, N_D, respectively. If $N_A > N_D$ and n_I is the intrinsic concentration, the position of the Fermi level with respect to the intrinsic level depends on

(a) $N_A - N_D$ (b) $N_A + N_D$

(c) $\dfrac{N_A - N_D}{n_i^2}$ (d) n_I

[1989]

Solution: (c)

For an n-type semiconductor

$$E_f = E_c - kT\log_e\left(\frac{\text{NAND}}{n_i^2}\right)$$

$$E_c - E_f = kT\log_e\frac{\text{NAND}}{n_i^2}$$

$$\therefore E_c - E_f \alpha \log_e\frac{\text{NAND}}{n_i^2}$$

Hence, the correct option is (c)

44. Consider two energy levels: E_1 E_2 eV above the Fermi level and E_2, E eV below the Fermi level. P_1 and P_2 are, respectively, the probabilities of E_1 being occupied by an electron and E_2 being empty. Then
(a) $P_1 > P_2$
(b) $P_1 = P_2$
(c) $P_1 < P_2$
(d) P_1 and P_2 depend on the number of free electrons **[1987]**

Solution: (c)

Given that

$P_1 = E_1$ Fermi level probability

$P_2 = E_2$ Fermi level probability

$$f(E) = \frac{1}{1+e^{(E\text{-}E_f)KT}}$$

where

$f(E)$ = Fermi-Dirac probability function

E_F = Fermi level

$P_1 < P_2$

Hence, the correct option is (c)

45. In an intrinsic semiconductor, the free electron concentration depends on
(a) Effective mass of electrons only
(b) Effective mass of holes only
(c) Temperature of the semiconductor
(d) Width of the forbidden energy band of the semiconductor **[1987]**

Solution: (c)

By mass action law

$n \times p = n_i^2$

ni = intrinsic carrier concentration

p = hole concentration

n = electron concentration

$n_i^2 \propto T^3$

$n_i \propto T^{3/2}$.

For intrinsic semiconductor

$n = p = n_i$

$n \propto T^{3/2}$

Hence, the correct option is (c)

46. According to the Einstein relation, for any semiconductor the ratio of diffusion constant to mobility of carriers
(a) Depends upon the temperature of the semiconductor
(b) Depends upon the type of the semiconductor
(c) Varies with life time of the semiconductor
(d) Is a universal constant **[1987]**

Solution: (a)

$$\text{Einsteis equation} \rightarrow \frac{D}{\mu} = \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T = \frac{T(^\circ\text{K})}{11600}$$

where

T = temperature in °K

V_T = thermal voltage

D = diffusion constant

μ = mobility

D_n = electron diffusion constant

D_p = hole diffusion constant

μ_n = electron mobility

μ_p = hole mobility

$V_T = T/11600$

$$\therefore \frac{D_n}{\mu_n} = T/11600$$

∴ It depends upon the temperature of the semiconductor.

Hence, the correct option is (a)

47. Direct band gap (DBG) semiconductors
(a) Exhibit short carrier life time and they are used for fabricating BJTs
(b) Exhibit long carrier life time and they are used for fabricating BJTs
(c) Exhibit short carrier life time and they are used for fabricating lasers
(d) Exhibit long carrier life time and they are used for fabricating BJTs **[1987]**

Solution: (c)

DBG semiconductors exhibit short carrier life time. They are used for fabricating lasers.

In DBG semiconductor during the recombination, the energy is released in the form of light.

Hence, the correct option is (c)

FIVE-MARKS QUESTIONS

1. An n-type silicon sample, having electron mobility μ_n twice the hole mobility μ_p, is subjected to a steady illumination such that the electron concentration doubles from its thermal equilibrium value. As a result, the conductivity of the sample increases by a factor of **[1991]**

Solution: Given sample in n-types silicon SC.

$\therefore\ \sigma_n = nq\,\mu n$

$n' = 2n$

$\sigma'_n = n'_q \mu_n = 2nq\mu_n$

$\sigma'_n = 2\sigma$

$\therefore$ New conductivity is twice the initial value.

2. Show that the minimum conductivity of an extrinsic silicon sample occurs when it is slightly p-type calculate the electron and hole concentration when where the conductivity is minimum given that $\mu_n = 1350$ cm W-sec, $\mu_p = 450$ cm-sec, and the intrinsic carrier concentration, $n_i = 1.5 \times 10^{10}$ cm^{-3}. **[1991]**

Solution: Given

$\mu_n = 1350$ cm^2/volt – sec

$\mu_p = 450$ cm^2 /volt – sec

$n_i = 1.5 \times 10^{10}$ / cm^3

$\sigma = nq\,\mu n + pq\,\mu_p$ = conductivity

$\sigma = q(n\,\mu_n + p\,\mu_p)$

for σ to be minimum

$$\frac{d\sigma}{dp} = 0$$

$$q\left[\mu_p + \mu_n \frac{du}{dp}\right] = 0$$

$$\mu_p + \mu_n \frac{du}{dp} = 0 \qquad \ldots\ldots(1)$$

By mass action law

$$np = ni^2 \qquad \ldots\ldots(2)$$

$$\frac{dx}{dp} = -\frac{ni^2}{p^2}$$

Substitute $\frac{du}{dp}$ in equation (1)

$$\mu_p + \mu_n\left(-\frac{n_i^2}{p^2}\right) = 0$$

$$p = n_i\sqrt{\frac{u_u}{\mu_p}} \qquad \ldots\ldots(3)$$

from equation (2) and (3)

$$n = n_i\sqrt{\frac{u_p}{\mu_n}}A \qquad \ldots\ldots(4)$$

$\sigma q\,[n\mu_n + p\mu_p]$

$$\sigma_{\min} = q\left[n_i\sqrt{\frac{u_p}{\mu_n}}\mu_n + n_i\sqrt{\frac{u_u}{\mu_p}}\mu_p\right]$$

$$\sigma_{\min} = 2n_i q\sqrt{\mu_n\mu_p}$$

from equation (2) and given data

$$n = n_i\sqrt{\frac{u_p}{\mu_n}} = 1.5\times10^{10}\sqrt{\frac{450}{1350}}$$

$P = 2.59 \times 10^{10}$/cm^3

from equation (4)

$$n = n_i\sqrt{\frac{u_p}{\mu_n}} = 1.5\times10^{10}\sqrt{\frac{450}{1350}}$$

$n = 0.866 \times 10^{10}$/cm^3

we know that for a semiconductor mobility of e^- is always greater than mobility of holes $\mu_n > \mu_p$

$$h = n_i\sqrt{\frac{\mu_p}{\mu_n}} \qquad P = n_i\sqrt{\frac{u_n}{\mu_p}}$$

$\therefore\ P > n$

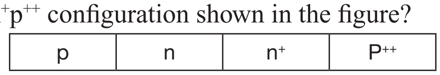
$\therefore$ It proves that minimum conductivity of extrinsic silicon occurs when it is slightly p type.

Chapter 2

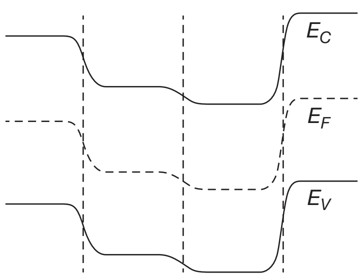
PN Junction

One-mark Questions

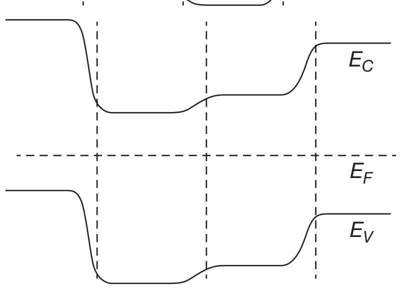
1. Which one of the following options describes correctly the equilibrium band diagram at $T = 300$ k of a Silicon pnn$^+$p^{++} configuration shown in the figure? **[2019]**



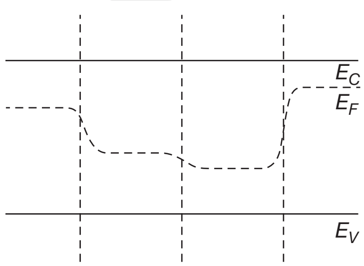
(A)



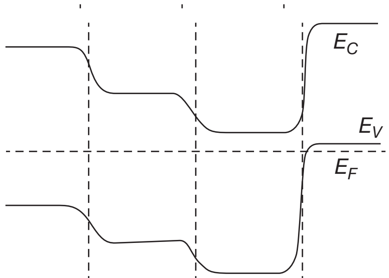
(B)



(C)



(D)



Solution: Fermi level for n & p type semiconductor are:

$$E_{F-N} = E_C - KT\ln\left(\frac{N_C}{N}\right)$$

$$E_{F-P} = E_V - KT\ln\left(\frac{N_V}{P}\right)$$

Flour the above equations we notice that when

in n-side, Fermi level is closer to E_C

in P-side, Fermi level is closer to E_V

in P^{++}, Fermi level penetrates into valence bond

Hence, the correct option is (D).

2. In a p-n junction diode at equilibrium, which one of the following statements is NOT TRUE? **[2018]**

(A) The hole and electron diffusion current components are in the same direction.

(B) The hole and electron drift current components are in the same direction.

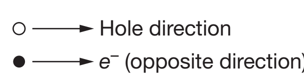
(C) On an average, holes and electrons drift in opposite direction.

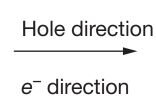
(D) On an average, electrons drift and diffuse in the same direction.

Solution: From the given statements

(i) $\left.\begin{array}{l} I_{drift} \\ I_{diffusion} \\ (I_p, I_n) \end{array}\right\}$ are in the same direction

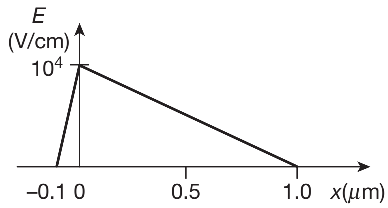
(ii) electrons and holes movement in opposite direction.





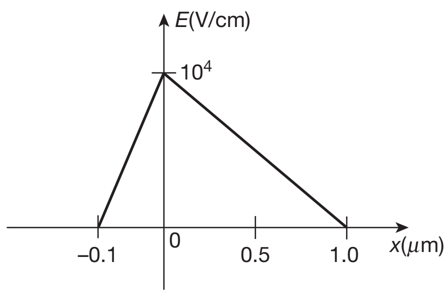
Hence, the correct option is (D)

3. The electric field profile in the depletion region of a p-n junction in equilibrium is shown in the figure. Which one of the following statements is NOT TRUE? **[2015]**



(A) The left side of the junction is n-type and the right side is p-type
(B) Both n-type and p-type depletion regions are uniformly doped
(C) The potential difference across the depletion region is 700 mV
(D) If the p-type region has a doping concentration of 10^{15} cm^{-3}, then the doping concentration in the n-type region will be 10^{16} cm^{-3}

Solution:



From the given options

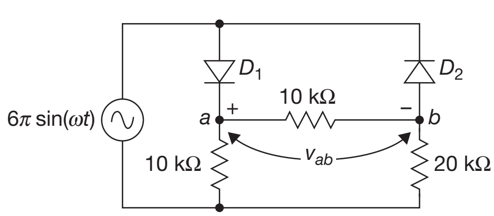
Built in potential $V_J = \dfrac{1}{2} \times 10^4 \times 10^2 \times 1.1 \times 10^{-6}$ = area

= 0.55 volts = 550 mV

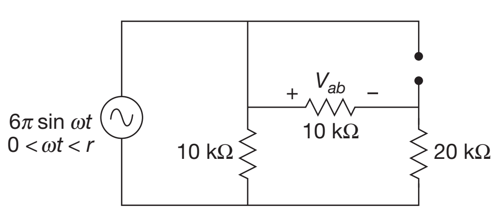
So option c is NOT correct

Hence, the correct option is (C).

4. In the circuit shown, assume that the diodes D_1 and D_2 are ideal. The average value of voltage V_{ab} (in Volts), across terminals 'a' and 'b' is ________. **[2015]**



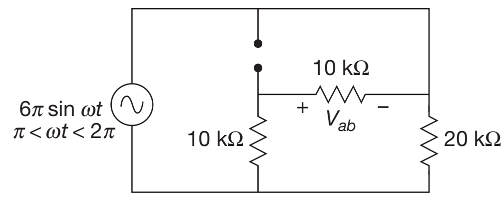
Solution: For positive half cycle circuit can be drawn as



$$\Rightarrow \quad V_{ab} = \frac{6\pi \sin \omega t \times 10}{30}$$

$$= 2\pi \sin \omega t V$$

For negative half cycle.



$$V_{ab} = \frac{6\pi \sin \omega t \times 10}{20} = 3\pi \sin \omega t \text{ V}$$

$$V_{\text{avg}} = \frac{1}{T}\int_0^T f(t)\,dt$$

$$= \frac{\omega}{2\pi}\left[\int_0^{\pi} 2\pi \sin \omega t\, dt + \int_{\pi}^{2\pi} 3\pi \sin \omega t\, dt\right]$$

$$= \frac{\omega}{2\pi}\left[2\pi \frac{(-\cos \omega t)_0^{\pi}}{\omega} + \frac{3\pi}{\omega}(-\cos \omega t)_{\pi}^{2\pi}\right]$$

= 2 + 3

= 5 V

Hence, the correct Answer is (4.85 to 5.15).

5. The built-in potential of an abrupt p-n junction is 0.75 V. If its junction capacitance (C_J) at a reverse bias (V_R) of 1.25 V is 5 pF, the value C_J (in pF) when V_R = 7.25 V is ________. **[2015]**

Solution: From the given data

Built-in voltage of an abrupt p-n junction is

$V_j = 0.75$ V

If $\quad V_R = 1.25$ V, $C_J = 5$ pF

then $\quad V_R = 7.25$ V, $C_J = ?$

we know

$$C = \frac{\varepsilon A}{W};\ C \ \alpha \ \frac{1}{W}$$

$$W \propto \sqrt{(V_R + V_j)}$$

$$\frac{C_{J_1}}{C_{J_2}} = \sqrt{\frac{V_j + V_{R_2}}{V_j + V_{R_1}}}$$

$$C_{J_2} = 5\,pF \times \sqrt{\frac{2}{8}}$$

$$C_{J_2} = 2.5 \text{ pF}$$

Hence, the correct Answer is (2.4 to 2.6).

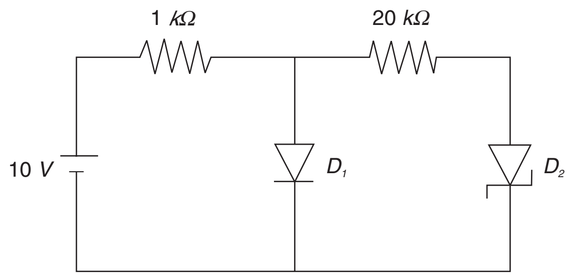
6. When the optical power incident on a photodiode is 10 μW and the responsivity is 0.8 A/W, the photocurrent generated (in μA) is______. **[2014]**

Solution: 8 μA

Photo current = incident power × responsivity

$= 10 \times 10^{-6} \times 0.8 = 8\ \mu A$.

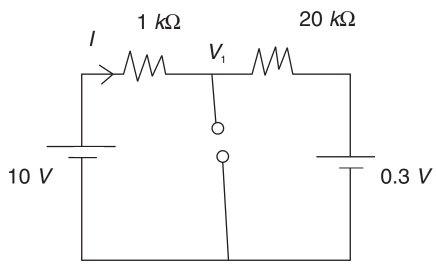
7. In the figure, assume that the forward voltage drops of the PN diode D_1 and Schottky diode D_2 are 0.7 V and 0.3 V, respectively. If ON denotes conducting state of the diode and OFF denotes non-conducting state of the diode, then in the circuit,



(a) both D_1 and D_2 are ON
(b) D_1 is ON and D_2 is OFF
(c) both D_1 and D_2 are OFF
(d) D_1 is OFF and D_2 is ON **[2014]**

Solution (c)

Let D_1 is off and D_2 is ON. Then



$$I = \frac{10 - 0.3}{21k} = 0.461 \text{ mA}$$

$\therefore\ V_1 = 9.539$ V

Both D_1 and D_2 are ON.

Hence, the correct option is (c)

8. In a forward biased $p - n$ junction, the sequence of events, that best describes the mechanism of current flow is

(a) injection, and subsequent diffusion and recombination of minority carriers
(b) injection, and subsequent drift and generation of minority carriers
(c) extraction, and subsequent diffusion and generation of minority carriers
(d) extraction, and subsequent drift and recombination of minority carriers **[2013]**

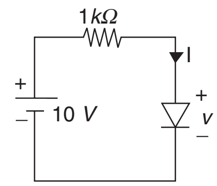
Solution (a)

First injection, then subsequent diffusion, and recombination of minority carriers take place.

Hence, the correct option is (a)

9. The i-v characteristics of the diode in the circuit given below are

$$i = \begin{cases} \dfrac{v - 0.7}{500} \text{ A}, & v \geq 0.7 \text{ V} \\ 0 \text{ A} & v < 0.7 \text{ V} \end{cases}$$



The current in the circuit is

(a) 10 mA (b) 9.3 mA
(c) 6.67 mA (d) 6.2 mA **[2012]**

Solution (d)

From the circuit

$-10 + 1000i + v = 0$

$i = 10 - v / 1000$

$v - 0.7/500 = 10 - v/1000$

$2v - 1.4 = 10 - v$

$3v = 11.4$

$v = 11.4/3$

$$i = \frac{10 - \dfrac{11.4}{3}}{1000} = 6.2 \text{ mA}$$

Hence, the correct option is (d)

10. A silicon $p - n$ junction is forward biased with a constant current at room temperature. When

the temperature is increased by 10°C, the forward bias voltage across the $p - n$ junction

(a) increases by 60 mV
(b) decreases by 60 mV
(c) increases by 25 mV
(d) decreases by 25 mV **[2011]**

Solution (b)

For 1°C rise in temperature, forward voltage decreased by 2.5 mV

So, forward voltage decreases by 25 mV.

Hence, the correct option is (b)

11. A Zener diode, when used in voltage stabilization circuits, is biased in

(a) reverse bias region below the breakdown voltage
(b) reverse breakdown region
(c) forward bias region
(d) forward bias constant current mode **[2011]**

Solution (c)

The Zener diode is reversed biased when used as a voltage regulator.

Hence, the correct option is (c)

12. Which of the following is Not associated with a *p-n* junction ?
(a) Junction capacitance
(b) Charge storage capacitance
(c) Depletion capacitance
(d) Channel length modulation **[2008]**

Solution (d)

This is associated with FET.

Hence, the correct option is (d)

13. In a *p-n* junction diode under reverse bias, the magnitude of electric field is maximum at
(a) the edge of the depletion region on the *p*-side
(b) the edge of the depletion region on the *n*-side
(c) the *p-n* junction
(d) the centre of the depletion region on the *n*-side **[2007]**

Solution (c)

Electric field is always max at the junction.

Hence, the correct option is (c)

14. The values of voltage (V_D) across a tunnel-diode corresponding to peak and valley currents are V_p and V_v, respectively. The range of tunnel-diode voltage V_D for which the slope of its I-V_D characteristics is negative would be
(a) $V_D < 0$ (b) $0 \le V_D < V_p$
(c) $V_p \le V_D < V_v$ (d) $V_D \ge V_v$ **[2006]**

Solution (c)

The slope is negative when $V_p \le V_D < V_v$

Hence, the correct option is (c)

15. A silicon $p - n$ junction at a temperature of 20°C has a reverse saturation current of 10 pico-amperes (pA). The reverse saturation current at 40°C for the same bias is approximately
(a) 30 pA (b) 40 pA
(c) 50 pA (d) 60 pA **[2005]**

Solution (b)

$$I_{(T2)} = I_{(T1)} 2^{\frac{T_2 - T_1}{10}}$$

$$I_{(40°)} = 10 \times 2^{20/10} = 40 \text{ PA}$$

Hence, the correct option is (b)

16. A silicon $p - n$ junction diode under reverse bias has depletion region of width 10 μm. The relative permittivity of silicon, $\varepsilon_r = 11.7$ and the permittivity of free space $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m. The depletion capacitance of the diode per square metre is
(a) 100 μ (b) 10 μ
(c) 1 μ (d) 20μ **[2005]**

Solution (b)

$$C = \frac{\varepsilon A}{d}$$

$$\frac{C}{A} = \frac{\varepsilon}{d} = \frac{11.7 \times 8.85 \times 10^{-12}}{10 \times 10^{-6}} \approx 10\ \mu$$

Hence, the correct option is (b)

17. Choose proper substitutes for *X* and *Y* to make the following statement correct. Tunnel diode and avalanche photodiode are operated in *X* bias and *Y* bias, respectively.
(a) *X*: reverse, *Y*: reverse
(b) *X*: reverse, *Y*. forward
(c) *X*: forward, *Y*: reverse
(d) *X*. forward, *Y* forward **[2003]**

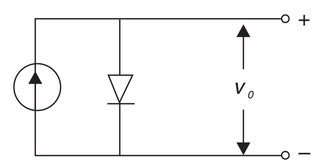
Solution (c)

Tunnel diode → forward biased

Avalanche photodiode → Reverse biased.

Hence, the correct option is (c)

18. In the figure, silicon diode is carrying a constant current of 1 mA. When the temperature of the diode is 20°C, V_D is found to be 700 mV. If the temperature rises to 40°C, V_D becomes approximately equal to
(a) 740 mV (b) 660 mV
(c) 680 mV (d) 700 mV **[2002]**



Solution (b)

For 1°C rise in temp, V_D decreases by 2 mV.

∴ for 20°C rise in temp, V_D decreases by 40 mV.

∴ $V_D = 660$ mV.

Hence, the correct option is (b)

19. For small signal ac operation, a practical forward biased diode can be modelled as
(a) a resistance and a capacitance
(b) an ideal diode and resistance in parallel
(c) a resistance and an ideal diode in series
(d) a resistance **[1998]**

Solution (d)

For small signal ac operation, a practical forward biased diode can be modelled as a resistance.

Hence, the correct option is (d)

20. The static characteristic of an adequately forward biased p--n junction is a straight line, if the plot is of

(a) log I vs. log V (b) log I vs. V
(c) I vs. log V (d) I vs. V

[1998]

Solution (b)

$$I = I_0 e^{V_D/\eta V_T}$$

$V_D = \eta V_T \ln (I/I_0)$

∴ Plot of log I vs V gives a straight line.

Hence, the correct option is (b)

21. The diffusion potential across a p-n junction
(a) decreases with increasing doping concentration
(b) increases with decreasing band gap
(c) does not depend on doping concentrations
(d) increases with increase in doping concentration **[1995]**

Solution (d)

The diffusion potential across a p-n junction increases with increase in doping concentration.

Hence, the correct option is (d)

22. A Zener diode works on the principle of
(a) tunnelling of charge carriers across the junction
(b) thermionic emission
(c) diffusion of charge carriers across the junction
(d) hopping of charge carriers across the junction **[1995]**

Solution (a)

A Zener diode works on the principle of tunnelling of charge carriers across the junction.

Hence, the correct option is (a)

23. The depletion capacitance, C_i, of an abruptly $p-n$ junction with constant doping on either side varies with $R.B.V_R$ as
(a) $C_J \propto V_R$ (b) $C_J \propto V_R^{-1}$
(c) $C_J \propto V_R^{-1/2}$ (d) $C_J \propto V_R^{-1/3}$

[1995]

Solution (c)

$$C_j \propto \frac{1}{\sqrt{w}}$$

$w \propto \sqrt{v}$

$C_j \propto 1/\sqrt{v}$

Hence, the correct option is (c)

24. In a junction diode
(a) the depletion capacitance increases with increase in the reverse bias
(b) the depletion capacitance decreases with increase in the reverse bias
(c) the depletion capacitance increases with increase in the forward bias
(d) the depletion capacitance is much higher than the depletion capacitance when it is forward biased **[1990]**

Solution (b)

$$C_D = \tau \cdot g = \tau \cdot \frac{I_f}{\eta V_T} = \frac{\tau \cdot I_0 (e^{V_D/\eta V_T} - 1)}{\eta V_T}$$

Also C ∝ 1/W

$\& W \propto \sqrt{|V_{RB} + V_0|}$ → reverse bias

∴ As reverse bias increases, W ↑ and hence C ↓.

Hence, the correct option is (b)

Two-mark Questions

1. In an ideal pn junction with an ideality factor of 1 at T = 300 K, the magnitude of the reverse-bias voltage required to reach 75% of its reverse saturation current, rounded off to 2 decimal places, is _____ mV.

[$K = 1.38 \times 10^{-23}$ jK^{-1}, $h = 6.625 \times 10^{-34}$ J, s, $q = 1.602 \times 10^{-19}$ C] **[2019]**

Solution: For $p-n$ junction diode

$$I = I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$$

$$I = -0.75 I_S$$

$$-0.75 \cancel{I_S} = \cancel{I_S}\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$$

$$1 - 0.75 = e^{\frac{V_{BE}}{V_T}}$$

$$\ln 0.25 = \frac{V_{BE}}{V_T}$$

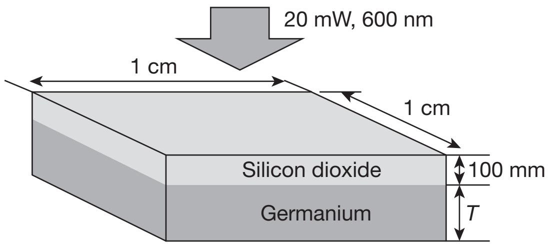
$$V_{BE} = V_T \ln 0.25$$

$$V_{BE} = -1.386 \times \frac{-1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}}$$

$$|V_{BE}| = 35.87 \text{ mV}$$

Hence, the correct answer is 35.87.

2. A Germanium sample of dimensions 1 cm × 1 am is illuminated with 20 mW, 600 nm laser light source as shown in the figure. The illuminated sample surface has a the 100 nm of loss-less. Silicon dioxide layer that reflects one-fourth of the incident light. From the remaining light, one-third of the power is reflected from the Silicon dioxide-Germanium interface, one-third is absorbed in the Germanium layer. and one-third is transmitted through the other side of the sample. If the absorption coefficient of Germanium at 600 nm is 3 × 10^4 cm^{-1} and the bandha is 0.666 eV, the thickness of the Germanium layer., rounded off to 3 decimal places, is _____ μm. **[2019]**



Solution:

$$P_i = 20 \text{ mV}$$

$$P_{r1} = \frac{P_i}{4}\left(S_i - 0_2\right)$$

$$P_t = \frac{1}{3}\left(P_i - \frac{P_i}{4}\right)$$

$$= \frac{P_i}{4}$$

$$P_a = \frac{1}{3}\left(P_i - \frac{P_i}{4}\right)$$

$$= \frac{P_1}{4}$$

$$P_{4E} = P_t + P_a = \frac{2P_i}{4}$$

$$P_t = P_o e^{-at}$$

$$\frac{P_i}{4} = \frac{2P_i}{4} e^{-at}$$

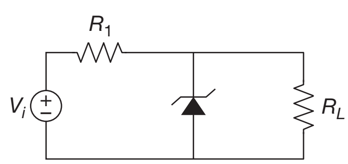
$$1 - 2e^{-at} = 0$$

$$e^{-at} = 0.5$$

$$a = \frac{-\ln(0.5)}{3 \times 10^4} \text{cm}$$

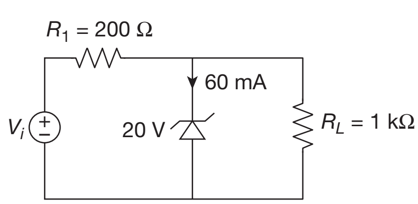
Hence, the correct answer is 0.231 μm.

3. In the circuit shown, the breakdown voltage and the maximum current of the Zener diode are 20 V and 60 mA, respectively. The values of R_1 and R_L are 200 Ω and 1 kΩ, respectively. What is the range of V_1 that will maintain the Zener diode in the 'on' state?" **[2019]**



(A) 20 V to 28 V (B) 24 V to 36 V
(C) 18 V to 34 V (D) 22 V to 34 V

Solution:



$$I_L = \frac{20-0}{1} = 20 \text{ mA}$$

$$I_R = I_z + I_L = 60 + 20 = 80 \text{ mA}$$

$$\text{Vimin} = V_2 \frac{\left(R_L + R_1\right)}{R_L}$$

$$= \frac{12}{10} \times 20$$

$$= 24 \text{ volt}$$

$$I_R = 80 \text{ mA}$$

$$\text{Vimax} = 200 \times 80 + 20$$

$$= 16 + 20$$

$$= 36 \text{ volt}$$

Hence, the correct option is (B)

4. The quantum efficiency (η) and respectively (R) at a wavelength λ (in μm) in a p-i-n photodetector are related by **[2019]**

(A) $R = \dfrac{1.24 \times \lambda}{\eta}$ (B) $R = \dfrac{\eta \times \lambda}{1.24}$

(C) $R = \dfrac{\lambda}{\eta \times 1.24}$ (D) $R = \dfrac{1.24}{\eta \times \lambda}$

Solution: $R = \dfrac{I_p}{P_o} = \dfrac{ng}{nf} = \dfrac{n\lambda}{1.24}$

Hence, the correct option is (B).

5. A *p-n* step junction diode with a contact potential of 0.65 V has a depletion width of 1μm at equilibrium. The forward voltage (in volts, correct to two decimal places) at which this width reduces to 0.6 μm is ______. **[2018]**

Solution:

contact potential $V_{01} = 0.65\text{V}$:

depletion width $W_1 = 1$ μm

forward voltage $V_{02} = ?$

reduced width $W_2 = 0.6$ μm

We know that

$$W = \sqrt{\frac{2\varepsilon_s}{q}\left[\frac{1}{N_A} + \frac{1}{N_D}\right]V_j}$$

$$W \propto \sqrt{V_j}$$

$$\frac{W_1}{W_2} = \sqrt{\frac{V_{o1}}{V_{o2}}}$$

$$\frac{1}{0.6} = \sqrt{\frac{0.65}{V_{o2}}}$$

$$V_{o2} = \frac{0.65}{2.77} = 0.234 \text{ V}$$

But, $V_{02} = V_{01} = V_{FB}$

$$0.234 = 0.65 - V_{FB}$$

$$V_{FB} = 0.416 \text{ V}$$

Hence, the correct answer is 0.4 to 0.43.

6. Red (R), Green (G) and Blue (B) Light Emitting Diodes (LEDs) were fabricated using $p - n$ junctions of three different inorganic semiconductors having different band-gaps. The builtin voltages of red, green and blue diodes are V_R, V_G and V_B, respectively. Assume donor and acceptor doping to be the same (N_A and N_D, respectively) in the P and n sides of all the three diodes.

Which one of the following relationships about the built-in voltages is TRUE? **[2018]**

(A) $V_R > V_G > V_B$
(B) $V_R < V_G < V_B$
(C) $V_R = V_G = V_B$
(D) $V_R > V_G < V_B$

Solution: For Red $V_R = 1.8\text{V}$

For Green $V_G = 2.2\text{V}$

For Blue $V_B = 5\text{V}$

From the above data

$$V_R < V_G < V_B.$$

Hence, the correct option is (B)

7. A solar cell of area 1.0 cm², operating at 1.0 sun intensity, has a short circuit current of 20 mA, and an open circuit voltage of 0.65 V. Assuming room temperature operation and thermal equivalent voltage of 26 mV, the open circuit voltage (in volts, correct to two decimal places) at 0.2 sun intensity is ________. **[2018]**

Solution:

Solar cell area = 1cm²

short circuit current $I_{sc1} = 20$ mA

open circuit voltage $V_{oc1} = 0.65\text{V}$

thermal equivalent voltage $V_T = 26\text{mV}$

Let open circuit voltage be V_{oc2} at 0.2 sun intensity

Open circuit voltage $V_{oc} = V_T \ln\left\{\frac{I_{sc}}{I_o}\right\}$

$I_o \rightarrow$ Intensity

$$V_{oc2} - V_{oc1} = V_T \ln\left\{\frac{I_{SC2}}{I_{SC1}}\right\}$$

$$V_{oc2} = V_{oc1} - 26 \times 10^{-3} \ln\frac{0.2}{1}$$

$$= 0.65 - 0.0418$$

$$= 0.608 \text{ V}$$

Hence, the correct answer is 0.59 to 0.63.

8. A junction is made between P-Si with doping density $N_{A1} = 10^{15}$ cm^{-3} and P Si with doping density $N_{A2} = 10^{17}$ cm^{-3}

Given: Boltzmann constant $K = 1.38 \times 10^{-23}$J. K^{-1}, electronic charge $q = 1.6 \times 10^{-19}$ C.

At room temperature (T = 300K), the magnitude of the built-in potential (in volts, correct to two decimal places) across this junction will be ________ . **[2018]**

Solution:

Boltzmann constant $K = 1.38 \times 10^{-23}$J. K^{-1}

electronic charge $q = 1.6 \times 10^{-19}$ C

Room temperature $T = 300$K

Now using the relation

$$V_O = V_T \ln\left\{\frac{P_1}{P_2}\right\}$$

$$V_O = 25.6 \times 10^{-3} \times \ln\left\{\frac{10^{17}}{10^{15}}\right\}$$

$$= 0.119 \text{ V}$$

Hence, the correct answer is 0.11 to 0.13.

9. For a particular intensity of incident light on a silicon pn junction solar cell, the photocurrent density (J_L) is 2.5 mA/cm² and the open-circuit voltage (V_{oc}) is 0.451 V. Consider thermal voltage (V_T) to be 25 mV. If the intensity of the incident light is increased by 20 times, assuming that the temperature remains unchanged, V_{oc} (in volts) will be ____________. **[2017]**

10. A region of negative differential resistance is observed in the current voltage characteristics of a silicon PN junction, if **[2015]**

(A) both the P-region and the N-region are heavily doped
(B) the N-region is heavily doped compared to the P-region
(C) the P-region is heavily doped compared to the N-region
(D) an intrinsic silicon region is inserted between the P-region and the N-region

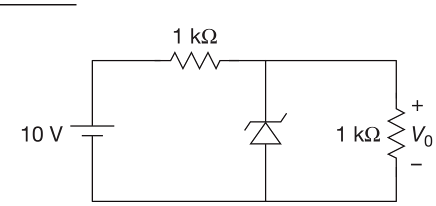
Solution: The tunnel diode is having negative resistance characteristics.

It is a heavily doped diode.

∴ both sides of P and N-region are heavily doped.

Hence, the correct option is (A).

11. In the circuit shown below, the Zener diode is ideal and the Zener voltage is 6 V. The output voltage V_o (in Volts) is ________ **[2015]**



Solution: From the given data

$$V_Z = 6\text{ V}$$

If zener diode is in ON position

$$V_o = V_Z = 6\text{ V.}$$

Check whether the zener diode is ON or OFF

If diode is ON,

$$V_L = \frac{1k \times V_{in}}{2k} = 6\text{ V}$$

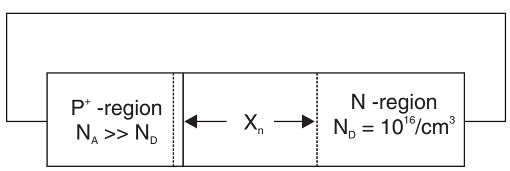
$$V_{\text{in}} = 12\text{ V}$$

The minimum required input voltage is 12 V, but given voltage is 10 V. So,zener diode is in OFF mode.

$$\therefore \quad V_o = \frac{V_{in} \times 1k}{2k} = \frac{V_{in}}{2} = 5\text{ V.}$$

Hence, the correct Answer is (5).

12. Consider an abrupt $p - n$ junction (at $T = 300$ K) shown in the figure. The depletion region width x_n on the N-side of the junction is 0.2 μrn and the permittivity of silicon ($\in_{sj}$) is 1.044×10^{-12} F/c m. At the junction, the approximate value of the peak electric field (in kV/cm) is________. **[2014]**

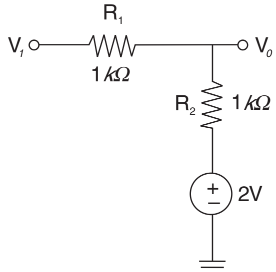


Solution: 30.651

$$\varepsilon_{\max} = \frac{q}{\varepsilon} W_N N_D$$

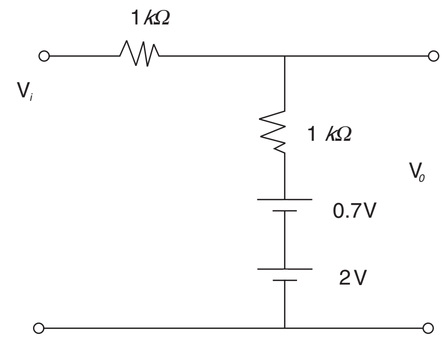
$$= \frac{1.6 \times 10^{-19}}{1.044 \times 10^{-2}} \times 0.2 \times 10^{-4} \times 10^{16} = 30.651\text{ kv/cm}$$

13. The diode in the circuit shown has $V_{on} = 0.7$ V but is ideal otherwise. If $V_i = 5\sin(\omega t)$ V, the minimum and maximum values of V_0 (in volts) are, respectively,



(a) −5 and 2.7 (b) 2.7 and 5
(c) −5 and 3.85 (d) 1.3 and 5 **[2014]**

Solution (c)



$i = 0$ when diode does not conduct. So, $V_0 = -5$ V.

Now, when diode conduct, then i_{max}

$$= \frac{5 - 2.7}{2} = \frac{2.3}{2} = 1.15\text{ mA}$$

$\therefore\ V_0 = 1.15 + 2.7 = 3.85$ V.

Hence, the correct option is (c).

14. The donor and acceptor impurities in an abrupt junction silicon diode are 1×10^{16} cm^{-3} and 5×10^{18} cm^{-3}, respectively. Assume that the intrinsic carrier concentration in silicon $n_1 = 1.5 \times 10^{10}$ cm^{-3} at 300K, $kT/q = 26\ m\text{V}$ and the permittivity of silicon $\in_{si} = 1.04 \times 10 - 12$ F/cm. The built-in potential and the depletion width of the diode under thermal equilibrium conditions, respectively, are

(a) 0.7 V and 1×10^{-4} cm
(b) 0.86 V and 1×10^{-4} cm
(c) 0.7 V and 3.3×10^{-5} cm
(d) 0.86 V and 3.3×10^{-5} cm **[2014]**

Solution (d)

$$V_{bi} = V_T \ln \frac{N_A N_D}{n_i^2}$$

$$26 \times 10^{-3} \ln \frac{10^{16} \times 5 \times 10^8}{2.25 \times 10^{20}} = 0.86\text{ V}$$

$$W = \sqrt{\frac{2\varepsilon}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V)}$$

$$= \sqrt{\frac{2 \times 1.04 \times 10^{-12}}{1.6 \times 10^{-19}}\left(1/10^{16} + \frac{1}{5 \times 10^{18}}\right)0.86}$$

$= 3.3 \times 10^{-5}$ cm.

Hence, the correct option is (d).

15. Compared to a $p - n$ junction with $N_A = N_D = 10^{14}$ cm^{-3}, which one of the following statements is TRUE for a $p - n$ junction with $N_A = N_D = 10^{20}$ cm^{-3}?

(a) Reverse breakdown voltage is lower and depletion capacitance is lower

(b) Reverse breakdown voltage is higher and depletion capacitance is lower
(c) Reverse breakdown voltage is lower and depletion capacitance is higher
(d) Reverse breakdown voltage is higher and depletion capacitance is higher **[2010]**

Solution (d)

Breakdown voltage $\propto \dfrac{1}{\text{doping}}$

$C \propto$ doping

$\therefore$ As doping increases, reverse breakdown voltage decreases whereas depletion capacitance will increase.

Hence, the correct option is (d).

16. Consider the following assertions.

S_1: For Zener effect to occur, a very abrupt junction is required

S_2: For quantum tunnelling to occur, a very narrow energy barrier is required.

Which of the following is correct?

(a) Only S_2 is true
(b) S_1 and S_2 are both true but S_2 is not a reason for S_1
(c) S_1 and S_2 are both true and S_2 is a reason for S_1
(d) Both S_1 and S_2 are false **[2008]**

Solution (a)

For quantum tunnelling, very narrow depletion width is desired. So, only S_2 is true.

Hence, the correct option is (a).

17. Group I lists four types of $p-n$ junction diodes. Match each device in Group I with one of the options in Group II to indicate the bias condition of that device in its normal mode of operation.

Group I	Group II
P. Zener diode	1. Forward bias
Q. Solar cell	2. Reverse bias
R. LASER diode	
S. Avalanche photodiode	

(a) P-1, Q-2, R-1, S-2
(b) P-2, Q-1, R-1, S-2
(c) P-2, Q-2, R-2, S-1
(d) P-2, Q-1, R-2, S-2 **[2007]**

Solution (b)

Solar cell and laser are forward biased.

Hence, the correct option is (b)

18. Group I lists four different semiconductor devices. Match each device in Group I with its characteristic property in Group II.

Group I

P. BJT   Q. MOS capacitor
R. LASER diode   S. JFET

Group II

1. Population inversion
2. Pinch-off voltage
3. Early effect
4. Flat-band voltage

(a) P-3, Q-1, R-4, S-2
(b) P-1, Q-4, R-3, S-2
(c) P-3, Q-4, R-1, S-2
(d) P-3, Q-2, R-1, S-4 **[2007]**

Solution (c)

Population inversion takes place in LASER

Pinch-off voltage is characteristic of JFET

Early effect in BJT

Hence, the correct option is (c)

19. A p^+n junction has a built-in potential of 0.8 V. The depletion layer width at a reverse bias of 1.2 V is 2 μrn. For a reverse bias of 7.2 V, the depletion layer width will be

(a) 4 μm   (b) 4.9 μm
(c) 8 μm   (d) 12 μm **[2007]**

Solution (a)

$$W \propto \sqrt{V_0 + V_{RB}}$$

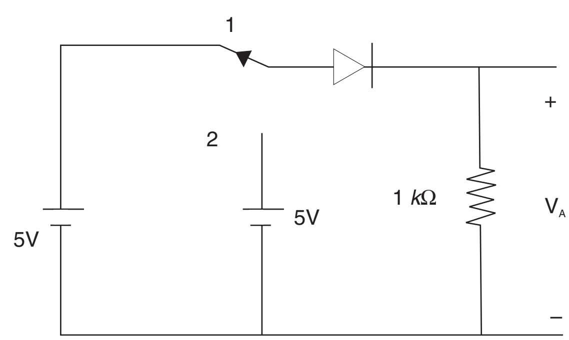
$$\therefore \frac{2}{x} = \frac{\sqrt{0.8+1.2}}{\sqrt{0.8+7.2}}$$

$$\frac{2}{x} = \sqrt{\frac{\cancel{2}}{\cancel{8}}\,{}^{1}\!/_{4}} \qquad x = 4$$

$\therefore$ Width = 4 μm

Hence, the correct option is (a)

20. In the circuit shown below, the switch was connected to position 1 at $t < 0$ and at $t = 0$, it is changed to position 2. Assume that the diode has zero voltage drop and a storage time t_s. For $0 < t \le t_s$, v_R is given by (all in volts)



(a) $V_R = -5$ (b) $V_R = +5$
(c) $0 \le V_R < 5$ (d) $-5 < V_R < 0$ **[2006]**

Solution (a)

Due to change in polarity, only current direction changes.

Initial $I = {}^{5}\!/_{1\,\text{k}} = 5$ mA

$\therefore\ V_R = 5$ V

Now new $V_R = -5$ V

Hence, the correct option is (a)

21. Find the correct match between Group 1 and Group 2

Group 1

E. Varactor diode F. PIN diode

G. Zener diode H. Schottky diode

Group 2

1. Voltage reference
2. High-frequency switch
3. Tuned circuits
4. Current controlled attenuator

(a) E-4, F-2, G-1, H-3
(b) E-2, F-4, G-1, H-3
(c) E-3, F-4, G-1, H-2
(d) E-1, F-3, G-2, H-4 **[2006]**

Solution (c)

Varactor diode is used in tuning circuits as it can act as capacitor.

PIN diode can be used as current controlled attenuator.

Hence, the correct option is (c)

22. In an abrupt $p - n$ junction, the doping concentrations on the p-side and n-side are $N_A = 9 \times 1016$ cm–3 and $N_D = 1 \times 1016$ cm–3, respectively. The $p - n$ junction is reverse biased and the total depletion width is 3 μm. The depletion width on the p-side is

(a) 2.7 μm (b) 0.3 μm
(c) 2.25 μm (d) 0.75 μm **[2004]**

Solution (b)

$W_N N_D = W_P N_A$

Let $W_N = $ x μm

$W_P = (3 - x)$ μm

$$x \times \cancel{10^{16}} = (3-x) \times q \times \cancel{10^{16}}$$

$x = 27 - 9x$

$\boxed{\text{x} = 2.7\ \mu\text{m}}$

$\therefore\ W_p = 0.3$ μm

Hence, the correct option is (b)

23. Consider an abrupt p junction. Let V_{bi} be the built-in potential of this junction and V_R be the applied reverse bias. If the junction capacitance (C_i) is 1 pF for $V_{bi} + V_R$ = 1 V, then for $V_{bj} + V_R = 4$ V, C_i will be

(a) 4 pF (b) 2 pF
(c) 0.25 pF (d) 0.5 pF **[2004]**

Solution (d)

$$C_j \propto \frac{1}{\sqrt{V_{RB} + V_0}}$$

$$\therefore \varepsilon_1 \frac{1}{x} = \sqrt{\frac{4}{1}} \quad x = \tfrac{1}{2} = 0.5 \text{ pF}$$

Hence, the correct option is (d)

24. Match items in Group-1 with items in Group-2, most suitably.

Group-1

P. LED

Q. Avalanche Photodiode

R. Tunnel diode

S. LASER

Group-2

1. Heavy doping
2. Coherent radiation
3. Spontaneous emission
4. Current gain

(a) P-1; Q-2; R-4; S-3
(b) P-2; Q-3; R-1; S-4
(c) P-3; Q-4; R-1; S-2
(d) P-2; Q-1; R-4; S-3 **[2003]**

Solution (c)

LED-spontaneous emission

LASER-coherent radiation.

Hence, the correct option is (c)

25. At 300 K, for a diode current of 1 mA, a certain germanium diode requires a forward bias of 0.1435 V, whereas a certain silicon diode requires a forward bias of 0.718 V. Under the conditions stated above, the closest approximation of the ratio of reverse saturation current in germanium diode to that in silicon diode is

(a) 1 (b) 5
(c) 4 × 103 (d) 8 × 103 **[2003]**

Solution (c)

$$I = I_0\left(e^{VD/\eta V_T} - 1\right)$$

For Ge → = 1

$S_i \to \eta = 2.$

Now $I_{0(Ge)} e^{(V_{D1}/\eta V_T - 1)} = I_{0(Si)} e^{(V_{D2}/\eta V_T - 1)}$

$$\frac{I_0(Ge)}{I_0(Si)} = \frac{e^{\left(\frac{0.718}{2\times 26\times 10^{-3}} - 1\right)}}{e^{\left(\frac{0.1435}{26\times 10^{-3}} - 1\right)}} = \frac{992211.23}{248}$$

≈ 4000.

Hence, the correct option is (c)

26. A particular green LED emits light of wavelength 5490 Å. The energy band gap of the semiconductor material used there is (Planck's constant = 6.626×10^{-34} J-s)

(a) 2.26 eV (b) 1.98 eV
(c) 1.17 eV (d) 0.74 eV **[2003]**

Solution (a)

$$E_G \ (eV) = \frac{1.24}{\lambda(\mu m)} = \frac{1.24}{5490\times10^{-4}} = 2.26\ eV$$

Hence, the correct option is (a)

27. A $p - n$ junction in series with a 100 ohms resistor is forwarded biased so that a current of 100 mA flows. If the voltage across this combination is instantaneously reversed at $t = 0$, current through diodes is approximately given by

(a) 0 mA (b) 100 mA
(c) 200 mA (d) 50 mA **[1998]**

Solution (b)

Due to change in polarity of voltage, only direction of current flow changes but magnitude remains the same.

Hence, the correct option is (b)

28. The built-in potential (diffusion potential) in a $p - n$ junction

(a) is equal to the difference in the Fermi-level of the two sides, expressed in volts
(b) increases with the increase in the doping levels of the two sides
(c) increases with the increase in temperature
(d) is equal to the average of the Fermi levels of the two sides **[1993]**

Solution (a) and (b)

$$V_T = \frac{kT}{q}\log e\frac{N_A N_D}{n_i^2}$$

$\therefore\ V_T \uparrow$ as $N_A, N_D \uparrow$

Also, V_T is equal to the difference in Fermi-level of two sides, expressed in volts.

Hence, the correct option is (a) and (b)

29. The small signal capacitance of an abrupt $P + n$ junction is 1 nf/cm_2 at zero bias. If the built-in voltage is 1 volt, the capacitance at a reverse bias voltage of 99 volts is

(a) 10 (b) 0.1
(c) 0.01 (d) 100 **[1991]**

Solution (b)

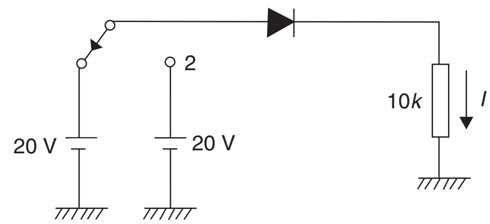
$$C \propto \frac{1}{\sqrt{V_{RB}+V_0}}$$

$$\frac{C_1}{C_2} = \frac{\sqrt{V_{RB2}+V_0}}{V_{RB1}+V_0}$$

$$\frac{1}{x} = \frac{\sqrt{100}}{\sqrt{1}} = x = 0.1\ \text{nF/cm}^2$$

Hence, the correct option is (b)

30. Referring to the below figure, the switch S is in position 1 initially and steady-state condition exists from time $t = 0$ to $t = t_0$. The switch is suddenly thrown into position 2. The current I through the 10 K resistor as a function of time t, from $t = 0$ is? (Give the sketch showing the magnitudes of the current at $t = 0$, $t = t_0$ and $t = \infty$). **[1991]**



Solution: 2 mA

Due to change in polarity, $J = \left[\frac{20}{10\times1000}\right] = 2$ mA

31. In a uniformly doped abrupt $p - n$ junction, the doping level of the n-side is four (4) times the doping level of the p side. The ratio of the depletion layer width of n-side verses p-side is

(a) 0.25 (b) 0.5
(c) 1.0 (d) 2.0 **[1990]**

Solution (a)

$$W_N N_D = W_P N_A$$

$$\frac{W_N}{W_P} = \frac{N_A}{N_D} = \frac{1}{4} = 0.25$$

Hence, the correct option is (a)

32. The switching speed of $P+ N$ junction (having a heavily doped P region) depends primarily on

(a) the mobility of minority carriers in the $P+$–region.
(b) the lifetime of minority carriers in the $P+$–region
(c) the mobility of majority carriers in the N-region
(d) the lifetime of majority carriers in the N-region **[1989]**

Solution: (d)

Switching speed depends upon the injected minority carriers. So, injected minority carries in N region.

Injected minority carriers are majority carriers.

Hence, the correct option is (d)

33. In a Zener diode

(a) only the P-region is heavily doped
(b) only the N-region is heavily doped
(c) both P- and N- regions are heavily doped
(d) both P- and P- regions are lightly doped **[1989]**

Solution (c)

Both regions are heavily doped.

Hence, the correct option is (c)

34. For p-n junction, match the type of breakdown with phenomenon

1. Avalanche breakdown
2. Zener breakdown
3. Punch through

A. Collision of carriers with crystal ions
B. Early effect
C. Rupture of covalent bond due to strong electric field.

(a) 1-B. 2-A. 3-C
(b) 1-C. 2-A. 3-B
(c) 1-A. 2-B. 3-C
(d) 1-A. 2-C. 3-B **[1988]**

Solution (d)

1-A, 2-C, 3-B

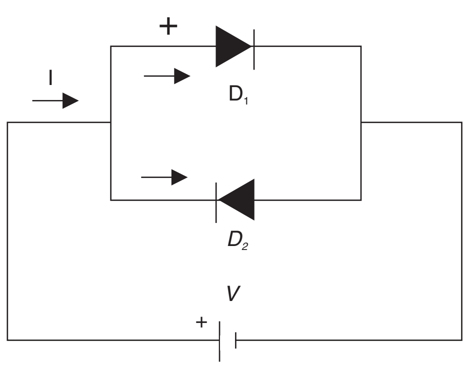
Avalanche breakdown → Collision of carriers with crystal ions.

Zener breakdown → Rupture of covalent bond due to strong electric field.

Punch through → Early effect.

Hence, the correct option is (d)

35. In the circuit shown below, the current voltage relationship when D_1 and D_2 are identical is given by (Assume Ge diodes)



(a) $V = \frac{KT}{q}\sinh\left(\frac{I}{2}\right)$

(b) $V = \frac{KT}{q}\ln\left(\frac{I}{I_O}\right)$

(c) $V = \frac{KT}{q}\sinh^{-1}\left(\frac{I}{2}\right)$

(d) $V = \frac{KT}{q}[\exp(-I)-1]$ **[1988]**

Solution (b)

$$V = \frac{KT}{q}\ln\left(\frac{I}{I_o}\right)$$

Diode D_1 is in forward bias

Diode D_2 is in reverse bias

So, the current through diode D_1 is forward current I_f and current through diode D_2 is

reverse current I_0.

So, total current $\approx I = I_f + I_o$

$$I = I_O\left(e^{V_d/nV_T} - 1\right) = I_O e^{V_d/nV_T} - I_O$$

$$I = \left(I_o e^{V_d/hV_T} - I_o\right) + I_o$$

$$I = I_O e^{V_d/hV_T}$$

$$e^{V_d hV_T} = \frac{I}{I_o}$$

$$\frac{V_d}{hV_T} = \ln\left(\frac{I}{I_o}\right)$$

$$V_d = hV_T \ln\left(\frac{I}{I_o}\right)$$

$$V_d = \frac{KT}{q}\ln\left(\frac{I}{I_o}\right) \qquad \{For\ Ge, h = 1\}$$

$\therefore V_d = V$

$$V = \frac{KT}{q}\ln\left(\frac{I}{I_o}\right)$$

Hence, the correct option is (b)

36. The diffusion capacitance of a $p-n$ junction

(a) decreases with increasing current and increasing temperature
(b) decreases with decreasing current and increasing temperature
(c) increases with increasing current and increasing temperature
(d) does not depend on current and temperature **[1987]**

Solution (b)

Decreases with decreasing current and increasing temperature.

$$Diffusion\ capaci\tan ce = C_D = \tau g = \frac{\tau}{r}$$

$$r = \frac{\eta V_T}{l_f}$$

$$C_D = \frac{Tl_f}{\eta V_T} = \frac{Tl_f}{\eta kT}$$

$C_D \propto I_f$

$C_D \propto$

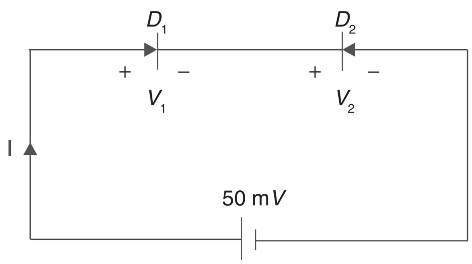
Hence, the correct option is (b)

Five-marks Question

1. For the circuit shown in figure. D_1 and D_2 are identical diodes with utility factor of unity. The thermal voltage $V_T = 25$ mV

(a) Calculate V_1 and V_2.

(b) If the reverse saturation current of D_1 and D_2 are 1 pA then compute the current I through the circuit



[2001]

Solution: n = 1 (for Ge diode)

Diode D_1 is forward bias and diode D_2 is reverse bias.

Cut in voltage of Ge diode

$Vy = 0.2$ V

$\therefore$ 0.2750 mV

So no biasing

Since both diodes are in series $\therefore$ forward current of D_1 is equal to reverse current of D_2

$\therefore I_{D_{F_1}} = I_{D_{r2}}$

$$I_0\left[e^{V_1/nV_T} - 1\right] = I_0\left[e^{-V_2/nV_T} - 1\right]$$

$$e^{-V_1/uV_T} - 1 = -e^{-V_2 . nV_T} + 1$$

$$e^{V_1/uV_T} + e^{0V_2/nV_T} = 2$$

$V_1 + V_2 = 50$ mV .... given

$V_2 = 50 - V_1$

and $n = 1$ .... given

$$\therefore e^{V_1/V_T} + e^{-(50-V_1)/V_T} = 2$$

$$e^{V_1/V_T} + e^{V_1/V_T} . e^{-50/V_T} = 2$$

$$e^{V_1/V_T}[1 + e^{-50/25}] = 2$$

$$e^{V_1/V_T} = \frac{2}{1 + e^{-2}}$$

$$\frac{V_1}{V_T} = \ln\left(\frac{2}{1 + e^{-2}}\right) = 0.566$$

$V_1 = V_T \times 0.566 = 25 \times 10^{-3} \times 0.566$

$V_1 = 14.15$ mV

$V_2 = 50 - 14.15 = 35.85$ mV

(b) $I = I_0\left[e^{V_1/nV_T} - 1\right]$

$$\left.\begin{aligned} I_0 &= 10^{-12} \text{ Amp} \\ n &= 1 \\ V_T &= 25 \text{ mV} \end{aligned}\right\} \text{given}$$

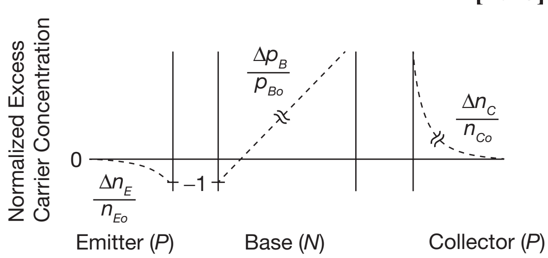
$I = 0.76$ p Amp

Chapter 3

Bipolar Junction Transistor

One-mark Questions

1. For a narrow base PNP BJT, the excess minority carrier concentrations (Δn_E for emitter, Δp_B for base, Δn_C for collector) normalized to equilibrium minority carrier concentrations (n_{E0} for emitter, p_{Bo} for base. n_{C0} for collector) in the quasi-neutral emitter, base and collector regions are shown in the figure. Which one of the following biasing modes is the transistor operating in? **[2017]**



X and *Y* axes are not to scale

(A) Forward active (B) Saturation
(C) Inverse active (D) Cut off

Solution: From the diagram given in problem, emitter-base junction is in reverse- and collector-base junction is in forward bias so it is operating in reverse active region.

Hence, the correct option is (C).

2. The Miller effect in the context of a Common Emitter amplifier explains **[2017]**

(A) an increase in the low-frequency cutoff frequency
(B) an increase in the high-frequency cutoff frequency
(C) a decrease in the low-frequency cutoff frequency
(D) a decrease in the high-frequency cutoff frequency

Solution: From the high frequency response analysis of CE amplifier, we can say that $f_H = \dfrac{1}{2\pi C_{\text{in}} R_{\text{in}}}$

$$C_{\text{in}} = C\pi + C_{\mu}[1 + g_m R_c]$$

$1 + g_m R_c$ is known as miller multiplier due to miller effect in *CE* amplifier input capacitance increases and Hence, there is a decrease in High frequency cut off frequency.

Hence, the correct option is (D).

3. An npn bipolar junction transistor (BJT) is operating in the active region. If the reverse bias across the base-collector junction is increased, then **[2017]**

(A) the effective base width increases and common-emitter current gain increases
(B) the effective base width increases and common-emitter current gain decreases
(C) the effective base width decreases and common-emitter current gain increases
(D) the effective base width decreases and common-emitter current gain decreases

4. If fixed positive charges are present in the gate oxide of an *n*-channel enhancement type MOSFET, it will lead to

(a) a decrease in the threshold voltage
(b) channel length modulation
(c) an increase in substrate leakage current
(d) an increase in accumulation capacitance

[2014]

Solution (a)

This decreases the threshold voltage

Hence, the correct option is (a)

5. An increase in the base recombination of a BJT increases

(a) the common emitter DC current gain β
(b) the breakdown voltage BV_{CEO}
(c) low temperature dry oxidation
(d) low energy ion-implantation

[2014]

Solution: (b)

Base recombination will lead to decrease in β and hence increase in $BV_{c\varepsilon 0.}$

Hence, the correct option is (b)

6. In CMOS technology, shallow *P*-well or *N*-well regions can be formed using
(a) low pressure chemical vapour deposition
(b) low energy sputtering
(c) low temperature dry oxidation
(d) low energy ion-implantation

[2014]

Solution (d)

Wells are formed using low level ion implantation.

Hence, the correct option is (d)

7. In MOSFET fabrication, the channel length is defined during the process of
(a) isolation oxide growth
(b) channel stop implantation
(c) poly-silicon gate patterning
(d) lithography step leading to the contact pads

[2014]

Solution (c)

Poly-silicon gate patterning defines the length.

Hence, the correct option is (c)

8. In a MOSFET operating in the saturation region, the channel length modulation effect causes
(a) an increase in the gate-source capacitance
(b) a decrease in the transconductance
(c) a decrease in the unity-gain cut-off frequency
(d) a decrease in the output resistance

[2013]

Solution (d)

Channel length modulation affects the output resistance.

Hence, the correct option is (d)

9. At room temperature, a possible value for the mobility of electrons in the inversion layer of a silicon *n*-channel MOSFET is
(a) 450 cm^2/V^{-s} (b) 1350 cm^2/V^{-s}
(c) 1800 cm^2/V^{-s} (d) 3600 cm^2/V^{-s}

[2010]

Solution (b)

This is a theoretical value of 1350 cm^2/V^{-s}.

Hence, the correct option is (b)

10. The drain current of a MOSFET in saturation is given by $I_D = K(V_{GS}-V_T)^2$ where K is a constant. The magnitude of the trans-conductance g_m is

(a) $\dfrac{K(V_{GS}-V_T)^2}{V_{DS}}$ (b) $2K(V_{GS}-V_T)$

(c) $\dfrac{I_D}{V_{GS}-V_{DS}}$ (d) $\dfrac{K(V_{GS}-V_T)^2}{V_{GS}}$

[2008]

Solution: (b)

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = 2k(V_{Gs}-V_T)$$

Hence, the correct option is (b)

11. The phenomenon known as 'Early Effect ' in a bipolar transistor refers to a reduction of the effective base-width caused by
(a) electron-hole recombination at the base
(b) the reverse biasing of the base-collector junction
(c) the forward biasing of emitter-base junction
(d) the early removal of stored base charge during saturation-to-cutoff switching

[2006]

Solution (b)

Electron hole recombination is responsible for early effect.

Hence, the correct option is (b)

12. If for a silicon *n-p-n* transistor, the base-to-emitter voltage (V_{BE}) is 0.7 V and the collector-to-base voltage (V_{CB}) is 0.2 V, then the transistor is operating in the
(a) normal active mode
(b) saturation mode
(c) inverse active mode
(d) cutoff mode

[2004]

Solution: (a)

$V_{B\varepsilon} > V_{CB}$

$\therefore$ Normal active mode.

Hence, the correct option is (a)

13. Consider the following statements S_1 and S_2.

S_1 The β of a bipolar transistor reduces if the base width is increased.

S_2 The β of a bipolar transistor increases if the doping concentration in the base is increased.

Which one of the following is correct?
(a) S1 is False and S_2 is True
(b) Both S_1 and S_2 are True
(c) Both S_1 and S_2 are False
(d) S_1 and True and S_2 is False

[2004]

Solution (d)

As base width increases, I_b increases.

$\beta = \dfrac{I_c}{I_b}$ $\therefore$ β will decrease.

Also, if doping in base is increased, then $I_b \uparrow$ $\therefore$ β will reduce.

Hence, the correct option is (d)

14. For an n-channel enhancement type MOSFET, if the source is connected at a higher potential than that of the bulk (i.e., $V_{SB} > 0$), the threshold voltage V_T of the MOSFET will

(a) remain unchanged (b) decrease
(c) change polarity (d) increase

[2003]

Solution (d)

Where $\varphi s = V_T \; ln\dfrac{NA}{V_i^e}$ and $V^e = \dfrac{\sqrt{2qN_A\varepsilon_{si}}}{Cox}$

As V_{SB} Positive, therefore voltage V_T increases.

Hence, the correct option is (d)

15. MOSFET can be used as a

(a) current-controlled capacitor
(b) voltage-controlled capacitor
(v) current-controlled inductor
(d) voltage-controlled inductor

[2001]

Solution: (b)

The MOSFET is a voltage-controlled capacitor.

Hence, the correct option is (b)

16. The effective channel length of a MOSFET in saturation decreases with increase in

(a) gate voltage (b) drain voltage
(c) source voltage (d) body voltage

[2001]

Solution (b)

Increase in drain voltage decreases the channel length.

Hence, the correct option is (b)

17. The break down voltage of a transistor with its base open is BV_{CEO} and that with emitter open is BV_{CBO}, then

(a) $BV_{CEO} = BV_{CBO}$
(b) $BV_{CEO} > BV_{CBO}$
(c) $BV_{CEO} < BV_{CBO}$
(d) BV_{CEO} is not related to $_{CBO}$

[1995]

Solution (c)

$$\beta v_{CEO} = \beta v_{CBO} \sqrt[n]{\frac{1}{\beta}}$$

Where β > 1

$\therefore \beta V_{CEO} < \beta V_{CBO}$.

Hence, the correct option is (c)

18. A BJT is said to be operating in the saturation region if

(a) both junctions are reverse biased
(b) base-emitter junction is reverse biased and base collector junction is forward biased
(c) base-emitter junction is forward biased and base-collector junction reverse biased
(d) both the junctions are forward biased

[1995]

Solution (d)

Both the junctions are forward biased.

Hence, the correct option is (d)

19. The Ebers Moll model is applicable to

(a) Bipolar junction transistors (BJT)
(b) NMOS transistors
(c) Unipolar junction transistors
(d) Junction field-effect

[1995]

Solution (a)

BJT are modelled by EM model

Hence, the correct option is (a)

20. The early-effect in a bipolar junction transistor is caused by

(a) Fast-turn-on
(b) Fast-turn-off
(c) large collector-base reverse bias
(d) large emitter-base forward bias

[1995 & 1999]

Solution (c)

As $V_{CB}\uparrow$, more reverse-biasing causes the base width to reduce, so if V_{CB} is changed, base width can be altered. This process is called early- effect

Hence, the correct option is (c)

21. The threshold voltage of an n-channel MOSFET can be increased by

(a) increasing the channel dopant-concentration
(b) reducing the channel dopant concentration
(c) reducing the GATE oxide thickness
(d) reducing the channel length

[1994]

Solution (a)

$$V_T = V_{T0} + \gamma\left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}\right]$$

$$\gamma = \frac{\sqrt{2qN_A\varepsilon}}{Cox} = \frac{\sqrt{2qN_A\varepsilon}}{3.45\times10^{-11}} \text{ to } x$$

So, as $N_A \uparrow \gamma \uparrow$ and hence $V_T\uparrow$

Hence, the correct option is (a)

22. The transit time of the current carriers through the channel of a JFET decides its_____ characteristic.

(a) source (b) drain
(c) GATE (d) source and drain

[1994]

Solution (b)

The drain decides the transit time.

Hence, the correct option is (b)

23. Channel current is reduced on application of a more positive voltage to the GATE of the depletion mode *n*-channel MOSFET. State True/ False. **[1994]**

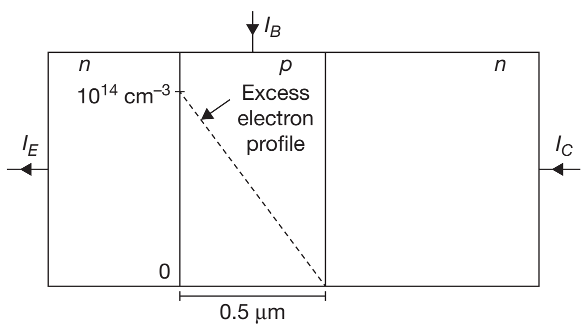
Solution: False

Because for + ve V_{as} current will increase due to forward biasing of Gate.

Two-marks Questions

1. The injected excess electron concentration profile in the base region of an NPN BJT, biased in the active region, is linear, as shown in the figure. If the area of the emitter base junction is 0.001 cm², μ_n = 800 cm²/(V – s) in the base region and depletion layer widths are negligible, then the collector current I_c(in mA) at room temperature is __________.

(Given: Thermal voltage V_T = 26 mV at room temperature, electronic charge $q = 1.6 \times 10^{-19}$C) **[2016]**



Solution: Area of the emitter base junction A = 0.001 cm²,

μ_n = 800 cm²/V–S

Charge $q = 1.6 \times 10^{-19}$C

Voltage V_T = 26 mV

As we know that the current density for NPN transistor is

$$J = q \cdot D_n \cdot \frac{dn}{dx} = \frac{I}{A}$$

$$I = J \cdot A = 1.6 \times 10^{-19} \times 800 \times 26 \times 10^{-3}$$

$$\times \frac{10^{14}}{0.5 \times 10^{-14}} \times 0.001$$

$$= 6.656 \text{ mA}$$

Hence, the correct Answer is (6.656 mA).

2. A BJT is biased in forward active mode. Assume V_{BE} = 0.7 V, kT/q = 25 mV and reverse saturation current I_s = 10^{-13} mA. The transconductance of the BJT (in mA/V) is____. **[2014]**

Solution: 5.76

$$g_m = \frac{I_c}{V_T} \approx \frac{I_E}{V_T}$$

$$I_E = I_s\left(e^{V_{B\varepsilon}/V_T}\right)$$

$$= 10^{-13} \times e^{\frac{0.7 \times 1000}{25}}$$

$$= 0.144 \text{ mA}$$

$$\therefore g_m = \frac{0.144}{26} \times 10^3 = 5.76 \text{ mA/V}$$

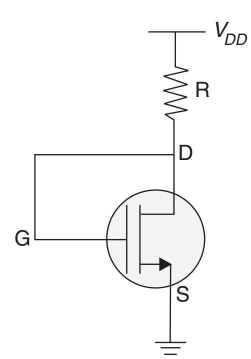
3. A depletion type *N*-channel MOSFET is biased in its linear region for use as a voltage controlled resistor. Assume threshold voltage V_{TH} = –0.5 V, V_{GS} = 2.0 V, V_{DS} = 5 V, W/L = 100, C_{ox} = 10–⁸ F/cm² and μ_n = 800 cm²V/-s. The value of the resistance of the voltage controlled resistor (in Ω) is___. **[2014]**

Solution (500)

$$\gamma d_s = \frac{1}{\mu_n C_0 \times \frac{w}{L}(V_{Gs} - V_T)}$$

$$= \frac{1}{800 \times 10^{-8} \times 100(2.5)} = 500\Omega$$

4. For the *n*-channel MOS transistor shown in figure, the threshold voltage V_{Th} is 0.8 V. Neglect channel length modulation effects. When the drain voltage V_D = 1.6 V, the drain current I_D was found to be 0.5 mA. If V_D is adjusted to be 2 V by changing the values of R and V_{DD}, the new value of I_D (in mA) is



(a) 0.625 (b) 0.75
(c) 1.125 (d) 1.5 **[2014]**

Solution (c)

$$I_D = k_n'(V_{GS} - V_T)^2$$

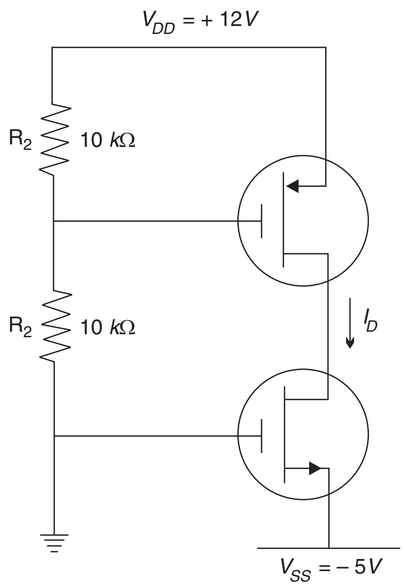
$$0.5 = k_n'(1.6-0.8)^2$$

$$k_n' = \frac{25}{32}\ \text{mA/V}^2$$

$$\text{Now } I_D' = \frac{25}{32}(2-0.8)^2 = 1.25\ \text{mA}$$

Hence, the correct option is (c)

5. For the MOSFET shown in the figure, the threshold voltage $|V_1|$ = 2 V and $K = \frac{1}{2}\mu C\left(\frac{W}{L}\right) = 0.1\ \text{mA/V}^2$. The value of I_D (in mA) is____. **[2014]**



Solution (0.9)

$I_{D1} = I_{D2}$

$I_D = 0.1\ (0-(-5)-2)^2$

$= 0.1(9) = 0.9$ mA

6. The slope of the I_D vs. V_{GS} curve of an n-channel MOSFET in linear region is $10^{-3}\ \Omega^{-1}$ at $V_{DS} = 0.1$ V. For the same device, neglecting channel length modulation, the slope of the $\sqrt{I_D}$ vs. V_{GS} (in $\sqrt{A}$/V) under saturation region is approximately______. **[2014]**

Solution: 0.0707

$$I_D = k_n\left[(V_{GS}-V_T)V_{DS} - \frac{V^2{}_{DS}}{2}\right]$$

$$\frac{\partial I_D}{\partial V_{GS}} = k_n[V_{DS}] = 10^{-3}$$

$k_n \times 0.1 = 10^{-3}$

$k_n = 10^{-2}$

for saturation region

$$I_D = \frac{k_n}{2}\left[V_{GS}-V_T\right]^2$$

$$\sqrt{I_D} = \sqrt{\frac{k_n}{2}} = \left[V_{GS}-V_T\right]$$

$$\frac{\partial\sqrt{I_D}}{\partial V_{GS}} = \sqrt{\frac{k_n}{2}} = \sqrt{\frac{10^{-2}}{2}} = 10^{-1}\times 0.707$$

$= 0.0707\sqrt{A}/V$

7. An ideal MOS capacitor has boron doping-concentration of 10^{15} cm–^{3} in the substrate. When a gate voltage is applied, a depletion region of width 0.5 μm is formed with a surface (channel) potential of 0.2 V. Given that $\varepsilon_0 = 8.854\times 10^{14}$ F/cm and the relative permittivities of silicon and silicon dioxide are 12 and 4, respectively, the peak electric field (in V/μm) in the oxide region is______. **[2014]**

Solution: 2.4

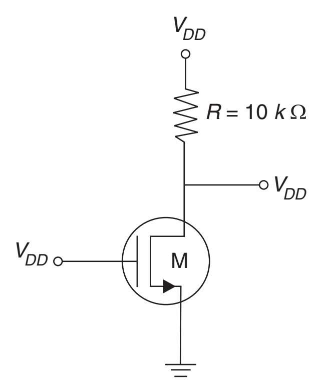
We know that peak electric field

$$E_{\max} = \frac{\varepsilon_s}{\varepsilon_{ox}}\cdot\varepsilon_s$$

$$E_s = \frac{0.2\times 2}{0.5\times 10^{-16}} = \frac{4}{5}\times 10^6\ \text{V/m}$$

$$E_{\max} = \frac{12}{\cancel{4}}\times\frac{\cancel{4}}{5} = 2.4\ \text{V}/\mu\text{m}$$

8. For the MOSFET M_1 shown in the figure, assume W/L = 2, V_{DD} = 2.0 V, $\mu n\ C_{OX}$ = 100 μA/V² and V_{TH} = 0.5 V. The transistor M_1 switches from saturation region to linear region when Vin (in volts) is_____.



[2014]

Solution (1.5)

I_D in saturation region

$$= \frac{k_n'}{2}(V_{Gs}-V_T)^2$$

$$= \frac{100}{2} \times 2(V_{Gs} - V_T)^2$$

At verge of linear region, $V_{Gs} - V_T = V_{Ds}$

$V_{DS} = V_0$

$I_D = 100\ V_0^2\ \mu A = 100 \times 10^{-6}\ V_0^2\ (A)$

Now $V_{DD} = 10^4\ I + V_0$

$V_{DD} = 2$ V

$$2 = \cancel{10^4} \times \cancel{10^{-4}} V_0^2 + V_0$$

$V_0^2 + V_0 - 2 = 0$

$(V_0 + 2)(V_0 - 1) = 0\ V_0 = 1\ V$

$V_{in} - V_T = 1$

$V_{in} = 1 + 0.5 = 1.5$ V

9. Consider two BJTs biased at the same collector current with area A_1 = 02 μm × 0.2 μm and A_2 = 300 μm. Assuming that all other device parameters are identical kT/q = 26 mV, the intrinsic carrier concentrations is 1 × 10^{10}cm–^{3}, and $q = 1.6 \times 10^{-19}$C, the difference between the base-emitter voltages (in mV) of the two BJTs (i.e., $V_{BE1} - V_{BE2}$) is______. **[2014]**

Solution: 380.28

$I_s \propto$ Area

Now $I_{c1} = I_{c2}$

$$I_{s1} e^{\frac{V_{BE1}}{\eta V_T}} = I_{s2} e^{V_{BE2}/\eta V_T}$$

$$e^{\frac{V_{BE1} - V_{BE2}}{V_T}} = \frac{I_{S2}}{I_{S1}} = \left(\frac{300 \times 300}{0.2 \times 0.2}\right)$$

$$V_{BE1} - V_{BE2} = 26 \ln\left(\frac{300 \times 300}{0.2 \times 0.2}\right) = 380.28 \text{ mV}$$

10. A BJT in a common-base configuration is used to amplify a signal received by a 50 Ω antenna. Assume kT/q = 25 mV. The value of the collector bias current (in mA) required to match the input impedance of the amplifier to the impedance of the antenna is________. **[2014]**

Solution: 0.5

γ_C = characteristics impedance = 50 Ω

$$I_c = \frac{V_T}{Ve} = \frac{25}{50} \times 10^{-3} = 0.5 \text{ mA}$$

11. The source of a silicon (n_I = 10^{10} per cm^3) n-channel MOS transistor has an area of 1 sq μrn and a depth of 1 μm. If the dopant density in the source is 10^{19} cm^{-3}, the number of holes in the source region with the above volume is approximately

(a) 10^7 (b) 100
(c) 10 (d) 0 **[2012]**

Solution (d)

Volume = area × depth

= 10–^{8} × 10–^{4} = 10–^{12} cm^3

$\therefore N_D = 10^{19}$ atoms/cm^3

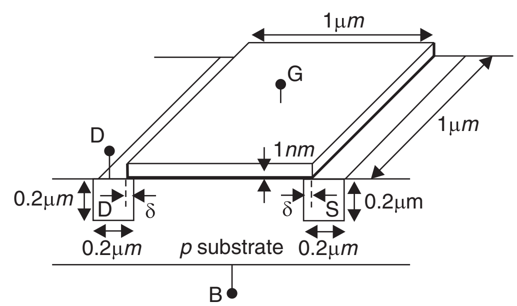
$$P = \frac{n_i^2}{N_D} = \frac{10^{20}}{10^{19}} = 10 \text{ atoms/cm}^3$$

Number of holes in 10^{-12} cm^3 volume $P = 10 \times 10^{-12}$ =10–^{11} ≈ 0.

Hence, the correct option is (d)

Common Data for Questions 11 and 12

In the three-dimensional view of a silicon n channel MOS transistor shown below, δ = 20 nm. The transistor is of width 1 μm. The depletion width formed at every p--n junction is 10 nm. The relative permittivities of Si and SiO_2, respectively, are 11.7 and 3.9, and $\varepsilon_0 = 8.9 \times$ 10–^{12} F/m.



12. The source-body junction capacitance is approximately

(a) 2 fF (b) 7 fF
(c) 2 pF (d) 7 pF **[2012]**

Solution (a)

Source body I_x^n capacitance

$$C_j = \frac{\varepsilon A}{d}$$

$\varepsilon = \varepsilon_\gamma$ (Si) × εo = 11.7 × 8.85 × 10^{-12} f/m

A = 1 μm × 0.2 μm = 0.2 × 10–^{12} m^2

d = 10 nm = 10–^{8} m

$$\therefore C_j = \frac{11.7 \times 8.85 \times 10^{-12} \times 0.2 \times 10^{-12}}{10^{-8}}$$

$C_j \cong 2$ ff

Hence, the correct option is (a)

13. The gate-source overlap capacitance is approximately

(a) 0.7 fF (b) 0.7 pF
(c) 0.35 fF (d) 0 .24 pF **[2012]**

Solution (a)

$$C_g = \frac{\varepsilon A}{d}$$

where $\varepsilon = \varepsilon_{(SiO_2)} \times \varepsilon_o$

$A = 1$ µm × δ = 1 µm × 20 nm = 20 × 10–[15] m

$d = 10–^{9}$ m

$$C_g = \frac{3.9 \times 8.85 \times 10^{-12} \times 20 \times 10^{-5}}{10^{-9}} = 0.7 \text{ ff}$$

Hence, the correct option is (a)

14. For a BJT, the common-base current gain α = 0.98 and the collector base junction reverse bias saturation current I_{CO} = 0.6 µA. This BJT is connected in the common emitter mode and operated in the active region with a base drive current I_B = 20 µA. The collector current I_C for this mode of operation is

(a) 0.98 mA (b) 0.99 mA
(c) 1.0 mA (d) 1.01 mA **[2011]**

Solution (d)

$I_c = \beta I_b + (1 + \beta)\, I_{c0}$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{0.02} = 49$$

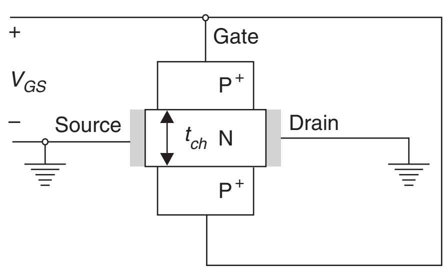
I_c = 49 × 20 + 50 × 0.6

= 980 + 30 = 1010 µA = 1.01 mA

Hence, the correct option is (d)

Common Data for Questions 14 and 15.

The channel resistance of an *n*-channel JFET shown in the figure below is 600 Ωwhen the full channel thickness (t_{ch}) of 10 µrn is available for conduction. The built-in voltage of the gate *P*+*N* junction (V_{bi}) is –1. When the gate to source voltage (V_{GS}) is 0 V, the channel is depleted by 1 µrn on each side due to the built-in voltage and hence the thickness available for conduction is only 8 µm.



15. The channel resistance when V_{GS} = 0 V is

(a) 480 Ω (b) 600 Ω
(c) 750 Ω (d) 1000 Ω **[2011]**

Solution: (c)

Hence, the correct option is (c)

16. The channel resistance when V_{GS} = –3 V is

(a) 360 Ω (b) 917 Ω
(c) 1000 Ω (d) 3000 Ω **[2011]**

Solution: (c)

Channel resistance ∝ channel width

V_{Gs} = 0 V, W = 1 V

V_{Gs} = –3 V, W = ?

$$\frac{1}{x} = \frac{\sqrt{-1}}{\sqrt{-3-1}} \qquad x = 2\ \mu\text{m}$$

∴ Net thickness for conduction will be 6 µm.

Now, $t_{ch1} \times R_1 = t_{ch2} \times R_2$

for Q. 3.41. 10 × 600 = 8 × R_2

R_2 = 750 Ω

Q. 3.42. 10 ×600 = 6 × R_2

R_2 = 1000 Ω

Hence, the correct option is (c)

17. In a uniformly doped BJT, assume that N_E, N_B and N_c are the emitter, base and collector dopings in atoms cm^{-3}, respectively. If the emitter injection efficiency of the BJT is close to unity, which one of the following conditions is TRUE?

(a) $N_E = N_B = N_C$
(b) $N_E >> N_B$ and $N_B > N_C$
(c) $N_E = N_B$ and $N_B < N_C$
(d) $N_E < N_B < N_C$ **[2010]**

Solution: (b)

Emitter is heavily doped to provide carriers.

Hence, the correct option is (b)

18. Consider the following two statements about the internal conditions in an *n*-channel MOSFET operating in the active region

S_1: The inversion charge decreases from source to drain

S_2: The channel potential increases from source to drain

Which of the following is correct?

(a) Only S_2 is true
(b) Both S_1 and S_2 are false
(c) Both S_1 and S_2 are true, but S_2 is not a reason for S_1
(d) Both S_1 and S_2 are true, and S_1 is a reason for S_2 **[2009]**

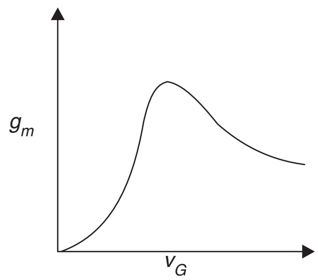
Solution (d)

Both are true but both have no relation.

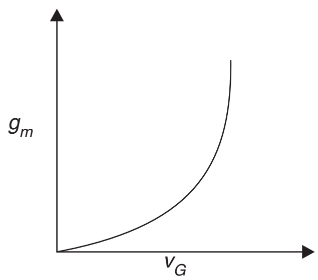
Hence, the correct option is (d)

19. The measured trans-conductance g_m of an NMOS transistor operating in the linear region is plotted against the gate voltage V_G at a constant drain voltage V_D. Which of the following figures represents the expected dependence of g_m on V_G? **[2008]**

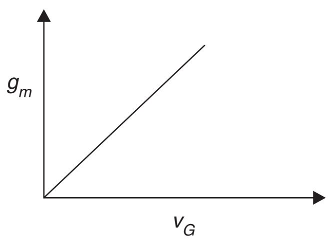
(a)



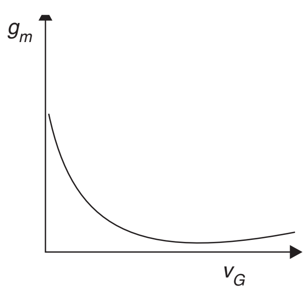
(b)



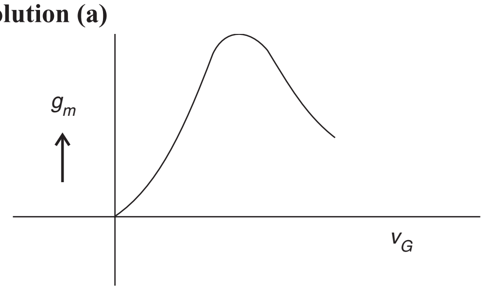
(c)



(d)



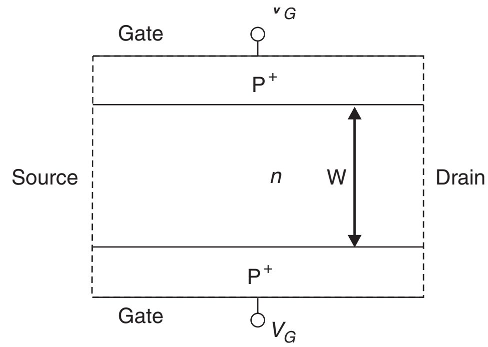
Solution (a)



Initially g_m increases as V_G increases but as more and more V_G is increased, mobility will get decreased. So, g_m will start reducing.

Hence, the correct option is (a)

20. The cross section of a JFET is shown in the following figure. Let V_G be –2 V and let V_p be the initial pinch-off voltage. If the width W is doubled (with other geometrical parameters and doping levels remaining the same), then the ratio between the mutual transconductances of the initial and the modified JFET is



(a) 4

(b) $\left(\dfrac{1-\sqrt{2/V_P}}{1-\sqrt{1/(2V_P)}}\right)$

(c) $\dfrac{1}{2}\left(\dfrac{1-\sqrt{2/V_P}}{1-\sqrt{1/(2V_P)}}\right)$

(d) $\dfrac{1-\left(2/\sqrt{V_P}\right)}{1-\left(1/\left(2\sqrt{V_P}\right)\right)}$

[2008]

Solution (b)

$$g_m = k_N'\left[1-\sqrt{\frac{V_{bi}-V_{Gs}}{Vp}}\right]$$

$k_{N'}$ = trans conductance parameter

$k_{N'} \propto w$

$$\therefore \frac{g_{m1}}{g_{m2}} = \frac{k_{n1}}{k_{n2}}\left[\frac{1-\sqrt{\dfrac{V_{bi}-V_{Gs1}}{Vp_1}}}{1-\sqrt{\dfrac{V_{bi}-V_{Gs2}}{Vp_2}}}\right]$$

$$Vp = \frac{qN_D a^2}{2\varepsilon} \quad 2a = \text{width of tranistor}$$

$\therefore$ Vp will be four times

$$\frac{g_{m1}}{g_{m2}} = \frac{1}{2}\left[\frac{1-\sqrt{\dfrac{0-(-2)}{Vp}}}{1-\dfrac{0-(-2)}{4Vp}}\right]$$

$$= \frac{1}{2}\left[\frac{1-\sqrt{2/Vp}}{1-\sqrt{1/2Vp}}\right]$$

Hence, the correct option is (b)

21. An MOS capacitor made using p-type substrate is in the accumulation mode. The dominant charge in the channel is due to the presence of

(a) holes
(b) electrons
(c) positively charged ions
(d) negatively charged ions **[2005]**

Solution (a)

The dominant charge depends on the substrate type.

Hence, the correct option is (a)

22. Consider the following statements S_1 and S_2.

S_1: The threshold voltage (V_T) of a MOS capacitor decreases with increase in gate oxide thickness

S_2: The threshold voltage (V_T) of a MOS capacitor decreases with increase in substrate doping concentration

Which one of the following is correct?
(a) S_1 is False and S_2 is True
(b) Both S_1 and S_2 are True
(c) Both S_1 and S_2 are False
(d) S_1 is True and S_2 is False **[2004]**

Solution (c)

$$V_T = V_{T0} + \gamma\left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}\right]$$

$$\gamma = \frac{\sqrt{2qN_A\varepsilon}}{C_{ox}} = \frac{\sqrt{2qN_A\varepsilon}}{3.45\times10^{-11}}t_{ox}$$

As $t_{ox}\uparrow \gamma\uparrow V_T\uparrow$
$N_A\uparrow \gamma\uparrow V_T\uparrow$
$\therefore$ Both statements are false.

Hence, the correct option is (c)

23. The drain of an n-channel MOSFET is shorted to the gate so that $V_{GS} = V_{DS}$. The threshold voltage (V_T) of MOSFET is 1 V. If the drain current (I_D) is 1 mA for V_{GS} = 2 V, then for V_{GS} = 3 V, I_D is
(a) 2 mA (b) 3 mA
(c) 9 mA (d) 4 mA **[2004]**

Solution: (d)

$I_D \propto (V_{Gs} - V_T)^2$

$$\frac{1}{I_{D2}} = \frac{(2-1)^2}{(3-1)^2} \qquad I_{D2} = 4\text{ mA}$$

Hence, the correct option is (d)

24. When the gate-to-source voltage (V_{GS}) of a MOSFET with threshold voltage of 400 mV, working in saturation, is 900 mV, the drain current is observed to be 1 mA. Neglecting the channel width modulation effect and assuming that the MOSFET is operating at saturation, the drain current for an applied V_{GS} of 1400 mV is
(a) 0.5 mA (b) 2.0 mA
(c) 3.5 mA (d) 4.0 mA **[2003]**

Solution (d)

$I_D = k(V_{GS} - V_T)^2$

$$\frac{I_{D1}}{I_{D2}} = \frac{(V_{GS1}-V_T)^2}{(V_{GS2}-V_T)^2}$$

$$\frac{1}{x} = \left(\frac{900-400}{1400-400}\right)^2 = \frac{1}{4}$$

$\therefore I_{D2} = 4$ mA

Hence, the correct option is (d)

25. An n-channel JFET has I_{DSSS} = 2 mA and V = –4 V. It has transconductance g_m (in mV/V). An applied GATE to source voltage V_{GS} of –2 V is
(a) 0.25 (b) 0.5
(c) 0.75 (d) 1.0 **[1999]**

Solution (b)

$$\text{gm} = \frac{-2I_{DSS}}{VP}\left[1-\frac{Vqs}{VP}\right]$$

$$= \frac{-\cancel{2}\times\cancel{2}}{(-\cancel{4})}\left[1-\frac{2}{4}\right] = 1/2 = 0.5\text{ mV/V}$$

Hence, the correct option is (b)

26. In a bipolar transistor at room temperature, if the emitter current is doubled, the voltage across its base-emitter junction
(a) doubles
(b) halves
(c) increases by about 20 mV
(d) decreases by about 20 mV **[1997]**

Solution (c)

$$I = I_{E0}[e^{V_{B\varepsilon}/\eta V_T} - 1]$$

for transistors, x = 1

$$\frac{I_1}{I_2} = \frac{I_{E0}\left[e^{\frac{V_{B\varepsilon1}}{\eta V_T}} - 1\right]}{I_{E0}\left[e^{\frac{V_{B\varepsilon2}}{\eta V_T}} - 1\right]} \qquad I_2 = 2I_1$$

$$\frac{1}{2} = \frac{e^{\frac{V_{B\varepsilon1}}{V_T}-1}}{e^{\frac{V_{B\varepsilon2}}{V_T}-1}}$$

$$\frac{1}{2} \approx \frac{e^{\frac{V_{B\varepsilon1}}{V_T}}}{e^{\frac{V_{B\varepsilon2}}{V_T}}}$$

$$\frac{1}{2} = e^{\frac{V_{B\varepsilon1}-V_{B\varepsilon2}}{V_T}}$$

$$-0.693 = \frac{V_B\varepsilon_1 - V_B\varepsilon_2}{V_T}$$

$V_T = 26 \times 10^{-3}$ V.

$V_B\varepsilon_2 - V_B\varepsilon_1 \cong 18$ mV.

$\therefore V_B\varepsilon$ increases by about 20 mV.

Hence, the correct option is (c)

27. If a transistor is operating with both of its junctions forward biased, but with the collector base forward bias greater than the emitter-base forward bias, then it is operating in the

(a) forward active mode
(b) reverse saturation mode
(c) reverse active mode
(d) forward saturation anode **[1996]**

Solution: (b)

Hence, the correct option is (b)

28. In a bipolar junction transistor (match the following).

List-I

A. The current gain increases
B. The collector breakdown voltage increases
C. The cut off frequency increases

List-II

1. The base doping is increased and the base width is reduced
2. The base doping is reduced and the base width is increased
3. The base doping the base width are reduced
4. The emitter area is increased and the collector area is reduced
5. The base doping and the base width are increased **[1995]**

Solution: (A → 3, B → 1, C → 1)

The current gain increases with the base doping base width.

The collector breakdown voltage increase causes the base doping increase and the base width is reduced.

The cut off frequency increases causes the base doping to increase and the base width is reduced

29. In a JFET

List-I

A The pinch-off voltage decreases
B. The transconductance increases
C. The transit time of the carriers in the channel is reduced

List-II

1. The channel doping is reduced
2. The channel length is increased
3. The conductivity of the channel is increased
4. The channel length is reduced
5. The GATE area is reduced **[1995]**

Solution: (A → 3, B → 3, C → 4)

(i) $V_p = \dfrac{qN_Da^2}{2\varepsilon}$

So, as $N_D\downarrow$, $V_P\downarrow$

(ii) $g_m\uparrow$ means conductivity increases.

(iii) Transit time $\propto \dfrac{1}{\text{length of channel}}$

30. In a transistor having finite β, forward bias across the base emitter junction is kept constant and the reverse bias across the collector base junction is increased. Neglecting the leakage across the collector base junction and the depletion region generations current, the base current will ___ (increase/decrease/remains constant). **[1992]**

Solution: (decrease)

As the reverse bias increases at CB (collector base) junction, the collector current (I_c) increases and the effective base width decreases. So the recombination in base decreases.

As reverse bias ↑, base width ↓, hence I_B will decrease.

31. An *n*-channel JFET has a pinch-off voltage $V_p = -5$ V, V_{DS}(max) = 20 V, and $g_m = 2$ mA/V. The min 'ON' resistance is achieved in the JFET for

(a) $V_{GS} = -7$ V and $V_{DS} = 0$ V
(b) $V_{GS} = 0$ V and $V_{DS} = 0$ V
(c) $V_{GS} = 0$ V and $V_{DS} = 20$ V
(d) $V_{GS} = -7$ V and $V_{DS} = 20$ V **[1992]**

Solution (b)

For minimum γ_{ds}, V_{as} should be +ve and large and V_{DS} should be minimum.

Hence, the correct option is (b)

32. The 'Pinch-off' voltage of a JFET is 5.0 volts, its 'cut-off' voltage is

(a) $(5.0)^{1/2}$ V (b) 2.5 V
(c) 5.0 V (d) $(5.0)^{3/2}$ V **[1990]**

Solution: (c)

Cut off voltage = pinch - off voltage = 5 V.

Hence, the correct option is (c)

33. Which of the following effects can be caused by a rise in the temperature

(a) increase in MOSFET current (I_{DS})
(b) increase in BJT current (I_C)
(c) decrease in MOSFET current (I_{DS})
(d) decrease in BJT current (I_C) **[1990]**

Solution: (b) and (c)

I_c - $\beta I_b + (1 + \beta)\, I_{CO}$

As temperature increases, I_{CO} also increases. So, the current (I_C) increases in BJT with rise in temperature.

∵ Mobility decreases as temperature increases.

$T\uparrow \rightarrow \mu\downarrow$

As temperature increases, I_c increases due to increase in leakage current whereas I_{DS} decreases due to reduction in mobility of carriers.

Hence, the correct option is (b) and (c)

34. In a MOSFET, the polarity of the inversion layer is the same as that of the
(a) charge on the GATE-EC-electrode
(b) minority carriers in the drain
(c) majority carriers in the substrate
(d) majority carriers in the source **[1989]**

Solution (d)

In MOSFET, majority carriers are responsible for action.

Hence, the correct option is (d)

35. In an *n*-channel JFET, V_{GS} is held constant. V_{DS} is less than the breakdown voltage. As V_{DS} is increased
(a) conducting cross-sectional area: of the channel '*S*' and the channel current density '*J*' both increase
(b) '*S*' decreases and '*J*' decreases
(c) '*S*' decreases and '*J*' increases
(d) '*S*' increases and '*J*' decreases **[1988]**

Solution (c)

As V_{DS} is increased, reverse biasing increases. So, cross-selection area *S* decreases and hence current density increases.

Hence, the correct option is (c)

36. In MOSFET devices, the *n*-channel type is better than the *P*-channel type in the following respects:
(a) it has better noise immunity
(b) it is faster
(c) it is TTL compatible
(d) it has better drive capability **[1988]**

Solution (b)

Due to high mobility of elections n-channel type MOSFET is faster.

Hence, the correct option is (b)

37. The pinch off voltage for an n-channel JFET is 4 V. When V_{GS} = 1 V, the pinch-off occurs for V_{DS} equal to
(a) 3 V (b) 5 V
(c) 4 V (d) 1 V

[1987]

Solution (a)

Given that,

$V_p = 4$ V

$V_{GS} = 1$ V

$|V_{DS}| = |V_p| - |V_{GS}|$

$|V_{DS}| = 4 - 1 = 3$V

Hence, the correct option is (a)

UNIT V

ANALOG ELECTRONICS

EXAM ANALYSIS

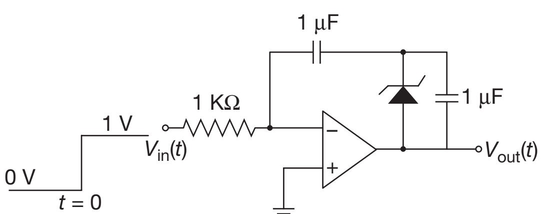
Exam Year	92	93	94	95	96	97	98	99	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14-1	14-2	14-3	14-4	15			16			17		18	19
																											Set 1	Set 2	Set 3	Set 1	Set 2	Set 3	Set 1	Set 2		
1 Mark Questions	-	6	5	10	2	3	13	4	8	3	4	5	4	3	2	2	1	-	3	2	2	2	4	3	3	3	4	3	4	3	3	6	3	3	3	4
2 Mark Questions	6	2	2	-	5	2	-	4	2	5	5	8	5	11	7	11	8	8	3	4	3	6	3	-	2	2	4	3	2	3	3	5	3	4	5	6
5 Mark Questions	1	1	2	2	2	4	4	1	1	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Total Marks	**17**	**15**	**19**	**20**	**22**	**27**	**33**	**18**	**17**	**23**	**14**	**21**	**14**	**25**	**16**	**24**	**17**	**16**	**9**	**10**	**8**	**14**	**10**	**3**	**7**	**7**	**12**	**9**	**8**	**9**	**9**	**16**	**9**	**11**	**13**	**16**
Chapter-wise marks distribution																																				
Operational Amplifiers	6	2	1	2	1	3	1	1	6	5	3	6	4	5	2	8	4	2	3	1	2	3	5	1	2	1		8			4		3		3	-
Diode Applications	2	1	-	-	2	-	1	2	-	2	2	3	4	2	6	3	1	6	-	4	1	4	-	-	1	1		1			6		1		3	2
BJT Analysis	2	4	4	2	4	2	7	1	5	1	2	4	1	7	6	4	4	2	6	3	3	2	3	1	3	5		5			4		4		-	-
FET and MOSFET	2	-	1	1	-	-	2	-	-	-	4	2	-	8	1	8	4	4	-	2	2	2	2	1	-	-		6			7		3		2	8
Frequency Response	-	1	-	2	3	-	1	4	-	2	-	-	2	-	-	-	-	-	-	-	-	2	-	-	-	-		-			-		-		-	-
Feedback Amplifiers	-	1	-	1	-	2	1	3	-	-	1	2	1	1	1	1	-	-	-	-	-	1	-	-	1	-		-			-		1		-	-
Oscillator Circuits	-	-	2	-	2	-	-	-	1	2	2	2	2	-	-	-	-	-	-	-	-	-	-	-	-	-		-			1		-		-	-
Power Amplifiers	-	1	1	2	-	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		-			-		-		-	-
Multi-vibrators and 555 Timers	-	-	-	-	-	-	-	-	-	1	-	2	-	2	-	-	4	2	-	-	-	-	-	-	-	-		-			1		-		-	-

Chapter 1

Operational Amplifiers

One-mark Questions

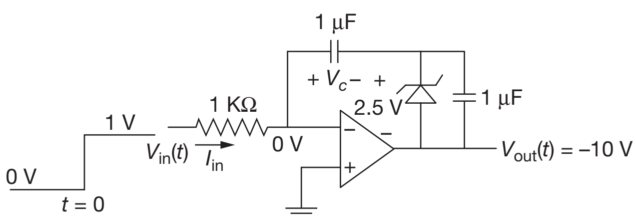
1. In the circuit shown below, the op-amp is ideal and Zener voltage of the diode is 2.5 volts. At the input, unit step voltage is applied, i.e., $v_{IN}(t) = u(t)$ volts. Also, at $t = 0$, the voltage across each of the capacitors is zero. **[2018]**



The time t, in milliseconds, at which the output voltage v_{OUT} crosses −10V is

(A) 2.5 (B) 5
(C) 7.5 (D) 10

Solution:



Input current $I_{in} = 1$ mA

Now the voltage can be calculated as

$$V_c = \frac{1}{C}\int idt$$

$$V_c = \frac{1}{1\times10^{-6}}\int 1\times10^{-3}t$$

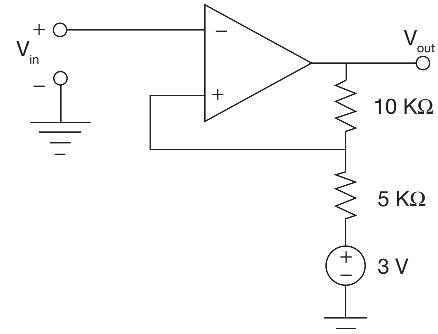
$$\Rightarrow \quad V_c = 1000t$$

$$1000\,t = 7.5$$

$$t = 7.5 \text{ msec}$$

Hence, the correct option is (C)

2. For the operational amplifier circuit shown in the figure, the output saturation voltage are ±15 V. The upper and lower threshold voltages for the circuit are, respectively, **[2017]**



(A) +5 V and −5 V (B) +7 V and −3 V
(C) +3 V and −7V (D) +3 V and −3 V

Solution: By KCL

$$\frac{15 - V_{UTP}}{10K} = \frac{V_{UTP} - 3}{5K}$$

$$15 - V_{UTP} = 2V_{UTP} - 6$$

$$3V_{UTP} = 21$$

$$\therefore V_{UTP} = 7\text{ V}$$

by KCL

$$\frac{-15 - V_{LTP}}{10K} = \frac{V_{LTP} - 3}{5K}$$

$$-15 - V_{LTP} = 2V_{LTP} - 6$$

$$3V_{LTP} = -9$$

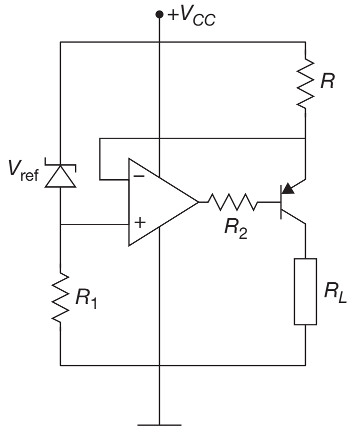
$$V_{LTP} = -3V.$$

∴ Upper threshold voltage = +7 V

Lower threshold Voltage= −3 V.

Hence, the correct option is (B).

3. Consider the constant current shown in the figure below. Let β represent the current gain of the transistor



The load current I_0 through R_L is **[2016]**

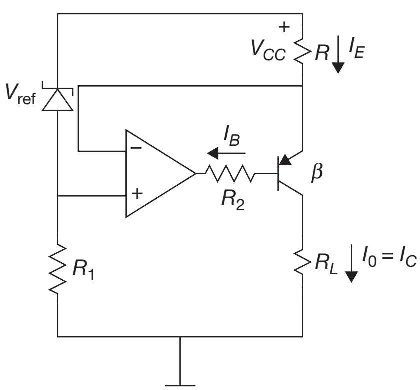
(A) $I_0 = \left(\dfrac{\beta+1}{\beta}\right)\dfrac{V_{ref}}{R}$

(B) $I_0 = \left(\dfrac{\beta}{\beta+1}\right)\dfrac{V_{ref}}{R}$

(C) $I_0 = \left(\dfrac{\beta+1}{\beta}\right)\dfrac{V_{ref}}{2R}$

(D) $I_0 = \left(\dfrac{\beta}{\beta+1}\right)\dfrac{V_{ref}}{2R}$

Solution: Re-draw the given circuit



$$I_E = I_B + I_C = (1+\beta)\cdot I_B$$

$$\frac{V_{\text{ref}}}{R} = (1+\beta)\cdot I_B$$

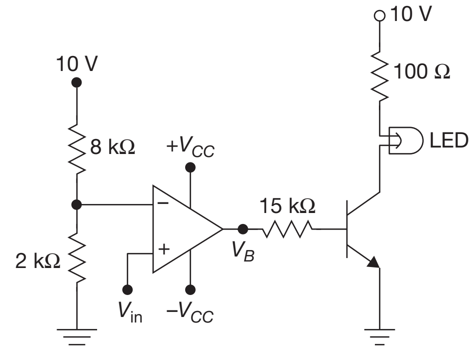
$$I_C = \beta\cdot I_B$$

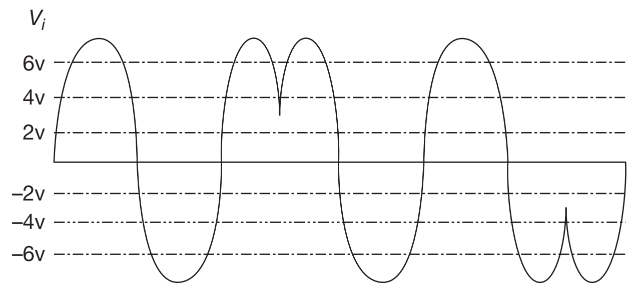
$$I_B = \frac{I_C}{\beta}$$

$$\therefore \quad I_C = I_O = \left(\frac{\beta}{1+\beta}\right)\cdot\frac{V_{\text{ref}}}{R}$$

Hence, the correct option is (B).

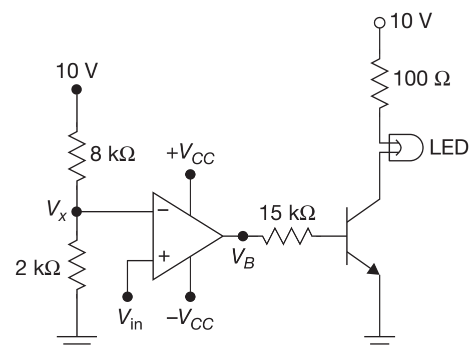
4. The following signal V_i of peak voltage 8 V is applied to the non inverting terminal of an ideal opamp. The transistor has $V_{BE} = 0.7$ V, $\beta = 100$; $V_{LED} = 1.5$ V, $V_{cc} = 10$ V and $-V_{cc} = -10$ V





The number of times the LED glows is ______. **[2016]**

Solution:



If V_B is positive then only the transistor is in ON state

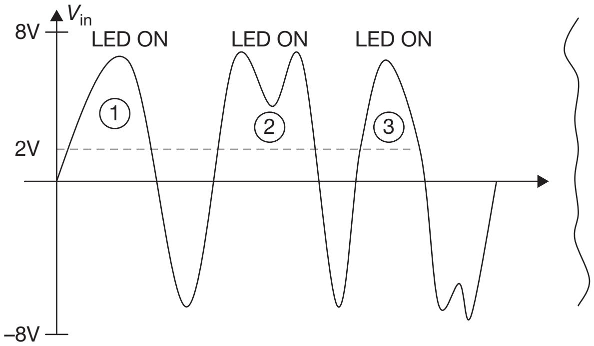
$\therefore \quad V_{in} > V_x$: LED glows

$\therefore \quad V_x = \dfrac{10x2}{10} = 2$ V

From the given V_{in}

$$V_{in} > 2\text{ V},\ V_B = +V_{\text{sat}}$$

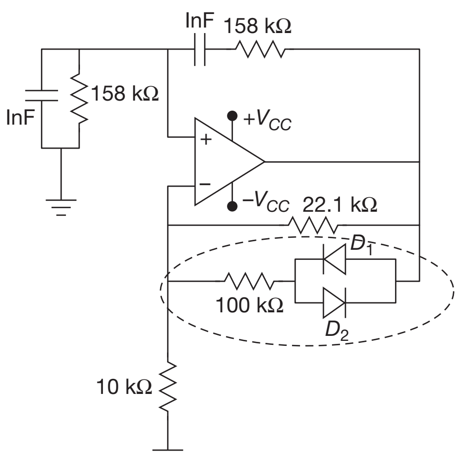
Then LED glows

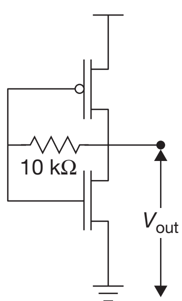


From the above data LED glows 3 times.

Hence, the correct Answer is (3).

5. Consider the oscillator circuit shown in the figure. The function of the network (shown in dotted lines) consisting of the 100 kΩ resistor in series with the two diodes connected back to back is to **[2016]**





(A) Introduce amplitude stabilization by preventing the op amp from saturating and thus producing sinusoidal oscillation of fixed amplitude.

(B) Introduce amplitude stabilization by forcing the opamp to swing between positive and negative saturation and thus producing square wave oscillations of fixed amplitude.

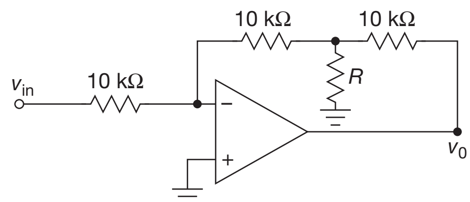
(C) Introduce frequency stabilization by forcing the circuit to oscillate at a single frequency.

(D) Enable the loop gains to take on a value that produces square wave oscillations.

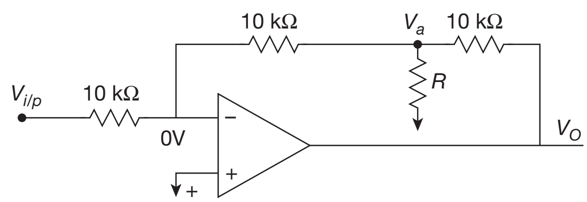
Solution: Here two diodes D_1 and D_2 are connected back to back. In case when diodes are connected back to back, the device is prevented since this condition introduces the stability avoiding the runoff or saturation of the device.

Hence, the correct option is (A).

6. In the circuit shown, assume that the op-amp is ideal. If the gain (v_o/v_{in}) is -12, the value of R (in kΩ) is ______ **[2015]**



Solution:



Write KCL at V_a

$$\frac{V_a}{R}+\frac{V_a-V_o}{10\,k\Omega}+\frac{V_a}{10\,k\Omega}=0$$

$$\Rightarrow \qquad V_a\left[\frac{1}{R}+\frac{1}{5}\right]=\frac{V_o}{10} \qquad \text{(a)}$$

KCL at 0 V

$$\frac{0-V_i}{10\,k\Omega}+\frac{0-V_a}{10\,k\Omega}=0$$

$$V_a=-V_i \qquad \text{(b)}$$

From both equations (a) and (b)

$$-V_i\left(\frac{5+R}{5R}\right)=\frac{V_o}{10}$$

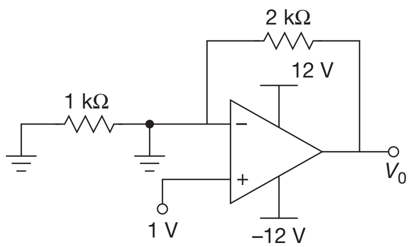
but $$\frac{V_o}{V_i}=-12$$

$$\frac{(5+R)10}{5R}=12$$

$$\Rightarrow \qquad R=1\text{ k}\Omega$$

Hence, the correct Answer is (1).

7. Assuming that the opamp in the circuit shown below is ideal, the output voltage V_o (in volts) is ________. **[2015]**



Solution: $V_- = 0$

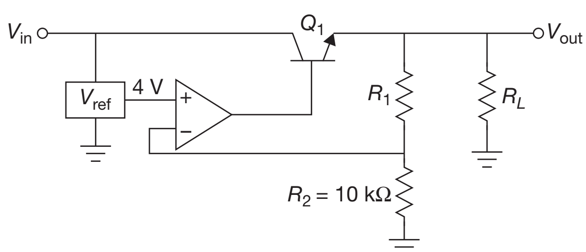
$V_+ = 1$ V

$V_+ > V_-$

so $V_o = +V_{sat} = +12$ V

Hence, the correct Answer is (11 to 12).

8. For the voltage regulator circuit shown, the input voltage (V_{in}) is 20 V ± 20% and the regulated output voltage (V_{out}) is 10 V. Assume the opamp to be ideal. For a load R_L drawing 200 mA, the maximum power dissipation in Q_1 (in Watts) is ________. **[2015]**

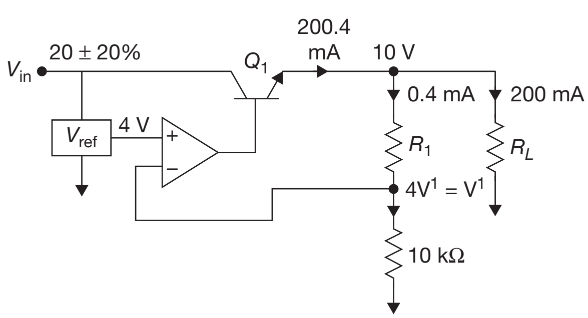


Solution: $P_{Q_1}(\max) = V_{CE(\max)} \times I_{C(\max)}$

$$V_{C\max} = 20 + 20\% = 24 \text{ V}$$

$$V_E = 10 \text{ V}$$

$$V_{CE\max} = 14 \text{ V}$$



$$\Rightarrow \quad V' = \frac{10 \times 10 \text{ k}\Omega}{10 \text{ k}\Omega} = 4 \text{ V}$$

$$R_1 = 15 \text{ k}\Omega$$

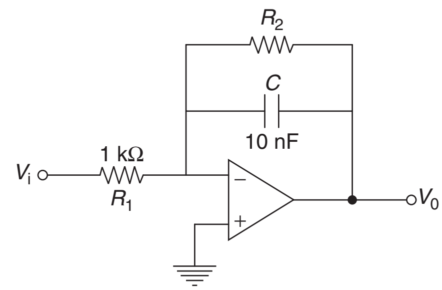
Current across $R_1 = \dfrac{10-4}{15} = 0.4$ mA

$$I_C = 200 \text{ ma} + 0.4 \text{ mA} = 200.4 \text{ mA}$$

$$P_{Q_1(\max)} = 14 \times 200.4 \times 10^{-3} = 2.805 \text{ W}$$

Hence, the correct Answer is (2.7 to 2.9).

9. In the low-pass filter shown in the figure, for a cut-off frequency of 5 kHz, the value of R_2 (in kΩ) is _____. **[2014]**



Solution: 3.183 kΩ

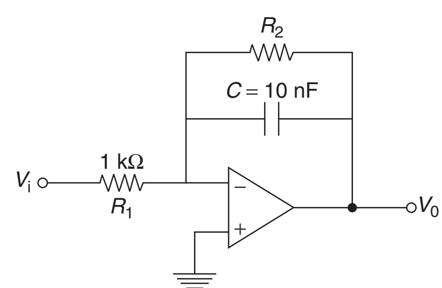
Output to input voltage ratio or gain

$$\frac{V_0}{V_i} = \frac{\cancel{Z}_f}{\cancel{Z}_1}$$

$$= -\frac{\left(R_2 \parallel \frac{1}{sC}\right)}{R_1}$$

$$= \left(\frac{\frac{R_2}{sC}}{R_2 + \frac{1}{sC}}\right) \times \frac{1}{R_1}$$

$$= -\left(\frac{R_2}{1 + sCR_2}\right) \times \frac{1}{R_1}$$



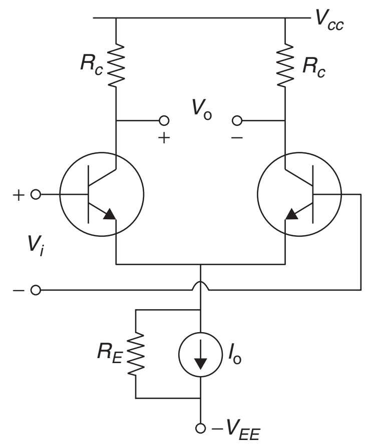
Cut-off frequency $= \dfrac{1}{CR_2}$ rad/sec.

$$2\pi \times 5k = \frac{1}{CR_2}$$

$$\Rightarrow 10\pi k = \frac{1}{10 \times 10^{-9} R_2}$$

$$\Rightarrow R_2 = \frac{10^4}{\pi}$$
$$= 3.183 \text{ k}\Omega$$

10. In the differential amplifier shown in the figure, the magnitudes of the common-mode and differential-mode gains are A_{cm} and A_d, respectively. If the resistance R_E is increased, then **[2014]**



(a) A_{cm} increases
(b) common-mode rejection ratio increases
(c) A_d increases
(d) common-mode rejection ratio decreases

Solution: (b)
The magnitudes of the common-mode and differential-mode gains are A_{CM} and A_d, respectively.

$$A_{CM} = \frac{-R_C}{2R_\varepsilon}$$

If R_ε is increased, then A_{CM} decreases

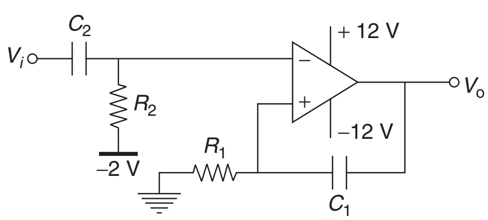
$$A_d = \frac{1}{2} g_m R_c$$

If R_ε is increased, then A_d does not get affected

$$\text{CMRR} = \frac{A_d}{A_{CM}} = \frac{1}{2} g_m R_c \left(-\frac{2R_\varepsilon}{R_C}\right)$$
$$= -g_m R_\varepsilon$$

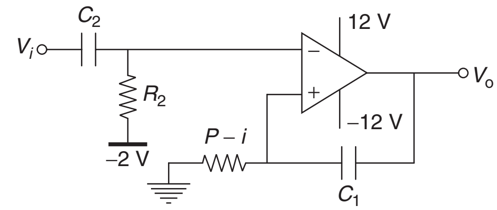
If R_ε is increased, then CMRR increased
Hence, the correct option is (b).

11. The circuit shown represents **[2014]**

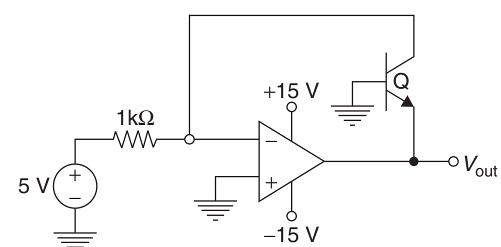


(a) A bandpass filter
(b) A voltage controlled oscillator
(c) An amplitude modulator
(d) A monostable multivibrator

Solution: (d)
The given circuit represents a mono stable multivibrator.



12. In the circuit shown below what is the output voltage (V_{out}) if a silicon transistor Q and an ideal op-amp are used? **[2013]**



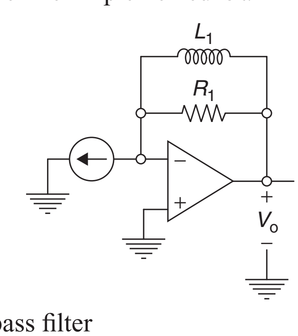
(a) − 15 V
(b) − 0.7 V
(c) + 0.7 V
(d) + 15 V

Solution: (b)
Since the transistor Q is a silicon transistor

$$V_{out} + V_{BE} = 0$$
$$V_{out} = -V_{BE}$$
$$= -0.7 \text{ V.}$$

Hence, the correct option is (b).

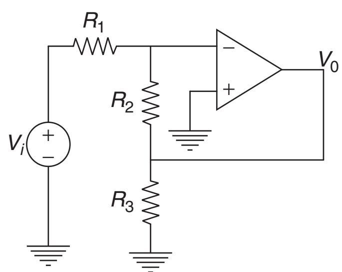
13. The circuit below implements a filter between the input current i_i and output voltage v_0. Assume that the op-amp is ideal. The filter implemented is a **[2011]**



(a) Low pass filter
(b) Band pass filter
(c) Band stop filter
(d) High pass filter

Solution: (d)
When $\omega = 0$
inductor acts as a SC
$\Rightarrow V_0 = 0$
and when $\omega = \infty$,
inductor acts as an OC
$\Rightarrow V_0 = i_1 R_1$
So, it acts as a high pass filter
Hence, the correct option is (d).

14. Assuming the op-amp to be ideal, the voltage gain of the amplifier shown below is **[2010]**

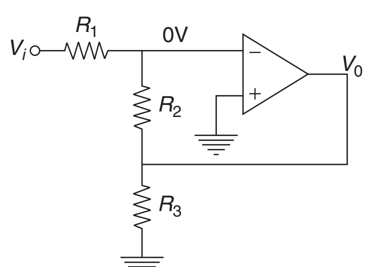


(a) $-\frac{R_2}{R_1}$ (b) $-\frac{R_3}{R_1}$

(c) $-\frac{R_2 \| R_3}{R_1}$ (d) $-\frac{R_2 + R_3}{R_1}$

Solution: (a)
Applying nodal analysis at non-inverting input

$$\frac{0 - V_i}{R_1} + \frac{0 - V_0}{R_2} = 0$$



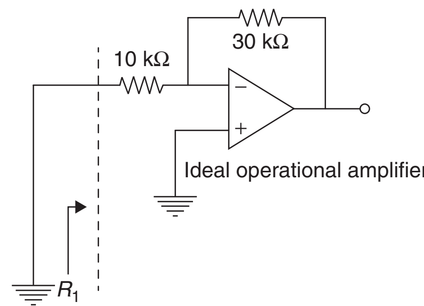
$$\Rightarrow \frac{V_i}{R_1} + \frac{V_0}{R_2} = 0$$

$$\Rightarrow V_0 = -V_i \frac{R_2}{R_1}$$

$$\Rightarrow \text{voltage gain } \frac{V_0}{V_i} = \frac{-R_2}{R_1}$$

Hence, the correct option is (a).

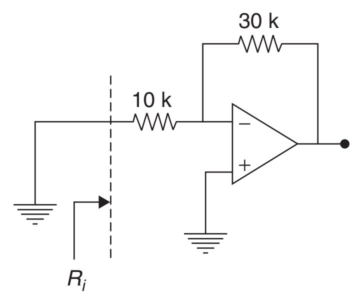
15. The input resistance R_i of the amplifier shown in the figure is **[2005]**



(a) $\frac{30}{4}$ kΩ (b) 10 kΩ

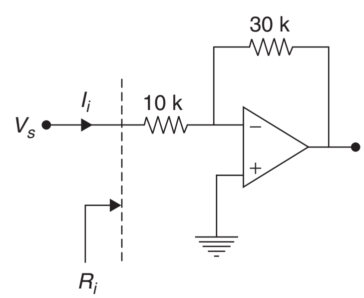
(c) 40 kΩ (d) infinite

Solution: (b)



Connect a V_s voltage source across inverting terminal of op-amp

$$I_i = \frac{Vs}{10k}$$



I/P resistance, $R_i = \frac{Vs}{I_i}$

$= 10$ kΩ

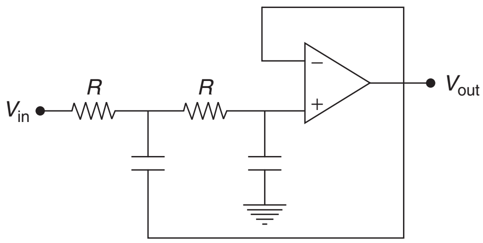
Hence, the correct option is (b).

16. An ideal op-amp is an ideal **[2004]**

(a) Voltage controlled current source
(b) Voltage controlled voltage source
(c) Current controlled current source
(d) Current controlled voltage source

Solution: (b)
An ideal op-amp is an ideal voltage controlled voltage source.

17. The circuit in the figure is **[2004]**

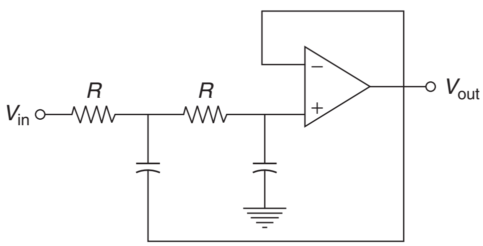


(a) low-pass filter (b) high-pass filter
(c) band-pass filter (d) band-reject filter

Solution: (a)

At $\omega = \infty$,

$$\frac{V_0}{V_{in}} = 0$$



$$V_0 = 0$$

At $\omega = 0$,

$$\frac{V_0}{V_{in}} = 1$$

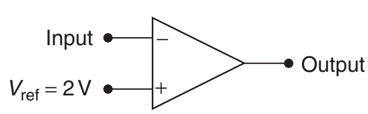
$$V_0 = V_{in}$$

So, the given circuit is a low-pass filter.
Hence, the correct option is (a).

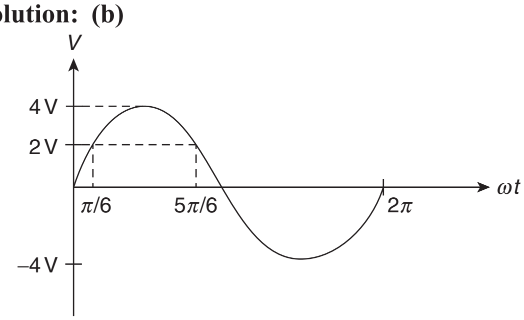
18. If the input to the ideal comparator shown in the figure is a sinusoidal signal of 8V (peak to peak) without any DC component, then the output of the comparator has a duty cycle of **[2003]**

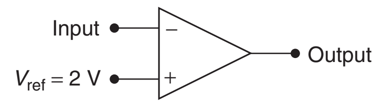
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{1}{12}$



Solution: (b)





$V_i = 4 \sin \omega t$
If $V_i = 2$ V
$2 = 4 \sin \omega t$

$$\Rightarrow \sin \omega t = \frac{1}{2}$$

$$\omega t = \frac{\pi}{6} \cdot \frac{5\pi}{6}$$

$$\text{duty cycle} = \frac{t_{ON}}{T} = \frac{5\pi/6 - \pi/6}{2\pi} = \frac{4\pi/6}{2\pi}$$

$$= \frac{1}{3}$$

Hence, the correct option is (b).

19. If the differential voltage gain and the common mode voltage gain of a differential amplifier are 48 dB and 2 dB, respectively, then its common mode rejection ratio is **[2003]**

(a) 23 dB (b) 25 dB
(c) 46 dB (d) 50 dB

Solution: (c)
Common mode rejection ratio is given by

$$\text{CMRR} = \frac{Ad}{Ac}$$

$A_d = 48$ dB
$A_c = 2$ dB
$(\text{CMRR})_{\text{dB}} = 20 \log A_d - 20 \log A_c$
$= 48 - 2$
$= 46$ dB
Hence, the correct option is (c).

20. A 741-type op-amp has a gain-bandwidth product of 1 MHz. A non-inverting amplifier using this op-amp and having a voltage gain of 20 dB will exhibit a −3 dB bandwidth of **[2002]**

(a) 50 kHz (b) 100 kHz

(c) $\frac{1000}{7.07}$ kHz (d) $\frac{1000}{7.07}$ kHz

Solution: (b)

Gain × $BW = 1 \times 10^6$

$$BW = \frac{1\times 10^6}{\text{Gain}} = \frac{10^6}{10}$$

$$= 10^5$$

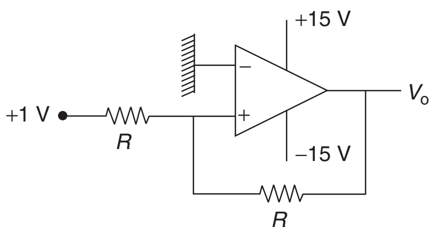
Hence, the correct option is (b).

21. The ideal OP AMP has the following characteristics: **[2001]**

(a) $R_i = \infty, A = \infty, R_0 = 0$
(b) $R_i = 0, A = \infty, R_0 = 0$
(c) $R_i = \infty, A = \infty, R_0 = \infty$
(d) $R_i = 0, A = \infty, R_0 = \infty$

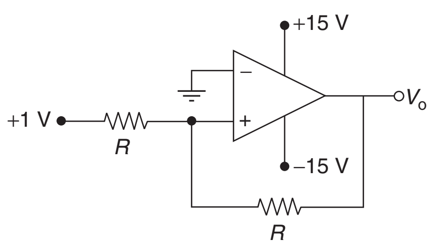
Solution: (a)
The ideal op-amp has the following characteristics:
In put impedance, $R_i = \infty$
Voltage gain, $A = \infty$
Output impedance, $R_0 = 0$
Hence, the correct option is (a).

22. In the circuit of the figure, V_o is **[2000]**



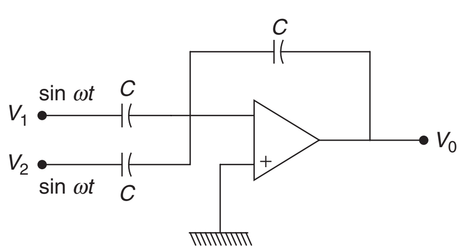
(a) − 1 V (b) 2 V
(c) + 1 V (d) + 15 V

Solution: (d)
Since I/P applied at the terminal O/P is +ve, and since feedback is positive, O/P voltage gets saturated.



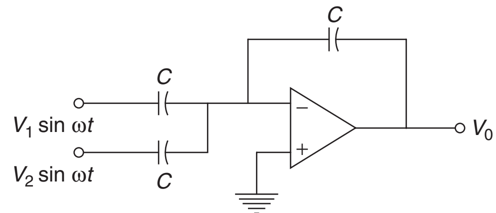
$\therefore V_0 = + 15$ V
Hence, the correct option is (d).

23. If the op-amp in the figure is ideal, then V_o is **[2000]**



(a) Zero (b) $(V_1 - V_2)\sin\omega t$
(c) $-(V_1 + V_2)\sin\omega t$ (d) $(V_1 + V_2)\sin\omega t$

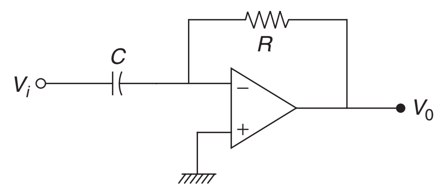
Solution: (d)
Applying super position theorem,



$$V_0 = V_1 \sin\omega t\left(\frac{C}{C}\right) - V_2 \sin\omega t\left(\frac{C}{C}\right)$$
$$= (V_1 + V_2)\sin\omega t$$

If V_i is a triangular wave, then V_0 will be a square wave.
Hence, the correct option is (d).

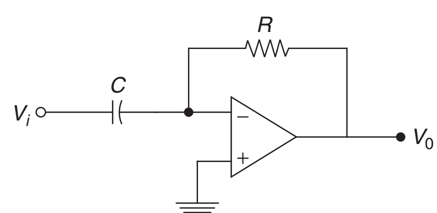
24. Assume that the op-amp or the figure is ideal. If V_i is a triangular wave, then V_0 will be **[2000]**



(a) square wave
(b) triangular wave
(c) parabolic wave
(d) sine wave

Solution: (a)
Op amp gain is given by

$$V_0 = V_i\left(\frac{-R}{X_c}\right)$$
$$= \frac{-V_i R}{X_c}$$
$$V_0 = -RC\frac{dV_i}{dt}$$

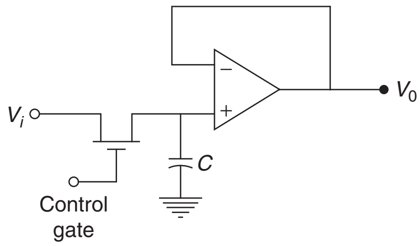


Hence, the correct option is (a).

25. The most commonly used amplifier in sample and hold circuits is **[2000]**
(a) a unity gain inverting amplifier
(b) a unity gain non-inverting amplifier
(c) an inverting amplifier with a gain of 10
(d) an inverting amplifier with a gain of 100

Solution: (b)

The most commonly used amplifier in sample and hold circuits is a unity gain non-inverting amplifier.



Hence, the correct option is (b).

26. The first dominant pole encountered in the frequency response of a compensated op-amp is approximately at **[1999]**

(a) 5 Hz (b) 10 kHz
(c) 1 MHz (d) 100 MHz

Solution: (a)

The first dominated pole encountered in the frequency response of a compensated op-amp is approximately at 5 Hz

Hence, the correct option is (a).

27. One input terminal of high gain comparator circuit is connected to ground and a sinusoidal voltage is applied to the other input. The output of comparator will be **[1998]**

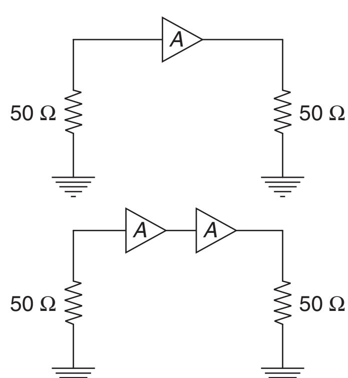
(a) a sinusoid
(b) a full rectified sinusoid
(c) a half rectified sinusoid
(d) a square wave

Solution: (d)

One I/P terminal of high gain comparator circuit is connected to ground and a sinusoid voltage is applied to the other I/P. The O/P of a comparator will be a square wave due to the fact that Op. Amp has a very high gain, i.e. O/P voltage gets saturated.

Hence, the correct option is (d).

28. An amplifier *A* has 6 dB gain and 50 Q input and output impedances. The noise figure of this amplifier as shown in the figure is 3 dB. A cascade of two such amplifiers as in the figure will have a noise figure of **[1997]**



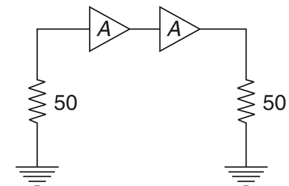
(a) 6 dB (b) 8 dB
(c) 12 dB (d) None of the above

Solution: (a)

Each amplifier has a noise figure of 3 dB

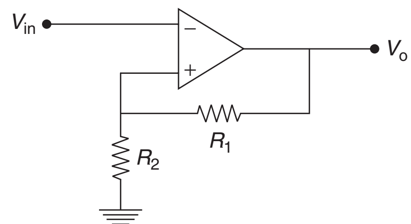
$\therefore$ the noise figure of the cascade combination = 3 dB + 3 dB

= 6 dB



Hence, the correct option is (a).

29. The circuit shown in the figure is that of **[1996]**

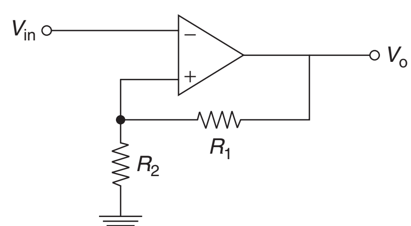


(a) a non-inverting amplifiers
(b) an inverting amplifier
(c) an oscillator
(d) a Schmitt trigger

Solution: (d)

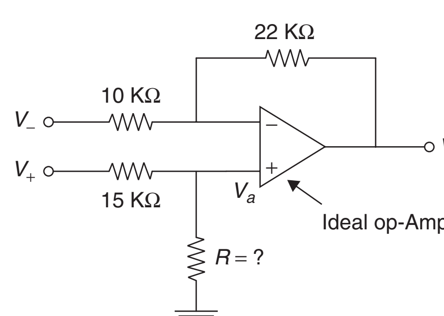
The given circuit has a positive feedback, so it has a very high O/P gain.

Any periodic waveform applied to I/P results in a square wave so wave. So, this circuit is a Schmitt trigger.



Hence, the correct option is (d).

30. In the given circuit figure, if the voltage inputs V_- and V_+ are to be amplified by the same amplification factor, the value of '*R*' should be



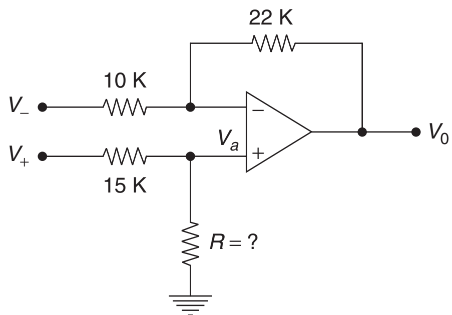
[1995]

Solution: 33 kΩ

Gain at inverting terminal

$$A^{-} = -\frac{R_f}{R_1}$$

$$A^{-} = \frac{-22K}{10k} = -2.2$$



$$\left|A^{-}\right| = 2.2$$

$$V_a = V + \left(\frac{R}{R+15}\right)$$

Gain at non-inverting terminal

$$A^{+} = 1 + \frac{R_f}{R_1}$$

$$\frac{V_0}{V_a} = 1 + \frac{R_f}{R_1} = \frac{V_o}{V + \left(\frac{R}{R+15}\right)}$$

$$\frac{V_0}{V_+} = \left(1 + \frac{22}{10}\right) \times \left(\frac{R}{R+15}\right)$$

$$\left|A^{+}\right| - 3.2\left(\frac{R}{R+15}\right)$$

$$\frac{3.2R}{R+15} = 2.2$$

$$3.2R = 2 - 2R + 33$$

$$R = 33\ \text{k}\Omega$$

31. An op-amp is used as a zero-crossing detector. If maximum output available from the op-amp is ± 12 V_{p-p} and the slew rate of the op-amp is 12 V/μ sec then the maximum frequency of the input signal that can be applied without causing a reduction in the *P–P* output is **[1995]**

Solution: 159 kHz

$V_{PP} = \pm 12$ V

$SR = 12\text{V/MSec}$

Let $V_0 = V_P \sin \omega t$

$$SR = \left.\frac{dV_0}{dt}\right|_{\max} = \left|V_p \omega \cos\omega t\right|_{\max} = V_p \omega_{\max}$$

$$SR = V_P \times 2\pi f_{\max}$$

$$f_{\max} = \frac{SR}{2\pi V_P}$$

$$= \frac{12\times 10^6}{2\pi \times 12} = 159\ \text{kHz}$$

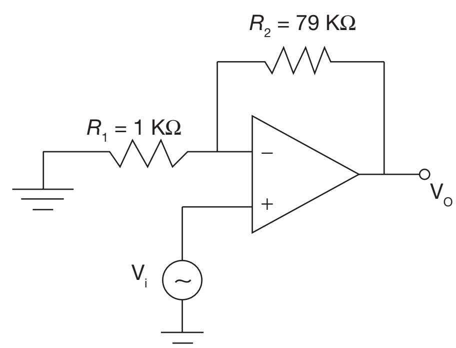
32. The frequency compensation is used in op-amps to increase its_____. **[1994]**

Solution: Stability

The frequency compensation used in Op Amp is used to increase the stability of op-amps.

Two-marks Questions

1. The amplifier circuit shown in the figure is implemented using a compensated operational amplifier (op-amp), and has an open-loop voltage gain $A_0 = 10^5$ V/V and an open-loop cut-off frequency, $f_c = 8$ Hz. The voltage gain of the amplifier at 15 kHz, in V/V, is ________. **[2017]**



Solution: $\beta = \frac{R_1}{R_1 + R_2} = \frac{1}{50}$

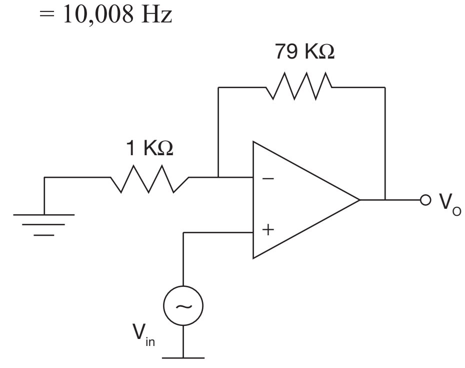
$$A_{cL} = \frac{A_{oL}}{1 + A_{oL}\beta} = \frac{1}{50}$$

Where $A_{cL:}$ closed loop gain

A_{oL}: open loop gain

$$\therefore A_{cL} = \frac{10^5}{1 + (10^5)\frac{1}{80}} \simeq 80$$

$$= 10{,}008\ \text{Hz}$$

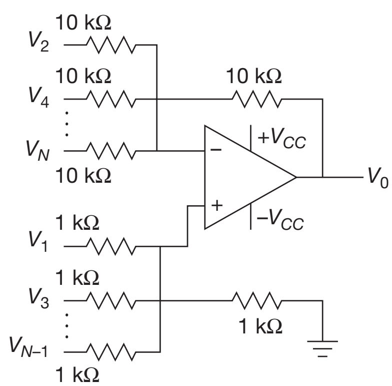


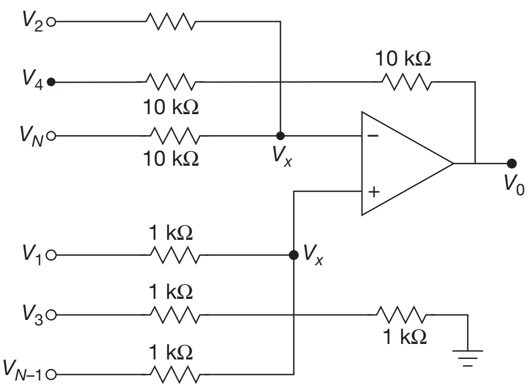
Gain at f = 15 KHz = 15000 Hz is $A_f = \dfrac{A_{cL}}{\sqrt{1+\left(\dfrac{f}{f_{cL}}\right)^2}}$

$$= \frac{80}{\sqrt{1+\left(\dfrac{15000}{10008}\right)^2}} \simeq 44.4$$

Hence, the correct answer is (43.3 to 45.3).

2. An ideal opamp has voltage sources V_1, V_3, V_5, V_{N-1} connected to the non inverting input and V_2, V_4, V_6, V_N connected to the inverting input as shown in the figure below ($+V_{cc}$ = 15 volt, $-V_{cc}$ = −15 volt) the voltages V_1, V_2, V_3, V_4, V_5, V_6, are $1, -\frac{1}{2}, \frac{1}{3}, -\frac{-1}{4}, \frac{1}{5}, \frac{-1}{6}$ volt respectively. As N approaches infinitely, the output voltage (in volt) is ______ **[2016]**





Solution: On applying virtual GND concept we have

$$V_+ = V_- = V_x$$

At non-inverting terminal (applying KCL)

$$\frac{V_x - V_1}{1k} + \frac{V_x - V_3}{1k} + ... + \frac{V_x - V_{N-1}}{1k} + \frac{V_x}{1k} = 0$$

$$V_1 + V_3 + V_5 + \ldots + V_{N-1} = V_x + \frac{N}{2}\cdot V_x$$

$$= \left[1 + \frac{N}{2}\right]\cdot V_x \qquad \text{(i)}$$

At inverting terminals

$$\frac{V_x - V_2}{10k} + \frac{V_x - V_4}{10k} + ... + \frac{V_x - V_N}{10k} + \frac{V_x - V_0}{10k} = 0$$

$$\therefore \quad V_2 + V_4 + \ldots + V_N = V_x\left[1 + \frac{N}{2}\right] - V_0 \qquad \text{(ii)}$$

But form (i) and (ii) we get

$$V_0 = \{V_1 + V_3 + \ldots\ldots + V_{N-1}\} -$$
$$\{V_2 + V_4 + \ldots. + V_N\}$$

Given; $V_j = \dfrac{1}{j}$; $j \to$ odd

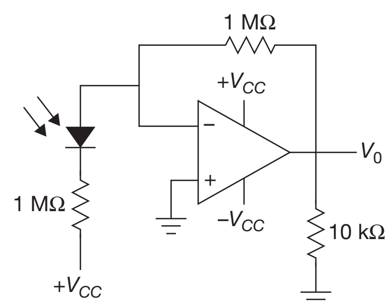
$V_j = \dfrac{-1}{j}$; $j \to$ even

$$\therefore \quad V_0 = \left\{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots\ldots + \frac{1}{\infty}\right\}$$

$$\therefore \quad V_0 = +V_{sat} = 15\text{ V}$$

Hence, the correct Answer is (14 V).

3. A $p - I - n$ photodiode of responsivity 0.8 A/W is connected to the inverting input of an ideal opamp as shown in the figure $+V_{cc}$ = 15 V, $-V_{cc}$ = −15 V, Load resistor R_L = 10 k Ω. If 10 μW of power is incident on the photodiode, then the value of the photocurrent (in μA) through the load is **[2016]**



Solution: Responsivity of photodiode R = 0.8 A/W

Photon current I_p = ?

We know that

$$R = \frac{I_p}{p_o}$$

$$I_p = R \cdot P_o = 0.8 \times 10\ \mu\text{A}$$

$$\frac{V_o - 0}{1M} = I_p$$

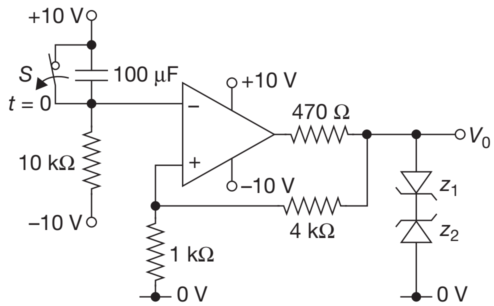
$$V_0 = 1M \times 8\ \mu = 8\text{ V}$$

The value of the photocurrent (in μA) through the load is

$$\therefore \quad I_L = \frac{V_0}{10k} = \frac{8}{10^4} = 800\ \mu\text{A}$$

Hence, the correct Answer is (800 μA).

4. In the op-amp circuit shown, the Zener diodes Z_1 and Z_2 clamp the output voltage V_0 to +5 V or –5 V. The switch S is initially closed and is opened at time $t = 0$.

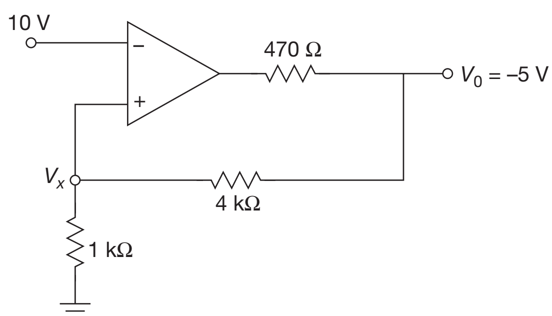


The time $t = t_1$ (in seconds) at which V_0 changes state is ______. **[2016]**

Solution: From the given data switch is closed initially and opened at $t = 0$.

for $t < 0$:

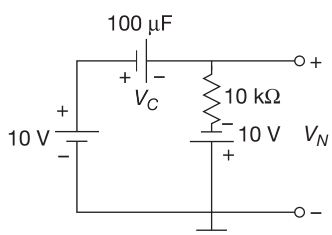
at $t = 0^-$:



$$V_x = \frac{-5 \times 1}{5} = -1\ \text{V}$$

$$V_c(0^-) = 0\ \text{V}$$

For $t > 0$:



$$t \to \infty$$

$$V_c(\infty) = 20\ \text{V}$$

$$V_c(t) = 20 + [0 - 20] \cdot e^{-t/\tau}$$

$$\tau = R \cdot C = 10\ \text{k}\Omega \times 100\ \mu\text{F}$$

$$\tau = 1\ \text{sec}$$

$$\therefore \quad V_c(t) = 20 - 20.\ e^{-t}$$

$$V_N = 10 - V_c$$

$$V_N = -10 + 20.\ e^{-t}.$$

Apply Virtual GND Concept

$$V_N = V_x$$

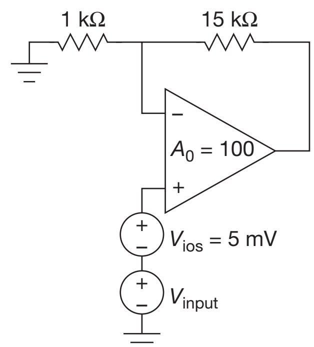
$$-10 + 20 \cdot e^{-t} = -1$$

$$20 \cdot e^{-t} = 9$$

$$t = 0.798\ \text{sec.}$$

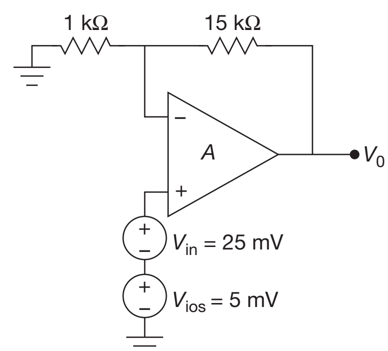
Hence, the correct Answer is (0.798 sec).

5. An op-amp has a finite open loop voltage gain of 100. Its input offset voltage V_{ios} (= +5 mV) is modelled as shown in the circuit below. The amplifier is ideal in all other respects. V_{input} is 25 mV.



The output voltage (in millivolts) is _____. **[2016]**

Solution:



$$A = 100,$$

we know for low gain op-amps

(since it is a non-inverting op-amp, therefore the gain can be given as)

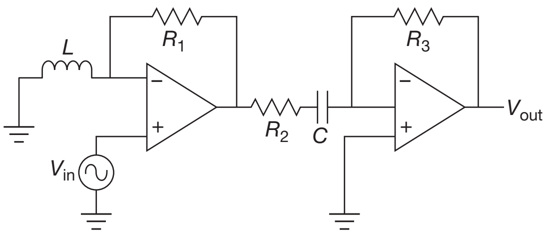
$$V_0 = \frac{\{1 + R_f/R_1\}}{1 + \frac{\{1 + R_f/R1\}}{A}} \times V_i$$

$$= \frac{16}{1 + \frac{16}{100}} \times 30\ \text{mV}$$

$$V_0 = 413.8\ \text{mV}$$

Hence, the correct Answer is (413.8 mV).

6. For the circuit shown in the figure, $R_1 = R_2 = R_3 = 1\ \Omega$, $L = 1\ \mu$H and $C = 1\ \mu$F. If the input $V_{in} = \cos(10^6 t)$, then the overall voltage gain (V_{out}/V_{in}) of the circuit is **[2016]**



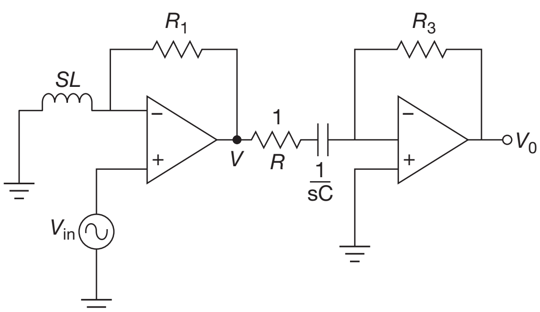
Solution: Resistance $R_1 = R_2 = R_3 = 1\ \Omega$,

Inductance $L = 1\mu H$

Capacitance $C = 1\mu F$

The input voltage is $V_{in} = \cos 10^6 t$ V

Angular frequency $\omega = 10^6$ rad/sec



After considering the virtual ground effect, we get (let V is denoted by V_x) then

$$\frac{V_x}{V_{in}} = \left(1 + \frac{R_1}{sL}\right)$$

$$\frac{V_o}{V_x} = \frac{-R_3}{\left(R_2 + \frac{1}{sC}\right)}$$

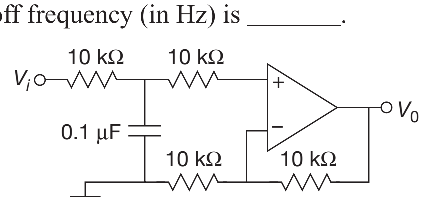
$$V_o = \frac{-R_3}{\left(R_2 + \frac{1}{sC}\right)} \times \left(1 + \frac{R_1}{sL}\right) \times V_{in}$$

Sub given values, we get

$$\frac{V_o}{V_{in}} = -1$$

Hence, the correct Answer is (–1).

7. In the circuit shown using an ideal opamp, the 3-dB cut-off frequency (in Hz) is ________. **[2015]**



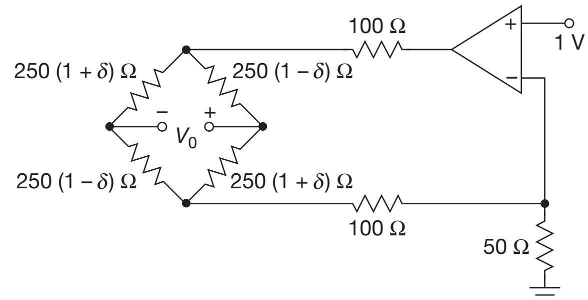
Solution: Cut-off frequency $f = \dfrac{1}{2\pi RC}$

$$= \frac{1}{2 \times \pi \times 10 \times 10^3 \times 0.1 \times 10^{-6}}$$

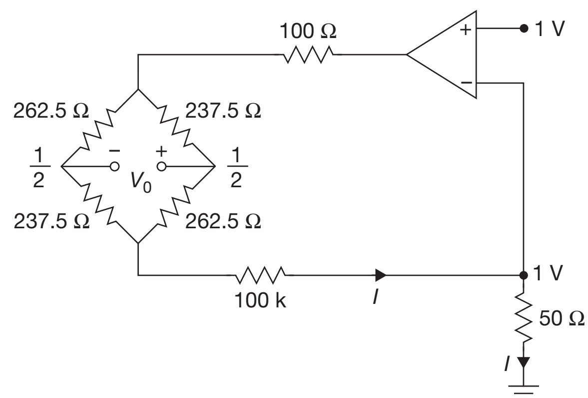
$= 159.15$ Hz

Hence, the correct Answer is (159 to 160).

8. In the circuit shown, assume that the opamp is ideal. The bridge output voltage V_0 (in mV) for $\delta = 0.05$ is ______ **[2015]**



Solution:



From the given circuit $V_+ = V_- = 1$ V

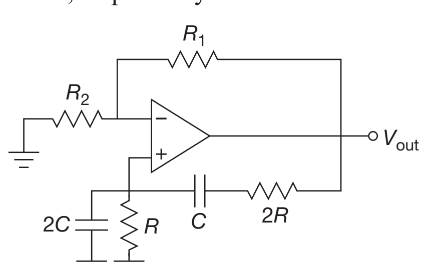
Current through 50 Ω is $= \dfrac{1}{50}$ Amp $= I$

$$V_o = \frac{1}{100}[262.5 - 237.5]$$

$$= \frac{25}{100} = 250 \text{ mV}$$

Hence, the correct Answer is (249 to 251).

9. The circuit shown in the figure has an ideal opamp. The oscillation frequency and the condition to sustain the oscillations, respectively are **[2015]**



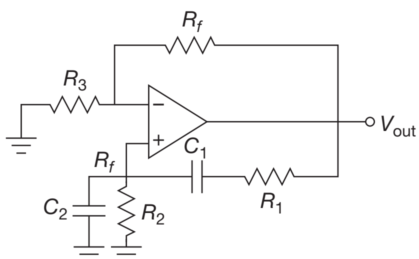
(A) $\dfrac{1}{CR}$ and $R_1 = R_2$

(B) $\dfrac{1}{CR}$ and $R_1 = 4R_2$

(C) $\dfrac{1}{2CR}$ and $R_1 = R_2$

(D) $\dfrac{1}{2CR}$ and $R_1 = 4R_2$

Solution: The given circuit is Wein Bridge Oscillator



The gain of the op–amp is $A = I + \dfrac{R_f}{R_3}$ and feedback factor $B = \dfrac{V_f}{V_o}$

Let $Z_1 = R_1 + 1/SC_1$ and $Z_2 = R_2 \parallel 1/SC_2$

$$V_f = \frac{Z_2 \times V_0}{Z_1 + Z_2} \qquad \because B = \frac{Z_2}{Z_1 + Z_2}$$

Sub all values mB we get finally

$$B = \frac{j\omega R_2C_1}{1 + j\omega(R_1C_1 + R_2C_2 + R_2C_1) - \omega_2 R_1R_2C_1C_2}$$

But B is real quantity so equate

$$1 - \omega_2 R_1R_2C_1C_2 = 0$$

Thus the frequency of oscillation,

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}} \quad \omega_0 = \frac{1}{2RC}$$

From the given data

$$R_1 = 2R,\ R_2 = R$$

$$C_1 = C,\ C_2 = 2C$$

And $$B = \frac{R_2C_1}{R_1C_1 + R_2C_2 + R_2C_1}$$

Substitute all values

$$B = \frac{RC}{2RC + 2RC + RC} = \frac{1}{5}$$

For sustained oscillation AB = 1

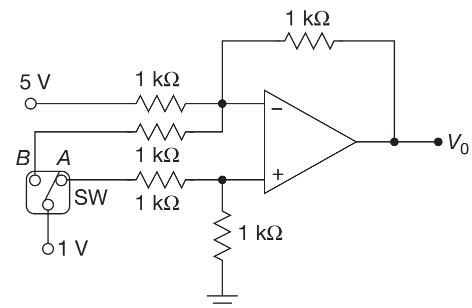
$$A = \frac{1}{B} \qquad \therefore\ 1 + \frac{R_f}{R_3} = 5$$

But $R_f = R_1$ and $R_3 = R_2$

$\therefore$ $R_1 = 4R_2$

Hence, the correct option is (D).

10. In the circuit shown, $V_0 = V_{0A}$ for switch SW in position A and $V_0 = V_{0B}$ for SW in position B. Assume that the opamp is ideal. The value of $\dfrac{V_{0B}}{V_{0A}}$ is ________. **[2015]**



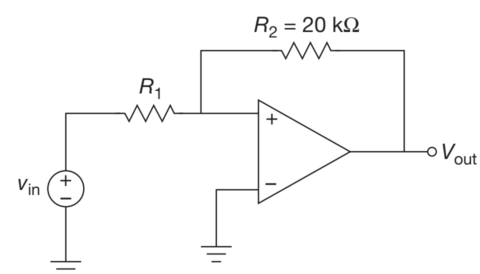
Solution: $V_{OB} = 5\left(\dfrac{-1}{1}\right) + 1\left(\dfrac{-1}{1}\right) = -6$ V

$$V_{OA} = \left(\frac{1\times1}{1+1}\right)\left(1+\frac{1}{1}\right)\ 5\left(\frac{1}{1}\right) = -4 \text{ V}$$

$$\frac{V_{OB}}{V_{OA}} = \frac{-6}{-4} = 1.5$$

Hence, the correct Answer is (1.5).

11. In the bistable circuit shown, the ideal opamp has saturation levels of $\pm$ 5 V. The value of R_1 (in k Ω) that gives a hysteresis width of 500 mV is _____. **[2015]**



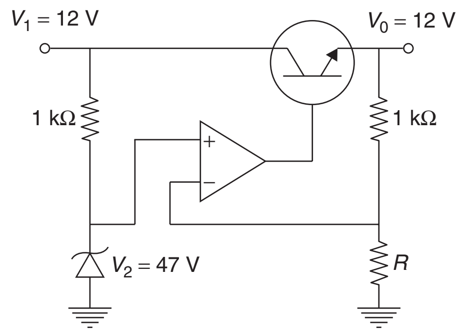
Solution: Hysteresis width = $V_{TH} - V_{TL}$

$$500 \text{ mV} = 5\left(\frac{R_1}{20k\Omega}\right) - (-5)\left(\frac{R_1}{20k\Omega}\right)$$

$$R_1 = 1 \text{ k}\Omega$$

Hence, the correct Answer is (1).

12. In the voltage regulator circuit shown in the figure, the op-amp is ideal. The BJT has $V_{BE} = 0.7$ V and $\beta = 100$, and the Zener voltage is 4.7 V. For a regulated output of 9 V, the value of R (in Ω) is______. **[2014]**



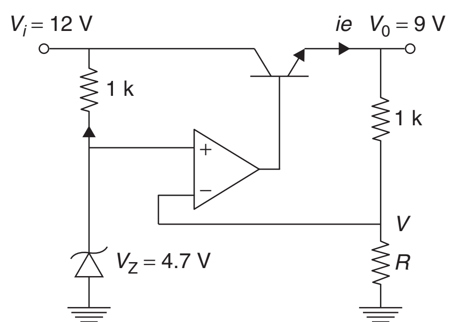
Solution:

Given

$V_{BE} = 0.7$ V

$B = 100$

$V_z = 4.7$ V



$V_i + 1k_i - 4.7 = 0$

$12 + i - 4.7 = 0$

$\Rightarrow i = 7.3$ mA

Since B is large enough

$I \cong ie = 7.3$ mA

$V = 4.7$ V (virtual ground)

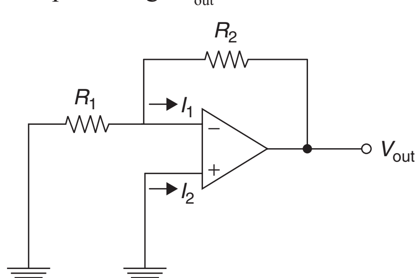
$V - ieR = 0$

$$\Rightarrow R = \frac{V}{ie}$$

$$= \frac{4.7}{7.3}$$

$\Rightarrow R = 0.64$ kΩ

13. In the circuit shown, the op-amp has finite input impedance, infinite voltage gain and zero input–offset voltage. The-output voltage V_{out} is **[2014]**



(a) $-I_2(R_1 + R_2)$

(b) I_2R_2

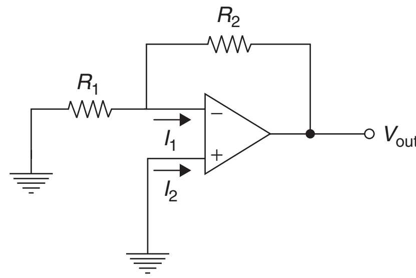
(c) I_1R_2

(d) $-I_1(R_1 + R_2)$

Solution: (c)

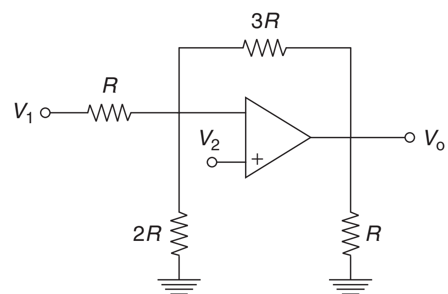
$V_{out} = I_1R_2$

Current does not enter the op-amp.



Hence, the correct option is (c).

14. Assuming that the op-amp in the circuit shown is ideal, V_o is given by **[2014]**



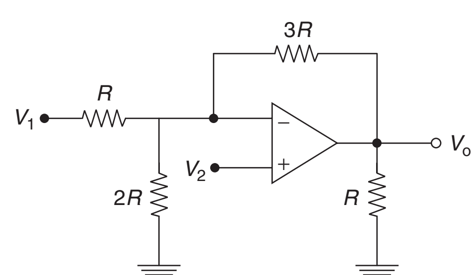
(a) $\frac{5}{2}V_1 - 3V_2$ (b) $2V_1 - \frac{5}{2}V_2$

(c) $-\frac{3}{2}V_i + \frac{7}{2}V_2$ (d) $-3V_1 + \frac{11}{2}V_2$

Solution: (d)

Applying nodal analysis,

$$\frac{V_2 - V_1}{R} + \frac{V_2}{2R} + \frac{V_2 - V_0}{3R} = 0$$



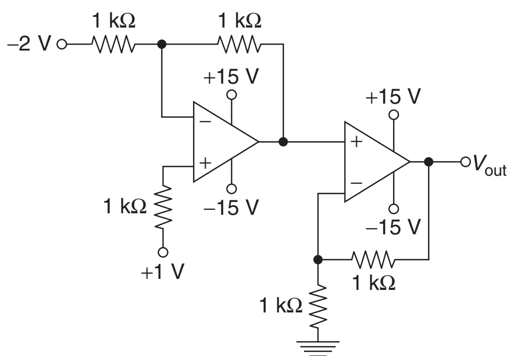
$$6V_2 - 6V_1 + 3V_2 + 2V_2 - 2V_0 = 0$$

$$\Rightarrow 11V_2 - 6V_1 - 2V_0 = 0$$

$$\Rightarrow V_0 = -3V_1 + \frac{11}{2}V_2$$

Hence, the correct option is (d).

15. In the circuit shown below, the op-amps are ideal. Then V_{out} in volts is **[2013]**



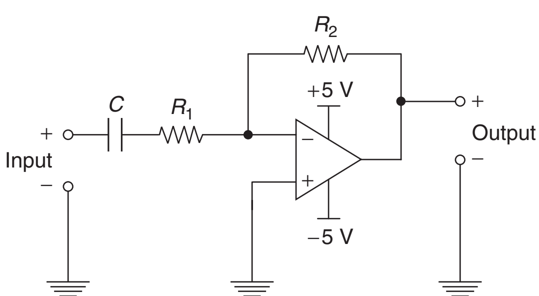
(a) 4 (b) 6 (c) 8 (d) 10

Solution: (c)

$V_1 = 1(1 + 1) - 2(-1)$
$= 2 + 2$
$= 4$ V
$V_{out} = 2V_1$
$= 2 \times 4$
$= 8$ V

Hence, the correct option is (c).

16. The circuit shown is a **[2012]**



(a) low pass filter with $f_{3dB} = \frac{1}{(R_1 + R_2)C}$ rad / s

(b) high pass filter with $f_{3dB} = \frac{1}{R_1 C}$ rad / s

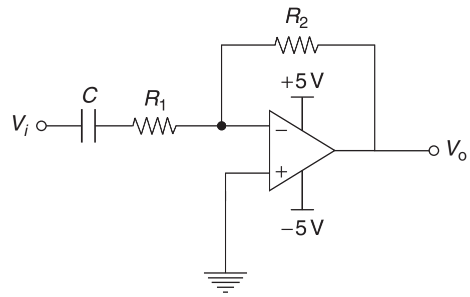
(c) low pass filter with $f_{3dB} = \frac{1}{R_1 C}$ rad / s

(d) high pass filter with $f_{3dB} = \frac{1}{(R_1 + R_2)C}$ rad / s

Solution: (b)

Non-inverting terminal gain

$$V_0(s) = -\left(\frac{R_2}{R_1 + \frac{1}{sc}}\right)V_i(s)$$



$$V_0(s) = \frac{-sCR_2}{1 + sCR_1}V_i(s)$$

thus cut-off frequency is $\frac{1}{R_1 C}$

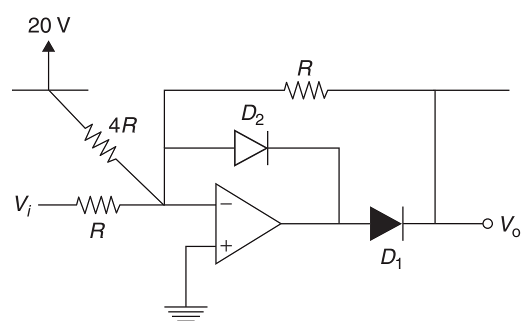
at $\omega = 0$
$V_0 = 0$
and, at $\omega = \infty$

$$V_0 = -V_i \frac{R_2}{R_1}$$

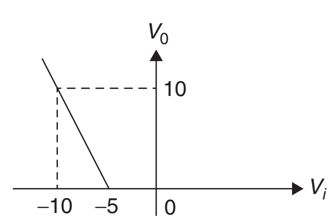
$\therefore$ it acts as a high-pass filter.

Hence, the correct option is (b).

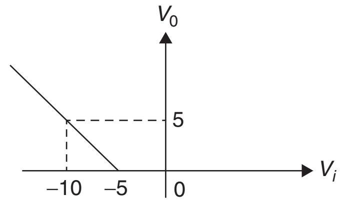
17. The transfer characteristic for the precision rectifier circuit shown below is (assume ideal op-amp and practical diodes) **[2010]**



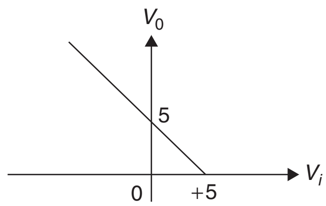
(a)



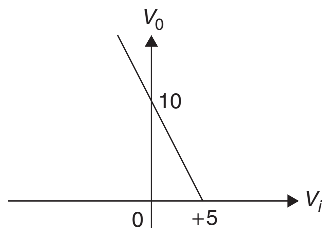
(b)



(c)



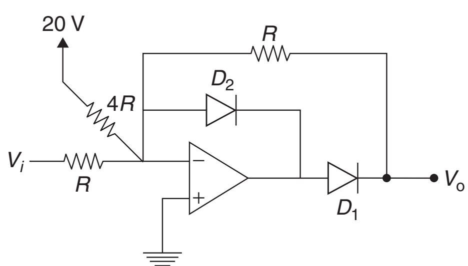
(d)



Solution: (b)

At $V_i = -10$ V

$$\frac{V_0 - 0}{R} = \frac{0 - 20}{4R} + \frac{0(-10)}{R}$$



$$V_0 = -5 + 10$$
$$= 5 \text{ V}$$

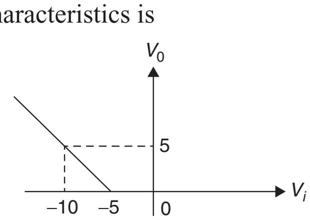
At $V_i = -5$ V

$$\frac{V_0 - 0}{R} = \frac{0 - 20}{4R} + \frac{0(-10)}{R}$$

$$V_0 = -5 + 5$$
$$= 0 \text{ V}$$

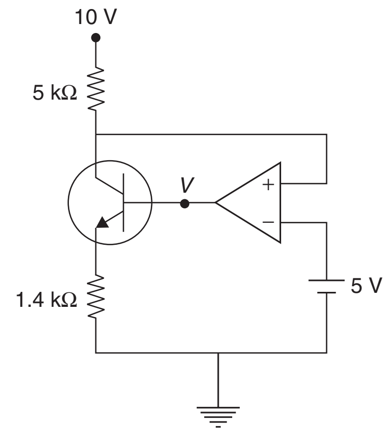
For $V_i > -5$ V, both diodes are conducting. So, $V_0 = 0$ V

$\therefore$ transfer characteristics is



Hence, the correct option is (b).

18. In the circuit shown below, the op-amp is ideal, the transistor has $V_{BE} = 0\ 6$ V and $\beta = 150$. Decide whether the feedback in the circuit is positive or negative and determine the voltage V at the output of the op-amp. **[2009]**



(a) Positive feedback, $V = 10$ V
(b) Positive feedback, $V = 0$ V
(c) Negative feedback, $V = 5$ V
(d) Negative feedback, $V = 2$ V

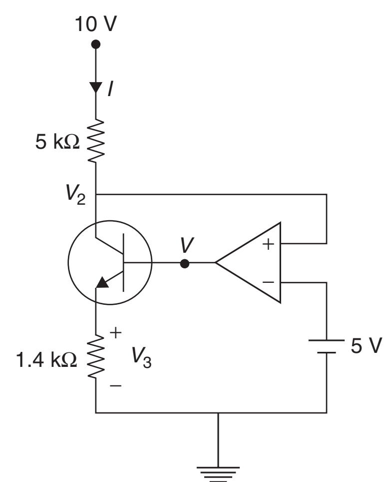
Solution: (d)

$V_2 = 5$ V (virtual grounds)

$10 - 5\text{I} - V_2 = 0$

$\Rightarrow 5 = 5\text{I}$

$\Rightarrow \text{I} = 1$ mA

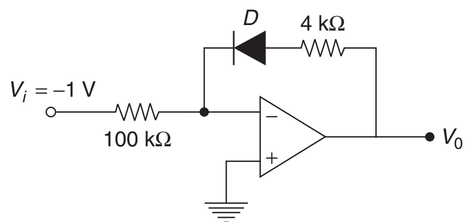


$$\Rightarrow V - V_{BE} - V_3 = 0$$
$$\Rightarrow V = V_{BE} + V_3$$
$$= 0.6 + 1.4\text{k} \times 1\text{m}$$
$$= 0.6 + 1.4$$
$$\Rightarrow V = 2 \text{ V}$$

Hence, the correct option is (d).

19. Consider the following circuit using an ideal op-amp. The I–V characteristics of the diode is described by the relation $I = I_0\left(e^{\frac{V}{V_T}} - 1\right)$ **[2008]**

where V_T = 25 mV, I_0 = 1 μA and V is the voltage across the diode (taken as positive for forward bias).



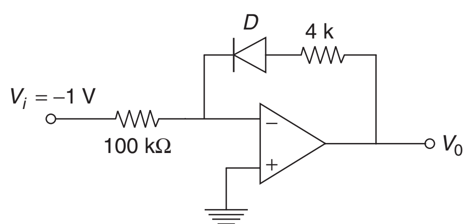
For an input voltage $V_i = -1$V, the output voltage V_0 is

(a) 0 V (b) 0.1 V
(c) 0.7 V (d) 1.1 V

Solution: (b)

$1 = 10\,(e^{V/Vt} - 1)$

$$= \frac{0-(-1)}{100k}$$



$$\Rightarrow 10^{-6}\left[e\frac{V}{25\times10^{-3}} - 1\right] = \frac{1}{10^5}$$

$\Rightarrow V = 0.06$ V

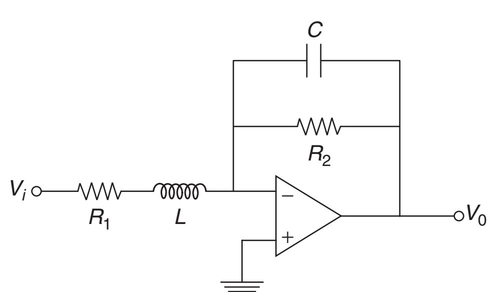
$\Rightarrow V = 0.06$ V

$$\Rightarrow \frac{V_0 - V}{4k} = \frac{1}{100k}$$

$\Rightarrow V_0 = 0.1$ V

Hence, the correct option is (b).

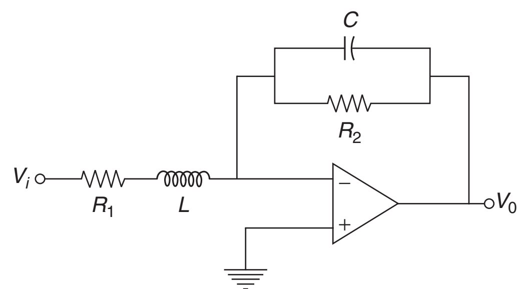
20. The op-amp circuit shown below represents a **[2008]**



(a) high-pass filter
(b) low-pass filter
(c) band-pass filter
(d) band-reject filter

Solution: (b)

At $\omega = 0$



$$V_0 = -V_i\frac{R_2}{R_1}$$

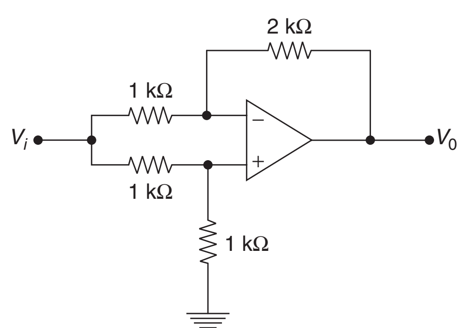
At $\omega = \infty$

$V_0 = 0$

∴ the circuit shown represents a low-pass filter.

Hence, the correct option is (b).

21. For the op-amp circuit shown in the figure, V_0 is **[2007]**



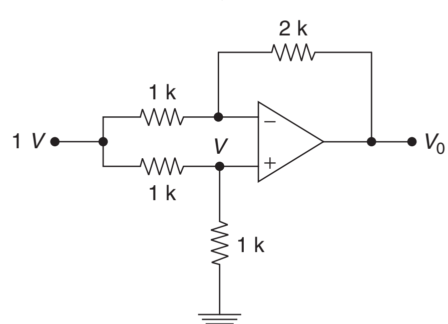
(a) –2 V (b) –1 V (c) –0.5 V (d) 0.5 V

Solution: (c)

Nodal analysis at non-inverting terminal

$$\frac{V-1}{1} + \frac{V}{1} = 0$$

$$2V = 1$$



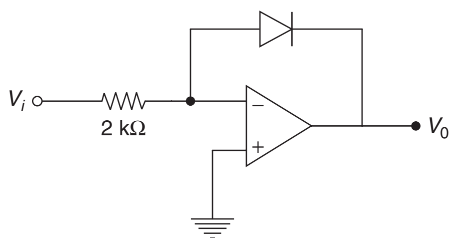
$V = 0.5$ V

Nodal analysis at inverting terminal

$$\frac{V-1}{1} + \frac{V-V_0}{2} = 0$$

$2V - 2 + V - V_0 = 0$
$3V = 2 + V_0$
$\Rightarrow 3 \times 0.5 = 2 + V_0$
$\Rightarrow 1.5 = 2 + V_0$
$\Rightarrow V_0 = -0.5$ V
Hence, the correct option is (c).

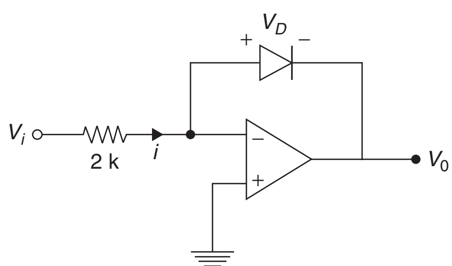
22. In the op-amp circuit shown assume that the diode current follows the equation $I = I_s \exp(V/V_T)$. For $V_i = 2$ V, $V_0 = V_{01}$, and for $V_i = 4$ V, $V_0 = V_{02}$. The relationship between V_{01} and V_{02} is **[2007]**



(a) $V_{02} = \sqrt{2}V_{01}$ (b) $V_{02} = e^2 V_{01}$
(c) $V_{02} = V_{01}\ln 2$ (d) $V_{01} - V_{02} = V_T \ln 2$

Solution: (d)

For $V_i = 2$ V
$V_0 = -V_D$
$V_0 = -V_T \ln\left(\frac{i}{1s}\right)$

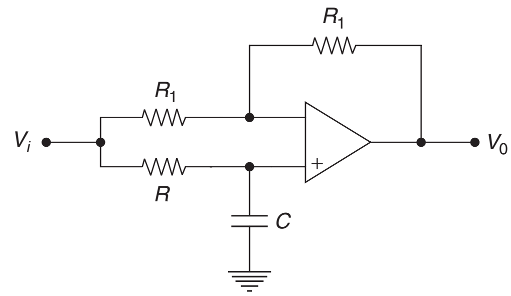


$2 = 2i$
$\Rightarrow i = 1$ mA
$$\Rightarrow V_0 = -V_T \ln\left(\frac{1m}{1s}\right) = V_{01} \quad (1)$$
$$V_0 = -V_D = -V_T \ln\left(\frac{i}{1s}\right)$$
$4 = 2_i$
$i = 2$ mA
$$V_0 = -V_T \ln\left(\frac{2m}{1s}\right) = V_{02} \quad (2)$$
$$\frac{1m}{1s} = e^{-V_{01}/V_T}$$
and $\frac{2m}{1s} = e^{-V_{02}/V_T}$

taking the ratio,
$$\frac{1}{2} = e\frac{(V_{02} - V_{01})}{V_T}$$

$\ln 2 = (V_{01} - V_{02})/ V_T$
$\Rightarrow V_{01} - V_{02} = V_T \ln 2$
Hence, the correct option is (b).

Linked Answer Questions 12 and 13.
Consider the op-amp circuit shown in the figure.



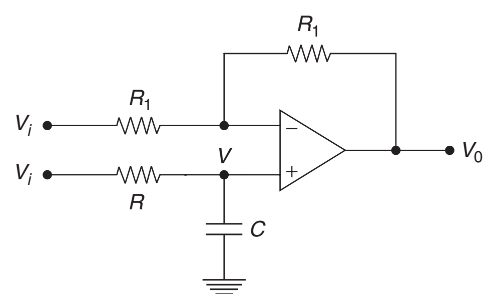
23. The transfer function $V_0(s)/V_i(s)$ is **[2007]**

(a) $\frac{1 - sRC}{1 + sRC}$ (b) $\frac{1 + sRC}{1 - sRC}$

(c) $\frac{1}{1 - sRC}$ (d) $\frac{1}{1 + sRC}$

Solution: (a)

Nodal analysis at non-inverting input
$$\frac{V - V_i}{R} + VsC = 0$$
$$V\left(\frac{1}{R} + sC\right) = \frac{V_i}{R}$$



$$\Rightarrow V = V_i/(1 + sCR) \quad (1)$$
$$\frac{V - V_i}{R_1} + \frac{V - V_0}{R_1} = 0$$
$$\Rightarrow 2V = V_i + V_o$$
$$\Rightarrow \frac{2V_i}{1 + sCR} = V_i + V_0$$
$$\Rightarrow \left(\frac{2}{1 + sCR} - 1\right) V_i = V_0$$
$$\Rightarrow \left(\frac{2 - 1 - sCR}{1 + sCR}\right) V_i = V_0$$

$$\Rightarrow \left(\frac{1-sCR}{1+sCR}\right)V_i = V_0$$

$$\Rightarrow \frac{V_0}{V_i} = \frac{1-sCR}{1+sCR}$$

Hence, the correct option is (a).

24. If $V_i = V_1 \sin(\omega t)$ and $V_o = V_2 \sin(\omega t + \phi)$, then the minimum and maximum values of ϕ (in radians) are, respectively **[2007]**

(a) $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ (b) 0 and $\frac{\pi}{2}$

(c) $-\pi$ and 0 (d) $-\frac{\pi}{2}$ and 0

Solution: (c)

$$V_i = V_1 \sin \omega t$$

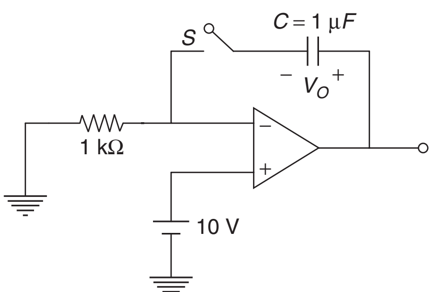
$$V_0 = V_2 \sin(\omega t + \phi)$$

$$\frac{V_0(s)}{V_i(s)} = \frac{1-sCR}{1+sCR}$$

$$\angle\frac{V_0}{V_I}(s) = \angle\frac{1-sCR}{1+sCR}$$

$\theta = -\tan^{-1}\omega CR - \tan^{-1}\omega CR$
$\theta = -2\tan^{-1}\omega CR$
Minimum value of $\theta = -\pi$ (at $\omega \to \infty$).
Maximum value of $\theta = 0$ (at $\omega = 0$).
Hence, the correct option is (c).

25. For the circuit shown in the following figure, the capacitor C is initially uncharged. At $t = 0$, the switch S is closed. The voltage V_C across the capacitor at $t = 1$ millisecond is______.



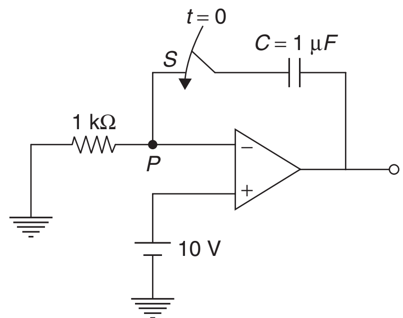
In the figure shown above, the op-amp is supplied with +15 V. **[2006]**

(a) 0 V (b) 6.3 V

(c) 9.45 V (d) 10 V

Solution: (d)

Capacitor is initially uncharged. At $t = 0$, switch 'S' is closed.

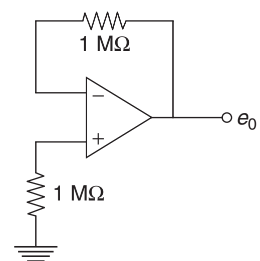


Applying kCL at node P

$$\frac{10}{1k} = C\frac{dV_C}{dt} = \frac{10^{-6}\times V_C}{1\times 10^{-3}}$$

$\Rightarrow V_C = 10$ V
Hence, the correct option is (d).

26. The voltage e_0 indicated in the figure has been measured by an ideal voltmeter. Which of the following can be calculated? **[2005]**

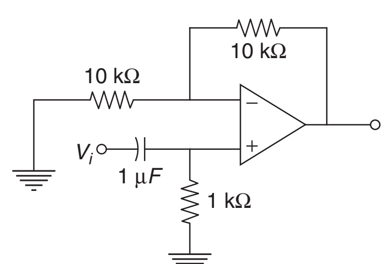


(a) Bias current of the inverting input only
(b) Bias current of the inverting and non-inverting inputs only
(c) Input offset current only
(d) Both the bias currents and the input offset current

Solution: (c)

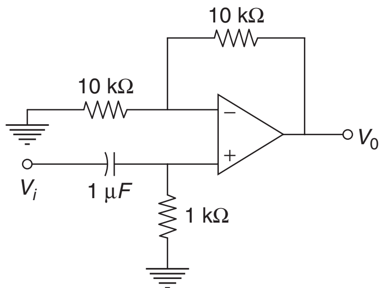
$V_1 = -I_{B1} \times 1\text{M}$
$V_2 = V_1 = -I_{B1} \times 1\text{M}$ (due to virtual ground)
Drop in feedback register 1M $= I_{R2} \times 1\text{M}$
$e_0 = V_2 + I_{B2} \times 1\text{M}$
$e_0 = -I_{B1} \times 1\text{M} + I_{B2} \times 1\text{M}$
$e_0 = (I_{B2} - I_{B1}) \times 1\text{M}$
where $(I_{B2} - I_{B1})$ is offset current.
Hence, the correct option is (c).

27. The op-amp circuit shown in the figure is a filter. The type of filter and its cut-off frequency are respectively **[2005]**



(a) high pass, 1000 rad/sec.
(b) low pass, 1000 rad/sec.
(c) high pass, 10000 rad/sec.
(d) low pass, 10000 rad/sec.

Solution: (c)
Since O/P is taken across 10 KΩ, it is a high filter



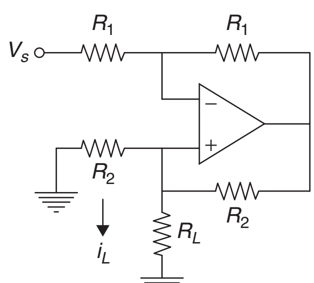
at $\omega = 0$
$V_0 = 0$
at $\omega = \infty$
$V_0 = 2V_i$
I/P is at non-inventing point

so, frequency $= \dfrac{1}{RC}$

$$= \frac{1}{1\times 10^3 \times 1\times 10^{-6}}$$

= 1000 rad/sec
Hence, the correct option is (c).

28. In the op-amp circuit given in the figure, the load current i_L is **[2004]**



(a) $-\dfrac{V_s}{R_2}$ (b) $\dfrac{V_s}{R_2}$

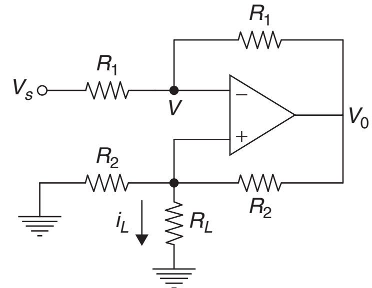
(c) $-\dfrac{V_s}{R_L}$ (d) $\dfrac{V_s}{R_1}$

Solution: (a)

Nodal analysis at inverting input

$$\frac{V - V_S}{R_1} + \frac{V - V_0}{R_1} = 0$$

$$\Rightarrow 2V = V_S + V_0 \qquad (1)$$



Nodal analysis at non-inverting terminal

$$\frac{V}{R_2} + \frac{V}{R_L} + \frac{V - V_0}{R_2} = 0$$

$$V\left(\frac{2}{R_2} + \frac{1}{R_L}\right) = \frac{V_0}{R_2} \qquad (2)$$

From equations (1) and (2)

$$V\left(\frac{2}{R_2} + \frac{1}{R_L}\right) = \frac{2V - V_S}{R_2}$$

$$\frac{2V}{R_2} + \frac{V}{R_L} = \frac{2V}{R_2} - \frac{V_S}{R_2}$$

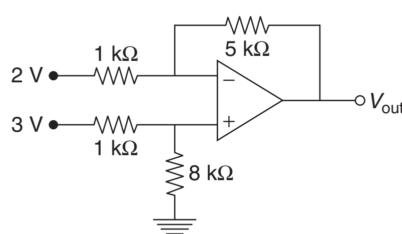
$$\Rightarrow \frac{V}{R_L} = \frac{-V_S}{R_2} \qquad (3)$$

$$V = i_L R_L$$

$$\Rightarrow iL = \frac{V}{R_L} = \frac{-V_s}{R_2}$$

Hence, the correct option is (a).

29. If the op-amp in the figure is ideal, the output voltage V_{out} will be equal to **[2003]**



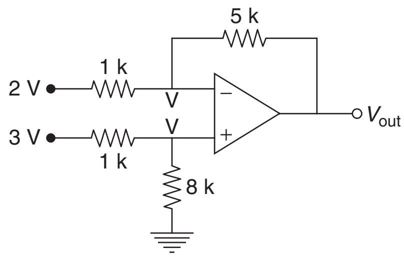
(a) 1 V (b) 6 V (c) 14 V (d) 17 V

Solution: (b)
Nodal analysis at non-inverting terminal of op-amp

$$\frac{V-3}{1} + \frac{V}{8} = 0$$

$8V - 24 + V = 0$
$9V = 24$

$$V = \frac{24}{9}$$



Nodal analysis at non-inverting terminal of op-amp

$$\frac{V-2}{1}+\frac{V-V_{out}}{5}=0$$

$$5V-10+V-V_{out}=0$$

$$6V=10+V_{out}$$

$$6\times\frac{24}{9}=10+V_{out}$$

$$16=10+V_{out}$$

$$V_{out}=6V$$

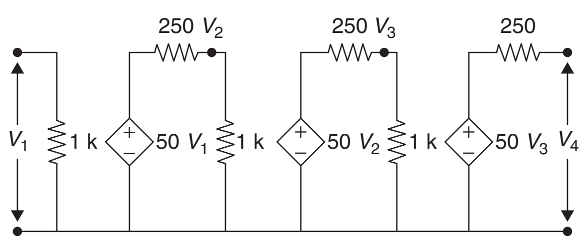
Hence, the correct option is (b).

30. Three identical amplifiers with each one having a voltage gain of 50, input resistance of 1 kΩ and output resistance of 250 Ω are cascaded. The open circuit voltage gain of the combined amplifier is **2003]**

(a) 49 dB (b) 51 dB
(c) 98 dB (d) 102 dB

Solution: (c)

$$A_V=\frac{V_4}{V_1}$$



$$\Rightarrow A_V=\frac{V_4}{V_3}\times\frac{V_3}{V_2}\times\frac{V_2}{V_1}$$

Voltage across 1k after first stage

$$=\frac{1000\times 50\,V_1}{1250}=40$$

Similarly, $\frac{V_3}{V_2}=40$

$\therefore A_v=40\times 40\times 50=8\times 10^4$

A_V in dB $=20\log(8\times 10^4)=98$ dB.

Hence, the correct option is (c).

31. An amplifier using an op-amp with a slew-rate SR = 1 V/μsec has a gain of 40 dB. If this amplifier has to faithfully amplify sinusoidal signals from dc to 20 kHz without introducing any slew-rate induced distortion, then the input signal level must not exceed______. **[2002]**

(a) 795 mV (b) 395 mV
(c) 79.5 mV (d) 39.5 mV

Solution: (c)

Slew rate = A. $2\pi f V_m$

$V=AV_m\sin\omega t$

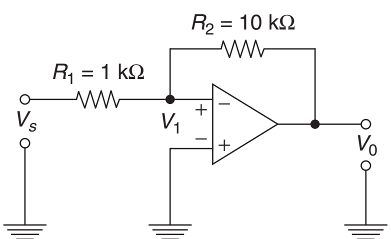
$$SR=\left.\frac{dV}{dt}\right|_{\max}=AV_m 2\pi f_m$$

$$V_m=\frac{SR}{A.2\pi f_m}=\frac{1\times 10^6}{100\ \ 2\pi\times 20\times 10^3}\quad\begin{bmatrix}20\log A=40\\ A=100\end{bmatrix}$$

$$=79.5\text{ mV}$$

Hence, the correct option is (c).

32. The inverting op-amp shown in the figure has an open-loop gain of 100. The closed-loop gain V_0/V_s is **[2001]**

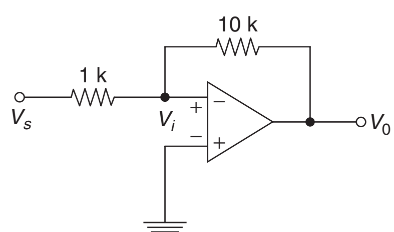


(a) –8 (b) –9 (c) –10 (d) –11

Solution: (b)

Open loop gain

$$\frac{V_0}{-V_i}=100$$



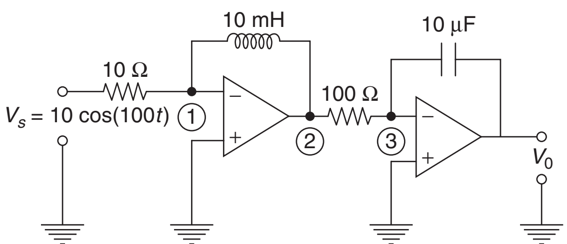
Nodal analysis at non-inverting terminal of the op-amp

$$\frac{V_i-V_S}{1}+\frac{V_i-V_0}{10}=0$$

$$10V_i - 10V_s + V_i - V_0 = 0$$
$$11V_i = 10\,V_s + V_0$$
$$\frac{-11V_0}{100} = 10V_s + V_0$$
$$\frac{-111}{100}V_0 = 10\,V_s$$
$$\Rightarrow \frac{V_0}{V_s} = \frac{-1000}{111} \cong -9$$

Hence, the correct option is (b).

33. In the figure assume the op-amps to be ideal. The output V_0 of the circuit is **[2001]**



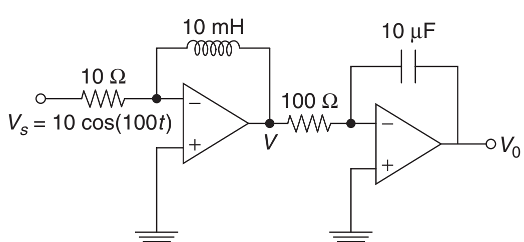
(a) $10\cos(100t)$

(b) $10\int_0^t \cos(100\tau)\,d\tau$

(c) $10^{-4}\int_0^t \cos(100\tau)\,d\tau$

(d) $10^{-4}\frac{d}{dt}\cos(100t)$

Solution: (a)



Output of first op-amp

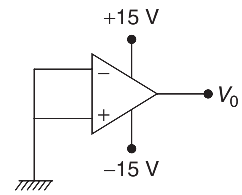
$$V = \frac{-L}{R}\frac{dV_s}{dt}$$
$$= \frac{-10m}{10} \times 10 \times 100\sin(100t)$$
$$= +\sin(100t)$$

Output of 2nd op-amp

$$V_0 = \frac{-1}{CR}\int V\,dt$$
$$= \frac{-1}{10\mu\,100}\left[\frac{-\cos 100t}{100}\right] = +10\cos 100t \text{ V}$$

Hence, the correct option is (a).

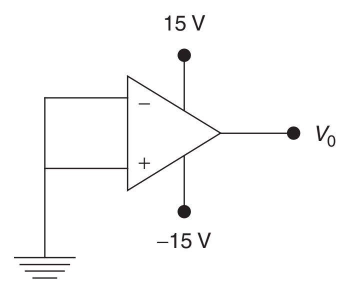
34. If the op-amp in the figure has an input offset voltage of 5 mV and an open-loop voltage gain of 10,000, then V_0 will be **[2000]**



(a) 0 V (b) 5 mV
(c) +15 V or −15 V (d) +50 V or −50 V

Solution: (c)

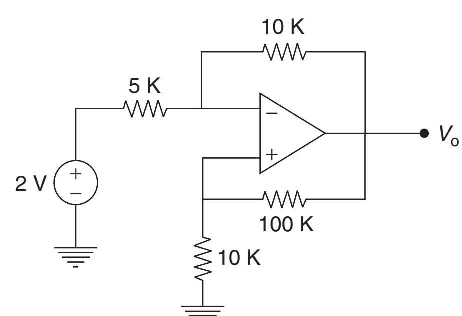
I/P offset voltage = 5 mV
Open loop gain = 10^4



O/P voltage = 5 m × 10^4
= 50 V
= 15 V (∴ O/P voltage can't go beyond ± 15 V)

Hence, the correct option is (c).

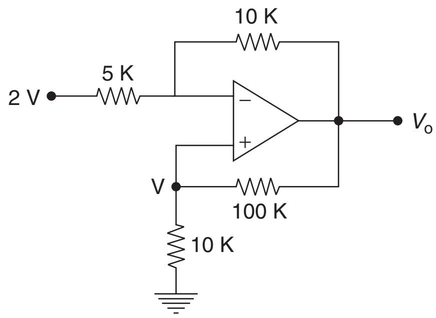
35. The output voltage V_0 of the circuit shown in the figure is **[1997]**



(a) −4 V (b) 6 V
(c) 5 V (d) −5.5 V

Solution: (d)

Applying nodal analysis,



$$\frac{V}{10}+\frac{V-V_0}{100}=0$$

$$11V = V_0 \qquad (1)$$

$$\frac{V-2}{5}+\frac{V-V_0}{10}=0$$

$$2V-4+V-V_0=0$$

$$3V = 4+V_O \qquad (2)$$

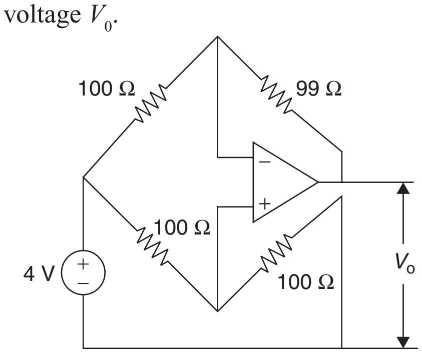
$$\frac{3V_0}{11}=4+V_0$$

$$\Rightarrow -\frac{8V_0}{11}=4$$

$$\Rightarrow V_0=-5.5\text{ V}$$

Hence, the correct option is (d).

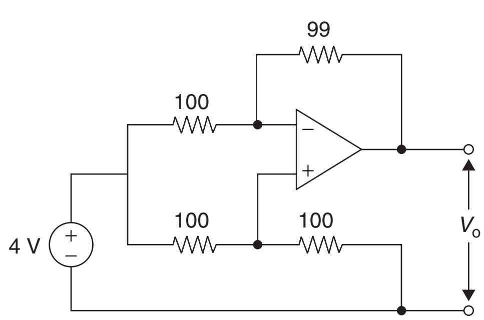
36. For the ideal op-amp circuit of figure, determine the output voltage V_0. **[1993]**



Solution: By calculating gain at the inverting terminal of the op-amp

$$(V_0)_1 = 4\times\left(\frac{-99}{100}\right)$$

$$= -3.96\ V$$

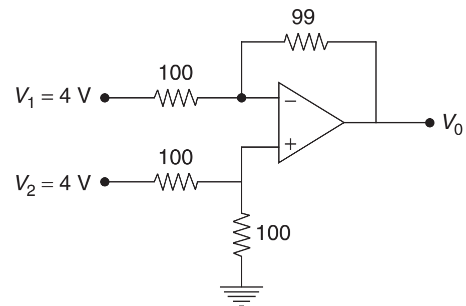


By calculating gain at the non-inverting terminal of the op-amp

$$(V_0)_2 = 4\times\left(\frac{100}{100+100}\right)\times\left(1+\frac{99}{100}\right)$$

$$= 2 \times 1.99$$

$$= 3.98\text{ V}$$



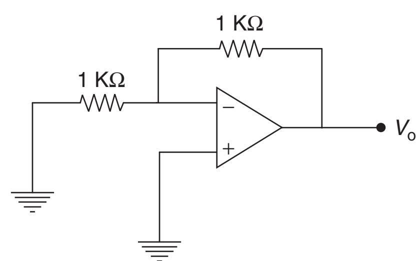
Total output voltage is

$V_0 = (V_0)_1 + (V_0)_2$

$= -3.96 + 3.98$

$= 0.02$ V.

37. An op-amp has an offset voltage of 1 mV and is ideal in all other respects. If this op-amp is used in the circuit shown in the figure, the O/P voltage will be (Select the nearest value) **[1992]**

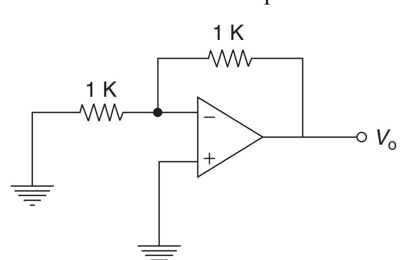


(a) 1 mV (b) 1 V (c) ±1 V (d) 0 V

Solution: (a)

I/P offset voltage = 1 mV

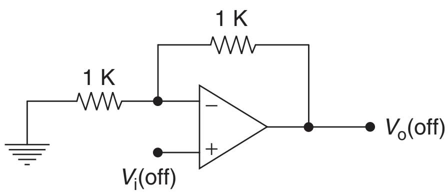
This voltage is applied to the terminal because off set is an error and if should be positive



$$V_0\text{ (off)} = V_i\text{ (off)}\left(1+\frac{R_f}{R_1}\right)$$

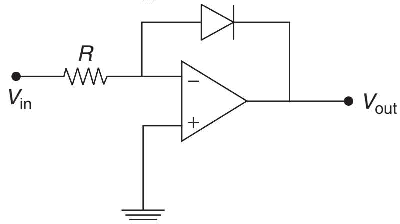
$$= 1\text{ m}\left(1+\frac{1k}{1k}\right)$$

$$= 2\text{ mV}$$



Hence, the correct option is (a).

38. The circuit of the figure uses an ideal op-amp for small positive values of V_{in}, the circuit works as **[1992]**

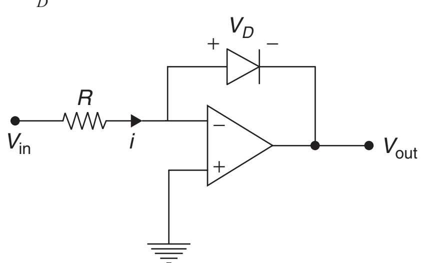


(a) a half-wave rectifier
(b) a differentiator
(c) a logarithmic amplifier
(d) an exponential amplifier

Solution: (c)

$V_{out} + V_D = 0$

$V_{out} = -V_D$ (1)



Diode current, $I_D \approx I_S\left(e^{\frac{V_d}{\eta V_T}} - 1\right)$

approximating, $I_D \approx I_S\, e^{V_D/\eta V_T}$

$$\Rightarrow V_D = \eta V_T \ln\left(\frac{I_D}{I_S}\right) \quad (2)$$

Output voltage of op-amp is

$$V_{out} = -V_D = -\eta V_T \ln\left(\frac{I_D}{I_s}\right) \quad (3)$$

Input voltage can be written as

$V_{in} = iR = I_D R$ $[\therefore I = I_D]$ (4)

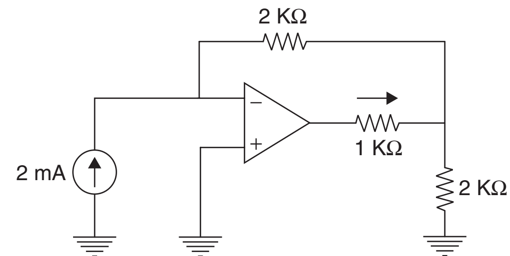
From equations (3) and (4)

$$V_{out} = -\eta V_T \ln\left(\frac{V_{in}}{I_S R}\right)$$

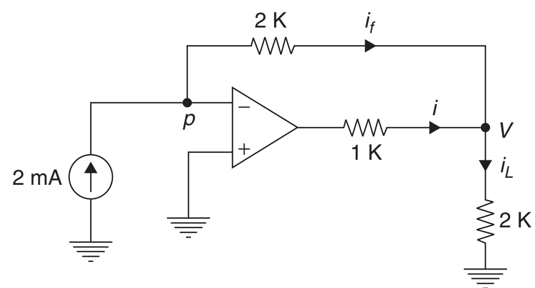
$\therefore$ the circuit acts as a logarithmic amplifier.

Hence, the correct option is (c).

39. Assume that the operational amplifier in figure is ideal, the current I through the 1 K ohm resistor is________. **[1992]**



Solution: If input current at inverting terminal of op-amp is 2 mA, applying nodal analysis at node P,



$$\frac{0 - V}{2k} - 2m = 0$$

V = −4V

$V = 2i_L = -4$

As current does not enter the op-amp

$\Rightarrow i_L = -2$ mA

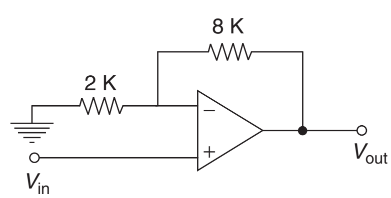
$i_L = i - i_f$

$\Rightarrow i = i_L + i_f$

$= -2m - 2m$

$i = -4$ mA

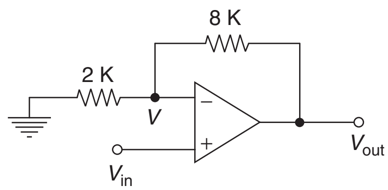
40. The op-amp of figure shown below has a very poor open loop voltage gain of 45 but is otherwise ideal. The gain of the amplifier equals: **[1990]**



(a) 5 (b) 20 (c) 4 (d) 4.5

Solution: (d)

$$\frac{V_{out}}{V_{in} - V} = 45 \quad (1)$$



Gain of the op-amp is given by

$$\frac{V_{\text{out}}}{V} = 1 + \frac{8k}{2k} = 5 \qquad (2)$$

$$\frac{V}{V_{\text{out}}} = \beta = \frac{1}{5} = 0.2$$

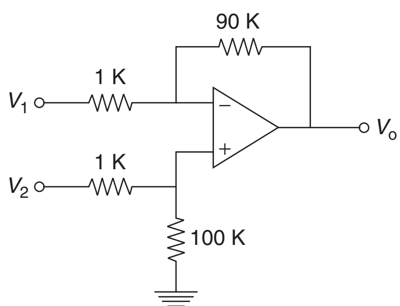
$$A = \frac{V_{\text{out}}}{V_{\text{in}} - V} = 45$$

$AB = 9$ (comparable to 1)

$$\therefore A_f = \frac{A}{1+AB} = \frac{45}{1+9} = 4.5$$

Hence, the correct option is (d).

41. The CMRR of the differential amplifier of the figure shown below is equal to **[1990]**



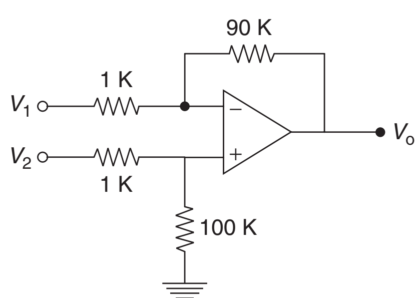
(a) ∞ (b) 0 (c) 1000 (d) 1800

Solution: (c)

Inverting gain of the op amp is given by

$$A_1 = \left.\frac{V_0}{V_1}\right|_{V_2} = 0$$

$$= \frac{-90k}{1k} = -90$$



Non-inverting gain is given by

$$A_2 = \left.\frac{V_0}{V_2}\right|_{V_1} = 0$$

$$= \left(1 + \frac{R_f}{R_1}\right)\left(\frac{100}{100+1}\right) = 90.09$$

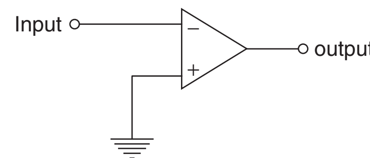
$$A_d = \frac{A_2 - A_1}{2} = \frac{90.09 - (-90)}{2} = 90$$

$$A_c = A_1 + A_2 = -90 + 90.09 = 0.09$$

$$\text{CMRR} = \left|\frac{A_d}{A_c}\right| = \frac{90}{0.09} = 1000$$

Hence, the correct option is (c).

42. If the input to the circuit of figure is a sine wave the output will be **[1990]**



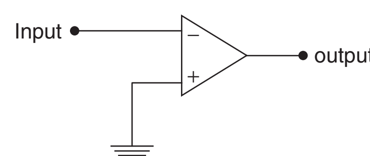
(a) A half-wave rectified sine wave
(b) A full-wave rectified sine wave
(c) A triangular wave
(d) A square wave

Solution: (d)

If I/P applied is sine wave

O/P = A (0 − sinωt)

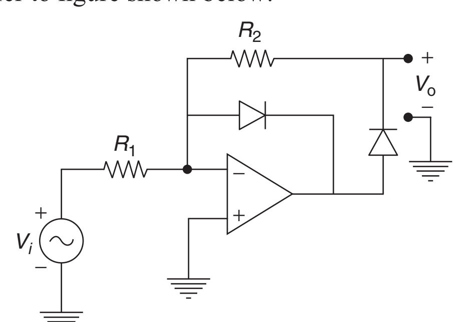
= −A sinωt



Since, op-amp was a very high voltage gain A, therefore the O/P voltage saturates and results in a square wave.

Hence, the correct option is (d).

43. Refer to figure shown below: **[1989]**



(a) For $V_i > 0$, $V_0 = -\dfrac{R_2}{R_1}V_i$

(b) For $V_i > 0$, $V_0 = 0$

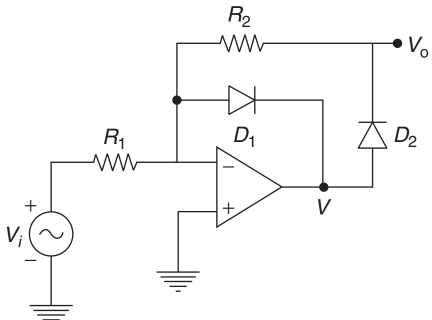
(c) For $V_i < 0$, $V_0 = -\dfrac{R_2}{R_1}V_i$

(d) For $V_i < 0$, $V_0 = 0$

Solution: (c)

Case 1: $V_i > 0$

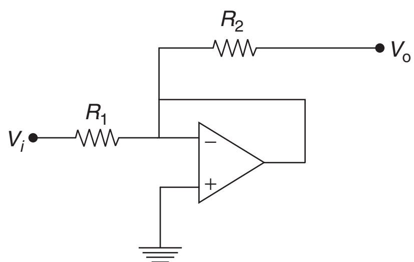
$\therefore D_1$ is on



V is *SC* (assuming op-amp ideas)

D_2 is OFF

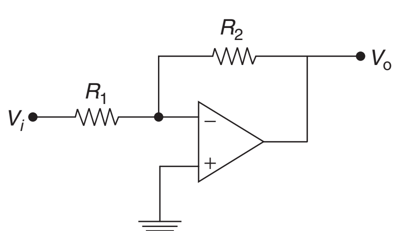
$V_0 = 0$ V



Case 2: $V_i < 0$

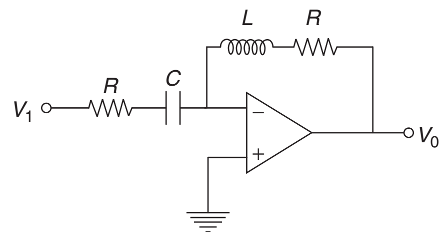
D_1 is OFF, D_2 is ON

$$V_0 = \frac{-R_2}{R_1} \times V_i$$



Hence, the correct option is (c).

44. The op-amp shown in the figure below is ideal. $R - \sqrt{L/C}$. The phase angle between V_0 and V_1 at $\omega = 1/\sqrt{LC}$ **[1988]**

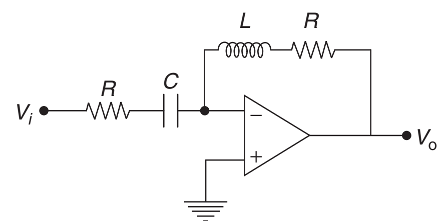


(a) $\dfrac{\pi}{2}$ (b) π

(c) $\dfrac{3\pi}{2}$ (d) 2π

Solution: (c)

$$\left.\begin{aligned} R &= \sqrt{\frac{L}{C}} \\ \omega &= \frac{1}{\sqrt{LC}} \end{aligned}\right\} \text{given}$$



Output to input voltage ratio is

$$\frac{V_o}{V_i} = -\frac{Z_f}{Z_1} = -\left(\frac{R + j\omega L}{R - \dfrac{j}{\omega c}}\right)$$

$$= \frac{\sqrt{R^2 + \omega^2 L^2}}{\sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}} \times \angle \tan^{-1}\frac{\omega L}{R} + \tan^{-1}\frac{1}{\omega CR} + \pi$$

Therefore the phase angle

$$\phi = \left(\tan^{-1}\frac{\omega L}{R} + \tan^{-1}\frac{1}{\omega CR}\right) + \pi$$

$$= \tan^{-1}\left(\frac{\dfrac{\omega L}{R} + \dfrac{1}{\omega CR}}{1 - \dfrac{\omega L}{R} \times \dfrac{1}{\omega CR}}\right) + \pi$$

$$= \tan^{-1}\left\{\frac{\omega^2 LC + 1}{\omega CR} \times \frac{1}{1 - \dfrac{L}{CR^2}}\right\} + \pi$$

$$= \tan^{-1}\left\{\frac{1+1}{\omega CR} \times \frac{1}{1 - \dfrac{L}{CR^2}}\right\} + \pi$$

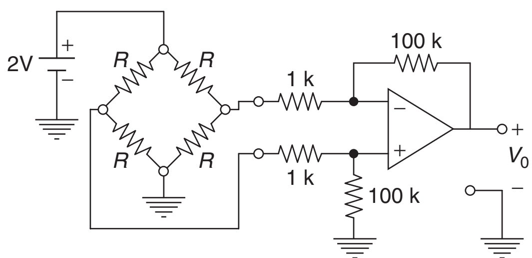
$$= \tan^{-1}\left\{\frac{2\sqrt{LC}}{CR}\times\frac{1}{0}\right\}+\pi$$

$$= \frac{\pi}{2}+\pi$$

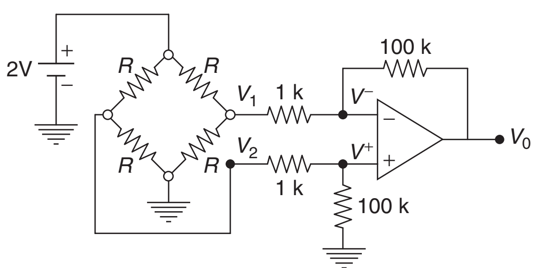
$$= \frac{3\pi}{2}$$

Hence, the correct option is (c).

45. In the figure shown below, if the CMRR of the operational amplifier is 60 dB, then the magnitude of the output voltage is: **[1987]**



Solution: 0.1 V



Apply nodal analysis at node voltages V_1 and V_2,

$$\frac{V_1-2}{R}+\frac{V_1}{R}=0$$

$$V_1 = 1\text{ V}$$

$$\frac{V_2-2}{R}+\frac{V_2}{R}=0$$

$$V_2 = 1\text{ V}$$

$$V_d = V_2 - V_1 = 0\text{ V}$$

$$V_C = \frac{V_2+V_1}{2} = \frac{1+1}{2} = 1\text{ V}$$

$$A^- = \left.\frac{V_0}{V_1}\right|_{V_2=0} = \frac{-R_f}{R_1} = -100$$

$$A^+ = \left.\frac{V_0}{V_2}\right|_{V_1=0} = \frac{V^+}{V_2}\left(1+\frac{R_f}{R_1}\right) = \left(\frac{100}{100+1}\right)\left(1+\frac{100}{1}\right) = 100$$

$$A_d = \frac{A^+ - A^-}{2} = \frac{100-(-100)}{2} = 100$$

$$\text{CMRR} = 60\text{ dB}$$
$$= 10^3$$

$$\text{CMRR} = 10^3 = \frac{A_d}{A_c}$$

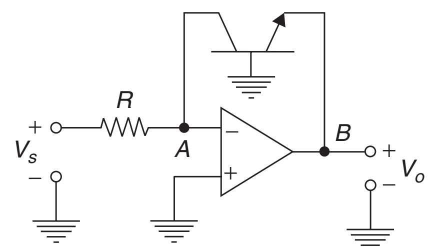
O/P voltage is given by,

$$V_0 = |A_c|V_c + |A_d|V_d$$

$$= \frac{A_d}{10^3}\times 1 + 0 = \frac{100}{10^3} = 0.1\text{ V}$$

Five-marks Questions

1. Assume that the op-amp in the figure, ideal



(a) Obtain an expression for V_o in terms of V_s, R and the reverse saturation current I_s of the transistor.

(b) If $R = 1\ \Omega$, $I_s = 1$ pA and the thermal voltage $V_T = 25$ mV, then than what is the value of the output voltage V_o for an input voltage $V_s = 1$V?

(c) Suppose that the transistor in the feedback path is replaced by a $p-n$ junction diode with a reverse saturation current of I_s the p-side of the diode is connected to node A and the n-side to node B. Then what is the expression for V_o in terms V_s, R and I_s? **[2001]**

Solution: (a) Current in resistance R is 1

$$i = \frac{V_S - 0}{R} = \frac{V_S}{R}$$

$$i = i_C = i_e = I_S e^{V_{be}/VT}$$

$$\frac{V_S}{R} = I_S e^{V_{be}/V_T}$$

$$V_{be} + V_0 = 0$$

$$V_0 = -V_{be}$$

$$\frac{V_S}{R} = I_S e^{-V_0/V_T}$$

$$-\frac{V_0}{V_T} = \ln\left(\frac{V_S}{RI_S}\right)$$

$$V_0 - V_T \ln\left(\frac{V_S}{RI_S}\right)$$

(b) $V_0 = -25\times10^{-3} \ln\left(\frac{1}{1\times10^{-12}}\right)$

$$V_0 = -0.69 \text{ V}$$

(c) $i = \frac{V_S}{R}$

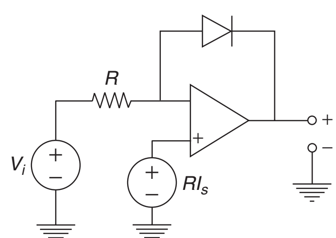
$$i = i_f = I_f e^{V_d/V_T} = V_s/R$$

$$\frac{V_d}{V_T} = \ln\left(\frac{V_S}{RI_S}\right)$$

$$V_0 + V_d = 0$$
$$V_0 = -V_d$$

$$V_0 = -V_T \ln\left(\frac{V_S}{RI_S}\right)$$

2. Consider the circuit given in figure using an ideal operational amplifier.



The characteristics of the diode are given by the relation $I = I_s\,[e^{qV/KT} - 1]$
Where V is the forward voltage across the diode.

(a) Express V_o as a function of V_j assuming $V_j > 0$.

(b) If $R = 100$ kΩ, $I_s = 1\,\mu$A and $\frac{KT}{q} = 25$ mV. Find the V_i for which $V_o = 0$. **[1997]**

Solution:

(a) $I = \frac{V_S - RI_S}{R}$

$$I_f = I_s\left[e^{\frac{Vd}{V_T}} - 1\right] = I = \frac{V_S}{R} - I_S$$

$$I_S e^{\frac{Vd}{VT}} - \frac{I}{S} = \frac{V_S}{R} - \frac{I}{S}$$

$$\frac{V_d}{V_T} = \ln\left(\frac{V_S}{RI_S}\right)$$

$$V_d = V_T \ln\left(\frac{V_S}{RI_S}\right)$$

$$RI_S - V_d - V_0 = 0$$

$$V_d = RI_S - V_0 = V_T \ln\left(\frac{V_s}{RI_S}\right)$$

$$V_0 = RI_S - \frac{KT}{q}\ln\left(\frac{V_S}{RI_S}\right) \qquad \left(V_T = \frac{KT}{q}\right)$$

(b) Using equation of V_0 of part (a)

$$0 = (100\times10^3)\times(1\times10^{-6})$$

$$-25\times10^{-3}\ln\left(\frac{V_i}{100\times10^3\times1\times10}\right)$$

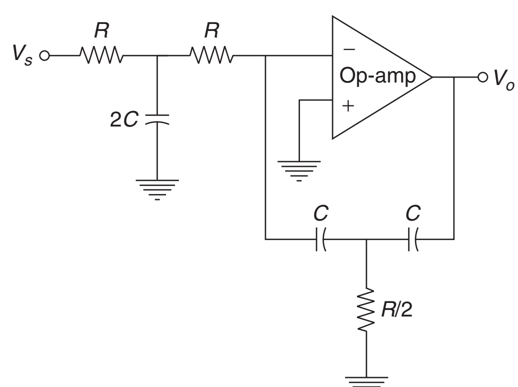
$\ln 10\ V_i = 4$

$$V_i = \frac{e^4}{10}$$

$V_i = 5.46$ V

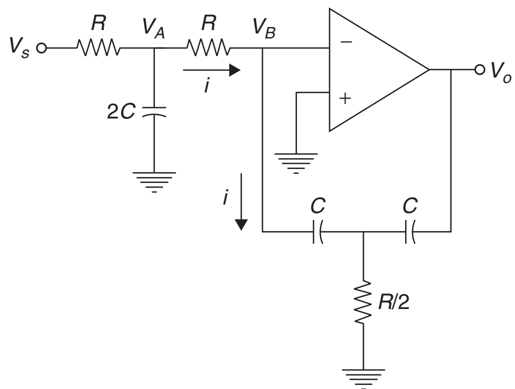
3. Shown that the system in figure is a double integrator.

In other words, prove that the transfer gain is given by $\frac{V_o(s)}{V_i(s)} = \frac{1}{(CR_s)^2}$ assuming ideal op-amp.



[1995]

Solution:



By virtual short:
$V_B = 0$
KCL at node A

$$\frac{V_A - V_S}{R} + \frac{V_A}{\frac{1}{2C_S}} + \frac{V_A - V_B}{R} = 0$$

$$2V_A + 2RC_s V_A = V_S$$

$$V_A = \frac{V_S}{2(RC_S + 1)} \qquad (1)$$

Apply KCL at node C

$$\frac{V_C - 0}{\frac{R}{2}} + \frac{V_C - V_B}{\frac{1}{C_s}} + \frac{V_C - V_0}{\frac{1}{C_s}} = 0 \qquad (V_B = 0)$$

$$V_C = \frac{RC_S V_0}{2(RC_S + 1)}$$

$$i = \frac{V_A - 0}{R} = \frac{0 - V_C}{1/C_S} = -C_S V_C$$

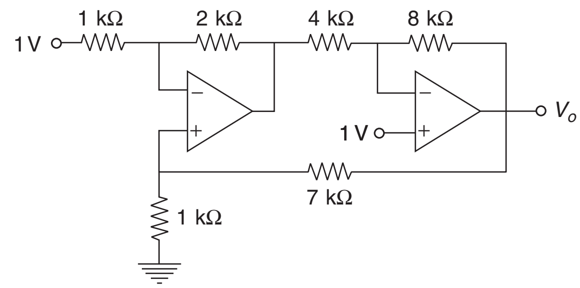
$$V_A = -RC_S V_C = \frac{V_s}{2(RC_S + 1)}$$

From equation (1)

$$\frac{V_s}{2(RC_S + 1)} = -RC_S\left[\frac{RC_S V_0}{2(RC_S + 1)}\right]$$

$$\frac{V_0}{V_S} - \frac{1}{(RC_S)^2}$$

4. Find the output voltage of the following circuit assuming ideal op-amp behaviour.

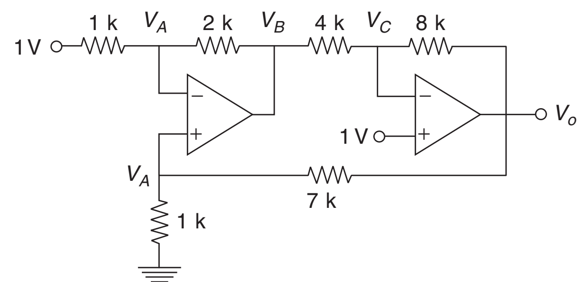


[1994]

Solution: By virtual short $V_C = 1$

$$V_A = V_0 \cdot \frac{1K}{1K + 7K} = \frac{V_0}{8}$$

$$V_A = \frac{V_0}{8} \qquad (1)$$



Apply KCL at mode A

$$\frac{V_A - 1}{1K} + \frac{V_A - V_B}{2K} = 0$$

$$2VA - 2 + V_A - V_B = 0$$

From equation (1) $V_A = \dfrac{V_0}{8}$

$$VB = \frac{3V_0}{8} - 2$$

Apply KCL at node C

$$\frac{V_C - V_B}{4} + \frac{V_C - V_0}{8} = 0$$

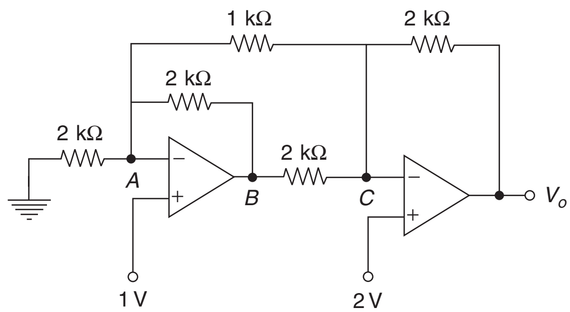
$$2V_C - 2V_B + V_C - V_0 = 0$$

$$V_C = 1\,\text{V and } V_B = \frac{3V_0}{8} - 2$$

$$\therefore\ 3 \times 1 - 2\left(\frac{3V_0}{8} - 2\right) - V_0 = 0$$

$$V_0 = 4\,\text{V}$$

5. Find the output, V_0 in the following circuit figure, as summing that the op-amps are ideal.



[1993]

Solution: By virtual short

$V_A = 1$ V $V_C = 2$ V

Apply KCL at node A

$$\frac{0-V_A}{2k} = \frac{V_A - V_B}{2k} + \frac{V_A - V_C}{1k}$$

$$\frac{0-1}{2k} = \frac{1-V_B}{2k} + \frac{1-2}{1k}$$

$V_B = 0$

Apply KCL at node C

$$\frac{V_C - V_B}{2k} + \frac{V_C - V_A}{1k} + \frac{V_C - V_0}{2k} = 0$$

$V_0 = 6$ V

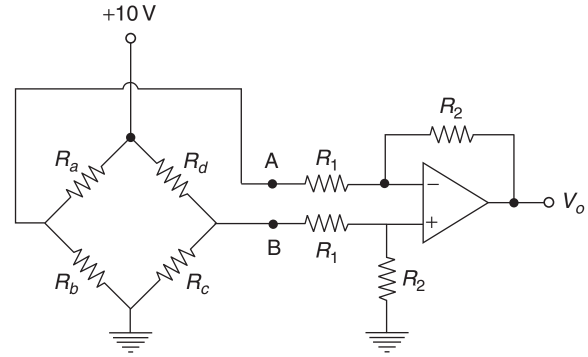
6. Consider the circuit shown in figure. The circuit uses an ideal operational amplifier. Assuming that the impedances at nodes A and B do not load the preceding bridge circuit, calculate the output voltage V_0.

(a) when $R_a = R_b = R_c = R_d = 10\ \Omega$.

(b) when $R_a = R_b = R_c = 10$ ohms and $R_d = 120\ \Omega$.

$R_2 = 12$ kΩ

$R_1 = 10$ kΩ



[1992: 5 Marks]

Solution:

(a) $$V_A = 10 \times \frac{R_b}{R_a + R_b}$$

$$= 10 \times \frac{100}{100+100} = 5\text{V}$$

$V_B = 5$V

Given op-amp is a differential amp

So $$V_0 = \frac{R_2}{R_1}(V_B - V_A) = \frac{12k}{10k}(5-5) = 0$$

(b) $$V_A = 10 \times \frac{R_b}{R_d + R_b} = 5\text{V}$$

$V_B = 9.55$ V

$$V_0 = \frac{12k}{10k}(4.55-5) = -0.55\text{V}$$

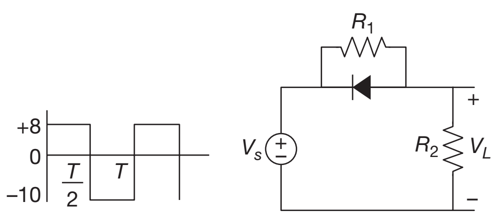
Chapter 2

Diodes Applications

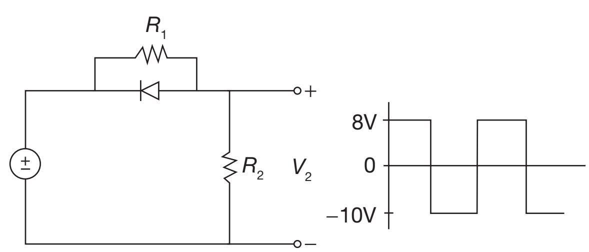
One-mark Questions

1. In the circuit shown V_s is a square wave of period T with maximum and minimum values of 8 V and –10 V. respectively. Assume that the diode is ideal and $R_1 = R_2$ = 50 Ω.

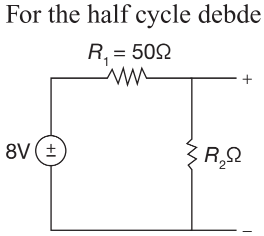
The average value of V_L is _____ volts (rounded off to 1 decimal place). **[2019]**



Solution:



For the half cycle debde is reverse biased

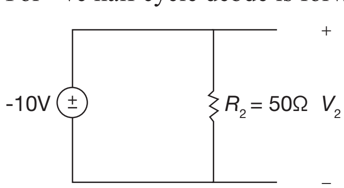


$$V_L = V_S \cdot \frac{R_2}{R_1 + R_2}$$

$$= 8 \cdot \frac{80}{50+50}$$

$$V_L = 4\text{ V}$$

For –ve half cycle debde is forward biased



$$V_L = V_S$$

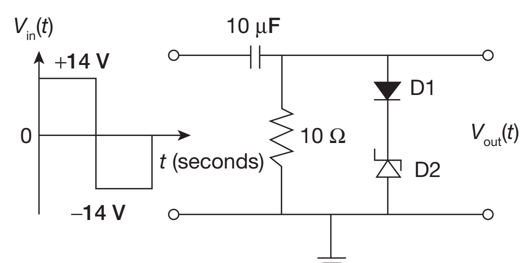
$$= -10\text{ V}$$

$$V_L(\text{avg}) = \frac{1}{T}\left[\int_0^{T/2} 4at + \int_{T/2}^{T} -10at\right]$$

$$= \frac{1}{T}\left[4 \times \frac{T}{2} - 10\left[T - \frac{T}{2}\right]\right]$$

$$= \frac{1}{T}[2T - 5T] = -\frac{3T}{T} = -3\text{ V}$$

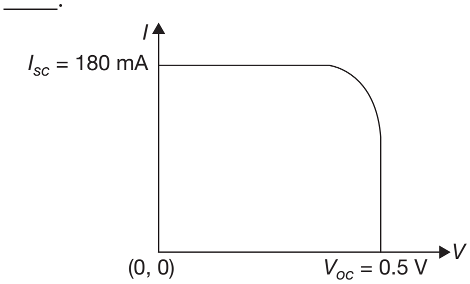
2. In the figure, D1 is a real silicon pn junction diode with a drop of 0.7 V under forward bias condition and D2 is a Zener diode with breakdown voltage of –6.8 V. The input $V_{in}(t)$ is a periodic square wave of period T, whose one period is shown in the figure. **[2017]**



Assuming 10 τ << T, where τ is the time constant of the circuit, the maximum and minimum values of the output waveform are respectively.

(A) 7.5 V and –20.5 V (B) 6.1 V and –21.9 V
(C) 7.5 V and –21.2 V (D) 6.1 V and –22.6 V

3. The figure shows the I–V characteristics of a solar cell illuminated uniformly with solar light of power 100 m W/cm². The solar cell has an area of 3 cm² and a fill factor of 0.7. The maximum efficiency (in %) of the device is _____. **[2016]**



Solution: Solar light power = 100 mW/cm^2

Solar cell area = 3 cm^2

Total power incident on the solar cell will be,

$$P_{in} = 3 \times 100 = 300 \text{ mW};$$

Now,

$$\text{Fill factor} = \frac{I_m V_m}{I_{SC} . V_{OC}}$$

$$\frac{\mathrm{I}_m V_m}{180 \text{ mA} \times 0.5 \text{ V}} = 0.7$$

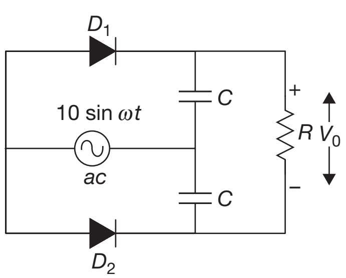
$$\mathrm{I}_m V_m = 63 \text{ mW}$$

$$\text{Maximum efficiency} = \frac{I_m V_m}{P_{in}} \times 100$$

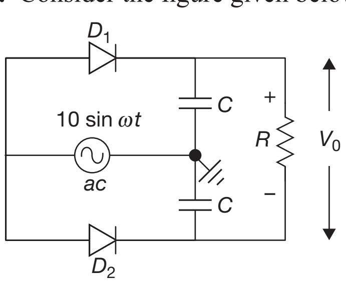
$$= \frac{63 \text{ mW}}{300 \text{ mW}} \times 100 = 21\%$$

Hence, the correct Answer is (21%).

4. The diodes $D1$ and $D2$ in the figure are ideal and the capacitors are identical. The product RC is very large compared to the time period of the AC voltage. Assuming that the diodes do not breakdown in the reverse bias, the output voltage V_0 (in volt) at the steady state is ____. **[2016]**



Solution: Consider the figure given below



Now for positive peak diode D_1 is ON and diode D_2 is ON, therefore

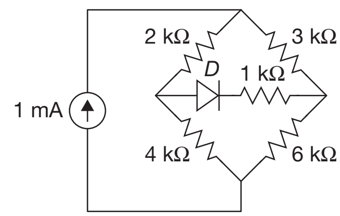
$$V_o = +V_c - V_c = 0$$

for negative peak diodes D_1 and D_2 are OFF, therefore

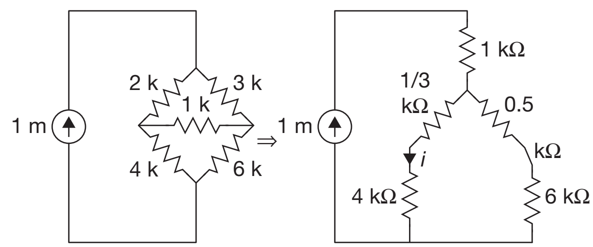
$$V_o = 0 \text{ V}$$

Hence, the correct Answer is (0 V).

5. The diode in the circuit given below has V_{ON} = 0.7 V but is ideal otherwise. The current (in mA) in the 4 kΩ resistor is ______. **[2015]**



Solution: Assume that diode is ON then



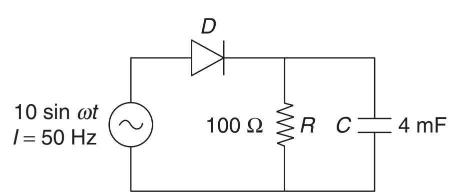
⇒ current across 4 k Ω is given by

$$= \frac{1 \times 10^{-3} \times \left(6 + \frac{1}{2}\right)}{\left(6 + \frac{1}{2}\right) + \left(4 + \frac{1}{3}\right)}$$

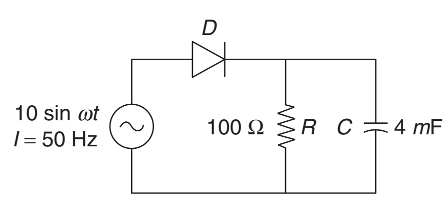
$$= \frac{3}{5} = 0.6 \text{ mA}$$

Hence, the correct Answer is (0.59 to 0.61).

6. The figure shows a half-wave rectifier. The diode D is ideal. The average steady-state current (in amperes) through the diode is approximately___________. **[2014]**



Solution: (0.1 A)

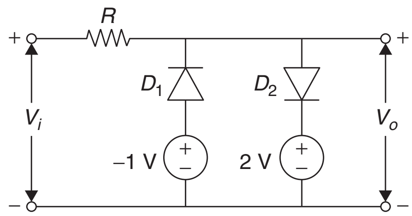


At steady state, capacitor is 0 C.
Steady-state current

$$= \frac{10}{100} = 0.1 \text{ A}$$

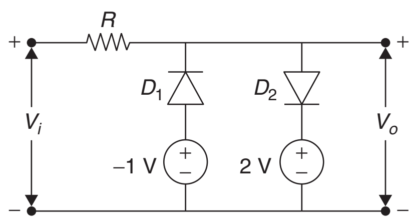
7. Two silicon diodes, with a forward voltage drop of 0.7 V, are used in the circuit shown in the figure. The range of input voltage V_i for which the output voltage $V_o = V_i$ is

[2014]

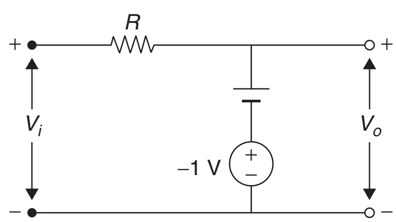


(a) $-0.3 \text{ V} < V_i < 1.3 \text{ V}$
(b) $-0.3 \text{ V} < V_i < 2 \text{ V}$
(c) $-1.0 \text{ V} < V_f < 2.0 \text{ V}$
(d) $-1.7 \text{ V} < V_f < 2.7 \text{ V}$

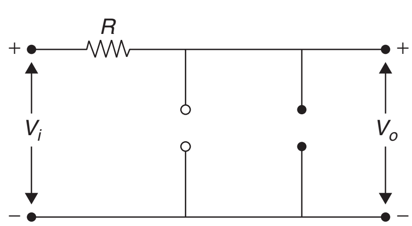
Solution: (d)



Case 1:
$V_i < -1.7$
$D_1 \rightarrow$ ON
$D_2 \rightarrow$ OFF



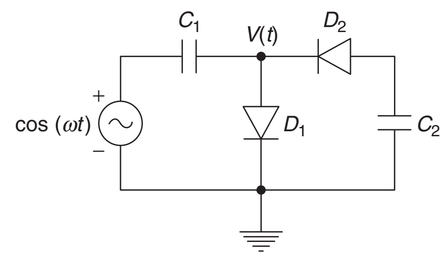
$V_0 = -1.7$ V
Case 2:
$-1.7 < V_i < 2.7$
$D_1 \rightarrow$ OFF
$D_2 \rightarrow$ OFF



$V_0 = V_i$
$\therefore$ the range of I/P voltage V_i for which the O/P voltage $V_0 = V_i$ is $-1.7 \text{ V} < V_i < 2.7\text{V}$.
Hence, the correct option is (d).

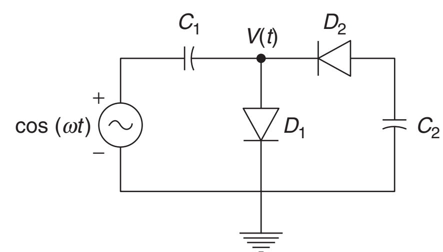
8. The diodes and capacitors in the circuit shown are ideal. The voltage $v(t)$ across the diode D_1 is

[2012]

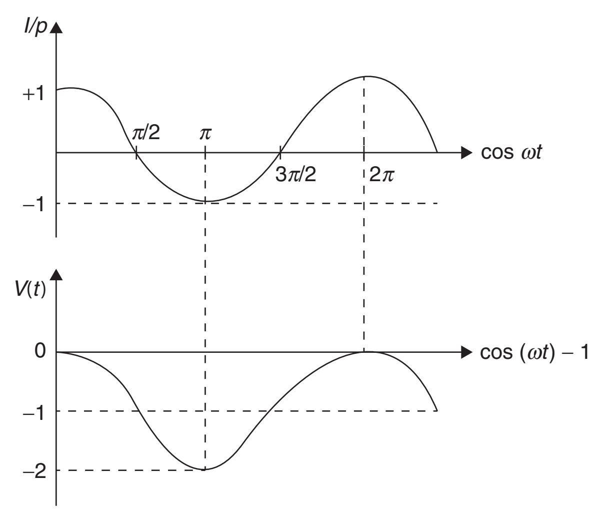


(a) $\cos(\omega t) - 1$
(b) $\sin(\omega t)$
(c) $1 - \cos(\omega t)$
(d) $1 - \sin(\omega t)$

Solution: (a)

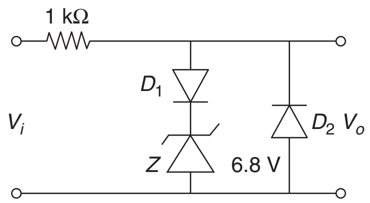


When excited cos ωt the clamping section clamps the positive peak to 0 V and negative peak to –2 V.
So whole cos ωt is lower by –1 V



Hence, the correct option is (a).

9. In the following limiter circuit, an input voltage $V_i = 10\sin 100\pi t$ is applied. Assume that the diode drop is 0.7 V when it is forward biased. The Zener breakdown voltage is 6.8 V. **[2008]**



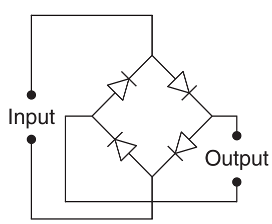
The maximum and minimum values of the output voltage respectively are

(a) 6.1 V, –0.7 V
(b) 0.7 V, –7.5 V
(c) 7.5 V, –0.7 V
(d) 7.5 V, –7.5 V

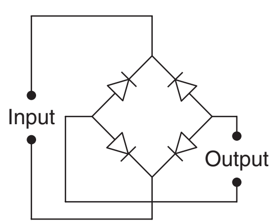
Solution: (c)
During +ve part of $V_i > 6.8$ V D_1 will be forward biased.
Thus net voltage = 6.8 + 0.7
= 7.5 V (max)
During –ve part of V_i D_2 will be forward biased
D_1 will be reverse biased
Thus net voltage = –0.7 (min.)
Hence, the correct option is (c).

10. The correct full wave rectifier circuit is **[2007]**

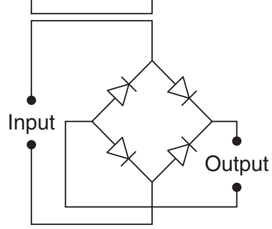
(a)



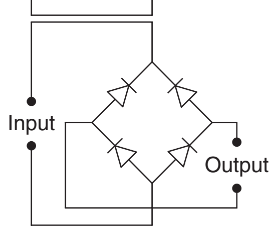
(b)



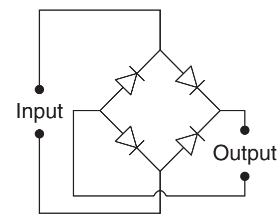
(c)



(d)

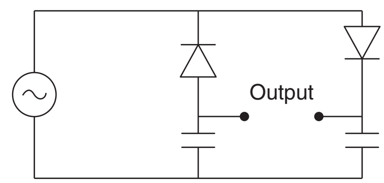


Solution: (c)
The correct full wave rectifier circuit is



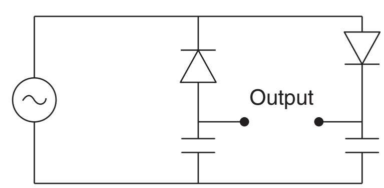
Hence, the correct option is (c).

11. The circuit shown in the figure is best described as a **[2003]**



(a) bridge rectifier
(b) ring modulator
(c) frequency discriminatory
(d) voltage doubler

Solution: (d)
The circuit shown in the figure is best described as a voltage doubler.



Hence, the correct option is (d).

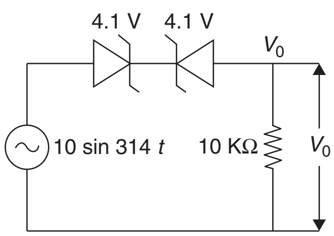
12. For full wave rectification, a four diode bridge rectifier is claimed to have the following advantages over a two diode circuit. **[1998]**

1. Less expensive transformer
2. Smaller size transformer and
3. Suitability for higher voltage application of these

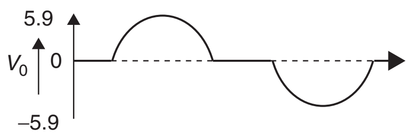
(a) only (1) and (2) are true
(b) only (1) and (3) are true
(c) only (2) and (3) are true
(d) (1), (2), as well as (3) are true

Solution: (d)
A four diode bridge rectifier uses the smaller size of transformer, which is less expensive transformer and these rectifiers are suitable for higher voltage applications, because of low P/V rating required of each diode.
Hence, the correct option is (d).

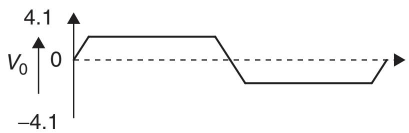
13. The wave shape of V_0 in the figure is **[1993]**



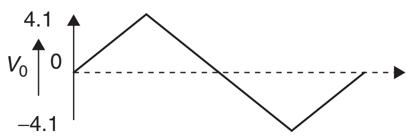
(a)



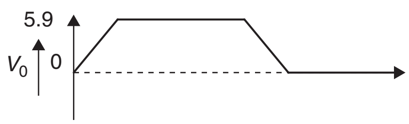
(b)



(c)



(d)

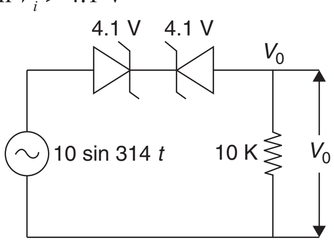


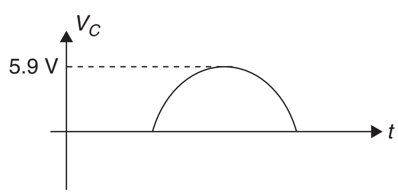
Solution: (a)

Case 1: During +ve half cycle

diode D_A is forward bias, so D_A is short circuit.

diode D_B is reverse bias, so Diode D_B is in conducting state when $V_i > 4.1$ V

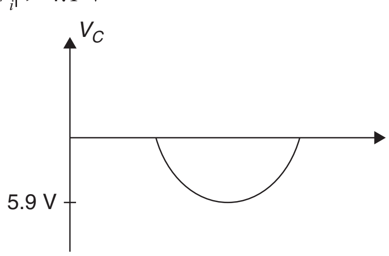


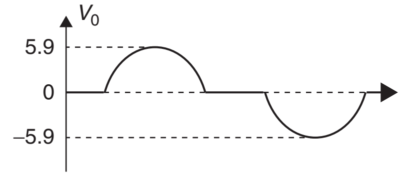


Case 2: During –ve half cycle

diode D_B is forward bias, so D_B is short circuit.

Diode D_A is reverse bias, so D_A is in conducting state when $|V_i| > 4.1$ V

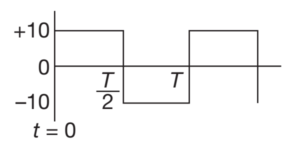


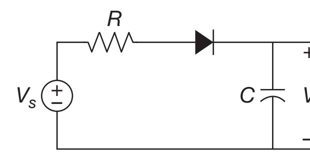


Hence, the correct option is (a).

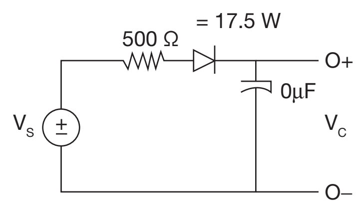
Two-marks Questions

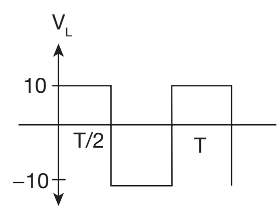
1. In the circuit shown, V_s is 10 V square wave of period, $T = 4$ ms with $R = 500\ \Omega$ and $C = 10\ \mu$F. The capacitor is initially uncharged at $t = 0$, and the diode is assumed to be ideal. The voltage across the capacitor (V_c) at 3 ms is equal to _____ volts (rounded off to one decimal place). **[2019]**





Solution:





For +ve half cycle D is forward biased

$$\therefore V_C = V_0 = 10\text{ V}$$

For –ve half cycle D is reverse biased

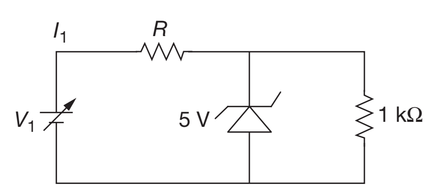
$$\therefore V_C = V_0\left[1 - e^{-t/RC}\right]$$

$$= 10\left[1 - e^{\frac{-2\times10^{-3}}{5\times10^{-3}}}\right]$$

$$= 10\left[1 - e^{-0.4}\right] = 3.3\text{ V}$$

Hence, the correct answer is 3.3 V

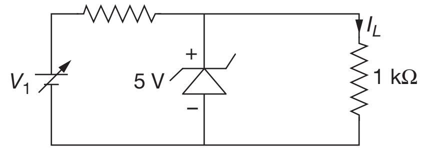
2. The circuit shown in the figure is used to provide regulated voltage (5 V) across 7the 1kΩ resistor. Assume that the Zener diode has a constant reverse breakdown voltage for a current range, starting from a minimum required Zener current, I_{Zmin} = 2 mA to its maximum allowable current. The input voltage V_1 may vary by 5% from its normal value of 6 V. The resistance of the diode in the breakdown region is negligible. **[2018]**



The value of R and the minimum required power dissipation rating of the diode, respectively, are

(A) 186 Ω and 10 m W
(B) 100 Ω and 40 mW
(C) 100 Ω and 10 mW
(D) 186 Ω and 40 m W

Solution:



Current $I_{S(min)} = 7$ mA
Current $I_L = 5$ mA
Current $I_Z = 2$ mA

We know that

$$I_{S(min)} = \frac{V_{I(min)} - 5}{R_{max}}$$

$$R_{max} = \frac{5.7 - 5}{7} = 100\ \Omega$$

Now we have

$$I_{S(max)} = \frac{6.3 - 5}{100} = 13\ \text{mA}$$

And

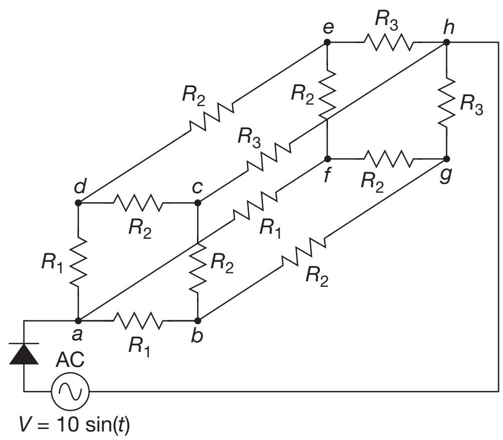
$$I_{Z(max)} = I_{s(max)} - I_L = 13 - 5 = 8\ \text{mA}$$

minimum power dissipation rating

$$P_{Z(min)} = V_Z \cdot I_{Z(max)}$$
$$= (5 \times 8)\ \text{mW}$$
$$= 40\ \text{mW}$$

Hence, the correct option is (B)

3. An AC voltage source $V = 10\sin(t)$ volts is applied to the following network assume that $R_1 = 3$ kΩ, $R_2 = 6$ kΩ and $R_3 = 9$ kΩ, and that the diode is ideal **[2016]**



RMS current I_{rms} (in mA) through the diode is ______.

Solution: AC voltage source $V = 10\sin(t)$

Let RMS current $I_{rms} = I$
Resistance $R_1 = 3$ k Ω
Resistance $R_2 = 6$ k Ω
Resistance $R_3 = 9$ k Ω,

$$V = \frac{I}{3}[R_1] + \frac{I}{6}[R_2] + \frac{I}{3}R_3$$

$$= \frac{I}{3}[3] + \frac{I}{6}[6] + \frac{I}{3}[9]$$

$$V = I[5\Omega]$$

$$\frac{V}{I} = 5\Omega$$

$$I_{rms} = \frac{\left(\frac{V_m}{R}\right)}{2}$$

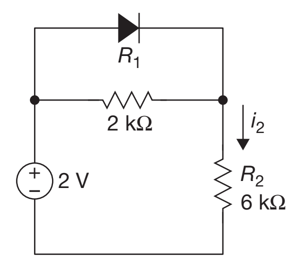
$$= \frac{\left(\frac{10}{5}\right)}{2} = 1\ \text{mA}$$

But key given

$$= \frac{1}{\sqrt{2}} = 0.7$$

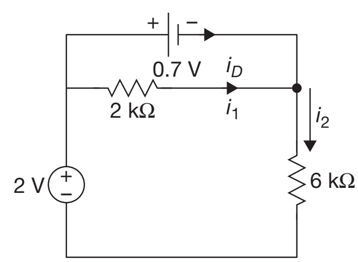
Hence, the correct Answer is (0.7).

4. Assume that the diode in the figure has $V_{on} = 0.7\ V$, but is otherwise ideal



The magnitude of the current i_2 (in mA) is equal to ____. **[2016]**

Solution: Let the diode is in ON state as shown below in figure



Let, $i_2 = i_1 + i_D =$

Applying KVL

$$i_2 = \frac{2-0.7}{6} = 0.216 \text{ mA}$$

$$i_1 = \frac{0.7}{2} \text{ mA} = 0.35 \text{ mA}$$

$$i_2 < i_1.$$

The Diode is forward biased and it required positive current to flow through it for its conduction.

So, diode current $i_D = -0.134$ mA

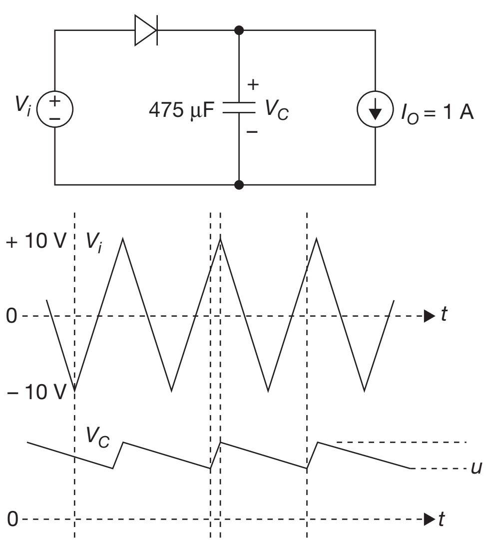
i_D is negative so, Diode is in OFF position,

$$\therefore \quad i_2 = \frac{2}{2+6} \text{ mA} = 0.25 \text{ mA}$$

Here, we obtained negative i_D so, the diode will remain off.

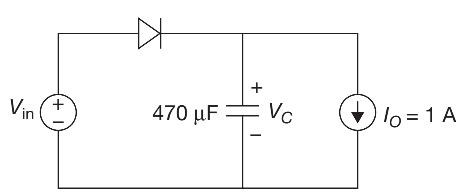
Hence, the correct Answer is (0.25 mA).

5. The figure shows a half wave rectifier with a 475 μF filter capacitor. The load draws a constant current I_0 = 1 A from the rectifier. The figure also shows the input voltage V_i, the output voltages V_c and the peak to peak voltage ripple u on V_c. The input voltage V_i is a triangle wave with an amplitude of 10 V and a period of 1 ms. **[2016]**



The value of the ripple u (in volts) is _____.

Solution:



Amplitude of input voltage V_m = 10 V

For HWR

$$V_{dc} = \frac{V_m}{\pi} = 3.18 \text{ V}$$

$$R_c = \frac{V_{dc}}{I_{dc}} = 3.18\ \Omega$$

$$f = \frac{1}{T} = \frac{1}{10^3} = 1 \text{ kHz}$$

For a HWR shunt capacitor filter

$$\gamma = \frac{V_{ac}(rms)}{V_{dc}} = \frac{1}{2\sqrt{3} f_c.R_L}$$

$$\gamma = 0.19$$

We know that

$$V_{ac} = 0.19 \times 3.18 = 0.6 \text{ V}$$

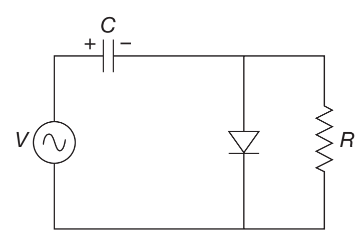
$$V_{ac} = \frac{V_r}{2\sqrt{3}} = \frac{u}{2\sqrt{3}}$$

$$\Rightarrow \quad u = 0.6 \times 2\sqrt{3}$$

$$= 2.07 \text{ V}$$

Hence, the correct Answer is (2.07 V).

6. If the circuit shown has to function as a clamping circuit, then which one of the following conditions should be satisfied for the sinusoidal signal of period T? **[2015]**



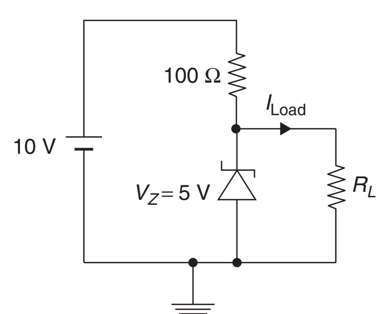
(A) $RC << T$ (B) $RC = 0.35$ T

(C) $RC \approx T$ (D) $RC >> T$

Solution: For clamping circuit RC >> T

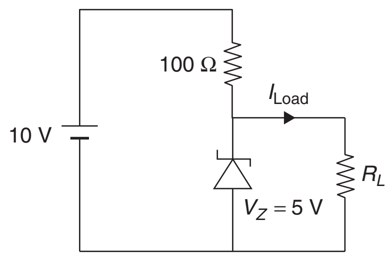
Hence, the correct option is (D).

7. In the circuit shown below, the knee current of the ideal Zener diode is 10 mA. To maintain 5 V across R_L, the minimum value of R_L in Ω and the minimum power rating of the Zener diode in mW, respectively, are **[2013]**



(a) 125 and 125 (b) 125 and 250
(c) 250 and 125 (d) 250 and 250

Solution: (b)



$$R_{L\min} = \frac{5}{I_{L\max}}$$

$$I_{100} = \frac{10-5}{100} = \frac{5}{100} = 50 \text{ mA}$$

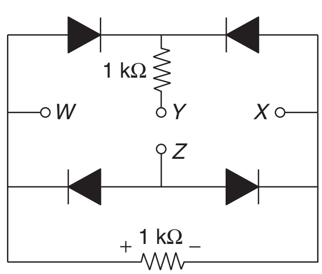
$$I_{L\max} = I_{100} - I_{\text{knee}}$$
$$= 40 \text{ mA}$$

$$R_{L\min} = \frac{5}{40} \times 100$$
$$= 125\ \Omega$$

Minimum power rating of Zener should be = 50 mA × 5 V = 250 mW

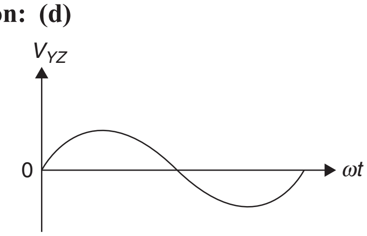
Hence, the correct option is (b).

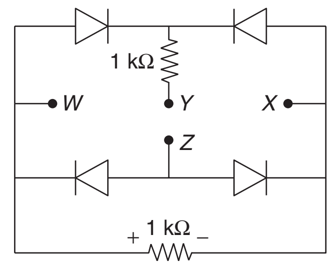
8. A voltage 1000 sinωt volts is applied across YZ. Assuming ideal diodes, the voltage measured across WX in volts, is **[2013]**



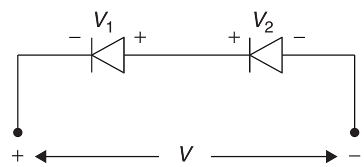
(a) sinωt
(b) (sinωt + |sin ωt|)/2
(c) (sinωt – |sin ωt|)/2
(d) 0 for all t

Solution: (d)



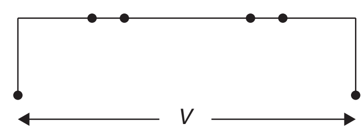


During the half cycle,
all diodes are OFF & hence



$|V_1| = |V_2|$
$V = 0$ V

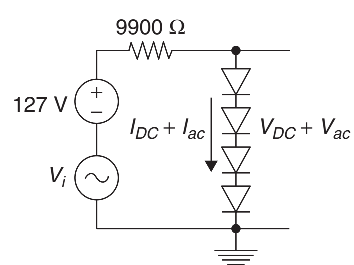
During –ve half cycle



$V = 0$ V

Hence, the correct option is (d).

Statement for Linked Answer Questions 9 and 10.
In the circuit shown below, assume that the voltage drop across a forward biased diode is 0.7 V. The thermal voltage $V_t = kT/q$ = 25 mV. The small signal input V_i = V cos(ωt) where V_p = 100 mV.

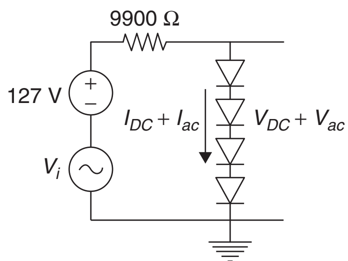


9. The bias current I_{DC} through the diodes is **[2011]**
(a) 1 mA (b) 1.28 mA
(c) 1 5 mA (d) 2 mA

Solution: (a)

Drop of voltage at one diode is 0.7 V.

$$I_{DC} = \frac{12.7 - (0.7 + 0.7 + 0.7 + 0.7)}{9900}$$
$$= 1 \text{ mA}$$



Hence, the correct option is (a).

10. The AC output voltage v_{ac} is **[2011]**

(a) 0.25 cos (ωt) mV (b) 1 cos (ωt) mV
(c) 2 cos(ωt) mV (d) 22 cos (ωt) mV

Solution: (b)

AC dynamic resistance,

$$\gamma_d = \frac{\eta V_T}{I_D}$$

$$= \frac{1 \times 25 \text{ mV}}{1 \text{ mA}} = 25\ \Omega$$

The AC dynamic resistance offered by each diode = 25 Ω

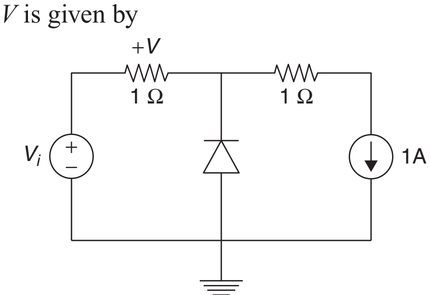
$$\therefore V_{ac} = V_i(ac)\left[\frac{4 \times 25\ \Omega}{9900 + 25}\right]$$

$$= 100 \times 10^{-3}\text{Cos}(\omega t)\left[\frac{100}{9925}\right]$$

V_s = Cos ωt mV

Hence, the correct option is (b).

11. In the circuit given below, the diode is ideal. The voltage V is given by **[2009]**

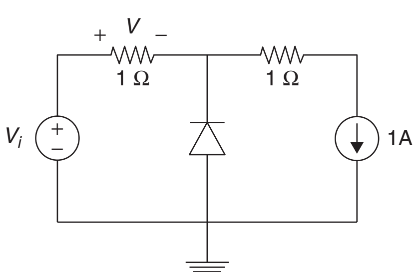


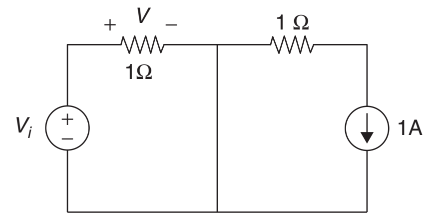
(a) min (V_i, 1) (b) max (V_i, 1)
(c) min ($-V_i$, 1) (d) max($-V_i$, 1)

Solution: (b)

Case 1:

$V_i < 0$

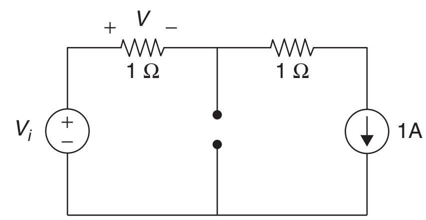




$V = V_i$; $V_i < 0$

Case 2:

$V_i > 0$



$V = 1$ V; $V_i > 0$

$\therefore V = \min(V_i, 1)$

Hence, the correct option is (b).

Common Data for Questions 12 and 13.

Consider a silicon *p-n* junction at room temperature having the following parameters:

Doping on the *n*-side = 1×10^{17} cm^{-3}

Depletion width on the *n*-side = 0.1 μm

Depletion width on the *p*-side = 1.0 μm

Intrinsic carrier concentration = 1.4×10^{10}cm^{-3}

Thermal voltage = 26 mV

Permittivity of free space = 8.85×10^{-14}F-cm^{-1}

Dielectric constant of silicon = 12

12. The built-in potential of the junction **[2009]**

(a) is 0.70 V
(b) is 0.76 V
(c) is 0.82 V
(d) cannot be estimated from the data given

Solution: (c)

$$V_0 = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$N_A x_{p0} = N_D x_{n0}$$

$$\Rightarrow N_A = \frac{10^{17} \times 0.1}{1} = 10^{16}\text{cm}^{-3}$$

$$V_0 = 0.026\ln\left[\frac{10^{17} \times 10^{16}}{(1.4 \times 10^{14})^2}\right]$$

$$= 0.28 \text{ V}$$

Hence, the correct option is (c).

13. The peak electric field in the device is **[2009]**

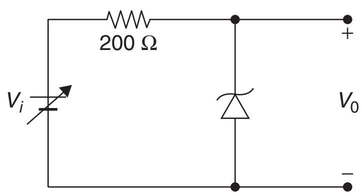
(a) 0.15 MVcm^{-1}, directed from *p*-region top-region
(b) 0.15 MVcm^{-1}, directed from *n*-region top-region
(c) 1.80 MVcm^{-1}, directed from *p*-region top-region
(d) 1.80 MVcm^{-1}, directed from *n*-region top-region

Solution: (b)

$$\varepsilon = \frac{q}{\in} N_d x_{n0}$$
$$= \frac{-q}{\in} N_a x_{p0}$$
$$= \frac{1.6\times10^{-19}\times10^{17}\times0.1\times10^{-4}}{12\times8.85\times10^{-14}}$$
$$= 0.15 \text{ MVcm}^{-1}$$

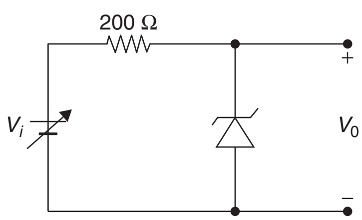
Directed from *n*-region to *p*-region.
Hence, the correct option is (b).

14. For the Zener diode shown in the figure, the Zener voltage at knee is 7 V, the knee current is negligible and the Zener dynamic resistance is 10 Ω. If the input voltage (V_i) range is from 10 to 16 V, the output voltage (V_0) ranges from **[2007]**



(a) 7.00 to 7.29 V (b) 7.14 to 7.29 V
(c) 7.14 to 7.43 V (d) 7.29 to 7.43 V

Solution: (b)
In addition to Zener diode voltage, some voltage drop also takes place due to dynamic resistance (10 Ω)



For $V_i = 10$ V

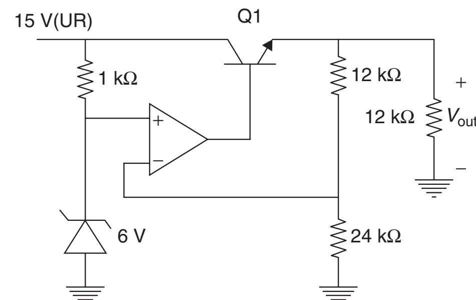
$$V_0 = 7 + (10-7)\left(\frac{10}{200+10}\right)$$
$$= 7.14 \text{ V}$$

For $V_i = 16$ V

$$V_0 = 7 + (16-7)\left(\frac{10}{200+10}\right)$$
$$= 7.43 \text{ V}$$

Hence, the correct option is (b).

Common Data for Questions 15 and **16**.
A regulated power supply, shown in figure below, has an unregulated input (UR) of 15 volts and generates a regulated output V_{out}. Use the component values shown in the figure



15. The power dissipation across the transistor shown in the figure is **[2006]**

(a) 4.8 watts (b) 5.0 watts
(c) 5.4 watts (d) 6.0 watts

Solution: (c)

Diode voltage
$V_A = 6$ V

$$V_{out} = \frac{6\times36}{24} = 9 \text{ V}$$

$$I_{out} = \frac{9}{10} = 0.9 \text{ A}$$

$$I = \frac{V_A}{24k} = \frac{6}{24k} = 0.25 \text{ mA}$$

Power dissipation across transistor = $V_{CE} \times i_C$
$V_{CE} = 15 - 9 = 6$ V
$\therefore i_C = I + i_{out} = 0.9$ A
$P_D = 6 \times 0.9 = 5.4$ W
$i_E = I + I_{out} \simeq 0.9$ A

Hence, the correct option is (c).

16. If the unregulated voltage increases by 20%, the power dissipation across the transistor Q1 **[2006]**

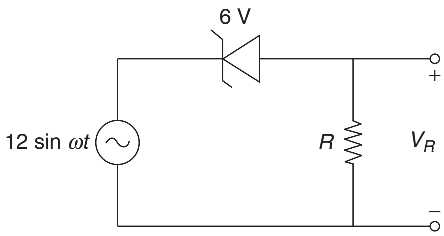
(a) increases by 20%
(b) increases by 50%
(c) remains unchanged
(d) decreases by 20%

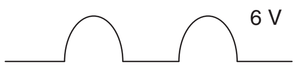
Solution: (b)
If unregulated power supply increases by 20% then
$V_{C\varepsilon} = 18 - 9 = 9$ V
$i_c = 0.9$ A
$P_D = 8.1$ W

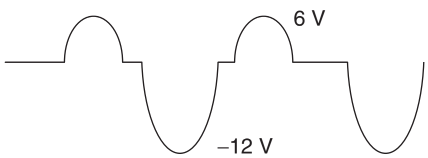
$$\% \text{ increase in power} = \left(\frac{8.1-5.4}{5.4}\right) = 50\%$$

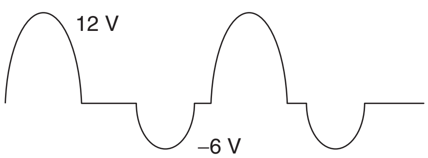
Hence, the correct option is (b).

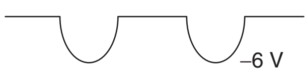
17. For the circuit shown below, assume that the Zener diode is ideal with a breakdown voltage of 6 volts. The waveform observed across R is **[2006]**



(a) 

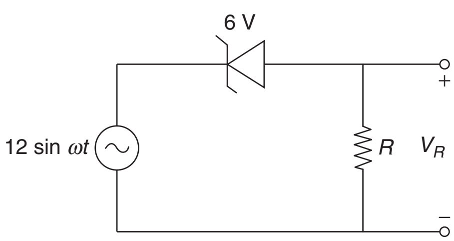
(b) 

(c) 

(d) 

Solution: (b)

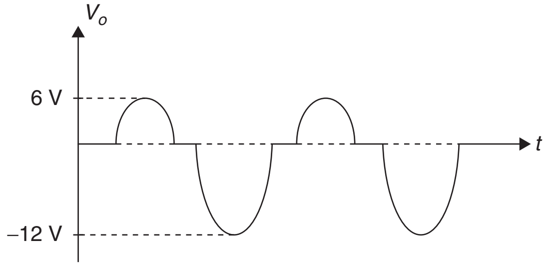
When I/P voltage increases from 0 to 6 V, diode is OFF.



When I/P is > 6 V, Zener is under break down.

$\therefore V_R = V_{\text{input}} - 6 = 12 \sin\omega t - 6$

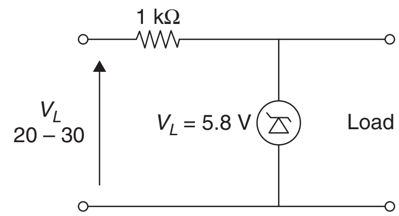
Maximum, it can go up to 6 V when I/P is –ve, diode is FB. In this case, O/P voltage follows I/P.



Hence, the correct option is (b).

18. The Zener diode in the regulator circuit shown in the figure has a Zener voltage of 5.8 V and a Zener knee current of 0.5 mA. The maximum load current drawn from this circuit ensuring proper functioning over the input voltage range between 20 and 30 V is

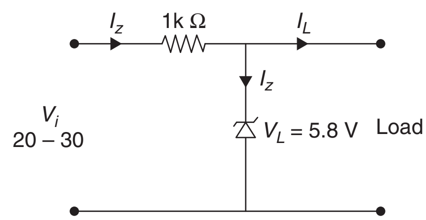
[2005]



(a) 23.7 mA (b) 14.2 mA
(c) 13.7 mA (d) 24.2 mA

Solution: (a)

$V_L = 5.8$ V



Maximum load current will be when $V_i = V_{\max} = 30$ V

Current through 1k resistor

$$\frac{30 - 5.8}{1k} = I_L + I_Z$$

$\Rightarrow 24.2 \text{ mA} = I_L + I_Z$

$I_L = 24.2 \text{ mA} - 0.5 \text{ mA}$

$= 23.7$ mA

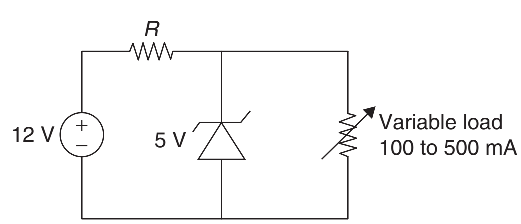
Hence, the correct option is (a).

19. In the voltage regulator shown in the figure, the load current can vary from 100 to 500 mA. Assuming that the Zener diode is ideal (i.e., the Zener knee current is negligibly small and Zener resistance is zero in the breakdown region), the value of R is **[2004]**

(a) 7 Ω (b) 70 Ω
(c) $\frac{70}{3}$ Ω (d) 14 Ω

Solution: (d)

$$\frac{12 - V}{R} = I_L + I_Z$$



When $I_L = 100$ mA

$$\Rightarrow \frac{12-5}{R} \geq 100 \text{ mA}$$

$$R \leq \frac{7 \times 1000}{100}$$

$R = (7 \times 1000/100) = 70$

When $I_L = 500$ mA,

$$\frac{12-5}{R} \geq 500 \text{ mA}$$

$$R \leq \frac{7 \times 1000}{500} = 14\ \Omega$$

$R = 14\ \Omega$ (choosing minimum one)

Hence, the correct option is (d).

20. In a full-wave rectifier using two ideal diodes, V_{dc} and V_m are the dc and peak values of the voltage respectively across a resistive load. If PIV is the peak inverse voltage of the diode, then the appropriate relationships for this rectifier are **[2004]**

(a) $V_{dc} = \frac{V_m}{\pi}, PIV = 2V_m$

(b) $V_{dc} = 2\frac{V_m}{\pi}, PIV = 2V_m$

(c) $V_{dc} = 2\frac{V_m}{\pi}, PIV = V_m$

(d) $V_{dc} = \frac{V_m}{\pi}, PIV = V_m$

Solution: (b)

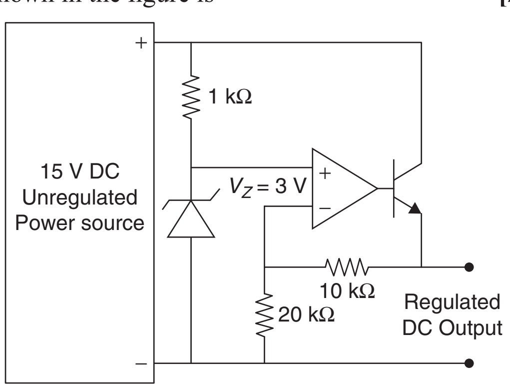
In a full-wave rectifier using two ideal diodes, V_{dc} and V_m are the dc and peak values of the voltage, respectively, across a resistive load. If P/V is the peak inverse voltage of the diode, then the appropriate relationship of this rectifier is

$$V_{dc} = \frac{2V_m}{\pi}$$

and $P/V = 2V_m$

Hence, the correct option is (b).

21. The output voltage of the regulated power supply shown in the figure is **[2003]**

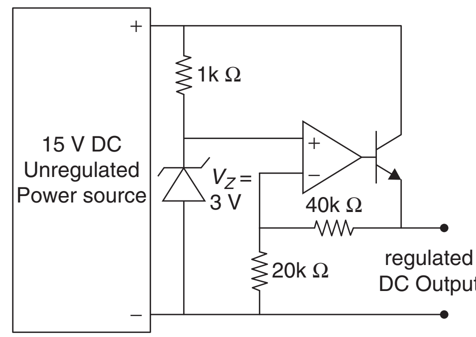


(a) 3 V (b) 6 V

(c) 9 V (d) 12 V

Solution: (c)

As voltage at non-inverting terminal is 3 V due to Zener diode, voltage at inverting terminal will be 3 V because of virtual ground.



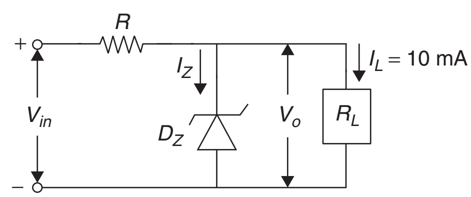
So, current in 20 k is $\frac{3}{20k}$

$$= \frac{3}{20} \text{ mA}$$

$$\therefore V_0 = \frac{3}{20k} \times 60k = 9 \text{ V}$$

Hence, the correct option is (c).

22. A Zener diode regulator in the figure is to be designed to meet the specifications: $I_L = 10$ mA, $V_0 = 10$ V and V_{in} varies from 30 V to 50 V. The Zener diode has $V_z = 10$ V and I_{zk} (knee current) $= 1$ mA. For satisfactory operation **[2002]**



(a) $R \leq 1800\ \Omega$

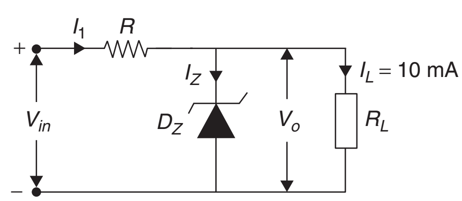
(b) $2000\ \Omega \leq R \leq 2200\ \Omega$

(c) $3700\ \Omega \leq R \leq 4000\ \Omega$

(d) $R > 4000\ \Omega$

Solution: (a)

$$\frac{V_{in} - v_0}{R} \geq I_z + I_L = I_1$$



When $V_{in} = 30$ V,

$$\frac{30-10}{R} \geq (10+1) \text{ mA}$$

$$\frac{20}{R} \geq 11 \text{ mA}$$

$\Rightarrow R \leq 1818\ \Omega$ (1)

When $V_{in} = 50$ V,

$$\frac{40}{R} \leq 11 \times 10^{-3}$$

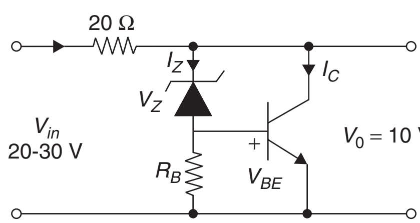
$$R \leq 3636 \qquad (2)$$

From equations (1) and (2)

$\Rightarrow R \leq 1818\ \Omega$

Hence, the correct option is (a).

23. The transistor shunt regulator shown in the figure has a regulated output voltage of 10 V, when the input varies from 20 to 30 V. The relevant parameters for the Zener diode and the transistor are: V_z = 9 5, V_{BE} = 0.5 V, β = 99. Neglect the current through R_B. Then the maximum power dissipated in the Zener diode (P_z) and the transistor (P_T) is **[2001]**



(a) $P_z = 75$ mW, $P_T = 7.9$ W
(b) $P_z = 85$ mW, $P_T = 8.9$ W
(c) $P_z = 95$ mW, $P_T = 9.9$ W
(d) $P_z = 115$ mW, $P_T = 11.9$ W

Solution: (c)

$V_{cc} = 10$ V

Case 1:

$$\frac{20-10}{20} = I = 0.5A$$

$I_c + I_b = 0.5$ A, also $I_c = \beta I_b$

On solving, we get $I_c = 0.495$ A

$P_T = (V_{cc} \times I_c)4.95$ W (not given in option)

Case 2:

$$\frac{30-10}{20} = I = 1A$$

$Ic + I_b = 0.5$ A, also $Ic = \beta I_b$

On solving, we get $Ic = 0.99$ A means $P_T = 9.9$ W

Hence, the correct option is (c).

24. A DC power supply has a no-load voltage of 30 V, and a full-load voltage of 25 V at a full-load current of 1 A. Its output resistance and load regulation, respectively, are **[1999]**

(a) 5 Ω and 20% (b) 25 Ω and 20%
(c) 5 Ω and 16.7% (d) 25 Ω and 16.7%

Solution: (a)

$$\text{voltage regulation} = \frac{\text{no load voltage-full load voltage}}{\text{full load voltage}}$$

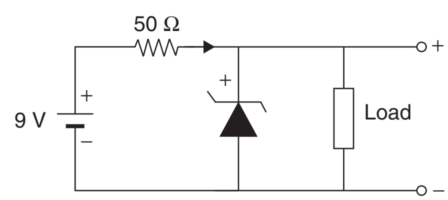
$$= \frac{V_{NL} - V_{FL}}{V_{FL}}$$

$$= \frac{30-25}{25} = \frac{1}{5} = 20\%$$

$$R_0 = \frac{V_{DCNL} - V_{DCFL}}{I_{DC}} = 5\ \Omega$$

Hence, the correct option is (a).

25. A Zener diode in the circuit shown below has a knee current of 5 mA, and a maximum allowed power dissipation of 300 mW. What are the minimum and maximum load currents that can be drawn safely from the circuit, keeping the output voltage V_f constant at 6 V? **[1996]**



(a) 0 mA, 180 mA (b) 5 mA, 110 mA
(c) 10 mA, 55 mA (d) 60 mA, 180 mA

Solution: (c)

Current through 50 Ω resistance = I

$$I = \frac{9-6}{50} = \frac{3}{50} = 60 \text{ mA}$$

Given that

$I_{z,\,min} = 5$ mA

$P_{z\max} = I_{z\max} V_z = 300$ mW

$$I_{z\max} = \frac{300 \times 10^{-3}}{V_z}$$

$$= \frac{300 \times 10^{-3}}{6} = 50 \text{ mA}$$

$I = I_{z\min} + I_{L\max} = I_{z\max} + I_{L\min}$

$60 = 50 + I_{L\min}$

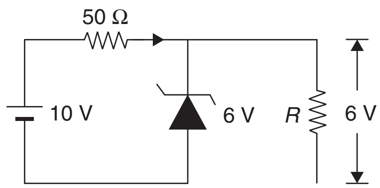
$I_{L\min} = 10$ mA

$60 = 5 + I_{L\,max}$

$I_{L\,max} = 55$ mA

Hence, the correct option is (c).

26. The 6 V Zener diode shown below has zero Zener resistance and a knee current of 5 mA. The minimum value of R so that the voltage across it does not fall below 6 V is **[1992]**



(a) 1.2 kΩ (b) 50 kΩ

(c) 80 kΩ (d) 0 kΩ

Solution: (c)

Knee current, $I_k = 5$ mA

For R to be minimum, I_L should be maximum and current across the Zener diode should be minimum i.e., I_k

Applying KVL,

$10 - 50I - 6 = 0$

$$\Rightarrow I = \frac{4}{50} A = 80 \text{ mA}$$

$I_L = I - I_K$
$= 80 \text{ m} - 5 \text{ m}$
$= 75 \text{ mA}$

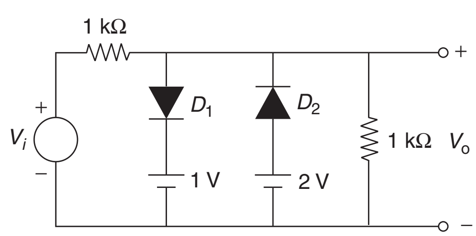
$$R = \frac{6}{I_L} = \frac{6}{75m} = 80\ \Omega$$

Hence, the correct option is (c).

Five-marks Questions

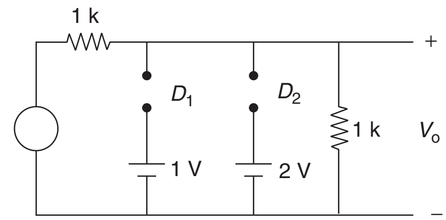
1. (A) Draw the transfer characteristics of the circuit of figure, assuming both D_1 and D_2 to be ideal

(B) How would the characteristics change if D_2 is ideal, but D_1 is no-ideal and has a forward resistance of 10 Ω and a reverse of infinity.



[1998]

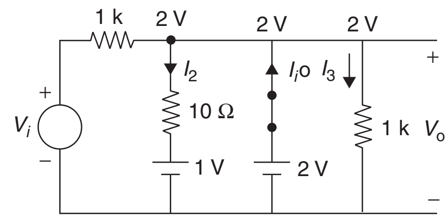
Solution: (A) Both the Diodes are ideal whom V_0 is between 1 V to 2 V $1\text{ V} \le V_0 \le 2\text{ V}$ in this case both the diodes D_1 and D_2 are forward bias



1 V and 2 V can not be in parallel and its violation of KVL

So this circuit does not exist.

(B) $V_0 \le 2$ V ($D_2 f, B$ and $D_1 f, B$)



Apply KCL

$I + I_1 = I_2 + I_3$

$$\frac{V_i - 2}{1k} + I_1 = \frac{2-1}{10} + \frac{2-0}{1k}$$

$$i_1 = \frac{104 - V_i}{1000}$$

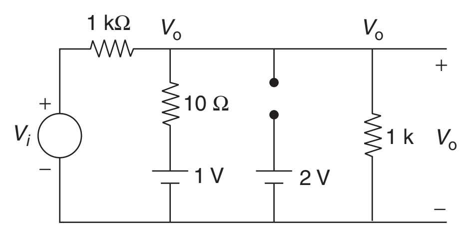
$i \ge 0$

$$\frac{104 - V_i}{1000} \ge 0$$

$V_i \le 104$

$V_i \le 104\text{V} \Rightarrow V_0 = 2\text{V}$

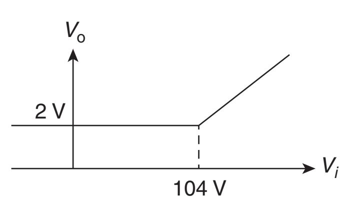
Case 2: $V_i \ge 104$ V, so $V_0 \ge 2$ V Diode D_1 is forward bias Diode D_2 is reverse bias



Apply KCL

$$\frac{V_0 - V_i}{1K} + \frac{V_0 - 1}{10} + \frac{V_0 - 0}{1K} = 0$$

$$V_0 = \frac{V_i + 100}{102}$$

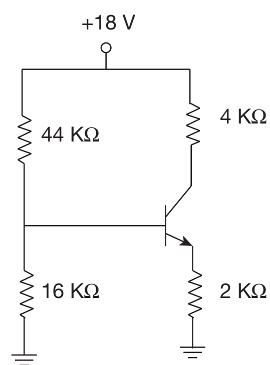


Chapter 3

BJT Analysis

One-mark Questions

1. Consider the circuit shown in the figure. Assume base-to-emitter voltage $V_{BE} = 0.8$ V and common-base current gain (α) of the transistor is unity. **[2017]**



The Value of the collector-to-emitter voltage V_{CE} (in volt) is ______________.

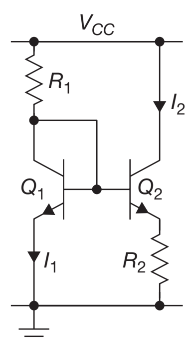
2. The Ebers Moll of a BJT is valid **[2016]**

(A) only in active mode
(B) only in active and saturation modes
(C) only in active and cut off modes
(D) in active, saturation and cut off modes

Solution: The Ebers Moll model is a model developed for BJTs. It is valid for all operating modes, active, saturation and cut off modes.

Hence, the correct option is (D).

3. Resistor R_1 in the circuit below has been adjusted so that $I_1 = 1$ mA. The bipolar transistors $Q1$ and $Q2$ are perfectly matched and have very high current gain, so their base currents are negligible. The supply voltage V_{cc} is 6 V. The thermal voltage kT/q is 26 mV.



The value of R_2 (in Ω) for which $I_2 = 100$ μA is ____. **[2016]**

Solution: Q_1 and Q_2 are perfectly matched, thus $I_1 = I_2$ and forms a current mirror. So base current $I_B \cong 0$.

Also,

$$I_1 \cong I_2 = 1 \text{ mA}.$$

Now using the relation for collector current

$$\frac{V_{cc} - V_{CE}}{R_2} = I_c$$

Substituting collector emitter $V_{CE} = 0.2$ V

$$R_2 = \frac{5.98}{1 \text{ mA}} = 598\ \Omega$$

Hence, the correct Answer is (598 Ω).

4. Which one of the following statements is correct about an AC coupled common emitter amplifier operating in the mid band region? **[2016]**

(A) The device parasitic capacitances behave like open circuits, whereas coupling and bypass capacitances behave like short circuits.

(B) The device parasitic capacitances, coupling capacitances and bypass capacitances behave like open circuits.

(C) The device parasitic capacitances, coupling capacitances and bypass capacitances behave like short circuits.

(D) The device parasitic capacitances behave like short circuits, whereas coupling and bypass capacitances behave like open circuits.

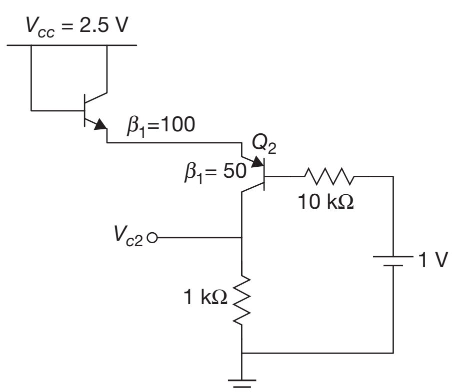
Solution:

Low frequency	Mid-band region	High frequency region
Coupling capacitors	Parasitic capacitances behaves like open circuit coupling and by pass capacitances behaves like short circuit	Junction capacitors

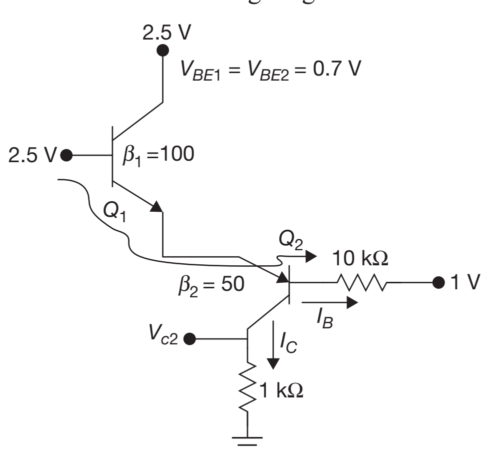
In the mid band region, an AC coupled amplifier performs different operations for different capacitors. The coupling capacitance and bypass capacitance becomes short circuited in the mid band region. While the parasitic capacitance behaves as open circuited. The junction capacitance is not considerable in mid band gap.

Hence, the correct option is (A).

5. Consider the circuit shown in the figure. Assuming $V_{BE1} = V_{EB2} = 0.7$ volt, the value of the DC voltage V_{c2} (in volt) is ____. **[2016]**



Solution: Consider the figure given below



$$\text{Voltage } V_{BE1} = V_{EB2} = 0.7 \text{ V}$$

Apply KVL, we get

$$-2.5 + 0.7 + 0.7 + 10K(I_B) + 1 = 0$$

The base current will be

$$I_B = \frac{2.5 - 0.7 - 0.7 - 1}{10k} = \frac{0.1}{10} \text{ mA}$$

And the collector current will be

$$I_C = \beta\, I_B$$

Since $\beta = 50$ since I_C flowing across $Q2$ will be

$$I_C = \frac{50 \times 0.1}{10} \text{ mA}$$

$$= 0.5 \text{ mA}$$

The required voltage is

$$V_{C2} = I_C \times 1k$$

$$= 0.5 \text{ mA} \times 1K = 0.5 \text{ V.}$$

Hence, the correct Answer is (0.5 V).

6. An npn BJT having reverse saturation current $I_S = 10^{-15}$ A is biased in the forward active region with $V_{BE} = 700$ mV. The thermal voltage (V_T) is 25 mV and the current gain (β) may vary from 50 to 150 due to manufacturing variations. The maximum emitter current (in μA) is ________. **[2015]**

Solution: $I_S = 10^{-15} \text{ A} = I_o$

$$V_{BE} = 0.7 \text{ V}$$

$$V_T = 25 \text{ mV} = 0.25 \text{ V}$$

$$\beta \to 50 \text{ to } 150$$

$$\Rightarrow \quad I_{E(\max)} = ?$$

$$I \approx I_o . e^{V_{BE}/\eta V_T} = I_C$$

Consider for small currents $n = 1$

$$I_C = 10^{-15} \times e^{0.7/0.025}$$

$$I_c = 1.446 \text{ mA}$$

$$I_E = (\beta + 1)\, I_B$$

$$I_E = \frac{(\beta+1)}{\beta}.I_C$$

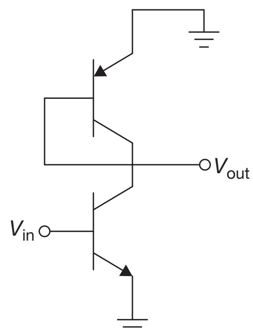
$$I_E \text{ max when } \beta = 50$$

$$I_E = 1.02 \times 1.446 \times 10^{-3}$$

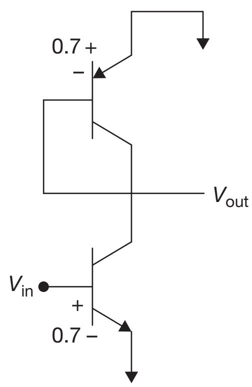
$$I_E = 1.475 \text{ mA}$$

Hence, the correct Answer is (1465 to 1485).

7. In the ac equivalent circuit shown, the two BJTs are biased in active region and have identical parameters with $\beta >> 1$. The open circuit small signal voltage gain is approximately ________. **[2015]**



Solution:



$$\Rightarrow \quad V_{in} = 0.7$$

$$V_{out} = -0.7$$

$$\frac{V_{out}}{V_{in}} = \frac{-0.7}{+0.7} = -1$$

Hence, the correct Answer is (–1.1 to –0.9).

8. A good current buffer has **[2014]**

(a) low input impedance and low output impedance.
(b) low input impedance and high output impedance.
(c) high input impedance and low output impedance.
(d) high input impedance and high output impedance.

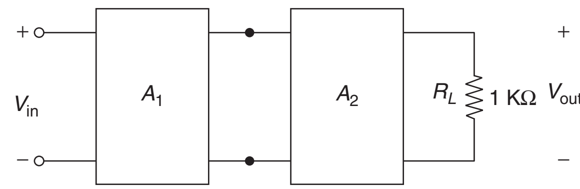
Solution: (b)

A good current buffer has low input impedance and high output impedance.
Hence, the correct option is (b).

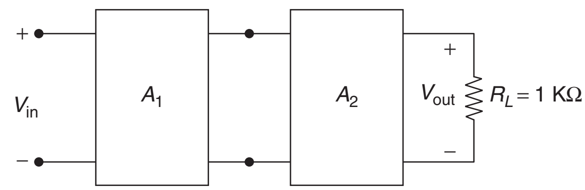
9. A cascade connection of two voltage amplifiers A_1 and A_2 is shown in the figure. The open-loop gain A_{vo}, input resistance R_{in}, and output resistance R_o for A_1 and A_2 are as follows:

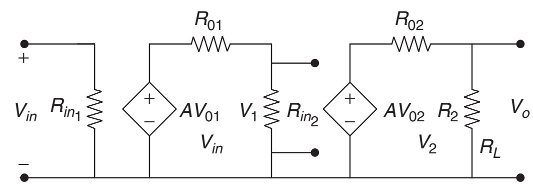
A_1: $A_{VO} = 10$, $R_{in} = 10$ kΩ, $R_o = 1$ kΩ
A_2: $A_{VO} = 5$, $R_{in} = 5$ kΩ, $R_o = 200$ kΩ

The approximate overall voltage gain v_{out}/v_{in} is **[2014]**



Solution: (34.72)





$$\frac{V_0}{A_{V_{02}} \times V_2} = \frac{RL}{RL + R0_2}$$

where $$V_2 = \frac{AV_{01} \times V_{in} R_{in_L}}{R_{in_2} + R_{01}}$$

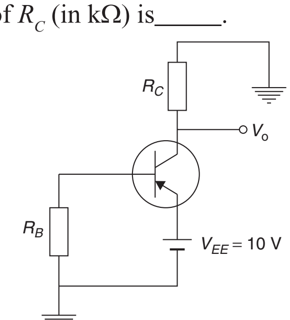
From equations (1) and (2), we get

$$\frac{V_0}{V_{in}} = AV_{02} \times AV_{01} \times \frac{R_L}{R_L + R0_2} \times \frac{R_{in_2}}{R_{in_2} + R0_1}$$

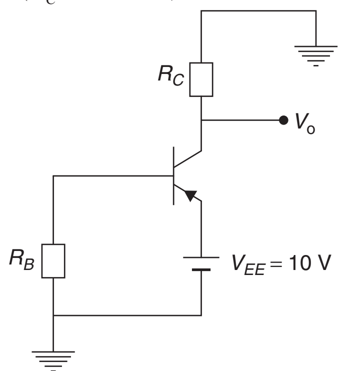
$$\Rightarrow \frac{V_0}{V_{in}} = 5 \times 10 \times \frac{1K}{1K + 0.2K} \times \frac{5K}{5K + 1K}$$

$$\Rightarrow \frac{V_0}{V_{in}} = 34.72$$

10. In the circuit shown, the PNP transistor has $|V_{BE}| = 0\ 7$V and $\beta = 50$. Assume that $R_B = 100$ kΩ. For V_0 to be 5 V, the value of R_C (in kΩ) is______. **[2014]**



Solution: ($R_C = 1.07\ k\Omega$)



Appling KVL in input loop, we get

$$10 = 0.7 + I_B R_B$$

$$I_B \frac{9.3}{100k} = 0.93 \text{ mA}$$

Hence, $k = \beta I_B = 4.65$ mA

$$\text{where } I_C = \frac{V_0 - 0}{R_C} = \frac{5}{R_C}$$

$$\therefore \quad R_C = \frac{5}{4.65}$$

$$R_C = 1.07\ k\Omega$$

11. If the emitter resistance in a common-emitter voltage amplifier is not bypassed, it will **[2014]**

(a) reduce both the voltage gain and the input impedance.
(b) reduce the voltage gain and increase the input impedance.
(c) increase the voltage gain and reduce the input impedance.
(d) increase both the voltage gain and the input impedance.

Solution: (b)

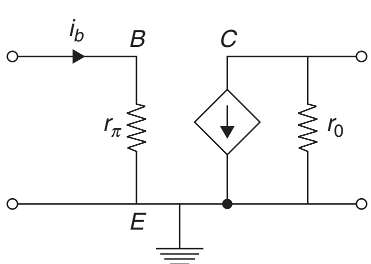
For an bypassed R_ε

$$R_i = \beta re + (1+\beta)R_\varepsilon$$

and $$AV = \frac{A_I R_L}{R_i}$$

Hence, the correct option is (b).

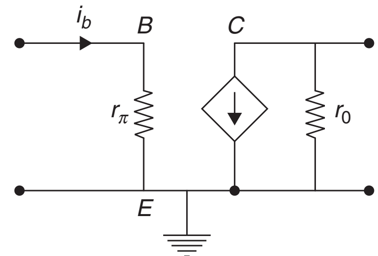
12. The current i_b through the base of a silicon NPN transistor is $1+0.1 \cos (10{,}000\ \pi t)$mA. At 300 K, the r_π in the small signal model of the transistor is **[2012]**



(a) 250 Ω
(b) 27.5 Ω
(c) 25 Ω
(d) 22.5 Ω

Solution: (c)

We know that



$$r_\pi = (\beta+1)r_e$$

$$r_\pi = (\beta+1)\frac{V_T}{I_e}$$

$$r_\pi = (\beta+1)\frac{V_T}{(\beta+1)I_b}$$

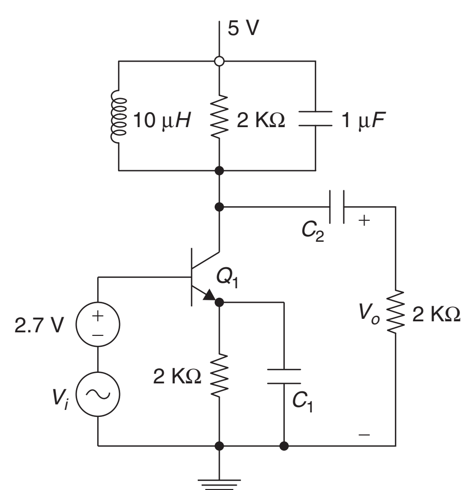
$$r_\pi = \frac{V_T}{I_b}$$

where I_b is dc current through base so I_b = 1mA, V_T = 2.5mV at room temperature So, $r_\pi = \dfrac{25\times10^{-3}}{1\times10^{-3}}$

$= 25\Omega$

Hence, the correct option is (c).

13. In the circuit shown below, capacitors C_1 and C_2 are very large and shorts at the input frequency, v_i is a small input. The gain magnitude $|v_0/v_1|$ at 10 M rad/s is **[2011]**



(a) maximum (b) minimum
(c) unity (d) zero

Solution: (a)

In a parallel RLC circuit

$L = 10\ \mu\text{H}$ and $C = 1\text{nF}$

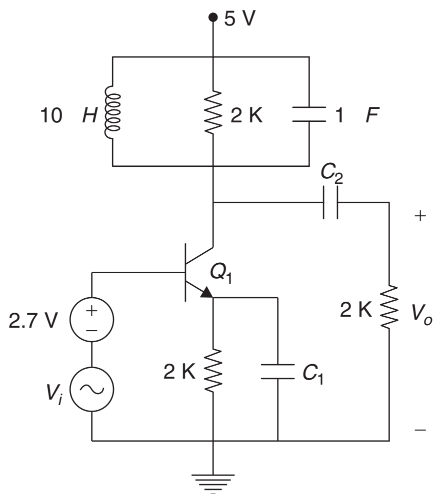
$$w_g = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{10 \times 10^{-6} \times 10^{-9}}}$$

$$= 10^7 \quad \text{rad/s}$$

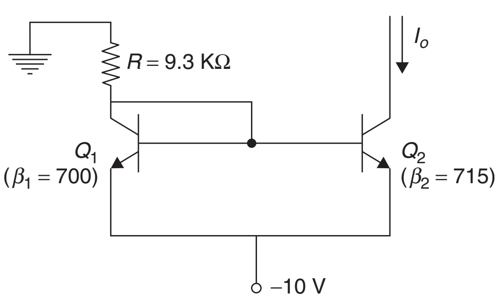
$$\simeq 10M \quad \text{rad/s}$$

So that for a tuned amplifier, gain is maximum at resonant frequency.



Hence, the correct option is (a).

14. In the silicon BJT circuit shown below assume that the emitter area of transistor Q1 is half that of transistor Q2. **[2011]**



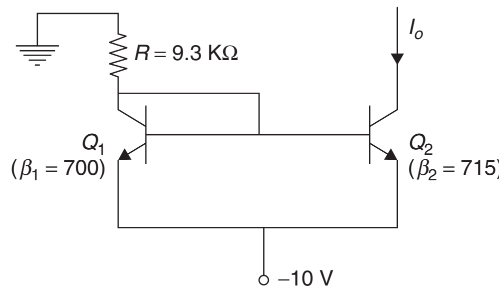
The value of current I_o is approximately **[2010]**

(a) 0.5 mA (b) 2 mA

(c) 9.3 mA (d) 15 mA

Solution: (b)

Since the emitter area of Q_1 is half of transistor Q_2,



$$I_1 = \frac{I_0}{2}$$

$$\Rightarrow I_0 = 2I_1$$

Now KVL for Q_1

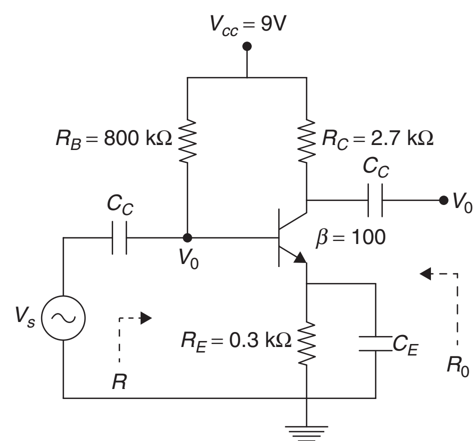
$9.3\ \text{K}\ (I_1) + 0.7 = 10$

$$\Rightarrow I_1 = 1\ \text{mA}$$

$$I_0 = 2I_1 = 2\ \text{mA}$$

Hence, the correct option is (b).

15. The amplifier circuit shown below uses a silicon transistor. The capacitors C_C and C_E can be assumed to be short at signal frequency and the effect of output resistance r_0 can be ignored. If C_E is disconnected from the circuit, which one of the statements is TRUE? **[2010]**



(a) The input resistance R_i increases and the magnitude of voltage gain A_v decreases

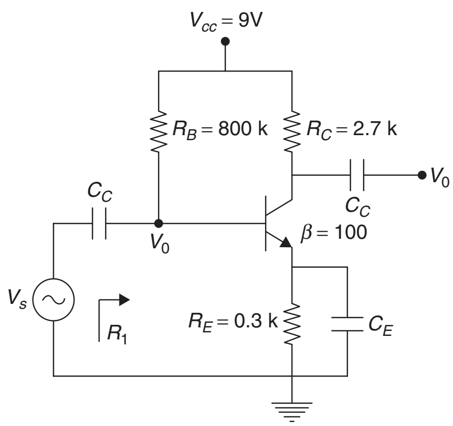
(b) The input resistance R_i decreases and the magnitude of voltage gain A_v increases

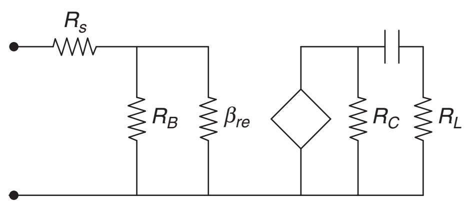
(c) Both input resistance R_i and the magnitude of voltage gain A_v decrease

(d) Both input resistance R_i and the magnitude of voltage gain A_v increase

Solution: (a)

Given circuit after removing C_ε will behave as current series feedback. Overall voltage gain will decrease as feedback signal comes into picture and since it is current – series feedback, input impedance increases.





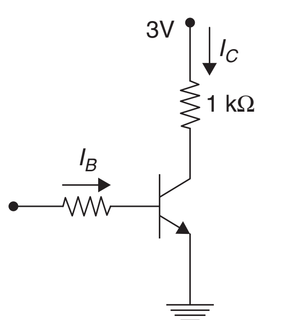
Hence, the correct option is (a).

16. The cascade amplifier is a multistage configuration of **[2005]**

(a) *CC–CB* (b) *CE–CB*
(c) *CB–CC* (d) *CE–CC*

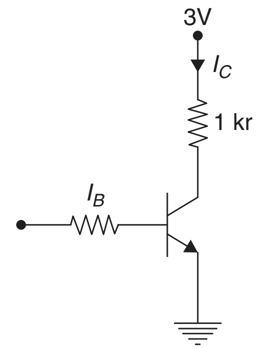
Solution: (b)
Cascade amplifier is a multistage configuration of *CE–CB*.
Hence, the correct option is (b).

17. Assuming $V_{CEsat} = 0.2$ V and $\beta = 50$, the minimum base current (I_B) required to drive the transistor in the figure to saturation is **[2004]**



(a) 56 μA (b) 140 μA
(c) 60 μA (d) 3 μA

Solution: (a)



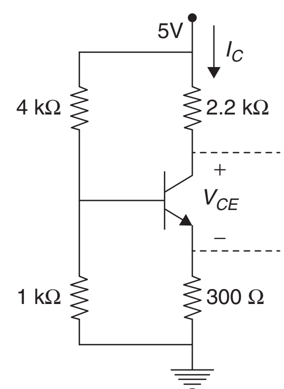
Collector current is given by

$$I_C = \frac{3 - 0.2}{1\text{ k}} = 2.8 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{2.8}{50} = 56\ \mu\text{A}$$

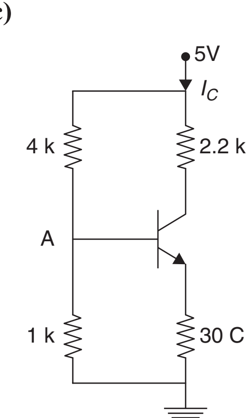
Thus, $I_{B\min} = 56$ µA is necessary to drive the transistor in saturation.
Hence, the correct option is (a).

18. Assuming that the β of the transistor is extremely large and $V_{BE} = 0.7$ V, I_C and V_{CE} in the circuit shown in the figure are **[2004]**



(a) $I_C = 1$ mA, $V_{CE} = 4.7$ V
(b) $I_C = 0.5$ mA, $V_{CE} = 3.75$ V
(c) $I_C = 1$ mA, $V_{CE} = 2.5$ V
(d) $I_C = 0.5$ mA, $V_{CE} = 3$ V.

Solution: (c)



Applying voltage divider at node A

$$V_A = \frac{5\times1}{5} = 1\text{ V}$$

$$I_\varepsilon = \frac{1-0.7}{300} = \frac{0.3}{300} = 1\text{ mA}$$

$I_C = I_\varepsilon$ as β is large
$\Rightarrow I_B = 0$
$\therefore V_{c\varepsilon} = 5 - 1(2.2 + 0.3)$
$= 2.5$ V
Hence, the correct option is (c).

19. Choose the correct match for input resistance of various amplifier configurations shown below: **[2003]**

Configuration	**Input resistance**
CB: Common Base	LO: Low
CC: Common Collector	MO: Moderate
CE: Common Emitter	HI: High

(a) *CB–LO, CC–MO, CE–HI*
(b) *CB–LO, CC–HI, CE–MO*
(c) *CB–MO, CC–HI, CE–LO*
(d) *CB–HI, CC–LO, CE–MO*

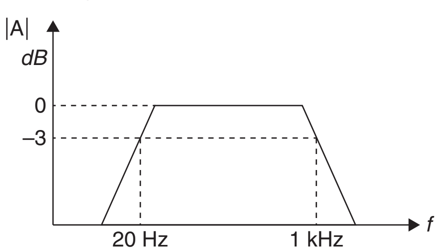
Solution: (b)

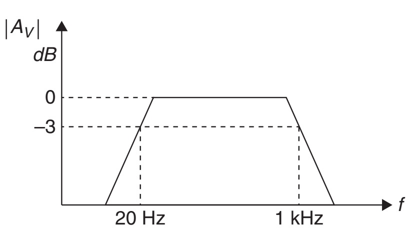
Configuration	I/P resistance
CB	low
CC	high
CE	moderate

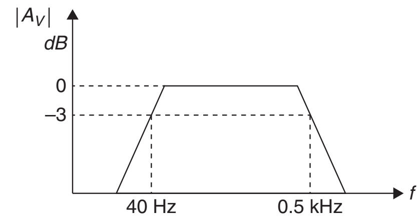
20. Generally, the gain of a transistor amplifier falls at high frequencies due to the **[2003]**
(a) internal capacitances of the device.
(b) coupling capacitor at the input.
(c) skin effect.
(d) coupling capacitor at the output.

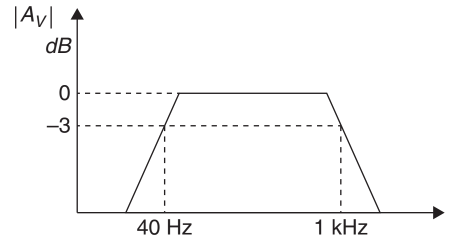
Solution: (a)
The gain of the transistor falls at high frequency due to internal capacitances of the device.

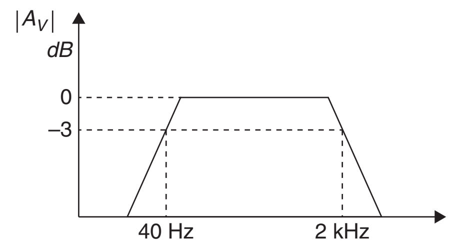
21. Three identical *RC*-coupled transistor amplifiers are cascaded. If each of the amplifiers has a frequency response as shown in the figure, the overall frequency response is as given in **[2002]**



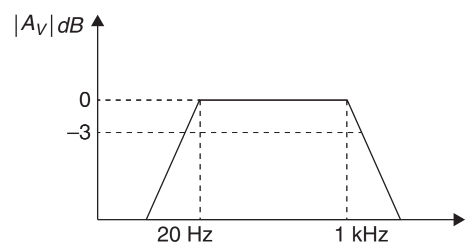
(a)


(b)


(c)


(d)


Solution: (b)
Frequency response of individual transistor amplifies

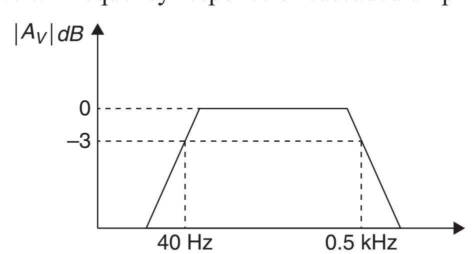


$$D = \sqrt{2^{1/n}-1}$$
$$= \sqrt{2^{1/3}-1}$$
$$= \sqrt{1.25-1} - \sqrt{0.25}$$
$$= 0.5$$

$$f_c' = \frac{f_c}{D} = \frac{20}{0.5} = 40\text{ Hz}$$

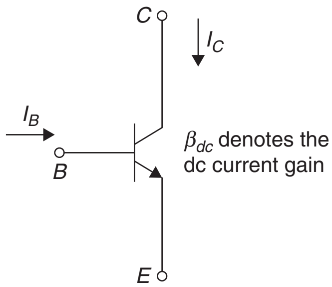
$f_H' = f_H \times D$
$= 1 \times 0.5$
$= 0.5$ kHz

Overall frequency response of cascaded amplifier:



Hence, the correct option is (b).

22. If the transistor in the figure is in saturation, then **[2002]**

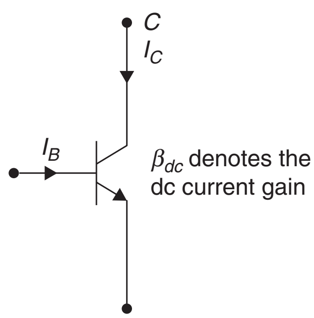


(a) I_C is always equal to $\beta_{dc} I_B$
(b) I_C is always equal to $-\beta_{dc} I_B$
(c) I_C is greater than or equal to $\beta_{dc} I_B$
(d) I_C is less than or equal to $\beta_{dc} I_B$

Solution: (d)

For transistor in saturation,

$$I_B \geq \left(\frac{I_c}{\beta}\right)$$



Hence, the correct option is (d).

23. The current gain of a BJT is **[2001]**

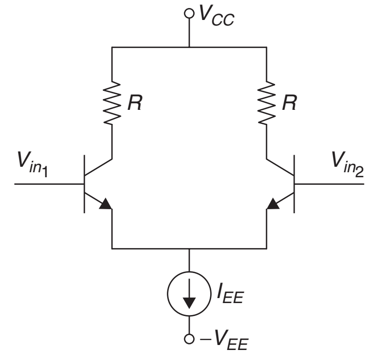
(a) $g_m r_o$ (b) $\dfrac{g_m}{r_o}$

(c) $g_m r_\pi$ (d) $\dfrac{g_m}{r_\pi}$

Solution: (c)

Current gain of BJT is $g_m \gamma_\pi$

24. In the differential amplifier of the figure, if the source resistance of the current source I_{EE} is infinite, then the common-mode gain is **[2000]**

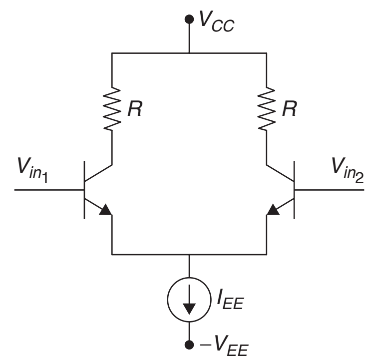


(a) zero (b) infinite

(c) indeterminate (d) $\dfrac{V_{in_1} + V_{in_2}}{2V_T}$

Solution: (a)

A common mode $= \left\{\dfrac{-R_c}{2R_\varepsilon}\right\}$



Given $R_\varepsilon = \infty$
thus, $A_{CM} = 0$
Hence, the correct option is (a).

25. Introducing a resistor in the emitter of a common emitter amplifier stabilizes the dc operating point against variations in **[2000]**
(a) only the temperature
(b) only the β of the transistor
(c) both temperature and β
(d) none of the above

Solution: (c)

Both variations in temperature as well as variation in β

26. The current gain of a bipolar transistor drops at high frequencies because of **[2000]**
(a) transistor capacitances
(b) high current effects in the base

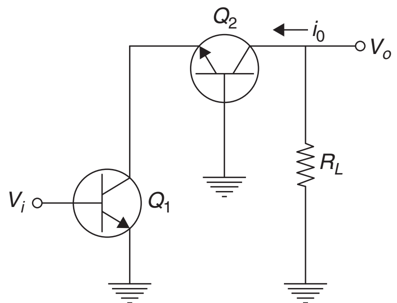
(c) parasitic inductive elements
(d) the early effect

Solution: (a)
At high frequency, due to transistor capacitances, gain starts reducing.

$$\therefore I_\varepsilon = \frac{V_\varepsilon}{R_\varepsilon} = \frac{4.3}{4.30}$$

$I_\varepsilon = I_c = 10$ mA
Hence, the correct option is (a).

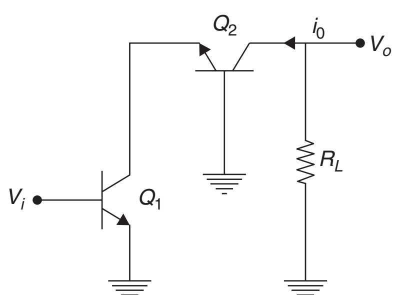
27. In the cascade amplifier shown in the figure, if the common-emitter stage (Q_1) has a transconductance g_{m1}, and the common base stage (Q_2) has a transconductance g_{m2}, then the overall transconductance $g(= i_0/V_i)$ of the cascade amplifier is **[1999]**



(a) g_{m1} (b) g_{m2}

(c) $\dfrac{g_{m1}}{2}$ (d) $\dfrac{g_{m2}}{2}$

Solution: (a)



$$g_m = g_m = \frac{i_0}{V_i}, \ i_0 \cong I\varepsilon_2 = Ic_1$$

$I_{C1} = \beta I_B,\ I_{\varepsilon 2} = I_{C1}$
$I_0 \cong \beta I_{B1},\ V_i = I_{B1}\gamma_\pi$

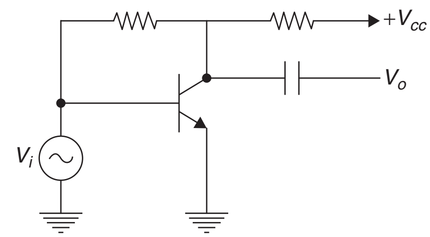
$$\frac{i_0}{V_i} = \frac{\beta I_{B1}}{I_{B1} r_\pi} = gm1$$

$$= \frac{I_{c1}}{V_i}(I_{c1} = \beta I_{B1})$$

In this cascade connection overall transconductance *gm* is equal to transconductance of the first stage.

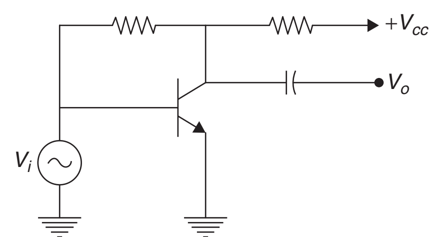
Thus it is equal to *gm*.
Hence, the correct option is (a).

28. The circuit of the figure is an example of feedback of the following type **[1998]**



(a) current series
(b) current shunt
(c) voltage series
(d) voltage shunt

Solution: (d)



The circuit of the figure is an example of feedback of the type voltage shunt.
Hence, the correct option is (d).

29. From the measurement of the rise time of the O/P pulse of an amplifier whose input is a small amplitude square wave, one can estimate the following parameter of the amplifier: **[1998]**
(a) gain-bandwidth product
(b) slew rate
(c) upper-3-dB frequency
(d) lower 3-dB frequency

Solution: (c)

$$BW = f_H = \frac{0.35}{t_r}$$

Upper 3- dB frequency
Hence, the correct option is (c).

30. A distorted sinusoid has the amplitude, $A_1, A_2, A_3, \ldots$ of the fundamental, second harmonic, third harmonic,… respectively. The total harmonic distortion is **[1998]**

(a) $\dfrac{A_2 + A_3 + ...}{A_1}$

(b) $\dfrac{\sqrt{A_2^2 + A_3^2 + ...}}{A_1}$

(c) $\frac{\sqrt{A_2^2 + A_3^2 + ...}}{\sqrt{A_1^2 + A_2^2 + A_3^2}}$

(d) $= \frac{\sqrt{A_2^{\,2} + A_3^{\,2} + ...}}{A_1}$

Solution: (b)

Total harmonic distortion $= \frac{\sqrt{A_2^{\,2} + A_3^{\,2} + ...}}{A_1}$

where A_1, A_2, A_3… are the fundamental, second harmonic, third harmonic, … respectively.
Hence, the correct option is (b).

31. The emitter coupled pair of BJTs gives a linear transfer relation between the differential O/P voltage and the differential input voltage V_{id} only when the magnitude of V_{id} is less 'α' times the thermal voltage, where 'α' is **[1998]**

(a) 4 (b) 3 (c) 2 (d) 1

Solution: (d)
The emitter coupled pair of *BJTs* gives a linear transfer relation between the differential O/P voltage and the differential I/P voltage V_{id} only.
When the magnitude of V_{id} is less 'α' times the thermal voltage, then α is '1'.
Hence, the correct option is (d).

32. A multistage amplifier has a low-pass response with three real poles at $s = -\omega_1$, $-\omega_2$ and ω_3. The approximate overall bandwidth B of the amplifier will be given by **[1998]**

(a) $S = -\omega_1 + \omega_2 + \omega_3$

(b) $\frac{1}{B} = \frac{1}{\omega_1} + \frac{1}{\omega_2} + \frac{1}{\omega_3}$

(c) $B = (\omega_1 + \omega_2 + \omega_3)^{\frac{1}{3}}$

(d) $B = \sqrt{\omega_1^2 + \omega_2^2 + \omega_3^3}$

Solution: (b)

$$\frac{1}{B} = \frac{1}{\omega_1} + \frac{1}{\omega_2} + \frac{1}{\omega_3}$$

Cascading of amplifier results in decrease of higher cut-off frequency and increase in lower cut-off frequency.
So, $f_H\downarrow$ and $f_L\uparrow$
$B\omega = f_H - f_L$
So, $BW\downarrow$.
Hence, the correct option is (b).

33. In a series regulated power supply circuit the voltage gain A_v of the 'pass' transistor satisfies the condition **[1998]**

(a) $A_v \to \infty$ (b) $1 << A_v << \infty$
(c) $A_v = 1$ (d) $A_v << 1$

Solution: (c)
In series regulated power supply, pass transistor is emitter follower type.
Thus, gain is nearly equal to me.
Hence, the correct option is (c).

34. The unit of $\frac{q}{kT}$ are **[1998]**

(a) V (b) V^{-1}
(c) J (d) J/K

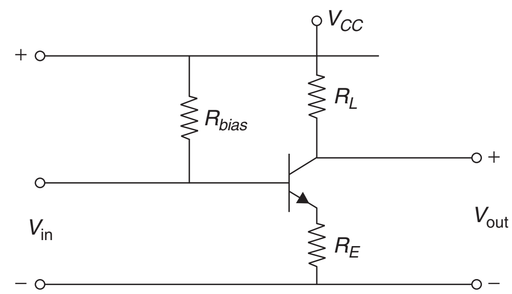
Solution: (b)

Thermal voltage $= V_T = \frac{kT}{q}$

So, $\frac{q}{kT} \to V^{-1}$

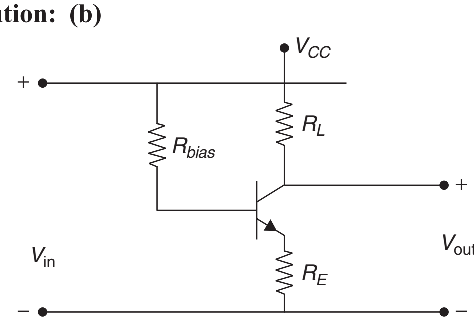
Hence, the correct option is (b).

35. In the BJT amplifier shown in the figure the transistor is biased in the forward active region. Putting a capacitor across R_E will **[1997]**



(a) decrease the voltage gain and decrease the I/P impedance.
(b) increase the voltage gain and decrease the I/P impedance.
(c) decrease the voltage gain and increase the I/P impedance.
(d) increase the voltage gain and increase the I/P impedance.

Solution: (b)



Putting a capacitor across will increase the voltage gain and decrease the I/P impedance.
This capacitor will be by pass R_ε thus '*ac*' signals will pass through capacitor C_ε.
Hence, the correct option is (b).

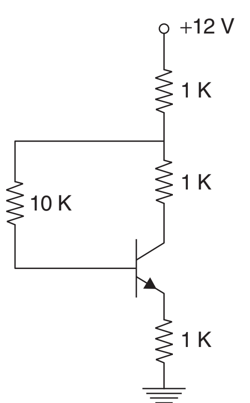
36. A cascade amplifier stage is equivalent to **[1997]**
(a) a common emitter stage followed by a common base stage.
(b) a common base stage followed by an emitter follower.
(c) an emitter follower stage followed by a common base stage.
(d) a common base stage followed by a common emitter stage.

Solution: (a)
A cascade connection is where common emitter configuration is followed by common base.
Hence, the correct option is (a).

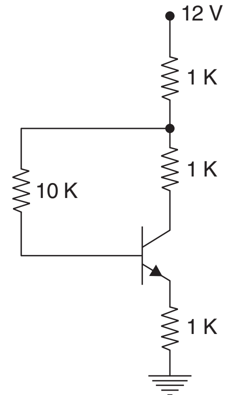
37. A BJT is said to be operating in the saturation Region if **[1995]**
(a) both the junctions are reverse biased.
(b) base-emitter junction is reverse biased and base-collector junction is forward biased.
(c) base-emitter junction is forward biased and base-collector junction is reverse-biased.
(d) both the junctions are forward biased.

Solution: (b)
Both the junctions are forward biased.
When emitter base (ε-B) junction and collector base (C-B) junction both are forward biased, then the *BJT* is said to be operating in saturation region.
Hence, the correct option is (b).

38. A transistor having $\alpha = 0.99$ and $V_{BE} = 0.7$ V is used in the circuit of the figure. The value of the collection current will be **[1995]**



Solution: (3.75 mA)



I_c cannot be 5.32 mA because
$I_c = 5.32$ mA will make $V_{c\varepsilon}$ negative which implies transistor is in saturation.
Through *kVL*,
$12\,I_B + 2\,I_c = 11.2$
$10\,I_B - I_c = 0.6$
Upon solving
$I_c \cong 3.75$ mA

39. In order to reduce the harmonic distortion in an amplifier its dynamic range has to be____________ **[1994]**

Solution: (Compressed)
In order to reduce the harmonic distortion is an amplifier its dynamic range has to be compressed.

40. A Common Emitter transistor amplifier has a collector current of 1.0 mA. When its base current is 25 μA at the room temperature, its input resistance is approximately equal to________ **[1994]**

Solution: (1 k|)
For common emitter configuration
I/P resistance = R_i

$$= \frac{V_T}{I_B} = \frac{25\times10^{-3}}{25\times10^{-6}} = 1\text{ k}'$$

41. The bandwidth of an n-stage tuned amplifier, with each stage having a bandwidth of B, is given by **[1993]**

(a) $\dfrac{B}{n}$ (b) $\dfrac{B}{\sqrt{n}}$

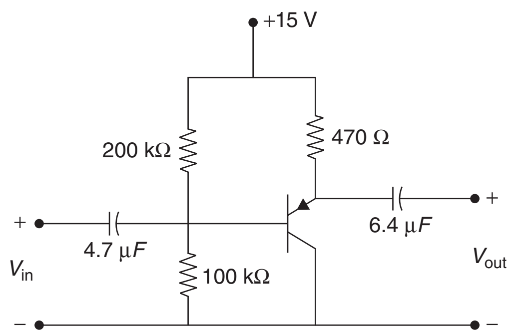
(c) $B\sqrt{2^{1/n}-1}$ (d) $\dfrac{B}{\sqrt{2^{1/n}-1}}$

Solution: (c)
The overall band width of an *n*- stage turned amplifier is

$$BW_n = B\sqrt{2^{1/n}-1}$$

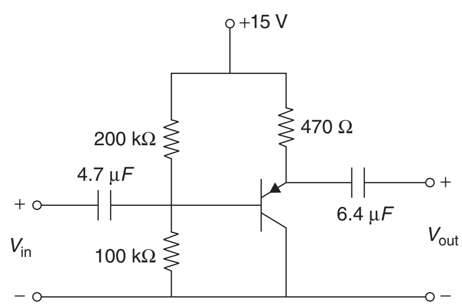
Hence, the correct option is (c).

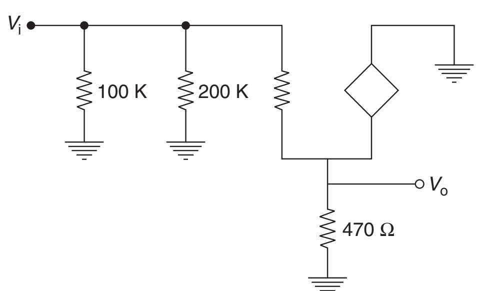
42. For the amplifier circuit of the figure, the transistor has a β of 800. The mid-band voltage gain $\frac{V_0}{V_i}$ of the circuit will be____ **[1993]**



(a) 0 (b) < 1 (c) ≈ 1 (d) 800

Solution: (c)





$$A_V = \frac{V_0}{V_i}$$

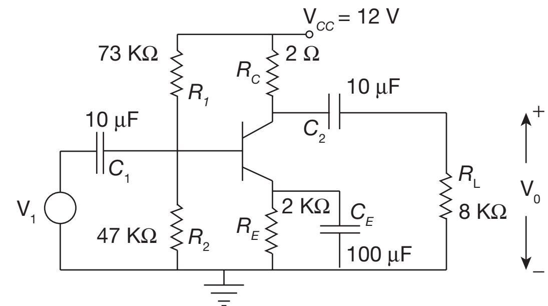
$$= \frac{l_B(1+h_{fe})R_\varepsilon}{l_B[h_{ie}+(1+h_{fe})]R_\varepsilon}$$

$$\cong 1$$

Hence, the correct option is (c).

Two-marks Questions

1. For the DC analysis of the common-Emitter amplifier shown, neglect the base current and assume that the emitter and collector currents are equal. Given that V_T = 25 mV, V_{BE} = 0.7 V and the *BJT* output resistance r_0 is practically infinite. Under these conditions the midband voltage gain magnitude, $A_v = |v_0/v_1| V/V$, is ____________. **[2017]**



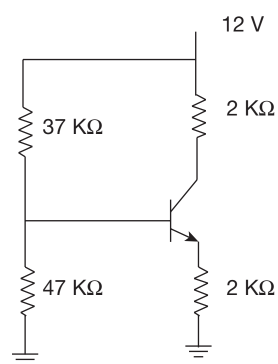
Solution: *DC analysis*: Capacitors are open circuited ac source is short circuited.

$$I_c = \frac{\frac{V_{cc}R_2}{R_1+R_2} - V_{BE}}{R_E}$$

$$I_c = \frac{\frac{12\times 47}{120} - 0.7}{2\text{ K}\Omega}$$

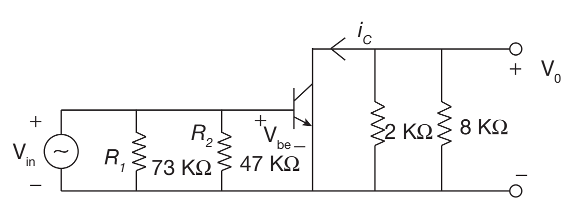
$$I_c = 2\,\text{mA}$$

$$g_m = \frac{I_c}{V_t} = \frac{2\,\text{mA}}{25\,\text{mV}} = \frac{2}{25}$$



A.C. analysis:

Capacitors are short circuited DC source is also short circuited.



$V_0 = -i_c(R_c \| R_L)$ $V_{in} = n_{be}$

Voltage $A_v = \frac{V_0}{V_{in}} = g_m (R_c \| R_L)$

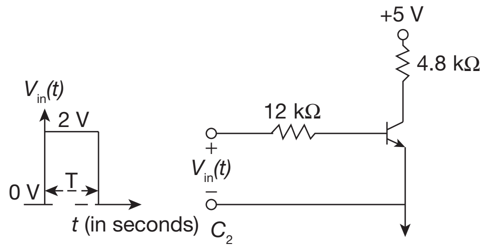
$$\therefore A_v = \frac{2}{25}[2K \| 8K]$$

$$\therefore A_v = \frac{\cancel{2}}{25}\left[\frac{2\times 8K}{1\cancel{0}\;_{5}}\right]$$

$\therefore A_v = 128.$

Hence, the correct answer is (127).

2. In the figure shown, the npn transistor acts as a switch.



For the input $V_{in}(t)$ as shown in the figure, the *transistor* switches between the cut-off and saturation regions of operation, when T is large. Assume collector-to-emitter voltage at saturation $V_{CE(sat)} = 0.2$ V and base-to-emitter voltage $V_{BE} = 0.7$ V. The minimum value of the common-base current gain (α) of the transistor for the switching should be ________. **[2017]**

Solution: Given that

$$V_{CE(sat)} = 0.2 \text{ V}$$

$$V_{BE} = 0.7$$

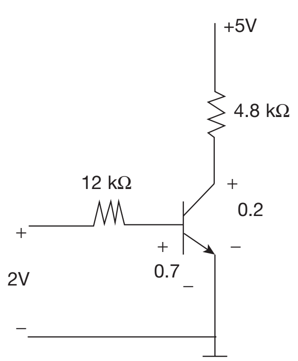
$$I_B = \frac{2-0.7}{12K\Omega} = 108.33\ \mu\text{A}$$

$$I_C = \frac{5-0.2}{4.8K\Omega} = \frac{4.8}{4.8K\Omega} = 1\text{ mA}$$

$$\beta = \frac{I_C}{I_B} = \frac{1\text{ mA}}{108.33\ \mu\text{A}} = 9.2310$$

$$\alpha = \frac{\beta}{\beta+1} = \frac{9.2310}{1+9.2310} = 0.9022$$

$$\therefore \quad \alpha = 0.9022$$



Hence, the correct answer is (0.89 to 0.91).

3. If the base width in a bipolar junction transistor is doubled, which one of the following statements will be TRUE? **[2015]**

(A) Current gain will increase.
(B) Unity gain frequency will increase.
(C) Emitter-base junction capacitor will increase.
(D) Early Voltage will increase.

Solution: If Base with in BJT is increased then early voltage also increases

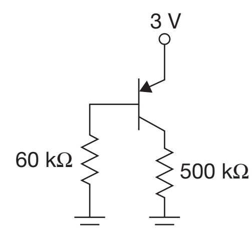
$$I_C = I_S\left[1+\frac{V_{CE}}{V_A}\right].e^{V/V_T} \text{ as Base width increases}$$

$\Rightarrow \; I_B\uparrow$ and $I_C\downarrow$

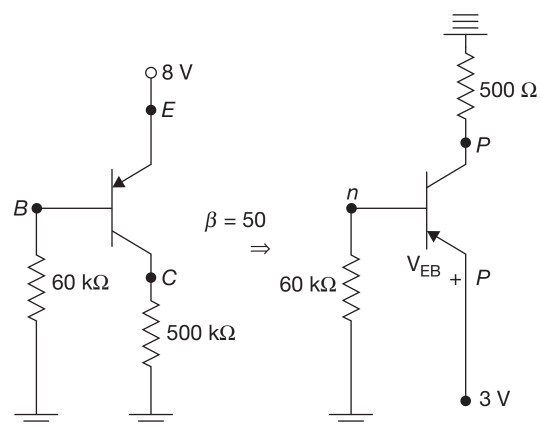
$\Rightarrow \; V_A\uparrow$

Hence, the correct option is (D).

4. In the circuit shown in the figure, the BJT has a current gain (β) of 50. For an emitter-base voltage V_{EB} = 600 mV, the emitter-collector voltage V_{EC} (in Volts) is ________. **[2015]**



Solution:



Given β = 50 and V_{EB} = 600 mV = 0.6 V

$$V_{EB} = V_E - V_B = 0.6 \text{ V}$$

But $\quad V_E = 3$ V

$$V_B = 2.4 \text{ V}$$

$$\therefore \quad I_B = \frac{2.4}{60}\text{ mA}$$

$$I_B = 0.04 \text{ mA}$$

$$I_C = \beta \cdot I_B = 2 \text{ mA}$$

$$V_C = R_C \cdot I_C$$

$$= 500 \times 2 \times 10^{-3}$$

$= 1$ V

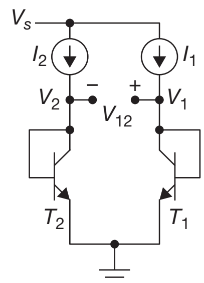
$V_{EC} = V_E - V_C$

$= 3 - 1$

$= 2$ V

Hence, the correct Answer is (2).

5. In the circuit shown, $I_1 = 80$ mA and $I_2 = 4$ mA. Transistors T_1 and T_2 are identical. Assume that the thermal voltage V_T is 26 mV at 27°C. At 50°C, the value of the voltage $V_{12} = V_1 - V_2$(in mV) is ________ **[2015]**



Solution: From the given data

$I_1 = 80$ mA and $I_2 = 4$ mA

$V_T = 26$ mV at 27°C

At 50°C, the value of the voltage $V_{12} = V_1 - V_2 = ?$

We know

$$I = I_o\left\{e^{V/\eta V_T} - 1\right\}$$

$$I \approx I_o\left\{e^{V/\eta V_T}\right\}$$

$$\frac{I_1}{I_2} = e^{(V_1 - V_2)/\eta . V_T}$$

We know

$$V_T = \frac{T}{11600}$$

at $T = 273 + 50° = 323°$ K

$$V_T = \frac{323}{11600} = 27.844\, mV$$

$$V_{12} = \eta V_T \cdot \ln\left(\frac{I_1}{I_2}\right) V$$

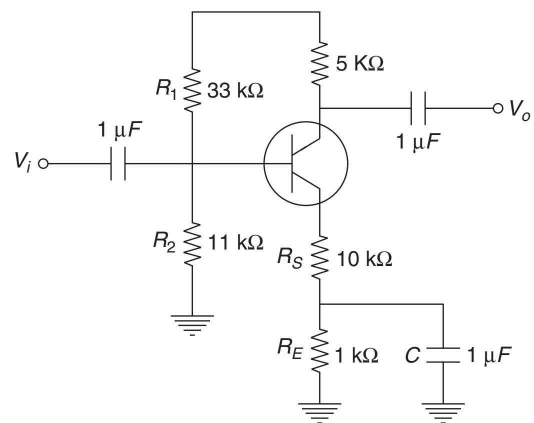
Let $\eta = 1$ (NOT given)

$$V_{12} = 27.844 \times 10^{-3} \cdot \ln\left(\frac{80}{4}\right)$$

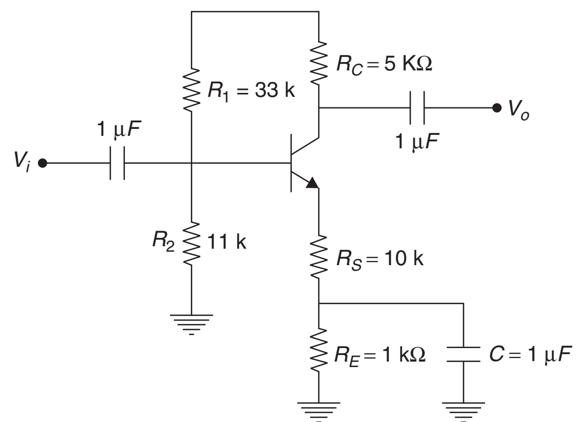
$= 83.413$ mV

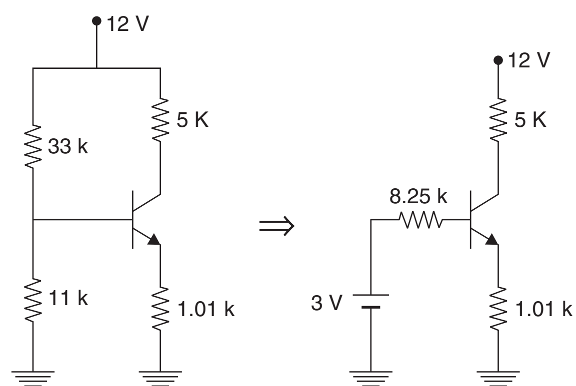
Hence, the correct Answer is (83.5 to 84.0).

6. For the amplifier shown in the figure, the BJT parameters are $V_{BE} = 0\ 7$ V, $\beta = 200$, and thermal voltage $V_T = 25$ mV. The voltage gain (v_0/v_i) of the amplifier is____. **[2014]**



Solution: (−223.6)





By applying KVL in input loop, we get

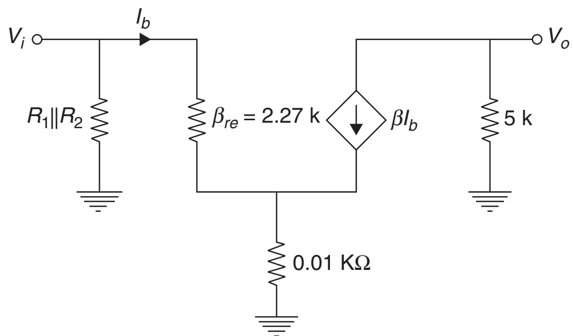
$$3 = 8.25\, I_B + 0.7 + 1.10 \text{KI}_\varepsilon$$

$$2.3 = I_\varepsilon\left[\frac{8.25\text{K}}{201} + 1.01\text{K}\right]$$

$$2.3 = I_\varepsilon \times 1 - 051\text{K}$$

$$l_\varepsilon = 2.2 \text{ mA}$$

AC equivalent model of amplifier is given as



where *re* is given by

$$r_e = \frac{VT}{I_\varepsilon}$$

$$r_e = \frac{25 \text{ mV}}{2.2 \text{ mA}} = 11.36\Omega$$

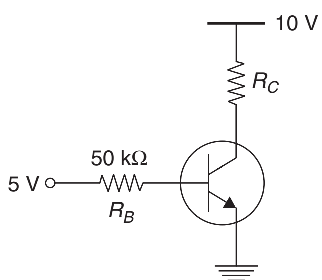
$$V_0 = -5\beta I_b \quad (1)$$

$$V_i = 2.27\text{K } I_b \ + 0 - 01\text{K}(I_b + \beta I_b) \quad (2)$$

From equations (1) and (2), we get

$$\frac{V_0}{V_i} = \frac{-5\text{K} \times 200}{4 - 28\text{K}} = -233.6$$

7. In the circuit shown, the silicon BJT has $\beta - 50$. Assume $V_{BF} = 0.7$ V and $V_{\text{CE(sat)}} = 0.2$ V. Which one of the following statements is correct? **[2014]**



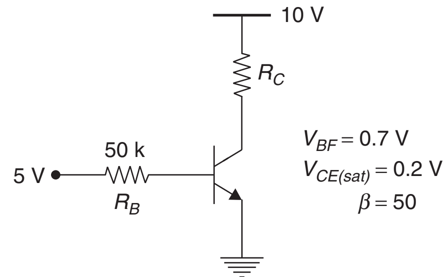
(a) For $R_C = 1$ kΩ, the BJT operates in the saturation region.

(b) For $R_C = 3$ kΩ, the BJT operates in the saturation region.

(c) For $R_C = 20$ kΩ, the BJT operates in the cut-off region.

(d) For $R_C = 20$ kΩ, the BJT operates in the linear region.

Solution: (b)

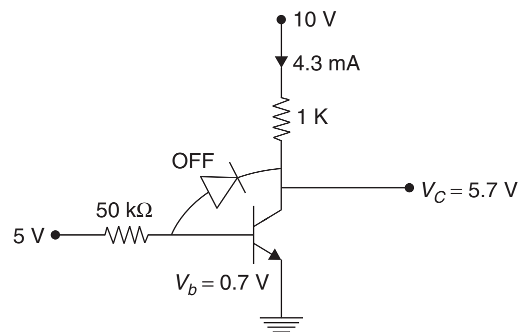


Assume the transistor is in active region

$$I_B \text{ active} = \frac{5 - 0.7}{50k} = 86 \text{ mA}$$

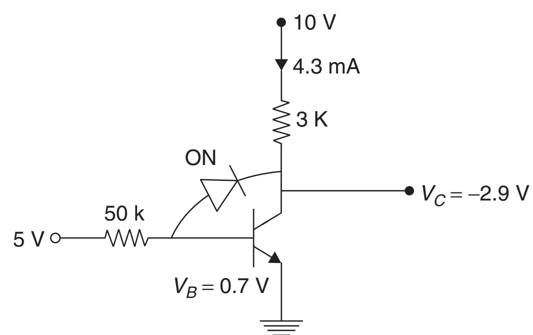
$$I_C \text{ active} = \beta I_B \text{ active}$$
$$= 50 \times 86 \text{ mA} = 4.3 \text{ mA}$$

Assume $R_e = 1$ kΩ



It is in active.

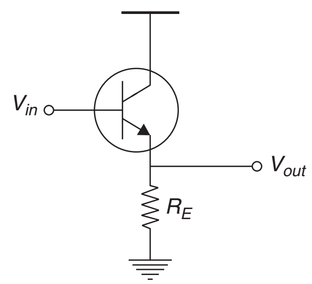
Assume $R_c = 3$kΩ



It is in saturation.

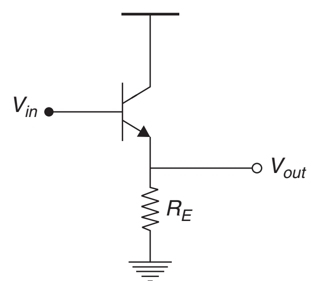
Hence, the correct option is (b).

8. Consider the common-collector amplifier in the figure (bias circuitry ensures that the transistor operates in forward active region, but has been omitted for simplicity) Let I_C be the collector current, V_{BE} be the base-emitter voltage and V_T be the thermal voltage. Also, g_m and r_o are the small-signal transconductance and output resistance of the transistor, respectively. Which one of the following conditions ensures a nearly constant small signal voltage gain for a wide range of values of R_E? **[2014]**

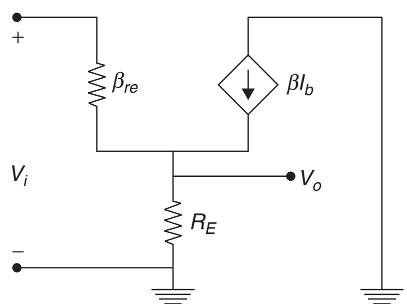


(a) $g_m R_E << 1$ (b) $I_C R_E >> V_T$
(c) $g_m r_o > 1$ (d) $V_{BE} >> V_T$

Solution: (b)



AC equivalent circuit for given common collector amplifier is



$$V_{in} = \beta_{re} I_b + (1+\beta) I_b R_\varepsilon$$
$$V_0 = (1+\beta) R_\varepsilon I_b$$

Then,

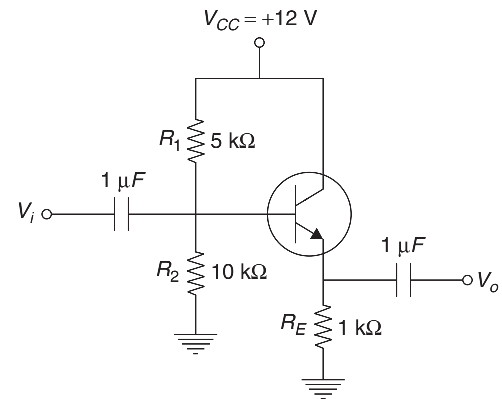
$$\frac{V_0}{V_{in}} = \frac{(1+\beta)R_\varepsilon}{\beta_{re} + (1+\beta)R_\varepsilon}$$
$$\frac{V_0}{V_{in}} = \frac{\beta R_\varepsilon}{\beta_{re} + \beta R_\varepsilon} = \frac{R_\varepsilon}{re + R_\varepsilon}$$

The condition for small signal voltage gain to be nearly constant is

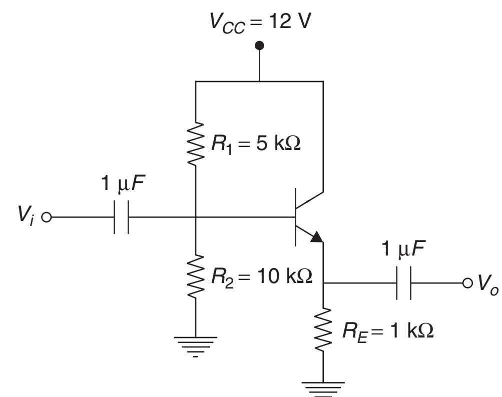
$$R_\varepsilon >> r_e$$
$$R_\varepsilon >> \frac{V_T}{I_C}$$
$$I_C R_\varepsilon >> V_T$$

Hence, the correct option is (b).

9. For the common collector amplifier shown in the figure the BJT has high β, negligible $V_{CE(sat)}$, and $V_{BF} = 0\ 7$ V. The maximum undistorted peak-to-peak output voltage v_o (in volts) is______. **[2014]**

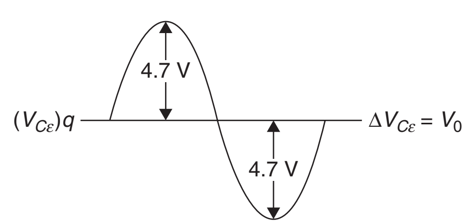


Solution: (9.4 V)
DC analysis



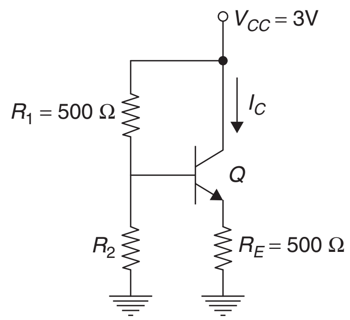
Applying voltage divider

$$(VB)_q = \frac{V_{Ce} \times R_2}{R_1 + R_2} = \frac{12 \times 10\ \text{K}\Omega}{15\ \text{K}\Omega}$$
$$= 8\ \text{V}$$
$$(V_\varepsilon)_Q = V_B - 0 - 7$$
$$= 8 - 0.7 = 7.3\ \text{V}$$
$$(V_{C\varepsilon}) = (V_C)_Q - (V_\varepsilon)_Q$$
$$= 12 - 7.3 = 4.7\ \text{V}$$



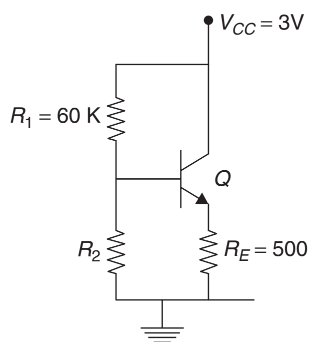
$$V_{0(p-p)} = 2 \times 4 - 7$$
$$= 9.4\ \text{V}$$

10. In the circuit shown below, the silicon npn transistor Q has a very high value of β. The required value of R_2 in kQ to produce $I_C = 1$ mA is **[2013]**



(a) 20 (b) 30 (c) 40 (d) 50

Solution: (c)

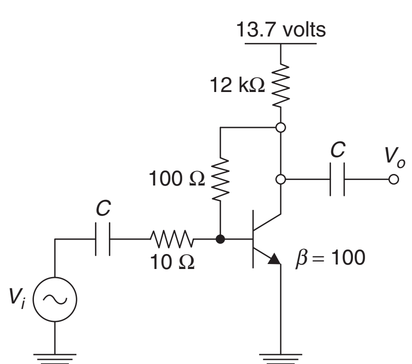


$$3 \cdot \frac{R_2}{60 + R_2} = 1.2$$

$$R_2 = 40k\Omega$$

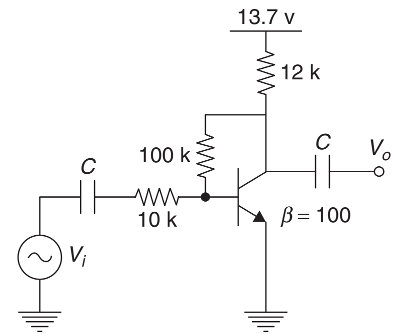
Hence, the correct option is (c).

11. The voltage gain A_v of the circuit shown below is **[2012]**



(a) $|A_v| \approx 200$ (b) $|A_v| \approx 100$
(c) $|A_v| \approx 20$ (d) $|A_v| \approx 10$

Solution: (d)



KVL in input loop,

$$13.7 - (I_C + I_B)12k - 100k(I_b) - 0.7 = 0$$

$$\Rightarrow I_B = 9.9 \text{ mA}$$

$$I_C = \beta/B = 0.99 \text{ mA}$$

$$1_\varepsilon = 1 \text{ mA}$$

$$\therefore r_e = \frac{26 \text{ mA}}{I_\varepsilon} = 26\Omega$$

$$Z_i = \beta r_e = 2.5k\Omega$$

$$\therefore A_V = \left(\frac{100k \,||\, 12k}{26}\right) = 412$$

$$Z_i^1 = Z_i \left\| \left(\frac{100k}{1+412}\right)\right.$$

$$= 221\Omega$$

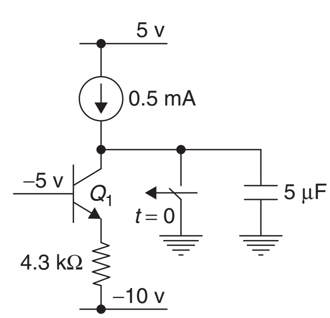
$$A_{VS} = A_V \frac{Z\delta^1}{Z_i^1 + R_S}$$

$$= 412\left(\frac{221}{221 + 10k}\right)$$

$$|A_{VS}| \approx 10$$

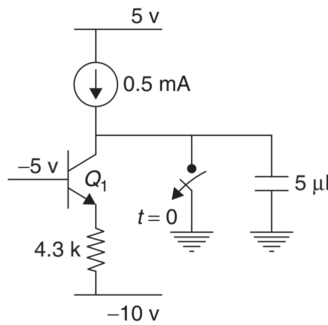
Hence, the correct option is (d).

12. For the BJT Q_1 in the circuit shown below, $\beta = \infty$, $V_{BEon} = 0.7$ V, $V_{CEsat} = 0.7$ V. The switch is initially closed. At time $t = 0$, the switch is opened. The time t at which Q_1 leaves the active region is **[2011]**



(a) 10 ms (b) 25 ms
(c) 50 ms (d) 100 ms

Solution: (c)



Apply KVL at the *BE* junction

$$1_\varepsilon = \frac{-5-0.7+10}{4.3\ \text{K}\Omega}$$

$$= \frac{4.3}{4.3\ \text{K}\Omega} = 1\ \text{mA}$$

Always, $1_\varepsilon = 1$ mA

At collector junction

$$I_{\text{cap}} + (0.5\ \text{mA}) = 1\ \text{mA} \quad (\because \beta = \infty;\quad I_\varepsilon = I_C)$$

$I_{\text{cap}} = 1 - 0.5 = 0.5$mA always constant

$$V_{C\varepsilon} = V_C - V_\varepsilon$$

$$\Rightarrow V_C = V_{C\varepsilon} + V_\varepsilon$$

$$= 0.7 + 4.3K \times 1 \times 10^{-3}$$

$$= 0.7 + 4.3 \quad (\because V_\varepsilon = 1_\varepsilon R_\varepsilon)$$

$$V_C = 5\text{V} = V_{\text{cap}}$$

$$V_{\text{cap}} = I_{\text{cap}} \frac{t}{C}$$

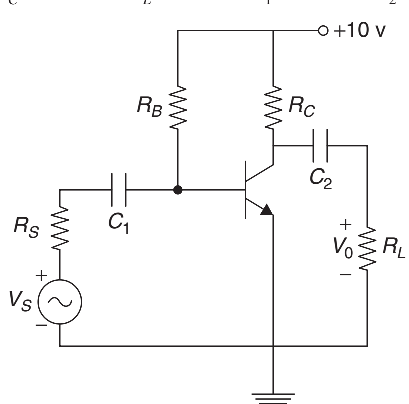
$$\Rightarrow t = \frac{V_{\text{cap}}(c)}{I_{\text{cap}}} = \frac{5 \times 5 \times 10^{-6}}{0.5 \times 10^{-3}} = 50\ \text{mA}$$

Hence, the correct option is (c).

Common Data for Questions 13 and 14.

Consider the common emitter amplifier shown below with the following circuit parameters:

$\beta = 100$, $g_m = 0.3861$ A/V, $r_o = \infty$, $r_\pi = 259\ \Omega$, $R_S = 1$ kΩ, $R_B = 93$ kΩ, $R_C = 250\ \Omega$, $R_L = 1$ kΩ, $C_1 = \infty$ and $C_2 = 4.7$ μF.



13. The resistance seen by the source V_s is **[2010]**

(a) 250 Ω (b) 1258Ω
(c) 93 kΩ (d) ∞

Solution: (b)

Base current

$$I_B = \frac{10-0.7}{93k}$$

$$= 100\ \mu\text{A}$$

$$I_c = \beta I_B$$

$$= 100 \times 100\ \mu\text{A}$$

$$I_\varepsilon = 10 - 1\ \text{mA}$$

$$\gamma_e = \frac{26\ \text{mV}}{10-1\ \text{mA}}$$

$$= 2.57\Omega$$

$$\beta_{re} = 100 \times 2.57 = 257\Omega$$

$$R_i = R_B \parallel \beta re$$

$$= 93K\Omega \parallel 257\Omega$$

$$= 257\Omega$$

$\therefore R_S$ (seen form R_S) = 1k Ω + 257 Ω = 1257 Ω

Hence, the correct option is (b).

14. The lower cut-off frequency due to C_2 is **[2010]**

(a) 33.9 Hz (b) 27.1 Hz
(c) 13.6 Hz (d) 16.9 Hz

Solution: (b)

Lower cut off frequency is given by

$$f_{L_{C_2}} = \frac{1}{2\pi(R_0 + R_L)C_2}$$

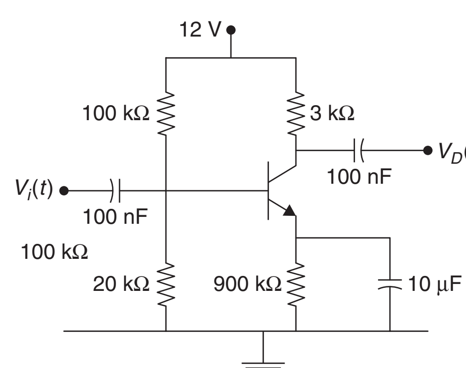
$$= \frac{1}{2\pi(250\Omega + 1K\Omega)4.7\ \mu\text{F}}$$

$$(\because\quad R_0 \simeq R_C = 250\Omega)$$

$$= 27.1\ \text{Hz}$$

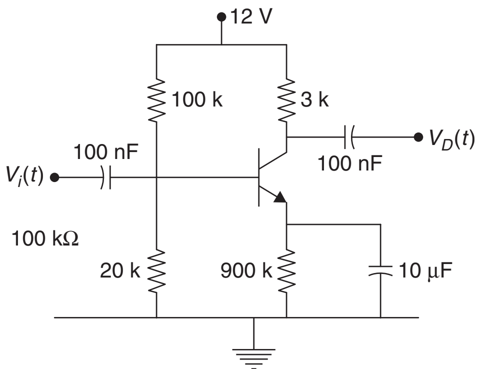
Hence, the correct option is (b).

15. A small signal source $v_i(t) = A \cos 20t + B \sin 10^6 t$ is applied to a transistor amplifier as shown below. The transistor has $\beta = 150$ and $h_{ie} = 3$ kΩ. Which expression best approximates $v_0(t)$? **[2009]**



(a) $v_0(t) = -1500\,(A\cos 20t + B\sin 10^6\,t)$
(b) $v_0(t) = -150\,(A\cos 20t + B\sin 10^6 t)$
(c) $v_0(t) = -1500\,B\sin 10^6 t$
(d) $v_0(t) = -150\,B\sin 10^6 t$

Solution: (d)



Gain

$$A_V = -h_{fe}\frac{R_L}{h_{ie}}$$

$$= -150\times\frac{3\text{ k}\Omega}{3\text{ k}\Omega}$$

$A_V = -150$

$\therefore\ V_0 = -150\ V_i(t)$

but $V_i(t) = A\cos 20t + B\sin 10^6 t$

where for $A\cos 20\omega t$,

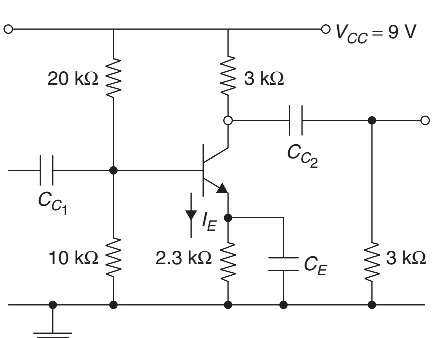
$$f = \frac{20}{2\pi} = 3.18\text{ Hz}$$

Since the coupling capacitors block low frequency signals, $A\cos 20t$ will not appear in output for

$$B\sin 10^6 t,\quad f = \frac{10^6}{2\pi}160\text{ kHz}$$

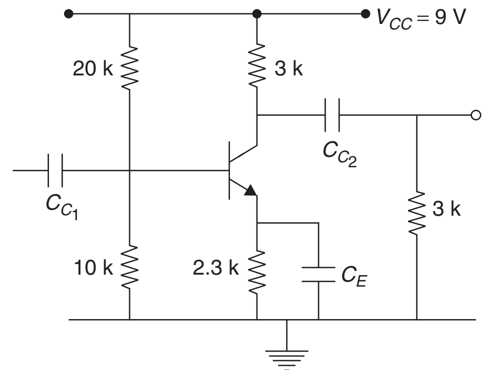
$\therefore$ $B\sin 10^6 t$ appear or $-150\,B\sin 10^6 t$
Hence, the correct option is (d).

Statement for Linked Answer Questions 16 and 17.
In the following transistor circuit, $V_{BE} = 0.7$ V, $r_e = 25$ mV/I_E, and β and all the capacitances are very large.



16. The value of DC current I_E is **[2008]**
(a) 1 mA (b) 2 mA
(c) 5 mA (d) 10 mA

Solution: (a)



Design is independent of I_B as β is very high,

$$I_B \simeq 0,\ V_B = \frac{9\times 10}{20+10} = 3\text{ V}$$

Thus applying Kirchhoff's law in base emitter, we get
$3 = 0.7 + I_\varepsilon \times 2.3\times 10^3$.
$\Rightarrow I_\varepsilon = 1$ mA
Hence, the correct option is (a).

17. The mid-band voltage gain of the amplifier is approximately **[2008]**
(a) −180 (b) −120
(c) −90 (d) −60

Solution: (d)

(3.53). Mid-band voltage gain $= -\dfrac{R_L'}{r_e}$

$Ie = 1$ mA

$$r_e = \frac{25}{I_\varepsilon} = 25\ \Omega$$

$R_L' = R_C \| R_L$
$= 3\text{ k}\|3\text{ k}$
$= 1.5$ k

Thus, $A_V = \dfrac{-1500}{25} = -60$

Hence, the correct option is (d).

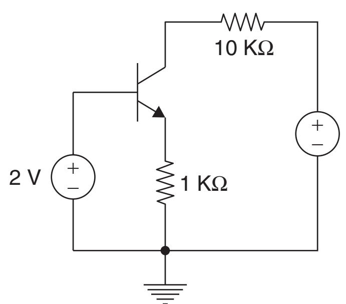
18. The *DC* current gain (β) of a BJT is 50. Assuming that the emitter injection efficiency is 0.995, the base transport factor is **[2007]**
(a) 0.980 (b) 0.985
(c) 0.990 (d) 0.995

Solution: (b)
Relation in alpha and beta is

$$\alpha = \frac{(\beta - 1)}{\beta} = \frac{49}{50}$$

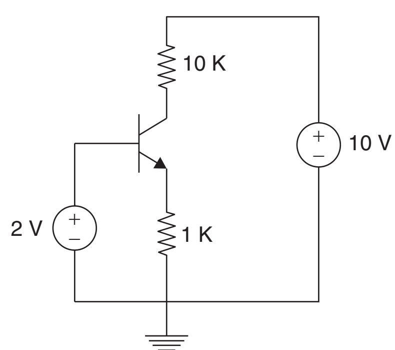
$= 0.98$
$\alpha = Rb$
where b = base transport factor
R = emitter injection efficiency
$b = 0.98/0.995$
$= 0.985$
Hence, the correct option is (b).

19. For the BJT circuit shown, assume that the β of the transistor is very large and V_{BE} = 0.7 V. The mode of operation of the BJT is **[2007]**



(a) cut-off (b) saturation
(c) normal active (d) reverse active

Solution: (b)

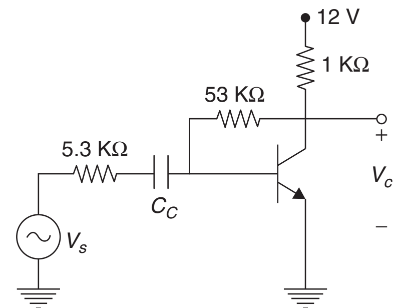


$V_{B\varepsilon} = 0.7$ V
$V_B = 2$ V
$\therefore V_\varepsilon = V_B - V_{B\varepsilon} = 1.3$ V

$$I_\varepsilon = \frac{V_\varepsilon}{R_\varepsilon} = \frac{1.3}{\text{k}} = 1.3 \text{ mA} \simeq I_C$$

$\therefore V_c = 10 - I_C R_c$
$= 10 - (1.3 \text{ mA})(10 \text{ k}\Omega)$
$V_C = -3$ V
$\therefore V_{BC} = V_B - V_C = 2 - (-3) = 5$ V
$\therefore B - E$ junction $= 0.7$ V $\Rightarrow FB$
$B - C$ junction $= 5$ V $\Rightarrow FB$
$\therefore$ saturation.
Hence, the correct option is (b).

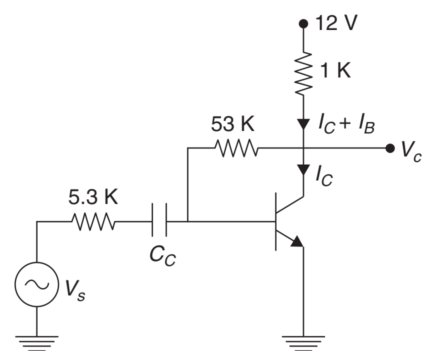
Common Data for Questions 20, 21 & 22.
In the transistor amplifier circuit shown in the figure below, the transistor has the following parameters:
$\beta_{DC} = 60$, $V_{BE} = 0.7$ V, $h_{ie} \to \infty$, $h_{fe} \to \infty$
The capacitance C_C can be assumed to be infinite.



In the above figure, the ground has been shown by the symbol

20. Under the DC conditions, the collector-to-emitter voltage drop is **[2006]**
(a) 4.8 volts (b) 5.3 volts
(c) 6.0 volts (d) 6.6 volts

Solution: (c)



$12 = (I_C + I_B)\,1\text{k} + I_B \times 53\text{k} + V_{B\varepsilon}$
$11.\,3 = (61\text{k} + 53\text{k})I_B$
$I_B = 99$ μA
$I_C = 5.95$ mA
$V_{C\varepsilon} \approx 12 - (1 \times 5.95)$
≈ 6 V
Hence, the correct option is (c).

21. If β_{DC} is increased by 10%, the collector-to-emitter voltage drop **[2006]**
(a) increases by less than or equal to 10%
(b) decreases by less than or equal to 10%
(c) increases by more than 10%
(d) decreases by more than 10%

Solution: (b)

Now $\beta_{dc} = 66$

$$I_B = \left(\frac{11.3}{120}\right) \text{mA}$$

$$I_C = \left(\frac{11.3}{120}\right) \times 66$$

$$I_C = 6.215 \text{ mA}$$
$$V_{C\varepsilon} = 12 - 6.215$$
$$= 5.78 \text{ V}$$

Thus $v_{c\varepsilon}$ decreases less than or equal to 10%.

Hence, the correct option is (b).

22. The small-signal gain of the amplifier V_C/V_S is **[2006]**

(a) −10 (b) −5.3 (c) 5.3 (d) 10

Solution: (b)

$1_{\varepsilon} = 6$ mA

$$\therefore r_e = \frac{26 \text{ mV}}{I_\varepsilon} = \frac{26 \text{ mv}}{6 \text{ mA}} = 4.33\ \Omega$$

$$\text{Now } A_V = \frac{-(R_\varepsilon \parallel z_2)}{r_e}$$

$$\text{where } Z_2 = \frac{53K}{1 - \dfrac{1}{A_V}} \simeq 53 \text{ k}\Omega$$

using Miller's theorem,

$$A_v = \frac{-(1\text{k} \parallel 53\text{k})}{4.33}$$
$$= -226.5$$

$$\text{Now } A_{Vs} = A_V \frac{z_i}{z_i + R_s}$$

where $z_i = z_1 \| \beta r_e$

$$Z_1 = \frac{53 \text{ k}}{1 - A_V} \text{ (using Miller's theorem)}$$

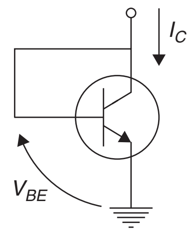
$$Z_1 = \frac{53 \text{ k}}{227.5} = 233\ \Omega$$

$$\therefore Z_1 = \frac{233(60 \times 4.33)}{233 + (60 \times 4.33)}$$
$$= 122.8\ \Omega$$

$$\Rightarrow A_{vs} = -226.5\left(\frac{122.8}{122.8 + 5.3 \text{ k}}\right)$$
$$= -5.13$$
$$A_{vs} \simeq -5.13$$

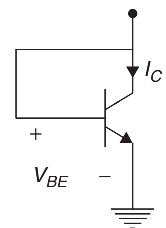
Hence, the correct option is (b).

23. For an npn transistor connected as shown in the figure, $V_{BE} = 0.7$ volts. Given that reverse saturation current of the junction at room temperature 300°K is 10^{-13} A, the emitter current is **[2005]**



(a) 30 mA (b) 39 mA
(c) 49 mA (d) 20 mA

Solution: (c)

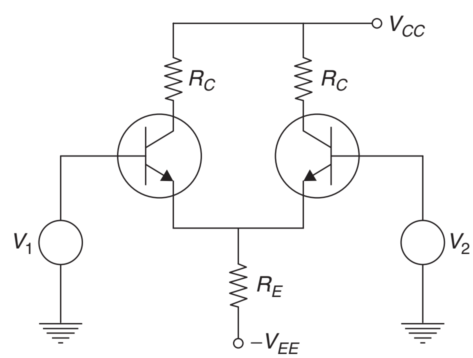


Since collector is connected to base, it is acting as forward bias diode

$$\therefore I_\varepsilon = I_s[e^{V_{BE}/\eta V_T} - 1]$$
$$= 10^{-13}[e^{0.7/1 \times 26 \text{ mv}} - 1]$$
$$= 49 \text{ mA}$$

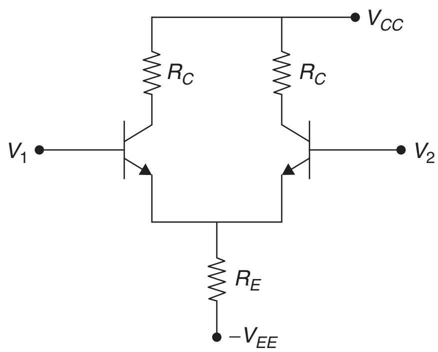
Hence, the correct option is (c).

24. In an ideal differential amplifier shown in the figure, a large value of (R_E) **[2005]**



(a) increases both the differential and common-mode gains.
(b) increases the common-mode gain only.
(c) decreases the differential-mode gain only.
(d) decreases the common-mode gain only.

Solution: (d)



Common mode gain is given by

$$A_{CM} = \frac{-2R_C}{R_\varepsilon}$$

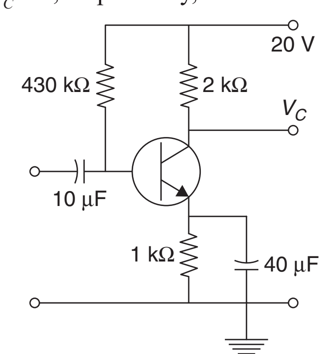
Differential gain is given by

$$Ad = \frac{1}{2}\,gmR_C$$

Large value of R_ε will decrease common mode gain and differential gain is not affected.

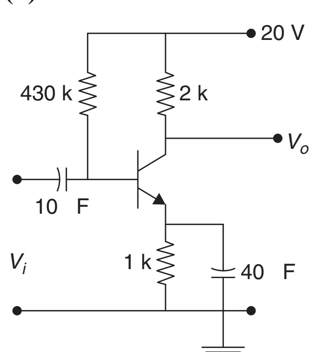
Hence, the correct option is (d).

25. The circuit using a BJT with $\beta = 50$ and $V_{BE} = 0.7$ V is shown in the figure. The base current I_B and collector voltage V_C are, respectively, **[2005]**



(a) 43 μA and 11.4 volts
(b) 40 μA and 16 volts
(c) 45 μA and 11 volts
(d) 50 μA and 10 volts

Solution: (b)



Emitter current is given by

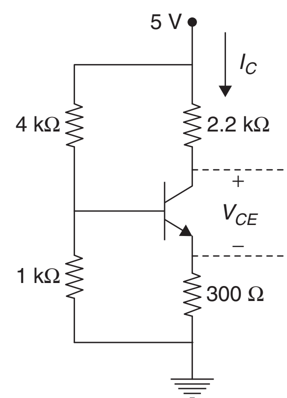
$I_\varepsilon = I_C + I_B$
$= (\beta + 1)\, I_B$
$z_0 = 430 \times I_B + 0.7 + 51 \times I_B$
$19.3 = 481\, I_B$
$I_B = 40.1\ \mu A$
$I_C = \beta I_B$
$= 50 \times 40.1\ \mu$A
$= 2$ mA

Collector voltage is given by

$V_C = V_{CC} - I_C R_C$
$= 20 - (2 \text{ mA} \times 2 \text{ k}\Omega)$
$= 16$ V

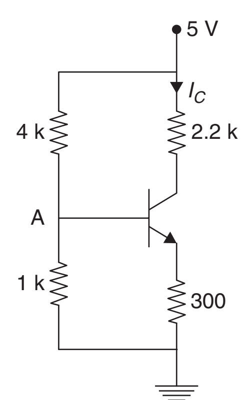
Hence, the correct option is (b).

26. Assuming that the β of the transistor is extremely large and $V_{BE} = 0.7$ V, I_C and V_{CE} in the circuit shown in the figure are **[2004]**



(a) $I_C = 1$ mA, $V_{CE} = 4.7$ V
(b) $I_C = 0.5$ mA, $V_{CE} = 3.75$ V
(c) $I_C = 1$ mA, $V_{CE} = 2.5$ V
(d) $I_C = 0.5$ mA, $V_{CE} = 3.$

Solution: (c)



Applying voltage divider at node A

$$V_A = \frac{5 \times 1}{5} = 1\text{ V}$$

$$I_{\varepsilon} = \frac{1-0.7}{300} = \frac{0.3}{300} = 1 \text{ mA}$$

$I_C = I_{\varepsilon}$ as β is large

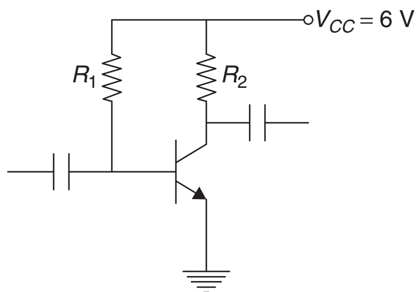
$\Rightarrow I_B = 0$

$\therefore V_{c\varepsilon} = 5 - 1(2.2 + 0.3)$

$= 2.5$ V

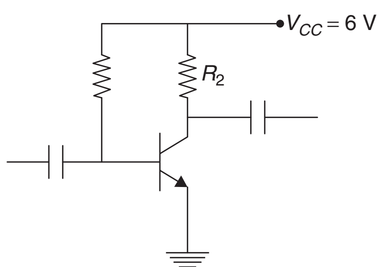
Hence, the correct option is (c).

27. In the amplifier circuit shown in the figure, the values of R_1 and R_2 are such that the transistor is operating at $V_{CE} = 3$V and $I_C = 1\ 5$ mA when its β is 150. For a transistor with β of 200, the operating point (V_{CE}, I_C) is **[2003]**



(a) (2 V, 2 mA)
(b) (3 V, 2 mA)
(c) (4 V, 2 mA)
(d) (4 V, 1 mA)

Solution: (a)



At the collector

$$R_z = \frac{V_{cc} - V_{c\varepsilon}}{I_c}$$

$$= \frac{6-3}{1.5\times10^{-3}}$$

$= 2$ kΩ

$$I_B = \frac{I_c}{\beta} = 0.01 \text{ mA}$$

$\beta = 150$

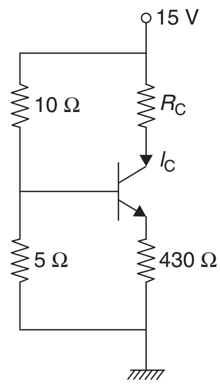
β changes to 200, $I_C = \beta \times I_B = 200 \times 0.01$ mA

$= 2$ mA

$V_{c\varepsilon} = 6 - 2_m \times 2$ kΩ $= 2$ V

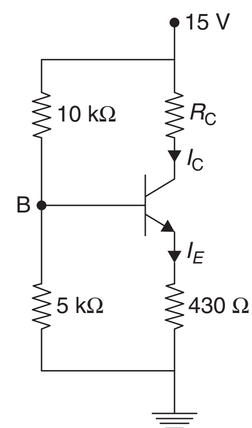
Hence, the correct option is (a).

28. In the circuit of the figure, assume that the transistor is in the active region. It has a large p and its base-emitter voltage is 0.7 V. The value of I_c is **[2000]**



(a) Indeterminate since R_c is not given
(b) 1 mA
(c) 5 mA
(d) 10 mA

Solution: (d)



Applying potential divider at B

$$V_B = \frac{15\times5}{5+10} = 5 \text{ V}$$

Since β is large, $I_B \simeq 0$,

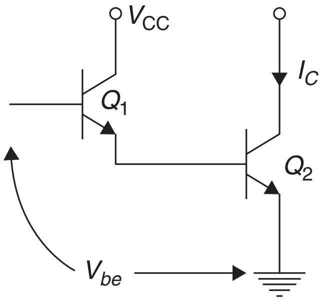
$V_{\varepsilon} = V_B - 0.7$

$= 5 - 0.7$

$= 4.3$ V

Hence, the correct option is (d).

29. A Darlington stage is shown in the figure. If the transconductance of Q_1 is g_{m1} and Q_2 is g_{m2}, then the overall transconductance given by g_m is given by **[1996]**



(a) g_{m1} (b) $0.5\, g_{m1}$
(c) g_{m2} (d) $0.5g_{m2}$

Solution: (d)

$$g_m = \frac{I_{e2}}{V_{be}}$$

Overall transconductance

$$g_m = \frac{I_{e2}}{2V_{be}}$$

$$= \frac{1}{2}\frac{I_{e2}}{V_{be}} = \frac{1}{2}\, g_{m2}$$

$$g_m = 0.5\, g_{m2}$$

Hence, the correct option is (d).

30. Match the following. **[1996]**

(a) Cascade amplifier
(b) Differential amplifier
(c) Darlington pair common-collector amplifier
(1) Does not provide current gain
(2) Is a wide band amplifier
(3) Has very low input impedance emitter amplifier and very high current gain
(4) Has very high input impedance and very high current gain
(5) Provides high common mode voltage rejection

Solution: (a–2, b–5, c–4)
Cascade amplifier–It provides a wide band amplifier.
Differential amplifier–Provides high common mode voltage rejection.
Darlington pair common-Collector amplifier–Has very high I/P impedance and very high current gain

31. Match the following: **[1994]**

List-I

A. The current gain of a BJT will be increased.
B. The current gain of a BJT will be reduced.
C. The break-down voltage of a BJT will be reduced.

List-II

1. The collector doping concentration is increased.
2. The base width is reduced.
3. The emitter doping concentration to base doping concentration ratio is reduced.
4. The base doping concentration is increased keeping the ratio of the emitter doping concentration to bade doping concentration constant.
5. The collector doping concentration is reduced.

Solution: (A–2, B–3, C–5)
As the base width of the *BJT* is reduced, then the recombination current (base current I_B) decreases as a result collector current (I_c) increases. So, the gain of the *BJT* increases.

$$\alpha = \frac{I_c}{I_\varepsilon}.$$

If the emitter doping concentration to base doping concentration ratio is reduced, then the emitter injection efficiency decreases, so the current gain (α) of *BJT* reduces.
If the collector doping concentration is increased then the breakdown (V_{BR}) of a *BJT* will be reduced.

32. α cut-off frequency of a bipolar junction transistor **[1993]**

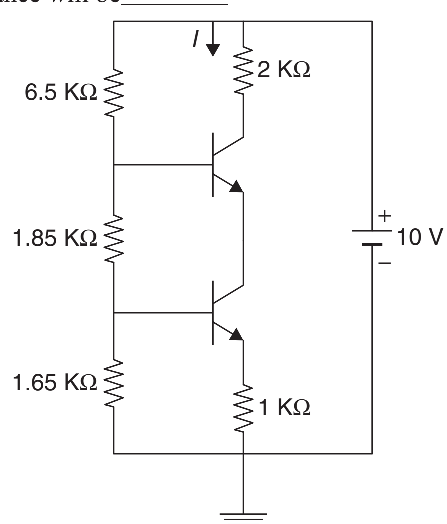
(a) increases with the increase in base width.
(b) increases with the increase in emitter width.
(c) increases with the increase in collector width.
(d) increases with decrease in the base width.

Solution: (d)
As base width decreases, recombination decreases. So collector current I_c increases

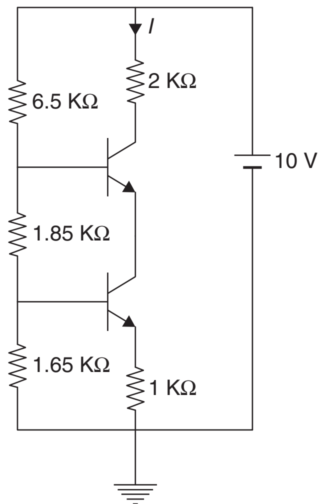
$$\alpha = \frac{I_c}{I_\varepsilon}.$$

So, α also increases.
Hence, the correct option is (d).

33. If the transistor in the figure has high value of β and V_{BE} of 0.65, the current I flowing through the 2 kilo ohms resistance will be________ **[1992]**



Solution: (1 mA)



Given that β is very large,

so $I_B = 0$

So, $V_{1.65} = \dfrac{10 \times 1.65}{1.65 + 1.85 + 6.5}$

= 1.65 V

Apply *KVL* at I/P mesh

$V_{1.65} = V_{be} + I_e R_e$

$= 0.6 + I_e \times 1\text{k}$

= 1.65

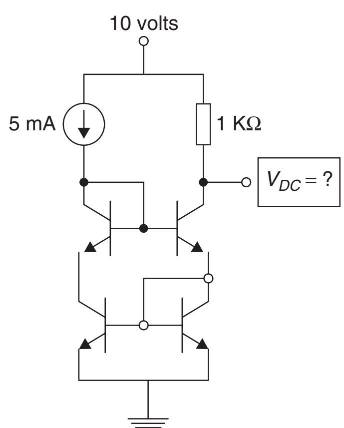
$I_e = 1$ mA

$\because \beta$ is very large

so, $I_c \simeq I_e$

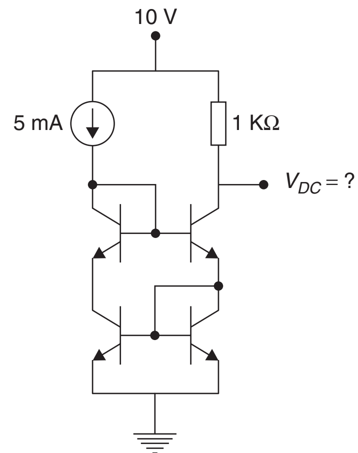
$I = I_c = 1$ mA

34. In the figure all transistors are identical and have a high value of beta. The voltage V_{DC} is equal to______. **[1991]**



Solution: (5 V)

Given that β is very large



So, $I_c = I_\varepsilon$

So, the current through 1 kΩ resistance

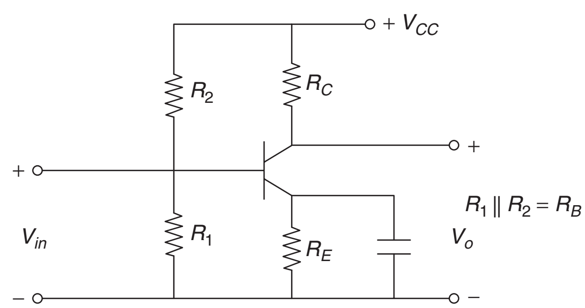
$I = 5$ mA

$V_{DC} = 10 - IR$

$= 10 - 5 \times 10^{-3} \times 1 \times 10^3$

$V_{DC} = 5$ V.

35. For good stabilized biasing of the transistor of the *CE* amplifier of figure, we should have **[1990]**



(a) $\dfrac{R_E}{R_B} << 1$

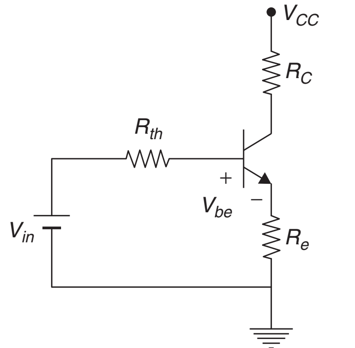
(b) $\dfrac{R_E}{R_B} >> 1$

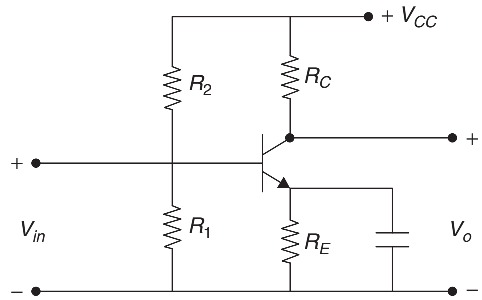
(c) $\dfrac{R_E}{R_B} << h_{FE}$

(d) $\dfrac{R_E}{R_B} >> h_{FE}$

Solution: (b)

$R_1 || R_2 = R_B$





Simplified self bias *circuit* using Thevenin's theorem. Thevenin open *circuit* voltage

$$V_{th} = \frac{V_{cc}R_1}{R_1 + R_2}$$

Thevenin interval resistance

$$R_{th} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = R_B$$

Applying *KVL* to I/P mesh

$V_{th} = I_B R + V_{be} + I_\varepsilon R_\varepsilon$

Put $I_\varepsilon = I_{BR} + V_{be} + I_\varepsilon R_\varepsilon$

$V_{th} = I_{BR} + V_B R_B + V_{be} + (I_B + I_C)\, R_\varepsilon$

Differentiate *wrt* I_c, keeping B and V_{be} constant

$$0 = (R_B + R_\varepsilon)\frac{\partial I_B}{\partial I_c} + R_\varepsilon + 0$$

$$\frac{\partial I_B}{\partial I_C} = \frac{-R_\varepsilon}{R_B + R_\varepsilon}$$

$$S = \frac{1+\beta}{1-\beta\dfrac{\partial I_b}{\partial I_c}}$$

$$S = \frac{1+\beta}{1+\beta\left(\dfrac{R_\varepsilon}{R_B + R_\varepsilon}\right)}$$

$$\beta >> 1$$

$$\beta\frac{R_\varepsilon}{R_B + R_\varepsilon} >> 1$$

$$S = \frac{\beta}{\beta\left(\dfrac{R_\varepsilon}{R_B + R_\varepsilon}\right)}$$

$$S = 1 + \frac{R_B}{R_\varepsilon}$$

For better stability, $S \cong 1$

So, $\dfrac{R_B}{R_\varepsilon} << 1$

$\dfrac{R_\varepsilon}{R_B} >> 1.$

Hence, the correct option is (b).

36. Which of the following statements are correct for basic transistor amplifier configurations? **[1990]**

(a) *CB* amplifier has low input impedance and low current gain.
(b) *CC* amplifier has low output impedance and a high current gain.
(c) *CE* amplifier has very poor voltage gain but very high input impedance.
(d) The current gain of *CB* amplifier is higher than the current gain of *CC* amplifiers.

Solution: (a, b)

In common base (*CB*) amplifiers I/P impedance (z_i) is low and current gain (α) is also low and current gain (α) is also low.

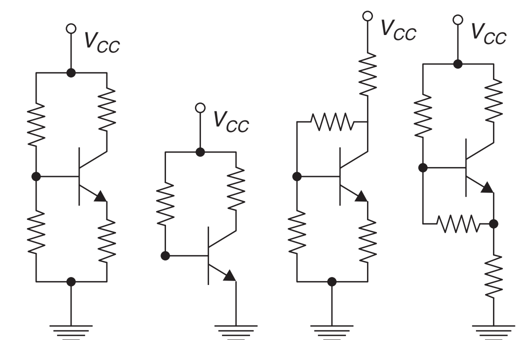
$$\alpha = \frac{I_c}{I_e}.$$

In common collector (*CC*) amplifiers O/P impedance (z_0) is low and current gain (γ) is high

$$\gamma = \frac{I_e}{I_b} = 1 + \beta$$

Hence, the correct options are (a, b).

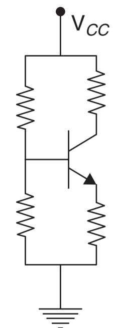
37. Of the four biasing circuits shown in figure, for a BJT, indicate the one which can have maximum bias stability **[1989]**



(a) (b) (c) (d)

Solution: (a)

In the *BIT* self-bias circuit or potential circuit provides the maximum bias stability.



38. The quiescent collector current I_C of a transistor is increased by changing resistances. As a result **[1988]**

(a) g_m will not be affected
(b) g_m will decrease
(c) g_m will increase
(d) g_m will increase or decrease depending upon bias stability.

Solution: (c)

$$g_m = \frac{I_c}{V_T}$$

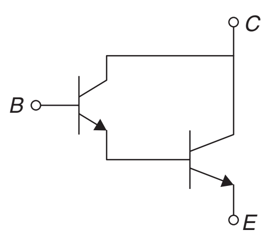
So, if $I_c \uparrow$ then $gm \uparrow$

$gm \propto I_c$

So, if the quiescent collector current I_c increases then transconductance *gm* also increases.

Hence, the correct option is (c).

39. Each transistor in the Darlington pair (see the figure below) has $h_{FE} = 100$. The overall h_{FE} of the composite transistor neglecting the leakage currents is **[1988]**



(a) 10,000 (b) 10,001
(c) 10,100 (d) 10,200

Solution: (c)

$$I_{\varepsilon 1} = (1+\beta)I_{B1}$$

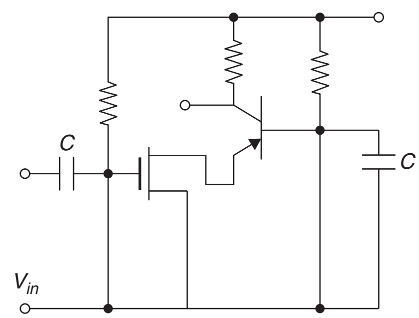
$$I_{\varepsilon 1} = (1+\beta)I_{B1}$$
$$= I_{B2}$$
$$I_{c2} = \beta I_{B2}$$
$$= \beta(1+\beta)I_{B1}$$

So, overall β of the composite transistor

$$\beta' = \frac{I_{C2}}{I_{B1}} = \beta(1+\beta)$$
$$= 100 \times (1+100) = 10{,}100$$

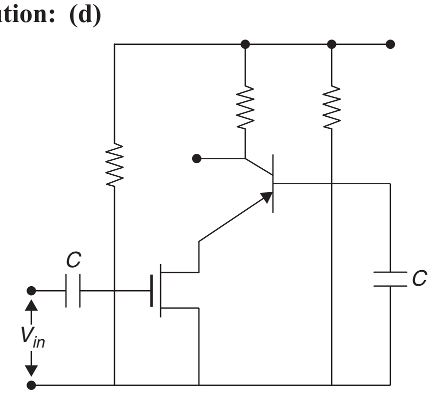
Hence, the correct option is (c).

40. The amplifier circuit shown below uses a composite transistor of a MOSFET and BIPOLAR in cascade. All capacitance are large. g_m of the MOSFET = 2 mA/V, and h_{fe} of the BIPOLAR = 99. The overall transconductance g_m of the composite transistor is **[1988]**



(a) 198 m A/V (b) 9.9 m A/V
(c) 4.95 m A/V (d) 1.98 m A/V

Solution: (d)



$$V_{in} = V_{gs}$$
$$gm' = \frac{1c}{V_{gs}}$$

$$I_c = \alpha I_\varepsilon = \frac{\beta}{1+\beta} I_\varepsilon$$

$$gm' = \left(\frac{\beta}{1+\beta}\right)\frac{I\varepsilon}{V_{gs}} = \frac{\beta I_\varepsilon}{(1+\beta)V_{gs}}$$

$$I_D = I_\varepsilon$$

$$gm' = \frac{\beta}{1+\beta} \times \frac{I_D}{V_{gs}} = \frac{\beta}{1+\beta}\,\text{gm}$$

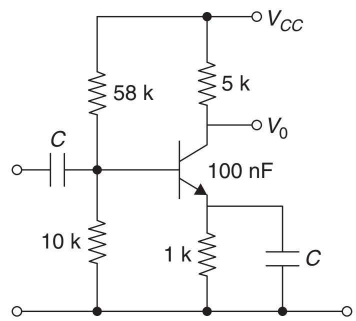
$$= \frac{99}{1+99} \times 2 \text{ mA/V}$$

$gm' = 1.98$ mA/V

Hence, the correct option is (d).

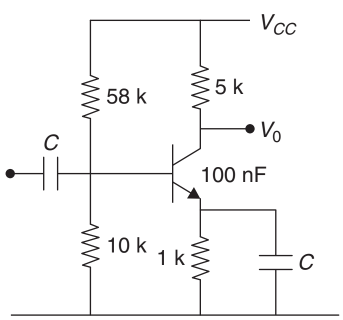
41. The transistor in the amplifier shown below has the following parameters:

$h_{fe} = 100$, $h_{ie} = 2$ kΩ, $h_{re} = 0$, $h_{oe} = 0.05$ mhos. C is very large. The output impedance is **[1988]**



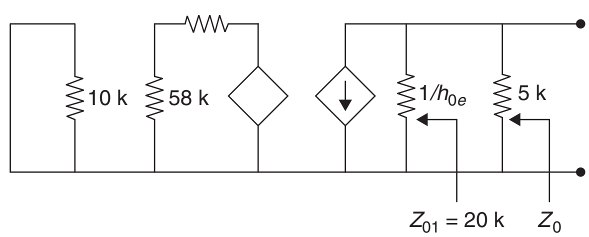
(a) 20 KΩ (b) 16 KΩ
(c) 5 KΩ (d) 4 KΩ

Solution: (d)



Output admittance,

$$y_0 = h_{0e} - \frac{h_{fe}h_{re}}{h_{ie} + R_s}$$



$$= 0.05 \times 10^{-3} - \frac{100 \times 0}{2 \times 10^3 + 10 \times 10^3}$$

$$y_0 = 0.05 \times 10^{-3}$$

$$z_{01} = \frac{1}{y_0} = \frac{1}{0.05 \times 10^{-3}} = 20 \text{ k}'$$

Output impedance,

$z_0 = z_{01} || 5\text{k} = 20\text{k}||5\text{k} = 4$ kΩ

Hence, the correct option is (d).

42. The configuration of cascade amplifier is **[1987]**

(a) $CE–CE$ (b) $CE–CB$
(c) $CC–CB$ (d) $CC–CC$

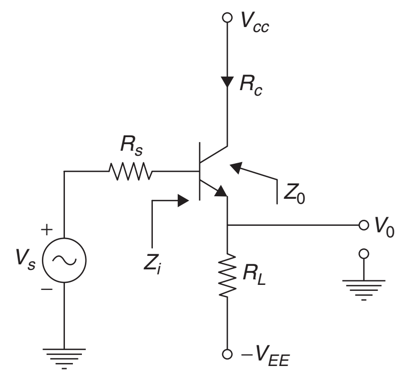
Solution: (b)

Cascade amplifier is the common emitter followed by common base configuration.

Hence, the correct option is (b).

Five-marks Questions

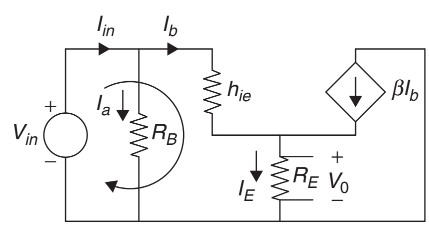
1. An emitter-follower amplifier is shown in the figure, Z_i is the impedance looking into the base of the transistor and Z_0 is the impedance looking into the emitter of the transistor **[2001]**



(A) Draw the small signal equivalent circuit of the amplifier.
(B) Obtain an expression for Z_i.
(C) Obtain an expression for Z_0.

Solution:

(A) Replace all the sources with internal impedance and short circuit all cap then draw small signal h model.



$I_E = I_b + \beta I_b = (1 + \beta)I_b$

$V_{in} = I_b h_{ie} + (1 + \beta)\, I_b R_E$ [KVL in Loop 1]

$V_0 = (1 + \beta)\, I_0 I_E$

$$A_V = \frac{V_0}{V_{in}} = \frac{(1+\beta)I_b R_E}{I_b[h_{ie} + (1+\beta)R_E]}$$

$$= \frac{(1+\beta)R_E}{h_{ie} + (1+\beta)R_E}$$

$\because\ (1+\beta)R_E < h_{ie}$

$A_V = 1$

(B) Input Impedance Z_{in}

$$I_{in} + I_a + I_b$$

$$Z_{in} = \frac{V_{in}}{I_{in}}$$

$$I_{in} = \frac{V_{in}}{R_B} + \frac{V_{in}}{h_{ie} + (1+\beta)R_E}$$

$$Z_{in} = \frac{V_{in}}{Z_{in}} = \frac{1}{\frac{1}{R_B} + \frac{1}{h_{ie} + (1+\beta)R_E}}$$

(C) Output impedance Z_0

Short circuit the input source and place 1 V source at V_0 and find I_0.

Apply KCL

$I_b + \beta I_b + I_0 = I_E$

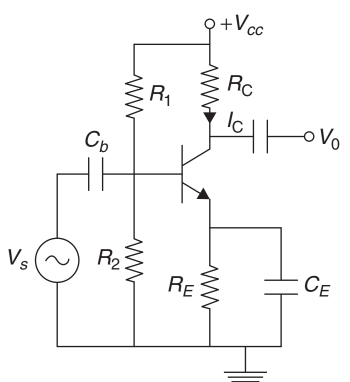
$$(1+\beta)\left(-\frac{1}{h_{ie}}\right) + I_0 = \frac{1}{R_E}$$

$$I_0 = \frac{1}{R_E} + \frac{(1+\beta)}{h_{ie}}$$

$$Z_0 = \frac{1}{I_0}$$

$$Z_0 = \frac{1}{\frac{1}{R_E} + \frac{1+\beta}{h_{ie}}}$$

2. For the amplifier of given figure, $l_c = 1.3$ mA, $R_c = 2$ kΩ, $R_E = 500$ Ω, $V_T = 26$ mV, $\beta = 100$, $V_{CC} = 15$ V, $V_s = 0.01 \sin(\omega t)$ and $C_b = C_e = 10$ μF.



(A) What is the small-signal voltage gain, $A_v = V_o/V_s$?

(B) What is the approximate A_v if C_e is removed?

(C) What will V_o be if C_b is short circuited?

[2000]

Solution:

(A) Voltage gain $= A_V = \frac{V_0}{V_i} = \frac{-R_C}{R_e}$

$$r_e = \frac{g_m}{I_e}$$

$$I_e = \left(\frac{1+\beta}{p}\right) I_C = \frac{1+100}{100} \times 1.3 \text{ mA}$$

$I_e = 1.313$ mA

$$r_e = \frac{26\times10^{-13}}{1.313\times10^{-13}} = 19.8\,\Omega \qquad \left[r_e = \frac{V_T}{I_e}\right]$$

$$A_V = \frac{-R_C}{r_c} = -101$$

(B) Voltage gain A_V when C_e is removed

$$A_V = \frac{-R_e}{R_e}$$

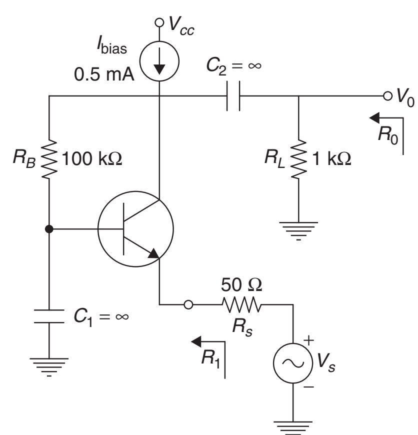
$A_V = -4$

(C) Output voltage, V_0

$V_0 = A_0 \cdot V_S$

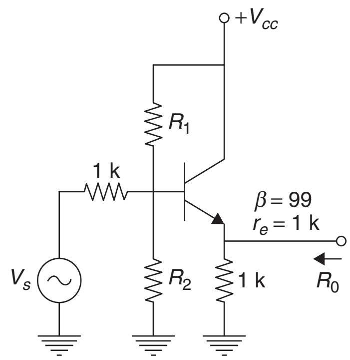
$V_0 = -1.01 \sin \omega t$

3. A bipolar junction transistor amplifier circuit is shown in the figure is. Assume that the current source I_{bias} is ideal, and the transistor has very large β, $r_b = 0$, and $r_0 \to \infty$. Determine the ac small-signal midband voltage gain (V_o/V_s). input resistance (R_1), and output resistance (R_0) of the circuit. Assume $V_T = 26$ mV.



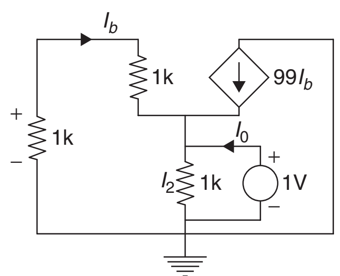
[1999]

4. In the circuit of figure. Determine the resistance R_0 seen by the output terminals ignore the effects of R_1 and R_2.



[1998]

Solution: Replace transistor with h – parameter model.



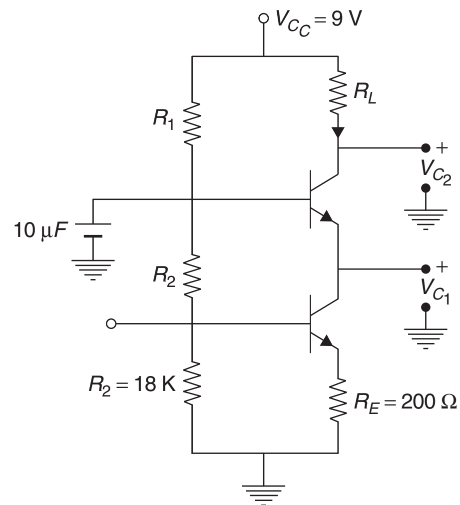
$$R_0 = \frac{1\text{V}}{I_0}$$

$$I_0 = \frac{0-1}{2k} = -0.5 \text{ mA}$$

$I_b + 99\,I_b + I_0 = I_L = 100\,I_b + I_0 = 1$ mA
$I_0 = 51$ mA

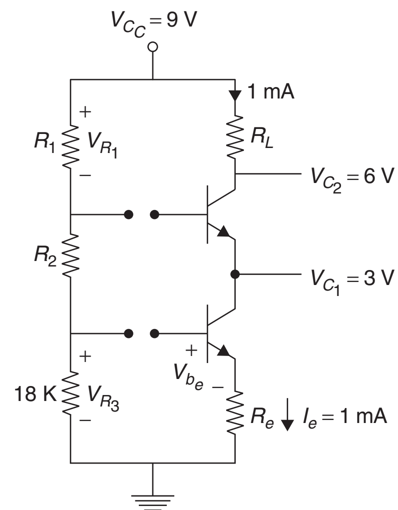
$$R_0 = \frac{1}{51 \text{ mA}} = 20\,\Omega$$

5. In the cascade amplifier circuit shown below, determine the values of R_1 R_2 and R_L. Such that the quiescent current through the transistors is 1 mA and the collector voltage V_{c1} = 3 V, and V_{c2} = 6 V. Take V_{BE} = 0.7 V. Transistor β to be high and base currents to be negligible



[1997]

Solution: Q current = I_C = 1 mA
β is very large $\therefore$ $I_b = 0$



$$R_L = \frac{V_{C_C} - V_{C_2}}{1\text{mA}} = \frac{9-6}{1 \text{ mA}} = 3\text{ k}\Omega$$

Apply KVL

$$V_{C_C} = V_{R_1} + V_{b_e} + V_{C_1} = V_{R_1} + 0.7 + 3$$

$$V_{R_1} = 5.3\text{V}$$

Apply KVL

$$V_{R_S} = V_{b_e} + I_e R_e = 0.9\text{V}$$

$$I_{R_3} = \frac{V_{R_3}}{R_3} = 50\ \mu\ \text{Amp}$$

$$\therefore\ I_b = 0$$

$$I_{R_1} = I_{R_2} = I_{R_3} = 50\ \mu\ \text{Amp}$$

$$V_{R_1} = I_{R_1} \times R_1$$

$$R_1 = \frac{V_{R_1}}{I_{R_1}} = 106\ \text{k}\Omega$$

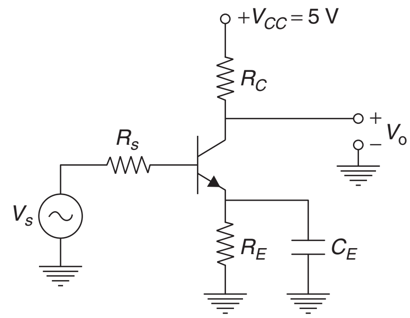
$$V_{C_C} = V_{R_1} + V_{R_2} + V_{R_3}$$

$$V_{R_2} = V_{C_C} - V_{R_1} - V_{R_3} = 2.8$$

$$V_{R_3} = R_2 I_{R_2}$$

$$\therefore\ R_2 = \frac{2.8}{50 \times 10^{-6}} = 56\ \text{k}\Omega$$

6. The transistor in the circuit shown in the figure is so biased (dc biasing N/W is not shown) that the dc collector current $I_c = 1$ mA. Supply is $V_{cc} = 5\text{V}$



The N/W components have following values, $R_c = 2\ \text{k}\Omega$, $R_s = 1.4\ \text{k}\ \Omega$, $R_E = 100\ \Omega$. The transistor has specifications, $\beta = 100$ and base spreading resistance $r'_{bb} = 100\Omega$.

Assume $\dfrac{KT}{q} = 25$ mV.

Evaluate input resistance R_1, for two cases. At a frequency of 10 kHz.

(A) C_E, the bypass capacitor across R_E is 25 μF.

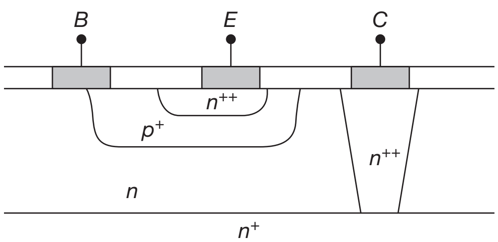
(B) The bypass capacitor C_E is removed leaving R_E unbypassed. **[1997]**

Chapter 4

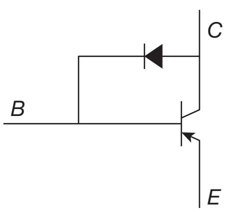
FET and MOSFET Analysis

One-mark Questions

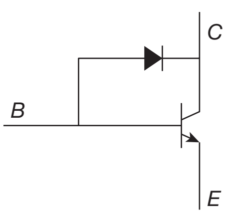
1. The correct circuit representation of the structure shown in the figure is **[2019]**



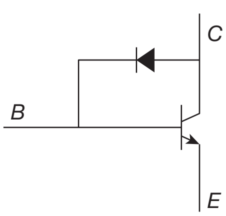
(A)



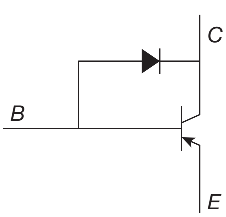
(B)



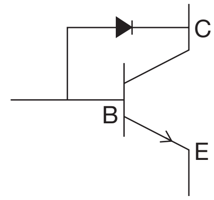
(C)



(D)

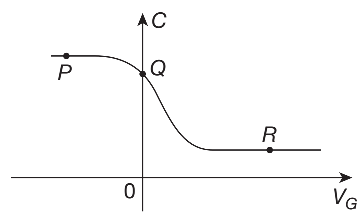


Solution:



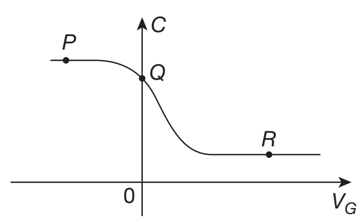
Hence the correct option is (B)

2. The figure shows the high-frequency C-V curve of a MOS capacitor (at $T = 300K$) with $\phi_{ms} = 0$ V and no oxide charges. The flat-band, inversion, and accumulation conditions are represented, respectively, by the points **[2019]**



(A) Q, R, P (B) P, Q, R
(C) Q, P, R (D) R, P, Q

Solution:



V_{us} –ve ⇒ accumulation ∴ P is accumulation

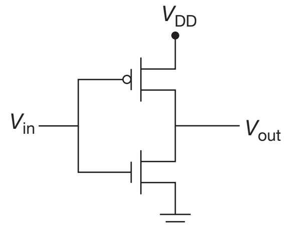
V_{us} inverses ⇒ inversion layer ∴ R is inversion

Hence, the correct option is (A)

3. A standard CMOS inverter is designed with equal rise and fall times ($\beta_n = \beta_p$). If the width of the pMOS transistor in the inverter is increased, what would be the effect on the LOW noise margin (NM_L) and the HIGH noise margin NM_H? **[2019]**

(A) NM_L decreases and NM_H increases.
(B) No change in the noise margins.
(C) Both NM_L and NM_H increase.
(D) NM_L increases and NM_H decreases.

Solution:



Given, rise time = full time

$$B_n = B_P$$

$$NK_L = V_{IH} - V_{OL}$$

$$NM_H = V_{OH} - V_{IH}$$

$$V_{OH} = V_{DD}$$

$$V_{OL} = O$$

$$V_{IH} = \frac{V_{DD} + V_{top} + K_r\left(2V_O + V_{TOP}\right)}{1 + K_r}$$

$$K_r = 1 + \frac{K_P}{K_n}$$

$$V_{IL} = \frac{2V_O - |V_{top}| - V_{DD} + K_r V_{TON}}{1 + K_r}$$

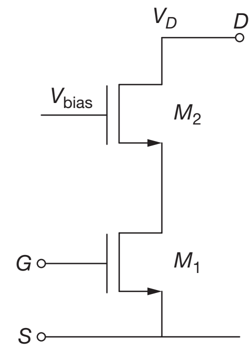
as width increases K_P increases & V_{IL} increases, $V_{IH} \uparrow$.

Thus increasing N_{ML} & decreasing N_{MH}.

Hence, the correct option is (D).

4. Two identical nMOS transistors M_1 and M_2 are connected as shown below. The circuit is used as an amplifier with the input connected between G and S terminals and the output taken between D and S terminals. V_{bias} and V_D and so adjusted that both transistors are in saturation.

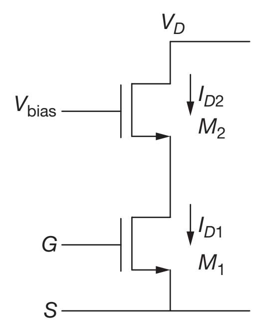
The transconductance of this combination is defined as $g_m = \dfrac{\partial i_D}{\partial v_{GS}}$ while the output resistance is $r_0 = \dfrac{\partial v_{DS}}{\partial i_D}$, where i_D is the current flowing into the drain of M_2. Let g_{m1}, g_{m2} be the transconductances and r_{01}, r_{02} be the output resistances of transistors M_2 and M_2, respectively.



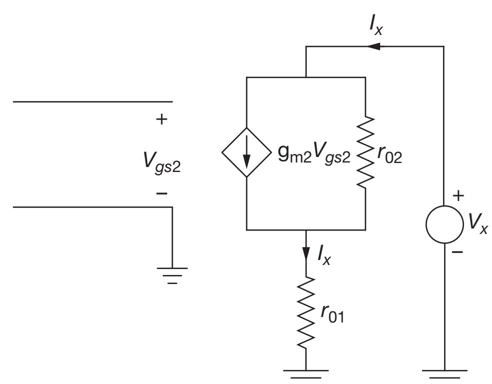
Which of the following statements about estimates for g_m and r_0 is correct? **[2018]**

(A) $g_m \approx g_{m1} \cdot g_{m2} \cdot r_{02}$ and $r_0 \approx r_{01} + r_{02}$
(B) $g_m \approx g_{m1} + g_{m2}$ and $r_0 \approx r_{01} + r_{02}$
(C) $g_m \approx g_{m1}$ and $r_0 \approx r_{01} \cdot g_{m2} \cdot T_{02}$
(D) $g_m \approx g_{m1}$ and $r_0 \approx r_{02}$

Solution: Consider the figure given below



$$g_m = \frac{\partial i_D}{\partial v_{GS}} = \frac{\partial i_{D1}}{\partial v_{GS}} \; ; g_{m1}$$



By Applying kirchoffs Voltage law in given circuit we get

$$V_x = (I_x - g_{m2}Vg_{s2})\, r_{02} + I_x\, r_{01}$$
$$V_x = I_x r_{02} - g_{m2}Vg_{s2} r_{02} + I_x\, r_{01}$$
$$V_x = I_x r_{02} - g_{m2} I_x r_{01} r_{02} + I_x\, r_{01}$$
$$\therefore \quad r_0 = V_x/I_x = r_{01} + r_{02} + r_{01} r_{02} g_m$$
$$r_0 \approx r_{01}\, r_{02}\, g_m$$

Hence, the correct option is (C)

5. Consider an n-channel MOSFET having width W, length L, electron mobility in the channel μ_n and oxide capacitance per unit area C_{ox}. If gate-to-source voltage $V_{GS} = 0.7$ V, drain-to-source voltage $V_{DS} = 0.1$ V, $(\mu_n C_{ox})$

= 100 μA/V^2, threshold voltage $V_{TH} = 0.3$ V and (W/L) = 50, then the trans conductance g_m (in mA/V) is ______________.

[2017]

6. An n-channel enhancement mode MOSFET is biased at $V_{GS} > V_{TH}$ and $V_{DS} > (V_{GS} - V_{TH})$, where V_{GS} is the gate-to-source voltage, V_{DS} is the drain-to-source voltage and V_{TH} is the threshold voltage. Considering channel length modulation effect to be significant, the MOSFET behaves as a **[2017]**

(A) Voltage source with zero output impedance
(B) Voltage source with non-zero output impedance
(C) Current source with finite output impedance
(D) Current source with infinite output impedance

7. Consider the following statements for a Metal Oxide Semiconductor Field Effect Transistor (MOSFET):

P: As channel length reduces, OFF state current increases.

Q: As channel length reduces, output resistance increases.

R: As channel length reduces, threshold voltage remains constant.

S: As channel length reduces, ON current increases.

Which of the above statements are incorrect?

[2016]

(A) P and Q (B) P and S
(C) Q and R (D) R and S

Solution: As we know that the drain current of MOSFET is $I_d = \mu_n C_{ox}\left(\frac{W}{L}\right)\left[(V_{GS} - V_t)^2 V_{DS} - \frac{V_{DS}{}^2}{2}\right]$

But $r \propto I_d\, si$

Hence, the correct option is (C).

8. A long channel NMOS transistor is biased in the linear region with V_{DS} = 50 mV and is used as a resistance. Which one of the following statements is NOT correct? **[2016]**

(A) If the device width W is increased, the resistance decreases.
(B) If the threshold voltage is reduced, the resistance decreases.
(C) If the device length L is increased, the resistance increases.
(D) If V_{GS} is increased, the resistance increases.

Solution: Drain to source resistance of NMOS transistor iV_{DS} = 50 mV

$$r_{ds} = \frac{1}{\mu_n \cdot c_{ox} \cdot \frac{w}{L}(V_{Gs} - V_T)})$$

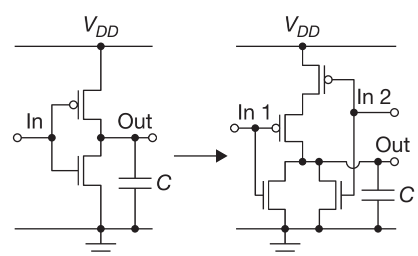
$$rds = vds/ids$$

$$r_{ds} \propto \frac{1}{V_{Gs}}$$

Therefore, as V_{GS} increases, r_{ds} decreases,

Hence, the correct option is (D).

9. Transistor geometries in a CMOS inverter have been adjusted to meet the requirement for worst case charge and discharge times for driving a load capacitor C. This design is to be converted to that of a NOR circuit in the same technology, so that its worst case charge and discharge times while driving the same capacitor are similar. The channel lengths of all transistors are to be kept unchanged. Which one of the following statements is correct? **[2016]**



(A) Widths of PMOS transistors should be doubled, while widths of NMOS transistors should be halved.
(B) Widths of PMOS transistors should be doubled, while widths of NMOS transistors should not be changed.
(C) Widths of PMOS transistors should be halved, while widths of NMOS transistors should not be changed.
(D) Widths of PMOS transistors should be unchanged, while widths of NMOS transistors should be halved.

Solution: From the given data

$$I_{D2} = 2I_{D1}$$

$$I_d = \mu_n C_{ox}\left(\frac{W}{L}\right)\left[(V_{GS} - V_t)^2 V_{DS} - \frac{V_{DS}{}^2}{2}\right]$$

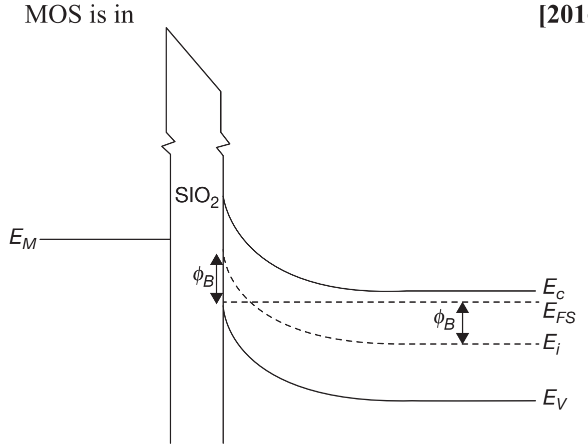
$$I_D \propto W$$

i.e., the width will increase with increase in drain current.

So width of PMOS transistors should be doubled, and widths of NMOS transistors should not be changed.

Hence, the correct option is (B).

10. The figure shows the band diagram of a Metal Oxide Semiconductor (MOS). The surface region of this MOS is in **[2016]**



(A) inversion (B) accumulation
(C) depletion (D) flat band

Solution: The given figure shows the band diagram of a MOS where the Fermi level is lying just below the conduction band with Fermi voltage of $\phi_{B.}$ this ϕ_B decides that surface region of this MOS is in inversion.

Hence, the correct option is (A).

11. The current in an enhancement mode NMOS transistor biased in saturation mode was measured to be 1 mA at a drain-source of 5 V. when the drain-source voltage was increased to 6 V while keeping gate-source voltage same, the drain current is increased to 1.02 mA. Assume that drain to source saturation voltage is much smaller than the applied drain-source voltage. The channel length modulation parameter λ (in V^{-1}) is ________. **[2015]**

Solution: From the given data

NMOS transistor is in saturation mode

So $\quad I_D = K\,[V_{GS} - V_T]^2\,(1 + \lambda\,.V_{DS})$

$I_D = 1$ mA at $V_{DS} = 5$ V

$I_D = 1.02$ mA at $V_{DS} = 6$ V

From the given data

$$V_{GS1} - V_T = V_{GS2} - V_T$$

$$\therefore \quad I_D = K^1(1 + \lambda.V_{DS})$$

$$\frac{I_{D1}}{I_{D2}} = \frac{1+\lambda V_{DS_1}}{1+\lambda V_{DS_2}}$$

$$\frac{1}{1.02} = \frac{1+\lambda(5)}{1+\lambda(6)}$$

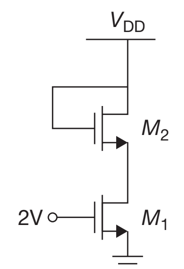
$$\Rightarrow \quad 1 + 6\,\lambda = 1.02 + 5.1\,\lambda$$

$$0.9\,\lambda = 0.2$$

$$\lambda = 0.022\ V^{-1}$$

Hence, the correct Answer is (0.018 to 0.026).

12. In the circuit shown, both the enhancement mode NMOS transistors have the following characteristics: $k_n = \mu_n C_{ox}(W/L) = 1\ \text{mA/V}^2$; $V_{TN} = 1$ V. Assume that the channel length modulation parameter λ is zero and body is shorted to source. The minimum supply voltage V_{DD} (in volts) needed to ensure that transistor M_1 operates in saturation mode of operation is ________. **[2015]**



Solution: From given circuit

$$V_{DD} = V_{D_2} = V_{G_2}$$

The transistor M_1 to be operated in saturation region

So $\quad V_{DS} = V_{GS} - V_{th}$

For M_1 transistor $V_S = 0$

$$V_S - V_D = V_S - V_G - V_{th}$$

$$\Rightarrow \quad V_D = V_G + V_{th}$$

$$= 2 + 1$$

$$= 3\text{ V}$$

Hence, the correct Answer is (2.9 to 3.1).

13. A MOSFET in saturation has a drain current of 1 mA for $V_{DS} = 0.5$ V. If the channel length modulation coefficient is 0.05 V^{-1}, the output resistance (in k Ω) of the MOSFET is _____ **[2015]**

Solution: Under channel modulation

$$I_D = I_{Dsat}\,(1 + \lambda V_{DS})$$

Output resistance

$$r_o = \frac{\partial V_{DS}}{\partial I_D}$$

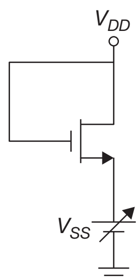
$$\frac{\partial I_D}{\partial V_{DS}} = I_{Dsat}\,\lambda$$

$$\Rightarrow \qquad r_o = \frac{1}{\lambda I_{Dsat}} = \frac{1}{0.05\times1\times10^{-3}}$$

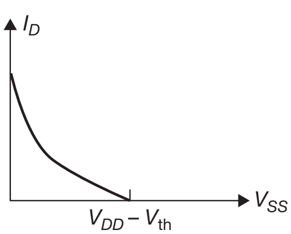
$$= \frac{10^5}{5} = 20\,k\Omega$$

Hence, the correct Answer is (19 to 21).

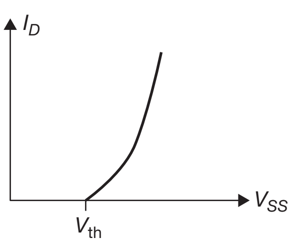
14. For the MOSFET in the circuit shown, the threshold voltage is V_{th}, where $V_{th} > 0$. The source V_{SS} is varied from 0 to V_{DD}. Neglecting the channel length modulation, the drain current I_D as a function of V_{SS} is represented by **[2015]**



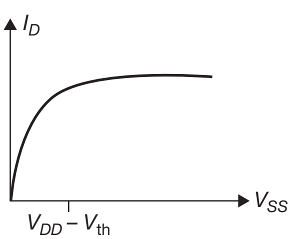
(A)



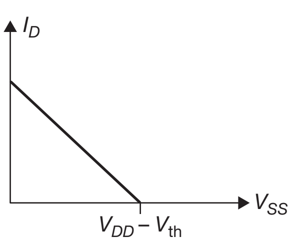
(B)



(C)



(D)



Solution: $V_D = V_G$ so FET is operating in saturation

So $\qquad V_{GS} = V_{SS} - V_{DD}$

$$\Rightarrow \qquad I_D = \frac{k_n'}{2}(V_{GS} - V_{th})^2$$

$$I_D = \frac{k_n'}{2}(V_{SS} - V_{DD} - V_{th})^2$$

If $V_{SS} = 0$ then I_D is max $I_D = \frac{k_n'}{2}(V_{DD} + V_{th})^2$ and $I_D \propto (V_{SS} - V_{DD} - V_{th})^2$

So, it varies non-linearly.

Hence, the correct option is (A).

15. In MOS capacitor with an oxide layer thickness of 10 nm, the maximum depletion layer thickness is 100 nm. The permittivities of the semi conductor and the oxide layer are ε_s and ε_{ox} respectively. Assuming $\varepsilon_s/\varepsilon_{ox} = 3$, the ratio of the maximum capacitance to the minimum capacitance of this MOS capacitor is ______. **[2015]**

Solution: From the given data

$$t_{ox} = 10\text{nm}$$

The maximum depletion thickness = 100 nm

$$\frac{\varepsilon_s}{\varepsilon_{ox}} = 3$$

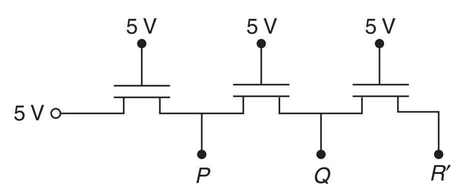
$$\therefore \quad \frac{C_{\max}}{C_{\min}} = \frac{(\varepsilon_{ox}/t_{ox})}{\dfrac{\dfrac{\varepsilon_{ox}}{t_{ox}} \times \dfrac{\varepsilon_s}{D_{\max}}}{\dfrac{\varepsilon_{ox}}{t_{ox}} + \dfrac{\varepsilon_s}{D_{\max}}}}$$

$$\therefore \quad \frac{C_{\max}}{C_{\min}} \approx 1 + \frac{\varepsilon_{ox}}{t_{ox}} \frac{D_{\max}}{\varepsilon_s}$$

$$= 1 + \frac{1}{3} \times \frac{100\times10^{-9}}{10\times10^{-9}} = 4.33$$

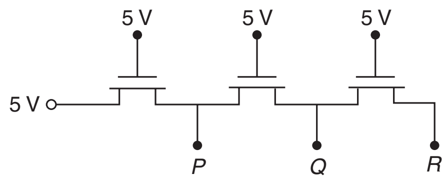
Hence, the correct Answer is (4.3 to 4.4).

16. In the following circuit employing pass transistor logic, all NMOS transistors are identical with a threshold voltage of 1 V. Ignoring the body-effect, the output voltages at P, Q and R are, **[2014]**



(a) 4 V, 3 V, 2 V
(b) 5 V, 5 V, 5 V
(c) 4 V, 4 V, 4 V
(d) 5 V, 4 V, 3 V

Solution: (c)



$V_T = IV$

For proper operation

$$V_{DS} = V_{GS} - V_T$$
$$V_D - V_S = V_G - V_S - V_T$$
$$\Rightarrow V_D = V_G - V_T$$

At p

$$\Rightarrow V_D = V_G - V_T$$
$$= 5 - 1$$
$$V_D = 4_V \quad \text{(at P)}$$

$$V_D \text{ at } Q = V_G - V_T$$
$$= 5 - 1 = 4 \text{ V}$$
$$V_D \text{ at } R = V_G - V_T$$
$$= 5 - 1 = 4 \text{ V}$$
$$\therefore\ V_{D(P)} = 4 \text{ V}$$
$$V_{D(S)} = 4 \text{ V}$$
$$V_{D(R)} = 4 \text{ V}$$

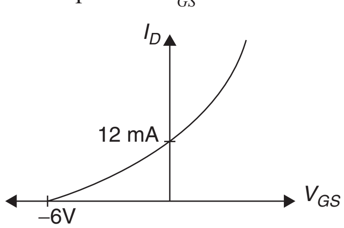
Hence, the correct option is (c).

17. An n channel depletion MOSFET has following two points on its $I_D - V_{GS}$ curve:
(i) $V_{GS} = 0$ at $I_D = 12$ mA and
(ii) $V_{GS} = -6$ volts at $Z_0 = \infty$
Which of the following Q-points will give the highest trans-conductance gain for small signals? **[2006]**

(a) $V_{GS} = -6$ V
(b) $V_{GS} = -3$ V
(c) $V_{GS} = 0$ V
(d) $V_{GS} = 3$ V

Solution: (d)

Since given device is depletion MOSFET n-channel the current will flow even for positive values of V_{GS}. We can see that for positive value of V_{GS} slope will be more. Hence current option is $V_{GS} = 3$ V.



Hence, the correct option is (d).

18. Two identical FETs, each characterized by the parameters g_m and r_d, are connected in parallel. The composite FET is then characterized by the parameters. **[1998]**

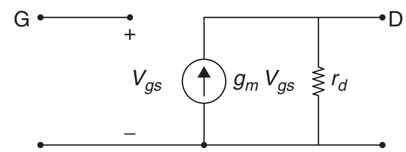
(a) $\frac{g_m}{2}$ and $2r_d$

(b) $\frac{g_m}{2}$ and $\frac{r_d}{2}$

(c) $2g_m$ and $\frac{r_d}{2}$

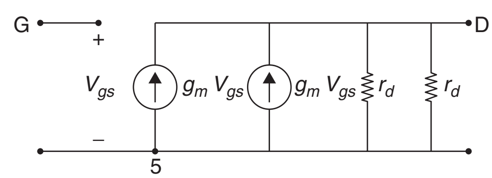
(d) $2g_m$ and $2r_d$

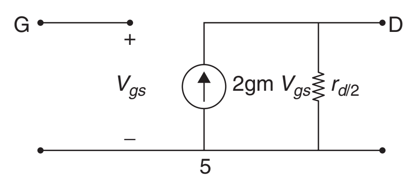
Solution: (c)



If two FETs are connected in parallel,

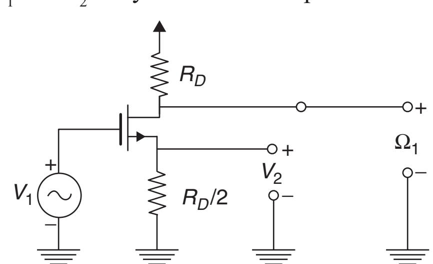
$$r_d' = \frac{rd}{2}$$
$$gm' = 2 \text{ gm}$$





Hence, the correct option is (c).

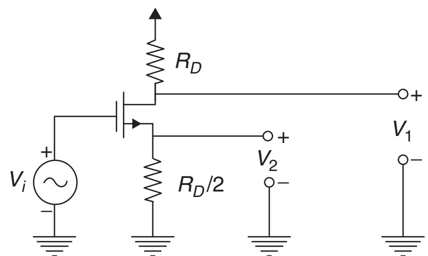
19. In the MOSFET amplifier of the figure, the signal output V_1 and V_2 obey the relationship **[1998]**



(a) $V_1 = \frac{V_2}{2}$ (b) $V_1 = -\frac{V_2}{2}$

(c) $V_1 = 2V_2$ (d) $V_1 = -2V_2$

Solution: (c)



$$V_1 = 2V_2$$
$$V_1 = I_D R_D$$
$$I_S = I_D$$
$$V_2 = I_S \frac{R_D}{2}$$
$$= \frac{I_D R_D}{2} = \frac{V_1}{2}$$

So, $V_1 = 2V_2$
Hence, the correct option is (c).

20. An n-channel JFET has $I_{DSS} = 1$ mA and $V_P = -5$ V. Its maximum transconductance is_______ **[1995]**

Solution: (0.4 ms)

$$g_{m,\max} = \left|\frac{2I_{DSS}}{V_P}\right|$$

$$= \frac{2\times1\times10^{-3}}{|-5|} = 0.4 \text{ ms}$$

21. The transit time of current carriers through the channel of an FET decides its ______ characteristics. **[1994]**

Solution: (switching)
The transit time of current carriers through the channel of an FET decides its switching characteristics.

Two-marks Questions

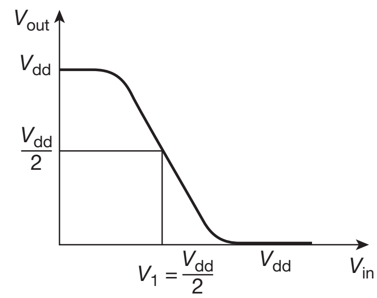
1. A CMOS inverter, designed to have a mid-point voltage V_1 equal to half of V_{dd}. As shown in the figure, has the following parameters:

$V_{dd} = 3$ V

$\mu_n C_{ox} = 100\ \mu\text{A/V}^2$; $V_{tn} = 0.7$ V for nmos

$\mu_p C_{ox} = 40\ \mu\text{A/V}^2$; $|V_{tp}| = 0.9$ V for pMOS

The ratio of $\left(\frac{W}{L}\right)_n$ to $\left(\frac{W}{L}\right)_P$ is equal to _____ (round off to 3 decimal places). **[2019]**



Solution: $Vdd = 3$ V

$\mu n C_{ox} = 100 \mu A / V^2$

$V_{tn} = 0.7$ V

$\mu_p C_{ox} = 40 \mu A / V^2$

$|V_{tp}| = 0.9$ V

using the current equation we get

$$I_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{\mu S} - V_t\right)^2$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n\left(V_{\mu s} - V_{tn}\right)^2 = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p\left(V_{\mu S} - V_{tp}\right)^2$$

$$\frac{\left(\frac{W}{L}\right)_n}{\left(\frac{W}{L}\right)_p} = \frac{40\times(1.5-0.9)^2}{100\times(1.5-0.7)^2}$$

$$\frac{\left(\frac{W}{L}\right)_n}{\left(\frac{W}{L}\right)_p} = \frac{4\times9}{16\times10} = 0.225$$

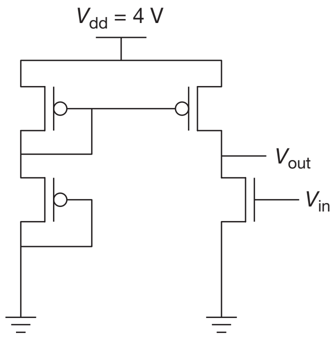
Hence, the correct answer is (0.225).

2. In the circuit shown, the threshold voltages of the pMOs $\left(|V_{tp}|\right)$ and nMOS (V_{tn}) transistors are both equal to 1 V. All the transistor have the same output resistance r_{ds} of 6 MΩ. The other parameter are listed below.

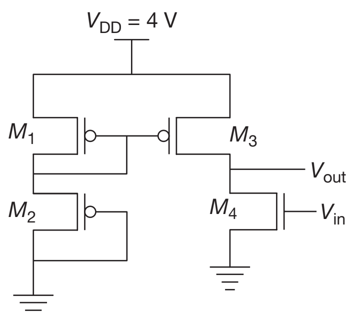
$$\mu_n C_{ox} = 60\ \mu\text{A/v}^2\ ; \frac{W}{L}\ \text{nMOS} = 5$$

$$\mu_n C_{ox} = 30\ \mu\text{A/v}^2\ ; \frac{W}{L}\ \text{nMOS} = 10$$

μ_n and μ_p are the carrier mobilities, and C_{ox} is the oxide capacitance per unit area. Ignoring the effect of channel length modulation and body bias, the gain of the circuit is _____ (rounded off to 1 decimal place) **[2019]**



Solution:



$$\mu_n C_{ox} = 60\mu\text{A/V}^2; \left(\frac{W}{L}\right)_n = 5$$

$$\mu_p C_{ox} = 30\mu\text{A/V}^2; \left(\frac{W}{L}\right)_p = 10$$

$$|V_{tp}| = V_{tn} = 1\text{ V}$$

$$r_{ds} = 6\,\text{M}\Omega$$

$$I_{DC} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p \left[V_{SG} - |V_{tp}|^2\right]$$

$$= \frac{1}{2} \times 30 \times 10 \times (2-1)^2$$

$$= 150\,\mu\text{A}$$

$$g_m = \sqrt{2 \times 1_{DC} \times \mu n\, C_{cox}\left(\frac{W}{L}\right)^n}$$

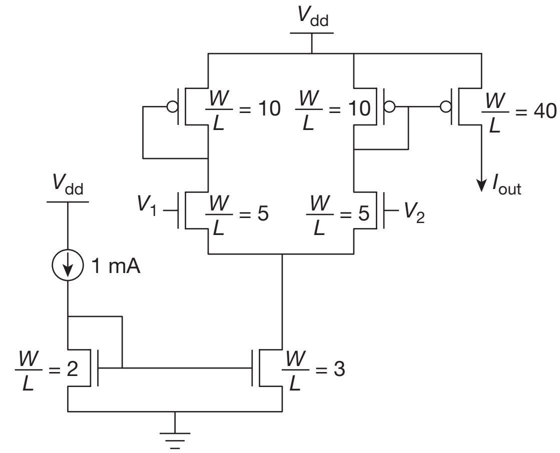
$$= \sqrt{2 \times 150 \times 60 \times 5}$$

$$= 300\text{ mA/V}$$

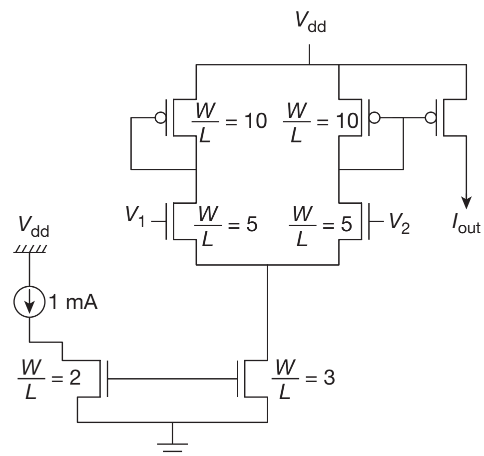
$$A_v = -g_m(rd \,||\, rd) = -300(6.1116)$$

$$= (-300)(3) = -900\text{ V/V}$$

3. In the circuit shown, $V_1 = 0$ and $V_2 = V_{dd}$. The other relevant parameters are mentioned in the figure. Ignoring the effect of channel length modulation and the body effect, the value of I_{out} is ______ mA (rounded off to 1 decimal place). **[2019]**



Solution:



$$I_{ref} = 1\text{ mA}$$

Current hours in the ratio $\left(\frac{W}{L}\right)$

$$\frac{I_{ref}}{I_o} = \frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2} = \frac{1}{I_o} = \frac{2}{3}$$

$$I_o = 1.5\text{ mA}$$

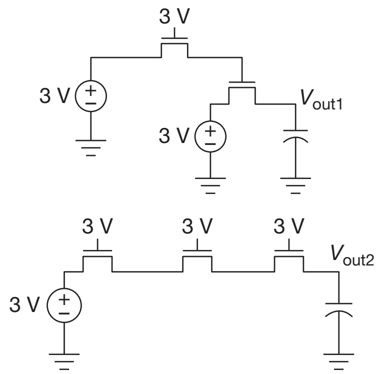
$$\frac{I_{out}}{I_o} = \frac{40}{10}$$

$$I_{out} = \frac{40}{10} \times 1.5$$

$$= 6\text{ mA}$$

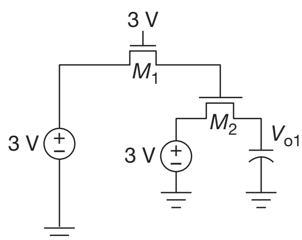
Hence, the correct answer is (6 mA).

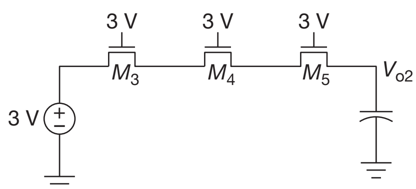
4. In the circuits shown, the threshold voltage of each nMOS transistor is 0.6 V, Ignoring the effect of channel length modulation and body bias, the values of Vout1 and Vout2. **[2019]**



(A) 1.8 and 2.4 (B) 2.4 and 1.2
(C) 1.8 and 1.2 (D) 2.4 and 2.4

Solution:





For M_1

$V_1 = 3 - 2.6 = 2.4$ V.

For M_2

$V_{o1} = V_1 - V_t = 2.4 - 0.6$

$V_{o1} = 1.8$ V

For M_3

$V_3 = 3 - 0.6 = 2.4$ V

For M_4

$V_4 = 3 - 0.6 = 2.4$ V.

For M_2

$V_5 = 3 - 0.6 = 2.4$

$V_{o2} = 2.4$ V.

Hence the correct option is (A)

5. Consider a long-channel MOSFET with a channel length 1 μm and width 10 μm. The device parameters are acceptor concentration $N_A = 5 \times 10^{16}$ cm^{-3}, electron mobility μ_n = 800 cm^2/V$_{-s,}$ oxide capacitance/area C_{ox} = 3.45 × 10^{-7} F/cm^2. Threshold voltage V_T = 0.7 V. The drain saturation current (I_{Dsat}) for a gate voltage of 5 V is _____ mA (rounded off to two decimal places). [ε_0 = 8.854 × 10^{-14} F/cm, ε_{st} = 11.9] **[2019]**

Solution:

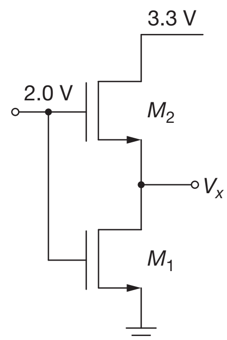
$$I_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[V_{\mu s} - V_T\right]^2$$

$$= \frac{1}{2} \times 800 \times 3.45 \times 10^{-7} \times 10\left[5 - 0.7\right]^2$$

$$= 25.5162 \text{ mA}$$

Hence, the correct answer is 25.562 mA.

6. In the circuit shown below, the (W/L) value for M_2 is twice that for M_1. The two nMOS transistors are otherwise identical. Threshold voltage V_T for both transistors is 1.0 V. Note that V_{GS} for M_2 must be > 1.0V.



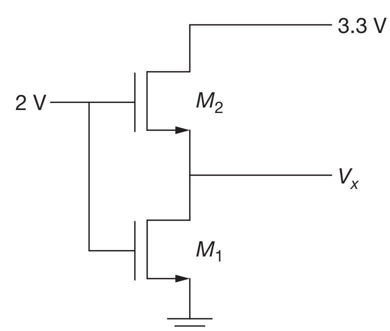
Current through the nMOS transistors can be modelled as:

$I_{DS} = \mu C_{ox}\left(\frac{W}{L}\right)((V_{GS} - V_T)\, V_{DS} - \frac{1}{2}\, V^2_{DS})$ for $V_{DS} \leq V_{GS} - V_T$

$I_{DS} = \mu C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_T)^2/2$ for $V_{DS} \geq V_{GS} - V_T$

The voltage (in volts, accurate to two decimal places) at V_x is ________. **[2018]**

Solution: Consider the figure given below



From the above figure we have

M_2: $V_{GS2} = 2 - V_x$

$V_{DS2} = 3.3 - V_x$

$V_{DS2} \geq V_{GS2} - V_t$

$3.3 - V_x \geq 1 - V_x$

Therefore M_2 operates in saturation region.

$$M_1: V_{GS1} = 2$$

$$V_{DS1} = V_x$$

$$V_x \le 1$$

We know that M_1 operates in linear region

$$I_{D1} = I_{D2}$$

$$\mu_n c_{ox}\left(\frac{\omega}{L}\right)\left[\left(V_{GS1} - V_T\right)\right]V_{DS} - \frac{V_{DS}{}^2}{2}$$

$$= \frac{1}{2}\ \mu_n c_{ox}\left(\frac{\omega}{L}\right)\left[\left(V_{GS2} - V_T\right)\right]^2$$

$$\mu_n c_{ox}\left(\frac{\omega}{L}\right)\left[(2-1)\right]V_x - \frac{V_x^2}{2}$$

$$= \frac{1}{2}\mu_n c_{ox}\left(\frac{2\omega}{L}\right)\left[2 - V_x - 1\right]^2$$

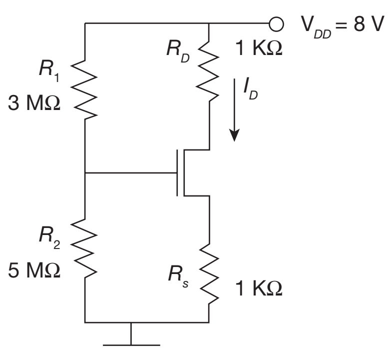
$$3V_x^2 - 6V_x + 2 = 0$$

$$V_x = 1 \pm \frac{1}{\sqrt{3}}\text{ V}$$

V_x cannot be $1 + \frac{1}{\sqrt{3}}$ V. So, V_x should be $1 - \frac{1}{\sqrt{3}} = 0.42$ V

Hence, the correct answer is 0.41 to 0.435.

7. For the circuit shown, assume that the NMOS transistor is in saturation. Its threshold voltage V_{tn} = 1V and its trans conductance parameter $\mu_n C_{ox}\left(\frac{W}{L}\right) = 1\text{ mA/V}^2$. Neglect channel length modulation and body bias effects. Under these conditions, the drain current I_D in mA is _______. **[2017]**

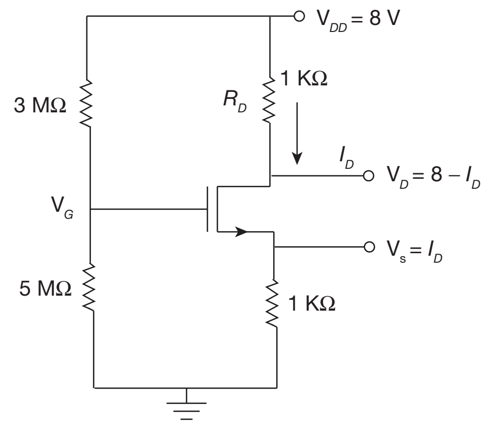


Solution:

$$V_{GS} = V_G - V_S \qquad V_G = \frac{V_{DD} \times R_2}{R_1 + R_2}$$

$$\therefore V_{GS} = 5 - I_D \qquad V_G = \frac{8 \times 5}{8} = 5\text{ V}$$

$$V_S = I_D,\quad V_D = 8 - I_D$$



Thus I_D is given by

$$I_D = \frac{1}{2}\mu_n Cox\frac{W}{L}(V_{GS} - V_{tn})^2$$

$$I_D = \frac{1}{2} \times 1 \times (5 - I_D - 1)^2$$

$$I_D = \frac{1}{2}(4 - I_D)^2$$

$$I_D = \frac{1}{2}(4 - I_D)^2$$

$$16 + I_D^2 \ - 8I_D = 2I_D$$

$$I_D^2 \ - 10I_D + 16 = 0$$

$$I_D^2 \ - 2I_D - 8I_D + 16 = 0$$

$$I_D(I_D - 2) - 8(I_D - 2) = 0$$

$$\therefore (I_D - 8)(I_D - 2) = 0$$

$$\therefore I_D = 2\text{ mA},\quad 8\text{ mA}$$

Given that nmos transistor is in saturation so

$$V_{DS} > V_{GS} - V_{tn}$$

$$For\ \ I_D = 2\text{ mA}$$

$$V_{GS} = 3,\quad V_{tn} = 1$$

$$V_{DS} = V_D - V_S = 6 - 2 = 4$$

$$V_{DS} > V_{GS} - V_{tn}$$

4 > 2 nmos transistor is in saturation for

$$I_D = 8\text{ mA}$$

$$V_{GS} = 5 - 8 = -3 \qquad V_{DS} = V_D - V_S$$

$$V_{tn} = 1\text{ V} \qquad V_{DS} = 0 - 8$$

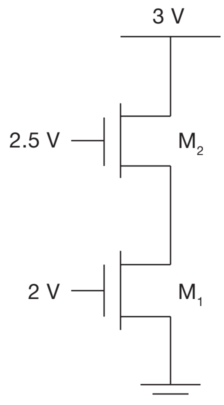
$$V_{DS} > V_{GS} - V_{tn} \qquad V_{DS} = -8$$

$$-8 > -4\text{ False}$$

∴ nmos transistor is not in saturation region.
∴ Drain current I_D = 2 mA

Hence, the correct answer is (1.9 to 2.1).

8. Assuming that transistors M_1 and M_2 are identical and have a threshold voltage of 1 V, the state of transistor M_1 and M_2 are respectively



(A) Saturation, Saturation
(B) Linear, Linear
(C) Linear, Saturation
(D) Saturation, Linear

9. Two n-channel MOSFETs, T1 and T2 are identical in all respects except that the width of T2 is double that of T1. Both the transistors are biased in the saturation region of operation. but the gate overdrive voltage (V_{GS} $-V_{TH}$) of T2 is double that of T1. where V_{GS} and V_{TH} are the gate-to-source voltage and threshold voltage of the transistors, respectively. If the drain current and transconductance of T1 are I_{D1} and g_{m1} respectively, the corresponding values of these two parameters for T2 are **[2017]**

(A) $8ID_1$ and $2g_{m1}$ (B) $8ID_1$ and $4g_{m_1}$
(C) $4ID_1$ and $4g_{m1}$ (D) $4ID_1$ and $2g_{m_1}$

10. Consider an n–channel metal oxide semiconductor field effect transistor (MOSFET) with a gate to source voltage of 1.8 V. Assume that $\frac{W}{L} = 4$, $\mu_n C_{ox} = 70 \times 10^{-6}$ AV^{-2}, the threshold voltage is 0.3 V, and the channel length modulation parameter is 0.09 V^{-1}. In the saturation region, the drain conductance (in micro Siemens) is ______. **[2016]**

Solution: Gate to source voltage $V_{GS} = 1.8$ V

$$\frac{W}{L} = 4,$$

$$\mu_n c_{ox} = 70 \times 10^{-6}\ A/V^2$$

Threshold voltage $V_T = 0.3$ V

Channel length modulation parameter $\lambda = 0.09V^{-1}$

Given transistor is in the Saturation region so

$$I_D = \frac{1}{2}k_n \cdot \frac{W}{L}(V_{GS} - V_T)^2 \cdot (1 + \lambda \cdot V_{DS})$$

But

$$V_{DS} = V_{GS} - V_T = 1.5\ V$$

$$g_d = \frac{\partial I_D}{\partial V_{DS}};\ \text{drain conductance}$$

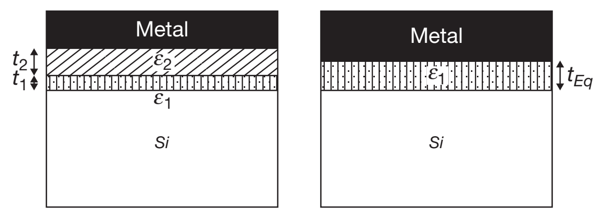
$$g_d = \frac{\partial I_D}{\partial V_{DS}} = \frac{1}{2}k_n \cdot \frac{W}{L} \times \left[(V_{GS} - V_T)^2 (0 + \lambda \cdot 1)\right]$$

$$g_d = \frac{1}{2} \times 70 \times 10^{-6} \times 4\left[(1.5)^2 (0 + 0.09 \times 1)\right]$$

$$= 140 \times 10^{-6}[0.2025] = 28.35 \times 10^{-6}\ \mho$$

Hence, the correct Answer is (28.35).

11. Figure I and II show two MOS capacitors of unit area. The capacitor in Figure I has insulator materials X (of thickness $t_1 = 1$ nm and dielectric constant $\varepsilon_1 = 4$) and Y (of thickness $t_2 = 3$ nm and dielectric constant $\varepsilon_2 = 20$). The capacitor in figure II has only insulator material X of thickness t_{Eq} If the capacitors are of equal capacitance, then the value of t_{Eq} (in nm) is ______. **[2016]**



Solution: Given

$$t_1 = 1\text{ nm and } \varepsilon_1 = 4$$

$$t_2 = 3\text{ nm and } \varepsilon_2 = 20$$

Capacitor in figure 2 has insulator material X of thickness $t_{Eq.}$

The oxide capacitance of MOS is

$$Cox = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{\varepsilon_r.\varepsilon_o}{t_{ox}}$$

$$Cox_1 = \frac{\varepsilon_o}{\left[\left(\frac{t_1}{\varepsilon_1}\right) + \frac{t_2}{\varepsilon_2}\right]}$$

And $$Cox_2 = \frac{\varepsilon_0 \times \varepsilon_1}{t_{Eq}}$$

Given $$Cox_1 = Cox_2$$

$$\frac{t_{Eq}}{\varepsilon_1} = \left[\frac{t_1}{\varepsilon_1} + \frac{t_2}{\varepsilon_2}\right]$$

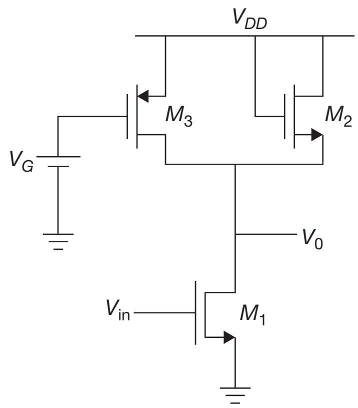
$$t_{Eq} = 4\left[\frac{1}{4} + \frac{3}{20}\right]\text{nm}$$

$$t_{Eq} = \left[1 + \frac{3}{5}\right]\text{nm}$$

$$t_{Eq} = 1.6\text{ nm}$$

Hence, the correct Answer is (1.6 nm).

12. In the circuit shown in the figure, the channel length modulation of all transistors is non-zero ($\lambda \neq 0$). Also, all transistors operate in saturation and have negligible body effect. The AC small signal voltage gain (V_0/V_{in}) of the circuit is **[2016]**



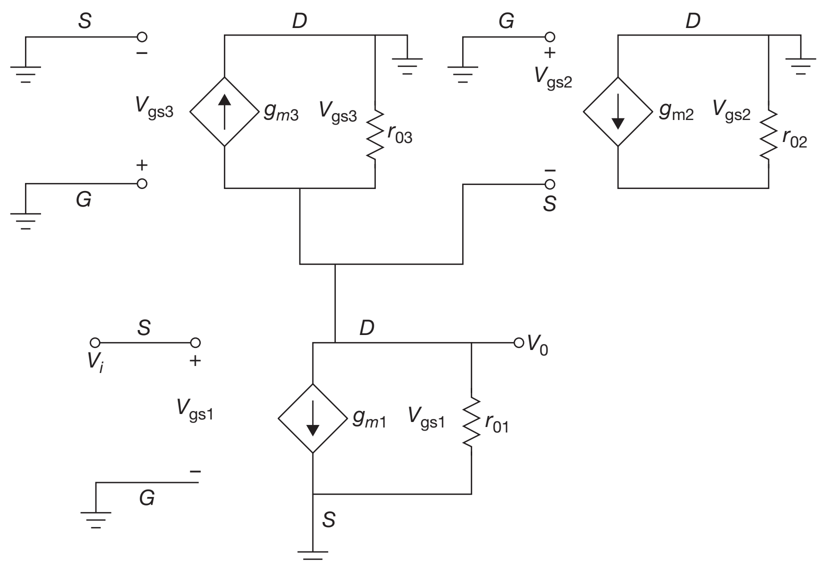
(A) $-g_{m1}(r_{01}\|r_{02}\|r_{03})$

(B) $-g_{m1}\left(r_{01}\parallel\dfrac{1}{g_{m3}}\parallel r_{03}\right)$

(C) $-g_{m1}\left(r_{01}\parallel\left(\dfrac{1}{gm_2}\parallel r_{02}\right)\parallel r_{03}\right)$

(D) $-g_{m1}\left(r_{01}\parallel\left(\dfrac{1}{gm_3}\parallel r_{03}\right)\parallel r_{02}\right)$

Solution: Apply AC analysis and replace all the transistor with small signal equivalent model



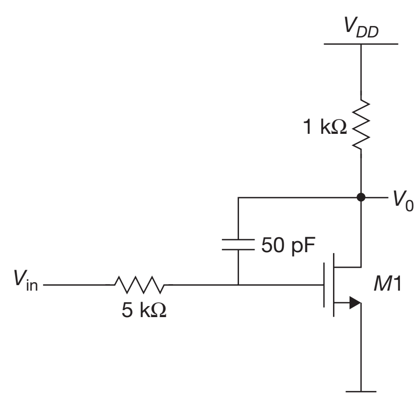
$$V_{ont} = -g_{m1}V_{gs1}\left[r_{01}\|r_{03}\|\left(\frac{1}{g_{m2}}\|r_{02}\right)\right]$$

$$V_{in} = V_{gs1}$$

$$\frac{V_{out}}{V_{in}} = \text{gain} = -g_{m1}\left(r_{01}\left\|\left(\frac{1}{g_{m2}}\|r_{02}\right)\right\|r_{03}\right)$$

Hence, the correct option is (C).

13. In the circuit shown in the figure, transistor M1 is in saturation and has transconductance $g_m = 0.01$ siemens. Ignoring internal parastitic capacitances and assuming the channel length modulation λ to be zero, the small input pole frequency (in kHz) is **[2016]**



Solution: Given $g_m = 0.01$

Voltage gain $A_v = -g_m R_D$

$= -0.01 \times 1000 = -10$

Due to the presence of capacitance of 50 pF, Miller effect will takes place resulting in input and output capacitance, C_{in} and C_{out} respectively.

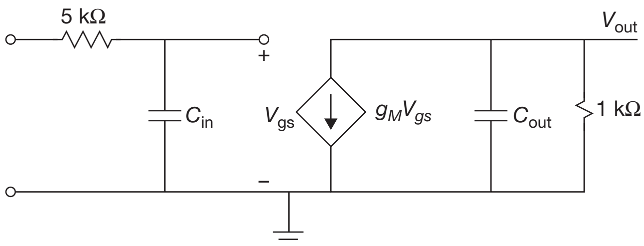
By using Miller's theorem.

$$C_{\text{in}} = C(1 - A_v)$$

and

$$\text{Cout} = C\left(1 - \frac{1}{A_v}\right)$$

$$C_{\text{in}} = (50 \times 10^{-12})\,(1 + 10) = 550 \text{ pF}$$



Input frequency will be

$$f_{\text{input}} = \frac{1}{2\pi RC}$$

$$= \frac{1}{2\pi \times 5 \times 10^{3} \times 550 \times 10^{-12}} = 57.88 \text{ kHz}$$

Hence, the correct Answer is (57.88 KHz).

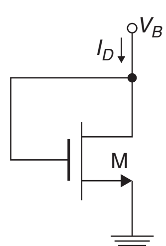
14. Which one of the following processes is preferred to form the gate dielectric (SiO_2) of MOSFETs? **[2015]**

(A) Sputtering
(B) Molecular beam epitaxy
(C) Wet oxidation
(D) Dry oxidation

Solution: Dry oxidation method is preferred to form the gate dielectric (Sio_2) of MOSFETS

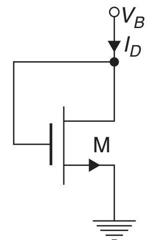
Hence, the correct option is (D).

15. The small-signal resistance (i.e., dV_B/dI_D) in kΩ offered by the n-channel MOSFET M shown in the figure below, at a bias point of $V_B = 2$ V, is (device data for M:device transconductance parameter $k_N = \mu_n C'\text{ox}(\text{W/L}) = 40$ μA/V², threshold voltage $V_{TN} = 1$ V, and neglect body effect and channel length modulation effects) **[2013]**



(a) 12.5 (b) 25 (c) 50 (d) 100

Solution: (b)



$$g_m = \frac{\partial I_D}{\partial V_{GS}}$$

$$V_B = V_G$$

$$V_S = 0$$

$$V_{GS} = V_G - V_S$$

$$= V_G - 0 = V_G = V_B$$

$$\frac{\partial V_B}{\partial I_D} = \frac{1}{g_m}$$

$$V_D = V_G$$

$$V_D - V_S = V_G - V_S$$

$$V_{DS} = V_{GS}$$

$$V_{DS} > V_{CS} - V_T$$

So, the given MOSFET is in saturation region

$$g_m = \mu n\, Coy \frac{W}{L}(V_{GS} - V_T)$$

$$g_m = 40 \times 10^{-6} \times (2 - 1) = 4 \times 10^{-5}$$

$$\frac{\partial V_B}{\partial I_D} = \frac{1}{g_m}$$

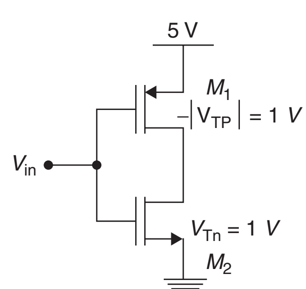
$$= \frac{1}{4 \times 10^{-5}}$$

$$= \frac{100 \times 10^{3}}{4}$$

$$= 25 \text{ kΩ}$$

Hence, the correct option is (b).

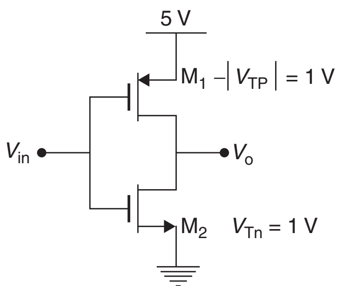
16. In the CMOS circuit shown, electron and hole mobilities are equal, and M_1 and M_2 are equally sized. The device M_1 is in the linear region if **[2012]**

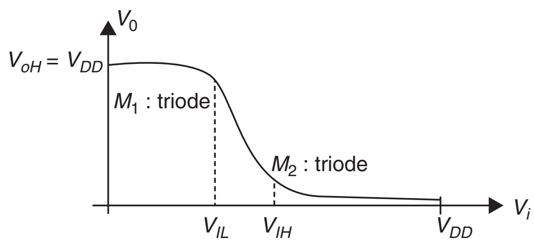


(a) $V_{jn} < 1.875$ V
(b) $1.875\ V < V_{in} < 3.125$ V
(c) V_{jn} 3.125 V
(d) $0 < V_{in} < 5$ V

Solution: (a)

The voltage transfer characteristics of the CMOS is





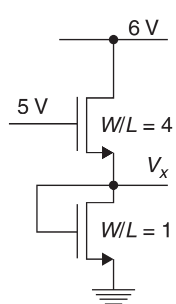
where $V_{IL} = \frac{1}{8}(3V_{DD} + 2V_t)$

$= 2.125$ V

Hence, approximately, $V_{in} < 1.875$ V. So the device M_1 is in the linear region.

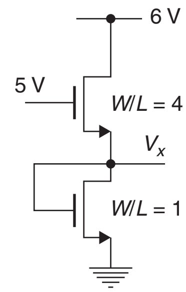
Hence, the correct option is (a).

17. In the circuit shown below, for the MOS transistors, $\mu_n C_{ox} = 100\ \mu A/V^2$ and the threshold voltage $V_T = 1$ V. The voltage V_x at the source of the upper transistor is **[2011]**



(a) 1 V
(b) 2 V
(c) 3 V
(d) 3.67 V

Solution: (c)



The transistor which has $\frac{W}{L} = 4$

$$V_{DS} = 6 - V_x, \text{ and,}$$
$$V_{GS} = 5 - V_x$$
$$V_{GS} = -V_T = 5 - V_x - 1$$
$$= 4 - V_x$$
$$V_{OS} \rangle V_{GS} - V_T$$

So that transistor is in saturation region, the transistor which has $\frac{W}{L} = 1$ drain is connected to gate so that transistor in saturation $V_{DS} > V_{GS} > V_T$ ($\therefore V_{DS} = V_{GS}$).

The current flow in both the transistor is same

$$\mu n Cox\left(\frac{W}{L}\right)_1\left(\frac{(V_{GS})_1 - V_T}{2}\right)^2$$

$$= \mu n\ Cox\left(\frac{W}{L}\right)_2\left(\frac{(V_{GS})_2 - V_T}{2}\right)^2$$

$$\frac{4(5 - V_x - 1)^2}{2} = \frac{1(V_x - 4)^2}{2} \quad (\because V_{GS} = V_x - 0)$$

$$4\left(V_x^2 - 8V_x + 16\right) = V_x^2 - 2V_x + 1$$

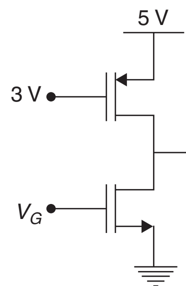
$$\Rightarrow 3V_x^2 - 30V_x + 63 = 0$$

$$\Rightarrow V_x = 3V$$

Hence, the correct option is (c).

Statement for Linked Answer Questions 18 and 19.

Consider the CMOS circuit shown, where the gate voltage V_G of the n-MOSFET is increased from zero, while the gate voltage of the p-MOSFET is kept constant at 3 V. Assume that, for both transistors, the magnitude of the threshold voltage is 1 V and the product of the transconductance parameter and the *(W/L)* ratio, i.e. the quantity $\mu C_{oX}(W/L)$, is 1 mA. V^{-2}.



18. Estimate the output voltage V_0 for $V_G = 1.5$ V. **[2009]**

(a) $4-\frac{1}{\sqrt{2}}V$ (b) $4+\frac{1}{\sqrt{2}}V$

(c) $4-\frac{\sqrt{3}}{2}V$ (d) $4+\frac{\sqrt{3}}{2}V$

Solution: (d)

$$\frac{1}{2}(V_{gs}-V_t)n^2 = (V_{gs}-V_t)_p V_{DS} - \frac{1}{2}V_{DS}^2$$

$$\Rightarrow \frac{1}{2}(1.5-1)^2 = (-2+1)\times V_{DS} - \frac{1}{2}V_{DS}^2$$

$$\Rightarrow V_{DS} = 4.875 \text{ V}$$

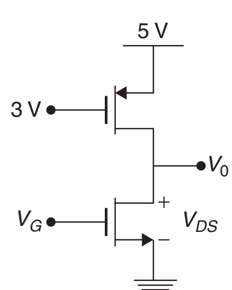
$$\Rightarrow V_0 = V_{DS} = 4.875 \text{ V} = \left(4+\frac{\sqrt{3}}{2}\right)\text{V}$$

Hence, the correct option is (d).

19. For small increase in V_G beyond 1 V, which of the following gives the correct description of the region of operation of each MOSFET? **[2009]**

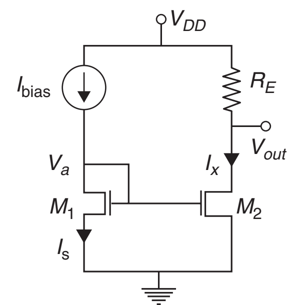
(a) Both the MOSFETs are in saturation region
(b) Both the MOSFETs are in triode region
(c) *n*-MOSFET is in triode and *p*-MOSFET is in saturation region
(d) *n*-MOSFET is in saturation and *p*-MOSFET is in triode region

Solution: (b)



Since the threshold voltage is *IV*, for small increase in *Va* beyond *IV*, *n*-MOSFET is in saturation and *p*-MOSFET is in triode region.
Hence, the correct option is (b).

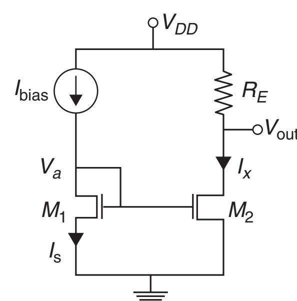
20. For the circuit shown in the following figure, transistors M_1 and M_2 are identical NMOS transistors. Assume that M_2 is in saturation and the output is unloaded. **[2008]**



The current I_x is related to I_{bias} as

(a) $I_x = I_{bias} + I_s$
(b) $I_x = I_{bias}$
(c) $I_x = I_{bias} - I_s$
(d) $I_x = I_{bias} - \left(V_{DD} - \frac{V_{out}}{R_E}\right)$

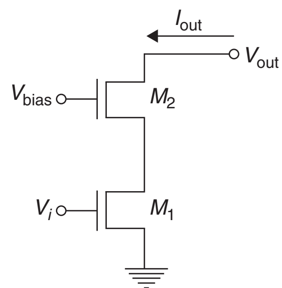
Solution: (b)
Given circuit is current mirror, since MOSFETS are identical



$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_1$$
$$\therefore I_x = I_{bias}$$

Hence, the correct option is (b).

21. Two identical NMOS transistors M_1 and M_2 are connected as shown below. V_{bias} is chosen so that both transistors are in saturation. The equivalent g_m of the pair is defined to be $\frac{\partial I_{out}}{\partial V_i}$ at constant V_{out}

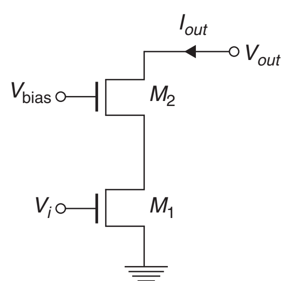


The equivalent g_m of the pair is **[2008]**

(a) the sum of individual g_m's of the transistors
(b) the product of individual g_m's of the transistors
(c) nearly equal to the g_m of M_1
(d) nearly equal to g_m/g_0 of M_2

Solution: (c)

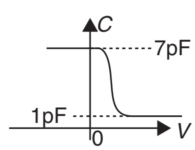
Since the current in both transistors are equal, g_m is decided by M_1



Hence, the correct option is (c).

Common Data Question 22, 23 and 24.

The figure shows the high-frequency capacitance-voltage (C–V) characteristics of a Metal/SiO_2/silicon (MOS) capacitor having an area of 1×10^{-4} cm². Assume that the permittivities ($\varepsilon_0\,\varepsilon_r$) of silicon and SiO_2 are 1×10^{-12} F/cm and 3.5×10^{-13} F/cm respectively.



22. Consider the following statements about the C–V characteristics plot:

S_1: The MOS capacitor has as n-type substrate.

S_2: If positive charges are introduced in the oxide, the C–V plot will shift to the left.

Then which of the following is true? **[2007]**

(a) Both S_1 and S_2 are true
(b) S_1 is true and S_2 is false
(c) S_1 is false and S_2 is true
(d) Both S_1 and S_2 are false

Solution: (b)

The MOS capacitor has a p-type substrate. If positive charges are introduced in the oxide, the C–V plot will shift to the right.

Hence, the correct option is (b).

23. The maximum depletion layer width in silicon is **[2007]**

(a) 0.143 μm
(b) 0.857 μm
(c) 1 μm
(d) 1.143 μm

Solution: (b)

Series capacitance is given by

$$\frac{1}{C_\alpha} = \frac{1}{C_{oy}} + \frac{1}{C_{si}}$$

$$C_{oy} = \frac{t_{oy}}{t_{oy}} \cdot A$$

$$C_{si} = \frac{t_{si}}{x_d} \cdot A$$

$$10^{12} = \frac{10^{12}}{7} + \frac{10^{12} \times 10^{-4}}{x_{d\max}}$$

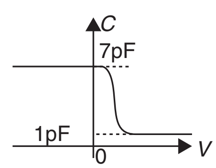
$$x_{d\max} = 0.857\ \mu\text{m}.$$

Hence, the correct option is (b).

24. The gate oxide thickness in the MOS capacitor is **[2007]**

(a) 50 μm (b) 143 μm
(c) 350 μm (d) 1 μm

Solution: (a)



Capacitance is given by

$$C = \frac{t_o t_r A}{d}$$

given, $C = 7\,pF$

$$A = 1\times10^{-4}$$

$$t_0 t_r = 3.5\times10^{-13}$$

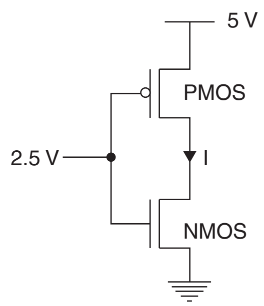
$$\text{thus, } d = \frac{t_0 t_r A}{C}$$

$$= 50 \text{ nm}$$

Hence, the correct option is (a).

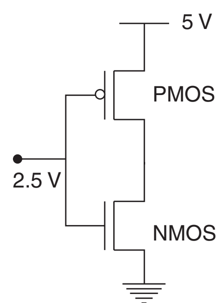
25. In the CMOS inverter circuit shown, if the transconductance parameters of the NMOS and PMOS transistors are $K_0 = K_p = \mu_0 C_{ox}\frac{W_n}{L_n} = \mu_p C_{ox}\frac{W_P}{L_P} = 40\ \mu A/V^2$

and their threshold voltages are $V_{THn} = |V_{THp}| = 1$ V, the current I is **[2007]**



(a) 0A (b) 25 μA
(c) 45 μA (d) 90 μA

Solution: (c)



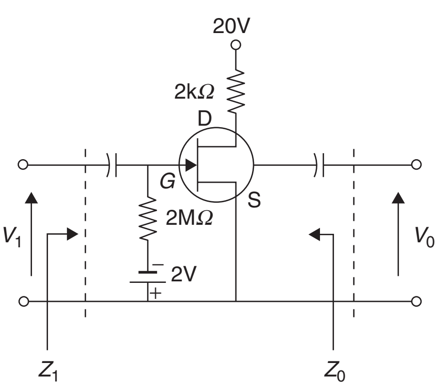
V_{GS} for each MOS is 2.5V
$V_T = IV$, device parameter,

$$k = 40m\frac{A}{V^2}$$

$$\text{so, } I_D = \frac{k}{2}(V_{GS} - V_T)^2$$
$$= 20(2.5-1)^2$$
$$= 45 \text{ mA}$$

Hence, the correct option is (c).

Common Data Questions 26, 27 and 28.
Given $r_d = 20$ kΩ, $I_{DSS} = 10$ mA, $V_p = -8$V



26. Transconductance in milli-Siemens (mS) and voltage gain of the amplifier are respectively **[2005]**

(a) 1.875 mS and 3.41
(b) 1.875 mS and –3.41
(c) 3.3 mS and –6
(d) 3.3 mS and 6

Solution: (a)

$$g_m = g_{mo}\left(1 - \frac{V_{GSQ}}{V_P}\right),$$

where

$$g_{mo} = \frac{2I_{DSS}}{V_P} = \frac{2\times 10}{8} = 2.5$$

$$g_m = 2.5\left(1 - \frac{2}{8}\right) = 1.875 \text{ ms}$$

$$\text{voltage gain} = -g_m(ra \,||\, R_D) - 1.875\left(\frac{20}{11}\right)$$
$$= 3.41$$

Hence, the correct option is (a).

27. I_D and V_{DS} under DC conditions are, respectively **[2005]**

(a) 5.625 mA and 8.75 V
(b) 7.500 mA and 5.00 V
(c) 4.500 mA and 11.00 V
(d) 6.250 mA and 7.50 V

Solution: (a)
Drain current is given by

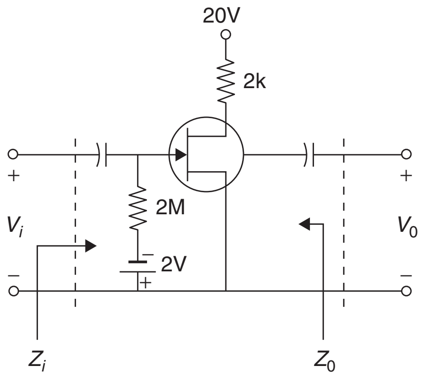
$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$
$$V_P = -8$$
$$I_{DSS} = 10 \text{ mA}$$
$$V_{GS} = -Z$$
$$I_D = 10m\left(1 - \frac{-2}{-8}\right) = 5.625 \text{ mA}$$
$$V_{DS} = V_D - 2K\, I_D = 8.75 \text{ V}$$

Hence, the correct option is (a).

28. Z_i and Z_0 of the circuit are respectively **[2005]**

(a) 2 MΩ and 2 kΩ
(b) 2 MΩ and $\frac{20}{11}k\Omega$
(c) infinity and 2 MΩ
(d) infinity and $\frac{20}{11}k\Omega$

Solution: (d)
Given

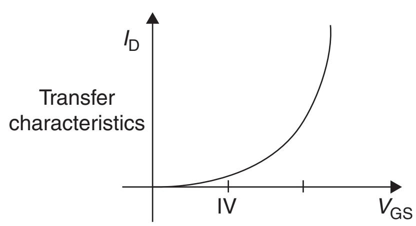


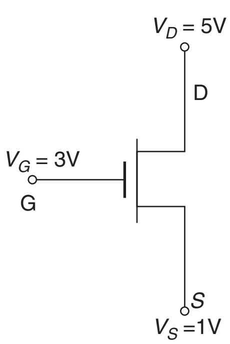
$$r_d = 20\ \text{K}\Omega$$
$$I_{DSS} = 10\ \text{mA}$$
$$V_P = -8\ \text{V}$$
$$Z_i = (R_D \,\|\, \infty)$$
$$= R_G = 2\ \text{m}\Omega$$

$$Z_0 = (R_D \,\|\, rd)$$
$$= (2K \,\|\, 20K) = \frac{20}{11}\ \text{K}\Omega$$

Hence, the correct option is (d).

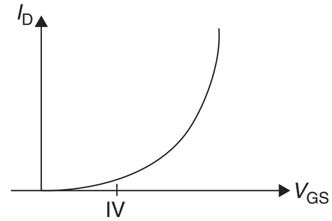
29. For an n-channel MOSFET and its transfer curve shown in the figure, the threshold voltage is **[2005]**

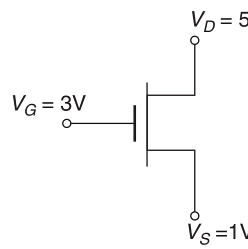




(a) 1 V and the device is in active region
(b) −1 V and the device is in saturation region
(c) 1 V and the device is in saturation region
(d) −1 V and the device is in active region

Solution: (b)
From graph,





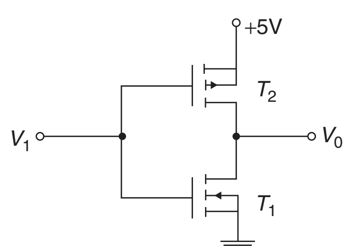
$$V_{th} = 1\ \text{V}$$
$$V_{GS} = 2\ \text{V}$$
$$V_{DS} = 4\ \text{V}$$

And $V_{DS} \geq (V_{GS} - V_T)$
$\therefore$ MOSFET is in saturation.
Hence, the correct option is (b).

30. The action of a JFET in its equivalent circuit can best be represented as a **[2003]**
(a) Current Controlled Current Source
(b) Current Controlled Voltage Source
(c) Voltage Controlled Voltage Source
(d) Voltage Controlled Current Source

Solution: (d)
JFET is voltage controlled current source. Input impedance of JFET is very high. Thus input current is zero.
Hence, the correct option is (d).

31. Consider the following statements in connection with the CMOS inverter in the figure, where both the MOSFETs are of enhancement type and both have a threshold voltage of 2V.
Statement 1: T_1 conducts when $V_i > 2$V
Statement 2: T_1 is always in saturation when $V_0 = 0$ V.

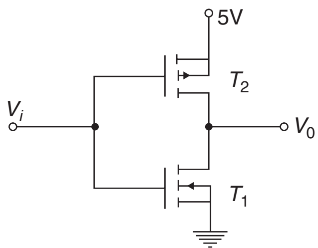


Which of the following is correct? **[2002]**
(a) Only Statement 1 is TRUE
(b) Only Statement 2 is TRUE

(c) Both the statements are TRUE
(d) Both the statements are FALSE

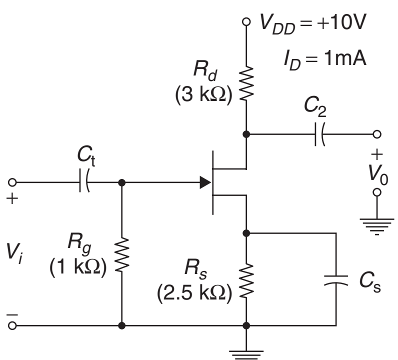
Solution: (a)
Statement 2 is false because V_{DS} will be less than $V_{GS} - V_T$

If $V_0 = 0$



Hence, the correct option is (a).

32. The voltage gain $A_v = \dfrac{V_0}{V_i}$ of the JFET amplifier shown in the figure is



[2002]

$I_{DSS} = 10$ mA $\quad V_p = -5$ V
(Assume C_1, C_2 and C_3 to be very large)

(a) +18 (b) –18 (c) +6 (d) –6

Solution: (d)

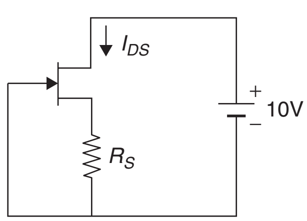
$V_G = 0 \Rightarrow V_S = I_D R_S = 1 \text{ mA} \times 2.5 \text{ K} = 2.5 \text{ V}$

$$V_{GS} = V_G - V_S = 0 - 2.5 = -2.5 \text{ V}$$

$$g_m = \frac{2 \times 10 \times 10^{-3}}{151}\left[1 - \left(\frac{-2.5}{-3}\right)\right] = 2 \text{ ms}$$

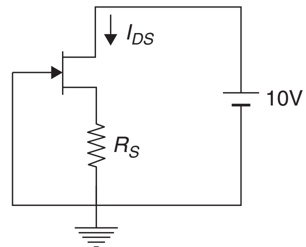
$$A_v = -g_m R_D = -2 \times 10^{-3} \times 3 \text{ K} = -6 \text{ V.}$$

33. The JFET in the circuit shown in the figure has an $I_{DSS} = 10$ mA and $V_P = -5$V. The value of the resistance R_S for a drain current $I_{DS} = 6.4$ mA is (Select the Nearest value) **[1992]**



(a) 150 Ω (b) 470 Ω
(c) 560 Ω (d) 1 kΩ

Solution: (a)
Using the formula for drain current,



$$I_{DS} = I_{DSS}\left(1 - \frac{V_{GS}}{Vp}\right)^2$$

$$6.4 \times 10^{-3} = 10 \times 10^{-3}\left(1 - \frac{V_{GS}}{-5}\right)^2$$

$$0.64 = \left(1 + \frac{V_{GS}}{5}\right)^2$$

$$\Rightarrow 1 + \frac{V_{GS}}{5} = 0.8$$

$$\Rightarrow \frac{V_{GS}}{5} = -0.2$$

$$\Rightarrow V_{GS} = -1 \text{ V}$$

$$\Rightarrow V_{GS} = -I_{DS} R_S$$

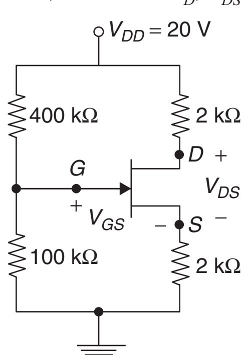
$$-1 = -6.4 \times 10^{-3} R_S$$

$$R_S = \frac{1}{6.4 \times 10^{-3}} = 156\Omega$$

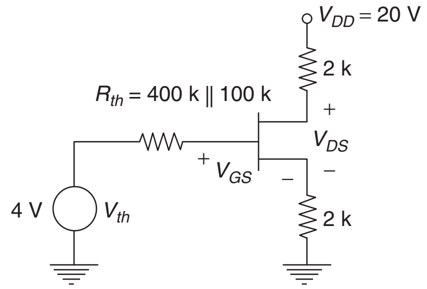
$$R_S \cong 150\Omega$$

Five-marks Questions

1. A JFET with $V_p = -4$V and $I_{DSS} = 12$ mA is used in the circuit shown in figure. Assuming the device to be operating in saturation, determine I_D, V_{DS} and V_{GS}. **[1996]**



Solution:



$$V_{th} = 20 \times \frac{100}{100+400} = 4V$$

$$R_{th} = 400 \,||\, 100K = \frac{\cancel{40}\overset{8}{}0 \times 1\cancel{00}}{\cancel{500}\,K} = 80K$$

$$\therefore\ I_4 = 0$$

Apply KVL to input mesh

$4 = V_{gs} + 2 \times 10^3 \times I_D$

$V_{GS} = G - 2\, I_D$ (1)

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$I_D = 12\left(1 - \frac{(4-2I_D)}{-4}\right)^2$$

$I_D = 5.33$ mA, 3 mA

Apply KVL to output mesh

$$20 = 2I_D + V_{DS} + 2I_{DS}$$

$$V_{DS} = 20 - 4I_{DS} \quad (2)$$

Substitute I_{DS} in equation (2)

$$V_{DS1} = 20 - 4\, I_{DS1} = 20 - 4 \times 5.33$$

$$U_{DS1} = -1.33\text{V}$$

But V_{DS} should be positive, So $I_{DS} \neq 5.33$ mA

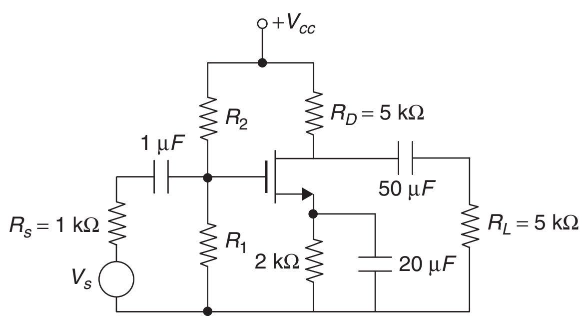
Substitute $I_{DS} = 3$ mA in equation (2)

$$V_{DS_2} = 8V$$

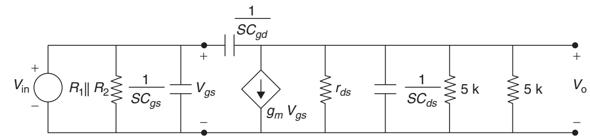
$$\therefore\ V_{GS} = 4 - 2I_D = 4 - 2 \times 3 = -2\text{V}$$

$$V_{GS} = -2\text{V}$$

2. In the JFET circuit shown in figure assume that $R_1 || R_2 = 1\text{M}\Omega$ and the total stray capacitance at the output to be 20 pF The JFET used has $g_m = 2$ mA/V, $C_{gs} = 20$ pF. and $C_{gd} = 2$ pF. Determine the upper cut-off frequency of the amplifier. **[1995]**



Solution:



$$(V_0 - V_{in})S\, C_{gd} + g_m\, V_{gs} + \frac{V_0}{r_{ds}} + V_0\ s\ \cos + \frac{V_0}{5k} + \frac{V_0}{5k} = 0$$

$$V_0\left[SC_{gd} + \frac{1}{r_{ds}} + s\ \cos + \frac{2}{5k}\right] = V_{in}(S\, C_{gd} - g_m)$$

$$A_V = \frac{V_0}{V_i}$$

$$= \frac{SC_{gd} - g_m}{SC_{gd} + \dfrac{1}{r_{ds}} + SC_{as} + \dfrac{2}{5k}}$$

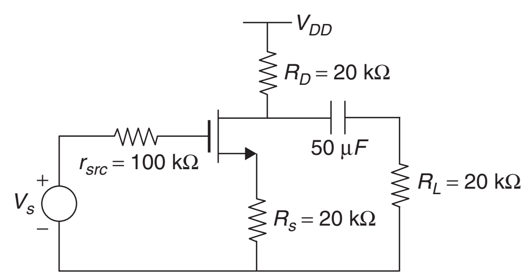
Upper cut off frequency.

$$f_H = \frac{1}{2\pi\tau} = \frac{1}{2\pi(r_{ds} \,||\, 5k \,||\, 5k)(C_{gd} + C_{ds})}$$

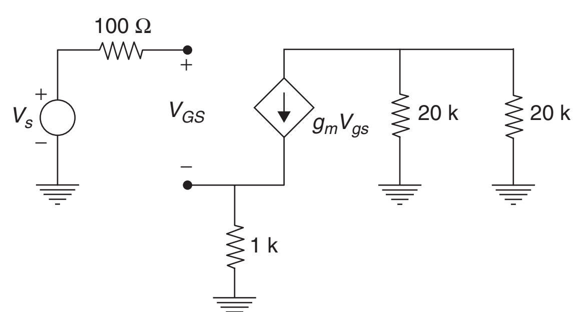
$$r_{ds} = \infty$$

$$f_H = 2.89 \text{ MHz}$$

3. In the MOSFET amplifier shown in the figure below has $\mu = 50$, $r_d = 10$ kΩ, $C_{gs} = 5$ pF, $C_{gd} = 1$ pF and $C_{ds} = 2$ pF. Draw a small signal equivalent circuit for the amplifier for midband frequencies and calculate its midband voltage gain. **[1994]**



Solution:



$\mu = 50$

$r_d = 10$ kΩ

$\mu = g_m r_d = g_m \times 10 \times 10^3 = 50$

$g_m = 5$ mA/V

$V_{in} - V_{gs} + g_m V_{gs} \times 1k$

$V_0 = -g_m V_{gs} (20 \| 20)$

$V_0 = -g_m V_{gs} (10 \text{ k})$

$$\frac{V_0}{V_{in}} = \frac{0 \, g_m V_{gs} [10 \text{ k}]}{V_{gs} + g_m V_{gs} \text{ 1k}} = \frac{-g_m [10 \text{ k}]}{1 + g_m \times 1\text{k}}$$

$$\frac{V_0}{V_{in}} = -8.3$$

Chapter 5

Frequency Response of Amplifier

One-mark Questions

1. The f_T of a BJT is related to its g_m, C_π and $C\mu$ as follows: **[1998]**

(a) $f_T = \dfrac{C_\pi + C_\mu}{g_m}$

(b) $f_T = \dfrac{2\pi(C_\pi + C_\mu)}{g_m}$

(c) $f_T = \dfrac{g_m}{C_\pi + C_\mu}$

(d) $f_T = \dfrac{g_m}{2\pi(C_\pi + C_\mu)}$

Solution: (d)

Unity gain bandwidth product of the *CE* transmitter

$$f_T = \frac{g_m}{2\pi(C_\pi + C_m)}$$
$$= \frac{g_m}{2\pi(C_e + C_c)}$$

Hence, the correct option is (d).

2. An n-p-n transistor has a beta cut-off frequency f_β of 1 MHz and Common Emitter short circuit low frequency current gain β_0 of 200. Its unity gain frequency f_T and the alpha cut-off frequency $f_{\alpha,}$ respectively, are **[1996]**

(a) 200 MHz, 201 MHz
(b) 200 MHz, 1999 MHz
(c) 199 MHz, 200 MHz
(d) 201 MHz, 200 MHz

Solution: (a)

$$f_T = \beta f_\beta$$
$$f_T = 200 \times 1$$
$$= 200 \text{ MHz}$$
$$f_\alpha = \frac{f_\beta}{1-\alpha} = (1+\beta) f_\beta$$
$$f_\alpha = (1+200)$$
$$= 201 \text{ MHz}$$

Hence, the correct option is (a).

3. An amplifier has an open-loop gain of 100 and its lower and upper-cut-off frequency of 100 Hz and 100 kHz, respectively, a feedback network with a feedback fact of 0.99, is connected to the amplifier. The new lower- and upper-cut-off frequencies are at_____ and _____ **[1995]**

Solution: (1 Hz)

$$1 + A\beta = 1 + 100 \times 0.99$$
$$= 1 + 99$$
$$= 100$$
$$f_L^1 = \frac{f_L}{1+A\beta} = \frac{100}{100} = 1 \text{ Hz}$$

4. An RC-coupled amplifier is assumed to have a single-pole low frequency transfer function. The maximum lower-cut-off frequency allowed for the amplifier to pass 50 Hz square wave with no more than 10% tilt is______. **[1995]**

Solution: (1.59 Hz)

$$\% \text{tilt} = \frac{\pi f_L}{f} \times 100\%$$

$$f_L = \frac{f \times \% \text{ tilt}}{\pi \times 100}$$

$$= \frac{50 \times 10}{\pi \times 100}$$

$$f_L = 1.59 \text{ Hz}$$

5. In a multi-stage RC-coupled amplifier the coupling capacitor: **[1993]**

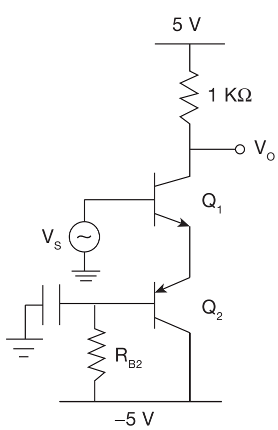
(a) Limits the low frequency response
(b) Limits the high frequency response
(c) Does not affect the frequency response
(d) Blocks the dc components without affecting the frequency response.

Solution: (d)

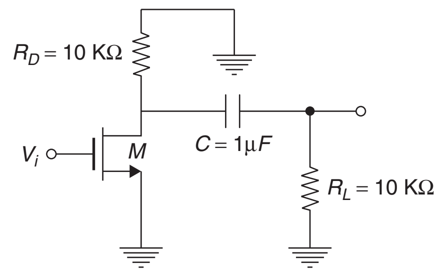
The low frequency of operation of a multistage *RC* coupled amplifier is limited by the coupling capacitor. Hence, the correct option is (d).

TWO-MARKS QUESTIONS

1. In the circuit shown, transistors Q_1 and Q_2 are biased at a collector current of 2.6 mA. Assuming that transistor current gains are sufficiently large to assume collector current equal to emitter current and thermal voltage of 26 mV, the magnitude of voltage gain $\frac{V_0}{V_s}$ in the mid-band frequency range is __________ (upto second decimal place). **[2017]**

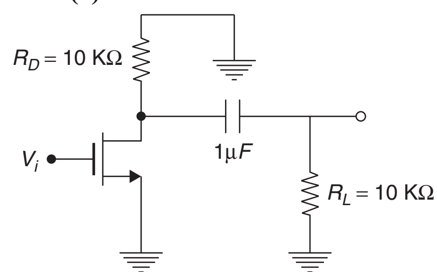


2. The ac schematic of an NMOS common-source stage is shown in the figure below, where part of the biasing circuits has been omitted for simplicity. For the n-channel MOSFET M, the transconductance g_m = 1 mA/V, and body effect and channel length modulation effect are to be neglected. The lower cut-off frequency in Hz of the circuit is approximately at **[2013]**



(a) 8 (b) 32 (c) 50 (d) 200

Solution: (a)



Lower cut off frequency is given by

$$f_L = \frac{1}{2\pi R_C}$$

$$= \frac{1}{2 \times 3.14 \times (10k + 10k) \times 1 \text{ mF}}$$

$$= 8 \text{ Hz}$$

Hence, the correct option is (a).

3. A bipolar transistor is operating in the active region with a collector current of 1 mA. Assuming that the β of the transistor is 100 and the thermal voltage (V_T) is 25 mV, the transconductance (g_m) and the input resistance (r_π) of the transistor in the common emitter configuration are **[2004]**

(a) g_m = 25 mA/V and r_π = 15.625 kΩ
(b) g_m = 40 mA/V and r_π = 4.0 kΩ
(c) g_m = 25 mA/V and r_π = 2.5 kΩ
(d) g_m = 40 mA/V and r_π = 2.5 kΩ

Solution: (d)

$$g_m = \frac{I_C}{V_T}$$

$$= \frac{1 \text{ mA}}{25 \text{ mV}}$$

$$= 0.04$$

$$= 40 \text{ mA/V}$$

$$h_{fe} = gm\ r\pi$$

$$\Rightarrow h_{fe} = \beta$$

$$r\pi = \frac{\beta}{g_m}$$

$$= \frac{100}{40 \times 10^{-3}} = 2.5 \text{ k}\Omega$$

Hence, the correct option is (d).

4. An npn BJT has $g_m = 38$ mA/V, $C_\mu = 10^{-14}$F, $C_\pi = 4 \times 10^{-13}$F, and DC current gain $\beta_0 = 90$. For this transistor f_T and f_β are **[2001]**
(a) $f_T = 1.64 \times 10^8$ Hz and $f_\beta = 1.47 \times 10^{10}$ Hz
(b) $f_T = 1.47 \times 10^{10}$ Hz and $f_\beta = 1.64 \times 10^8$ Hz
(c) $f_T = 1.33 \times 10^{12}$ Hz and $f_\beta = 1.47 \times 10^{10}$ Hz
(d) $f_T = 1.47 \times 10^{10}$ Hz and $f_\beta = 1.33 \times 10^{12}$ Hz

Solution: (a)

$$f_T = \frac{g_m}{2\pi(C_\mu + C_\pi)}$$

$$= \frac{38\times10^{-3}}{2\pi(10^{-14}+4\times10^{-13})}$$

$$= \frac{38\times10^{-3}}{2\pi\times10^{-13}(4.1)}$$

$$= 1.47\times10^{10}\,\text{Hz}$$

$$f_\beta = \frac{f_T}{\beta_0}$$

$$= \frac{1.47\times10^{10}}{90}$$

$$= 1.64\times10^{8}\,\text{Hz} \quad (\beta_0 = h_{fc})$$

Hence, the correct option is (a).

5. An amplifier is assumed to have a single-pole high-frequency transfer function. The rise time of its output response to a step function input is 35 nsec. The upper –3 dB frequency (in MHz) for the amplifier to a sinusoidal input is approximately at **[1999]**
(a) 4.55 (b) 10 (c) 20 (d) 28.6

Solution: (b)

$$tr \times BW = 0.35$$

$$BW = \frac{0.35}{35\times10^{-9}} = 10.$$

Hence, the correct option is (b).

6. An n-p-n transistor (with $C = 0.3$ pF) has a unity – gain cut-off frequency f_T of 400 MHz at a dc bias current I_c = 1 mA. The value of its C_μ (in pF) is approximately (V_T = 26 mV) **[1999]**
(a) 15 (b) 30 (c) 50 (d) 96

Solution: (a)

$$f_T = \frac{1}{2\pi RC}$$

$$\frac{1}{R} = g_m = \frac{I_C}{V_T}$$

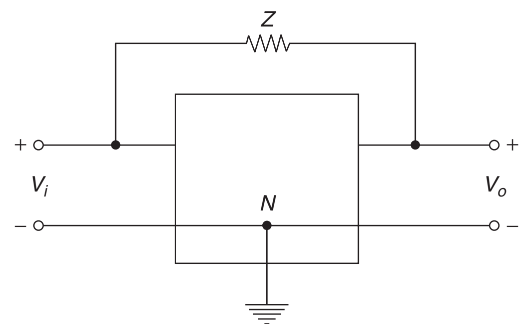
$$f_T = \frac{I_C}{2\pi V_T \times C\mu}$$

$$C\mu = \frac{1\text{ mA}}{2\pi\times 26\text{ mV}\times 400\times10^6}$$

$$C\mu = 15\text{ pF}$$

Hence, the correct option is (a).

7. In the circuit shown in the figure, N is a finite gain amplifier with a gain of K, a very large input impedance, and a very low output impedance. The input impedance of the feedback amplifier with the feedback impedance Z connected as shown will be **[1996]**



(a) $Z\left[1-\frac{1}{K}\right]$

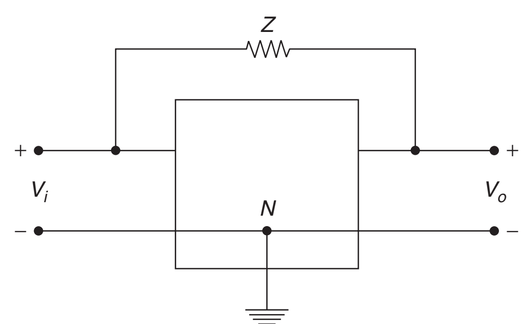
(b) Z(1 – K)

(c) $\left[\frac{Z}{K-1}\right]$

(d) $\left[\frac{Z}{1-K}\right]$

Solution: (d)

Miller theorem states that if a series impedance Z is connected between the input and output terminal of a network, then it can be replaced with a shunt impedance Z_1 in the side and by a shunt impedance Z_2 in the output section of the network



If $V_i > V_2$ then I_1 flows

$$I_1 = \frac{V_1 - V_2}{Z}$$

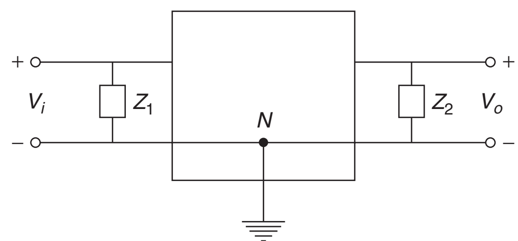
$$= \frac{V_1[1 - V_2 / V_1]}{Z}$$

$$= V_1 \frac{[1 - A_V]}{Z}$$

$$\frac{V_1}{I_1} = Z_1$$

$$= \frac{Z}{1 - A_V}$$

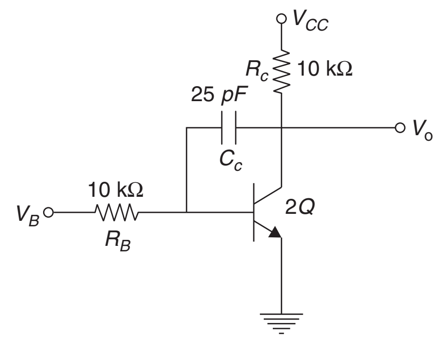
$$Z_1 = \frac{Z}{1 - K}$$



Hence, the correct option is (d).

Five-marks Questions

1. A common-emitter amplifier with an external capacitor C_c connected across the base and the collector of the transistor is shown in figure.



Transistor data $g_m = 5$ mA/V, $r_\pi = 20$ kΩ, $C_\pi = 1.5$ pF and $C_\mu = 0.5$ pF determine the upper cut-off frequency f_H of the amplifier. **[1996]**

Solution: Net feedback, $C_f = C + C_m = 25.5\ \mu$F

Apply Miller's theorem to C_f,

$$K = -g_m \quad R_C = -50$$

$$C_m = C_f(1 - k) = 1300.5 \text{ pF}$$

$$f_H = \frac{1}{2\pi C_m (R_f \parallel r\pi)}$$

$$f_H = 18.38 \text{ kHz}$$

Chapter 6

Feedback Amplifiers

One-mark Questions

1. A good trans impedance amplifier has **[2018]**

(A) low input impedance and high output impedance.
(B) high input impedance and high output impedance.
(C) high input impedance and low output impedance.
(D) low input impedance and low output impedance.

Solution:

Hence, the correct option is (D)

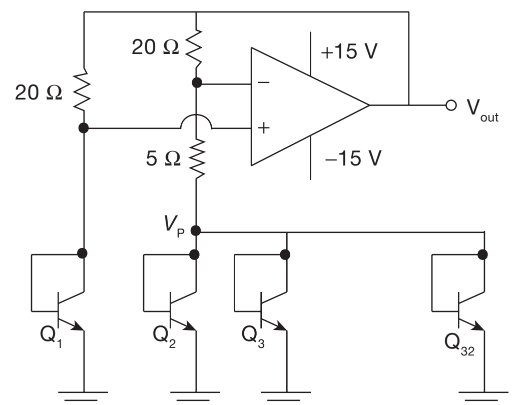
2. A good trans conductance amplifier should have **[2017]**

(A) high input resistance and low output resistance.
(B) low input resistance and high output resistance.
(C) high input and output resistances.
(D) low input and output resistances.

Solution: For a trans conductance amplifier input resistance is high and output resistance is also high. trans conductance amplifier can also be called voltage controlled current source, *i.e.,* vccs. An amplifiers is VC, when input resistance is high and an amplifies is cc when output resistance is high.

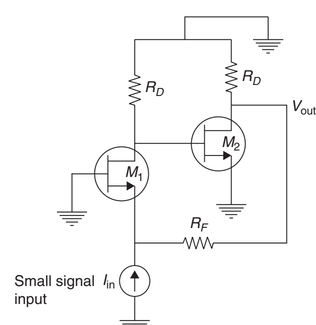
Hence, the correct option is (C).

3. In the voltage reference circuit shown in the figure, the op-amp is ideal and the transistors Q_1, Q_2, …., Q_{32} are identical in all respects and have infinitely large values of common-emitter current gain (β). The Collector current (I_c) of the transistors is related to their base-emitter voltage (V_{BE}) by the relation $I_c = I_s \exp (V_{BE}/V_T)$, where I_s is the saturation current. Assume that the voltage Vp shown in the figure is 0.7 V and the thermal voltage $V_T = 26$ mV. **[2017]**



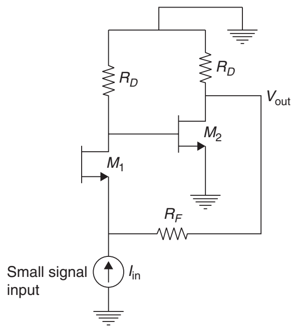
The output voltage V_{out} (in volts) is _____________.

4. In the ac equivalent circuit shown in the figure, if i_{in} is the input current and R_F is very large, the type of feedback is **[2014]**



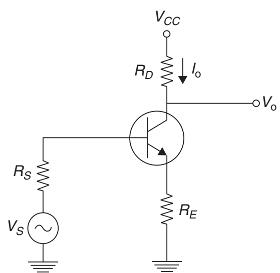
(a) voltage-voltage feedback
(b) voltage-current feedback
(c) current-voltage feedback
(d) current-current feedback

Solution: (b)
Sampling → current
Mixing → voltage



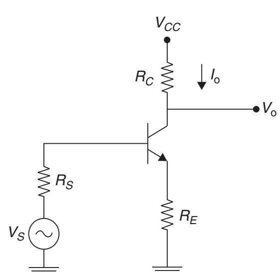
So, the given feedback is of type voltage–current feedback.
Hence, the correct option is (b).

5. The feedback topology in the amplifier circuit (the base bias circuit is not shown for simplicity) in the figure is **[2014]**



(a) Voltage shunt feedback
(b) Current series feedback
(c) Current shunt feedback
(d) Voltage series feedback

Solution: (a)
The feedback topology in the amplifier circuit is current series because

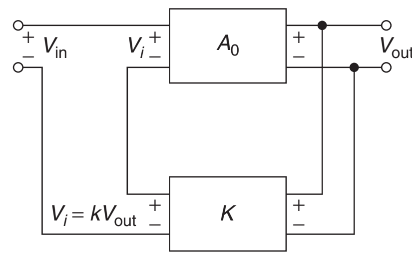


Sampling $\rightarrow$ current
Mixing $\rightarrow$ voltage
So, current series feedback.
Hence, the correct option is (a).

6. The desirable characteristics of a transconductance amplifier are **[2014]**
(a) high input resistance and high output resistance
(b) high input resistance and low output resistance
(c) low input resistance and high output resistance
(d) low input resistance and low output resistance

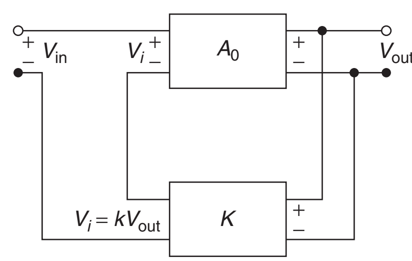
Solution: (a)
The desirable characteristics of a transconductance amplifier are high input resistances and high output resistances.
Hence, the correct option is (a).

7. In a voltage-voltage feedback as shown below, which one of the following statements is TRUE if the gain k is increased? **[2013]**



(a) The input impedance increases and output impedance decreases
(b) The input impedance increases and output impedance also increases
(c) The input impedance decreases and output impedance also decreases
(d) The input impedance decreases and output impedance increases

Solution: (a)
The given configuration is a voltage–series feedback configuration. So, the input impedance increases
$R_{if} = R_i(1 + A_0 k)$ and the output impedance decreases.



$$R_{of} = R_i(1 + A_0 k)$$

Hence, the correct option is (a).

8. In a transconductance amplifier, it is desirable to have **[2007]**
(a) a large input resistance and a large output resistance
(b) a large input resistance and a small output resistance
(c) a small input resistance and a large output resistance
(d) a small input resistance and a small output resistance

Solution: (a)
Transconductance amplifier has voltage at input end and current at output end. Thus, input resistance must be high and output resistance must be high.
Hence, the correct option is (a).

9. The input impedance (Z_i) and the output impedance (Z_o) of an ideal trans-conductance (voltage controlled current source) amplifier are **[2006]**

(a) $Z_i = 0, Z_0 = 0$
(b) $Z_i = 0, Z_0 = \infty$
(c) $Z_i = \infty, Z_0 = 0$
(d) $Z_i = \infty, Z_0 = \infty$

Solution: (d)
For a voltage controlled current source input is voltage. So input impedance is to be as high as possible output is current so output impedance is to be kept as high as possible so that full current can go to load

i. e., $Z_i = \infty$

$$Z_0 = \infty$$

Hence, the correct option is (d).

10. The effect of current shunt feedback in an amplifier is to **[2005]**

(a) increase the input resistance and decrease the output resistance
(b) increase both input and output resistances
(c) decrease both input and output resistances.
(d) decrease the input resistance and increase the output resistance

Solution: (a)
With current shunt negative feedback,

$$R_{if} = \frac{R_i}{1+A\beta}$$

$$\text{and} \quad R_{of} = R_0(1+A\beta)$$

Hence, the correct option is (a).

11. Voltage series feedback (also called series-shunt feedback) results in **[2004]**

(a) increase in both input and output impedances
(b) decrease in both input and output impedances
(c) increase in input impedance and decrease in output impedance
(d) decrease in input impedance and increase in output impedance

Solution: (c)
Voltage series feedback results in increase in input impedance and decrease in output impedance,

$$R_{if} = R_i(1+A\beta)$$

and

$$R_{of} = \frac{R_0}{1+A\beta}$$

Hence, the correct option is (c).

12. In a negative feedback amplifier using voltage-series (i.e. voltage-sampling, series mixing) feedback. **[2002]**

(a) R_i decreases and R_o decreases
(b) R_i decreases and R_o increases
(c) R_i increases and R_o decreases
(d) R_i increases and R_o increases

(R_i and R_o denote the input and output resistances, respectively)

Solution: (c)
Voltage – series negative feedback increases input impedances and decreases output impedance,

$$R_{if} = R_i(1+A\beta)$$

$$R_{of} = \frac{R_0}{1+A\beta}$$

Hence, the correct option is (c).

13. Negative feedback in an amplifier **[1999]**

(a) reduces gain.
(b) increases frequency and phase distortions.
(c) reduces bandwidth.
(d) increases noise.

Solution: (a)

$$A_f = \frac{A}{1+A\beta} \quad \text{(for −ve feedback)}$$

Thus, gain reduces.
Hence, the correct option is (a).

14. In a shunt–shunt negative feedback amplifier, as compared to the basic amplifier. **[1998]**

(a) both input and output impedance decrease.
(b) input impedance decreases but output impedance increases.
(c) input impedance increases but output impedance decreases.
(d) both input and output impedance increase.

Solution: (a)
In a shunt–shunt negative feedback amplifier, as compared to the basic amplifier, both input and output impedances decrease

$$R_{if} = \frac{R_i}{1+A\beta}$$

$$R_{of} = \frac{R_0}{1+A\beta}$$

Hence, the correct option is (a).

15. To obtain very high input and output impedances in a feedback amplifier, the mostly used is **[1995]**

(a) voltage-series
(b) current-series

(c) voltage-shunt
(d) current-shunt

Solution: (b)
Current series feedback amplifiers have very high input and very high output impedances

$$R_{if} = R_i(1+A\beta) = R_i(1+Gm\beta)$$
$$R_{of} = R_0(1+A\beta) = R_0(1+Gm\beta)$$

Hence, the correct option is (b).

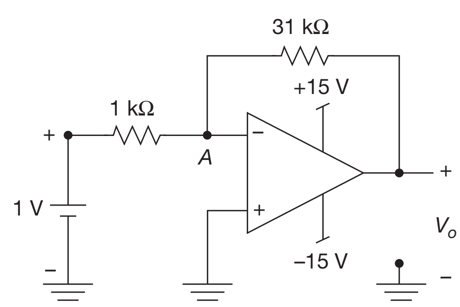
16. Negative feedback in amplifiers **[1993]**

(a) improves the signal to noise ratio at the input
(b) improves the signal to noise ratio at the output
(c) does not affect the signal to noise ratio at the output
(d) reduces distortion

Solution: (a, d)
Negative feedback in amplifiers
(b) improves the signal to noise ratio at the output
(d) reduces distortion.
Hence, the correct options are (a and d).

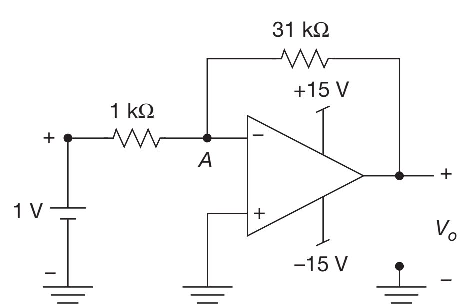
Two-marks Questions

1. An op-amp based circuit is implemented as shown below.



In the above circuit, assume the op-amp to be ideal. The voltage (in volts, correct to one decimal place) at node A, connected to the negative input of the op-amp as indicated in the figure is ________. **[2018]**

Solution: Consider the figure given below

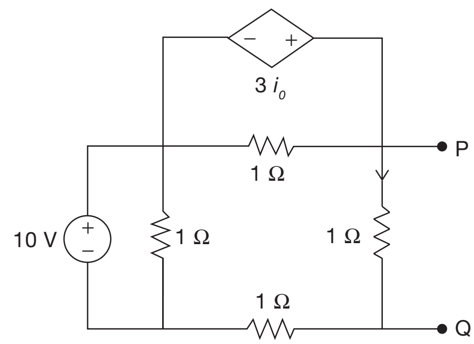


If we apply Kirchhoff's current law in above circuit we get

$$\frac{1-V_A}{1} = \frac{V_A - V_0}{31}$$

The required voltage is $V_A = 0.5$ V
Hence, the correct answer is 0.4 to 0.6.

2. Consider the circuit shown in the figure.



The thevenin equivalent resistance (in Ω) across P-Q is __________. **[2017]**

3. An amplifier without feedback has a voltage gain of 50, input resistance of 1 kΩ and output resistance of 2.5 kΩ. The input resistance of the current-shunt negative feedback amplifier using the above amplifier with a feedback factor of 0.2 is **[2003]**

(a) 1/11 kΩ (b) 1/5 kΩ
(c) 5 kΩ (d) 11 kΩ

Solution: (a)

$$R_{if} = \frac{R_i}{1+A\beta}$$
$$\beta = 0.2, \quad A = 50$$
$$\Rightarrow R_{if} = \frac{1k}{1+50\times 0.2} = \frac{1}{11}\,\text{k}\Omega$$

Hence, the correct option is (a).

4. An amplifier has an open-loop gain of 100. An input impedance of 1 kΩ, and an output impedance of 100 Ω.A feedback network with a feedback factor of 0.99 is connected to the amplifier in a voltage series feedback mode. The new input and output impedances, respectively, are **[1999]**

(a) 10 Ω and 1 Ω
(b) 10 Ω and 10 Ω
(c) 100 kΩ and 1 Ω
(d) 100 kΩ and 1 kΩ

Solution: (d)
Feedback mode is voltage series – input impedances increases. Output impedance decreases.

$$Z_{if} = Z_i(1 + A\beta)$$

$$Z_{of} = \frac{Z_0}{1 + A\beta} = \frac{100}{1+99} = 1\Omega$$

$$A = 100, \quad \beta = 0.99, \quad Z_i = 1\,\text{k}\Omega, \quad Z_0 = 100\,\Omega$$

$$Z_{if} = 1k(1+99) = 100\text{ k }\Omega$$

Hence, the correct option is (d).

5. Negative feedback in
(1) voltage series configuration
(2) current shunt configuration **[1997]**
(a) increases input impedance
(b) decreases input impedance
(c) increases closed loop gain
(d) leads to oscillation.

Solution: (a)
Voltage series configuration – increases the impedance $R_{if} = R_i(1 + A\beta)$

Current shunt configuration – decreases the input impedances $R_{if} = \dfrac{R_i}{1 + A\beta}$

Hence, the correct option is (a).

6. Two non-inversing amplifiers, one having a unity gain and the other having a gain of twenty, are made using identical operational amplifiers. As compared to the unity gain amplifier, the amplifier with gain twenty has **[1991]**
(a) less negative feedback
(b) greater input impedance
(c) less bandwidth
(d) none of the above.

Solution: (c)
For identical operational amplifier, gain-bandwidth product is constant i.e.,

$$G \times BW = \text{constant}$$

$$A_1 \times BW_1 = A_2 \times BW_2$$

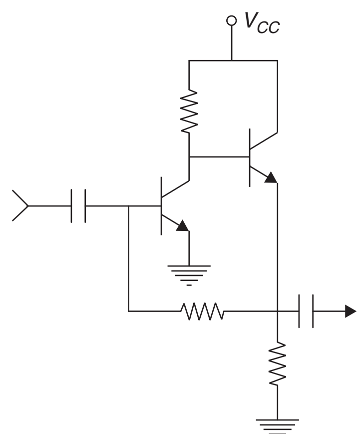
$$BW_2 = \frac{A_1 \times BW_1}{A_2} = \frac{1 \times BW_1}{20}$$

$$= \frac{BW_2}{20}$$

So, as compared to the unity gain amplifier with gain twenty has less BW.
Hence, the correct option is (c).

7. The feedback amplifier shown in the figure has: **[1989]**
(a) current-series feedback with large input impedance and large output impedance.
(b) voltage-series feedback with large input impedance and low output impedance.
(c) voltage-shunt feedback with low input impedance and low output impedance.
(d) current-shunt feedback with low input impedance and output impedance.

Solution: (c)



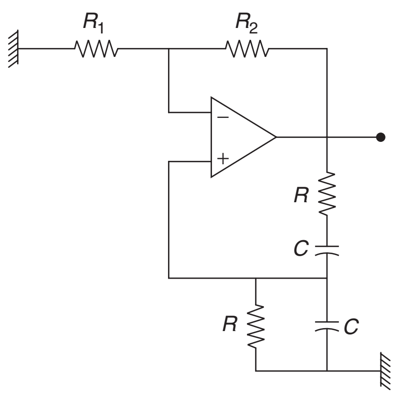
Emitter is output node, so it is voltage sampler voltage shunt feedback.
Hence, the correct option is (c).

Chapter 7

Oscillator Circuits

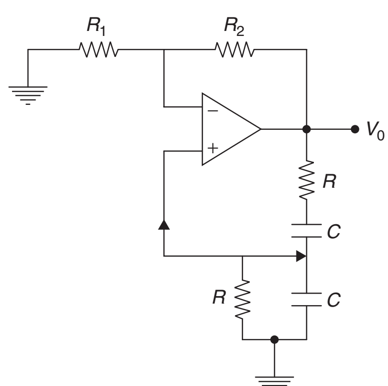
One-mark Questions

1. The configuration of the figure is a **[2000]**



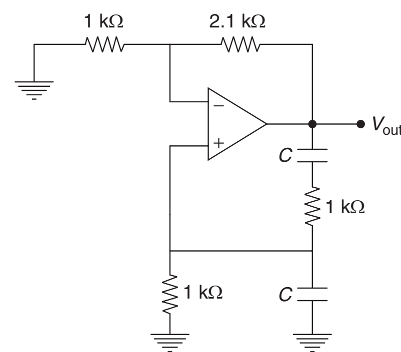
(a) Precision integrator
(b) Hartley oscillator I
(c) Butterworth high pass filter
(d) Wien-bridge oscillator

Solution: (d)
The given configuration is a Wien-bridge oscillator.



Two-marks Questions

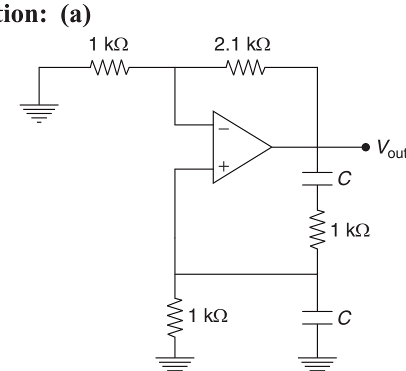
1. The value of C required for sinusoidal oscillations of frequency 1 kHz in the circuit of the figure is **[2004]**



(a) $\dfrac{1}{2\pi}\mu F$ (b) $2\pi\mu$F

(c) $\dfrac{1}{2\pi\sqrt{6}}\mu F$ (d) $2\pi\sqrt{6}\mu F$

Solution: (a)



Given circuit is a Wien-bridge oscillator oscillation frequency given by

$$W = \frac{1}{RC}$$

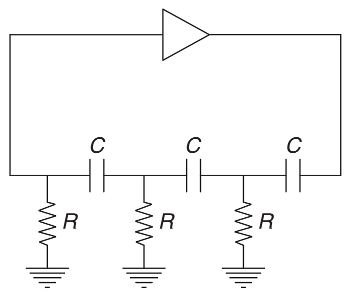
$$\Rightarrow C = \frac{1}{WC}$$

$$= \frac{1}{2\pi 10^3 \times 1 \times 10^3}$$

$$= \frac{1}{2\pi}\mu\text{F}$$

Hence, the correct option is (a).

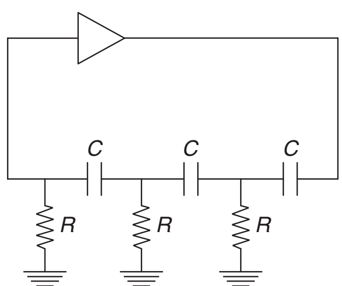
2. The oscillator circuit shown in the figure has an ideal inverting amplifier. Its frequency of oscillation (in Hz) is **[2003]**



(a) $\dfrac{1}{(2\pi\sqrt{6}RC)}$ (b) $\dfrac{1}{(2\pi RC)}$

(c) $\dfrac{1}{(\sqrt{6}RC)}$ (d) $\dfrac{1}{\sqrt{6}(2\pi RC)}$

Solution: (a)



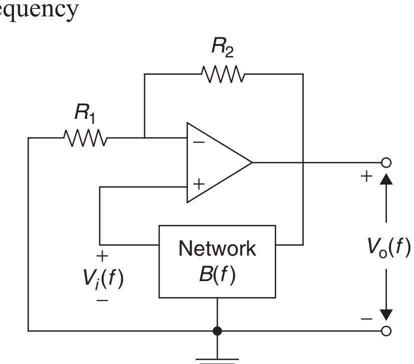
Frequency of oscillation for this oscillator is

$$f = \frac{1}{2\pi\sqrt{6}RC}$$ where RC = phase shift oscillator.

Hence, the correct option is (a).

3. The circuit in the figure employs positive feedback and is intended to generate sinusoidal oscillation. If at a frequency f_0,

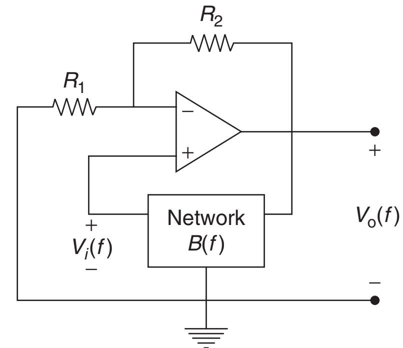
$$B(f) = \frac{V_f(f)}{V_0(f)} = \frac{1}{6}\angle 0^\circ$$ then to sustain oscillation at this frequency **[2002]**



(a) $R_2 = 5R_1$ (b) $R_2 = 6R_1$

(c) $R_2 = \dfrac{R_1}{6}$ (d) $R_2 = \dfrac{R_1}{5}$

Solution: (a)



$$\beta = \frac{1}{6}$$

and $$\beta = \frac{R_1}{R_1 + R_2}$$

$$\frac{\beta V_0 - 0}{R_1} + \frac{\beta V_0 - V_0}{R_2} = 0$$

$$\Rightarrow \beta\left(\frac{R_1 + R_2}{R_1 R_2}\right) = \frac{1}{R_2}$$

$$\beta = \frac{R_1}{R_1 + R_2}$$

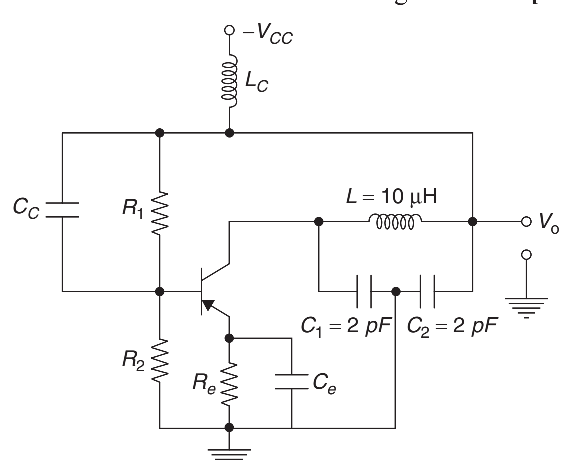
$$\frac{1}{6} = \frac{R_1}{R_1 + R_2}$$

$$\Rightarrow R_1 + R_2 = 6R_1$$

$$\Rightarrow R_2 = 5R_1$$

Hence, the correct option is (a).

4. The oscillator circuit shown in the figure is **[2001]**

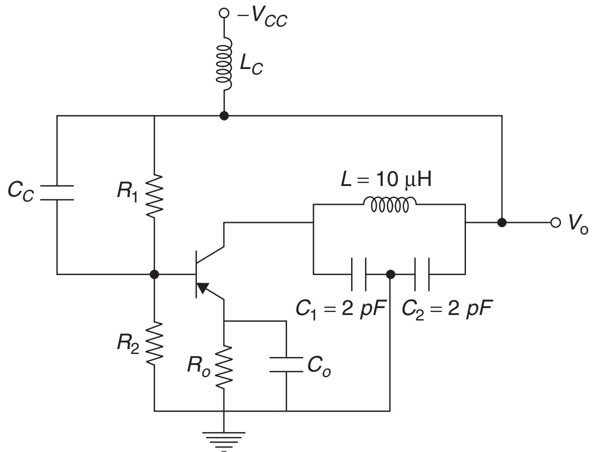


(a) Hartley oscillator with $f_{\text{oscillation}} = 79.6$ MHz

(b) Colpitts oscillator with $f_{\text{oscillation}} = 50.3$ MHz

(c) Hartley oscillator with $f_{\text{oscillation}}$ = 159.2 MHz
(d) Colpitts oscillator with $f_{\text{oscillation}}$ = 159.2 MHz

Solution: (b)
Given oscillator is a Colpitts oscillator



$$f = \frac{1}{2\pi\sqrt{LCer}}$$

$$Cer = \frac{C_1C_2}{C_1 + C_2} = \frac{2\times 2}{4}$$

$$= 1pf$$

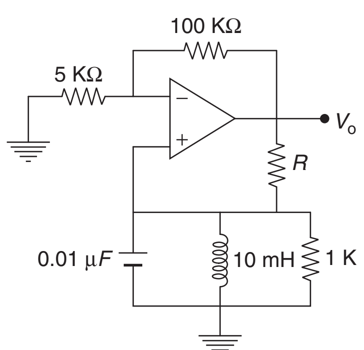
$$f = \frac{1}{2\pi\sqrt{10\times 10^{-6}\times 10^{-12}}}$$

$$= \frac{1\times 10^{9}}{2\pi\sqrt{10}} = 50.3 \text{ MHz}$$

Hence, the correct option is (b).

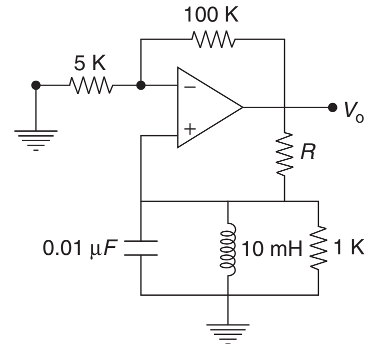
5. Value of R in the oscillator shown in the given figure is chosen in such a way that it just oscillates at an angular frequencies of 'ω'. The value of 'ω' and the required value of R will, respectively, be

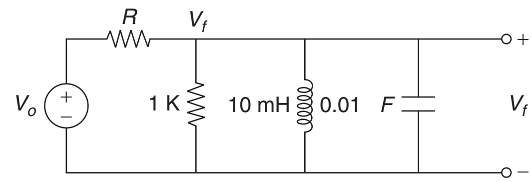
[1996]



(a) 10^5 rad/sec, $2 \times 10^4\ \Omega$
(b) 2×10^4 rad/sec, $2 \times 10^4\ \Omega$
(c) 2×10^4 rad/sec, $10^5\ \Omega$
(d) 10^5 rad/sec, 10^5 Q

Solution: (b)





$$\omega = \frac{1}{\sqrt{LC}}$$

$$\omega = \frac{1}{\sqrt{10\times 10^{-3}\times 0.01\times 10^{-6}}}$$

$$\omega = 10^5 \text{ rad/sec}$$

Apply nodal analysis

$$\frac{V_f - V_0}{R} + \frac{V_f}{1K} + \frac{V_f}{LS} + V_f CS = 0$$

$$\Rightarrow V_t\left[\frac{1}{R} + \frac{1}{1K}\right] + V_f\left[WC - \frac{1}{WL}\right] = \frac{V_0}{R}$$

$$\text{for oscillation, } WC - \frac{1}{WL} = 0$$

$$V_f\left[\frac{1}{R} + \frac{1}{1K}\right] = \frac{V_0}{R}$$

$$\Rightarrow \beta = \frac{V_f}{V_0} = \frac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{K}}$$

$$A = 1 + \frac{R_f}{R_i} = 1 + \frac{100k}{5k} = 21$$

$$\beta = \frac{1}{A} = \frac{1}{21}$$

$$= \frac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{1K}} = \frac{\frac{1}{R}}{\frac{R + 1K}{R\times 1K}}$$

$$R \pm 1k = 21\times 1k$$

$$R = 20\ \text{k}\Omega$$

$$= 2\times 10^{4}\,\Omega$$

Hence, the correct option is (b).

6. Match the following **[1994]**

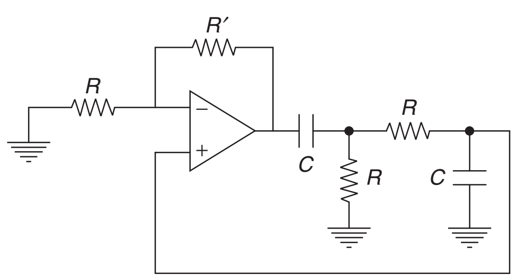
List - I	List - II
(a) Hartley	(1) Low-frequency oscillator
(b) Wien-bridge	(2) High-frequency oscillator
(c) Crystal	(3) Stable-frequency oscillator
	(4) Relaxation oscillator
	(5) Negative Resistance oscillator

Solution: (a-2, b-1, c-3)

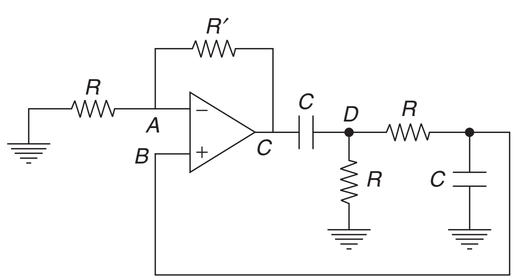
(a) Hartley oscillator	(2) High-frequency oscillator
(b) Wien-bridge oscillator	(1) Low-frequency oscillator
(c) Crystal oscillator	(3) Stable-frequency oscillator

Five-marks Questions

1. Find the value of R' in the circuit of figure. For generating sinusoidal oscillations. Find the frequency of oscillations. **[1998]**



Solution:



By virtual short

$$V_B = V_A$$

Apply KCL at node A

$$\frac{V_A - O}{R} + \frac{V_A - V_C}{R'} = 0$$

$$V_C = V_A\left(1 + \frac{R'}{R}\right) \quad (1)$$

Apply KCL at node B.

$$V_A = V_B$$

$$V_C = V_B\left(1 + \frac{R'}{R}\right) \quad (2)$$

$$\frac{V_B - O}{\frac{1}{SC}} + \frac{V_B - V_D}{R} = 0$$

$$V_D = V_B(1 + RCs) \quad (3)$$

Apply KCL at node D

$$\frac{V_D - V_C}{\frac{1}{SC}} + \frac{V_D}{R} + \frac{V_D - V_B}{R} = 0$$

$$V_D(2 + RCS) = V_C RCS + V_B \quad (4)$$

From equations (1), (3) and (4) we have

$$V_B(1 + RCS)(2 + RCS) = V_B\left(1 + \frac{R^1}{R}\right)(RCS) + V_B$$

$$2 + 3RCS + R^2C^2S^2 - \left(1 + \frac{R^1}{R}\right)RCS - 1 = 0$$

$$1 + R^2C^2S^2 + \left(2 - \frac{R^1}{R}\right)RCS = 0$$

So for oscillations.

$$2 - \frac{R^1}{R} = 0$$

$$R^1 = 2R$$

for frequency of oscillation

$1 + R^2C^2S^2 = 0$

Put $S = j\omega$

$1 - R^2C^2\omega^2 = 0$

$$\omega = \frac{1}{RC}$$

Chapter 8

Power Amplifiers

One-mark Questions

1. Crossover distortion behaviour is characteristic of **[1999]**

(a) Class A output stage
(b) Class B output stage
(c) Class AB output stage
(d) Common-base output stage

Solution: (b)
Crossover distortion behaviour is a characteristic of class B output stage amplifier and this distortion can be reducing by adopting push-pull configuration.
Hence, the correct option is (b).

2. A power amplifier delivers 50 W output at 50% efficiency. The ambient temperature is 25 °C. If the maximum allowable junction temperature is 150° C, then the maximum thermal resistance θg_c that can be tolerated is______ **[1995]**

Solution: (2.5)
Power dissipated $(P_D) = 50 \times 50\%$

$$\frac{P_o}{P_{in}} = \frac{1}{2}$$

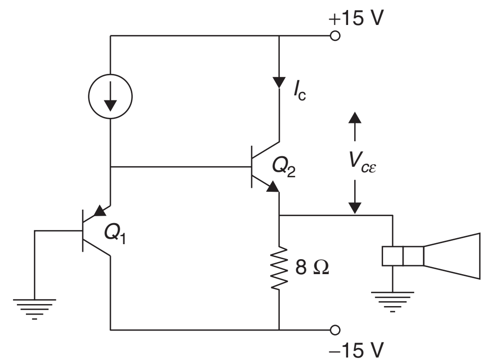
$$P_{in} = 2P_o = 2 \times 50 = 100\ W$$

$$P_D = P_{in} - P_o = \frac{T_j - T_A}{\theta}$$

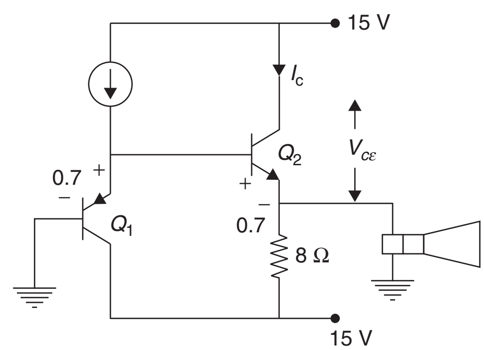
$$\theta = \frac{T_j - T_A}{P_D} = \frac{150 - 25}{50} = \frac{125}{50}$$

$$\theta = 2.5\ °\text{C/W}$$

3. The circuit shown in the figure supplies power to on 8 Ω speaker, LS. The values of I_C and V_{CE} for this circuit will be I_C = ____ and V_{CE} **[1995]**



Solution: (15 V)



$$I_C = \frac{15}{8} A$$

$$V_{c\varepsilon} = 15\text{ V}$$

$$V_{\varepsilon_1 B_1} = 0.7\text{ V}$$

$$V_{\varepsilon_1} - V_{B_1} = 0.7\text{ V}$$

$$V_{\varepsilon_1} = 0.7\text{ V} \quad [\because VB_1 = 0]$$

$$V_{\varepsilon_1} = V_{B_2} = 0.7\text{ V}$$

$$V_{B_2\varepsilon_2} = 0.7\text{ V}$$

$$V_{B_2} - V_{\varepsilon_2} = 0.7\text{ V}$$

$$0.7 - V_{\varepsilon_2} = 0.7$$

$$V_{\varepsilon_2} = 0\text{ V}$$

Apply KVL at emitter terminal

$$0 = I_\varepsilon R_\varepsilon - 15$$
$$I_\varepsilon = \frac{15}{Re} = \frac{15}{8}$$
$$V_{c\varepsilon} = V_C - V_\varepsilon = 15 - 0$$
$$= 15\,\text{V}$$

4. A Class A transformer coupled, transistor power amplifier is required to deliver a power output of 10 watts. The maximum power rating of the transistor should not be less than **[1994]**

(a) 5 W (b) 10 W (c) 20 W (d) 40 W

Solution: (c)

$$\frac{P_{D\max}}{P_{AC\max}} = 2$$

For class A transformer coupled amplifier, $P_{D\max} = 2 \times P_{AC\max} = 20\,\text{W}$ $\therefore$ Power rating $\geq 20\,\text{W}\cdot$

5. In a transistor push-pull amplifier **[1993]**

(a) there is no dc present in the output
(b) there is no distortion in the output
(c) there is no even harmonics in the output
(d) there is no odd harmonics in the output

Solution: (c)

The output of push pull amplifier consists of only odd harmonics terms.

$$I_o = 2k[B_1 \cos wt + B_3 \cos 3wt + B_5 \cos 5wt + \ldots]$$

Hence, the correct option is (c).

Two-marks Questions

6. In case of class A amplifiers the ratio (efficiency of transformer coupled amplifier)/(efficiency of a transformer less amplifier) is **[1987]**

(a) 2.9 (b) 1.36 (c) 1.0 (d) 0.5

Solution: (a)

Class A amplifier ratio

$$= \frac{\text{efficiency of transformer coupled amplifier}}{\text{efficiency of transformer less amplifier}}$$

$$= \frac{50\%}{25\%} = 2$$

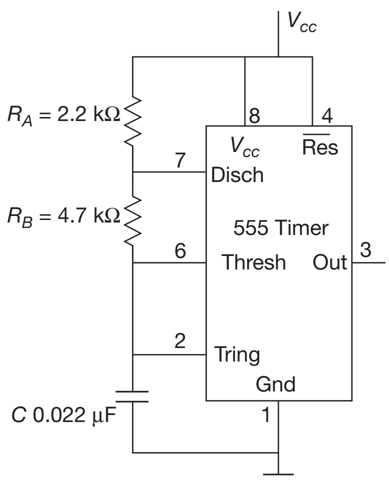
Hence, the correct option is (a).

Chapter 9

Multi-vibrators and 555 Timers

One-mark Questions

1. In the Astable multivibrator circuit shown in the figure, the frequency of oscillation (in kHz) at the output pin 3 is _____. **[2016]**



Solution: Resistance $R_A = 2.2\ \text{k}\Omega$

Resistance $R_B = 4.7\ \text{k}\Omega$

Capacitance $C = 0.022\ \mu f$

For Astable multivibrator frequency can be expressed as

$$f = \frac{1}{0.69(R_A + 2R_B)C}$$

$$= \frac{1}{0.69 \times 11.6 \times 10^3 \times 0.022 \times 10^{-6}}$$

= 5.67 kHz

Hence, the correct Answer is (5.67 kHz).

2. Consider the following two statements:

Statement 1: A stable multi-vibrator can be used for generating square wave.

Statement 2: Bi-stable multi-vibrator can be used for storing binary information. **[2001]**

(a) Only statement 1 is correct
(b) Only statement 2 is correct
(c) Both the statements 1 and 2 are correct
(d) Both the statements 1 and 2 are incorrect

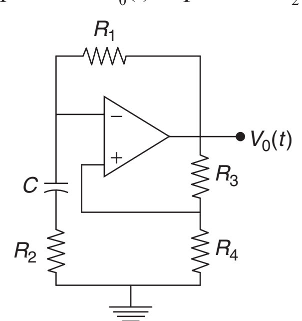
Solution: (c)

A stable multi-vibrator can be used for generating square wave Bistable multi-vibrator can be used for storing binary information.

Hence, the correct option is (c).

Two-marks Questions

1. In the following astable multi-vibrator circuit, which properties of $v_0(t)$ depend on R_2?

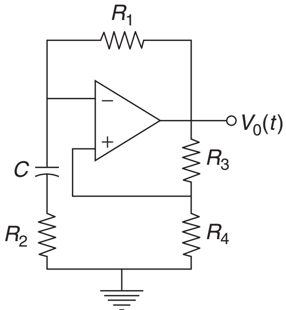


[2009]

(a) Only the frequency
(b) Only the amplitude
(c) Both the amplitude and the frequency
(d) Neither the amplitude nor the frequency

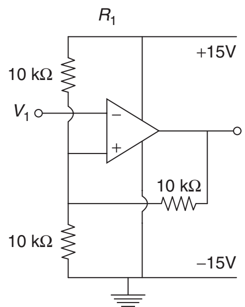
Solution: (a)

In the given a stable multi-vibrator circuit, only the frequency of $V_0(t)$ depends on R_2.



Hence, the correct option is (a).

2. Consider the Schmidt trigger circuit shown below:

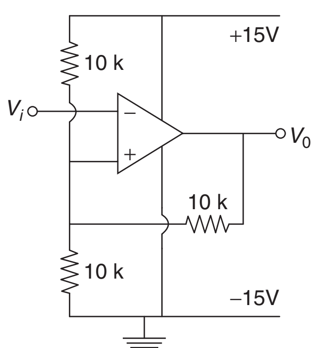


A triangular wave which goes from –12 V to 12 V is applied to the inverting input of the op-amp. Assume that the output of the op-amp swings from +15 V to –15 V. The voltage at the non-inverting input switches between **[2008]**

(a) –12 V and +12 V
(b) –7.5 and +7.5 V
(c) –5 V and +5 V
(d) 0 V and 5 V

Solution: (c)

Let V_A be the voltage at non-inverting terminal KCL at this node,



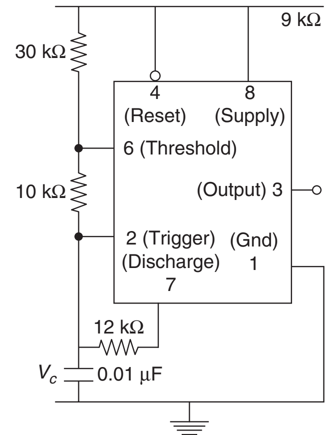
$$\frac{V_A - 15}{10k} + \frac{V_A + 15}{10k} + \frac{V_A - V_0}{10k} = 0$$

$$\Rightarrow V_A = \frac{V_0}{3}$$

Since output voltage swings between +15 V and –15 V, V_A swings between +5 V and –5 V.

Hence, the correct option is (c).

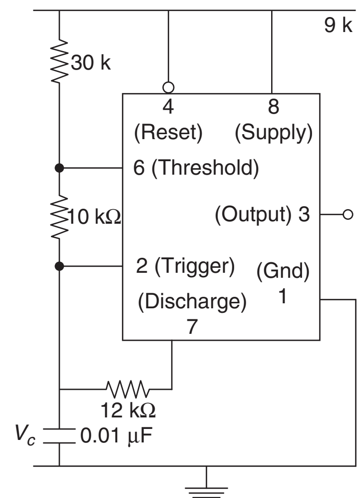
3. An astable multi-vibrator circuit using IC 555 timer is shown below. Assume that the circuit is oscillating steadily.



The voltage V_C across the capacitor varies between **[2008]**

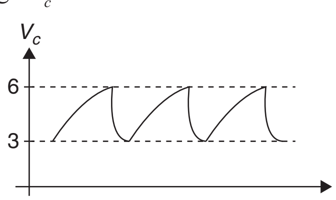
(a) 3 V and 5 V
(b) 3 V and 6 V
(c) 3.6 V and 6 V
(d) 3.6 V and 5 V

Solution: (b)



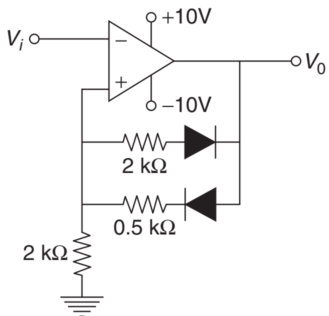
The voltage V_C varies between V_{ce} to $\frac{2V_{cc}}{3}$ given that, $V_{cc} = 9$ V

∴, voltage V_c varies between 3 V and 6 V.

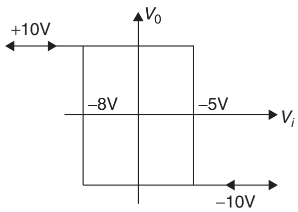


Hence, the correct option is (b).

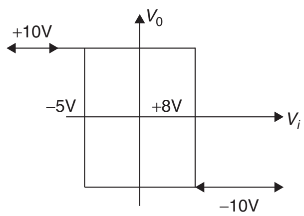
4. Given the ideal operational amplifier circuit shown in the figure indicates the correct transfer characteristics assuming ideal diodes with zero cut-in voltage.



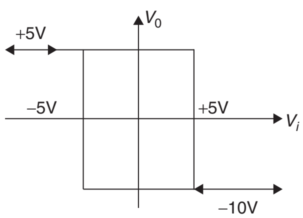
(a)



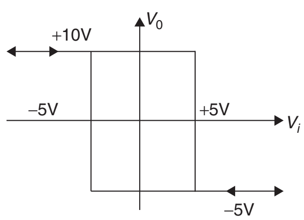
(b)



(c)

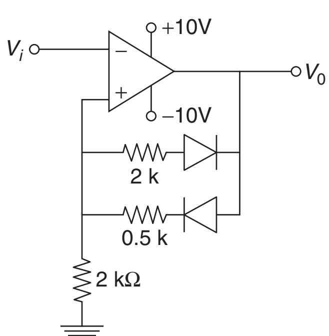


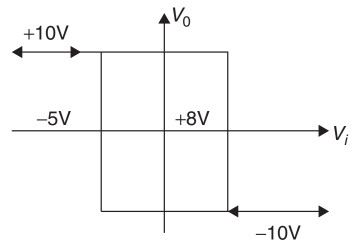
(d)



[2005]

Solution: (a)
When lower diode is ON,





$$V_{0_T} = 10 \times \frac{2}{2+0.5} = 8\text{V}$$

$\Rightarrow$ if $V_i > 8$ then V_0 is negative

When upper diode is ON,

$$V_{\varepsilon_1} = V_{B_2} = 0.7\,\text{V}$$

Hence, the correct option is (a).

5. An ideal saw-tooth voltage waveform of frequency 500 Hz and amplitude 3 V is generated by charging a capacitor of 2 μF in every cycle. The charging requires **[2003]**

(a) constant voltage source of 3 V for 1 ms
(b) constant voltage source of 3 V for 2 ms
(c) constant current source of mA for 1 ms
(d) constant current source of 3 mA for 2 ms

Solution: (d)

$$0 = I_\varepsilon R_\varepsilon - 15$$

$$I_\varepsilon = \frac{15}{Re} = \frac{15}{8}$$

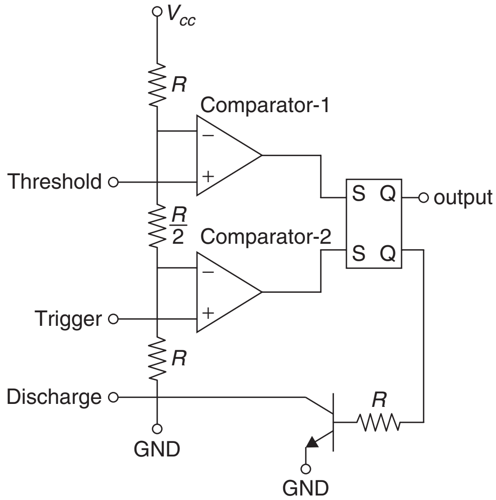
$$V_{c\varepsilon} = V_C - V_\varepsilon = 15 - 0$$

$$= 15\,\text{V}$$

Hence, the correct option is (d).

Five-mark Questions

1. Implement a monostable multi-vibrator using the timer circuit shown in figure. Also determine an expression for ON time T of the output pulse. **[1998]**



Solution: $$V_{th} = V_{CC}\left[\frac{R+\frac{R}{2}}{R+\frac{R}{2}+R}\right] = \frac{3}{5}V_{CC}$$

$$V_{trig} = \frac{2V_{CC}}{5}$$

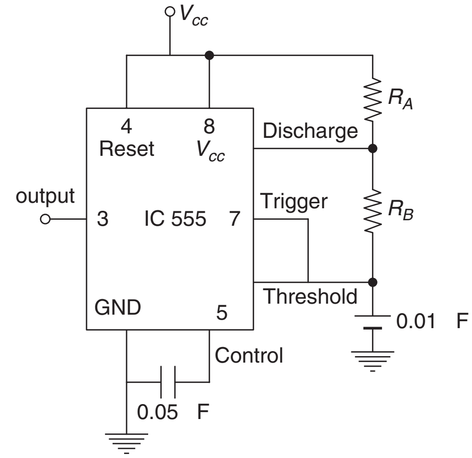
T_{on} = time to change from 0 to $\frac{3\,V_{CC}}{5}$ $\frac{3V_{CC}}{5} = V_{CC}|$ $[1-e^{-ti/R_AC_A}]$

$$e^{-ti/R_AC_A} = 1-\frac{3}{5} = 0.4$$

$$\frac{-ti}{R_AC_A} = \ln(0.4) = -0.916$$

$$T_{on} = 0.916\,R_AC_A$$

2. An IC-555 chip has been used to construct a pulse generator. Typical pin connections with components is shown below is figure for such an application. However it is desired to generate a square pulse of 10 kHz.



Evaluate values of R_A and R_B if the capacitor has the values of 0.01 μF for the configuration chosen. If necessary you can suggest modifications in the external circuit configuration. **[1997]**

Solution: $f = 10$ kHz

$$T = \frac{1}{f} = 100\ \mu\text{ sec}$$

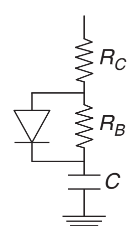
$T_{on} + T_{off} = 100\ \mu$ sec

$$T_{off} = T_{on} = \frac{100}{2} = 50\ \mu\text{ sec}$$

$T_H = 0.7\,(R_A + R_B)C$
$T_L = 0.7\,R_BC$

Since it is not possible to get square wave with this configuration.

∴ Diode is connected across R_B



Since Diode on during charging and diode off during discharging.

$T_H = 0.7\,R_{AC}$
$T_C = 0.7\,R_{AC}$

$$R_A = R_B = \frac{50\times10^{-6}}{0.7\times10^{-8}} = 7.14\text{k}\Omega$$

UNIT VI

DIGITAL CIRCUITS

EXAM ANALYSIS

Exam Year	92	93	94	95	96	97	98	99	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14-1	14-2	14-3	14-4	15			16			17		18	19
																											Set 1	Set 2	Set 3	Set 1	Set 2	Set 3	Set 1	Set 2		
1 Marks Ques.	4	4	6	6	1	9	9	4	3	3	3	5	6	2	-	2	-	1	2	3	4	1	1	4	2	2	1	2	1	2	7	2	2	2	3	3
2 Marks Ques.	-	-	1	-	-	-	1	1	4	4	3	7	7	5	6	7	8	6	2	2	1	2	2	1	3	4	3	2	1	7	4	4	3	3	3	3
5 Marks Ques.	-	1	-	2	4	1	1	1	1	1	1																									
Total Marks	**4**	**09**	**08**	**16**	**21**	**14**	**16**	**17**	**16**	**16**	**21**	**19**	**20**	**12**	**12**	**16**	**16**	**13**	**6**	**7**	**6**	**5**	**5**	**6**	**8**	**10**	**7**	**6**	**3**	**16**	**15**	**10**	**8**	**8**	**9**	**9**
Chapter wise marks distribution																																				
Number Systems	-	1	-	-	-	1	1	-	-	1		1	-	5	1	2	1	2	-	-	-	-	-	-	1	-		0			1		0		0	0
Boolean Algebra	-	-	-	-	-	-	2	3	-	-		-	3	2	2	-	3	-	2	-	-	1	2	3	1	-		2			0		1		1	0
Logic Gates	-	-	2	2	1	-	2	2	3	3	3	-	3	2	2	-	2	-	2	1	-	-	1	2	-	-		5			2		1		0	2
Combinational Circuits	1	-	-	1	-	1	-	2	-	2		2	5	2	1	2	2	2	6	2	1	1	-	-	3	3		1			4		5		1	0
Sequential Circuits	1	1	1	2	-	1	3	2	4	2		-	3	4	4	4	4	4	4	2	5	3	-	-	1	5		4			3		0		1	1
Logic Families	1	-	2	-	-	4	2	2	-	2	-	3	1	4	-	2	2	1	-	-	1	-	-	-	-	1		0			1		2		2	0
Memories	1	-	2	1	1	-	-	2	-	3	-	2	-	2	-	-	-	-	-	-	-	2	-	-	2	-		0			0		2		1	0
ADC and DAC	-	-	1	1	-	-	1	1	4	-	1	3	1	-	2	4	4	-	-	-	-	-	-	-	-	-		1			1		0		0	0

Chapter 1

Number Systems

One-mark Questions

1. The number of bytes required to represent the decimal number 1856357 in packed BCD (Binary Coded Decimal) form is________. **[2014]**

Solution:

Given number is $(1856357)_{10}$.

In BCD one digit is represented by 4 bits ,so

1 8 5 6 3 5 7

$$= (\underbrace{0001\ 1000}_{1\text{ byte}}\ \underbrace{0101\ 0110}_{1\text{ byte}}\ \underbrace{0011\ 0101}_{1\text{ byte}}\ \underbrace{0111}_{1\text{ byte}})_{BCD} = 4\text{bytes}$$

2. $X = 01110$ and $Y = 11001$ are two 5-bit binary numbers represented in 2's complement format. The sum of X and Y represented in 2's complement format using 6 bits is

(a) 100111 (b) 001000
(c) 000111 (d) 101001 **[2007]**

Solution: (c)

2's complement

$$\begin{cases} X = 01110 & \text{MSB} = 0 \Rightarrow +\text{ve number} = +14 \\ Y = 11001 & \text{MSB} = -1 \Rightarrow +\text{ve no} \end{cases}$$

Now as Y is –ve , sum of X & $Y = X + (-Y)$

$= X + 2\text{'}sc\,(Y)$

Sum = X + 2's comp of Y

$+\underline{11001}$

Neglect ← $1\rfloor\underline{00111}$ $= \underline{000111}$

↓

+ve result (0 added to make 6th bit)

Hence, the correct option is (c).

3. Decimal 43 in hexadecimal and *BCD* number system is, respectively,

(a) $B2$, 01000011 (b) $2B$, 01000011
(c) $2B$, 0011 0100 (d) $B2$, 0100 0100 **[2005]**

Solution: (b)

$(43)_{10}$

(i) *BCD*: 4 →0100 3→0011

$\Rightarrow (43)_{10} = (0100\ 0011)_{BCD}$

(ii) Hexadecimal: 43 = 32 + 11 ⇒ $(2B)_{16}$

($2B$, 0100 0011)

Hence, the correct option is (b).

4. The range of signed decimal numbers that can be represented by 6-bit 1's complement number is

(a) −31 to +31 (b) −63 to + 63
(c) −64 to + 63 (d) −32 to + 3 **[2004]**

Solution: (a)

2, s complement form, MSB is sign bit and reset of 5 bit is mag bits.

$\Rightarrow$ range $= -(2^5 - 1)$ to $+(2^5 - 1)$

$= -31$ to 31

Hence, the correct option is. (a)

5. 4-bit 2's complement representation of a decimal number is 1000. The number is

(a) +8 (b) 0
(c) −7 (d) −8 **[2002]**

Solution: (d)

2's complement = 1000——, MSB = 1 ⇒ – ve number.

Take inverse of 2's comp:

Subt 1: 1000

$\underline{-1}$

$\underline{0111}$

Take 1's comp = 1000 = 8 (mag) and sign = –ve

⇒ number of (–8)

Hence, the correct option is (d).

6. The 2's complement representation of −17 is

(a) 101110 (b) 101111
(c) 111110 (d) 110001 **[2001]**

Solution: (b)

2's complement of –17

+ve 17 = 010001(6 bit representation)

Take 1's comp = 101110

add 1 $\underline{+\quad\quad 1}$

$\underline{101111}$ = (–17)

Hence, the correct option is (b).

7. An equivalent 2's complement representation of the 2's complement number 1101 is

(a) 110100 (b) 001101
(c) 110111 (d) 111101 **[1998]**

Solution:(d)

2's complement representation = 1101

Now to extend the number into more number of bytes, the MSB (sign) is copied to left side

8 bit rep = 1 1 1 1 0 1

8 bit = 1 1 1 1 1 1 0 1

Hence, the correct option is (d).

8. A signed integer has been stored in a byte using the 2's complement format. We wish to store the same integer in a 16 bit word. We should:

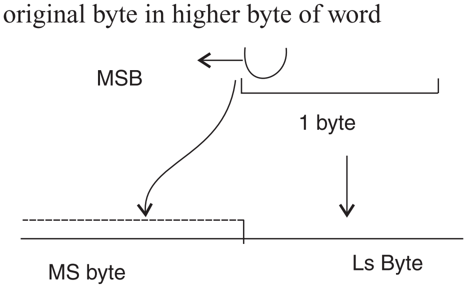
(a) copy the original byte to the less significant byte of the word and fill the more significant byte with zeros.
(b) copy the original byte to the more significant byte of the word and fill the less significant byte with zeros.
(c) copy the original byte to the less significant byte of the word and make each bit of the more significant byte equal to the most significant bit of the original byte.
(d) copy the original byte to the less significant bytes well as the more significant byte of the word. **[1997]**

Solution: (c)

In 2's complete form , if MSB =1⇒ –ve number

=0 ⇒+ve number

∴To extended it to 16-bit word , copy the original byte (less significance) of word and copy the MSB (sign bit) of original byte in higher byte of word



In 2's complement form, if MSB 1 ⇒ –ve number

= 0 ⇒ +ve number

∴To extend it to 16-bit word, copy the original byte to the lower byte (less significant) of word and copy the MSB (sign bit) of original byte in higher byte of word

Hence, the correct option is (c).

9. 2's complement representation of a 16-bit number (one sign bit and 15 magnitude bits) is FFFF. Its magnitude in decimal representation is

(a) 0 (b) 1
(c) 32,767 (d) 65, 535 **[1993]**

Solution: (b)

2's complement 16 bit no (1 sign + 15mag bit) = FFFF = 1111 1111 1111 1111

2's complement of 111 1111 1111 1111 is 000 0000 0001 and its decimal equivalent is 1 with negative sign.

∴ Decimal number is –1 and its mag is 1.

Hence, the correct option is (b).

10. The subtraction of a binary number *Y* from another binary number *X*, done by adding the 2's complement of *Y* to *X*, results in a binary number without overflow. This implies that the result is:

(a) negative and is in normal form.
(b) negative and is in 2's complement form.
(c) positive and is in normal form.
(d) positive and is in 2's complement form. **[1987]**

Solution: (b)

For X Y operation, using 2's component form, if result has

i) overflow, then it is discarded and result is *positive* and in normal format.
ii) no overflow, then result is negative and in 2's complement form.

Hence, the correct option is (b).

Two-marks Questions

1. The two numbers represented in signed 2's complement form are $P = 11101101$ and $Q = 11100110$. If Q is subtracted from P, the value obtained in signed 2's complement form is

(a) 10000011 (b) 00000111
(c) 11111001 (d) 111111001 **[2008]**

Solution: (b)

$P = 11101101\ \ (-\text{ve}) = -19$

$Q = 11100110\ \ (-\text{ve}) = -26$

Then $P - Q = (-\text{ve})\,|P| - (-\text{ve}\,|Q|) = -ve\,|P| + |Q|$

$$= |Q| - |P| \text{ (mag)}$$
$$= 26 - 19 = 7$$
$$= 000111$$

Hence, the correct option is (b).

2. A new Binary Coded Pentary (BCP) number system is proposed in which every digit of a base-5 number is represented by its corresponding 3-bit binary code. For example, the base-5 number 24 will be represented by its BCP code 010100. In this numbering system, the BCP code 100010011001 corresponds to the following number in base-5 system:

(a) 423 (b) 1324
(c) 2201 (d) 4231 **[2006]**

Solution: (d)

BPC (base –5)

Carry digit $\rightarrow$ 3-bit binary code

$\Rightarrow 2 \rightarrow 010$

$4 \rightarrow 100$ $(24)_{10} = (\underline{010}\ \underline{100})_5$

Then for $\left.\underset{4}{\overset{100}{\downarrow}}\right| \left.\underset{2}{\overset{010}{\downarrow}}\right| \left.\underset{3}{\overset{011}{\downarrow}}\right| \underset{1}{\overset{001}{\downarrow}}$ (make pair of 3-bits)

$(100010011001)_5 = (4231)_{10}$

Hence, the correct option is (d).

3. 11001, 1001 and 111001 correspond to the 2's complement representation of which of the following sets of number?

(a) 25, 9 and 57, respectively
(b) −6, −6 and −6, respectively
(c) −7, −7 and −7, respectively
(d) −25, −9 and −57, respectively **[2004]**

Solution: is (c)

$$11001 \xrightarrow{\text{inv 2;s}} -00111 = -7$$
$$1001 \xrightarrow{\text{inv 2;s}} -0111 = -7$$
$$111001 \longrightarrow -000111 = -7$$

*(For inv. 2's complement, subtract (+1) and then take 1's comp)

$\Rightarrow (\underline{6})\ -7, -7, -7$

Hence, the correct option is (c).

Chapter 2

Boolean Algebra

One-mark Questions

1. A function $F(A, B, C)$ defined by three Boolean variables A, B and C when expressed as sum of products is given by

$$F = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B\cdot\bar{C} + A\cdot\bar{B}\cdot\bar{C}$$

Where, $\bar{A}$, $\bar{B}$ and $\bar{C}$ are the complements of the respective variables. The product of sums (POS) form of the function F is. **[2018]**

(A) $F = (A + B + C)\,(A + \bar{B} + C)\,(\bar{A} + B + C)$

(B) $F = (\bar{A} + \bar{B} + \bar{C})\,(\bar{A} + B + \bar{C})\,(A + \bar{B} + \bar{C})$

(C) $F = (A + B + \bar{C})\cdot(A + \bar{B} + \bar{C})\cdot(\bar{A} + B + \bar{C})$ $(\bar{A} + \bar{B} + C)\left(\bar{A} + \bar{B} + \bar{C}\right)$

(D) $F = (\bar{A} + \bar{B} + C)\cdot(\bar{A} + B + C)\cdot(A + \bar{B} + C)$ $(A + B + \bar{C})\,(A + B + C)$

Solution: sum of products function is given as

$$F = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B\cdot\bar{C} + A\cdot\bar{B}\cdot\bar{C}$$

The K- map for the above function is given below

A \ BC	00	01	11	10
0	1	0	0	1
1	1	0	0	0

From the above K- map the product of sums (POS) form of the function F is

$$F = (A + B + \bar{C})\cdot(A + \bar{B} + \bar{C})\cdot$$
$$(\bar{A} + B + \bar{C})\cdot(\bar{A} + \bar{B} + \bar{C})\cdot(\bar{A} + \bar{B} + C)$$

Hence, the correct option is (C)

2. The Boolean expression $F(X, Y, Z) = \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XY\bar{Z} + XYZ$ converted into the canonical product of sum (POS) form is **[2015]**

(A) $(X + Y + Z)(X + Y + \bar{Z})(\bar{X} + Y + \bar{Z})$

(B) $\left(X + \bar{Y} + Z\right)\left(\bar{X} + Y + \bar{Z}\right)\left(\bar{X} + \bar{Y} + Z\right)$ $\left(\bar{X} + \bar{Y} + \bar{Z}\right)$

(C) $\left(X + Y + Z\right)\left(\bar{X} + Y + \bar{Z}\right)\left(X + \bar{Y} + Z\right)$ $\left(\bar{X} + \bar{Y} + \bar{Z}\right)$

(D) $\left(X + \bar{Y} + \bar{Z}\right)\left(\bar{X} + Y + Z\right)\left(\bar{X} + \bar{Y} + Z\right)$ $\left(X + Y + Z\right)$

Solution: $F(X, Y, Z) = \bar{x}y\bar{z} + x\bar{y}\bar{z} + xy\bar{z} + xyz$

$= \sum m(2, 4, 6, 7) = \prod M(0, 1, 3, 5)$

$= (X + Y + Z)(X + Y + \bar{Z})\,(X + \bar{Y} + \bar{Z})(\bar{X} + Y + \bar{Z}$
$)$

Hence, the correct option is (A).

3. A function of Boolean variables X, Y and Z is expressed in terms of the min-terms as

$$F(X, Y, Z) = \Sigma\,(1, 2, 5, 6, 7)$$

Which one of the product of sums given below is equal to the function $F(X, Y, Z)$? **[2015]**

(A) $\left(\bar{X} + \bar{Y} + \bar{Z}\right).\left(\bar{X} + Y + Z\right).\left(X + \bar{Y} + \bar{Z}\right)$

(B) $\left(X + Y + Z\right).\left(X + \bar{Y} + \bar{Z}\right).\left(\bar{X} + Y + Z\right)$

(C) $\left(\bar{X} + \bar{Y} + Z\right)\cdot\left(\bar{X} + Y + \bar{Z}\right)\cdot\left(X + \bar{Y} + Z\right)\cdot$ $\left(X + Y + \bar{Z}\right)\cdot\left(X + Y + Z\right)$

(D) $\left(X + Y + \bar{Z}\right)\cdot\left(\bar{X} + Y + Z\right)\cdot\left(\bar{X} + Y + \bar{Z}\right)\cdot$ $\left(\bar{X} + \bar{Y} + Z\right)\cdot\left(\bar{X} + \bar{Y} + \bar{Z}\right)$

Solution: $F(x, y, z) = \sum m(1, 2, 5, 6, 7)$

$= \prod M\,(0, 3, 4)$

$$= (x+y+z)\,(\bar{x}+\bar{y}+\bar{z})\,(\bar{x}+y+z)$$

Hence, the correct option is (B).

4. The Boolean expression

$(X+Y)(X+\bar{Y})+\overline{(\bar{X}\bar{Y})}+\bar{X}$ simplifies to

(a) X

(b) Y

(c) XY

(d) $X+Y$ **[2014]**

Solution: (a)

$y=(x+y)(x+\bar{y})(\overline{\bar{x}\,\bar{y}+\bar{x}})$

$=(x\cdot x+y\cdot x+x\bar{y}+y\cdot\bar{y})(\overline{\bar{x}(\bar{y}+1)})$ $\quad xx=x$, $\quad y\bar{y}=0$

$=(x+xy+x\bar{y})(\bar{\bar{x}})$ $\quad \bar{y}+1=1$

$=x(1)x=x$

Hence, the correct option is (a).

5. For an n-variable Boolean function, the maximum number of prime implicants is

(a) $2(n-1)$ (b) $n/2$

(c) 2^n (d) $2^{(n-1)}$ **[2014]**

Solution: (d)

Hence, the correct option is (d).

6. In the sum of products function $f(X, Y, Z) = \sum(2, 3, 4, 5)$ the prime implicates are

(a) $\bar{X}Y,\ X\bar{Y}$

(b) $\bar{X}Y,\ X\bar{Y}\bar{Z},\ X\bar{Y}Z$

(c) $\bar{X}Y\bar{Z},\ \bar{X}YZ,\ X\bar{Y}$

(d) $\bar{X}Y\bar{Z},\ \bar{X}YZ,\ X\bar{Y}\bar{Z},\ X\bar{Y}Z$ **[2012]**

Solution: (d)

z \ xy	00	01	11	10
0	c	1 (2)	(6)	1 (4)
1	1	1 (3)	(7)	1 (5)

$F(x, y, z) = \{(1, 2, 3)\}$

$\Rightarrow PI = \bar{x}y + x\bar{y}$

Hence, the correct option is (d).

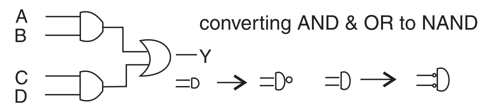
7. The Boolean function $Y = AB + CD$ is to be realized using only 2-input NAND gates. The minimum number of gates required is

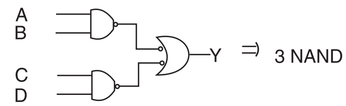
(a) 2 (b) 3

(c) 4 (d) 5 **[2007]**

Solution: (b)

$y = AB + CD$





Hence, the correct option is (b).

8. The number of distinct Boolean expressions of 4 variables is

(a) 16 (b) 256

(c) 1024 (d) 65536 **[2003]**

Solution: (d)

For 4 variables:

For n var 2^{2^n} function

$\Rightarrow 2^{2^4} = 2^{16} = 1024 \times 2^6 = 65536$

Hence, the correct option is (d).

9. The logical expression $Y = A + \bar{A}B$ is equivalent to

(a) $y = AB$ (b) $y = \bar{A}B$

(c) $y = \bar{A} + B$ (d) $y = A + B$ **[1999]**

Solution: (d)

$Y = A + \bar{A}B$ by distribution law:

$A + \bar{A}B = (\underbrace{A+\bar{A}}_{1})(A+B)$

$Y = (A+\bar{A})(A+B)$

$Y = A + B$

Hence, the correct option is (d).

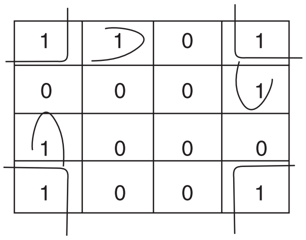
10. The K-map for a Boolean function is shown in the figure. The number of essential prime implicants for this function is

CD \ AB	00	01	11	10
00	1	1	0	1
01	0	0	0	1
11	1	0	0	0
10	1	0	0	1

(a) 4 (b) 5

(c) 6 (d) 8 **[1998]**

Solution: (a)



Quad : 1

Pair : 3

$\therefore$ to cover all 1's, *Quad* + 3 Pair

$\Rightarrow$ 4 *PI*

Hence, the correct option is (a).

11. Two 2's complement numbers having sign bits x and y are added and the sign bit of the result is z. Then, the occurrence of overflow is indicated by the Boolean function.

(a) xyz (b) $\bar{x}\,\bar{y}\,\bar{z}$

(c) $(\overline{X}\,\overline{Y}Z + XY\overline{Z})$ (d) $xy + yz + zx$

[1998]

Solution: (c)

$2n$ 2's complement, if no having sign bit X & Y are added, then overflow occurs; if X and Y are the same (same sign) and resultant sign changes

e.g., $X = Y = 0$ (+ve both), then Z should be 0 if $Z = 1 \Rightarrow$ overflow

$X = Y = 1$ (–ve) $\Rightarrow$ (–ve) + (–ve): –ve ($Z = 1$) but $Z = 0$ overflow

$\Rightarrow$ overflow: $(\overline{X}\,\overline{Y}Z + XY\overline{Z})$

Hence, the correct option is (c).

12. The number of Boolean functions that can be generated by n variables is equal to:

(a) 2^{2n-1} (b) 2^{2^n}

(c) 2^{n-1} (d) 2^n **[1990]**

Solution: (b)

For n variable function, as n can be 0 or 1

$\Rightarrow 2^n$ input combination is possible

For (2) I/P , (2) 0/P $\xrightarrow{} (2^n)$ combination

$\Rightarrow 2^{2^n}$ function possible

(As if $n = 2$ $(\overline{A}, \overline{B})$, 4 input: $\overline{A}\overline{B}, A\overline{B}, \overline{A}B, AB$)

For 4 input, 2^4 function can be made (16).

Hence, the correct option is (b).

Two-marks Questions

1. Which one of the following gives the simplified sum of products expression for the Boolean function $F = m_0 + m_2 + m_3 + m_4$, where m_0, m_2, m_3 and m_5 are minterms corresponding to the inputs A, B and C with A as the MSB and C as the LSB ? **[2017]**

(A) $\overline{A}B + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C$

(B) $\overline{A}\,\overline{C} + \overline{A}B + A\overline{B}C$

(C) $\overline{A}\,\overline{C} + A\overline{B} + A\overline{B}C$

(D) $\overline{A}BC + \overline{A}\,\overline{C} + A\overline{B}C$

Solution: Given that

$F = m_0 + m_2 + m_3 + m_5$

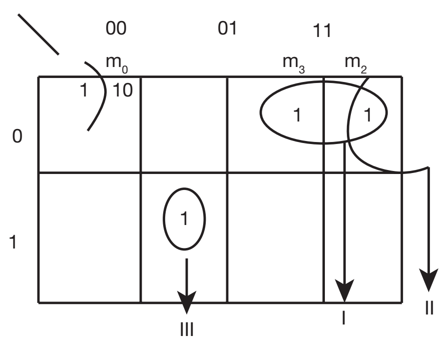
$\therefore F = I + II + III$

$I = \overline{A}B$

$II = \overline{A}\,\overline{C}$

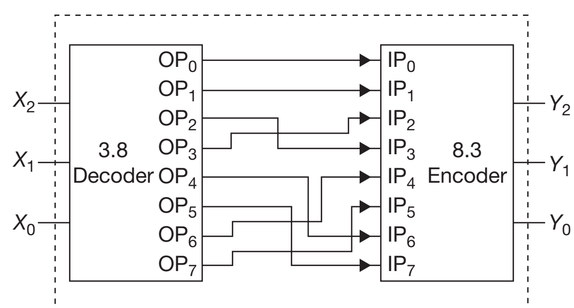
$III = A\overline{B}C$

$\therefore$ Simplified $F = \overline{A}B + \overline{A}\,\overline{C} + A\overline{B}C$.



Hence, the correct option is (B).

2. Identify the circuit below **[2016]**



(A) Binary to gray code converter

(B) Binary to XS3 converter

(C) Gray to binary converter

(D) XS3 to binary converter

Solution: Consider the table given below

X_2	X_1	X_0	Decoder Output	Encoder Input	Y_2	Y_1	Y_0
0	0	0	OP_0	IP_0	0	0	0
0	0	1	OP_1	IP_1	0	0	1
0	1	0	OP_2	IP_3	0	1	1
0	1	1	OP_3	IP_2	0	1	0
1	0	0	OP_4	IP_6	1	1	0
1	0	1	OP_5	IP_7	1	1	1
1	1	0	OP_6	IP_4	1	0	0
1	1	1	OP_7	IP_5	1	0	1

Binary to gray code converter except last 2 conditions.

Hence, the correct option is (A).

3. Following is the K–map of a Boolean function of five variables P, Q, R, S and X. The minimum sum of product (SOP) expression for the function is **[2016]**

RS \ PQ	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	0	1
10	0	0	0	0

$x = 0$

RS \ PQ	00	01	11	10
00	0	1	1	0
01	1	0	0	1
11	1	0	0	1
10	0	1	1	0

$x = 1$

(A) $\bar{P}\,\bar{Q}\,S\,\bar{X} + P\bar{Q}\,S\,\bar{X} + Q\bar{R}\,\bar{S}\,X + QR\,\bar{S}\,X$

(B) $\bar{Q}\,S\,\bar{X} + Q\,\bar{S}\,X$

(C) $\bar{Q}\,S\,X + Q\,\bar{S}\,\bar{X}$

(D) $\bar{Q}\,S + Q\,\bar{S}$

Solution: Given,

RS \ PQ	00	01	11	10
00				
01	1			1
11	1			1
10				

$X = 0$

RS \ PQ	00	01	11	10
00		1	1	
01	1			
11	1			
10		1	1	

$X = 1$

From the given K-maps

$$F(PQRSX) = S.\bar{Q}\bar{X} + \bar{S}.QX$$

Hence, the correct option is (B).

(from the given K-maps

When $X = 0$

$$F_1(PQRSX) = SQ'X'$$

and $$F_2(PQRSX) = S'QX$$

Hence, $$F(PQRSX) = SQ'X' + S'QX$$

options with conditions of $X = 0$ and $X = 1$ and K-Maps option (B) gives the correct result)

Hence, the correct option is (B).

4. Consider the Boolean function
$F(w, x, y, z) = wy + xy + \bar{w}xyz + \bar{w}\bar{x}y + xz + \bar{x}\,\bar{y}\,\bar{z}$.

Which one of the following is the complete set of essential prime implicants?

(a) $w, y, xz, \bar{x}\,\bar{z}$ (b) w, y, xz

(c) $y, \bar{x}\,\bar{y}\,\bar{z}$ (d) $y, xz, \bar{x}\,\bar{z}$ **[2014]**

Solution:(d)

$f(w,\ x,\ y,\ z) = wy + xy + \bar{w}xyz + \bar{w}\bar{x}y + xz + \bar{x}\bar{y}\bar{z}$

For essential PI, we reduce f

$$f = wy + xy + zx\underbrace{(1 + \bar{w}y)}_{=1} + \bar{x}\bar{w}y + \bar{x}\bar{y}\bar{z}$$

$$= wy + xy + xz + \bar{x}\bar{w}y + \bar{x}\bar{y}\bar{z}$$

$$= y(w + \bar{x}\bar{w}) + xy + xz + \bar{x}\bar{y}\bar{z}$$

$$= yw + y\bar{x} + xy + xz + \bar{x}\bar{y}\bar{z}$$

$$= y(w + \underbrace{\bar{x} + x}_{=1}) + xz + \bar{x}\bar{y}\bar{z}$$

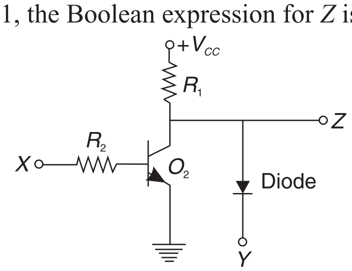
$$= y + xz + \bar{x}\bar{y}\bar{z} = xz + \underbrace{(y + \bar{y})}_{=1}(y + \bar{x}\bar{z})$$

$$f = \underline{xz + y + \bar{x}\bar{z}}$$

$$\Rightarrow PI: y,\ xz,\ \bar{x}\bar{z}$$

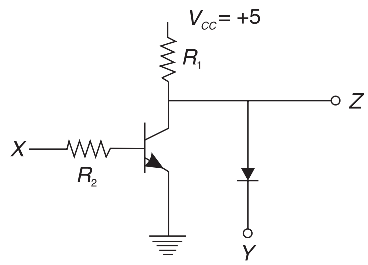
Hence, the correct option is (d).

5. In the circuit shown below, Q_1 has negligible collector-to-emitter saturation voltage and the diode drops negligible voltage across it under forward bias. If V_{cc} is +5 V, X and Y are digital signals with 0 V as logic 0 and V_{cc} as logic 1, the Boolean expression for Z is



(a) XY (b) $\overline{X}Y$
(c) $X\overline{Y}$ (d) $\overline{X}\overline{Y}$ **[2013]**

Solution: (b)



Input x and y output: z

a) $x = 0$ $y = 0 \Rightarrow Q =$ is OFF $z = 0$
b) $x = 0$ $y = 5 \Rightarrow Q$: OFF $z = 5$
c) $x = 5$ $y = 0 \Rightarrow Q$: ON $z = 0$
d) $x = 5$ $y = 5 \Rightarrow Q$: ON $z = 0$

X	Y	Z
0	0	0
0	1	1
1	0	0
1	1	0

$\Rightarrow Z = \overline{X}Y$

Hence, the correct option is (b).

6. If $X = 1$ in the logic equation

$[X + Z\{\overline{Y} + (\overline{Z} + X\overline{Y})\}]\{\overline{X} + \overline{Z}(X + Y)\} = 1,$

Then
(a) $Y = Z$
(b) $Y = \overline{Z}$
(c) $Z = 1$
(d) $Z = 0$ **[2009]**

Solution:(d)

$$\left[x + z\left\{\overline{y} + (\overline{z} + x\overline{y})\right\}\right]\left[\overline{x} + \overline{z}(x + y)\right] = 1$$

put $x = 1$

$$\Rightarrow \left[\underbrace{1 + z\{\ \}}_{=1}\right]\left[0 + \overline{z}(\underline{1 + y}_{=1})\right] = 1$$

$$\Rightarrow \overline{z} = 1 \Rightarrow z = 0$$

Hence, the correct option is (d).

7. The Boolean expression

$Y = \overline{A}\overline{B}\overline{C}D + \overline{A}BC\overline{D} + A\overline{B}\overline{C}D + AB\overline{C}\overline{D}$ can be minimized to

(a) $Y = \overline{A}\overline{B}\overline{C}D + \overline{A}B\overline{C} + A\overline{C}D$
(b) $Y = \overline{A}\overline{B}\overline{C}D + BC\overline{D} + A\overline{B}\overline{C}D$
(c) $Y = \overline{A}BC\overline{D} + \overline{B}\overline{C}D + A\overline{B}\overline{C}D$
(d) $Y = \overline{A}BC\overline{D} + \overline{B}\overline{C}D + AB\overline{C}\overline{D}$ **[2007]**

Solution: (d)

$$y = \overline{A}\underline{\overline{B}\overline{C}D} + \overline{A}BC\overline{D} + A\overline{B}\overline{C}D + A\underline{B\overline{C}\overline{D}}$$

$$= (\overline{A} + A)\overline{B}\overline{C}D + \overline{A}BC\overline{D} + A\overline{B}\overline{C}D$$

$$y = \overline{B}\overline{C}\overline{D} + \overline{A}BC\overline{D} + A\overline{B}\overline{C}D$$

Hence, the correct option is (d).

8. The Boolean expression for the truth table shown is

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

(a) $B(A + C)(\overline{A} + \overline{C})$
(b) $B(A + \overline{C})(\overline{A} + C)$
(c) $(A + \overline{C})(\overline{A} + C)$
(d) $\overline{B}(A + C)(\overline{A} + \overline{C})$ **[2005]**

Solution: (a)

From truth table $f = \overline{A}BC + AB\overline{C}$

$$= B(\overline{A}C + A\overline{C})$$

or

$$f = B(\overline{A}C + A\overline{C} + A\overline{A} + C\overline{C})$$

$$= B(A + C)(\overline{A} + \overline{C}) \qquad \{A\overline{A} = 0 \quad C\overline{C} = 0\}$$

Hence, the correct option is (a).

9. The Boolean expression $AC + B\overline{C}$ is equivalent to

(a) $\overline{A}C + B\overline{C} + AC$
(b) $\overline{B}C + AC + B\overline{C} + \overline{A}C\overline{B}$
(c) $AC + B\overline{C} + \overline{B}C + ABC$
(d) $ABC + \overline{A}B\overline{C} + AB\overline{C} + A\overline{B}C$ **[2004]**

Solution: (d)

$$y = AC + B\overline{C}$$

By k-MAP:

C \ AB	00	01	11	10
0		1	1	
1			1	1

or

$$y = AC(B + \overline{B}) + B\overline{C}(A + \overline{A})$$

$$y = ABC + A\bar{B}C + AB\bar{C} + \bar{A}B\bar{C}$$

Hence, the correct option is (d).

10. If the functions W, X, Y and Z are as follows

$$W = R + \bar{P}Q + \bar{R}S$$

$$X = PQ\bar{R}\bar{S} + \bar{P}\bar{Q}\bar{R}\bar{S} + P\bar{Q}\bar{R}\bar{S}$$

$$Y = RS + \overline{PR + P\bar{Q} + \bar{P}Q}$$

$$Z = R + S + \overline{PQ + \bar{P}\bar{Q}R + P\bar{Q}\bar{S}}$$

then

(a) $W = Z, X = \bar{Z}$ (b) $W = Z, X = y$
(c) $W = Y$ (d) $W = Y = \bar{Z}$

[2003]

Solution: (a)

$$W = R + \bar{P}Q + \bar{R}S$$

$$X = PQ\bar{R}\bar{S} + \bar{P}\bar{Q}\bar{R}\bar{S} + P\bar{Q}\bar{R}\bar{S}$$

$$Y = RS + \overline{PR + \bar{P}\bar{Q} + \bar{P}Q}$$

$$Z = R + S + \overline{PQ + \bar{P}\bar{Q}R + P\bar{Q}\bar{S}}$$

Now

$$W = R + \bar{R}S + \bar{P}Q = \underbrace{(R + \bar{R})}_{=1}(R + S) + \bar{P}Q \qquad (1)$$

$$W = R + S + \bar{P}Q$$

$$X = PQ\bar{R}\bar{S} + (\bar{P} + P)\bar{Q}\bar{R}\bar{S} = PQ\underline{\bar{R}\bar{S}} + \bar{Q}\underline{\bar{R}\bar{S}} \qquad (2)$$

$$= \bar{R}\bar{S}(\bar{Q} + PQ) = \bar{R}\bar{S}\underbrace{(\bar{Q} + Q)}_{=1}(\bar{Q} + P)$$

$$= \bar{R}\bar{S}\bar{Q} + \bar{R}\bar{S}P$$

$$Y = RS + \overline{PR + \bar{Q}\underbrace{(P + \bar{P})}_{=1}} =$$

$$RS + \overline{PR + \bar{Q}} = RS + \cancel{\overline{\underbrace{(P + \bar{P})}_{=1}(\bar{P} + R)}}$$

$$= \cancel{RS + \overline{\bar{P} + R}} = RS + \overline{PR}\cdot Q = SR + (\bar{R} + \bar{P})Q$$

$$= Y = SR + \bar{R}Q + \bar{P}Q$$

$$Z = R + S + \overline{P(Q + \bar{S}) + \bar{P}\bar{Q}\bar{R}} \qquad 3$$

$$= R + S + \overline{P(\bar{S} + Q)}(P + QR)$$

$$= R + S + (\bar{P} + S\bar{Q})(P + Q + R)$$

$$= R + S + PS\bar{Q} + \bar{P}Q + \bar{P}R + S\bar{Q}R$$

$$= R + S + \bar{P}(Q + R) = \underline{R} + S + \bar{P}Q$$

$$+ \underline{\bar{P}R} = R + S + \bar{P}Q \quad (\bar{P} + 1 = 1) \qquad 4$$

From (1) (2) (4) $w = z$. From (2) and (4) $x = \bar{z}$.

Hence, the correct option is (a).

11. The minimized form of the logical expression $(\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C})$ is

(a) $\bar{A}\bar{C} + B\bar{C} + \bar{A}B$
(b) $A\bar{C} + \bar{B}C + \bar{A}B$
(c) $\bar{A}C + \bar{B}C + \bar{A}B$
(d) $A\bar{C} + \bar{B}C + A\bar{B}$ **[1999]**

Solution: (a)

$$Y = (\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C})t$$

$$= \bar{A}(\bar{B} + B)\bar{C} + \bar{A}BC + AB\bar{C} \qquad B + \bar{B} = 1$$

$$= \bar{A}\bar{C} + \bar{A}BC + AB\bar{C} = \bar{A}(\bar{C} + BC) + AB\bar{C}$$

$$= \bar{A}(\bar{C} + B)(\bar{C} + C) + AB\bar{C} \qquad \text{(distribution law)}$$

$$= \bar{A}\bar{C} + \bar{A}B + AB\bar{C} = 1$$

$$= \bar{C}(\bar{A} + AB) + \bar{A}B = \bar{C}(\bar{A} + A)(\bar{A} + B) + \bar{A}B$$

$$Y = \bar{C}\bar{A} + \bar{C}B + \bar{A}B$$

Hence, the correct option is (a).

FIVE-MARKS QUESTIONS

1. Given the Boolean function F in three variables R, S and T as

$$F = \bar{R}S\bar{T} + R\bar{R}T + RST$$

(a) Express F in the minimum sum-of-products form.
(b) Express F in the minimum product-of-sums form.
(c) Assuming that both true and complement forms of the input variables are available, draw a circuit to implement F using the minimum number of 2-input NAND gates only. **[1996]**

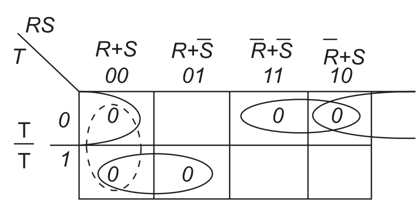
Solution: (a) $F = \bar{R}S\bar{T} + R\bar{S}T + RST$

$$= \bar{R}S\bar{T} + RT(\bar{S} + S)$$

$$= \bar{R}S\bar{T} + RT$$

(b) $F = \bar{R}S\bar{T} + R\bar{S}T + RST$

$\Sigma m(2, 5, 7)$



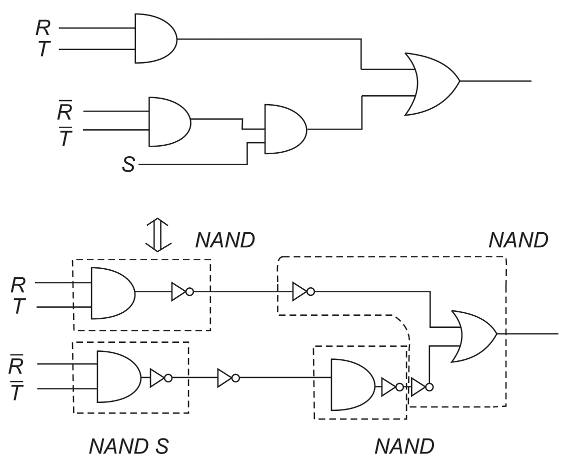
To possible solves

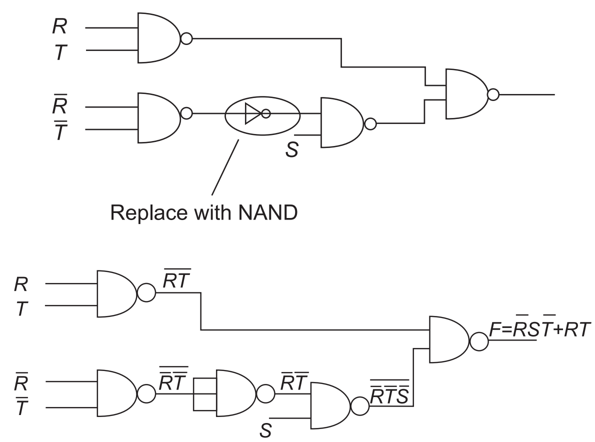
Solution 1 $\rightarrow F = (R + S)(R + \bar{T})(\bar{R} + T)$

Solution 2 $\rightarrow F = (S + T)(R + \bar{T})(\bar{R} + T)$

(c) NAND – NAND $\rightarrow$ AND-OR

$$F = \bar{R}S\bar{T} + RT$$





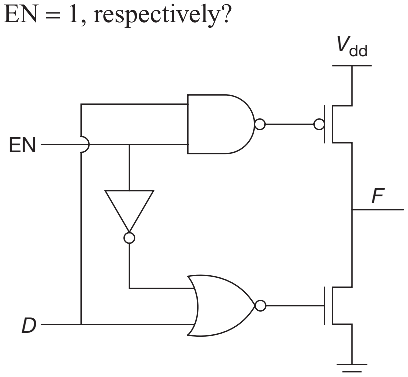
Hence, the correct option is (a)

Chapter 3

Logic Gates

One-mark Questions

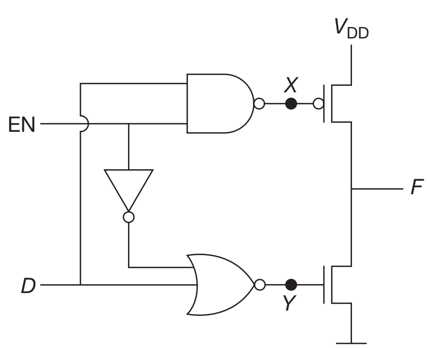
1. In the circuit shown, what are the values of F for EN = 0 and EN = 1, respectively? **[2019]**



(A) 0 and 1 (B) Hi-Z and D

(C) Hi-Z and $\overline{D}$ (D) 0 and D

Solution:



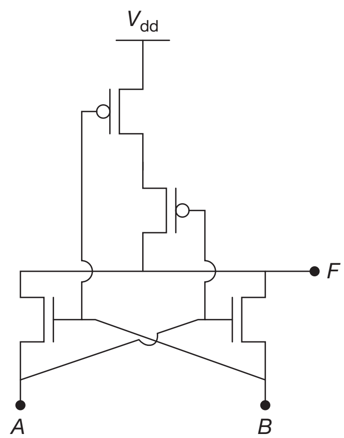
$X = \overline{E_N \cdot D}$

$Y = \overline{\overline{E_N} + D}$

When $EN = 0$	When $EN = 1$
$X = \overline{0 \cdot D} = 1$	$X = \overline{1 \cdot D} = \overline{D}$
$Y = \overline{\overline{0} + D} = 0$	$Y = \overline{\overline{1} + D} = D$
$\therefore F$ is Hight-z	$\therefore F$ is D

Hence, the correct option is (B).

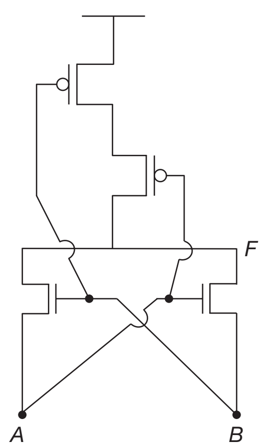
2. In the circuit shown. A and B are the inputs and F is the output. What is the functionality of the circuit? **[2019]**



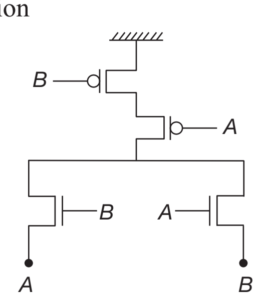
(A) XNOR (B) SRAM Cell

(C) XOR (D) Latch

Solution:



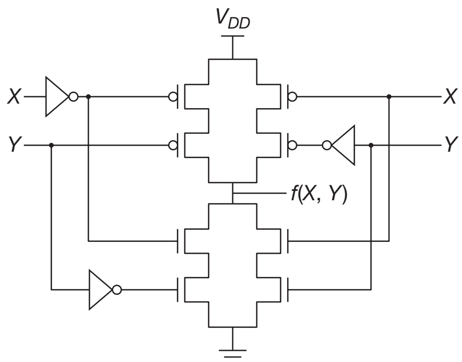
On simplification



A	B	F
0	0	1
0	1	0
1	0	0
1	1	1

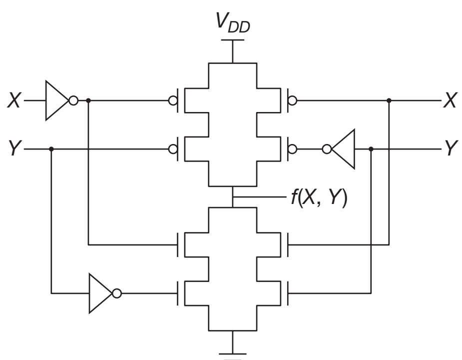
$\therefore$ F is an XOR circuit.

3. The logic function $f(X, Y)$ realized by the given circuit is **[2018]**



(A) NOR (B) AND
(C) NAND (D) XOR

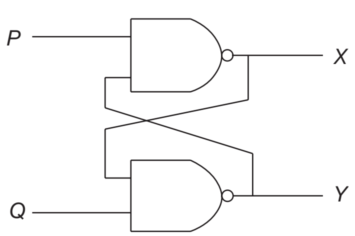
Solution:



$f(X, Y) = \overline{\bar{X}\bar{Y} + XY} = \bar{X} \odot \bar{Y} = X \oplus Y$

Hence, the correct option is (D)

4. In the latch circuit show, the NAND gates have non-zero, but unequal propagation delays. The present input condition is: $P = Q = 0$, If the input condition is changed simultaneously to $P = Q = 1$, the outputs X and Y are **[2017]**



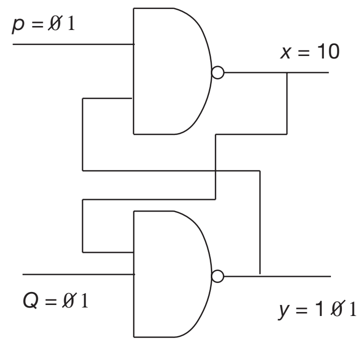
(A) $X = 1, Y = 1$
(B) Either $X = 1, Y = 0$ or $X = 0, Y = 1$
(C) Either $X = 1, Y = 1$ or $X = 0, Y = 0$
(D) $X = 0, Y = 0$

Solution: When $p = 0, Q = 0 \Rightarrow x = 1, y = 1$

when $p = 1, Q = 1 \Rightarrow x = 1, y = 0$

(or)

when $p = 1, Q = 1 \Rightarrow x = 0, y = 1$



Hence, the correct option is (B).

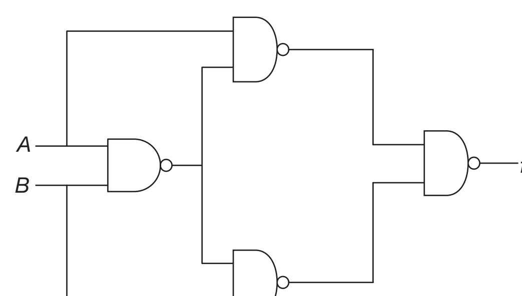
5. The minimum number of 2-input NAND gates required to implement a 2 input XOR gate is **[2016]**

(A) 4 (B) 5
(C) 6 (D) 7

Solution: For XOR gate

$$f = A\bar{B} + \bar{A}.B$$

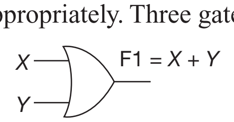
For the implementation of XOR gate, number of NAND gates required can be obtained as



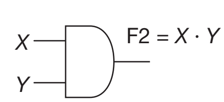
From above figure to design a XOR gate minimum 4 NAND gates are required.

Hence, the correct option is (A).

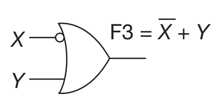
6. A universal logic gate can implement any Boolean function by connecting sufficient number of them appropriately. Three gates are shown. **[2015]**



Gate 1



Gate 2



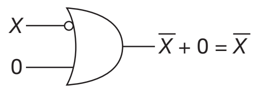
Gate 3

Which one of the following statements is TRUE?

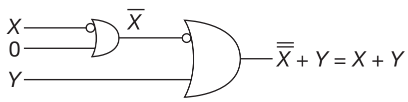
(A) Gate 1 is a universal gate.
(B) Gate 2 is a universal gate.
(C) Gate 3 is a universal gate.
(D) None of the gates shown in a universal gate.

Solution: A Universal logic gate can implement any Boolean function by connecting sufficient number of gates.

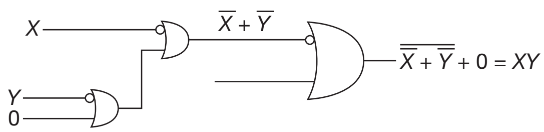
{AND, NOT}, {OR, NOT} are universal gates



NOR gate



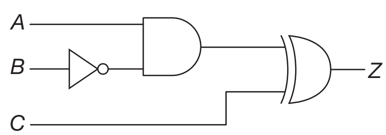
OR gate



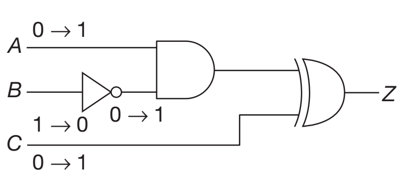
AND gate

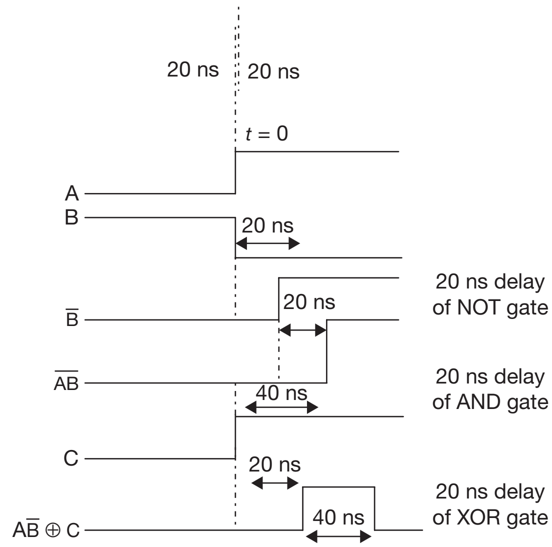
Hence, the correct option is (C).

7. All the logic gates shown in the figure have propagation delay of 20 ns. Let $A = C = 0$ and $B = 1$ until time $t = 0$. At $t = 0$, all the inputs flip (i.e., $A = C = 1$ and $B = 0$) and remain in that state. For $t > 0$, output $Z = 1$ for a duration (in ns) of **[2015]**



Solution: All the logic gates have same propagation delay = 20 ns.





Hence, the correct Answer is (40).

8. A 3-input majority gate is defined by the logic function $M(a, b, c) = ab + bc + ca$. Which one of the following gates is represented by the function $M\left(\overline{M(a,b,c)}, M(a,b,\bar{c}), c\right)$? **[2015]**

(A) 3-input NAND gate
(B) 3-input XOR gate
(C) 3-input NOR gate
(D) 3-input XNOR gate

Solution: 3 input Majority gate $M(a, b, c) = ab + bc + ac$.

$$M\left(\overline{M(a,b,c)}, M(a,b,\bar{c}), c\right) = ?$$

$$= \overline{M(a,\ b,\ c)}\ .\ M(a,b,\bar{c})$$
$$+\ \overline{M(a,\ b,\ c)}\,\mathrm{c} + M(a,b,\bar{c}).c$$

$$= (ab + bc + ac)'(ab + bc' + ac')$$
$$+ (ab + bc + ac)'c. + (ab + bc' + ac')\,c$$

$$= (a' + b')\,(b' + c')(a' + c')(ab + bc' + ac')$$
$$+ (a' + b')(b' + c')(a' + c')\,c$$
$$+ (ab + bc' + ac')\,c$$

$$= (a'c' + b')\,(a' + c')\,(ab + bc' + ac')$$
$$+ (a'c' + b')\,(a' + c').c + abc$$

$$= (a'b' + a'c' + b'c')\,(ab + bc' + ac')$$
$$+ (a'b' + a'c' + b'c')c + abc$$

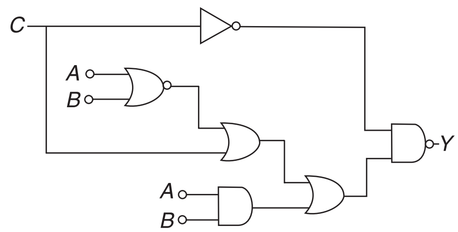
$$= a'bc' + ab'c' + a'b'c + abc$$

$= \sum m(1, 2, 4, 7) = a \oplus b \oplus c \rightarrow$ 3 input XOR gate

Odd no. of 1's in the minterms = XOR gate

Hence, the correct option is (B).

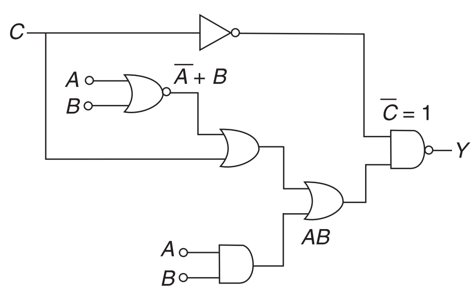
9. In the circuit shown in the figure, if $C = 0$, the expression for Y is



(a) $Y = A\overline{B} + \overline{A}B$ (b) $Y = A + B$

(c) $Y = \overline{A} + \overline{B}$ (d) $Y = AB$ **[2014]**

Solution: (a)



$(C = 0)$

$$\overline{Y} = (\overline{A+B} + \underbrace{\cancel{C}}_{=0} + AB)(\overline{C}) = 1$$

$$\overline{Y} = (\overline{A+B} + AB)$$

$$Y = \overline{\left(\overline{A+B} + AB\right)}$$

$$= (A+B)\overline{AB}$$

$$Y = (A+B)(\overline{A}+\overline{B}) = \underline{\overline{A}B + A\overline{B}}.$$

Hence, the correct option is (a).

10. A bulb in a staircase has two switches, one switch being at the ground floor and the other one at the first floor. The bulb can be turned ON and also can be turned OFF by any one of the switches irrespective of the state of the other switch. The logic of switching of the bulb resembles

(a) an AND gate (b) an OR gate

(c) an XOR gate (d) a NAND gate **[2013]**

Solution: (c)

(1) Bulb can be turned ON/OFF by any one of switch

$\Rightarrow$ for (0,1) and (1,0) input, output = 1, else zero

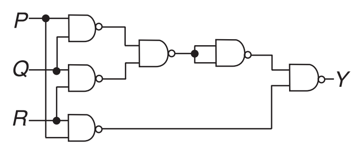
$\Rightarrow A$	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

$\Rightarrow Y = A\overline{B} + \overline{A}B$

$\Rightarrow Y = A \oplus B$

Hence, the correct option is (c).

11. The output Y in the circuit below is always '1' when



(a) two or more of the inputs P, Q, R are '0'

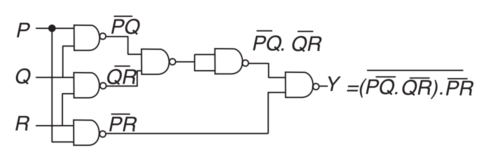
(b) two or more of the inputs P, Q, R are '1'

(c) any odd number of the inputs P, Q, R is '0'

(d) any odd number of the inputs P, Q, R is '1'

[2011]

Solution: (b)



$$Y = \overline{\overline{PQ} \cdot \overline{QR}} + PR = PQ + QR + PR$$

$Y = PQ + QR + PR$

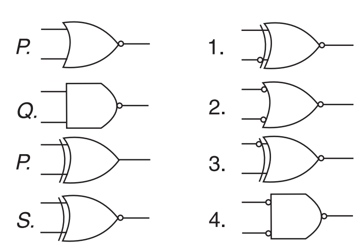
For $y = 1$, either PQ or QR or $PR = 1$ (at least any one)

$\Rightarrow$ 2 or more of $P, Q, R = 1$.

Hence, the correct option is (b).

12. Match the logic gates in Column-A with their equivalents in Column-B.

Column-A Column-B



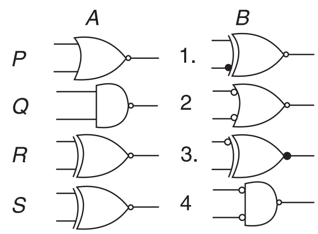
(a) P–2, Q–4, R–1, S–3

(b) P–4, Q–2, R–1, S–3

(c) P–2, Q–4, R–3, S–1

(d) P–4, Q–2, R–3, S–1 **[2010]**

Solution: (d)



P : NOR$=\bar{A}\bar{B} = \overline{A+B}$ (4)

$Q : NAND(\overline{A+B}) = \bar{A}+\bar{B}' = (\overline{AB})$ (2)

R: X – OR = (3)

S: X – NOR = (1)

$(\overline{\bar{A}\oplus\bar{B}}) = \bar{\bar{A}}\,\bar{\bar{B}} + \bar{A}\bar{B} = A\bar{B} + \bar{A}\bar{B} = A\oplus B$

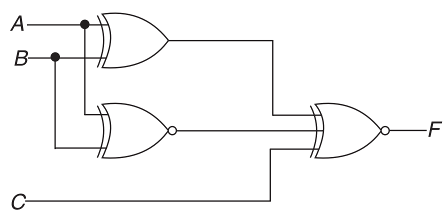
↓

$A\oplus B = \bar{A}\bar{B} + \bar{\bar{A}}B = A\,\bar{\oplus}\,B$

$\Rightarrow P-4,\ Q-2,\ R-3,\ \bar{S}-1.$

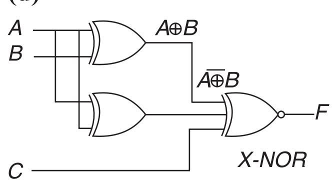
Hence, the correct option is (d).

13. For the output F to be 1 is the logic circuit shown, the input combination should be



(a) $A = 1, B = 1, C = 0$ (b) $A = 1, B = 0, C = 0$
(c) $A = 0, B = 1, C = 0$ (d) $A = 0, B = 0, C = 1$
[2010]

Solution: (d)



$F = A\oplus B\,\bar{\oplus}\,A\,\bar{\oplus}\,B\,\bar{\oplus}\,C$

If $f = 1 \Rightarrow$ odd number of input = 1

If $C = 0 \Rightarrow$ both $A\oplus B\ \&\ A\,\bar{\oplus}\,B = 1 \rightarrow$ (not possible)

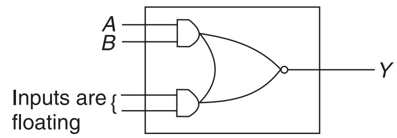
$C = 1 \Rightarrow$ either $A\oplus B = 1$ or $A\,\bar{\oplus}\,B = 1$

$$\Rightarrow \text{either } C=1 \begin{cases} A = 0, B = 1 \\ \text{or } A = 1, B = 0 \\ \text{or } A = 1, B = 1 \\ \text{or } \underline{A = 0, B = 0} \end{cases}$$

$\Rightarrow (A = 0 = B, C = 1)$

Hence, the correct option is (d).

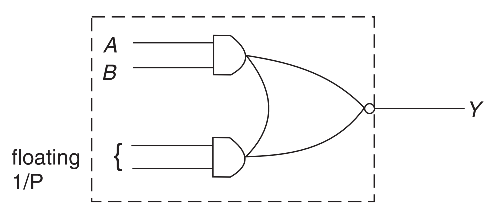
14. The figure shows the internal schematic of a TTL AND-OR-Invert (AOI) gate. For the inputs shown in the figure, the output Y is



(a) 0 (b) 1
(c) AB (d) $\overline{AB}$ **[2004]**

Solution:(a)

TTL (A0I) gate



Now for TTL, floating input considers 1.

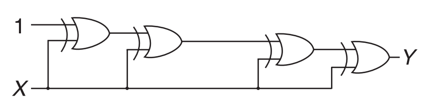
$y = \overline{(1\cdot 1)+(A\cdot B)} = \overline{1+AB}$ $\quad AB+1=1$

$y = \bar{1} = 0$

$y = 0.$

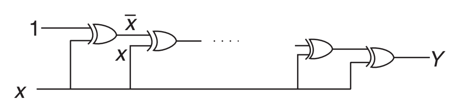
Hence, the correct option is (a).

15. If the input to the digital circuit (in the figure) consisting of a cascade of 20 XOR gates is X, then the output Y is equal to



(a) 0 (b) 1
(c) $\bar{X}$ (d) X **[2002]**

Solution: (b)



Now $X\oplus 1 = \bar{X}\cdot 1 + X\cdot\bar{1} = X + X\cdot 0$ (1st X-OR)

$X\oplus 1 = \bar{X}$

For example 2nd X-OR:

$X\oplus X = \bar{\bar{X}}\cdot X + \bar{X}\cdot\bar{X} = X\cdot X + \bar{X}\bar{X} = X + \bar{X}$

$= 1$

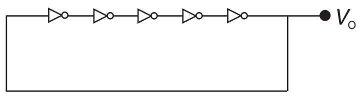
∴ For X-OR of odd number, output = $\bar{X}$

Even number of output = 1

⇒ for y = output of 20th X-OR, y = 1.

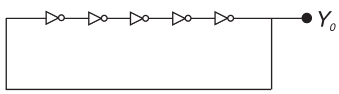
Hence, the correct option is (b).

16. For the ring oscillator shown in the figure, the propagation delay of each inverter is 100 pico sec. What is the fundamental frequency of the oscillator output?



(a) 10 MHz (b) 100 MHz
(c) 1GHz (d) 2GHz **[2001]**

Solution: (c)



TPD = 100 p sec (1 inv)

As $f = \dfrac{1}{2Ntpd}$ (N = number of inv in ring oscillator)

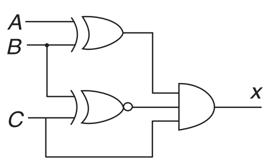
$= \dfrac{1}{2\times 5\times 100\times 10^{-12}}$ $(N = 5)$

$= \dfrac{1}{1000\times 10^{-12}} = \dfrac{1}{10^{-9}}$

$f = 10^9 = 1\text{ GHz}$

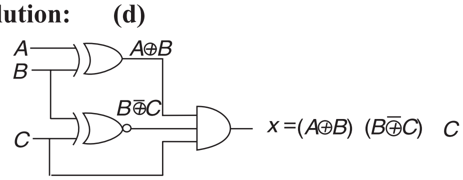
Hence, the correct option is (c).

17. For the logic circuit shown in the figure, the required input condition (A, B, C) to make the output (X) = 1 is



(a) 1, 0, 1 (b) 0, 0, 1
(c) 1, 1, 1 (d) 0, 1, 1 **[2000]**

Solution: (d)



For $x = 1$,

(a) $c = 1$

(b) $A \oplus B = 1 \quad \Rightarrow B \neq A$

(c) $B \overline{\oplus} C = 1 \quad \Rightarrow B = C = 1$ (from a)

$c = 1, \quad B = 1, \quad A \neq B = 0$ (from b)

$\Rightarrow (A,\ B,\ C) = (0,\ 1,\ 1)$ (from d)

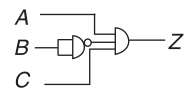
Hence, the correct option is (d).

18. The minimum number of 2-input NAND gates required to implement the Boolean function $Z = A\bar{B}C$, assuming that A, B and C are available, is

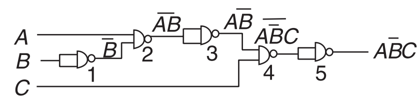
(a) two (b) three
(c) five (d) six **[1998]**

Solution: (c)

$Z = A\bar{B}C$



For NAND gates



5 NAND

Hence, the correct option is (c).

19. For the identity $AB + \bar{A}C + BC = AB + \bar{A}C$, the dual form is

(a) $(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$

(b) $(\bar{A}+\bar{B})(A+\bar{C})(\bar{B}+\bar{C}) = (\bar{A}+\bar{B})(A+\bar{C})$

(c) $(A+B)(\bar{A}+C)(B+C) = (\bar{A}+\bar{B})(A+C)$

(d) $\bar{A}\bar{B} + A\bar{C} + \bar{B}\bar{C} = \bar{A}\bar{B} + A\bar{C}$ **[1998]**

Solution: (a)

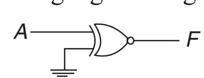
$AB + \bar{A}C + BC = AB + \bar{A}C$

In duel form: $\bullet \leftrightarrow +$

$\Rightarrow (A+B)\cdot(\bar{A}+C)\cdot(B+C) = (A+B)\cdot(\bar{A}+C)$ (a)

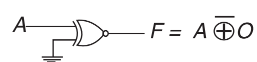
Hence, the correct option is (a).

20. The output of the logic gate in figure is



(a) 0 (b) 1
(c) $\bar{A}$ (d) A **[1997]**

Solution: (c)



Now $F = \bar{A}\bar{O} + A\cdot O$

$= \bar{A}\cdot 1 + O$

$F = \bar{A}$

Hence, the correct option is (c).

21. The Boolean function $A + BC$ is a reduced form of

(a) $AB + BC$ (b) $(A + B)\,.\,(A + C)$
(c) $\bar{A}B + A\bar{B}C$ (d) $(A + C)\cdot B$

[1997]

Solution: (b)

$y = A + BC$

Checking all equations:

(a) $AB + BC = B(A+C) \neq A + BC$

(b) $(A+B)(A+C) = (A+BC)$

(c) $\overline{A}B + A\overline{B}C \neq A + BC$

(d) $(A+C)B \neq A + BC$

Hence, the correct option is (b).

22. The minimum number of NAND gates required to implement the Boolean function $A + A\overline{B} + A\overline{B}C$ is equal to

(a) Zero (b) 1

(c) 4 (d) 7 **[1995]**

Solution: (a)

$$y = A + A\overline{B} + A\overline{B}C$$

For minimum NAND gates reduce it

$$y = A + A\overline{B}(1+C) \quad (1+C=1)$$

$$= A + A\overline{B}$$

$$y = A(1+\overline{B}) = A$$

$$Y = A$$

$\Rightarrow$ number of NAND is required is zero.

Hence, the correct option is (a).

23. The output of a logic gate is '1' when all its inputs are at logic '0'. The gate is either

(a) a NAND or an EX-OR gate

(b) a NOR or an EX-NOR gate

(c) an OR or an EX-NOR gate

(d) an AND or an EX-OR gate **[1994]**

Solution: (b)

Output $y = 1$ when all inputs $= 0$

Then

a) $y =$ EXNOR (for all input same, output = 1)

b) $y =$ NAND (NAND =1, when all input = 0)

c) $y =$ NOR (NOR = 1, input =0)

(NOR and EXNOR)

Hence, the correct option is (b).

24. A ring oscillator consisting of 5 inverters is running at a frequency of 1.0 MHz. The propagation delay per gate is_____n sec. **[1994]**

Solution:

For ring oscillator of 5 inverters ($N = 5$) mini frequency of operation is

$$f = \frac{1}{2Ntpd}\; TPD = \text{propagam delay of 1 inverter}$$

$$\Rightarrow TPD = \frac{1}{2Nf}$$

$$N = 5$$

$$\Rightarrow TPD = \frac{1}{2\times 50\times 10^{6}}$$

$$= \frac{1}{10^{7}} = 10^{-7} = 100\times 10^{-9}\ \text{sec}$$

$$TPD = 100\ n\text{sec}$$

25. Boolean expression for the output of XNOR (equivalence) logic gate with inputs A and B is

(a) $A\,\overline{B} + \overline{A}\,B$ (b) $\overline{A}\overline{B} + AB$

(c) $(\overline{A}+B)(A+\overline{B})$ (d) $(\overline{A}+\overline{B})(A+B)$

[1993]

Solution: (b) and (c)

XNOR

$$= \frac{\overline{A\oplus B}}{\overline{A\overline{B}+\overline{A}B}} = AB + \overline{A}\,\overline{B} \qquad \text{(b)}$$

$$\overline{\overline{A\overline{B}}+\overline{\overline{A}B}}$$

$$= (\overline{A}+B)(A+\overline{B}) \qquad \text{(c)}$$

Hence, the correct option is (b) and (c).

26. For the logic circuit shown in the figure, the output is equal to

$$y = \overline{AB} + \overline{BC} + \overline{A} + \overline{C}$$

$$= \overline{A} + \overline{B} + \overline{B} + \overline{C} + \overline{A} + \overline{C} \qquad (\overline{A} + \overline{A} = \overline{A})$$

$$y = \overline{A} + \overline{B} + \overline{C}$$

Hence, the correct option is (b).

27. Indicate which of the following logic gates can be used to realized all possible combinational Logic functions:

(a) OR gates only

(b) NAND gates only

(c) EX-OR gates only

(d) NOR gates only **[1989]**

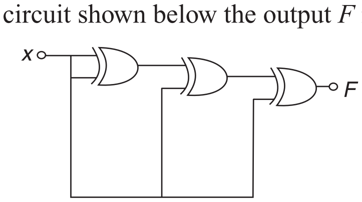
Solution: (b) and (d)

For realizing all logic function: UNIVERSAL GATES used

$\Rightarrow$ NAND and NOR.

Hence, the correct option is (b) and (d).

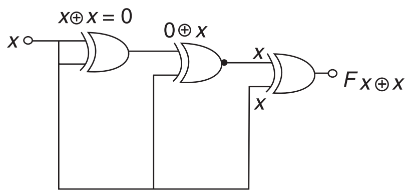
28. For the circuit shown below the output F is given by



(a) $F = 1$ (b) $F = 0$

(c) $F = X$ (d) $F = \bullet X$ **[1988]**

Solution: (b)



$x \oplus x = x\bar{x} + x\bar{x}$
$= 0 + 0$

$x \oplus x = 0$

Then $x \oplus 0 = \bar{x}.0 + \bar{0}.x$

$= 0 + x \cdot 1$

$x \oplus 0 = x$

$\Rightarrow F = x \oplus x$

$F = 0$

Hence, the correct option is (b).

29. Minimum number of 2-input NAND gates required to implement the function, $F = (\bar{X} + \bar{Y})(Z + W)$ is

(a) 3 (b) 4
(c) 5 (d) 6 **[1988]**

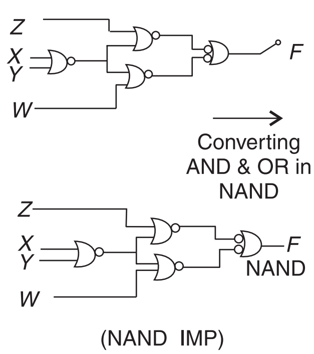
Solution: (b)

$F = (\bar{x} + \bar{y})(z + w)$

For NAND gate implementation $(\overline{AB})$

$F = (\overline{x \cdot y})(\bar{z} + w)$

$= \bar{x}\bar{y}z + \bar{x}\bar{y}w$

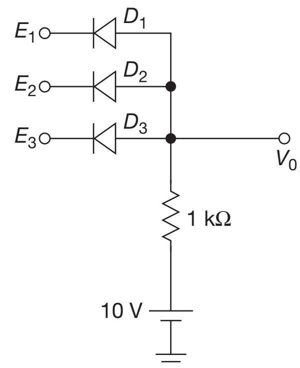


$\Rightarrow$ NAND gates

Hence, the correct option is (b).

Two-marks Questions

1. In the circuit shown, diodes D_1, D_2 and D_3 are ideal, and the inputs E_1, E_2 and E_3 are "0 V" for logic '0' and "10 V" for logic '1'. What logic gate does the circuit represent? **[2015]**



(A) 3-input OR gate (B) 3-input NOR gate
(C) 3-input AND gate (D) 3-input XOR gate

Solution: If any of the input E_1, E_2, E_3, is zero. The corresponding diode gets forward biased, and outputs V_0 = Voltage Across Forward biased diode = 0.

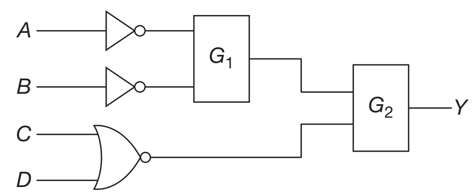
So Any input = 0, output = 0.

If No input = 0, i.e., All input are 1, All diodes will be reverse biased, V_0 = Logic 1,

So it is a 3 input AND gate.

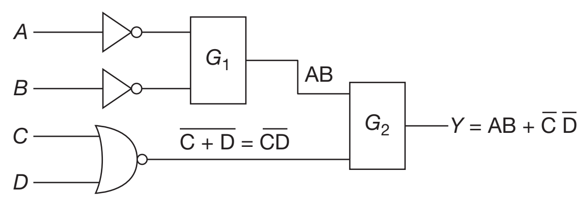
Hence, the correct option is (C).

2. In the figure shown, the output Y is required to be $Y = AB + \bar{C}\,\bar{D}$. The gates G_1 and G_2 must be, respectively. **[2015]**



(A) NOR, OR (B) OR, NAND
(C) NAND, OR (D) AND, NAND

Solution:



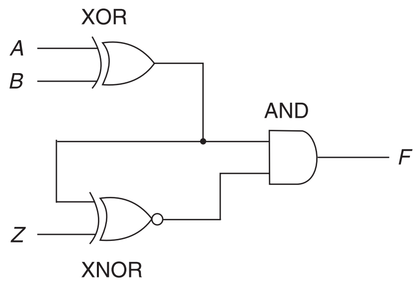
NOR gate output is $\overline{C + D} = \bar{C}\,\bar{D}$

This term present in output of G_2 so we can assume G_2 is OR gate, and other input of G_2 is AB.

For G_1 gate, inputs are $\bar{A}, \bar{B}$, and output is AB, so G_1 is NOR gate $\overline{\bar{A} + \bar{B}} = AB$

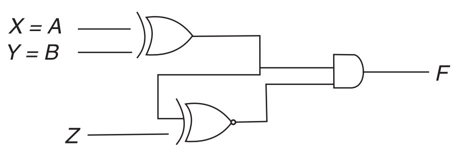
Hence, the correct option is (A).

3. The output F in the digital logic circuit shown in the figure is



(a) $F = \bar{X}YZ + X\bar{Y}Z$ (b) $F = \bar{X}Y\bar{Z} + XY\bar{Z}$
(c) $F = \bar{X}\bar{Y}Z + XYZ$ (d) $F = \bar{X}\bar{Y}\bar{Z} + XYZ$ **[2014]**

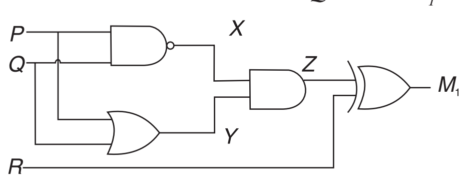
Solution: (a)



$$F = (A \oplus B)\cdot(A \oplus B \,\overline{\oplus}\, Z)$$
$$= (A \oplus B)\big((A \oplus B)\cdot Z + A \,\overline{\oplus}\, B \cdot \bar{Z}\big)$$
$$= A \oplus B \cdot Z + \underbrace{(A \oplus B)(A \,\overline{\oplus}\, B)}_{=0}\cdot \bar{Z}$$
$$= (A \oplus B)Z$$
$$F = (AB + A\bar{B})Z$$
$$F = (\bar{A}BZ + A\bar{B}Z)$$
$$F = \underline{\bar{X}YZ + X\bar{Y}Z}.$$

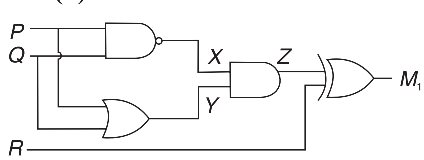
Hence, the correct option is (a).

4. Which of the following Boolean expressions correctly represents the relation between P, Q, R and M_1?



(a) M_1 = (P OR Q) XOR R
(b) M_1 = (P AND Q) XOR R
(c) M_1 = (P NOR Q) XOR R
(d) M_1=(P XOR Q) XOR R **[2008]**

Solution: (d)

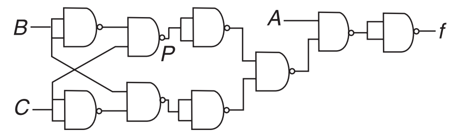


$$X = \overline{P\cdot Q}$$
$$Y = P + Q$$
$$Z = X \cdot Y = \overline{PQ}(P+Q)$$
$$= (\bar{P} + \bar{Q})(P+Q)$$
$$= P\bar{Q} + \bar{P}Q$$

$Z = P \oplus Q$ and $M_1 = Z \oplus R = \underline{P \oplus Q \oplus R}$.

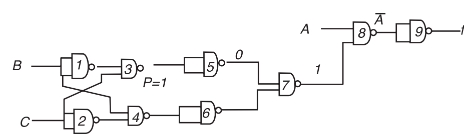
Hence, the correct option is (d).

5. The point P in the following figure is stuck-at-1. The output f will be



(a) $\overline{AB\bar{C}}$ (b) $\bar{A}$
(c) $AB\bar{C}$ (d) A **[2006]**

Solution: (d)



If P is stuck at 1 ⇒ output of NAND (5) = 0

⇒ For NAND (7), output = 1

$$\Rightarrow f = \overline{(\overline{A.1})\cdot \bar{A}} = \overline{\bar{A}\bar{A}} = \bar{\bar{A}}$$

$\underline{f = A}$.

Hence, the correct option is (d).

6. The number of product terms in the minimized sum-of-product expression obtained through the following K-map is (where 'd' denotes don't care states)

1	0	0	1
0	d	0	0
0	0	d	1
1	0	0	1

(a) 2 (b) 3
(c) 4 (d) 5 **[2005]**

Solution: (a)

∴ For SOP expression:

⇒1 quad + 1 pair is taken as two product terms.

1	0	0	1
0	d	0	0
0	0	d	1
1	0	0	1

Hence, the correct option is (a).

7. A Boolean function f of two variables x and y is defined as follows: $f(0, 0) = f(0, 1) = f(1, 1) = 1$; $f(1, 0) = 0$. Assuming complements of x and y are not available, a minimum cost solution for realizing f using only 2-input NOR gates and 2-input OR gates (each having unit cost) would have a total cost of

(a) 1 unit (b) 4 unit

(c) 3 unit (d) 2 unit **[2004]**

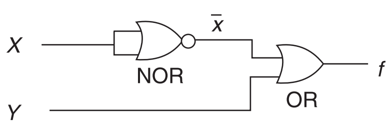
Solution: (d)

$$f(0,0) = f(0,1) = f(1,1) = 1 \ ; \quad f(1,0) = 0$$

X	Y	F
0	0	1
0	1	1
1	1	1
1	0	0

$$f = \bar{x}(y+\bar{y}) + xy$$
$$= \bar{x} + xy$$
$$f = \bar{x} + y$$

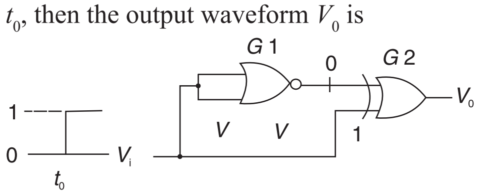
As $\bar{X}$ is not available

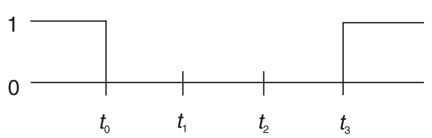


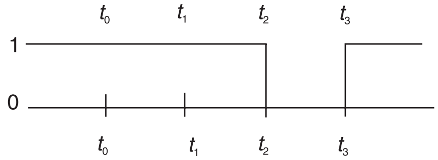
2 gates $\Rightarrow$ 2 unit.

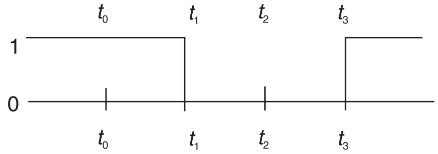
Hence, the correct option is (d).

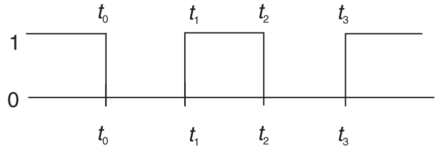
8. The gates G_1 and G_2 in the figure have propagation delays of 10 nsec and 20 nsec, respectively. If the input V_i makes an abrupt change from logic 0 to 1 at time $t = t_0$, then the output waveform V_0 is



(a) 

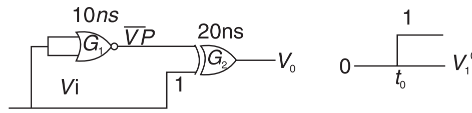
(b) 

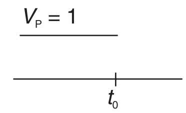
(c) 

(d) 

[2002]

Solution: (b)





(a) Before to ($t <$ to)

$V_i = 0$, $G_1 = 1$

$G_2 = 01(1 \oplus 0)$

(b) $t >$ to, $Vi = 1$

Now, output of G_1 becomes $\overline{1+1} = 0$ (after 10 ns)

$\therefore$ Till $t_0 - \left(\dfrac{t_0 + t_1}{t_2}\right)$ V_0 remains $(G_2 = 0 \oplus 1 = 1)$ after

$t_0 + t_1$, $G_1 = 0$ (after TPD = 10 ns)

$\Rightarrow G_2 = 0 \oplus 1 = 1$, but after TPD = 20ns

$\Rightarrow$ After $\dfrac{(t_0 + t_1) + 20ns}{t_3}$, $V_0 = 1$

V_0 remain 0 from $(t_0 + t_1) - (t_0 + t_3)$

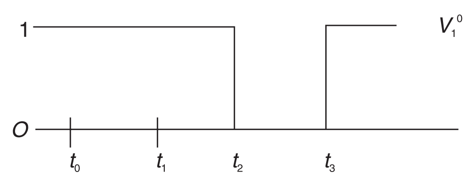
$(t_2 - t_3) \rightarrow V_0 = 0$

$\Rightarrow V_0 \quad t < t_0 = 1$

$t_0 < t < t_2 = 1$

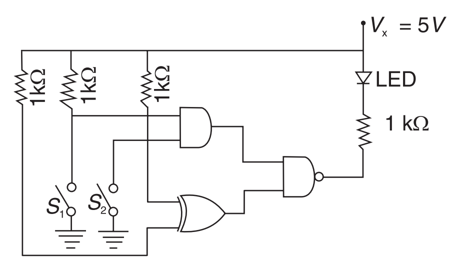
$t_2 < t_3 = 0$

$t_3 < t = 1$



Hence, the correct option is (b).

9. In the figure, the LED



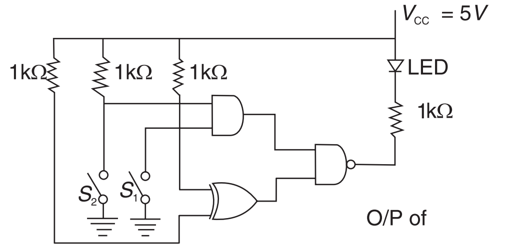
(a) emits light when both S_1 and S_2 are closed.

(b) emits light when both S_1 and S_2 are open.

(c) emits light when only of S_1 and S_2 is closed

(d) does not emit light, irrespective of the switch positions. **[2001]**

Solution: (d)



LED glows when n side is low ($P \rightarrow +5$)

$\Rightarrow$ output of NAND = 0

$\Rightarrow$ Output AND and XOR = 1

$\Rightarrow$ Input of AND = 1, 1

Input of XOR = 0, 1 or 1, 0

For AND gate: S_1 and S_2 should be open (input = high (+5 Vcc)) but when S_1 and S_2 are open, then input to X-OR = 121 $\Rightarrow$ output = 0

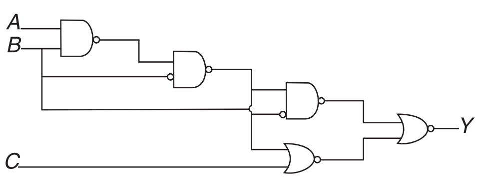
$\therefore$ If S_1S_2 both open $\rightarrow$

S_1S_2 both close $\rightarrow$

$$\left.\begin{matrix}\text{NAND} \neq 0\\ \text{NAND} \neq 0\end{matrix}\right\}\text{LED does not glow}$$

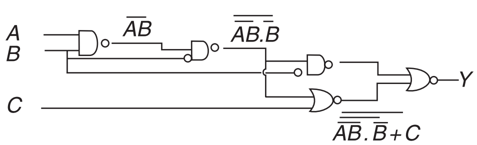
Hence, the correct option is (d).

10. For the logic circuit shown in the figure, the simplified Boolean expression for the output Y is



(a) $A + B + C$ (b) A
(c) B (d) C **[2000]**

Solution:



$$y = ((\overline{\overline{\overline{AB}\;\overline{B}\;\overline{B}}}) + \overline{\overline{\overline{AB}\;\overline{B}} + \overline{C}})$$

Now $\overline{\overline{AB}.B} = (\overline{\overline{A}+\overline{B}})\overline{B} = \overline{\overline{AB}} + \overline{B} = (\overline{\overline{A}+1})(\overline{B})$

$$= \overline{\overline{B}} = B \qquad \overline{A}+1 = 1$$

$$\Rightarrow y = \overline{\overline{(B\cdot\overline{B})} + \overline{B+\overline{C}}} = \overline{\overline{O} + \overline{B+\overline{C}}} = 87 \quad (\because B\cdot\overline{B}) = 0$$

$$y = \overline{1 + \overline{BC}} = \overline{1} = 0$$

$$y = 0$$

FIVE-MARKS QUESTIONS

1. The truth table for the output Y in terms of three inputs A, B and C are given in Table. Draw a logic circuit realization using only NOR gates.

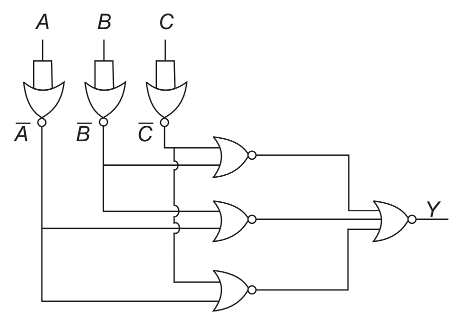
A	0	1	0	1	1	1	0	1
B	0	0	1	1	0	0	1	1
C	0	0	0	0	1	1	1	1
Y	1	1	1	0	1	0	0	0

[1993]

Solution: $y = \pi m\,(3, 5, 6, 7) = y\,(\text{CBA}) = y(\text{ABC})$

C \ BA	00	01	11	10
0			0	
1		0	0	0

$$y = (\overline{A}+\overline{B})(\overline{B}+\overline{C})(\overline{C}+\overline{A})$$



2. The operating condition (ON = 1, OFF = 0) of three pumps (x, y, z) are to be monitored. $x = 1$ implies that pump X is on. It is required that the indicator (LED) on the panel should glow when a majority of the pumps fail.

	$\overline{z}$	z
$\overline{x}\ \overline{y}$		
$\overline{x}\ y$		
$x\ y$		
$x\ \overline{y}$		

Figure (a)

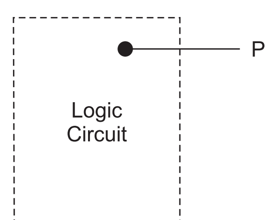


Figure (b)

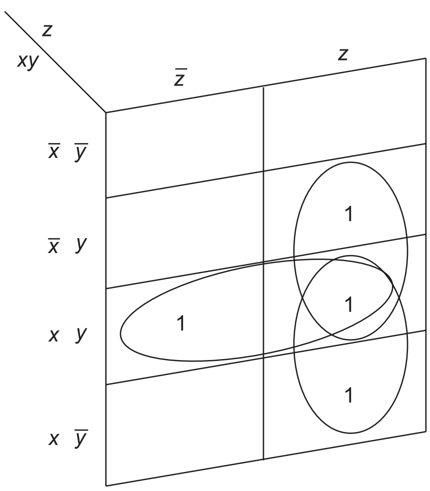
(a) Enter the logical values in the K-map in the format shown in figure (a). Derive the minimal Boolean sum-of-products expression whose output is zero when a majority of the pumps fail.

(b) The above expression is implemented using logic gates and point P is the output of this circuit, as shown in figure (b). P is at 0 V when a majority of the pumps fail and is at 5 V otherwise. Design a circuit to drive the LED using this output. The current through the LED should be 10 mA and the using this output. The current through the LED should be 10 mA and the voltage drop across it is 1 V. Assume that P can source or sink 10 mA and a 5 V supply is available. [2000]

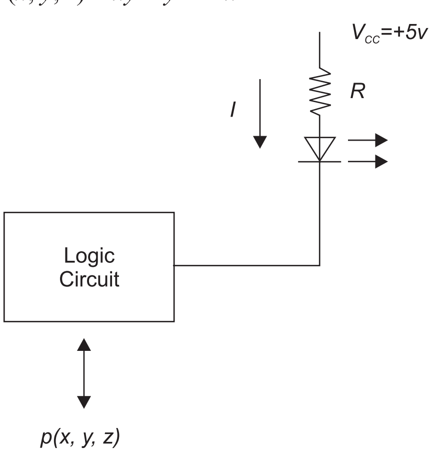
Solution:

	X	Y	Z	P
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

(a) $P(x, y, z) = \bullet m\,(3, 5, 6, 7)$



(b) $P(x, y, z) = xy + yz + zx$



$= xy + yz + xz$

when majority of pumps fails P is at '0' therefore current 'I' flows through the ckt.

$$\left.\begin{aligned} I &= 10 \text{ mA} \\ V_D &= 1\text{V} \end{aligned}\right\} \text{given}$$

$$R = \frac{5-1}{10 \text{ m}} = \frac{4}{10\times10^{-3}} = 400\Omega$$

3. For digital block shown in Figure (a), the output $Y = f(S_3, S_2, S_1\, S_0)$ where S_3 is MSB and S_0 is LSB. Y is given in terms of minterms as

$Y = \Sigma m(1, 5, 6, 7, 11, 12, 13, 15)$ and its complements $Y = \text{l}m(0, 2, 3, 4, 8, 9, 10, 14)$.

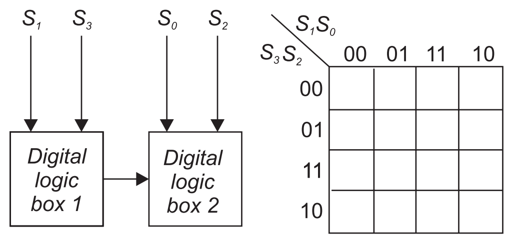


Figure (a) **Figure (b)**

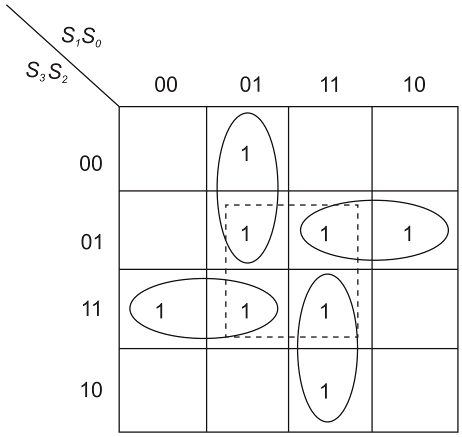
(a) Enter the logical values in the given Karnaugh map [figure (b)] for the output Y.
(b) Write down the expression for Y in sum-of-products from using minimum number of terms.
(c) Draw the circuit for the digital logic boxes using four 2-input NAND gates only for the each of the boxes. **[2001]**

Solution:

(a)

S_3S_2 \ S_1S_0	00	01	11	10
00	0	1	0	0
01	0	1	1	1
11	1	1	1	0
10	0	0	1	0

(b)



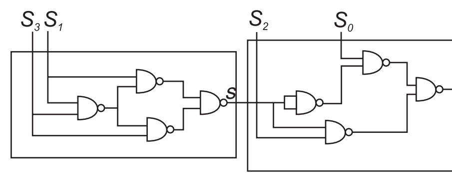
$$y = \bar{S}_3\bar{S}_1S_0 + \bar{S}_3S_2S_1 + S_3S_2\bar{S}_1 + S_3S_1S_0$$

In solution quad has not been included because it is Reduntant term.

(c). $y = S_3S_2\bar{S}_1 + \bar{S}_3S_2S_1 + \bar{S}_3\bar{S}_1S_0 + S_3S_1S_0$

$$= S_2(S_3\bar{S}_1 + \bar{S}_3S_1) + S_0(\bar{S}_3\bar{S}_1 + S_3S_1)$$

$$= S_2(S_3 \oplus S_1) + S_0(\overline{S_3 \oplus S_1}) = S_2S + S_0\bar{S}A$$

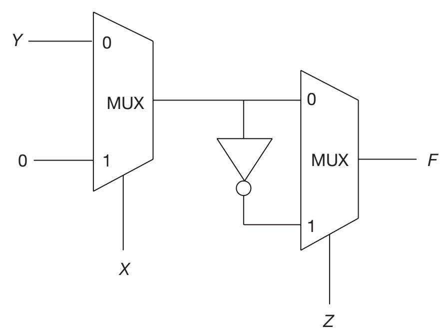


Chapter 4

Combinational Circuits

One-mark Questions

1. Consider the circuit shown in the figure **[2017]**

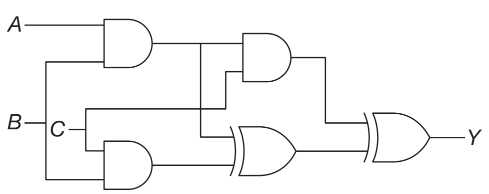


The Boolean expression F implemented by the circuit is

(A) $\overline{X}\,\overline{Y}\,\overline{Z} + XY + \overline{Y}Z$ (B) $\overline{X}Y\overline{Z} + XZ + \overline{Y}Z$

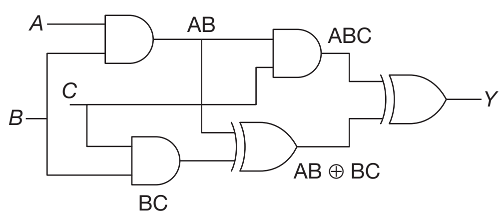
(C) $\overline{X}Y\overline{Z} + XY + \overline{Y}Z$ (D) $\overline{X}\,\overline{Y}\,\overline{Z} + XZ + \overline{Y}Z$

2. The output of the combinational circuit given below is **[2016]**



(A) $A + B + C$ (B) $A(B + C)$
(C) $B(C + A)$ (D) $C(A + B)$

Solution:



$$Y = ABC \oplus AB \oplus BC$$

$$= ABC \oplus \left[AB\left(\overline{BC}\right) + \overline{AB}\left(BC\right)\right]$$

$$= ABC \oplus \left[AB\overline{C} + \overline{A}BC\right]$$

$$= ABC \oplus B\left(A \oplus C\right)$$

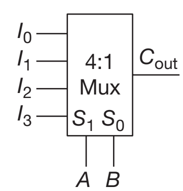
$$= ABC\left[\overline{B} + AC + \overline{A}\ \overline{C}\right] + \overline{ABC}\left(AB\overline{C} + \overline{A}.BC\right)$$

$$= ABC + \overline{A}BC + AB\overline{C}$$

$$= BC + AB = B\left(C + A\right)$$

Hence, the correct option is (C).

3. A 4 : 1 multiplexer is to be used for generating the output carry of a full adder. A and B are the bits to be added while C_{in} is the input carry and C_{out} is the output carry. A and B are to be used as the select bits with A being the more significant select bit. **[2016]**



Which one of the following statements correctly describes the choice of signals to be connected to the inputs I_0, I_1, I_2, and I_3 so that the output is C_{out}?

(A) $I_0 = 0, I_1 = C_{in}, I_2 = C_{in}$ and $I_3 = 1$
(B) $I_0 = 1, I_1 = C_{in}, I_2 = C_{in}$ and $I_3 = 1$
(C) $I_0 = C_{in}, I_1 = 0, I_2 = 1$ and $I_3 = C_{in}$
(D) $I_0 = 0, I_1 = C_{in}, I_2 = 1$ and $I_3 = C_{in}$

Solution: In a 4:1 multiplexer, the no of select lines will be 2 which are A and B, observing the carry output will make it easier to find the min terms.

Carry output of full adder will have min terms

$$\sum m\left(3,5,6,7\right)$$

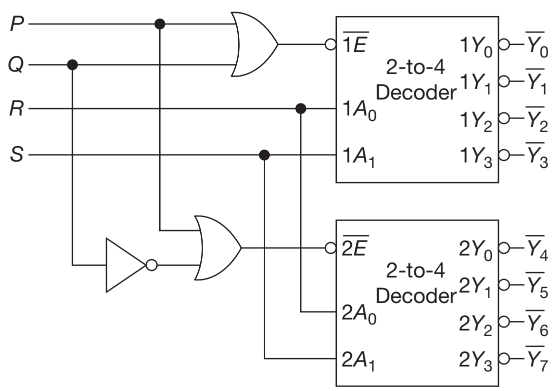
To implement it, we can use tabular method:

C_{in} \ AB	I_0 00	I_1 01	I_2 10	I_3 11
0	0	2	4	(6)
1	1	(3)	(5)	(7)
	0	C_{in}	C_{in}	1

$$I_0 = 0$$
$$I_1 = I_2 = C_{in}$$
$$I_3 = 1$$

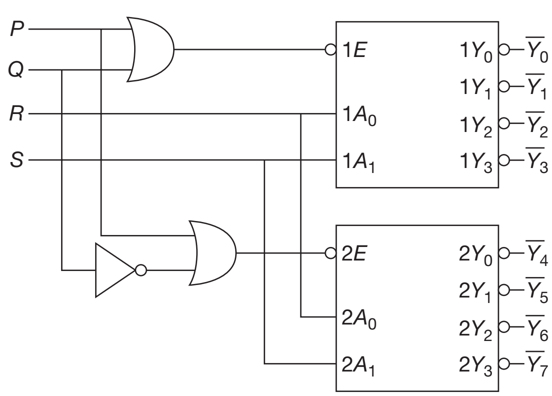
Hence, the correct option is (A).

4. A1-to-8 demultiplexer with data input D_{in}, address inputs S_0, S_1, S_2 (with S_0 as the LSB) and $\overline{Y}_0$ to $\overline{Y}_7$ as the eight demultiplexed outputs, is to be designed using two 2-to-4 decoders (with enable input $\overline{E}$ and address inputs A_0 and A_1) as shown in the figure. D_{in}, S_0, S_1 and S_2 are to be connected to P, Q, R and S, but not necessarily in this order. The respective input connections to P, Q, R and S terminals should be **[2015]**



(A) S_2, D_{in}, S_0, S_1
(B) S_1, D_{in}, S_0, S_2
(C) D_{in}, S_0, S_1, S_2
(D) D_{in}, S_2, S_0, S_1

Solution:



The OR gate output is zero, when inputs are zero. Then decoders will be enabled.

$P = Q = 0$, 1st Decoder will work with inputs S, R (A_1 A_0)

$P = 0$, $Q = 1$, 2nd Decoder will work with input S, R (A_1 A_0)

So, Q, S, R are inputs $S_2 S_1 S_0$ of Demultiplexer

P is $D_{in,}$

$P, Q, R, S = D_{in}, S_2, S_0, S_1$

Hence, the correct option is (D).

5. In a half-subtractor circuit with X and Y as inputs, the borrow (M) and difference ($N = X - Y$) are given by

(a) $M = X \oplus Y$, $N = XY$
(b) $M = XY$, $N = X \oplus Y$
(c) $M = \overline{X}Y$, $N = X \oplus Y$
(d) $M = X\overline{Y}$, $N = \overline{X \oplus Y}$ **[2014]**

Solution: (c)

HALF SUBT.:

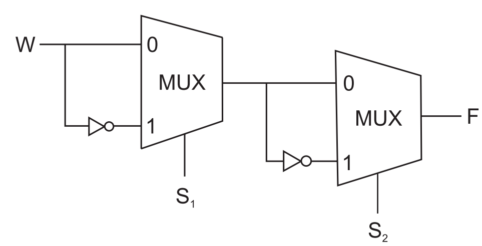
$N = X - Y$

X	Y	N	M
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$\begin{bmatrix} N = X \oplus Y \\ M = \overline{X}.Y \end{bmatrix}.$$

Hence, the correct option is (c).

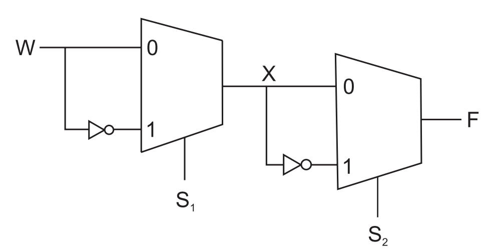
6. Consider the multiplexer based logic circuit shown in the figure.



Which one of the following Boolean functions is realized by the circuit?

(a) $F = W\overline{S_1}\,\overline{S_2}$
(b) $F = WS_1 + WS_2 + S_1 S_2$
(c) $F = \overline{W} + S_1 + S_2$
(d) $F = W \oplus S_1 \oplus S_2$ **[2014]**

Solution: (d)



Output × of MUX 1: $X = S_1\overline{W} + \overline{S_1}W = S_1 \oplus W$

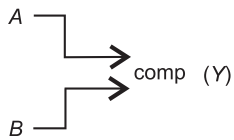
$F = S_2\overline{X} + \overline{S_2}X = S_2 \oplus X = \underline{S_1 \oplus W \oplus S_2}.$

Hence, the correct option is (d).

7. The output Y of a 2-bit comparator is logic 1 whenever the 2-bit input A is greater than the 2-bit input B. The number of combination for which the output is logic 1 is

(a) 4 (b) 6
(c) 8 (d) 10 **[2012]**

Solution: (b)



$Y = 1$ when $(A > B)$

If $B = 00$. Then A can be: 01, 10, 11

$B = 01, A = 10, 11$

$B = 10 \quad A = 11$

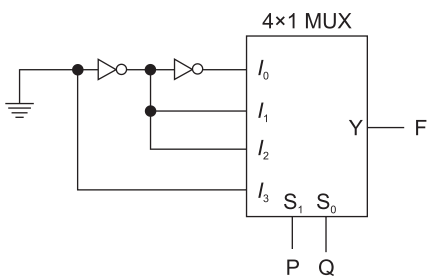
$B = 11$, then A cannot be $>B$

$\Rightarrow A = (01, 10, 11), (10, 11), (11) \quad \Rightarrow$ 6 Combination.

$B = (00) \qquad (010) \qquad (10)$

Hence, the correct option is (b).

8. The logic function implemented by the circuit below is (ground implies a logic `0')



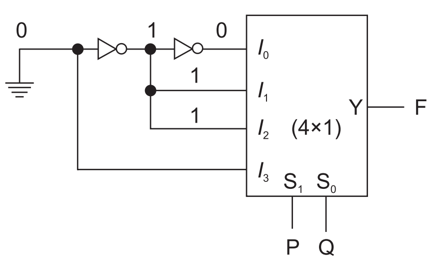
(a) $F = \text{AND}(P, Q)$

(b) $F = \text{OR}(P, Q)$

(c) $F = \text{XNOR}(P, Q)$

(d) $F = \text{XOR}(P, Q)$ **[2011]**

Solution: (d)



$$F = PQI_3 + P\bar{Q}I_2 + \bar{P}QI_1 + \bar{P}\bar{Q}I_0$$

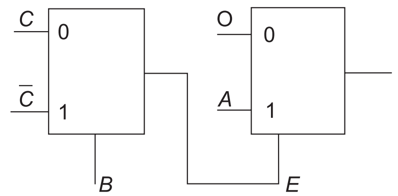
$$= \cancel{PQ\cdot 0} + P\bar{Q}\cdot 1 + \bar{P}Q\cdot 1 + \cancel{\bar{P}\bar{Q}\cdot 0}$$

$$F = P\bar{Q} + \bar{P}Q = P \oplus Q$$

$$F = XOR(P, Q).$$

Hence, the correct option is (d).

9. The Boolean function f implemented in the figure using two input multiplexers is

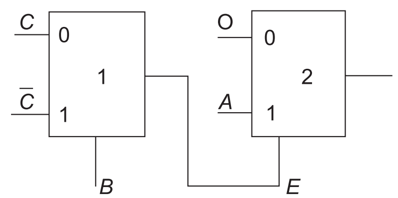


(a) $A\bar{B}C + AB\bar{C}$ (b) $ABC + A\bar{B}\bar{C}$

(c) $\bar{A}BC + \bar{A}\bar{B}\bar{C}$ (d) $\overline{AB}C + \bar{A}B\bar{C}$

[2005]

Solution: (a)



$\Rightarrow$ From MUX 1: E = $C\bar{B} + \bar{C}B$

$$f = \bar{E}\cdot 0 + E\cdot A$$

$$= 0 + (C\bar{B} + \bar{C}B)A$$

$$f = AB\bar{C} + A\bar{B}C.$$

Hence, the correct option is (a).

10. Without any additional circuitry, an 8:1 MUX can be used to obtain

(a) some but not all Boolean functions of 3 variables
(b) all functions of 3 variables but none of 4 variables
(c) all functions of 3 variables and some but not all of 4 variables
(d) all functions of 4 variables **[2003]**

Solution: (c)

8:1 MUX

$\Rightarrow$ 3 select lines $(2^{(3)})$

$\Rightarrow$ All function of 3 & some 4 variable function.

Hence, the correct option is (c).

11. A 2-bit binary multiplier can be implemented using

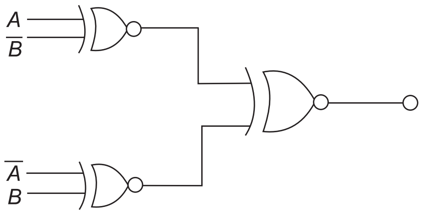
(a) 2 input ANDs only
(b) 2 input X-ORs and 4-input AND gates only
(c) Two (2) input NORs and one XNOR gate
(d) XOR gates and shift registers. **[1997]**

Solution: (a)

2-bit binary multiplier can be implemented using 2 input AND gote only.

Hence, the correct option is. (a)

12. The output of the circuit shown in figure is equal to

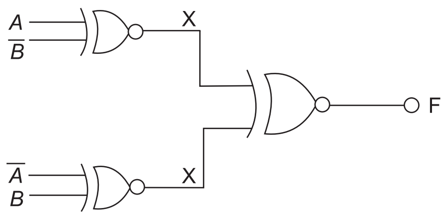


(a) 0 (b) 1

(c) $\overline{A}B + A\overline{B}$ (d) $\overline{(A\oplus B)}\oplus\overline{(A\oplus B)}$

[1995

Solution: (b)



$\Rightarrow F = X\overline{\oplus}X$

$F = 1$

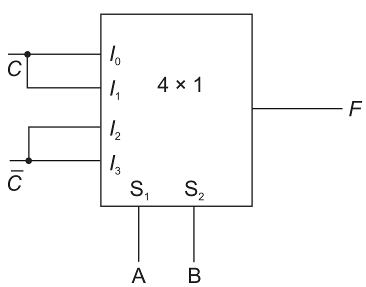
If $A\oplus\overline{B} = X$ $(\overline{A}\overline{B}+\overline{A}B)$

Then $\overline{A}\oplus B = X$, also

$=(\overline{A}\overline{B}+AB)$ (X-NOR of same number $= 1$)

Hence, the correct option is (b).

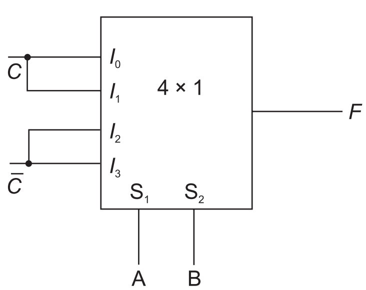
13. The logic realized by the circuit shown in figure is



(a) $F = A\oplus C$ (b) $F = A\oplus C$

(c) $F = B\oplus C$ (d) $F = B\oplus C$ **[1992]**

Solution: (b)



For 4 × 1 MUX,

$$F = C(\overline{A}\overline{B}+\overline{A}B)+\overline{C}(A\overline{B}+AB)$$

$$= C\overline{A}(\overline{B}+B)+\overline{C}A(\overline{B}+B)$$

$$= C\overline{A}+\overline{C}A$$

$$F = C\oplus A.$$

So, the answer is (b).

Hence, the correct option is (b).

14. The minimal function that can detect a 'divisible by 3' 8421 *BCD* code digit (representation is $D_8\, D_4\, D_2\, D_1$) is given by

(a) $D_8\, D_1 + D_4\, D_2 + D_8D_2D_1$

(b) $D_8D_1 + D_4D_2\overline{D}_1 + \overline{D}_4D_2D_1 + \overline{D}_8\overline{D}_4\overline{D}_2\overline{D}_1$

(c) $D_8D_1 + D_4D_2 + \overline{D}_8\overline{D}_4\overline{D}_2\overline{D}_1$

(d) $D_4D_2\overline{D}_1 + D_4D_2D_1 + D_8D_4D_2D_1$ **[1990]**

Solution: (b)

To detect a number divisible by three:

	D_8	D_4	D_2	D_1	Y
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	1

0 is divisible

$$Y = \overline{D}_8\overline{D}_4\overline{D}_2\overline{D}_1 + \overline{D}_8\overline{D}_4D_2D_1$$

$$+\overline{D}_8D_4D_2\overline{D}_1 + D_8\overline{D}_4D_1.$$

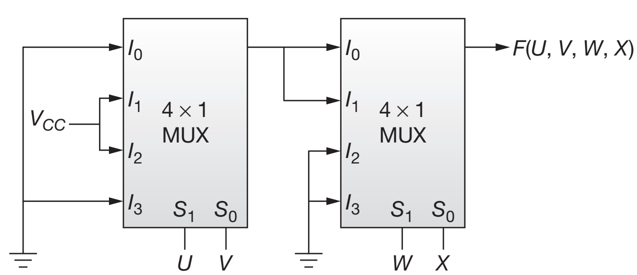
(BCD – till a)

Hence, the correct option is (b).

Two-marks Questions

1. A four-variable Boolean function is realized using 4 × 1 multiplexers as shown in the figure **[2018]**

The minimized expression for F (U, V, V, X) is



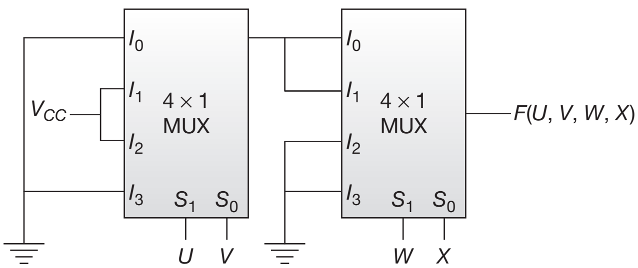
(A) $(UV+\bar{U}\bar{V})\bar{W}$

(B) $UV+\bar{U}\bar{V})(\bar{W}\bar{X}+\bar{W}X)$

(C) $(U\bar{V}+\bar{U}V)\bar{W}$

(D) $U\bar{V}+\bar{U}V)(\bar{W}\bar{X}+\bar{W}X)$

Solution: Consider the figure given below



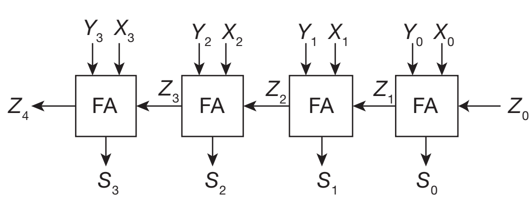
From the above figure, the minimized expression for F (U, V, V, X) will be

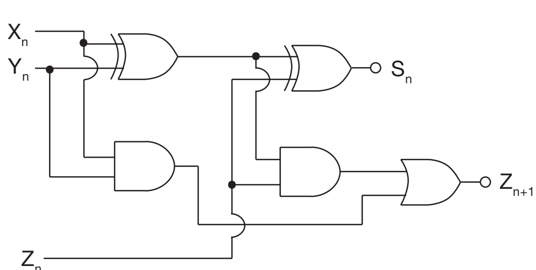
$$F=\left(\bar{U}\bar{V}+U\bar{V}\right)\left(\bar{W}\right)$$

Hence, the correct option is (C)

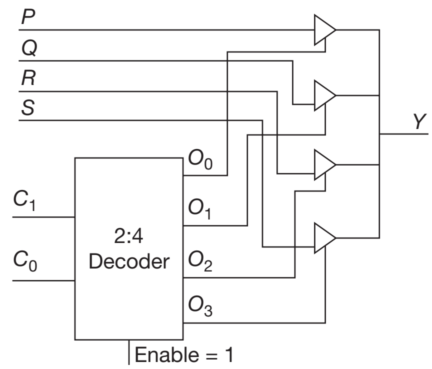
2. Figure I shows a 4-bit ripple carry adder realized using full adders and Figure II shows the circuit of a full-adder (FA). The propagation delay of the XOR, AND and OR gate in Figure II are 20 ns, 15 ns and 10 ns, respectively. Assume all the inputs to the 4-bit adder are initially reset to 0.

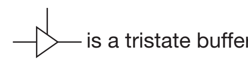
At t = 0, the inputs to the 4-bit adder are changed to $X_3X_2X_1X_0$ = 1100, $Y_3Y_2Y_1Y_0$= 0100 and Z_0 =1. The output of the ripple carry adder will be stable at t (in ns) = ____________. **[2017]**





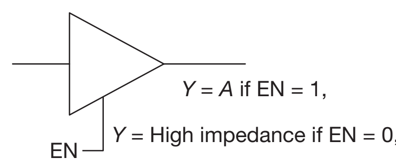
3. The functionality implemented by the circuit below is **[2016]**





(A) 2 to 1 multiplexer
(B) 4 to 1 multiplexer
(C) 7 to 1 multiplexer
(D) 6 to 1 multiplexer

Solution:



C_1C_0 of decoder will activate (=1), one of the output O_0 to O_3. So that corresponding tristate buffer will give output same as input, other tristate buffers will be disabled.

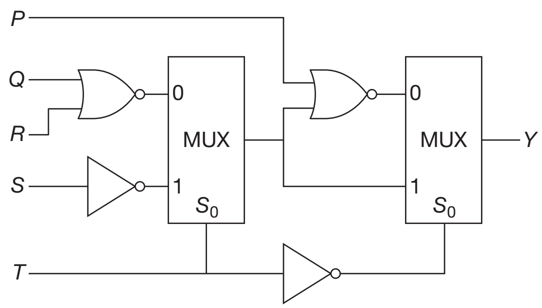
C_1	C_0	O_0	O_1	O_2	O_3	Y
0	0	1	0	0	0	P
0	1	0	1	0	0	Q
1	0	0	0	1	0	R
1	1	0	0	0	1	S

The two inputs of decoder are handling 4 outputs simultaneously disabling the unused buffers therefore; it will behave as multiplexer.

Hence, the correct option is (B).

4. For the circuit shown in the figure, the delays of NOR gates, multiplexers and inverters are 2 ns, 1.5 ns and 1 ns respectively. If all the inputs P, Q, R, S and T are applied at the same time instant, the maximum propagation delay (in ns) of the circuit is ___________. **[2016]**

Solution:



Since, the multiplexer has two input lines therefore, the number of select lines is 1, i.e., T.

Hence, the MUX works in 2 cases;

(i) when $T = 0$

(delay is caused by two multiplexers and one inverter)

delay $= t_{\text{NOR}} + t_{\text{MUX}} + t_{\text{MUX}} = 2 + 1.5 + 1.5 = 5$ ns

(ii) when $T = 1$

(delay is caused by both the multiplexers, inverter T and a NOR gate)

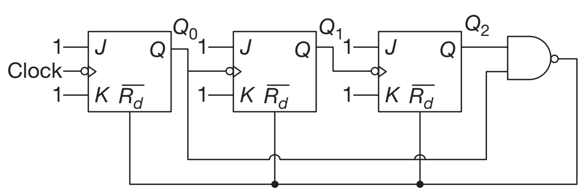
delay $= t_{\text{inv}} + t_{\text{MUX}} + t_{\text{NOR}} + t_{\text{MUX}} = 1 + 1.5 + 2 + 1.5 = 6$ ns

So the maximum delay = 6 ns.

Hence, the correct Answer is (6 ns).

5. The circuit shown consists of J-K flip-flops, each with an active low asynchronous reset $\left(\overline{R_d}\ input\right)$. The counter corresponding to this circuit is **[2015]**

(A) a modulo-5 binary up counter
(B) a modulo-6 binary down counter
(C) a modulo-5 binary down counter
(D) a modulo-6 binary up counter



Solution: All the flip-flops are in toggle mode $Q_{n+1} = \overline{Q}_n$

$$J = 1, K = 1, Q_{n+1} = \overline{Q}_n,$$

so for every clk pulse flip-flop will change its state.

Clk input is connected Q_0 (LSB).

Sequence will be $Q_2 Q_1 Q_0$

Q is connected to falling edge so it's n up counter

NAND gate is connected to $\overline{\text{Rd}}$ input (reset)

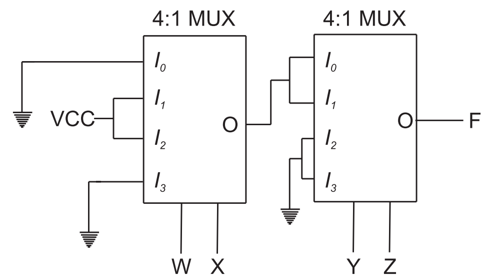
So when NAND output is zero all flip-flops will go back to reset state.

NAND output = 0, when $Q_2, Q_0 = 1, 1$

So $Q_2 Q Q_0 = 101$ or 111, in UP counter 101 state occurs first, so counter counts up 101, at the same moment it go to 000 state, so total states we can see are from 000 to 100 only. Modulo-5-up counter

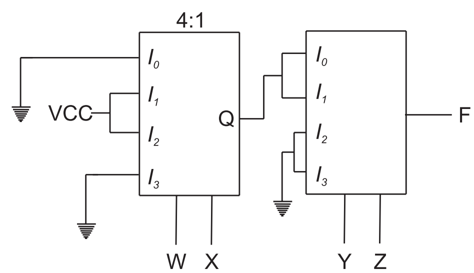
Hence, the correct option is (A).

6. In the circuit shown, W and Y are MSBs of the control inputs. The output MSBs is given by



(a) $F = W\overline{X} + \overline{W}X + \overline{Y}\overline{Z}$
(b) $F = W\overline{X} + \overline{W}X + \overline{Y}Z$
(c) $F = W\overline{X}\overline{Y} + \overline{W}X\overline{Y}$
(d) $F = (\overline{W} + \overline{X})\overline{Y}\overline{Z}$ **[2014]**

Solution: (c)



$VCC = +1$

gmd = 0

$W_1 Y \rightarrow$ MSB of S.L.

$$\Rightarrow Q = WX \cdot 0 + W\overline{X} \cdot 1 + \overline{W}X \cdot 1 + \overline{W} \cdot \overline{X} \cdot 0$$

$$Q = W\overline{X} + \overline{W}X$$

$$Q = W \oplus X$$

$$F = (\overline{Y}\,\overline{Z} + \overline{Y}Z)(Q) + \cancel{(Y\overline{Z} + YZ)}.0$$

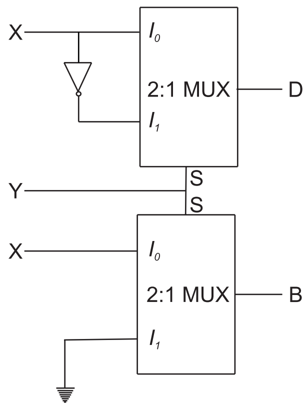
$$= \overline{Y} \cdot Q = \overline{Y}(W\overline{X} + \overline{W}X)$$

$$F = W\overline{X}\overline{Y} + \overline{W}X\overline{Y}.$$

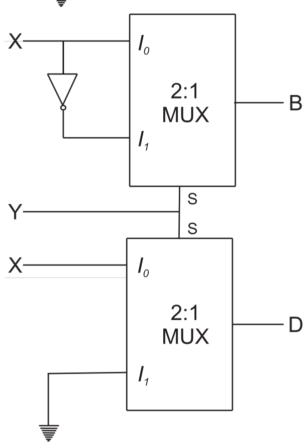
Hence, the correct option is (c).

7. If X and Y are inputs and the difference ($D = X - Y$) and the borrow (B) are the outputs, which one of the following diagrams implements a half-subtractor?

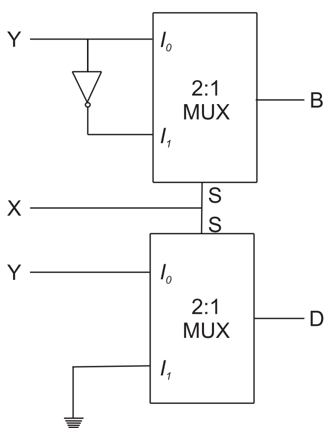
(a)



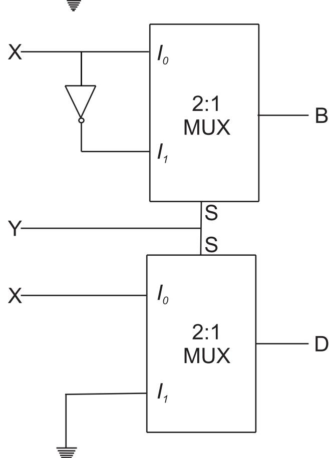
(b)



(c)



(d)



[2014]

Solution: (a)

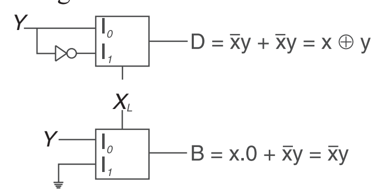
For 2 bit subtraction: Diff

$D = X \oplus Y \qquad = (X - Y)$

Borrow $B = \bar{X}Y$

Checking all MUX:

(a)



(b) $D = Y\bar{X} + \bar{Y}X$ ✓

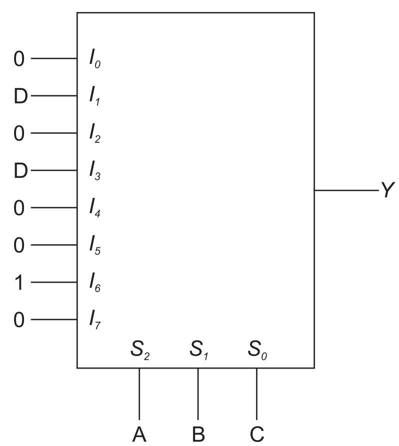
$B = YD + \bar{Y}X \neq$ Borrow ×

(c) $B = X\bar{Y} + \bar{Y}X = X \oplus Y \neq$ Borrow ×

(d) $B = Y\bar{X} + \overline{XY} \neq$ Borrow ×

Hence, the correct option is (a).

8. An 8-to-1 multiplexer is used to implement a logical function Y as shown in the figure. The output



(a) $Y = A\bar{B}C + A\bar{C}D$

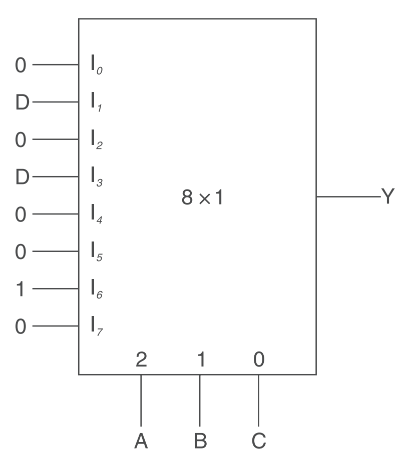
(b) $Y = \bar{A}BC + A\bar{B}D$

(c) $Y = AB\bar{C} + \bar{A}CD$

(d) $Y = \bar{A}\bar{B}D + A\bar{B}C$

[2014]

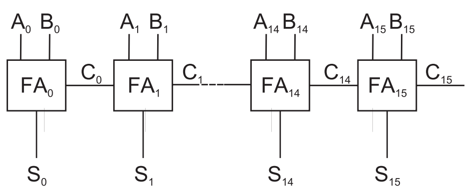
Solution: (c)



$$= ABC\cdot 0 + AB\bar{C}\cdot 1 + A\bar{B}C\cdot D + 0 + \bar{A}BC\cdot D +$$
$$\bar{A}B\bar{C}\cdot D + \bar{A}\bar{B}C\cdot D + 0$$
$$= AB\bar{C} + \bar{A}BCD + \bar{A}B\bar{C}\cdot 0 + \bar{A}\bar{B}CD$$
$$= B\bar{C}(A+X) + \bar{A}CD(\underset{=1}{B+\bar{B}})$$
$$Y = AB\bar{C} + \bar{A}CD.$$

Hence, the correct option is (c).

9. A 16-bit ripple carry adder is realized using 16 identical full adders (FA) as shown in the figure. The carry-propagation delay of each FA is 12 ns and the sum-propagation delay of each FA is 15 ns. The worst case delay (in ns) of this 16-bit adder will be__________.



[2014]

Solution:

In ripple carry adder, carry out of 1 FA becomes carry in of next FA.

$\therefore$ For generation of G_5, P.d. (carry) of all 16 FA added:

= 16 × 12 ns

= 192 ns.

For sum generation, output sum of a FA uses carry out of previous

FA $\Rightarrow$ 16 FA P.d. (sum) added

= 16 × 15 ns

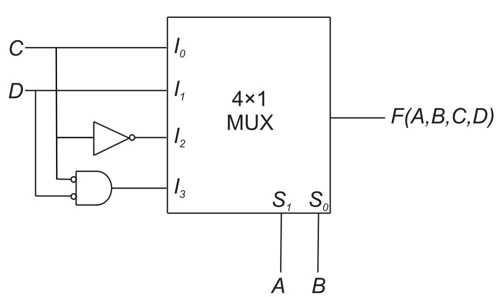
= 240 ns

As final sum pd > carry pd $\Rightarrow$ Worst case delay

= Pd sum

= 240 ns.

10. The Boolean function realized by the logic circuit shown is



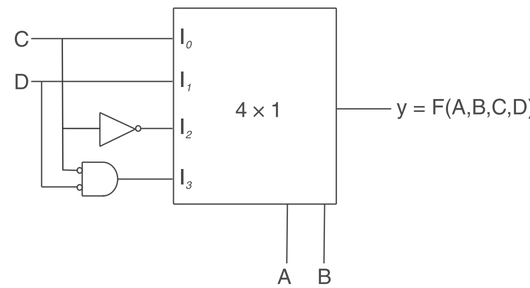
(a) $F = \sum m(0, 1, 3, 5, 9, 10, 14)$

(b) $F = \sum m(2, 3, 5, 7, 8, 12, 13)$

(c) $F = \sum m(1, 2, 4, 5, 11, 14, 15)$

(d) $F = \sum m(2, 3, 5, 7, 8, 9, 12)$ **[2010]**

Solution: (d)



$$f = ABI_3 + A\bar{B}I_2 + \bar{A}BI_1 + \bar{A}\bar{B}I_0$$
$$= AB(\bar{C}\bar{D}) + A\bar{B}\bar{C} + \bar{A}BD + \bar{A}\bar{B}C$$

Now f should be converted in canonical form

$$f = \underset{(1100)}{AB\bar{C}\bar{D}} + \underset{(1001)}{A\bar{B}\bar{C}D} + \underset{(1000)}{A\bar{B}\bar{C}\bar{D}} +$$
$$\underset{(0111)}{\bar{A}BCD} + \underset{(0101)}{\bar{A}B\bar{C}D} + \underset{(0011)}{\bar{A}\bar{B}CD} + \underset{(0010)}{\bar{A}\bar{B}C\bar{D}}$$
$$\Rightarrow f = m_{12} + m_a + m_8 + m_7 + m_5 + m_3 + m_2$$
$$f = \sum m(2,3,5,7,8,9,12) .$$

Hence, the correct option is (d).

Common Data for Question 11 and 12.

Two products are sold from a vending machine, which has two push buttons P_1 and P_2. When a button is pressed, the price of the corresponding product is displayed in a 7-segment display.

If no buttons are pressed, '0' is displayed, signifying 'Rs. 0'

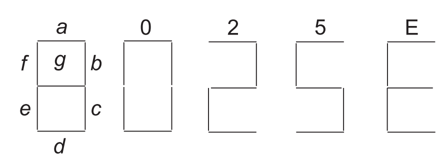
If only P_1 is pressed, '2' is displayed, signifying 'Rs. 2'

If only P_2 is pressed, '5' is displayed, signifying 'Rs. 5'

If both P_1 and P_2 are pressed, 'E' is displayed, signifying 'Error.'

The names of the segments in the 7-segment display, and the glow of the display for '0',

'2', '5' and 'E' are shown below.



Consider

(i) push button pressed/not pressed in equivalent to logic 1/0, respectively.

(ii) a segment glowing/not glowing in the display is equivalent to logic 1/0, respectively.

11. If segments a to g are considered as functions of P_1 and P_2, then which of the following is correct?

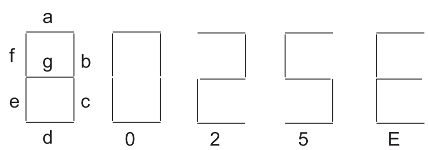
(a) $g = \overline{P}_1 + P_2$, $d = c + e$

(b) $g = P_1 + P_2$, $d = c + e$

(c) $g = \overline{P}_1 + P_2$, $e = b + c$

(d) $g = P_1 + P_2$, $e = b + c$ **[2009]**

Solution:



12. What are the minimum numbers of NOT gates and 2-mput OR gates required to design the logic of the driver for this 7-segment display?

(a) 3 NOT and 4 OR (b) 2 NOT and 4 OR

(c) 1 NOT and 3 OR (d) 2 NOT and 3 OR **[2009]**

Solution: (a)

For g, it gives, when P_2 pressed (P_1 not) for 5 = $P_2\overline{P}_1$,

$$g = P_1\overline{P}_2 + \overline{P}_1\underline{P_2} + P_1\underline{P_2}$$

$$= P_1\overline{P}_2 + P_2$$

$$= (P_1 + P_2)\underbrace{(\overline{P}_2 + P_2)}_{=1}$$

when P_1 pressed (not Q_2) for 2 = $P_1\overline{P}_2$ when both pressed (E) = P_1P_2

$$g = P_1 + P_2$$

Similarly for d: for 0 $(\overline{P_1 P_2})$, for 2, 5, E

$$\Rightarrow d = \overline{P_1P_2} + P_1\overline{P}_2 + \overline{P}_1P_2 + P_1P_2$$

also, for C: for 0, 5

$$\Rightarrow c = \overline{P}_1\overline{P}_2 + P_2\overline{P}_1$$

for e: 0, 2, E

$$e = \overline{P_1P_2} + P_1\overline{P}_2 + P_1P_2$$

$$\Rightarrow c + e = \overline{P_1\underline{P_2}} + P_1\overline{P}_2 + P_1P_2 + \overline{\underline{P_1}P_2} + P_2\overline{P}_1$$

$$e + c = \overline{P_1P_2} + P_1\overline{P}_2 + P_2\overline{P}_1 + P_2P_1 = d$$

$$\Rightarrow d = c + e, \quad g = P_1 + P_2.$$

Hence, the correct option is (a).

13. What are the minimum number of 2-to-1 multiplexers required to generate a 2-input AND gate and a 2-input Ex-OR gate?

(a) 1 and 2 (b) 1 and 3

(c) 1 and 1 (d) 2 and 2 **[2009]**

Solution: (d)

Logic of driver for 7-seg display:

(1) $g = P_1 + P_2$

(2) $c = \overline{P_1P_2} + P_2\overline{P}_1 = \overline{P}_1$

(3) $e = \overline{P_1P_2} + P_1\overline{P}_2 + P_1P_2 = \overline{P}_2 + P_1P_2 = \overline{P}_2 + P_1$

(4) $\underline{a} = \overline{P_1P_2} + \overline{P}_1P_2 + P_1\overline{P}_2 + P_1P_2 = 1 = \underline{d}$

(5) $b = \overline{P_1P_2} + P_1\overline{P}_2 = \overline{P}_2$

(6) $f = \overline{P}_1\overline{P}_2 + P_2\overline{P}_1 + P_1P_2 = \overline{P}_1P_2$

$\Rightarrow \overline{P}_1$ & $\overline{P}_2$ req

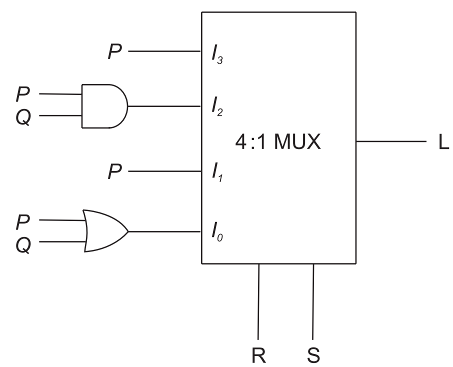
(2 NOT gates)

$g, e, f \rightarrow$ 10R each (3 OR)

2 NOT & 3 OR.

Hence, the correct option is (d).

14. For the circuit cuit shown in the following figure, I_0–I_3 are inputs to the 4 : 1 multiplexer. R (MSB) and S are control bits.



The output Z can be represented by

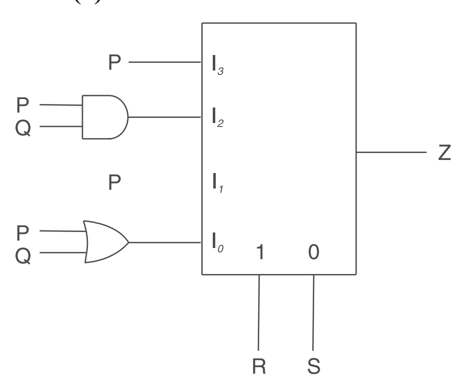
(a) $PQ + P\overline{Q}S + \overline{Q}\overline{R}\overline{S}$

(b) $P\overline{Q} + PQ\overline{R} + \overline{P}\overline{Q}\overline{S}$

(c) $P\overline{Q}\overline{R} + \overline{P}QR + PQRS + \overline{Q}\overline{R}\overline{S}$

(d) $PQ\overline{R} + PQR\overline{S} + P\overline{Q}\overline{R}S + \overline{Q}\overline{R}\overline{S}$ **[2008]**

Solution: (a)



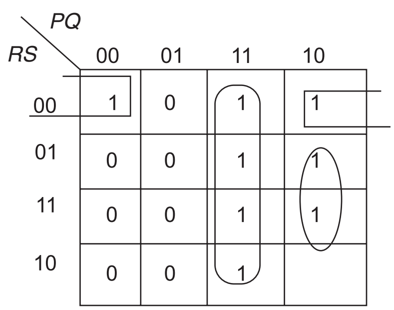
$$z = RSI_3 + R\bar{S}\overset{I_2}{\overline{PQ}} + \bar{R}SI_1 + \bar{R}\bar{S}I_0$$
$$= RSP + R\bar{S}PQ + \bar{R}SP + \bar{R}\bar{S}(P + \bar{Q})$$
$$= RSP + R\bar{S}PQ + \bar{R}SP + P\bar{R}\bar{S} + \bar{Q}\bar{R}\bar{S}$$

Solving by *k*-map

Quad PQ

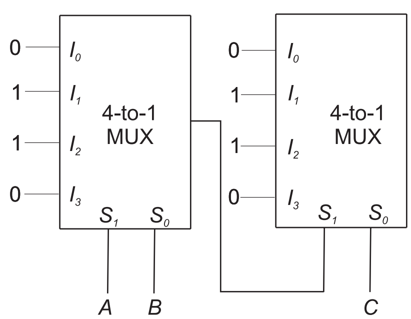
Pair : $\bar{R}\bar{S}\bar{Q}$, $P\bar{Q}S$

$$\Rightarrow z = \underline{PQ + P\bar{Q}S + \bar{Q}\bar{R}\bar{S}} \qquad \text{(a)}$$



Hence, the correct option is (a).

15. In the following circuit, X is given by



(a) $X = A\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C + ABC$

(b) $X = \bar{A}BC + A\bar{B}C + AB\bar{C} + \bar{A}\bar{B}\bar{C}$

(c) $X = AB + BC + AC$

(d) $X = \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{C}$ **[2007]**

Solution: (a)

Output of MUX (1):

$$S_1 = AB\underset{=0}{I_3} + \bar{A}BI_2 + A\bar{B}I_1 + \bar{A}\bar{B}\underset{=0}{I_0}$$

$$S_1 = (\bar{A}B + A\bar{B})$$

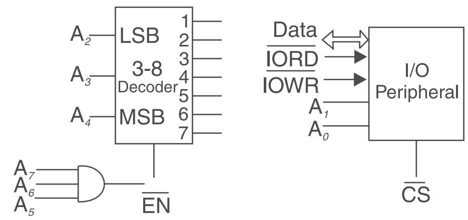
Output of MUX (2):

$$X = S_1S_0\underset{=0}{I_3} + S_1\bar{S}_0\underset{=1}{I_2} + \bar{S}_1S_0\underset{=1}{I_1} + \bar{S}_1\bar{S}_0\underset{=0}{I_0}$$
$$= S_1\bar{S}_0 + \bar{S}_1S_0$$
$$\Rightarrow X = S_1 \oplus S = \underline{A \oplus B \oplus C}$$
$$S_0 = C \quad \text{(given)}$$

$$S_1 = A \oplus B \qquad \text{(from 1)}$$
$$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

Hence, the correct option is (a).

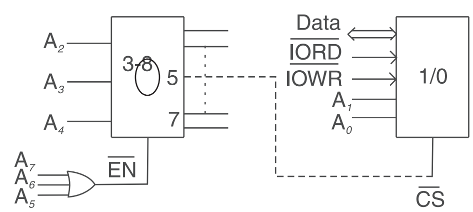
16. An I/O peripheral device shown in the figure below is to be interfaced to an 8085 microprocessor. To select the I/O device in the I/O address range D4 H-D7 H, its chip-select $(\overline{CS})$ should be connected to the output of the decoder shown in the figure



(a) output 7 (b) output 5

(c) output 2 (d) output 0 **[2006]**

Solution: (b)



A	7	6	5	4	3	2	1	0
(O_4) 4 =	1	1	0	1	0	1	0	0
(D_7) 4 =	1	1	0	1	0	1	1	1

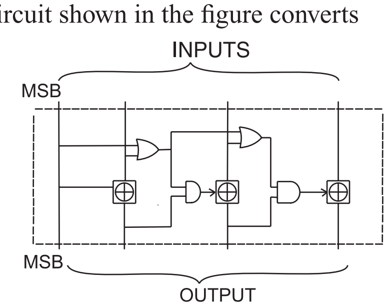
Input of decoder = 101 (for D_4 to D_7)

= 5

$$\Rightarrow \frac{\text{Output}}{\text{(b)}} 5 \quad \text{to} \quad \overline{CS}$$

Hence, the correct option is (b).

17. The circuit shown in the figure converts



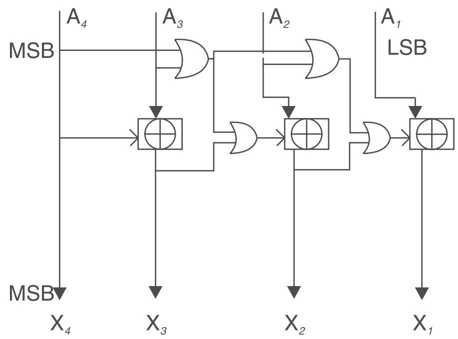
(a) BCD to binary code

(b) Binary to excess –3 code

(c) Excess –3 to Gray code

(d) Gray to Binary code **[2003]**

Solution: (d)



$X_4 = A_4$

$X_3 = A_4 \oplus A_3 = X_4 \oplus A_3$

$X_2 = A_2 \oplus \left[X_3 (A_4 + A_3) \right]$

$X_1 = A_1 \oplus X_2 (A_4 + A_3 + A_2)$

By drawing T. T. find outputs

For example: For $A_4A_3A_2A_1$ = 1010, output = 1100 (GARY CODE) .

Hence, the correct option is (d).

18. The minimum number of 2-to-1 multiplexers required to realize a 4-to-1 multiplexer is

(a) 1 (b) 2

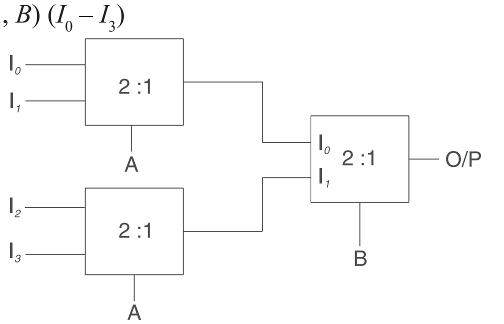
(c) 3 (d) 4 **[2004]**

Solution: (c)

(2 × 1) MUX to realize (4 × 1) MUX

(4 × 1) MUX: 2 select lines, 4 input

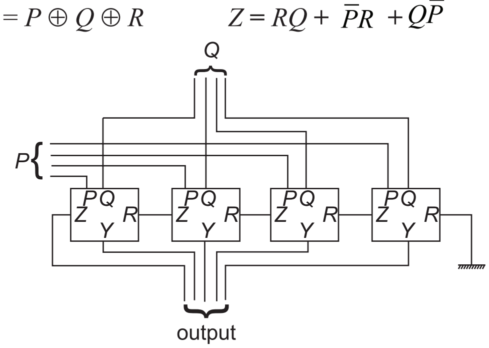
(A, B) $(I_0 - I_3)$



Hence, the correct option is (c).

19. The circuit shown in the figure has 4 boxes each described by inputs P, Q, R and outputs Y, Z with

$Y = P \oplus Q \oplus R$ $\qquad Z = RQ + \overline{P}R + Q\overline{P}$



The circuit acts as a

(a) 4-bit adder giving $P + Q$

(b) 4-bit subtractor giving $P - Q$

(c) 4-bit subtractor giving $Q - P$

(d) 4-bit adder giving $P + Q + R$ **[2003]**

Solution: (b)

$y = P \oplus Q \oplus R \quad z = RQ + \overline{P}R + Q\overline{P}$

For example, P =1101 Q = 1001

$\Rightarrow y_n = P_n \oplus Q_n \oplus R_n, \quad z = R_nQ_n + \overline{P}_nR_n + Q_n\overline{P}_{nt}$

& $R_{n+1} = z_n$

$z_4 = y_5(MSB)$

and output $y_5y_4y_3y_2y_1 = 00010 \Rightarrow$

4-bit subtractor

$Q \rightarrow$ input (4-bit)

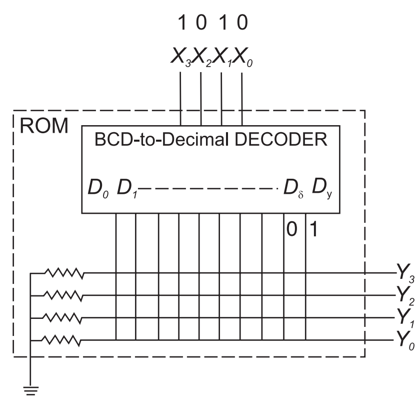
$R \rightarrow$ Cin

$z \rightarrow$ borrow out

Complete truth table can be checked for all 2^3 (4-bit inputs)

Hence, the correct option is (b).

20. If the input X_3, X_2, X_1, X_0 to the ROM in the figure are 8 4 2 1 BCD numbers, then the outputs Y_3, Y_2, Y_1 Y_0 are



(a) gray code numbers (b) 2 4 2 1 BCD numbers

(c) excess-3 code numbers (d) none of the above **[2002]**

Solution: (b)

		4	2	1	(6, 7, 8, 9)	(D_4 – D_9)	($D2$, 3, 8, 9)	(1, 3, 4, 7, 9)
	X_3	X_2	X_1	X_0	Y_3	Y_2	Y_1	Y_0
D_0	0	0	0	0	0	0	0	0
D_1	0	0	0	1	0	0	0	1
D_2	0	0	1	0	0	0	1	0
D_3	0	0	1	1	0	0	1	1
D_4	0	1	0	0	0	1	0	0
D_5	0	1	0	1	0	1	0	1
D_6	0	1	1	0	1	1	0	0
D_7	0	1	1	1	1	1	0	1
D_8	1	0	0	0	1	1	1	0
D_9	1	0	0	1	1	1	1	1

BCD – (0 to 9)

⇒ 9 is given by $y_3y_2y_1y_0 = 1111$

$1 \rightarrow 000\frac{1}{1}\ y_0 \rightarrow 1$

$2 \rightarrow 000\frac{1}{2}0\ \ y_1 \rightarrow 2$

$4 \rightarrow 0 - 00\ y_2 \rightarrow 4$

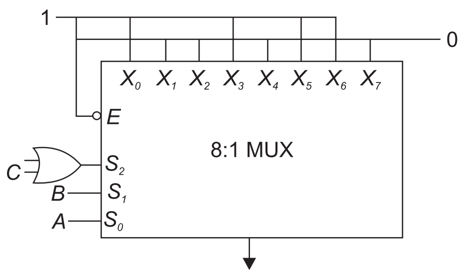
$6 \rightarrow 1\frac{1}{4}00\ y_2 \rightarrow 4$

$\Rightarrow y_3 \rightarrow 2$

$\Rightarrow y_3y_2y_1y_0 \rightarrow 2421.$

Hence, the correct option is (b).

21. In the TTL circuit in the figure, S_2 and S_0 are select lines and X_7 and X_0 are input lines. S_0 and X_0 are LSBs. The output Y is



(a) indeterminate

(b) A ⊕ B

(c) $\overline{A\oplus B}$

(d) $\bar{C}\,(\overline{A\oplus B}) + C(A \oplus B)$ **[2001]**

Solution:(c)

For 8 × 1 MUX: *(For TTL, flowing input = 1)

Output y ⇒ input to $S_2 = C + (\underline{1}) = C + 1 = 1$

⇒ from 8 outputs, lower 4 are 0 $(\bar{S}_2 = 0)$

$$\Rightarrow y = S_2(\underset{=1}{ABI_3} + \underset{0}{B\bar{A}I_2} + \underset{0}{\bar{B}AI_1} + \underset{=1}{\bar{B}\bar{A}I_0})$$

$$= 1(AB + \bar{A}\bar{B})$$

$$y = A\overline{\oplus}B$$

Hence, the correct option is (c).

22. For a binary half-subtractor having two inputs A and B, the correct set of logical expressions for the outputs D (= A minus B) and 1 (= borrow) are

(a) $D = AB + \bar{A}B,\ X = \bar{A}B$

(b) $D = \bar{A}B + A\bar{B} + A\bar{B},\ X = A\bar{B}$

(c) $D = \bar{A}B + A\bar{B},\ X = \bar{A}B$

(d) $D = AB + \bar{A}\bar{B},\ X = A\bar{B}$ **[1999]**

Solution: (c)

Half subsector: $(A - B)$

A	B	D	X
0	0	0	0
1	0	1	0
0	1	1	1
1	1	0	0

$$\left.\begin{aligned} \Rightarrow D &= \bar{A}B + A\bar{B} \\ X &= \bar{A}B \end{aligned}\right\}\quad (\underline{c})$$

Hence, the correct option is (c).

Five-marks Questions

1. A ROM is to be used to implement the Boolean functions given below:

$F_1(A, B, C, D) = ABCD + \bar{A}\bar{B}\bar{C}\bar{D}$

$F_2(A, B, C, D) = (A + B) + (\bar{A} + \bar{B} + \bar{C})$

$F_3(A, B, C, D) = \sum 13,15 + \sum_{\phi} 3,5$

(a) What is the minimum size of the ROM required?

(b) Determine the data in each location of the ROM. **[1995]**

Solution: 4 inputs (ABCD) and 3 outputs ($F_1F_2F_3$)

So size of ROM = $2^4 \times 3 = 48$

(b) $f_1(A, B, C, D) = ABCD + \bar{A}\bar{B}\bar{C}\bar{D}$

$= \sum m(0, 15)$

$$f_2(ABCD) = (A + B)(\bar{A} + \bar{B} + C)$$

$$= (A + B + C)(A + B + \bar{C})(\bar{A} + \bar{B} + C)$$

$$= (A + B + C + D)(A + B + C + \bar{D})$$

$$(A + B + \bar{C} + D)(A + B + \bar{C} + \bar{D})$$

$$= (\bar{A} + \bar{B} + C + D)(\bar{A} + \bar{B} + C + \bar{D})$$

$= \pi m(0, 1, 2, 3, 12, 13)$

$= \sum m\ (4, 5, 6, 7, 8, 9, 10, 11, 14, 15)$

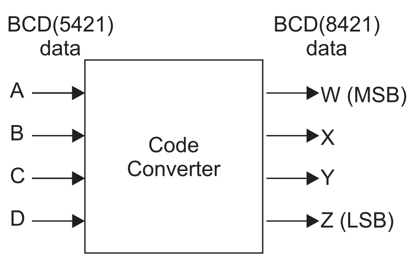
$F_3(ABCD) = \sum m(13,15) + \sum(3,5)$

	A	B	C	D	F_1	F_2	F_3
0	0	0	0	0	1	0	0
1	0	0	0	1	0	0	0
2	0	0	1	0	0	0	0
4	0	1	0	0	0	1	0
5	0	1	0	1	0	1	1
6	0	1	1	0	0	1	0
7	0	1	1	1	0	1	0
8	1	0	0	0	0	1	0

9	1	0	0	1	0	1	0
10	1	0	1	0	0	1	0
11	1	0	1	1	0	1	0
12	1	1	0	0	0	0	0
13	1	1	0	1	0	0	1
14	1	1	1	0	0	1	0
15	1	1	1	1	1	1	1

2. A 'code converter' is to be designed to convert from the BCD (5421) to the normal BCD (8421). The input BCD combinations for each digit are given below. A block diagram of the converter is shown in figure.

Decimal	BCD (5421)			
	A	B	C	D
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	1	0	0	0
6	1	0	0	1
7	1	0	1	0
8	1	0	1	1
9	1	1	0	0



(a) Draw K-maps for outputs W, X, Y and Z.
(b) Obtain minimized expression for the output W, X, Y and Z. **[1995]**

Solution:

Decimal	BCD(5421)				BCD(8421)			
	A	*B*	*C*	*D*	*W*	*X*	*Y*	*Z*
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	1	0	0	0	1	0
3	0	0	1	1	0	0	1	1
4	0	1	0	0	0	1	0	0
5	1	0	0	0	0	1	0	1
6	1	0	0	1	0	1	1	0
7	1	0	1	0	0	1	1	1
7	1	0	1	0	0	1	1	1
8	1	0	1	1	1	0	0	0
9	1	1	0	0	1	0	0	1

$W = \sum m(11, 12) + \sum d(5, 6, 7, 13, 14, 15)$

$X = \sum m(4, 8, 9, 10) + \sum d(5, 6, 7, 13, 14, 15)$

$Y = \sum m(2, 3, 9, 10) + \sum d(5, 6, 7, 13, 14, 15)$

$Z = \sum m(1, 3, 8, 10, 12) + \sum d(5, 6, 7, 13, 14, 15)$

(a) $\underline{W}$

CD \ AB	00	01	11	10
00			1	
01		×	×	
11		×	×	1
10		×	×	

$\underline{X}$

CD \ AB	00	01	11	10
00		1		1
01		×	×	1
11		×	×	
10		×	×	1

$\underline{Y}$

CD \ AB	00	01	11	10
00				
01		×	×	1
11	1	×	×	
10	1	×	×	1

$\underline{Z}$

AB \ CD	00	01	11	10
00			1	1
01	1	×	×	
11	1	×	×	
10		×	×	1

(b) $W = AB + ACD$

$X = \bar{A}B + A\bar{B}\bar{C} + A\bar{B}\bar{D}$

or

$\bar{A}B + A\bar{B}\bar{C} + AC\bar{D}$

or

$\bar{A}B + A\bar{C}D + A\bar{B}\bar{D}$

$Y = \bar{A}C + A\bar{C}D + C\bar{D}$

$Z = \bar{A}D + A\bar{D}$

Solution: (a)

Decimal	BCD (5421)				BCD (8421)			
	A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	1	0	0	0	1	0
3	0	0	1	1	0	0	1	1
4	0	1	0	0	0	1	0	0
5	0	1	0	1	0	1	0	1
6	0	1	1	0	0	1	1	0
7	0	1	1	1	0	1	1	1
8	1	0	1	1	1	0	0	0
9	1	1	0	0	1	0	0	1

$W = \sum m(11, 12) + \sum d(8, 9, 10, 13, 14, 15)$

$X = \sum m(4, 5, 6, 7) + \sum d(8, 9, 10, 13, 14, 15)$

$Y = \sum m(2, 3, 6, 7) + \sum d(8, 9, 10, 13, 14, 15)$

$Z = \sum m(1, 3, 5, 7, 12) + \sum d(8, 9, 10, 13, 14, 15)$

w

CD \ AB	00	01	11	10
00				
01				
11	1	×	×	×
10	×	×	1	×

x

CD \ AB	00	01	11	10
00				
01	1	1	1	1
11		×	×	×
10	×	×		×

y

CD \ AB	00	01	11	10
00			1	1
01			1	1
11		×	×	×
10	×	×		×

z

CD \ AB	00	01	11	10
00	1		1	
01		1	1	
11	1	×	×	×
10	×	×		×

$W = A$

$X = \bar{A}B$

$Y = \bar{A}C$

$Z = AB + BD + \bar{A}CD + \bar{B}\bar{C}\bar{D}$

Hence, the correct option is (a).

3. It is desired to generate the following three Boolean functions.

$$F_1 = a\bar{b}c + \bar{a}b\bar{c} + bc$$

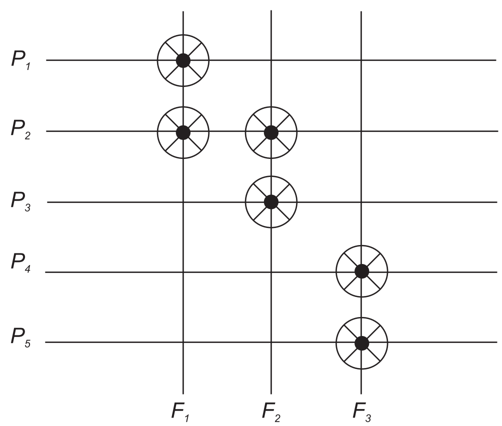
$$F_2 = a\bar{b}c + ab \pm a\bar{b}\bar{c}$$

$$F_3 = \bar{a}\bar{b}\bar{c} + abc \pm \bar{a}c$$

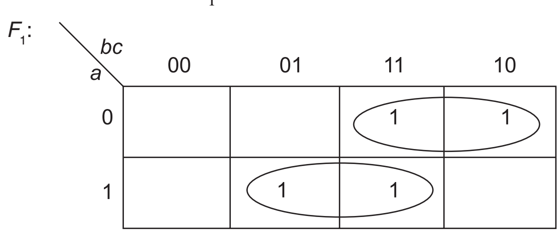
By using an OR gate array as shown in figure where P_1 to P_5 are the product terms in one or more of the variables a, $\bar{a}$, $\bar{b}$, c and $\bar{c}$.

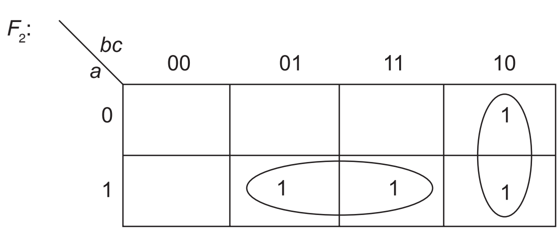
Write down the terms $P_v P_2$, P_3, P_4 and P_5

Solution: First simplifying $F_1F_2F_3$

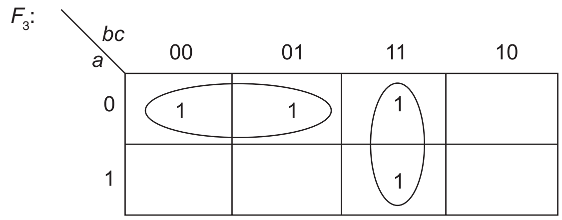


$$F_1 = \bar{a}b + ac \qquad (1)$$





$$F_1 = b'c + ac \qquad (2)$$



$$F_3 = \bar{a}\bar{b} + bc \qquad (3)$$

From the OR gate array

$$F_1 = P_1 + P_2 \qquad (4)$$

$$F_2 = P_2 + P_3 \qquad (5)$$

$$F_3 = P_4 + P_5 \qquad (6)$$

Comparing (1) and (4)

(2) and (5)

(3) and (6)

$P_1 = \bar{a}b$

$P_2 = ac$

$P_3 = b\bar{c}$

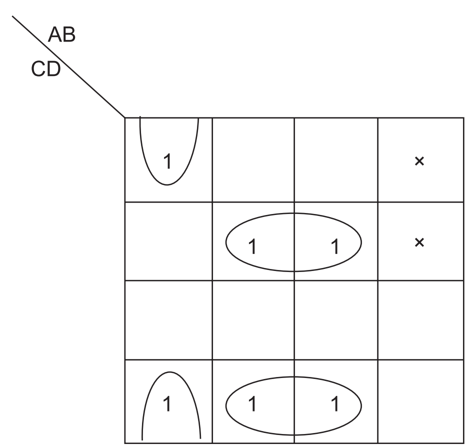
$P = bc,\ P_5 = \overline{ac}$

or

$P_4 = \overline{ac}, P_5 = bc$

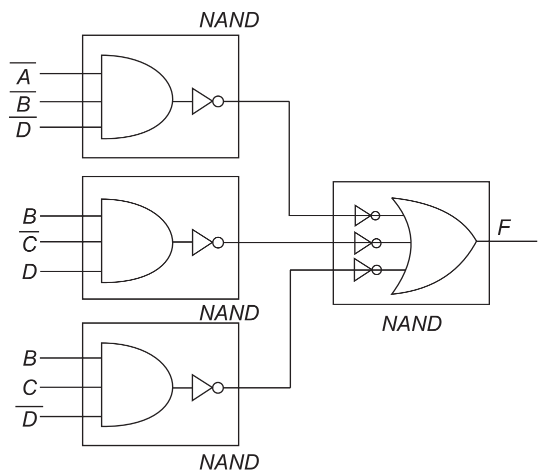
4. In certain application, four inputs A, B, C, D (both true and complement forms available are fed to logic circuit, producing an output F which operates a relay). The relay turns on when $F(ABCD) = 1$ for the following states of the inputs ($ABCD$): '0000', '0010', '0101', '0110', '1101' and '1110'. States '1000' and '1001' do not occur, and for the remaining states, the relay is off. Minimize F with the help of a Karnaugh and realize it using a minimum number of 3-input NAND gates. **[1999]**

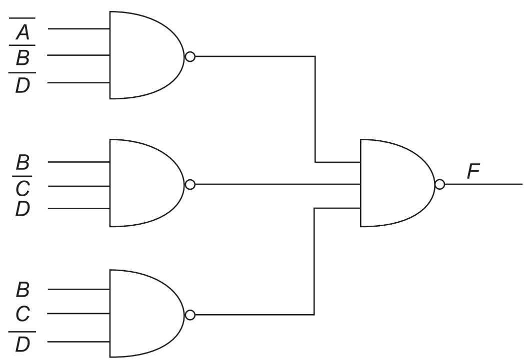
Solution:



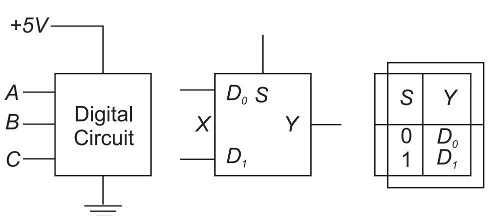
$$F = \bar{A}\bar{B}\bar{D} + B\bar{C}D + B\bar{C}D$$

To realize with NAND gates only first draw AND-OR ckt then replace all AND OR gates with NAND gates





5. The inputs to a digital circuit shown in the figure is are the external signals A, B and C. ($\bar{A}$, $\bar{B}$ and $\bar{C}$ are not available). The +5V power supply (logic 1) and the ground (logic 0) are also available. The output of the circuit is $X = \bar{A}B + \bar{A}\bar{B}\bar{C}$.



(a) Write down the output function in its canonical *SOP* and *POS* forms.

(b) Implement the circuit using only two 2:1 multiplexers shown in the figure where S is the data-select line, D_0 and D_1 are the input data lines and Y is the output line. The function table for the multiplexer is in given table. **[2002]**

Solution:

(a) Solving k map for minterm(POS)

A \ BC	00	01	11	10
0	1		1	1
1				

$$X = \bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C}$$

Solving k map for max term (POS)

A \ BC	00	01	11	10
0		0		
1	0	0	0	0

$$X = (A+B+\bar{C})(\bar{A}+B+C)(\bar{A}+B+\bar{C})($$
$$(\bar{A}+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)$$

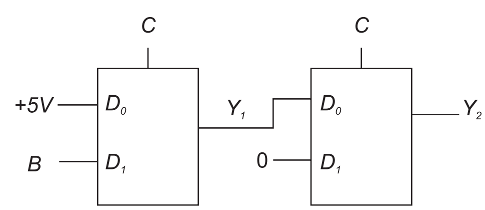
(b) $X = \bar{A}\bar{C} + \bar{A}B = \bar{A}(B+\bar{C})$

$$y_1 = \bar{C}.1 + C.B = \bar{C} + BC = (C+\bar{C})(\bar{C}+B)$$

$$y_1 = B + \bar{C}$$

$$y_2 = \bar{A}y_1 + A0 = \bar{A}y_1$$

$$y_2 = \bar{A}(B+\bar{C}) = X$$

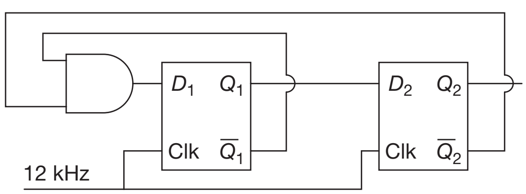


Chapter 5

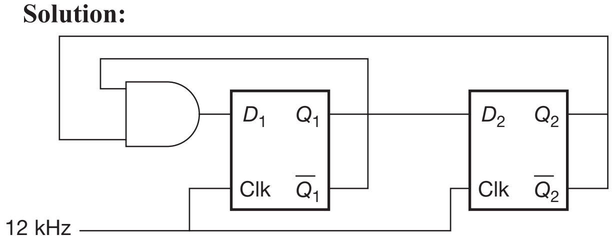
Sequential Circuits

One-mark Questions

1. In the circuit shown, the clock frequency, i.e., the frequency of the CLK signal, is 12 kHz. The frequency of the signal at Q_2 is ______ kHz **[2019]**



Solution:



$$D_1 = \overline{Q_1} \cdot \overline{Q_2}$$
$$D_2 = Q_1$$

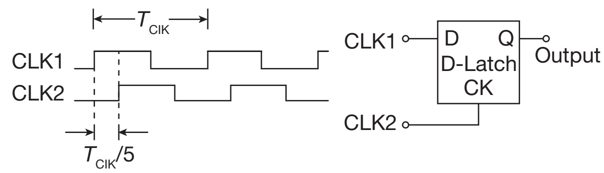
Clk	Present	State	Input		Next	State
	Q_1	Q_2	D_1	D_2	Q_1^+	Q_2^+
0	0	0	1	0	1	0
1	1	0	0	1	0	1
2	0	1	0	0	0	0
3	0	0	–	–	–	–

$\therefore$ There are only 3 states.

So, output frequency will be $\dfrac{12}{3} = 4$ KHz

Hence, the correct answer is (4).

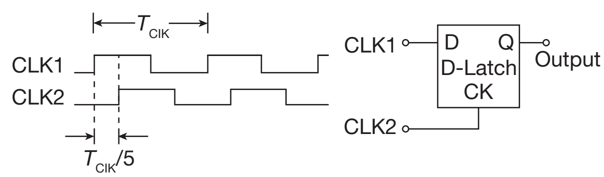
2. Consider the D-Latch shown in the figure, which is transparent when its clock input CK is high and has 0 propagation delay. In the figure, the clock signal CLK1 has a 50% duty cycle and CLK2 is 1/4 period delayed version of CLK1. The duty cycle at the output of the latch in percentage is ______________. **[2017]**



Solution: $T_{\text{ON}} = \dfrac{T_{\text{CLK}}}{2} - \dfrac{T_{\text{CLK}}}{5} = \dfrac{3T_{\text{CLK}}}{10}$

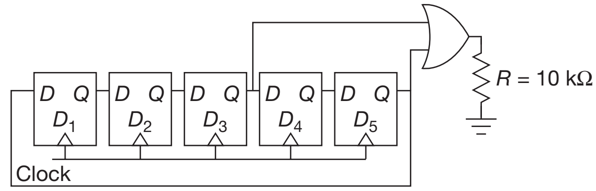
$$\therefore \text{ Output Duty Cycle} = \frac{T_{ON}}{T_{CLK}} = \frac{\frac{3T_{CLK}}{10}}{T_{CLK}} = \frac{3}{10}$$

$$\therefore \text{ Output Duty Cycle in \%} = \frac{3}{10} \times 100\% = 30\%$$



Hence, the correct answer is (29.9 to 30.1).

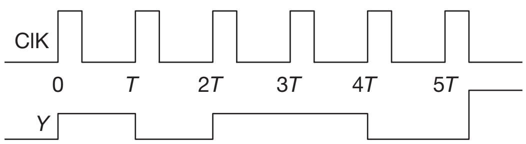
3. Assume that all the digital gates in the circuit shown in the figure are ideal, the resistor R = 10 kΩ and the supply voltage is 5 V. The D flip flops D_1, D_2, D_3, D_4 and D_5 are initialized with logic values 0, 1, 0, 1, and 0, respectively. The clock has a 30% duty cycle. **[2016]**



The average power dissipated (in mW) in the resistor R is ______ .

Solution: All the flip flops are provided same positive edge clock and it will behave as a counter.

Clk	Q_1	Q_2	Q_3	Q_4	Q_5	$Q_3 + Q_5 = Y$
0	0	1	0	1	0	0
1	0	0	1	0	1	1
2	1	0	0	1	0	0
3	0	1	0	0	1	1
4	1	0	1	0	0	1
5	0	1	0	1	0	0



Average power can be calculated using

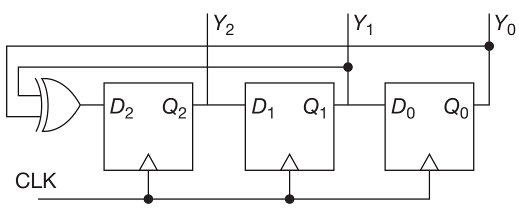
$$P_{Avg} = \frac{1}{T}\int_0^T \frac{V^2(t)}{R}\,dt$$

$$= \frac{1}{5T}\left(\int_0^T \frac{25}{10} + \int_T^{2T} 0dt + \int_{2T}^{4T} \frac{25}{10} + \int_{4T}^{5T} 0dt\right)\text{mW}$$

$$= \frac{1}{5}\left(\frac{25}{10} + \frac{25}{10} \times 2\right) = 1.5 \text{ mW}$$

Hence, the correct Answer is (1.5 mW).

4. A three bit pseudo random number generator is shown. Initially, the value of output $Y = Y_2 Y_1 Y_0$ is set to 111. The value of output Y after three clock cycles is **[2015]**



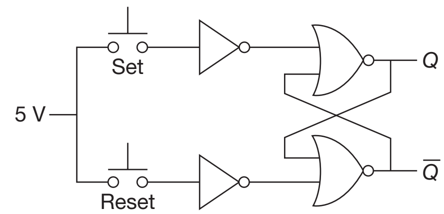
(A) 000 (B) 001
(C) 010 (D) 100

Solution:

Clk	Q_2	Q_1	Q_0	D_2: $Q_1 \oplus Q_0$	D_1: Q_2	D_0: Q_1
0	1	1	1	0	1	1
1	0	1	1	0	0	1
2	0	0	1	1	0	0
3	1	0	0			

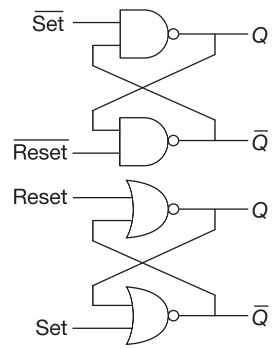
Hence, the correct option is (D).

5. An SR latch is implemented using TTL gates as shown in the figure. The set and reset pulse inputs are provided using the push-button switches. It is observed that the circuit fails to work as desired. The SR latch can be made functional by changing **[2015]**



(A) NOR gates to NAND gates
(B) Inverters to buffers
(C) NOR gates to NAND gates and inverters to buffers
(D) 5 V to ground

Solution: TTL implements NAND gates.

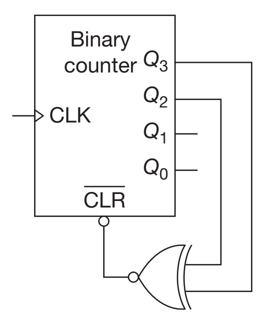


For NOR gate Reset, set are connected as inputs to Q, $\overline{Q}$ NOR gates.

In the given circuit set, Reset are connected to Q, $\overline{Q}$ gates, so by replacing NOR gates with NAND gates, we can get the correct functionality of SR latch

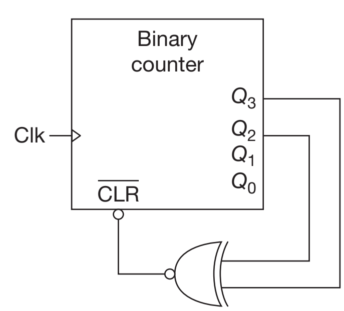
Hence, the correct option is (A).

6. The figure shows a binary counter with synchronous clear input. With the decoding logic shown, the counter works as a **[2015]**



(A) mod-2 counter (B) mod-4 counter
(C) mod-5 counter (D) mod-6 counter

Solution:



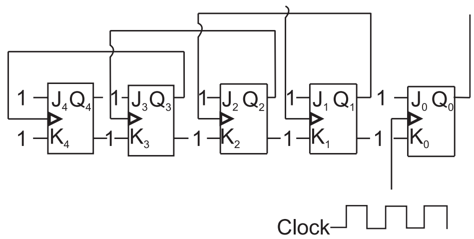
Given circuit is Binary Counter with Synchronous clear.

CLK	Q_3	Q_2	Q_1	Q_0	CLK = $Q_3 \odot Q_2$
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	0	0	0	1

So no. of states = Modulus = 5

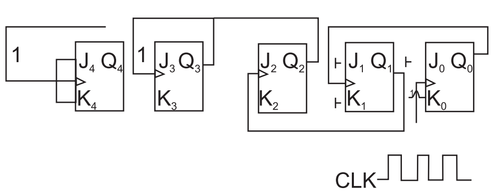
Hence, the correct option is (C).

7. Five JK flip-flops are cascaded to form the circuit shown in the figure. Clock pulses at a frequency of 1 MHz are applied as shown. The frequency (in kHz) of the waveform at Q_3 is_____.



[2014]

Solution:



Output of Q_0 = clk of Q_1

Q_1 = clk of Q_2 and so on

& clk → Q_0

⇒ ASYN counters ⇒ $2n$ Asyn counter of clk freq = f then freq of

$$Q_1 = f/2$$

$$Q_2 = f/2 \times 2 \quad (f/2\ 2)$$

$$f_3 = \frac{f_{clk}}{8} = \frac{1\text{ MHz}}{8} = \frac{1000\text{ K}}{8}$$

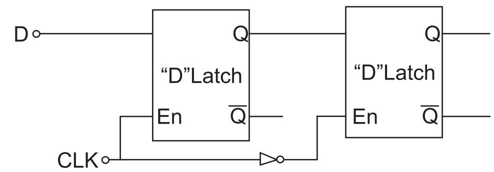
$$f_3 = 6.25\text{ KHz}$$

$$Q_3 = \frac{f}{2\times2\times2}\,(f/2^3)$$

$$Q_4 = f/24$$

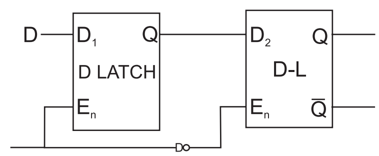
$$Q_5 = f/25$$

8. The circuit shown in the figure is a



(a) toggle Flip Flop
(b) JK Flip Flop
(c) SR Latch
(d) Master-Slave D Flip Flop **[2014]**

Solution: (d)



D_1 – LATCH

D_1 = input 0

clk → +ve

D_2 – LATCH

$D_2 = Q_1$ (output of D_1)

clk → −ve (of D_1)

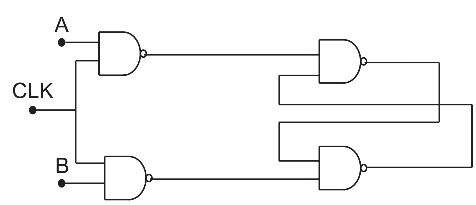
⇒ When D changes, Q_1, changes on +ve clock cycle. and as Q_1 changes (D_2), Q_2 change, but on –ve clock

⇒ input D goes to Q_2 only once during a complete clk cycle

⇒ This is MS-FF

Hence, the correct option is (d).

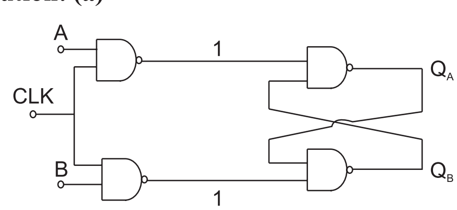
9. Consider the given circuit



In this circuit, the race around

(a) does not occur
(b) occurs when CLK = 0
(c) occurs when CLK = 1 and $A = B = 1$
(d) occurs when CLK = 1 and $A = B = 0$ **[2012]**

Solution: (a)



RACE AROUND condition

Because when output toggles between 0 and 1 at any instance, we cannot decide the output (1 or 0)

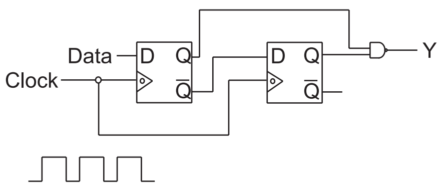
(a) From figure, we can see it is R-S FF

So in RS FF:

R	S	Q_{n+1}	
0	0	Q_n	
0	1	1	⇒No race around condition
1	0	0	
1	1	Not valid	

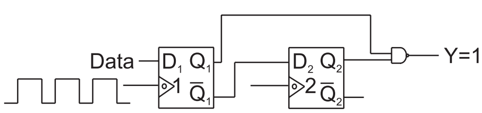
Hence, the correct option is (a).

10. When the output Y in the circuit below is '1', it implies that data has



(a) changed from '0' to '1'
(b) changed from '1' to '0'
(c) changed in either direction
(d) not changed **[2011]**

Solution: (a)



$y = 1 \Rightarrow Q_1 \cdot Q_2 = 1$ $(Q_1 = 1 = Q_2)$

As $Q_{n+1} = D_n$ (for D FF)

⇒ Before clk pulse, both D_1 and $D_2 = 1$

$D_1 = 1$ $\quad$ $D_2 = 1$

As $D_2 = \overline{Q_1}$ $\quad$ & $\quad$ $D_2 = 1$ (before clk)

$\Rightarrow \overline{Q_1} = 1$ & $Q_1 = 0$

As $Q_1 = 0$, this means D_1 was D (before going to 1)

⇒ Data has changed from 0 to 1.

Hence, the correct option is (a).

11. A master-slave flip-flop has the characteristic that

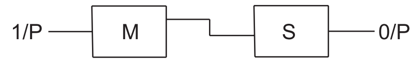
(a) change in the input immediately reflected in the output
(b) change in the output occurs when the state of the master is affected
(c) change in the output occurs when the state of the slave is affected
(d) both the master and the slave states are affected at the same time **[2004]**

Solution: (c)

Master-slave FF:

– Change in input does not reflect output change immediately

– Change in output depends on state change in slave



Hence, the correct option is (c).

12. Choose the correct one from among the alternatives A, B, C, D after matching an item from Group 1 with the most appropriate item in Group 2.

Group-1

P. Shift register

Q. Counter

R. Decoder

Group-2

1. Frequency division
2. Addressing in memory chips
3. Serial to parallel data conversion

(a) $P-3, Q-2, R-1$ $\quad$ (b) $P-3, Q-1, R-2$
(c) $P-2, Q-1, R-3$ $\quad$ (d) $P-1, Q-2, R-2$ **[2004]**

Solution: (b)

P. Shift Reg → serial to || data conv. (3)

Q. Counter → freq division (1)

R. Decoder → Memory Chips (2)

$\Rightarrow P-3, Q-1, R-2$

Hence, the correct option is (b).

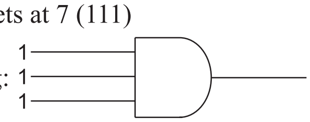
13. A 0 to 6 counter consists of 3 flip flops and a combination circuit of 2 input gate(s). The combination circuit consists of

(a) one AND gate
(b) one OR gate
(c) one AND gate and one OR gate
(d) two AND gates **[2003]**

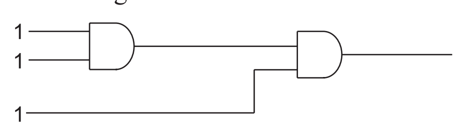
Solution: (d)

0 to 6 counter ⇒ Resets at 7 (111)

⇒ 31/P AND gate reg:



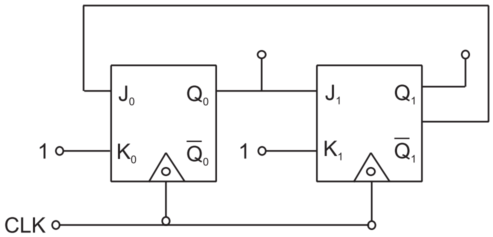
but with 21/P gate:



⇒ 2 AND req.

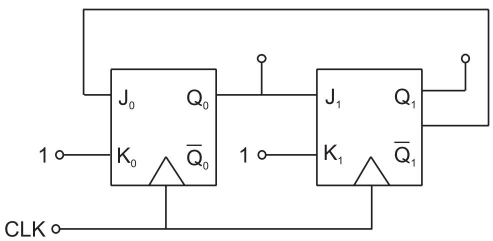
Hence, the correct option is (d).

14. Figure shows a mod-K counter. Here K is equal to



(a) 1 (b) 2
(c) 3 (d) 4 **[1998]**

Solution: (c)



Initially cleared: $Q_0\,Q_1 = 00$

$\overline{Q}_0\overline{Q}_1 = 11$

$\rightarrow 10 \rightarrow$ Q_0

Next clk: $J_0K_0 = 11 \Rightarrow Q_0 = 1$

$J_1K_1 = 01 \Rightarrow Q_1 = 0,\ \overline{Q}_1 = 1$

Next clk: $J_0K_0 = 11 \Rightarrow Q_0 = 0$ (toggle)

$Q_0 \rightarrow 10 \rightarrow 01$

$J_1K_1 = 11 \Rightarrow Q_1 = 1$ (toggle)

Next clk: $J_0K_0 = 01 \Rightarrow Q_0 = 0$ (reset)

$J_1K_1 = 01 \Rightarrow Q_1 = 0$ (reset)

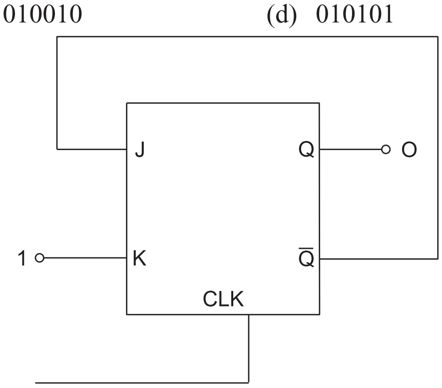
$\Rightarrow$ 00 $\longrightarrow$ 10 $\longrightarrow$ 01

Mod – 3 counter.

Hence, the correct option is (c).

15. In a J-K flip-flop we have $J = \overline{Q}$, and $K = 1$ (see figure). Assuming the flip-flop was initially cleared and then clocked for 6 pulses, the sequence at the Q output will be

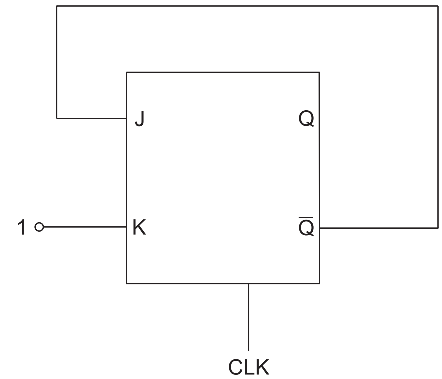
(a) 010000 (b) 011001
(c) 010010 (d) 010101



[1997]

Solution: (d)

$J = \overline{Q},\ K = 1$



Initially cleared $Q = 0$

Ck	J	K	Q	$\overline{Q}$
0	–	–	0	1
1	1	1	1	0 (toggle)
2	0	1	0	1
3	1	1	1	0
4	0	1	0	1
5	1	1	1	0
6	0	1	0	1

$\Rightarrow Q = 010101.$

Hence, the correct option is (d).

16. An R-S latch is
(a) combinatorial circuit
(b) synchronous sequential circuit
(c) one bit memory element
(d) one clock delay element **[1995]**

Solution: (c)

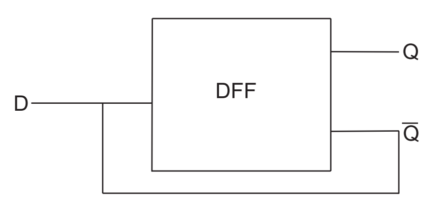
R–S latch
(a) Combinational Circuit – No (seq)
(b) Syn. Seq. Ckt – No (No clk, $\therefore$ not sync)
(c) 1 – bit mem. Element – Yes
(d) 1 clk delay – No (No clk in latch)

Hence, the correct option is (c).

17. A switch-tail ring counter is made by using a single D flip-flop. The resulting circuit is a
(a) SR flip-flop (b) JK flip-flop
(c) D flip-flop (d) T flip-flop **[1995]**

Solution: (d)

* Switch Tail $\rightarrow$ Ripple counter like



Now for On,

$D = \overline{Q}_n$ and $Q_{n+1} = D = \overline{Q}_n$

Toggle $\Rightarrow$ TFF

Hence, the correct option is (d).

18. Synchronous counters are ________than the ripple counters. **[1994]**

Solution:

Synchronous counter: All FF work simultaneously (have individual clk connection)

Ripple counter: output of 1 FF acts as clk to next FF (output ripples through FF connections)

$\therefore$ Synch. Are FASTER than ripple.

19. A pulse train with a frequency of 1 MHz is counted using a modulo-1024 ripple counter built with J-K flip flops. For proper operation of the counter, the maximum permissible propagation delay per flip flop stage is ________n sec. **[1993]**

Solution:

Modulo 1024 ripple counter

$\Rightarrow 2^{10}$, i.e., 10 FF ($N = 10$)

As $f_{clk} = 1$ mHz

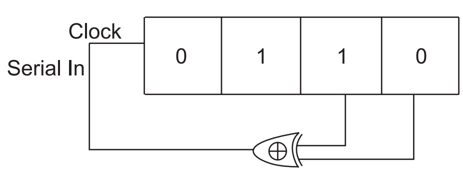
and we know $f_{clk} < \dfrac{1}{NTp \cdot d}$ (N = number of FF)

or $TPD < \dfrac{1}{N \cdot f_{clk}}$

$$\Rightarrow TPD_{\max} = \frac{1}{10 \times 1 \times 10^6} = 10^{-7} = 100 \times 10^{-9}$$

= 100 n sec.

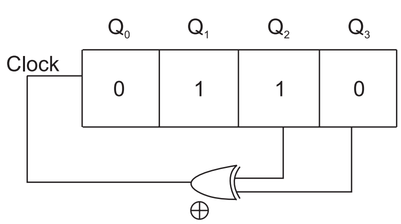
20. The initial contents of the 4-bit serial-in-parallel-out, right-shift, Shift Register shown in the figure is 0110. After three clock pulses are applied, the contents of the Shift Register will be



(a) 0000 (b) 0101

(c) 1010 (d) 1111 **[1992]**

Solution: (c)



Clk	Q_0	Q_1	Q_2	Q_3
0	0	1	1	0
1	1	0	1	1
2	0	1	0	1
3	1	0	1	0

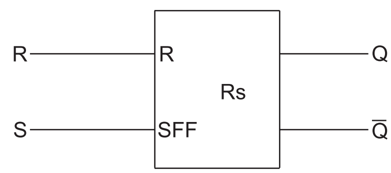
$$Q_{0n+1} = Q_{2n} \oplus Q_{3n}$$

Hence, the correct option is (c).

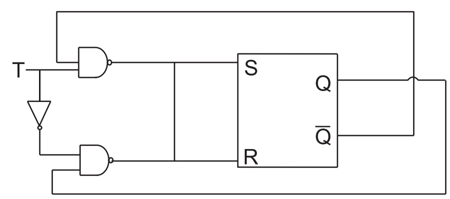
21. An S–R FLIP–FLOP can be converted into a T FLIP-FLOP by connecting _____to Q and _________ to $\overline{Q}$ **[1991]**

Solution:

S–R FF to T FF



$T = S\overline{Q}_n + RQ_n$



For T FF,

$T = S\overline{Q}_n + RQ_n$

$\Rightarrow R$ is conn. to Q

S to $\overline{Q}$.

22. A 4-bit modulo-16 ripple counter uses JK flip-flops. If the propagation delay of each FF is 50 ns, the maximum clock frequency that can be used is equal to:

(a) 20 MHz (b) 10 MHz

(c) 5 MHz (d) 4 MHz **[1990]**

Solution: (c)

For ripple counter, total delay

= $NTPD$

= 4 × 50 ns

and max clock frequency:

$$f < \frac{1}{NTPD}$$

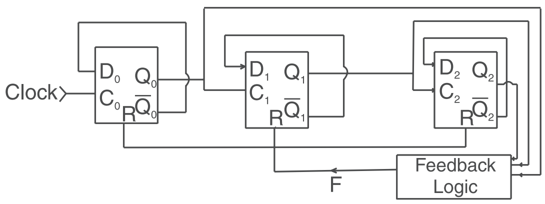
$$f_{\max} = \frac{1}{4 \times 50 \times 10^{-9}} = \frac{1}{200 \times 10^{-9}}$$

$$= 5 \times 10^6$$

$$f = 5\ MHz.$$

Hence, the correct option is (c).

23. A ripple counter using negative edge-triggered D-flip flops is shown in figure below. The flip-flops are cleared to '0' at the *R* input. The feedback logic is to be designed to obtain the count sequence shown in the same figure. The correct feedback logic is:



Count sequence in the order of $Q_1Q_2Q_3$

→000→001→010→011→100→101→

(a) $F = \overline{Q_2Q_1\bar{Q}_0}$ (b) $F = Q_2\bar{Q}_1\bar{Q}_0$

(c) $F = \bar{Q}_2\bar{Q}_1Q_0$ (d) $F = \bar{Q}_2\bar{Q}_1\bar{Q}_0$

[1987]

Solution: (a)

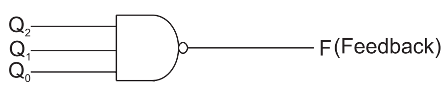
Ripple counter sequence:

→000 → 001 → 010 → 011 → 100 → 101 ←

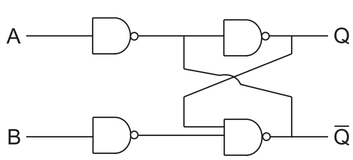
This means, after 101, next state $\underset{(Q_2Q_1Q_0)}{110}$ should be cleared (000)

(mod-6 counter)

$\Rightarrow Q_0Q_1Q_2$ in NAND gate (to R I/P)



24. The circuit given below is a



(a) J–K flip-flop
(b) Johnson's counter
(c) R–S latch
(d) None of above **[1988]**

Solution: (c)

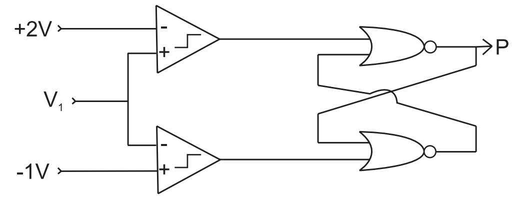
If

A	B	Q_{n+1}
0	0	Q_n (previous o/p)
0	1	0
1	0	1
1	1	Not valid

∴ From PS-NS table, it is RS LATCH.

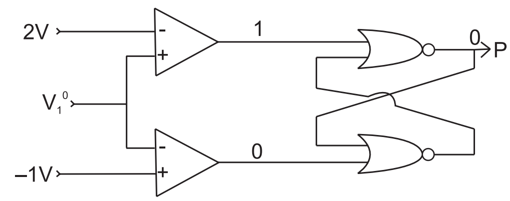
Hence, the correct option is (c).

25. Choose the correct statements relating to the circuit of figure



(a) For $V_i = -2v$, $P = 0$
(b) For $V_i = +3v$, $P = 0$
(c) For $V_i = 0v$, $P = 0$ always
(d) For $V_i = 0v$, P can be either 0 or 1. **[1987]**

Solution: (b)



For comparator: output is +1, if $V_+ > V_-$ 0, if $V_- > V_+$

$\Rightarrow$ In case a) $V_i = 2V$, ∴ for compl, $V_+ = V_- \Rightarrow$ NO output.

$V_i = +3V \Rightarrow$ For Comp 1, $V_+ > V_- \Rightarrow$ output = +1

Comp 2, $V_- > V_+ \Rightarrow$ output = 0

∴ For *P*: P → due to 1 from comp 1,

$P = 0$

$V_i = 0 \Rightarrow C_1$: output = 0

C_2: output = 0

But output *P* will depend on previous state output

$V_i = 0$ P → on previous state

Hence, the correct option is (b).

Two-marks Questions

1. A traffic signal cycles from Green to Yellow, Yellow to Red and Red and Red to Green. In each cycle, Green is turned on for 70 seconds, Yellow is turned on for 5 seconds and the Red is turned on for 75 seconds. This traffic light has to be implemented using a finite state machine (FSM). The only input to this FSM is a clock of 5 seconds period. The minimum number of flip-flops required to implement this FSM is _______. **[2018]**

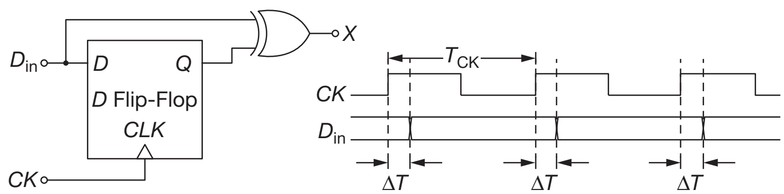
Solution: We know that in order to turn Green for 70 seconds we need 2 flip flops

We know that in order to turn Red for 75 seconds we need 2 flip flops

1 flipflop is required to turn yellow colour totally we need 5 such flipflops to have traffic signal cycles from Green to Yellow, Yellow to Red and Red to Green

Hence, the correct answer is 5.

2. In the circuit shown below, a positive edge – triggered D Flip-flop is used for sampling input data D_{in} using clock CK. The XOR gate outputs 3.3 volts for logic HIGH and 0 volts for logic LOW levels. The data bit and clock periods are equal and the value of $\Delta T/T_{ck}$ = 0.15, where the parameters ΔT and T_{CK} are shown in the figure. Assume that the Flip and the XOR gate are ideal.



If the probability of input data bit (D_{in}) transition in each clock period is 0.3, the average value (in volts, accurate to two decimal places) of the voltage at note X, is _______ **[2018]**

Solution:

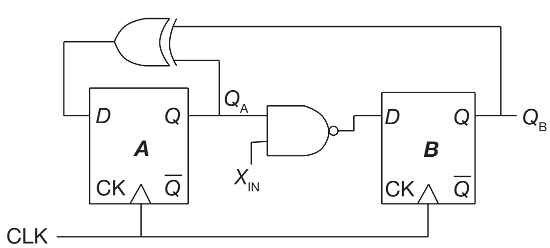
$$V_{x(avg)} = \left[0.3\times 3.3\left[1-\frac{\Delta T}{T_{CK}}\right]\right]\times(0.7\times 0)\,\mathrm{V}$$

$$= 0.3 \times 3.3 \times [1 - 0.15]$$

$$= 3.3 \times 3.3 \times 0.85 = 0.8415\ \mathrm{V}$$

Hence, the correct answer is 0.82 to 0.86.

3. A finite state machine (FSM) is implemented using the D flip-flops A and B, and logic gates, as shown in the figure below. The four possible states of the FSM are $Q_A\,Q_B$ = 00, 01, 10 and 11.



Assume that X_{IN} is held at a constant logic level throughout the operation of the FSM. When the FSM is initialized to the state $Q_A\,Q_B$ = 00 and clocked, after a few clock cycles, it starts cycling through. **[2017]**

(A) all of the four possible states if $X_{IN} = 1$
(B) three of the four possible states if $X_{IN} = 0$
(C) only two of the four possible states if $X_{IN} = 1$
(D) only two of the four possible states if $X_{IN} = 0$

Solution: From the given circuit diagram.

$$D_A = Q_A \oplus Q_B$$
$$D_B = \overline{Q.X_{in}}$$

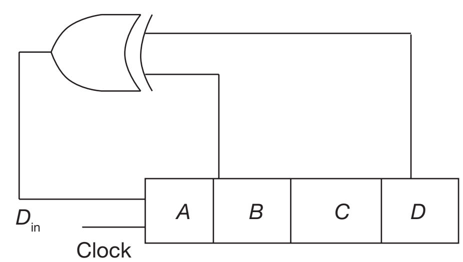
Let us take $X_{in} = 0$

Present	State	D_A	D_B	next	State
Q_A	Q_B			Q_A	Q_B
0	0	0	1	0	1
0	1	1	1	1	1
1	1	0	1	0	1
0	1	1	1	1	1
1	1	0	1	0	1

∴ If X_{in} = 0 the given circuit will have two states, 01, 11.

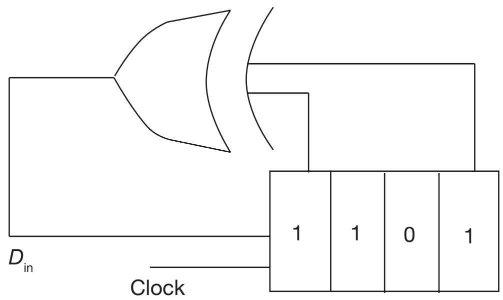
Hence, the correct option is (D).

4. A 4-bit shift register circuit configured for right-shift operation, i.e., $D_{in} \to A, A \to B, B \to C, C \to D$, is shown. If the present state of the shift register is $ABCD$ = 1101, the number of clock cycles required to reach the state $ABCD$ = 1111 is ______________. **[2017]**



Solution: $D_{in} \to A, A \to B, B \to C, C \to D$

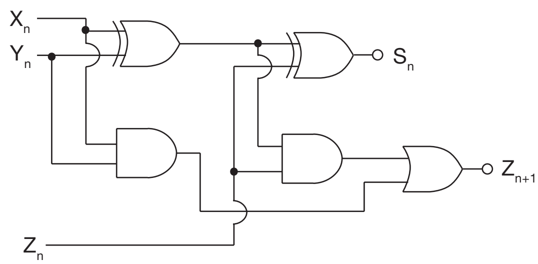
	A	B	C	D
	1	1	0	1
1	0	1	1	0
2	0	0	1	1
3	1	0	0	1
4	0	1	0	0
5	0	0	1	0
6	0	0	0	1
7	1	0	0	0
8	1	1	0	0
9	1	1	1	0
10	1	1	1	1

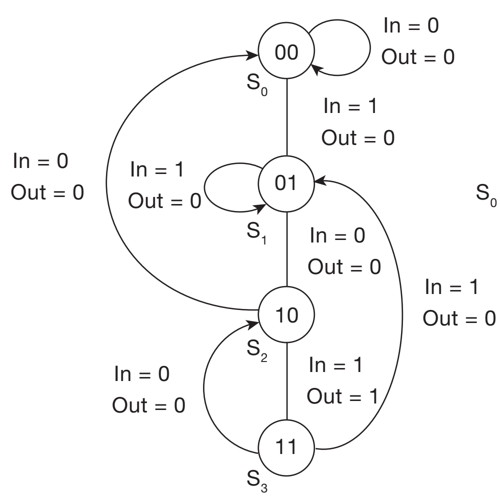


$\therefore$ 10 clock pulses are required to get state of $ABCD$ = 1111

Hence, the correct answer is (10).

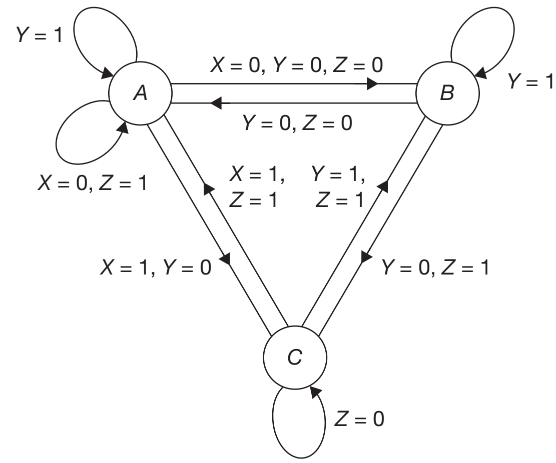
5. The state diagram of a finite state machine (FSM) designed to detect an overlapping sequence of three bits is shown in the figure. The FSM has an input 'In' and an output 'Out'. The initial state of the FSM is S_0.





If the input sequence is 10101101001101, starting with the left-most bit, then the number of times 'Out' will be 1 is ____________. **[2017]**

6. The state transition diagram for a finite state machine with states A, B and C, and binary inputs X, Y and Z is shown in the below figure. **[2016]**



Which one of the following statements is correct?

(A) Transitions from state A are ambiguously defined.
(B) Transitions from State B are ambiguously defined.
(C) Transitions from State C are ambiguously defined.
(D) All of the state transitions are defined unambiguously.

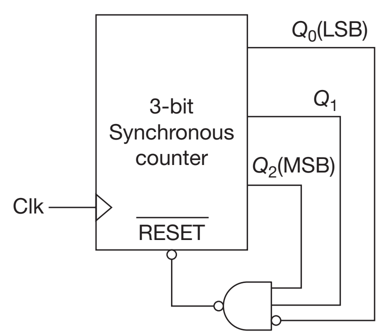
Solution: xyz = 001 condition is not defined so transitions from state C are ambiguously defined.

Hence, the correct option is (C).

7. For the circuit shown in the figure, the delay of the bubbled NAND gate is 2 ns and that of the counter is assumed to be zero. **[2016]**

If the clock (CLK) frequency is 1 GHz, then the counter behaves as a

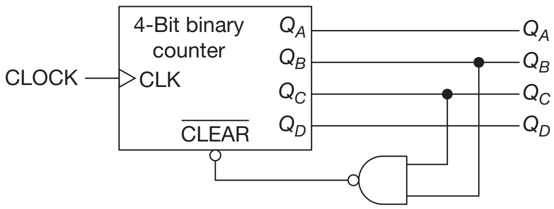
(A) mod 5 counter (B) mod 6 counter
(C) mod 7 counter (D) mod 8 counter



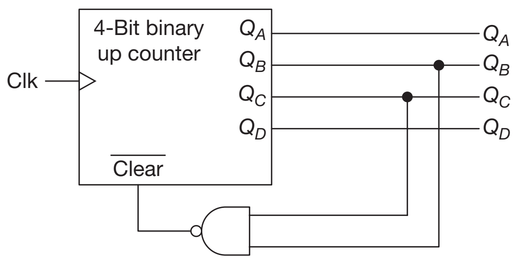
Solution: Here, NAND gate is used, which will behave to reset the counter. The counter will begin with all bits as 0. The clock used is positive edge triggered, thus the change in bit will occur during transition from 0 to 1. The MSB Q0 will be complemented in every clock pulse and Q1 will change when a transition of 0 to 1 occurs in Q0. The LSB Q3 will change due to transition from 0 to 1 occurring in Q2. Hence, the counter will start counter until all the bits become 1. As soon as all bits are 1, the output of NAND will become 0 and the bits will be reset. The counter will behave as mod 8 counter. So Option D.

Hence, the correct option is (D).

8. A mod-n counter using a synchronous binary up-counter with synchronous clear input is shown in the figure. The value of n is ________. **[2015]**



Solution:



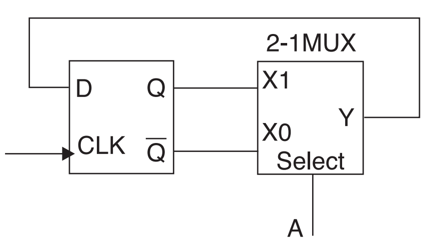
The NAND gate shown is connected to synchronous clear, i.e., clear will be applied after clock pulse.

Clk	Q_A	Q_B	Q_C	Q_D	$\overline{\text{Clear}}$
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	0	0	0	

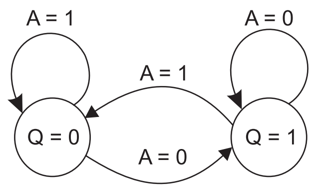
As clear is synchronous input, clear will be zero in 6th clock pulse, but output is cleared in next clock pulse so total no. of states = Modulus = 7.

Hence, the correct answer is (7).

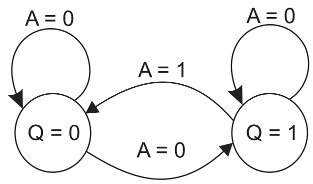
9. The state transition diagram for the logic circuit shown in



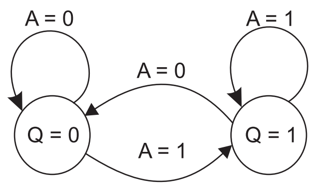
(a)



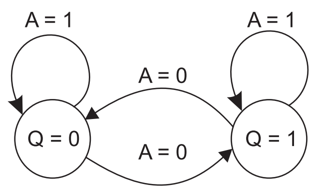
(b)



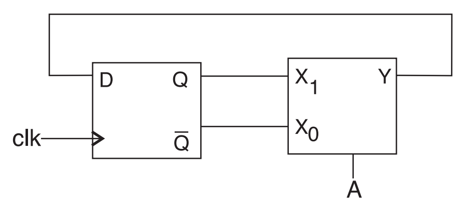
(c)



(d)



Solution: (d)



Stall diagram

Q can be 0 or 1

(a) If $A = 0$, (i) $Q = 0 \Rightarrow X_1 = 0$

$X_0 = \overline{0} = 1$

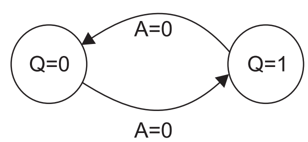
$\Rightarrow Y = 1\ (= X_0)$

$\Rightarrow D = y = 1$ and $Q = D = 1$ (Q inserts)

$A = 0$ (ii) $Q = 1 \Rightarrow X_1 = 1, X_0 = 0$

$\Rightarrow D = Y = X_0 = 0$

$\Rightarrow Q = 0$ (inserts)



(b) If $A = 1$

(i) $Q = 0 \Rightarrow X_1 = 0$

$Y_0 = 1$

and $Y = X_I = 0$

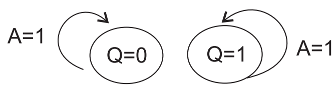
$\Rightarrow D = Y = 0$

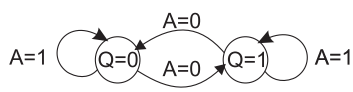
$\Rightarrow Q = D = 0$ (no change)

(ii) $Q = 1 \Rightarrow X_1 = (1), X_0 = 0$

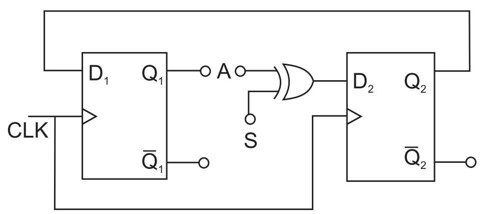
and $Y = X_1 = 1$

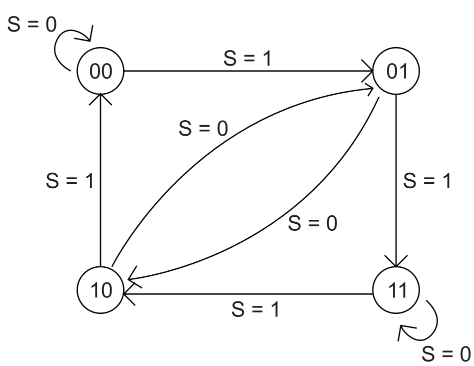
$\Rightarrow D = Y = 1$ & $Q = D = 1$ (no change)





10. The digital logic shown in the figure satisfies the given state diagram when Q_1 is connected to input A of the XOR gate.





Suppose the XOR gate is replaced by an XNOR gate. Which one of the following options preserves the state diagram?

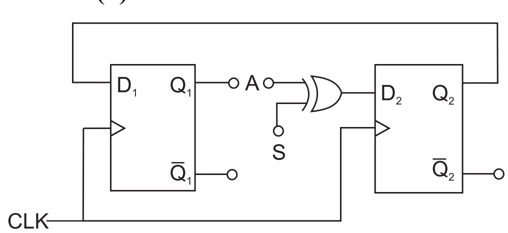
(a) Input A is connected to $\overline{Q}_2$

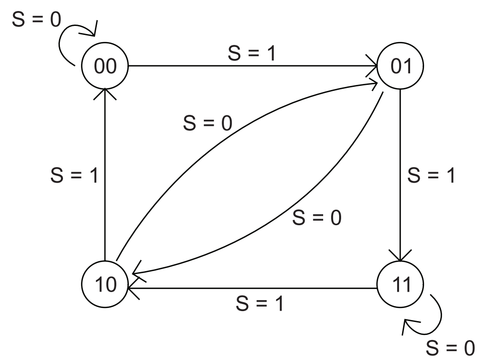
(b) Input A is connected to Q_2

(c) Input A is connected to $\overline{Q}_1$ and S is complemented

(d) Input A is connected to $\overline{Q}_1$ **[2014]**

Solution: (d)





When XOR $\rightarrow$ replaced by XNOR, then A = ?

From the diagram, 2b input D_2 remains same

$\Rightarrow (Q_2, D_1, 2Q_1)$

For X–OR: $D_2 = A \oplus S = Q_1 \oplus S = \overline{Q}_1(s) + Q_1\overline{S}$ (1)

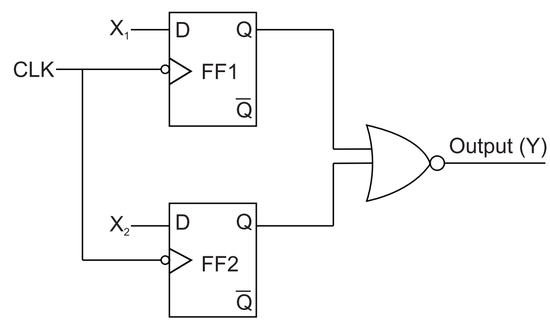
X–NOR: $D_2 = \overline{A \oplus S} = \overline{A}\,\overline{S} + A(S)$ (2)

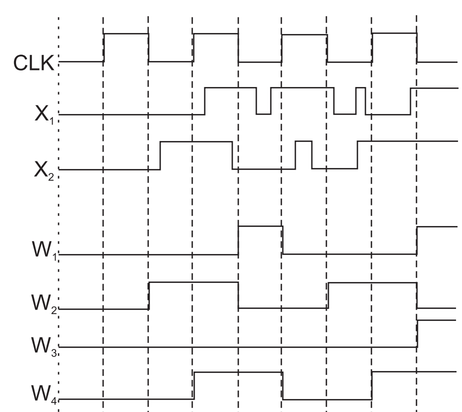
Comparing 1 and 2: $A = \overline{Q}_1$ (Weff of S)

or $\overline{A} = Q_1\ (\overline{S})$

$\Rightarrow A$ is connected to $\overline{Q}_1$ (d)

11. In the circuit shown, choose the correct timing diagram of the output (Y) from the given waveforms W_1, W_2, W_3 and W_4.

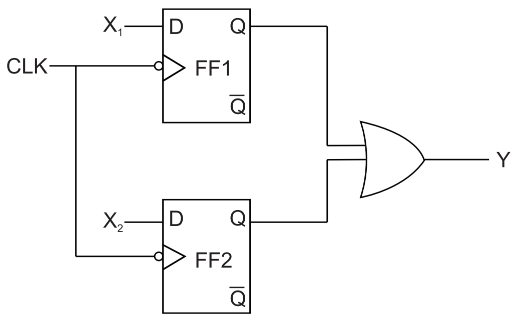




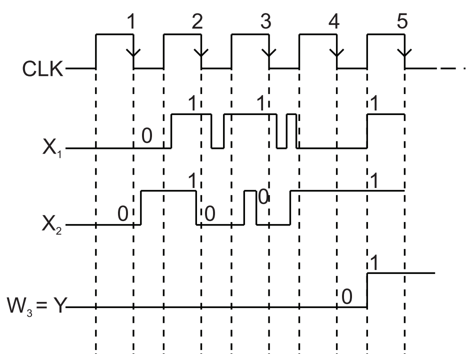
(a) W_1 (b) W_2

(c) W_3 (d) W_4 **[2014]**

Solution:



DFF: $Q_{n+1} = D_n$ & $Y = Q_1.Q_2$



$\Longrightarrow$ ©

(1) At clk 1, $X_1 = 0, X_2 = 0$
$\Rightarrow Q_1\ Q_2 = 0 \Rightarrow Y = 0$

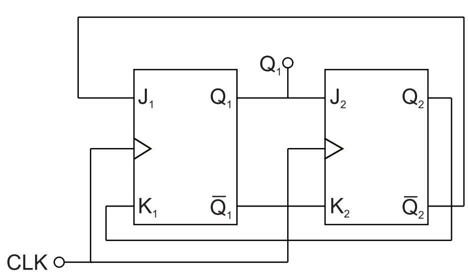
(2) $X_1 = 1, X_2 = 0 \Rightarrow Q_2 = 0$
$\Rightarrow Y = Q_1 . Q_2 = 0$

(3) $X_1 = 1, X_2 = 1 \Rightarrow Y = Q_1 \cdot Q_2 = 1$

(4) $X = 1, X2 = 1 \Rightarrow Y = Q1\ Q2 = 0$

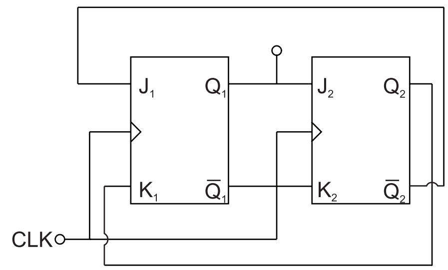
$Q_1 = 1, Q_2 = 1$

12. The outputs of the two flip-flops Q_1 Q_2 in the figure shown are initialized to 0, 0. The sequence generated at Q_1 upon application of clock signal is



(a) 01110... (b) 01010...
(c) 00110… (d) 01100... **[2014]**

Solution: (d)



$J_1 = \overline{Q}_2 \qquad J_2 = Q_1$

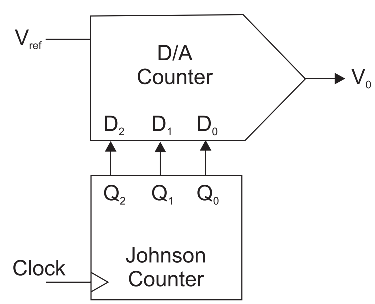
$K_1 = Q_2 \qquad K_2 = \overline{Q}_1$

$Q_1 = Q_2 = 0$ (intially)

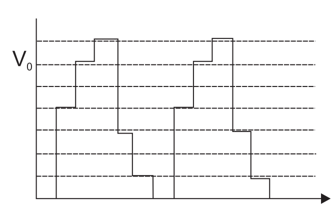
	PS		*NS*			
clk	Q_1	Q_2	J_1K_1	J_2K_2	Q_1	Q_2
0					0	0
1	0	0	10	01	1	0
2	1	0	10	10	1	1
3	1	1	01	10	0	1
4	0	1	01	01	0	0

Hence, the correct option is (d).

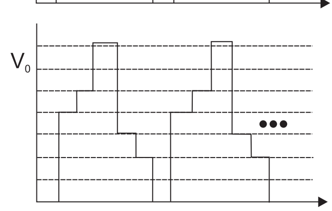
13. The output of a 3-stage Johnson (twisted-ring) counter is fed to a digital-to-analogue (D/A) converter as shown in the figure below. Assume all states of the counter to be unset initially. The waveform which represents the D/A converter output V_0 is



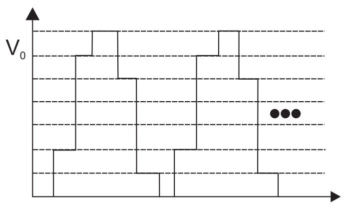
(a)



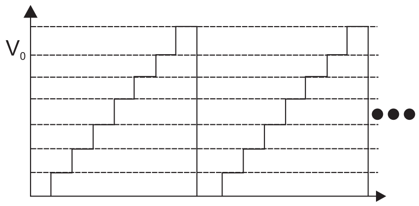
(b)



(c)

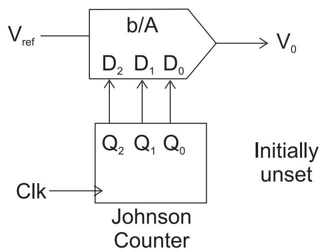


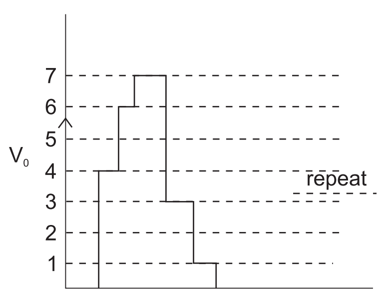
(d)



[2011]

Solution: (a)





Johnson counter is finished using counter with states:

Q_2	Q_2	Q_2 $(= D_2\ D_1\ D_0)$	V_0
0	0	0	0
1	1	0	4
1	1	1	6
1	1	1	7
0	1	1	3
0	0	1	1
0	0	0	0

Hence, the correct option is (a).

14. Two D flip-flops are connected as a synchronous counter that goes through the following $Q_B\ Q_A$ sequence 00 ® 11 ® 01 ® 10 ® 00 ®...

The connections to the inputs D_A and D_B are

(a) $D_A = Q_B$, $D_B = Q_A$

(b) $D_A = \bar{Q}_A, D_B = \bar{Q}_B$

(c) $D_A = (Q_A \bar{Q}_B + \bar{Q}_A Q_B), D_B = Q_A$

(d) $D_A = (Q_A Q_B + \bar{Q}_A \bar{Q}_B), D_B = \bar{Q}_B$

[2011]

Solution: (d)

D FF synchronous counter:

DFF: $Q_{n+1} = D_n$

$Q_n Q_A: 00 \to 11 \to 01 \to 10$ (PS)

It this is PS then NS $Q_B Q_A$:

$11 \to 01 \to 10 \to 00$ (NS)

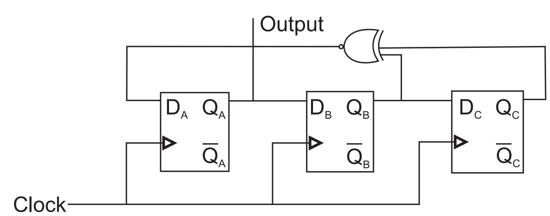
	$Q_B Q_A$ (PS)	$Q_B Q_A$ (NS) $\Rightarrow$	$D_B D_A$
	00	11	11
$\Rightarrow$	11	01	01
	01	10	10
	10	00	00

$$\Rightarrow D_B = \overline{Q_B Q_A} + \overline{Q_B Q_A} = \overline{Q_B}(\overline{Q_A} + \overline{Q_A}) = D_B = \overline{Q}_B$$

$$D_A = \bar{Q}_B \bar{Q}_A + Q_B Q_A$$

Hence, the correct option is (d).

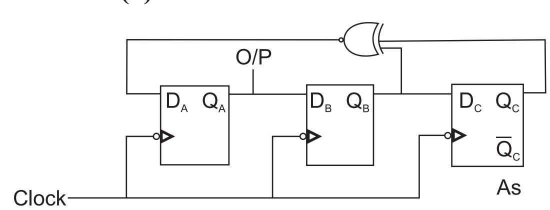
15. Assuming that all flip-flops are in reset conditions initially, the count sequence observed at Q_A in the circuit shown is



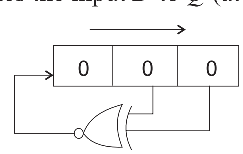
(a) 0010111... (b) 0001011...

(c) 0101111... (d) 0110100... **[2010]**

Solution: (d)



As D FF copies the input D to Q (at clk)



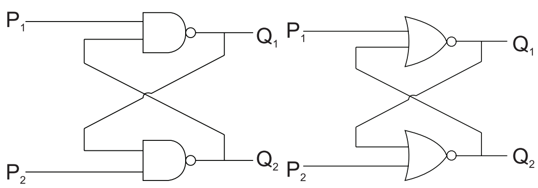
$Q_A = 01101000$.............(d)

Output $= Q_A$, initially $Q_A Q_B Q_C = 0$

clk	$Q_B \oplus Q_C$ D_A	D_B	D_C	Q_A	Q_B	Q_C
0	0	0	0	0	0	0
1	1	0	0	1	0	0
2	1	1	0	1	1	0
3	0	1	1	0	1	1
4	1	0	1	1	0	1
5	0	1	0	0	1	0
6	0	0	1	0	0	1
7	0	0	0	0	0	0

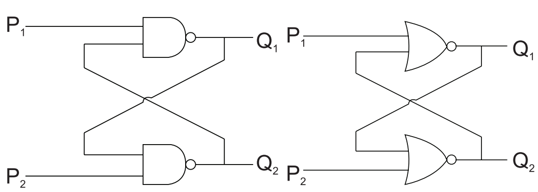
Hence, the correct option is (d).

16. Refer to the NAND and NOR latches shown in the figure. The inputs (P_1 P_2) for both the latches are first made (0, 1) and then, after a few seconds, made (1, 1). The corresponding stable outputs (Q_1, Q_2) are



(a) NAND: first (0, 1) then (0, 1) NOR: first (1, 0) then (0, 0)
(b) NAND: first (1, 0) then (1, 0) NOR: first (1, 0) then (1, 0)
(c) NAND: first (1, 0) then (1, 0) NOR: first (1, 0) then (0, 0)
(d) NAND: first (1, 0) then (1, 1) NOR: first (0, 1) then (0, 1) **[2009]**

Solution: (c)



NAND LATCH

Initially: $P_1P_2 = (0, 1)$ NAND: As $P_1 = 0$,

$Q_1 = 1$ $(Q_1, Q_2) = (1, 0)$

Now $\overline{P_2 \cdot Q_1} = \bar{1}\cdot 1 = 0 = Q_2$

NOR: $P_2 = 1 \Rightarrow Q_2 = 0$

$(Q_1, Q_2) = (1, 0)$

$Q_1 = \overline{P_1 + Q_2} = \bar{0} = 1$

Then $P_1P_2 = (1, 1)$ NAND: $Q_1 = \overline{P_1 - Q_2} = \overline{Q_2}$ &

$Q_2 = 0$ (Case state)

$Q_1 = \bar{0} = 1$

$Q_2 = \overline{P_2 \cdot Q_1} = \overline{1 \cdot 1} = 0$ $\quad (Q_1 = 1, LS)$

$(Q_1, Q_2) = (1, 0)$

NOR: $P_1 = 1 \Rightarrow Q_1 = 0$

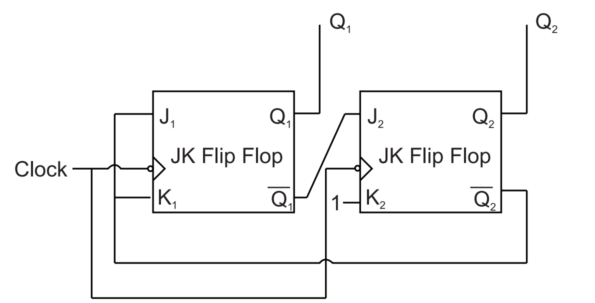
$P_2 = 1 \Rightarrow Q_2 = 0$ $\quad (Q, 0)$

$\Rightarrow$ (C) NAND: (1, 0) then (1, 0).

NOR (1, 0) then (0, 0).

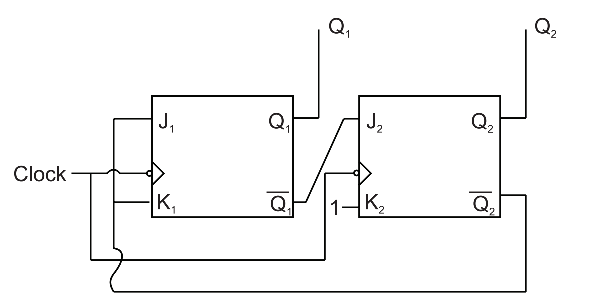
Hence, the correct option is (c).

17. What are the counting states (Q_1, Q_2,) for the counter shown in the figure below?



(a) 11, 10, 00, 11, 10, ...
(b) 01, 10, 11, 00, 01, ...
(c) 00, 11, 01, 10, 00, ...
(d) 01, 10, 00, 01, 10, ... **[2009]**

Solution: (a)



Clk initially: $Q_1Q_2 = 0_0$ $(Q_1Q_2 = 1_1)$ $0_0 \to 1_1$

(1) $\Rightarrow J_1K_1 = 1$ & Q_1 toggle to 1 $(\overline{Q_1} = 0)$

$\Rightarrow J_2 = 1$ (LS of $\overline{Q_1}$), $x_2 = 1 \Rightarrow Q_2 = 1(\overline{Q_2} = 0)$

(2) $\overline{Q_1} = 0 \Rightarrow J_2 = 0 \Rightarrow Q_1 = 1$ (no change)

$00 \to 11 \to 10$

$\overline{q_2} = 0 \Rightarrow J_1K_1 = 0$ $\quad Q_2 = 0$ (reset)

(1) Last state $Q_1Q_2 = 10$

$\overline{Q_2} = 1$ $J_1K_1 \Rightarrow Q_1$ toggle to 0

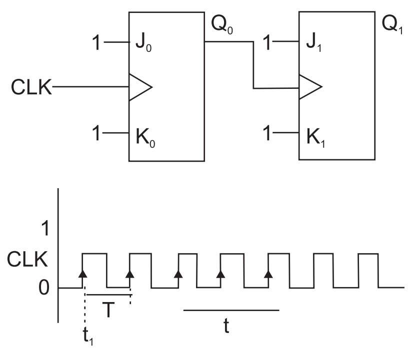
$\overline{Q_1} = 0 = J_2,\ K_2 = 1 \Rightarrow Q_2$ reset to 0

clk	J_1K_1	J_2K_2	Q_1Q_2
0	11	11	00
1	11	11	11
2	00	01	10
3	11	01	00

$00 \rightarrow 11 \rightarrow 10 \rightarrow 00 \rightarrow 11\ldots$ so on.

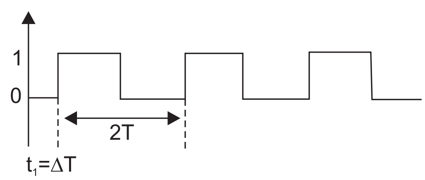
Hence, the correct option is (a).

18. For each of the positive edge-triggered J-K flip flop used in the following figure, the propagation delay is ΔT.

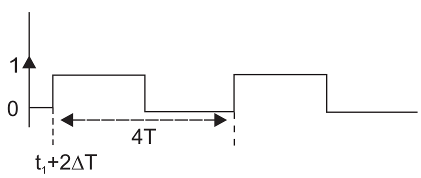


Which of the following waveforms correctly represents the output at Q_1?

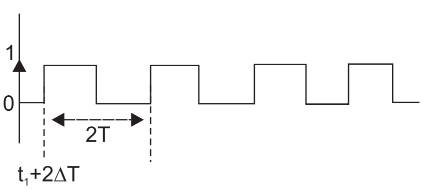
(a)



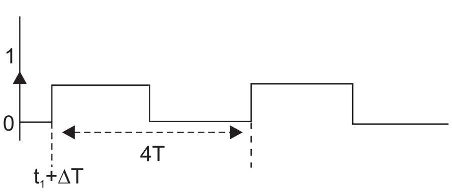
(b)



(c)

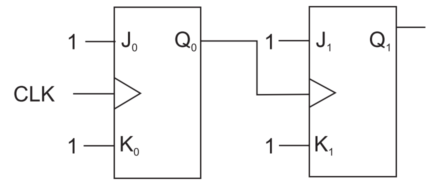


(d)



[2008]

Solution: (b)

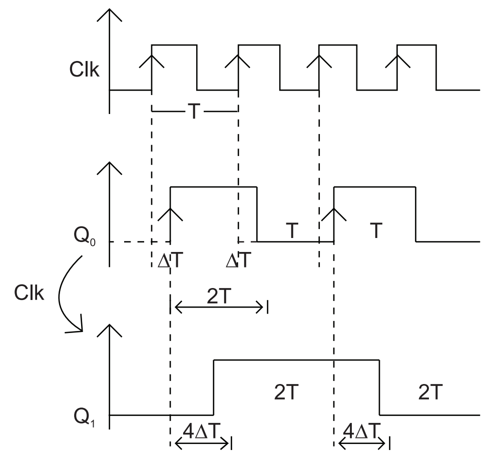


At $JK = 1$, FF toggles for each edge of clock.

$\therefore$ Q_0 toggles at ΔT delay $T = 2\text{T}\ (+1\Delta T)$.

Q_1 toggles according to Q_0 (Q_0 is clk for Q_1) + ΔT

$T = 4T\ (+2\ \Delta T)$.

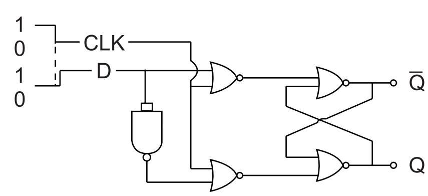


($T = 2T$, delay $= \Delta T$)

($T = 4T$, delay $= 2\Delta T$)

Hence, the correct option is (b).

19. For the circuit shown in the figure, D has a transition from 0 to 1 after CLK changes from 1 to 0. Assume gate delays to be negligible



Which of the following statements is true?

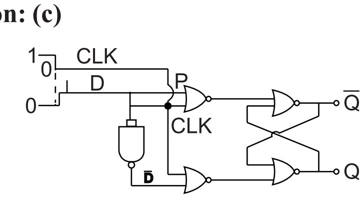
(a) Q goes to 1 at the CLK transition and stays at 1.

(b) Q goes to 0 at the CLK transition and stays at 0.

(c) Q goes to 1 at the CLK transition and goes to 0 when D goes to 1.

(d) Q goes to 0 at the CLK transition and goes to 1 when D goes to 1. **[2008]**

Solution: (c)



At clk transition $1 \to 0$, $(D = 0)$

$\overline{D + Clk} = 1 \Rightarrow \overline{Q} = 0$

$\overline{\overline{D} + Clk} = 0 \qquad Q = 1$

After that, at D transition: $D = 1$ (Clk = 0)

$\overline{\overline{D} + Clk} = \overline{0 + 0} = 1$

& $\overline{D + Clk} = \overline{1 + 0} = 0, Q = \overline{1} = 0 = \overline{Q} = 1 \Rightarrow \overline{Q} = 1$

$\Rightarrow Q$ goes to 1 at clk transition, and goes' to Q at D transition.

Hence, the correct option is (c).

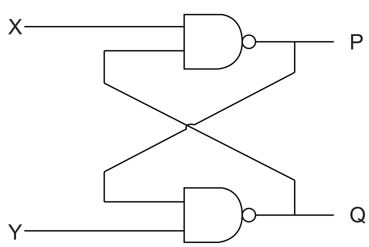
20. The following binary values were applied to the X and Y inputs of the NAND latch shown in the figure in the sequence indicated below:

$X = 0, Y = 1;$

$X = 0, Y = 0;$

$X = 1, Y = 1.$

The corresponding stable P, Q outputs will be



(a) $P = 1, Q = 0; P = 1,$ $Q = 0; P = 1, Q = 0$ or $P = 0, Q = 1$
(b) $P = 1, Q = 0; P = 0,$ $Q = 1$ or $P = 0, Q = 1; P = 0, Q = 1$
(c) $P = 1, Q = 0; P = 1,$ $Q = 1; P = 1, Q = 0$ or $P = 0, Q = 1$
(d) $P = 1, Q = 0; P = 1, Q = 1; P = 1, Q = 1$

[2007]

Solution: (c)

NAND later

clk	X	Y	P	Q
1	0	1	1	0
2	0	0	1	1
3	1	1	$\overline{Q}$	$\overline{P}$

:

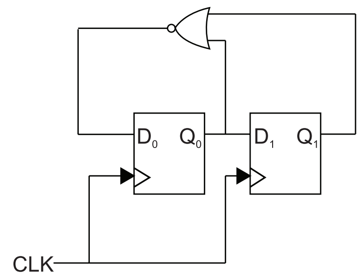
$\Rightarrow$ If PQ (earlier) = 10 then $PQ = \overline{QP} = 10$

If PQ (previous state) = 01 $\Rightarrow PQ = 01$

$\Rightarrow$ If $X_4 = 11$, PQ depends on last state 1, 0 or 01

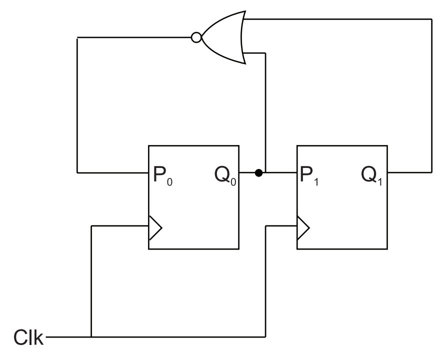
Hence, the correct option is (c).

21. For the circuit shown, the counter state $(Q_1 Q_0)$ follows the sequence



(a) 00, 01, 10, 11, 00 …
(b) 00, 01, 10, 00, 01 …
(c) 00, 01, 11, 00, 01 ...
(d) 00, 10, 11, 00, 10 ... **[2007]**

Solution: (b)



Initially, $Q_0 Q_1 = 00$

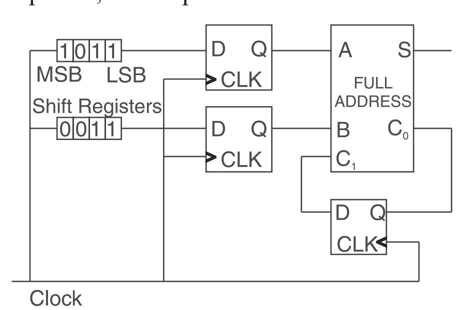
and $D_0 = \overline{Q_0 + Q_1} = \overline{Q_0}\,\overline{Q_1}$ $D_1 = Q_0$

D_0	D_1	Q_0	Q_1
0	0	0	0
1	0	1	0
0	1	0	1
0	0	0	0
1	0	1	0

$\Rightarrow$ $Q_1 Q_0 : 00, 01, 10, 00$

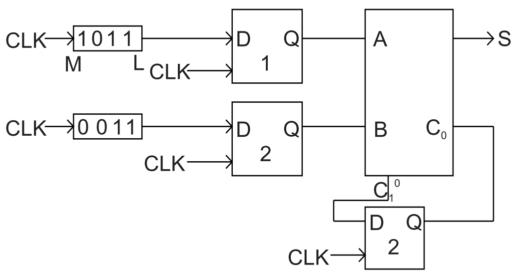
Hence, the correct option is (b).

22. For the circuit shown in the figure below, two 4-bit parallel-in serial-out shift registers loaded with the data shown are used to feed the data to a full adder. Initially, all the flip-flops are in clear state. After applying two clock pulses, the outputs of the full adder should be



(a) $S = 0$ $C_0 = 0$
(b) $S = 0$ $C_0 = 1$
(c) $S = 1$ $C_0 = 0$
(d) $S = 1$ $C_0 = 1$ **[2006]**

Solution: (d)



Initially $Q_1Q_2 = 0$

$Q_3 = 0$

$S_0 = 0$

$C_0 = 1$

At 1st clk pulse:

$D_1 = 1, Q_1 = 1$

$D_2 = 1, Q_2 = 1$

$(C_0 = 1)\ D_3 = 1, Q_3 = 1$

$\Rightarrow S = 1 + 1 + 1 = 1$

$C_0 = 1$

After 2nd pulse

$D_1 = D_2 = 1 \Rightarrow Q_1 = Q_2 = 1$

As C_0 W as 1, $D_3 = 1 = Q_3$

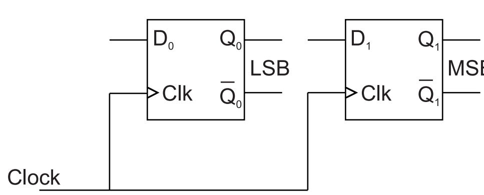
$\Rightarrow 5 = 1 + 1 + 1 = 1, C_0 = 1.$

Hence, the correct option is (d).

23. Two D-flip-flops, as shown below, are to be connected as a synchronous counter that goes through the following Q_1Q_0 sequence

$$00 \longrightarrow 01 \longrightarrow 11 \longrightarrow 10 \longrightarrow 00 \longrightarrow .$$

The inputs D_0 and D_1, respectively, should be connected as

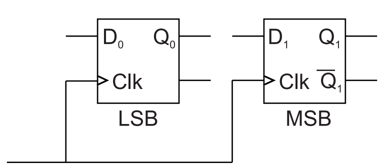


(a) $\bar{Q}_1$ *and* Q_0 (b) $\bar{Q}_0$ *and* Q_1

(c) $\bar{Q}_1Q_0$ *and* $\bar{Q}_1Q_0$ (d) $\bar{Q}_1\bar{Q}_0$ *and* Q_1Q_0

[2006]

Solution: (a)



1 2 3 4

Seq $Q_1Q_0 = 00 \rightarrow 0.1 \rightarrow 11 \rightarrow 10 \rightarrow 00 \rightarrow$

Initially $Q_0Q_1 = 00$

at Clk 1, $Q_0 = 1 \Rightarrow D_0 = 1 \Rightarrow D_0 \rightarrow \bar{Q}_1$ (as Q 1 = 0)

2, $Q_0 = 1 \Rightarrow D_0 = 1$ again $= \bar{Q}_1$

$\Rightarrow D_0 = \bar{Q}_1$

Clk 1: $Q_1 = 0 \Rightarrow D_1 = 0 \Rightarrow Q_0$

Clk 2: $Q_1 = 1 = \Rightarrow D_1 = 1$

$\Rightarrow D_1 \rightarrow Q_0$

$\Rightarrow$ (a) $(\bar{Q}_1, Q_0)$.

Hence, the correct option is (a).

24. The present output Qn of an edge triggered JK flip-flop is logic 0. If $J = 1$, then $Qn+1$

(a) cannot be determined

(b) will be logic 0

(c) will be logic 1

(d) will race around **[2005]**

Solution (c)

$Qn = 0$ (present output)

Now $J = 1$ (set), but $k = ?$

(a) If $k = 1 \Rightarrow JK = 11$ toggle $\Rightarrow Q_{n+1} = \bar{Q}_n = 1$

$\Rightarrow$ for $J = 1$ $(Q_n = 0)$

(b) $k = 0 \Rightarrow JK = 10$ (set)

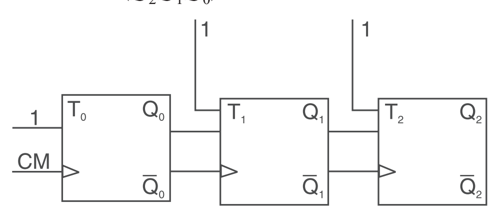
$Q_{n+1} = 1.$

$Q_{n+1} = 1$

Hence, the correct option is (c).

25. The given figure shows a ripple counter using positive edge triggered flip-flops.

If the present state of the counter is $Q_2Q_1Q_0 = 011$, then its next state $(Q_2Q_1Q_0)$ will be



(a) 010 (b) 100

(c) 111 (d) 101 **[2005]**

Solution: (b)

$Q_2Q_1Q_0 = 011$ (ripple counter)

Next state: $Q_0 \rightarrow$ toggles 1 to 0 = 0 (the edge trigger)

$\bar{Q}_0 = 0$ to 1 (–1)

For Q_1: $\bar{Q}_0$ is clk (0 to 1) $\Rightarrow$ + ve edge clock

$\Rightarrow Q_1$ toggles (1 to 0) =0.

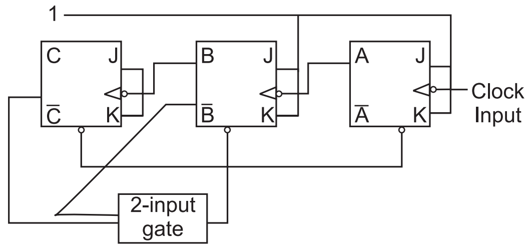
Q_2: $\bar{Q}_1$ is clk (0 to 1) $\Rightarrow$ + ve edge

$\Rightarrow Q_2$ (0 to 1) = 1

$\Rightarrow Q_2Q_1Q_0 = 100.$

Hence, the correct option is (b).

26. In the modulo-6 ripple counter shown in the figure, the output of the 2-input gate is used to clear the *J–K* flip-flops.



The 2-input gate is

(a) a NAND gate (b) a NOR gate
(c) an OR gate (d) an AND gate **[2004]**

Solution: (c)

Mod–6 ⇒ 0 to 5 (101)

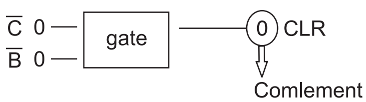
At CBA, counter is reset.

110

$\Rightarrow\ A = 0 \quad \bar{A} = 1$

$B = 1 \quad \bar{B} = 0$

$C = 1 \quad \bar{C} = 0$



$\Rightarrow \bar{C}$ OR $\bar{B} = \overline{CB} = \overline{CLR}$

$CLR = CB$ (required)

⇒ an OR gate.

Hence, the correct option is (c).

27. A 4 bit ripple counter and a 4 bit synchronous counter are made using flip flops having a propagation delay of 10 ns each. If the worst case delay in the ripple counter and the synchronous counter be R and S respectively, then

(a) $R = 10$ ns, $S = 40$ ns
(b) $R = 40$ ns, $S = 10$ ns
(c) $R = 10$ ns, $S = 30$ ns
(d) $R = 30$ ns, $S = 10$ ns **[2003]**

Solution: (b)

Ripple counter: delay all 4 added

$\Rightarrow R = 4 \times 10 = 40$ ns

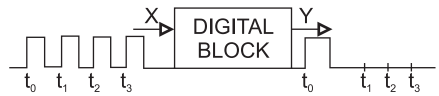
Synch counter: Only 1 delay (as all FF works simultaneously)

= 5

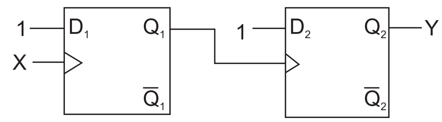
$\Rightarrow R = 40$ ns, $S = 10$ ns.

Hence, the correct option is (b).

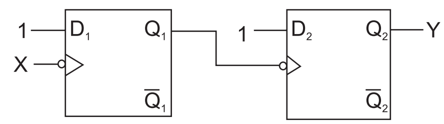
28. The digital block in the figure is realized using two positive edge triggered D-flip-flops. Assume that for $t < t_0$, $Q_1 = Q_2 = 0$. The circuit in the digital block is given by:



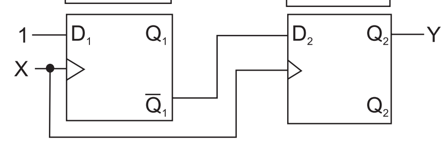
(a)



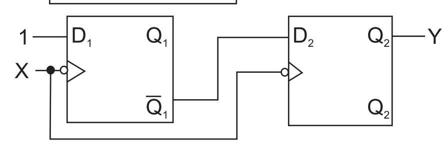
(b)



(c)



(d)



[2001]

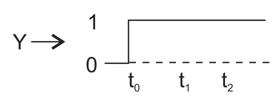
Solution: (c)

$Q_1Q_2 = 00\ (t < t_0)$

$Y\ Q_1 = D_1 = 1$ (when clk arrives)

When no clk $\Rightarrow Q = D \Rightarrow Q_1 = 1$ always

$\therefore\ Q_2\ D_2 = 1$ (fixed, as clk $(Q_1) = 1$ always)



X when clk, $Q_1 = D_1 = 1, \bar{Q}_1 = 0$ (clk for Q_2)

$\therefore Q_2$ (–ve edge triggered) = 1 always

(Both the edge triggered) at the edge,

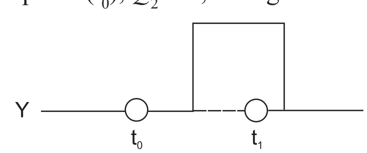
$Q_1 = D_1(=1) = 1$

For D_2, $D_2 = \bar{Q}_1 = 0$ (at the edge)

$\Rightarrow Q_2 = 0$

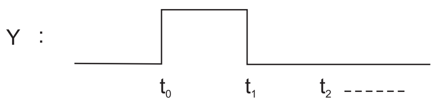
But initially $Q_1 = 0 \Rightarrow \bar{Q}_1 = 0\ D_2$

$\therefore$ At 1st pulse (t_0), $Q_2 = 1$, then goes to 0



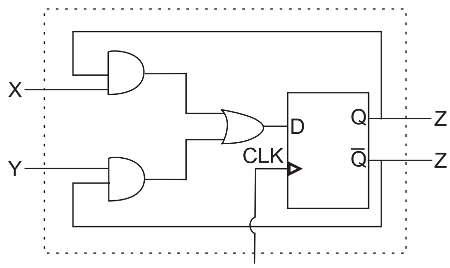
Same as (c),

But it is –ve edge triggered



Hence, the correct option is (c)

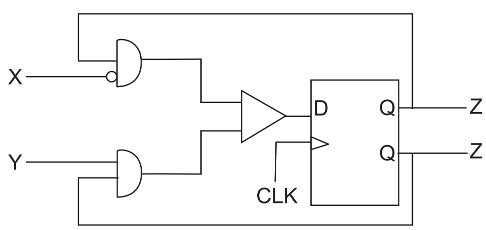
29. A sequential circuit using D Flip-Flop and logic gates is shown in the figure, where X and Y are the inputs and Z is the output. The circuit is



(a) $S-R$ Flip-Flop with inputs $X = R$ and $Y = S$
(b) $S-R$ Flip-Flop with inputs $X = S$ and $Y = R$
(c) $J-K$ Flip-Flop with inputs $X = J$ and $Y = K$
(d) $Q_1 - K$ Flip-Flop with inputs $X = K$ and Y $= J$

[2000]

Solution: (d)



X	Y	Q_{n+1}
0	0	Z_n
0	1	$Z_n + \overline{Z_n} = 1$
1	0	$0 + 0 = 0$
1	1	$\overline{Z} + 0 = \overline{Z}$

$\Rightarrow$ when $XY = 01$, $Q = 1$ (set) Y = set

10, $Q = 0$ (reset) X = Reset

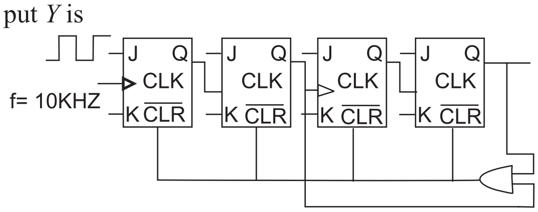
11 $Q = \overline{z}$ toggle $\Rightarrow$ JK FF

$\Rightarrow X$ = Reset $= K$

$Y = J$ (set).

Hence, the correct option is (d).

30. In the figure, the J and K inputs of all the four Flip-Flips are made high. The frequency of the signal at output Y is



(a) 0.833 kHz (b) 1.0 kHz
(c) 0.91 kHz (d) 0.77 kHz **[2000]**

Solution: (b)

$JK = 11$, for all 4 FF.

$\Rightarrow$ All 4 FF toggle for each clk pulse.

For output frequency, find number of states

IF all are initially cleared (0000), then

	Q_1			Q_4
	0	0	0	0
clk 1	1	0	0	0
2	0	1	0	0
3	1	1	0	0

(clk at falling edge: $1 \rightarrow 0$)

Now $C\overline{LR} = \overline{Q_4 Q_2} \Rightarrow CLR = Q_4 Q_2 = 11$

$\Rightarrow$ clears at $Q_4 Q_3 Q_2 = 1010$

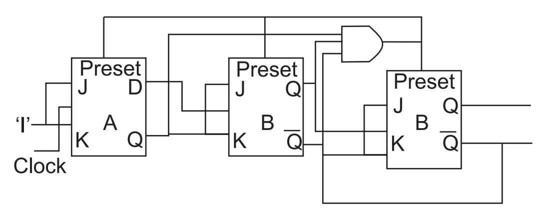
$0000 \rightarrow 0001 \rightarrow 0110 \rightarrow 0011 \rightarrow \ldots 1010$

$\Rightarrow$ 0 to 9 (10 states) $\downarrow$ cleared

$$\Rightarrow f_{\text{output}} = \frac{F}{10} = \frac{10K}{10} = 1\text{ KHz.}$$

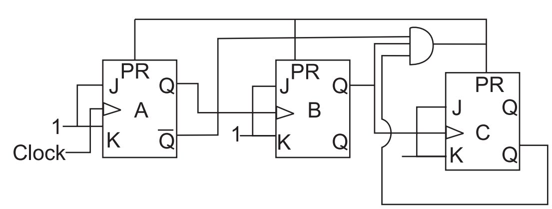
Hence, the correct option is (b).

31. The ripple counter shown in the figure works as a



(a) mod-3 up counter
(b) mod-5 up counter
(c) mod-3 down counter
(d) mod-5 down counter **[1999]**

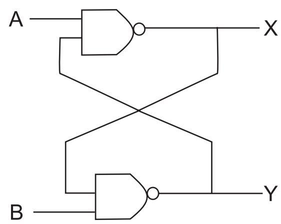
Solution: (d)



$PR \quad \overline{Q}_A \overline{Q}_B \overline{Q}_C$

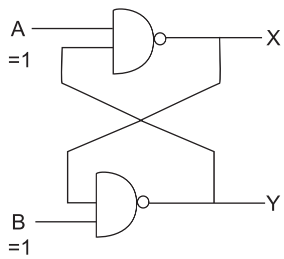
$\Rightarrow$ at 010 counter is reset.

32. In the figure given below, $A = 1$ and $B = 1$. The input B is now replaced by a sequence 101010..... the outputs x and y will be



(a) fixed at 0 and 1, respectively
(b) $x = 1010$ .....while $y = 0101$ ......
(c) $x = 1010$......and $y = 1010$......
(d) fixed at 1 and 0, respectively **[1998]**

Solution: (a)



I/P $B \to 101010\ldots$

$\to$ when $A = 1$, $B = 1$

$X = \overline{A \cdot Y} = \overline{Y}$

nly $Y = \overline{B \cdot X} = \overline{X}$

$\to$ When $A = 1$ (fixed) $B = 0$,

$Y = \overline{O \cdot X} = (1)$

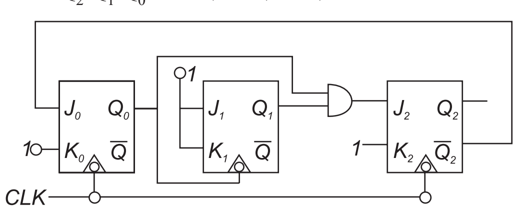
$X = \overline{A \cdot Y} = (0)$

$$\begin{array}{l} \to A = 1 \;\; X = \overline{Y} = \bar{1} = (0) \\ \quad B = 1 \;\; Y = \overline{X} = \bar{0} = (1) \end{array} \Rightarrow \begin{array}{l} X = 0 \text{ fixed} \\ Y = 1 \text{ fixed} \end{array}$$

Hence, the correct option is (a).

Five-marks Question

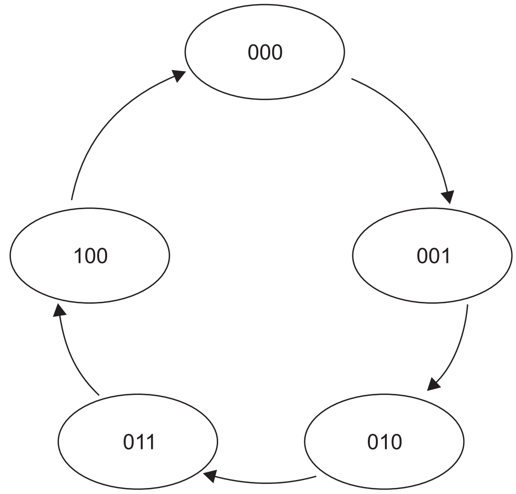
1. The mod-5 counter shown in figure counts through states $Q_2 Q_1 Q_0$ = 000, 001, 010, 011 and 100.



(a) Will the counter lockout if it happens to be in any one of the unused states ?

(b) Find the maximum rate at which the counter will operate satisfactorily. Assume the propagation delays of flip-flop and AND gate to be t_F and t_A. **[1998]**

Solution:



Unused states are

To check lock out condition if we give unused $\left.\begin{matrix} 101 \\ 110 \\ 111 \end{matrix}\right\}$ states as input to the ckt and corresponding next state is one of the used state the ckt is said to be lock out free or No. Lock out condition exist"

For 101:

θ_2	θ_1	θ_0	J_0	K_0	J_1	K_1	J_2	K_2	θ_{2^+}	θ_{1^+}	θ_{0^+}
1	1	0	0	1	1	1	0	1	0	0	0

used state

$\therefore$ No Lock out

For 111:

θ_2	θ_1	θ_0	J_0	K_0	J_1	K_1	J_2	K_2	θ_{2^+}	θ_{1^+}	θ_{0^+}
1	1	1	0	1	1	1	1	1	0	0	0

used state

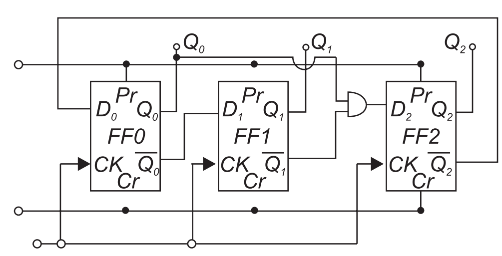
No Lock out

$\therefore$ Counter is lock out free.

(b) Minimum time Required = $2t_F + t_A$

Max clock rate = $\dfrac{1}{2t_F + t_A}$

2. A sequence generator is shown in figure. The counter status ($Q_0\ Q_1\ Q_2$) is initialized to 010 using preset/clear inputs.



The clock has a period of 50 ns and transitions take place at the rising clock edge.

(a) Give the sequence generated at Q0 till it repeats.

(b) What is the repetition rate for the generated sequence? **[1997]**

Solution:(a) $D_0 = \bar{\theta}_2$

$D_1 = \bar{\theta}_0$

$D_2 = \bar{\theta}_1 . \theta_0$

D_0	D_1	D_2	θ_0	θ_1	θ_2
1	1	0	0	1	0
1	0	0	1	1	0
1	0	1	1	0	0
0	0	1	1	0	1
0	1	0	0	0	1
1	1	0	0	1	0

So sequence at ϑ_0 is

0 → 1 → 1 → 1 → 0

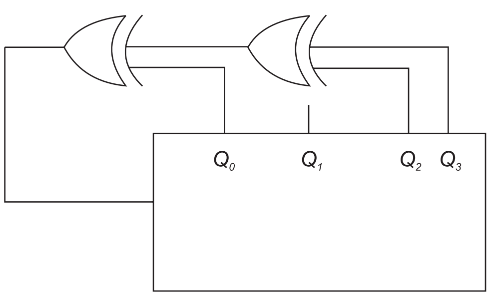
(b) No. of states = 5

Talk = 50 n sec

$$\text{Repetition rate} = \frac{1}{5 \times \text{talk}} = \frac{1}{5 \times 50 \times 10^{-9}}$$

$$= 4 \text{ MHz}$$

3. A 4-bit shift register, which shifts 1 bit to the right at every clock pulse, is initialized to values (1000) for (Q_o Q_1 Q_2 Q_3). The D input is derived from Q_0, Q_2 and Q_3 through two XOR gates as shown in figure.



[1996]

(a) Write the 4-bit values (Q_0 Q_1, Q_2 Q_3) after each clock pulse till the pattern (1000) reappears on (Q_0 Q_1 Q_2 Q_3).

(b) To what values should the shift register be initialized so that the pattern (1001) occurs after the first clock pulse?

Solution: (a)

$D = \theta_0 \oplus \theta_2 \oplus \theta_3$

	θ_0	θ_1	θ_2	θ_4
	1	0	0	0
1[st] clk	0	0	0	1
2[nd] clk	0	0	1	1
3[rd] clk	0	1	1	0
4[th] clk	1	1	0	1
5[th] clk	1	0	1	0
6[th] clk	0	1	0	0
7[th] clk	1	0	0	0

So after 7[th] clk pulse pattern 1000 will reappear on $\vartheta_0 \vartheta_1 \vartheta_2 i_3$

(b)

θ_0	θ_1	θ_2	θ_3
1	1	0	0
1	0	0	1

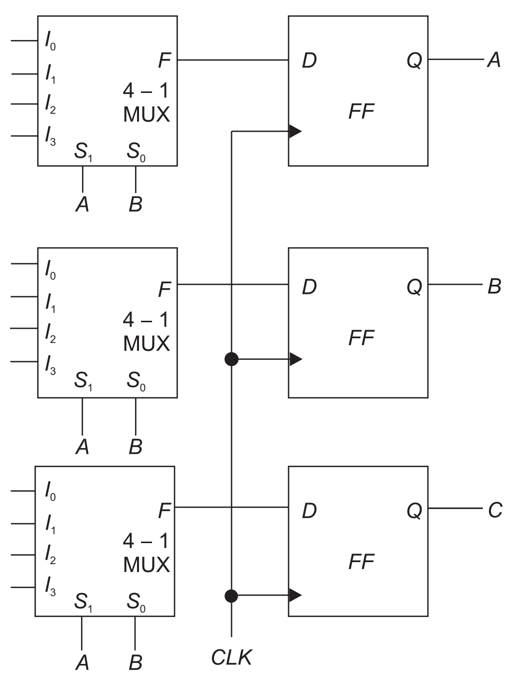
$\therefore$ 1100 should be their initially so that after 1[st] clk 1001 patterns comes

4. A state machine is required to cycle through the following sequence of states:

ABC

→ 000 → 010 → 111 → 100 → 011 → 101

One possible implementation of the state machine is shown below. Specify what signals should be applied to each of the multiplexer inputs.



[1996]

Solution:

Present State			Next State			FF input		
A	B	C	A	B	C	D_A	D_B	D_C
0	0	0	0	1	0	0	1	0
0	1	0	1	1	1	1	1	1
1	1	1	1	0	0	1	0	0
1	0	0	0	1	1	0	1	1
0	1	1	1	0	1	1	0	1
1	0	1	0	0	0	0	0	0

$$D_A = \Sigma m(2, 3, 7) + \Sigma d(1, 6)$$
$$D_B = \Sigma m(0, 2, 4) + \Sigma d(1, 6)$$
$$D_C = \Sigma m(2, 3, 4) + \Sigma d(1, 6)$$

$D_A =$

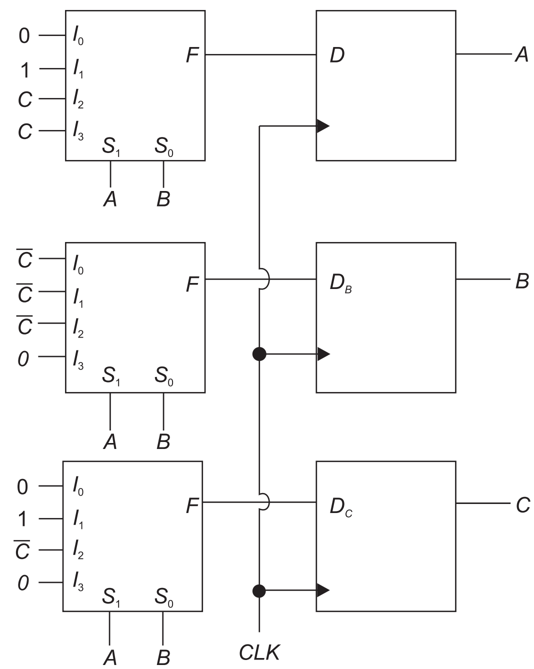
	I_0	I_1	I_2	I_3
C	0	(2)	4	6
C	1	(3)	5	(7)
	0	1	0	C

$D_B =$

	I_0	I_1	I_2	I_3
C	(0)	(2)	(4)	6
C	1	3	5	7
	C	C	C	0

$D_C =$

	I_0	I_1	I_2	I_3
C	0	(2)	(4)	6
C	1	(3)	5	7
	0	1	C	0

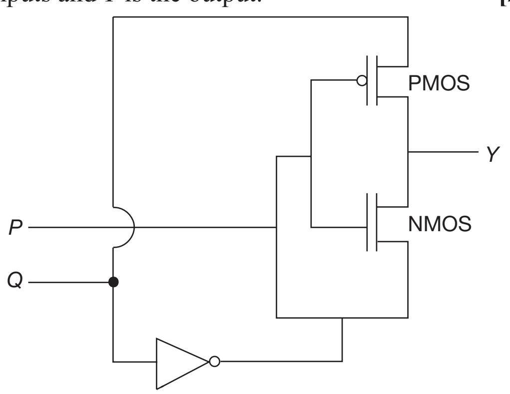


Chapter 6

Logic Families

One-mark Questions

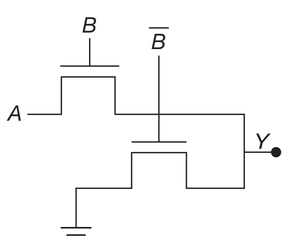
1. For the circuit shown in the figure P and Q are the inputs and Y is the output. **[2017]**



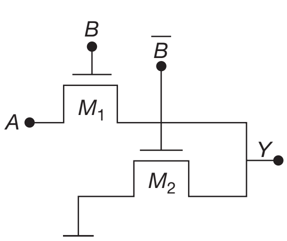
The logic implemented by the circuit is

(A) XNOR (B) XOR
(C) NOR (D) OR

2. The logic functionality realized by the circuit shown below is **[2016]**



(A) OR (B) XOR
(C) NAND (D) AND



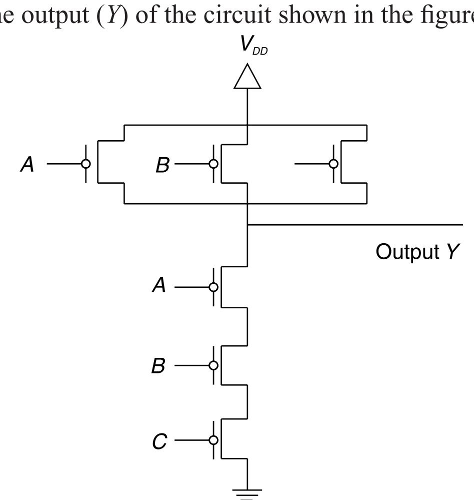
Solution:

A	B	Y	M_1	M_2	Y
0	0	0	OFF	ON	GND
0	1	0	ON	OFF	A
1	0	0	OFF	ON	GND
1	1	1	ON	OFF	A

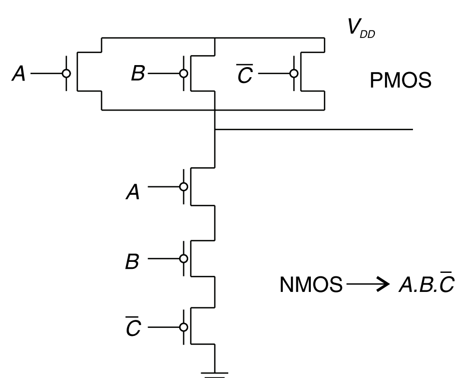
∴ From the above truth table output = AB ⇒ AND gate

Hence, the correct option is (D).

3. The output (Y) of the circuit shown in the figure is



(a) $\bar{A}+\bar{B}+C$ (b) $A+\bar{B}\cdot\bar{C}+A\cdot\bar{C}$
(c) $\bar{A}+B+\bar{C}$ (d) $A\cdot B\cdot\bar{C}$ **[2014]**



$\rightarrow$ CMOS logic

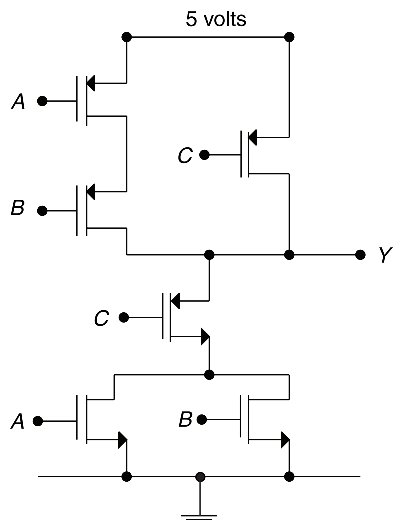
$$\Rightarrow y = \overline{A.B.\overline{C}}$$

$$= \overline{A} + \overline{B} + \overline{\overline{C}}$$

$$y = \overline{A} + \overline{B} + C.$$

Hence, the correct option is (a).

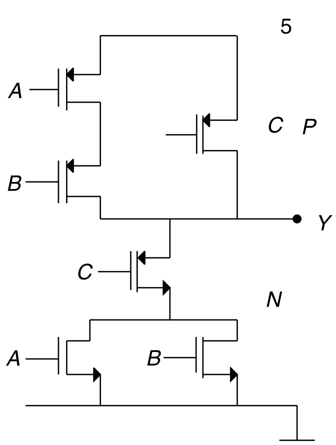
4. In the circuit shown



(a) $Y = \overline{A}\,\overline{B} + \overline{C}$ (b) $Y = (A + B)C$

(c) $Y = (\overline{A} + \overline{B})\overline{C}$ (d) $Y = AB + C$ **[2012]**

Solution: (a)



$\rightarrow A + B$ (as $A \parallel B$)

$\rightarrow C$ series with $(A + B)$ in N channel.

$\Rightarrow$ $C(A + B)$ (reverse in P)

$\Rightarrow$ $Y = C(A + B)$

$$y = \overline{C(A + B)}$$

$$= \overline{C} + \overline{A + B}$$

$$y = \overline{C} + \overline{A}\,\overline{B}$$

Hence, the correct option is (a).

5. The full forms of the abbreviations TTL and CMOS in reference to logic families are

(a) Triple Transistor Logic and Chip Metal Oxide Semiconductor

(b) Tristate Transistor Logic and Chip Metal Oxide Semiconductor

(c) Tristate Transistor Logic and Complementary Metal Oxide Semiconductor

(d) Tristate Transistor Logic and Complementary Metal Oxide Silicon **[2009]**

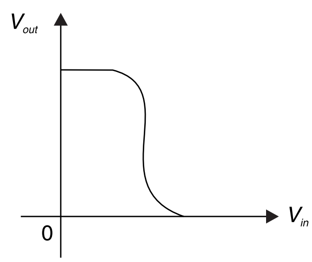
Solution: (c)

TTL $\rightarrow$ Tristate Transistor logic

CMOS $\rightarrow$ Complementary metal oxide S.C.

Hence, the correct option is (c).

6. Given figure is the voltage transfer characteristic of



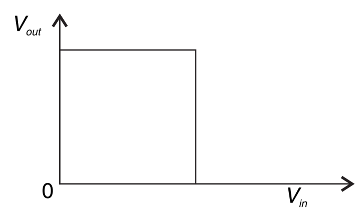
(a) an NMOS inverter with enhancement mode transistor as load

(b) an NMOS inverter with depletion mode transistor as load

(c) a CMOS inverter

(d) a BJT inverter **[2004]**

Solution: (c)



At Vin = 0 at Vin = High

Vout = High Vout = 0

$\Rightarrow$ Very close to be ideal inverter $\Rightarrow$ CMOS.

Hence, the correct option is (c).

7. The output of the 74 series GATE of TTL gates is taken from a BJT in

(a) totem pole and common collector configuration

(b) either totem pole or open collector configuration

(c) common base configuration

(d) common collector configuration **[2003]**

Solution: (b)

The output of the 74 series GATE of TTL gates is taken from a BJT in either totem pole or open collector configuration.

Hence, the correct option is (b).

8. A Darlington emitter-follower circuit is sometimes used in the output stage of a TTL gate in order to
(a) increase its I_{OL}
(b) reduce its I_{OH}
(c) increase its speed of operation
(d) reduce power dissipation **[1999]**

Solution: (c)

Darlington EF: It has low $R0$ which results in short time constant for charging for capacitive load (at output). Small charging time results in higher speed of operation.

Hence, the correct option is (c).

9. Commercially available ECL Gates use two ground lines and one negative supply in order to
(a) reduce power dissipation
(b) increase fan-out
(c) reduce loading effect
(d) eliminate the effect of power line glitches or the biasing circuit. **[1999]**

Solution: (d)

ECL uses two ground lines and one negative supple to eliminate the power line glitches or effects of biasing.

Hence, the correct option is (d).

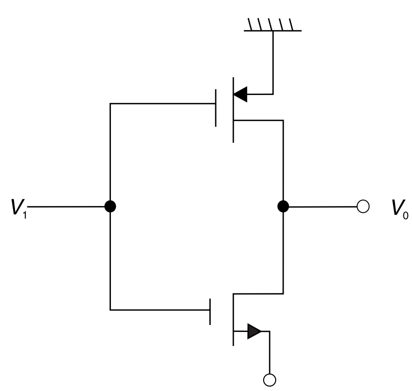
10. The noise margin of an TTL gate is about
(a) 0.2 V (b) 0.4 V
(c) 0.6 V (d) 0.8 V **[1998]**

Solution: (b)

TTL noise margin ~ 0.4 V.

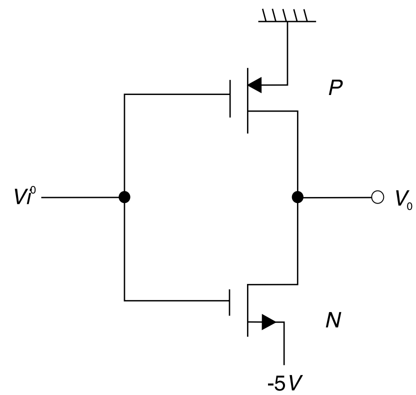
Hence, the correct option is (b).

11. The threshold voltage for each transistor in figure is 2 V. For this circuit to work as an inverter, Vi must take the values



(a) $-5\ V$ and $0\ V$ (b) -5 V and 5 V
(c) $0\ V$ and $5\ V$ (d) $-3\ V$ and $3\ V$
[1998]

Solution: (a)



$Vth = 2V$.

For inverter, $V_i = ?$

=	V_i	V_0	
	-5	0	
	0	-5	inverter (a)

If $V_i = -5 \Rightarrow N \to ON\,(V_D > V_{th})$

$P \to$ OFF

and $V_0 = 5 - 5 = 0$

$V_i = 0 \Rightarrow P \to$ ON $(V_D < V_{th})$

$N \to$ OFF

$V_0 = -5$

Hence, the correct option is (a).

12. In standard TTL the 'totem pole' stage refers to
(a) the multi-emitter input stage.
(b) the phase splitter.
(c) the output buffer.
(d) open collector output stage. **[1997]**

Solution: (c)

TTL: An output stage having two transistors connected such that only one is ON at a time is called TOTEM POLE stage.

80 Tetempole $\to$ output buffer.

Hence, the correct option is (c).

13. The inverter 74 AL S01 has the following specifications:

$I_{OH\ max} = -0.4$ mA, $I_{OL\ max} = 8$ mA, $I_{IH\ max} = 20\mu$A, $I_{IL\ max} = -0.1$ mA.

The fan out based on the above will be
(a) 10 (b) 20
(c) 60 (d) 100 **[1997]**

Solution: (b)

74 AL SOL $I_{OH} = -0.4$ mA $\quad I_{OL\,max} = 8$ mA

$\quad I_{iH\,max} = 20$ μA $\quad I_{iL\,max} = -0.1$ mA

FANOUT:

FO for high level:

$$\frac{\text{I}_{\text{OH max}}}{\text{I}_{\text{IH max}}} = \frac{0.4\text{ m}}{20\mu} = \frac{0.4\times10^{-3}}{20\times10^{-6}} = \frac{0.4}{2\cancel{0}}\times 10\cancel{00}$$

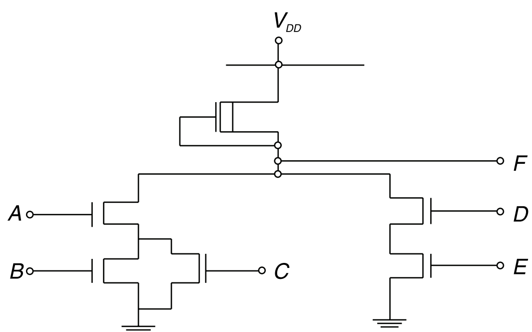
$$(\text{FO})_{\text{low level}} = \frac{\text{I}_{\text{DL max}}}{\text{I}_{\text{IL max}}} = \frac{8\ \cancel{m}}{0.1\ \cancel{m}} = 80$$

F.O. = min [(F))H, (FO)L] = min (20, 80)

F.O = 20.

Hence, the correct option is (b).

14. For the NMOS logic gate shown in figure, the logic function implemented is



(a) $\overline{ABCDE}$

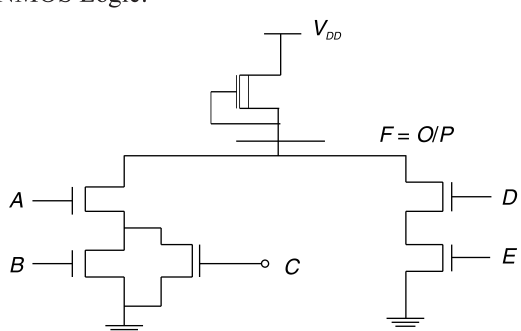
(b) $(AB+\overline{C})\cdot(\overline{D+E})$

(c) $\overline{A\cdot(B+C)+D\cdot E}$

(d) $(\overline{A+B})\cdot C+\overline{D}\cdot\overline{E}$ **[1997]**

Solution: (c)

NMOS Logic:



D and $E \rightarrow$ series $\Rightarrow$ DE

Only $B||C \rightarrow B + C$

and A series $(B||C)$

$\Rightarrow$ $A(B + C)$

$\Rightarrow$ $A(B + C) + DE$

and upper NMOS $\rightarrow$ inverter

$\Rightarrow$ $F = \overline{A(B + C) + DE}.$

Hence, the correct option is (c).

15. The gate delay of an NMOS inverter is dominated by charge time rather than discharge time because

(a) the driver transistor has a larger threshold voltage than the load transistor

(b) the driver transistor has larger leakage currents compared to the load transistor

(c) the load transistor has a smaller W/L ratio compared to the driver transistor

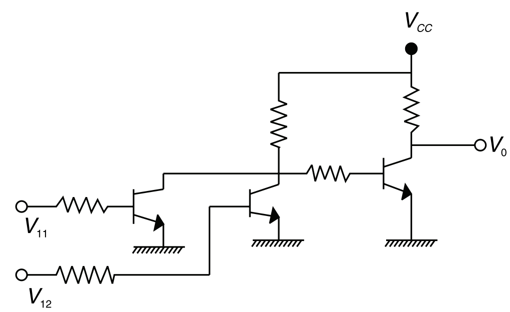
(d) none of the above. **[1997]**

Solution: (c)

In NMOS inverter, if load transistor has smaller (W/L) ratio as compared to driver, then inverter is dominated by charge time.

Hence, the correct option is (c).

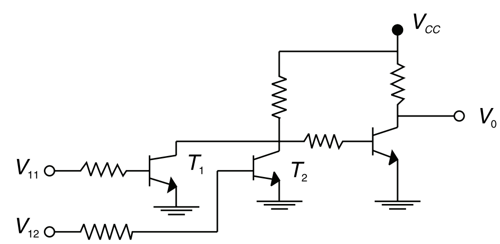
16. Figure shows the circuit of a gate in the Resistor Transistor Logic (RTL) family. The circuit represents a



(a) NAND (b) AND

(c) NOR (d) OR **[1992]**

Solution: (d)



RTL logic family

V_{i_1} V_{i_2} V_0

(1) LOW LOW T_1, T_2 – OFF $\Rightarrow$ LOW V_0

(2) LOW HIGH T_2 – ON $\Rightarrow$ HIGH V_0

(3) H L T_1 – ON $\Rightarrow$ V_0 HIGH

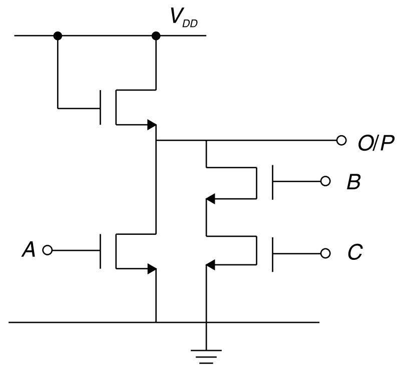
(4) L H $T_1, T_2 \rightarrow$ ON, HIGH

$\Rightarrow$ V_0 is low only when both

T_1 and T_2 are off $\Rightarrow$ $No = (V_{i_1} + V_{i_2}) \Rightarrow$ (d) or gate.

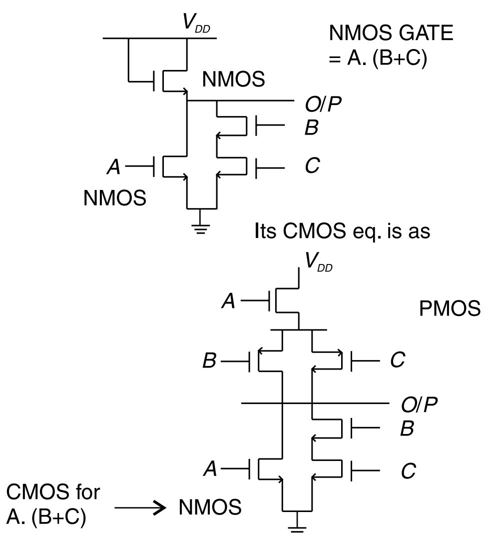
Hence, the correct option is (d).

17. The CMOS equivalent of the following nMOS gate (figure) is (draw the circuit).

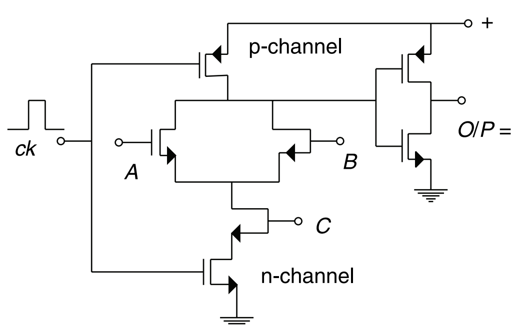


[1991]

Solution:

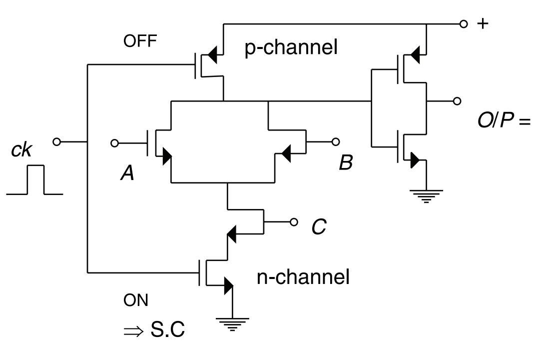


18. In the figure, the Boolean expression for the output in terms of inputs A, B and C when the clock 'ck' is high is given by



[1991]

Solution:

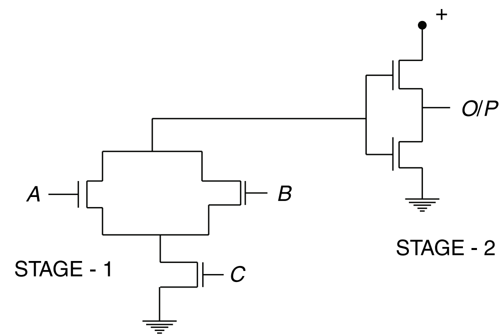


When clock is HIGH,

a) For N-channel MOS, It is ON

b) P-MOS-off

$\therefore$ Circuit can be redrawn as



From Stage 1: $2t$ is $(A + B)$. C and stage 2 is inverter (CMOS)

$\Rightarrow$ output = $(A + B)\,C$.

19. Among the digital IC-families ECL, TTL and CMOS:

(a) ECL has the least propagation delay

(b) TTL has the largest fan-out

(c) CMOS has the biggest noise margin

(d) TTL has the lowest power consumption

[1989]

Solution: (a)

In ECL TTL, CMOS

least prop delay: ECl is fastest logic family.

Hence, the correct option is (a).

20. A logic family has threshold voltage V_R = 2 V, minimum guaranteed output high voltage $V_{OH} = 4\ V$, minimum accepted input high voltage $V_{IH} = 3\ V$, maximum guaranteed output low voltage $V_{OL} = 1\ V$, and maximum accepted input low voltage $V_{IL} = 1.5\ V$. Its noise margin is

(a) $2\ V$ (b) $1\ V$

(c) $1.5\ V$ (d) $0.5\ V$ **[1989]**

Solution: (d)

$V_R = 2V$ (threshold)

$V_{OH} = 4V$

$V_{OL} = 1V$

$V_{IH} = 3V$

$V_{IL} = 1.5V$

Noise margin is calculated as $(NM)_4 = V_{OH} - V_{IH} = 4 - 3 = 1V$ (noise m is High level voltage)

$(NM)_L = NM$ in low voltage level $= V_{IL} - V_{OL} = 1.5 - 1 = 0.5V$

$NM = \min\,[NM_H - NM_L] = \min\,[1], 0.8] = 0.5$ V.

(NM represents the amplitude of noise voltage that may cause the logic level to change.)

Hence, the correct option is (d).

21. Fill in the blanks of the statements below concerning the following Logic Families:

Standard TTL (74 XXLL), low power TTL (74L XX), low power schottky

TTL (74L SXX), schottky TTL(74 SXX), emitter coupled logic (ECL), CMOS.

(a) Among the TTL Families, __________ family requires considerably less power than the standard TTL (74 XX) and also has comparable propagation delay.

(b) Only the __________ family can operate over a wide range of power supply voltages. **[1987]**

Solution:
TTL family list:

FAMILY	DELAY (ns)	POWER (mw)
1. Basic	10	10
2. Low-power (L)	35	1
3. SCHOTTKY(S)	3	18
4. Low-power Schottky (LS)	9	2
5. Advance scholtky (AS)	1.5	1Z
6. Adv. low-power schottky (ALS)	4	1

ECl: power dissipation $\simeq$ 25 mw

CMOS: 0.002 mv

Prop delay $\simeq$ 2ns

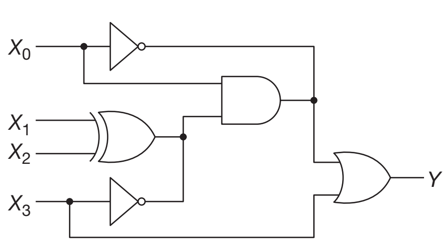
Now Low Power (LS) has delay comp. to basic & power less than it

(a) Low Power (LS) schottky

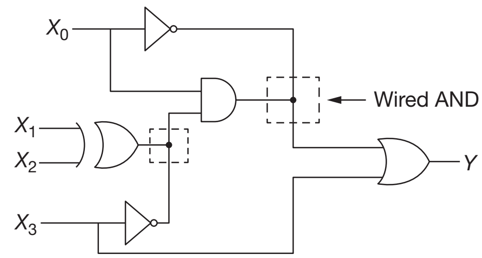
(b) CMOS

Two-marks Questions

1. The logic gates shown in the digital circuit below use strong pull-down nMOS transistors for LOW logic level at the outputs. When the pull-downs are off, high-value resistors set the output logic levels to HIGH (i.e. the pull-ups are weak). Note that some nodes are intentionally shorted to implement "wired logic". Such shorted nodes will be HIGH only if the outputs of all the gates whose outputs are shorted are HIGH. **[2018]**



Solution: Consider the circuit diagram given below



From the above figure the output will be

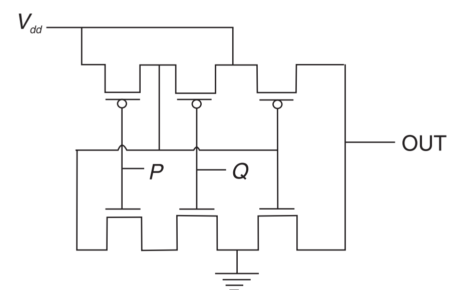
$$Y = \overline{X_0}\left[\left(X_1 \oplus X_2\right)\cdot X_0\right] + X_3$$

$$Y = X_3$$

Output Y will be high when Input X_3 is High. Therefore there are 8 such possible combinations.

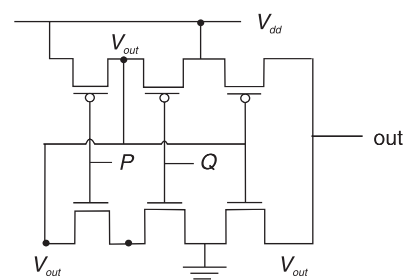
Hence, the correct answer is 8.

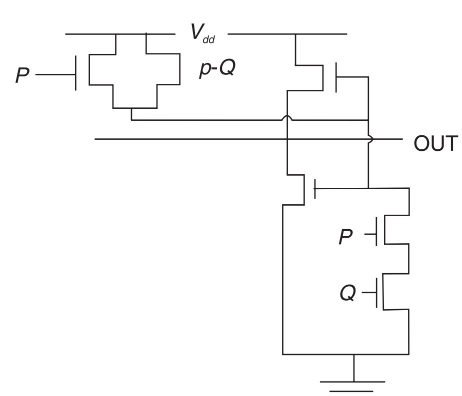
2. The logic function implemented by the following circuit at the terminal OUT is



(a) P NOR Q (b) P NAND Q
(c) P OR Q (d) P AND Q **[2008]**

Solution: (d)





From the circuit,

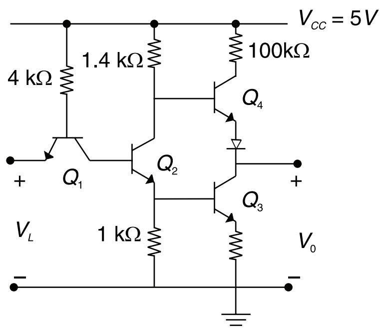
In NMOS $\rightarrow PQ \rightarrow$ series

$P \rightarrow P \| Q$

$\Rightarrow$ P AND Q.

Hence, the correct option is (d).

3. The circuit diagram of a standard TTLNOT gate is shown in the figure. When Vi = 2.5 V, the modes of operation of the transistors will be



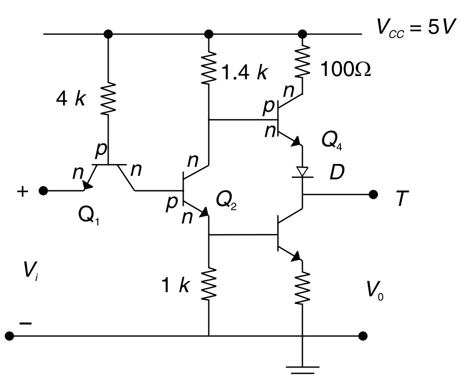
(a) $Q1$: reverse active;
$Q2$: normal active;
$Q3$: saturation;
$Q4$: cut-off

(b) $Q1$: reverse active
$Q2$: saturation;
$Q3$: saturation;
$Q4$: cut-off

(c) $Q1$: normal active;
$Q2$: cut-off;
$Q3$: cut-off;
$Q4$: saturation

(d) $Q1$: saturation;
$Q2$: saturation;
$Q3$: saturation;
$Q4$: normal active **[2007]**

Solution: (b)



Vi = 2.5 V (TTL NOT gate)

$Q1 : EB : RB$

$BC : FB$ Reverse Active

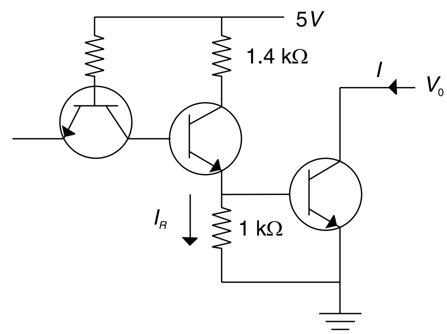
$Q2$: Active

$Q4$: Cutoff

$Q3$: Sat.

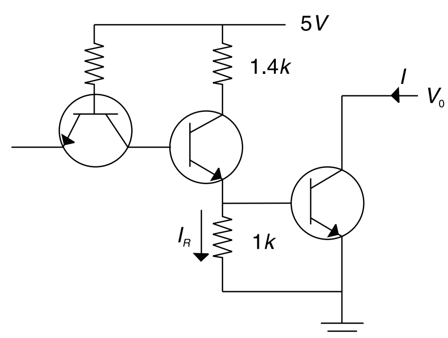
Hence, the correct option is (b)

4. The transistors used in a portion of the TTL gate shown in the figure have a β = 100. The base-emitter voltage is 0.7 V for a transistor in active region and 0.75 V for a transistor in saturation. If the sink current I = 1 mA and the output is at logic 0, then the current IR will be equal to



(a) 0.65 mA (b) 0.70 mA
(c) 0.75 mA (d) 1.00 mA **[2005]**

Solution: (c)



β = 100

VBE = 0.7 V (active region)

VBE sat = 0.75 V

Isat = 1 mA

Output = O(logic), IR = ?

Jsink = 1 mA = Ic sat

(i.e. BJT in sat mode)

VBE sat = 0.75

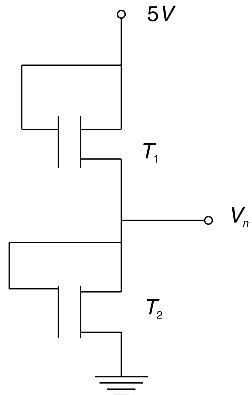
and 0.75 = JR. 1k

$$J_R = \frac{0.75}{1000} = 0.75 \text{ mA}$$

Hence, the correct option is (c).

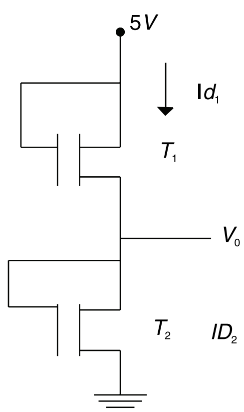
5. Both transistors T_1 and T_2 shown in the figure have a threshold voltage of 1 volts. The device parameters K_1

and K_2 of T_1 and T_2 are, respectively, 36 μA/V_2 and 9 μA/V_2. The output voltage V_0 is



(a) 1 V (b) 2 V
(c) 3 V (d) 4 V **[2005]**

Solution: (c)



$V_{th} = 1$ V
$K_1 = 36$ μA/V²
$K2 = 9$ μA/V²
$V_0 = ?$
As $I_{D1} = I_{D2}$ (sat mode)
$\Rightarrow K_1\,(Va^s_1 - V_t)^2 = K_2(Va^s_2 - V_t)^2$
or $\sqrt{k_1}\,|V_a s_1 - V_t| = \sqrt{k_2}\,|V_a s_2 - V_t|$
$Va^s_1 = 5 - V_0$, $Va^s_2 = (V_0 - 0)$gmd
$K_1(Va^s_1 - V_t)^2 = K_2\,(Va^s_2 - V_t)^2 \quad V_t = 1$
$\overset{4}{\cancel{36}}(5 - V_0 - 1)^2 = \cancel{9}(V_0 - 1)^2 \quad Va^s_1 = 5 - V_0$
$4(4 - V_0)2 = (V_0 - 1)2 \quad VG^s_2 = (V_0 - 0)$
$2(4 - V_0) = (V_0 - 1)$
$8 - 2V_0 = V_0 - 1$
$3V_0 = 9$
$V_0 = 3V.$
Hence, the correct option is (c).

6. The DTL, TTL, ECL and CMOS family GATE of digital ICs are compared in the following 4 columns

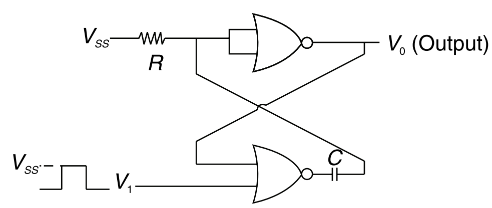
	(P)	(Q)	(R)	(S)
Fanout is minimum	DTL	DTL	TTL	CMOS
Power consumption is minimum	TTL	CMOS	ECL	DTL
Propagation delay is minimum	CMOS	ECL	TTL	TTL

The correct column is
(a) P (b) Q
(c) R (d) S **[2003]**

Solution: (b)

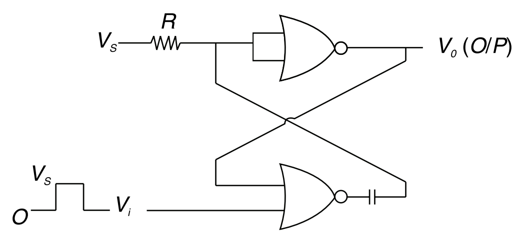
CMOS: Power Consumption is lowest
ECL: Program delay is minimum
$\Rightarrow$ (*Q*).
Hence, the correct option is (b).

7. The circuit in the figure has two CMOS NOR-gates. This circuit functions as a-



(a) flip-flop
(b) Schmidt trigger
(c) monostable multi-vibrator
(d) astable multi-vibrator **[2002]**

Solution: (c)



This circuit will act like a mono-stable multi-vibrator.)
Hence, the correct option is (c).

8. Given that for a logic family,
V_{OH} is the minimum output high-level voltage.
V_{OL} is the maximum output-low-level voltage.
V_{IH} is the minimum acceptable input high-level voltage and
V_{IL} is the maximum acceptable input low-level voltage.
The correct relationship is
(a) $V_{IH} > V_{0H} > V_{IL} > V_{OL}$
(b) $V_{OH} > V_{IH} > V_{IL} > V_{OL}$
(c) $V_{IH} > V_{OH} > V_{OL} > V_{IL}$
(d) $V_{OH} > V_{IH} > V_{OL} > V_{IL}$ **[1987]**

Solution: (b)

V_{OH}: (V HIGH) min value (2.4 V) min value V_{IH}: (V in high) min ($\simeq$ 2 V) min value.

V_{OL}: (Vo low) max (0.4V) max value V_{IL}: (V in low) max (0.8 V) max value

From general value: $V_{OH} > V_{IH} > V_{IL} > V_{OL}$.

Hence, the correct option is (b).

9. In the output stage of a standard TTL, we have a diode between the emitter of the pull-up transistor and the collector of the pull-down transistor. The purpose of this diode is to isolate the output node from the power supply Vcc. **[1994]**

Solution:

This is TRUE, to isolate output from power Vcc, a diode is attached bho E of pull-up and C of pull-down transfer (TRUE)

Chapter 7

Memories

One-mark Questions

1. In a DRAM, **[2017]**
(A) periodic refreshing is not required
(B) information is stored in a capacitor
(C) Information is stored in a latch
(D) Both read and write operation can be performed simultaneously

2. An 8085 microprocessor based system uses a 4 K × 8 bit RAM whose starting address is AA00 H. The address of the last byte in this RAM is
(a) 0FFFH (b) 1000H
(c) B9FFH (d) BA00H
[2001]

Solution: (d)

$4k \times 8$ bit RAM, starting add: AAOOH

8085 $\rightarrow$ 16 and lines total.

$4k \rightarrow 2^{12}$, 12 odd lines : $A_0 - A_{11}$

$4k = 4 \times 1024 = 4096$ bytes

$A_{15} - A_9$ are fixed (1010 101)

$= 1000\ H$

Starting add : AAOO

\+ number of bytes : 1000

Last bytes BA00H.

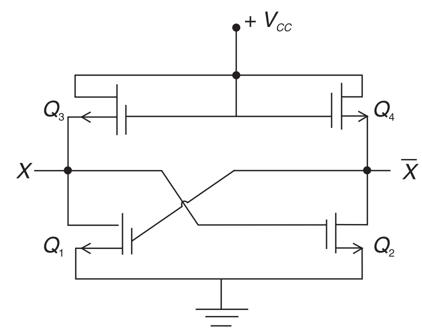
Hence, the correct option is (d)

3. Each cell of a static Random Access Memory contains
(a) 6 MOS transistors
(b) 4 MOS transistors and 2 capacitors
(c) 2 MOS transistors and 4 capacitors
(d) 1 MOS transistor and 1 capacitors **[1996]**

Solution:(a)

Each cell of a static RAM contains six MOS transistors. Each bit on static RAM is stored on four transistors out of which two are PMOS and two are NMOS, that from cross-coupled inverters. Remaining two transistors are used to control reading from or writing into cell.

SRAM Cell:



4-transistor for storing each bit on SRAM + 2 transistors for controlling read from or writing into cell.

Hence, the correct option is (a).

4. The minimum number of MOS transistors required to make a dynamic RAM cell is
(a) 1 (b) 2
(c) 3 (d) 4 **[1995]**

Solution: (b)

From 7.3, it can be seen that DRAM cell required one transistor.

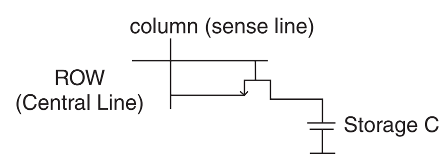
Hence, the correct option is (b).

5. A dynamic RAM consists of
(a) 6 transistors
(b) 2 transistors and 2 capacitors
(c) 1 transistor and 1 capacitor
(d) 2 capacitors only **[1994]**

Solution: (c)

A dynamic RAM consists of 1 transistor and 1 capacitor.

DRAM : A Dynamic RAM cell (MOS) is:



$\Rightarrow$ 1 transistor + 1 capacitor.

Hence, the correct option is (c).

6. A PLA can be
   (a) as a microprocessor
   (b) as a dynamic memory
   (c) to realize a sequential logic
   (d) to realize a combinational logic. **[1994]**

   Solution: (d)

   PLA is a type of fixed architecture logic devices with programmable AND gates followed by programmable OR gates. The PLA is used to implement complex combinational circuits.

   PLA $\rightarrow$ complex complain cpt as arrays of prog AND & OR.

   Hence, the correct option is (d).

7. Choose the correct statements (s) from the following:
   (a) PROM contains a programmable AND array and a fixed OR array.
   (b) PLA contains a fixed AND array and a programmable OR array.
   (c) PROM contains a fixed AND array and a programmable OR array
   (d) PLA contains a programmable AND array and a programmable OR array. **[1992]**

   Solution: (c) & (d)

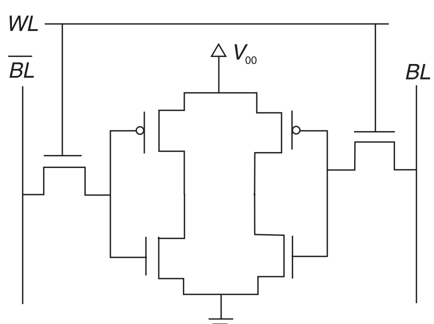
   (a) PROM $\rightarrow$ Prog. AND and fixed OR ✗ (has fusible diodes at each inter seem) $\Rightarrow$ Prog. OR
   (b) PLA $\rightarrow$ fixed AND, Prog. OR ✗ (Prog. AND & prog OR)
   (c) PROM $\rightarrow$ fixed AND, Prog. OR ✓
   (d) PLA $\rightarrow$ Prog. AND & OR ✓

   Hence, the correct option is (c) and (d).

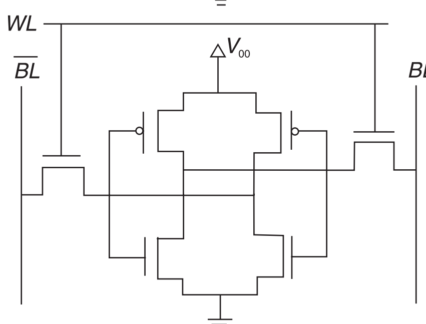
Two-marks Questions

1. If WL is the Word Line and BL the Bit Line, an SRAM cell is shown in

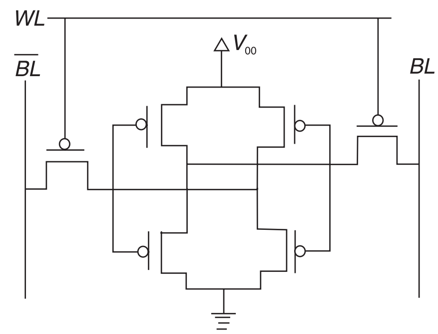
(a)



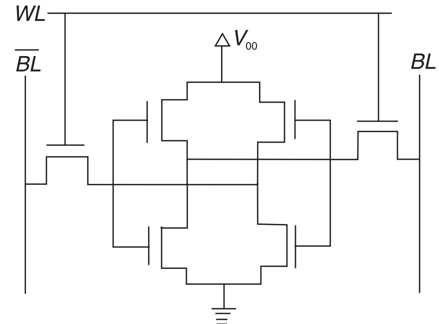
(b)



(c)



(d)



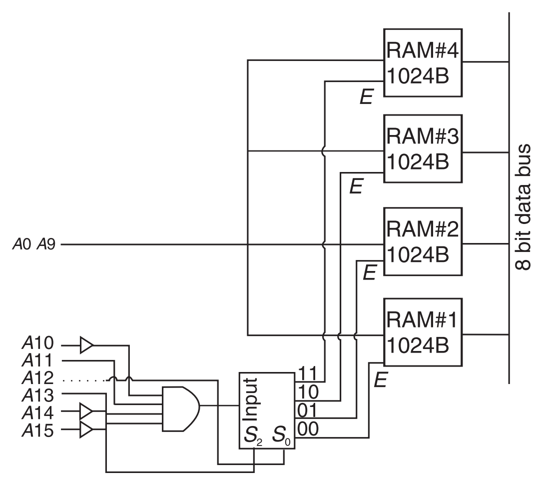
[2014]

Solution: (b)

For the construction of SRAM we require 4 MOSFET (2 PMOS and 2 NMOS) with interchanged outputs connected to each CMOS inverter, This condition is followed in option (b)

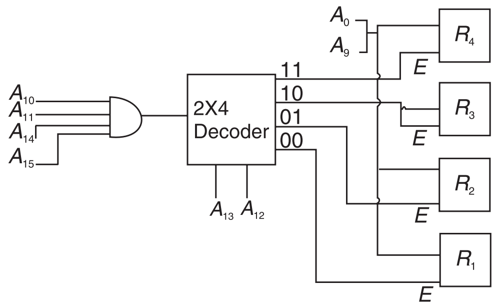
Hence, the correct option is (b).

2. There are four chips each of 1024 bytes connected to a 16 big address bus as shown in the figure below. RAMs 1, 2, 3 and 4 respectively are mapped to address



(a) 0C00H-0FFFH, 1C00H-1FFFH, 2C00H-2FFFH, 3C00H-3FFFH
(b) 1800H-1FFFH, 2800H-2FFFH, 3800H-3FFFH, 4800H-4FFFH
(c) 0500H-08FFH, 1500H-18FFH, 3500H-38FFH, 5500H-58FFH
(d) 0800H-0BFFH, 1800H-1BFFH, 2800H-2BFFH, 3800H-3BFFH **[2013]**

Solution: (d)



$S_1 = A_{13}$

$S_0 = A_{12}$

$A_{10} - A_{15} \to$ decoder input

$A_0 - A_9 \to$ add lines for RAM

$\Rightarrow A_{15}\, A_{14} = 00$

$A_{11}\, A_{10} = 10$

(a) $A_{12}\, A_{13} = 00 \Rightarrow S_1 S_{12} = 00 \to R_1$ selected.

$A_{15}A_{14}A_{13}A_{12}$ $A_{11}A_{10}A_9 \ldots\ldots\ldots\ldots A_0$

0 0 0 0 1 0 0 0

1 0 11 1

(R1) i.e. 0800H – 08FF H

(b) $A_13A_{12} = 01$ $(S_1S_2 = 01) \to R_2$

A_{15} A_{14} A_{13} A_{12} A_{11} A_{10} A_9 ...... A_0

0 0 0 1 0 0 0

1 1.........1

$R_2 \Rightarrow$ 1800A – 18FF

(c) R_3 : $A_{13}\, A_{12} = 10 \Rightarrow A_{15}\, A_{14}\, A_{13}\, A_{12}\, A_{11}\, A_{10} =$

0010 10 0 0

1 1

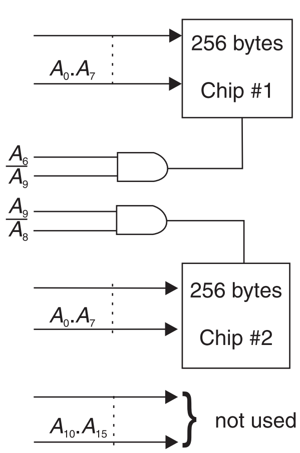
$\Rightarrow$ 2800H – 2BFFH R_3

(d) $R_4 : A_{13}A_{12} = 1 \Rightarrow$ 0011 100 0

0011 101 1 $\Rightarrow$ 3800H – 3 BFFH]

Hence, the correct option is (b).

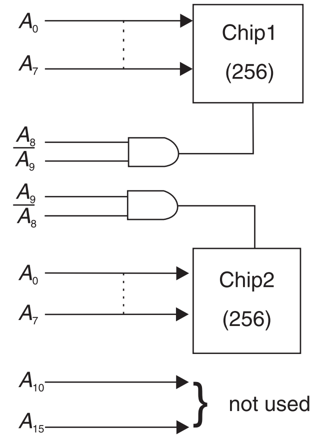
3. What memory address range is NOT represented by chip 1 and chip 2 in the figure. A_0 to A_{15} in this figure are the address lines and CS means Chip select.



(a) 0100 – 02 FF (b) 1500 – 16 FF

(c) F900 – FAFF (d) F800 – F9FF **[2005]**

Solution: (d)



For chip 1 : $A_9A_8 = 01$

$A_7 - A_0$ = OO – FF

$\Rightarrow A_9 - A_0$: 01 0000 0000 to

01 FFFF FFFF

For chip 2, $CS = A_9\overline{A_8}$

$\Rightarrow A_9\, A_8 = 10$

$\Rightarrow A_9 - A_0 = 10$ 0000 0000 to

10 FFFF FFFF

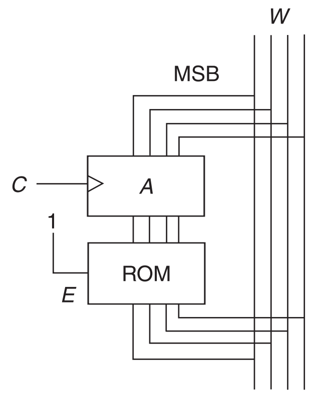
0 0000 0000 to 10 FFFF FFFF add are converted by chip 1 and chip 2

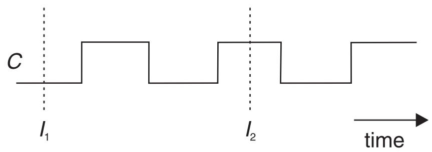
$\therefore$ F800 – F9FF are not represented.

Hence, the correct option is (d).

4. In the circuit shown in the figure, A is parallel-in, parallel-out 4 bit register, which loads at the rising edge of the clock C. The input lines are connected to a 4 bit bus, W. Its output acts as the input to a 16 × 4 ROM whose output is floating when the enable input E is 0. A partial table of the contents of the ROM is as follows

Address	Data
0	0011
2	1111
4	0100
6	1010
8	1011
10	1000
11	0010
14	1000

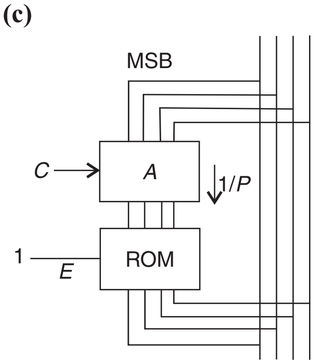




The clock to the register is shown, and the data on the W bus at time t_1 is 0110. The data on the bus at time t_2 is

(a) 1111 (b) 1011
(c) 1000 (d) 0010 **[2003]**

Solution: (c)



ROM Add:		Data
0		0011
2		1111
4		0100
(6)	→	1010
8		1011
(10)	→	1000
11		0010
14		1000

At t_1, $W = 0110$

∴ Input to ROM at $t_1 = 0110 =$ add. 6

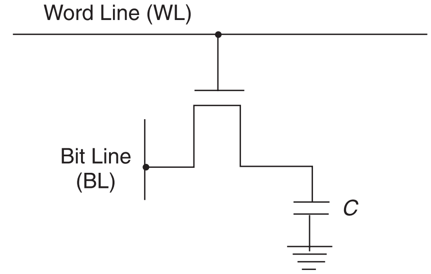
∴ at add 6, data = 1010 (w)

At t_2, W becomes 1010, ∴ A gets input 1010 and supplies to ROM.

∴ At address 1010 (= 10), data = 1000.

Hence, the correct option is (c).

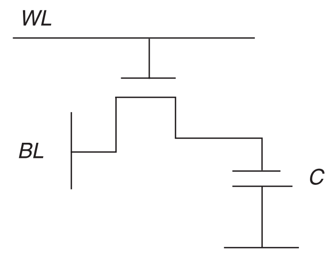
5. In the DRAM cell in the figure, the V_t of the NMOSFET is 1 V. For the following three combinations of WL and BL voltages.



(a) 5 V; 3 V; 7 V (b) 4 V; 3 V; 4 V
(c) 5 V; 5 V; 5 V (d) 4 V; 4 V; 4 V

[2001]

Solution:(b)



$Vt = 1V$ (VMOS)

Hence, the correct option is (b).

6. If CS = $\overline{A}_{15}\ A_{14}\ A_{13}$ is used as the chip select logic of a 4K RAM in an 8085 system, then its memory range will be

(a) 3000 H – 3 FFF H
(b) 7000 H – 7 FFF H
(c) 5000 H – 5 FFF H and 6000 H – 6 FFF H
(d) 6000 H – 6 FFF H and 7000 H – 7 FFF H

[1999]

Solution:(d)

$\overline{CS} = A_{15}A_{14}A_{13}$ of $4k$ RAM (8085)

$4k \Rightarrow 212$, i.e. 12 add lines $A_0 - A_{11}$

and $\overline{CS} = 011$

$$
\begin{array}{cccccccl}
\Rightarrow & A_{15} & A_{14} & A_{13} & A_{12} & A_{11} & & \ldots\ldots\ldots\ldots\ldots A_0 \\
 & 0 & 1 & 1 & 0 & \rightarrow 0 & & \ldots\ldots\ldots\ldots\ldots 0 \\
 & & & & & \searrow 1 & & \ldots\ldots\ldots\ldots\ldots 1 \\
 & & & & 1 & \rightarrow 0 & & \ldots\ldots\ldots\ldots\ldots 0 \\
 & & & & & \searrow 1 & & \ldots\ldots\ldots\ldots\ldots 1
\end{array}
$$

$A_{12} = \Rightarrow A_{15}A_{14}A_{13}A_{12} = 0110 = 6$

$\Rightarrow$ Range: 6000 – 6FFF

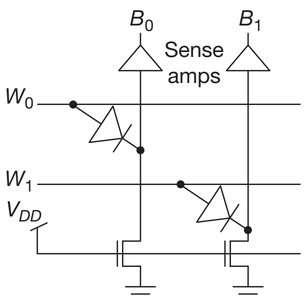
$A_{12} = 1 \quad \Rightarrow A_{15}A_{14}A_{13}A_{12} = 0111 = 7 \Rightarrow 7000 – 7FFF$

$\Rightarrow$ Total range: 6000 – 6FFF and 7000 – 7FFF.

Hence, the correct option is (d).

Two-mark Questions

1. A 2 × 2 ROM array is built with the help of diodes as shown in the circuit below. Here WO and W1 are signals that select the word lines and B0 and B1 are signals that are output of the sense amps based on the stored data corresponding to the bit lines during the read operation. **[2018]**



$$\begin{array}{c c} & \begin{matrix} B_0 & B_1 \end{matrix} \\ \begin{matrix} W_0 \\ W_1 \end{matrix} & \begin{bmatrix} D_{00} & D_{01} \\ D_{10} & D_{11} \end{bmatrix} \end{array}$$

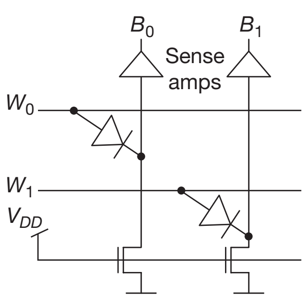
Bits stored in the ROM array

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

During the real operation, the selected word line goes high and the other word line is in a high impedance state. As per the implementation shown in the circuit diagram above, what are the bits corresponding to D_{ij} (where $I = 0$ or 1 and $j = 0$ or 1) stored in the ROM?

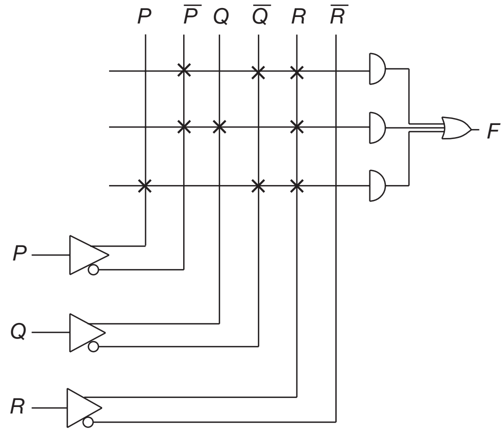
Solution: Consider the figure



Bit stored in ROM array is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Hence, the correct option is (A).

2. A programmable logic array (PLA) is shown in the figure.



The Boolean function F implemented is **[2017]**

(A) $\overline{P}\,\overline{Q}R + \overline{P}QR + P\overline{Q}\,\overline{R}$

(B) $\left(\overline{P}+\overline{Q}+R\right)\left(\overline{P}+Q+R\right)\left(P+\overline{Q}+\overline{R}\right)$

(C) $\overline{P}\,\overline{Q}R + \overline{P}QR + P\overline{Q}R$

(D) $\left(\overline{P}+\overline{Q}+R\right)\left(\overline{P}+Q+R\right)\left(P+\overline{Q}+R\right)$

3. Consider a discrete memory less source with alphabet $S = \{s_0, s_1, s_2, s_3, s_4, \ldots.\}$ and respective probabilities of occurrence $P = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots\right\}$. The entropy of the source (in bits) is _____. **[2016]**

Solution: The entropy of discrete memory less source is given as

$$H = P_1 \log_2 \frac{1}{P_1} + P_2 \log_2 \frac{1}{P_2} + \ldots.$$

$$H = \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \frac{1}{8}\log_2 8 + \ldots.$$

$$= \frac{1}{2} + \frac{1}{4} \times 2 + \frac{1}{8}.3 + \frac{1}{16}\pi{\cdot}4 \ldots.$$

$$= \frac{1}{2} + \frac{1}{2^2}2 + \frac{1}{2^3}3 + \frac{1}{2^4}4 \ldots.$$

$$= \sum^{\alpha} n\left(\frac{1}{2}\right)^n \Rightarrow \text{AGP}$$

$n = 0$

$ab, (b + d)br, (a + 2d)br^2$

$a = 1, b = 1/2, d = 1, r = 1/2$

$$s = \frac{ab}{1-r} + \frac{dbr}{(1-r)^2}$$

$$\Rightarrow \frac{1\times\frac{1}{2}}{1-\frac{1}{2}} + = \frac{1\times\frac{1}{2}\times\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2} = 2.$$

Hence, the correct option is (2).

Chapter 8

ADC and DAC

One-mark Questions

1. A digital system is required to amplify a binary-encoded audio signal. The user should be able to control the gain of the amplifier from a minimum to a maximum in 100 increments. The minimum number of bits required to encode, in straight binary, is

(a) 8 (b) 6
(c) 5 (d) 7 **[2004]**

Solution: (d)

Step size = 100

$\Rightarrow$ 100 different voltages can be represented

$\Rightarrow$ If n is the number of bits, $2n \geq 100$

$64 \geq 100 \Rightarrow \min n = 7$

Hence, the correct option is (d).

2. The minimum number of comparators required to build an 8-bit flash ADC is

(a) 8 (b) 63
(c) 255 (d) 256 **[2003]**

Solution: (c)

$$\text{8-bit flash type} \Rightarrow 2^n - 1 = 2^8 - 1$$
$$= 256 - 1$$
$$\text{255 comparators.}$$

Hence, the correct option is (c).

3. The number of comparators required in a 3-bit comparator type ADC is

(a) 2 (b) 3
(c) 7 (d) 8 **[2002]**

Solution: (c)

3-bit comparator type: $(2^n - 1)$ comparators

$$: 2^3 - 1$$
$$: 7.$$

Hence, the correct option is (c).

4. An 8 bit successive approximation analogue to digital converter has full scale reading of 2.55 V and its conversion time for an analogue input of 1 V is 20 µs. The conversion time for a 2 V input will be

(a) 10 µs (b) 20 µs
(c) 40 µs (d) 50 µs **[2000]**

Solution: (b)

Full scale $V = 2.55\ V$

Conversion time for $1V = 20$ µs

Conversion time for $2V = 20$ µs (same as it depends on the number of bits only).

Hence, the correct option is (b).

5. The number of comparators in a 4-bit flash ADC is

(a) 4 (b) 5
(c) 15 (d) 16 **[2000]**

Solution: (c)

Flash ADC:

For n output bits, $(2^n - 1)$ comparators required.

$\Rightarrow$ For 4-bits, $(2^n - 1) = 16 - 1$

$= 15$ comparator required.

Hence, the correct option is (c).

6. The resolution of a 4-bit counting ADC is 0 5 volts. For an analogue input of 6.6 volts, the digital output of the ADC will be

(a) 1011 (b) 1101
(c) 1100 (d) 1110 **[1999]**

Solution:(d)

Resolution = 0.5 (4-bit)

$V_{in} = 6.6\text{V}$

V_0 of ADC = input V/Resolution (conv in binary)

$$= \frac{6.6}{015} = 13.2 \simeq 14 \text{ (rounding of to higher digit)}$$

$\Rightarrow 14 = 8 + 4 + 2 = (1110)2.$

Hence, the correct option is (d).

7. The advantage of using a dual slope ADC in a digital voltmeter is that

(a) its conversion time is small
(b) its accuracy is high
(c) it gives output in BCD format
(d) it does not require a comparator **[1998]**

Solution: (b)

Dual-slope ADC : high accuracy.

The advantage of using a dual slope ADC in a digital voltmeter is its high accuracy.

Hence, the correct option is (b).

8. For an ADC, match the following: if

List-I

(A) Flash converter
(B) Dual slope converter
(C) Successive Approximation Converter

List-II

(1) requires a conversion time of the order of a few seconds
(2) requires a digital-to-analogue converter
(3) minimizes the effect of power supply interference.
(4) requires a very complex hardware
(5) is a tracking A/D converter. **[1995]**

Solution:

(a) Flash → req. comparative and 0 coding circuit ⇒ (4) complex H/w
(b) Dual-slope: minimum effect of power supply (3)
(c) Successive approx: $2n$ turn req. DAC for A to D conversion (2)

(a) – 4, (b) – 3, (c) – 2

9. (a) Successive approximation

(b) Dual-slope

(b) Parallel comparator

Maximum conversion time for 8 bit ADC in clock cycles

(1) 1	(2) 2
(3) 16	(4) 256
(5) 512	**[1994]**

Solution: (a)

Successive approximation: $T_{max} = nT_{clk}$

n = number of bits = 8 T_{clk}

(b) Dual-slope: $2^{n^H}\, T_{clk} = 2^9 T_{clk}$

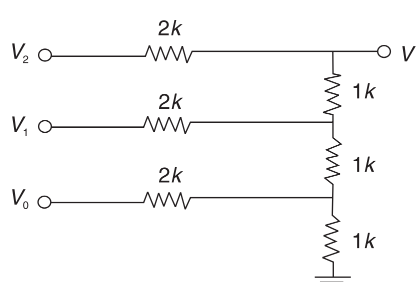
(c) Parallel comparator = 572 T_{clk}

⇒ $= 1T_{clk}$.

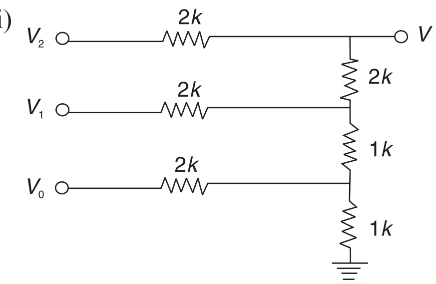
Hence, the correct option is (a).

10. Which of the resistance networks of figure can be used as 3 bit R-2R ladder DAC. Assume V_0 corresponds to LSB.

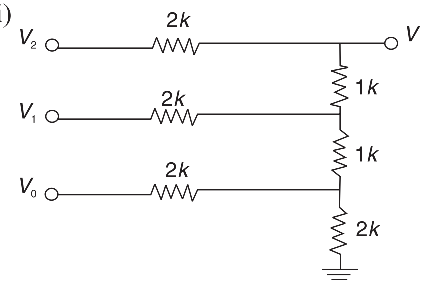
(i)



(ii)



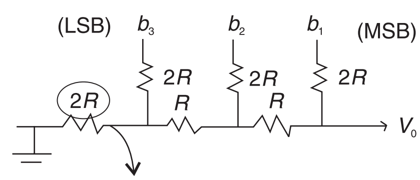
(iii)



(a) Both (i) and (ii)	(b) Both (i) and (ii)
(c) Only (iii)	(d) Only (ii) **[1990]**

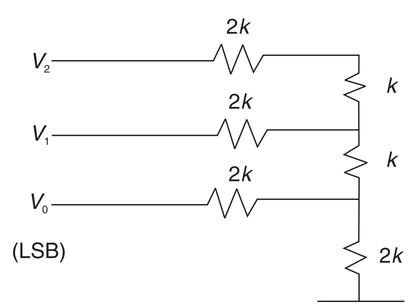
Solution: (c)

3 bit $R - 2R$ ladder – $R22R$ are used general circuit:



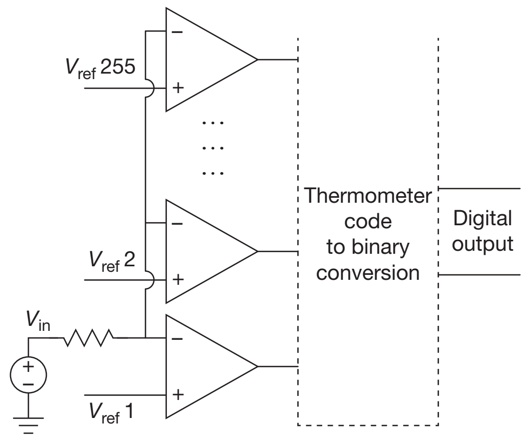
i.e., with LSB, $2R$ is used (with gmd)

⇒ (c)



Two-marks Questions

1. In an N bit flash ADC, the analog voltage is fed simultaneously to $2^N - 1$ comparators. The output of the comparators is then encoded to a binary format using digital circuits. Assume that the analog voltage source V_{in} (whose output is being converted to digital format) has a source resistance of 75 Ω as shown in the circuit diagram below and the input capacitance of each comparator is 8 pF. The input must settle to an accuracy of $\frac{1}{2}$ LSB even for a full scale input change for proper conversion. Assume that the time taken by the thermometer to binary encoder is negligible. **[2016]**



If the flash ADC has 8 bit resolution, which one of the following alternatives is closest to the maximum sampling rate?

(A) 1 megasamples per second
(B) 6 megasamples per second
(C) 64 megasamples per second
(D) 256 megasamples per second

Solution: Given that accuracy= 0.5LSB for N bit flash counter even for Full scale voltage, then using Nyquist theorem where *Fs* is greater and equal to 2 Fm. So for $V_{in} = 2^N - 1$,

Hence, the correct option is (A).

2. Consider a four bit *D* to *A* converter. The analog value corresponding to digital signals of values 0000 and 0001 are 0 V and 0.0625 V respectively. The analog value (in Volts) corresponding to the digital signal 1111 is ______. **[2015]**

Solution: 4-bit A/D converter

For input 0000 → output Voltage = 0 V

For input 0001 → output Voltage = 0.0625 V

$V_0 = K$(input in decimal)

$$0.0625 = K(1) \Rightarrow K = 0.0625$$

For input 1111 = $(15)_{10}$

$$V_0 = 0.0625 \times 15 = 0.9375$$

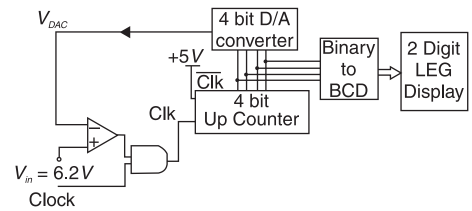
Hence, the correct Answer is (0.93 to 0.94).

Common Data for Questions 1 and 2

In the following circuit, the comparator output is logic '1' if $V_1 > V_2$ and is logic '0' otherwise. The D/A conversion is done as per the relation

$V_{DAC} = \sum_{n=0}^{3} 2^{n-1} b_n$ volts, where b_3 (MSB), b_2, b_1 and b_0 (LSB) are the counter outputs.

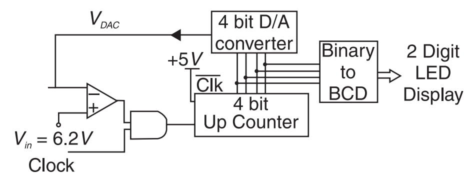
The counter starts from the clear state



3. The stable reading of the LED displays is

(a) 06 (b) 07
(c) 12 (d) 13 **[2008]**

Solution: (d)



$$V_{DAC} = \sum_{n=0}^{3} 2^{n-1} b_n$$

b_3 = MSB

b_0 = LSB Counter is cleared initially

*Counter will count, till clk (V_{in} AND comp. output) is high, i.e.,

$V_{DAC} < V_+$

$(V - 1)$ (62)

b_3	b_2	b_1	b_0	V_{DAC}
0	0	0	0	0
0	0	0	1	0.5
0	0	1	0	1.5
0	0	1	1	
0	1	0	0	
1	1	0	1	6.5
1	1	1	0	7

At this point, $V_{DAC} > V_+ \Rightarrow$ counter stops

∴ Corresponding stable reading is $6.5 \times t_c$

$= 6.5 \times 2$

$= 13.$

Hence, the correct option is (d).

4. The magnitude of the error between V_{DAC} and V_{in} at steady state in volts is

(a) 0.2 (b) 0.3
(c) 0.5 (d) 1.0 **[2008]**

Solution: (b)

Now V_{DAC} at stable state = 6.5

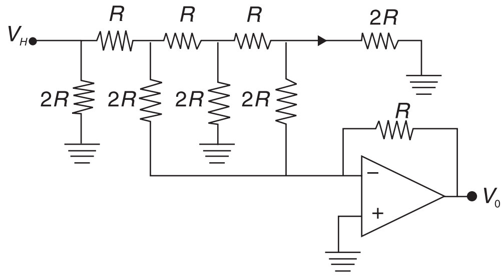
and $V_{in} = 6.2$

⇒ error = 6.5 – 6.2 = 0.3.

Hence, the correct option is (b).

Statement for Linked Answer Questions 5 and 6:

In the digital-to-analogue converter circuit shown in the figure below, $V_R = 10$ V and $R = 10$ kΩ.

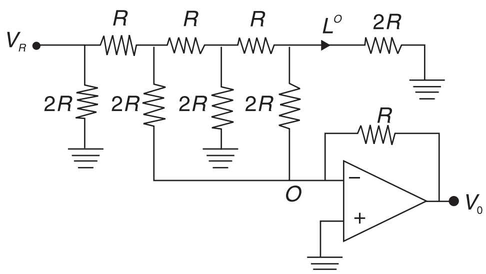


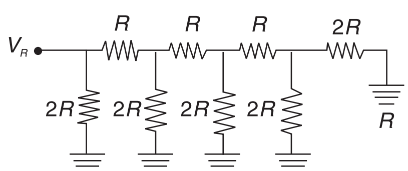
5. The current I is

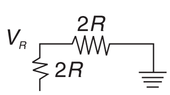
(a) 31.25 μA (b) 62.5 μA
(c) 125 μA (d) 250 μA **[2007]**

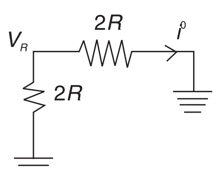
Solution: (b)

$VR = 10$V, $R = 10$ kΩ $V+ = 0$, $V- = 0$

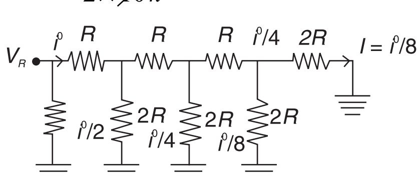








$$\Rightarrow\ i = \frac{V_R}{2R} = \frac{\cancel{10}}{2\times \cancel{10}k} = 0.5 \text{ mA}$$



$$I = \frac{i}{2\times 2\times 2} = \frac{0.5}{8} = 6.25\,\mu A$$

Hence, the correct option is (b).

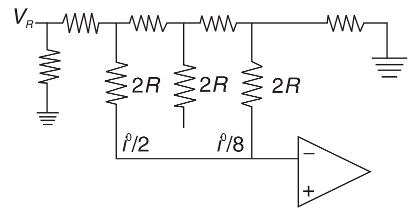
6. The voltage V_0 is

(a) − 0.781 V (b) − 1.562 V
(c) − 3.125 V (d) − 6.250 V

[2007]

Solution: (c)

Inverting terminal:



$$\text{Inverting} = \frac{i}{2} + \frac{i}{8} = \frac{5i}{8} = \frac{5\times 0.5}{8}\text{mA}$$

$= 3.12.5$ μA

Now $V_0 = -R \times I_{inv}$

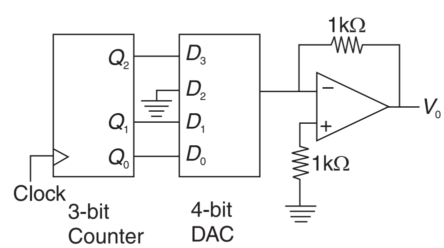
$= -\ 10k \times 3.125$ μA

$= -\ 3.12.5 \times 104 \times 10^{-6}$

$V_0 = -\ 3.125$ V.

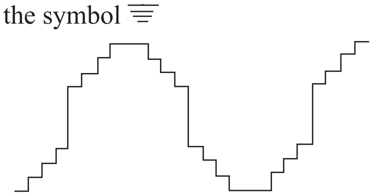
Hence, the correct option is (c).

7. A 4-bit D/A converter is connected to a free-running 3-bit UP counter, as shown in the following figure. Which of the following waveforms will be observed at V_0?

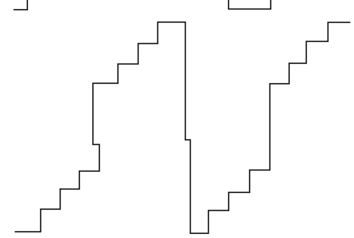


In the figure shown above, the ground has been shown by the symbol

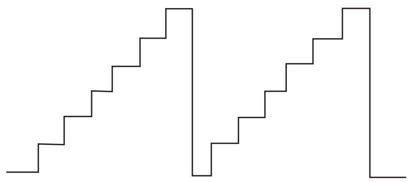
(a)



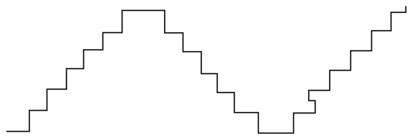
(b)



(c)

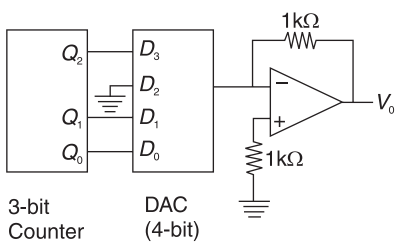


(d)



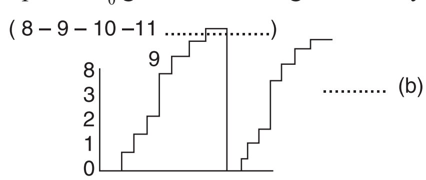
[2006]

Solution: (b)

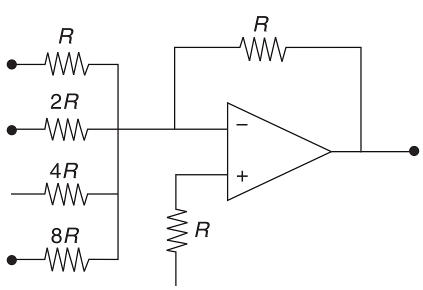


AK	Counter			DAC				
	θ_2	θ_1	θ_1	D_3	D_2	D_1	D_0	
1	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	1	1
3	0	1	0	0	0	1	0	2
4	0	1	1	0	0	1	1	3
5	1	0	0	1	0	0	0	8
6	1	0	1	1	0	0	1	9
7	1	1	0	1	0	1	0	10
8	1	1	1	1	0	1	1	11

Now due to comparative: V_0 will rise till 4th pulse, by on 5th pulse, V_0 goes to 8 then again rises by 1 step.



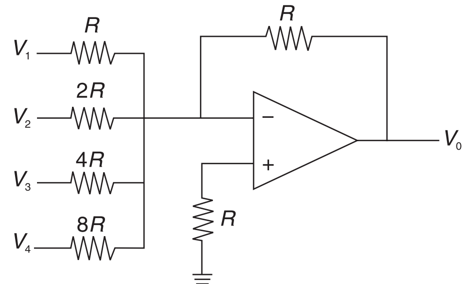
8. The circuit shown in the figure is a 4-bit DAC



The input bits 0 and 1 are represented by 0 and 5 V, respectively. The OP AMP is ideal, but all the resistances and the 5 V inputs have a tolerance of ± 10%. The specification (rounded to the nearest multiple of 5%) for the tolerance of the DAC is

(a) ±35% (b) ±20%
(c) ±10% (d) ±5% **[2003]**

Solution: (a)



(4 bit DAC)

$5V$ ilp

$(0 \to 0)$

$(1 \to 5\text{V})$

$$V_0 = \left[\frac{v_1}{1R} + \frac{V_2}{2R} + \frac{V_3}{4R} + \frac{V4}{8R}\right]^R$$

$$= -\left(V_1 + \frac{V_2}{2} + \frac{V_3}{4} + \frac{V_4}{8}\right)$$

$5V$ input, $R \to 10\%$ tolerance

$\Rightarrow V_0 = -5\,[1 + 0.5 + 0.25 + 0.125] = -9.375$

$(V_{0\,\max}$ due to tolerance)

Tolerance of DAC:

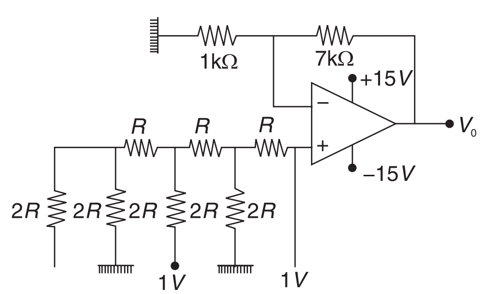
$$V_D = \frac{-110}{90} \times \left(5.5 + \frac{5.5}{2} + \frac{5.5}{4} + \frac{5.5}{8}\right)$$

$$= \frac{-11}{9} \times 5.5 = -12.604$$

$T = 34.44\ \Omega\ 350/0.$

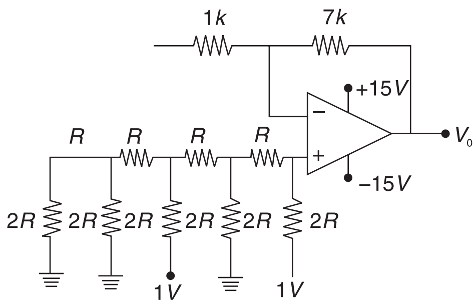
Hence, the correct option is (a).

9. For the 4 bit DAC shown in the figure, the output voltage V0 is

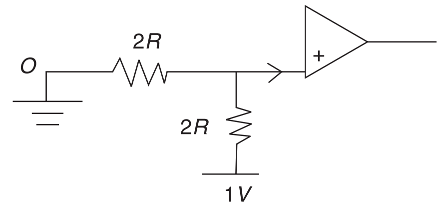


(a) 10 V (b) 5 V
(c) 4 V (d) 8 V **[2000]**

Solution: (b)



At non-inv terminal:



$$\left(1+\frac{7\cancel{k}}{1\cancel{k}}\right) = 8 \Rightarrow V_0 = \left(0+\frac{1}{8}+\frac{1}{2}+0\right)\times 8 = \frac{5}{\cancel{8}}\times\cancel{8}$$

$V^0 = 5V$.

Hence, the correct option is (b).

UNIT VII

MICROPROCESSOR

EXAM ANALYSIS

Exam Year	92	93	94	95	96	97	98	99	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14-1	14-2	14-3	14-4	15			16			17		18	19
																											Set 1	Set 2	Set 3	Set 1	Set 2	Set 3	Set 1	Set 2		
1 Marks Ques.	2	2	-	2	2	3	2	-	2	-	-	-	-	-	-	-	-	1	1	-	-	1	-	-	-	-	-	2	-	2	3	2	2	3	3	3
2 Marks Ques.	-	-	-	-	-	-	-	-	1	-	1	1	4	2	1	3	1	-	1	1	-	-	-	1	-	1	1	2	1	1	1	3	4	2	2	-
5 Marks Ques.	-	1	1	-	1	-	-	-	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Total Marks	**2**	**7**	**5**	**2**	**7**	**3**	**2**	**-**	**4**	**-**	**2**	**2**	**8**	**4**	**2**	**6**	**2**	**1**	**3**	**2**	**-**	**1**	**-**	**2**	**-**	**2**	**2**	**6**	**2**	**4**	**5**	**8**	**10**	**7**	**7**	**3**
Chapter wise marks distribution																																				
Basics of 8085	-	1	-	1	-	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		-			-		-		-	-
Instructions of 8085	1	-	1	1	-	-	-	2	-	-	2	6	4	-	-	2	-	2	2	-	-	-	-	-	-	-		2			2		1		1	-
Memory Interfacing	-	-	-	-	1	-	-	-	-	-	-	-	-	-	2	-	-	1	-	-	-	2	2	1	-	-		-			1		-		-	-
8085 Interfacing	-	-	-	-	-	-	-	-	-	-	-	2	-	-	-	-	-	-	-	-	-	-	1	-	1	-		-			-		-		-	-
8085 Interrupts	-	-	-	-	-	1	-	2	-	-	-	-	-	-	-	-	-	1	-	-	-	-	-	-	-	-		-			-		-		-	-
8085 Programming	-	-	-	1	1	-	-	-	-	-	-	-	2	4	-	-	-	-	-		1	1	2	-	1	-		1			-		1		1	-

Chapter 1

Basics of 8085

One-mark Questions

1. An I/O processor controls the flow of information between
(a) cache memory and I/O devices
(b) main memory and I/O devices
(c) two I/O devices
(d) cache and main memories **[1998]**

Solution: (b)

An I/O processor controls the flow of information between main memory and I/O devices.

Hence, the correct option is (b).

2. An instruction used to set the carry flag in a computer can be classified as
(a) data transfer (b) arithmetic
(c) logical (d) program control **[1998]**

Solution: (c)

An instruction used to set the carry flag in a computer can be classified as logical instruction.
Hence, the correct option is (c).

3. An 'assembler' for a microprocessor is used for
(a) assembly of processors in a production line.
(b) creation of new program using different modules.
(c) translation of a program from assembly language to machine language.
(d) translation of a higher level language into English text. **[1995]**

Solution: (c)

An assembler for a microprocessor is used for the translation of a program from assembly language to machine language.

Hence, the correct option is (c).

4. In a microcomputer, wait states are used to
(a) make the processor wait during a DMA operation.
(b) make the processor wait during an interrupt processing.
(c) make the processor wait during a power shutdown.
(d) interface slow peripherals to the processor. **[1993]**

Solution: (d)

In a microcomputer, wait states are used to interface slow peripherals to the processor.

Hence, the correct option is (d).

5. In register index addressing mode, the effective address is given by
(a) the index register value.
(b) the sum of the index register value and the operand.
(c) the operand.
(d) the difference of the index register value and the operand. **[1988]**

Solution: (a)

In register index addressing mode, the effective address is given by the register index value.

Hence, the correct option is (a).

Two-mark Questions

1. In an 8085 system, a PUSH operation requires more clock cycles than a POP operation. Which one of the following options is the correct reason for this? **[2016]**
(A) For POP, the data transceivers remain in the same direction as for instruction fetch (memory to processor), whereas for PUSH their direction has to be reversed.
(B) Memory write operations are slower than memory read operations in an 8085 based system.
(C) The stack pointer needs to be pre decremented before writing registers in a PUSH, where as a POP operation uses the address already in the stack pointer.
(D) Order of registers has to be interchanged for a PUSH operation, whereas POP uses their natural order.

Solution: For push operation

T_1	T_2	T_3
6 clock cycles	3 clock cycles	3 clock cycles
Fetch and Decode	Write	Write

Total 6 + 3 + 3 = 12 clk cycles.

For pop operation

T_1	T_2	T_3
4 clock cycles	3 clock cycles	3 clock cycles
Fetch and Decode	read	read

For a PUSH operation, more cycles are required because after fetching and decoding (4), data will be written onto the stack with 3 clock each (of write operation), resulting in (4 + 2 + 3 + 3 = 12 clock) while in POP, operations are fetch, decoding and read. But here fetch + decode takes only 4 clocks resulting in (4 + 3 + 3 = 10 clock) so Push takes more clock cycles to execute the POP.

Hence, the correct option is (C).

2. A 16 Kb (= 16,384 bit) memory array is designed as a square with an aspect ratio of one (number of rows is equal to the number of columns). The minimum number of address lines needed for the row decoder is ______. **[2015]**

Solution: 16K bit memory array is to be designed with an aspect ratio of one

16 K bit = $2^4 \cdot 2^{10} = 2^{14} = 2^7 \cdot 2^7 = 2^7 \times 128$

So no. of address lines = 7,

So no. of outputs of decoder = $2^7 = 128$

no. of data lines = 128

Hence, the correct Answer is (7).

Chapter 2

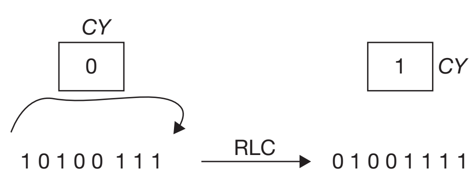
Instructions of 8085 Microprocessor

One-mark Questions

1. In an 8085 microprocessor, the contents of the accumulator and the carry flag are A7 (in hex) are 0, respectively. If the instruction RLC is executed, then the contents of the accumulator (in hex) and the carry flag, respectively, will be **[2016]**

(A) 4E and 0 (B) 4E and 1
(C) 4F and 0 (D) 4F and 1

Solution: Accumulator = A7H, CY = 0



After RLC (circular left shift without carry),

$$\text{Accumulator} = 4F_H$$

$$\text{CY flag} = 1$$

Hence, the correct option is (D).

2. The total number of memory accesses involved (inclusive of the op-code fetch) when an 8085 processor executes the instruction LDA 2003 is

(a) 1 (b) 2
(c) 3 (d) 4 **[1996]**

Solution: (d)

Instruction LDA 2003 requires four machine cycles.

Machine cycle 1 $\rightarrow$ op-code fetch (LDA)

Machine cycle 2 $\rightarrow$ memory read (03)

Machine cycle 3 $\rightarrow$ memory read (20)

Machine cycle 4 $\rightarrow$ memory read [(2003)] to accumulator

Hence, the correct option is (d).

3. When a CPU is interrupted, it

(a) stops execution of instructions.
(b) acknowledges interrupt and branches to a sub-routine.
(c) acknowledges interrupt and continues.
(d) acknowledges interrupt and waits for the next instruction from the interrupting device. **[1995]**

Solution: (d)

CPU here is a microprocessor. The options are concerned with INTR only as remaining interrupts do not require interrupt acknowledgment.

Hence, the correct option is (d).

4. In a microprocessor, the register that holds the address of the next instruction to be fetched is

(a) accumulator
(b) program counter
(c) stack pointer
(d) instruction register **[1993]**

Solution: (b)

Program counter stores the address of the next instruction to be executed.

Hence, the correct option is (b).

5. In a microprocessor system, the stack is a used for:

(a) storing the program return address whenever a sub-routine jump instruction is executed
(b) transmitting and receiving input–output data.
(c) storing all important CPU register contents whenever an interrupt is to be serviced.
(d) storing program instructions for interrupt service routines. **[1989]**

Solution: (a)

Stack is a temporary set of memory locations in the main memory. These memory locations are used to store the binary information temporarily during the execution of a program. Whenever a sub-routine jump instruction (CALL) is executed, the stack is used for storing the program return address.

Hence, the correct option is (a).

Two-marks Questions

1. In an 8085 microprocessor, the shift register which stores the result of an addition and the overflow bit are, respectively **[2015]**

(A) B and F (B) A and F
(C) H and F (D) A and C

Solution: Addition will be performed with respect to accumulator, and result will be stored in accumulator.

ADD B,this instruction will add register B with accumulator and store result in accumulator.

As per the result, the flags will be affected in flag register. So overflow bit is stored in flag register.

Hence, the correct option is (B).

2. In an 8085 microprocessor, which one of the following instructions changes the content of the accumulator? **[2015]**

(A) MOV B, M
(B) PCHL
(C) RNZ
(D) SBI BEH

Solution: In 8085 Microprocessor

MOV B, M → move contents of Memory location to B

PCHL → copy the contents of HL to PC.

RNZ → return to main program on No Zero result

SBI BEH → Subtract BEH from Accumulator, and store result in Accumulator.

Hence, the correct option is (D).

3. An 8085 assembly language program is given below. Assume that the carry flag is initially unset. The content of the accumulator after the execution of the program is

MVI	A.07H
RLC	
MO V	B, A
RLC	
RLC	
ADD	B
RRC	

(a) 8CH (b) 64H
(c) 23H (d) 15H **[2011]**

Solution: (c)

MVI A, 07 H → A = 07

RLC → Rotate accumulator 1 bit left without carry

C 00000111

A = 00001110 = OE H

MOV B, A B = OEH

RLC 00001110 = 00011100

RLC 00011100 = 00111000

A = 38H

ADD B → A + B

00111000

<u>00001110</u>

01000110

RRC → Rotate acc right 1 bit without carry

A = 01000110 = 00100011

A = 23H

Hence, the correct option is (c).

4. For the 8085 assembly language program given below, the content of the accumulator after the execution of the program is

300	MVIA, 45
3002	MOV B, A
3003	STC
3004	CMC
3005	RAR
3006	XRA B

(a) 00H (b) 45H
(c) 67H (d) E7H **[2010]**

Solution: (c)

MVI A, 45 → A ← 45H

MOV B, A → B ← 45H

TC → CY = 1

CMC → CY = 0

RAR → Rotate accumulate 1 bit right through carry

carry

0 0100 010 1

A = 001000101

XRA B → A Ex or with B

(00100010) ⊕ (01000101)

= 01100111

A = 67H

Hence, the correct option is (c).

5. An 8085 executes the following instructions

2710 LXI H, 30A0H

2713 DAD H

2714PCHL

All addresses and constants are in Hex. Let PC be the contents of the program counter and HL be the contents of the HL register pair just after executing PCHL. Which of the following statements is correct?

(a) PC = 2715H HL = 30A0H
(b) PC = 30A0H HL = 2715H
(c) PC = 6140H HL = 6140H
(d) PC = 6140H HL = 2715H **[2008]**

Solution: (c)

LXI H, 30 40H → HL = 30A 0H

DAD H → HL ← HL + HL

```
 30 A0
 30 A0
 -----
 61 40H
```

PCHL → PC ← HL

Thus, PC = 6140 H HL = 6140H

Hence, the correct option is (c).

6. If in addition following code exists from 0109H onwards,

ORI 40 H

ADDM,

what will be the result in the accumulator after the last instruction is executed?

(a) 40H (b) 20H
(c) 60H (d) 42H **[2005]**

Solution: (c)

At 0109, A → 00H

```
ORI 40H → A ←  01000000
               00000000
          A ←  01000000 (40H)
```

Add M

M → HL → 20H

A ← A + M ← 20 + 40

A ← 60H

Hence, the correct option is (c).

7. The following program starts at location 0100H.

LXISP, 00FF

LXIH, 0107

MVI A, 20H

SUB M

The content of accumulator when the program counter reaches 0109H is

(a) 20H (b) 02H
(c) 00H (d) FFH **[2005]**

Solution: (c)

```
0100 LXI  SP,  00FF
     0103 LXI  H, 0107
0106 MVI  A,   20H
                  → 0107 → 20H
0108 SUB  M    → A ←A – M
0109
```

M contains HL pair address data

H1 → 0107

At HL 0107, operand 20H is stored thus

A ← 20H – 20H

A ← 00H

Hence, the correct option is (c).

8. It is desired to multiply the numbers OAH by OBH and store the result in the accumulator. The numbers are available in registers B and C, respectively. A part of the 8085 program for this purpose is given below:

MVI A, 00H

Loop:-----------

HLTEND

The sequence of instructions to complete the program would be

(a) JNZ LOOP, ADD B, DCRC
(b) ADD B, JNZ LOOP, DCR C
(c) DCR C, JNZ LOOP, ADD B
(d) ADD B, DCR C, JNZ LOOP **[2004]**

Solution: (d)

ADD B, DCR C, JNZ LOOP

MVIA, 00	A = 00
LOOP ADD B	$A = A + B$
DCRC	C = C - 1 Loop will till C = 0.
JNZLOOP	So, loop will execute 11 times (OB).

Common data for Questions 10 and 11.Consider an 8085 microprocessor system

Hence, the correct option is (d).

9. Consider the sequence of 8085 instructions given below.

LXI H, 9258H, MOV A, M,

Which one of the following is performed by this sequence?

(a) Contents of location 9258 are moved to the accumulator.
(b) Contents of location 9258 are compared with the contents of the accumulator.
(c) Contents of location 9258 are complemented and stored in location 9258.
(d) Contents of location 5892 are complemented and stored in location 589 **[2004]**

Solution: (c)

LXI H, 9258 $\rightarrow$ HL $\leftarrow$ 9258H

MOV A, M $\rightarrow$ A $\leftarrow$ contents of add. 9258

CMA $\rightarrow$ complement accumulator

MOV M $\rightarrow$ complement of A is stored in M (9258 H)

Hence, the correct option is (c).

10. The number of memory cycles required to execute the following 8085 instructions:

(I) LDA 3000H
(II) LXI D, F0F1H

would be:

(a) 2 for (I) and 2 for (II)
(b) 4 for (I) and 3 for (II)
(c) 3 for (I) and 3 for (II)
(d) 3 for (I) and 4 for (II) **[2004]**

Solution: **(b)**

Memory cycles

LDA3000H $\rightarrow$ $\text{Fetch}, \underbrace{\text{Read}, \text{Read},}_{\text{address}} \underbrace{\text{Read}}_{\text{data}}$

LXI D, F0F1H $\rightarrow$ Fetch, Read, Read

Hence, the correct option is (b).

11. In an 8085 microprocessor, the instruction CMP *B* has been executed while the content of the accumulator is less than that of register *B*. As a result

(a) carry flag will be set but zero flag will be reset.
(b) carry flag will be reset but zero flag will be set
(c) both carry flag and zero flag will be reset.
(d) both carry flag and zero flag will be set. **[2003]**

Solution: (a)

CMP B $\rightarrow$ contents of B and A are compared and the result is indicated by flag.

$A < B$. $\therefore$ CY = 1, $Z = 0$.

Hence, the correct option is (a).

12. The contents of register (B) and accumulator (A) of 8085 microprocessor are 49 H and 3 AH, respectively. The contents of A and the status of carry flag (CY) and sign flag (S) after executing SUB B instructions are

(a) A = F1, CY =1, S = 1
(b) A = 0F, CY =1, S = 1
(c) A = F0, CY = 0, S = 0
(d) A = 1F, CY = 1, S = 1 **[2000]**

Solution: (a)

A $\rightarrow$ 3 AH $\rightarrow$ 00111010

B $\rightarrow$ 49H $\rightarrow$ 01001001

SUBA $\rightarrow$ 11110001

CY = 1, S = 1, A = F1

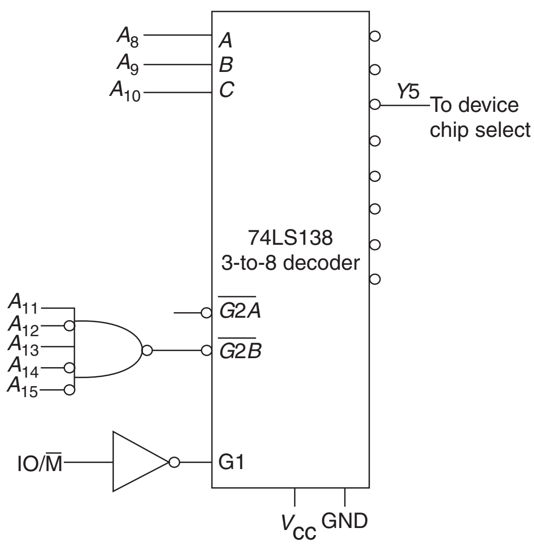
Hence, the correct option is (a).

Chapter 3

Memory Interfacing

One-mark Questions

1. In the circuit shown below, the device connected to Y5 can have address in the range



(a) 2000 - 20 FF (b) 2D00 - 2DFF
(c) 2E00 - 2EFF (d) FD00 - FDFF

[2010]

Solution: (b)

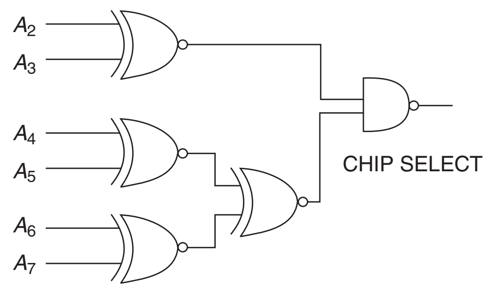
To connect Y5, input CBA should be 101. Possible range will be

A_{15}	A_{14}	A_{13}	A_{12}	A_{11}	A_{10}	A_9	A_8	A_7	A_0
0	0	.1	0	1	1	0	1	0	... 0
0	0	1	0	1	1	0	1	1	1

Therefore, the device can have address in the range 2000 - 2DFF.

Hence, the correct option is (b).

2. The decoding circuit shown in the below-given figure has been used to generate the active low chip select signal for a microprocessor peripheral. (The address lines are designated as A0 to A7 for IO addresses.) The peripheral will correspond to IO addresses in the range:



(a) 60 H to 63H (b) A4H to A7H
(c) 50 H to AFH (d) 70H to 73H

[1997]

Solution: (a)

For active low chip signal, NAND gate output should be '0'.

NAND gate output will be '0', when all the inputs are '1'. So, possible combinations given in options are

A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0	
0	1	1	0	0	0	0	0	60 H to 63 H
0	1	1	0	0	0	1	1	
1	0	1	0	0	1	0	0	A4 H to A7 H
1	0	1	0	0	1	1	1	
0	1	0	1	0	0	0	0	50 H to AF H
1	0	1	0	1	1	1	1	
0	1	1	1	0	0	0	0	70 H to 73 H
0	1	1	1	0	0	1	1	

A_2 and A_3 should be the same. So, only options (a), (c) and (d) satisfy this.

Option (c) cannot be correct, because all chip select signals are not the same.

Option (d) cannot be correct because it gives logic '1' output. So, the correct option is (a).

Hence, the correct option is (a).

3. In an 8085 microprocessor system with memory mapped I/O,

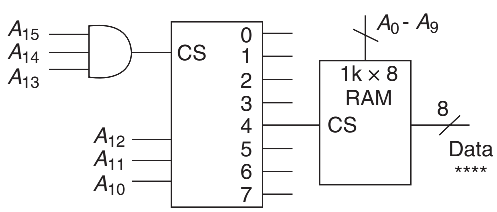
(a) I/O devices have 16-bit addresses.
(b) I/O devices are accessed using IN and OUT instructions.
(c) there can be a maximum of 256 input devices and 256 output devices.
(d) arithmetic and logic operations can be directly performed with the I/O data. **[1992]**

Solution: (a) and (d)

In memory mapped I/O, the I/O devices are also treated as memory locations. Under assumption, they will be given 16-bit address. Microprocessor uses related instructions to communicate with I/O devices. Arithmetic and logic operations can be directly performed with the I/O data.

Hence, the correct option is (a and d).

4. An 8-bit microprocessor has 16-bit address bus $A_0 - A_{15}$. The processor has a 1 kB byte memory chip as shown. The address range for the chip is



(a) F00FH TO F40EH
(b) F100 TO F4FFH
(c) F000H TO F3FFH
(d) F700H TO FAFFH **[1988]**

Solution: (c)

1 kB $\rightarrow A_9 \rightarrow A_0$; 2^{10} = 1 kB = 1024 B

A_{15}	A_{14}	A_{13}	A_{12}	A_{11}	A_{10}	A_9	A_8	A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0
1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
F				0				0				0			
1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1
F				3				F				F			

∴ Range is F000H – F3FFH.

Since $A_{12}\,A_{11}\,A_{10} \rightarrow 100$ such that output 4 is selected.

Hence, the correct option is (c).

5. A microprocessor with a 16-bit address bus is used in a linear memory selection configuration (i.e., address bus lines are directly used as chip selects of memory chips) with 4 memory chips. The maximum addressable memory space is

(a) 64 k (b) 16 k
(c) 8 k (d) 4 k **[1988]**

Solution: (b)

Among 16 address lines, as there are 4 chips, $A_{15}\,A_{14}\,A_{13}\,A_{12}$ are used as chip select lines. So, remaining are 12.

$\therefore 2^{12}$ = 4 kB; for each chip 4 × 4 kB = 16 kB.

Hence, the correct option is (b).

Two-marks Questions

1. An 8*k* byte ROM with an active low Chip Select input $(\overline{CS})$ is to be used in an 8085 microprocessor based system. The ROM should occupy the address range 1000H to 2FFFH. The address lines are designated as A_{15} to A_0, where A_{15} is the most significant address bit. Which one of the following logic expressions will generate the correct $\overline{CS}$ signal for this ROM? **[2016]**

(A) $A_{15} + A_{14} + (A_{13}\,A_{12} + \overline{A_{13}}\;\overline{A_{12}})$

(B) $A_{15}\,A_{14}\,(A_{13} + A_{12})$

(C) $\overline{A_{15}}\;\overline{A_{14}}\;(A_{13}\overline{A_{12}} + \overline{A_{13}}A_{12})$

(D) $\overline{A_{15}} + \overline{A_{14}} + A_{13}\,A_{12}$

Solution: For this ROM to generate the $\overline{CS}$ signal, the input at $\overline{CS}$ pin must be 0. The capacity of ROM is

$$8k \text{ Byte ROM} = 8k \times 8 = 2^{13} \times 8$$

13 Address lines and 8 data lines.

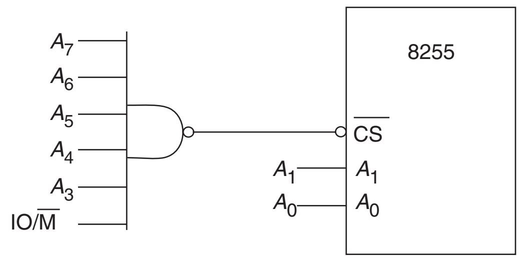
A_{15}	A_{14}	A_{13}	A_{12}	A_{11}	A_{10}	A_9	A_8	A_7	A_6	–	–	–	A_0	
0	0	0	1	0	0	0	0	0	0	–	–	–	0	⇒ 1000H
0	0	1	0	1	1	1	1	1	1	–	–	–	1	⇒ 2FFFH

By using higher order bits $(\overline{CS})$ chip select input has to be connected.

For above range $\overline{CS} = 0 = A_{15} + A_{14} + (A_{13}\;A_{12} + \overline{A_{13}}\,.\overline{A_{12}})$

Hence, the correct option is (A).

2. An 8255 chip is interfaced to an 8085 microprocessor system as an I/O mapped I/O as shown in the figure. The address lines A_0 and A_1 of the 8085 are used by the 8255 chip to decode internally its three ports and the control register. The address lines A_3 to A_7 as well as the IO/ $\overline{\text{M}}$ signal are used for address decoding. The range of addresses for which the 8255 chip would get selected is



(a) F8H–FBH (b) F8H–FCH
(c) F8H–FFH (d) F0H–F7H

[2007]

Solution: (c)

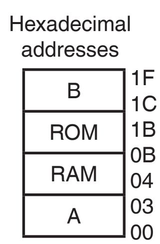
O/P of NAND gate is 0 if A_7 to A_3 & IO / $\overline{M} = i$

	A_7	A_6	A_5	A_3	A_2	A_1	A_0	
Starting address	1	1	1	1	1	0	0	0 → F8H
Final address	1	1	1	1	1	1	1	1 → FFH

Hence, the correct option is (c).

Five-marks Questions

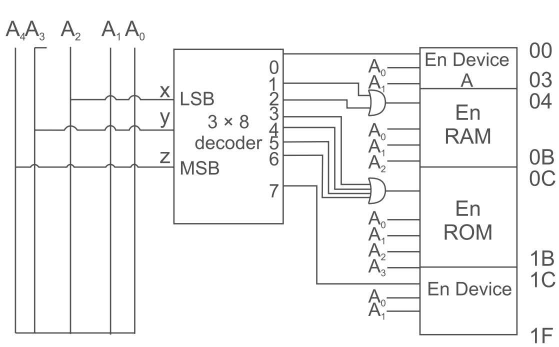
1. A microprocessor has five address lines [A_0- A_4] and eight data lines [D_0 - D_7]. An input device A, an output device B, a ROM and a RAM are memory mapped to the microprocessor at the addresses as shown in figure. Devices A and B have four addressable registers each; RAM has 8 bytes and ROM has 16 bytes. (a) Indicate the address lines to be connected to each device and memory.



[1993]

Solution:

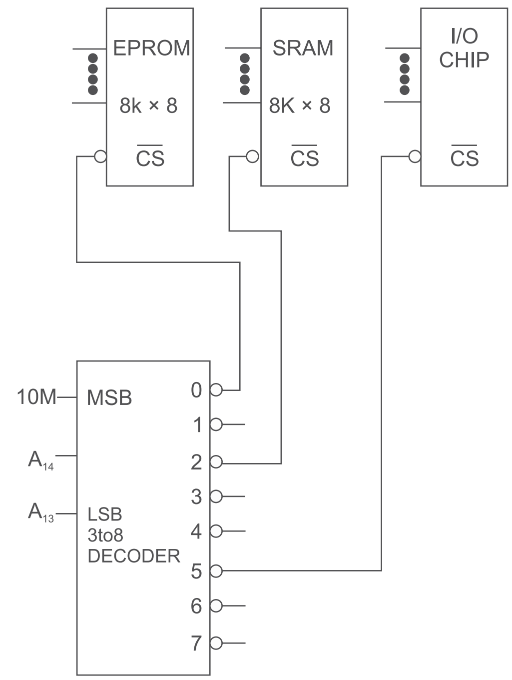
Device A has 2 add line required. Device B requires 2 address lines. ROM requires 3 add lines. RAM require 4 address lines



2. Consider the decoder circuit shown in figure for providing chip select signals to an EPROM, a RAM and an I/O chip with four addressable registers from a de multiplexed 8085 address bus.

(i) Specify all the memory address ranges to which the EPROM will respond.

(ii) Specify all the memory address ranges to which the RAM will respond.



Solution:

A15 = 0	A15 = 1	A_{15}	A_{14}	A_{13}	A_{12}	A_{11}	A_{10}	A_9	A_8	A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0
0000 H to 1FFF H	8000 H to 9FFF H	X X	X X	0 0	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1
4000 H to 5FFF H	C000 H to DFFF H	X X	1 1	0 0	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1	0 1

EPROM address range: 0000H to 1FFFH
or
8000H to 9FFFH

SRAM address range: 4000H to 5FFFH
or
C000H to DFFFH

Chapter 4

Microprocessor 8085 Interfacing

One-mark Question

1. For a microprocessor system using I/O-mapped I/O, the following statement(s) is not true.
   (a) Memory space available is greater.
   (b) Not all data transfer instructions are available.
   (c) I/O and memory address spaces are distinct.
   (d) I/O address space is greater. **[1988]**

   Solution: (d)

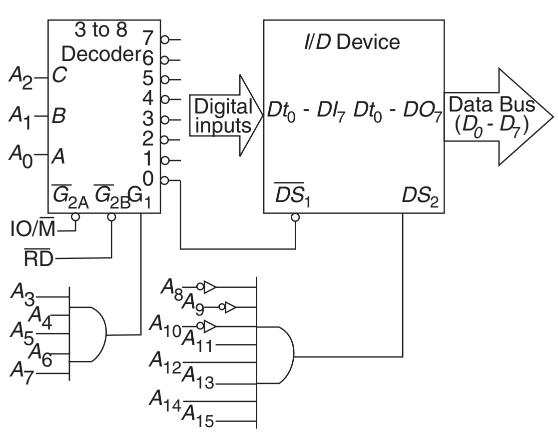
   Number of I/O devices = 256

   Memory capacity = 64 kB

   Hence, the correct option is (d).

Two-marks Questions

1. For the 8085 microprocessor, the interfacing circuit to input 8-bit digital data (DI_0 - DI_7) from an external device is shown in the figure. The instruction for correct data transfer is



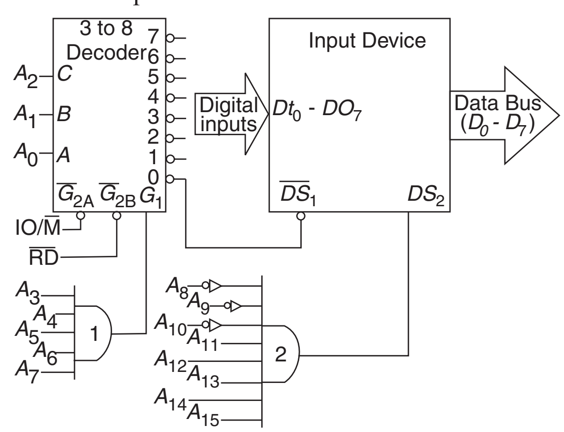
   (a) MVI A, F8H (b) IN F8H
   (c) OUTF8H (d) LDAF8F8H

   [2014]

   Solution: (d)

   To transfer, the data should be present in accumulator and proper instruction should be executed.

   - For the proper working and gates 1 and 2 should produce output as 1.



   - Since $\overline{DS}_1$ is connected to output 0 of decoder, so input selection should be 000. Therefore, address lines have inputs as

A_{15}	A_{14}	A_{13}	A_{12}	A_{11}	A_{10}	A_9	A_8	A_7	A_6	A_5	A_4	A_3	A_2	A_1	A_0
1	1	1	1	1	0	0	0	1	1	1	1	1	0	0	0

   That is, accumulator should be fed with the contents of address F8F8H, i.e., LDA F8F8H.

   Hence, the correct option is (d).

2. The 8255 programmable peripheral interface is used as described below.
   (I) An A/D converter is interfaced to a microprocessor through an 8255. The conversion is initiated by a signal from the 8255 on Port C. A signal on Port C causes data to be strorbed into Port A.

(II) Two computers exchange data using a pair of 8255s. Port A works as a bidirectional data port supported by appropriate handshaking signals.

The appropriate modes of operation of the 8255 for (I) and (II) would be
(a) Mode 0 for (I) and Mode 1 for (II)
(b) Mode 1 for (I) and Mode 0 for (II)
(c) Mode 2 for (I) and Mode 0 for (II)
(d) Mode 2 for (I) and Mode 1 for (II) **[2004]**

Solution: (c)

Options are incorrect since port A can be operated as bidirectional port only in mode-2. (b) can be correct if it is mode 1 for (I) and mode 2 for (II).

Hence, the correct option is (c).

Five-marks Questions

1. Write down the sequence of Instructions which are actually (till a HLT instruction), if the program begins with the location 1FF5H.

Address (HEX)	8085 Instructions
1FF5	XRA A
1FF6	LXI H.2000H
1FF9	PCHL
1FFA	HLT
1FFB	LXI H, 2001H
1FFE	ANI00H
2000	LXI H, FFFFH
2003	INXH
2004	JZ2100H
2007	HLT
2100	LXI H, 1 FFFH
2103	MOV A, M
2104	INR A
2105	HLT

[1994]

2. The program given below is run on an 8085/based microcomputer system. Determine the contents of the registers:

PC, SP, B, C, H, L after a halt instruction is executed.

LOC

2000 START LXI SP.1000H
LXI H, 2F37H
XRA A
MOV A, H
INX H
PUSH H
CZ 20FFH i
JMP 3000H
HLT
20FF ADD H
RZ
POP B
PUSHB
RM
3000 HLT **[1991]**

Solution:

2000:LXI SP_1, 1000H → SP ← 1000H
2003:LXI H, 2F37H → HL ← 2F37H
2006:XRA A → A ←00H, Z21
2007:MOV A, H → A ← 2F
2008:IMX H → HL ← 2F38H
2009:Push H → SP ← OFFE
200A:CZ20FFH → As Z = 1, SP ← 0FFC
→ PC ← 20FF
due to call on zero
20FF:ADD, H → A + M → A → 2F + 2F → 5
2100:RZ → Z = 0
2101:POP B → SP ← 0FFE and BC ← 200D
2103:RM → If S = 1 in previous addition
3000:HLT → But S = 0, so condition false

After execution
PC = 3000
SP = 0FFE
B = 20
C = OD
H = 2F
L = 38

Chapter 5

Microprocessor 8085 Interrupts

One-mark Questions

1. In a microprocessor, the service routine for a certain interrupt starts from a fixed location of memory which cannot be externally set, but the interrupt can be delayed or rejected. Such an interrupt is
(a) non-maskable and non-vectored
(b) maskable and non-vectored
(c) non-maskable and vectored
(d) maskable and vectored **[2009]**

Solution: (d)

Interrupt which has a fixed address location is said to be vectored and which can be delayed or rejected is known as maskable.

Hence, the correct option is (d).

2. The number of hardware interrupts (which require an external signal to interrupt) present in an 8085 microprocessor is
(a) 1 (b) 4
(c) 5 (d) 13 **[2000]**

Solution: (c)

1. TRAP
2. RST 7.5
3. RST 6.5
4. RST 5.5
5. INTR

Hence, the correct option is (c).

3. In the 8085 microprocessor, the RST6 instruction transfers the program execution to the following location:
(a) 30H (b) 24H
(c) 48H (d) 60H **[2000]**

Solution: (a)

Hexadecimal of $(6 \times 8 = 48)$ is 30H.

Hence, the correct option is (a).

4. In an 8085 μP system, the RST instruction will cause an interrupt
(a) only if an interrupt service routine is not being executed
(b) only if a bit in the interrupt mask is made 0
(c) only if interrupts have been enabled by an El instruction
(d) None of the above **[1997]**

Solution: (c)

RST instruction will cause an interrupt only if interrupts have been enabled by an El instruction at the beginning of program.

Hence, the correct option is (c).

Chapter 6

Microprocessor 8085 Programming

One-mark Questions

1. The clock frequency of an 8085 microprocessor is 5 MHz. If the time required to execute an instruction is 1.4 µs, then the number of *T*-states needed for executing the instruction is **[2017]**

(A) 1 (B) 6
(C) 7 (D) 8

Solution: $f_{\mu p} = 5$ MHz $T = \dfrac{1}{f_{\mu p}} = 0.2$ µsec

$1T \rightarrow 0.2$ µsec

to complete an instruction it took 1.4 µsec, therefore by linear relation

$T_{execution} = 1.4\ \mu$

No. of T states = $T/T_{execution} = 1.4/0.2 = 7$

Hence, the correct option is (C).

2. For 8085 microprocessor, the following program is executed.

```
    MVI A, 05H;
    MVI B, 05H;
PTR: ADD B;
    DCR B;
JNZ PTR;
ADI 03H;
HLT.
```

At the end of program, accumulator contains

(a) 17H (b) 20H
(c) 23H (d) 05H **[2013]**

Solution: (a)

MVI A, 05H $\rightarrow$ A = 05H

MVI B, 05H $\rightarrow$ B = 05H

PTR: ADD B $\rightarrow A \leftarrow A + B$

DCR B

JNZ PTR Loop will execute 5 times till value of B becomes 0

$A = \underline{A + B} = \underline{05 + 05 + 04 + 03 + 02 + 01 + 00}$

In loop due to DCR B value of B is decreased

$A = 14$ H

A01 03H $A = A + 03$

$A = 17$H.

Hence, the correct option is (a).

3. The following instructions have been executed by an 8085 µP:

ADDRESS (HEX)	INSTRUCTION
6010	LXI H, 8A79H
46013	MOVA,L
6014	ADDH
6015	DAA
6016	MOVH,A
6017	PCHL

From which address will the next instruction be fetched?

(a) 6019
(b) 0379
(c) 6979
(d) None of the above **[1997]**

Solution: (c)

6010 LXI H, 8A79H $\rightarrow$ HL $\leftarrow$ 8A79H

6013 MOV A, L $\rightarrow$ A = 79

6014 ADD H $\rightarrow$ A $\leftarrow$ A + H A $\leftarrow$ 8A + 79

$$\begin{array}{r} 10001010 \\ 01111001 \\ \hline 1>00000011 \\ \hline \end{array}$$

A = 03H CY = 1 AC = 1

6015 DAA Decimal adjust after accumulator

Since CY = 1, AC = 1

A = 03 + 66 = 69H

6016 MOV H, A → H = 69H

6017 PCHL → Exchange contents of PC & HL

HL = 6979H

Thus, PC becomes 6979H.

Thus, next instruction would be fetched from 6979H.

Hence, the correct option is (c).

4. The following sequence of instructions is executed by an 8085 microprocessor:

1000 LXI SP, 27FF

1000 CALL 1006

1006 POPH

The contents of the stack pointer (SP) and the HL register pair on completion of execution of these instructions are:

(a) SP = 27 FF, HL = 1003
(b) SP = 27 FD, HL = 1003
(c) SP = 27 FF, HL = 1006
(d) SP = 27 FD, HL = 1006 **[1996]**

Solution: (c)

LXI SP, 27 FF → SP ← 27FF H

1003 CALL 1006 → SP → SP-2 SP ← 27FC H

PC gets pushed onto stack

PC = 1006

1006 POP H SP ← SP + 2 SP ← 27FF H

1006 gets pepped into HL

Thus HL = 1006H

SP = 27FFH

Hence, the correct option is (c).

5. The following program is run on an 8085 microprocessor:

Memory Address is in HEX	**Instruction**
2000	LXI SP, 1000
2003	PUSH H
2004	PUSH D
2005	CALL 2050
2008	POP H
2009	HLT

At the completion of execution of the program, the program counter of the 8085 contains______, and the stack pointer contains_______. **[1992]**

Solution: (200A H, 0FFE H)

LXI SP, 1000 → SP ← 1000

PUSH H SP ← SP-2 SP ← 0FFEH
(push HL pair into stack)

PUSH D SP ← SP-2 SP ← 0FFCH
(push DE pair into stack)

CALL 2050H SP ← SP-2 SP ← 0FFAH
(push PC into stack)

After returning from sub-routine

SP ← SP + 2 SP ← 0FFC

POP H SP ← SP + 2 SP ← 0FFE H

PC would contain next address of HLT

HLT takes 1 byte; thus PC = 200A H

SP = 0FFE H

Two-marks Questions

1. The following FIVE instructions were executed on an 8085 microprocessor.

MVI A, 33 H
MVI B, 78 H
ADD B
CMA
ANI 32 H

The accumulator value immediately after the execution of the fifth instruction is **[2017]**

(A) 00 H (B) 10 H
(C) 11 H (D) 32 H

Solution: Given instructions

MVI A, 33_H
MVI B, 78_H
ADD B
CMA
ANI 32_H
A = 33_H
+ $B = 78_H$ ADD B
$A = AB_H \Rightarrow A = A + B$
$A = 10101011$
Complement A since CMA
Then A = 01010100.
AND A with 32 H, i.e., 00110010
A : 01010100
32 H : 00110010
And Operation $A = 00010000$
$A = 00010000$
∴ A = 10 H

Hence, the correct option is (B).

2. Which one of the following 8085 microprocessor programs correctly calculates the product of two 8-bit numbers stored in registers B and C? **[2015]**

(A)
```
        MVI A, 00H
        JNZ LOOP
        CMP C
LOOP    DCR B
        HLT
```

(B)
```
        MVI A, 00H
        CMP C
LOOP    DCR B
        JNZ LOOP
        HLT
```

(C)
```
        MVI A, 00H
LOOP    ADD C
        DCR B
        JNZ LOOP
        HLT
```

(D)
```
        MVI A, 00H
        ADC C
        JNZ LOOP
LOOP    INR B
        HLT
```

Solution: We have to multiply register B and C, we can do it By adding B register C number of times, or adding Register C, B number of times.

Choice (C) MVI A, 00H $\to$ A = 00H reset Accumulator

Loop: ADD C $\to$ Add C to Accumulator

DCR B $\to$ decrement B

JNZ $\to$ till B becomes zero Add C

HLT

Hence, the correct option is (C).

3. An 8085 microprocessor executes `STA 1234H‘ with starting address location 1FFEH (STA copies the contents of the accumulator to the 16-bit address location). While the instruction is fetched and executed, the sequence of values written at the address pins $A_{15}-A_8$ is:

(a) 1FH, 1FH.20H, 12H
(b) 1FH.FEH, 1FH.FFH, 12H
(c) 1FH.1FH.12H.12H
(d) 1FH, 1FH, 12H.20H.12H **[2014]**

Solution: (a)

IFFE STA 1234

1FFEH $\to$ STA OP code stored

1FFFH $\to$ ‘12’ operand stored

2000H $\to$ ‘34’ operand stored

STA 1234 does storing value of accumulator at address 1234 H

Thus starting from 1FFEH,

	$A_{15}-A_8$	A_7-A_0
for fetch	1F	FE
for first memory read	1F	FF
for second memory read	20	00
for storing accumulator	12	34

Thus values of $A_{15}-A_8$

1FH, 1FH, 20H, 12H

Hence, the correct option is (a).

4. After execution of line 7 of the program, the status of the CY and Z flags will be

(a) CY = 0, Z = 0 (b) CY = 0, Z = 1
(c) CY = 1, Z = 0 (d) CY = 1, Z = 1 **[2007]**

Solution: (b)

ANI 9BH $\to$ A = A.9B

= (11101010). (10011011)

A = 10001010

A = 8AH

CPI 9FH $\to$ 8A < 9FH Accumulator compare immediate

CY = 1, Z = 0

STA 3010 H $\to$ Store accumulator to 3010 H

thus CY = 1, Z = 0 after line 7.

Hence, the correct option is (b).

Common Data for Questions 3 and 4

An 8085 assembly language program is given below.

Line1:MVI A, B5H
2 : MVIB, OEH
3 : XRI69H
4 : ADDB
5 : ANI9BH
6 : CPI9FH
7 : STA3010H
8 : HLT

5. The contents of the accumulator just after execution of the ADD instruction in line 4 will be

(a) C3H (b) EAH
(c) DCH (d) 69H **[2007]**

Solution: (b)

MVI A, B5 H $\to A = 10110101$

MVI B, OE H $\to B = 00001110$

XRI 69 H $\to A = A \oplus 69$

$A = 10110101 \oplus 01101001$

$A = 11011100$

ADD B $\to A = A + B$

$$\begin{array}{r r} & 11011100 \\ & 00001110 \\ \hline A = & 11101010 \\ \hline \end{array}$$

After ADD, A = EA H.

Hence, the correct option is (b).

6. Following is the segment of an 8085 assembly language program:

LXI SP, EFFFH CALL3000H 3000 H: LXIH.3CF4H PUSH PSW SPHL POP PSW RET

On completion of RET execution, the contents of SP is

(a) 3CFOH (b) 3CF8H
(c) EFFDH (d) EFFFH **[2006]**

Solution: (b)

LXI SP, EFFF H → SP ← EFFF H

CALL 3000H → SP ← SP-2
SP = EFFD H

3000H: LXI H, 3 CFH H → HL – 3CFH H

PUSH PSW → SP ← SP–2 SP = EFFBH

SPHL → SP ← 3CFH

POP PSW → SP ← SP + 2 SP = 3CF6

RET → Pop + Jump SP ← SP + 2
SP = 3CF8H

Hence, the correct option is (b).

7. Consider the following assembly language program.

```
        MVI     B, 87H
        MOV     A, B
START:  JMP     NEXT
        MVI     B, 00H
        XRA     B
        OUT     PORT 1
        HLT
NEXT:   XRA     B
        JP      START
        OUT     PORT 2
        HLT
```

The execution of the above program in an 8085 microprocessor will result in

(a) an output of 87H at PORT1
(b) an output of 87H at PORT2
(c) infinite looping of the program execution with accumulator data remaining at 00H.
(d) infinite looping of the program execution with accumulator data alternating between 00H and 87H **[2002]**

Solution: (b)

MVI B, 87 H → B = 87H

MOV A, B → A = 87H

START: JMP NEXT

NEXT: XRA B → A = A ⊕ B
= (10000111) ⊕ (10000111)

A = 00H

Sign bit = 0

∴ Control goes back to start (JP conditional Jump only when sign bit = 0)

from start goes back to NEXT

XRAB → A ⊕ B = (00000000) ⊕ (10000111)

A = 37H

Sign bit = 1

∴ This time, it would not jump to start.

Out Port 2 → output of A = 87 H on port 2.

Hence, the correct option is (b).

UNIT VIII

COMMUNICATION

EXAM ANALYSIS

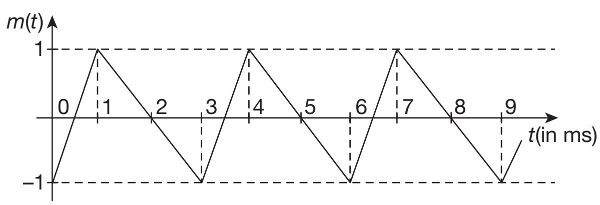
Exam Year	92	93	94	95	96	97	98	99	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14-A	14-B	14-C	14-D	15			16			17		18	19
																											Set 1	Set 2	Set 3	Set 1	Set 2	Set 3	Set 1	Set 2		
1 Marks Ques.	-	-	4	8	1	2	10	3	1	2	3	3	5	3	-	3	1	3	3	2	4	1	3	3	4	3	-	1	2	4	2	3	3	2	3	2
2 Marks Ques.	5	2	1	-	3	1	-	4	4	4	4	14	10	7	14	11	11	6	4	4	3	5	4	4	4	4	6	4	4	3	5	2	3	2	3	3
5 Marks Ques.	-	-	-	-	-	-	-	-	-	1																										
Total Marks	**2**	**1**	**-**	**3**	**1**	**1**	**2**	**5**	**5**	**3**	**3**	**7**	**4**	**6**	**3**	**1**	**4**	**2**	**4**	**3**	**2**	**-**	**-**	**-**	**-**	**-**	**12**	**9**	**10**	**10**	**12**	**7**	**9**	**6**	**9**	**8**
Chapter wise marks distribution																																				
Analog Communication Systems	-	2	3	3	2	-	2	3	4	1	3	6	5	4	6	2	3	2	2	2	1	-	-	3	3	1		7			4		4		2	1
Random Signals and Noise	1	-	-	1	1	1	4	-	-	2	1	4	3	4	4	3	3	4	3	1	2	2	3	3	3	5		3			2		1		1	2
Digital Communication Systems	3	-	2	4	1	2	4	4	1	3	3	7	7	2	3	9	5	2	2	3	3	3	2	-	2	1		6			9		4		2	1
Information Theory	-	1	-	-	-	-	-	-	-	-	-	-	-	-	1	-	1	1	-	-	1	1	2	1	-	-		1			3		1		1	-

Chapter 1

Analog Communication Systems

One-mark Questions

1. The baseband signal $m(t)$ shown in the figure is phase-modulated to generate the PM signal $\phi(t) = \cos(2\pi f_c t + km(t))$. The time t on the x-axis in the figure is in milliseconds. If the carrier frequency is f_c = 50 kHz and $k = 10\pi$, then the ratio of the minimum instantaneous frequency (in kHz) is to the maximum instantaneous frequency (in kHz) is _____ (rounded off to 2 decimal places). **[2019]**



Solution: $\phi(t) = \cos\left(2\Pi f_c t + km(t)\right)$

$$\theta_i(t) = 2\Pi f_c t + km(t)$$

$$f_i = \frac{1}{2\Pi}\frac{d}{dt}\theta_i(t)$$

$$= \frac{1}{2\Pi}\frac{d}{dt}\theta_i(t)$$

$$= \frac{1}{2\Pi}\frac{d}{dt}\left[2\Pi f_c t + km(t)\right]$$

$$= \frac{1}{2\Pi}\left[2\Pi f_c + k\right]\frac{d}{dt}m(t)$$

$$= f_c + \frac{k}{2\Pi}\frac{d}{dt}m(t)$$

$$f_c = 50 \text{ KHz}$$

$$k = 10\Pi$$

$$\therefore f_{\max} = 50\text{ k} + \frac{10\Pi}{2\Pi}\cdot\frac{2}{1\text{ ms}} = 60\text{ k}\left[\frac{dm(t)}{dt}\Big|_{\max} = 2\right]$$

$$t_{\min} = 50\text{ k} - \frac{10\Pi}{2\Pi}\cdot 1\times 10^3 = 45\text{ k}\left[\frac{dm(t)}{dt}\Big|_{\max} = -1\right]$$

$$\frac{t_{\max}}{f_{\min}} = \frac{60\text{ k}}{45\text{ k}} = 0.75$$

2. Consider the following amplitude modulated signal:

$S(t) = \cos(2000\ \pi t) + 4\cos(2400\ \pi t) + \cos(2800\ \pi t)$.

The ratio (accurate to three decimal places) of the power of the massage signal to the power of the carrier signal is _______. **[2018]**

Solution: amplitude modulated signal is given as

$S(t) = \cos(2000\ \pi t) + 4\cos(2400\ \pi t) + \cos(2800\ \pi t)$

$$S_{AM}(t) = \frac{\mu A_C}{2}\cos[2\pi(f_c - f_m)t] + A_C\cos(2\pi f_C t) + \frac{\mu A_C}{2}\cos[2\pi(f_C - f_m)t]$$

$$S(t) = A_c \cdot \cos 2\pi fct + \frac{Ac\mu}{2}\cdot\cos\pi(fc + fm)t + \frac{Ac\mu}{2}\cdot\cos 2\pi(fc - fm)t$$

$$P_t = P_c + P_{SB}$$

From the given data

$$A_c = 4 \text{ and } \frac{Ac\mu}{2} = 1$$

$$P_C = \frac{AC}{2} = 8$$

$$P_{SB} = 2\left\{\frac{Ac\mu}{2\sqrt{2}}\right\}^2 = \left(\frac{Ac\mu}{2}\right)^2$$

$$= (1) = 1$$

$$\frac{P_{SB}}{P_C} = \frac{1}{8} = 0.125$$

Hence, the correct answer is 0.12 to 0.13.

3. The block diagram of a frequency synthesizer consisting of a phase locked loop (PLL) and divided by N counter (comprising ÷2, ÷4, ÷8, ÷16 outputs) is sketched below. the synthesizer is excited with a 5 kHz signal (input 1). The free running frequency of the PLL is set to 20 kHz. Assume that the commutator switch makes contacts repeatedly in the order 1 – 2 – 3 – 4. **[2016]**

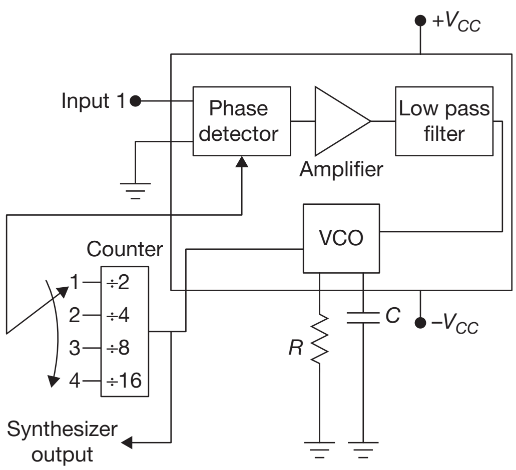
The corresponding frequencies synthesized are

(A) 10 kHz, 20 kHz, 40 kHz, 80 kHz
(B) 20 kHz, 30 kHz, 80 kHz, 160 kHz
(C) 80 kHz, 40 kHz, 20 kHz, 10 kHz
(D) 160 kHz, 80 kHz, 40 kHz, 20 kHz

Solution: Running freq is set to be 20 kHz. Here freq divider is used with N counter. O/p freq becomes N times the given freq so the outputs are 2 × 5 = 10 Hz, 4 × 5 = 20 Hz, 8 × 5 = 40 Hz, 16 × 5 = 80 Hz

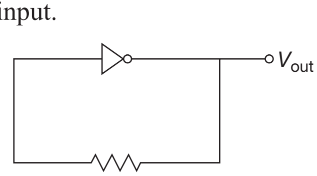
Hence, the correct option is (A).

4. What is the voltage V_{out} in the following circuit? **[2016]**



(A) 0 V
(B) (V_T of PMOS + V_T of NMOS)/2
(C) Switching threshold of inverter
(D) V_{DD}

Solution: The given circuit is inverter connected from output to input.



An inverter is the circuit which provides the complemented output. Here an inverter circuit consisting of NOT gate and resistance is present; therefore, there will be a switching threshold.

Hence, the correct option is (C).

5. A superheterodyne receiver operates in the frequency range of 58 MHz–68 MHz. The intermediate frequency f_{IF} and local oscillator frequency f_{LO} are chosen such that $f_{IF} \leq f_{LO}$. It is required that the image frequencies fall outside the 58 MHz–68 MHz band. The minimum required f_{IF} (in MHz) is ______. **[2016]**

Solution: Frequency range of receiver is 58 MHz to 68 MHz.

$$f_{Lo} = f_s + \text{IF}$$

$$f_{si} = f_{Lo} + \text{IF}$$

$$f_{si} = f_s + 2\text{IF}.$$

As per question f_{si} should be outside the range.

$$f_{si} \geq 68 \text{ MHz}.$$

$$f_s + 2\text{IF} \geq 68 \text{ MHz}.$$

Assume

$$f_s = 59 \text{ MHz}.$$

$$2\text{IF} \geq 68 - 58$$

$$\text{IF} \geq 5 \text{ MHz}.$$

Hence, the correct Answer is (5).

6. The amplitude of a sinusoidal carrier is modulated by a single sinusoid to obtain the amplitude modulated signal $S(t) = 5\cos 1600\pi t + 20\cos 1800\pi t + 5\cos 2000\pi t$. The value of the modulation index is ______. **[2016]**

Solution: AM

$$S(t) = A_c \cos 2pf_c t\,[1 + k_a\, m(t)]$$

$$S(t) = A_c \cos 2pf_c t\,[1 + k_a A_m \cos 2\pi f_m t]$$

$$S(t) = A_c \cos 2pf_c t + \frac{A_c\mu}{2}\cos 2\pi(f_c + f_m)\,t + \frac{A_c\mu}{2}\cos 2\pi\,(f_c - f_m)\,t$$

Comparing the above standard equation with given equation, we get

$$f_c = 900 \text{ Hz},$$

$$f_m = 100 \text{ Hz}$$

$$A_c = 20$$

$$\frac{A_c\mu}{2} = 5$$

$$A_c\mu = 10$$

$$\mu = \frac{10}{A_c} = \frac{10}{20}$$

$$\mu = \frac{1}{2} = 0.5\,.$$

Hence, the correct Answer is (0.5).

7. For a superheterodyne receiver, the intermediate frequency is 15 MHz and the local oscillator frequency is 3.5 GHz. If the frequency of the received signal is greater than the local oscillator frequency, then the image frequency (in MHz) is ______ . **[2016]**

Solution: Intermediate frequency freq (f_{IF})= 15MHz ;

local oscillator frequency (f_{L0}) = 3.5 GHz = 3500 MHZ

As we know that

$$(f_s - f_{L0} = f_{IF})$$

$$f_s = f_{L0} + f_{IF}$$

$$= 3500 \text{ MHz} + 15 \text{ MHz}$$

$$f_s = 3515 \text{ MHz}$$

and

$$f_{si} = f_s - 2f_{IF}$$

Up conversion (frequency of received signal is greater than LO frequency)

$$f_{si} = 3515 - 2 \times 15$$

$$= 3485 \text{ MHz}$$

Hence, the correct Answer is (3485 MHz).

8. Consider the signal $s(t) = m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c.t)$ where $\hat{m}(t)$ denotes the Hilbert transform of $m(t)$ and the bandwidth of $m(t)$ is very small compared to f_c. The signal $s(t)$ is a **[2015]**

(A) high-pass signal

(B) low-pass signal

(C) band-pass signal

(D) double sideband suppressed carrier signal

Solution: Signal $s(t) = m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)$

This is the equation of SSB-SC so it will look like band-pass signal.

Hence, the correct option is (C).

9. Consider sinusoidal modulation in an AM system. Assuming no overmodulation, the modulation index (H) when the maximum and minimum values of the envelope, respectively, are 3V and 1V, is ______. **[2014]**

Solution:

Modulation index for AM modulated wave is given by

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

$$\mu = \frac{3-1}{3+1}$$

$$\mu = \frac{2}{4} = 0.5$$

10. A modulated signals is $y(t) = \mathrm{m}(t)\cos(4000\pi t)$, where the baseband signal $m(t)$ has frequency components less than 5 kHz only. The minimum required rate (in kHz) at which $y(t)$ should be sampled to recover $m(t)$ is ________. **[2014]**

Solution:

The minimum sampling rate is given by Nyquist rate

$f_s = 2f_m$

$f_s = 2 \times 5$

$f_s = 10$ kHz

11. The phase response of a pass band waveform at the receiver is given by $\phi(f) = -2\pi a(f - f_c) - 2p\beta f$

where f_c is the centre frequency, and α and β are positive constants. The actual signal propagation delay from the transmittance to receiver is

(a) $\dfrac{a-\beta}{a+\beta}$ (b) $\dfrac{a\beta}{a+\beta}$

(c) α (d) β **[2014]**

Solution: (c)

Group delay is given by

$$= \frac{-1}{2\pi}\frac{d\phi}{df} = \frac{-1}{2\pi}[-2\pi\alpha]$$

Group delay = α

Hence, the correct option is (c).

12. Consider an FM signal

$$f(t) = \cos\left[2\pi f_c t + \beta_1 \sin 2\pi f_1 t + \beta_2 2\pi f_2 t\right]$$

The maximum deviation of the instantaneous frequency from the carrier frequency f_c is

(a) $\beta_1 f_1 + \beta_2 f_2$ (b) $\beta_1 f_2 + \beta_2 f_1$

(c) $\beta_1 + \beta_2$ (d) $f_1 + f_2$ **[2014]**

Solution: (a)

Maximum frequency deviation is

$$= \frac{1}{2\pi}\cdot\left|\frac{d\phi}{dt}\right|_{\max}$$

φ = Phase deviation

$$= \frac{1}{2\pi}\{\beta_1 2\pi f_1 \cos 2\pi f_1 t + \beta_2 2\pi f_2 \cos 2\pi f_2 t\}\Big|_{\max}$$

$$= \frac{1}{2\pi}(2\pi\beta_1 f_1 + 2\pi\beta_2 f_2)$$

$= \beta_1 f_1 + \beta_1 f_2$

Hence, the correct option is (a).

13. In a double side-band (DSB) full carrier AM transmission system, if the modulation index is doubled, then the ratio of total sideband power to the carrier power increases by a factor of _____. **[2014]**

Solution:

Ratio of P_{SB} to PC

$$g_+(t) = e^{-at} e^{+j(w_C t + \Delta w)t}$$

$$\left(\frac{P_{SB}}{P}\right)_1 = \frac{\mu^2 T}{2}$$

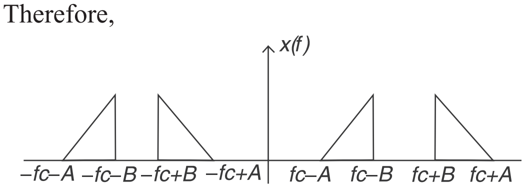
and $\left(\frac{P_{SB}}{P_C}\right)_2 = \frac{\mu_0^2}{2}$

where $\mu_2 = 2\mu$

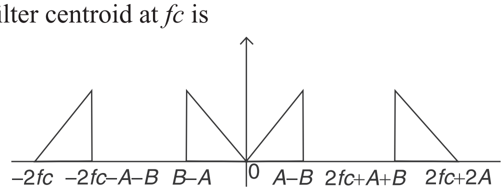
Hence, $\left(\dfrac{\frac{P_{SB}}{P_C}}{\frac{P_{SB}}{P_C}}\right) = \dfrac{1}{4}$

Four times.

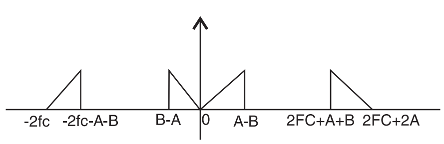
Therefore,



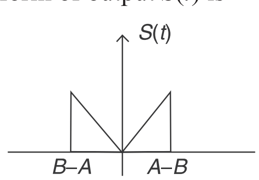
The Fourier transform of output $y(t)$ of the high pass filter centroid at fc is



Now, the output of second multiplier is $y(t).w(t) = z(t)$



After passing through the low pass filter centred at f_c the Fourier transform of output $S(t)$ is



Thus, bandwidth = $(A - B) - 0$

= 100 – 40

= 60 Hz

14. The List-I (lists the attributes) and the List-II (list of the modulation systems) are given. Match the attribute to the modulation system that best meets it.

List-I

A. Power efficient transmission of signals

B. Most bandwidth efficient transmission of voice signals

C. Simplest receiver structure

D. Bandwidth efficient transmission of signals with significant dc component

List-II

1. Conventional AM
2. FM
3. VSB
4. SSB-SC

	A	B	C	D
(a)	4	2	1	3
(b)	2	4	1	3
(c)	3	2	1	4
(d)	2	4	3	1

[2011]

Solution:(b)

AM has simple receiver and VSB is bandwidth efficient transmission of signals with significant dc component. SSB – SC is most bandwidth efficient transmission of voice signal.

Hence, the correct option is (b).

15. Suppose that the modulating signal is $m(t) = 2\cos(2\pi f_m t)$ and the carrier is $x_C(t) = A_C \cos(2\pi f_c t)$. Which one of the following is a conventional AM signal without over-modulation?

(a) $x(t) = A_c\, m(t)\cos(2\pi f_c t)$

(b) $x(t) = A_c\left[1 + m(t)\right]\cos(2\pi f_c t)$

(c) $x(t) = A_C \cos(2\pi f_c t) + \frac{A_C}{4} m(t)\cos(2\pi f_c t)$

(d) $x(t) = A_C \cos(2\pi f_m t)\cos(2\pi f_c t) + A_C \sin(2\pi f_m t)\sin(2\pi f_c t)$ **[2010]**

Solution:(c)

Conventional AM signal is

$x(t) = A_C[1 + m(t)] \cos(w_c t)$

$= A_C \cos(w_c t) + A_C m(t) \cos w_c t$

In option b,

Modulation index = $\frac{2}{1} = 2$.

So, it is conventional A.M signal but with over modulation.

In option c,

$$x(t) = A_C \cos(2\pi f_c t) + \frac{A_C m(t)}{4}\cos(2\pi f_c t)$$

Here, modulation index

$$= \frac{2}{4} = \frac{1}{2}$$

Therefore, it is conventional AM signal without over modulation.

Hence, the correct option is (c).

16. Consider an angle modulated signal $x(t) = 6\cos\left[2\pi \times 10^6 t + 2\sin(8000\pi t) + 4\cos(8000\pi t)\right]$ V.

The average power of $x(t)$ is

(a) 10 W (b) 18 W

(c) 20 W (d) 28 W **[2010]**

Solution:(b)

Average power of angle modulated signal $x(t)$ is $\frac{A_C^2}{2}$

Here $A_C = 6$

Therefore, average power $= \frac{6^2}{2} = 18W$

Hence, the correct option is (b).

17. For a message signal $m(t) = \cos(2\pi f_m t)$ and carrier of frequency f_c, which of the following represents a single side-band (SSB) signal?

(a) $\cos(2\pi f_m t)\cos(2\pi f_c t)$

(b) $\cos(2\pi f_c t)$

(c) $\cos\left[2\pi(f_c + f_m)t\right]$

(d) $\left[1 + \cos(2\pi f_m t)\right]\cos(2\pi f_c t)$ **[2009]**

Solution:(c)

$\cos(2\pi (f_c + f_m)(t)$ represents only USB of A.M – SSB/SC signal.

Hence, the correct option is (c)

18. Consider the amplitude modulated (AM) signal $A_c \cos\omega_c t + 2\cos\omega_m t \cos\omega_c t$. For demodulating the signal using envelope detector, the minimum value of A_c should be

(a) 2 (b) 1

(c) 0.5 (d 0 **[2008]**

Solution: (a)

Modulated signal

$$\phi_{AM}(t) = A_C \cos\omega_c t + 2\cos\omega_m t + \cos\omega_c t$$
$$= (AC + 2\cos\omega_m t)\cos w_c t$$

Condition for envelope detection of an Aμ signal is

$= A + 2\cos\omega_m t \geq 0$

$A_C - 2 \geq 0$

$A_C \geq 2$

Therefore, minimum value of A_C should be 2.

Hence, the correct option is (a).

19. Find the correct match between Group-1 and Group-2.

Group-1

P. $\{1 + km(t)\}A\sin(\omega_c t)$

Q. $km(t)A\sin(\omega_c t)$

R. $A\sin\{\omega_c t + km(t)\}$

S. $A\sin\left[\omega_c t + k\int_{-\infty}^{t} m(t)dt\right]$

Group-2

W. Phase modulation

X. Frequency modulation

Y. Amplitude modulation

Z. DSB-SC modulation

(a) P-Z, Q-Y, R-X, S-W

(b) P-W, Q-X, R-Y, S-Z

(c) P-X, Q-W, R-Z, S-Y

(d) P-Y, Q-Z, R-W, S-X **[2005]**

Solution: (d)

Correct combination will be as followed

P. $\{1 + km(t)\}A\sin(\omega_c t)$ Amplitude modulation

Q. $km(t)A\sin(\omega_c t)$ DSB-SC modulation

R. $A\sin\{\omega_c t + km(t)\}$ Phase modulation

S. $A\sin\left[\omega_c t + k\int_{-\infty}^{t} m(t)dt\right]$ Frequency modulation

Hence, the correct option is (d).

20. Which of the following analogue modulation scheme requires the minimum transmitted power and minimum channel band-width?

(a) VSB (b) DSB-SC

(c) SSB (d) AM **[2005]**

Solution: (c)

Transmission single side band modulated signal requires same bandwidth as original message and takes minimum power for transmission as carrier is suppressed here.

Hence, the correct option is (c)

21. An AM signal is detected using an envelope detector. The carrier frequency and modulating signal frequency are 1 MHz and 2 kHz respectively. An appropriate value for the time constant of the envelope detector is

(a) 500 μ sec (b) 20 μ sec

(c) 0.2 μ sec (d) 1 μ sec **[2004]**

Solution:(b)

Fc = carrier frequency

Fm = message signal frequency

For envelop detector time RC constant is given by

$$\frac{1}{f_c} < \text{RC} \leq \frac{1}{f_m}$$

$1\,\mu$ sec $<RC''$ 0.5 ms

Hence, the correct option is (b).

22. An AM signal and a narrow-band FM signal with identical carriers, modulating signals and modulation indices of 0.1 are added together. The resultant signal can be closely approximated by
(a) broadband FM
(b) SSB with carrier
(c) DSB-SC
(d) SSB without carrier **[2004]**

Solution: (b)

Standard equation for narrow band FM is given by

$$S(t) = A_C \cos w_c t + \frac{A_c \beta}{2} \cos(w_c + w_m)$$

$$t - \frac{A_C \beta}{2} \cos 9 w_c - w_m)$$

$$S(t)_{AM} = A_C \cos w_c t + \frac{A_C m_a}{2}$$

$$\cos(w_c + w_m)t + \frac{A_C m_a}{2} \cos(\omega_c - \omega_m)$$

$S(t) + S(t)_{AM}$ = carrier + USB = SSB with carrier.

Note: $S(t) + S(t)_{AM} = SSB + SC$

Hence, the correct option is (b).

23. The input to a coherent detector is DSB–SC signal plus noise. The noise at the detector output is
(a) the in-phase component
(b) the quadrature-component
(c) zero
(d) the envelope **[2003]**

Solution: (a)

The coherent detector rejects the quadrature component of noise and therefore noise at output has in phase component only. The in-phase component of noise and output are additive at output of detector.

Hence, the correct option is (a).

24. A 1 MHz sinusoidal carrier is amplitude modulated by a symmetrical square wave of period 100 μ sec. Which of the following frequencies will NOT be present in the modulated signal?
(a) 990 kHz (b) 1010 kHz
(c) 1020 kHz (d) 1030 kHz **[2002]**

Solution: (c)

Given

1 MHz $\Rightarrow$ 1000 kHz = f_c (carrier frequency)

100 $\mu s \Rightarrow$ 10 kHz = f_m (message signal frequency)

Frequencies Present = 1000 ± 10 ($f_c \pm f_m$)

= 970,1030.

Half wave symmetric wave . So, only odd harmonics are present.

Hence, the correct option is (c).

25. A band limited signal is sampled at the Nyquist rate. The signal can be recovered by passing the samples through
(a) an RC filter
(b) an envelope detector
(c) an PLL
(d) an ideal low-pass filter with the appropriate bandwidth **[2001]**

Solution: (d)

Ideal low pass filter with the appropriate BW can recover a sampled band limited signal.

Hence, the correct option is (d).

26. The amplitude modulated wave form $s(t) = A_c[1 + K_a m(t)] \cos\omega_c t$ is fed to an ideal envelope detector. The maximum magnitude of K_a m(t) is greater than 1.

Which of the following could be the detector output?
(a) $A_c m(t)$ (b) $A_c^2 \left[1{+}K_a m(t)\right]^2$
(c) $[Ac|\,1 + K_a m(t)|]$ (d) $Ac|1 + K_a m(t)|^2$ **[2000]**

Solution:(c)

When the modulation index of AM wave is less than unity the output of the envelope detector is envelope of the AM wave but when the modulation index is greater than unity then the output of the envelope detector is not envelope but mode of the envelope of the AM wave. Thus, the detector output in given case would be $A_C[1 + k_a m(t)]$.

Hence, the correct option is (c).

27. The input to a channel is a bandpass signal. It is obtained by linearly modulating a sinusoidal carrier with a single-tone signal. The output of the channel due to this input is given by

$y(t) = (1/100)\cos(100t - 10^{-6})\cos(10^6 t - 1.56)$.

The group delay (t_g) and the phase delay (t_p) in seconds of the channel are
(a) $t_g = 10^{-6}$, $tp = 1.56$
(b) $t_g = 1^{.56}$, $tp = 10^{-6}$
(c) $t_g = 10^{-8}$, $tp = 1.56 \times 10^{-6}$
(d) $t_g = 10^8$, $tp = 1.56$ **[1999]**

Solution: (c)

Given

$$y(t) = \left(\frac{1}{100}\right) \cos(100t - 10^{-6}) \cos(10^6 t - 1.56)$$

Now comparing with

$y(t) = \cos(t - t_0)\cos[w_c(t - t_p)]$

$t_p = 1.56 \times 10^{-6}$ Hz

$t_g = 10^{-8}$ Hz

Hence, the correct option is (c).

28. A modulated signal is given by $s(t) = m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)$ where the baseband signal $m_1(t)$ and $m_2(t)$ have bandwidths of 10 kHz and 15 kHz, respectively. The bandwidth of the modulated signal, in kHz, is

(a) 10 (b) 15

(c) 25 (d) 30 **[1999]**

Solution: (b)

Given modulation technique is DSB-SC with f_{max}=15 KHz, so bandwidth can be given as

Bandwidth = $2{*}f_{max}$ = 30 kHz

Hence, the correct option is (b).

29. A modulated signal is given by $s(t) = e^{-at}\cos[(\omega_c + \varnothing\omega)t]u(t)$, where a, ω_c and $\varnothing\omega$ are positive constants, and $\omega c >> \varnothing\omega$. The complex envelope of $s(t)$ is given by

(a) $\exp(-at)\exp[j(\omega_c + \varnothing\omega)t]u(t)$

(b) $\exp(-at)\exp(j\varnothing\omega t)u(t)$

(c) $\exp(j\varnothing\omega t).u(t)$

(d) $\exp[j(\omega_c + \varnothing\omega)t]$ **[1999]**

Solution:(d)

Complex envelope

$\bar{g}(t) = g_+(t)^{-jw_c t}$

where $g + (t)$ is pre envelope given as

$$g_+(t) = g(t) + jg(t)$$

and $g(t) = e^{-at}\sin[(w_c + \Delta_w)t]$

$$g_+(t) = e^{-at}e^{+j(w_c t + \Delta w)t}$$

Hence, $\tilde{g}(t) = e^{-at}e^{-jw_c t}u(t)$

$$\tilde{g}(t) = e^{-at}e^{-j\Delta wt}u(t)$$

Hence, the correct option is (d).

30. The image channel selectivity of superheterodyne receiver depends upon

(a) IF amplifier only

(b) RF and IF amplifiers only

(c) Preselector, RF and IF amplifier

(d) Preselector and RF amplifiers only **[1998]**

Solution: (d)

The image rejection should be achieved before IF stage because once it enters into IF amplifier it becomes impossible to remove it from wanted signals. So, image channel selectivity depends upon pre-selector and RF amplifiers only. The IF amplifier helps in rejection of adjacent channel frequency and not image frequency.

Hence, the correct option is (d).

31. A DSB-SC signal is generated using the carrier $\cos(\omega c t + \theta)$ and modulating signal $x(t)$. The envelope of the DSB-SC signal is

(a) $x(t)$

(b) $|x(t)|$

(c) only positive portion of $x(t)$

(d) $x(t)\cos\theta$ **[1998]**

Solution: (b)

The 'envelope' of the DSB-SC (double side band–suppress carrier) signal depends on the $|x(t)|$ and not on $x(t)$.

Hence, the correct option is (b).

32. The image channel rejection in a superheterodyne receiver comes from

(a) IF stages only

(b) RF stages only

(c) detector and RF stages only

(d) detector, RF and IF stages **[1996]**

Solution: (b)

Image rejection should be achieved in IF amplifier. It becomes impossible to remove it from wanted signal. So, the image channel rejection in a super heterodyne receiver comes from RF stages only.

Hence, the correct option is (b).

33. The image (second) channel selectivity of a superheterodyne communication receiver is determined by

(a) antenna and preselector

(b) the preselector and RF amplifier

(c) the preselector and IF amplifier

(d) the RF and IF amplifier **[1995]**

Solution: (b)

The image rejection should be achieved IF stage because once it enters into IF amplifier it becomes impossible to remove it from wanted signal.

So, the image channel selectivity depends upon preselective and RF amplifiers only. If amplifier helps in rejection of adjacent channel frequency and not image frequency.

Hence, the correct option is (b).

34. A PLL can be used to demodulate

(a) PAM signals (b) PCM signals

(c) FM signals (d) DSB–SC signals **[1995]**

Solution: (c)

PLL (Phase Locked Loop) is used to demodulate the FM signals.

Hence, the correct option is (c).

35. A PAM signal can be detected by using

(a) an ADC (b) an integrator

(c) a band pas filter (d) a high pass filter

[1995]

Solution: (b)

An integrator circuit is used in PAM receiver circuit.

Hence, the correct option is (b).

36. $v(t) = 5[\cos(10^6\pi t) - \sin(10^3\pi t) \times \sin(10^6\pi t)]$ represents

(a) DSB suppressed carrier signal

(b) AM signal

(c) SSB upper sideband signal

(d) Narrow band FM signal **[1994]**

Solution: (d)

Carrier and upper side band are in phase and lower side band is out of phase with carrier and upper side-band.

$$v(t) = 5\cos(10^5\pi t) - \frac{5}{2}\cos$$

$$(10^6 - 10^3)\pi t + \frac{5}{2}\cos(10^6 + 10^3)$$

So given signal is narrow band FM signal.

Hence, the correct option is (d).

37. A 10 MHz carrier is frequency modulated by a sinusoidal signal of 500 Hz, the maximum frequency deviation being 50 kHz. The bandwidth required, as given by the Carson rule is__________ **[1994]**

Solution:

Δf = frequency deviation

f_m = message signal frequency

Bandwidth is calculated by with Carson's rule

$\text{BW} = 2(\Delta f + f_m)$

$\text{BW} = 2(50 + 0.5)$

$\text{BW} = 101$ kHz

Two-marks Questions

1. Let $c(t) = A_c \cos(2\pi f_c^t)$ and $m(t) = \cos(2\pi f_m^t)$. It is given that $f_c \gg 5 f_m$. The signal $c(t) + m(t)$ is applied to the input of a non-linear device, whose output $v_o(t)$ is related to the input $v_i(t)$ as $v_o(t) = a v_i(t) + b v_o^2(t)$, where a and b are positive constants. The output of the non-linear device is passed through an ideal ban-pass filter with centre frequency f_c and bandwidth $3f_m$, to produce an amplitude modulated (AM) wave. It is desired to have the sideband power of the AM wave to be half of the carrier power, then a/b is: **[2018]**

(A) 0.25 (B) 0.5

(C) 1 (D) 2

Solution: $V_i(t) = A_c \cos(2\pi f_c t) + \cos(2\pi f_m t)$

$$V_0(t) = aV_i(t) + b\,v_i^2(t)$$

$$= a A_c \cos(2\pi f_c t) + a\cos(2\pi f_m t)$$

$$+ b[A_c^2 \cos^2(2\pi f_c t) + \cos^2(2\pi f_m t)$$

$$+ 2A_c \cos 2\pi(f_c t)\cos(2\pi f_m t)$$

After BPF,

$$y(t) = a A_c \cos 2\pi f_c t + 2b\cos(2\pi f_c t)\cos(2\pi f_m t)$$

$$= a A_c \left[1 + \frac{2b}{a}\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

$$\mu = \frac{2b}{a}$$

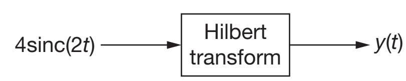
$$P_{SB} = \frac{\mu^2}{2} \quad P_c = \frac{P_C}{2}$$

$$\mu^2 = 1,\ \mu = 1$$

$$\frac{2b}{a} = 1 \Rightarrow \frac{a}{b} = 2$$

Hence, the correct option is (D).

2. The input 4 sin$C$ $(2t)$ is fed to a Hilbert transformer to obtain $y(t)$, as shown in the figure below:

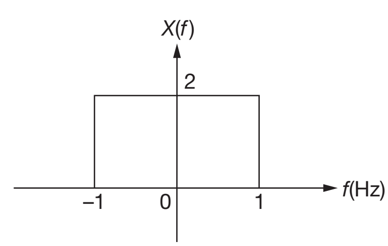


Here, $\text{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}$. The value (accurate to two decimal places) of $\int_{-\infty}^{\infty} |y(t)|^2\,dt$ is ________. [2018]

Solution: $4\sin c(2t) \xrightarrow{FT} \frac{4}{2}\text{rect}\left[\frac{f}{2}\right]$

$$\int_{-\infty}^{\infty} |X(f)|^2\,df = 2(2^2) = 8$$

$$\int_{-\infty}^{\infty} |y(t)|^2\,dt = 8$$



Hence, the correct answer is 8 to 8.

3. The unmodulated carrier power in an AM transmitter is 5 kW. This carrier is modulated by a sinusoidal modulating signal. The maximum percentage of modulation is 50%. If it is reduced to 40% then the maximum unmodulated carrier (in kW) that can be used without overloading the transmitter is _______________. **[2017]**

Solution:

$$P_c = 5 \text{ kW for } \mu_{\max} = 0.5$$

$$P_{t(\max)} = P_c\left[1+\frac{(0.5)^2}{2}\right]$$

$$= 5.625 \text{ kW}$$

$$\mu = 0.4$$

$$P_{c(\max)} = \left[1+\frac{\mu^2}{2}\right]$$

$$= P_{t(\max)}$$

$$\Rightarrow P_{c(\max)} = 5.208 \text{ kW}$$

Hence, the correct answer is (5.1 to 5.2).

4. A modulating signal given by $x(t) = 5 \sin (4\pi 10^3 t - 10\,\pi \cos 2\,\pi\, 10^3\, t)$ V is fed to a phase modulator with phase deviation constant $kp = 5$ rad/V. If the carrier frequency is 20 kHz, the instantaneous frequency (in kHz) at t = 0.5 ms is ___________. **[2017]**

Solution: $x(t) = 5 \sin (4000\pi t - 10\pi \cos 200\pi t)$ V

PM signal,

$s(t) = A_c \cos [\omega_c t + Kp \times (t)]$

Instantaneous angle,

$\theta(t) = \omega_c t + Kp \times (t)$

Instantaneous frequency is

$$\omega_i(t) = \frac{d}{dt}\theta(t)$$

$$= w_c + k_p \frac{d}{dt}x(t)$$

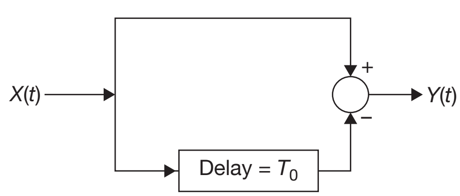
$$fi(t) = \frac{\omega_i(t)}{\theta\pi}$$

At $t = 0.5$ ms

$$f_i(t) = 70 \text{ kHz}$$

Hence, the correct answer is (70).

5. A wide sense stationary random process $X(t)$ passes through the LTI system shown in the figure. If the autocorrelation function of $X(t)$ is $Rx(\tau)$, then the autocorrelation function $R_y(\tau)$ of the output $Y(t)$ is equal to **[2016]**



(A) $2R_x(\tau) + R_x(\tau - T_0) + R_x(\tau + T_0)$
(B) $2R_x(\tau) - R_x(\tau - T_0) - R_x(\tau + T_0)$
(C) $2R_x(\tau) + 2R_x(\tau - 2T_0)$
(D) $2R_x(\tau) - 2R_x(\tau - 2T_0)$

Solution: $Y(t) = x(t) - x(t - T_0)$

The autocorrelation of output y can be given as

$R_y(\tau) = E\,[y(t)\, y(t + \tau)]$

$= E\,[[x(t) - x(t - T_0)]\,[x(t + \tau) - x(t - T_0 + \tau]$

$R_y(\tau) = E[x(t)\, x(t + \tau) - x(t)\, x(t - T_0 + \tau)$
$- x(t - T_0)\, x(t + \tau)$

$R_y(\tau) = [R_x(\tau) - R_x(\tau - T_0) - R_x(\tau + T_0) + R_x(\tau)$

$R_y(\tau) = 2R_x(\tau) - R_x(\tau - T_0) - R_x(\tau + T_0)$

Hence, the correct option is (B).

6. The complex envelope of the band pass signal $x(t) = -\sqrt{2}\left(\frac{\sin(\pi t/5)}{\pi t/5}\right)\sin\left(\pi t - \frac{\pi}{4}\right)$, cantered about $f = \frac{1}{2}$ Hz, is **[2015]**

(A) $\left(\frac{\sin(\pi t/5)}{\pi t/5}\right)e^{j\frac{\pi}{4}}$

(B) $\left(\frac{\sin(\pi t/5)}{\pi t/5}\right)e^{-j\frac{\pi}{4}}$

(C) $\sqrt{2}\left(\frac{\sin(\pi t/5)}{\pi t/5}\right)e^{j\frac{\pi}{4}}$

(D) $\sqrt{2}\left(\frac{\sin(\pi t/5)}{\pi t/5}\right)e^{-j\frac{\pi}{4}}$

Solution: Band pass signal $x(t)$

$$= -\sqrt{2}\left[\frac{\sin(\pi t/5)}{(\pi t/5)}\right] Sin\left(\pi t - \frac{\pi}{4}\right) \text{ can be as}$$

$$x(t) = -\sqrt{2}\left[\frac{\sin(\pi t/5)}{(\pi t/5)}\right]\left[\sin \pi t \cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cdot\cos \pi t\right]$$

$$= \frac{\sin(\pi t/5)}{(\pi t/5)}[\cos \pi t - \sin \pi t]$$

$$\because \sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

So $x(t) = X_c(t) \cos 2\,\pi f e^t - X_c(t) \sin 2\,\pi f_c t$

(low pass representation of Band pass single or L-SSB representation)

Where $X_c(t) = X_s(t) = \dfrac{\sin(\pi t/5)}{(\pi t/5)}$

So complex envelope

$$X_{ce}(t) = X_c(t) + j\, X_s(t)$$

$$= \frac{\sin(\pi t/5)}{(\pi t/5)}[1+j]$$

$$= \sqrt{2}\frac{\sin(\pi t/5)}{(\pi t/5)}e^{j\frac{\pi}{4}}$$

Hence, the correct option is (C).

7. A random binary wave y (t) is given by

$$y\,(\mathrm{t}) = \sum_{n=-\infty}^{\infty} X_n p(t - nT - \phi)$$

where $p(t) = u(t) - u(t - T)$, $u(t)$ is the unit step function and ϕ is an independent random variable with uniform distribution in $[0, T]$. The sequence $\{X_n\}$ consists of independent and identically distributed binary valued random variables with $P\{X_n = +1\} = P\{X_n = -1\} = 0.5$ for each n.

The value of autocorrelation **[2015]**

$$R_{yy}\left(\frac{3T}{4}\right) \overset{\Delta}{=} E\left[y(t)\,y\left(t - \frac{3T}{4}\right)\right] \text{ equals \_\_\_\_\_\_\_\_}$$

Solution: $y(t) = \sum_{n=-\infty}^{+\infty} X_n p(t - nT - \phi)$

If $\quad p(t) = u(t) - u(t - T)$

So amplitude (A) of $p(t) = 1$

and we know that

$$\text{Autocorrelation } R_{XX}(\tau) = \begin{cases} A^2\left(1 - \frac{|\tau|}{T}\right), & |\tau| < T \\ 0, & |\tau| > 0 \end{cases}$$

So for $\quad R_{yy}\left(\frac{3T}{4}\right) = A^2\left(1 - \frac{3T/4}{T}\right)$

$$= 1.\left(1 - \frac{3}{4}\right)$$

Since $A = 1$

$$= \frac{1}{4} = 0.25$$

Hence, the correct Answer is (0.24 to 0.26).

8. A message signal $m(t) = A_m \sin(2\pi f_c t)$ is used to modulate the phase of a carrier $A_c \cos(2\pi f_c t)$ to get the modulated signal $y(t) = A_c \cos(2\pi f_c t + m(t))$. The bandwidth of $y(t)$ **[2015]**

(A) depends on A_m but not on f_m.
(B) depends on f_m but not A_m.
(C) depends on both A_m and f_m.
(D) does not depend on A_m or f_m.

Solution: $\Phi_{pm}(t) = A \cos[\omega_c t + k_p f(t)]$

Where

$\Phi_{pm}(t) =$ Phase modulated signal

$m(t)$ or $f(t) =$ message signal

$$\theta_i(t) = \omega_c t + k_p m(t)$$

here $\quad k_p = 1$

So $\quad \theta_i(t) = \omega_c t + m(t)$

$$\text{Bandwidth} = 2(\Delta\omega + \omega_m) = 2(\Delta f + f_m)\pi$$

$$\omega_i(t) = \frac{d\theta_i(t)}{dt} = \omega_c + A_m 2\pi f_m \cos(2\pi f_m t)$$

$$\Delta\omega = A_m f_m . 2\pi \cos 2\pi f_m t$$

So bandwidth depends upon A_m and f_m.

Hence, the correct option is (C).

9. The directivity of an antenna array can be increased by adding more antenna elements, as a large number of elements **[2015]**

(A) improves the radiation efficiency.
(B) increases the effective area of the antenna.
(C) results in a better impedance matching.
(D) allows more power to be transmitted by the antenna.

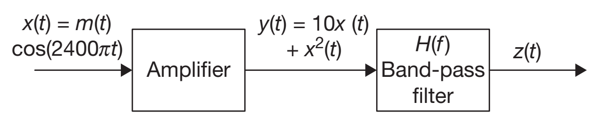
Solution: $\quad D = \frac{4\pi}{\lambda^2} A_e$

$$D \propto A_e$$

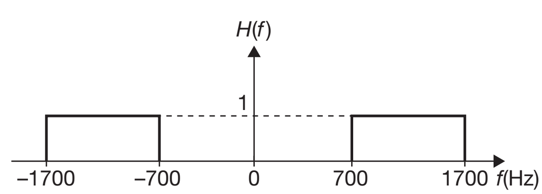
So directivity increases as the effective area of antenna increases.

Hence, the correct option is (B).

10. In the system shown in Figure (a), $m(t)$ is a low-pass signal with bandwidth W Hz. The frequency response of the band-pass filter $H(f)$ is shown in Figure (b). If it is desired that the output signal $z(t) = 10x(t)$, the maximum value of W (in Hz) should be strictly less than ________. **[2015]**



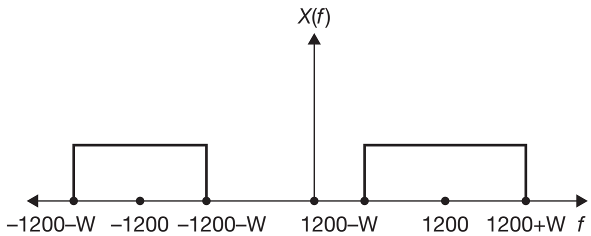
(a)

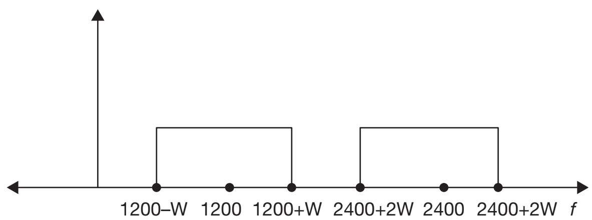


(b)

Solution: From the figure.

$x(t)$ have frequency spectrum





Now for $Y(f)$ one sided spectrum will be

Because $x^2(t)$ contains twice frequency components of $x(t) = 2 \times (1200 \pm W)$

$$= (2400 \pm W)$$

And $\quad z(t) = 10\, x(t)$

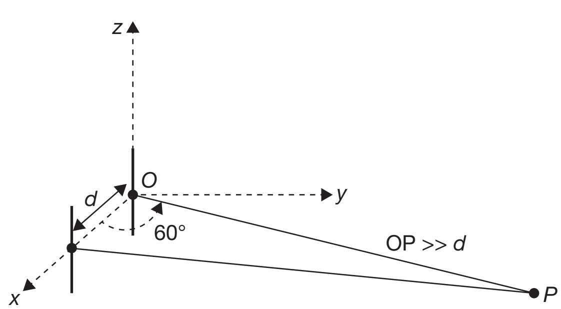
So Band pass filter should completely reject the $x^2(t)$ spectrum.

So $\quad 2400 - 2W = 1700$

$$W = 350 \text{ Hz}$$

Hence, the correct Answer is (349 to 350).

11. Two half-wave dipole antennas placed as shown in the figure are excited with sinusoidally varying currents of frequency 3 MHz and phase shift of $\pi/2$ between them (the element at the origin leads in phase). If the maximum radiated E-field at the point P in the x-y plane occurs at an azimuthal angle of 60°, the distance d (in meters) between the antennas is ________ **[2015]**



Solution: $\Psi = \beta d \cos\theta + \phi$

Given that $\quad \phi = -\dfrac{\pi}{2}$

$$\theta = 60°$$

$$f = 3 \text{ MHz}$$

$$\Rightarrow \quad \lambda = \frac{3\times10^8}{3\times10^6} = 100\text{m}$$

maximum field occurs at $\Psi = 0$

$$\beta d \cos\theta + \phi = 0$$

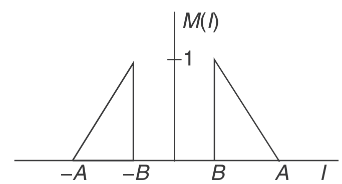
$$\frac{2\pi}{100}\times d\cos 60° - \frac{\pi}{2} = 0$$

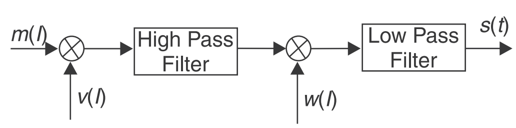
$$\Rightarrow \quad d\times\frac{1}{2}\times\frac{2\pi}{100} = +\frac{\pi}{2}$$

$$d = 50 \text{ m}$$

Hence, the correct Answer is (49 to 51).

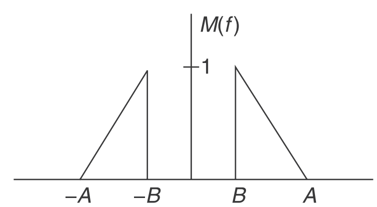
12. In the figure, $M(f)$ is the Fourier transform of the message signal $m(t)$ where $A = 100$ Hz and $B = 40$ Hz. Given $v(t) = \cos(2\pi f_c t)$ and $w(t) = \cos(2\pi(f_c + A)t)$, where $f_c > A$. The cut-off frequencies of both the filters are f_c.





The bandwidth of the signal at the output of the modulator (in Hz) is _____. **[2014]**

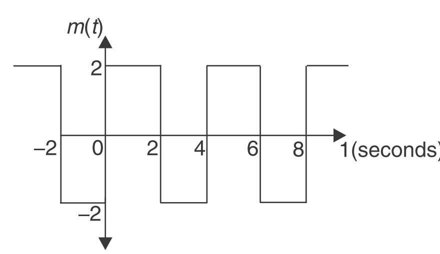
Solution:



After passing through first multiplier, the output is

$m(t) \times v(t) = x(t)$

13. The signal $m(t)$ as shown is applied both to a phase modulator (with k_p as the phase constant) and a frequency modulator with (k_f as the frequency constant) having the same carrier frequency



The ratio kp/kf (in rad/Hz) for the same maximum phase deviation is

(a) 8π (b) 4π

(c) 2π (d) π **[2012]**

Solution: (b)

For phase modulator

$$\phi(t) = 2\pi f_c t + k_p m(t)$$

maximum phase deviation is

$$(\phi_b)_{\max} = k_p \max[m(t)] = 2k_p$$

for frequency modulator

$$\phi(t) = 2\pi f_c t + 2\pi k_f \int m(t)dt$$

$$(\phi'_D)_{\max} = 2\pi k_f \left[\int m(t)dt\right]_{\max}$$

$$(\phi'_D)_{\max} = 2\pi k_f \left[\int_0^2 m(t)dt\right]$$

$$(\phi'_D)_{\max} = 2\pi k_f \left[\int_0^1 2.dt\right]$$

$$(\phi'_D)_{\max} = 8\pi k_f$$

given

$$(\phi'_D)_{\max} = (\phi_0)_{\max}$$

8πkf = 2kp

$$\frac{R_p}{k_f} = 4\pi$$

Hence, the correct option is (b).

14. A message signal $m(t)$ = cos 2000 πt + 4 cos 4000 πt modulates the carrier $c(t) = \cos 2\pi f_c t$ where f_c = 1 MHz to produce an AM signal. For demodulating the generated AM signal using an envelope detector, the time constant RC of the detector circuit should satisfy

(a) 0.5 ms < RC < 1 ms

(b) 1μs < < RC < 0.5 ms

(c) RC << 1μs

(d) RC >> 0.5 ms **[2011]**

Solution: (b)

Time constant for envelop detector is given as

$$\frac{1}{f_c} << RC << \frac{1}{f_m}$$

$$\frac{1}{1\text{ MHz}} << RC << \frac{1}{2\text{ kHz}}$$

1 μs << RC << 0.5

Hence, the correct option is (b).

15. A message signal given by

$m(t) = \left(\frac{1}{2}\right)\cos\omega_1 t - \left(\frac{1}{2}\right)\sin\omega_2 t$ is amplitude-modulated with a carrier of frequency ω_c to generate

$s(t) = [1 + m(t)]\cos\omega_c t$

What is the power efficiency achieved by this modulation scheme?

(a) 8.33 % (b) 11.11 %

(c) 20 % (d) 25 % **[2009]**

Solution: (c)

Power efficacy is given as = $\eta = \frac{m_a^2}{2 + m_a^2} \times 100$

$$ma_1 = \frac{V_m}{V_C} = \frac{1}{2} = ma_2$$

$$m_a = \sqrt{ma_a^2 + ma_2^2}$$

$$\eta = \frac{0.25 \times 2}{2 + 0.25 \times 2} \times 100\%$$

$$= 20\%$$

Hence, the correct option is (c).

16. Consider the frequency modulated signal

$10\cos\left[2\pi \times 10^5 t + 5\sin(2\pi \times 1500t) + 7.5\sin(2\pi \times 1000t)\right]$ with carrier frequency of 10^5 Hz.

The modulation index is

(a) 12.5 (b) 10

(c) 7.5 (d) 5 **[2008]**

Solution: (b)

Modulation index

$$m_f = \frac{f}{f_m}$$

where

f = maximum frequency deviation.

f_m = maximum frequency component

given that

f_m = 1500 Hz

Deviation,

Δθ = 5 sin (2π × 1500t) + 7.5 sin (2π × 1000t)

$$\Delta w = \frac{d\theta}{dt} = 5 \times 2\pi \times 1500\cos(2\pi \times 500t)$$

$$+ 7.5 \times 2\pi \times 1000\cos(2\pi \times 1000t)$$

$$\Delta w_{\max} = 2\pi\{7500 + 7500\}$$

$$\delta = \frac{\Delta w_{\max}}{2\pi} = 1500\text{ Hz}$$

$$m_f = \frac{\delta}{f_m} = \frac{15000}{1500} = 10$$

Hence, the correct option is (b).

17. The signal $\cos\omega_c t + 0.5 \cos\omega_m t \text{in}\omega_c t$ is
(a) FM only
(b) AM only
(c) both AM and FM
(d) neither AM nor FM **[2008]**

Solution: (c)

The signal $\cos w_c t + 0.5 \cos w_m t \cos w_c t$ is either AM or narrow-band FM signal.

Hence, the correct option is (c).

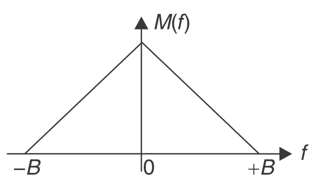
18. A Hilbert transformer is a
(a) linear system
(b) non-casual system
(c) time-varying system
(d) low-pass system **[2007]**

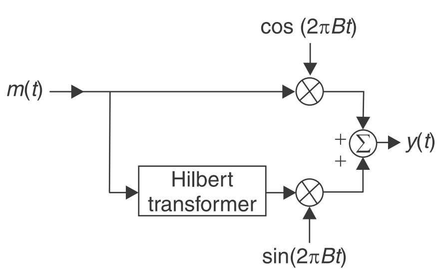
Solution: (a)

The Hilbert transform of $u(t)$ can be thought of as the convolution of $u(t)$ with the function $1/(\pi t)$, which is a linear process.

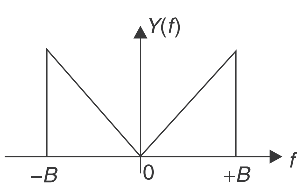
Hence, the correct option is (a).

19. In the following scheme, if the spectrum $M(f)$ of $m(t)$ is as shown, then the spectrum $Y(f)$ of $y(t)$ will be

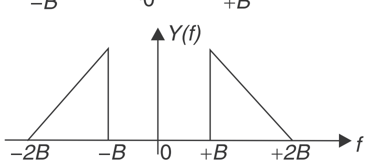




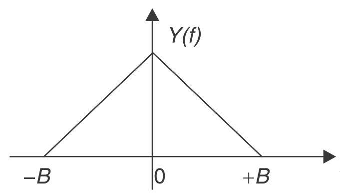
(a)



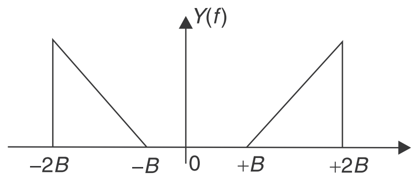
(b)



(c)



(d)



[2007]

Solution: (a)

Hillert transformer is used for SSB generation and sum of quadrature components given LSB.

Hence, the correct option is (a).

20. The diagonal clipping in Amplitude Demodulation (using envelope detector) can be avoided if RC time-constant of the envelope detector satisfies the following condition (here W is message bandwidth and is ω carrier frequency both in rad/sec)

(a) $RC < \frac{1}{W}$ (b) $RC > \frac{1}{W}$

(c) $RC < \frac{1}{\omega}$ (d) $RC > \frac{1}{\omega}$ **[2006]**

Solution: (a)

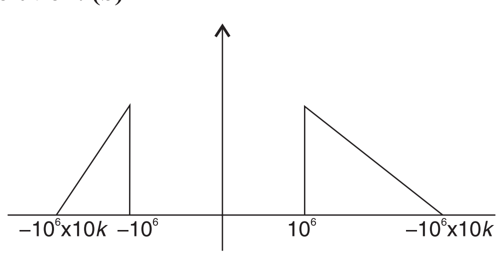
To avoid diagonal clipping detector time constant should follow the condition $\frac{1}{f_c} RC \leq \frac{1}{W}$ where fc denotes carrier frequency and W represents message signal bandwidth.

Hence, the correct option is (a).

21. A message signal with bandwidth 10 kHz is Lower-Side Band SSB modulated with carrier frequency $f_{c1} = 10^6$ Hz. The resulting signal is then passed through a Narrow-Band Frequency Modulator with carrier frequency $f_{c2} = 10^9$ Hz.

The bandwidth of the output would be
(a) 4×10^4 Hz (b) 2×10^6 Hz
(c) 2×10^9 Hz (d) 2×10^{10} Hz **[2006]**

Solution: (b)



From figure

$fm = 10^6$ Hz

(as 10k is small in comparison to 10^6)

BW = $2fm = 2 \times 10^6$ Hz

Hence, the correct option is (b).

Common Data for Questions 11 and 12:

Let $g(t) = p(t)* p(t)$, where * denotes convolution and $p(t) = u(t) - u(t - 1)$ with $u(t)$ being the unit step function.

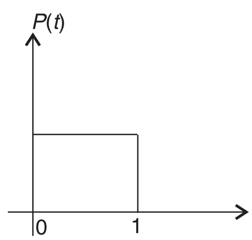
22. The impulse response of filter matched to the signal is given as:

(a) $s(1 - t)$ (b) $-s(1 - t)$
(c) $-s(t)$ (d) $s(t)$ **[2006]**

Solution: (c)

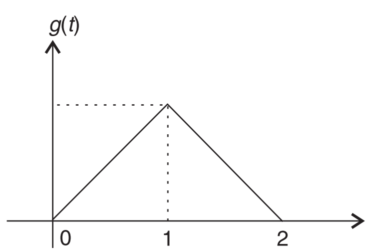
Signal $p(t)$ can be written as

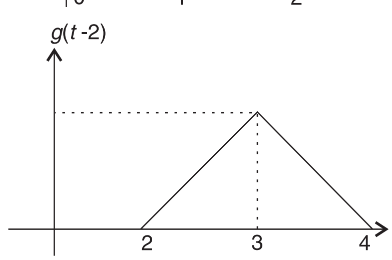
$P(t) = \mu(t) - \mu(t - 1)$



Signal $g(t)$ can be written as

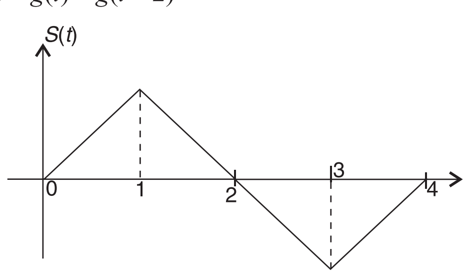
$g(t) = p(t) * p(t)$





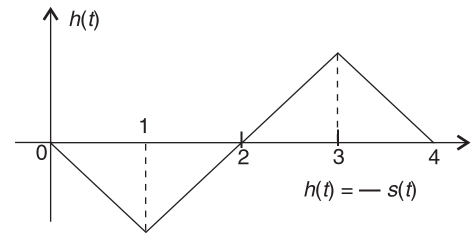
Then signal $s(t)$ can be written as

$s(t) = g(t) - g(t - 2)$



Impulse response of match filter $h(t) = s(T - t)$

$h(t) = -s(t)$



Hence, the correct option is (c).

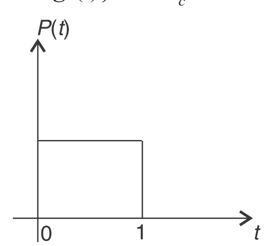
23. An Amplitude Modulated signal is given as in the interval . One set of possible values of the modulating signal and modulation index would be

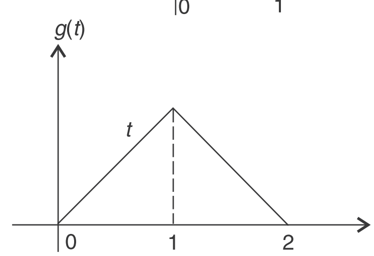
(a) t, 0.5 (b) t, 1.0
(c) t, 2.0 (d) t^2, 0.5 **[2006]**

Solution: (a)

Given signal

$X_{AM}(t) = 100 \{P(t) + 0.5g(t)\} \cos w_c t$





$s(t) = 100 \{1 + m(t)\} \cos w_c t, p(t) = 1$

$m(t) = 0.5g(t)$, comparing AM equation with

i) = $0.5t$

modulating signal = t {from equation of line}

modulation index = 0.5

Hence, the correct option is (a).

Common Data for Questions 24 and 25

Consider the following Amplitude Modulated (AM) signal, where $f_m < B$

24. The average side-band power for the AM signal given above is

(a) 25 (b) 12.5
(c) 6.25 (d) 3.125 **[2006]**

Solution: (c)

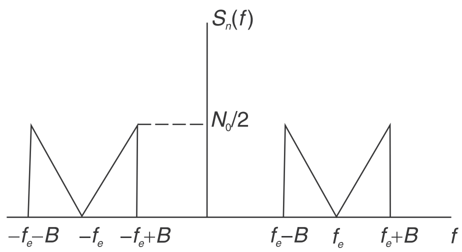
Power of signal will be given by

$$P_s = \frac{P_C m_a^2}{2} = \frac{A_C^2}{2} \times \frac{m_a^2}{2}$$

$$P_s = \frac{100}{2} \times \frac{0.25}{2} = 6.25 \text{ watt}$$

Hence, the correct option is (c).

25. The AM signal gets added to a noise with Power Spectral Density $Sn(f)$ given in the figure below. The ratio of average sideband power to mean noise power would be:



(a) $\dfrac{25}{8\,N_0B}$ (b) $\dfrac{25}{4\,N_0B}$

(c) $\dfrac{25}{2\,N_0B}$ (d) $\dfrac{25}{N_0B}$ **[2006]**

Solution: (b)

Power of signal will given by

$$P_s = \frac{25}{4}$$

$P_N = N_0B$

$$\therefore SNR = \frac{P_S}{P_N} = \frac{25}{4N_0B}$$

Hence, the correct option is (b).

26. A device with input $x(t)$ and output $y(t)$ is characterized by: $y(t) = x^2(t)$.

An FM signal with frequency deviation of 90 kHz and modulating signal bandwidth of 5 kHz is applied to this device. The bandwidth of the output signal is

(a) 370 kHz (b) 190 kHz

(c) 380 kHz (d) 95 kHz **[2005]**

Solution: (a)

f_m = message signal frequency

Δf = frequency deviation

$$\text{BW} = 2(\Delta f + fm) = 2(180 + 5)$$
$$= 370 \text{ kHz}$$

Note: When *fm* signal is applied to doubles frequency derivation doubles but *fm* remains the same.

Hence, the correct option is (a).

27. A carrier is phase modulated (PM) with frequency deviation of 10 kHz by a single tone frequency of 1 kHz. If the single tone frequency is increased to 2 kHz, assuming that phase deviation remains unchanged, the band width of the PM signal is

(a) 21 kHz (b) 22 kHz

(c) 42 kHz (d) 44 kHz **[2005]**

Solution: (d)

For phase modulated signal, phase deviation can be given as

$\Delta\varphi = mf$

m_f remains the same for both cases

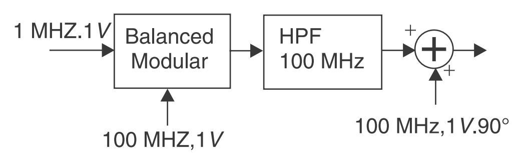
$$\frac{\Delta f}{2} = \frac{10}{1}$$

$\Delta f = 20$

$$\text{BW} = 2(\Delta f + f_m) = 2(20 + 2)$$
$$= 44 \text{ k}$$

Hence, the correct option is (d).

28. A 100 MHz carrier of 1V amplitude and a 1 MHz modulating signal of 1 V amplitude are fed to a balance modulator. The output of the modulator is passed through an ideal high-pass filter with cut-off frequency of 100MHz. The output of the filter is added with 100MHz signal of 1V amplitude and 90^0 phase shift as shown in the figure. The envelope of the resultant signal is



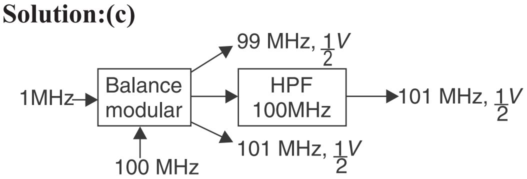
(a) constant

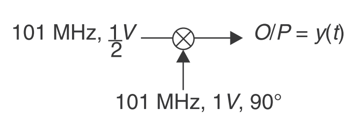
(b) $A \sin\{\omega_c t + \text{km}(t)\}$

(c) $\sqrt{5/4 - \sin\left(2\pi \times 10^6 t\right)}$

(d) $\sqrt{5/4 + \cos\left(2\pi \times 10^6 t\right)}$ **[2004]**

Solution:(c)





Given

$$y(t) = \frac{1}{2}\cos\left\{2\pi \times 101 \times 10^6\right\}t + \sin\left\{2\pi \times 100 + 10^6 t\right\}$$

$$=\frac{1}{2}\begin{Bmatrix}\cos(2\pi\times100\times10^{6})t\\ \cos10^{6}t2\pi-\sin(2\pi\times100\times10^{6})\end{Bmatrix}$$

$$t\sin2\pi\times10^{6}t+\sin2\pi\times100\times10^{6}t$$

$$=\frac{1}{2}\cos2\pi\times100\times10^{6}t[\cos2\pi\times10^{6}t]$$

$$+\frac{1}{2}\sin2\pi\times100\times10^{6}t[-\sin2\pi\times10^{6}+2]$$

$= A\cos 2\pi \times 10^6 t + B \sin\cos 2\pi \times 10^6 t$

Envelope

$$=\sqrt{A^2+B^2}$$

$$=\sqrt{\frac{1}{4}\cos^{2}2\pi\times10^{6}t+\frac{1}{4}(\sin2\pi\times10^{6}t-2)}$$

Envelope

$$=\sqrt{\frac{5}{4}-\sin2\pi\times10^{6}t}$$

Hence, the correct option is (c).

29. Two sinusoidal signals of same amplitude and frequencies 10 kHz and 10.1 kHz are added together. The combined signal is given to an ideal frequency detector. The output of the detector is

(a) 0.1 kHz sinusoid
(b) 20.1 kHz sinusoid
(c) a linear function of time
(d) a constant **[2004]**

Solution:(a)

Let denote the output frequency as

$$s(t)=A\cos\{2\pi\times10\times10^{3}t\}+A\cos\{2\pi\times10.kt\}$$

$$T_1=\frac{1}{10k}=100\mu s$$

$$T_2=\frac{1}{10.1}=99\mu s$$

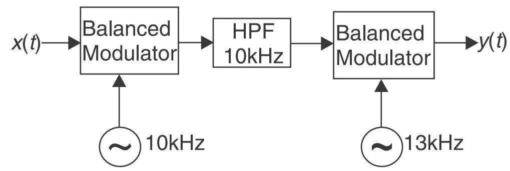
$$\frac{T_1}{T_2}=\text{rational}$$

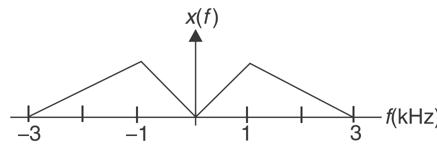
$\therefore s(t)$ will be periodic with period RCM of T_1 and T_2 = 9900 μs, 10,000 μs

$\therefore$ frequency of elector = 0.1 kHz.

Hence, the correct option is (a).

30. Consider a system shown in figure. Let $X(f)$ and $Y(f)$ denote the Fourier transforms of $x(t)$ and $y(t)$, respectively. The ideal HPF has the cutoff frequency 10 kHz.





The positive frequencies where $Y(f)$ has spectral peaks are

(a) 1 kHz and 24 kHz
(b) 2 kHz and 24 kHz
(c) 1 kHz and 14 kHz
(d) 2 kHz and 14 kHz **[2004]**

Solution:(b)

Peaks of $x(t)$ are at $1k$ and $-1k$ initially output of balanced modulator $= f_c \pm 1k$ and $f_c \pm -1k$

$f_c \pm 1k = 11k, 9k$

$f_c \pm 1k = 9k, 11k$

Output of HPF with $f_c = 10k$ will be $11k$ frequency component.

$y(f) = 13k \pm 11k$

$= 24k$ and $2k$

Hence, the correct option is (b).

31. A DSB-SC signal is to be generated with a carrier frequency $f_c = 1$ MHz using a non-linear device with the input–output characteristic $V_0 = a_0 v_i + a_1 v_i^3$ where a_0 and a_1 are constants. The output of the non-linear device can be filtered by an appropriate band-pass filter. Let $V_i = A_c^i \cos\left(2\ f_c^i t\right) + m(t)$ where $m(t)$ is the message signal. Then the value of f_c^i (in MHz) is

(a) 1.0 (b) 0.333
(c) 0.5 (d) 3.0 **[2003]**

Solution: (c)

Given

$V_0 = a_0 v_i + a_1 v_i^3$

$$= a_0[A_c^1\cos w_c^i t]+a_0 m(t)+a_1 A_c^{c^3}\cos 3w_c^i t$$

$$+a_1 m^3(t)+3a_1 m^2(t)A_c^i\cos w_c^i t+3a_1 A_c^i\cos^2 w_c^i t$$

For DSB SC we are concerned with only last term

$m(t)\cos w_c t \rightarrow f_c = 1$ MHz

For $\cos^2$ term $2w_c^i = 2\pi\times 1\,\text{MHz}$

Hence, the correct option is (c).

Common Data for Questions 32 and 33

Let $m(t) = [(4\pi\times 10^3)t]$ be the message signal and $c(t) = 5\cos[(2\pi\times 10^6)t]$ be the carrier.

32. $c(t)$ and $m(t)$ are used to generate an AM signal. The modulation index of the generated AM signal is 0.5. Then the quality $\dfrac{\text{Total sideband power}}{\text{Carrier power}}$ is

(a) 1/2 (b) 1/4
(c) 1/3 (d) 1/8 **[2003]**

Solution: (d)

Pc= carrier power

m_a = modulation index

A_c = amplitude of Carrier signal

$$P_1 = P_C\left[1+\frac{m_a^2}{2}\right], P_C = \frac{A_C^2}{2}$$

$$\text{Sideband power} = \frac{A_C^2}{2}.\frac{m_a^2}{2}$$

$$\frac{P_S}{P_C} = \frac{\left(\frac{A_C^2 m_a^2}{4}\right)}{\left(\frac{A_C^2}{2}\right)} = \frac{1}{8}$$

Hence, the correct option is (d).

33. $c(t)$ and $m(t)$ are used to generate an FM signal. If the peak frequency deviation of the generated FM signal is three times the transmission bandwidth of the AM signal, then the coefficient of the term $\cos[2\pi(1008 \times 10^3 t)]$ in the FM signal (in terms of the Bessel coefficients) is

(a) $5J_4(3)$ (b) $\frac{5}{2}J_8(3)$
(c) $\frac{5}{2}J_8(4)$ (d) $5J_4(6)$ **[2003]**

Solution: (d)

Given

FM modulated waveform is given by

$$X_{FM}(t) = A_C \sum_{n=-\infty}^{\infty} j_n(\beta)\cos(w_c + nw_m)t$$

BW = $2w_n$, $\Delta w = 3 \times Bw = 6w_n$

$w_c + nw_n = 2\pi \times 1008 \times 10^3$ (as given in a question)
$= 2\pi \times 1008 \times 10^3$

$\therefore \eta = 4$

Bessel coefficient $5j_4(6)$

Hence, the correct option is (d).

34. Choose the correct one from among the alternative a, b, c, d after matching an item in Group 1 with the most appropriate item in Group 2.

Group-1

P. Ring modulator Q. VCO
R. Foster-Seely discriminator S. Mixer

Group-2

1. Clock recovery
2. Demodulation of FM
3. Frequency conversion
4. Summing the two inputs
5. Generation of FM
6. Generation of DSB-SC

(a) P – 1; Q – 3; R – 2; S – 4
(b) P – 6; Q – 5; R – 2; S – 3
(c) P – 6; Q – 1; R – 3; S – 2
(d) P – 5; Q – 6; R – 1; S – 3 **[2003]**

Solution:(b)

Combination is given as followed

P. Ring modulator	Generation of DSB-SC
Q. VCO	Generation of FM
R. Foster-Seely discriminator	Demodulation of FM
S. Mixer	Frequency conversion

Hence, the correct option is (b).

35. A superheterodyne receiver is to operate in the frequency range 550–1650 kHz, with the intermediate frequency of 450 kHz. Let $R = \dfrac{C_{max}}{C_{min}}$ denote the required capacitance ratio of the local oscillator and I denote the image frequency (in kHz) of the incoming signal. If the receiver is tuned to 700 kHz, then

(a) $R = 4.41$, $I = 1600$
(b) $R = 2.10$, $I = 1150$
(c) $R = 3.0$, $I = 1600$
(d) $R = 9.0$, $I = 1150$ **[2003]**

Solution:(a)

f_{IF} = intermediate frequency

f_{is} = Image frequency

$$R = \frac{C_{max}}{C_{min}} = \left(\frac{f_{max}}{f_{min}}\right)^2 = \left(\frac{1650+450}{550+450}\right)^2 = 4.41$$

$1 = f_s + 2f_{if}$
$= 700 + 2 \times 450 = 1600$

Hence, the correct option is (a).

36. An angle-modulated signal is given by

$S(t) = \cos 2\pi(2 \times 10^6 t + 30\sin 150t + 40\cos 150t)$.

The maximum frequency and phase deviations of $s(t)$ are

(a) 10.5 kHz, 140π rad
(b) 6 kHz, 80π rad

(c) 10.5 kHz, 100π rad
(d) 7.5 kHz, 100π rad **[2002]**

Solution:(d)

Given

$s(t) = \cos[2\neq \times 10^6 \times 2t + 2\neq \times 30 \sin 150t + 2\neq \times 40 \cos 150t]$

$$\frac{d\theta}{dt} = 2\pi \times 2\times 10^6 + 2\pi \times 4500 \cos$$

$$150t + 2\pi \times 6000(-\sin 150t)$$

$$\Delta w = w_1 - w_c = 2\pi[4500^2 + 6000^2]^{\frac{1}{2}}$$

$\Delta w = 2\pi \times 7.5k$ rad/sec

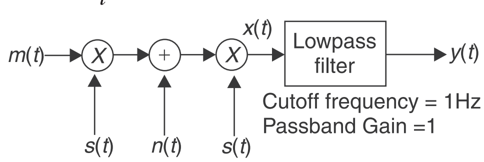
$$\Delta f = \frac{\Delta w}{2\pi} = 7.5\,\text{kHz}$$

$$\Delta\phi = 2\pi\sqrt{30^2 + 40^2}$$

$$\Delta\phi = 100\pi$$

Hence, the correct option is (d).

37. In the figure $m(t) = \dfrac{2\sin 2\pi t}{t}$, $s(t) = \cos 200\pi t$ and $n(t) = \dfrac{\sin 199\pi t}{t}$. The output $y(t)$ will be



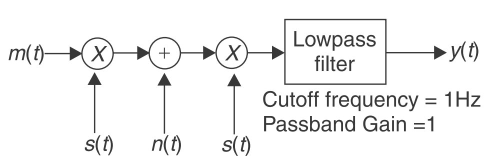
(a) $\dfrac{\sin 2\pi\,\text{t}}{\text{t}}$

(b) $\dfrac{\sin 2\pi\,\text{t}}{\text{t}} + \dfrac{\sin \pi\,\text{t}}{\text{t}} \cos 3\pi\,\text{t}$

(c) $y(t) = \dfrac{\sin 2t}{2t} + \dfrac{\sin 0.5\pi t\ +\ \cos 1.5\pi t}{t}$

(d) $\dfrac{\sin 2\pi\,\text{t}}{\text{t}} + \dfrac{\sin 0.5\pi\,\text{t}}{\text{t}} \cos 0.75\pi\,\text{t}$ **[2002]**

Solution:



Input at LPF $x(t) = [m(t).s(t) + n(t)]\, x(t)$

$x(t) = m(t).s^2(t) + n(t)\, s(t)$

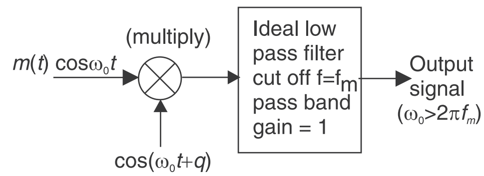
$$x(t) = \frac{1}{t}[\sin 202\pi t + \sin 199\pi t]\cos 200\pi t$$

$$\text{Frequency present } = \frac{402}{2}, \frac{2}{2}.\frac{2}{2}, \frac{398}{2}, \frac{1}{2}, \frac{399}{2}$$

Now passing from LPF with $f_c = 1$ Hz

$$y(t) = \frac{\sin 2t}{2t} + \frac{\sin 0.5\pi t\ +\ \cos 1.5\pi t}{t}$$

38. A message $m(t)$ band limited to the frequency f_m has a power of P_m. The power of the output signal in the figure is



(a) $\dfrac{P_m\cos\theta}{2}$ (b) $\dfrac{P_m}{4}$

(c) $\dfrac{P_m\sin^2\theta}{4}$ (d) $\dfrac{P_m\cos\theta}{2}$ **[2000]**

Solution: (d)

Output signal $= \dfrac{m(t)}{2}\cos\theta$

$m(t)$ has power P_m

Power of a $m(t) \to a^2.P_m$

Here $a = \dfrac{1}{2}\cos\theta$

Power of output signal $= \dfrac{P_m}{4}\cos^2\theta$

Hence, the correct option is (d).

39. The Hilbert transform of $\cos\omega_1 t + \sin\omega_2 t$ is

(a) $\sin\omega_1 t - \cos\omega_2 t$ (b) $\sin\omega_1 t + \cos\omega_2 t$
(c) $\cos\omega_1 t - \sin\omega_2 t$ (d) $\sin\omega_1 t + \sin\omega_2 t$ **[2000]**

Solution: (a)

A Hilbert transform produces a 90o phase shift

$$\sin\omega_2 t \xleftrightarrow{H.T} -\cos\omega_2 t$$

$$\cos\omega_1 t \xleftrightarrow{H.T} \sin\omega_1 t$$

Hence, the correct option is (a).

40. In an FM system, a carrier of 100 MHz is modulated by a sinusoidal signal of 5 kHz. The bandwidth by Carson's approximation is 1 MHz. If $y(t)$ = (modulated wave form)^3, then by using Carson's approximation, the bandwidth of $y(t)$ around 300 MHz and the spacing of spectral components are, respectively.

(a) 3 MHz, 5 kHz (b) 1 MHz, 15 kHz
(c) 3 MHz, 15 kHz (d) 1 MHz, 5 kHz **[2000]**

Solution: (a)

Frequency modulated waveform is given by

$x(t) = A_C \cos[w_c t + \beta \sin w_m t]$

$y(t) = k \cos[3w_c t + 3\beta \sin w_m t]$

After passing from $y(t) = [x(t)]_3$

$\beta' = 3\beta$ β= modulation index

$\Delta f' = 3\times\Delta f = \Delta f >> f_m$ Δf = frequency deviation

$\text{BW} = 2\nabla f' \times 3 = 3\,\text{MHz}$

$f_m = 5$ kHz

Hence, the correct option is (a).

41. An FM signal with a modulation index 9 is applied to a frequency tripler. The modulation index in the output signal will be

(a) 0 (b) 3
(c) 9 (d) 27 **[1996]**

Solution: (d)

In a frequency multiplier circuit, the modulation index is multiplied by η. So

$\beta'_{FM} = \eta\beta_{FM}$ β_{FM}= modulation index

$\beta' = 3 \times 9 = 7$

Hence, the correct option is (d).

42. Match List-I with List-II and select the correct answer using the code given below the lists:

List-I	**List-II**
A. SSB	1. Envelope detector
B. AM	2. Integrate and dump
C. BPSK	3. Hilbert transform
D.	4. Ratio detector
	5. PLL

Codes:

	A	B	C
(a)	3	1	2
(b)	3	2	1
(c)	2	1	3
(d)	1	2	3

[1994]

Solution:(a)

Hibert transform → SSB

AM → Envelope Detector

BPSK → Integrate and dump

Hence, the correct option is (a).

43. Which of the following demodulator(s) can be used for demodulating the signal

$x(t) = 5(1 + 2\cos 200\pi t)\cos 20000\pi t$

(a) envelope demodulator
(b) square-law demodulator
(c) synchronous demodulator
(d) none of the above **[1993]**

Solution:(c)

Given that $x(t) = 5(1 + 2\cos 200\,\pi t)\cos 20000\,\pi t$,

standard equation for AM signal is $X_{AM}(t) = AC(1 + m \cos w_m t)\cos w_c t$.

By comparing $m = 2$,

this is the case of over modulation. So, synchronous modulator only can detect.

Hence, the correct option is (c).

44. A superheterodyne radio receiver with an intermediate frequency of 455 kHz is tuned to a station operating at 1200 kHz. The associated image frequency is ____ kHz. **[1993]**

Solution:

IF = intermediate frequency

f_{is}= Image frequency

$f_{si} = f_s + 2IF$

$F_{si} = 1200 + 2(455) = 2110$ KHz.

45. The maximum power efficiency of an AM modulator is

(a) 25% (b) 50%
(c) 33% (d) 100% **[1992]**

Solution: (b)

Power efficiency of AM modulator is given by

$$\eta = \frac{\mu^2}{2+\mu^2},\ \mu_{\max} = 1,$$

where is modulation index

$$\eta_{\max} = \frac{1}{2+1} = \frac{1}{3}$$

Hence, the correct option is (b).

46. A 4 GHz carrier is DSB--SC modulated by a low pass message signal with maximum frequency of 2 MHz. The resultant signal is to be ideally sampled. The minimum frequency of the sampling impulse train should be

(a) 4 MHz (b) 8 MHz
(c) 8 GHz (d) 8.004 GHz
[1990]

Solution: (b)

Fc = carrier frequency

f_m = message signal frequency

f_L = lower frequency

f_H = higher frequency

f_c = 4 GHz = 4000 MHz

F_m = 2 MHz, $f_H = f_c + f_m$

$F_L = f_c - f_m$

$\text{BW} = F_H - F_L = 2f_m$

BW = 2 × 2 = 4 MHz

Hence, the correct option is (b).

47. In commercial TV transmission in India, picture and speech signals are modulated respectively as

(Picture)		(Speech)
(a) VSB	and	VSB
(b) VSB	and	SSB

(c) VSB and FM

(d) FM and VSB **[1990]**

Solution: (c)

In commercial TV transmission in India, picture signal is modulated using VSB (vestigial sideband) modulation and speech or audio signal is modulated using FM modulation.

Hence, the correct option is (c).

48. A carrier $A_c \cos\omega_c t$ is frequency modulated by a signal $E_m \cos\omega_m t$. The modulation index is mf. The expression for the resulting FM signal is

(a) $A_c \cos\left[\omega_c t + m_f \sin\omega_m t\right]$

(b) $A_c \cos\left[\omega_c t + m_f \cos\omega_m t\right]$

(c) $A_c \cos\left[\omega_c t + 2\pi m_f \sin\omega_m t\right]$

(d) $A_c \cos\left[\omega_c t + \dfrac{2\pi m_f E_m}{\omega_m} \cos\omega_m t\right]$ **[1989]**

Solution:(a)

w_c = carrier frequency

w_m = carrier frequency

mf = modulation index

K_f = frequency sensitivity

$$X_{Fm}(t) = A_C \cos[w_c t + k_f \int m(t)dt]$$

$$X_{Fm}(t) = A_C \cos[w_c t + k_f \int E_m \cos w_n t dt]$$

$$x_{Fm}(t) = A_C \cos\left[w_c t + \frac{k_f E_m}{w_m} \sin w_m t\right]$$

Modulation index

$$m_f = \frac{k_f E_m}{w_m}$$

$X_{Pm}(t) = A_C \cos[w_c t + m_f \sin w_m t]$

Hence, the correct option is (a).

49. Which of the following schemes suffer(s) from the threshold effect?

(a) AM detection using envelope detection

(b) AM detection using synchronous detection

(c) FM detection using a discriminator

(d) SSB detection with synchronous detection **[1989]**

Solution: (c)

FM detection using a discriminator suffers from the threshold effect.

Hence, the correct option is (c).

50. A signal $x(t) = 2\cos(\pi .10^4 t)$ volts is applied to an FM modulator with the sensitivity constant of 10 kHz/volt. Then the modulation index of the FM wave is

(a) 4 (b) 2

(c) 4/π (d) 2/π **[1989]**

Solution:(a)

A_m = amplitude of message signal

w_m = carrier frequency

K_f = frequency sensitivity

$$\text{Modulation index } = m_f = \frac{k_f A_m}{w_m} = \frac{\Delta w}{w_m}$$

$$m_1 = \frac{k_f A_m}{w_m} = \frac{2\pi \times 10 \times 10^3 \times 2}{\pi \times 10^4} = 4$$

Hence, the correct option is (a).

51. In a super heterodyne AM receiver, the image channel selectivity is determined by

(a) The preselector and RF stages

(b) The preselector, RF and IF stages

(c) The IF stages

(d) All the stages **[1987]**

Solution:(a)

The image rejection should be achieved before IF stage because once it enters into IF amplifier, it becomes impossible to remove it from wanted signal. So image channel selectivity depends upon pre-selector and RF amplifiers only. The IF helps in rejection of adjacent channel frequency and not image frequency.

Hence, the correct option is (a).

52. In a radar receiver the antenna is connected to the receiver through a waveguide. Placing the preamplifier on the antenna side of the waveguide rather than on the receiver side leads to

(a) a reduction in the overall noise figure.

(b) a reduction in interference.

(c) an improvement in selectivity characteristics

(d) an improvement in directional characteristics **[1987]**

Solution:(a)

A pre-amplifier is a very large gain amplifier with low noise figure. Noise figure of cascaded amplifier can be given as:

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots \frac{F_n - 1}{G_1 G_2 G_3 G_{n-1}}$$

Therefore, placing the pre-amplifier on the antenna side of the wave guide will result in the reduction of overall noise figure of the system.

Hence, the correct option is (a).

Five-marks Questions

1. A signal $x(t) = \exp(-2\pi Bt)\,u(t)$ is the input to an ideal low pass filter with bandwidth B Hz. The output is denoted by $y(t)$. Evaluate

$$\int_{-\infty}^{\infty} [y(t) - x(t)]^2\, dt. \qquad [1988]$$

Solution: given that, $x(t) = \exp(-2\pi Bt)\,u(t)$ taking fourier transform on both side,

$$x(f) = \frac{1}{2\pi B + j\omega} = \frac{1}{2\pi(B + jf)}$$

So, $H(f) = 1$ for $-B'' f'' B$

$= 0$ for otherwise

for $-B'' t'' B$

$= 0$ for otherwise

$$Y(f) = X(f).H(f) = \frac{1}{2\pi(B + jf)} \text{ for } -B'' t'' B$$

$= 0$ for otherwise

We know that,

$$\int_{-\infty}^{\infty} y\ (t)dt \quad \int_{-\infty}^{\infty} |y(t)|\ dt \quad \int \frac{1}{B^2 \quad f^2} df \quad \frac{1}{\ }$$

$$\int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} |x(f)^2 df| = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \frac{1}{B^2 + f^2} df = \frac{1}{4\pi B}$$

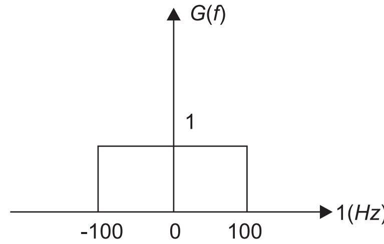
$$\int_{-\infty}^{\infty} x(t)y(t)dt = \int_{-\infty}^{\infty} x(t)y^*(t)dt = \frac{1}{4\pi} \int_{-B}^{B} \frac{1}{B^2 + t^2} dt = \frac{1}{8\pi B}$$

$$\int_{-\infty}^{\infty} [y(t) - x(t)]^2 dt$$

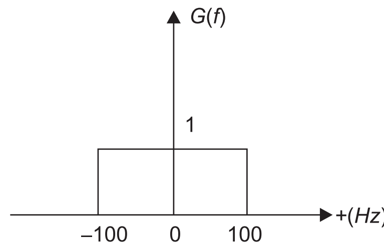
$$= \int_{-\infty}^{\infty} y^2(t)dx + \int_{-\infty}^{\infty} x^2(t)dt - 2\int_{-\infty}^{\infty} x(t).y(t)dt$$

$$= \frac{1}{8\pi B} + \frac{1}{4\pi B} - 2.\frac{1}{8\pi B} = \frac{1}{8\pi B}$$

2. A baseband signal $g(t)$ band limited to 100 Hz modulates a carrier of frequency f_c Hz. The modulated signal $g(t)\cos 2\pi f_c t$ is transmitted over a channel whose input x and output y are related by $y = 2x + x^2$. The spectrum of $g(t)$ is shown in figure. Sketch the spectrum of the transmitted signal and the spectrum of the received signal.



Solution:



$x(t) = g(t)\,.\,\cos(2\pi f_c t)$

$$x(t) = \frac{1}{2}[G(f - f_c) + G(f + f_c)]$$

Received signal

$y(t) = 2x + x^2 = y_1(t) + y_2(t)$

$y_1(t) = 2x(t) \leftrightarrow 2x(f) = y_1(f)$

$$y_2(f) = x^2(t) = g^2(t).\cos^2 2\quad fct \qquad \cos 4\quad f\ t$$

$$2\pi\ fct = \frac{g^2(t)}{2} + \frac{g^2(t)}{2}\cos 4\pi f_c t$$

$$= y_3(t) + y_4(t)$$

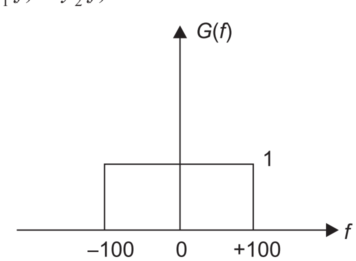
$$y_3(t) = \frac{g^2(t)}{2} = \frac{1}{2}[G(g) * G(f)]$$

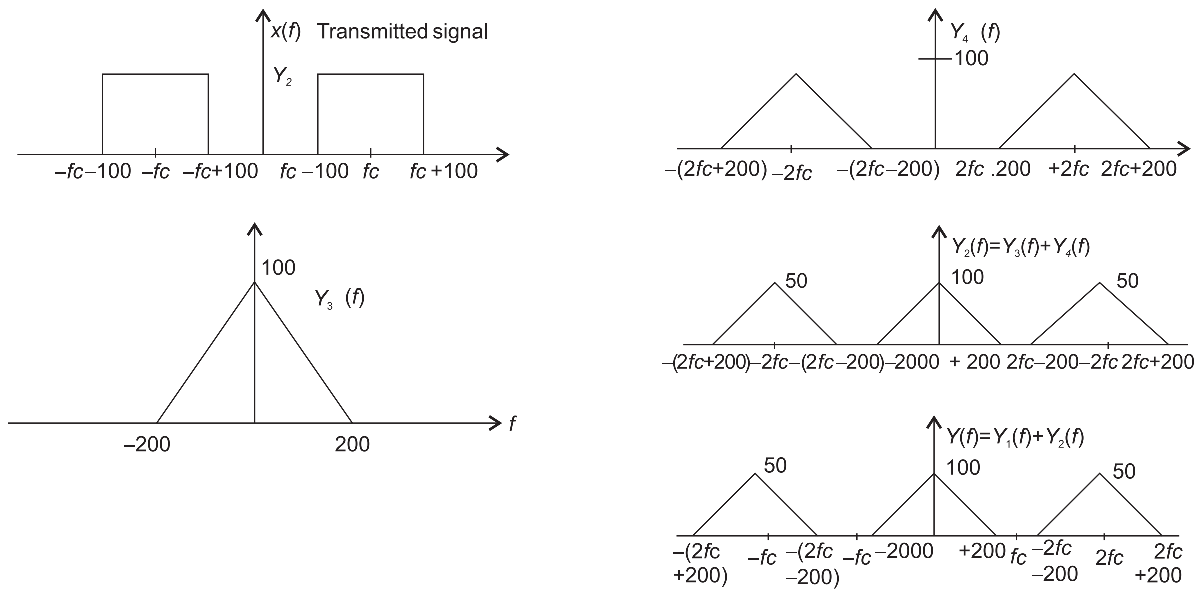
$$x_4(t) = g^2(t)\cos(2\pi\, 2 f_c t) = y_3(t).\cos(2\pi\, 2 f_c t)$$

$$y_4(t) = \frac{1}{2}[y_3(f - 2f_c) + y_3(f + 2f_c)$$

$y_2(f) = y_3(f) + y_4(f)$

$y_2(f) = y_1(f) + y_2(f)$.





x(f) Transmitted signal
Y2
–fc–100 –fc –fc+100 fc –100 fc fc +100
100
Y3 (f)
–200
200
f
Y4 (f)
100
–(2fc+200) –2fc –(2fc–200) 2fc .200 +2fc 2fc+200
Y2(f)=Y3(f)+Y4(f)
50
100
50
–(2fc+200)–2fc–(2fc–200)–2000 + 200 2fc–200–2fc 2fc+200
Y(f)=Y1(f)+Y2(f)
50
100
50
–(2fc +200)
–fc
–(2fc –200)
–fc
–2000
+200
fc
–2fc –200
2fc
2fc +200

Chapter 2

Random Signals and Noise

One-mark Questions

1. Let the random variable X represent the number of times a fair coin needs to be tossed till two consecutive heads appear for the first time. The expectation of X is ______. **[2015]**

Solution: Given that X is a random variable representing "The number of times a fair coin needs to be tossed till two consecutive heads appear for the first time".

$\therefore$ The possible values of X are 2, 3, 4, 5, ……

$$P(X=2) = P(HH) = P(H)\,P(H) = \frac{1}{2}\times\frac{1}{2}=\frac{1}{2^2}$$

$$P(X=3) = P(THH) = P(T)\,P(H)\,P(H)$$

$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{2^3}$$

$$P(X=4) = P(TTHH) = P(T)\,P(T)\,P(H)\,P(H)$$

$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{2^4}$$

$\therefore$ Expectation of $X = E(x) = \sum_{x=2}^{\infty} x\,P(X=x)$

$$= 2P(X=2) + 3\,P(X=3) + 4P(X=4) + \ldots\infty$$

$$=2\times\frac{1}{2^2}+3\times\frac{1}{2^3}+4\times\frac{1}{2^4}+\ldots.\infty$$

$$\therefore\quad E(X)=\frac{1}{2^2}\left[2+3\times\frac{1}{2}+4\times\frac{1}{2^2}+5\times\frac{1}{2^3}+\ldots\infty\right]\quad(1)$$

Consider $2+3\times\frac{1}{2}+4\times\frac{1}{2^2}+5\times\frac{1}{2^3}+\ldots.\infty$

which is an arithmetic geometric progression with $a = 2$, $d = 1$ and $r=\frac{1}{2}$

$\therefore$ The sum of infinite number of terms in AGP

$$=\frac{a}{1-r}+\frac{dr}{(1-r)^2}$$

$$=\frac{2}{\left(1-\frac{1}{2}\right)}+\frac{1\times\left(\frac{1}{2}\right)}{\left(1-\frac{1}{2}\right)^2}$$

$$=4+\frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{4}\right)}$$

$$= 4 + 2$$
$$= 6$$

$\therefore$ Substituting in (1), we have

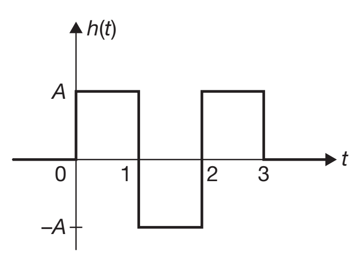
$$E(X)=\frac{1}{4}\times 6=\frac{3}{2}=1.5$$

Hence, the correct Answer is (1.5).

2. A zero mean white Gaussian noise having power spectral density $\frac{N_0}{2}$ is passed through an LTI filter whose impulse response $h(t)$ is shown in the figure. The variance of the filtered noise at $t = 4$ is **[2015]**

(A) $\frac{3}{2}A^2N_0$ (B) $\frac{3}{4}A^2N_0$

(C) A^2N_0 (D) $\frac{1}{2}A^2N_0$



Solution: For a zero mean white Gaussian noise variance is infinite.

So for finite variance of white Gaussian noise we should have some BIBO system.

$$\sigma^2=\int_{-\infty}^{+\infty}\frac{N_0}{2}\,|H(f)|^2\,df$$

By Parseval's relation

$$=\frac{N_o}{2}\int_{-\infty}^{+\infty}|h(t)|^2\,dt$$

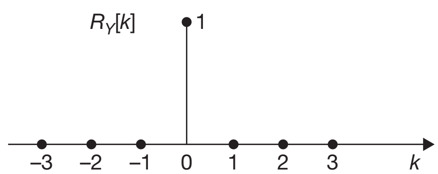
$$=\frac{N_o}{2}\left[\int_0^1|h(t)|^2\,dt+\int_1^2|h(t)|^2\,dt+\int_2^3|h(t)|^2\,dt\right]$$

$$=\frac{N_o}{2}\left[A^2+A^2+A^2\right]\ =\frac{3A^2N_o}{2}$$

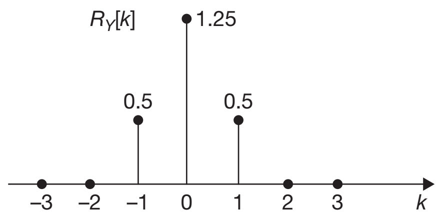
Hence, the correct option is (A).

3. $\{X_n\}_{n=-\infty}^{n=\infty}$ is an independent and identically distributed (i.i.d) random process with X_n equally likely to be +1 or –1. $\{Y_n\}_{n=-\infty}^{n=\infty}$ is another random process obtained as $Y_n = X_n + 0.5\,X_{n-1}$. The autocorrelation function of $\{Y_n\}_{n=-\infty}^{n=\infty}$, denoted by $R_y[k]$, is **[2015]**

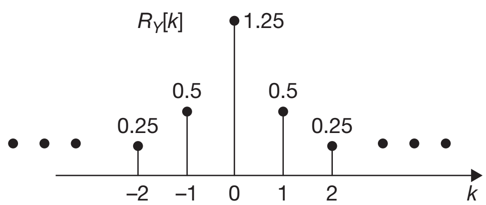
(A)



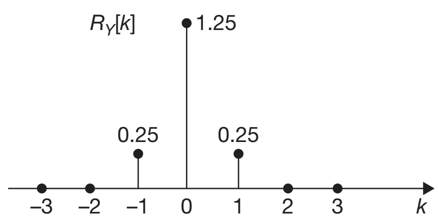
(B)



(C)



(D)



Solution: $R_y(k) = R_y(n, n + k)$

$= E[Y[n] \cdot Y[n + k)]$

$Y[n] = X[n] + 0.5\,x[n – 1]$

$R_y(k) = E[(X[n] + 0.5\,X[n – 1]) \cdot (x[n + k]$
$+ 0.5\,x[n + k – 1])]$

$= E[x[n]\,x[n + k] + 0.5\,x[n – 1]\,x[n + k]$
$+ 0.5\,x[n]\,x[n + k – 1] + 0.25\,x[n – 1]\,x[n + k – 1]$

$= R_x(k) + 0.5\,R_x[k + 1] + 0.5\,R_x(k – 1) + 0.25\,R_x(k)$

$R_y(k) = 1.25\,R_x(k) + 0.5\,R_x(k + 1) + 0.5\,R_x(k – 1)$

$R_x(k) = E[x[n]\,x[n + k]]$

for $k = 0$ $E[x^2[n]] = I^2\,\frac{1}{2} + (–1)^2\,\frac{1}{2} = 1$
For $k \neq 0$

$R_x(k) = E[x[n]] \cdot E[X(n + k)] = 0$

So $R_y(k) = 1.25\,R_x(k) + 0.5\,R_x(k – 1) + 0.5\,R_x(k + 1)$

For $R_y(0) = 1.25\,R_x(0) + 0.5\,\cancel{R_x}(–1) + 0.5\,\cancel{R_x}(+ 1)$

$= 1.25$

$R_y(1) = 1.25\,\cancel{R_x}(1) + 0.5\,R_x(0) + 0.5\,\cancel{R_x}(+ 2)$

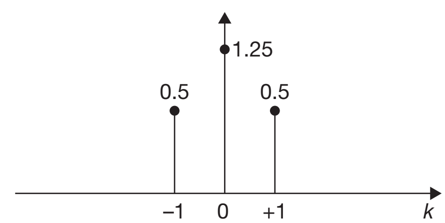
$= 0.5$

$R_y(– 1) = 1.25\,\cancel{R_x}(– 1) + 0.5\,\cancel{R_x}(– 2) + 0.5\,R_x(0)$

$= 0.5$

$R_y(2) = 1.25\,\cancel{R_x}(2) = 0.5\,\cancel{R_x}(1) + 0.5\,\cancel{R_x}(3)$

So all values of $R_y(k)$ are zero except at k =0, 1 and –1. So



Hence, the correct option is (B).

4. Let x_1, x_2, and x_3 be independent and identically distributed random variables with the uniform distribution on [0, 1]. The probability $P\{x_1$ is the largest$\}$ is_________ **[2014]**

Solution:

Probability $P(x_1) = P(x_2) = P(x_3)$

$P_1 + P_2 + P_3 = 1$ {total probability theorem}

$P(x_1) + P(x_2) + P(x_3) = 1$

$3p\,(x_1) = 1$

$P(x_1) = 1/3 = 0.3733$

5. Let X be a random variable which is uniformly chosen from the set of positive odd numbers less than 100. The expectation, $E[X]$, is **[2014]**

Solution:

$$E(x)=\frac{1+2+3+\ldots 99}{50}$$

$$E(n)=\frac{2500}{50}$$

$E(Y) = 50$

6. Let X be a zero mean unit variance Gaussian random variable. $E[|X|]$ is equal to ______. **[2014]**

Solution:

For Gaussian random variables

$$f_x(x) = \frac{1}{\sigma_x\sqrt{2\pi}}\left(\frac{\mu - x}{2\sigma^2}\right)^2$$

for mean (μ) = 0 and variance (σ^2_X) = 1

$$f_x(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$$

$$E[|x|] = \int_{-\infty}^{\infty}(x) f_x(x)dx$$

$$= \int_{-\infty}^{\infty}|x|.\frac{1}{\sqrt{2\pi}}e^{-x^2/2}dx$$

$$= \int_{-\infty}^{0}-x.\frac{1}{\sqrt{2\pi}}e^{-x^2/2}dx + \int_{0}^{\infty}x.\frac{1}{\sqrt{2\pi}}e^{-x^2/2}dx$$

$$= 2\int_{0}^{\infty}x.\frac{1}{\sqrt{2\pi}}e^{-x^2/2}dx + \frac{2}{\sqrt{2\pi}}\int_{0}^{\infty}x.e^{\frac{-x^2}{2}}dx$$

Let $\frac{-x^2}{2} = t$

$$\frac{-2\times dx}{2} = dt$$

Hence $\frac{2}{\sqrt{2\pi}}\int_{0}^{\infty}-e^{-t}dt = 0.8$

$\therefore\ E[|x|] = 0.8$

7. If calls arrive at a telephone exchange such that the rate of arrival of any call is independent of the time of arrival of earlier or future calls, the probability distribution function of the total number of call in a fixed time interval will be

(a) Poisson
(b) Gaussian
(c) Exponential
(d) Gamma **[2014]**

Solution: (a)

Given system follows property of Poisson distribution.

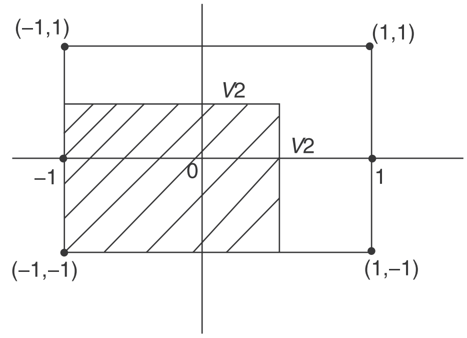
Hence, the correct option is (a)

8. Two independent random variables X and Y are uniformly distributed in the interval $[-1, 1]$. The probability that max $[X, Y]$ is less than 1/2 is

(a) 3/4
(b) 9/16
(c) 1/4
(d) 2/3 **[2013]**

Solution: (b)

$-1 \le x \le 1$ and $-1 \le y \le 1$ is the entire rectangle, region for maximum of $\{x, y\}$ is less than ½ is shaded region inside the rectangle.

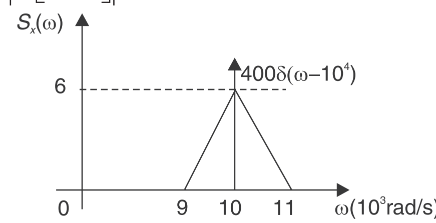


$$P(\max(x, y) < 1/2\} = \frac{\text{Area of shaded region}}{\text{Area of entire rectangle}}$$

$$= \frac{\frac{3}{2}\times\frac{3}{2}}{2\times 2} = \frac{9}{16}$$

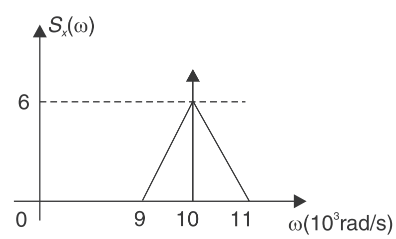
Hence, the correct option is (b)

9. A power spectral density of a real process $X(t)$ for positive frequencies is shown below. The values of $E[X^2(t)]$ and $\left|E\left[X(t)\right]\right|$, respectively, are



(a) $6000/\pi$, 0
(b) $6400/\pi$, 0
(c) $6400/\pi$, $20/(\pi/\sqrt{2})$
(d) $6000/\pi$, $20/(\pi\sqrt{2})$ **[2012]**

Solution: (a)



We know

$$P = E[x^2(t)] = 2\times\frac{1}{2\pi}\int_{9\times10^3}^{11\times10^3}S_x(w)dw$$

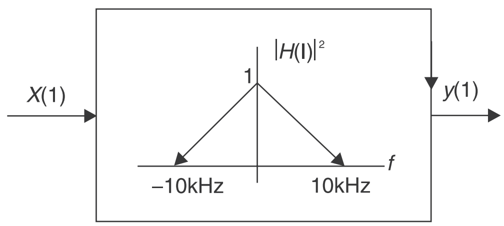
$$E[x^2(t)] = \frac{1}{\pi}\left[400 + \frac{1}{2}\times 6\times 2\times 10^3\right]$$

$$= \frac{6400}{\pi}$$

at $w = 0$ there is no any frequency component available. So, the value of the process will be zero.

Hence, the correct option is (a)

10. A white noise process $x(t)$ with two–sided power spectral density 1×10^{-10} W/Hz is input to a filter whose magnitude squared response is shown below.



The power of the output process $y(t)$ is given by

(a) 5×10^{-7} W (b) 1×10^{-6} W
(c) 2×10^{-6} W (d) 1×10^{-5} **[2009]**

Solution: (b)

PSD of white noise = 1×10^{-10} w/Hz

Output PSD = $|H(f)|^2$ input PSD

$G_0(f) = K.|H(f)|2$

Output Noise power N_0 =

$$\int_{-f_0}^{f_0} G_0(f)dt = k \times (\text{Area under } |H(f)|^2 \text{ cur})$$

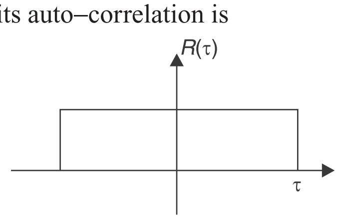
$$= k \times 2\left(\frac{1}{2} \times \text{base} \times \text{height}\right)$$

$$= kf_0 \times 1$$
$$= 1 \times 10^{-10} \times 10 \times 10^3 = 10^{-6}\, w$$

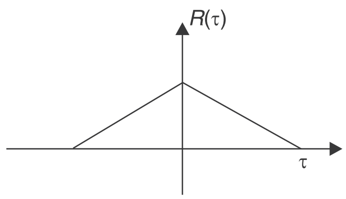
Hence, the correct option is (b)

11. If the power spectral density of stationary random process is a sinc–squared function of frequency, the shape of its auto–correlation is **[2009]**

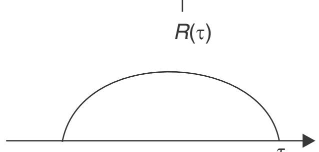
(a)



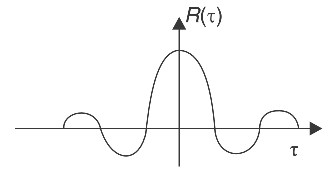
(b)



(c)



(d)



Solution: (b)

PSD = $FT\{R(\tau)\}$

Which is $(\text{sinc})^2$ function

Hence, the correct option is (b)

12. If E denotes expectation, the variance of a random variable X is given by

(a) $E[x^2] - E^2[x]$ (b) $E[x^2] + E^2[x]$
(c) $E[x^2]$ (d) $E^2[x]$ **[2007]**

Solution: (a)

Variance can be given as

$$\sigma_x^2 = E[x^2] - E^2(x)$$

A.C. power = Total power – D.C. power

Hence, the correct option is (a)

13. If $R(\tau)$ is the auto–correlation function of a real, wide–sense stationary random process, then which of the following is NOT true?

(a) $R(\tau) = R(-\tau)$
(b) $|R(\tau)| \leq (0)$
(c) $R(\tau) = -R(-\tau)$
(d) The mean square value of the process is $R(0)$ **[2007]**

Solution: (c)

Auto-correlation function is an even function.

Hence, the correct option is (c)

14. If $S(f)$ is the power spectral density of a real, wide-sense stationary random process, then which of the following is Always true?

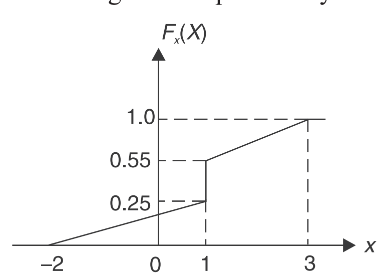
(a) $S(0) \geq S(f)$ (b) $S(f) \geq 0$
(c) $S(-f) = -S(f)$ (d) $\int_{-\infty}^{\infty} S(f)df = 0$ **[2007]**

Solution: (b)

Power Spectral Density is always positive function.

Hence, the correct option is (b)

15. The distribution function $F_x(x)$ of a random variable X is shown in the figure. The probability that $X = 1$ is



(a) zero (b) 0.25
(c) 0.55 (d) 0.30 **[2004]**

Solution: (d)

Given

$P(x = 1) = F(x = 1^+) - F_x(x = 1-)$

$= 0.5 - 0.25$

$= 0.30$

Hence, the correct option is (d)

16. The noise at the input to an ideal frequency detector is white. The detector is operating above threshold. The power spectral density of the noise at the output is

(a) raised-cosine (b) flat
(c) parabolic (d) Gaussian **[2003]**

Solution: (c)

When the detector is operating above the threshold them PSD of the noise at the output of ideal frequency detector is parabolic.

Hence, the correct option is (c)

17. The PDF of a Gaussian random variable X is given by $P_x(x) = \frac{1}{3\sqrt{2\pi}} e^{\frac{-(x-4)^2}{18}}$. The probability of the event $\{X = 4\}$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{3\sqrt{2\pi}}$

(c) 0 (d) $\frac{1}{4}$ **[2001]**

Solution: (c)

For a Gaussian random variable, probability is defined for an interval, for a point it is always zero.

Hence, the correct option is (c)

18. A probability density function is given by $P(x) = K\exp(-x^2/2). -\infty < x < \infty$. The value of K should be

(a) $\frac{1}{\sqrt{2\pi}}$ (b) $\sqrt{\frac{2}{\pi}}$

(c) $\frac{1}{2}\sqrt{\pi}$ (d) $\frac{1}{\pi\sqrt{2}}$ **[1998]**

Solution: (a)

We know that this is a form of a Gaussian pulse. So, we can compare it by a Gaussian PDF.

$$P_x(x) = \frac{1}{\sqrt{2\pi a^2}} \exp\left[\frac{-(x-\mu)^2}{2a^2}\right]$$

when $\sigma = 1$ and $\mu = 0$

$$P_x(x) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2}\right]$$

We have given

$$P_x(x) = K.\exp\left[-\frac{x^2}{2}\right]$$

By comparing

$$\left[k = \frac{1}{\sqrt{2\pi}}\right]$$

Hence, the correct option is (a)

19. The amplitude spectrum of a Gaussian pulse is

(a) uniform
(b) a sine function
(c) Gaussian
(d) an impulse function **[1998]**

Solution: (c)

Since Fourier transform (Time domain to frequency domain transform) of Gaussian signal is also a Gaussian Signal, so spectrum of amplitude will be a Gaussian also.

$$e^{-\pi t^2} \xleftrightarrow{F.T.} e^{-\pi f^2}$$

Hence, the correct option is (c)

20. The ACF of a rectangular pulse of duration T is

(a) a rectangular pulse of duration T
(b) a rectangular pulse of duration $2T$
(c) a triangular pulse of duration T
(d) a triangular pulse of duration $2T$ **[1998]**

Solution: (d)

ACF of a rectangular pulse of duration T is

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t).x(t+\tau).dt$$

Here by solving through this formula, auto–correlation of rectangular pulse of duration T is a triangular pulse of duration $2T$.

Hence, the correct option is (d)

21. The probability density function of the envelope of narrow band Gaussian noise is

(a) Poisson (b) Gaussian
(c) Rayleigh (d) Rician **[1998]**

Solution: (c)

The probability density function of the envelope of narrow band Gaussian noise follows Rayleigh density function.

Hence, the correct option is (c)

22. For a narrow band noise with Gaussian quadrature components, the probability density function of its envelope will be

(a) uniform (b) Gaussian
(c) exponential (d) Rayleigh **[1995]**

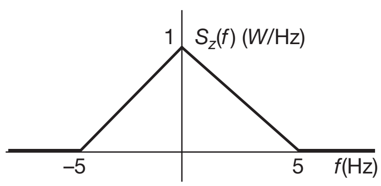
Solution: (d)

As we have discussed in 2.2 80 Ω a narrow band noise with Gaussian quadrature components the probability density function of its envelop will be a Rayleigh function.

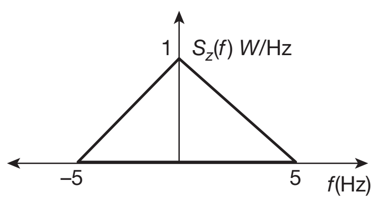
Hence, the correct option is (d)

TWO-MARKS QUESTIONS

1. Let a random process $Y(t)$ be described as $Y(t) = h(t) * X(t) + Z(t)$, where $X(t)$ is a white noise process with power spectral density $S_x(f) = 5$ W/Hz. The filter $h(t)$ has a magnitude response given by $|H(f)| = 0.5$ for $-5 \le f \le 5$, and zero elsewhere. $Z(t)$ is stationary random process, uncorrelated with $X(t)$, with power spectral density as shown in the figure. The power in $Y(t)$, in watts, is equal to _______ W (rounded off to two decimal places). **[2019]**



Solution:



$$P[y(t)] = P[x(t)\cdot h(t)] + P[z(t)]$$

$$P_z(t) = \text{Area under the curve}$$

$$= \frac{1}{2} \times 10 \times 1 = 5 \text{ w}$$

$$P_x(t)\cdot H(t) = \int_{-\infty}^{\infty} |H(t)|^2 S\times(f)\,df$$

$$\left|H(H)^2 \cdot S_x(f)\right| = 0.5^2 \times 5 = 1.25$$

$$P_x(t)\cdot H(t) = \int_{-\infty}^{\infty} 1.25\,dt$$

$$= \int_{-5}^{5} 1.25\,dt = 1.25[5-(-5)]$$

$$= 1.25 \times 10 = 12.5 \text{ w}$$

$$P_y(t) = 5 + 12.5 \text{ w}$$

$$= 17.5 \text{ w}$$

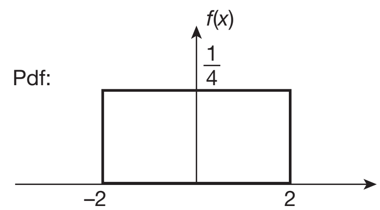
2. A random variable X takes values -1 and $+1$ with probabilities 0.2 and 0.8, respectively. It is transmitted across a channel which adds noise N, so that the random variable at the channel output is $Y = X + N$. The noise N is independent of X, and is uniformly distributed over the interval [–2, 2]. The receiver makes a decision

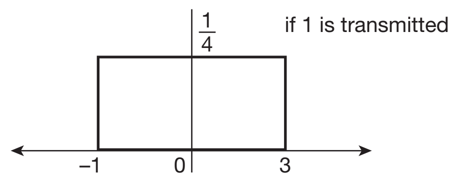
$$\hat{X} = \begin{cases} -1, & \text{if } Y \le \theta \\ +1, & \text{if } Y > \theta \end{cases}$$

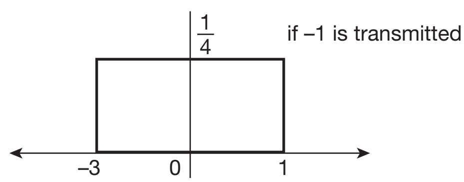
where the threshold $\theta \in [-1, 1]$ is chosen so as to minimize the probability of error Pr $[\hat{X} \ne X]$. The minimum probability of error, rounded off to 1 decimal place, is **[2019]**

Solution: $P(X = -1) = 0.2$

$P(X = 1) = 0.8$



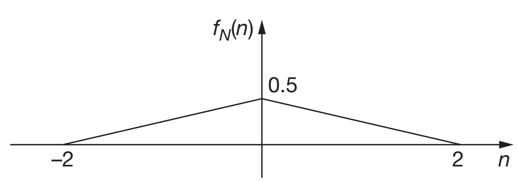




$$P_e = P(1)\cdot P_{e1} + P(-1)P_{e-1}$$

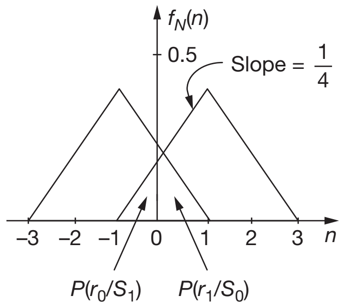
$$= 2 \times 0.2 \times \frac{1}{4} = \frac{0.4}{4} = 0.1$$

3. A binary source generates symbols $X \in \{-1, 1\}$ which are transmitted over a noisy channel. The probability of transmitting $X = 1$ is 0.5. Input to the threshold detector is $R = X + N$. The probability density function $f_N(n)$ of the noise N is shown below.



If the detection threshold is zero, then the probability of error (correct to two decimal places) is _______ **[2018]**

Solution: If we assume that S_0 & S_1 be transmitted symbols with values (–1, 1) respectively and r_0 and r_1 be the received symbols. Then we have



Probability of error can be calculated as given below

$$P_e = P(S_1)\,P(r_0IS_1) + P(S_0)\,P(r_1IS_0)$$

$$P(r_0IS_1) = P(r_1IS_0) = \frac{1}{2}\times 1\times\frac{1}{4} = \frac{1}{8}$$

We know that $P(S_0) = P(S_1) = \dfrac{1}{2}$

$$\Rightarrow \qquad P_e = \frac{1}{2}\left(\frac{1}{8}+\frac{1}{8}\right) = 0.125$$

Hence, the correct answer is 0.12 to 0.14.

4. Consider a white Gaussian noise process $N(t)$ with two-sides power spectral density $S_N(f) = 0.5$ W/H_z as input to a filter with impulse response $0.5e^{-t^2/2}$ (where t is in seconds) resulting in output $Y(t)$. The power in $Y(t)$ in watts is **[2018]**

(A) 0.11 (B) 0.22
(C) 0.33 (D) 0.44

Solution: We know that power spectral density of noise input, is expressed as

$$S_N(f) = 0.5 \text{ W/Hz}$$

Power of output can be calculated as

$$y(t) = \int_{-\infty}^{\infty} S_N(f)|H(f)|^2\,df$$

$$= 0.5\int_{-\infty}^{\infty} |H(f)|^2\,df = 0.5\int_{-\infty}^{\infty} |h(t)|^2\,dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty}\left(\frac{1}{2}e^{\frac{-t^2}{2}}\right)^2 dt = 0.22 \text{ W}$$

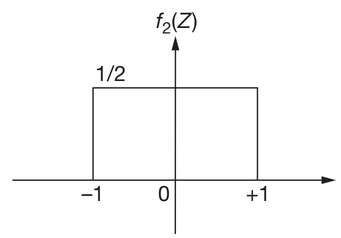
Hence, the correct option is (B)

5. A random variable X takes values −0.5 and 0.5 with probabilities $\frac{1}{4}$ and $\frac{3}{4}$, respectively. The noisy observation of X is $Y = X + Z$, where Z has uniform probability density over the interval (–1, 1). X and Z are independent. If the MAP rule-based detector outputs $\hat{X}$ as

$$\hat{X} = \begin{cases} -0.5, & Y < \propto \\ 0.5, & Y \geq \propto, \end{cases}$$

then the value of $\propto$ (accurate to two decimal places) is ______. **[2018]**

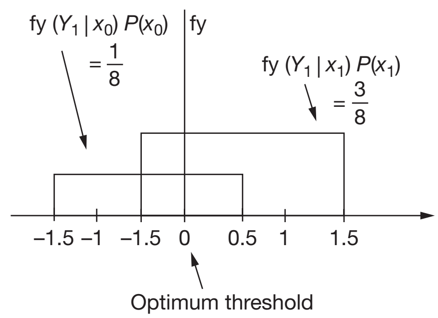
Solution: Consider the figure given below



$$P(x_0) = \frac{1}{4};\ P(x_1) = \frac{3}{4}$$

$$fy\,(Y_1|x_0)\,P(x_0) \underset{x_1}{\overset{x_0}{\gtrless}} fy(Y_1|x_1)\,P(x_1)$$

$$\alpha = -0.5$$

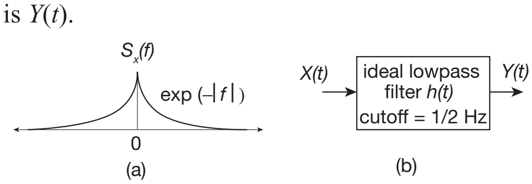


Hence, the correct answer is –0.5.

6. Let $X(t)$ be a wide sense stationary random process with the power spectral density $S_x(f)$ as shown in Figure (a), where f is in Hertz (Hz). The random process $X(t)$ is input to an ideal lowpass filter with the frequency response.

$$H(f) = \begin{cases} 1, & |f| \leq \frac{1}{2}\text{Hz} \\ 0, & |f| > \frac{1}{2}\text{Hz} \end{cases}$$

as shown in figure (b). The output of the low-pass filter is $Y(t)$.



Let E be the expectation operator and consider the following statements:

I. $E[X(t)] = E[Y(t)]$

II. $E[X^2(t)] = E[Y^2(t)]$

III. $E[Y^2(t)] = 2$

Select the correct option: **[2017]**

(A) Only I is true

(B) Only II and III are true

(C) Only I and II are true

(D) Only I and III are true

Solution: The frequency response of the LPF is given by

$E[y(t)] = H(0)E[x(t)]$

$H(0) = 1$

$\Rightarrow E[y(t)] = E[x(t)]$

$E[y^2(t)] \neq E[x^2(t)]$

As LPF does not allow 100% power from input to output

$$E[x^2(t)] = \int_0^{\infty} s_x(f)df = 2 \text{ W}$$

As $E[x^2(t)] \neq E[y^2(t)]$

$E[y^2(t)] \neq 2$

Hence, statement-1 is only correct

Hence, the correct option is (A).

7. A voice-grade AWGN (additive white Gaussian noise) telephone channel has a bandwidth of 4.0 kHz and two sided noise power spectral density $\frac{\eta}{2} = 2.5 \times 10^{-5}$ Watt per Hz. If information at the rate of 52 kbps is to be transmitted over this channel with arbitrarily small bit error rate, then the minimum bit-energy E_b(in mJ/bit) necessary is **[2016]**

Solution: Power spectral density $\frac{\eta}{2} = 2.5 \times 10^{-5}$ Watt

$N = 4.5 \times 10^{-5}$ Watt/Hz

Bandwidth $B = 4$ KHz

Bit rate $R_b = 52$ kbps

As we know that the capacity of channel

$C = B\log_2[1 + S/N]$

$$E_b = \frac{S}{R_b} = 31.503$$

We know that

$$\frac{S}{N} = 8191$$

$S = 819.1 \times 2$

$$E_b = \frac{819.1 \times 2}{R_b} = 31.503.$$

Hence, the correct Answer is (31.503).

8. Consider a random process $x(t) = 3V(t) - 8$, where $V(t)$ is a zero-mean stationary random process with autocorrelation $R_v(\tau) = 4e^{-5|\tau|}$. The power in $X(t)$ is _____. **[2016]**

Solution: As we know that power is the mean square value

Given $R_v(\tau) = 4e^{-5|\tau|}$.

And $V(t)$ is a zero-mean stationary random process so $E[V(t)] = 0$

$E[x^2(t)] = \text{power}$

$E[(3V(t) - 8)^2]$

$E[9V^2(t) + 64 - 48\, V(t)]$

$9E[V^2(t)] + E[64] - 48\, E[V(t)]$

$\Rightarrow$ $E[V(t)] = 0, E[64] = 64.$

$E[V^2(t)] = R(0) = 4 \cdot e^{-5(0)} = 4.$

$\Rightarrow$ $9 \times 4 + 64.$

$\Rightarrow$ 100.

Hence, the correct Answer is (100).

9. Let X be a real–valued random variable with $E[X]$ and $E[X^2]$ denoting the mean values of X and X^2, respectively. The relation which always holds

(a) $(E[x])^2 > E[x^2]$

(b) $E[x^2] \geq (E[x])^2$

(c) $E[x^2] = (E[x])^2$

(d) $E[x^2] > (E[x])^2$ **[2014]**

Solution: (b)

Variance $\sigma_x^2 = E(x^2) - [E(x)]^2$

$\therefore \sigma^2_x$ion means be negative.

$\therefore E(x^2) \geq [E(x)]^2$

Hence, the correct option is (b)

10. Consider a random process $X(t) = \sqrt{2}\sin(2\pi + \phi)$, where the random phase ϕ is uniformly distributed in the interval $[0, 2\pi]$. The autocorrelation $E[X(t_1)\, X(t_2)]$ is

(a) $\cos(2\pi(t_1 + t_2))$

(b) $\sin(2\pi(t_1 - t_2))$

(c) $\sin(2\pi(t_1 + t_2))$

(d) $\cos(2\pi(t_1 - t_2))$ **[2014]**

Solution: (d)

Given $X(t) = \sqrt{2}\sin(2\pi + \phi)$ and ϕ is uniformly distributed random variable in interval $[0, 2\pi]$

$$E\left[X(t_1) * X(t_2)\right] = \int_0^{2\pi} \sqrt{2}\sin(2t\pi\theta) * $$

$$\sqrt{2}\sin(2\pi t_2+\theta)f_\theta(\theta)d\theta$$

$$= 2\int_0^{2\pi} \sin(2\pi t_1+\theta)\sin(2\pi t_2+\theta)\frac{1}{2\pi}d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \sin(2\pi(t_1+t_2)+2\theta)d\theta + \frac{1}{2\pi}$$

$$\int_0^{2\pi} \cos(2\pi(t_1-t_2))d\theta$$

$$\int_0^{2\pi} \sin(2\pi(t_1+t_2)+2\theta)d\theta = 0$$

as it is integration in one time period

$$\frac{1}{2\pi}\int_0^{2\pi} \cos(2x(t_1-t_2))d\theta = \cos(2\pi(t_1-t_2))$$

So, $E\left[X(t_1) * X(t_2)\right] = \cos(2\pi(t_1-t_2))$

Hence, the correct option is (d)

11. The input to a 1–bit quantizer is a random variable X with PDF $f_x(x) = 2e^{-2x}$ for $x \geq 0$ and $f_x(x) = 0$ for $x < 0$. For outputs to be of equal probability, the quantizer threshold should be ____. **[2014]**

Solution:

Assume quantization voltage is V_{th} for outputs of quantizer to have equal probability by →

$P(x \leq V_{th}) = P(x \geq V_{th})$

$$\int_{-\infty}^{V_{th}} f_x(x)a_x = \int_{V_{th}}^{d} f_x(x)dx$$

$$\int_0^{V_{th}} 2e^{-2x}dy = \int_{V_{th}}^{\infty} 2e^{-2x}dx$$

$$e^{-2xV_{th}} = \frac{1}{2}$$

$$V_{th} = 0.346$$

12. The power spectral density of a real stationary random process $X(t)$ is given by

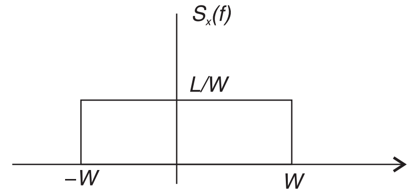
$$S_x(f) = \begin{cases} \frac{1}{W}, & |f| \leq W \\ 0 & |f| > W \end{cases}$$

The value of the expectation

$E\left[\pi x(t) \cdot x\left(t - \frac{1}{4W}\right)\right]$ is _______. **[2014]**

Solution:

$$S_x(f) = \begin{cases} 1/w & |f| \leq w \\ 0 & |f| > w \end{cases}$$



$\therefore R_x(\tau) \leftrightarrow S_x(f)$

$= s \sin e\,[2\,w\tau]$

Also

$$E\left[\pi \times (t).x\left(t - \frac{1}{4w}\right)\right] = \pi E\left[x(t).x\left(t - \frac{1}{4w}\right)\right]$$

$$= \pi R_x\left(\frac{1^{‘}}{4w}\right) = \pi .2 \sin e\left(\frac{1}{2}\right)$$

We know that

$$\sin e(x) = \frac{\sin \pi x}{\pi x}$$

From equation (1), we get

$$\therefore 2\pi\frac{\sin \pi/2}{\pi/2} = 4$$

$$= \frac{P^2}{2}E[\cos(2\pi(t_1 - t_2)] - \cos[2\pi(t_1 + t_2 + 2a)]$$

$$= \frac{P^2}{2}\cos[2\pi(t_1 - t_2)]$$

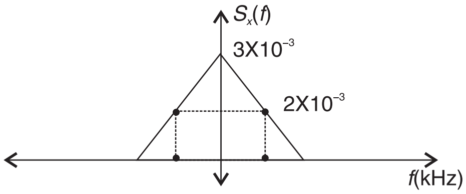
13. A real band–limited random process $X(t)$ has two–sided power spectral density

$$S_x(f) = \begin{cases} 10^{-6}(3000 - |f| \text{ Watts/Hz}) & \text{for } |t| \leq 3\text{kHz} \\ 0 & \text{otherwise} \end{cases}$$

where f is the frequency expressed in Hz. The signal $x(t)$ modulates a carrier $\cos 16000\pi t$ and the resultant signal is passed through an ideal band–pass filter of unity gain with centre frequency of 8 kHz and band–width of 2 kHz. The output power (in watts) is ________. **[2014]**

Solution:

$$S_x(f) = \begin{cases} 10^{-6}(3000 - |f| \text{watts/Hz for } |f| \leq 3 \text{ kHz} \\ 0 & \text{otherwise} \end{cases}$$



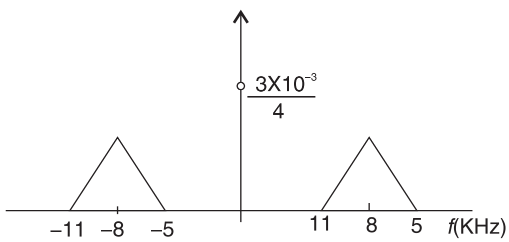
if $x(l)$ PSD $S_x(f)$ $\longleftrightarrow$

$$2\,fx(t) \xleftrightarrow{PSD} Sy(t)$$

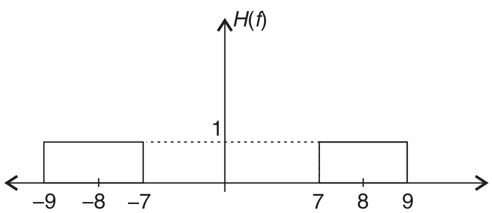
Then

$$x(l)\cos 2\pi\, fe^{t} \xleftrightarrow{PSD} \frac{S_x(f-f_c)+S_x(f+f_c)}{4}$$

$\therefore$ PSD of $x(t)$ cos $2A \times 80\ wt$



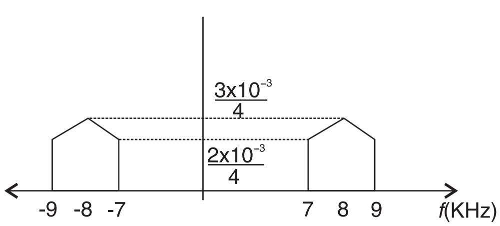
Now, BPF is given by



We know that

$$S_x(f) \to H(f) \to |H(f)|^2\, S_x(f)$$

$\therefore$ PSD of BPF output

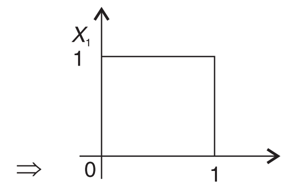


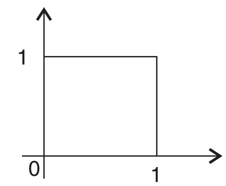
Output power = total area of output PSD

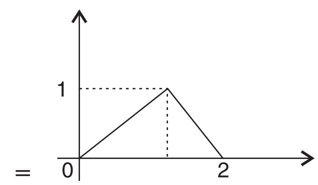
$$= 2\left[(2000)\left(\frac{2\times10^{-3}}{4}\right) + 2\left(\frac{1}{2}\times1000\times\frac{10^{-3}}{4}\right)\right]$$

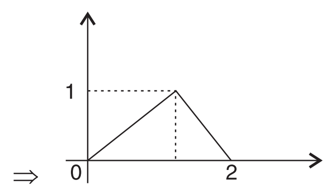
$$= 2\left[1+\frac{1}{4}\right] = 2.5 \text{ watts}$$

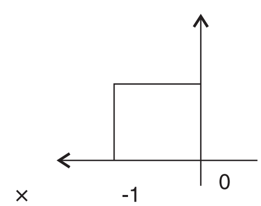
14. Let X_1, X_2 and X_3 be cdistributed random variables with the uniform distribution on [0, 1]. The probability $P\{X_1, + X_2 \le X_3\}$ is ______. **[2014]**











Solution:

$x_1 + x_2 \le x_3$

$x_1 + x_2 - x_3 \le 0$

As $P[x_1 + x_2 - x_3 \le 0] = P[y \le 0]$

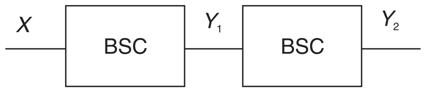
and y is another random variable whose probability density function is given by convolution of x_1, x_2 and x_3.

$$\text{So, } P(y \le 0) = \int_{-1}^{0} \frac{y^2}{2}\,dy$$

$$P(y \le 0) = \left[\frac{y^2}{6}\right]_{-1}^{0}$$

$P(y$ ττ $0) = 0.16$

15. A binary random variable X takes the value of 1 with probability 1/3. X is input to a cascade of two independent identical binary symmetric channels (BSCs) each with crossover probability 1/2. The output of BSCs is the random variables Y_1 and Y_2 as shown in the figure.



The value of $H(Y_1) + H(Y_2)$ in bits is ______. **[2014]**

Solution:

$P(x = 1) = 1/3$ and $P(x = 0) = 2/3$

So $H(y_1) = P(y_1)\log\frac{1}{P(y)}$

$H(y_2) = P(y_2)\log\frac{1}{P(y_2)}$

where $P(y = 1) = P(y_1 = 0)$

$= \frac{1}{3}\times\frac{1}{2} + \frac{2}{3}\times\frac{1}{2} = \frac{1}{2}$

Similarly

$P(y_2 = 1) = P(y_2 = 0) = y_2$

Here $H(y_1) + H(y_2) = \frac{1}{2}\log_2 2 + \frac{1}{2}\log,$

$2 + \frac{1}{2}\log_2 2 + \frac{1}{2}\log_2 2$

$H(y_1) + H(y_n) = 2$ bits

16. Consider a communication scheme where the binary values signal X satisfies $P\{X = 1\} = 0.75$ and $P\{X = -1\} = 0.25$. The received signal $Y = X + Z$, where Z is a Gaussian random variable with zero mean and variance σ^2. The received signal Y is fed to the threshold detector. The output of the threshold detector X is

$$\hat{X} = \begin{cases} +1 & Y > \tau \\ -1 & Y \le \tau \end{cases}$$

To achieve a minimum probability of error $P\{\hat{X} \ne X\}$, the threshold τ should be

(a) strictly positive
(b) zero
(c) strictly negative
(d) strictly positive, zero, or strictly negative depending on the nonzero value of σ^2 **[2014]**

Solution: (c)

Assume hypothesis H_1 and H_0 are given as $H_1 : X = +1; H_0 : X = -1$ and their probabilities $P(H_1) = 0.75$; $P(H_0) = 0.25$

Let received sigma $\gamma = X + Z$ where Z follow normal distribution as $N(0, -2)$; with $f_z(Z) = \frac{1}{\sigma\sqrt{2\pi}}e^{-z^2/2\sigma^2}$

Then received signal $\gamma = \begin{cases} 1+Z & if X = 1 \\ -1+Z & if X = -1 \end{cases}$

$$f_\gamma\left(y/H_1\right) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(\gamma-1)^2/2\sigma^2}$$

and $f_\gamma\left(y/H_0\right) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(\gamma+1)^2/2\sigma^2}$

Let y_{opt} denote an optimal threshold, then probability of error for optimum threshold will be calculated as

$$\left.\frac{f_\gamma\left(y/H_1\right)}{f_\gamma\left(y/H_0\right)}\right|_{y=y_{opt}} = \frac{P(H_0)}{P(H_1)}$$

$$\left.e^{-[(\gamma-1)-(\gamma+1)]^2/2\sigma^2}\right|_{y_{opt}} = \frac{P(H_0)}{P(H_1)}$$

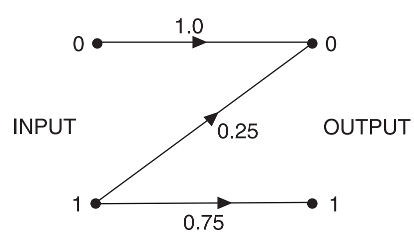
$$e^{+2y_{opt}/\sigma^2} = \frac{P(H_0)}{P(H_1)}$$

So, y_{opt} can be given as

$y_{opt} = \frac{\sigma^2}{2}\ln\frac{P(H_0)}{P(H_1)} = \frac{-\sigma^2}{2} = -0.55\sigma^2$, here, y_{opt} is less than zero, so negative.

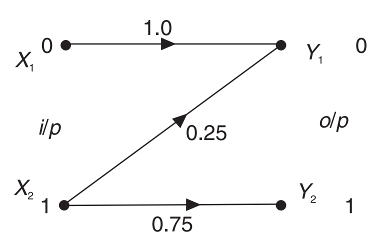
Hence, the correct option is (c)

17. Consider the Z–channel given in the figure. The input is 0 or 1 with equal probability.



If the output is 0, the probability that the input is also 0 equals ____________. **[2014]**

Solution:



Given $P(x) = [0.5 \quad 0.5]$

$$P\left[\frac{y}{x}\right] = \begin{array}{c} \\ x_1 \\ x_2 \end{array}\begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 1 & 0 \\ 0.25 & 0.75 \end{bmatrix} \end{array}$$

Then

$$P[x, y] = \begin{bmatrix} 0.5 & 0 \\ 0.125 & 0.375 \end{bmatrix}$$

and $P(y_1) = 0.625$

$P(y_2) = 0.375$

and the conditional probability matrix

$P\left(\dfrac{x}{y}\right)$ is given by

$$P\left[\frac{x}{y}\right] = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.8 & 0 \\ 0.33 & 1 \end{bmatrix} \end{array}$$

Hence $P\left[\dfrac{x_1}{y_1}\right] = P\left(\dfrac{0}{0}\right) = 0.8$

18. Consider a discrete–time channel $Y = X + Z$, where the additive noise Z is signal–dependent. In particular, given the transmitted symbol $X \in \{-a, +a\}$ at any instant, the noise sample Z is chosen independently from a Gaussian distribution with mean βX and unit variance. Assume a threshold detector with zero threshold at the receiver. When $\beta = 0$, the BER was found to be

$Q(a) = 1 \times 10^{-8}$

$$\left(Q(v) = \frac{1}{\sqrt{2\pi}} \int_{v}^{\infty} e^{-u^2/2} du, \text{ and for } v > 1. \text{ use} \right.$$

$$\left. Q(v) = e^{-v^2/2} \right)$$

When $\beta = -0.3$, the BER is closest to

(a) 10^{-7} (b) 10^{-6}

(c) 10^{-4} (d) 10^{-2} **[2014]**

Solution: (c)

Given $X \in [-\alpha, -\alpha]$ and $P(x = -a) = P(x = a) = 0.5$

Let $\gamma = X + Z$ be the received signal and Z follows normal distribution as $N(\beta X, 1)$ as

$$f_z(Z) = \frac{1}{\sqrt{2\pi}} e^{-(Z-\beta X)^2}, \quad y = \begin{cases} -a + Z & if\ X = -a \\ a + Z & f\ X = +a \end{cases}$$

$H_1 : X = +a; H_0 : X = -a$, and threshold $=0$

$$f_\gamma\left({}^{y}\!/\!_{H_1}\right) = \frac{1}{\sqrt{2\pi}} e^{-0.5(\gamma - a(1+\beta))^2}$$

and $$f_\gamma\left({}^{y}\!/\!_{H_0}\right) = \frac{1}{\sqrt{2\pi}} e^{-0.5(\gamma + a(1+\beta))^2}$$

Bit error rate is given by

$$P_e = P(H_1)P(e/H_1) + P(H_0)P(e/H_0)$$

$$= 0.5 \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}} e^{-0.5(y-a(1+\beta))^2} dy + 0.5 \int_{-0}^{\infty} \frac{1}{\sqrt{2\pi}}$$

$$e^{-0.5(y+a(1+\beta))^2} dy = Q(a(1+\beta))$$

When $\beta = 0$, $P_e = Q(a) = 1 \times 10^{-8}$

$$= e^{-a^2/2} \rightarrow a = 6.07$$

$\beta = -0.3$, $P_e = Q(6.07(1-0.3)) = Q(4.249)$

$$P_e = e^{-(4.249)^2/2} = 1.2 \times 10^{-4}$$

$$P_e \approx 10^{-4}$$

Hence, the correct option is (c)

19. Let U and V be two independent zero mean Gaussian random variables of variances $\dfrac{1}{4}$ and $\dfrac{1}{9}$ respectively. The probability

$P(3V \geq 2U)$ is

(a) 4/9 (b) 1/2

(c) 2/3 (d) 5/9 **[2013]**

Solution: (b)

$P(3V - 2U) = P(3V - 2U \geq 0)$

$= P(W \geq 0)$

Where $W = 3V - 2U$

$U = N(0, 1/4)$

↓ ↓

Mean Variance

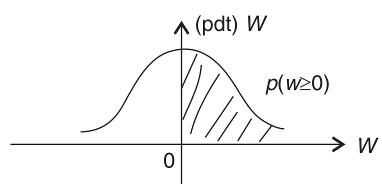
$V = N(0, 1/a)$

$W = 3V - 2U$

$$W = N\left(0, 0 \times \frac{1}{4} + 4 \times \frac{1}{9}\right)$$

$= N(0, 2.7)$

Since W is Gaussian variable with 0 mean,



$(P(w \geq 0) = 1/2 =$ area for $w \geq 0]$.

Hence, the correct option is (b)

20. Consider two identically zero–mean random variables U and V. Let the cumulative distribution functions of U and $2V$ be $F(x)$ and $G(x)$, respectively. Then, for all values of x

(a) $F(x) - G(x) \leq 0$
(b) $F(x) - G(x) \geq 0$
(c) $(F(x) - G(x)) \cdot x \leq 0$
(d) $(F(x) - G(x)) \cdot x \geq 0$ **[2013]**

Solution: (d)

$F(x) = P(x \leq x)$

$G(x) = P(2x \leq x)$

$\quad = P(x \leq x/2)$

For position values of x

$F(x) - G(x) > 0.$

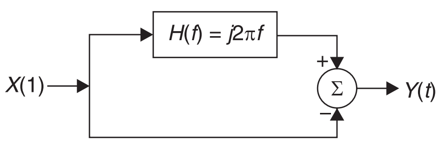
For negative values of x

$F(x) - G(x) < 0$

but $[F(x) - G(x)], X \geq 0$ for all values of x.

Hence, the correct option is (d)

21. $X(t)$ is a stationary random process with autocorrelation function $R_x(\tau) = \exp(-\pi\tau^2)$. This process is passed through the system below. The power spectral density of the output process $Y(t)$ is



(a) $(4\pi^2 f^2 + 1)\exp(-\pi f^2)$
(b) $(4\pi^2 f^2 - 1)\exp(-\pi f^2)$
(c) $(4\pi^2 f^2 + 1)\exp(-\pi f)$
(d) $(4\pi^2 f^2 - 1)\exp(-\pi f)$ **[2011]**

Solution: (a)

$$y(t) = \frac{d}{dt}x(t) - x(t)$$

$$y(f) = jwx(f) - x(f)$$

$$\quad = [j2\pi f - 1]\, x(f)$$

$$\left(H(f) = \frac{y(f)}{x(f)}\right)$$

$$\text{PSD} = |H(f)|^2 \cdot S_x(f)$$

$$= |(j2\pi f - 1)|^2 . S_x(f)$$

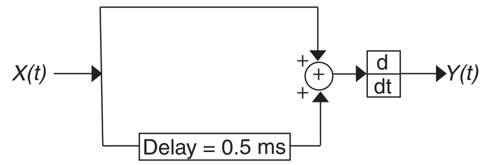
$$S_x(f) = FT\{R_x(\tau)\} = e^{-\pi t^2}$$

Since Fourier of Gaussian is also Fourier,

$$S_y(f) = [y\pi^2 f^2 + 1].e^{-\pi f^2}$$

Hence, the correct option is (a)

22. $X(t)$ is a station process with the power spectral density $S_X(f) > 0$ for all f. The process is passed through a system shown below.



Let $S_Y(f)$ be the power spectral density of $Y(t)$, which one of the following statements is correct?

(a) $S_Y(f) > 0$ for all f
(b) $S_Y(f) = 0$ for $|f| > 1$ kHz
(c) $S_Y(f) = 0$ for $f = nf_0, f_0 \neg = 2$ kHz, n any integer
(d) $S_Y(f) = 0$ for $f = (2n + 1) f_0, f_0 = 1$ kHz, n any integer **[2010]**

Solution: (d)

$y_1(t) = x(t) + x(t - 0.5 \times 10^{-3})$

$y_1(f) = x(f)\, [1 + e^{-j2\pi}\, (0.5 \times 10^{-3})$

$$H_1(f) = \frac{y_1(f)}{x(f)} = 1 + e^{-ej\pi f \times 10^{-3}}$$

$$H(f) = H_1(f)\,.\,H_2(f)$$

$$= j2\pi f\left[1 + e^{-j(\pi f \times 10^{-3})}\right]$$

$$S_y(f) = |H(f)|^2 S_x(f)$$

$$= 4\pi^2 t^2\left[2 + 2\cos(\pi f \times 10^{-3})\right] S_x(f)$$

For $f_0 = 1$ kHz and $f = (2n + 1) f_0$

$\pi f \times 10^{-3}$ is odd multiple of π

$\therefore\ S_y(f) = 0$

Hence, the correct option is (d)

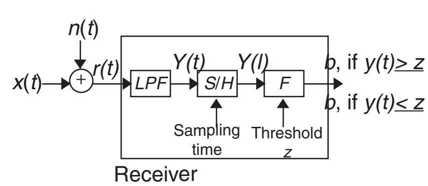
Common Data Questions 23 and 24

Consider a baseband binary PAM receiver shown below. The additive channel noise $n(t)$ is white with power spectral density $S_N(f) = N_0/2 = 10^{-20}$ W/Hz. The low–pass filter is ideal with unity gain and cut–off frequency 1 MHz. Let Y_k represent the random variable $y(t_k)$.

$Y_k = N_k$ if transmitted bit $b_k = 0$

$Y_k = a + N_k$ if transmitted bit $b = 1$

where N_k represents the noise sample value. The noise sample has noise probability density function, $p_{Nk}(n) = 0.5\alpha e^{-\alpha|n|}$ (This has mean zero and variance $2/\alpha^2$.) Assume transmitted bits to be equiprobable and threshold z is set to $a/2 = 10^{-6} V$.



23. The value of the parameter α (in V–¹) is

(a) 10^{10} (b) 10^7

(c) 1.414 × 10–^{10} (d) 2 × 10–^{20} **[2010]**

Solution: (b)

$S_{N0}(f) = S_N(f)\ |H(f)|^2$

$S_{N0}(f) = 10^{-20}\ |H(f)|^2$

Output noise power $= \int_{-\infty}^{\infty} S_{N0}(f)df$

$= 10^{-20}\ \dfrac{w}{H_2} \times 2\times 10^6\ \text{Hz}$

$= 2 \times 10^{-14}\ w$

Power = mean square value

$[\alpha/\alpha^2 = 2 \times 10^{-14})$

$\alpha = 10^7$

Hence, the correct option is (b)

24. The probability of bit error is

(a) $0.5 \times e^{-3.5}$ (b) $0.5 \times e^{-5}$

(c) $0.5 \times e^{-7}$ (d) $0.5 \times e^{-10}$ **[2010]**

Solution: (d)

When $a \to 1$

$r_k = a + N_k$

Threshold $z = 1/2 = 10^{-6}$

$a = 2 \times 10^{-6}$

for error to occur, $y_k < 10^{-6}$

$2 \times 10^{-6} + N_k < 10^{-6}$

$N_k < -10^{-6}$

$$P(0/1) = \int_{-\infty}^{-10^{-6}} P_{NK}(n)dn$$

$$\int_{-\infty}^{-10^{-6}} 0.5\alpha.e^{\alpha n}dn;\ \alpha = 107$$

$P(0/1) = 0.5e^{-10}$

When a '0' is transmitted

$y_k = N_k$

For error to occur

$y_k > 10^{-6}$

$$P(1/0) = \int_{10^{-6}}^{\infty} P_{NK}(n)dn = 0.5\times 10^{-6}$$

Since both bits are equiprobable,

$P(0) = P(1) = 1/2$

The probability of bit error

$= P(1) . P(0/1) + P(0) . P(1/0)$

$$= \frac{P(0/1) + P(1/0)}{2} = 0.5\times 10^{10}$$

Hence, the correct option is (d)

25. Consider two independent random variables X and Y with identical distributions. The variables X and Y take values 0, 1 and 2 with probabilities $\frac{1}{2}, \frac{1}{4}$, and $\frac{1}{4}$ respectively. What is the conditional probability $P(X + Y = 2|X - Y = 0)$?

(a) 0 (b) $\frac{1}{16}$

(c) $\frac{1}{6}$ (d) 1 **[2009]**

Solution: (c)

$$P(x+y=2/x-y=0) = \frac{(P * x+y=2, x-y=0)}{P(x-y=0)}$$

$$P(x+y=2, x-y=0) = P(x=1, y=1) = \frac{1}{16}$$

$P(x - y = 0) = P(x = 0, y = 0) + P(x = 1, y = 1) + P(x = 2, y = 2)$

$= P(x = 0) .P(y = 0) + P(x = 1)\ P(y = 1) + P(x = 2)\ P(y = 2) = 3/8$

$$P(x+y=2/x-y=0) = \frac{1}{16}\times\frac{8}{3} = \frac{1}{16}$$

Hence, the correct option is (c)

26. A discrete random variable X takes values from 1 to 5 with probabilities as shown in the table. A student calculates the mean X as 3.5 and her teacher calculates the variance of X as 1.5. Which of the following statements is true?

K	1	2	3	4	5
$P(X = K)$	0.1	0.2	0.4	0.2	0.1

(a) Both the student and the teacher are right

(b) Both the student and the teacher are wrong

(c) The student is wrong but the teacher is right

(d) The student is right and but the teacher is wrong **[2009]**

Solution: (b)

Mean $\bar{x} = \Sigma\ xi\ P(xi)$

$= 1 \times 0.1 + 2 \times 0.2 + 3 \times 0.4 + 4 \times 0.2 + 5 \times 0.$

$= 3$

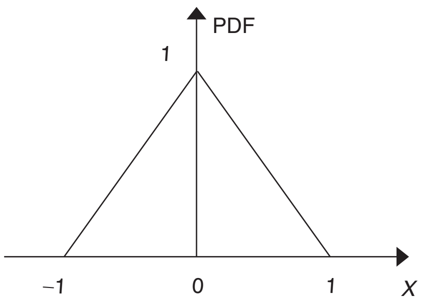
σ^2 variance $= \overline{x^2} - \overline{x}^2 = \Sigma x_i^2\, p(x_i) - (\Sigma\, x_i P(x_i))^2$

$\sigma^2 = [(1 \times 0.1) + 4 \times 0.2 + 9 \times 0.4 + 16 \times 0.2 \times 25 \times 0.1)] - (3)^2$

$[\sigma^2 = 1.2]$

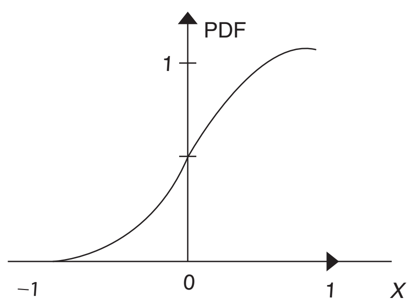
Hence, the correct option is (b)

27. The Probability Density Function (PDF) of a random variable X is as shown below. **[2008]**

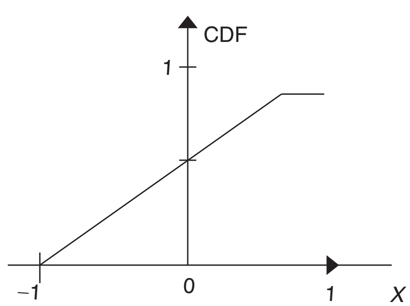


The corresponding Cumulative Distribution Function (CDF) has the form

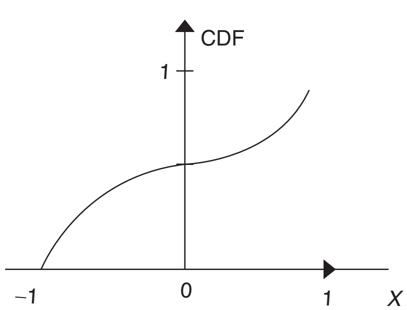
(a)



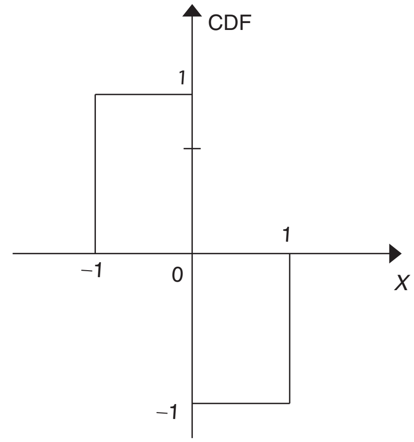
(b)



(c)



(d)



Solution: (a)

Density function is given as

$f_x(x) = (t + 1)u(t + 1) – 2tu(t) + (t – 1)u(t – 1)$

CDF function is calculated as integration of density function.

$$F_x(x) = \int_{-\infty}^{x} f_x(x)dx$$

integral of increasing ramp signed is increasing parabola and integral of decreasing ramp signal is decreasing parabola.

Hence, the correct option is (a)

28. $P_x(x) = M \exp(-2|x|) + N \exp(-3|x|)$ is the probability density function for the real random variable X over the entire x–axis. M and N are both positive real numbers. The equation relating M and N is

(a) $M + \frac{2}{3}N = 1$ (b) $2M + \frac{1}{3}N = 1$

(c) $M + N = 1$ (d) $M + N = 3$ **[2008]**

Solution: (a)

PDF is given as

$P_x(x) = m \exp(-2|x|) + N \exp(-3|x|)$

$$\int_{-\infty}^{\infty} P_x(x)dx = 1$$

$$\int_{-\infty}^{\infty} \left\{me^{-2|x|} + Ne^{-3|x|}\right\} dx = 1$$

$$\int_{0}^{\infty} (me^{-2x} + Ne^{-3x})dx = \frac{1}{2}$$

$$\frac{m}{2} + \frac{N}{3} = \frac{1}{2}$$

$$m + \frac{2N}{3} = 1$$

Hence, the correct option is (a)

29. Noise with double–sided power spectral density of K over all frequencies is passed through a RC low pass filter with 3 dB cut–off frequency of f_c. The noise power at the filter output is

(a) K (b) Kf_c
(c) $K\pi f_c$ (d) ∞ **[2008]**

Solution: (c)

Filter transfer function is given by

$$H(f) = \frac{1}{1 + j2\pi fR_c} = \frac{1}{1 + jf/f_c}$$

$$\text{Output PSD} = |H(f)|^2 \cdot \text{input PSD} = \frac{f_C^2}{f^2 + f_C^2}.K$$

$$\text{Output noise power} = \int_{-\infty}^{\infty} (o/p\ PSD).df$$

$$= k\int_{-\infty}^{\infty} \frac{fc^2}{fc^2 + f^2}\partial f$$

$$= k\pi f_c$$

(By substitution $f = f_c \tan Q$)

Hence, the correct option is (c)

30. A zero–mean white Gaussian noise is passed through an ideal low–pass filter of bandwidth 10 kHz. The output is then uniformly sampled with sampling period t_s = 0.03 msec. The samples so obtained would be

(a) correlated
(b) statistically independent
(c) uncorrelated
(d) orthogonal **[2006]**

Solution: (b)

If white noise is sampled, no matter how closely in time the samples are taken, they are uncorrected. If white noise is Gaussian then samples are statically independent.

Hence, the correct option is (b)

Common Data for Questions 31 and 32

The following two questions refer to wide sense stationary stochastic processes.

31. It is desired to generate a stochastic process (as voltage process) with power spectral density

$$s(\omega) = \frac{16}{16 + \omega^2}$$

By driving a linear–time–invariant system by zero mean white noise (as voltage process) with power spectral density being constant equal to 1, the system which can perform the desired task could be

(a) first order low pass R–L filter
(b) first order high pass R–C filter
(c) tuned L–C filter
(d) series R–L–C filter **[2006]**

Solution: (a)

Given

$$S(w) = \frac{16}{16 + w^2}$$

$$H(s) = \frac{4}{4 + s}$$

This is a low pass filter of RL type.

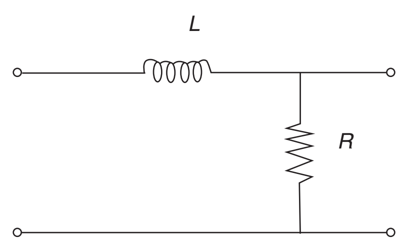
Hence, the correct option is (a)

32. The parameters of the system obtained in the above question would be

(a) first order R–L low pass filter would have
$R = 4\ \Omega$ $L = 1\ H$
(b) first order R–C high pass filter would have
$R = 4\ \Omega$ $C = 0.25$F
(c) tuned L–C filter would have $L = 4\ H$
$C = 4\ F$
(d) series R–L–C low pass filter would have
$R = 1\Omega$, $L = 4H$, $C = 4F$ **[2006]**

Solution: (a)

For given network



$$H(jw) = \frac{R}{R + jwL}$$

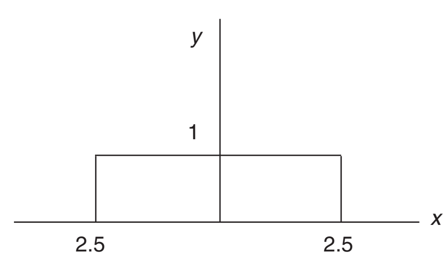
So $R = 4$, $L = 1$

Hence, the correct option is (a)

33. A uniformly distributed random variable X with probability density function

$$f_x(x) = \frac{1}{10}\left(u\left(x+5\right) - u\left(x-5\right)\right)$$

where $u(.)$ in the unit step function is passed through a transformation given in the figure below. The probability density function of the transformed random variable Y would be



(a) $f_y(y) = \frac{1}{5}\left(u\left(y+2.5\right) - u\left(y-2.5\right)\right)$

(b) $f_y(y) = 0.5\delta(y) + 0.5\delta(y-1)$

(c) $f_y(y) = 0.25\delta(y+2.5) + 0.25\delta(y-2.5) + 0.5\delta(y)$

(d) $f_y(y) = 0.25\delta(y+2.5) + 0.25\delta(y-2.5) + \frac{1}{10}\left(u\left(y+2.5\right) - u\left(y-2.5\right)\right)$

[2006]

Solution: (c)

Sample space of random variable $x = (-\infty, \infty)$after transformation sample space of random variable $y = (0, 1)$

Hence $f_x(y) = A\delta(y) + p[\delta(y-1)]$

Hence, the correct option is (c)

34. Noise with uniform power spectral density of N_0 W/Hz is passed through a filter $H(\omega) = 2\exp(-j\omega t_d)$ followed by an ideal low pass filter of bandwidth B Hz. The output noise power in watts is

(a) $2N_0B$ (b) $4N_0B$
(c) $8N_0B$ (d) $16N_0B$ **[2005]**

Solution: (b)

Output noise power is calculated by calculating output power spectral density as

$S_y = |H(w)|^2 S_x$

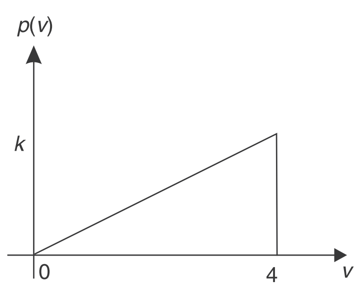
$SN_{out} = |H(w)|^2 SN_{input}$

$= 4N_0$

$P_N = \text{B.W.} \times SN_{out} = 4N_0B.$

Hence, the correct option is (b)

35. An output of a communication channel is a random variable v with the probability density function as shown in the figure. The mean square value of v is



(a) 4 (b) 6
(c) 8 (d) 9 **[2005]**

Solution: (c)

Probability density function can be written as

$f_x(x) = y = mx + c = x/8$

Total probability = 1

Area = 1

$\frac{k \times 4}{2} = 1$

$k = 1/2$

$$E[x^2] = \int_0^4 x^2 f_x(x)dx$$

$$= \int_0^4 x^2\left(\frac{x}{8}\right)dx = \left.\frac{x^4}{4\times 8}\right|_0^4$$

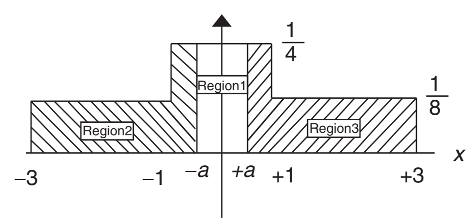
$E[x^2] = 8$

Hence, the correct option is (c)

Common Data for Questions 36 and 37.

A symmetric three-level midtread quantizer is to be designed assuming equiprobable occurrence of all quantization levels.

36. If the probability density function is divided into three regions as shown in the figure, the value of a in the figure is



(a) 1/3 (b) 2/3
(c) 1/2 (d) 1/4 **[2005]**

Solution: (b)

Probability of occurrence of each region is

$$\int_{-1}^{a} f_x(x)dx = 1/3$$

$$\frac{1}{4}\int_{-a}^{a} xdx = \frac{1}{3} \Rightarrow \frac{2a}{4} = \frac{1}{3}$$

$[a = 2/3]$

37. The quantization noise power for the quantization region between $-a$ and $+a$ in the figure is

(a) $\frac{4}{81}$ (b) $\frac{1}{9}$

(c) $\frac{5}{81}$ (d) $\frac{2}{81}$ **[2005]**

Solution: (a)

Noise power =

$$E[x^2] = \int_{-a}^{a} x^2 f_x(x)dx = 2\int_0^a x^2.\frac{1}{4}.dx$$

$$E[x^2] = 2\int_0^{2/3} \frac{1}{4}x^2 dx = \frac{1}{2}\left[\frac{x^3}{3}\right]_0^{2/3}$$

$$E[x^2] = \frac{1}{6}\times\frac{8}{27} = \frac{4}{81}$$

38. A 1 mW video signal having a bandwidth of 100 MHz is transmitted to a receiver through a cable that has 40 dB loss. If the effective one–sided noise spectral density at the receiver is 10^{-20} watt/Hz, then signal–to–noise ratio of the receiver is

(a) 50 dB (b) 30 dB
(c) 40 dB (d) 60 dB **[2004]**

Solution: (a)

$$\text{SNR} = \frac{P_S}{N_0 B} = \frac{10^{-3}}{10^{-20}\times 100\times 10^6} = 10^9$$

$(\text{SNR})_{\text{dB}} = 10 \log_{10} (\text{SNR}) = 10 \log_{10} (10^9) = 90$ dB

Cable loss = 40 dB

SNR at $x = (90 - 40)$dB = 50 dB

Hence, the correct option is (a)

39. A random variable X with uniform density in the interval 0 to 1 is quantized as follows:

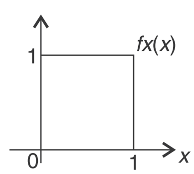
If 0 ″ X ″ 0.3, $x_q = 0$

If 0 .3 < X ″ 1, $x_q = 0.7$

Where x_q is the quantized value of X. The root–mean square value of the quantized noise is

(a) 0.573 (b) 0.198
(c) 2.205 (d) 0.266 **[2004]**

Solution: (b)



Quantization error = Sampled Value – Quantized value $= x - x_q$

Mean square value of quantization Noise

$= E[(x - x_q)^2]$

$$= \int_0^1 (x - x_q)^2 f_x(x)dx$$

$$= \int_0^1 (x - x_q)^2 /dx = \int_0^{0.3} (x-0)^2 dx + \int_{0.3}^1 (x - 0.7)^2$$

$dx = 0.03$

Root mean square value of quantization noise

$= \sqrt{0.039} = 0.198$

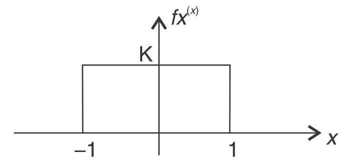
Hence, the correct option is (b)

40. Let X and Y be two statistically independent random variables uniformly distributed in the ranges (–1, 1) and (–2, 1) respectively. Let $Z = X + Y$. Then the probability that ($Z \le -2$) is

(a) zero (b) $\frac{1}{6}$

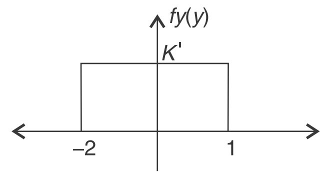
(c) $\frac{1}{3}$ (d) $\frac{1}{12}$ **[2003]**

Solution: (d)

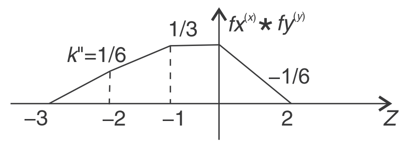


Area under curve $f_x(x) = 1$

$2 \times k = 1 = k = 1/2$



$3 \times k' = 1$,k' = 1/3



$k'' = 1/6$

Since it lies in mid

$P\{2 \le -2] =$ area of graph $[2 \le -2]$

$$= \frac{1}{2}\times\frac{1}{6}\times 1 = \frac{1}{12}$$

Common Data for Questions 2.32 and 2.33.

Let X be the Gaussian variable obtained by sampling the process at $t = t_i$ and let

$$Q(-) = \int_{-}^{\infty} -\frac{1}{\sqrt{2}} e^{\frac{x^2}{2}} dy$$

Auto correlation function $R_{xx}(\tau) = 4\left(e^{-0.2|\tau|} + 1\right)$ and mean = 0

Hence, the correct option is (d)

41. The probability that $[x \le 1]$ is

(a) $1 - Q(0.5)$ (b) $Q(0.5)$

(c) $Q\left(\frac{1}{2\sqrt{2}}\right)$ (d) $1 - Q\left(\frac{1}{2\sqrt{2}}\right)$ **[2003]**

Solution: (d)

Probability function is given by

$$P(x \le 1) = F_x(1) = 1 - Q\left(\frac{x-\mu}{a}\right)\Bigg|_{x=1}$$

Here $\mu = 0$

$R_x(0) = \sigma^2$

$\sigma^2 = 8$

$\Rightarrow \sigma = 2\sqrt{2}$

$$\therefore P(x \le 1) = 1 - Q\left(\frac{1}{2\sqrt{2}}\right)$$

Hence, the correct option is (d)

42. Let Y and Z be the random variables obtained by sampling $X(t)$ at $t = 2$ and $t = 4$, respectively. Let $W = Y - Z$. The variance of W is

(a) 13.36 (b) 9.36

(c) 2.64 (d) 8.00 **[2003]**

Solution: (c)

Variance can be given by

$$\sigma_w^2 = E(y-z)^2]$$

$\sigma^2_w = E(y^2) + E(z^2) - 2E(y.z)$

$\sigma^2_w = \sigma^2_y + a^2_z - 2R_{xx}(2)$

$+8 + 8 - 2[ye^{-0.2121} + 1]$

$\sigma^2_w = 2.64$

Hence, the correct option is (c)

43. If the variance σ^2_x of $d(n) = x(n) - x(n-1)$ is one–tenth the variance and $\sigma^2 x$ of a stationary zero–mean discrete–time signal $x(n)$, then the normalized autocorrelation function $R_{xx}(k)/\sigma_x^2$ at $k = 1$ is

(a) 0.95 (b) 0.90

(c) 0.10 (d) 0.05 **[2002]**

Solution: (a)

The variance for the signal is given by

$= \sigma_x^2 \; E\{[x(n) - x(n-1)]^2\}$

$= E[x^2(n)] + E[x^2(n-1)] - 2E[x(n)\,\eta(n-1)]$

$$= \frac{ax^2}{10} = ax^2 + ax^2 - 2R_{xx}(1)$$

$$2R_{xx}(1) = \frac{19}{10}\sigma x^2$$

$$\left[\frac{R_{xxx}(1)}{\sigma x^2} = \frac{19}{20} = 0.95\right]$$

Hence, the correct option is (a)

44. The PSD and the power of a signal $g(t)$ are, respectively, $S_g(\omega)$ and P_g. The PSD and the power of the signal $ag(t)$ are, respectively,

(a) $a^2S_g(\omega)$ and a^2P_g (b) $a^2S_g(\omega)$ and aP_g

(c) $aS_g(\omega)$ and a^2P_g (d) $aS_g(\omega)$ and aP_g **[2001]**

Solution: (a)

We know that

$$S_x(w) — \boxed{H(w)} — S_y(w)$$

$S_y(w) = |H(w)|^2 S_x(w)$

Hence, the correct option is (a)

45. The power spectral density of a deterministic signal is given by $[\sin(f)/(f)]^2$, where 'f' is frequency. The auto-correlation function of this signal in the time domain is

(a) a rectangular pulse

(b) a delta function

(c) a sine pulse

(d) a triangular pulse **[1997]**

Solution: (d)

Auto–correlation function and power spectral density make the Fourier transform pair

$$R_x(\tau) \xrightarrow{F.T.} G_x(w)$$

$$R_x(\tau) == F^{-1}\left[\sin C^2\left(\frac{f}{\pi}\right)\right]$$

Since we know that Fourier transform of triangular signal is a square of sin C function, so inverse Fourier transform will be a triangular signal.

Hence, the correct option is (d)

46. The auto-correlation function of an energy signal has

(a) no symmetry

(b) conjugate symmetry

(c) odd symmetry

(d) even symmetry **[1996]**

Solution: (b, d)

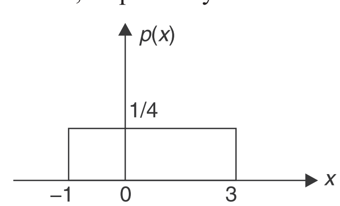
Auto-correlation function has property of conjugate symmetric and even symmetric for energy signal.

$[R_x(\tau) = R_x(-\tau)]$ even symmetry

$[R_x(\tau) = R_x^*(-\tau)]$ conjugate symmetry

Hence, the correct option is (b, d)

47. For a random variable 'X' following the probability density function, $p(x)$, shown in figure, the mean and the variance are, respectively



(a) 1/2 and 2/3 (b) 1 and 4/3
(c) 1 and 2/3 (d) 2 and 4/3 **[1992]**

Solution: (b)

We know that mean is given as

$$\mu_x = E(x) = \int_{-\infty}^{\infty} x P_x(x)\,dx$$

$$= \int_{-1}^{3} x \frac{1}{4}\,dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2}\right]_{-1}^{3}$$

$[\mu_x = 1]$

Variance is given as

$$\sigma x^2 = E[(x-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu_n)^2 P_x(x)dx$$

$$\sigma x^2 = \frac{1}{4}\int_{-2}^{3}(x^2+1-2x)dx = \frac{1}{4}\left[\frac{x^2}{3}+x-\frac{2x^2}{2}\right]_{-1}^{3}$$

$$\left[\sigma x^2 = \frac{4}{3}\right]$$

Hence, the correct option is (b)

48. Zero mean Gaussian noise of variance N is applied to a half wave rectifier. The mean squared value of the rectifier output will be

(a) Zero (b) $N/2$
(c) $N\sqrt{z}$ (d) N **[1989]**

Solution: (b)

Half wave rectifier is given as

$y = x$ for $x \geq 0$

$= 0$ for $x < 0$

$$\text{So } f(y) = \frac{1}{2}\delta(y) + \frac{1}{\sqrt{2\pi N}} e^{\left(-y^2/2N\right)}$$

$$E(y^2) = \int_0^{\infty} y^2 + (y)dy$$

$$= \int_0^{\infty} y^2\left[\frac{1}{2}\delta(y) + \frac{1}{\sqrt{2\pi N}} e^{\frac{-y^2}{2N}}\right]dy$$

$$= 0 + \int_0^{\infty} \frac{y^2}{\sqrt{2\pi N}} e^{\frac{-y^2}{2N}}\,dy$$

Let that $\dfrac{y}{\sqrt{N}} = t$

$dy = \sqrt{N}\,dt$

$$E[y^2] = \frac{1}{\sqrt{2\pi N}}\int_0^{\infty} N t^2 e^{\frac{-t^2}{2}} \sqrt{N}\,dt$$

$$= \frac{N}{\sqrt{2\pi}}\int_0^{\infty} t^2 e^{\frac{-t^2}{2}}\,dt$$

$$E[y^2] = \frac{N}{2}$$

Hence, the correct option is (b)

49. Events A and B are mutually exclusive and have a nonzero probability. Which of the following statement(s) are true?

(a) $P(A \cup B) = P(A) + P(B)$
(b) $P(B^C) > P(A)$
(c) $P(A \cap B) = P(A)P(B)$
(d) $P(B^C) < P(A)$ **[1988]**

Solution: (a)

Since events are mutually exclusive, so $P(A \cap B)$

$P(A \cap B) = P(A) + P(B) + P(A \cap B)$

$[P(A \cap B) = P(A) + P(B)]$

Hence, the correct option is (a)

50. The variance of a random variable X is σ^2_X. Then the variance of $-kx$ (where k is a positive constant) is

(a) σ^2_X (b) $-k\sigma^2_X$
(c) $k\sigma^2_X$ (d) $k^2\sigma^2_X$ **[1987]**

Solution: (d)

Given that variance of =var $(x) = \sigma^2_X$

Since var $(x) = E(x^2)$

So var$(-kx) = E((-kx)^2)$

$a^2 = E[k^2x^2]$

$a^2 = k^2E[x^2]$

$a^2 = k^2\sigma^2_x$

Hence, the correct option is (d)

51. White Gaussian noise is passed through a linear narrow band filter. The probability density function of the envelope of the noise at the filter output is

(a) Uniform (b) Poisson
(c) Gaussian (d) Rayleigh **[1987]**

Solution: (d)

We know that narrow band re-presentation of Noise is

$n(t) = n_c(t)\cos w_ct - n_s(t)\sin w_ct$

envelope for the given signal will be

$$R(t) = \sqrt{n_c^2(t) + n_s^2(t)}$$

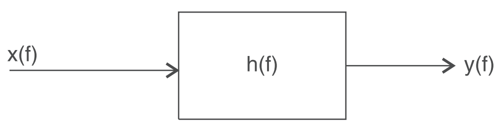
Where $n_c(t)$ and $n_s(t)$ are two independent zero mean Gaussian processes with same variance, resulting envelope will be Rayleigh variable.

Hence, the correct option is (d)

Five-marks Questions

1. White Gaussian noise with zero mean and double-sided power spectral density η/2 is the input x(t) to a linear system with impulse response h(t) = exp(-t/RC)u(t). The output is y(t). Evaluate E[x(t + τ) y(t)] for τ > 0. (Note: u(t) is the unit step function) **[1988]**

Solution:



$$E[x(t-\tau).y(t)] = R_{xy}(\tau)$$

The cross correlation between input process $x(t)$ and output process $y(t)$ is

$$R_{xy}(\tau) = h(\tau) * R_{xx}(\tau)$$

$$S_{xy}(\omega) = H(\omega), S_{xx}(\omega)$$

$$S_{yx}(\omega) = S_{xy}(-\omega) = S_{xy}(-\omega).H(-\omega)$$

$$S_{yx}(\omega) = H^*(\omega)S_{xx}(\omega)$$

$$R_{yx}(\tau) = h(-\tau) * R_{xx}(\tau)$$

Autocorrelation function of $x(t)$ i.e. white noise is an Impulse function.

$$R_{xx}(\tau) = \frac{\eta}{2}\delta(\tau)$$

$$h(-\tau) = \exp\left(\frac{\tau}{RC)}\right).u(-\tau)$$

$h(-\tau)$ is defined for $\tau = (-\infty, 0)$

$$R_{xx}(\tau) * h(-\tau) = \frac{\eta}{2}\exp\left(\frac{\tau}{RC}\right).u(-\tau)$$

$R_{xx}(\tau) = 0$ for $\tau > 0$

Chapter 3

Digital Communication Systems

One-mark Questions

1. Which one of the following statements about differential pulse code modulation (DPCM) is true? **[2017]**

(A) The sum of message signal sample with its prediction is quantized

(B) The message signal sample is directly quantized, and its prediction is not used

(C) The difference of message signal sample and a random signal is quantized

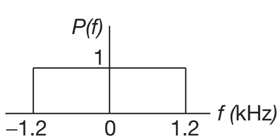
(D) The difference of message signal sample with its prediction is quantized

Solutions: Unlike in PCM, to conserve the channel bandwidth, the difference of message signal sample with its prediction is quantized in DPCM.

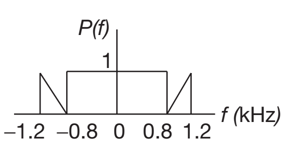
Hence, the correct option is (D).

2. In a digital communication system, the overall pulse shape $p(t)$ at the receiver before the sampler has the Fourier transform $P(f)$. If the symbols are transmitted at the rate of 2000 symbols per second, for which of the following cases is the inter symbol interference 0? **[2017]**

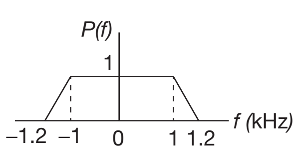
(A)



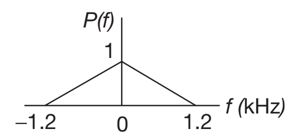
(B)



(C)



(D)



Solution: For a pulse, which is free from inter symbol interference (ISI), If $P(t)$ is having spectrum $P(f)$

Then $\sum_{K=-\infty}^{\infty} p(f-kR_s) = \text{constant}$

$R_s = 2$ KSpa

This condition is met by pulse given in option B.

Hence, the correct option is (B).

3. Which one of the following statements about differential pulse code modulation (DPCM) is true? **[2017]**

(A) The sum of message signal sample with its prediction is quantized.

(B) The message signal sample is directly quantized, and its prediction is not used.

(C) The difference of message signal sample and a random signal is quantized.

(D) The difference of message signal sample with its prediction is quantized.

Solution: Unlike in PCM, to conserve the channel bandwidth, the difference of message signal sample with its prediction is quantized in DPCM.

Hence, the correct option is (D).

4. Consider a wireless communication link between a transmitter and a receiver located in free space, with finite and strictly positive capacity. If the effective areas of the transmitter and the receiver antennas, and the distance between them are all doubled, and everything else remains unchanged, the maximum capacity of the wireless link **[2017]**

(A) increases by a factor of 2.
(B) decreases by a factor of 2.
(C) remains unchanged.
(D) decreases by a factor of $\sqrt{2}$.

Solution: $\frac{P_R}{P_T} = \left(\frac{\lambda}{4\pi R}\right)^2 DD$

$P_R \rightarrow$ Received power
$P_T \rightarrow$ Transmitted power
$D_t \rightarrow$ Transmitting antenna directivity
$D_r \rightarrow$ Receiving antenna directivity
$\lambda \rightarrow$ wave length

$$d = \frac{4K}{\lambda^2} A_e,$$

where $A_e \rightarrow$ effective area

$$\therefore \quad \frac{P_R}{P_T} = \frac{1}{\lambda^2 R^2} A_{et} A_{er}$$

Given that A_{et}, A_{er} and R each is doubled. Then, $\frac{P_R}{P_T}$ remains constant

Hence, the correct option is (C).

5. A sinusoidal message signal is converted to a PCM signal using a uniform quantizer. The required signal-to-quantization noise ratio (SQNR) at the output of the quantizer is 40 dB. The minimum number of bits per sample needed to achieve the desired SQNR is _______. **[2017]**

Solution: For the sinusoidal input

SQNR = $6n + 1.8$

(n : no. of bits/ sample)

$40 \leq 6n + 1.8$

$$n \leq \frac{38.2}{6} = 7$$

Hence, the correct answer is (7).

6. Consider binary data transmission at a rate of 56 kbps using base band binary pulse amplitude modulation (PAM) that is designed to have a raised cosine spectrum. The transmission bandwidth (in kHz) required for a roll off factor of 0.25 is _____. **[2016]**

Solution: Data transmission rate R_b = 56 kpbs,

Roll off factor $\alpha = 0.25$.

The transmission bandwidth (in kHz)

$$\text{BW} = \frac{R_b}{2}[1 + \alpha]$$

$$= \frac{56}{2}[1 + 0.25] \text{ kHz}$$

$$= 35 \text{ kHz}.$$

Hence, the correct Answer is (35).

7. A discrete memory-less source has an alphabet $\{a_1, a_2, a_3, a_4\}$ with corresponding probabilities $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right\}$. The minimum required average code word length in bits to represent this source for error-free reconstruction is ____. **[2016]**

Solution: The discrete memory-less source has following probabilities:

$$P_{a1} = 1/2, P_{a2} = 1/4, P_{a3} = 1/8, P_{a4} = 1/8$$

The entropy can be expressed as

$$H = P_{a_1} \log_2 \frac{1}{p_{a_1}} + P_{a_2} \log_2 \frac{1}{p_{a_2}}$$

$$+ P_{a_3} \log_2 \frac{1}{p_{a_3}} + P_{a_4} \log_2 \frac{1}{p_{a_4}}.$$

$$= \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \frac{1}{8}\log_2 8 + \frac{1}{8}\log_2 8$$

$H = 1.75$ bits per word

Hence, the correct Answer is (1.75).

8. A speech signal is sampled at 8 kHz and encoded into PCM format using 8 bits/sample. The PCM data is transmitted through a baseband channel via 4-level PAM. The minimum bandwidth (in kHz) required for transmission is _____. **[2016]**

Solution: Speech signal frequency F_s = 8khz,

No. of levels $M = 4$

No. of bits = $n = 8$

$M = 2^n$ then $n = \text{Log}_2 M$

and Bit rate $r_b = n \cdot F_s$

Bit rate = sampling rate × no. bits per sample

= 8k samples/sec × 8 bits/sample

r_b = 64 kbps

$$(\text{BW})_{\min} = \frac{r_b}{2\log_2 M}$$

M = no of level PAM.

$$(\text{BW})_{\min} = \frac{64}{2\log_2 4}$$

$$(\text{BW})_{\min} = 16 \text{ kHz}$$

Hence, the correct Answer is (16).

9. An analog voltage in the range 0 to 8 V is divided in 16 equal intervals for conversion to 4-bit digital output. The maximum quantization error (in V) is ______. **[2014]**

Solution:

The maximum quantization error

$$Q_e = \frac{\Delta}{2}$$

$$A = \frac{\text{Dynamic range}}{2^n \text{ or } L}$$

$$\Delta = \frac{8}{2^4}$$

$$\Delta = \frac{8}{16} = 0.5$$

$$Q_e = \frac{0.5}{2}$$

$$= 0.25$$

10. The bit rate of digital communication system is R Kbits/s. The modulation used is 32-QAM. The minimum bandwidth required for $|s|$ free transmission is

(a) $R/10$ Hz (b) $R/10$ kHz
(c) $R/5$ Hz (d) $R/5$ kHz **[2013]**

Solution: (b)

$2^n = m$

$n = \log_2 m = \log_2 32 = 5$

$$R_b' = \frac{R_b}{n} = \frac{R}{5} \text{ kbps}$$

$$(BW)_{\min} = \frac{R_b'}{2} = \frac{R}{10} \text{ kHz}$$

Hence, the correct option is (b)

11. In a baseband communications link, frequencies up to 3500 Hz are used for signalling. Using a raised cosine pulse with 75% excess bandwidth and for no inter-symbol interference, the maximum possible signalling rate in symbols per second is

(a) 1750 (b) 2625
(c) 4000 (d) 5250 **[2012]**

Solution: (d)

Frequency used for signalling is $f = 3500$ Hz

Excess BW used in B $= 0.75 \times f = 2625$

We know that if B is the BW available, then

$$\frac{R_b}{2} \leq B$$

That is, if P_D is data rate then $\frac{R_b}{2}$ is the minimum BW required for transmission.

R_b ″ $2B$

R_b ″ 5250

$(R_b)_{\max} = 5250$ symbol/sec

Hence, the correct option is (d)

12. An analog signal is band-limited to 4kHz, sampled at the Nyquist rate and the samples are quantized into 4 levels. The quantized levels are assumed to be independent and equally probable. If we transmit two quantized samples per second, the information rate is

(a) 1 bit/sec (b) 2 bit/sec
(c) 3 bit/sec (d) 4 bit/sec **[2011]**

Solution: (d)

Quantization levels are equiprobable hence

H $= \log_2 4 = 2$ bits/sample

$r = 2$ sample/sec

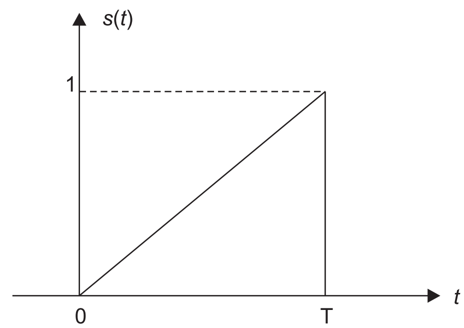
Hence information rate $= rH$

$R = 2 \times 2$

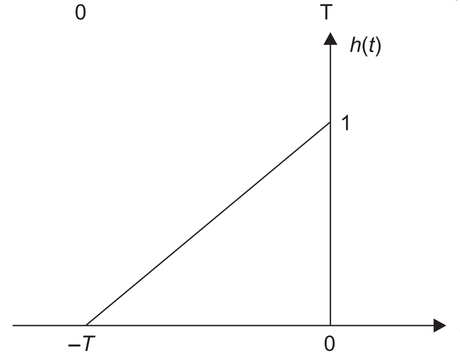
$= 4$ bits/sec

Hence, the correct option is (d)

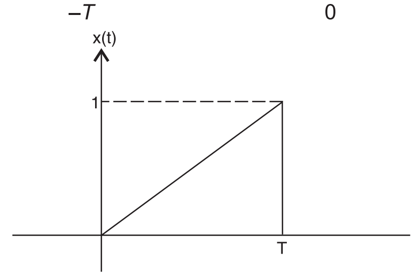
13. Consider the pulse shape $s(t)$ as shown. The impulse response $h(t)$ of the filter matched to this pulse is **[2010]**



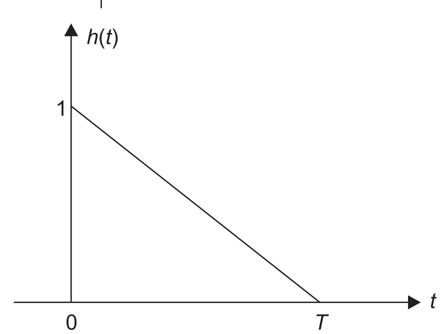
(a)



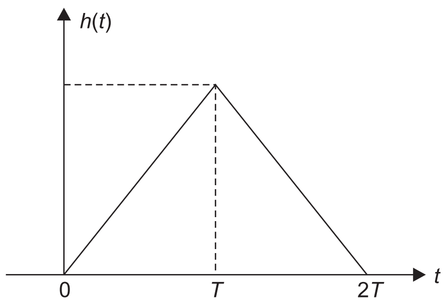
(b)



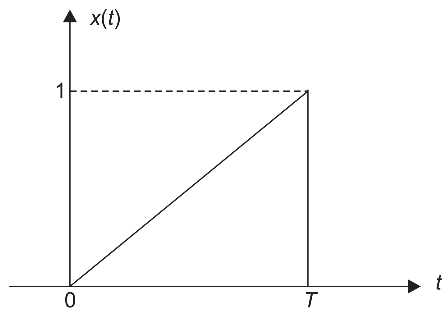
(c)



(d)

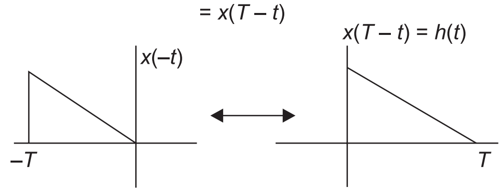


Solution: (c)



Impulse response of matched filter

$= x(T - t)$



Hence, the correct option is (c)

14. Refractive index of glass is 1.5. Find the wavelength of a beam of light with frequency of 10^{14} Hz in glass. Assume velocity of light is 3×10^8 m/s in vacuum

(a) 3 mm (b) 3 mm
(c) 2 mm (d) 1 mm **[2005]**

Solution: (c)

In vacuum $C = f\lambda$

$$C = 3 \times 10^8$$

$$f = 10^{14}$$

$$\lambda = \frac{3\times10^8}{10^{14}}$$

$$\lambda = 3 \times 10^{-6}\text{m}$$

In class $\lambda_g = \dfrac{\lambda}{\text{h}} = \dfrac{3\times10^{-6}}{1.5}$

$= 2\mu\text{m}$

Hence, the correct option is (c)

15. In a PCM system, if the code word length is increased from 6 to 8 bits, the signal to quantization noise ratio improves by the factor

(a) 8/6 (b) 12
(c) 16 (d) 8 **[2004]**

Solution: (c)

SNRQ α 2^{2h}

α 4^n

as $n = 2$

So SNRQ increases by 16.

Hence, the correct option is (c)

16. In the output of a DM speech encoder, the consecutive pulses are of opposite polarity during time interval $t_1 \leq t \leq t_2$. This indicates that during this interval

(a) the input to the modulator is essentially constant
(b) the modulator is going through slope overload
(c) the accumulator is in saturation
(d) the speech signal is being sampled at the Nyquist rate **[2004]**

Solution: (a)

In between two adjacent sampled value, if the base band signal changes an amount less than step size the output of the DM is a sequence of alternative positive and negative pulses. This small change in base band signal indicated that the base band is almost constant.

Hence, the correct option is (a)

17. At a given probability of error, binary coherent FSK is inferior to binary coherent PSK by

(a) 6 dB (b) 3 dB
(c) 2 dB (d) 0 dB **[2003]**

Solution: (b)

For given probability of error, binary coherent FSK is inferior to binary coherent PSK by 3dB.

Hence, the correct option is (b)

18. Consider a sample signal

$$y(t) = 5\times10^{-6}x(t)\sum_{n=-\infty}^{+\infty}\delta(t - n\text{T}_s) \text{ where}$$

$x(t) = 10\cos(8\pi \times 10^3)\,t$ and $\text{T}_s = 100\mu\text{sec}$

When $y(t)$ is passed through an ideal low-pass filter with a cutoff frequency of 5 kHz, the output of the filter is

(a) $5 \times 10^{-6}\cos(8\pi \times 10^3)t$
(b) $5 \times 10^{-5}\cos(8\pi \times 10^3)t$
(c) $5 \times 10^{-1}\cos(8\pi \times 10^3)t$
(d) $10\cos(8\pi \times 10^3)$ **[2002]**

Solution: (c)

$$y(t) = 5\times10^{-6}x(t)\sum_{n=-\infty}^{\infty}\delta(t - nT_s)$$

$$= 5\times10^{-6}x(t)\left\{\frac{1}{T_s} + \frac{2}{T_s}(\cos w_s t + \cos 2w_s t + ...)\right\}$$

$$= 5\times10^{-6}\left\{\frac{x(t)}{T_s} + \frac{2}{T_s}(x(t)\cos w_s t + x(t)\right.$$

$$\left.\cos 2w_s t + ...)\right\}$$

Given

cut off frequency of low pass fitter = 5 kHz

$T_s = 100 \times 10^{-6}$

$$f_s = \frac{1}{100} \times 10^6 = 10 \text{ kHz}$$

$$f_m = \frac{\theta\cancel{\pi} \times 10}{2\cancel{\pi}} = 4 \text{ Hz}$$

Only first component gets passed, other will be suppressed because $= f_s - f_{m1} f_s + f_{m1} 2f_s - f_{m,} 2f_s + f_m > 5$ Hz

$$y(t) = \frac{5 \times 10^{-6} \times (t)}{T_s} = \frac{5 \times \cancel{10^{-6}} \times 10 \cos(8\pi \times 10^3)}{100 \times \cancel{10^{-6}}}$$

$$= 5 \times 10^{-1} \cos (2\pi \times 10^3).$$

Hence, the correct option is (c)

19. For a bit-rate of 8 Kbps, the best possible values of the transmitted frequencies in a coherent binary FSK system are

(a) 16 kHz and 20 kHz
(b) 20 kHz and 32 kHz
(c) 20 kHz and 40 kHz
(d) 32 kHz and 40 kHz **[2002]**

Solution: (d)

For fsk

$f_1 = nf_b$

$f_2 = mf_b$

So $f_1 = 4 \times 8 = 32$ kHz

$f_2 = 5 \times 8 = 40$ kHz

Hence, the correct option is (d)

20. In a PCM system with uniform quantization, increasing the number of bits from 8 to 9 will reduce the quantization noise power by a factor of

(a) 9 (b) 8
(c) 4 (d) 2 **[1998]**

Solution: (c)

$$\text{SNRQ}\ \alpha\ 2^{2^n}$$

$$\frac{\text{SNRQ}_1}{\text{SNRQ}_1} = \frac{2^{2\times 8}}{2^{2\times 9}}$$

$(\text{SNRQ})_2 = 4(\text{SNRQ})_1$

So, increases by a factor of 4.

Hence, the correct option is (c)

21. Flat top sampling of low pass signals

(a) gives rise to aperture effect
(b) implies oversampling
(c) leads to aliasing
(d) introducing delay distortion **[1998]**

Solution: (a)

Flat top sampling of low pass signals gives rise to aperture effect.

Hence, the correct option is (a)

22. Quadrature multiplexing is

(a) the same as FDM
(b) the same as TDM
(c) a combination of FDM and TDM
(d) quite different from FDM and TDM

[1998]

Solution: (d)

Quadrature carrier multiplexing utilizes carrier phase shifting and synchronous detection to permit two DSB signals to occupy the same frequency band. It is the scheme where same carrier frequency is used for two different DSB signals. It is also known as QAM. So, quadrature multiflexing is quite different from FDM and TDM.

Hence, the correct option is (d)

23. Compression in PCM refers to relative compression of

(a) higher signal amplitudes
(b) lower signal amplitudes
(c) lower signal frequencies
(d) higher signal frequencies **[1998]**

Solution: (a)

Compression is PCM refer to relative compression of higher signal frequencies.

Hence, the correct option is (a)

24. The line code that has zero DC component for pulse transmission of random binary data is

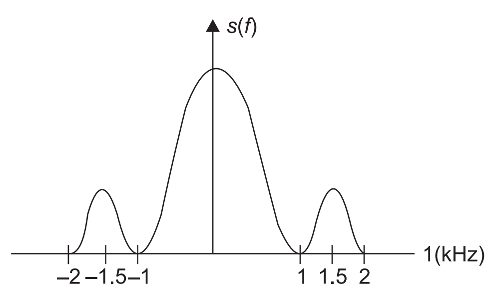
(a) more-return to zero (NRZ)
(b) return to zero (RZ)
(c) alternate mark inversion (AMI)
(d) none of the above **[1997]**

Solution: (c)

Alternate mark inversion (AMI) code has zero DC component for pulse transmission of random binary data.

Hence, the correct option is (c)

25. A deterministic signal has the power spectrum given in figure. The minimum sampling rate needed to completely represent signal is



(a) 1 kHz (b) 2 kHz
(c) 3 kHz (d) none of these **[1997]**

Solution: (b)

In sine function maximum power contain in main lobe. So, we consider only main lobe

BW = 2 kHz

$f_s = 2 \times \text{BW}$

$f_s = 4$ kHz

BW = 2(600 + 150)

= 2 × 750

= 1500 Hz

Hence, the correct option is (b)

26. If the number of bits per sample in a PCM system is increased from a n to $n + 1$, the improvement in signal to quantization nose ratio will be

(a) 3 dB (b) 6 dB
(c) 2 n dB (d) n dB **[1995]**

Solution: (b)

We know that

$(\text{SNRQ})_{\text{dB}} = 1.8 + 6n$

for n bit

$(\text{SNRQ})_{\text{dB}} = 1.8 + 6n$

for $n + 1$ bit

$$(\text{SNRQ})'_{\text{dB}} = 1.8 + 6(n+1)$$

$$= 1.8 + 6n + 6$$

So, SNR increases by 6 dB

Hence, the correct option is (b)

27. A 1.0 kHz signal is flat top sampled at the rate of 1800 samples/sec and the samples are applied to an ideal rectangular LPF with cut-off frequency of 1100Hz, then the output of the filter contains

(a) only 800 Hz component
(b) 800 Hz and 900 Hz components
(c) 800 Hz and 1000 Hz components
(d) 800 Hz, 900 Hz and 100 Hz components **[1995]**

Solution: (c)

$$f_{out} = \frac{M(t)}{T_s} + \frac{2}{T_s}$$

$(m(t) \cos w_s t + m(t) \cos 2\, w_s t)$

$f_m = 1$ kHz

$f_s = 1.8$ kHz

$f_{\text{cut off}} = 1.1$ kHz

So, at output frequencies present $= f_m, f_s - f_m$

= 1000 Hz and 800 Hz components

Hence, the correct option is (c)

28. The signal to quantization noise ratio in an n-bit system

(a) depends upon the sampling frequency employed
(b) is independent of the value of 'n'
(c) increases with increasing value of 'n'
(d) decreases with the increasing value of 'n' **[1995]**

Solution: (c)

We know that

$$\text{SNRQ} = \frac{3}{2} 2^{2^n}$$

So SNRQ depends on n

Hence SNRQ increases as increasing value of n.

Hence, the correct option is (c)

29. For a given data rate, the bandwidth B_p of a BPSK signal and the bandwidth B_0 of the OOK signal are related as

(a) $B_P = B_0/4$
(b) $B_P = B_0/2$
(c) $B_P = B_0$
(d) $B_{Pv} = 2B_0$ **[1995]**

Solution: (c)

$$\text{SNRQ}\ \alpha\ 2^{2^n}$$

$$\frac{(\text{SNRQ})_1}{(\text{SNRQ})_2} = \frac{2^{2^n}}{2^{2(n+1)}} = \frac{\cancel{2^{2^n}}}{\cancel{2^{2^n}}.2^2}$$

$(\text{SNRQ}_2) = (\text{SNRQ}) \times 4$

So SNRQ increases by factor = 4

So this is independent of n.

Hence, the correct option is (c)

30. Increased pulse width in the flat top sampling leads to

(a) attenuation of high frequencies in reproduction
(b) attenuation of low frequencies in reproduction
(c) greater aliasing errors in reproduction
(d) no harmful effects in reproduction **[1994]**

Solution: (a)

Increased pulse width in the flat-top sampling leads to greater attenuation of high frequencies in reproduction. This effect is known as aperture effect.

Hence, the correct option is (a)

31. The bandwidth required for the transmission of a PCM signal increases by a factor of _____ when the number of quantization levels is increased from 4 to 64. **[1994]**

Solution:

$$\text{BW of PCM system} = \frac{\eta f_s}{2}$$

BW α n

$L_1 = 4$, $L_2 = 64$

$\eta_1 = \log_2 4$, $\eta_2 = \log_2 64$

$\eta_1 = 2$, $\eta_2 = 6$

$$\frac{BW_1}{BW_2} = \frac{\eta_1}{\eta_2} = \frac{\cancel{2}}{\cancel{6}\;3}$$

$BW_2 = 3 \times BW_1$

Increases by a factor of 3.

Two-marks Questions

1. A single bit, equally likely to be 0 and 1, is to be sent across an additive white Gaussian noise (AWGN) channel with power spectral density $N_0/2$ Binary signaling with $0 \rightarrow p(t)$ and $1 \rightarrow q(t)$, is used for the transmission, along with an optimal receiver that minimizes the bit-error probability.

Let $\phi_1(t)$, $\phi_2(t)$ from an orthonormal signal set.

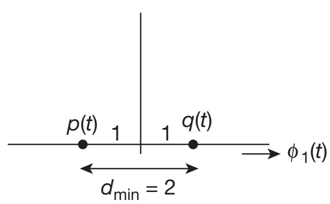
If we choose $p(t) = \phi_1(t)$ and $p(t) = -\phi_1(t)$ we would obtain a certain bit-error probability $P_{b.}$

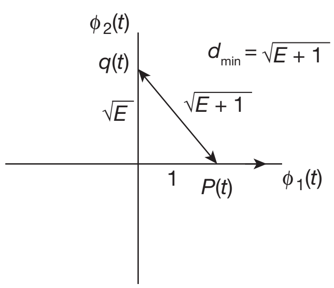
If we keep $p(t) = \phi_1(t)$, but take $q(t) = \sqrt{E}\,\phi_2(t)$, for what value of E would we obtain the same bity-error probability P_b? **[2019]**

(A) 0 (B) 3

(C) 1 (D) 2

Solution: $P(0) = P(1) = \dfrac{1}{2}$





For probability of error to be same

$$d_1 = d_2$$
$$2 = \sqrt{E+1}$$
$$4 = E+1$$
$$E = 3$$

Hence, the correct option is (B)

2. In binary frequency shift keying (FSK) the given signal waveforms are

$u_0(t) = 5\cos(20000\pi t)$; $0 \le t \le T$, and

$u_1(t) = 5\cos(22000\,\pi t)$; $0 \le t \le T$,

Where T is the bit-duration interval and t is in seconds. Both $u_0(t)$ and $u_1(t)$ are zero outside the interval $0 \le t \le$ T. With a matched filter (correlator) based receiver, the smallest positive value of T (in milliseconds) required to have $u_0(t)$ and $u_1(t)$ uncorrelated is **[2017]**

(A) 0.25 ms (B) 0.5 ms

(C) 0.75 ms (D) 1.0 ms

Solution: $u_0(t) = 5\cos 20000\pi t$

$u_1(t) = 5\cos 22000\pi t$

FSK gets uncorrelated when

$$f_1 - f_2 = n\frac{R_b}{2}; \quad n = 1, 2, 3....$$

$$R_b = 2\left(\frac{f_1 - f_2}{n}\right) = \frac{2000}{n} \text{ bits/s}$$

$$R_{b(\max)} = 200 \text{ bits/see}$$

$$T_{b(\min)} = \frac{1}{R_{b(\max)}} = 0.5 \text{ ms}$$

Hence, the correct option is (B).

3. A digital communication system uses a repetition code for channel encoding/decoding. During transmission, each bit is repeated three times (0 is transmitted as 000, and 1 is transmitted as 111). It is assumed that the source puts out symbols independently and with equal probability. The decoder operates as follows: In a block of three received bits, if the number of zeros exceeds the number of ones, the decoder decides in favour of a 1. Assuming a binary symmetric channel with crossover probability $p = 0.1$, the average probability of error is _____. **[2016]**

Solution: Using the error probability for digital system when each bit is repeated 3 times. All the symbols are independent and have equal probabilities.

$$P_e = P^3 + 3P^2(1 - P)$$

$$P = 0.1$$

$$P_e = 0.028.$$

Hence, the correct option is (0.028).

4. An analog pulse $s(t)$ is transmitted over an additive white Gaussian noise (AWGN) channel. The received signal is $r(t) = s(t) + n(t)$, where $n(t)$ is additive white Gaussian noise with power spectral density $\dfrac{N_0}{2}$. The received signal is passed through a filter with impulse response $h(t)$. Let E_s and E_h denote the energies of the pulse $s(t)$ and the filter $h(t)$ respectively. When the signal to noise ratio (SNR) is maximized at the output of the filter (SNR_{max}) which of the following holds? **[2016]**

(A) $E_s = E_h$; $\text{SNR}_{\max} = \dfrac{2E_s}{N_0}$

(B) $E_s = E_h$; $\text{SNR}_{\max} = \dfrac{E_s}{2N_0}$

(C) $E_s > E_h$; $\text{SNR}_{\max} > \dfrac{2E_s}{N_0}$

(D) $E_s < E_h$; $\text{SNR}_{\max} = \dfrac{2E_h}{N_0}$

Solution: $\text{SNR}_{\max} = \dfrac{2E_b}{N_o}$ $\quad [\because E_b = E_s]$

Hence, the correct option is (A).

5. An information source generates a binary sequence $\{\alpha_n\}$. α_n can take one of the two possible values –1 and +1 with equal probability and are statistically independent and identically distributed. This sequence is precoded to obtain another sequence $\{\beta_n\}$, as $\beta_n = \alpha_n + k\alpha_{n-3}$. The sequence $\{\beta_n\}$ is used to modulate a pulse $g(t)$ to generate the base band signal **[2016]**

$$X(t) = \sum_{n=-\infty}^{\infty} \beta_n g(t - nT),$$

where

$$g(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}.$$

If there is a null at $f = \dfrac{1}{3T}$ in the power spectral density of $X(t)$, then k is ______.

Solution:

$$S_x(f) = \frac{|G(f)|^2}{T}\left[\sum_{m=-\infty}^{\infty} R_b(\tau) e^{j2\pi f \tau T}\right]$$

$$R_b(\tau) = E[\beta_n \beta_{n-\tau}]$$

$$= E[(\alpha_n + k\alpha_{n-3})(\alpha_{n-\tau} + k_{\alpha n-\tau-3})]$$

$$= (1 + k^2)R(\tau) + KR(\tau + 3) + KR(\tau - 3)$$

$$R_b(\tau) = \begin{bmatrix} 1+k^2 & \tau = 0 \\ k & \tau = \pm 3 \\ 0 & otherwise \end{bmatrix}$$

PSD, $S_b(f) = 1 + k^2 + 2K\cos(2\pi f 3T)$

Null occurs at $f = \dfrac{1}{3T}$

$\Rightarrow \quad f = \dfrac{1}{3T}$, $S_b = 1 + K^2 + 2K\cos 2\pi\left(\dfrac{1}{3T}\right) \times 3T$

i.e., $\quad 1 + k^2 + 2k = 0$

$\Rightarrow \quad k = -1$

Hence, the correct Answer is (–1).

6. An ideal band pass channel 500 Hz–2000 Hz is deployed for communication. A modem is designed to transmit bits at the rate of 4800 bits/s using 16 – QAM. The roll off factor of a pulse with a raised cosine spectrum that utilizes the entire frequency band is __________. **[2016]**

Solution: Bit rate R_b = 4800 bits/s

Bandwidth BW = 2000Hz – 500Hz

= 1500 Hz

Number of Levels

$$m = 16,\ m = 2^n$$

where n is the number of bits and

$$n = \log_2 m$$

Let the roll-off factor of a pulse be $\alpha = ?$

Now using the relation

$$\text{BW} = \frac{R_b(1+\alpha)}{\log_2 m}$$

$$1500 = \frac{R_b(1+\alpha)}{\log_2 16}$$

$$\alpha = 0.25.$$

Hence, the correct Answer is (0.25).

7. A binary communication system makes use of the symbols 'zero' and 'one'. There are channel errors. Consider the following events:

x_0: a 'zero' is transmitted

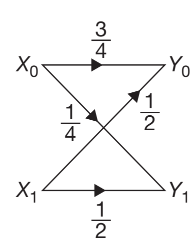
x_1: a 'one' is transmitted

y_0: a 'zero' is received

y_1: a 'one' is received

The following probabilities are given: $P(x_0) = \dfrac{1}{2}$ $P(y_0/x_0) = \dfrac{3}{4}$ and $P(y_0/x_1) = \dfrac{1}{2}$. The information in bits that you obtain when you learn which symbol has been received (while you know that a 'zero' has been transmitted) is _____. **[2016]**

Solution: It is given that,



$P(x_0) = 1/2$

$P(y_0/x_0) = 3/4 \qquad P(y_0).P(x_0) = P(x_0, y_0)$

$P(y_1/x_0) = 1/4 \qquad \frac{3}{4} \times 1/2 = P(x_0, y_0)$

$\frac{3}{8} = P(x_0, y_0)$

$P(x_0, y_1) = P(x_0)\, P(y_1)$

$= 1/2 \times 1/4 = 1/8.$

(Using the expression of entropy for conditional probability)

$$H(y/x_0) = P(y_0/x_0) \log_2 \frac{1}{P(y_0/x_0)} + P(y_1/x_0) \log_2 \frac{1}{P(y_1/x_0)}$$

$$= \frac{-3}{4} \log_2 \frac{3}{4} - \frac{1}{4} \log_2 \frac{1}{4}$$

$H(y/x_0) = 0.81$ bits per symbol.

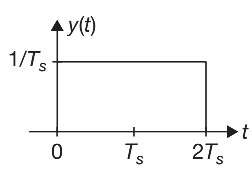
Hence, the correct Answer is (0.81).

8. A binary baseband digital communication system employs the signal

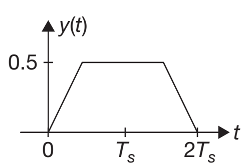
$$p(t) = \begin{cases} \frac{1}{\sqrt{T_s}}, 0 \le t \le T_s \\ 0, \quad \textit{otherwise} \end{cases}$$

for transmission of bits. The graphical representation of the matched filter output $y(t)$ for this signal will be **[2016]**

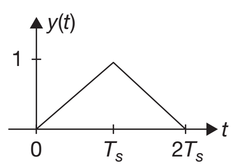
(A)



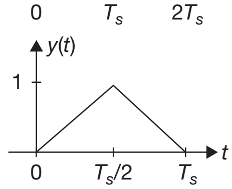
(B)



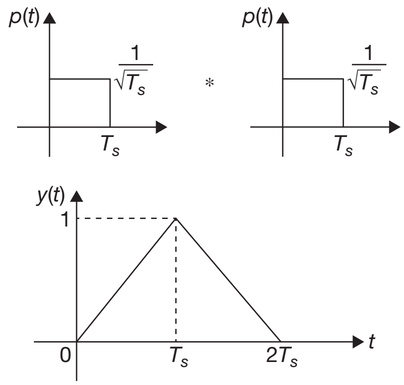
(C)



(D)



Solution: Convolution of 2 pulse with equal width = triangular.



Since, the convolution of two rect pulses is always triangular with width($= 2T_s$) equal to the sum of the duration of two rect pulses. So option (C).

Hence, the correct option is (C).

9. Consider a four-point moving average filter defined by the equation $y[n] = \sum_{i=0}^{3} \alpha_1 x[n-i]$. The condition on the filter coefficients that results in a null at zero frequency is **[2015]**

(A) $\alpha_1 = \alpha_2 = 0;\ \alpha_0 = -\alpha_3$
(B) $\alpha_1 = \alpha_2 = 1;\ \alpha_0 = -\alpha_3$
(C) $\alpha_0 = \alpha_3 = 0;\ \alpha_1 = \alpha_2$
(D) $\alpha_1 = \alpha_2 = 0;\ \alpha_0 = \alpha_3$

Solution: Given $Y[n] = \sum_{i=0}^{3} \alpha_i x[n-i]$

$$= \alpha_0\, x[n] + \alpha_1\, x\,[n-1] + \alpha_2\, x[n-2] + \alpha_3\, x[n-3]$$

Given filter represents high pass filter because null is at zero frequency.

From options, let us consider $\alpha = \alpha_2 = 0; \alpha_0 \ -\alpha_3$

Then $\quad Y[n] = -\alpha_3 \left[x[n] - x[n-3] \right]$

$$H[z] = -\alpha_3\, [1 - z^{-3}]$$

$$H[e^{jn}] = -\alpha_3\, [1 - e^{-j3n}]$$

At $\Omega = 0$ then $H(e^0) = -\alpha_3\, [1-1]$

$= 0$

So option A is satisfied.

Hence, the correct option is (A).

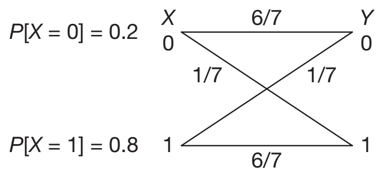
10. The modulation scheme commonly used for transmission from GSM mobile terminals is **[2015]**

(A) 4-QAM
(B) 16-PSK
(C) Walsh–Hadamard orthogonal codes
(D) Gaussian minimum shift keying (GMSK)

Solution: The modulation scheme commonly used for transmission from GSM mobile terminals is Gaussian minimum shift keying (GMSK).

Hence, the correct option is (D).

11. The input X to the Binary Symmetric Channel (BSC) shown in the figure is '1' with probability 0.8. The cross-over probability is 1/7. If the received bit $Y = 0$, the conditional probability that '1' was transmitted is ______. **[2015]**



Solution: $P[x] = [0.2 \;\; 0.8]$

$$P\left[\frac{Y}{X}\right] = \begin{array}{c} \\ x_1 \\ x_2 \end{array}\begin{array}{c} \begin{array}{cc} Y_1 & Y_2 \end{array} \\ \begin{bmatrix} 6/7 & 1/7 \\ 1/7 & 6/7 \end{bmatrix} \end{array}$$

$$\text{Joint probability } P[X, Y] = \begin{array}{c} \\ X_1 \\ X_2 \end{array}\begin{array}{c} \begin{array}{cc} Y_1 & Y_2 \end{array} \\ \begin{bmatrix} \frac{6}{7}\times 0.2 & \frac{1}{7}\times 0.2 \\ \frac{1}{7}\times 0.8 & \frac{6}{7}\times 0.8 \end{bmatrix} \end{array}$$

$$= \begin{array}{c} \\ X_1 \\ X_2 \end{array}\begin{array}{c} \begin{array}{cc} Y_1 & Y_2 \end{array} \\ \begin{bmatrix} 0.171 & 0.0285 \\ 0.114 & 0.685 \end{bmatrix} \end{array}$$

$$P(Y_1) = 0.171 + 0.114 = 0.285$$

$$P[Y_2] = 0.0285 + 0.685 = 0.7130$$

$$P[X/Y] = \begin{array}{c} \\ X_1 \\ X_2 \end{array}\begin{array}{c} \begin{array}{cc} Y_1 & Y_2 \end{array} \\ \begin{bmatrix} 0.6 & 0.0399 \\ 0.4 & 0.960 \end{bmatrix} \end{array}$$

By dividing columns of $P(X, Y)$ by $P(Y_1)$ and $P(Y_2)$

So $P\left(\frac{X_2}{Y_1}\right) = 0.4$

Hence, the correct Answer is (0.39 to 0.41).

12. The transmitted signal in a GSM system is of 200 kHz bandwidth and 8 users share a common bandwidth using TDMA. If at a given time 12 users are talking in a cell, the total bandwidth of the signal received by the base station of the cell will be at least (in kHz) ______. **[2015]**

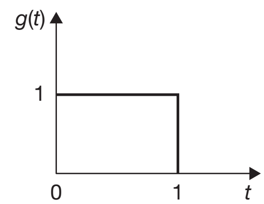
Solution: 8 users share a channel of bandwidth 200 kHz.

So for 12 users two channels will be used so bandwidth will be $2 \times 200 = 400$ kHz.

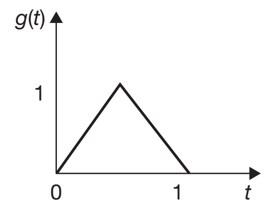
Hence, the correct Answer is (400).

13. Consider a binary, digital communication system which uses pulses $g(t)$ and $-g(t)$ for transmitting bits over an AWGN channel. If the receiver uses a matched filter, which one of the following pulses will give the minimum probability of bit error? **[2015]**

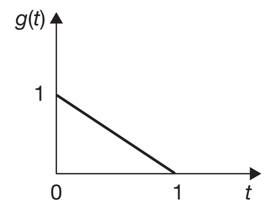
(A)



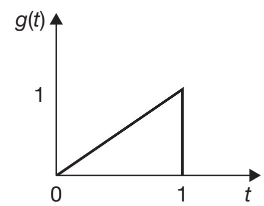
(B)



(C)



(D)



Solution: For a matched filter receiver the probability of error

$$= Q\left(\sqrt{\frac{2E}{N_o}}\right)$$

The probability of error will be minimum for which energy is maximum

So for option (A) energy $E = \int_0^1 (1)^2\, dt = 1$

For option (B) energy $E = 2\int_0^{1/2} (2t)^2\, dt$

$$= 2\left[\frac{4t^3}{3}\right]_0^{1/2}$$

$$= \frac{1}{3}$$

For option (C) and (D) energy $E = \int_0^1 (t)^2\, dt$

$$= \left[\frac{t^3}{3}\right]_0^1 = \frac{1}{3}$$

Hence, the correct option is (A).

14. A sinusoidal signal of amplitude A is quantized by a uniform quantizer. Assume that the signal utilizes all the representation levels of the quantizer. If the signal to quantization noise ratio is 31.8 dB, the number of levels in the quantizer is ________. **[2015]**

Solution: $(\text{SNR})_{\text{dB}} = 6N + 1.8$

Where N = Number of bits used to encode

$$31.8 = 6N + 1.8$$

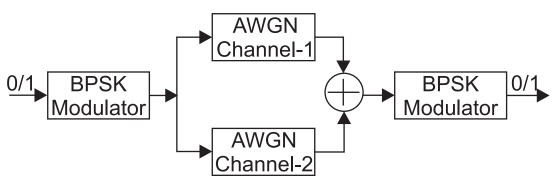
$$6N = 30$$

$$N = 5$$

So no. of levels $= 2^5 = 32$.

Hence, the correct Answer is (32).

15. Let $Q(\sqrt{\gamma})$ be the BER of a BPSK system over an AWGN channel with two-sided noise power spectral density $N_0/2$. The parameter γ is a function of bit energy and noise power spectral density. A system with two independent and identical AWGN channels with noise power spectral density $N_0/2$ is shown in figure. The BPSK demodulator receives the sum of outputs of both the channels.



If the BER of this system is $Q(b\sqrt{\gamma})$ then the value of b is _____. **[2014]**

Solution:

BER for the given system is $Q\sqrt{2r}$

$b = \sqrt{2}$

$= 1.414$.

16. Coherent orthogonal binary FSK modulation is used to transmit two equiprobable symbol waveforms $s_1(t) = 2\cos 2\pi f_1 t$ and $s_2(t) = 2\cos 2\pi f_2 t$, where α= 4 mV. Assume an AWGN channel with two-sided noise power spectral density $\frac{N_0}{2} = 0.5\times 10^{-12}$ W/Hz. Using an optimal receiver and relation the bit error probability for a data rate of 5000 kbps is

(a) $Q(2)$ (b) $Q(2\sqrt{2})$

(c) $Q(4)$ (d) $Q\left(4\sqrt{2}\right)$ **[2014]**

Solution: (c)

$$P_e = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$R_b = 500 \times 10^3$ bps

$$\frac{N_0}{2} = 0.5\times 10^{-12}$$

$$\alpha = 4 \times 10^{-3}\ V$$

Then $T_b = \dfrac{1}{R_b} = 2\times 10^{-6}$ sec

$$E_b = \frac{2\times 10^{-6}\times 16\times 10^{-6}}{2}$$

$$= 16 \times 10^{-12}$$

$$P_e = \frac{1}{2} erfc\left[\sqrt{\frac{16\times 10^{-12}}{2\times 10^{-12}}}\right]$$

$$= \frac{1}{2} erfc\sqrt{8}$$

$$P_e = \frac{1}{2} erfc\left(\frac{\sqrt{8}\times\sqrt{2}}{\sqrt{2}}\right)$$

$$= \frac{1}{2} erfc\left(\frac{\sqrt{16}}{\sqrt{2}}\right)$$

$$= \frac{1}{2} erfc\left(\frac{4}{\sqrt{2}}\right)$$

$$P_e = \theta(y)\left[\because \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right) = Q(x)\right]$$

$$P\left(\frac{y=0}{x=1}\right) = \frac{1}{4}\times\frac{1}{4}\times 1 = \frac{1}{16}$$

$$P_e = \frac{1}{2}\times 0 + \frac{1}{2}\times\frac{1}{8}$$

$$= \frac{1}{16}$$

Hence, the correct option is (c)

17. In a PCM system, the signal $m(t) = \{\sin(100\pi t + \cos(100\pi t))\}$ is sampled at the Nyquist rate. The samples are processed by a uniform quantizer with step size 0.75 V. The minimum data rate of the PCM system in bits per second is **[2014]**

Solution:

$m(t) = \sin(100\pi t) + \cos(100\pi t)$

Maximum peak of $m(t)$ is $\sqrt{2}$

$$\Delta = \frac{V_{\max} - V_{\min}}{2^n}$$

$$0.75 = \frac{2\sqrt{2}}{2^n}$$

$$0.75 = \frac{2\sqrt{2}}{2^n}$$

$n = 2$

$R_b = nf_s$

$= 200$ bits/sec

18. An M-level PSK modulation scheme is used to transmit independent binary digits over a band-pass channel with bandwidth 100 kHz. The bit rate is 200 kbps and the system characteristic is a raised-cosine spectrum with 100% excess bandwidth. The minimum value of M is ________.

[2014]

Solution:

$$B = \frac{R_b}{\log_2 m}(1+\alpha)$$

$$100 = \frac{200\times 2}{\log_2 m}$$

$\log_2 m = 4$

$m = 16$

19. If the detection threshold is 1, the BER will be

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{1}{16}$ **[2013]**

Solution: (d)

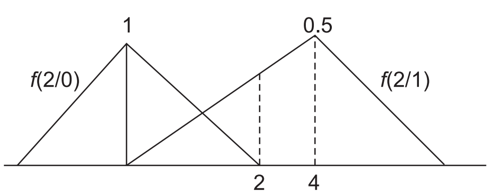
BER is given as

$$P_e = P(0)P\left(\frac{1}{0}\right)+P(1)P\left(\frac{0}{1}\right)$$

If detection threshold = 1 then

$$P(0)=P(1)=\frac{1}{2}$$

$$P\left(\frac{y=1}{x=0}\right)=0$$



Hence, the correct option is (d)

20. The optimum threshold to achieve minimum bit error rate (BER) is

(a) $\frac{1}{2}$ (b) $\frac{4}{5}$

(c) 1 (d) $\frac{3}{2}$ **[2013]**

Solution: (b)

Optimum threshold is given by the point of intersection of two PDF curves

$$f\left(\frac{2}{0}\right)=1-|2|\ \ |2|''\ 1$$

$$f\left(\frac{2}{1}\right)=\frac{2}{4};\quad 0<2<2$$

The point of intersection which decides optimum threshold

$$1-2=\frac{2}{4}$$

$$1=2+\frac{2}{4}$$

$$2=\frac{4}{5}$$

21. A binary symmetric channel (BSC) has a transition probability of 1/8. If the binary transmit symbol X is such that $P(x = 0) = 9/10$, then the probability of error for an optimum receiver will be

(a) 7/80 (b) 63/80

(c) 9/10 (d) 1/10 **[2012]**

Solution:

$$P(x=0)=\frac{9}{10}$$

$$P(x=1)=1-\frac{9}{10}$$

$$=\frac{1}{10}$$

Transition Probability $= P\left(\frac{1}{0}\right)=P\left(\frac{0}{1}\right)=\frac{1}{8}$

$$P_e = P(0)P\left(\frac{1}{0}\right)+P(1)P\left(\frac{0}{1}\right)$$

$$=\frac{+9}{10}\times\frac{1}{8}+\frac{1}{10}\times\frac{1}{8}$$

$$=\frac{1}{8}$$

22. A BPSK scheme operating over an AWGN channel with noise power spectral density of $N_0/2$, uses equiprobable signals $s_1(t)=\sqrt{\frac{2E}{T}}\sin(\omega_o t)$ and $s_2(t)=-\sqrt{\frac{2E}{T}}\sin(\omega_o t)$ over the symbol internal (0, T). If the local oscillator in a coherent receiver is ahead in phase 45° with respect to the received signals, the probability of error in the resulting system is

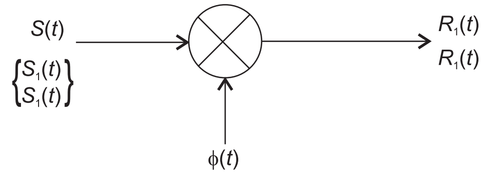
(a) $Q\left(\sqrt{\frac{2E}{N_o}}\right)$ (b) $Q\left(\sqrt{\frac{E}{N_o}}\right)$

(c) $Q\left(\sqrt{\frac{E}{2N_o}}\right)$ (d) $Q\left(\sqrt{\frac{E}{4N_o}}\right)$

[2012]

Solution: (b)

BPSK receiver is shown as



Given BPSK signals are $S_1(t)=\sqrt{\dfrac{2E}{T}}\sin(\omega_c t)$ and $S_2(t)=-\sqrt{\dfrac{2E}{T}}\sin(\omega_c t)$ for the interval of $0\le t\le t$ duration. Let ϕ (t) represent the local oscillator generated signal which is 45° leading with respect to the received signal. Then $\phi(t)=\sqrt{\dfrac{2}{T}}\sin(\omega_c t+45)\quad 0\le t\le T$ assuming ϕ(t) is of Unit energy. Let $R_1(t)$ and $R_2(t)$ be the received signal output, given as

$$=R_1(t)=\int_0^T S_1(t)\phi(t)\,dt$$

$$=\int_0^T\sqrt{\frac{2E}{T}}\sin(\omega_c t)\sqrt{\frac{2}{T}}\sin(\omega_c t+45°)\,dt=\sqrt{\frac{E}{2}}$$

Similarly,

$$R_2(t)=-\sqrt{\frac{E}{2}}\,.$$

Let decision threshold be given as $\gamma=\sqrt{\dfrac{E}{2}}+N$. Then probability of error while transmitting S_1 and receiving it as s_2 is

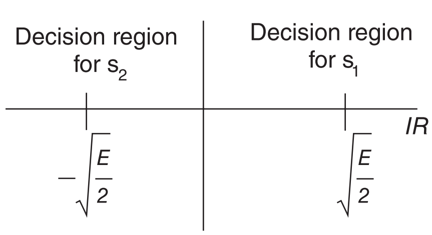
$$P(S_2/S_1)=P(\gamma<0)=P\left(\sqrt{\frac{E}{2}}+N<0\right)$$

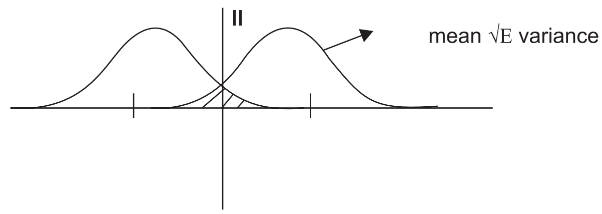
$$=P\left(N<-\sqrt{\frac{E}{2}}\right)$$

And

$$P(S_2/S_1)=P(\gamma<0)=P\left(\sqrt{\frac{E}{2}}+N<0\right)$$

$$=P\left(N<-\sqrt{\frac{E}{2}}\right)$$





$$P\left(N<\sqrt{\frac{E}{2}}\right)=\int_{-\infty}^{0}\frac{1}{\sqrt{2\pi\frac{N_0}{2}}}\exp\left(\frac{\left(x+\sqrt{\frac{E}{2}}\right)^2}{2\frac{N_0}{2}}\right)dx$$

$$=\int_{-\infty}^{0}\frac{1}{\sqrt{\pi N_0}}\exp\frac{\left(x+\sqrt{\frac{E}{2}}\right)^2}{N_0}dx$$

Let $\dfrac{\left(x+\sqrt{\frac{E}{2}}\right)^2}{N_0}=Z$. Then $dx=\sqrt{\dfrac{N_0}{2}}dz$,

So,

$$P\left(N<\sqrt{\frac{E}{2}}\right)=\int_{\sqrt{\frac{E}{N_0}}}^{\infty}\frac{1}{2\pi}\exp\left(-\frac{Z^2}{2}\right)dz$$

$$=Q\left(\sqrt{\frac{E}{N_0}}\right)$$

As symbols are equiprobable

so $P(e)=\dfrac{1}{2}\left(P(S_2/S_1)+P(S_1/S_2)\right)$

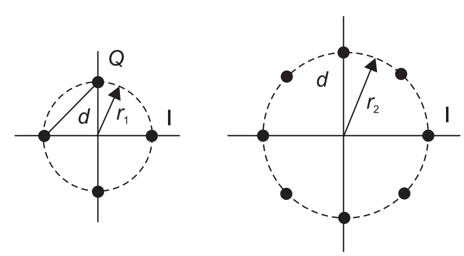
so

$$P(e)=\frac{1}{2}Q\left(\sqrt{\frac{E}{N_0}}\right)+\frac{1}{2}Q\left(\sqrt{\frac{E}{N_0}}\right)=Q\left(\sqrt{\frac{E}{N_0}}\right)$$

Hence, the correct option is (b)

Common Data Questions 23 & 24

A four-phase and an eight-phase signal constellation are shown in the figure below.



23. For the constraint that the minimum distance between pairs of signals points be d for both constellations, the radii r_1 and r_2 of the circles are

(a) $r_1 = 0.707\, d, r_2 = 2.782\, d$
(b) $r_1 = 0.707\, d, r_2 = 1.932\, d$
(c) $r_1 = 0.707\, d, r_2 = 1.545\, d$
(d) $r_1 = 0.707\, d, r_2 = 1.307\, d$ **[2011]**

Solution: (d)

For m c

$$d = 2\sin\left(\frac{\pi}{m}\right)\sqrt{E_s}$$

Distance of any point from origin is $\sqrt{E_s}$

For 4-ary, $r_1 = \sqrt{E_{s1}}$

8-ary, $r_2 = \sqrt{E_{s2}}$

For 4 ary $m = 4$ $\quad d_1 = 2\sin\left(\frac{\pi}{4}\right)r_1$

8 ray $m = 8$ $\quad d_2 = 2\sin\left(\frac{\pi}{4}\right)r_2$

if $d_1 = d_2 =$ d then

$$2\sin\left(\frac{\pi}{4}\right)r_1 = d$$

$$r_1 = \frac{d}{\sqrt{2}} = 0.707d$$

$$2\sin\left(\frac{\pi}{8}\right)r_2 = d$$

$$r_2 = \frac{d}{2\sin\left(\frac{\pi}{6}\right)}$$

$$r_2 = 1.307d$$

Hence, the correct option is (d)

24. Assuming high SNR and that all signals are equally probable, the additional average transmitted signal energy required by the 8-PSK signals to achieve the same error probability as the 4-PSK signals is

(a) 11.90 dB (b) 8.73 dB
(c) 6.79 dB (d) 5.33 dB **[2011]**

Solution: (d)

$$P_e \alpha \sqrt{E_s}$$

$$\frac{\sqrt{E_{S_1}}}{\sqrt{E_{S_2}}} = \frac{r_1}{r_2} = \frac{0.707\,d}{1.307\,d}$$

$$\frac{E_{S_2}}{E_{S_1}} = \left(\frac{1.307}{0.707}\right)^2 = 3.42$$

To active same error 2nd must have 3.42 time than 1st.

The value in dB = 10 log (3.42)

= 5.33 dB

Hence, the correct option is (d)

25. The Nyquist sampling rate for the signal

$$s(t) = \frac{\sin(500\ \ t)}{t} \times \frac{\sin(700\ \ t)}{t}$$

is given by

(a) 400 Hz (b) 600 Hz
(c) 1200 Hz (d) 1400 Hz **[2010]**

Solution: (c)

$$s(t) = \frac{1}{2}\left[\frac{2\sin(500\pi t)\sin(700\pi t)}{\pi^2 t^2}\right]$$

$$= \frac{1}{2\pi^2 t^2}\left[\cos(700\pi t - 500\pi t) - \cos(700\pi t + 500\pi t)\right]$$

$$= \frac{1}{2\pi 4^2}[\cos 200\pi t - \cos 1200\pi t]$$

Maximum frequency component

$$f_m = \frac{1200\cancel{\pi}}{2\cancel{\pi}} = 600$$

Nyquist sampling rate = $2f_m$ = 1200 Hz

Hence, the correct option is (c)

Common data for Questions 26 and 27

The amplitude of a random signal is uniformly distributed between –5V and 5V.

26. If the signal to quantization noise ratio required in uniformly quantizing the signal is 43.5 dB, the step size of the quantization is approximately

(a) 0.0333 V (b) 0.05 V
(c) 0.0667 V (d) 0.10 V **[2009]**

Solution: (c)

SNR = 43.5 dB

$1.76 + 6.02n = 43.5$

$n = 6.94$

$\oplus$ 7

$$\text{Step size } \Delta = \frac{V_H - V_L}{2^n}$$

$$= \frac{5-(-5)}{2^7}$$

= 0.07

$\oplus$ 0.0667

Hence, the correct option is (c)

27. If the positive values of the signals are uniformly quantized with a step size of 0.05 V, and the negative values are uniformly quantized with a step size of 0.1 V, the resulting signal to quantization noise ratio is approximately

(a) 46 dB (b) 43.8 dB
(c) 42 dB (d) 40 dB **[2009]**

Solution: (b)

For the samples

$S = 0.05$

$Q = 2^n$

$$Q = \frac{5}{0.5} = 100$$

$Q = 2^n$

$n = 7$

SNR = $1.76 + 6n$

= 43.76 dB

for one samples

$$s = 0.1,\ \theta = \frac{5}{0.1} = 10$$

$\vartheta = 2^n$, $n = 4$

SNR = $1.76 + 6n$ = 25.76 dB

Best SNR = 43.8 dB.

Hence, the correct option is (b)

28. Consider a Binary Symmetric Channel (BSC) with probability of error being p. To transmit a bit, say 1, we transmit a sequence of three 1s. The receiver will interpret the received sequence to represent 1 if at least two bits are 1. The probability that the transmitted bit will be received in errors is

(a) $p^3 + 3p^2(1-p)$ (b) p^3
(c) $(1-p)^3$ (d) $p^3 + p^2(1-p)$
[2008]

Solution: (a)

P_e = if two of received bit 0 or 3 of 3 received bit 0

$$= {}^3C_2 P.P(1-P) + {}^3C_3 P.P.P$$

$= 3P^2(1-P) + P^3$

Hence, the correct option is (a)

29. Four message band limited to W, W, $2W$ and $3W$, respectively, are to be multiplexed using Time Division Multiplexing (TDM). The minimum bandwidth required for transmission of this TDM signal is

(a) W (b) $3W$
(c) $6W$ (d) $7W$ **[2008]**

Solution: (d)

$f_{s1} = 2 \times w = 2w$

$f_{s2} = 2 \times w = 2w$

$f_{s3} = 2 \times 2w = 4w$

$f_{s4} = 2 \times 3w = 6w$

$f_s = 2w + 2w + 4w + 6w$

$= 14w$

For minimum bandwidth $n = 1$

$R_b = nf_s$

$= 1 \times 14$

$= 14w$

$$(\text{BW})_{\min} = \frac{R_b}{2}$$

$(\text{BW})_{\min} = 7$W.

Hence, the correct option is (d)

Common Data for Questions 30, 31 and 32.

A speech signal, band width limited to 4 kHz and peak voltage varying between +5 V and –5 V, is sampled at the Nyquist rate. Each sample is quantized and represented by 8 bits.

30. If the bits 0 and 1 are transmitted using bipolar pulses, the minimum bandwidth required for distortion free transmission is

(a) 64 kHz (b) 32 kHz
(c) 8 kHz (d) 4 kHz **[2008]**

Solution: (b)

While using the bipolar pulses to transmit the bits 0 and 1 the minimum bandwidth required for distortion-free transmission is four times the theoretical bandwidth (Nyquist bandwidth)

$f_m = 4$ kHz

Nyquist bandwidth

$f_{s\,\min} = 2f_m = 8$ kHz

Minimum bandwidth of bipolar signalling is

$4f_{s\min} = Pb/2$

$= 4 \times 8$

$= 32$ kHz

Hence, the correct option is (b)

31. Assuming the signals to be uniformly distributed between its peak to peak value, the signal to noise ratio at the quantizer output is

(a) 16 dB (b) 32 dB
(c) 48 dB (d) 64 dB **[2008]**

Solution: (c)

SNR = $1.76 + 6.02\,n$

$\oplus$ 6n

$\oplus\ 6 \times 8$

= 48 dB

Hence, the correct option is (c)

32. The number of quantization levels required to reduce the quantization noise by a factor of 4 would be

(a) 1024 (b) 512
(c) 256 (d) 64 **[2008]**

Solution: (b)

Quantization noise power $P_N = \dfrac{\Delta^2}{12}$

$$P'_N = \frac{P_N}{4}$$

$$\frac{\Delta^2}{12} = \frac{\Delta^2}{12\times 4}$$

$$s' = \frac{1}{s^2}$$

$$s = \frac{V_{PP}}{2^n}$$

$$\frac{V_{PP}}{2^n} = \frac{V_{PP}}{2.2^n} = \frac{V_{pp}}{2.28}$$

$2^n = 2^9 = 512.$

Therefore, number of quantization levels required to reduce the quantization noise by a factor 4 would be 512.

Hence, the correct option is (b)

33. In delta modulation, the slope overload distortion can be reduced by

(a) decreasing the step size
(b) decreasing the granular noise
(c) decreasing the sampling rate
(d) increasing the step size **[2007]**

Solution: (d)

For no slope overload

$$\Delta f_s \geq \left|\frac{d}{dt} m(t)\right|_{\max}$$

So by increasing step size slope, overload is reduced.

Hence, the correct option is (d)

34. The raised cosine pulse $p(t)$ is used for zero $|S|$ in digital communications. The expression for $p(t)$ with unity roll-off factor is given by

$$p(t) = \frac{\sin 4\pi wt}{4\pi wt(1-16w^2t^2)}$$

The value of $p(t)$

at $t = \dfrac{1}{4\text{w}}$ is

(a) –0.5 (b) 0
(c) 0.5 (d) ∞ **[2007]**

Solution: (c)

$$p(t) = \frac{\sin 4 \quad \text{wt}}{4 \quad \text{wt}(1-16\text{w}^2t^2)}.$$

$$p\left(\frac{1}{4w}\right) = \frac{\sin \cancel{4\pi} w \times \frac{1}{\cancel{4w}}}{\cancel{4\pi} w \times \frac{1}{\cancel{4w}}\left(1-\frac{\cancel{16w^2}}{\cancel{16w^2}}\right)}$$

At it is in percentage form

$$p\left(\frac{1}{2}\right) = \frac{\cos 4\pi wt}{1-48w^2t^2}$$

by L hospital rule

$$p(t) = \frac{\cos\pi}{1-3}$$

$$= 0.5$$

Hence, the correct option is (c)

35. During transmission over a certain binary communication channel, bit errors occur independently with probability p. The probability of AT MOST one bit in error in a block of n bits is given by

(a) p^n (b) $1 - p^n$
(c) $np(1-p)^{n-1}+(1-p)^n$ (d) $1-(1-p)^n$

[2007]

Solution: (c)

Out of n bit at most one error of probability is

$${}^nC_0(P)^0(1-P)^n + {}^nC_1(P)^1(1-P)^{n-1}$$

$= (1-P)^n + nP(1-P)^{-1}$

36. In a GSM system, eight channels can co-exist in 200 kHz bandwidth using TDMA. A GSM based cellular operator is allocated 5 MHz bandwidth. Assuming a frequency reuse factor of $1/5$, i.e. a five-cell repeat pattern, the maximum number of simultaneous channels that can exists in one cell is

(a) 200 (b) 40
(c) 25 (d) 5 **[2007]**

Solution: (b)

Allocated BW = 5 mHz

Frequency reduced factor = 1/5

BW allocated = 5 × 1/5

= 1 mHz

Number of simultaneous channel $= \dfrac{1\text{ mHz}}{200\text{ mHz}} \times 8$

$= 40$

Hence, the correct option is (b)

37. In a Direct Sequence CDMA system, the chip rate is 1.2288×10^6 chips per second. If the processing gain is desired to be AT LEAST 100, the data rate

(a) must be less than or equal to 12.288×10^3 bits per sec
(b) must be greater than 12.288×10^3 bits per sec
(c) must be exactly equal to 12.288×10^3 bits per sec
(d) can take any values than 122.88×10^3 bits per sec **[2007]**

Solution: (a)

Processing gain $= \dfrac{R_C}{R_b}$

$$\frac{R_C}{R_b} \geq 100$$

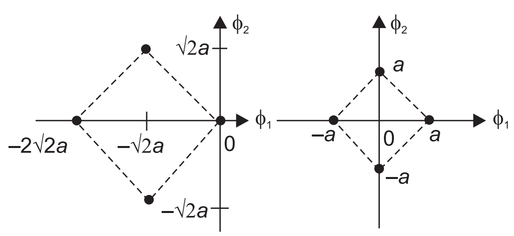
$$R_b \leq \frac{R_C}{100}$$

R_b ″ 12.288×10^3 bits per second.

Hence, the correct option is (a)

Common Data for Question 38 & 39

Two 4-ray signal constellations are shown. It is given that ϕ_1 and ϕ_2 constitute an orthonormal basis for the two constellations. Assume that the four symbols in both the constellations are equiprobable. Let $N_0/2$ denote the power spectral density of white Gaussian noise.



Constellation 1 Constellation 2

38. The ratio of the average energy of Constellation 1 to the average energy of Constellation 2 is
(a) $4a^2$ (b) 4
(c) 2 (d) 8 **[2007]**

Solution: (b)

Average energy of constellation 1 is

$$E_1 = \frac{0 + 4a^2 + 4a^2 + 8a^2}{4} = 4a^2$$

$$E_2 = \frac{a^2 + a^2 + a^2 + a^2}{4} = \frac{4a^2}{4}$$

$$= a^2$$

$$\frac{E_1}{E_2} = 4$$

Hence, the correct option is (b)

39. If the constellation are used for digital communication over an A WGN channel, then which of the following statements is true?
(a) Probability of symbol error for Constellation 1 is lower
(b) Probability of symbol error for Constellation 1 is higher
(c) Probability of symbol error is equal for both the constellations
(d) The value of N_0 will determine which of the two constellations has a lower probability of symbol error **[2007]**

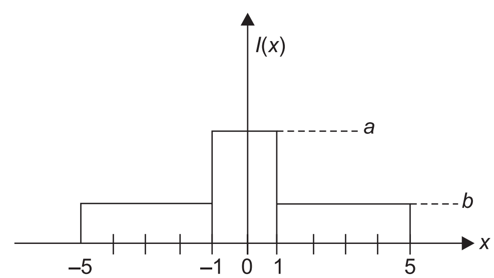
Solution: (a)

The probability of error decreases with increase in average energy. As constellation 1 has more energy than that of constellation 2, so the probability of symbol error for constellation 1 is lower.

Hence, the correct option is (a)

Common Data for Questions 40 and 41

An input to a 6-level quantizer has the probability density function $f(x)$ as shown in the figure. Decision boundaries of the quantizer are chosen so as to maximize the entropy of the quantizer output. It is given that three consecutive decision boundaries are '–1', '0' and '1'.



40. The values of a and b are
(a) $a = 1/6$ and $b = 1/12$
(b) $a = 1/5$ and $b = 3/40$
(c) $a = 1/4$ and $b = 1/16$
(d) $a = 1/3$ and $b = 1/24$ **[2007]**

Solution: (a)

To maximize the entropy, probability must be equal for each symbol.

$$\int_1^5 b\,dx = \frac{1}{3}$$

$b \times 4 = 1/3$

$$\boxed{b = \frac{1}{12}}$$

$$a\left(\int_{-1}^{0} dx + \int_{0}^{1} dx\right) = \frac{1}{3}$$

$$2a = \frac{1}{3}$$

$$a = \frac{1}{6}$$

Hence, the correct option is (a)

41. Assuming that the reconstruction levels of the quantizer are the mid-points of the decision boundaries, the ratio of signal power to quantization noise power is

(a) $\frac{152}{2}$ (b) $\frac{64}{3}$

(c) $\frac{76}{3}$ (d) 28 **[2007]**

Solution: (d)

$$P_s = \int_{-5}^{-1} x^2 . \frac{1}{12}dx + \int_{-1}^{1} x^2 \times \frac{1}{6}dx + \int_{1}^{5} x^2 x \frac{1}{12}dx$$

$$= \frac{1}{12}\left(\frac{x^3}{3}\right)_{-5}^{-1} + \frac{1}{6}\left(\frac{x^3}{3}\right)_{-1}^{1} + \frac{1}{12}\left(\frac{x^3}{3}\right)_{1}^{5}$$

$$= \frac{1}{36}(-1+125) + \frac{1}{16}(1+1) + \frac{1}{36}(124)$$

$$= \frac{124}{36} + \frac{1}{18} \times 2 + \frac{124}{36}$$

$= 7$ volt^2

$$P_N = \frac{\Delta^2}{12} = \frac{(5-(-5))^2}{12 \times 6^2}$$

$\oplus$ 0.25

$$\text{SNRQ} = \frac{7}{0.25} = 28$$

Hence, the correct option is (d)

42. The minimum sampling frequency (in samples/sec) required to reconstruct the following signal from its samples without distortion

$$x(t) = 5\left(\frac{\sin 2\pi 1000t}{\pi t}\right)^3 + 7\left(\frac{\sin 2\pi 1000t}{\pi t}\right)^2$$

would be

(a) 2×10^3
(b) 4×10^3
(c) 6×10^3
(d) 8×10^3 **[2006]**

Solution: (c)

$$x(t) = 5\sin\left(\frac{2\pi \times 10^3 t}{\pi t}\right)^3 + 7\left(\frac{\sin 2\pi \times 10^3 t}{\pi t}\right)^2$$

$f_s = (2f_m) \times 3$
$= (2 \times 1000) \times 3$
$= 6 \times 10^3$ Hz

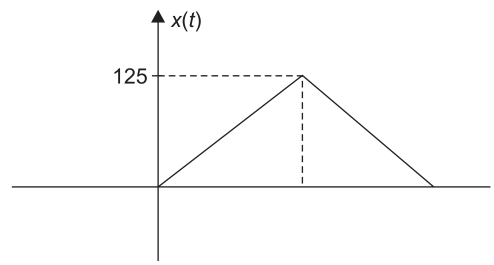
Hence, the correct option is (c)

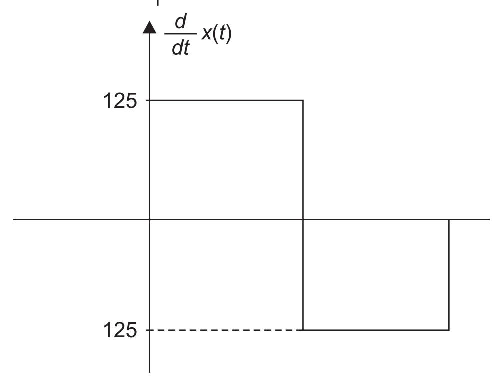
43. The minimum step-size required for a Delta-Modulator operating at 32K samples/sec to track the signal (here $u(t)$ is the unit step function) $x(t) = 125t(u(t) - u(t-1)) + (250 - 125t)(u(t-1) - u(t-2))$ so that slope-overload is avoided would be

(a) 2^{-10} (b) 2^{-8}
(c) 2^{-6} (d) 2^{-4} **[2006]**

Solution: (b)

$$\Delta f_s \geq \left|\frac{d}{dt}m(t)\right|_{\max}$$



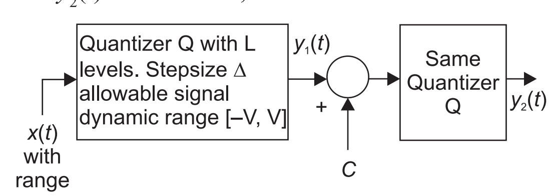


$$\Delta_{\min} = \frac{(m(t))_{\max}}{f_s} = \frac{\cancel{125}}{32 \times \cancel{1000}_{8}}$$

$= 2^{-8}$

Hence, the correct option is (b)

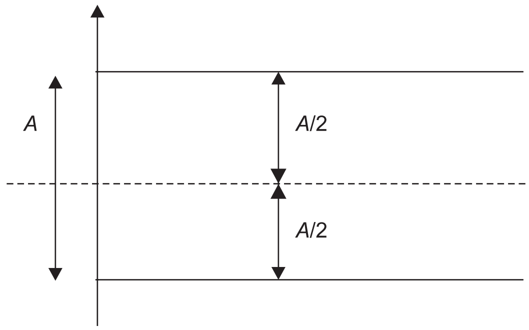
44. In the following figure the minimum value of the constant 'C', which is to be added to $y_1(t)$ such that $y_1(t)$ and $y_2(t)$ are different, is



$$\left[\frac{-v}{2}, \frac{v}{2}\right]$$

(a) Δ (b) Δ/2
(c) $\Delta^2/12$ (d) Δ/L **[2006]**

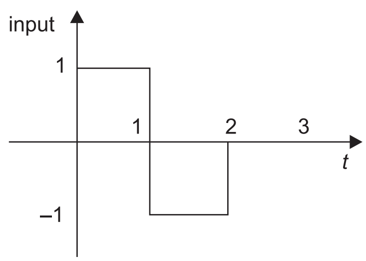
Solution: (b)



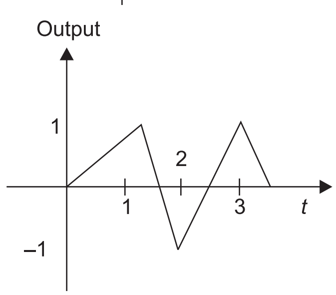
For $y_1(t)$ and $y_2(t)$ to be different minimum step size of $A/2$ is needed, else they will be same.

Hence, the correct option is (b)

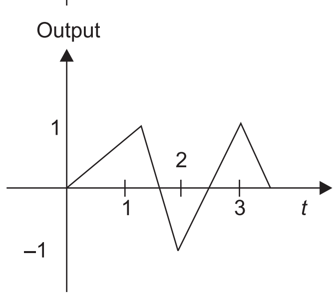
45. A signal as shown in the figure is applied to a matched filter. Which of the following does represent the output of this matched filter? **[2005]**



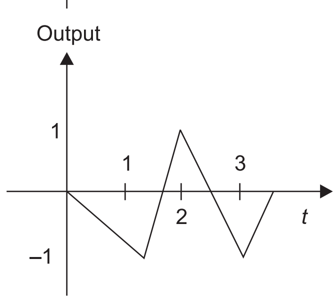
(a)



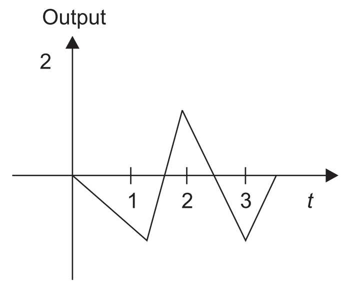
(b)



(c)

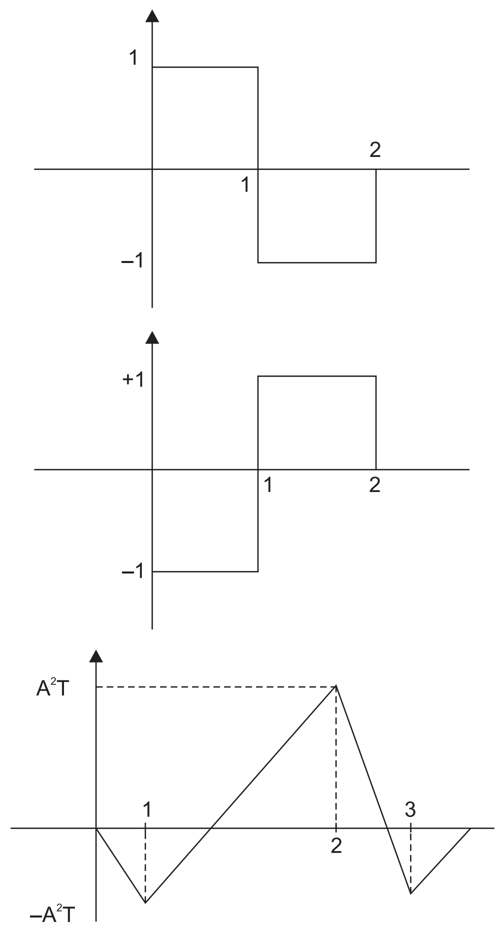


(d)



Solution: (c)

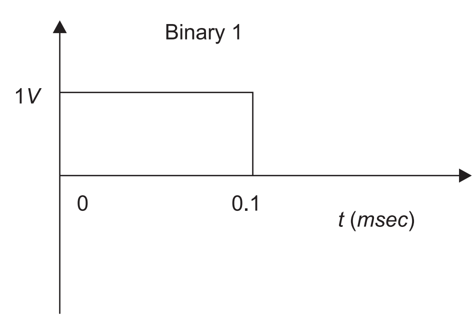
$y(t) = x(t) \times x(T - t)$



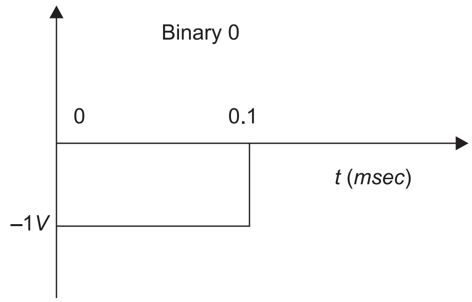
Hence, the correct option is (c)

46. A source produces binary data at the rate of 10 kbps. The binary symbols are represented as shown in the figure

(a)



(b)



The source output is transmitted using two modulation schemes, namely binary PSK (BPSK) and quadrature PSK (QPSK). Let B_1 and B_2 be the bandwidth requirements of BPSK and QPSK, respectively. Assuming that the bandwidth of the above rectangular pulses is 10 kHz, B_1 and B_2 are

(a) $B_1 = 20$ kHz, $B_2 = 20$ kHz
(b) $B_1 = 10$ kHz, $B_2 = 20$ kHz
(c) $B_1 = 20$ kHz, $B_2 = 10$ kHz
(d) $B_1 = 10$ kHz, $B_2 = 10$ kHz **[2004]**

Solution: (c)

Bit rate $= R_b = 10$k

$Bt = 2R_b = 20k$

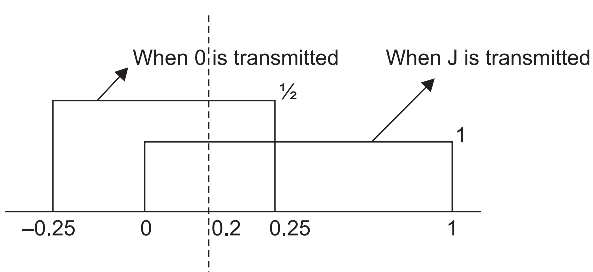
$B_2 = R_b = 10k$

Hence, the correct option is (c)

47. Consider a binary digital communication system with equally likely 0's and 1's. When binary 0 is transmitted the detector input can lie between the levels –0.25V and +0.25V with equally probability: when binary 1 is transmitted, the voltage at the detector can have any value between 0 and 1V with equal probability. If the detector has a threshold of 0.2 V (i.e. if the received signal is greater than 0.2 V, the bit is taken as 1), the average bit error probability is

(a) 0.15 (b) 0.2
(c) 0.05 (d) 0.5 **[2004]**

Solution: (a)



$k \times 1/2 = 1$

$$P_e\left(\frac{1}{0}\right) = \int_{0.2}^{0.25} 2$$

$= 2 \times (0.25 – 0.2)$

$= 2 \times 0.05$

$= 0.1$

$$P_e\left(\frac{0}{1}\right) = \int_0^{0.2} 1.dx = 0.2$$

$$(P_e)_{ay} = \frac{0.1+0.2}{2}$$

$= 0.15$

Hence, the correct option is (a)

48. Choose the correct one from among the alternatives a, b, c, d after matching an item from Group 1 with the most appropriate item in Group 2.

Group-1	**Group-2.**
1. FM	P. Slope overload
2. DM	Q.μ -law
3. PSK	R. Envelope detector
4. PCM	S. Capture effect
	T. Hilbert transform
	U. Matched filter

(a) $1 – T, 2 – P, 3 – U, 4 – S$
(b) $1 – S, 2 – U, 3 – P, 4 – T$
(c) $1 – S, 2 – P, 3 – U, 4 – Q$
(d) $1 – U, 2 – R, 3 – S, 4 – Q$ **[2004]**

Solution: (c)

1. FM- capture effect
2. DM- slope overload
3. PSK- matched filter
4. PCM-.μ-law

Hence, the correct option is (c)

49. Three analog signals, having bandwidths 1200 Hz, 600 Hz and 600 Hz, are sampled at their respective Nyquist rates, encoded with 12 bit words, and time division multiplexed. The bit rate for the multiplexed signal is

(a) 115.2 kbps (b) 28.8 kbps
(c) 57.6 kbps (d) 38.4 kbps **[2004]**

Solution: (c)

Sampling frequency

$f_s = 2(1200 + 600 + 600)$

$= 4800$ Hz

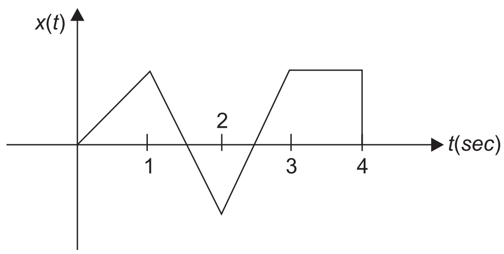
$R_b = nf_s$

$= 12 \times 4.8$

$= 57.6$ kpbs

Hence, the correct option is (c)

50. Consider the signal $x(t)$ shown in the figure. Let $h(t)$ denote the impulse response of the filter matched to $x(t)$, with $h(t)$ being non-zero only in the interval 0 to 4 sec. The slope of $h(t)$ in the interval $3 < t < 4$ sec is



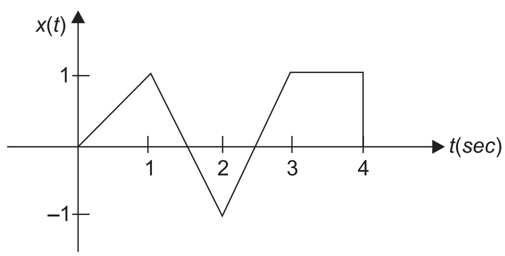
(a) $\frac{1}{2}\text{sec}^{-1}$ (b) $-1\ \text{sec}^{-1}$

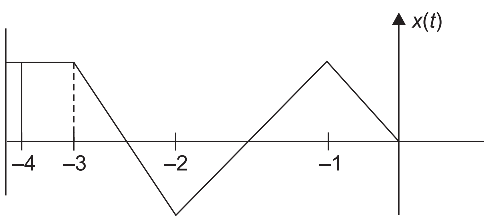
(c) $-\frac{1}{2}\text{sec}^{-1}$ (d) $1\ \text{sec}^{-1}$ **[2004]**

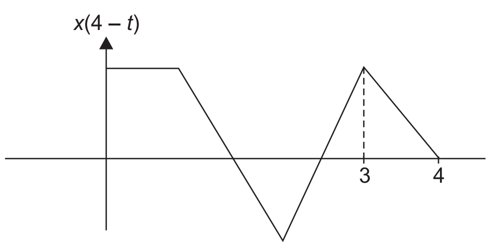
Solution: (b)

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$

$H(t) = x(T - t)$



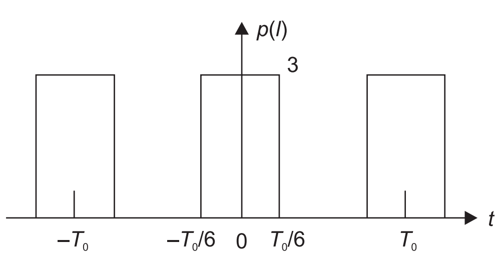




Hence slope between $3 < t < 4 = -1$.

Hence, the correct option is (b)

51. Let $x(t) = 2\cos(800\ \ t) + \cos(1400\ \ t).x(t)$ be sampled with the rectangular pulse train shown in the figure. The only spectral components (in kHz) present in the sampled signal in the frequency range 2.5 kHz to 3.5 kHz are



(a) 2.7, 3.4 (b) 3.3, 3.6

(c) 2.6, 2.7, 3.3, 3.4, 3.6 (d) 2.7, 3.3 **[2003]**

Solution: (d)

Exponential Fourier series coefficient is given as

$$C_n = \frac{1}{T_0}\Big|_{-T_0/6}^{T_0/6} Ae^{-jnw_0 t}$$

$$= \frac{A}{\pi n}\sin\left(\frac{n\pi}{3}\right)$$

from c_n is clear that 1, 2, 4, 5, 7 …. Harmonics are present.

Frequency components are = $10^3.2 \times 10^3.4 \times 10^3$.

$x(t)$ has frequency components $0.7t$, $0.4k$

$(PH) \pm (t)$ gives $(1 \pm 0.7)k$, $(2 \pm 0.7)k$, $(4 \pm 0.7)k$, $(1 \pm 0.4)k$, (2 ± 0.4)

Frequencies present in range of $2.5k$ to $3.5k$ are 2.7, 3.3

Hence, the correct option is (d)

52. A sinusoidal signal with peak-to-peak amplitude of 1536 V is quantized into 128 levels using a mid-rise uniform quantizer. The quantization-noise power is

(a) 0.768 V (b) $48 \times 10^{-6}\ V^2$

(c) $12 \times 10^{-6}\ V^2$ (d) 3.072 V **[2003]**

Solution: (c)

$V_{P-P} = 1.5366$

$L = 128$

$n = \log_2 12.8 = 7$

$$P_N = \frac{\Delta^2}{12} = \frac{\left(\frac{V_{p-p}}{L}\right)^2}{12}$$

$$= \frac{\left(\frac{1.5366}{128}\right)^2}{12}$$

$= 12 \times 10^{-6} V^2$

Hence, the correct option is (c)

53. If E_b, the energy per bit of a binary digital signal, is 10^{-5} watt-sec and the one-sided power spectral density of the white noise, $N_0 = 10^{-6}$ W/Hz, then the output SNR of the matched filter is

(a) 26 dB (b) 10 dB

(c) 20 dB (d) 13 dB **[2003]**

Solution: (d)

Signal to noise ratio is given as

$$\text{SNR} = \frac{2E_B}{N_0}$$

$$= \frac{2\times 10^{-5}}{10^{-6}} = 20$$

$(\text{SNR})_{\text{dB}} = 10\ 10\text{g}_{10}\ 20$

$= 13$ dB

Hence, the correct option is (d)

54. The input to a linear delta modulator having a step-size $\Delta = 0.628$ is a sine wave with frequency f_m and peak amplitude E_m. If the sampling frequency $f_s = 40$ kHz, the combination of the sine-wave frequency and the peak amplitude, where slope overload will take place, is

	$\geq f_m$	p_m
(a)	0.3 V	8 kHz
(b)	1.5 V	4 kHz
(c)	1.5 V	2 kHz
(d)	3.0 V	1 kHz

[2003]

Solution: (b)

For no slop over load to occur

$$\Delta f_s \geq \left|\frac{d}{dt}m(t)\right|_{\max}$$

for slope over load to occur

$$\Delta f_s < \left|\frac{d}{dt}m(t)\right|_{\max}$$

$\Delta f_s = 0.625 \times 40 \times 10^3$

$= 25.12\ k$

Solving by options

$2\pi \times 1.5 \times 4 = 37.7$

which is greater than Δf_s

55. If S represents the carrier synchronization at the receiver and ρ represents the bandwidth efficiency, then the correct statement for the coherent binary PSK is

(a) ρ = 0.5, S is required
(b) ρ = 1.0, S is required
(c) ρ = 0.5, S is not required
(d) ρ = 1.0, S is not required **[2003]**

Solution: (a)

For BPSK

$$\text{bandwidth efficiency} = \frac{\text{bit rate}}{\text{BW}}$$

$$= \frac{\cancel{f_b}}{2\cancel{f_b}}$$

$= 0.5$

For coherent BPSK synchronization is required at the receiver.

Hence, the correct option is (a)

56. A signal is sampled at 8 kHz and is quantized using 8-bit uniform quantizer. Assuming SNR_q for a sinusoidal signal, the correct statement for PCM signal with a bit rate of R is

(a) $R = 32$ kbps, $\text{SNR}_q = 25.8$ dB
(b) $R = 64$ kbps, $\text{SNR}_q = 49.8$ dB
(c) $R = 64$ kbps, $\text{SNR}_q = 55.8$ dB
(d) $R = 32$ kbps, $\text{SNR}_q = 49.8$ dB **[2003]**

Solution: (b)

Bit rate $= nf_s$

$= 2 \times 8k = 64$ kbps

SNR $= 1.76 + 6n$

$= 1.76 + 6 \times 8$

$= 49.8$

Hence, the correct option is (b)

57. A signal $x(t) = 100\cos(24\pi \times 10^3)t$ is ideally sampled with a sampling period of 50 μsec and then passed through an ideal low pass filter with cutoff frequency of 15 kHz. Which of the following frequencies is/are present at the filter output?

(a) 12 kHz only (b) 8 kHz only
(c) 12 kHz and 9 kHz (d) 12 kHz and 8 kHz **[2002]**

Solution: (d)

$x(t) = 100\cos(24\pi \times 10^3)t$

$f_m = \text{k kHz}$

$$f_s = \frac{1}{50}\times 10^6 = 20\,\text{kHz}$$

$f_{\text{cut off}} = 15$ Hz

$$y(t) = \frac{y(t)}{T_s} + \frac{2}{T_s}\left(x(t)\cos w_s t + x(t)\cos 2w_s t + \ldots.\right)$$

Passed component $= f_{m1}, f_s - f_m$

$= 12$ Hz, 8 Hz

it will suppress bear other components having frequency > 15 Hz.

Hence, the correct option is (d)

58. A video transmission system transmits 625 picture frames per second. Each frame consists of a 400 × 400 pixel grid with 64 intensity levels per pixel. The data rate of the system is

(a) 16 Mbps (b) 100 Mbps
(c) 600 Mbps (d) 6.4 Gbps **[2001]**

Solution: (c)

Data rate = 625 × 400 × 400 × 6

= 600 Mbps

Hence, the correct option is (c)

59. The Nyquist sampling interval for the signal $\sin c(700t) + \sin c(500t)$ is

(a) $\frac{1}{350}$ sec (b) $\frac{\quad}{350}$ sec

(c) $\frac{1}{700}$ sec (d) $\frac{\quad}{175}$ sec **[2001]**

Solution: (c)

$f(t) = \sin c(700\,t) + \sin c(500\,t)$

$$= \frac{\sin 700\pi t}{700\pi t} + \frac{\sin 500\pi t}{500\pi t}$$

$$f_s = \frac{700\cancel{\pi}}{\cancel{2}\cancel{\pi}} \times \cancel{2}$$

$$T_S = \frac{1}{f_s}$$

$$T_S = \frac{1}{700}\text{sec}$$

Hence, the correct option is (c)

60. During transmission over a communication channel, bit error occur independently with probability p. If a block of n bits is transmitted, the probability of at most one bit error is equal to

(a) $1 - (1 - p)^n$

(b) $p + (n - 1)(1 \text{ c } p)$

(c) $np\,(1 - p)^{n-1}$

(d) $(1 - p)n + \text{np}(1 - p)^{n-1}$ **[2001]**

Solution: (d)

Probability of error = P

Probability no error = $1 - P$

Out of n bit probability of at most one bit error = either there is no error or 1 error

$$= ^{\eta} C_0 (P)^0 (1-P)^n + ^n C_1 (P)^1 (1-P)^{n-1}$$

$$= (1 - P)^n + np(1 - P)^n$$

Hence, the correct option is (d)

61. In a digital communication system employing Frequency Shift Keying (FSK), the 0 and 1 bit are represented by sine waves of 10 kHz and 25 kHz, respectively. These waveforms will be orthogonal for a bit interval of

(a) 45 μsec (b) 200 μsec

(c) 50 μsec (d) 250 μsec **[2000]**

Solution: (b)

f_{m0} = 10 kHz = 2 × 5 kHz

f_{m1} = 25 kHz = 5 × 5 kHz

$$T_b = \frac{1}{5\text{ kHz}} \times \frac{1000}{1000}$$

$$T_b = 200\ \mu s$$

Hence, the correct option is (b)

62. The Nyquist sampling frequency (in Hz) of a signal given by $6 \times 10^4 \text{sinc}^3(400)*10^6 \text{sinc}^3(100t)$ is

(a) 200 (b) 300

(c) 1500 (d) 1000 **[1999]**

Solution: (c)

Given

$s(t) = 6 \times 10^4 \sin c^3 (400\,t) \times 10^6 \sin c^3 (100\,t)$

$f_{m1} = 400 \times 3 = 1200\ H_2$

Total $P = P_{m1} + P_{m2} + P_{m3} = 150$

$f_{m2} = 100 \times 3 = 300\text{Hz } 3 = 150\text{Hz}$

Hence, the correct option is (c)

63. The peak-to-peak input to an 8-bit PCM coder is 2 volts. The signal power-to-quantization noise power ratio (in dB) for an input of $0.5\cos(\omega_m t)$ is

(a) 47.8 (b) 43.8

(c) 95.6 (d) 99.6 **[1999]**

Solution: (b)

$$\text{Signal power } = \frac{V_M^2}{2} = \frac{(0.5)^2}{2} = \frac{0.25}{2}$$

$$= 0.125$$

$$\Delta = \frac{V_{\max} - V_{\min}}{L}$$

$$= \frac{V_{PP}}{L}$$

$$= \frac{2}{2^8} = \frac{1}{2^7}$$

$$\text{Noise power } P_N = \frac{\Delta^2}{12}$$

$$= \left(\frac{1}{2^{14}}\right) \times \frac{1}{12}$$

$$\text{SNRQ} = \frac{0.125}{1} \times 2^{14} \times 12$$

$(\text{SNRQ})_{\text{dB}} = 10 \log 0.125 \times 12 \times 2^{14}$

= 43.8 dB

Hence, the correct option is (b)

64. The input to a matched filter is given by

$$s(t) = \begin{cases} 10\sin(2\quad \times 10^6) & 0 < t < 10^{-4}\text{sec} \\ 0 & \text{Otherwise} \end{cases}$$

The peak amplitude of the filter output is

(a) 10 volts (b) 5 volts

(c) 10 millivolts (d) 5 millivolts

[1999]

Solution: (d)

Output of matched filter = energy of signal

$$= \int_{-\infty}^{\infty} |x(t)|\,dt$$

$$= \int_0^{10^{-4}} 100\sin^2(2\pi \times 10^6)t\ dt$$

$$= \frac{100}{2} \int_{0}^{10^{-4}} (1 - \cos 2 \times 2\pi \times 10^6 +)$$

$$= (50T)_0^{10^{-4}} + 0$$

$= 50 \times 10^{-4}$

$= 5$ mill V_0H

Hence, the correct option is (d)

65. Four independent messages have bandwidths of 100Hz, 100Hz, 200Hz, and 400Hz, respectively. Each is sampled at the Nyquist rate, and the samples are Time Division Multiplexed (TDM) and transmitted. The transmitted sample rate (in Hz) is

(a) 1600 (b) 800
(c) 400 (d) 200 **[1999]**

Solution: (a)

$f_{m1} = 100$ Hz

$f_{m2} = 100$ Hz

$f_{m3} = 200$ Hz

$f_{m4} = 400$ Hz

sample rate $= 2(f_{m1} + f_{m2} + f_{m3} + f_{m4})$

$= 2(100 + 100 + 200 + 400)$

$= 2 \times 800$

$= 1600$ Hz

Hence, the correct option is (a)

66. The number of bits in a binary PCM system is increased from n to $n + 1$. As a result, signal to quantization noise ratio will improve by a factor

(a) $(n + 1)/^n$
(b) $2^{(n+1)}$
(c) $2^{(n+1)}$
(d) which is independent of n **[1996]**

Solution: (d)

Signal to quantization ratio is given as

$$(SNRQ)_1 = \frac{3}{2} 2^{2n}$$

SNR with $n = (n + 1)$

$$(SNRQ)_2 = \frac{3}{2} 2^{2(n+1)} = 4(SNRQ)_1$$

Hence, the correct option is (d)

67. The bit stream 01001 is differentially encoded using 'Delay and Ex OR' scheme for DPSK transmission. Assuming the reference bit as a '1' and assuming phases of '0' and 'π' for 1's and 0's respectively, in the encoded sequence, the transmitted phase sequence becomes

(a) π,0, π, π, 0 (b) 0, π, π, 0, 0
(c) 0, π, π,π, 0 (d) π, π, 0, π, π
[1992]

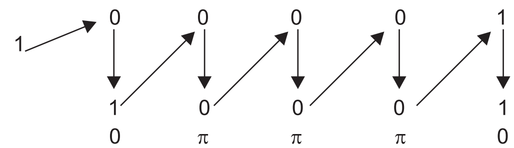
Solution: (c)

Given that

reference bit = 1

logic 0 = π

logic 1 = 0°



Hence, the correct option is (c)

68. Coherent demodulation of FSK signal can be detected using

(a) correlation receiver
(b) bandpass filter and envelope detectors
(c) matched filter
(d) discriminator detection **[1992]**

Solution: (a)

Coherent demodulation of FSK signal can be detected using correlation receiver.

Hence, the correct option is (a)

69. Source encoding in a data communication system is done in order to

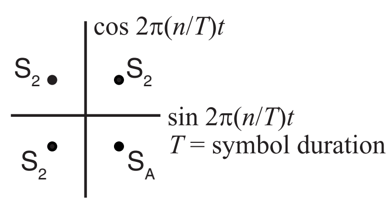
(a) enhance the information transmission
(b) bandpass filters and envelope rate detectors
(c) conserve the transmitted power
(d) discriminator detection **[1992]**

Solution: (a)

The purpose of source encoding in a data communication system is to increase the information transmission rate and the purpose of channel encoding is to decrease the probability of error. The channel encoding helps in detection and correction of errors.

Hence, the correct option is (a)

70. For the signal constellation shown in the figure, the type of modulation is **[1991]**



Solution:

For given signal phase shift in each symbol is 90°.

So modulation type is QPSK.

71. A signal has frequency components from 300 Hz to 1.8 kHz. The minimum possible rate at which the signal has to be sampled is _______. **[1991]**

Solution:

Given

$f_H = 1800$ Hz

$f_L = 300$ Hz

BW = 1800-300

= 1500 Hz

$$k = \frac{f_H}{BW} = \frac{1800}{1500}$$

$k = 1$

$$(f_s)_{\min} = \frac{2f_H}{k}$$

$$= \frac{2\times1800}{1}$$

= 3600 samples/sec

72. A 4 GHz carrier is DSB-SC modulated by a low pass message signal with maximum frequency of 2 MHz. The resultant signal is to be ideally sampled. The minimum frequency of the sampling impulse train should be

(a) 4 MHz (b) 8 MHz
(c) 8 GHz (d) 8.004 GHz **[1990]**

Solution: (b)

Given that

$f_c = 4$ GHz

$f_m = 2$ mHz

$f_{\max} = f_c + f_m = 4000 + 2 = 2002$

$f_{\min} = f_c - f_m = 4000 - 2 = 3998$

B.W. $= (f_{\max} - f_{\min})$

= (4002 – 3998)

= 4 mHz

$f_s = 2 \times 4 = 8$ mHz

Hence, the correct option is (b)

73. In a BPSK signal detector, the local oscillator has a fixed phase error of 20°.This phase error deteriorates the SNR at the output by a factor of

(a) cos 20° (b) $\cos^2 20°$
(c) cos70° (d) $\cos^2 70°$ **[1990]**

Solution: (b)

Output power charges by cos 2 ϕ.

So $\phi = 20°$

Output power changes by = $\cos^2 30$.

Hence, the correct option is (b)

74. A signal has frequency components from 300 Hz to 1.8 kHz. The minimum possible rate at which the signal has to be sampled is **[1991]**

Solution:

Given

$f_H = 1800$ Hz

$f_L = 300$ Hz

BW = 1800 – 300

= 1500 Hz

$$k = \frac{f_H}{BW} = \frac{1800}{1500}$$

$k = 1$

$$(f_s)_{\min} = \frac{2f_H}{k}$$

$$= \frac{2\times1800}{1}$$

= 3600 samples/sec

75. In binary data transmission DPSK is preferred to PSK because

(a) a coherent carrier is not required to be generated at the receiver
(b) for a given energy per bit, the probability of error is less
(c) the 180° phase shifts of the carrier are unimportant
(d) more protection is provided against impulse noise **[1989]**

Solution: (a)

A coherent carrier is required in PSR but in DPSK coherent carrier is not required. So DPSW is preferred over PSK.

Hence, the correct option is (a)

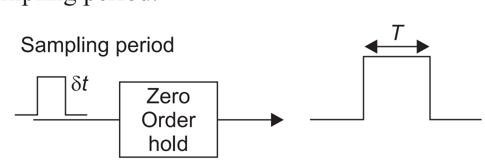
76. The transfer function of a zero-order hold is

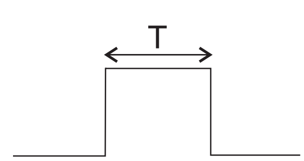
(a) $\dfrac{1-\exp(-Ts)}{s}$ (b) 1/s
(c) 1 (d) $\dfrac{1}{[1-\exp(-T_s)]}$

[1988]

Solution: (a)

A zero order hold systems holds the input signal value for a period of T, i.e., for an input of short duration (Δ) pulse, it produces an output pulse of duration T, the sampling period.





input $x(t) = \delta(t)$

$x(s) = 1$

$y(t) = 4(t) - 4(t - T)$

$$y(s) = \frac{1}{s} - \frac{e^{-Ts}}{s}$$

$$y(s) = \frac{1 - e^{-Ts}}{s}$$

$$\frac{y(s)}{x(s)} = H(s)$$

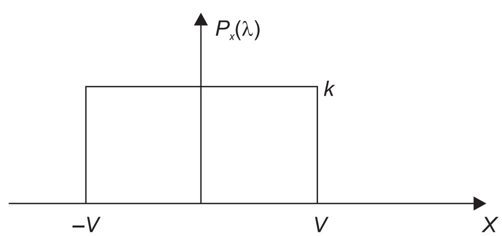
$$H(s) = \frac{1 - e^{-Ts}}{s}$$

Hence, the correct option is (a)

77. A signal having uniformly distributed amplitude in the interval (–V to +V) is to be encoded using PCM with uniform quantization. The signal to quantizing noise ratio is determined by the

(a) dynamic range of the signal
(b) sampling rate
(c) number of quantizing levels
(d) power spectrum of signal **[1988]**

Solution: (c)



Area under the curve = 1

$k \times 2V = 1$

$$k = \frac{1}{2V}$$

$$\text{SNRQ} = \frac{\text{Signal power}}{\text{noise power}}$$

We know that

$$\text{noise power} = \frac{\Delta^2}{12}$$

$$\Delta = \frac{V_{\max} - V_{\min}}{L}$$

$L = 2^n$

$$\Delta = \frac{V - (-V)}{2^n} = \frac{2V}{2^n}$$

$$P_N = \left(\frac{2V}{2^n}\right)^2 \times \frac{1}{12} \qquad (1)$$

Signal power

$$= \int_{-V}^{V} x^2 \times \frac{1}{2V} dx$$

$$= \frac{1}{2V}\left(\frac{x^3}{3}\right)_{-V}^{V}$$

$$= \frac{1}{6V}\left(V^3 + V^3\right)$$

$$= \frac{V^2}{3}$$

$$SNRQ = \frac{\frac{V^2}{\cancel{3}}}{\left(\frac{RV}{2^n}\right)^2} \times \cancel{12}\ \cancel{4}$$

$\text{SNRQ} = 2^{2^n}$

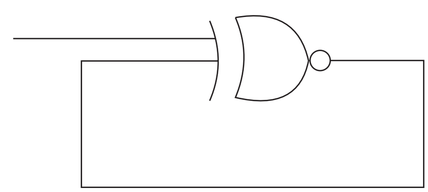
Hence signal to quantization noise ratio is determined by the number of quantization levels.

Hence, the correct option is (c)

78. The message bit sequence to a DPSK modulator is 1, 1, 0, 0, 1, 1. The carrier phase during the reception of the first two message bits is π, π. The carrier phase for the remaining four message bits is

(a) π, π, 0, π
(b) 20, 0, 0, π, π
(c) 0, π, π, π
(d) π, π, 0, 0 **[1988]**

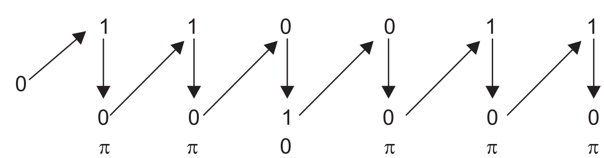
Solution: (c)



Logic 1 = 0°

Logic 0 = 180° = π

Let reference bit = 0



0, π, π, π

Hence, the correct option is (c)

79. In a digital communication system, transmission of successive bits through a noisy channel is assumed to be independent events with error probability P. The probability of at most one error in the trans mission of an 8-bit sequence is

(a) $\dfrac{7(1-p)}{8}+\dfrac{7(1-p)}{8}+\dfrac{p}{8}$

(b) $(1-p)^8 + 8p(1-p)^7$

(c) $(1-p)^8 + p(1-p)^7$

(d) $(1-p)^8 + p(1-p)^7$. **[1988]**

Solution: (b)

Probability of error = P

If there is no error then probability = $1 - P$,

Probability of at most one error in the transmission of an 8 bit sequence is = No error + 1 error

$$= {}^8C_0(P)^0(1-P)^8 + {}^8C_1P^1(1-P)^7$$

$$= (1-P)^8 + 8P(1-P)^7$$

Hence, the correct option is (b)

80. Companding in PCM system lead to improved signal to quantization noise ratio.

This improvement is for

(a) lower frequency components only
(b) higher frequency components only
(c) lower amplitudes only
(d) higher amplitudes only **[1987]**

Solution: (c)

Companding results in making SNR uniform, throughout the signal irrespective of amplitude levels, since in uniform quantization, step size is same the quantization noise power is uniform, throughout the signal.

Thus, higher amplitude of signal will have better SNR than the lower amplitudes.

Hence companding is used for improving SNR at lower amplitudes.

Hence, the correct option is (c)

Chapter 4

Information Theory

One-mark Questions

1. A linear Hamming code is used to map 4-bit messages to 7-bit codewords. The encoder mapping is linear. If the massage 0001 is mapped to the codeword 0000111, and the massage 0011 is mapped to the codeword 1100110, then the massage 0010 is mapped to **[2019]**

(A) 0010011 (B) 1100001

(C) 1111000 (D) 1111111

Solution:

$$\begin{array}{lll} C_1: & 0\ 0\ 0\ 1 & 0\ 0\ 0\ 0\ 1\ 1\ 1 \\ C_2: \oplus & 0\ 0\ 1\ 1 & 1\ 1\ 0\ 0\ 1\ 1\ 0 \\ \hline & 0\ 0\ 1\ 0 & 1\ 1\ 0\ 0\ 0\ 0\ 1 \end{array}$$

Hence, the correct option is (B).

2. Let (X_1, X_2) be independent random variables. X_1 has mean 0 and variance 1, while X_2 has mean 1 and variance 4. The mutual information I (X_1; X_2) between X_1, and X_2 in bits is __________. **[2017]**

Solution: For two independent random variables,

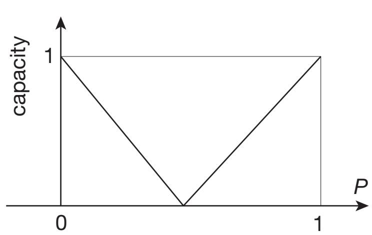
$I\,(x : y) = H(x) = H\,(x/y)$

$H\,(x/y) = H\,(x)$ for independent X and $y \Rightarrow I(x : y) = 0$

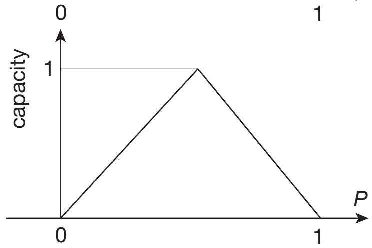
Hence, the correct answer is (0).

3. Which one of the following graphs shows the shannon capacity (channel capacity) in bits of a memoryless binary symmetric channel with crossover probability P? **[2017]**

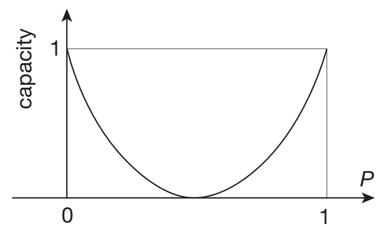
(A)



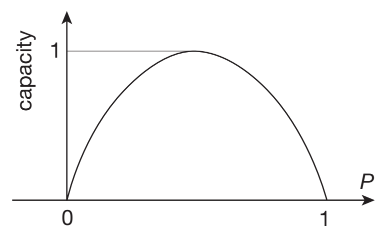
(B)



(C)



(D)



Solution: The capacity of memoryless BSC is expressed as

$$C = 1 + p\log_2 p + (1-p)\log_2(1-p)$$

Hence, the correct option is (C).

4. In a code-division multiple access (CDMA) system with N = 8 chips, the maximum number of users who can be assigned mutually orthogonal signature sequences is ______. **[2014]**

Solution:

Maximum number of users who can be assigned mutually orthogonal signature sequences is = 8.

5. The capacity of a Binary Symmetric Channel (BSC) with cross-over probability 0.5 is _____. **[2014]**

Solution:

Channel capacity $C = P\log_2 P + (1-P)\log_2 (1-P) + 1$

$C = 0.$

It is the case of channel with independent input and output, hence $C = 0$.

6. The capacity of a band-limited additive white Gaussian noise (AWGN) channel is given by

$C = W \log_2\left(1 + \frac{P}{\sigma^2 W}\right)$ bits per second (bps), where W is the channel bandwidth, P is the average power received and σ^2 is the one-side power spectral density of the AWGN.

For a fixed $\frac{P}{\sigma^2} = 1000$, the channel capacity (in kbps) with infinite bandwidth ($W \rightarrow {}^\circ$) is approximately

(a) 1.44 (b) 1.08
(c) 0.72 (d) 0.36 **[2014]**

Solution: (a)

Channel capacity is given by

$$C = W \log_2\left(1 + \frac{P}{\sigma^2 W}\right)$$

Hence, the correct option is (a)

7. A source alphabet consists of N symbols with the probability of the first two symbols being the same. A source encoder increases the probability of the first symbol by a small amount ε and decreases that of the second by ε . After encoding, the entropy of the source

(a) increases
(b) remains the same
(c) increases only if $N = 2$
(d) decreases **[2012]**

Solution: (d)

We know that Entropy is maximum when symbols are equal probable, so if probability will change from equal to non–equal, entropy will decrease.

Hence, the correct option is (d)

Two-mark Questions

1. Consider a binary channel code in which each codeword has a fixed length of 5 bits. The Hamming distance between any pair of distinct code words in this code is at least 2. The maximum number of code words such a code can contain is ______. **[2018]**

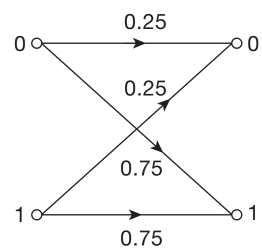
Solution: Possible number of coder words in which the Hamming distance is atleast 2 are as follows

A	B	C	D	E
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

The maximum possible code words are 16.

Hence, the correct answer is 16.

2. Consider a binary memoryless channel characterized by the transition probability diagram shown in the figure. **[2017]**



The channel is

(A) lossless (B) noiseless
(C) useless (D) deterministic

Solution: $P\left[\left(\frac{y}{x}\right)\right] = \begin{pmatrix} 0.25 & 0.75 \\ 0.25 & 0.75 \end{pmatrix}$

If mutual information $I(X;Y) = 0$ for every possible input distribution, then the channel is called as useless (or) zero-capacity channel.

$$Let[P(x)] = [\alpha(1-\alpha)]$$
$$H(x) = -\alpha \log_2 \alpha - (1-\alpha)\log_2(1-\alpha) \text{ bits/symbol}$$
$$[P(y)] = [P(x)]\left[P\left(\frac{y}{x}\right)\right] = [0.25 \;\; 0.75]$$

$$\left[P(X,Y)\right] = \begin{bmatrix} \frac{\alpha}{4} & \frac{3\alpha}{4} \\ \frac{(1-\alpha)}{4} & \frac{3(1-\alpha)}{4} \end{bmatrix}$$

$$\left[P\left(\frac{X}{Y}\right)\right] = \frac{\left[P(X,Y)\right]}{\left[P(Y)\right]_d} = \begin{bmatrix} \alpha & \alpha \\ 1-\alpha & 1-\alpha \end{bmatrix}$$

$$H\left(\frac{X}{Y}\right)=-\sum_{1}\sum_{i}P\left(X_iY_i\right)$$

$$\text{Log } P\left(\frac{X_i}{Y_i}\right)\text{bits/symbol}$$

$$=-\alpha\log_2\alpha-(1-\alpha)\log_2(1-\alpha)\text{bits/symbol}$$

$$I=(X;Y)=H(X)-H\left(\frac{X}{Y}\right)=0$$

Hence, the given binary memoryless channel is a useless channel.

Hence, the correct option is (C).

3. If the vectors $e_1 = (1, 0, 2)$ $e_2 = (0, 1, 0)$ and $e_3 = (-2, 0, 1)$ form an orthogonal basis of the three dimensional real space R^3, then the vector $u = (4, 3, -3) \in R^3$ can be expressed as **[2016]**

(A) $u=-\frac{2}{5}e_1-3e_2-\frac{11}{5}e_3$

(B) $u=-\frac{2}{5}e_1-3e_2+\frac{11}{5}e_3$

(C) $u=-\frac{2}{5}e_1+3e_2+\frac{11}{5}e_3$

(D) $u=-\frac{2}{5}e_1+3e_2-\frac{11}{5}e_3$

Solution: Given vectors are $e_1 = (1, 0, 2)$, $e_2 = (0, 1, 0)$ and $e_3 = (-2 , 0, 1)$

Given $\quad u = (4, 3, -3)$

Let, $\quad u = ae_1 + be_2 + ce_3 \quad (1)$

$\therefore (4, 3, -3) = a(1, 0, 2) + b(0, 1, 0) + c(-2, 0, 1)$

$= (a - 2c, \text{b}, 2a + c)$

$\therefore (4, 3, -3) = (a - 2c, b, 2a + c)$

Comparing the corresponding values on both sides,

$$4 = a - 2c \quad\Rightarrow\quad a - 2c = 4 \qquad (2)$$

$$3 = b \quad\Rightarrow\quad b = 3$$

$$-3 = 2a + c \quad\Rightarrow\quad 2a + c = -3 \qquad (3)$$

Solving (2) and (3), we get

$$a=\frac{-2}{5}$$

and

$$c=\frac{-11}{5}$$

substituting the values of a, b and c in (1), we get

$$u=\frac{-2}{5}e_1+3e_2-\frac{11}{5}e_3$$

Hence, the correct option is (D).

4. A source emits bit 0 with probability $\frac{1}{3}$ and bit 1 with probability $\frac{2}{3}$. The emitted bits are communicated to the receiver. The receiver decides for either 0 or 1 based on the received value R. It is given that the conditional density functions of R are as **[2015]**

$$f_{R|0}(r)=\begin{cases}\frac{1}{4}, & -3\le x\le 1\\ 0, & \text{otherwise}\end{cases}$$

and

$$f_{R/2}(r)=\begin{cases}\frac{1}{6}, & -1\le x\le 5\\ 0, & \text{otherwise}\end{cases}$$

The minimum decision error probability is **[2015]**

(A) 0 (B) 1/12

(C) 1/9 (D) 1/6

Solution: The conditional density function for

$$f_{R/0}(r)=\begin{cases}\frac{1}{4} & -3\le x\le 1\\ 0 & \text{otherwise}\end{cases}$$

$$f_{R/1}(r)=\begin{cases}\frac{1}{6}, & -1\le x\le 5\\ 0, & \text{otherwise}\end{cases}$$

So error region will be

$$P_{R/0}(r_1)=\int_{-3}^{-1}\frac{1}{4}dx=\frac{1}{2}$$

for function 1

$$P_{R/1}(r_1)=\int_{-1}^{1}\frac{1}{4}dx=\frac{1}{2}$$

and $P_{R/1}(r_2)=\int_{1}^{5}\frac{1}{6}dx=\frac{2}{3}$

for function 2

$$P_{R/0}(r_2)=\int_{-1}^{1}\frac{1}{6}dx=\frac{1}{3}$$

So minimum decision error probability

$$=P_{R/1}(r_1)\cdot P_{R/0}(\text{r}_2)=\frac{1}{2}\cdot\frac{1}{3}=\frac{1}{6}$$

Hence, the correct option is (D).

5. A fair coin is tossed repeatedly until a ‘head’ appears for the first time. Let L be the number of tosses to get this first ‘head’. The entropy $H(L)$ in bits is ______. **[2014]**

Solution:

$P_1=\frac{1}{2}$ (probability of getting first head).

Probability to get head in 2nd time $= \frac{1}{2}.\frac{1}{2} = \frac{1}{4}$

$P_3 = \frac{1}{2}.\frac{1}{2}.\frac{1}{2} = \frac{1}{8}$

Entropy $H = \sum_{i=1}^{N} P_i \log_2\left(\frac{1}{P_i}\right)$.

H ⊕ 2

6. Let U and V be two independent and identically distributed random variables such that $P(U = +1) = P(U = -1) = \frac{1}{2}$. The entropy $H(U + V)$ in bits is

(a) 3/4 (b) 1
(c) 3/2 (d) $\log_2 3$ **[2013]**

Solution: (c)

U	V	U + V	
1	1	2	$P(U+V=2) = \frac{1}{4}$
+1	−1	0	$P(U+V=0) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$
−1	+1	0	$PCV+V=-2 = \frac{1}{4}$
−1	−1	−2	

$$H(U+V) = \frac{3}{2}$$

Hence, the correct option is (c)

7. A communication channel with AWGN operating at a signal to noise ratio SNR >> 1 and bandwidth B has capacity C_1. If the SNR is doubled keeping B constant, the resulting capacity C_2 is given by

(a) $C_2 \approx 2C_1$ (b) $C_2 \approx C_1 + B$
(c) $C_2 \approx C_1 + 2B$ (d) $C_2 \approx C_1 + 0.3B$ **[2009]**

Solution: (b)

Given

SNR >> 1

$$C_1 = B\log_2\left(1 + \frac{S}{N}\right) = B\log_2\left(\frac{S}{N}\right)$$

when SNR is doubled

$$C^1 = B\log_2\left(\frac{2S}{N}\right) = C_1 + B$$

$C^1 = C_1 + B$

Hence, the correct option is (b)

8. A memoryless source emits n symbols each with a probability p. The entropy of the source as a function of n

(a) increases as log n
(b) decreases as log $(1/n)$
(c) increases as n
(d) increases as n log n **[2008]**

Solution: (a)

Entropy $H(m) = -\sum_{i=1}^{n} P_i \log(a)$ bits

If

$H(m) = \log \eta$

$\therefore \sum_{i=1}^{n} \frac{1}{n} = 1$

Hence, the correct option is (a)

9. A source generates three symbols with probabilities 0.25, 0.25, 0.50 at a rate of 3000 symbols per second. Assuming independent generation of symbols, the most efficient source encoder would have average bit rate of

(a) 6000 bits/sec (b) 4500 bits/sec
(c) 3000 bits/sec (d) 1500 bits/sec **[2006]**

Solution: (b)

Bit rate = $H.R_b$

$$H = \Sigma P_i \log_2\left(\frac{1}{P_i}\right) = 1.5$$

Bitrate = 1.5 × 3000

= 4500 bps

Hence, the correct option is (b)

10. A binary source has symbol probabilities 0.8 and 0.2. IF extension coding (blocks of 4 symbols) is used, the lower and upper bounds on the average code word length are

(a) lower _____. (b) higher _____. **[1991]**

Solution:

$$\text{Entropy} = H = -\sum_{i=1}^{n} P_i \log_2(P_i)$$

H = [0.8 log2 (0.8) + 0.2 log2 (0.2)]

H = 0.72 bits/symbol

For the extended coding scheme, using blocks of 4 symbols, the entropy is given by 4 × 0.72 = 2.8876 bits/4 symbols

The bounds on the average code word length are

H (x) ≤•L H (x) + 1

(a) Lower bound = 2.8876 bits
(b) Upper bound = 3.8876 bits

11. An image uses 512 × 512 picture elements. Each of the picture elements can take any of the 8 distinguishable intensity levels. The maximum entropy in the above image will be

(a) 2097152 bits (b) 786432 bits
(c) 648 bits (d) 144 bits **[1990]**

Solution: (b)

Number of symbols that one image pixel can have = M = 8.

Maximum Entropy associated with one image pixel

H_{max} = log² M= log²8 = 3

Maximum entropy associated with image = 512 × 512 × H_{max}

= 512 × 512 × 3

= 786432

Hence, the correct option is (b)

12. A source produces 4 symbols with probability $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ and $\frac{1}{8}$. For this source, a practical coding scheme has an average codeword length of 2 bits/symbols. The efficiency of the code is

(a) 1 (b) 7/8
(c) 1/2 (d) 1/4 **[1989]**

Solution: (b)

Code efficiency =

$$\eta = \frac{\text{(Entropy)H}}{\text{(Average code length)L}} \times 100\%$$

$$\text{Entropy} = \text{H} = -\sum_{i=1}^{n} \text{P}_i \log_2(\text{P})$$

$$\text{H} = -\left[\frac{1}{2}\log_2\left(\frac{1}{2}\right) + \frac{1}{4}\log_2\left(\frac{1}{4}\right) + \frac{1}{8}\log_2\left(\frac{1}{8}\right) + \frac{1}{8}\log_2\left(\frac{1}{8}\right)\right]$$

$$H = \frac{1}{2} + \frac{1}{4}\times 2 + \frac{1}{8}\times 3 + \frac{1}{8}\times 3$$

$$\text{H} = 1 + \frac{3}{4} = \frac{7}{4}$$

$$\text{Code efficiency} = \eta = \frac{\text{H}}{\text{L}} \times 100\%$$

$$\eta = \frac{\frac{7}{4}}{2} \times 100\%$$

$$\eta = \frac{7}{8}$$

Hence, the correct option is (b)

UNIT IX

ELECTROMAGNETIC THEORY

EXAM ANALYSIS

Exam Year	92	93	94	95	96	97	98	99	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14-1	14-2	14-3	14-4	15			16			17		18	19
																											Set 1	Set 2	Set 3	Set 1	Set 2	Set 3	Set 1	Set 2		
1 Marks Ques.	-	-	5	5	3	2	12	3	4	4	4	2	2	1	2	2	2	2	2	4	4	1	2	2	2	4	1	2	1	2	4	2	2	2	3	3
2 Marks Ques.	2	6	2	-	4	1	-	4	4	4	4	7	6	6	8	7	5	3	2	3	5	2	3	4	3	3	3	2	1	7	4	4	3	3	3	3
5 Marks Ques.	-	2	4	3	3	3	5	3	2	3	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Total Marks	**6**	**22**	**29**	**20**	**26**	**19**	**37**	**26**	**22**	**27**	**22**	**16**	**14**	**13**	**18**	**16**	**12**	**8**	**6**	**10**	**14**	**5**	**8**	**10**	**8**	**10**	**7**	**6**	**3**	**16**	**12**	**10**	**8**	**8**	**9**	**9**
Chapter wise marks distribution																																				
Basics	-	6	3	2	1	-	1	1	-	-	2	3	2	-	-	1	1	5	-	3	6	-	1	2	4	8		3			11		5		1	3
Uniform Plane Waves	-	4	1	2	4	-	6	2	4	3	4	7	2	1	7	5	2	-	3	3	3	4	-	2	2	-		5			3		2		1	-
Transmission Lines	2	2	3	-	2	1	1	4	1	3	2	2	8	8	4	4	4	2	3	3	2	1	4	3	1	-		4			3		1		2	-
Waveguide	-	-	1	-	-	1	-	1	3	2	1	2	1	2	4	4	2	1	-	1	2	-	1	3	1	-		1			2		1		-	2
Antenna	4	-	1	1	4	2	4	2	3	4	3	2	1	2	3	2	3	-	-	-	1	-	2	-	-	2		-			4		1		1	1

Chapter 1

Basics

One-mark Questions

1. In the table shown. List-I and II. Respectively, contain terms appearing on the left-hand side and the right-hand side of Maxwell's equations (in their standard form). Match the left-hand side with the corresponding right-hand side. **[2019]**

List-I	List-II
1. $\nabla \cdot D$	P. 0
2. $\nabla \times E$	Q. ρ
3. $\nabla \cdot B$	R. $-\frac{\partial B}{\partial t}$
4. $\nabla \times H$	S. $J+\frac{\partial D}{\partial t}$

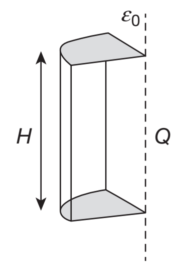
(A) 1 – R, 2 – Q, 3 – S, 4 – P
(B) 1 – Q, 2 – S, 3 – P, 4 - R
(C) 1 – P, 2 – R, 3 – Q, 4 – S
(D) 1 – Q, 2 – R, 3 – P, 4 – S

Solution: The differential four of the 4 Maxwell's equations are:

$$\nabla \cdot D = P$$
$$\nabla \cdot B = O$$
$$\nabla \times E = \frac{-\partial B}{\partial t}$$
$$\nabla \times H = J + \frac{\partial D}{\partial T}$$

Hence the correct option is (D)

2. What is the electric flux $\left(\int \bar{E} \cdot d\,\hat{a}\right)$ through a quarter-cylinder of height H (as shown in the figure) due to an infinitely long line charge along the axis of the cylinder with a charge density of Q? **[2019]**



(A) $\frac{4H}{Q\varepsilon_0}$ (B) $\frac{H\varepsilon_0}{4Q}$

(C) $\frac{HQ}{\varepsilon_0}$ (D) $\frac{HQ}{4\varepsilon_0}$

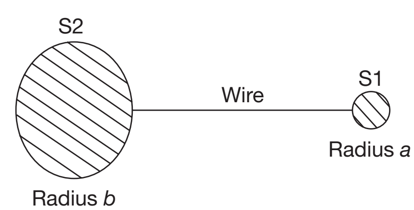
Solution: The electric flux leaving the cylinder of height, H

$$= \int E.da = \frac{Q}{2\Pi E_o p}\left(\frac{2\Pi pH}{4}\right)$$

$$= \int E.da = \frac{QH}{4E_o}$$

Hence, the correct option is (D).

3. Two conducting spheres S1 and S2 of radii a and $b(b>a)$ respectively, are placed far apart and connected by a long, thin conducting wire, as shown in the figure.



For some charge placed on this structure, the potential and surface electric field on S1 are V_a and E_a and that on S2 V_b and E_b, respectively. Then, which of the following is CORRECT? **[2017]**

(A) $V_a = V_b$ and $E_a < E_b$
(B) $V_a > V_b$ and $E_a > E_b$

(C) $V_a = V_b$ and $E_a > E_b$
(D) $V_a > V_b$ and $E_a = E_b$

4. Concentric spherical shells of radii 2 m, 3 m and 8 m carry uniform surface charge densities of 20 nC/m^2, –4 nC/m^2 and $\rho_{s,}$ respectively. The value of ρ (nC/m^2) required to ensure that the electric flux density $\vec{D} = \vec{0}$ at radius 10 m is ______. **[2016]**

Solution: The electric flux density $\overline{D} = 0$ at $r = 10$ m when the total charge enclosed by the sphere of radius 10 m, is equal to zero, then

$$20 \times 10^{-9} \times 16\pi - 4 \times 10^{-9} \times 64\pi + \rho_s \times 256\pi = 0$$

$$\Rightarrow \qquad \rho_s = -0.25 \text{ nC/m}^2$$

Hence, the correct Answer is (–0.25 nC/m^2).

5. A uniform and constant magnetic field $B = \hat{Z}B$ exists in the $\hat{Z}$ direction in vacuum. A particle of mass m with a small charge q is introduced in to this region with an initial velocity $V = \hat{x}v_x + \hat{z}v_z$. Given that B, m, q, v_x and v_z are all non-zero, which one of the following describes the eventual trajectory of the particle? **[2016]**

(A) Helical motion in the $\hat{z}$-direction
(B) Circular motion in the xy plane
(C) Linear motion in the $\hat{z}$-direction
(D) Linear motion in the $\hat{x}$-direction

Solution: When the field is directed in the 'z' direction according to the right hand thumb rule, the motion of the particle is in helical trajectory in the z-direction.

At a fixed point in the z-direction the, the particle moves in the x–y plane.

Hence, the correct option is (A).

6. Faraday's law of electromagnetic induction is mathematically described by which one of the following equations? **[2016]**

(A) $\nabla \cdot \vec{B} = 0$ (B) $\nabla \cdot \vec{D} = \rho_v$

(C) $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$ (D) $\nabla \times \vec{H} = \sigma \vec{E} + \dfrac{\partial \vec{D}}{\partial t}$

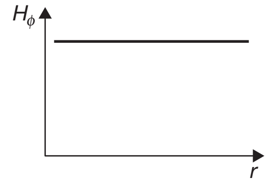
Solution: From the basic four equations, faraday's law of EMI can be best represented by (C) showing that the time varying electric field produces an orthogonal space varying magnetic field and vice versa

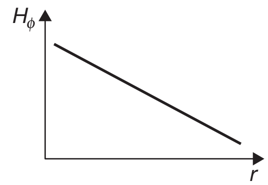
$$\nabla x \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

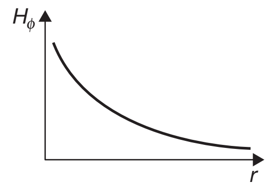
One of the Maxwell's equations based on Faraday's law of electro-magnetic induction.

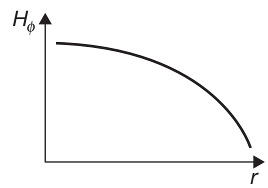
Hence, the correct option is (C).

7. Consider a straight, infinitely long, current carrying conductor lying on the z-axis. Which one of the following plots (in linear scale) qualitatively represents the dependence of H_Φ on r, where H_Φ is the magnitude of the azimuthal component of magnetic field outside the conductor and r is the radial distance from the conductor? **[2015]**

(A) 

(B) 

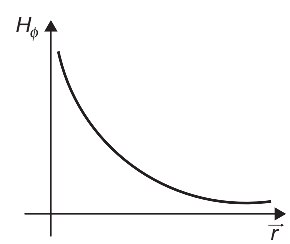
(C) 

(D) 

Solution: Given that conductor is lying on z-axis

$$H_\varphi = \int_L \frac{I\, \vec{dl} \times \vec{r}}{4\pi \left|\vec{r}\right|^3}$$

$$H_\varphi \propto \frac{1}{r^2}$$



Hence, the correct option is (C).

8. A vector $\vec{P} = x^3 y\, \vec{a}_x - x^2y^2\, \vec{a}_y - x^2yz\, \vec{a}_z$. Which one of the following statements is TRUE? **[2015]**

(A) $\vec{P}$ is solenoidal, but not irrotational
(B) $\vec{P}$ is irrotational, but not solenoidal
(C) $\vec{P}$ is neither solenoidal nor irrotational
(D) $\vec{P}$ is both solenoidal and irrotational

Solution: Given $\overline{P} = x^3y\,\overline{a}_x - x^2y^2\,\overline{a}_y - x^2yz\,\overline{a}_z$

$$\text{Div}\,\overline{P} = \nabla.\overline{P} = \frac{\partial}{\partial x}(x^3y) + \frac{\partial}{\partial y}(-x^2y^2) + \frac{\partial}{\partial z}(-x^2yz)$$

$$= 3x^2y - 2x^2y - x^2y$$

$$= 0$$

$\therefore \overline{P}$ is solenoidal (1)

$$\text{Curl}\ \overline{P} = \nabla \times \overline{P} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3y & -x^2y^2 & -x^2yz \end{vmatrix}$$

$$= \bar{i}\left(\frac{\partial}{\partial y}(-x^2yz) - \frac{\partial}{\partial z}(-x^2y^2)\right) - \bar{j}\left(\frac{\partial}{\partial x}(-x^2yz)\right.$$

$$\left. - \frac{\partial}{\partial z}(x^3y)\right) + \bar{k}\left(\frac{\partial}{\partial x}(-x^2y^2) - \frac{\partial}{\partial y}(x^3y)\right)$$

$$= (-x^2z + 0)\,\bar{i} + 2xyz\,\bar{j} + (-2xy^2 - x^3)\,\bar{k}$$

$$\Rightarrow \text{curl}\ \overline{P} = -x^2\,2\,\bar{i} + 2xyz\,\bar{j} - (x^3 + 2xy^2)\,\bar{k}$$

$\therefore$ curl $\overline{P} \neq \overline{0}$

$\therefore$ $\overline{P}$ is NOT irrotational (2)

From (1) and (2),

$\overline{P}$ issolenoidal, but not irrotational.

Hence, the correct option is (A).

9. The magnitude of the gradient for the function f(x, y, z) = $x^2 + 3y^2 + z^3$ at the point (1, 1, 1) is__________. **[2014]**

Solution: (7)

$F(x, y, z) = x^2 + 3y^2 + z^3$

$$\nabla F = \frac{\partial F}{\partial x}ax + \frac{\partial F}{\partial y}ay + \frac{\partial F}{\partial z}az$$

$$= 2xax + 6yay + 3z^2az$$

$$\nabla F(1, 1, 1) = 2ax + 6ay + 3az$$

$$\nabla F(1, 1, 1) = \sqrt{2^2 + 6^2 + 3^2}$$

$$= \sqrt{4 + 36 + 9}$$

$$|\nabla F|_{(1,1,1)} = 7$$

10. The directional derivative of $f(x, y) = \frac{xy}{\sqrt{2}}(x+y)$ at (1, 1) in the direction of the unit vector an angle of $\frac{\pi}{4}$ with y-axis, is given by____ **[2014]**

Solution: (2.121)

$$F(x, y) = \frac{xy}{\sqrt{2}}(x+y)$$

$$\underline{\nabla F} = \frac{\partial F}{\partial x}a\hat{x} + \frac{\partial F}{\partial y}a\hat{y}$$

$$= \frac{(2xy + y^2)}{\sqrt{2}}a\hat{x} + \frac{(2xy + x^2)}{\sqrt{2}}a\hat{y}$$

$$\Delta F(1,1) = \frac{3}{\sqrt{2}}a\hat{x} + \frac{3}{\sqrt{2}}a\hat{y}$$

11. Consider a closed surface S surrounding a volume V. If is the position vector of a point inside S, with $\hat{n}$ the unit normal on S, the value of the integral $\oiint_S 5\vec{r}\cdot\hat{n}\,dS$ is

(a) 3 V (b) 5 V
(c) 10 V (d) 15V **[2011]**

Solution: (d)

$$\oint_S\oint 5\vec{r}.\hat{n}\,ds = \iiint_v \nabla.5\vec{r}\,dv \text{ [applying divergence theorem]}$$

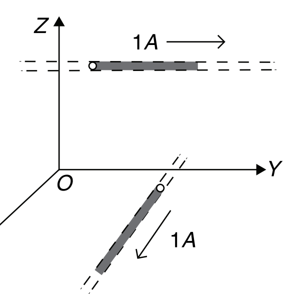
$$= 5\iiint_v \nabla.\vec{r}\,dv$$

$$= 5 \times 3$$

$$= 15\text{ V}$$

Hence, the correct option is (d).

12. Two infinitely long wires carrying current are as shown in the figure below. One wire is in the y-z plane and parallel to the y-axis. The other wire is in the x-y plane and parallel to the x-axis. Which components of the resulting magnetic field are non-zero at the origin?



(a) x, y, z components (b) x, y components
(c) y, z components (d) x, z components
[2009]

Solution: (d)

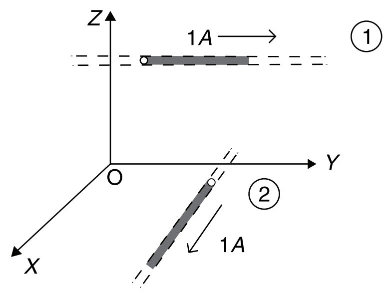
$H(1) = ay - az$ [at origin]

$= -ax$

$H(2) = ax \times -ay$

$= -az$

$\therefore$ x, z components will be present.



Hence, the correct option is (d).

13. For static electric and magnetic fields in an inhomogeneous source free medium, which or the following represents the correct form of two of Maxwell's equations?

(a) $\nabla . E = 0$
$\nabla \times B = 0$

(b) $\nabla . E = 0$
$\nabla . B = 0$

(c) $\nabla \times E = 0$
$\nabla \times B = 0$

(b) $\nabla \times E = 0$
$\nabla \times B = 0$ **[2008]**

Solution: (d)

Maxwell's equations are for static field,

$\nabla \cdot D = \rho_v$ $\quad$ $\nabla \cdot E = 0$

$\nabla \cdot B = 0$ $\quad$ $\nabla \cdot H = J$

Hence, the correct option is (d).

14. If C is a closed curve enclosing a surface S, then the magnetic field intensity $\vec{H}$, the current density $\vec{J}$ and the electric flux density $\vec{D}$ are related by

(a) $\iint_s \vec{H} \bullet \vec{ds} = \oint_C \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \bullet \vec{dl}$

(b) $\int_C \vec{H} \bullet \vec{dl} = \oiint_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \bullet \vec{ds}$

(c) $\oiint_S \vec{H} \bullet \vec{ds} = \int_C \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \bullet \vec{dl}$

(d) $\oint_C \vec{H} \bullet \vec{dl} = \iint_s \vec{J} + \frac{\partial \vec{D}}{\partial t} \bullet \vec{ds}$ **[2007]**

Solution: (d)

Maxwell Fourth equation i.e.

$$\oint_C \vec{H} . \vec{dl} = \int\int_S \left(\vec{J} + \frac{\vec{dD}}{dt} \right) . \vec{ds}$$

Hence, the correct option is (d).

15. The Unit of $\nabla \times$ H is

(a) Ampere
(b) Ampere/meter
(c) Ampere/meter2
(d) Ampere-meter

[2003]

Solution: (c)

$$\nabla = \frac{\partial}{\partial x} ax + \frac{\partial}{\partial y} ay + \frac{\partial}{\partial z} az$$

unit of ∇ is m^{-1} (per meter)

unit of H = Ampere/meter

$$\nabla \times H = \frac{1}{\text{meter}} \times \frac{\text{Ampere}}{\text{meter}}$$

So, unit of $\nabla \times H$ is Ampere/(meter)^2

Hence, the correct option is (c).

16. An electric field on a plane is described by its potential $V = 20(r^{-1} + r^{-2})$ where r is the distance from the source. The field is due to

(a) a monopole
(b) a dipole
(c) both a monopole and a dipole
(d) a quadrupole **[1999]**

Solution: (c)

$$V = 20(r^{-1} + r^{-2})$$

$$= \frac{20}{r} + \frac{20}{r^2}$$

$$V\,[\text{monopole}] = \frac{\theta}{4\pi E r}$$

$$V\,[\text{dipole}] = \frac{\theta d \cos\theta}{4\pi E r^2}$$

Therefore, the field is due to both a monopole and a dipole.

Hence, the correct option is (c).

17. The Maxwell's equation, $\nabla \times \text{H} = \text{J} + \frac{\partial \vec{\text{D}}}{\partial t}$ is based on

(a) Ampere's law
(b) Gauss's law
(c) Faraday's law
(d) Coulomb's law

[1998]

Solution: (a) Ampere's Law

Maxwell 4th equation, $\nabla \times \vec{H} = \vec{j} + \frac{\vec{dD}}{dt}$ is based upon modified ampere's Law,

Hence, the correct option is (a)

18. A metal sphere with 1 m radius and a surface charge density of 10 Coulombs/m^2 is enclosed in a cube of 10 m side. The total outward electric displacement normal to the surface of the cube is

(a) 40 π Coulombs
(b) 10 π Coulombs
(c) 5 π Coulombs
(d) None of the above **[1996]**

Solution: (a)

Using Gauss Law, $\oint D.\overrightarrow{ds} = Q_{enclosed}$

$$Q_{enclosed} = \rho_s \times surface\ area$$
$$= 10\ \text{c/m}^2 \times 4\pi r^2\ \text{m}^2$$
$$= 40\ \pi\ Coulombs$$

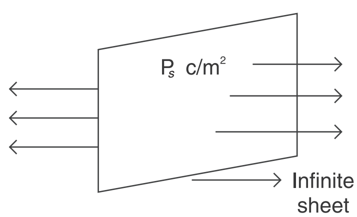
Hence, the correct option is (a).

19. In the infinite plane, y = 6 m, there exists a uniform surface charge density of (1/6000) μC/m². The associated electric field strength is
(a) 30 $\hat{\imath}$ V/m (b) 3 $\hat{\jmath}$ V/m
(c) 30 $\hat{k}$ V/n (d) 60 $\hat{\jmath}$ V/m **[1995]**

Solution: (b)

Sheet charges and uniform fields.

$$\vec{D} = \frac{\rho_s}{2}\overrightarrow{aN} \text{ and } \vec{E} = \frac{\rho_s}{2E}\overrightarrow{aN}$$



Here, $\overrightarrow{aN} = \overrightarrow{ay}$ and $\rho_s = \dfrac{1}{6000}\mu\text{c/m}^2$

$$\vec{E} = \frac{1\times10^{-6}}{6000\times 8.854\times10^{-12}\times 2}\overrightarrow{ay}$$

$$\vec{E} = 3\hat{j}\ \text{v/m}$$

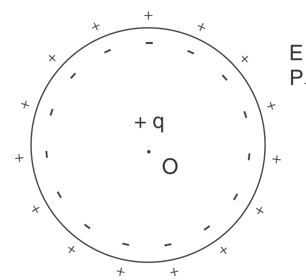
Hence, the correct option is (b).

20. The electric field strength at distant point. P, due to a point charge, + q, located at the origin is 100 μV/m If the point charge is now enclosed by a perfectly conducting metal sheet sphere whose center is at the origin, then the electric field strength at the point, P, outside the sphere, becomes
(a) zero (b) 100 μV/m
(c) –100 μV/m (d) 50 μV/sm **[1995]**

Solution: (b)

Since, in conductor charge induces equal and opposite charge on surface then field remain same.

i.e. 100 μV/m



Hence, the correct option is (b).

21. $\oint_C \vec{A}.\vec{dl} = \int_S \_\_\_\ \vec{ds}$ **[1994]**

Solution: $(\nabla\times\vec{A})$

By using stoke's theorem, $\oint_C \vec{A}.dl = \int_S (\nabla\times\vec{A}).\overrightarrow{ds}$

22. An electrostatic field is said to be conservative when….

$$\oint \vec{E}\cdot\overrightarrow{dl} = 0 \quad (1)$$

i.e. the closed line integral of the field is zero because for the open line integral, the field will be, $\int \vec{E}.\overrightarrow{dl} = V$ [i.e. voltage across the line]

Now, apply stokes theorem on equation (1)

i.e., $\oint_C \vec{E}.\overrightarrow{dl} = \int_s (\nabla\times\vec{E})\overrightarrow{ds}$ thus, $\nabla\times\vec{E} = 0$ **[1994]**

Solution: (b)

the curl of the field is equal to zero. An electrostatic field is said to be conservative when the closed line integral of the field is zero.

$$\oint \vec{E}.\overrightarrow{dl} = 0$$

Stoke's theorem

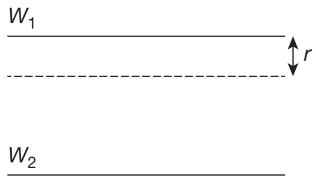
$$\int_c \vec{E}\cdot\overrightarrow{dl} = \int_s (\nabla\times\vec{E})\cdot\overrightarrow{ds}$$

So, $\nabla\times\vec{E} = 0$

Hence, the correct option is (b).

Two-marks Questions

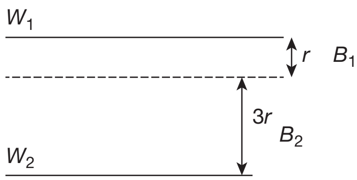
1. Two identical copper wires W_1 and W_2, placed in parallel as shown in the figure. Carry currents I and $2I$, respectively, in opposite direction. If the two wires are separated by a distance of $4r$, then the magnitude of the magnetic field $\vec{B}$ between the wires at a distance r from W_1 is **[2019]**



(A) $\dfrac{\mu_0^2 I^2}{2\pi r^2}$ (B) $\dfrac{6\mu_0 I}{5\pi r}$

(C) $\dfrac{\mu_0 I}{6\pi r}$ (D) $\dfrac{5\mu_0 I}{6\pi r}$

Solution:



$$B = B_1 + B_2$$

$$B_1 = \frac{\mu_0}{2\Pi}\frac{I}{r}$$

$$B_2 = \frac{\mu_0}{2\Pi}\frac{2I}{3r}$$

$$B = B_1 + B_2$$

$$= \frac{\mu_0}{2\Pi}\frac{I}{r}\left[1+\frac{2}{3}\right]$$

$$= \frac{5\mu_0 I}{6\Pi r}$$

Hence, the correct option is (D).

2. The expression for an electric field in free space is $E = E_0(\hat{x}+\hat{y}+j2\hat{z})e^{-j(\omega t-kx+ky)}$, where x, y, z represent the spatial coordinates, t represents time, the ω, k are constants. This electric field. **[2017]**

(A) Does not represent a plane wave.
(B) represents a circularly polarized plane wave propagating normal to the z-axis.
(C) represents an elliptically polarized plane propagating along the x–y plane.
(D) represents a linearly polarized plane wave.

Solution: Direction of propatation between $+x$ and $+y$

$e^{-j\omega t}e^{-jkx}e^{-jky}$

The magnitudes of electric field z-component and $x-y$ components are not equal

$$|E_{zo}| = 2E_0$$

$$|E_{xo}| = \sqrt{2}E_0$$

and the phase difference between the two components $= 90°$

$\therefore$ The field represents an elliptically polarized plane wave propagating along the $x-y$ plane.

Hence, the correct option is (C).

3. If the vector function $\vec{F} = \hat{a}_x(3y-k_1z)+\hat{a}_y(k_2x-2z)-\hat{a}_z(k_3y+z)$ is irrotational, then the values of the constants k_1, k_2 and k_3, respectively, are **[2017]**

(A) 0.3, –2.5, 0.5 (B) 0.0, 3.0, 2.0
(C) 0.3, 0.33, 0.5 (D) 4.0, 3.0, 2.0

Solution: Given $\bar{F} = \hat{a}_x(3y - k_1z) + \hat{a}_y(k_3y + z)$ is irrotational

$\Rightarrow$ curl $\bar{F} = \bar{0}$

$$\Rightarrow \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 3y-k_1z & k_2x-2z & -k_3y-z \end{vmatrix} = \bar{0}$$

$\Rightarrow \bar{i}\,(-k_3+2) - \bar{j}\,(0+k_1) + \bar{k}\,(k_2-3) = \bar{0}$

$\Rightarrow -k_3+2 = 0,\ k_1 = 0$ and $k_2 - 3 = 0$

$\Rightarrow k_3 = 2;\ k_1 = 0$ and $k_2 = 3$.

Hence, the correct option is (B).

4. The current density in a medium is given by

$$\vec{J} = \frac{400\sin\theta}{2\pi\left(r^2+4\right)}\ \hat{a}_r\ \text{Am}^{-2}.$$

The total current and the average current density flowing through the portion of a spherical surface $r = 0.8$ m, $\dfrac{\pi}{12} \le \theta \le \dfrac{\pi}{4}$, $0 \le \phi \le 2\pi$ are given respectively, by **[2016]**

(A) 15.09 A, 12.86 Am^{-2}
(B) 8.73 A, 13.65 Am^{-2}
(C) 12.86 A, 9.23 Am^{-2}
(D) 10.28 A, 7.56 Am^{-2}

Solution: Radius of spherical surface $r = 0.8$ m.

Current density is given as

$$\bar{J} = \frac{400\sin\theta}{2\pi(r^2+4)}\hat{a}_r\ \text{Am}^{-2}$$

Total current flowing through the given spherical surface, can be calculated using

$$I = \iint_s \bar{J}\ d\bar{s}$$

$$= \int_{Q=0}^{2\pi}\int_{\theta=\pi/2}^{\pi/4} \frac{400\sin\theta}{2\pi(r^2+4)}a_r\, r^2\sin\theta d\theta d\phi\, a_r$$

$r = 0.8$

$= 15.09$ A.

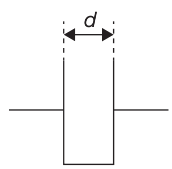
Average current density through the surface, will be

$$J_{av} = \frac{I}{\iint\limits_s ds} = \frac{15.09}{\int\limits_{Q=0}^{2\pi}\int\limits_{\theta=\pi/2}^{\pi/4} r^2 \sin\theta d\theta d\phi}$$

$$= 12.86 \text{ A/m}^2.$$

Hence, the correct option is (A).

5. The parallel plate capacitor shown in the figure has movable plates. The capacitor is charged so that the energy stored in it is E, when the plate separation is d, the capacitor is then isolated electrically and the plates are moved such that the plate separation becomes $2d$.



At this new plate separation, what is the energy stored in the capacitor, neglecting fringing effects? **[2016]**

(A) $2E$

(B) $\sqrt{2}\,E$

(C) E

(D) $\frac{E}{2}$

Solution: The capacitor is isolated and fringing effects are neglected then the energy of capacitor is

$$E = \frac{1}{2}CV^2$$

$$E = \frac{1}{2}\frac{Q^2}{C}$$

Q = constant

$$E \propto \frac{1}{C} \quad (1)$$

$$C = \frac{\varepsilon A}{d}$$

ε, A are constant

$$C \propto \frac{1}{d} \quad (2)$$

From (1) and (2)

$$E \propto d$$

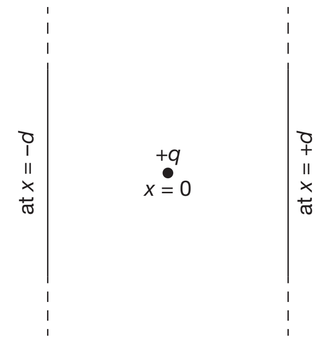
$$\frac{E_1}{E_2} = \frac{d_1}{d_2}$$

$$\frac{E}{E_2} = \frac{d}{2d}$$

$$E_2 = 2E$$

Hence, the correct option is (A).

6. A positive charge q is placed at $x = 0$ between two infinite metal plates placed at $x = -d$ and at $x = +d$ respectively. The metal plates lie in the yz-plane.



The charge is at rest at $t = 0$, when a voltage $+V$ is applied to the plate at $-d$ and voltage $-V$ is applied to the plate at $x = +d$. Assume that the quantity of the charge q is small enough that it does not perturb the field set up by the metal plates. The time that the charge q takes to reach the right plate is proportional to **[2016]**

(A) $\frac{d}{V}$

(B) $\frac{\sqrt{d}}{V}$

(C) $\frac{d}{\sqrt{V}}$

(D) $\sqrt{\frac{d}{V}}$

Solution: Charge transit time

$$t = \frac{d}{V}$$

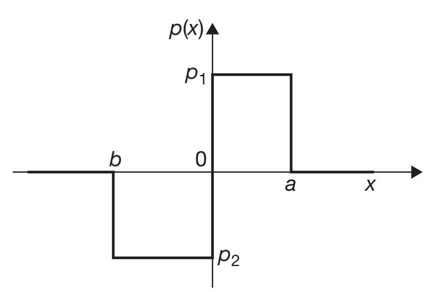
Where the velocity

$$v \propto \sqrt{v}$$

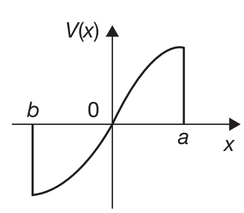
$$\therefore \quad t \propto \frac{d}{\sqrt{v}}$$

Hence, the correct option is (C).

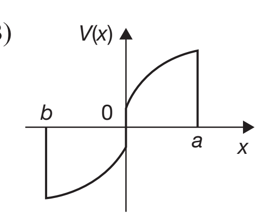
7. Consider the charge profile shown in the figure. The resultant potential distribution is best described by **[2016]**

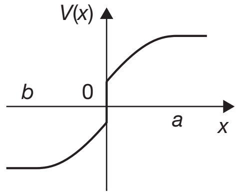


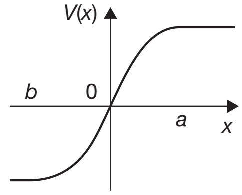
(A)



(B)



(C) 

(D) 

Solution: The resultant potential distribution for the given charge profile can be better described by (d) because the input is having unit step change with a discontinuity outside the given interval from b to a, thus its potential distribution with a linear change from b to a and constant values at the remaining values.

Hence, the correct option is (D).

8. In a source free region in vacuum, if the electrostatic potential $\phi = 2x^2 + y^2 + cz^2$, the value of constant c must be ________. **[2015]**

Solution: In source free region $\nabla.E = 0$

Where $E = -\nabla.\phi$

$$E = -\left[\frac{\partial}{\partial x}\left(2x^2+y^2+cz^2\right)\hat{a}_x + \frac{\partial}{\partial y}\left(2x^2+y^2+cz^2\right)\hat{a}_y + \frac{\partial}{\partial z}\left(2x^2+y^2+cz^2\right)\hat{a}_z\right]$$

$$E = -\left[4x\hat{a}_x + 2y\hat{a}_y + 2cz\,\hat{a}_z\right]$$

Source free region $\nabla.E = 0$

$$\Rightarrow \quad -[4 + 2 + 2c] = 0$$

$$2c = -6$$

$$c = -3$$

Hence, the correct Answer is (–3.1 to –2.9).

9. The force on a point charge $+q$ kept at a distance d from the surface of an infinite grounded metal plate in a medium of permittivity $\in$ is

(a) 0

(b) $\dfrac{q^2}{16\pi \in d^2}$ away from the plate

(c) $\dfrac{q^2}{16\pi \in d^2}$ towards the plate

(d) $\dfrac{q^2}{4\pi \in d^2}$ towards the plate **[2014]**

Solution: (c)

Direction if force will be $+q$ to $-q$ therefore, towards to the plate, $F = \dfrac{q^2}{16\pi E d^2}$

10. If $\vec{r} = xa_x + ya_y + za_z$ and $|r| = r$ then div($r^2\nabla$ (ln$r$))_____. **[2014]**

Solution: (3)

$$\vec{r} = xa_x + ya_y + za_z \qquad (1)$$

$$|\vec{r}| = r$$

$$\text{div}(r^2\nabla(\ln r))$$

$$= \text{div}\left\{r^2 \cdot \frac{1}{r^2}\left(\frac{\partial r}{\partial x}a\hat{x} + \frac{\partial x}{\partial y}a\hat{y} + \frac{\partial r}{\partial z}a\hat{z}\right)\right\}$$

$$= \text{div}\left\{r^1\frac{dr}{\partial x}a\hat{x} + r\frac{\partial r}{\partial y}a\hat{y} + r\frac{\partial r}{\partial z}a\hat{z}\right\} \qquad (2)$$

Put equation (1) in equation (2)

So, $\text{div}(r^2\nabla(\ln r)) = \text{div}[r(a\hat{x} + a\hat{y} + a\hat{z})]$

$$= \frac{\partial r}{\partial x} + \frac{\partial r}{\partial y} + \frac{\partial r}{\partial z}$$

$$\text{div}\left|r^2\,1\text{n}\right| = 3$$

11. Given the vector

$\vec{A} = (\cos x)(\sin y)\hat{a}_x + (\sin x)(\cos y)\hat{a}_y$ where

$\hat{a}_x, \hat{a}_y$ denote unit vectors along x, y directions, respectively. The magnitude of curl of $\vec{A}$ is______ **[2014]**

Solution: (0)

$$\nabla\times\vec{A} = \begin{vmatrix} a\hat{x} & a\hat{y} & a\hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ Ax & Ay & Az \end{vmatrix}$$

$$\vec{A} = (\cos x)(\sin y)a\hat{x} + (\sin x)(\cos y)a\hat{y}$$

$Ax = (\cos x)\,(\sin y)$

$Ay = (\sin x)\,(\cos y)$

$Az = 0$

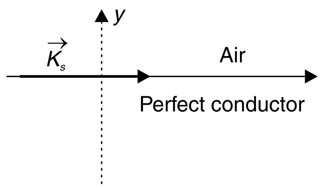
$$\nabla\times\vec{A} = \begin{vmatrix} a\hat{x} & a\hat{y} & a\hat{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \cos x\sin y & \sin x\cos y & 0 \end{vmatrix}$$

$$= az^1(\cos x\cos y - \cos x\cos y) = 0$$

$$\therefore \left|\nabla\times\vec{A}\right| = 0$$

12. A region shown below contains a perfect conducting half-space and air. The surface current $\vec{K}s$ on the surface of the perfect conductor is $\vec{K}s = \text{x}^2$ amperes per meter.

The tangential $\vec{H}$ field in the air just above the perfect conductor is



(a) $-(x+z)$ amperes per meter

(b) $(\hat{x}+\hat{z})$ amperes per meter

(c) $-\hat{z}^2$ amperes per meter

(d) $\hat{z}^2$ amperes per meter **[2014]**

Solution: (d)

Properly of magnetic boundary conditions states

$$H_{1t} - H_{2t} = \vec{ks}$$

Where, $\vec{K}s$ is surface current.

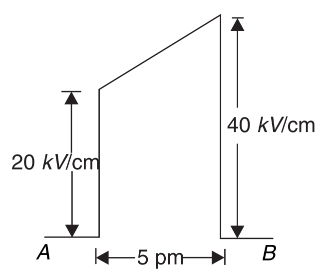
Here, $H_{2t} = 0$

$$H_{1t} = 2\hat{x}\times\hat{y}$$

$$H_{1t} = 2\hat{z}\ \text{A/m}$$

Hence, the correct option is (d).

13. The electric field (assumed to be one-dimensional) between two points A and B is shown Let Ψ_A and Ψ_B be the electrostatic potentials at A and B, respectively. The value of $\Psi_A \Psi_B$ in Volts is________



[2014]

Solution: (–15)

$$E = -\frac{dv}{dx}$$

$$= 20\times10^5\ \text{r/m} + \frac{20\times10^5}{5\times10^{-6}}x$$

$$E = (20\times10^5 + 4\times10^{11}x)v/m$$

$$\int_{\psi A}^{\psi B} dv = -\int_0^{5\ \mu m} E\,.dx$$

$$\psi_B - \psi_A = -\left[20\times10^5 x + \frac{4\times10^{11}r^2}{2}\right]_0^{5\times10^{-6}}$$

$$= -\left[20\times10^5\times(5\times10^{-6}) + \frac{4\times10^{11}\times(5\times10^{-6})^2}{2}\right]$$

$$\psi_B - \psi_A = -[10+5]$$

$$\psi_B - \psi_A = -15V$$

14. Given $\vec{F} = z\hat{a}_x + x\hat{a}_y + y\hat{a}_z$. If S represents the portion of the sphere $x^2 + y^2 + z^2 = 1$ for$z \geq 0$, then $\int_s \nabla\times\vec{F}\cdot\vec{ds}$ ______. **[2014]**

Solution: (3.14)

$$\vec{F} = za\hat{x} + xa\hat{y} + ya\hat{z}$$

$$\nabla\times\vec{F} = \begin{vmatrix} a\hat{x} & a\hat{y} & a\hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z & x & y \end{vmatrix}$$

$$\vec{A} = \nabla\times\vec{F} = a\hat{x} + a\hat{y} + a\hat{z}$$

$$\int_S (\nabla\times\vec{F}).ds = \int\int \vec{A}.dydz\,ax$$

$$+\int\int \vec{A}.dxdz\,ay + \int\int \vec{A}.dxdy\,az$$

For, $z \geq 0$

$$\int\int_S (\nabla\times\vec{A}).ds = \int\int (a\hat{x} + a\hat{y} + a\hat{z}).(a\hat{z}\,dxdy)$$

$$= \int_S dxdy$$

$$\int\int_S (\nabla\times\vec{A}).ds = 3.14$$

15. If $\vec{E} = -(2y^3 - 3yz^2)$ is the electric $\hat{x} - (6xy^2 - 3xz^2)\hat{y} + (6xyz)\hat{z}$ field in a source free region, a valid expression for the electrostatic potential is

(a) $xy^3 - yz^2$ (b) $2xy^3 - xyz^2$

(c) $y^3 + xyz^2$ (d) $2xy^3 - 3xyz^3$

[2014]

Solution: (d)

$$\vec{E} = -(2y^3 - 3yz^2)\hat{x} - (6xy^2 - 3xz^2)\hat{y} + (6xyz)\hat{z}$$

$$E = -\nabla V$$

For, $V = 2xy^3 - 3xyz^2$

$$\frac{\partial v}{\partial x} = 2y^3 - 3yz^2$$

$$\frac{\partial v}{\partial y} = 6xy^2 - 3xz^2$$

$$\frac{\partial v}{\partial z} = -6xyz$$

which satisfy given E.

Hence, the correct option is (d).

16. The direction of vector A is radially outward from the origin, with $|A| = kr^n$ where $r^2 = x^2 + y^2 + z^2$ and k is a constant. The value of n for which $\nabla.A = 0$ is

(a) –2 (b) 2

(c) 1 (d) 0 **[2012]**

Solution: (a)

$$\nabla.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 Ar)$$

$$+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}A\phi$$

(Spherical coordinates)

Here, $|A| = kr^n$

$\vec{A} = kr^n\,\hat{a}r$ [radically outward]

So, $\nabla.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 kr^n) + 0 + 0$

$$\nabla.\vec{A} \Rightarrow \frac{k}{r^2}\frac{\partial}{\partial r}(r^{n+2})$$

$\nabla.\vec{A} = 0$ only if r^{n+2} = constant

So, $\frac{\partial}{\partial r}(\text{constant}) = 0$

$r^{n+2} = 1$ at $n + 2 = 0$ i.e. $n = -2$

Hence, the correct option is (a).

Common Data for Questions 17 and 18

An infinitely long uniform solid wire of radius a carries a uniform dc current of density $\vec{j}$.

17. The magnetic field at a distance r from the center of the wire is proportional to

(a) r for $r <$ a and $1/r^2$ for r $> a$

(b) 0 for $r <$ a and $1/r$ for r $> a$

(c) r for $r <$ a and $1/r$ for r $> a$

(d) 0 for $r <$ a and $1/r^2$ for r $> a$ **[2012]**

Solution: (c)

For infinitely long uniform wire,

$|B|$ is given as,

$$|B| = \frac{\mu_0 I}{2\pi r}$$

For, $r < a$, $I = J\pi r^2$

So, $|B| = \frac{\mu_0 J_r}{2}$

So, $|B| \propto r$

For $r > aI = J\pi a^2$

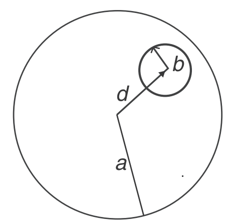
So, $|B| = \frac{\mu_0 J\pi a^2}{2\pi r}$

$$= \frac{\mu_0 Ja^2}{\pi r}$$

$$|B| \propto \frac{1}{r}$$

Hence, the correct option is (c).

18. A hole of radius b(b < a) is now drilled along the length of the wire at a distance d from the center of the wire as shown below.



The magnetic field inside the hole is

(a) uniform and depends only on d

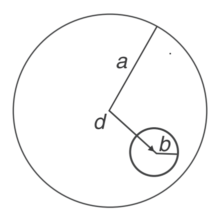
(b) uniform and depends only on b

(c) uniform and depends on both b and d

(d) non uniform **[2012]**

Solution: (c)

The magnetic field inside the hole will be uniform and depend upon both b and d.



Hence, the correct option is (c).

19. The electric and magnetic fields for a TEM wave of frequency 14 GHz in a homogeneous medium of relative permittivity ε_r and relative permeability $\mu_r = 1$are given by

$$E_z = Ee^{j(wt-\beta y)}4\hat{z}$$

$$H = 3e^{j(\omega t - 280\pi y)}u_z \text{ A/m}$$

Assuming the speed of light in free space to be 3×10^4 m/s, intrinsic impedance of free space to be 120π the relative permittivity ε_r of the medium and the electric field amplitude E_p are

(a) $\varepsilon_r = 3$ $E_p = 120\pi$ (b) $\varepsilon_r = 3$ $E_p = 360\pi$
(c) $\varepsilon_r = 9$ $E_p = 360\pi$ (d) $\varepsilon_r = 9$ $E_p = 120\pi$ **[2011]**

Solution: (d)

$$\vec{E} = E_p e^{j(wt-280\pi y)} 4\hat{z} \text{ v/m}$$

$$\vec{H} = 3e^{j(wt-280\pi y)}\overrightarrow{4x} \text{ A/m}$$

$V = 3 \times 10^8$ m/s [Free space]

η [Free space] = 120π

General equation is given as,

$$E_z = Ee^{j(wt-\beta y)}4z$$

$$H_x = He^{j(wt-\beta y)}4x$$

$$\eta = \frac{|E|}{|H|} = \frac{E_p}{3} = \frac{120\pi}{\sqrt{\in r}}$$

$$\beta = \frac{2\pi}{\lambda} = 280\pi \Rightarrow \frac{1}{140} \text{ meter}$$

$$\lambda = \frac{v}{f}$$

$V = \lambda.f$

$$= \frac{1}{140} \times 14 \times 10^9$$

$V = 1 \times 10^8$ m/sec

$$V = \frac{C}{\sqrt{\in r}} \Rightarrow \frac{3\times 10^8}{\sqrt{\in r}} = 1\times 10^8$$

$$\in r = 9$$

So, $E_p = \dfrac{3\times 120\pi}{\sqrt{\in r}}$

$$E_p = 120\pi$$

Hence, the correct option is (d).

20. If a vector field $\oint_C \vec{A}\cdot\vec{dl} = \int\int_{S_C} \vec{V}\cdot\vec{dS}$ is related to another vector field $\vec{A}$ through $\vec{V} = \vec{V}\times\vec{A}$, which of the following is true? Note C and S_C refer to any closed contour and any surface whose boundary is C

(a) $\oint_C \vec{V}\cdot\vec{dl} = \int\int_{S_C} \vec{A}\cdot\vec{dS}$

(b) $\oint_C A.\vec{dl} = \int\int_S \vec{V}.ds$

(c) $\oint_C \nabla\times\vec{V}\cdot\vec{dl} = \int\int_{S_C} \nabla\times\vec{A}\cdot\vec{dS}$

(d) $\oint_C \vec{A}\cdot\vec{dl} = \int\int_{S_C} \vec{V}\cdot\vec{dS}$ **[2009]**

Solution: (b)

$\vec{V} = \nabla\times\vec{A}$ (given)

$\oint_C A.\vec{dl} = \int\int_S (\nabla\times\vec{A})ds$ [using stoke's theorem]

$$\oint_C \nabla\times\vec{A}\cdot\vec{dl} = \int\int_{S_C} \vec{V}\cdot\vec{dS}$$

Hence, the correct option is (b).

21. A magnetic field in air is measured to be

$$B = B_0\left(\frac{x}{x^2+y^2}\hat{y} - \frac{y}{x^2+y^2}\hat{x}\right)$$

What current distribution leads to this field?[Hint: the algebra is trivial in cylindrical coordinates]:

(a) $\vec{J} = -\dfrac{B_0\hat{z}}{\mu_0}\left(\dfrac{1}{x^2+y^2}\right), r \neq 0$

(b) $\vec{J} = -\dfrac{B_0\hat{z}}{\mu_0}\left(\dfrac{2}{x^2+y^2}\right), r \neq 0$

(c) $\vec{J} = 0, r \neq 0$

(d) $\vec{J} = -\dfrac{B_0\hat{z}}{\mu_0}\left(\dfrac{1}{x^2+y^2}\right), r \neq 0$ **[2009]**

Solution: (c)

$$\vec{B} = B_0\left[\frac{x}{x^2+y^2}\hat{y} - \frac{y}{x^2+y^2}\hat{x}\right] \quad (1)$$

Convert rectangular coordinates to cylindrical coordinates.

i.e., $x = \rho\cos\phi$

$$\overrightarrow{ax} = \cos\phi\,\overrightarrow{a\rho} - \sin\phi\,\overrightarrow{a\phi}$$

$$y = \rho\sin\phi$$

$$\overrightarrow{ay} = \sin\phi\,\overrightarrow{a\rho} + \cos\phi\,\overrightarrow{a\phi}$$

$$z = z$$

$$\overrightarrow{az} = \overrightarrow{az}$$

put these values in equation (1)

$$\vec{B} = B_0\overrightarrow{a\phi}$$

$$\vec{H} = \frac{\vec{B}}{\mu}\overrightarrow{a\phi} = \text{constant i.e. } \frac{B_0}{\mu}\overrightarrow{ap}$$

So, $\vec{J} = \nabla\times\vec{H} = \nabla\times\left(\dfrac{B_0}{\mu}\overrightarrow{a\phi}\right)$

$$\vec{J} = 0$$

Hence, the correct option is (c).

22. A parallel plate air-filled capacitor has plate area of 10^{-4} m^2 and plate separation of 10^{-3} *m*. It is connected to a 0.5 *V*, 3.6 GHz source. The magnitude of the displacement current is

($\varepsilon_0 = 1/36\pi\times10^{-9}$F/m)

(a) 10 mA (b) 100 mA
(c) 10A (d) 1.59 mA **[2004]**

Solution: (a)

Displacement current is equal to,

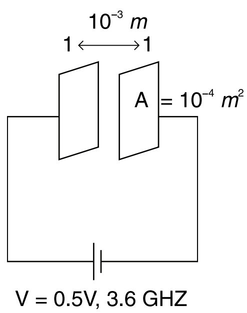
$Id = AJ_d$

A = area of plate

Id = current density

$$J_d = \frac{dD}{dt} = \frac{\in dE}{dt}$$

$E = v/d$



$$|I_d| = A \times 2\pi \times F \times \frac{\in \times V}{d}$$

$$= 10^{-4} \times 2\pi \times 3.6 \times 10^9 \times \frac{1}{36\pi} \times 10^{-9} \times \frac{0.5}{10^{-3}}$$

$$I_d = 10 \text{ mA}$$

Hence, the correct option is (a).

23. If the electric field intensity is given by $E = (xu_x + yu_y + zu_z)$volt/m the potential difference between X (2, 0, 0) and Y (1, 2, 3) is

(a) +1 volt (b) −1 volt
(c) +5 volt (d) +6 volt **[2003]**

Solution: (c)

$E = (xVx + yVy + zVz)$ v/m, x(2, 0 ,0) and y(1, 2, 3)

$$V = -\int E.dl$$

$$= -\left\{\int_1^2 x\,dx\,\overrightarrow{Vx} + \int_2^0 y.dy\,\overrightarrow{Vy} + \int_3^0 z.dz\,\overrightarrow{Vz}\right\}$$

= 5v

Hence, the correct option is (c).

24. The electric field on the surface of a perfect conductor is 2 V/m. The conductor is immersed in water with $\in = 80 . \in_0$ The surface charge density on the conductor is

(a) 0 C/m²
(b) 2 C/m,?
(c) 1.8×10^{-11} C/m²
(d) 1.41×10^{-9}C/m²

($\in = 10^{-9}$)/(36 π)F/m)s **[2002]**

Solution: (d)

$D = \in E$

also, D = ρs [Gauss Law]

$\in E = \rho s = \in r \in o \rho_s$

$$\rho_s = 2 \times 80 \times \frac{1}{36} \times \frac{10^{-9}}{\pi}$$

$$\rho_s = 1.41 \times 10^{-4} \text{ c/m}^2$$

Hence, the correct option is (d).

25. Match List-I with List-II and select the correct answer using the code given below the Lists:

List-I

A. $\nabla.\vec{H} = \vec{J}$

B. $\int_C \vec{E}.\vec{dI} = -\int_S B.\vec{ds}$

C. $\nabla \cdot \vec{J} = \frac{\partial \rho}{\partial t}$

List-ll

1. Continuity equation
2. Faraday's Law
3. Ampere's Law
4. Gauss's Law
5. Biot-Savart Law

Codes:

	A	B	C
(a)	3	2	1
(b)	2	1	3
(c)	2	3	1
(d)	1	2	3

[1994]

Solution: (a)

(A) $\nabla.\vec{H} = J \rightarrow$ Ampere Law

(B) $\oint_C E.\vec{dl} = -\oint_S B.\vec{ds}$

$$\nabla \times \vec{E} = -\frac{dB}{dt} \rightarrow \text{Faraday Law}$$

(C) $\nabla \cdot \vec{J} = \frac{\partial \rho}{\partial t}$ continuity equation.

Hence, $A-3$, $B-2$, $C-1$
Hence, the correct option is (a).

26. Given $V = x\cos^2 y\hat{i} + x^2e^2\hat{j} + z\sin^2 y\hat{k}$ and S the surface of unit cube with one corner at the origin and edges parallel to the coordinate axis, the value of the integral $\iint_c \vec{V}.\hat{n}dS$ is_____. **[1993]**

Solution: $V = \int\int \vec{V}.\vec{n}\,ds = \frac{1}{2}\sin(2)$

We have given, $V = x\cos^2 yi + x^2e^z j + z\sin^2 yk$ can be written as,

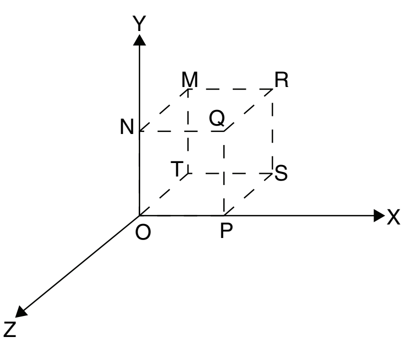
$V = x\cos^2 \overrightarrow{ax} + x^2e^z\overrightarrow{ay} + z\sin^2\overrightarrow{az}$

Now, $\int\int_C \vec{V}.\vec{n}\,ds$ will be equal to,

$$\int_{z1}^{z2}\int_{y1}^{y2} \vec{V}_x\, dydz\,\overrightarrow{ax} + \int_{z1}^{z2}\int_{x1}^{x2} \vec{V}_y\, dxdz\,\overrightarrow{ay} + \int_{y1}^{y2}\int_{x1}^{x2} \vec{V}_z\, dxdy\,\overrightarrow{az}$$

Here, $\vec{V}x = x\cos^2 y$

$\vec{V}y = x^2e_z$



$$A = \int_{z1}^{z2}\int_{y1}^{y2} \overrightarrow{Vx}\, dydz\,\overrightarrow{ax}$$

at, $x = 0$, $A_1 = 0$.

At $x = 1$, $A_2 = \int_0^{-1}\int_0^1 \cos^2 y\, dydz$

$$A_2 = +\frac{1}{2}\left[\frac{1+\sin(2)}{2}\right]$$

So, $A = \frac{1}{2}\left[\frac{1+\sin(2)}{2}\right]$ (1)

$B = \int_{z1}^{z2}\int_{x1}^{x2} \overrightarrow{Vy}\, dxdz\,\overrightarrow{ay}$ aty $= 0$,

$$B_1 = \int_0^{-1}\int_0^1 x^2e^z\, dxdy$$

$$B_1 = \frac{1}{3}[e-1]$$

at, y = 1, $B_2 = -\int_0^{-1}\int_0^1 x^2e^z\, dxdy$

$$B_2 = -\frac{1}{3}[e-1]$$

$B = B_1 + B_2 = 0$ (2)

$$C = \int_{y1}^{y2}\int_{x1}^{x2} \vec{V}_z\, dxdy\,\overrightarrow{az}$$

at, $z = 0$, $C_1 = \int_0^1\int_0^1 0.dxdy = 0$

at, zF $-$ 1, $C_2 = -\int_0^1\int_0^1 \sin^2 y\, dxdy$

$$C_2 = -\frac{1}{2}\left[1-\frac{\sin(2)}{2}\right]$$

$C = C_1 + C_2 = -\frac{1}{2}\left[1-\frac{\sin(2)}{2}\right]$ (3)

Added up equations (1), (2), (3)

$V = A + B + C$

$$= \frac{1}{2}\left[1+\frac{\sin(2)}{2}\right] + 0 - \frac{1}{2}\left[1-\frac{\sin(2)}{2}\right]$$

$$V = \int\int \vec{V}.\vec{n}\,ds = \frac{1}{2}\sin(2)$$

27. For a uniformly charged sphere of radius R and charge density ρ, the ratio of magnitude of electric fields at distances R/2 and 2R from the centre, i.e.,

$\frac{E(r = R/2)}{E(r = 2R)}$ is _____ **[1993]**

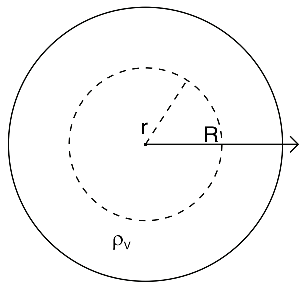
Solution: (2)

[For $r < R$]

$\oint D.ds = 2$

$D\,.\,S = 2$

$$D = \frac{\rho_v \times \frac{4}{3}\pi r^3}{4\pi r^2} = \frac{\rho_v .r}{3}$$



$$E = \frac{D}{t} = \frac{\rho_{v.r}}{3E}$$

Here, $r = \frac{R}{2}$

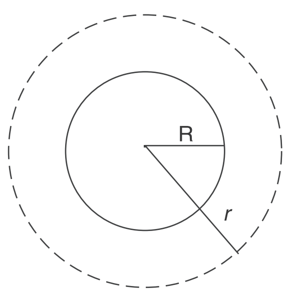
$$E = \frac{\rho_{v.R}}{6E}$$

(For r>R] $D = \dfrac{\rho_v \times \frac{4}{3}\pi R^3}{4\pi R^2} = \dfrac{\rho_v R^3}{3r^2}$

$$E = \frac{\rho_v R^3}{3Er^2}$$

Here, $r = 2R$

$$E = \frac{\rho_v R}{12t}$$



So, $\dfrac{E[r = R/2]}{E[r = 2R]} = 2$

28. A long solenoid of radius R, and having N turns power unit length carries a time dependent current $I(t) = I_0 \cos(\omega t)$. The magnitude of induced electric field at a distance $R/2$ radially from the axis of the solenoid is

(a) $\frac{R}{2}\mu_0 N I_0 \omega \sin(\omega t)$

(b) $\frac{R}{4}\mu_0 N I_0 \omega \cos(\omega t)$

(c) $\frac{R}{4}\mu_0 N I_0 \omega \cos(\omega t)$

(d) $R\mu_0 N I_0 \omega \sin(\omega t)$ **[1993]**

Solution: (c)

$I(t) = I_0 \cos(wt)$

We know, $H = NI$

So, $H = NI_0 \cos(wt)$

$B = H_0 H$

$B = \mu_0 N I_0 \cos(wt)$

According to maxwell second equation,

$$\oint E.\vec{dl} = -\int \frac{\vec{dB}}{dt}.\vec{ds}$$

So, $E.2\pi \dfrac{R}{2} = \mu_0 N I_0 \dfrac{d}{dt}[\cos(wt)] \times \dfrac{\pi R^2}{y}$

$$E = \frac{\mu_0 N I_0 w R \sin w(t)}{y}$$

Hence, the correct option is (c).

29. The electric field strength at a far-off point P due to a point charge, $+q$ located at the origin, O is 100 millivolts/meter. The point charge is now enclosed by a perfectly conducting hollow metal sphere with its centre and the origin, O. The electric field strength at the point, P

(a) remains unchanged in its magnitude and direction.
(b) remains unchanged in its magnitude but reverse in direction.
(c) would be that due to a dipole formed by the charge, $+q$, at O and $-q$ induced.
(d) would be zero. **[1989]**

Solution: (d)

According to Gauss Law:—The total displacement or electric flux through any closed surface surrounding charges is equal to the amount of charge enclosed.

$$\oint D.ds = 2$$

Here, total enclosed charge is, $-q + q = 0$ thus, $D = 0$

also, $E = \dfrac{D}{E} = 0$

Hence, the correct option is (d).

30. Which of the following field equations indicate that the free magnetic charge do not exist

(a) $H = \frac{1}{\mu}\nabla \times A$

(b) $H = \oint \frac{1 dI \times R}{4\pi R^2}$

(c) $\nabla.H = 0$

(d) $\nabla \times H = J$ **[1990]**

Solution: (c)

As we know, $\oint B.ds = 0$ or, $\nabla.B = 0$ [Solenoidal in nature] B = μ*H* thus, $\nabla.(\mu H) = 0$

$\nabla.H = 0$

Hence, the correct option is (c).

31. Vector potential is a vector
(a) whose curl is equal to the magnetic flux density
(b) whose curl is equal to the electric field intensity
(c) whose divergence is equal to the electric potential
(d) which is equal to the vector product E × H **[1988]**

Solution: (a)

The term vector potential physical stands for work done per basic cause (i.e. current element in magnetic field). Hence, $\vec{A} = \frac{w}{I.\vec{dl}}$

is vector so, potential is a vector. Whose curl will be equal to, $\nabla \times \vec{A} = \vec{B}$ also,

$\nabla \times \vec{A} = \mu\vec{H}$

Hence, the correct option is (a).

32. On either side of a charge-free interface between two media
(a) the normal components of the electric field are equal
(b) the tangential component of the electric field are equal
(c) the normal components of the electric flux density are equal
(d) the tangential components of the electric flux density are equal **[1988]**

Solution: (b) and (c)

Conditions at Boundary surfaces.
(a) $Et_1 = Et_2$ (b) $Ht_1 = Ht_2$
(c) $Bn_1 = Bn_2$ (d) $Dn_1 = Dn_2$
Hence, the correct option is (b) and (c).

Five-marks Questions

1. Given that $\vec{D} = r^2\vec{a}_r + 2Sin\theta\vec{a}_\theta$ in spherical coordinate system, where *D* is the electric flux density, find the charge density *p*? **[1987]**

Solution: Given, $\vec{D} = r^2 a_r + 2\sin\theta a_\theta$ and

$$\vec{D} = D_r a^{-1} r + D\vec{a}_\theta + D_\phi \vec{a}_\phi$$

So,

$Dr = r^2$

$D\vartheta = 2 \sin \vartheta$

Dø = 0

and,

$$P_V = \nabla.\vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 Dr) + \frac{1}{r\sin\theta}.\frac{\partial}{\partial\theta}(D_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}$$

$$= \frac{1}{r^2}4r^3 + \frac{2}{r\sin\theta}2\sin\theta.\cos\theta$$

$$= 4r + \frac{4}{r}\cos\theta$$

$$= 4\left[r + \frac{1}{r}\cos\theta\right]$$

2. Evaluate the integral, $\int \vec{r}\cdot\vec{dr}$, where *C* is the helical path described by, *x* = ϲost, *y* = sint, *z* = *t*, joining the points given by *t* = 0 and *t* = π/2 **[1994]**

Solution: In Cartesian coordinates,

$\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$

$d\vec{r} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$

$$I = \int_C \vec{r}.d\vec{r} = \int_C (xdx + ydx + zdz) = \frac{1}{2}\left[x^2 + y^2 + z^2\right]_P^Q$$

with, *t* = *OP* $(y_1 y_1 z)$ = (1, 0, 0)

with, $t = \frac{\pi}{2} Q(x, y, z) = (0, 1, \pi/2)$

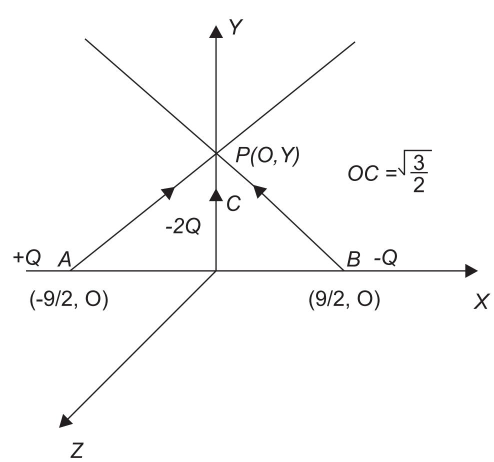
$$I = \frac{1}{2}\left[x^2\right]_1^a + \frac{1}{2}\left[y^2\right]_0^1 + \frac{1}{2}\left[z^2\right]_0^{\pi/2} = \frac{\pi^2}{8}$$

3. Three electrostatic point charges are located in the *xy*-plane as given below:

+Q at (–*a*/2, 0), + Q at (*a*/2, 0) and
– 02Q at (0, > /3/2)

Calculate the coordinate of the point, *P*, on the *y*-axis, where the potential due to these charges is zero. Also, calculate the magnitude of the electric field strength at *P*. At the point, *P* what is the angle between the equipotential passing through *P* and the *y*-axis? **[1995]**

Solution:



point, charge location $+QA\left(\frac{-9}{2},0\right)$, $+QB\left(\frac{9}{5},0\right)$ and $-2Q\,C\left(0,\frac{\sqrt{3}}{2}a\right)$

(i) Let, $P(a, y)$ be the point on y-axis, where $V = V_1 + V_2 + V_3$ due to these change is zero, $V = V_1 + V_2 + V_3 = 0$

$$\frac{Q}{4\pi E_0}\left[\frac{1}{AP}+\frac{1}{BP}-\frac{2}{CP}\right]=0$$

But $AB = BP$, so,

$$\frac{1}{AP}+\frac{1}{AP}-\frac{2}{CP} \text{ or } AP = CP$$

$$\sqrt{\frac{a^2}{4}+y^2} = y-\frac{\sqrt{3}}{2}a, \Rightarrow y=\frac{a}{2\sqrt{3}}$$

$$P(a,y)=P\left(0,\frac{a}{2\sqrt{3}}\right)$$

(ii) $\underline{V}$ at point $P(0, y)$

$$V=\frac{Q}{4\pi E_0}\left[\frac{1}{AP}+\frac{1}{BP}-\frac{2}{CP}\right]$$

$$=\frac{Q}{2\pi E_0}\left[\frac{1}{AP}-\frac{1}{CP}\right]$$

$$V=\frac{Q}{2\pi E_0}\left[\frac{1}{\sqrt{\frac{a^2}{4}+y^2}}-\frac{1}{\sqrt{y-\frac{\sqrt{3}}{2}a}}\right]$$

$$\vec{E}=-\nabla V=-\left[\frac{\partial}{\partial x}V\hat{a}_x+\frac{\partial}{\partial y}V\hat{a}_y+\frac{\partial}{\partial z}V\hat{a}_z\right]$$

$$\frac{\partial V}{\partial x}=0$$

$$\frac{\partial V}{\partial z}=0$$

Solving

$$\frac{\partial v}{\partial y}=\frac{Q}{2\pi E_0}\left[\begin{array}{c}-\frac{1}{2}\left(\frac{a^2}{4}+y^2\right)^{-3/2} \\ (2y)+\left(y-\frac{\sqrt{3}}{2}a\right)^{-2}\end{array}\right]$$

$$\vec{E}=\frac{-Q}{2\pi E_0}\left[\begin{array}{c}-y\left(\frac{a^2}{4}+y^2\right)^{-3/2} \\ +\left(y-\frac{\sqrt{3}}{2}a\right)^{-2}\end{array}\right]\vec{a}_y$$

At $P(a, y) = P\left(o,\frac{a}{2\sqrt{3}}\right)$

$$\vec{E}=\frac{-Q}{2\pi E_0}\left[\begin{array}{c}\frac{-a}{2\sqrt{3}}\left(\frac{a^2}{4}+\frac{a^2}{12}\right)^{-3/2} \\ +\left(\frac{a}{2\sqrt{3}}-\frac{\sqrt{3}}{2}a\right)^{-2}\end{array}\right]\vec{a}_x$$

$$E=E_y=\frac{3Q}{4\pi E_0 a^2}$$

(iii) The direction of $\vec{E}$ at P is the direction of the normal to the equipotential surface ($V = 0$) at that points in the direction of the decreasing values of V. The direction of $\vec{E}$ is in the *–ve y* direction, the angle between the equipotential surface and y-axis is zero.

4. Given an irrotational vector field

$\vec{\mathrm{F}}=(k_1xy+k_2z^3)\vec{a}_x+$
$(3x^2-k_3z)\vec{a}_y+(3xz^2-y)\vec{a}_z$

Find $\nabla\,\vec{\mathrm{F}}$ at (1, 1, – 2). **[1998]**

Solution: Given

$$\vec{F}=(k_1xy+k_2z^3)\hat{a}_x+(3x^2-k_3z)\hat{a}_y+(3yz^2-y)\hat{a}_a$$

$$\nabla.F=\frac{\partial}{\partial y}F_x+\frac{\partial}{\partial y}F_y+\frac{\partial}{\partial z}F_z$$

$$= k_1y + 0 + 6xz = k_1y + 6xz$$

at (1, 1, −2)

$$\nabla.\vec{F}=k_1(1)+6(1)(-2)=k_1-12.$$

5. Given E= $10e^{-j(4x-kt)}$ – V/m in free space.

(a) Write all the four Maxwell's equations in free space.
(b) Find $\Delta \times$ E.
(c) Find H. **[2000]**

Solution: Given, $E=10e^{-4j(4x-kt)}a_y V/m$

represents uniform plane wave travelling in x direction with velocity $v - k/4$ and $E_x = 0$, $E_2 = 0$

So,

$$\frac{\partial}{\partial y}E_y=-j40e^{-j(4x-kt)}$$

$$\nabla\times\vec{E}=\begin{vmatrix}a_x & a_y & a_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ E_x & E_y & E_z\end{vmatrix}=\begin{vmatrix}a_x & a_y & a_z\\ \frac{\partial}{\partial y} & 0 & 0\\ 0 & E_y & 0\end{vmatrix}$$

$$\nabla \times \vec{E} = \vec{a}_x(0) - a_y(0) + a_2 \cdot \frac{\partial}{\partial y} E_y$$

$$= j40e^{-j(4x-kt)} = 40e^{j(4x-kt+90)}$$

(c) For uniform plane wave, $\eta = \dfrac{E}{H}$, $\quad E = E_y \hat{a}_y$

Since, wave is travelling in x direction so, the direction of $\vec{H}$ is in z-direction.

$\hat{y} + \hat{z} = \hat{x}$

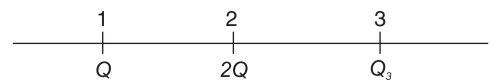
$$\eta = \eta_0 = 120\pi = \frac{E_y}{Az}$$

$$H_2 = \frac{E_y}{120\pi} = \frac{10}{120\pi} e^{-j(4x-kt)}$$

$$\vec{H} = H_2 \vec{a}_z = \frac{1}{12\pi} e^{-j(4x-kt)} \vec{a}_2$$

6. A system of three electric charges lying in a straight line is in equilibrium. Two of the charges are positive with magnitudes Q and $2Q$, and are 50 cm apart. Determine the sign, magnitude and position of the third charge. **[2001]**

Solution:



Let Q_3 be the third change located at the distance x.

From Q

$$V_1 = \frac{2Q}{4\pi E(50)} + \frac{Q_3}{4\pi E(x)} \tag{1}$$

$$V_2 = \frac{Q}{4\pi E(50)} + \frac{Q_3}{4\pi E(x-50)} \tag{2}$$

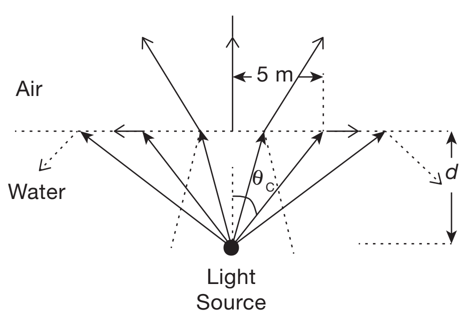
Chapter 2

Uniform Plane Waves

One-mark Questions

1. The permittivity of water at optical frequencies is 1.75 ε_0. It is found that an isotropic light source at a distance d under water forms an illuminated circular area of radius 5 m, as shown in the figure. The critical angle is θ_c.

The value of d (in meter) is ________________. **[2017]**



Solution:

$$\theta_C = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left[\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right] = 49.1°$$

$$\tan 49.1 = \frac{5}{d} = 1.547$$

$$\Rightarrow \quad d = 4.33 \text{ m}$$

Hence, the correct answer is (4.30 to 4.36).

2. Let the electric field vector of a plane electromagnetic wave propagating in a homogeneous medium be expressed as $E = \hat{x}E_x e^{-j(\omega t - \beta z)}$ where the propagation constant β is a function of the angular frequency ω. Assume that β(ω) and E_x are known and are real. From the information available, which one of the following CANNOT be determined? **[2016]**

(A) The type of polarization of the wave
(B) The group velocity of the wave
(C) The phase velocity of the wave
(D) The power flux through the $z = 0$ plane

Solution: To find out the power through $z = 0$ plane, the plane area, medium intrinsic impedance $\eta\left(=\sqrt{\frac{\mu}{\varepsilon}}\right)$ are required.

Hence, the correct option is (D).

3. If a right-handed circularly polarized wave is incident normally on a plane perfect conductor, then the reflected wave will be **[2016]**

(A) right-handed circularly polarized
(B) left-handed circularly polarized
(C) elliptically polarized with a tilt angle of 45°
(D) horizontally polarized

Solution: When a right hand circularly polarized wave is incident normally on a plane perfect conductor, i.e., σ = ∞ then orientation changes by 180° at any of the two fields present. Therefore, option (B) is correct

Hence, the correct option is (B).

4. In the electric field component of a plane wave travelling in a lossless dielectric medium is given by $\vec{E}(z, t) = \hat{a}_y 2\cos\left(10^8 t - \frac{z}{\sqrt{2}}\right)$ V/m. The wavelength (in m) for the wave is _______ **[2015]**

Solution: Given equation

$$\vec{E}(z, t) = \hat{a}_y 2\cos\left(10^8 t - \frac{z}{\sqrt{2}}\right) \text{ V/m}$$

$$\beta = \frac{1}{\sqrt{2}}$$

$$\omega = 10^8$$

$$\beta = \frac{2\pi}{\lambda} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \lambda = 2\sqrt{2}\pi$$

$$= 8.88\text{m}$$

Hence, the correct Answer is (8.85 to 8.92).

5. The electric field of a uniform plane electromagnetic wave is

$$\vec{E} = \left(\vec{a}_x + j4\,\vec{a}_y\right) \exp\,[j(2\pi \times 10^7 t - 0.2z)]$$

The polarization of the wave is **[2015]**

(A) right handed circular
(B) right handed elliptical
(C) left handed circular
(D) left handed elliptical

Solution: From the given equation

$$|E_x| \neq |E_y|$$

So it is elliptically polarized wave

Generalized expression for left elliptically polarized electric field is

$$E = \left(E_x\,\hat{a}_x + E_y \hat{a}_y\right) e^{j(\omega t - \beta z)}$$

Hence, the correct option is (D).

6. A coaxial-cable with an inner diameter of 1 mm and outer diameter of 2.4 mm is filled with a dielectric of relative permittivity 10.89. Given $\mu_0 = 4\pi\ 10^{-7}$H/m, $\varepsilon_0 = \dfrac{10^{-9}}{36\pi}$ F/m the characteristic impedance of the cable as

(a) 330 Ω (b) 16 Ω
(c) 143.3 Ω (d) 43.4 Ω **[2012]**

Solution: (b)

$$z_0 = \frac{136}{\sqrt{\epsilon_r}} \log \frac{D}{d}$$

$\in_r = 10.89$, $D = 2.4$ mm, $d = 1$ mm

$$z_0 = \frac{188}{\sqrt{10.89}} \log\left(\frac{2.4}{1}\right)$$

$Z_0 = 15.89\ \Omega$

Hence, the correct option is (b)

7. The electric field of a uniform plane electromagnetic wave in free space, along the positive x direction, is given by $\vec{E} = 10(a_y + ja_z)e^{-j25x}$. The frequency and polarization of the wave, respectively, are

(a) 1.2 GHz and left circular
(b) 4 Hz and left circular
(c) 1.2 GHz and right circular
(d) 4 GHz and right circular **[2012]**

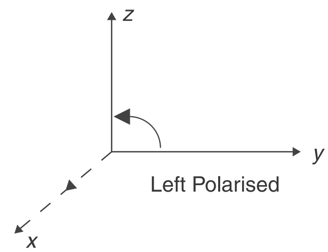
Solution: (a)

$$\vec{E} = 10e^{-j25x} a\hat{y} + j10e^{-j25x} a\hat{z}$$

$$E_y = 10e^{-j25x} a\hat{y}$$

$$E_z = 10e^{(-j25x + \frac{\pi}{2})} a\hat{z}$$

Both component are of equal magnitude with phase difference 90°.



So, polarization of the wave is left circular.

$$\beta = \frac{2\pi}{\lambda} = 25$$

$$\lambda = \frac{2\pi}{25}$$

$$f = \frac{C}{\lambda} = \frac{3 \times 10^8 \times 25}{2\pi}$$

$f = 1.2$ GHz

Hence, the correct option is (a)

8. A plane wave propagating in air with $(-8\hat{a}_x + 6\hat{a}_y - 5\hat{a}_z)e^{j(\omega t - 3x - 4y)}\ V/m$ is incident on a perfectly conducting slab positioned at $x \leq 0$ the field of the reflected waves is

(a) $(-8\hat{a}_x - 6\hat{a}_y - 5\hat{a}_z)e^{j(\omega t + 3x + 4y)}\ V/m$

(b) $(-8\hat{a}_x - 6\hat{a}_y - 5\hat{a}_z)e^{j(\omega t + 3x + 4y)}\ V/m$

(c) $(-8\hat{a}_x - 6\hat{a}_y - 5\hat{a}_z)e^{j(\omega t + 3x + 4y)}\ V/m$

(d) $(-8\hat{a}_x - 6\hat{a}_y - 5\hat{a}_z)e^{j(\omega t + 3x + 4y)}\ V/m$ **[2012]**

Solution: (c)

$$\vec{E}_i = (8a\hat{x} + 6a\hat{y} + 5a\hat{z})e^{j(\omega t + 3x + 4y)} \quad v/m$$

For perfectly conducting slab, $E_t = 0$

$E_i = -E_r$

$\Gamma_x = -1$

Direction of propagation of reflected wave for normal components reversed.

$\vec{E}_{n_r} = -x$ direction

So, $\vec{E}_r = -\vec{E}_i$ [with $-x$ direction]

$$\vec{E}_i = (8a\hat{x} + 6a\hat{y} + 5a\hat{z})e^{j(wt + 3x + 4y)} \quad v/m$$

Hence, the correct option is (c)

9. Consider the following statements regarding the complex Poynting vector $\vec{P}$ for the power radiated by a point source in an infinite homogenous and lossless medium.

Re ($\vec{P}$) denotes the real part of, S denotes a spherical surface whose centre is at the point source, and $\hat{n}$ denotes the unit surface normal on S. Which of the following statements is TRUE?

(a) Re ($\vec{P}$) remains constant at any radial distance from the source

(b) Re ($\vec{P}$). increases with increasing radial distance from the source

(c) $\oiint_s Re(\vec{P}).\hat{n}\,dS$ remains constant at any radial distance from the source

(d) $\oiint_s Re(\vec{P}).\hat{n}\,dS$ decreases with increasing radial distance from the source **[2011]**

Solution: (c)

$\oiint_s R_e(\vec{p}).\hat{n}\,ds$ remains constant at any radial distance from the source.

Hence, the correct option is (c)

10. The electric field component of a time harmonic plane EM wave travelling in a nonmagnetic lossless dielectric medium has amplitude of 1 V/m. If the relative has permittivity of the medium is 4, the ≤≤ magnitude ofthe time average power density vector (in W/m^2) is

(a) $\dfrac{1}{30\pi}$ (b) $\dfrac{1}{60\pi}$

(c) $\dfrac{1}{120\pi}$ (d) $\dfrac{1}{240\pi}$ **[2010]**

Solution: (c)

$$|E| = |V|m$$

$$\in_r = 4$$

$$P_{av} = \frac{E^2}{2\eta}$$

$$\eta = \frac{120\pi}{\sqrt{\in_r}} = 60\pi\ \Omega$$

$$P_{avg} = \frac{(1)^2}{2\times 60\pi} = \frac{1}{120\pi}\text{ w/m}^2$$

Hence, the correct option is (c)

11. A plane wave of wavelength λ is travelling in a direction making an angle 30° with positive x-axis and 90° with positive y-axis The E field of the plane wave can be represented as (E_0: s a constant)

(a) $\vec{E} = \hat{y}E_0 e^{i\left(\omega t - \frac{\pi}{\lambda}x - \frac{\sqrt{3}\pi}{\lambda}z\right)}$

(b) $\vec{E} = \hat{y}E_0 e^{i\left(\omega t - \frac{\pi}{\lambda}x + \frac{\sqrt{3}\pi}{\lambda}z\right)}$

(c) $\vec{E} = \hat{y}E_0 e^{i\left(\pi t + \frac{\sqrt{3}\pi}{\lambda}x + \frac{\pi}{\lambda}z\right)}$

(d) $\vec{E} = \hat{y}E_0 e^{i\left(\omega t - \frac{\sqrt{3}\pi}{\lambda}x - \frac{\pi}{\lambda}z\right)}$ **[2007]**

Solution: (a)

$$\tan 30° = \frac{\beta z}{\beta x} = \frac{1}{\sqrt{3}}$$

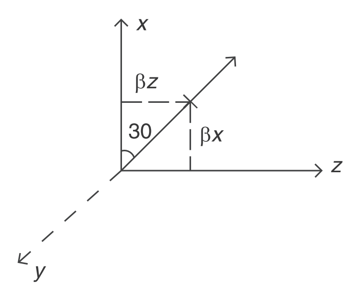
$$\beta x = \sqrt{3}\beta z$$

$$\beta = \frac{2\pi}{w} = \sqrt{\left(\frac{\sqrt{3}\pi}{\lambda}\right)^2 + \left(\frac{\pi}{\lambda}\right)^2}$$

$$= \sqrt{(Bx)^2 + (Bz)^2}$$

$$E(x,z,t) = E_0 e^{-j\beta_x x} e^{-j\beta_z . z} e^{jwt}\hat{y}$$

$$E(x,z,t) = \hat{y}E_0 e^{j}\left(wt - \frac{\sqrt{3}\pi}{\lambda}x - \frac{\pi}{\lambda}z\right)$$



Hence, the correct option is (a)

12. The electric field of an electromagnetic wave propagating in the positive z-direction is given by

$$E = \hat{a}_x \sin(\omega t - \beta z) + \hat{a}_y \sin(\omega t - \beta z + \pi/2)$$

The wave is

(a) linearly polarized in the z-direction

(b) elliptically polarized

(c) left-hand circularly polarized

(d) right-hand circularly polarized **[2006]**

Solution: (c)

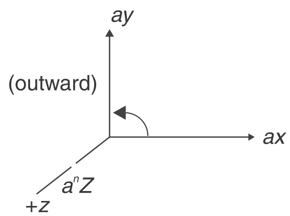
$$E = a\hat{x}\sin(wt - \beta z) + a\hat{y}\sin(wt - \beta z + \pi/2)$$

$$E_x = a\hat{x}\sin(wt - \beta z)$$

$$E_y = a\hat{y}\sin(wt - \beta z + \pi/2)$$

E_x and E_y both are out of phase by 90° with equal magnitude and, $a\hat{x} + a\hat{y} = a\hat{z}$

So, left circularly polarized



Hence, the correct option is (c)

13. The magnetic field intensity vector of a plane wave is given by

$R(x, t) = 10 \sin(50000t + 0\ 004x + 30)\ \hat{a}_y$, where $\hat{a}_y$ denotes the unit vector in y direction'. The wave is propagating with a phase velocity

(a) 5×10^4 m/s (b) 3×10^3 m/s
(c) 1.25×10^7 m/s (d) 3×106 m/s **[2005]**

Solution: (c)

$$\vec{H}(x,t) = 10\sin(50000t + 0.004x + 30)a\hat{y}$$

$w = 50{,}000$

$\beta = -0.004$

$V_p = -1.25 \times 10^7$ m/s

$$V_p = \frac{w}{\beta} = \text{phase velocity}$$

$$= \frac{5\times10^4}{-4\times10^{-3}}$$

$V_p = -125 \times 10^7$m/s

Hence, the correct option is (c)

14. The depth of penetration of electromagnetic. wave in a medium having conductivity σ at a frequency of 1 MHz is 25 *cm*. The depth of penetration at a frequency of 4 MHz will be

(a) 6.25 cm (b) 12 50 cm
(c) 50.00 cm (d) 100.00 cm **[2003]**

Solution: (b)

Hence, the correct option is (b)

15. If a plane electromagnetic wave satisfies the equation $\frac{\partial^2 E}{\partial Z^2} = \sigma^2 \frac{\partial^2 E}{\partial t^2}$ the wave propagates in the

(a) x-direction
(b) z-direction
(c) y-direction
(d) xy plane at an angle of 45° between the x and z directions **[2001]**

Solution:(b)

For the wave equation $\frac{\partial^2 E_x}{\partial z^2} = C^2 \frac{\partial^2 E_x}{\partial t^2}$,

the wave will propagates in the z-direction.

Hence, the correct option is (b)

16. Identify which one of the following will NOT satisfy the wave equation.

(a) $50e^{i(\omega t - 3z)}$
(b) $\sin[\omega(10z + 5t)]$
(c) $\cos(y^2 + 5t)$
(d) $\sin(x)\cos(t)$ **[1999]**

Solution: (c)

$\cos(y^2 + 5t)$ does not satisfy wave equation.

Hence, the correct option is (c)

17. The intrinsic impedance of copper at high frequency is

(a) purely resistive
(b) purely inductive
(c) complex with a capacitive component
(d) complex with an inductive component **[1998]**

Solution: (d)

Intrinsic impedance, $\eta = \sqrt{\frac{jw\mu}{\sigma + jw \in}}$

For copper, $\sigma >> wt$

Hence, the correct option is (d)

18. The wavelength of a wave with propagation constant $(0.1\pi + j0.\ 2\pi)\text{m}^{-1}$is

(a) $\frac{2}{\sqrt{0.05}}\ m$ (b $10\ m$
(c) $20\ m$ (d) $30\ m$ **[1998]**

Solution: (b)

Propagation constant of a wave is,

$\chi = \alpha + j\beta$

given, $\chi = 0.1\pi + j0.2\pi$

$$\beta = \frac{2\pi}{\lambda} = 0.2\pi$$

$\lambda = 10$ m

Hence, the correct option is (b)

19. The depth of penetration of a wave in a lossy dielectric increases with increasing

(a) conductivity (b) permeability
(c) wavelength (d) permittivity **[1998]**

Solution: (c)

Depth of penetration or skin depth for lossy dielectric is,

$$\vec{E} = E_0 e^{f(wt\text{-}bz)}(a\hat{x} + a\hat{y})$$

$$\delta = \frac{1}{\sqrt{f}}$$

$$\delta \propto \sqrt{\lambda}$$

Hence, the correct option is (c)

20. The polarization of a wave with electric field vector $\vec{E} = E_0 e^{j(t-†z)}(\vec{a}_x + \vec{a}_y)$ is

(a) linear
(b) elliptical
(c) left hand circular
(d) right hand circular **[1998]**

Solution: (a)

$$\vec{E} = E_0 e^{f(wt-\beta z)}(ax + ay)$$

the phase difference between E_x and E_y component is zero

$\therefore$ the polarization of a wave is linear
Hence, the correct option is (a)

21. The time averaged Poynting vector, in W/m^2,for a wave with $\vec{E} = 24e^{j(\omega t-\beta z)}\vec{a}_y$ V/m in free space is

(a) $-\frac{2.4}{\pi}\vec{a}$ (b) $\frac{2.4}{\pi}\vec{a}_z$

(c) $\frac{4.8}{\pi}\vec{a}_z$ (d) $-\frac{4.8}{\pi}\vec{a}_z$ **[1998]**

Solution: (a)

$$P_{avg} = \frac{1}{2\eta}|E|^2 = -\overrightarrow{az}$$

Direction of power = $-\overrightarrow{az}$ since, wave is travelling in $-\overrightarrow{az}$ direction

$\vec{E} = 24e^{j(wt+\beta z)}ay$ v/m

$|E| = 24$

$\eta = 120\pi$ ohm

$$P_{avg} = \frac{(24)^2}{2\times 120\pi}(-\overrightarrow{az})$$

$$P_{avg} = -\frac{2.4}{\pi}\overrightarrow{az}\ w/m^2$$

Hence, the correct option is (a)

22. A loop is rotating about the y-axis in a magnetic field $B = B_0 \cos(\omega t + \phi)\hat{a}\ T$. The voltage in the loop is

(a) zero
(b) due to rotation only
(c) due to transformer action only
(d) due to both rotation and transformer action

[1998]

Solution: (d)

The voltage in the loop is due to two reactions.

(i) Time varying magnetic field is $V_B = -\int_C \frac{\overrightarrow{dB}}{dt}.\overrightarrow{ds}$

(ii) Voltage induced in a loop moving with velocity 'v' in steady magnetic field.

$$V_m = \oint_L (\vec{V}\times\vec{B})\overrightarrow{dl}$$

$$V_{total} = V_m + V_B$$

Hence, the correct option is (d)

23. The intrinsic impedance of a lossy dielectric medium is given by

Icon

(a) $\frac{j\omega\mu}{\sigma}$ (b) $\frac{j\omega\in}{\mu}$

(c) $\sqrt{\frac{j\omega\mu}{(\sigma+\omega\in)}}$ (d) $\sqrt{\frac{\mu}{\in}}$ **[1995]**

Solution: (c)

The intrinsic impedance of a lossy dielectric medium is given as,

$$\eta = \sqrt{\frac{jw\mu}{\sigma + jw\in}}$$

Hence, the correct option is (c)

24. Copper behaves as a

(a) conductor always
(b) conductor or dielectric depending on the applied electric field strength
(c) conductor or dielectric depending on the frequency
(d) conductor or dielectric depending on the elec tric current density **[1995]**

Solution: (a)

Loss tangent factor for copper is $\frac{\sigma}{wt} >> 1$ as, σ for copper = 5.8 × 10^7 mho/m

$\in$ = 8.85 × 10^{-12} F/m

$$\eta = \sqrt{\frac{jw\mu}{\sigma}} = \sqrt{\frac{w\mu}{\sigma}}e^{j\pi/4} = \sqrt{\frac{w\pi}{2\sigma}} + j\sqrt{\frac{w\mu}{2\sigma}}$$

Thus, copper behaves as a conductor always.
Hence, the correct option is (a)

25. A plane electromagnetic wave traveling along the +zdirection, has its electric field given by $E_x = 2\cos(\omega t)$ and $E_y = 2\cos(\omega t + 90)°$ the wave is

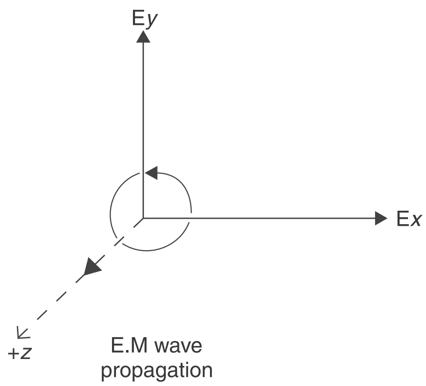
(a) linearly polarized
(b) right circularly polarized
(c) left circularly polarized
(d) elliptically polarized **[1994]**

Solution: (a)

$E_x = 2 \cos wt$

$E_y = 2 \cos (wt + 90°)$

Since E.M wave has two planar components of E-field both out of phase by 90° with equal amplitude so, wave is circularly polarized.



Also from figure, we can say wave is left circularly polarized.

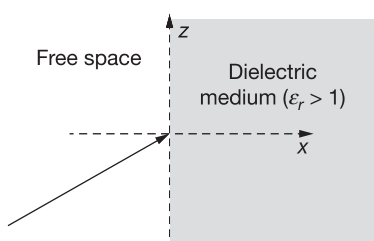
Hence, the correct option is (a)

Two-marks Questions

1. A uniform plane wave traveling in free space and having the electric field

$$\vec{E} = (\sqrt{2}\hat{a}_x - \hat{a}_2)\cos[6\sqrt{3}\pi \times 10^8 t - 2\pi(x - \sqrt{2}z)]\text{V/m}$$

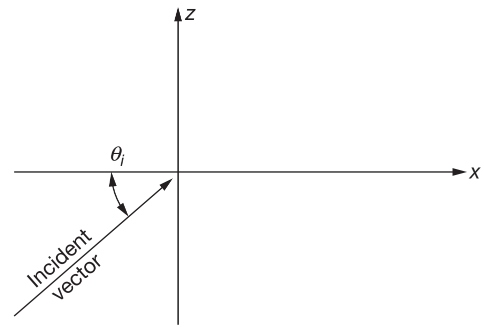
Is incident on a dielectric medium (relative permittivity > 1, relative permeability = 1) as shown in the figure and there is no reflected wave.



The relative permittivity (correct to two decimal places) of the dielectric medium is ________. **[2018]**

Solution: We know that uniform plane wave is travelling with electric field vector being parallel to the plane of incidence. Therefore it is parallel polarization. We know that wave is not reflected otherwise it represents total transmission. Angle of incidence, θ_i = Brewster angle θ_{13}

$$= \tan^{-1}\left(\frac{n_2}{n_1}\right)$$



$\because \quad n_1 = 1$

$$\theta_i = \tan^{-1}\frac{\beta_z}{\beta_x}$$

$$= \tan^{-1}(\sqrt{2})$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}(\sqrt{2})$$

$$n_2 = \sqrt{2}$$

$$\sqrt{\varepsilon_{r2}} = \sqrt{2}$$

$$\varepsilon_{r2} = 2$$

$\because \quad n_1 = 1$

Hence, the correct answer is 1.9 to 2.1.

2. The distance (in meters) a wave has to propagate in a medium having a skin depth of 0.1 m so that the amplitude of the wave attenuates by 20 dB, is **[2018]**

(A) 0.12 (B) 0.23

(C) 0.46 (D) 2.3

Solution: We know that the amplitude of e–field in the direction of propagation can be expressed as

$$E_z = E_0 e^{-\alpha Z}$$

here E_0 represents the maximum, amplitude that occurs at $Z = 0$.

Now we have

$$\therefore \quad 20 \log_{10}\left[\frac{E_0}{E_0 e^{-z/8}}\right] = 20$$

$$\because \quad \delta = \frac{1}{\alpha} = 0.1 \text{ m}$$

$$\log_{10}\left[e^{10z}\right] = 1$$

$$\Rightarrow \quad z = 0.23 \text{ m}$$

Hence, the correct option is (B)

3. The electric field of a uniform plane wave travelling along the negative z direction is given by the following equation

$$\bar{E}_W^i = \left(\hat{a}_x + j\hat{a}_y\right) E_0 e^{jkz}.$$

This wave is incident upon a receiving antenna placed at the origin and whose radiated electric field towards the incident wave is given by the following equation

$$E_a = \left(\hat{a}_x + 2\hat{a}_y\right) E_I \frac{1}{r} e^{-jkr}$$

The polarization of the incident wave, the polarization of the antenna and losses due to the polarization mismatch are, respectively, **[2016]**

(A) Linear, Circular (Clockwise) – 5dB
(B) Circular (clockwise), Linear –5dB
(C) Circular (clockwise), Linear, –3dB
(D) Circular (anti clockwise), Linear, –3dB

Solution: Polarization of incident wave,

$$\overline{E_w^i} = (ax + ja_y)E_0 e^{jkz}$$

Direction of propagation '$-z$'

$$|E_{xo}| = |E_{yo}|$$

E_y leads E_x by 90°

Right (clock wise) circular polarization.

Polarization of antenna,

$$\overline{Ea} = (ax + 2ay)E_I \frac{1}{r} e^{-jkr}$$

$$|E_{XO}| = E_I$$

$$|E_{YO}| = 3E$$

E_x and E_y are in phase

Linear polarization: The linearly polarized antenna simply picks up the in phase component of circularly polarized wave which has two orthogonal linearly polarized waves 90° out of phase. As a result LP antenna has a polarization mismatch loss of 0.5 (–3 dB) PLF (linear to circular) = 0.5 = –3 dB.

Hence, the correct option is (C).

(From the given eqn, we see that both Ex and Ey both are in phase which shows that there is linear polarization in it with mismatch loss of –3 dB option (c))

4. The electric field intensity of a plane wave travelling in free space is given by the following expression $E(x, t) = a_y\, 24\, \pi\cos(\omega t - k_0 x)$ (V/m)

In this field, consider a square area 10 cm × 10 cm on a plane $x + y = 1$. The total time-averaged power (in mW) passing through the square area is ________. **[2015]**

Solution: $P_{\text{avg}} = \dfrac{|E_o|^2}{2\eta}\,\hat{a}_n$

$$= \frac{(24\pi)^2}{2\times 120\pi}\hat{a}_x$$

$$= 7.53\,\hat{a}_x$$

Wave is propagating along x-axis so field is in y-z plane in 10 cm × 10 cm square

$$P = \int P_{avg}.ds$$

$$= \int 7.53\hat{a}_x \cdot \left(\frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}\right) ds$$

Where $\dfrac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}$ is unit vector

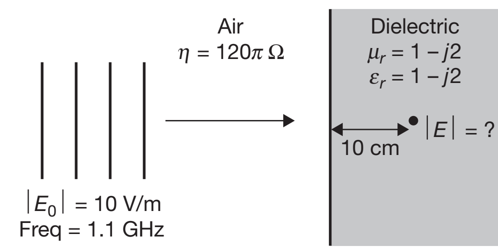
$$= \frac{7.53}{\sqrt{2}} \int_0^{10^{-1}} \int_0^{10^{-1}} dy\, dz$$

$$= 5.32 \times 10^{-2}\ \text{W}$$

$$= 53.2\ \text{mW}$$

Hence, the correct Answer is (53 to 54).

5. Consider a uniform plane wave with amplitude (E_0) of 10 V/m and 1.1 GHz frequency travelling in air, and incident normally on a dielectric medium with complex relative permittivity (ε_r) and permeability (μ_r) as shown in the figure.



The magnitude of the transmitted electric field component (in V/m) after it has travelled a distance of 10 cm inside the dielectric region is ________. **[2015]**

Solution:

Air	Dielectric
$\eta_1 = 120\,\pi\,\Omega$	$\mu_r = 1 - j_2$
$E_o = 10$ V/m	$\varepsilon_r = 1 - j_2$
$f = 1.1$ GHz	$\eta_2 = 120\,\pi\,\Omega$

$E = E_o\, e^{-\gamma z}$ for decaying electric field

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}\ \ [\sigma\ \ll j\ \omega\varepsilon]$$

$$= \sqrt{j\omega\mu(j\omega\varepsilon)}$$

$$= j\omega\sqrt{\mu_r\,\varepsilon_r}\sqrt{\mu_o\varepsilon_o}$$

$$\alpha + j\beta = j\omega\sqrt{\mu_o\,\varepsilon_o}\ (1 - j_2)$$

From above equation $\alpha = 2\,\omega\sqrt{\mu_o\,\varepsilon_o}$

$$= \frac{2\times 2\pi\times 1.1\times 10^9}{3\times 10^{10}}$$

$$= 0.4607\ \text{cm}$$

Electric field strength after 10cm

$$E = 10e^{-10 \times 0.4602}$$

$$= 0.099 \text{ V/m}$$

Hence, the correct Answer is (0.08 to 0.12).

6. The electric field of a plane wave propagating in a lossless non-magnetic medium is given by the following expression **[2015]**

$$E(z, t) = a_x 5 \cos(2\pi \times 10^9 t + \beta z)$$

$$+ a_y 3 \cos(2\pi \times 10^9 t + \beta z - \frac{\pi}{2})$$

The type of the polarization is

(A) Right-hand Circular
(B) Left-hand Elliptical
(C) Right-hand Elliptical
(D) Linear

Solution: From given wave equation

$|E_x| \neq |E_y|$ so it is elliptically polarized wave at

$$\omega t + \beta z = 0$$

then $$E(z, t) = \hat{a}_x 5$$

$$\omega t + \beta_z = \frac{\pi}{2}$$

$$E(z, t) = \hat{a}_y 3$$

So it is left-hand elliptically polarized.
Hence, the correct option is (B).

7. If the electric field of a plane wave is

$$\vec{E}(z,t) = \hat{x} 3\cos(\omega t - kz + 30^\circ)$$

$$- \hat{y} 4\sin(\omega - kz + 45^\circ)(mV/m)$$

the polarization state of the plane wave is

(a) left elliptical (b) left circular
(c) right elliptical (d) right circular

[2014]

Solution: (a)

$$\vec{E}(z,t) = \hat{x}3\cos(wt - kz + 30°)$$

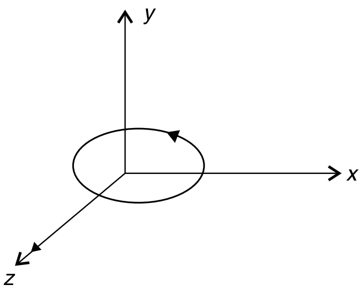
$$-\hat{y}4\sin(wt - kz + 45°)(\text{mv/m})$$

$$\vec{E}_x = 3\cos(wt - kz + 30°)$$

$$\vec{E}_y = -4\sin(wt - kz + 45°)$$

$$\vec{E}_y = -4\cos(wt - kz + 35°)$$

Phase difference between two components is 105° with unequal magnitude.



So, given wave is left elliptical polarized.
Hence, the correct option is (a)

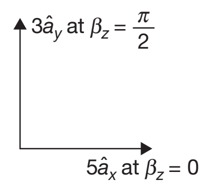
8. Assume that a plane wave in air with an electric field $\vec{E} = 10\cos(\omega t - 3x - \sqrt{3}z)\hat{a}_y$ V/mis incident on a non-magnetic dielectric slab of relative permittivity 3 which covers the region $z > 0$. The angle of transmission in the dielectric slab is________degrees. **[2014]**

Solution: 30°

$$\vec{E} = 10\cos(wt - 3x - \sqrt{3z})\overrightarrow{ay}$$

$$\tan\theta_1 = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\theta_1 = 60°$$



Using snell's law,

$$\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2$$

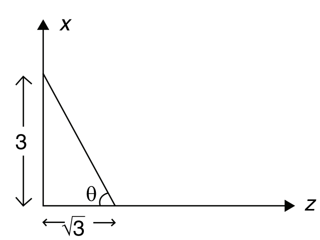
$$\sqrt{\epsilon_1}\sin 60° = \sqrt{\epsilon_2}\sin\theta_2$$

$$t_1 = 1,\ t_2 = 3$$

$$\sin\theta_2 = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{1}{2}$$

$$\theta_2 = 30°$$

v/m



9. The angle of incidence and the expression for are

(a) $60^\circ\ and\ \frac{E_o}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi \times 10^4 (x+z)}{3\sqrt{2}}}\ V/m$

(b) $60^\circ\ and\ \frac{E_o}{\sqrt{2}}(\hat{a}_x + \hat{a}_z)e^{-j\frac{\pi \times 10^4 z}{3}}\ V/m$

(c) $45^\circ\ and\ \frac{E_o}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi \times 10^4 (x+z)}{3\sqrt{2}}}\ V/m$

(d) 60° and $\dfrac{E_o}{\sqrt{2}}(a_x - a_z)e^{-j\frac{\pi\times10^4 z}{3}}\ V/m$ **[2013]**

Solution: (c)

$\sqrt{\epsilon_{r1}}\sin\theta_1 = \sqrt{\epsilon_{r2}}\sin\theta_t$

$\theta_t = 19.2°$

$\sin\theta_i = \sqrt{4.5}\sin(19.2°)$

$\sin\theta_i = 2.12 \times 0.3289$

$= 44.2° \simeq 45°$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{600\times10^{-6}} = \frac{\pi\times10^4}{3}\sqrt{Bx^2 + By^2}$$

and direction of E_i component is a_x and $-a_z$ so,

Thus, 45° and $\dfrac{E_0}{\sqrt{2}}(ax - az)e^{-j\frac{\pi\times10^4}{3\sqrt{2}}(x+z)}$ v/m

Hence, the correct option is (c)

10. The expression for

60° *and* $\dfrac{E_o}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times10^4 z}{3}}\ V/m$ is

(a) $E_r = \dfrac{0.23E_0}{\sqrt{2}}(a\hat{x} + a\hat{z})e^{\frac{-j\pi\times10^4(x-z)}{3\sqrt{2}}}$ v/m

(b) $-\dfrac{E_0}{\sqrt{2}}(\hat{a}_x + \hat{a}_z)e^{j\frac{\pi\times10^4 z}{3}}\ V/m$

(c) $0.44\dfrac{E_0}{\sqrt{2}}(\hat{a}_x + \hat{a}_z)e^{-j\frac{\pi\times10^4(x-z)}{3}}\ V/m$

(d) $\dfrac{E_0}{\sqrt{2}}(\hat{a}_x + \hat{a}_z)e^{-j\frac{\pi\times10^4(x+z)}{3}}\ V/m$ **[2013]**

Solution: (a)

$$\frac{E_r}{E_i} = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_2} + \sqrt{\epsilon_1}} = \frac{1-\sqrt{4.5}}{1+\sqrt{4.5}} = \frac{-1.12}{3.12}$$

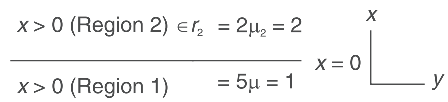
$$\frac{E_r}{E_i} = -0.359$$

$|E_r| = 0.23$

$$E_r = \frac{0.23E_0}{\sqrt{2}}(ax + az)e^{\frac{-j\pi\times10^4(x-z)}{3\sqrt{2}}}\ \text{v/m}$$

Hence, the correct option is (a)

11. A current sheet $\vec{H}_2 = 1.5\hat{u}_x + 30\hat{u}_y - 10\hat{u}_z\ A/m$ lies on the dielectric interface $x = 0$ between two dielectric media with $\varepsilon_{r1} = 1$, $\mu_{r1} = 1$ in Region-1 ($x < 0$) and $\varepsilon_{r2} = 2$, $\mu_{r2} = 2$ in Region-2 ($x > 0$) If the magnetic field in Region-1 at $x = 0$ is $\vec{H}_1 = 3\hat{u}_x + 30\hat{u}_y$ A/m, the magnetic field in Region-2 at $x = 0 +$ is



(a) $\vec{H}_2 = 1.5u_x + 30u_y - 10u_z\ A/m$

(b) $\vec{H}_2 = 3\hat{u}_x + 30\hat{u}_y - 10\hat{u}_z\ A/m$

(c) $\vec{H}_2 = 1.5\hat{u}_x + 40\hat{u}_y\ A/m$

(d) $\vec{H}_2 = 3\hat{u}_x + 30\hat{u}_y + 10\hat{u}_z\ A/m$ **[2011]**

Solution: (a)

From property of magnetic field boundary relation.

$B_{n1} = B_{n2}$

$B \times 1 = B \times 2$ [normal component]

$\mu_1 H_{x1} = \mu_2 H_{x2}$

$\vec{H}_1 = 3\hat{u}x + 3ou\hat{y}$ A/m

$H_{x2} = \dfrac{3\times1}{2}4x = 1.54x$

$H_{t1} - H_{t2} = -\vec{J}_s \times \overrightarrow{an}$

$30\ uy - H_{t2} = -10u\hat{y}\times4\hat{x}$

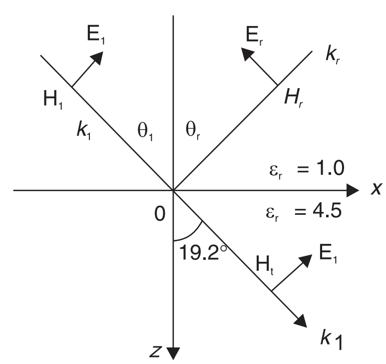
$H_{t2} = 30uy - 10uz$

So, $\vec{H}_2 = \vec{H}_{n1} + \vec{H}_{t2}$

$\vec{H}_2 = 1.5u\hat{x} + 30uy - 10u\hat{z}$

Common Data for Questions 12 and 13

A monochromatic plane wave of wavelength $\lambda = 600$ μrn is propagating in the direction as shown in the figure below. $\vec{E}_i$, $\vec{E}_r$, and $\vec{E}_t$ denote incident, reflected, and transmitted electric field vectors associated with the wave.



Hence, the correct option is (a)

12. A plane wave having the electric field component $\vec{E}_i = 24\cos(3\times10^8 t - \beta y)\hat{a}_z$ V/m and travelling in free space is incident normally on a lossless medium with $\mu = \mu_0$ and $\in = 9 \in_0$ which occupies the region $y \geq 0$. The reflected magnetic field component is given by

(a) $\frac{1}{10\pi}\cos(3\times10^8 t + {}^2 y)\hat{a}_x$ A/m

(b) $\frac{1}{20\pi}\cos(3\times10^8 t + {}^2 y)\hat{a}_x$ A/m

(c) $-\frac{1}{20\pi}\cos(3\times10^8 t + \beta y)\hat{a}_x$ A/m

(d) $-\frac{1}{10\pi}\cos(3\times10^8 t + \beta y)a_x$ A/m **[2010]**

Solution: (a)

$$\vec{E}_i = 24\cos(3\times10^8 t - \beta y)\hat{a}z \text{ v/m}$$

$$a\hat{z}\times\hat{a}[Hi] = a\hat{y} \Rightarrow a\hat{z} + a\hat{x} = a\hat{y}$$

$$Hi = \frac{1}{\eta}\vec{E}_i\, a\hat{x} = \frac{1}{5\pi}\cos(3\times10^8 t - py)a\hat{x}$$

$$\frac{H_r}{H_i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = \frac{\sqrt{\epsilon_2} - \sqrt{\epsilon_1}}{\sqrt{\epsilon_2} + \sqrt{\epsilon_1}} = \frac{\sqrt{9} - \sqrt{1}}{\sqrt{9} + \sqrt{1}} = \frac{1}{2}$$

$$H_r = \frac{1}{2}H_i$$

$$H_r = \frac{1}{10\pi}\cos(3\times10^8 t + \beta y)ax$$

Hence, the correct option is (a)

13. A uniform plane wave in the free space is normally incident on an infinitely thick dielectric slab (dielectric constants $\varepsilon_r = 9$) The magnitude of the reflection coefficient is

(a) 0 (b) 0.3

(c) 0.5 (d) 0.8 **[2008]**

Solution: (c)

$\varepsilon_{r1} = 9$ $t_{r2} = 9$

$$\Gamma = \left|\frac{\sqrt{\epsilon_{r1}} - \sqrt{\epsilon_{r2}}}{\sqrt{\epsilon_{r1}} + \sqrt{\epsilon_{r2}}}\right| = \left|\frac{1-3}{1+3}\right|$$

$$\Gamma = 0.5$$

Hence, the correct option is (a)

14. The $\vec{H}$ field (in A/m) of a plane wave propagating in free space is given by

$$\vec{H} = x\frac{5\sqrt{3}}{\eta_0}\cos(\omega - \beta) + y\frac{5}{\eta_0}\sin\left(\omega - \beta z + \frac{\pi}{2}\right)$$

The time average power flow density in watts is

(a) $\frac{\eta_0}{100}$ (b) $\frac{100}{\eta_0}$

(c) $50\eta_0^2$ (d) $\frac{50}{\eta_0}$ **[2007]**

Solution: (d)

For free space, $P = \frac{E^2}{2\eta_0}$ or, $\frac{\eta_0 H^2}{2}$

$$H = \frac{5\sqrt{3}}{\eta_0}\cos(wt - \beta z)\hat{x} + \frac{5}{\eta_0}\sin\left(wt - pz + \frac{\pi}{2}\right)\hat{y}$$

$$H = \sqrt{Hx^2 + Hy^2}$$

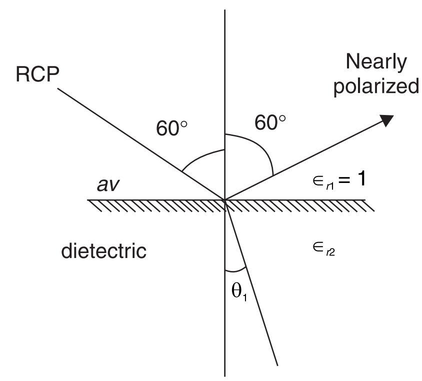
$$= \frac{5}{\eta_0}\sqrt{(\sqrt{3})^2 + 1}$$

$$H = \frac{10}{\eta_0}$$

$$P = \frac{\eta_0 \times 100}{2\eta_0^2} = 50 \text{ watt}$$

Hence, the correct option is (d)

15. A Right Circularly Polarized (RCP) plane wave is incident at an angle of 60° to the normal, on an air-dielectric interface. If the reflected wave is linearly polarized, the relative dielectric constant $\in_{r2}$ is



(a) $\sqrt{2}$ (b) $\sqrt{3}$

(c) 2 (d) 3 **[2007]**

Solution: (d)

Brewster's angle of incidence is given as,

$$\tan\theta\ B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\tan 60° = \sqrt{\frac{\epsilon_2}{1}} = \sqrt{3}$$

$t_2 = 3$

Hence, the correct option is (d)

16. A medium of relative permittivity $\in_{r_2} = 2$ forms an interface with free-space. A point source of electromagnetic energy is located in the medium at a depth of 1 meter from the interface. Due to the total internal reflection, the transmitted beam has a circular cross-section over the interface. The area of the beam cross-section at the interface is given by

(a) $2\pi m^2$ (b) $\pi^2 m^2$

(c) $\pi/2\ m^2$ (d) πm^2 **[2006]**

Solution: (d)

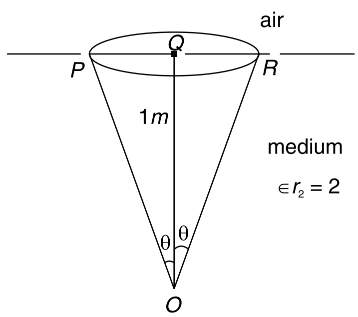
$$\sqrt{\in_{r_2}}\sin\theta = 1$$

$$\sin\theta = \frac{1}{\sqrt{2}}$$

$$\theta = 45°$$

$$O\theta = 1\text{ m}$$

$$P\theta = 1\text{ m}$$

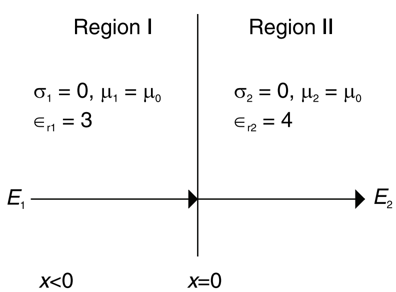


Area of circle at interface is, $\pi(p\theta)^2$

$A = \pi\ m^2$

Hence, the correct option is (d)

17. A medium is divided into regions I and II about x = 0 plane, as shown in the figure below. An electromagnetic wave with electric field $\vec{E}_1 = 4\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$ is incident normally on the interface from region I. The electric field $\vec{E}_2$ in region-II at the interface is



(a) $E_2 = E_1$

(b) $4\hat{a}_x + 0.75\hat{a}_y - 1.25$

(c) $3\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$

(d) $-3\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$ **[2006]**

Solution: (c)

$$\vec{E}_1 = 4\overrightarrow{ax} + 3\overrightarrow{ay} + 5\overrightarrow{az}$$

$$\vec{E}_{t_1} = \vec{E}_{t_2}$$

$$\vec{E}_{t_1} = 3\overrightarrow{ay} + 5\overrightarrow{az} = \vec{E}_{t_2}$$

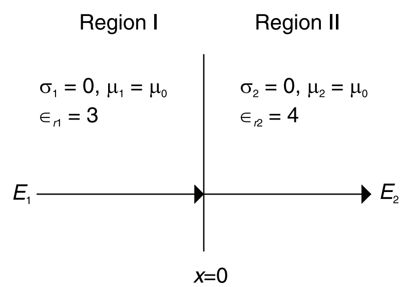
$$D_{n_1} = D_{n_2}$$

$$t_1\vec{E}_{n_1} = t_2\vec{E}_{n_2}$$

$$\vec{E}_{n_2} = \frac{3}{4}\times 4\overrightarrow{\sigma_x} = 3\overrightarrow{ax}$$

$$\vec{E}_2 = \vec{E}_{n_2} + \vec{E}_{t_2}$$

$$\vec{E}_2 = 3\overrightarrow{ax} + 3\overrightarrow{ay} + 5\overrightarrow{az}$$



Hence, the correct option is (c)

18. When a plane wave travelling in free space is incident normally on a medium having $\in_r = 4.0$, the fraction of power transmitted into the medium is given by

(a) 8/9 (b) 1/2

(c) 1/3 (d) 5/6 **[2006]**

Solution: (a)

$$P_i = P_t + P_r$$

$$P_r = |\Gamma^1|^2 P_i$$

$$P_t = (1-\Gamma^2)P_i$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\in_1} - \sqrt{\in_2}}{\sqrt{\in_1} + \sqrt{\in_2}}$$

$$\in_1 = 1, \in_2 = 4$$

$$\Gamma = -\frac{1}{3}$$

$$\frac{P_t}{P_i} = 1 - \left(\frac{1}{3}\right)^2 = \frac{8}{9}$$

Hence, the correct option is (a)

19. If $\vec{H} = \left(\frac{k}{w\mu}\right)(a\hat{y} + ja\hat{x})e^{j*kz-wt)}$ and

$\hat{H} = \left(\frac{k}{\omega\mu}\right)(\hat{a}_y + j\hat{a}_x)e^{jkz-j\omega t}$ the time averaged Poynting vector is

(a) null vector (b) $\left(\frac{k}{\omega\mu}\right)\hat{a}_z$

(c) $\left(\frac{2k}{\omega\mu}\right)\hat{a}_z$ (d) $\left(\frac{k}{2\omega\mu}\right)a_z$

[2004]

Solution: (a)

$$\vec{E} = (a\hat{x} + ja\hat{y})e^{j(kz-wt)}$$

$$\vec{H} = \left(\frac{k}{w\mu}\right)(a\hat{y} + ja\hat{x})e^{j*kz-wt)}$$

$$P_{avg} = \frac{1}{2}R_e[\vec{E}\times\vec{H}]$$

$$= \frac{1}{2}R_e\left[(ax + jay)e^{j(kz-wt)}\right.$$

$$\left.\times\left(\frac{k}{w\mu}\right)(ay - jax)e^{j(kz-wt)}\right]$$

$$= \frac{1}{2}\left(\frac{k}{w\mu}\right)\left[(a\hat{x} + a\hat{y}) +\right.$$

$$\left. j(a\hat{x} + a\hat{x}) + j(a\hat{y} + a\hat{y}) + (a\hat{y} + a\hat{x})\right]$$

$$= \frac{1}{2}\left(\frac{k}{w\mu}\right)[a\hat{z} + 0 + 0 - a\hat{z}]$$

$P_{avg} = 0$

Hence, the correct option is (a)

20. Medium 1 has the electrical permittivity $\varepsilon 1 = 1.5\varepsilon_0$ farad/m and occupies the region to the left of $x = 0$ plane Medium 2 has the electrical permittivity $\varepsilon_2 = 2.5\varepsilon_0$ farad/m and occupies the region to the right of $x = 0$ plane. If E_1 in medium 1 is $E_1 = (2u_x - 3u_y + 1u_z)$ volt/m, then E_2 in medium 2 is

(a) $(2.0u_x - 7.5u_y + 2.5u_z)$volt/m
(b) $(2\text{–}0u_x - 2.0u_y + 0.6u_z)$volt/m
(c) $(1.2u_x - 3.0u_y + 1.0u_z)$volt/m
(d) $(1.2u_x - 2.0u_y + 0\ 6u_z)$volt/m **[2003]**

Solution: (c)

$$E_{t_1} = E_{t_2}$$

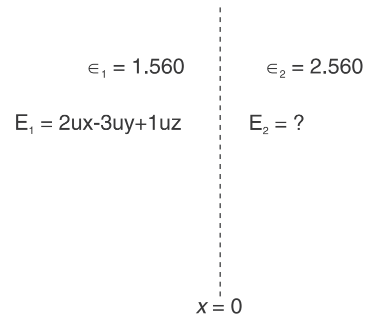
$$E_{t_1} = -3uy + uz = E_{t_2}$$

$$\in_1 E_{n_1} = \in_2 E_{n_2}$$

$$1.5 \in_0 .2ux = 2.5 \in_0 E_{n_2}$$

$$E_{n_2} = 1.2\,u_x$$

So, $E_2 = 1.2ux - 3uy + uz$



Hence, the correct option is (c)

21. A uniform plane wave travelling in air is incident on the plane boundary between air and another dielectric medium with $\varepsilon_r = 4$. The reflection coefficient for the normal incidence, is

(a) Zero (b) $0.5 \angle 180°$
(c) $0.333 \angle 0°$ (d) $0.333\angle 180°$ **[2003]**

Solution: (d)

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\dfrac{1}{\sqrt{\in_2}} - \dfrac{1}{\sqrt{\in_1}}}{\dfrac{1}{\sqrt{\in_2}} + \dfrac{1}{\sqrt{\in_1}}}$$

$\in_2 = 4,\ \in_1 = 1$

$$\Gamma = \frac{\frac{1}{2} - 1}{\frac{1}{2} + 1} = -\frac{1}{3} = 0.333 < 180°$$

Hence, the correct option is (d)

22. If the electric field intensity associated with a uniform plane electromagnetic wave travelling in a perfect dielectric medium is given by $E(z, t) = 10\cos(2\pi 10^7 t - 0.1\pi z)$. volt/m, the velocity of the travelling wave is

(a) 3.00×10^8 m/sec (b) 2.00×10^8 m/sec
(c) 6.28×10^7 m/sec (d) 2.00×10^7 m/sec **[2003]**

Solution: (b)

$E(z, t) = 10\cos(2\pi \times 10^7 t - 0.1\pi z)$ v/m

$w = 2\pi \times 10^7$

$\beta = 0.1\pi$

$$V = \frac{w}{\beta} = \frac{2\pi \times 10^7}{0.1\pi}$$

$V = 2 \times 10^8$ m/s

Hence, the correct option is (b)

23. A plane wave is characterized by $\vec{E} = (0.5\hat{x} + \hat{y}e^{jwt-jkz})e^{j\omega t-jkz}$. This wave is

(a) linearly polarized
(b) circularly polarized
(c) elliptically polarized
(d) unpolarized **[2002]**

Solution: (c)

$\vec{E} = 10.5\hat{x} + \hat{y}e^{j\pi/2})e^{(wt-kz)}$

Here, $E_x = 0.5e^{j(wt-kt)}\hat{x}$

$E_y = ej(wt - kz + \pi/2)\hat{y}$

E_x and E_y are out of phase by 90° with unequal magnitude so, this wave is elliptically polarized

Hence, the correct option is (c)

24. Distilled water at 25°C is characterized byσ = 1.7 × 10^{-4} mho/m and $\in$ = 78 $\in_0$ at a frequency of 3 GHz. Its loss tangent tan $\delta$is

(a) 1.3 × 10^{-5} (b) 1.3 × 10^{-3}
(c) 1.7 × 10^{-4}/78 (d) 1.7 × 10"4/(78$\in_0$)
($\in$=10^{-9}/(36π)F/m) **[2002]**

Solution: (a)

Loss tangent, $\dfrac{\sigma}{w\in}$

$$= \frac{1.7\times10^{-4}\times36\pi}{2\times\lambda\times3\times10^{9}\times78\times10^{9}}$$

= 1.3 × 10^{-5}

Hence, the correct option is (a)

25. A material has conductivity of 10^{-2} mho/m and a relative permittivity of 4. The frequency at which the conduction current in the medium is equal to the displacement current is

(a) 45 MHz (b) 90 MHz
(c) 450 MHz (d) 900 MHz
[2001]

Solution: (a)

Conduction current = |σ*E*|

displacement current = |*jw*$\in$*E*|

2π*f*$\in$*E* = σ*E*

$$f = \frac{\sigma}{2\pi\times\in_0\in r} = \frac{9\times10^{9}\times2\times10^{-2}}{4}$$

f = 45 mHz

Hence, the correct option is (a)

26. A uniform plane wave in air impinges at 45° angle on a lossless dielectric material with dielectric constant $\in_r$. The transmitted wave propagates in a 30° direction with respect to the normal. The value of $\in_r$ is

(a) 1.5 (b) $\sqrt{1.5}$
(c) 2 (d) $\sqrt{2}$ **[2000]**

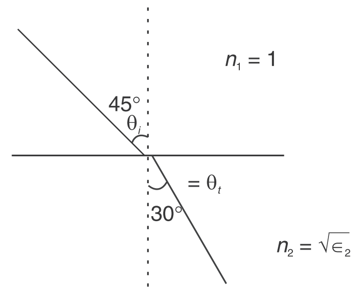
Solution: (c)

$\eta_1 \sin\theta_i = \eta_2 \sin\theta_t$

$$\frac{1}{\sqrt{\in_2}} = \frac{\sin 30°}{\sin 45°}$$

$$\sqrt{\in_2} = \frac{d}{\sqrt{2}}$$

$\in_2 = 2$



Hence, the correct option is (c)

27. Two coaxial cables 1 and 2 are filled with different dielectric constants $\in_{r_1}$ and $\in_{r_1}$ respectively. The ratio of the wavelengths in the two cables, (λ_1/λ) is

(a) $\sqrt{\in_{r_1}/\in_{r_2}}$ (b) $\sqrt{\in_{r_2}/\in_{r_1}}$

(c) $\in_{r_1}/\in_{r_2}$ (d) $\in_{r_2}/\in_{r_1}$ **[2000]**

Solution: (b)

Velocity, $V \propto \dfrac{1}{\sqrt{t}}$

[$V = f\lambda$]

So, $\dfrac{\lambda_1}{\lambda_2} = \sqrt{\dfrac{\in_2}{\in_1}}$

Hence, the correct option is (b)

28. A plane wave propagating through a medium $\in_{r'}$ = 8, μ_r = 2, and σ = 0 has its electric field given by

$\vec{E}$ = 0.5 sin(10^8t βz)*V/m*. The wave impedance, in ohms is

(a) 377 (b) 198.5 ∠ 180°
(c) 182.9 ∠ 14° (d) 188.5 **[1999]**

Solution: (d)

$E_r = B$, $\mu r = 2$, σ = 0

$\vec{E} = 0.5\sin(10^8 t - \beta z)$

$$\eta = \sqrt{\frac{jw\mu}{\sigma + jw\in}}$$

w = 10^8

$$\eta = \sqrt{\frac{\mu r\mu_0}{\in r\, t_0}} = 120\pi\sqrt{\frac{\mu r}{\in r}}$$

$$\eta = 120\pi\sqrt{\frac{2}{8}}$$

$$\eta = 60\pi = 188.5\Omega$$

Hence, the correct option is (d)

29. A uniform plane wave in air is normally incident on an infinitely thick slab If the refractive index of the glass slab is 1.5, then the percentage of the incident power that is reflected from the air-glass interface is

(a) 0% (b) 4%
(c) 20% (d) 100% **[1996]**

Solution: (b)

Reflection coefficient, $\Gamma^1 = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

For given problem, η_1 (air) = 1

η_2(dielectric slab) = 1.5.

$$\Gamma = \frac{1.5-1}{1.5+1} = \frac{1}{5}$$

$$|\Gamma| = \frac{1}{5}$$

$$\frac{P_r}{P_i} = |\Gamma|^2 = \frac{1}{25}$$

$$\% P_r = \frac{1}{25} \times P_i \times 100 = 4\% \text{ of } P_i$$

Hence, the correct option is (b)

30. Some unknown material has a conductivity of 10^6 mho/m and a permeability of 4π 10^{-7} H/m.The skin depth for the material at 1 GHz is

(a) 15.9 μm (b) 20 9 μrn
(c) 25.9 μm (d) 30.9 μm **[1996]**

Solution: (a)

$\sigma = 10^6$ mho/m

$H = 4\pi \times 10^{-7}$ H/m

$f = 1$ GHz

Skin depth, $\delta = \sqrt{\frac{2}{w\mu\sigma}} = \sqrt{\frac{1}{\pi f \mu\sigma}}$

$$= \sqrt{\frac{1}{3.14 \times 1 \times 10^9 \times 4\pi \times 10^{-7} \times 10^6}}$$

$\delta = 15.9$ μm

Hence, the correct option is (a)

31. A material is described by the following electrical parameters at a frequency of 10 GHz: $\sigma = 10^6$ mho/m, $\mu = \mu 0$ and $\in/\in 0 = 10$. The material at this frequency is considered to be

$$\left(\in_0 = \frac{1}{36\pi} \times 10^{-9} \text{ F/m}\right)$$

(a) a good conductor
(b) a good dielectric
(c) neither a good conductor, nor a good dielectric
(d) a good magnetic material **[1993]**

Solution: (a)

Loss tangent $\frac{\sigma}{wt}$ for given problem is,

$$\frac{\sigma}{wt} = \frac{10^6}{2\pi \times 10 \times 10^9 \times \frac{10}{36\pi} \times 10^{-9}}$$

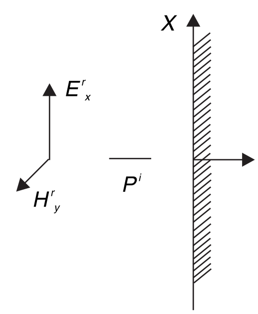
$$\frac{\sigma}{wt} = 1.798 \times 10^5$$

which is very large value as compare to 1

So, the material at this frequency is a good conductor

Hence, the correct option is (a)

32. A plane wave is incident normally on a perfect conductor as shown in figure. Here, E_X^i, H_y^i and $\vec{P}^i$ are electric field, magnetic field and Poynting vector, respectively, for the incident wave. The reflected waver should have



(a) $E_x^r = E_x^i$ (b) $H_y^r = -H_y^i$
(c) $\vec{P}^r = -\vec{P}^i$ (d) $E_x^r = E_x^i$ **[1993]**

Solution: (a) and (c)

For a perfect conductor, E is zero with in a perfect conductor.

$$E_x^i + E_x^r = E_x^t$$

E_x^t =0 For perfect conductor.

$$E_x^i = -E_x^r$$

and, $H_x^r = H_x^i$

So, $P^r = -P^c$

Hence, the correct option is (a) and (c)

33. The electric field component of a uniform plane electromagnetic wave propagating in the Y-direction in a lossless medium will satisfy the equation

(a) $\frac{\partial^2 E_x}{\partial y^2} = \mu \in \frac{\partial^2 E_x}{\partial t^2}$

(b) $\frac{\partial^2 E_y}{\partial x^2} = \mu \in \frac{\partial^2 E_y}{\partial t^2}$

(c) $\frac{\partial^2 E_x}{\partial y^2} = \mu E \frac{\partial^2 E_x}{\partial t^2}$

(d) $\frac{\sqrt{E_x^2 + E_Z^2}}{H_x^2 + H_z^2} = \sqrt{\mu / \in}$ **[1991]**

Solution: (d)

Since E.M wave is propagating in *y*-direction than *Ey* and *Hy* component must be zero.

Ey and *Hy* = 0

E.M wave equation given as,

$$\frac{\partial^2 E_x}{\partial y^2} = \mu \in \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{E_x}{H_z} = -\eta \text{ and } \frac{E_z}{H_x} = \eta$$

$$\frac{E}{H} = \frac{\sqrt{E_x^2 + E_z^2}}{H_x^2 + H_z^2} = \eta = \sqrt{\frac{\mu}{\in}}$$

Hence, the correct option is (d)

34. The incoming solar radiation at a place on the surface of the earth is 1.2 KW/m². The amplitude of the electric field corresponding to this incident power is nearly equal to

(a) 80m*V/m* (b) 2.5 *V/m*

(c) 30 *V/m*

(d) 950 *V/m* **[1990]**

Solution: (d)

$$\text{Power} = \frac{E^2}{2\eta}$$

E = Electric field intensity

η = Intrinsic impedance

$$E = \sqrt{2\eta p}$$

η = 120π

P = 1.2 *kW/m²*

$$E = \sqrt{2 \times 120\pi \times 1.2 \times 10^3}$$

$$E = 950\, v/m$$

Hence, the correct option is (d)

35. In a good conductor the phase relation between the tangential components of electric field E_t and the magnetic field H_t is as follows

(a) E_t and H_t are in phase

(b) E_t and H_t are out of phase

(c) H_t leads E_t by 90°

(d) E_t leads H_t by 45° **[1988]**

Solution: (d)

η = intrinsic impedance

$$= \frac{E_t}{H_t} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega \in}}\ 2$$

For a good conductor

σ>>ω∈

$$\frac{E_t}{H_t} = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} e^{j\pi/4}$$

So, E_t leads H_t by an angle of 45°.

Hence, the correct option is (d)

36. The skin depth of copper at a frequency of 3 GHz is 1 micron (10^{-6} meter). At 12 GHz, for a non-magnetic conductor whose conductivity is 1/9 times that of copper, the skin depth would be

(a) $\sqrt{9 \times 4}$ microns (b) $\sqrt{9 \times 4}$ microns

(c) $\sqrt{4/9}$ microns (d) $1/\sqrt{9 \times 4}$ microns

[1989]

Solution: (b)

We know that

For a good conductor

$$\text{Skin depth} = \delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$\delta \propto \frac{1}{\sqrt{f\sigma}}$$

$$\frac{\delta_2}{\delta_1} = \sqrt{\frac{f_1\sigma_1}{f_2\sigma_2}}$$

Given that,

δ_1 = 1 micron

f_1 = 3 GHz

f_2 = 12 GHz

$$\frac{\sigma_2}{\sigma_1} = \frac{1}{9}$$

$$\frac{\delta_2}{1} = \sqrt{\frac{3}{12} \times \frac{9}{1}} = \sqrt{\frac{9}{4}}$$

$$\delta_2 = \sqrt{\frac{9}{4}}\ microns$$

Hence, the correct option is (b)

37. For an electromagnetic wave incident from one medium to a second medium, total reflection takes place when

(a) The angle of incidence is equal to the Brewster angle with E field perpendicular to the plane of incidence

(b) The angle of incidence is equal to the Brewster angle with E field parallel to the plane of incidence

(c) The angle of incidence is equal to the critical angle with the wave moving from the denser medium to a rarer medium

(d) The angle of incidence is equal to the critical angle with the wave moving from a rarer medium to a denser medium **[1987]**

Solution: (c)

Total internal inflection takes place when incident angle is greater than critical angel θ*c*.

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}}$$

For total internal reflection

$\theta_i \geq \theta_c$

$$\theta_i \geq \sin^{-1}\left(\sqrt{\frac{\xi_{r2}}{\xi_{r1}}}\right)$$

For total internal reflection to take place, the wave should move from a denser medium to a rarer medium and the angle of incidence should be greater than or equal to the critical angle.

Hence, the correct option is (c)

Five-marks Questions

1. A medium has a breakdown strength of 16 kV/m rms. Its relative permeability is 1.0 and relative permittivity is 4.0. A plane electromagnetic wave is transmitted through the medium. Calculate the maximum possible power flor density and the associated magnetic field. **[2001]**

Solution: $\mu_r = 1, E_r = 4.$

given, $E = E_{brms} = 16$ kv/m

$$\eta = \sqrt{\frac{\mu_r}{E_r}}120\pi = 60\pi, \; P = \frac{E_{brms}^2}{\eta}$$

$$\text{So, } P_{max} = \frac{(16\times10^3)^2}{60\pi} = 1.36\times10^6 \text{ w/m}^2$$

$$H_{brms} = \frac{E_{brms}}{\eta} = 84.88 \text{ A/m}$$

2. The electric field vector of a wave is given as

$$\vec{E}=E_0 e^{j(\omega t+3x-4y)}\frac{8\vec{a}_x+6\vec{a}_y+5\vec{a}_z}{\sqrt{125}} \text{ V/m}$$

Its frequency is 10 GHz

(i) Investigate if this wave is a plane wave,

(ii) Determine its propagation constant, and

(iii) Calculate the phase velocity in *y*-direction **[1998]**

Solution:

$$\vec{E} = E_0^{J(\omega t+3x-4y)}\frac{8a_x+6a_y+5a_z}{\sqrt{125}}\text{V/m}$$

$f = 10 \times 10^9$ Hz

(a) $P_{avg1} = \dfrac{|E_1|^2}{2\eta_0} = 0.997 \text{ w/m}^2$

(b) $\varepsilon = \varepsilon' - j\varepsilon'' \;\; \varepsilon' = 9\varepsilon_0$

$\varepsilon'' = 0.09\varepsilon_0 = 10^{-3}\varepsilon'.$

$$\alpha = \frac{\omega}{2}E''\sqrt{\frac{40}{E'}} = 6.29\times10^{-3}\, n/m$$

Skin depth = α = 1/α = 159 m

(c) $E_2 = E_1 e^{-\alpha x}$

$x = 5f = 5/\alpha$

$E_2 = E_1 e^{-\alpha 5/\alpha} = E_1 e^{-5}$

$$\eta_2 = \sqrt{\frac{\mu_0}{E^1}} = \sqrt{\frac{\mu_0}{9E_0}} = \frac{1}{3}\times120\pi = 40\pi$$

$$P_2 = \frac{E_2^2}{2\eta_2} = \frac{339.24}{80\pi}e^{-10}$$

$P_2 = 3e^{-10}$ w/m²

(i) $\beta_x = -3, \beta_y = +4, \beta_z = 0$

$\hat{\eta} = \dfrac{8\hat{a}_x}{\sqrt{125}} + \dfrac{6\hat{a}_y}{\sqrt{125}} + \dfrac{5\hat{a}_z}{\sqrt{125}}$, which conclude that the given field vector represents a plane wave in direction of x^n.

(ii) $\beta_x^2 + \beta_y^2 + \beta_z^2 = \beta^2(\cos^2 A + \cos^2 B + \cos^2 C)$

$$= \beta^2\left(\frac{64}{125}+\frac{36}{125}\right) = \frac{4}{5}\beta^2$$

$$\frac{4}{5}\beta^2 = (-3)^2 + (4)^2 = 25^2, \Rightarrow \;\; \beta = 5.59$$

$\gamma = \alpha + {}_j\beta = 0 + j5.59$

$= 5.39\,j$

(iii) Phase velocity in the *y*-direction.

$$V_y = \frac{\omega}{\beta_y} = \frac{\omega}{\beta\cos\beta} = \frac{2\pi\times10^{10}}{4} = 1.57\times10^{10}\text{ m/sec}$$

3. A plane wave with $\eta = \sqrt{\dfrac{jw\mu}{\sigma}} = \sqrt{\dfrac{w\mu}{\sigma}}e^{j\pi/4} = \sqrt{\dfrac{w\pi}{2\sigma}} + j\sqrt{\dfrac{w\mu}{2\sigma}}$ is incident normally on a thick plane conductor lying in the X-Y plane. Its conductivity is 6 × 10⁶ S/m and surface impedance is 5 × 10⁻⁴ ∠45°Ω. Determine the propagation • constant and the skin depth in the conductor. **[1998]**

Solution: Given, $E_y = 10e^{j(\omega t - \beta z)}$ for good conductor, σ >> ω*E*

$$Z_s = \sqrt{\frac{j\omega\mu}{v + j\omega E}} \cong \sqrt{\frac{j\omega\mu}{\sigma}} = R_s + jxs$$

$$R_s = 5\times10^{-4}\angle 45°$$

$$R_s = x_s = \sqrt{\frac{\omega/4}{2\sigma}} = \frac{5\times10^{-4}}{\sqrt{2}}$$

Propagation constant,

$$\alpha = \frac{\sqrt{j\omega\sigma}}{2}\beta = (R_s)\sigma = \frac{5\times10^{-4}}{\sqrt{2}}\times6\times10^{6}$$

$\Rightarrow \alpha = \beta = 2|2|$

$\gamma = \alpha + J\beta = 2|2| + j2|2|$

Skin depth, $d = \frac{1}{\alpha} = \frac{1}{2|2|} = 0.471$ mm,

4. Match the following descriptions with each of the diagrams given in figure. Fields are near the interface, but on opposite sides of the boundary. Vectors are drawn to scale.

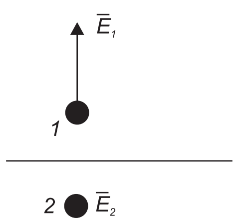
(a) Medium 1 and medium 2 are dielectrics with $\varepsilon_1 > \varepsilon_1$.

(b) Medium 1 and medium 2 are dielectrics with e., $\varepsilon_1 < \varepsilon_1$.

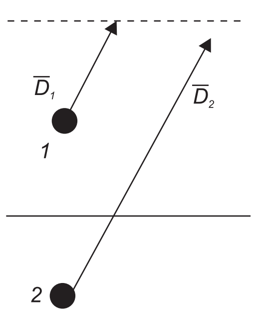
(c) Medium 2 is a perfect conductor

(d) Impossible

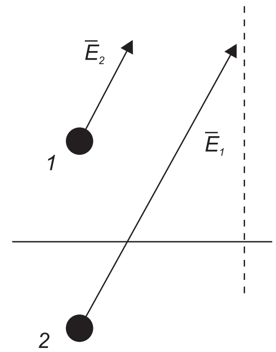
(e) Medium 1 is a perfect conductor. **[1993]**



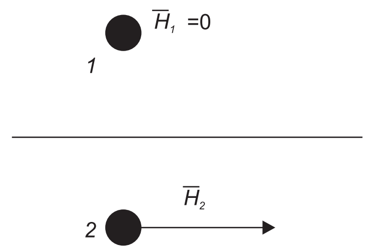
(i)



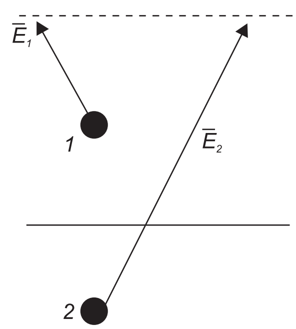
(ii)



(iii)



(iv)



(v)

Solution: Fig.1 $DN_1 - DN_2 = P_S$

So $DN_1 = PS.$ $\Rightarrow E_{N1} = P_S/E$

$E_{t1} = E_{t2} = 0$

$E_1 = E_{t1} + E_{N1} = 0 + P_S/E = P_S/E$

Fig.2 $Dt_2 > Dt_1$

As, $E_{t1} = E_{t2}$

$E_2 > E_1$

Fig.3 $\varepsilon_1 > E_1$

Fig.4 $H_1 = 0, J = H_2$ A/m

Fig 5. $E_{N2} > E_{N1}, Et_1 \uparrow Et_2$.

which is not a possible, boundary condition.

$$V_3 = \frac{Q}{4\pi E_x} + \frac{2Q}{4\pi E(x-50)} \qquad (3)$$

at equilibrium P.E. $= QV_1 + 2QV_2 + Q_3V_3 = 0$

from equation (1) (2) and (3)

$$A\left[\frac{2Q}{4\pi E(50)}+\frac{Q_3}{4\pi E(x)}\right]$$

$$+2Q\left[\frac{Q}{4\pi E(50)}+\frac{Q_3}{4\pi E(x-50)}\right]$$

$$+Q_3\left[\frac{Q}{4\pi E(x)}+\frac{2Q}{4\pi E(x-50)}\right]=0 \quad (4)$$

Solving equation (4), $Q = -50Q^3$

or $Q_3 = \frac{-1}{50}Q$

so, $x(x-50) = (x+25) \Rightarrow x^2 - 50x - x + 25 = 0$ (5)

Solving (5) $x = \frac{-51 \pm \sqrt{(-51)^2 - 4(1)(25)}}{2}$

$= 30.5$ of 0.5 cm

$x = 0.5$ cm as $(0.5 << 50.5)$

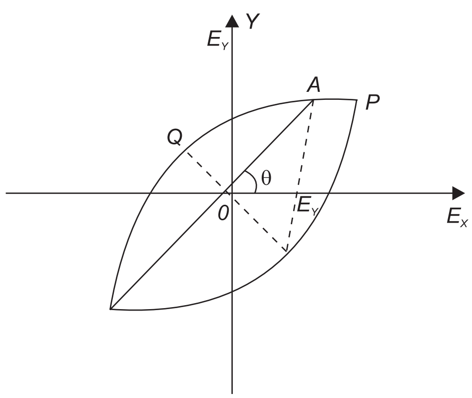
So, $x = 50.5$ cm.

5. A wave travelling in the $+z$ -direction is the resultant of two linearly polarized components, $E_x = 3\cos\omega t$, and $Ey = 2\cos(\omega t + 45°)$. Determine

(a) the axial ratio, and

(b) the angle between the major axis of the polarization ellipse and the $+x$ axis. **[1994]**

Solution: Wave is travelling in $+z$ direction.



$E_x = 3\cos\omega t$

$E_y = 2\cos(\omega t + 45°)$

t varies from 0 to $T = \frac{2\pi}{\omega}$

$OA_{max} = OP =$ semi major axis of ellipse.

$OA_{min} = OQ$

$$OA^2 = E^2 = E_x^2 + E_y^2$$

$$E^2 = (3\cos\omega t)^2 + (2\cos(\omega t + 45°))^2$$

$(OA)^2 = E^2 = 6.5 + 4.5\cos(2\omega t) - 2\sin 2\omega t$ (1)

OA is max, when, $\frac{dE}{dt} = 0,$ and Solving (1)

$\tan(2\omega t) = -4/9$

$\Rightarrow \omega t = -11.98°$

So, $E_x = 3\cos(11.98) = 2.934$

$E_y = 2\cos(-11.98 + 45) = 1.676$

$$E = \sqrt{E_x^2 + E_y^2} = 3.378$$

at, $\omega t = 78.02$

$E_y = 0.6227$ and $E_y = -1.08$

$$E = \sqrt{E_x^2 + E_y^2} = 1.246$$

So, $E_{max} = 3.3378$

$E_{min} = 1.246$

$$\text{Axial ratio} = \frac{E_{\max}}{E_{\min}} = \frac{3.378}{1.246} = 2.71$$

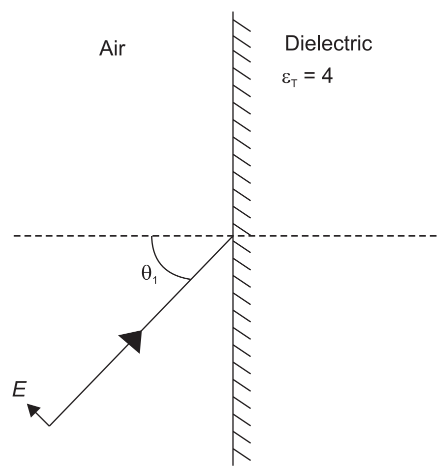
(b) $\tan\theta = \frac{E_y}{Ex}$ at $Emax$

$$\tan\theta = \frac{1.676}{2.934} \Rightarrow \vartheta = 29.73°$$

6. A uniform plane wave having parallel polarization is obliquely incident on an air-dielectric interface as shown in figure. If the wave has an electric field $E = 10$ V/m. Find

(i) the angle of incidence 9, for which there is no reflection of the wave, and

(ii) the surface charge density at the interface.



Solution: θB Brewster angle,

$$\tan(\theta_b) = \sqrt{\frac{E_{r2}}{E_{r1}}} = 2$$

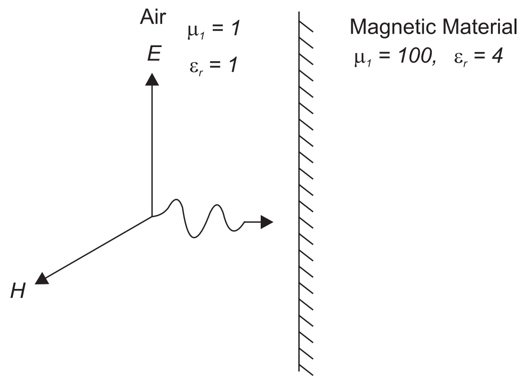
θb = 6.3.4°

(ii) $E_t = E_i$

$$\frac{D_t}{E_2} = \frac{D_i}{E_1} \Rightarrow D_t = \frac{E_2}{E_1}D_i = \frac{E_0 E_r}{E_0} = 4D_i$$

$P_s = P_t - D_i = 3D_i = 3E_0 E_i = 0.26$ nc/m^2

7. A uniform plane wave is normally incident from air on an infinitely thick magnetic material with relative permeability 100 and relative permittivity 4 (Figure). The wave has an electric field of 1 V/meter (rms). Find the average Poynting vector inside the material. **[1997]**



Solution:

$$\eta_1 = \sqrt{\frac{\mu_{r_1}}{E_{r_1}}}(120\pi) = 120\pi$$

$$\eta_2 = \sqrt{\frac{\mu_{r2}}{E_{r2}}}(120\pi) = 600\pi$$

Reflection coefficient.

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{600\pi - 120\pi}{600\pi + 120\pi} = \frac{2}{3}$$

$$P_{\arg i} = \frac{E_i^2}{\eta_1} = \frac{1}{120\pi}$$

$$\frac{(P_{avg})i}{(P_{avg})i} = 1 - (\Gamma)^2 = 1 - \left(\frac{2}{3}\right)^2 = \frac{3}{9}$$

$$(P_{avg})t = \frac{5}{9} \times \frac{1}{120\pi}\vec{a}_2 = 1.47\vec{a}_2\ m_{xy}/m^2$$

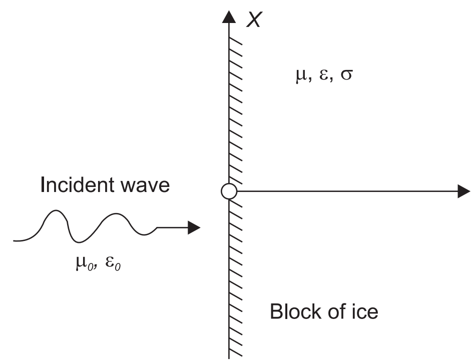
8. A plane wave in free space with $\vec{E} = (\sqrt{\pi})$ $(10.0\hat{x} + 11.8\hat{y})\exp[j(4\pi \times 10^8 t - kz)]$.

where $\hat{x}$ and y are unit vectors in the x and y-directions, respectively, is incident normally on a semi-infinite block of ice as shown in Figure. For ice, $\mu = \mu_0$, $\sigma = 0$ and $\varepsilon = 9_{e0}(1 - j0.001)$.

(a) Calculate the average power density associated with the incident wave.

(b) Calculate the skin depth in ice.

(c) Estimate the average power density at a distance 5 times the skin depth in the ice block, measured from the interface.



Solution: from given data,

$\vec{E} = 24e^{j(\omega t - \beta z)}\vec{a}_y$ $\qquad \omega = 4\pi \times 10^8$

$E_y = 11.8\sqrt{\pi}$ $\qquad \beta = k = \omega\sqrt{H\omega}$

$|E|_2 = |E_x|^2 + |E_y|^2 = 751.6,\ \eta = \eta_0 = 120\pi$

Chapter 3

Transmission Lines

One-mark Questions

1. The voltage of an electromagnetic wave propagating in a coaxial cable with uniform characteristic impedance is $V(\ell) = e^{-\gamma\ell + j\omega t}$ volts. Where ℓ is the distance along with length of the cable in metres, $\gamma = (0.1 + j40)\ \text{m}^{-1}$ is the complex propagation constant, and $\omega = 2\pi \times 10^9$ rad/s is the angular frequency. The absolute value of the attenuation in the cable in dB/metre is ______________. **[2017]**

Solution: $\gamma = 0.1 + j40^{-1}\ \text{m}^{-1}$

$$\gamma = \alpha + j\beta\ \ \text{m}^{-1}$$

$\alpha \longrightarrow$ attenuation constant (Np/m or dB/m)

$\beta \longrightarrow$ Phase constant (rad/m or degree/M)

$\therefore\ \ \alpha = 0.1$ Np/m

1 Np = 8.686 dB

$\therefore\ \alpha = 0.8686$ dB/m

Hence, the correct answer is (0.85 to 0.88).

2. A two wire transmission line terminates in television set. The VSWR measured on the line is 5.8. The percentage of power that is reflected from the television set is __________. **[2017]**

Solution: $VSWR = 5.8$

Reflection Coefficient, $K = \dfrac{VSWR - 1}{VSWR + 1}$

$$= \frac{5.8 - 1}{5.8 + 1} = \frac{4.8}{6.8} = 0.7$$

Percentage of power that is reflected

$$|k|^2 \times 100 = (0.7)^2 \times 100 = 49\%$$

Hence, the correct answer is (48.5 to 49.5).

3. The propagation constant of a loose transmission line is $(2 + j5)\ \text{m}^{-1}$ and its characteristic impedance is $(50 + j0)\Omega$ at $\omega = 10^6$ rad s^{-1}. The values of the line constants L, C, R, G are respectively **[2016]**

(A) L = 200 μH/m, C = 0.1 μF/m, R = 50 Ω/m, G = 0.02 S/m

(B) L = 250 μH/m, C = 0.1 μF/m, R = 100 Ω/m, G = 0.04 S/m

(C) L = 200 μH/m, C = 0.2 μF/m, R = 100 Ω/m, G = 0.02 S/m

(D) L = 250 μH/m, C = 0.2 μF/m, R = 50 Ω/m, G = 0.04 S/m

Solution: Propagation constant of a loose transmission line is $(2 + j5)\ \text{m}^{-1}$.

Characteristic impedance is $(50 + j0)\Omega$.

Angular frequency $\omega = 10^6$ rad s^{-1}.

As Z is real, line is a lossless line,

$$\Rightarrow \qquad Z = \sqrt{\frac{L}{C}} = 50$$

$$\beta = \omega\sqrt{LC}$$

$$5 = 10^6 \times \sqrt{LC}$$

$$\Rightarrow \qquad \sqrt{LC} = 5 \times 10^{-6}$$

$$\sqrt{LC} \times \sqrt{\frac{L}{C}} = L = 250\ \mu\text{H/m}$$

$$\Rightarrow \qquad \sqrt{LC} = 5 \times 10^{-6}$$

$$\Rightarrow \qquad C = \frac{25 \times 10^{-12}}{25 \times 10^{-5}}$$

$$= 0.1\ \mu\text{F/m}.$$

Assuming the line to be a low loss distortion less line.

$$\alpha = \sqrt{RG} = 2$$

$$RG = 4$$

$$\frac{R}{L} = \frac{G}{C}$$

or $$\frac{R}{G} = \frac{L}{C} = 2500$$

$$\Rightarrow \quad RG \times \frac{R}{G} = R^2 = 10^4$$

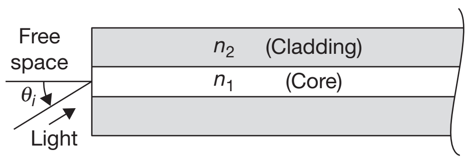
$$R = 100\,\Omega$$

$$RG = 4$$

$$G = 4/100 = 0.04 \text{ S/m.}$$

Hence, the correct option is (B).

4. Light from free space is incident at an angle θ_i to the normal of the facet of a step index large core optical fibre. The core and cladding refractive indices are $n_1 = 1.5$ and $n_2 = 1.4$ respectively



The maximum value of θ_i (in degrees) for which the incident light will be guided in the core of the fibre is ____. **[2016]**

Solution: The refractive index of core $n_1 = 1.5$

Refractive index of core $n_2 = 1.4$

Using the law of optical refraction, we get

$$\sin\theta_i = \sqrt{n_1^2 - n_2^2}$$

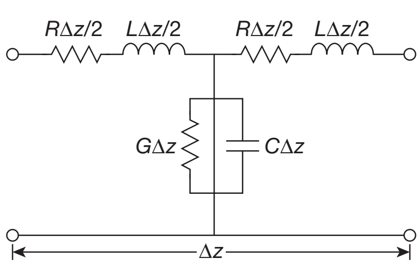
$$\theta_i = \sin^{-1}\left\{\sqrt{(1.5)^2 - (1.4)^2}\right\}$$

$$= \sin^{-1}\{0.538\} = 32.58°$$

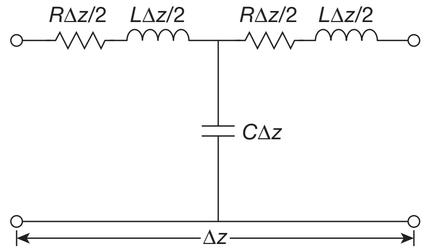
Hence, the correct Answer is (32.58°).

5. A coaxial cable is made of two brass conductors. The spacing between the conductors is filled with Teflon ($\varepsilon_r = 2.1$, $\tan\delta = 0$). Which one of the following circuits can represent the lumped element model of a small piece of this cable having length Δz? **[2015]**

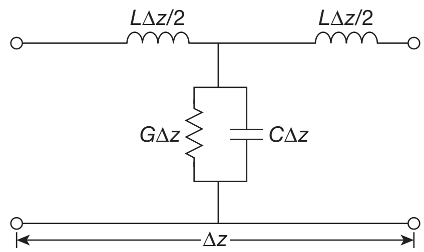
(A)



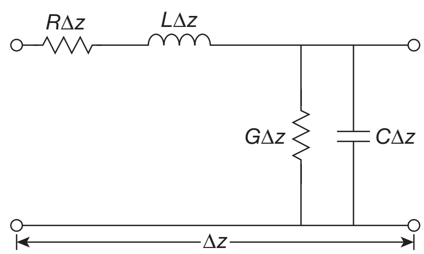
(B)



(C)



(D)



Solution: Given that loss tangent $\tan\delta = 0 = \dfrac{\sigma}{\omega\varepsilon}$

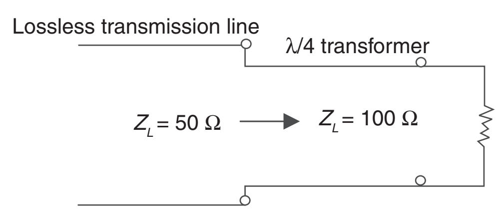
Loss tangent $\tan\delta = 0 = \dfrac{\sigma}{\omega\varepsilon}$

$\Rightarrow \quad \sigma = 0$

So, conductance G = 0.

Hence, the correct option is (B).

6. To maximize power transfer, a lossless transmission line is to be matched to a resistive load impedance via a $\lambda/4$transformer as shown,



The characteristic impedance (in Ω)of the $\lambda/4$ transformer is____. **[2014]**

Solution: (70.72)

For $\lambda/4$ transformer,

$$z_{in} = \frac{z_0^2}{z_L}$$

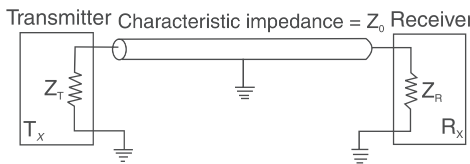
Here, $z_{in} = 50\ \Omega$ and $z_L = 100\ \Omega$

$$z_0 = \sqrt{z_{in} \cdot z_L}$$

$= \sqrt{50 \times 100}$

$z_0 = 70.72\ \Omega$

7. In the following figure, the transmitter Transmission sends a wideband modulated *RF* signal via a coaxial cable to the receiver *Rx* The output impedance Z_T of Transmission, the characteristic impedance Z_0, of the cable and the input impedance Z_R of R_x. are all real.



Which one of the following statements is TRUE about the distortion of the received signal due to impedance mismatch?

(a) The signal gets distorted if $Z_R \neq Z_0$, irrespective of the value of Z_T

(b) The signal gets distorted if $Z_T \neq Z_0$, irrespective of the value of Z_R.

(c) Signal distortion implies impedance mismatch at both ends: $Z_T \neq Z_0$ and $Z_R \neq Z_0$.

(d) Impedance mismatches do NOT result in signal distortion but reduce power transfer efficiency.

[2014]

Solution: (c)

For given problem, following statements *(a)* and *(b)* cannot be true because transmission line can be matched for $z_R \neq z_0$ and $z_T \neq z_0$ by using quarter wavelength transformer.

Statement (d) also not true because signal distortion occurs due to mismatching at both ends i.e. $z_T \neq z_0$ and $z_R \neq z_0$

Therefore (c) is true about the distortion of the received signal.

Hence, the correct option is (c).

8. The return loss of a device is found to be 20 dB. The voltage standing wave ratio (VSWR) and magnitude of reflection coefficient are respectively

(a) 1.22 and 0.1 (b) 0.81 and 0.1

(c) –1 22 and 0.1 (d) 2.44 and 0.2

[2013]

Solution: (a)

Return Loss = $-20 \log_{10}|\Gamma +$ dB

$-20 \log_{10}|\Gamma| = 20$ dB

$\log_{10}|\Gamma| = -1$

$|\Gamma| = 10^{-1} = 0.1$

$$\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1.1}{0.9}$$

VSWR = 1.22

Hence, the correct option is (a).

9. A transmission line of characteristic impedance 50 Ω is terminated by a 50 Ω load. When excited by a sinusoidal voltage source at 10 *GHz*, the phase difference-between two points spaced 2 mm apart on the line is found to be π/4 radians. The phase velocity of the wave along the line is

(a) 0.8×10^8 m/s (b) 1.2×10^8 m/s

(c) 1.6×10^8 m/s (d) 3×10^8 m/s

[2011]

Solution: (c)

$\beta . l = \dfrac{\pi}{4}$ [for l = 2 mm]

$$\beta = \frac{\pi}{4 \times 2 \times 10^{-3}}$$

$$\beta = \frac{1000\pi}{8} \text{ rad/m}$$

$w = 2\pi \times 10 \times 109$ rad/sec

Velocity, $V = \dfrac{w}{\beta} = \dfrac{2\pi \times 10 \times 10^9}{1000\pi} \times 8$

V = 1.6 × 108 m/s

Hence, the correct option is (c).

10. A transmission line has a characteristic impedance of 50 Ω and a resistance of 0 1 Ω/m. If the line is distortionless, the attenuation constant (in Np/m) is

(a) 500 (b) 5

(c) 0.014 (d) 0.002 **[2010]**

Solution: (d)

For distortionless transmission line,

$$\frac{L}{R} = \frac{C}{G}$$

$$z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}}$$

attenuation constant, $\propto = \sqrt{RG} = \sqrt{R} . \dfrac{\sqrt{R}}{Z_0}$

$$\propto = \frac{R}{Z_0} = \frac{0.1}{50}$$

$\propto = 0.002$ Np/m

Hence, the correct option is (d).

11. The VSWR can have any value between

(a) 0 and 1 (b) – 1 and + 1

(c) 0 and ∞ (d) 1 and ∞ **[2002]**

Solution: (d)

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

For, $|\Gamma| = 0$, $VSWR = 1$

For, $|\Gamma| = 1$, $VSWR = \infty$

So, $VSWR$ can have any value between (1 and ∞)

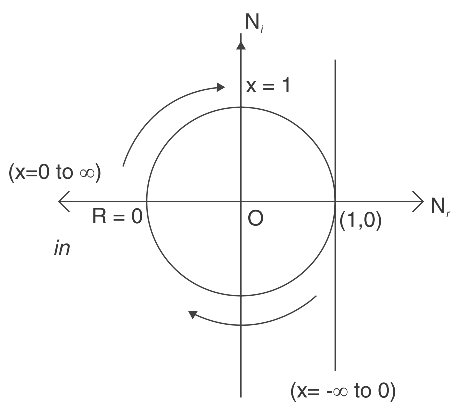
Hence, the correct option is (d).

12. In an impedance Smith chart, a clockwise movement along a constant resistance circle gives rise to

(a) a decrease in the value of reactance
(b) an increase in the value of reactance
(c) no change in the reactance value
(d) no change in the impedance value **[2002]**

Solution: (b)

In clockwise direction along constant resistance circle gives rise to an increase in the value of reactance.



Hence, the correct option is (b).

13. A transmission line is distortion less if

(a) $RL = \frac{1}{GC}$ (b) RL = GC
(c) LG = RC (d) RG -= LC **[2001]**

Solution: (c)

Condition for distortion less is,

$$\frac{R}{L} = \frac{G}{C}$$

Rise time constant = Fall time constant

$RC = LG$

Hence, the correct option is (c).

14. The magnitudes of the open-circuit and short-circuit input impedances of a transmission line are 100 Ω and 25 Ωrespectively. The characteristic impedance of the line is,

(a) 25 Ω (b) 50 Ω
(c) 75Ω (d) 100Ω **[2000]**

Solution: (b)

$z_{sc} = 25\ \Omega$

$Z_{oc} = 100\ \Omega$

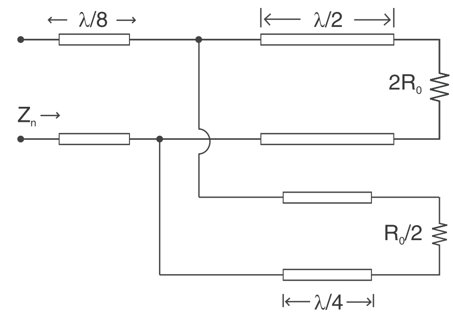
$$z_0 = \sqrt{z_{sc} \cdot z_{oc}}$$

$$= \sqrt{25 \times 100}$$

$$z_0 = 50\Omega$$

Hence, the correct option is (b).

15. All transmission line section in Figure, have a characteristic impedance $R_0 + j0$. The input impedance Z_{in} equals



(a) $\frac{2}{3}R_0$ (b) R_0

(c) $\frac{3}{2}R_0$ (d) $2R_0$ **[1998]**

Solution: (b)

$$R_1\left[l = \frac{\lambda}{2}\right] = 2R_0$$

$$R_2\left[l = \frac{\lambda}{4}\right] = \frac{z_0^2}{z_L}$$

$$= \frac{R_0^2}{R_0/2}$$

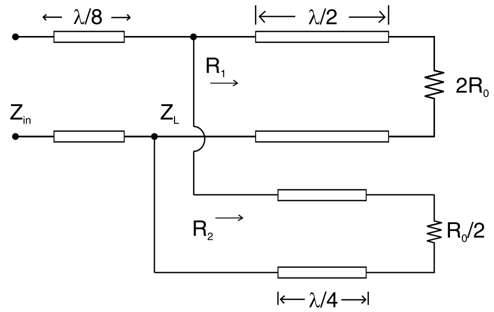
$R_2 = zR_0$

$$z_L = (R_1 \| R_2) = \frac{2R_0 \times 2R_0}{4R_0} = R_0$$

$$z_{in} = [l = \lambda/8] = z_0\left[\frac{z_L + jz_0}{z_0 + jz_L}\right]$$

$$z_{in} = R_0\frac{[R_0 + jR_0]}{(R_0 + jR_0)}$$

$$z_{in} = R_0$$



Hence, the correct option is (b).

16. A transmission line of 50 Ω characteristic impedance is terminated with a 100 Ω resistance The minimum impedance measured on the line is equal to

(a) 0Ω (b) 25Ω
(c) 50 Ω (d) 100Ω **[1997]**

Solution: (b)

$z_0 = 50\ \Omega$

$z_L = 100\ \Omega$

$$z_{in(\min)} = \frac{z_0^2}{z_L} = \frac{50 \times 50}{100}$$

$$z_{in(\min)} = 25\ \Omega$$

Hence, the correct option is (b).

17. A lossless transmission line having 50 Ω characteristic impedance and length λ/4 is short circuited at one end and connected to an ideal voltage source of 1 *V* at the other end. The current drawn from the voltage sources is

(a) 0 (b) 0.02 A
(c) ∞ (d) none of the these

[1996]

Solution: (a)

For Quarter wave transformer, $z_{in} = \dfrac{z_0^2}{z_L}$

$z_L = 0\ \Omega$ [short circuited]

$z_{in} = \infty$ = open circuit

$$I_s = \frac{V_s}{z_{in}} = \frac{V_s}{\infty} = 0\ Amp$$

Hence, the correct option is (a).

18. The capacitance per unit length and the characteristic impedance of a lossless transmission line are *C* and Z_0 respectively. The velocity of a travelling wave on the transmission line is

(a) Z_0C (b) $1/(Z_0C)$
(c) Z_0/C (d) C/Z_0 **[1996]**

Solution: (b)

$$z_0 = \sqrt{\frac{L}{C}},\ V = \frac{1}{\sqrt{LC}}$$

$$\sqrt{L} = z_0\sqrt{C}$$

So,

$$V = \frac{1}{z_0 C}$$

Hence, the correct option is (b).

19. A load impedance, (200 + *j*0) Ω is to be matched to a 50 Ω lossless transmission line by using a quarter wave linetransformer (Q W T). The characteristic impedance of the QWT required is......... **[1994]**

Solution: (100)

$z_L = (200 + j0)$

$z_{in} = 50\ \Omega$

For quarter wave transformer,

$$z_{in} = \frac{z_0^2}{z_L}$$

$$z_0^2 = \sqrt{200 \times 50}$$

$$z_0 = 100\Omega$$

Two-marks Questions

1. There are two photolithography systems : one with light source of wavelength λ_1 = 156nm (system 1) and another with light source of wavelength λ_2 = 325nm (system 2). Both photolithography systems are otherwise identical. If the minimum feature sizes that can be realized using System 1 and System 2 are $L_{\min 1}$ and $L_{\min 2}$ respectively, the ratio $L_{\min 1}/L_{\min 2}$ (correct to two decimal places) is ______. **[2018]**

Solution: We know that

$$L \propto \lambda$$

Therefore

$$\frac{L_{\min 1}}{L_{\min 2}} = \frac{\lambda_1}{\lambda_2}$$

$$\frac{156\ \text{nm}}{325\ \text{nm}} = 0.48$$

Hence, the correct answer is 0.47 to 0.51.

2. A lossy transmission line has resistance per unit length *R* = 0.05 Ω/m. The line is distortionless and has characteristic impedance of 50 Ω. The attenuation constant (in Np/m, correct to three decimal places) of the line is ______. **[2018]**

Solution: resistance per unit length $R = 0.05\ \Omega/\text{m}$

Distortionless line $\rightarrow \dfrac{R}{L} = \dfrac{G}{C}$

characteristic impedance $Z_0 = 50\ \Omega$

We know that

$$Z_0 = \sqrt{\frac{L}{C}} = 50$$

$$\Rightarrow \frac{L}{C} = 2500$$

$$\frac{L}{C} = \frac{R}{G} = 2500$$

$$\Rightarrow \qquad G = \frac{0.05}{2500} = 2 \times 10^{-5}\ \mho/\text{m}$$

Alternation constant can be calculated as

$$\alpha = \sqrt{RG}$$

$$= \sqrt{0.05 \times 2 \times 10^{-5}} = 0.001\ \text{Np/m}$$

$$= 1\ \text{mNp/m}$$

Hence, the correct answer is 0.001.

3. An optical fiber is kept along the $\hat{z}$ direction. The refractive indices for the electric fields along $\hat{x}$ and $\hat{y}$ directions in the fiber are $n_x = 1.5000$ and $n_y = 1.5001$, respectively ($n_x \neq n_y$ due to the imperfection in the fiber cross-section). The free space wavelength of a light wave propagating in the fiber is 1.5 µm. If the light-wave is circularly polarized at the input of the fiber, the minimum propagation distance after which it becomes linerly polarized, in centimeters, is ______________. **[2017]**

Solution: The phase difference between the electric field x and y components is $\dfrac{\pi}{2}$ (circular polarization).

The wave polarization changes to linear, when the wave travels a minimum distance such that the phase difference between the E_x and E_y gets chaged from $\dfrac{\pi}{2}$ to π.

$$Z_{\min} k_x - Z_{\min} K_y = \frac{\pi}{2}$$

$$Z_{\min}\left[\frac{\omega}{\mu_{px}} - \frac{\omega}{\mu_{py}}\right] = \frac{\pi}{2}$$

$$\therefore \omega = 2\pi f$$

$$V_P = \frac{c}{n}$$

$$n = \sqrt{\varepsilon_r}$$

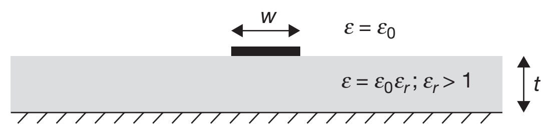
$$n_x = 1.5\ \mu\text{m}$$

$$m_y = 1.5001\ \mu\text{m}$$

$$Z_{\min} = 0.375\ \text{cm}$$

Hence, the correct answer is (0.36 to 0.38).

4. A lossless micro strip transmission line consists of a trace of width w. It is drawn over a practically infinite ground plane and is separated by a dielectric slab of thickness t and relative permittivity $\varepsilon_r > 1$. The inductance per unit length and the characteristic impedance of this line are L and z_0, respectively. **[2016]**



Which one of the following inequalities is always satisfied?

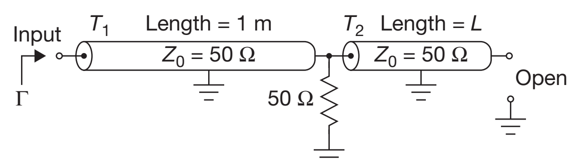
(A) $Z_0 > \sqrt{\dfrac{Lt}{\varepsilon_0 \varepsilon_r w}}$ (B) $Z_0 < \sqrt{\dfrac{Lt}{\varepsilon_0 \varepsilon_r w}}$

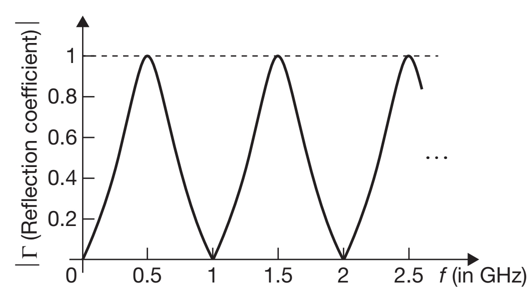
(C) $Z_0 > \sqrt{\dfrac{LW}{\varepsilon_0 \varepsilon_r t}}$ (D) $Z_0 < \sqrt{\dfrac{LW}{\varepsilon_0 \varepsilon_r t}}$

Solution: From the given fig, where relative permittivity are varying, the expression for characteristics impedance can be better inferred from option (B).

Hence, the correct option is (B).

5. A microwave circuit consisting of lossless transmission lines T_1 and T_2 is shown in the figure. The plot shows the magnitude of the input reflection coefficient T_L as a function of frequency f. The phase velocity of the signal in the transmission lines is 2×10^8 m/s. **[2016]**





The length L (inn meters) of T_2 is _____.

Solution: We know that

$$Z_{oc} = jZ_0 \cot(\beta L)$$

Effective Load impedance for 1 m long line is

$$Z_L = 50 \parallel [-jZ_0 \cot \beta L]$$

for $\qquad \gamma = 0,\ Z_L = 50$

i.e. $\qquad -jZ_0 \cot \beta L = \infty$

$$\beta L = \pi$$

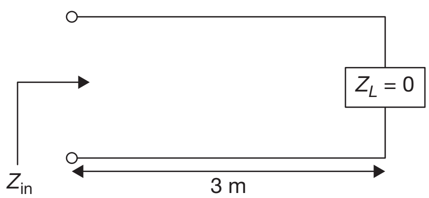
$$\omega\sqrt{LC} \times L = \pi$$

$$2\pi \times 10^9 \times \frac{1}{2\times10^8} \times L = \pi$$

$$\Rightarrow \qquad L = \frac{1}{10} = 0.1 \text{ m}$$

Hence, the correct Answer is (0.1).

6. Consider the 3 m long lossless air-filled transmission line shown in the figure. It has a characteristic impedance of 120 π Ω, is terminated by a short circuit, and is excited with a frequency of 37.5 MHz. What is the nature of the input impedance (Z_{in})? **[2015]**



(A) Open (B) Short
(C) Inductive (D) Capacitive

Solution: l = 3 m

$$\eta = 120\,\pi\,\Omega$$

$$f = 37.5 \text{ MHz}$$

$$Z_L = 0$$

$$Z_{in} = Z_o\left[\frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}\right]\Omega$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{C}{f} = \frac{3\times10^8}{37.5\times10^6} = 8 \text{ m}$$

$$\beta l = \frac{2\pi}{8}\times 3 = 3\pi/4$$

$$Z_{in} = -jZ_o \tan\beta l$$

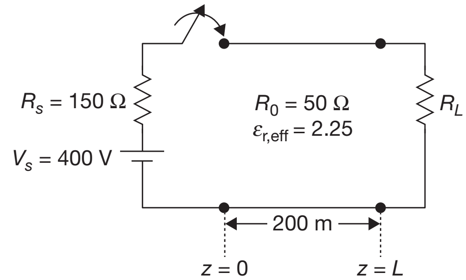
$$= -jZ_o \tan\frac{3\pi}{4}$$

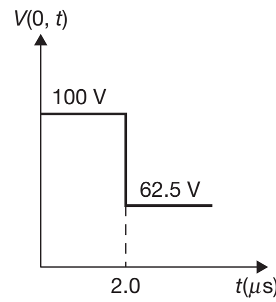
$$= -jZ_o$$

So Z_{in} represents capacitive.

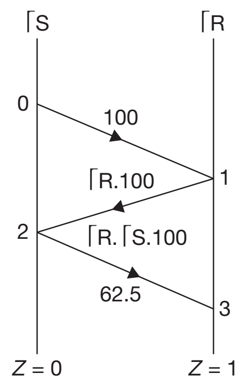
Hence, the correct option is (D).

7. A 200 m long transmission line having parameters shown in the figure is terminated into a load R_L. The line is connected to a 400 V source having source resistance R_S through a switch, which is closed at $t = 0$. The transient response of the circuit at the input of the line ($z = 0$) is also drawn in the figure. The value of R_L (in Ω) is ______. **[2015]**





Solution:



Given $V(t = 2\,\mu s\,,\, Z = 0) = 62.5$

$$62.5 = V(t = 0, z = 0) + V(t = 1, z = 0) + V\,(t = 2, z = 0)$$

$$62.5 = 100 + \Gamma_R\,.\,100 + \Gamma_R\,.\,\Gamma_S.\,100$$

$$\Gamma_R = \frac{R_L - 50}{R_L + 50}$$

and

$$\Gamma_S = \frac{150 - 50}{150 + 50} = \frac{1}{2}.$$

$$0.625 = 1 + \Gamma_R + 0.5\,\Gamma_R$$

$$1.5\,\Gamma_R = 0.625 - 1$$

$$\Gamma_R = \frac{-1}{4}.$$

$$\therefore \qquad R_L = 30\,\Omega\,.$$

Hence, the correct Answer is (29 to 31).

8. A coaxial capacitor of inner radius 1 mm and outer radius 5 mm has a capacitance per unit length of 172 pF/m. If the ratio of outer radius to inner radius is doubled, the capacitance per unit length (in pF/m) is ______. **[2015]**

Solution: Co-axial capacitor $C = \dfrac{2\pi\varepsilon}{\ln\left(b/a\right)}$ F/m

$$C \,\alpha\, \frac{1}{\ln\left(b/a\right)}$$

$$\Rightarrow \quad \frac{172}{C_2} = \frac{\ln(10)}{\ln(5)}$$

$\Rightarrow$ $C_2 = 120.22$ pF/m

Hence, the correct Answer is (120 to 120.4).

9. The input impedance of a $\dfrac{\lambda}{8}$ section of a lossless transmission line of characteristic impedance 50Ω is found to be real when the other end is terminated by a load $Z_L = (R + jX)\ \Omega$. If X is 30Ω the value of R (in Ω) is______. **[2014]**

Solution: (40)

For $l = \lambda/8$

$$z_{in} = z_0 \left[\frac{z_1 + jz_0}{z_0 + jz_L}\right]$$

$z_0 = 50\ \Omega$, $z_L = R + jx = R + j30$

z_{in} = real

$$z_{in} = 50\left[\frac{R + j30 + j50}{50 + jR - 30}\right] = 50\left[\frac{R + j80}{20 + jR}\right]$$

$$z_{in} = 50\frac{[R + j80][20 - jR]}{(20^2 + R^2)}$$

$$= \frac{50[20R + 80R]}{(20^2 + R^2)} + \frac{j50[1600 - R^2]}{(20^2 + R^2)}$$

Since, z_{in} is real so, its imaginary part must be zero.

$1600 - R^2 = 0$

$R = 40\Omega$

10. For a parallel plate transmission line, let v be the speed of propagation and Z be the characteristic impedance. Neglecting fringe effects, a reduction of the spacing between the plates by a factor of two results in

(a) halving of v and no change in Z
(b) no changes in v and halving of Z
(c) no change in both v and z
(d) halving of both v and z **[2014]**

Solution: (b)

For parallel plate,

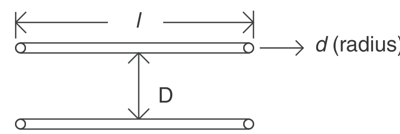
$$C = \frac{\pi l \in}{\ln(D/d)}$$

$$L = \frac{\mu}{\lambda} l . \ln\left(\frac{D}{d}\right)$$

$$\eta = \sqrt{\frac{L}{C}} = \sqrt{\frac{\frac{\mu}{\pi}\ln\left(\frac{D}{d}\right)}{\frac{\pi \in}{\ln}\left(\frac{D}{d}\right)}} = \frac{\ln\left(\frac{D}{d}\right)}{\pi}\sqrt{\frac{\mu}{\in}}$$

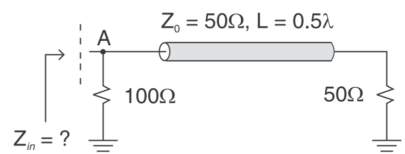
$$V = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu t}}$$

Now, reduction in D results no change in V but having of z.



Hence, the correct option is (b).

11. In the transmission line shown, the impedance Z_{in} (in ohms) between node A and the ground is



[2014 : 2 Marks, Set-2]

Solution: (33.33)

$z_{in} = (z_1 \parallel z_2)$

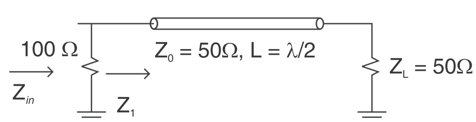
$z_2 = 100\ \Omega$

For, $l = \lambda/2$, $\beta l = \pi$

$z_{in^1} = z_L$

$\therefore z_1 = 50\ l$

$$z_{in} = \frac{100 \times 50}{100 + 50} = 33.3\Omega$$



12. A transmission line with a characteristic impedance of 100 Ω is used to match a 50 Ω section to a 200 Ωsection. If the matching is to be done both at 429 MHz and 1 GHz, the length of the transmission line can be approximately

(a) 82.5 cm (b) 1.05 m
(c) 1.58 m (d) 1.75 m **[2012]**

Here, $z_i = 50\ \Omega$, $z_L = 200\ \Omega$

$$z_0 = \sqrt{50 \times 200} = 100\Omega$$

For making, quarter wavelength transformer

For given, frequency, $\lambda = \frac{c}{f}$ and, l will be, $\lambda/4$

13. A transmission line of characteristic impedance 50 Ω is terminated in a load impedance Z_L. The VSWR of the line is measured as 5 and the first of the voltage maxima in the line is observed at a distance of λ/4 from the load. The value of Z_L is

(a) 10 Ω
(b) 250 Ω
(c) (19.23 + j46.15)Ω
(d) (19.23 – j46.15)Ω **[2011]**

Solution: (a)

The distance between maxima and minima is λ/4.

Thus minima at load and reflection coefficient are negative.

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = 5 = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$= -\frac{2}{3}$$

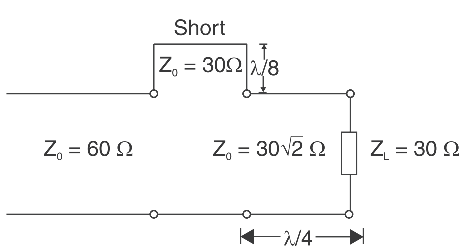
$$= \frac{zl - zo}{zl + zo}$$

$$-\frac{2}{3} = \frac{zl - 50}{zl + 50}$$

$$zl = 10$$

Hence, the correct option is (a).

14. In the circuit shown, all the transmission line sections are lossless. The Voltage Standing Wave Ratio (VSWR) on the 60 Ω line is



(a) 1.00 (b) 1.64
(c) 2.50 (d) 3.00 **[2010]**

Solution: (b)

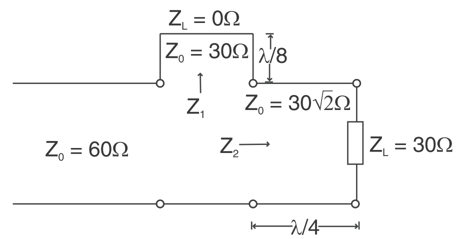
For $l = \lambda/8$, $z_L = 0\ \Omega$

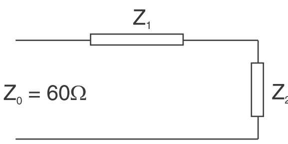
$$z_{in} = z_0 \frac{[z_L + j z_0]}{[z_0 + j z_L]}$$

$z_l = jz_0 = j\ 30\ \Omega$

For $l = \lambda/y$, $z_L = 30\ \Omega$

$$z_{in} = \frac{z_0^2}{z_L} = \frac{30 \times 30 \times 2}{30} = 60\Omega$$





$$Z_L^1 = Z_1 + Z_2$$
$$= 60 + j30$$

$$\Gamma = \frac{z_L^1 - z_0}{z_L^1 + z_0} = \frac{60 + j30 - 60}{60 + j30 + 60}$$

$$\Gamma = \frac{j30}{120 + j30} = \frac{j}{4 + j}$$

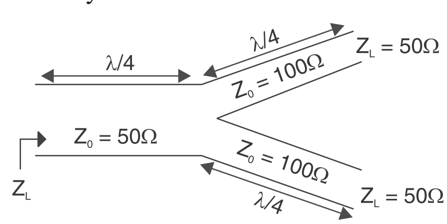
$$|\Gamma| = \frac{1}{\sqrt{17}}$$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$VSWR = 1.64$

Hence, the correct option is (b).

15. A transmission line terminates in two branches each of length λ/4, as shown. The branches are terminated by 50 Ω loads. The lines are lossless and have the characteristic impedances shown. Determine the impedance Z_0 as seen by the source.

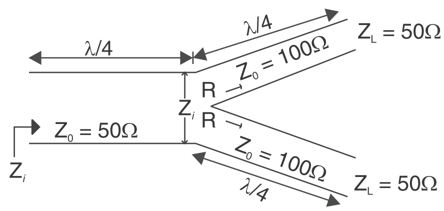


(a) 200Ω (b) 100Ω
(c) 50 Ω (d) 25 Ω **[2009]**

Solution: (d)

$R = 200\ \Omega$

$z_i = 25\ \Omega$



Hence, the correct option is (d).

16. One end of a loss-less transmission line having the characteristic impedance of 75Ω and length of 1 cm is short-circuited. At 3 GHz, the input impedance at the other end of the transmission line is

(a) 0 (b) Resistive

(c) Capacitive (d) Inductive **[2008]**

Solution: (d)

For short-circuit load, $z_L = 0$

$z_{in} = j\, z_0 \tan(\beta l)$

$$\beta = \frac{2\pi}{\lambda} \text{ and } \lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^9} = 10\,\text{cm}$$

$$\beta = \frac{2\pi}{10} = \frac{\pi}{5} \text{ rad/cm}$$

$l = 1$ cm

$$\beta . l = \frac{\pi}{5} = 36°$$

Z_{in} = J75 tan 36°

$z_{in} = j54.49\Omega$

Input impedance is inductive

Hence, the correct option is (d).

17. In the design of a single mode step index optical fiber close to upper cut-off, the single-mode operation is NOT preserved if .

(a) radius as well as operating wavelength are halved

(b) radius as well as operating wavelength are doubled

(c) radius is halved and operating wavelength is doubled

(d) radius is doubled and operating wavelength is halved **[2008]**

Solution: (d)

Cut-off frequency, $F_c = \dfrac{2\pi a}{\lambda}\sin\theta$

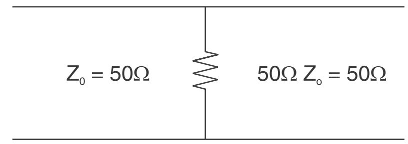
a = radius of core, λ = wave length

For single mode step index, F_c must lie between $0 < F_c <$ 2.405.

Out of four options, (d) radius doubled and λ halved lead to $F_c > 2.405$.

Hence, the correct option is (d).

18. A load of 50 Ωis connected in shunt in a 2-wire transmission line of Z_0 = 50Ω as shown in the figure. The 2-port scattering parameter matrix (S-matrix) of the shunt element is



(a) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{4} & -\frac{3}{4} \\ -\frac{3}{4} & \frac{1}{4} \end{bmatrix}$

[2007]

Solution: (b)

Since, $z_L = z_0$, so impedance are matched thus no reflection occurs at load and source.

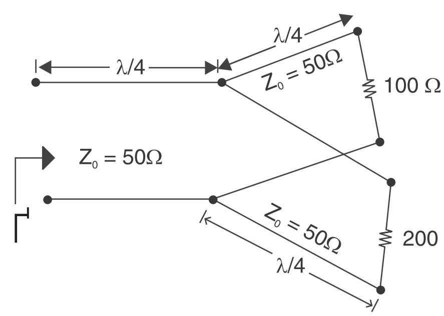
$S_{11} = S_{22} = 0$

S = [0 1]

[1 0]

Hence, the correct option is (b).

19. The parallel branches of a 2-wire transmission line are terminated in 100 Ω and 200 Ω resistors as shown in the figure. The characteristic impedance of the line is Z_0 = 50 Ω and each section has a length of λ/4. The voltage reflection coefficient Γat the input is



(a) $-i\dfrac{7}{5}$ (b) $\dfrac{-5}{7}$

(c) $j\dfrac{5}{7}$ (d) $\dfrac{5}{7}$ **[2007]**

Solution: (d)

For, $l = \dfrac{\lambda}{4}$

$$z_{in} = \frac{z_0^2}{z_L}$$

$$R_1 = \frac{50 \times 50}{100} = 25\Omega$$

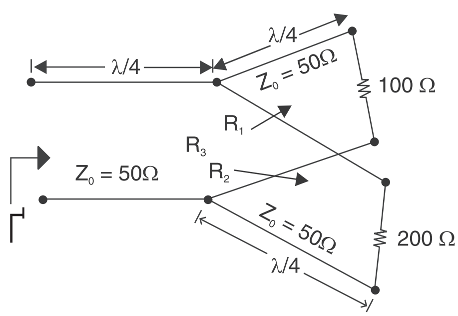
$$R_2 = \frac{50\times 50}{200} = \frac{25}{2}\Omega$$

$$R_3 = (R_1 \| R_2) = \frac{25\times\frac{25}{2}}{25\times\frac{25}{2}} = \frac{25}{3}\Omega$$

$$z_{in} = \frac{50\times 50}{25/3} = 300\Omega$$

$$\Gamma = \frac{z_{in} - z_0}{z_{in} + z_0} = \frac{300-50}{300+50}$$

$$\Gamma = \frac{250}{350} = \frac{5}{7}$$



Hence, the correct option is. (d)

Common data for Questions 20 & 21:

A 30-Volts battery with zero source resistance is connected to a coaxial line of characteristic impedance of 50 Ohms at t = 0 second and terminated in an unknown resistive load. The line length is such that it takes 400 us for an electromagnetic wave to travel from source end to load end and vice-versa. At t = 400 μs, the voltage at the load end is found to be 40 Volts.

20. The load resistance is

(a) 25 Ohms (b) 50 Ohms
(c) 75 Ohms (d) 100 Ohms **[2006]**

Solution: (d)

$v_t = v_r + v_i$

$v_i = 30\ v,\ v_t = 40\ v$

$v_r = 10\ v$

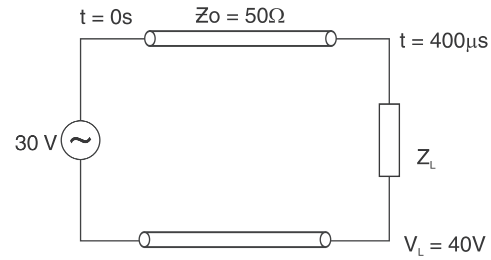
$$\Gamma = \frac{V_r}{V_i} = \frac{10}{30} = \frac{1}{3}$$

$$\Gamma_{[\text{at load}]} = \frac{z_L - z_0}{z_L + z_0} = \frac{1}{3}$$

$3 \times (z_L - 50) = z_L + 50$

$3z_L - 150 = z_L + 50$

$z_L = 100\ \Omega$



Hence, the correct option is (d).

21. The steady-state current through the load resistance is

(a) 1.2 Amps (b) 0.3 Amps
(c) 0.6 Amps (d) 0.4 Amps **[2006]**

Solution: (b)

$$I_L = \frac{V_L}{Z_L} = \frac{30}{100}$$

$I_L = 0.3$ Amps

Hence, the correct option is (b).

22. Characteristic impedance of a transmission line is 50Ω. Input impedance of the open-circuited line is Z_{oc}. = 100 + *j* 150 Ω. When the transmission line is short-circuited, then value of the input impedance will be

(a) 50 Ω (b) 100 + *j*150Q.Ω
(c) 7.69 + *j*11.54 Ω (d) 7.69 – *j* 11.54 Ω **[2005]**

Solution: (d)

$z_0 = 50\ \Omega$

$z_{oc} = 100 + j\ 150\ \Omega$

$z_0 = \sqrt{z_{oc} \cdot z_{sc}}$

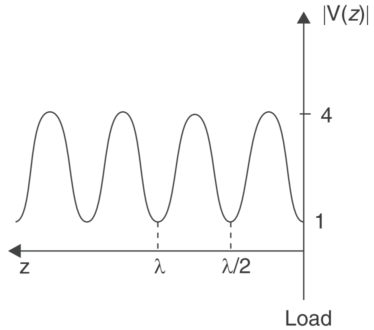
$$z_{sc} = \frac{z_0^2}{z_{oc}} = \frac{50 \times 50}{100 + j150}$$

$$z_{sc} = \frac{50}{2+j3} = \frac{50(2-j3)}{13}$$

$$z_{sc} = 7.69 - j11.54\Omega$$

Common Data for Questions 23 and 24

Voltage standing wave pattern in a lossless transmission line with characteristic impedance 50 ohms and a resistive load is shown in the figure.



Hence, the correct option is (d).

23. The value of the load resistance is

(a) 50 Ω (b) 200 Ω

(c) 12.5 Ω (d) 0Ω **[2005]**

Solution: (c)

$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{4}{1} = 4$$

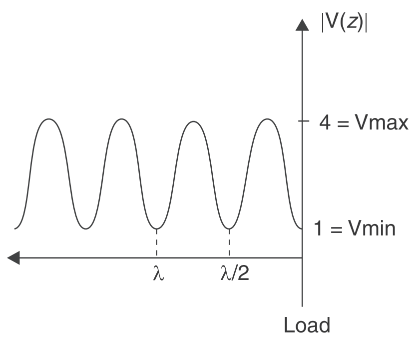
$$VSWR = \frac{1+\Gamma}{1-\Gamma} = 4$$

$$1 + \Gamma = 4 - 4\Gamma$$

$$|\Gamma| = \frac{3}{5}$$

Since, V_{min} at load then, $z_L < z_0$

$$VSWR = \frac{z_0}{z_L} = 4 \Rightarrow z_L = \frac{50}{4} = 12.5\Omega$$



Hence, the correct option is (c).

24. The reflection coefficient is given by

(a) –0.6 (b) –1

(c) 0.6 (d) 0 **[2005]**

Solution: (a)

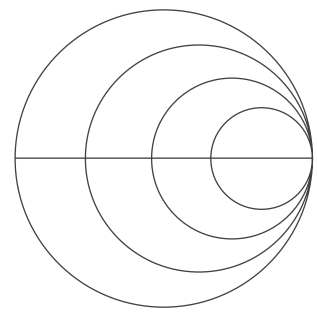
$$\frac{(1+\Gamma)}{(1-\Gamma)} = \frac{z_0}{z_L} = \frac{1}{4}$$

$$4 + 4\Gamma = 1 - \Gamma$$

$$\Gamma = \frac{-3}{5} = -0.6$$

Hence, the correct option is (a).

25. Many circles are drawn in a Smith chart used for transmission line calculations. The circles shown in the figure represent



(a) unit circles

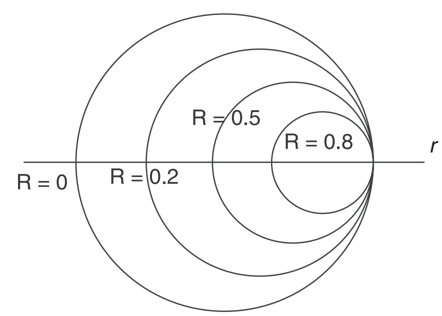
(b) constant resistance circles

(c) constant reactance circles

(d) constant reflection coefficient circles **[2005]**

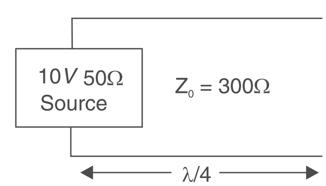
Solution: (b)

The circles shown in the figure represent constant resistance circles.



Hence, the correct option is (b).

26. Consider a 300 Ω, quarter-wave long (at 1 GHz) transmission line as shown in the figure It is connected to a 10V, 50 Ωsource at one end and is left open circuited at the other end. The magnitude of the voltage at the open circuit end of the line is



(a) 10 V (b) 5V

(c) 80 V (d) 60/7 V **[2004]**

Solution: (c)

$$I = \frac{10V}{50\Omega} = \frac{1}{5} \text{ Amp}$$

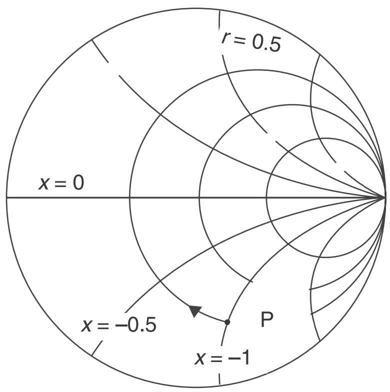
$$v_L = z_0 I$$

$$= \frac{300 \times 1}{5}$$

$$v_L = 60\ v$$

Hence, the correct option is (c).

27. Consider an impedance $Z = R + jX$ marked with point P in an impedance Smith chart as shown in the figure. The movement from point P along a constant resistance circle in the clockwise direction by an angle 45° is equivalent to



(a) adding an inductance in series with Z
(b) adding a capacitance in series with Z
(c) adding an inductance in shunt across Z
(d) adding a capacitance in shunt across Z **[2004]**

Solution: (a)

On movement in clockwise direction by an angle 45° on constant resistance circle is equivalent to increase in reactance which is addition of inductance in series with z.

$Z = R_0$

$Z = R_0 + jX$ [in clockwise direction]

Hence, the correct option is (a).

28. A plane electromagnetic wave propagating in free space is incident normally on a large slab of loss-less, non-magnetic, dielectric material with $\epsilon > \epsilon_0$. Maxima and minima are observed when the electric field is measured in front of the slab. The maximum electric field is found to be 5 times the minimum field. The intrinsic impedance of the medium should be

(a) 120 πΩ (b) 60 πΩ
(c) 600 πΩ (d) 24 πΩ **[2004]**

Solution: (d)

$E_{max} = 5\, E_{min}$

$$VSWR = \frac{E_{\max}}{E_{\min}} = 5$$

$$\frac{1+|\Gamma|}{1-|\Gamma|} = 5 \Rightarrow |\Gamma| = \frac{2}{3}$$

$$|\Gamma| = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

$\epsilon_1 = 1,\ \epsilon_2 = ?$

$$\frac{1-\sqrt{\epsilon_2}}{1+\sqrt{\epsilon_2}} = \frac{-2}{3}$$

$[\eta_2 < t\eta_1]$

$\epsilon_2 = 25$

$$\eta_2 = \frac{120\pi}{\sqrt{\epsilon_2}} = \frac{120\pi}{5}\,\Omega$$

$$\eta_2 = 24\pi\,\Omega$$

Hence, the correct option is (d).

29. A lossless transmission line is terminated in a load which reflects a part of the incident power. The measured $VSWR$ is 2. The percentage of the power that is reflected back is

(a) 57.73 (b) 33.33
(c) 0.11 (d) 11.11 **[2004]**

Solution: (d)

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = 2$$

$$1+|\Gamma| = 2 - 2|\Gamma|$$

$$|\Gamma| = \frac{1}{3}$$

Pref = $|\Gamma|^2$ Pinc

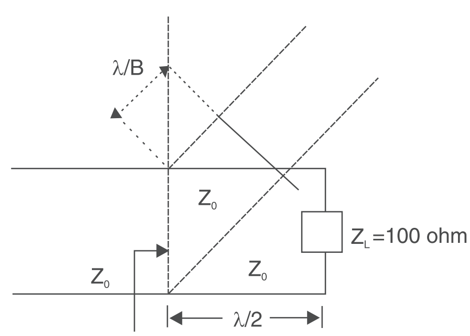
$$\%P_{ref} = \frac{|\Gamma|^2 P_{inc} \times 100}{P_{inc}}$$

$$= \frac{1}{9} \times 100\%$$

$$= 11.11\%$$

Hence, the correct option is (d).

30. A short-circuited stub is shunt connected to a transmission line as shown in the figure. If Z_0=50 Ω, the admittance Y seen at the junction of the stub and the transmission line is



(a) (0.01 – j0 02)mho
(b) (0.02 – j0.01)mho

(c) (0.04 + j0 02)mho
(d) (0 02 + j0) mho **[2003]**

Solution: (a)

$z_0 = 50\ \Omega$

y_s = admittance at junction of stub

β.ls = λ/8

$$\beta.ls = \frac{2\pi}{\lambda}.\frac{\lambda}{8} = \frac{\pi}{4}$$

$$z_{ins} = z_0\left[\frac{z_L\cos\frac{\pi}{4} + jz_0\sin\frac{\pi}{4}}{z_0\cos\frac{\pi}{4} + jz_L\sin\frac{\pi}{4}}\right]$$

Here, $z_L = 0$ [short circuit stub]

$$z_{ins} = z_0\ j\tan\frac{\pi}{4}$$

$z_{ins} = j\ z_0$ = j 50 Ω

$$y_s = \frac{1}{z_{ins}} = -j\,0.02\,mho$$

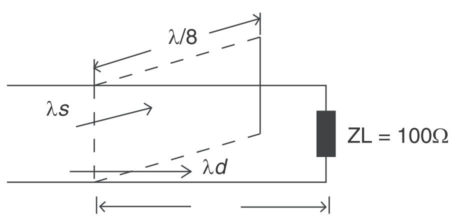
y_d = admittance at junction of transmission line

lt = λ/2

$$\beta.lt = \frac{2\pi}{\lambda}\ .\ \frac{\lambda}{2} = \pi$$

$z_{int} = z_L = 100\ \Omega$

$$y_d = \frac{1}{z_{\text{int}}} = 0.01\,mho$$



Hence, the correct option is (a).

31. A uniform plane electromagnetic wave incident normally on a plane surface of a dielectric material is reflected with a *VSWR* of 3. What is the percentage of incident power that is reflected?
(a) 10% (b) 25%
(c) 50% (d) 75% **[2001]**

Solution: (b)

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

1+|Γ'| = 3[1–|Γ'|

4|Γ| = 2

$$|\Gamma| = \frac{1}{2} = 0.5$$

$$\%\ P_r = \frac{|\Gamma|^2 P_i}{P_i}\times 100 = (0.5)^2\times 100$$

$$\%\ P_r = 25\%$$

Hence, the correct option is (b).

32. In a twin-wire transmission line in air, the adjacent voltage maxima are at 12.5 cm and 27.5 cm. The operating frequency is
(a) 300 MHz (b) 1 GHz
(c) 2GHz (d) 6.28 GHz **[1999]**

Solution: (b)

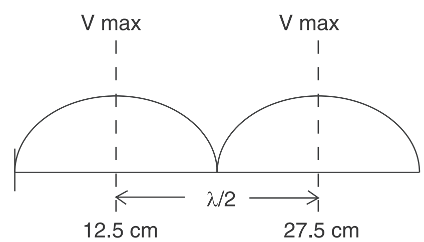
λ/2 = (27.5 – 12.5) cm

$$\frac{1}{2} = 15\text{CM}$$

λ = 30 cm = 0.3 m

$$f = \frac{c}{\lambda} = \frac{3\times 10^8\ \text{m/s}}{0.3\,\text{m}}$$

f = 1 GHz



Hence, the correct option is (b).

33. In air, a lossless transmission line of length 50 cm with L = 10uH/mC = 40pF/m is operated at 25 MHz. Its electrical path length is
(a) 0.5 meters (b) λ meters
(c) π /2 radians (d) 180 degrees
[1999]

Solution: (c)

$$V = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\times 10^{-6}\times 40\times 10^{-12}}}$$

V = 0.5 × 10⁸ m/s

$$\lambda = \frac{v}{f} = \frac{0.5\times 10^8}{25\times 10^6} = 2\,\text{m}$$

$$\beta = \frac{2\pi}{\lambda} = \pi$$

Electrical path length is $\beta l = \frac{\pi \times 50}{100}$

$$\beta l = \frac{\pi}{2} \text{ radians}$$

Hence, the correct option is (c).

34. If a pure resistance load, when connected to a lossless 75 ohm line, produces a *VSWR* of 3 on the line, then the load impedance can only be 25 ohms. True/False (Give Reason) **[1994]**

Solution: (FALSE)

For pure resistance load, say R_L

Lossless line, $R_0 = 75\ \Omega$

$$VSWR = \frac{R_L}{R_0}$$

(If, $R_L = R_0$)

or, $VSWR = \frac{R_0}{R_L}$ (If, $R_0 > R_L$)

For, $VSWR = 3$

So, $R_L = 3R_0 = 225\Omega$ $[R_L > R_0]$

or $R_L = \frac{R_0}{3} = 25\Omega$ $[R_L < R_0]$

Now, $\frac{1+|\Gamma|}{1-|\Gamma|} = 3$

$1+|\Gamma| = 3-3|\Gamma||$

$4|\Gamma| = 2$

$$|\Gamma| = \frac{1}{2}$$

Now, $\Gamma = \frac{1}{2}$ or $-\frac{1}{2}$

For, $\Gamma = \frac{1}{2}$, $z_L = 225\ \Omega$

and for $\Gamma = \frac{-1}{2}$, $z_L = 25\ \Omega$

Therefore, statement the load impedance can only be 25 Ω is false.

35. Consider a transmission line of characteristic impedance 50 ohm. Let it be terminated at one end by ($+j$ 50) ohm. The *VSWR* produced by it in the transmission line will be

(a) +1 (b) 0
(c) ∞ (d) $+j$ **[1993]**

Solution: (c)

$z_0 = 50$ ohm

$Z_L = j50$ ohm

$$\Gamma = \frac{z_L - z_0}{z_L + z_0} = \frac{j50 - 50}{j50 + 50} = \frac{j-1}{j+1}$$

$|\Gamma| = 1$

$$VSWR = \frac{1+|\Gamma^1|}{1-|\Gamma^1|} = \infty$$

Hence, the correct option is (c).

36. A transmission line whose characteristic impedance is a pure resistance

(a) must be a lossless line
(b) must be a distortionless line
(c) may not be a lossless line
(d) may not be a distortionless line **[1992]**

Solution: (b) and (c)

For a distortion less line

RC = LG

$\alpha - \sqrt{RG}$

$\beta = \omega\sqrt{LC}$

$$Z_o = R_o = \sqrt{\frac{L}{C}}$$

R = 0

G = 0

α = 0

$\beta = \omega\sqrt{LC}$

$$Z_o = R_o = \sqrt{\frac{L}{C}}$$

A lossless line is always a distortion less line but a distortion less line may or may not be lossless line.

Hence, the correct option is (b) and (c).

37. The input impedance of a short circuited lossless transmission line quarter wave long is **[1991]**

Solution: (c)

$l = \lambda/y$ [quarter wave transformer]

$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{y} = \frac{\pi}{2}$$

$$z_{in} = z_0 \frac{[z_L \cos\pi/2 + jz_0 \sin\pi/2]}{[z_0 \cos\pi/2 + jz_L \sin\pi/2]} = \frac{z_0^2}{z_L}$$

$z_L = 0$ [short-circuited]

$z_{in} = \infty$

Hence, the correct option is (c).

38. A 50 ohm lossless transmission line has a pure reactance of (j 100) ohms as its load. The *VSWR* in the line is

(a) 1/2 (Half) (b) 2 (Two)
(c) 4 (Four) (d) ∞ (Infinity)

[1989]

Solution: (d)

Reflection coefficient $= \Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$$= \frac{j100 - 50}{j100 + 50}$$

$$|\Gamma| = \frac{\sqrt{(100)^2 + (50)}}{(100)^2 + (50)} = 1$$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1}{1-1} = \frac{2}{0} = \infty$$

Hence, the correct option is (d).

39. A Two wire transmission line of characteristic impedance Z_0 is connected to a load of impedance $Z_L (Z_L \neq Z_0)$ Impedance matching cannot be achieved with
(a) a quarter-wavelength transformer
(b) a half-wavelength transformer
(c) an open-circuited parallel stub
(d) a short-circuited parallel stub **[1988]**

Solution: (b)

For $z_0 \neq z_L$

a half-wavelength transform ($\lambda/2$) can not be used for impedance matching because for, $l = \lambda/2$

$$\beta.l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

$$z_{\text{in}} = z_0 \frac{(z_L \cos\pi + z_0 \sin\pi)}{(z_0 \cos\pi + z_L \sin\pi)}$$

$$z_{in} = z_L$$

Hence, the correct option is (b).

40. A transmission line of pure resistive characteristic impedance is terminated with an unknown load. The measured value of $VSWR$ on the line is equal to 2 and a voltage minimum point is found to be at the load. The load impedance is then
(a) Complex
(b) Purely capacitive
(c) Purely resistive
(d) Purely inductive **[1987]**

Solution: (c)

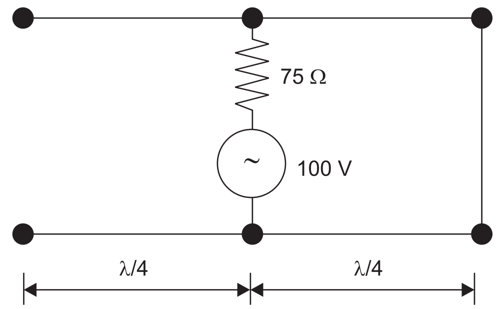
Given, voltage minimum (V_{min}) point is atload.

If V_{min} or V_{max} occurs at the load for a lossless transmission line then load impedance Z_L is purely resistive.

Hence, the correct option is (c).

Five-marks Questions

1. A $\dfrac{\lambda}{2}$ - section of a 600 Ω transmission line, short circuited at one end and open circuited at the other end, is shown in figure. A 100 V/ 75 Ω. generator is connected at the mid point of the section as shown in the figure. Find the voltage at the open-circuited end of the line.



[1997]

Solution: The current through generator,

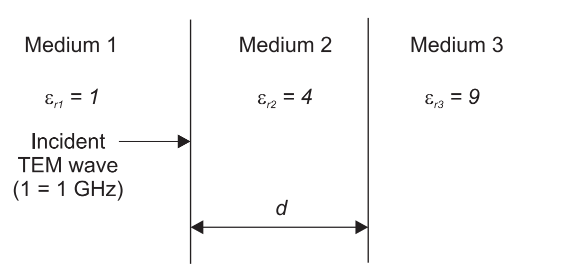
$$t_g = \frac{100}{75} = \frac{4}{3}\text{amp}, \ \beta = \frac{2\pi}{\lambda}; \ l = \frac{\lambda}{4}$$

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

V_2 at a distance l is given by,

$$N_1 = V_g \cos\beta l + jIg\, R_0 \sin\beta l$$
$$= jIg\, R_0 = j800 = 800 \angle 90^0$$

2. The three regions shown in figure all lossless and non-magnetic. Find
(a) Wave impedance in medium 2 and 3.
(b) d such that medium 2 acts as a quarter wave ($\lambda/4$) transformer.
(c) Reflection coefficient (Γ) and voltage standing wave ratio (VSWR) at the interface of the mediums 1 and 2, when $d = \lambda/4$



[2000]

Solution: For a lossless line, $\alpha = 0$, and $\mu = \mu 0 = 4\pi \times 10^{-7}$

(a) $\eta_2 = \sqrt{\dfrac{\mu_0}{E_0}} \times \dfrac{1}{\sqrt{E_{r_2}}} = \dfrac{120\pi}{\sqrt{4}} = 60\pi\ \Omega$

$$\eta_3 = \sqrt{\frac{\mu_0}{E_0}} \times \frac{1}{\sqrt{E_{r_3}}} = \frac{120\pi}{\sqrt{9}} = 40\pi\Omega$$

(b) $V_2 = \dfrac{1}{\sqrt{\mu_2 E_2}} = \dfrac{C}{\sqrt{E_{r2}}} =$

$$\frac{3\times 10^8}{\sqrt{4}} = 1.5\times 10^8 \text{ m/sec}$$

$$V_2 = f\lambda,\ \lambda = \frac{V_2}{f} = 0.15 \text{ m.}$$

$$d = \frac{\lambda}{4} = 3.75 \text{ cm}$$

(c) $z_0 = \eta_2,\ z_1 = \eta_3$

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{\eta_2^2}{\eta_3} = \frac{(60\pi)^2}{(40\pi)} = 90\pi\Omega$$

$$\Gamma = \frac{Z_L - Z_0}{Z_1 + Z_0},\ ZL = Z_{\text{in}} = 90\pi$$

$$Z_0 = \eta_1 = \sqrt{\frac{\mu_0}{E_0}} \times \frac{1}{\sqrt{E_{r_1}}} = 120\pi$$

$$\Gamma = \frac{90\pi - 120\pi}{90\pi + 120\pi} = -1/7$$

and $\text{VSWR} = \dfrac{1+|\Gamma|}{1-|\Gamma|} = \dfrac{4}{3}$

Chapter 4

Waveguide

One-mark Questions

1. A two port network has scattering parameters given by $[S]=\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22}\end{bmatrix}$. If theport-2 of the two-port is short circuited, thes$_{11}$, parameter for the resultant one-port network is

(a) $\dfrac{s_{11}-s_{11}s_{22}+s_{12}s_{21}}{1+s_{22}}$

(b) $\dfrac{s_{11}+s_{11}s_{22}-s_{12}s_{21}}{1+s_{22}}$

(c) $\dfrac{s_{11}+s_{11}s_{22}+s_{12}s_{21}}{1-s_{22}}$

(d) $\dfrac{s_{11}-s_{11}s_{22}+s_{12}s_{21}}{1-s_{22}}$ **[2014]**

Solution: (b)

$V_1^- = S_{11}V_1^+ + S_{12}V_2^+$

$V_2^- = S_{21}V^+ + S_{22}V_2^+$

Γ_L = at load on port 2

$$\Gamma_2 = \frac{V_2^+}{V_2^-}$$

$V_1^- = S_{11}V_1^+ + S_{12}\Gamma_2 V_2^-$

$V_2^- = S_{21}V_1^+ + S_{22}\Gamma_2 V_2^-$

$V_2^-(1-S_{22}\Gamma_2) = S_{21}V_1^+$

$$V_1^- = S_{11}V_1^+ + S_{12}\frac{\Gamma_2 S_{21}V_1^+}{1-S_{22}\Gamma_2}$$

$$\frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_2}{1-S_{22}\Gamma_2}$$

$$\Gamma_2 = \frac{z_L - z_0}{z_L + z_0}$$

$Z_L = 0$

$\Gamma_2 = -1$

$$\text{So, } \frac{V_1^-}{V_1^+} = S_{11} - \frac{S_{12}S_{21}}{1+S_{22}}$$

$$\frac{V_1^-}{V_1^+} = \frac{S_{11}+S_{11}S_{22}-S_{12}S_{21}}{1+S_{22}}$$

Hence, the correct option is (b)

2. Which one of the following field patterns represents a TEM wave travelling in the positive x direction?

(a) $E = +8\hat{y}, H = -4\hat{z}$

(b) $E = -2\hat{y}, H = -3\hat{z}$

(c) $E = +2\hat{z}, H = +2\hat{y}$

(d) $E = -3\hat{y}, H = +4\hat{z}$ **[2014]**

Solution:(b)

For, $E_y + H_z = TEMx$ (+ve)

$E_{-z} \times H_y = TEMx$ (+ve)

$E_{-y} \times H_{-z} = TEMx$ (+ve × direction)

Hence, the correct option is (b)

3. Consider an air filled rectangular waveguide with a cross-section of 5 cm × 3 cm. For this waveguide, the cut-off frequency (in MHz) of TE$_{21}$ mode is________. **[2014]**

Solution: (7810)

$a \times b = 5$ cm × 3 cm

$$f_c = \frac{V}{2}\sqrt{\left(\frac{2}{5}\right)^2 + \left(\frac{1}{3}\right)^2} \times 100$$

For TE$_{21}$,

$V = 3 \times 10^8$ m/s

$$f_c = \frac{3\times10^8}{2}\sqrt{\left(\frac{2}{5}\right)^2 + \left(\frac{1}{3}\right)^2} \times 100$$

$f_c = 7810$ MHz

4. The modes in a rectangular waveguide are denoted by TE_{mn}/TM_{mn} where m and n are the eigen numbers along the larger and smaller dimensions of the waveguide respectively. Which one of the following statements is TRUE?

(a) The TM_{10} mode of the waveguide does not exist
(b) The TE_{10} mode of the waveguide does not exist
(c) The TM_{10} and the TE_{10} modes both exist and have the same cut-off frequencies
(d) The TM_{10} and the TM_{01} modes both exist and have the same cut-off frequencies **[2011]**

Solution: (a)

For TM mode, TM_{00}, TM_{01}, TM_{10} does not exist

For TE mode

TE_{00} does not exist but TE_{01} and TE_{10} exists

Hence, the correct option is (a)

5. Which of the following statements is true regarding the fundamental mode of the metallic waveguides shown?

P Coaxial Q Cylindrical R Rectangular

(a) Only P has no cutoff-frequency
(b) Only Q has no cutoff-frequency
(c) Only R has no cutoff-frequency
(d) All three have cutoff-frequencies **[2009]**

Solution: (a)

For the three given metallic waveguides P, Q, R only P, i.e., coaxial cable support TEM mode which means P has no cut off frequency while d and R both have cut off frequency.

Hence, the correct option is (a)

6. The phase velocity of an electromagnetic wave propagating in a hollow metallic rectangular waveguide in the TE_{10} mode is

(a) equal to its group velocity
(b) less than the velocity of light in free space
(c) equal to the velocity of light in free space
(d) greater than the velocity of light in free space

[2004]

Solution: (d)

Inside the waveguide phase velocity is greater and group velocity is smaller than c

Hence, the correct option is (d)

7. The phase velocity for the TE_{10}-mode in an air-filled rectangular waveguide is

(a) less than c
(b) equal to c
(c) greater than c
(d) none of the above

Note: (c is the velocity of plane waves in free space) **[2002]**

Solution: (c)

$$V_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$V_p > C$

Hence, the correct option is (c)

8. The phase velocity of waves propagating in a hollow metal waveguide is

(a) greater than the velocity of light in free space.
(b) less than the velocity of light in free space
(c) equal to the velocity of light in free space
(d) equal to the group velocity **[2001]**

Solution: (a)

$$V_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{t}\right)^2}}$$

For wave propagation, $f > Fc$

$V_p > C$

Hence, the correct option is (a)

9. The dominant mode in a rectangular waveguide is TE_{10}, because this mode has

(a) no attenuation
(b) no cut-off
(c) no magnetic field component
(d) the highest cut-off wavelength **[2001]**

Solution: (d)

For TE_{10}

$\lambda c = 2a$

which is highest cut-off wavelength.

Hence, the correct option is (d)

10. A TEM wave is incident normally upon a perfect conductor. The E and H fields at the boundary will be, respectively,

(a) minimum and minimum
(b) maximum and maximum
(c) minimum and maximum
(d) maximum and minimum **[2000]**

Solution: (a)

Electric field equation, $E = E_0 e^{-\alpha z}\cos(wt - \beta z)ax$

For good conductor, $\gamma = \alpha + j\beta$

Attenuation constant, $\alpha = \sqrt{\pi f \mu \sigma}$

For good conductor, $\sigma >> 1$

Which lead exponential damping of the wave ($e^{-\alpha z}$ factor).

Therefore, E and H field at the boundary of perfect conductor will be minimum

Hence, the correct option is (a)

11. Assuming perfect conductors of a transmission line, pure TEM propagation is NOT possible in
(a) coaxial cable
(b) air-filled cylindrical waveguide
(c) parallel twin-wire line in air
(d) semi-infinite parallel plate waveguide **1999]**

Solution: (b)

Pure TEM propagation is not possible in waveguide like rectangular or cylindrical waveguide since, for a TEM wave,

$E_z = 0$ and Hz = 0

This makes E_x, E_y, H_x, H_y vanish and hence a TEM wave cannot exist inside a waveguide.

Hence, the correct option is (b)

12. A rectangular air filled waveguide has a cross section of 4 cm × 10 cm. The minimum frequency which can propagate in the waveguide is
(a) 1.5 GHz (b) 2 0 GHz
(c) 2.5 GHz (d) 3.0 GHz **[1997]**

Solution: (none of the above)

$b \times a = 4$ cm × 10 cm

$$f_{c\,\min} = \frac{c}{2a} = \frac{3\times10^8}{2\times 10\times10^{-2}}$$

$f_{c\text{ min}} = 1.5$ GHz

The minimum frequency which can propagate will be $f \geq fc$ min

i.e. 1.5 GHz

13. The interior of a $\frac{20}{3}$ cm × $\frac{20}{4}$ cm rectangular wave-guide is completely filled with a dielectric of $\in_r = 4$.Waves of free space wave lengths shorter than .....can be propagated in the TE_{11} mode.

[1994]

Solution: (8)

$m = 1$ and $n = 1$

$$\lambda_c = \frac{v}{f_c}$$

$$V = \frac{c}{\sqrt{\in_r}} = \frac{3\times10^8}{\sqrt{4}} = 1.5\times10^8 \text{ m/s}$$

$$f_c = \frac{V}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$a = \frac{20}{3}\text{cm},\ b = \frac{20}{4}\text{cm}$$

$fc = 1.875 \times 10^9$ Hz

$$\lambda_c = \frac{1.5\times10^8}{1.872\times10^9} = 8 \text{ cm}$$

∴ cut off wavelength, $\lambda c = 8$ cm

Two-marks Questions

1. A rectangular waveguide of width w and height h has cut-off frequencies for TE_{10} and TE_{11} modes in the ratio 1 : 2. The aspect ratio w/h, rounded off to two decimal places, is _____. **[2019]**

Solution:

$$f_C = \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\cdot\frac{C}{2}$$

$$\frac{f_{CTE10}}{f_{CTE}} = \frac{1}{2}$$

$$\frac{\frac{C}{2}\cdot\frac{1}{w}}{\frac{C}{2}\sqrt{\left(\frac{1}{w}\right)^2 + \left(\frac{1}{h}\right)^2}} = \frac{1}{2}$$

$$\frac{\frac{1}{w}}{\sqrt{\left(\frac{1}{w}\right)^2 + \left(\frac{1}{h}\right)^2}} = \frac{1}{2}$$

$$\left(\frac{1}{w}\right)^2 + \left(\frac{1}{h}\right)^2 = 4\left(\frac{1}{w}\right)^2$$

$$\left(\frac{1}{h}\right)^2 = 3\left(\frac{1}{w}\right)^2$$

$$\frac{w}{h} = \sqrt{3} = 1.732$$

2. The dispersion equation of a waveguide. Which relates the wave number k to the frequency ω is $k(\omega) = \left(\frac{1}{c}\right)\sqrt{\omega^2 - \omega_0^2}$

Where the speed of light $c = 3 \times 10^8$ m/s, and ω_0 is a constant. If the group velocity is 2×10^8 m/s, then the phase velocity is **[2019]**
(A) 2×10^8 m/s (B) 3×10^8 m/s
(C) 1.5×10^8 m/s (D) 4.5×10^8 m/s

Solution:

$$K(\omega) = \left(\frac{1}{C}\right)\sqrt{\omega^2 - \omega_0^2}$$

$$C = \sqrt{V_g V_p}$$

$$V_p = \frac{C^2}{V_g} = \frac{\left(3\times10^8\right)^2}{2\times108} = 4.5\times10^8 \text{ m/s}$$

Hence, the correct option is (D).

3. The cutoff frequency of TE_{01} mode of an air filled rectangular waveguide having inner dimensions a cm × b cm ($a > b$) is twice that of the dominant TE_{10} mode. When the waveguide is operated at a frequency which is 25% higher than the cutoff frequency of the dominant mode, the guide wavelength is found to be 4 cm. The value of b (in cm, correct to two decimal places) is ______ **[2018]**

Solution:

If K_c is the cut off phase cause and c is speed of light. Then the frequency can be expressed as

$$K_{CTE01} = 2\,K_{CTE10}$$

$$\frac{\pi}{b} = \frac{2\pi}{a} \quad \because K_c$$

$$\Rightarrow \quad b = \frac{a}{2}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$4 = \frac{(3\times10^{10}/f)}{\sqrt{1-\left(\frac{1}{1.25}\right)^2}}$$

$$f = \frac{3\times10^{10}}{2.4} = 12.5\ \text{GH}_z$$

$$1.25 f_c = 12.5\ \text{GH}_z$$

$$\Rightarrow \quad f_c = 10\ \text{GH}_z (\text{TE}_{10})$$

$$\frac{C}{2a} = 10\times10^9$$

$$\frac{3\times10^{10}}{2a} = 10^{10}$$

$$a = 1.5\ \text{cm}$$

$$b = \frac{a}{2} = 0.75\ \text{cm}$$

Hence, the correct answer is 0.7 to 0.8.

4. Standard air-filled rectangular waveguides of dimensions a = 2.29 cm and b = 1.02 cm are designed for radar applications. It is desired that these waveguides operate only in the dominant TE_{10} mode with the operating frequency at least 25% above the cutoff frequency of the TE_{10} mode but not higher than 95% of the next higher cutoff frequency. The range of the allowable operating frequency f is **[2017]**

(A) 8.19 GHz ≤ f ≤ 13.1 GHz
(B) 8.19 GHz ≤ f ≤ 12.45 GHz
(C) 6.55 GHz ≤ f ≤ 13.1 GHz
(D) 1.64 GHz ≤ f ≤ 10.24 GHz

Solution: Range of operating frequency:

$$1.25F_{CTE10} \le f \le 0.95F_{CTE01}$$

$$F_{CTE10} = \frac{c}{2a} = \frac{3\times10^{10}}{2\times2.29} = 6.55\ \text{GHz}$$

$$F_{CTE20} = \frac{C}{a} = 13.1\ \text{GHz}$$

$$\therefore 8.1875\ \text{GHz} \le F \le 12.445\ \text{GHz}$$

Hence, the correct option is (B).

5. Consider an air filled rectangular waveguide with dimensions a = 2.286 cm and b = 1.016 cm. At 10 GHz operating frequency, the value of the propagation constant (per meter) of the corresponding propagating mode is __________. **[2016]**

Solution: The two dimensions of waveguide are a = 2.286 cm and b = 1.016 cm.

Frequency $\quad f$ = 10GHz

So, $\quad \lambda$ = 0.03 m

Guide propagation constant,

$$\gamma_g^2 = \gamma^2 + k_c^2$$

Where, $\gamma = j\omega\sqrt{\mu\in}$, intrinsic propagation constant

$k_c = \dfrac{2\pi}{\lambda_c}$, cut off phase constant

$$\left(\lambda c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}\cdot\frac{c}{2}\right)$$

So, λc is obtained from above expression

$$\Rightarrow \quad \gamma_g = \sqrt{\gamma^2 + k_c^2}$$

$$= 157\ \text{m}^{-1}$$

Hence, the correct Answer is (57 m^{-1}).

6. Consider an air filled rectangular waveguide with dimensions a = 2.286 and b = 1.016 cm. The increasing order of the cut off frequencies for different modes is

(A) $TE_{01} < TE_{10} < TE_{11} < TE_{20}$
(B) $TE_{20} < TE_{11} < TE_{10} < TE_{01}$
(C) $TE_{10} < TE_{20} < TE_{01} < TE_{11}$
(D) TE_{10} ,$< TE_{11} < TE_{20} < TE_{01}$

Solution: Cut off frequency, can be expressed as

$$f_c = \frac{k_c c}{2\pi}$$

Where, $\quad k_c$ = cut off phase constant
$\quad c$ = speed of light

$$TE_{10} \to k_c = \frac{\pi}{a} = \frac{\pi}{2.286} \quad (m = 1, n = 0))$$

$$TE_{20} \to k_c = \frac{2\pi}{a} = \frac{\pi}{1.143} \quad (m = 2, n = 0)$$

$$TE_{01} \to k_c = \frac{\pi}{b} = \frac{\pi}{1.016} \quad (m = 0, n = 1)$$

$$TE_{11} \to k_c = \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2} = \frac{\pi}{0.862} \quad (m = n = 1)$$

$$\therefore \quad f_{cTE10} < f_{cTE20} < f_{cTE01} < f_{cTE11}$$

Hence, the correct option is (C).

7. The longitudinal component of the magnetic field inside an air-filled rectangular waveguide made of a perfect electric conductor is given by the following expression

$H_z(x, y, z, t) = 0.1 \cos(25\,\pi x) \cos(30.3\,\pi y) \cos(12\,\pi \times 10^9 t - \beta z)$ (A/m)

The cross-sectional dimensions of the waveguide are given as $a = 0.08$m and $b = 0.033$ m. the mode of propagation inside the waveguide is **[2015]**

(A) TM_{12} (B) TM_{21}
(C) TE_{21} (D) TE_1

Solution: General magnetic field expression is given as

$$H_z(x, y, z, t) = H_o \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos(\omega t - \beta z)\text{A/m}$$

So by comparing given equation with generalized equation

$$\frac{m\pi}{a} = 25\pi$$

$$\Rightarrow \quad m = 25 \times 0.08 = 2$$

$$\frac{n\pi}{b} = 30.3\pi$$

$$n = 30.3 \times 0.033$$

$$= 0.99 \,\square\, 1$$

H_z field is present only for transverse electric field only.

So mode is TE_{21}

Hence, the correct option is (C).

8. An air-filled rectangular waveguide of internal dimensions a cm × b cm ($a > b$) has a cutoff frequency of 6 GHz for the dominant TE_{10} mode. For the same waveguide, if the cutoff frequency of the TM_{11} mode is 15 GHz, the cutoff frequency of the TE_{01} mode in GHz is _______. **[2015]**

Solution: $f_C = 6 \text{ GHz} = \dfrac{3\times10^{10}}{2a} = TE_{10}$

$$\Rightarrow \quad a = 2.5\text{cm}$$

For the same waveguide f_C for TM_{11} is 15 GHz

$$\lambda_C = \frac{2ab}{\sqrt{a^2+b^2}} = \frac{3\times10^{10}}{15\times10^9} = 2$$

$$b = 1.09 \text{ cm}$$

then cut-off frequency of $T_{E_{01}} = \dfrac{3\times10^{10}}{\lambda_C}$

$$\lambda_C = 2b$$

$$= 2.18\text{cm}$$

$$f_C \text{ of } TE_{01} = \frac{3\times10^{10}}{2.18} = 13.7 \text{ GHz}$$

Hence, the correct Answer is (13.5 to 13.8).

9. For a rectangular waveguide of internal dimensions $a \times b$ ($a > b$), the cut-off frequency for the TE_{11} mode is the arithmetic mean of the cut-off frequencies for TE_{10} mode and TE_{20} mode. If $a = \sqrt{5}$ cm, the value of b (in cm) is_____. **[2014]**

Solution: (2)

$$\text{Given } f_{c\,TE_{11}} = \frac{f_{c\,TE_{11}} + f_{c\,TE_{20}}}{2} \qquad (1)$$

$$f_{c\,TE_{10}} = \frac{V}{2a}$$

$$f_{c\,TE_{20}} = \frac{V}{a}$$

$$f_{c\,TE_{11}} = \frac{V}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

Put all above values in equation (1)

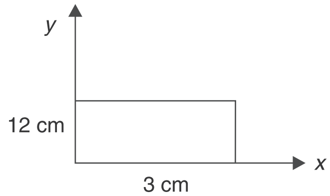
$$\frac{V}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = \frac{\frac{V}{2a} + \frac{V}{a}}{2}$$

$$a = \sqrt{5} \text{ cm}$$

$$b = \frac{2a}{\sqrt{5}}$$

$$b = 2 \text{ cm}$$

10. The magnetic field along the propagation direction inside a rectangular waveguide with the cross-section shown in figure is Hz = 3 cos (2.094 × 10^2×) cos (2.618 × 10^2 y) cos (6.283 × 10^{10} t-z)



The phase velocity v_p of the wave inside the waveguide satisfies

(a) $v_p > c$
(b) $v_p = c$
(c) $0 < v_p < c$
(d) $v_p = 0$ **[2012]**

Solution: (a)

Since, $V_p = \dfrac{c}{\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}}$

So, $V_p > C$ always

Hence, the correct option is (a)

11. A rectangular waveguide of internal dimensions (a = 4 cm and b = 3 cm) is to be operated in TE11 mode. The minimum operating frequency is

(a) 6.25 GHz (b) 6.0 GHz
(c) 5.0 GHz (d) 3.75 GHz **[2008]**

Solution: (a)

$a \times b$ = 4 cm × 3 cm

For TE_{11} mode, $f_c = \dfrac{c}{2}\sqrt{\left(\dfrac{1}{a}\right)^2+\left(\dfrac{1}{b}\right)^2}$

$$f_c = \frac{3\times10^8}{2}\sqrt{\left(\frac{100}{4}\right)^2+\left(\frac{100}{3}\right)^2}$$

f_c = 6.25 GHz

Hence, the correct option is (a)

12. An air-filled rectangular waveguide has inner dimensions of 3 cm × 2 cm. The wave impedance of the TE_{20} mode of propagation in the waveguide at a frequency of 30 GHz is (free space impedance η_0 = 377 Ω)

(a) 308 Ω (b) 355Ω
(c) 400 Ω (d) 461 Ω **[2007]**

Solution: (c)

For TE_{20}, $\lambda_c = \dfrac{z_a}{m} = a$

a = 3 cm

λ_c = 3 cm

$$f_c = \frac{c}{\lambda_c} = \frac{3\times10^8}{3\times10^{-12}} = 10\,\text{GHz}$$

$$\eta_{TE} = \frac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

η_0 = 377 Ω, f = 30 GHz

$$\eta_{TE20} = \frac{377}{\sqrt{1-\left(\frac{10}{30}\right)^2}}$$

$$\eta_{TE20} = 400\Omega$$

Hence, the correct option is (c)

13. The $\vec{E}$ field in a rectangular waveguide of inner dimensions $a \times b$ is given by

$$\vec{E} = \frac{\omega\mu}{h^2}\left(\frac{\pi}{a}\right)H_0 \sin\left(\frac{2\pi r}{a}\right)\sin(\omega t-\beta z)\hat{y}$$

where H_0 is a constant, and a and b are thedimensions along the x-axis and the y-axisrespectively The mode of propagation inthe waveguide is

(a) TE_{20} (b) TM_{11}
(c) TM_{20} (d) TE_{10} **[2007]**

Solution: (a)

$$\vec{E} = \frac{w\mu}{h^2}\left(\frac{\pi}{a}\right)H_0 \sin\left(\frac{2\pi x}{a}\right)\sin(wt-\beta z)\hat{y} \quad (1)$$

$\vec{E}_y$ in general is,

$$\vec{E} = \frac{-x}{h^2}c\left(\frac{\eta\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right)x.\cos\left(\frac{n\pi}{b}\right)ye^{j(wt-xz)} \quad (2)$$

Compare equation (1) and (2)

Here, m = 2 and n = 0

So, mode is TE_{20}

Hence, the correct option is (a)

14. A rectangular waveguide having TF_{10} mode as dominant mode is having a cutoff frequency of 18-GHz for the TE_{30} mode. The inner broad-wall dimension of the rectangular waveguide is

(a) 5/3 cms (b) 5 cms
(c) 5/2 cms (d) 10 cms **[2006]**

Solution: (c)

$F_c = \dfrac{c}{2}\sqrt{\left(\dfrac{m}{a}\right)^2+\left(\dfrac{n}{b}\right)^2}$ For TE_{mn}

For TE_{mo} $f_c = \dfrac{c_m}{2a}$

For TE_{10}, $f_c = \frac{c}{2a}$

$$\lambda C_m = \frac{C}{F_c} = \frac{2a}{m}$$

$$\lambda_{C_1} = \lambda_{C_3}$$

$$\lambda_{C_1} = \frac{c}{F_c} = \frac{3\times10^8}{18\times10^9} = \frac{1}{60}$$

$$\lambda_{C_3} = \frac{2a}{3} = \frac{1}{60}$$

$$a = \frac{1}{40}\text{ m}$$

For TE_{10}, $a = 2.5$ cm

Hence, the correct option is (c).

15. In a microwave test bench, why is the microwave signal amplitude modulated at 1 kHz?
(a) To increase the sensitivity of measurement
(b) To transmit the signal to a far-off place
(c) To study amplitude modulation
(d) Because crystal detector fails at microwave frequencies **[2006]**

Solution:(d)

Since crystal detector fails at microwave frequencies, microwave signal amplitude modulated at 1 KHz.

Hence, the correct option is (d)

16. A rectangular metal wave guide filled with a dielectric material of relative permittivity $\in_r = 4$ has the inside dimensions 3.0 cm x 1.2 cm. The cut-off frequency for the dominant mode is
(a) 2.5 GHz (b) 5.0 GHz
(c) 10.0 GHz (d) 12.5 GHz **[2003]**

Solution:(a)

$$f_c = \frac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{V}{2a}$$

$$V = \frac{c}{\sqrt{tr}} = \frac{3\times10^8}{\sqrt{4}}$$

For dominant node TE_{10}, $m = 1$, $n = 0$

$V = 1.5 \times 10_8$ m/s

$a = 3$ cm

$$f_c = \frac{1.5\times10^8}{2\times3} = 2.5\,\text{GHz}$$

Hence, the correct option is (a).

17. A rectangular waveguide has dimensions 1 cm ×0.5 cm. Its cut-off frequency is
(a) 5 GHz (b) 10 GHz
(c) 15 GHz (d) 12 GHz **[2000]**

Solution:(c)

Cut-off frequency for TE_{10} [dominant mode] is,

$$f_c = \frac{c}{2a}$$

$a = 1$ cm

$$f_c = \frac{3\times10^8}{2\times10^{-2}}$$

$f_c = 15$ GHz

Hence, the correct option is (c).

18. Choose the correct statements

For a wave propagating in an air filled rectangular wave guide
(a) Guided wavelength is never less than the free space wavelength.
(b) Wave impedance is never less than the free space impedance.
(c) Phase velocity is never less than the free space velocity.
(d) TEM mode is possible if the dimensions of the wave guide are properly chosen. **[1990]**

Solution: (a) and (b)

(i) λ_g [guided wavelength] $= \dfrac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

$lg>\lambda$

(ii) η_{TM} [Impedance for TM mode] $= \eta_0\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

where, $\eta_0 = 120\pi$

$\eta + m < \eta_0$

η_{TE} [TE mode] $= \dfrac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

$\eta_{TE} > \eta_0$

(iii) $V_p = \dfrac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

$V_p > c$

Hence, the correct option is (a and b)

19. For a normal mode EM wave propagating in a hollow rectangular wave guide

(a) the phase velocity is greater than the group velocity
(b) the phase velocity is greater than velocity of light in free space
(c) the phase velocity is less than the velocity of light in free space
(d) the phase velocity may be either greater than or less than the group velocity **[1988]**

Solution: (a) and (b)

Phase velocity, $V_p = \dfrac{c}{\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}}$

Group velocity, $V_g = c\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}$

For normal mode, $f > f_c$

So, $V_p > c$ and $V_g < c$

$V_p > c > V_g$

20. The cut off frequency of a waveguide depends upon
(a) The dimensions of waveguide
(b) The dielectric property of the medium in the waveguide
(c) The characteristic imoedance of the waveguide
(d) The transverse and axial components of the fields **[1987]**

Solution: (a) adn (b)

$$f_c = \frac{V_o}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

where, $v_o = \dfrac{c}{\sqrt{\mu_r \in_r}}$

$$f_c = \frac{c}{\sqrt{\mu_r \in_r \times 2}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

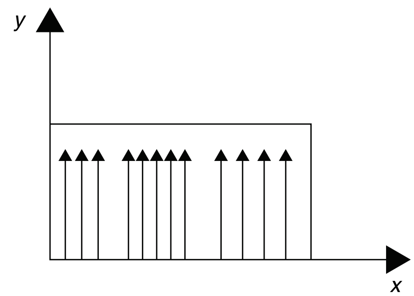
So, f_c depends on 'a' and 'b' (dimensions of waveguide)

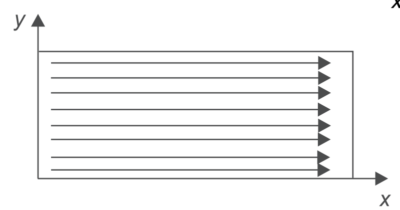
$$f_c \propto \frac{1}{\sqrt{\mu_r \in_r}}$$

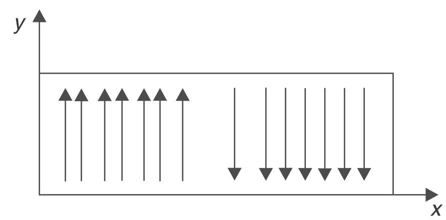
So, f_c depends upon the dielectric property of the medium in the waveguide.

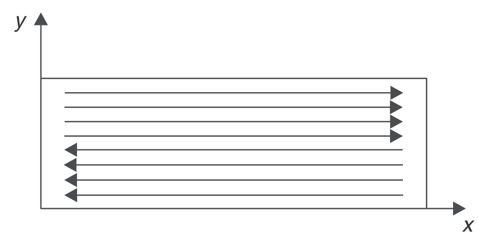
Hence, the correct option is (a and b)

21. Which one of the following does represent the electric field lines for the TE_{02} mode in the cross-section of a hollow rectangular metallic waveguide?

(a)


(b)


(c)


(d)


Solution: (d)

For the given figures.
(a) TE_{10} (b) TE_{01}
(c) TE_{20} (d) TE_{02}

Hence, the correct option is (d)

Five-marks Questions

1. Consider a parallel plate waveguide with plate separation d as shown in figure. The electric and magnetic fields for the TEM mode are given by

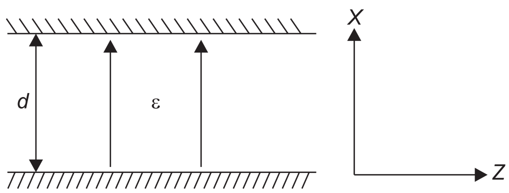
$$E_x = E_0 e^{-jkz+j\omega t},\ H_y = \frac{E_0}{\eta} e^{-jkz+j\omega t}$$

Where $k = T <$ one.

(a) Determine the surface charge densities ρ_s on the plates at $x = 0$ and $x = d$.

(b) Determine the surface current densities $\vec{J}_s$ on the same plates.

(c) Prove that ρ_s and $\vec{J}_s$ satisfy the current continuity condition.



Solution: Given that wave is propagating in +2 direction.

$$E_x = E_0 e^{+j(\omega t - kz)}$$

$$H_y = \frac{E_0}{\eta} e^{+j(\omega t - kz)}$$

(a) $\vec{E} = \dfrac{P_S}{E} a_x$ or $E_x = \dfrac{P_S}{E}$

$$P_S = \in E_x = \in E_0 \cos(\omega t - kz)$$

(b) $J_S = \vec{\eta} \times \vec{H} =$ for surface $x = 0$,

$$\vec{J}_S = \vec{a}_x \times \vec{H} = H_y \hat{a}_z = \frac{E_0}{\eta} \cos(\omega t - kz)\hat{a}_2$$

$k = \eta\omega E$

$$\Rightarrow \frac{1}{\eta} = \frac{\omega E}{K}$$

$$\vec{J}_S = \omega \in \frac{E_0}{K} \cos(\omega t - kz)\vec{a}_2$$

at, $x = d$, $\vec{J}_S = -\dfrac{E_0}{\eta} \cos(\omega t - kz) a_2$

(c) From current continuity eg^n. for time varying fields given by

$$\nabla.\vec{J} = \frac{-\partial}{\partial t} P \Rightarrow \frac{d}{dz} JSJ = \frac{-\partial}{\partial t} P.$$

$$\frac{d}{dz}\left[\frac{\in \omega E_0}{k} \cos(\omega t - kz)\right] = \omega \in E_0 \sin(\omega t - kz)$$

$$\Rightarrow \frac{-\partial}{\partial t} P = \frac{\partial}{\partial t}(\in E_0 \cos(\omega t - kz)$$

$$= \omega \in E_0 \sin(\omega t - kz),$$

So, $\dfrac{d}{dz} JSz = \dfrac{-\partial}{\partial t} P.$

Thus current continuity equation verified.

2. A rectangular hollow metal waveguide has dimensions $a = 2.29$ cm and $b = 1.02$ cm. Microwave power at 10 GHz is transmitted through the waveguide in the TE_{10} mode.

(a) Calculate the cut-off wavelength and the guide wavelength for this mode.

(b) What are the other (TE or TM) modes that can propagate through the waveguide?

(c) If $a = b = 2.29$ cm, what are the modes which can propagate through the waveguide? **[2001]**

Solution: $a = 2.29$, $b = 1.02$ cm, $c = 10$ GHz

(a) $\lambda = 20 = 4.58$ cm

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10 \times 10^9} = 3 \text{ cm}$$

$$f_c = \frac{V_0}{\lambda_C} = 6.55 \text{ GHz}$$

$$\bar{\lambda} = \frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = 3.97 \text{ cm},$$

(b) for TE_{10}, $f_{c1} = \dfrac{V_0}{2a} = 6.55$ GHz

for TE_{01}, $f_{c2} = \dfrac{V_0}{2b} = 14.7$ GHz,

$fc_2 > f$ so only TE_{10} mode can propagate through the given rectangular waveguide.

(c) $a = b = 2.29$ cm

$f = 10$ GHz

for TE_{10}, $f_{c10} = \dfrac{V_0}{2a} = \dfrac{3 \times 10^{10}}{2 \times 2.29} = 6.55$ GHz

for TE_{01} $f_{c01} = \dfrac{V_0}{2b} = \dfrac{3 \times 10^8}{2 \times 2.29} = 6.55$ GHz

for TE_{11} or TM_{11}

$$f_{C11} = \frac{V_0}{2a}\sqrt{1^2 + 1^2} = \frac{V_0}{\sqrt{2}a} = 9.226 \text{ GHz}$$

for TE_{20} and TE_{02}

$$f_{C20} = f_{C02} = \frac{V_0}{a} = \frac{3 \times 10^{10}}{2.29} = 13.1 \text{ GHz}$$

So, only TE_{10}, TE_{01}, TE_{11} and TM_{11} are the possible mode that propagates through the rectangle wave guide.

3. A 100 m section of an air-filled rectangular waveguide operating in the TE_{10} mode has a cross-sectional dimension of 1.071 cm x 0.5 cm. Two pulsed carriers of 21 GHz and 28 GHz are simultaneously launched at one end of the waveguide section. What is the time delay difference between the two pulses at the other end of the waveguide? **[1999]**

Solution: Time delay = $T = \dfrac{\bar{\beta}}{\omega} z$

$$f_c = \frac{V_0}{2a} = 14 \text{ GHz}$$

$$\bar{\beta} = \beta\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 2\pi\sqrt{\mu E}\sqrt{f^2 - f_C^2}$$

for $f = f_1 = 21$ GHz

$$\bar{\beta} = 2\pi\sqrt{\mu\varepsilon}\sqrt{[(21)^2 - (14)^2]\times 10^{18}}$$
$$= 2\pi\sqrt{\mu\varepsilon}\, 15.65\times 10^9$$

$$T_1 = \frac{\bar{\beta}}{\omega}\times 100 = 2\pi\sqrt{\mu\varepsilon}\times\frac{13.65\times 10^{11}}{2\pi\times 21\times 10^9}$$
$$= 0.745\sqrt{\mu\varepsilon}\times 100$$

for $f = f_2 = 28$ GHz =

$$2\pi\sqrt{\mu E}\sqrt{[(28)^2 - (14)^2]\times 10^{18}}$$

$$\bar{\beta}_2 = 2\pi\sqrt{\mu E}\, 24.2\times 10^9$$

$$T = \frac{-}{}\times 100 = 0.866\sqrt{\ F}\times 100$$

Time delay difference,

$$T_d = T_2 - T_1 = \sqrt{\mu E}(0.866 - 0.745)\times 100$$

$$= \frac{1}{\cdot\times\ \ 8}\times 0.121\times 100 = 0.04\ \mu\,\text{sec}$$

4. A rectangular waveguide with inner dimensions 6 cm x³ has been designed for a single mode operation. Find the possible frequency range of operation such that the lowest frequency is 5% above the cut-off and the highest frequency is 5% below the cutoff of the next higher mode. **[1998]**

Solution: Given, $b = 3$ cm, $a = 6$ cm, $a = 2b$,

Lowest TE_{10}

$$f_{c_1} = \frac{V_0}{2a} = 2.5\ \text{GHz}$$

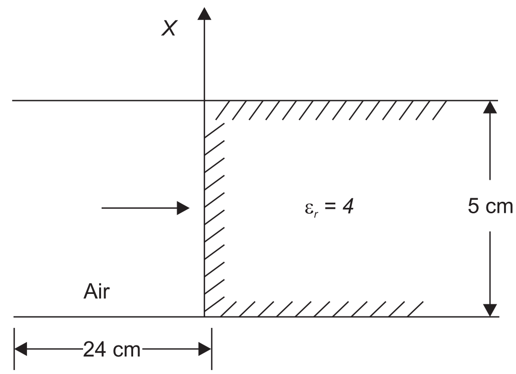
Next higher mode TE_{01} or TE_{20}

$f_{lowest} = 1.05\ f_{c_1} = 2.625$ GHz,

$$f_{c_2} = \frac{V_0}{a} = 5\ \text{GHz}$$

$f_{highest} = 0.95\ f_{c_2} = 4.75$ GHz,

5. The region between a pair of parallel perfectly conducting planes of infinite extent in the y and z direction is partially filled with a dielectric as shown in figure. A 30 GHz TE_{10} wave is incident on the air dielectric interface as shown. Find VSWR at the interface.



[1998]

Solution: Given, $a = 5$ cm

TE_{10}, $f = 30$ GHz

If medium – 1

$$f_c = \frac{V_0}{2a} = 3\ \text{GHz}$$

$$\eta_1 = \frac{\eta_0}{\left(1-\frac{f_c}{f}\right)^2} = \frac{120\pi}{\left(1-\left(\frac{3}{30}\right)^2\right)^{1/2}} = 378.89\Omega$$

Medium – 2, $\mu = \mu_0$, $E = 4E_0$

$$V_P = \frac{E}{\sqrt{E_r}} = \frac{3\times 10^8}{\sqrt{4}} = 1.5\times 10^8\ \text{m/sec}$$

$$f_c = \frac{V_0}{2a} = 1.5\ \text{GHz}$$

$$\eta_0 = \sqrt{\frac{\mu}{E}} = \frac{120\pi}{\sqrt{4}} = 60\pi$$

$$\eta_2 = \frac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = 188.73\Omega,$$

Reflection coeff. $\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \dfrac{188.73 - 378.89}{188.73 + 378..89}$

$|\Gamma| = 0.335$

$$\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|} = 2$$

6. In an air-filled rectangular waveguide, the vector electric field is given by

$$\vec{E} = \cos(2\pi y)\exp\left[-j\left(\frac{40\pi}{3}\right)z + j\omega t\right]\hat{i}_x V/m$$

Find vector magnetic field and the phase velocity of the wave inside the waveguide. **[1996]**

Solution:

$$\vec{E} = \cos(2xy)\exp\left[-j\left(\frac{40\pi}{3}\right)2 + j\omega t\right]_{\text{V/m}}^{\hat{i}_x}$$

$$\vec{\beta} = \frac{40\pi}{3}$$

$$\bar{\eta} = \frac{\omega\mu}{\bar{\beta}} = \frac{\omega \times 4\pi \times 10^{-7}}{40\pi/3} = 3\omega \times 10^{-8}$$

$$H_y = \frac{E_x}{\eta} = \frac{1}{3\omega \times 10^{-8}}\left\{\cos(2xy)e\left\{-J\left(\frac{40\pi}{3}\right)z + j\omega t\right\}\right.$$

$$\vec{H} = H_x j_y$$

7. A rectangular hollow metal waveguide is required to be so designed to propagate a 9375 MHz signal in its TE_{10}-mode that the guide-wavelength equals the cut-off wavelength. Calculate the value of 'a' (breadth or the wider dimension of the waveguide). Take $b = a/2$. Also, calculate the cut-off frequency of the next higher order mode. **[1995]**

Solution: For rectangular waveguide.

TE_{10} mode. $f = 9375$ MHz

$b = 9/2$

For, follow waveguide,

$v_0 = 3 \times 10^8$ m/sec

$$\lambda_0 = \frac{V_0}{f} = 3.2 \text{ cm}$$

guide wavelength, $\bar{\lambda} = \lambda_C =$ cutoff wavelength

$$\lambda_0 = \frac{\bar{\lambda}\lambda_c}{\sqrt{\bar{\lambda}^2 + \lambda_C^2}} = \frac{\lambda_C^2}{\sqrt{2}\lambda_C} = \frac{\lambda_C}{\sqrt{2}} = 3.2\sqrt{2}.$$

$$\lambda_C = 2a = 3.2\sqrt{2}, \Rightarrow a = 2.26,$$

Since $ba = 2b$

$$f_c = \frac{V_0}{2a}\sqrt{m^2 + (4n)^2}$$

For TE_{10} and TE_{20}, $f_c = \dfrac{V_0}{9} = 13.257$ GHz

8. A rectangular hollow metal wave guide of internal cross-section of 7.366 cm × 3.556 cm carries a 3 GHz signal in the TE_{10}-mode. Calculate the maximum power handling capability of the waveguide assuming the maximum permissible electric field inside the wave-guide to be 30 kV/vm. **[1994]**

Solution:

$$f_c = \frac{V}{2a} = \frac{3 \times 10^8 \times 10^2}{2 \times 7.366} = 2.036 \text{ Ghz}$$

$$f_c = \frac{2.036}{3} = 0.679$$

$$\left(\frac{f_c}{f}\right)^2 = 0.46,$$

$$\eta = \frac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{120\pi}{\sqrt{1-0.46}} = 513.42$$

$$P_{\text{maxd}} = \frac{E_{\text{max}}^2}{2\eta} = 8.766 \times 10^9 \text{ w/m}^2$$

$$P_{\text{max}} = P_{\text{maxd}}\, X\, \text{Area} = 23 \text{ mw}$$

Chapter 5

Antenna

One-mark Questions

1. Radiation resistance of a small dipole current element of length ℓ at a frequency of 3 GH_Z is 3 ohms. If the length is changed by 1%, then the percentage change in the radiation resistance, rounded off to two decimal places, is ______ %. **[2019]**

Solution:

Radiation resistance of a small de'pole is given by

$$R_r = 80\Pi^2\left(\frac{l}{\lambda}\right)^2$$

length is changed by 1%

$$\therefore l_2 = l_1 + 1\%\ l_1$$
$$= l_1 + 0.01 l_1$$
$$= l_1(1 + 0.01)$$
$$l_2 = 1.01 l_1$$

$$Rr = 80\Pi^2\left(\frac{l_1}{\lambda}\right)^2;\ Rr_1 = 80\Pi^2\left(\frac{l_2}{\lambda}\right)^2$$

Taking ratio

$$\frac{Rr_2}{Rr_1} = \frac{\cancel{80\Pi}^2\left(\frac{l_2}{\cancel{\lambda}}\right)^2}{\cancel{80\Pi}^2\left(\frac{l_1}{\cancel{\lambda}}\right)^2} = \left(\frac{l_2}{l_1}\right)^2 = \left(\frac{1.01\cancel{l}_1}{\cancel{l}_1}\right)^2$$

$$= 1.0201$$

$$R_{r_2} = 1.0201 R_{r_1}$$

Percentage change in radiation resistance:

$$= \frac{R_{r_2} - R_{r_1}}{R_{r_1}} \times 100$$

$$= \frac{1.0201\ \cancel{R_{r_1}} - 1\cancel{R_{r_1}}}{\cancel{R_{r_1}}} \times 100$$

$$= 0.0201 \times 100 = 2.01\%$$

2. For an antenna radiating in free space, the electric field at a distance of 1 km is found to be 12mV/m. Given that intrinsic impedance of the free space is 120 $\pi\Omega$, the magnitude of average power density due to this antenna at a distance of 2 km from the antenna (in nW/m^2) is____. **[2014]**

Solution: (47.7)

E at a distance of 1 km = 12 mv/m

E at distance of 2 km, = 6 mv/m

$$P_{avg} = \frac{1}{2}\frac{E^2}{\eta}$$

$$= \frac{1}{2} \times \frac{(6 \times 10^{-3})^2}{377}$$

$= 47.7$ nw/m2

3. Match Column-A with Column-B.

Column-A

1. Point electromagnetic source
2. Dish antenna
3. Yagi-Uda antenna

Column-B

P. Highly directional   Q. End fire

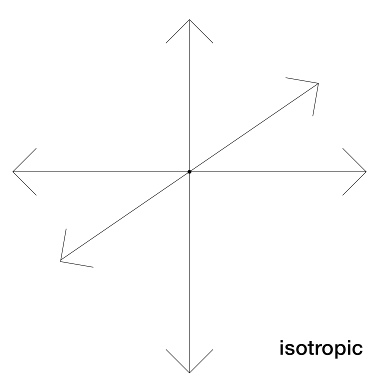
R. Isotropic

	P	Q	R
(a)	1	2	3
(b)	2	3	1
(c)	2	1	3
(d)	3	2	1

[2014]

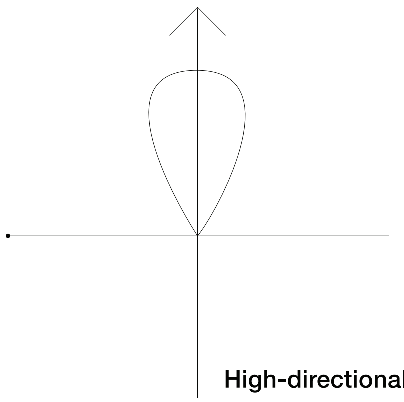
Solution: (b)

1. Point electromagnetic source.



isotropic

2. Dish antenna



So, Ans → $P - 2$; Q – 3; $R - 1$

Hence, the correct option is (b).

4. The radiation pattern of an antenna in spherical co-ordinates is given by

F(θ) = $\cos^4\theta$, 0 ≤ θ ≤ π/2

The directivity of the antenna is

(a) 10 dB (b) 12.6 dB
(c) 11.5 dB (d) 18 dB **[2012]**

Solution: (a)

$$D = \frac{4\pi U_{\max}}{\pi_{rad}}$$

$$f(\theta) = \cos^4\theta,\ 0'' \ \pi/2$$

$$\pi_{rad} = \int_0^{2\pi}\int_0^{\pi/2} f(\theta)\sin\theta\, d\phi$$

$$\pi_{rad} = \int_0^{2\pi}\int_0^{\pi/2} \cos^4\theta\sin\theta\, d\phi\, d\theta$$

$$\pi_{rad} = \frac{2\pi}{5}$$

$$D = \frac{4\pi U_{\max}}{\left(\frac{2\pi}{5}\right)} = 10U_{\max}$$

$$D = 10[f(\theta)]_{\max} = 10$$

D(dB) = 10 $\log_{10}^{(10)}$

= 10 dB

Hence, the correct option is (a).

5. For a Hertz dipole antenna, the Half Power Beam Width (HPBW) in the E-plane is

(a) 360° (b) 180°
(c) 90° (d) 45° **[2008]**

Solution: (c)

For a Hertz dipole antenna,

HPBW = 90°

Hence, the correct option is (c).

6. A transmission line is feeding 1 Watt of power to a horn antenna having a gain of 10 *dB*. The antenna is matched to the transmission line. The total power radiated

(a) 10 Watts (b) 1 Watt
(c) 0.1 Watt (d) 0.01 Watt **[2006]**

Solution: (b)

Gain of amplifier = 10 dB

But gain of antenna = Directive gain so, radiated power will be 1 *watt*

Hence, the correct option is (b).

7. Consider a lossless antenna with a directive gain of + 6dB. If 1 mW of power is fed to it the total power radiated by the antenna will be

(a) 4 mW (b) 1 mW
(c) 7 mW (d) 1/4 mW **[2004]**

Solution: (b)

For a lossless antenna, its efficiency is 100%. So, total power radiated will be 1 mw

Hence, the correct option is (b).

8. The line-of-sight communication requires the transmit and receive antennas to face each other. If the transmit antenna is vertically polarized, for best reception the receiver antenna should be

(a) horizontally polarized
(b) vertically polarized
(c) at 45° with respect to horizontal polarization
(d) at 45° with respect to vertical polarization **[2002]**

Solution: (b)

For best reception receiver antenna must have same polarization as transmit antenna

Hence, the correct option is (b).

9. The frequency range for satellite communication is

(a) 1 kHz to 100 kHz
(b) 100 kHz to 10 kHz
(c) 10 MHz to 30 MHz
(d) 1 GHz to 30 GHz **[2000]**

Solution: (d)

Frequency range for satellite communication is, (1GHz to 30 GHz)

Hence, the correct option is (d).

10. If the diameter of a λ/2 dipole antenna is increased from $\frac{\lambda}{100}$ to $\frac{\lambda}{50}$, then its

(a) bandwidth increases
(b) bandwidth decreases
(c) gain increases
(d) gain decreases **[2000]**

Solution: (b)

$$\text{Bandwidth} \propto \frac{1}{\text{diameter}}$$

Hence, the correct option is (b).

11. The vector $\vec{H}$ in the far field of an antenna satisfies

(a) $\nabla \cdot \vec{H} = 0$ *and* $\nabla \times \vec{H} = 0$

(b) $\nabla \cdot \vec{H} \neq 0$ *and* $\nabla \times \vec{H} \neq 0$

(c) $\nabla \cdot \vec{H} = 0$ *and* $\nabla \times \vec{H} \neq 0$

(d) $\nabla \cdot \vec{H} = 0$ *and* $\nabla \times \vec{H} = 0$ **[1998]**

Solution: (c)

We know,

$\nabla \times \vec{H} = J_C + J_D$

J_C = Conventional current density

J_d = Displacement current density

So, $\nabla \times \vec{H} \neq 0$

$\nabla . \vec{B} = 0$

So, $\nabla . \vec{H} = 0$

Hence, the correct option is (c).

12. The radiation resistance of a circular loop of one turn is 0.01 Ω. The radiation resistance of five turns of such a loop will be

(a) 0.002 Ω (b) 0.01 Ω

(c) 0.05 Ω (d) 0.25 Ω **[1998]**

Solution: (d)

$$R_r = 80\pi \left(\frac{ndl}{\lambda}\right)^2$$

$$R_r \propto n^2$$

$$\frac{R_{r1}}{R_{r2}} = \frac{\eta_1^2}{\eta_2^2}$$

$$\frac{0.01}{R_{r2}} = \frac{1}{(5)^2}$$

$$R_{r2} = 0.25\Omega$$

Hence, the correct option is (d).

13. An antenna in free space receives 2 μW of power when the incident electric field is 20 m V/m rms. The effective aperture of the antenna is

(a) 0.005 m^2 (b) 0.05 m^2

(c) 1.885 m^2 (d) 3.77 m^2 **[1998]**

Solution: (c)

$$A_e = \frac{P_r}{P_d}$$

$P_r = 2 \times 10^{-6} w$

$= 1.061 \times 10^{-6}$

$$A_e = \frac{2\times10^{-6}}{1.061\times10^{-6}}$$

$A_e = 1.885$ m^2

Hence, the correct option is (c).

14. The far field of an antenna varies with distance r as

(a) $1/r$ (b) $1/r2$

(c) $1/r3$ (d) $1/\sqrt{r}$ **[1998]**

Solution: (a)

$$E \propto \frac{1}{r}$$

Hence, the correct option is (a).

15. An antenna, when radiating, has a highly directional radiation pattern. When the antenna is receiving, its radiation pattern

(a) is more directive

(b) is less directive

(c) is the same

(d) exhibits no directivity at all **[1995]**

Solution: (c)

An antenna is a reciprocal device, whose characteristics remain same when it is transmitting or receiving.

Hence, the correct option is (c).

16. For a dipole antenna.

(a) the radiation intensity is maximum along the normal to the dipole axis

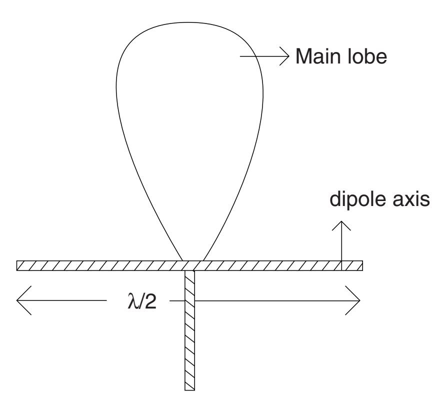
(b) the current distribution along its length is uniform irrespective of the length .

(c) the effective length equals its physical length

(d) the input impedance is independent of the location of the feed-point **[1994]**

Solution: (a)

The radiation intensity is maximum along the normal to the dipole axis.



Hence, the correct option is (a).

Two-marks Questions

1. A half wavelength dipole is kept in the x–y plane and oriented along 45° from the x-axis. Determine the direction of null in the radiation for $\theta \le \phi \le \pi$. Here the angle $\phi(0 \le \phi \le 2\pi)$ is measured from the z-axis and the angle is measured from the x-axis in the x-y plane. **[2017]**

(A) $\theta = 90°$, $\phi = 45°$ (B) $\theta = 45°$, $\phi = 90°$
(C) $\theta = 90°$, $\phi = 135°$ (D) $\theta = 45°$, $\phi = 135°$

Solution: Assuming the dipole is centrefed, the max radiation goes at right angles to its axis and no radiation (null) along its axis.

$\therefore$ for $0 \le \theta \le \pi$, the null exists for $\phi = 45°$ and $\phi = 90°$

Hence, the correct option is (A).

2. An antenna pointing in a certain direction has a noise temperature of 50 K. The ambient temperature is 290 K. The antenna is connected to a pre amplifier that has a noise figure of 2 dB and an available gain of 40 dB over an effective band width of 12 MHz. The effective input noise temperature T_e for the amplifier and the noise power P_{a0} at the output of the preamplifier, respectively, are **[2016]**

(A) $T_e = 169.36$ K and $P_{a0} = 3.73 \times 10^{-10}$ W
(B) $T_e = 170.8$ K and $P_{a0} = 4.56 \times 10^{-10}$ W
(C) $T_e = 182.5$ K and $P_{a0} = 3.85 \times 10^{-10}$ W
(D) $T_e = 160.62$ K and $P_{a0} = 4.6 \times 10^{-10}$ W

Solution: $F = 2\text{dB} = 10^2$

(Bandwidth) BW=12 MHz, and Gain is 40 dB = 10^4

Effective input noise temperature,

$$T_e = T_o [F - 1]$$

Where F is noise figure, T_o = ambient temp = 290 K

$$= 290\,[1.58 - 1]$$

$$= 290 \times 0.58$$

$$= 169.36° \text{ K}$$

Output noise power,

$$P_{ao} = KAT_eB$$

$$P_{ao} = (50 + 169.36) \times 1.38 \times 10^{-23} \times 10^4 \times 12 \times 10^6$$

$$= 3.63 \times 10^{-10} \text{ W}$$

Hence, the correct option is (A).

3. Two lossless X band horn antennas are separated by a distance of 200 λ The amplitude reflection coefficients at the terminals of the transmitting and receiving antennas are 0.15 and 0.18, respectively. The maximum directivities of the transmitting and receiving antennas (over the isotropic antenna) are 18 dB and 22 dB respectively. Assuming that the input power in the lossless transmission line connected to the antenna is 2 W and that the antennas are perfectly aligned and polarization matched, the power (in mw) delivered to the load the receiver is _____. **[2016]**

Solution:

$R = 200\lambda$	$D_{ot} = 63.1$
$\lvert\Gamma_t\rvert = 0.15$	$D_{0r} = 158.5$
$\lvert\Gamma_r\rvert = 0.18$	
$P_i = 2$ W	$\therefore \left[D(dB) = 10\log_{10} D\right]$

$$P_t = P_i\left[1 - \lvert\Gamma_t\rvert^2\right] = 1.955 \text{ W}$$

Friis transmission formula

$$P_r = P_t\left(\frac{\lambda}{4\pi R}\right)^2 D_{Ot}D_{Or}$$

$$= 3.1 \text{ mW}$$

Power delivered to load,

$$P_L = P_r\left[1 - \Gamma_r^2\right] \approx 3 \text{ mW}$$

Hence, the correct Answer is (3 mW).

4. The far zone power density radiated by a helical antenna is approximated as

$$\vec{W}_{\text{rad}} = \vec{W}_{\text{average}} = \approx \hat{a}_r\, C_0 \frac{1}{r^2}\cos^4\theta.$$

The radiated power density is symmetrical with respect to ϕ and exists only in the upper hemisphere; $0 \le \theta \le \frac{\pi}{2}$; $0 \le \phi \le 2\pi$; C_0, is a constant. The power radiated by the antenna (in watts) and the maximum directivity of the antenna, respectively are **[2016]**

(A) 1.5 C_0, 10 dB
(B) 1.256 C_0, 10dB
(C) 1.256 C_0, 12 dB
(D) 1.5 C_0, 12 dB

Solution: The radiated power can be expressed as

$$p_{rad} = \iint \overline{w}_{rad} . d\bar{s}$$

$$= c_0 \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} a_r \frac{1}{r^2}\cos^4\theta . a_r r^2 \sin\theta\, d\theta\, d\phi$$

$$= 1.256\, C_o.$$

The radiated power density is also symmetrical with respect to ϕ and present only in upper hemis so

$$D_0 = 4\pi\frac{U_{\max}}{P_{rad}}$$

$$4\pi \frac{C_0}{1.256C_0} \quad [\because U = r^2 W, U_{\max} = C_0]$$

$$D_o = 10$$

$$D_o(\text{dB}) = 10 \text{ dB}.$$

Hence, the correct option is (B).

5. A radar operating at 5 GHz uses a common antenna for transmission and reception. The antenna has a gain of 150 and is aligned for maximum directional radiation and reception to a target 1 km away having radar cross section of 3 m^2. If it transmits 100 kW, then the received power (in μW) is__________. **[2016]**

Solution: Gain of antenna $G = 150$

Frequency of radar $f = 5$ GHz; $P_t = 100$ kW

Distance of target $r = 1$ km;

Radar cross sectional area $A = 3$ m^2

$$\lambda = c/f = (3 \times 10^8)/(5 \times 10^9)$$

$$= 0.06 \text{ m}$$

Received power,

$$p_r = \frac{p_t G^2 \lambda^2 S}{(4\pi)^3 r^4}$$

$$= (100 \times 10^3 \times 150^2 \times 0.06^2 \times 3)/(4 \times 3.14)^3(1000)^4$$

$$= 0.012 \ \mu\text{W}$$

6. In spherical coordinates, let a_θ, a_ϕ denote unit vectors along the θ φ, directions.

$$E = \frac{100}{r}\sin\theta \cos(\omega - \beta)\,\hat{a}_\theta \ \ and$$

$$H = \frac{0.265}{r}\sin\theta \cos(\omega t - \beta r)\hat{a}_\phi \ \ A/m$$

represent the electric and magnetic field components of the EM wave at large distances r from a dipole antenna, in free space The average power (W) crossing the hemispherical shell located at $r = 1$ km, $0 \le \theta \le \varphi/2$ is____ **[2014]**

Solution: (55.5)

$$P_{avg} = \frac{E_0^2}{2\eta}$$

Power radiated, $= \int_0^{2\pi}\int_0^{\pi/2} \frac{E_0^2}{2\eta}.r^2 \sin\theta \, d\theta d\phi$

Here, η = 377 Ω

$$r = 1 \text{ km}$$

$$E_0 = \frac{100}{r}\sin\theta$$

Putting this in the integral, we get

$$= \int_0^{2\pi}\int_0^{\pi/2} \frac{10000}{2\eta r^2}.r^2 \sin^3\theta \, d\theta d\phi$$

$$= 2\pi \int_0^{\pi/2} \frac{10000}{2\eta}\sin^3\theta \, d\theta$$

$$= 55.5$$

7. At 20 GHz, the gain of a parabolic dish antenna of 1 meter diameter and 70% efficiency is

(a) 15 dB (b) 25dB

(c) 35 dB (d) 45 dB **[2008]**

Solution: (d)

$f = 20$ GHz

$$\lambda = \frac{3 \times 10^8}{20 \times 10^9} = 1.5 \, cm$$

η = 70%, $D = 1$ m

$$G = \eta\pi^2 \left(\frac{D}{\lambda}\right)^2$$

$$G = 0.7\pi^2 \left|\frac{1 \times 20 \times 10^9}{3 \times 10^8}\right|^2$$

$G = 30705.43$

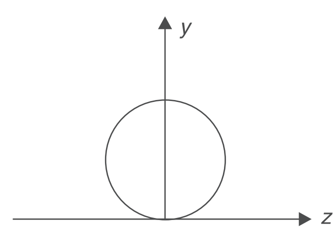
$G(\text{dB}) = 10 \log_{10} (30705.43)$

$= 45$ dB

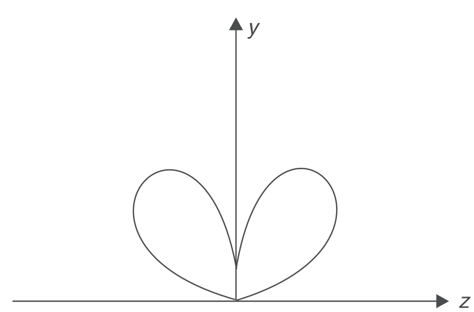
Hence, the correct option is (d).

8. A $\frac{\lambda}{2}$ dipole is kept horizontally at a height of $\frac{\lambda_0}{2}$ above a perfectly conducting infinite ground plane. The radiation pattern in the plane of the dipole ($\vec{E}$ plane) looks approximately as

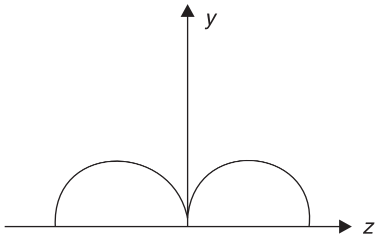
(a)



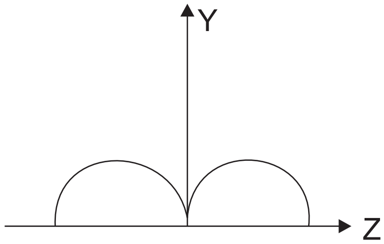
(b)



(c)



(d)



[2007]

Solution: (b)

For, (a) $\rightarrow d = \lambda/4$

(b) $\rightarrow d = \lambda/2$

(c) $\rightarrow d = \lambda$

Hence, the correct option is (b).

9. A mast antenna consisting of a 50 meter long vertical conductor operates over a perfectly conducting ground plane. It is base-fed at a frequency of 600 kHz The radiation resistance of the antenna in Ohms is

(a) $\frac{2\pi^2}{5}$ (b) $\frac{\pi^2}{5}$

(c) $\frac{4\pi^2}{5}$ (d) $20\ \pi^2$ **[2006]**

Solution: (a)

$$R_{rad} = 80\pi^2\left(\frac{dl}{\lambda}\right)^2\ \Omega$$

For conducting ground, $R_{rad} = 40\pi^2\left(\frac{dl}{\lambda}\right)^2\ \Omega$

$f = 600$ kHz

$$\lambda = \frac{3 \times 10^8}{20 \times 10^9} = 1.5\ cm$$

$\lambda = 500$ m, l = 50 m

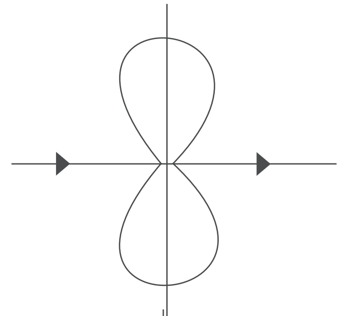
$$R_{rad} = \frac{40\pi^2}{\left(\frac{50}{500}\right)^2}$$

$$R_{rad} = \frac{2\pi^2}{5}$$

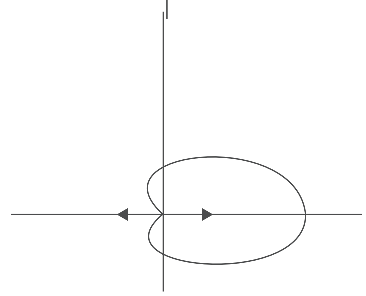
Hence, the correct option is (a).

10. Two identical and parallel dipole antennas are kept apart by a distance of $\lambda/4$ in the H-plane. They are fed with equal currents but the right most antenna has a phase shift of +90°. The radiation pattern is given as

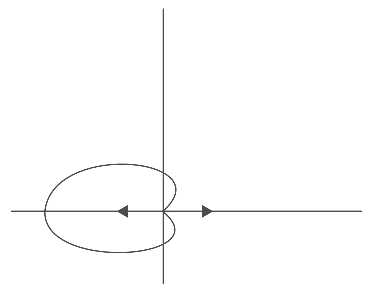
(a)



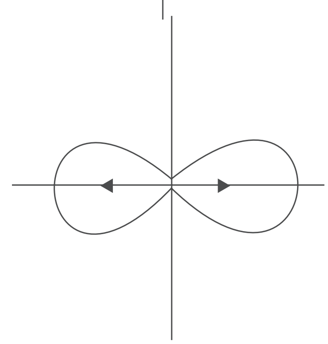
(b)



(c)

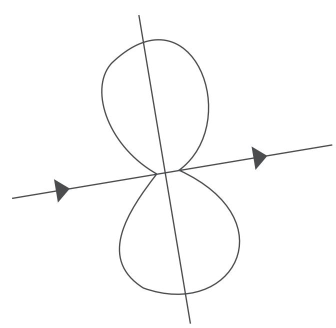


(d)



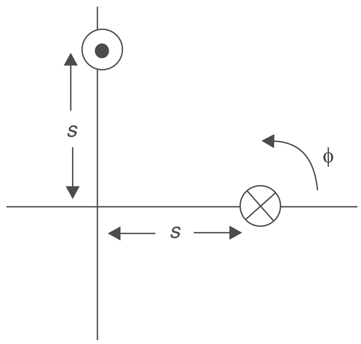
[2005]

Solution: (a)



Hence, the correct option is (a).

11. Two identical antennas are placed in the $\varphi = \pi/2$ plans as shown in the figure The elements have equal amplitude excitation with 180° polarity difference, operating a: wavelength λ. The correct value of the magnitude of the far-zone resultant electric field strength normalized with that of a single element, both computed for φ = 0, is



(a) $2\cos\left(\frac{2\pi s}{\lambda}\right)$ (b) $=2\sin\left(\frac{\pi s}{\lambda}\right)$

(c) $2\cos\left(\frac{\pi s}{\lambda}\right)$ (d) $2\sin\left(\frac{\pi s}{\lambda}\right)$ **[2003]**

Solution: (d)

$\psi = \beta d \sin\theta \cos\phi + \delta$

θ = 90°, $d = \sqrt{2}s$

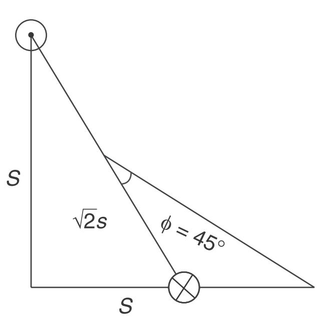
φ = 45°, δ = 180°

$$\psi = \frac{2\pi}{\lambda}\sqrt{2}\cos 45° + 180°$$

Normalized array factor $= 2\left|\cos\frac{\psi}{2}\right|$

$$= 2\left\{\cos\left(\frac{\pi}{\lambda}\sqrt{2}s\cos 45° + 90°\right)\right\}$$

$$2\sin\left(\frac{2\pi s}{\lambda}\right)$$



Hence, the correct option is (d).

12. A person with a receiver is 5 km away from the transmitter. What is the distance that this person must move further to detect a 3 dB decrease in signal strength?

(a) 942 m (b) 2070 m
(c) 4978 m (d) 5320 m **[2002]**

Solution: (b)

Signal strength at distance R, $P_1 = \dfrac{P}{4\pi R_1^2}$

R = 5000 m

$$\frac{P_1}{P_2} = \left(\frac{\lambda}{5000}\right)^2 = \frac{P_1}{P_1/2}$$

$$\lambda = 5000\sqrt{2}$$

Extra distance = (5000√2 – 5000) m = 2071 m

Hence, the correct option is (b).

13. In a uniform linear array, four isotropic radiating elements are spaced λ/4apart. The progressive phase shift between the elements required for forming the main beam at 60° off the end-fire is:

(a) –πradians (b) –π/2 radians
(c) –π/4 radians (d) –π/8 radians **[2001]**

Solution: (c)

Phase difference, $\psi = \beta d\cos\theta + \alpha$

For an array to be end fire, ψ = 0

So, $\beta d\cos\theta + \alpha = 0$

$a = -\beta d\cos\theta$

$\alpha = -\beta d$ [maximum]

So, $\psi = \beta d\cos\theta - \beta d$

$= \beta d(\cos\theta - 1)$

$d = \dfrac{\lambda}{4}$ and θ = 60°

$$\psi = \frac{2\pi}{\lambda}\times\frac{\lambda}{4}[\cos 60 - 1]$$

$$\psi = -\frac{\pi}{4}\text{ radians}$$

Hence, the correct option is (c).

14. A medium wave radio transmitter operating at a wavelength of 492 m has a tower antenna of height 124 m. What is the radiation resistance of the antenna?

(a) 25Ω (b) 36.5 Ω
(c) 50Ω (d) 73Ω **[2001]**

Solution: (b)

Here, L = 124 m i.e. nearly about λ/4

$$L \simeq \frac{492}{4} = 123\text{ m}$$

So, given antenna is quarter wave monopole antenna, whose R_a = 36.5 Ω

Hence, the correct option is (b).

15. For an 8 feet (2.4m) parabolic dish antenna operating at 4 GHz, the minimum distance required for far field measurement is closest to

(a) 7.5 cm (b) 15 cm
(c) 15 m (d) 150 m **[2000]**

Solution: (d)

$F = 4$ GHz

$$\lambda = \frac{3\times10^8}{4\times10^9}$$

$\lambda = 7.5$ cm

$l = 2.4$ m

(distance), $d = \dfrac{2l^2}{\lambda}$

$d = 1.536$ m 150 cm

Hence, the correct option is (d).

16. A transmitting antenna radiates 251 W isotropically. A receiving antenna, located 100 m away from the transmitting antenna has an effective aperture of 500 cm^2 The total power received by the antenna is

(a) 10 μW (b) 1 μW
(c) 20 μW (d) 100 μW **[1999]**

Solution: (d)

Intensity [at d = 100 m],

$$I = \frac{251}{4\pi(100)^2}\,\text{w/m}^2$$

Power = $I \times A_e$

$Ae = 500$ cm^2

$$\text{power} = \frac{251}{4\pi(100)^2}\times500\times10^{-4}\ \text{watt}$$

$$P = 100\mu w$$

Hence, the correct option is (d).

17. A parabolic dish antenna has conical beam 2° wide. The directivity of the antenna is approximately

(a) 20 dB (b) 30 dB
(c) 40 dB (d) 50 dB **[1997]**

Solution: (c)

$$\text{Directivity} = \frac{41253}{BW(\theta)\,BW(\phi)}$$

$BW(\theta) = BW(\phi) = 2°$

$$D = \frac{41253}{(2)^2} = 10{,}000$$

$D\,dB = 10\log(10)^4$

$D = 40\,dB$

Hence, the correct option is (c).

18. A transverse electromagnetic wave with circular polarization is received by a dipole antenna. Due to polarization mismatch, the power transfer efficiency from the wave to the antenna is reduced to about

(a) 50% (b) 35.3%
(c) 25% (d) 0% **[1996]**

Solution: (d)

The power transfer efficiency from the wave to antenna is reduced to 0%.

Hence, the correct option is (c).

19. A 1 km long microwave link uses two antennas each having 30 dB gain If the power transmitted by one antenna is 1 W at 3 GHz the power received by the other antenna is approximately

(a) 98.6 μW (b) 76.8 μW
(c) 63.4 μW (d) 55.2 μW **[1996]**

Solution: (c)

$$Pd\ (\text{at } d = 1\text{ km}) = \frac{P_tG_t}{4\pi R^2}$$

$$P_d = \frac{1}{4000\pi}$$

$P_t = 1$ w, $G_t = 1000$

$R = 1$ km = 1000 cm

$$(A_e)_r = \frac{G_r\lambda^2}{4\pi}$$

$$P_r = \frac{G_r\lambda^2}{4\pi}\cdot\frac{1}{4000\pi}$$

$P_r = (\text{Ae})rP_d$

$G_r = 1000$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^9} = 10\text{ cm}$$

$$P_r = 63.3\mu w$$

Hence, the correct option is (c).

20. Two dissimilar antennas having their maximum directivities equal,

(a) must have their beam widths also equal
(b) cannot have their beam widths equal because they are dissimilar antenna
(c) may not necessarily have their maximum power gains equal
(d) must have their effective aperture areas (capture areas) also equal **[1992]**

Solution: (a)

Directivity, $D = \dfrac{4\pi}{\Omega^2}$ [Beam width]

So, If $D_1 = D_2$

then, $\Omega1 = \Omega2$

Therefore, Beam width also equal.

Hence, the correct option is (a).

21. The beamwidth between firs nulls of a uniform linear array of N equally spaced (element spacing = d), equally excited antennas, is determine by
(a) N alone and not by d
(b) d alone and not by N
(c) the ratio, (N/d)
(d) the product, (Nd) **[1992]**

Solution: (d)

BWFN of a uniform linear array of N equally spaced is,

For end fire array, $BWFN = \frac{4}{3}\sqrt{\frac{2\lambda}{Nd}}$

For Broadside array, $BWFN = \frac{2\pi}{Nd}$

So, we need the product (Nd) to determine $BWFN$.

Hence, the correct option is (d).

22. In a board side array of 20 isotropic radiators, equally spaced at distance of $\lambda/2$, the beam width between first nulls is
(a) 51.3 degrees (b) 11.46 degrees
(c) 22.9 degrees (d) 102.6 degrees
[1991]

Solution: (b)

$$BWFN = \frac{2\lambda}{\eta d}\text{ radian}$$

Here, $\eta = 20$ and $d = \frac{\lambda}{2}$

$$BWFN = \frac{2\lambda}{20.\lambda/2} = \frac{1}{5}\text{ radian}$$

$$= \frac{180}{\pi}\times\frac{1}{5}\text{ degrees}$$

$BWFN = 11.46$ degrees

Hence, the correct option is (b).

23. Two isotropic antennas are separated by a distance of two wavelengths. If both the antennas are fed with currents of equal phase and magnitude, the number of lobes in the radiation pattern in the horizontal plane are
(a) 2 (b) 4
(c) 6 (d) 8 **[1990]**

Solution: (d)

Phase difference, $\psi = \alpha + \beta d\cos\theta$

α = current phase difference = 0

$d = 2\lambda$

$$\therefore \psi = \frac{2\pi}{\lambda}.2\lambda\cos\theta = 4\pi\cos\theta$$

$$|E_T| = 2E_0\cos\frac{\psi}{2}$$

Say $E_0 = 1$

$$2\cos\frac{\psi}{2} = 2\cos\left[\frac{4\pi\cos\theta}{2}\right] = 2\cos(2\pi\cos\theta)$$

For θ: 0 to 2π

$|E_T|$ is maximum at $\theta = 0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$

So, total number of lobes are 8.

Hence, the correct option is (d).

24. The electric field E and the magnetic field H of a short dipole antenna satisfy the condition
(a) the r component of E is equal to zero
(b) both r and θ components of H are equal to zero
(c) the θ component of E dominates the r component in the far-field region
(d) the θ and φ components of H are of the same order of magnitude in the near field region **[1988]**

Solution: (b) and (c)

For short dipole antenna, components of E and H are Er, $E\theta$ and $H\varphi$

Hence, the correct option is (b) and (c).

Five-marks Questions

1. A thin-dipole antenna is (λ/15) long, where X is the operating wavelength. If the dipole has a loss resistance of 1.5 ohm determine its
(i) radiation resistance, and
(ii) terminal resistance **[1987]**

Solution:(i) $y = \frac{\lambda}{15} < \frac{\lambda}{10}$

So, the given dipole antenna can be considered as a short antenna, so the current distribution is uniform with average equal to $lm/2$.

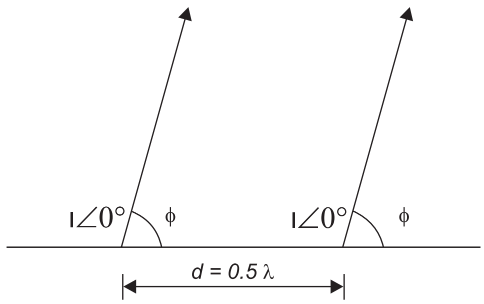
$$P_r = 80\pi^2\left(\frac{l}{\lambda}\right)^2\left(\frac{I_m}{2\sqrt{2}}\right)^2 = \frac{(I_m)^2}{(\sqrt{2})^2}R_x$$

$$\text{So, } R_x = 20\pi^2\left(\frac{l}{\lambda}\right)^2$$

(ii) Terminal resistance of the Antenna

$$= R_{10ss} + R_{rad} = 1.5 + 20\pi^2\left(\frac{l}{\lambda}\right)^2$$

2. Consider an array of two non-directional radiators with spacing $d = 0.5\lambda$. Determine the directions of maximum radiation when the radiators are excited as shown in figure. Calculate the phase shift required for turning the direction of the maximum radiation by 90°, keeping the separation, d, unchanged.



[1993]

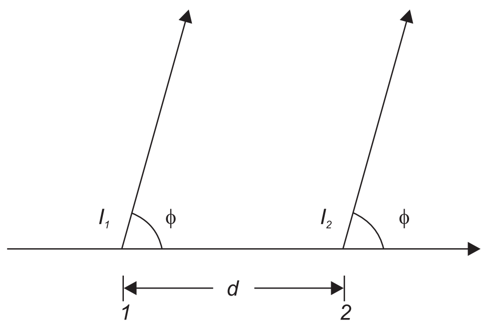
Solution: $l_2 = kl_1 < \alpha$

Given, $I_2 = I_1 = I\angle 0°$

So, α = 0

$k = 1$

$d = 0.5\lambda$



for n element with $k = 1$

$$\frac{E_T}{E_1} = \left| \frac{\sin\left(\eta \dfrac{\psi}{2}\right)}{\sin\left(\dfrac{\psi}{2}\right)} \right|$$

$$\psi = \beta d \cos\phi = \alpha = \frac{2\pi}{\lambda}.\frac{\lambda}{2}\cos\phi + 0 = \pi \cos\phi$$

for $n = 2$

$$\frac{E_T}{E_2} = \left| 2\cos\left(\frac{\psi}{2}\right) \right| = \left| 2\cos\left(\frac{\pi}{2}\cos\phi\right) \right|$$

for maximum radiation,

$$\frac{\pi}{2}\cos\phi = 0, \Rightarrow \phi = \pm\pi/2$$

for turning the direction of the maximum radiation by 90°

$$y = \frac{\lambda}{15} < \frac{\lambda}{10}$$

ϕ : = 0 and $\pi/2$

3. Two spacecrafts are separated by 3000 km. Each has a paraboloidal reflector antenna of 0.85 m diameter operating at a frequency of 2 GHz with an aperture efficiency of 64%. If the spacecraft A's receiver requires 1 pW for a 20 dB signal-to-noise ratio, what transmitter power is required on the spacecraft B to achieve this signal-to-noise ratio? **[1994]**

Solution: R = 3000 km, D = 0.85 m

$e_A = e_B = 64\%$

$f = 2 \times 10^9$ Hz

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{2\times10^9} = 15\text{ cm} = 0.15\text{ cm}$$

20 dB (SNR) = 10 $\log_{10}$ (SNR)

SNR = 100

$P_A = 1PW = 10^{-12}$ W

if gain of 2nd Antenna

$$G_B = \frac{4\pi}{\lambda^2}.\frac{\pi D^2}{4} = \frac{\pi^2 D^2}{\lambda^2}$$, power density

of 2nd Antenna

$$W = \frac{e_B.P_B.G_B}{4\pi R^2}$$, So power transmitted by 2nd Antenna,

$$P_B = \frac{W\times 4\pi R^2}{e_B}\frac{\lambda^2}{\pi e_B D^2} = \frac{4WR^2\lambda^2}{\pi e_B D^2}$$

So, power received at transmitting Antenna,

$$P_A = W \times w_A \times \frac{\pi D^2}{4},\ W = \frac{P_A \times 4}{e_A \pi D^2}$$

So,

$$P_B = \frac{4R^2\lambda^2}{\pi D^2} \times \frac{4P_A}{e_A \pi D^2} = \frac{16 P_A R^2 \lambda^2}{\pi^2 e_A e_B D^4} = 1.5335\text{ W}$$

4. Two dipoles are so fed and oriented in free space that they produce the following electromagnetic waves:

$E_x = 10e^{i(\omega t - z\omega\pi/3)}$volts/meter

$E_y = j10e^{\,i(\omega t - z\omega\pi/3)}$ volts/meter

(a) Write down the expression for the corresponding magnetic field strength vector.

(b) Calculate the frequency of the wave.

(c) Give the complete description of the polarization of the wave. **[1995]**

Solution: given, $E_x 10e^{J(\omega t - \pi/3z)}$

$$E_y = J10e^{J(\omega t - 2\pi/3)} = 10e^{J(\omega t - 2\pi/3 + \pi/2)}$$

em wave is travelling in the positive z-direction β = π/3.

for free space, $\eta_0 = 120\pi\Omega$

(a) $\eta = -\dfrac{E_y}{H_x} \Rightarrow$

$$H_x = -\frac{E_y}{\eta} = -\frac{1}{12\pi} e^{J(\omega t - 2\pi/3 - \pi l2)}$$

$$\eta = \frac{E_x}{H_y} \Rightarrow H_y = \frac{E_y}{\eta} = \frac{10e^{J(\omega t - 2\pi/3)}}{120\pi}$$

$H_2 = 0,$

(b) $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{\pi/3} = 6\text{m}$

$$f = \frac{C}{\lambda} = \frac{3\times 10^8}{6} = 50 \text{ MHz}$$

(c) let, $Z = 0$, $E_x = 10 \cos \omega t = E_0 \cos \omega t$

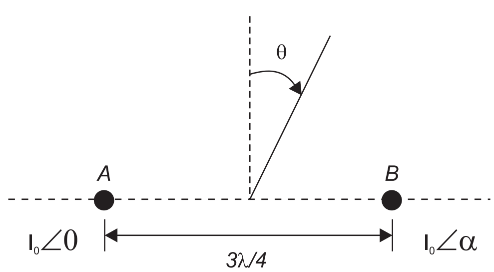
$E_y = -10 \sin \omega t = -E_0 \sin \omega t = E_0 \cos \omega t$

So, $E_0 = 10 = \sqrt{E_x^2 + E_y^2}$,

$\phi := \pi/\mathbf{2}$

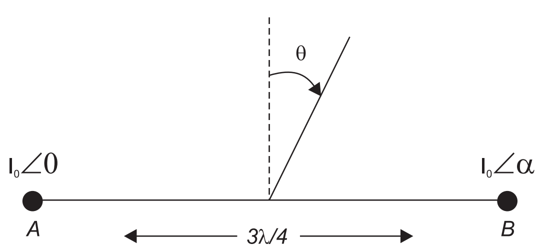
So the polarization is left polarization.

5. Two isotropic antennas A and B form an array as shown in figure. The currents fed to the two antennas are $I_0\angle 0$ and $I_0\angle$ respectively. What should be the value of a so that the radiation pattern has a null at $e = 30°$. Find the direction of the maximum radiation for that value of a and draw the radiation pattern. (λ. is the wavelength of operation).



[1996]

Solution:



$$\psi = \beta d \cos\phi + \alpha\,;\ \psi = \frac{2\pi}{\lambda},\ d = \frac{3\lambda}{4}$$

$$\psi = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} \cos\phi + \alpha = \frac{3\pi}{2} \cos\phi + \alpha$$

$$\frac{E_T}{E_A} = \frac{2\sin\frac{\psi}{2}.\cos\frac{\psi}{2}}{\sin\frac{\psi}{2}} = 2\cos\frac{\psi}{2}$$

$$= 2\cos\left(\frac{3\pi}{4}\cos\phi + \frac{\alpha}{2}\right) \quad (1)$$

$\varphi = 30°$

$\phi = 90 - \theta = 60°$

from (1) $\dfrac{3\pi}{4} \times \dfrac{1}{2} \times \dfrac{\alpha}{2} = \dfrac{\pi}{2}$

or, $\alpha = \pi/4$

So, $\dfrac{E_T}{E_A} = 2\cos\left\{\dfrac{3\pi}{4}\cos\phi + \dfrac{\pi}{8}\right\}$

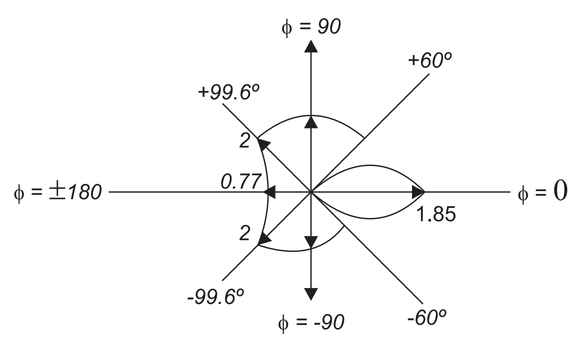
for maximum radiation,

$$\cos\left[\frac{3\pi}{4}\cos + \frac{\pi}{\theta}\right] = 1$$

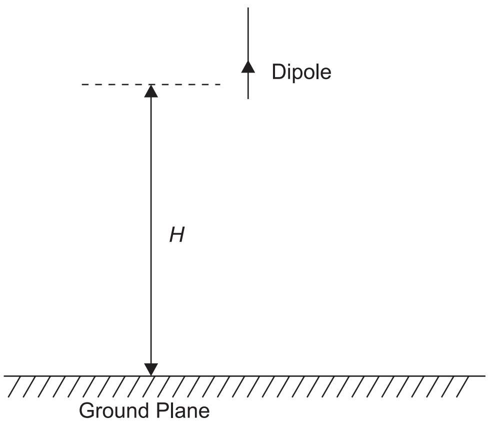
$$\Rightarrow \frac{3\pi}{4}\cos\phi + \frac{\pi}{8} = 0 \text{ or } \pm\pi$$

$\phi_m = -9.6°$ and $-170.4°$

φ	ψ / 2	E_t/E_a
0	$\frac{7\neq}{8}$ or $-\frac{\pi}{8}$	1.85
60°	π/2	0
90°	π/8	1.85
99.6	0	2
180°	$\frac{-5\pi}{8}$	0.77



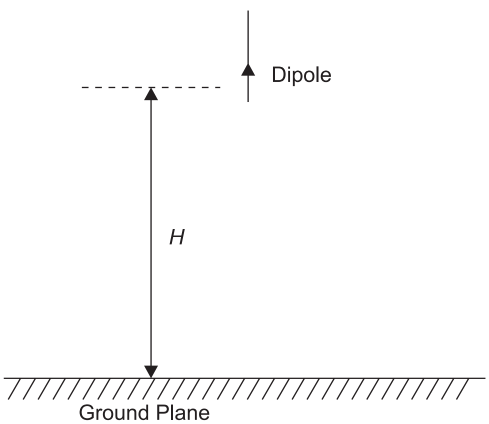
6. A dipole antenna has a sin 9 radiation pattern, where the angle 8 is measured from the axis of the dipole. The dipole is vertically located above an ideal ground plane (Figure). What should be the height of the dipole, H in terms of wave length so as to get a null in the radiation pattern at an angle of 45° from the ground plane? Find the direction of maximum radiation also.



[1997]

Solution:

A dipole antenna having a sin θ radiation pattern can be considered as an horizontal dipole,



$$\beta = \frac{2\pi}{\lambda},\ \beta d = \frac{2\pi}{\lambda}2H$$

$$\frac{E_T}{E_A} = \frac{2\sin\left(\frac{\psi}{2}\right)\cos\left(\frac{\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} = 2\cos\left(\frac{\psi}{2}\right)$$

$$\psi = \beta d\cos\phi + \alpha = \frac{4\pi H}{\lambda}\cos\phi$$

$$\frac{E_T}{E_A} = 2,$$

$$\psi = 0 \text{ or } \phi := \pi/2$$

$$\frac{E_T}{E_A} = 0,\ \psi = \pi$$

for null

$$\psi = \pi = \frac{4\pi H}{\lambda}\cos\phi,\ \phi := 45^0$$

$$\frac{4\pi H}{\lambda}\cos 45° = \pi, \Rightarrow H = \frac{\lambda}{2\sqrt{2}}$$

for maximum radiation,

$\psi = 0$,

$$\frac{4\pi H}{\lambda}\cos\phi = 0,\ \phi = \frac{\pi}{2}$$

7. The average power of an omnidirectional antenna varies as the magnitude of cos(θ), where θ is is the azimuthal angle. Calculate the maximum directive gain of the antenna' and the angles at which it occurs. **[1999]**

Solution: Given that,

$$\phi(\theta,\phi) = \phi_m\,|\cos\phi\,|$$

total radiated power,

$$W_k \int \phi(\theta,\phi)\,d\Omega$$

$$W_r = \int \phi_m \cos\theta.\sin\theta\ d\theta.d\phi$$

$$= \frac{\phi_m}{2}\int_0^{\pi}\sin 2\theta.d\theta \int_0^{2\pi} d\phi = \pi\phi_m$$

$$\phi_{avg} = \frac{W_r}{4\pi} = \frac{\phi_m}{4}$$

Directive gain of the antenna,

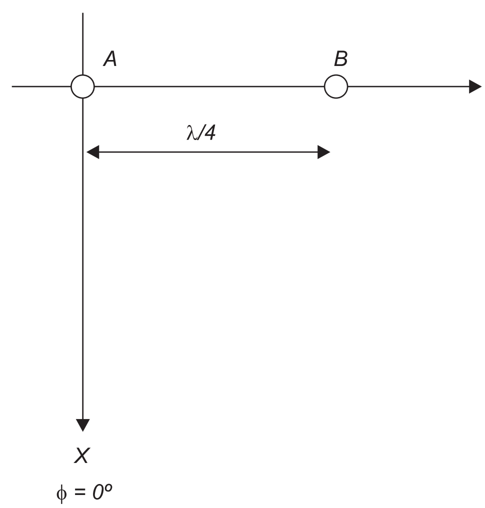
$$g_a(\theta,\phi) = \phi\frac{(\theta,\phi)}{\phi_{avg}} = 4\cos\theta$$

$g_{a\ \max} = 4$, this occur for θ = 0° or 180°

8. Consider a linear array of two half-wave dipoles A and B as shown in figure. The dipoles are λ/4 apart and are excited in such a way that the current on element B lags that on element A by 90° in phase.

(a) Obtain the expression for the radiation pattern for E in the XY plane, i.e., (θ = 90°).

(b) Sketch the radiation pattern obtained in (a).



[2002]