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Optimization Under Uncertainty

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Investment Planning for Electricity Generation

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1 Introduction

The demand for electricity depends on a multitude of factors, ranging from weather and time of the day to the economic industrial activity. This demand always has to be satisfied. However, keeping up peak production at all times would be unnecessarily expensive. Furthermore, there are different methods available for energy generation. The trade-off between these technologies is key in efficient energy production, as each of the available technologies has a different balance between investment and operating costs. A strategic investment allows for cost minimization whilst guaranteeing that demand can be satisfied in each scenario. In this case study, variations of the problem are considered, where randomness is introduced in different aspects of the model in order to mimic the stochasticity of the energy demand curve. First, only the demand is considered to be random. Second, uncertainty is introduced in the availability of the technologies and importing of energy is allowed. Lastly, randomness is introduced in the duration of the different demand levels. For each of these models, it is attempted to find the optimal initial investment and use of the different technologies, with the objective to minimize expected cost while satisfying demand.

2 Problem Formulation

This contribution is based on the problem discussed in Louveaux and Smeers 1988. In the article, a multistage programming model is formulated to optimize the investment plan. In our investigation, a two-stage recourse model will be applied; where the first stage deals with the investment planning, and the second stage deals with the distribution of the technologies over the different segments of the load-curve. Since the load curve is not know in advance, it is instead modelled using random variables. The used approximation of the load-curve is a piecewise constant curve. Hence, during T_j time units, demand is D_j, $j = 1, 2, .., k$. For now, it is assumed $T_1, T_2, .., T_k$ are fixed time units, whereas demand $D_1, D_2, .., D_k$ are random variables. The demand is specified as $D_h = \sum_{j=1}^{h} \xi_j$, $h = 1, 2, .., k$. That is, the demand related to the j-th demand block is ξ_j, which is defined as the j-th "mode". It is assumed that there is no technology available to start with. In our case study, 4 technologies are considered, which can be used to satisfy demand in 3 different modes. The load curve for a general case with k modes and the relationship with the random variables used to reconstruct it is shown in Figure 1.

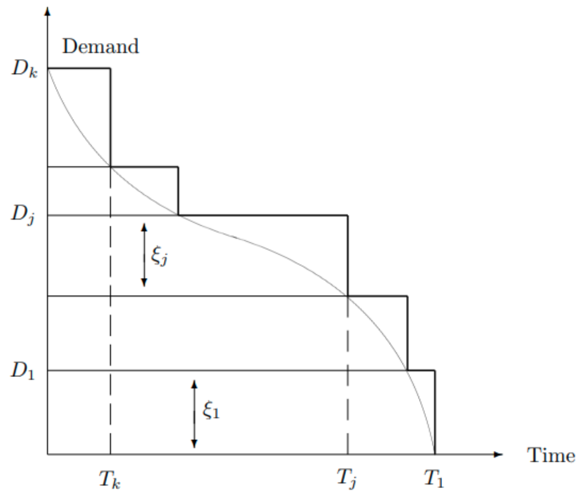


Figure 1: Approximation of the Load Curve for k Modes

The variables are defined as follows.

n number of technologies
k number of modes

x_i capacity for technology i, $i = 1, .., n$
y_{ij} capacity of technology i effectively used in mode j, $j = 1, .., k$
I_j energy imported in mode j, $j = 1, .., k$

ξ_j demand for mode $j, j = 1, .., k$
$\xi_j^{\max}$ maximum possible value for $\xi_j, j = 1, .., k$

T_j duration of mode j, $j = 1, .., n$
c_i investment and maintenance cost for technology i per unit capacity, $i = 1, .., n$
q_i production cost of technology i per unit capacity per unit of time, $i = 1, .., n$
c_{max} maximum allowed investment cost

3 Methodology

3.1 Base Model

In the base model, only the electricity demands ($\xi_j, j = 1, 2, 3$) are random. The distribution of the demand variables and their means, are given in Table 1. Furthermore, the available technologies and their corresponding investment (c_i) and production costs (q_i), can be found in Table 2.

Prob.	ξ_1	ξ_2	ξ_3
0.3	3	2	1
0.4	5	3	2
0.3	7	4	3
$\bar{\xi}$	5	3	2

Table 1: Distribution of Energy Demand

Tech.	c_i	q_i
x_1	10	4.0
x_2	7	4.5
x_3	16	3.2
x_4	6	5.5

Table 2: Available Technologies

It is valuable to note that technology 3 has a relatively high investment costs, but a low production costs, while for technology 4 the opposite holds. Technology 1 and 2 are more balanced in the trade-off between production and investment costs. Theoretically, one could expect technology 3 to be used in modes with a long duration, as there the production costs add up rapidly over time, whereas technology 4 appears more suitable to satisfy short peak demands. Lastly, note that the probability of demand decreasing over the modes, $P(\xi_1 \geq \xi_2 \geq \xi_3)$, is 82%, since:

$$P(\xi_1 \geq \xi_2 \geq \xi_3) = 1 - P(\xi_1 < \xi_2) - P(\xi_1 < \xi_3) - P(\xi_2 < \xi_3) = 1 - 0.3^2 - 0 - 0.3^2 = 0.82. \quad (1)$$

Hence, in many of the realizations, the modes will be decreasing in duration. This is a fact that will be useful in analyzing how the technologies are used over the different modes, as the demand of the mode is of natural importance in order to select the optimal technology for it.

3.1.1 Objective

The main objective of the problem is to minimize all expenses and to satisfy the demand under the given constraints. The expenses can be set up in two parts; the immediate investment in the technologies, and the use of these technologies to satisfy the realized demand

$$\min_{x \geq 0} \left\{ \sum_{i=1}^{n} c_i x_i + E_\xi \min_{y \geq 0} \{ \sum_{i=1}^{n} \sum_{j=1}^{k} q_i T_j y_{ij} : \text{2nd Constraints} \} : \text{1st Constraints} \right\}. \quad (2)$$

3.1.2 Constraints

The first stage constraints are given by the following equations.

$$\sum_{i=1}^{n} x_i \geq \sum_{j=1}^{k} \xi_j^{\max}, \quad (3)$$

that is, the total available capacity has to be enough to cover maximum demand. This constraint ensures that even in the worst possible scenario, demand can always be satisfied with the available technologies.

$$\sum_{i=1}^{n} c_i x_i \leq c_{\max}, \quad (4)$$

this is the budget constraint. The initial investment can not exceed the planned budget for technological investment.

The second stage constraints are as follows

$$\sum_{j=1}^{k} y_{ij} \leq x_i \; \forall \; i = 1, .., n, \tag{5}$$

this set of constraints ensures that technologies can not use more capacity over the different modes than was invested in for each technology.

$$\sum_{i=1}^{n} y_{ij} \geq \xi_j \; \forall \; i = j, .., k, \tag{6}$$

this set of constraints ensures that technologies are used in such a way that demand is satisfied in every mode.

This model is then presented in matrix form. For the exact matrices used we refer to the Section A, where the numerical values are also displayed. The objective function is then given by

$$\min_{x \geq 0} cx + qy. \tag{7}$$

With the first set of constraints being represented by

$$Ax \leq b, \tag{8}$$

and the second set of constraints being represented by

$$Tx + Wy \leq h(\xi). \tag{9}$$

3.2 Random Technology Availability and Energy Import

In this section we extend our model by including a more realistic representation of the energy production process and its market. The effective output of an installed technology varies according to factors peculiar to each technique of electricity generation. We attempt to capture these uncertainty by introducing a random parameter α_i, for $i = 1, \ldots, n$, that dampens the operational availability of each of the technology according to

$$\begin{aligned} \alpha_1 &\sim \mathcal{U}(0.6, 0.9) \\ \alpha_2 &\sim \mathcal{U}(0.7, 0.8) \\ \alpha_3 &\sim \mathcal{U}(0.5, 0.8) \\ \alpha_4 &\sim \mathcal{U}(0.9, 1.0). \end{aligned}$$

For an installed capacity x_i, the actual amount available is then given by $\alpha_i x_i$. We further introduce the possibility to import electricity, a transaction that has no investment cost by high production cost ($q_I = 10$).

The new feature can be seen as an exaggerated version of the forth technology, so we would expect it to be used mostly in the later modes of the load curve. We define the recourse variable for the imported electricity as $I = \begin{pmatrix} I_1 & I_2 & I_3 \end{pmatrix}$, where I_i corresponds to the amount of electricity imported to cover demand in mode i.
The most important consequences for our modeling approach, however, resides in the relaxation of the constraints for maximum capacity coverage in the first stage and the mode use of the available technology in the second stage, which can now be supplemented by the I_i's. It is always possible to import electricity, even if the installed capacity is not able to cover demand, or if you do not have capacity at all.
The new objective function can be expressed as

$$\min_{x\geq 0}\left\{\sum_{i=1}^{n} c_i x_i + E_\xi \min_{y\geq 0}\{\sum_{i=1}^{n}\sum_{j=1}^{k} q_i T_j y_{ij} + \sum_{j=1}^{k} q_I T_j I_j : \text{2nd Constraints}\} : \text{1st Constraints}\right\}. \tag{10}$$

The additional specifications have consequences for the solution method we use and its matrices can be inspected in Section A.2 of the Appendix. The probability space is also different, the distributions of the α_i's span a continuous probability space so that we cannot calculate the exact solution for each of the possible outcomes. Clearly, with a less efficient electricity output, $\alpha_i < 1$, the expected costs rise and the is a shift away from the less performing technologies, particularly the investment on x_3.

3.3 Random Mode Duration

In this section, the model is extended to allow for randomness in the duration of the different modes. For T_2 and T_3, the deterministic distributions are replaced by random variables τ_2 and τ_3. The distribution and means of these variables can be found in Table 3.

Prob.	τ_1	τ_2
0.6	5	0.5
0.4	7.5	1.75
$\bar{\tau}$	6	1

Table 3: Distribution of Mode Durations

This change in parameters does not yield a change in the constraints compared to the model discussed in Section 3.2. It only results in a change of the objective function, where T_2 and T_3 are replaced by τ_2 and τ_3 respectively. In matrix form the model is expressed as:

$$\min_{x\geq 0} cx + q(\tau)y, \tag{11}$$

with the first set of constraints being represented by

$$Ax \leq b, \tag{12}$$

and the second set of constraints being represented by

$$T(\alpha)x + Wy \leq h(\xi). \tag{13}$$

The exact matrices and their values are again available in the Appendix.

3.4 Solving the Models

To get a solution for the models, a few different methods are considered. Initially, the EV and EEV are constructed such that an upper and lower bound can be established for the two stage solution. This can then be used to considered whether it is worth it to solve the TS problem to gain a more accurate solution or not. Furthermore, the value of the stochastic solution (VSS) can be calculated. Once the EV and EEV are calculated, it can be considered to solve the TS problem if the bounds are not strict enough. The WS solution is also considered, as it gives insight through the value of perfect information.

3.4.1 Expected Value Solution

For the EV solution, all random variables in the model are replaced by their respective expectations. This reduces the models into a non-stochastic one stage linear program, and hence it can be solved by any LP-solver. The solution of this model is a feasible solution for the stochastic model, but not necessarily optimal. The expected value of the EV solution actually constructs a lower bound of the TS objective value. This model is given by

$$\begin{aligned} \min_{x,y\geq 0} \ & cx + q(\bar{\tau})y \\ \text{Such that: } & Ax \leq b \\ & T(\bar{a})x + Wy \leq h(\bar{\xi}), \end{aligned} \tag{14}$$

where $\bar{\xi} = E[\xi]$, $\bar{\alpha} = E[\alpha]$, and similarly $\bar{\tau} = E[\tau]$.

An interesting realization is that, for the EV of the base model, the demand is known to be $5 + 3 + 2 = 10 < 14 = \sum_{j=1}^{k} \xi_j^{\max}$. Hence, constraint 3 could be relaxed to $\sum_{i=1}^{n} x_i \geq 10$, and demand would still be supplied with certainty. However, by relaxing this constraint, the solution x^{EV} would not necessarily be feasible for all realizations of the random variables. For the purpose of analysis, constraint 3 is not relaxed, such that an upper and lower bound can be constructed. Once the possibility of importing energy gets introduced, this constraint is no longer relevant and has no effect on the other models.

3.4.2 Expectation of the Expected Value Solution

For the expectation of the expected value, the x vector is fixed to be equal to x^{EV}. Where x^{EV} is the optimal investment for the EV problem. Next, for each scenario an LP is established and the optimal recourse action y^s is selected for each realization of the random variables. For each of these scenarios, an objective value is obtained. The average of all these scenarios weighed by their probabilities yields the EEV. Hence, N LP's are solved of the form

$$\begin{aligned} \min_{y \geq 0} cx^{\text{EV}} + q(\tau^s)y^s \\ \text{Such that: } T(a^s)x^{\text{EV}} + Wy^s \leq h(\xi^s). \end{aligned} \tag{15}$$

Here for each s there is a realization $(\xi^s, \alpha^s, \tau^s)$ of the random variables. The constraint $Ax \leq b$ can be excluded, as any x^{EV} satisfies this constraint. The obtained objective values EEV^S are then weighed by their respective probabilities yielding

$$\text{EEV} \approx \sum_{i=1}^{N} \text{EEV}^i \cdot P(s = i).$$

However, for the model where stochastic availability of the technologies is introduced, it is not possible to directly create a LP for each state. Since the availabilities of the technologies are continuously uniformly distributed, it is infeasible to create a state for each possible realization. Instead, a discrete approximation of the uniform distributions could be created. For this, a lower bound could be created using Jensen's inequality, and an upper bound using the Edmundson-Madanksy (EM) inequality. However, for four random variables, having two partitions each would already result in $3^4 = 81$ combinations, on top of the $3^3 = 27$ combinations for ξ, this would yield $81 \cdot 27 = 2187$ states to solve. The high number of states makes it difficult to solve the problem in little time. This leaves us with the option to either consider the approximation without multiple partitions, or use simulation. Decreasing the partitions could lead to inaccurate estimates of the distributions. However, simulation will also only give an estimate of the solution, and might need a high number of samples to give a good estimate.

For this model simulation is used. The samples used for the simulation are given by $s^i = (\,\xi_1^i\ \xi_2^i\ \xi_3^i\ \ \alpha_1^i\ \alpha_2^i\ \alpha_3^i\ \alpha_4^i\,)$. For each of these samples, a set of constraints is added to the LP where the random parameters are replaced by the sample, each with their unique recourse actions y^s. This method will also be applied for the model with stochastic mode durations, where the sample also includes values for τ_2 and τ_3 to replace T_2 and T_3. In sampling, the costs are not adjusted based on the probability of occurrence, but instead are assigned costs $\frac{1}{N}$, as the occurrence of the samples already adjust for the likelihood of the realization occurring.

3.4.3 Two-Stage Solution

For the base model with only random demand, a large scale deterministic equivalent (LSDE) of the LP can be established. This LSDE contains each realization $s = 1, ..., S$, with its own corresponding optimal set of recourse actions y^s, where S is the total number of possible unique realizations. This set of constraints then has the recourse action costs corresponding to y^s as a value weighed by the probability of the realization occurring. By adjusting the costs for the probability of the event occurring, an optimal expected objective value is calculated.

In the previous section it was discussed that for the case with stochastic availability, not all states could be included. This also prevents the use of LSDE for the more advanced models. Instead, simulation is applied again. N samples are obtained, each for which a set of constraints is added, and their own corresponding state related recourse variables $y^s, s = 1, ..., N$. Hence the model is then given by

$$\begin{aligned} \min_{x,\tilde{y}\geq 0} \; & cx + \tilde{q}\tilde{y} \\ \text{Such that: } & Ax \leq b \\ T(a^1)x + Wy^1 & \leq h(\xi^1) \\ T(a^2)x + Wy^2 & \leq h(\xi^2) \\ & \vdots \\ T(a^N)x + Wy^N & \leq h(\xi^N). \end{aligned} \tag{16}$$

Given the distribution of the random variables, the solution technique that we use is appropriate. Nevertheless, for more realistic distributions with heavier tails, a much larger sample size is necessary to construct a good estimate and the computing time necessary for the solution becomes limiting.

3.4.4 Wait-and-See Solution

For the WS solution it is assumed that there is perfect information. Hence, the realizations of the random variables are available before deciding the initial investment x. Thus, the model is allowed to select a different x for each realization. Defining S as the number of total possible realizations in the base model, this implies that S LP's are to be solved. These are then again weighed by the probability of their occurrence. Consequently, the model is given by S LPs of the shape

$$\begin{aligned} \min_{x\geq 0, y^s\geq 0} \; & cx + qy^s \\ \text{Such that: } & Tx + Wy^s \leq h(\xi^s), \end{aligned} \tag{17}$$

with $s = 1, .., S$. The objective value of this solution is then denoted by WS^s. It follows that the WS value is given by

$$WS = \sum_{i=1}^{S} \text{WS}^i P(s = i). \tag{18}$$

As the complete state space can not be used in the case of random technological availability, sampling is used instead. Hence, the WS model is given by N LPs of the shape

$$\min_{y^s \geq 0} cx + q(\tau^s)y^s$$
$$\text{Such that: } T(\alpha^s)x + Wy^s \leq h(\xi^s). \tag{19}$$

Each objective value for sample s is given by WS^s. Then an approximation of the WS value is given by

$$WS \approx \frac{1}{N}\sum_{s=1}^{N} \text{WS}^s. \tag{20}$$

3.5 Sensitivity Analysis

In order to consider the uncertainties surrounding the model, sensitivity analysis is carried out. A lot of factors in the model are stochastic already, many of these could easily be extended to include broader distributions within the considered models. However, one thing that has not been analyzed is the effect of a difference in imported energy costs. To do so, the energy prices will be varied and the new objective value will be considered. We assume that the original investment x is fixed as given by the TS solution, but the recourse action can be adjusted for the higher energy price. For the purposes of sensitivity analysis, only the model including all stochasticity will be considered.

4 Results

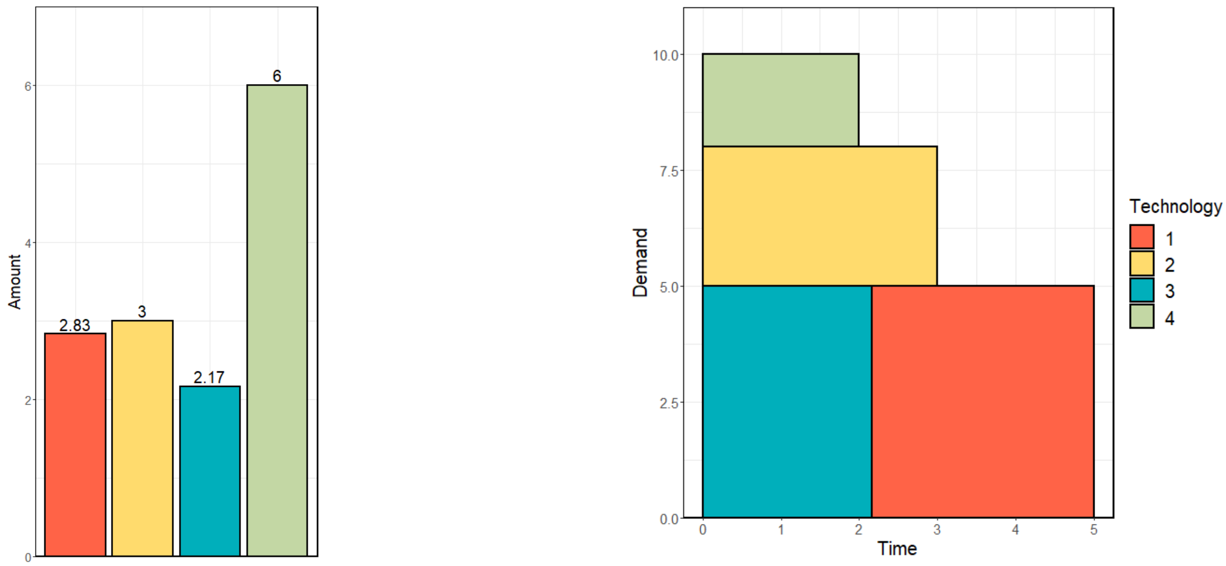
We will present the results of the different models we have developed in a series of graphs and tables. We illustrate the initial investment on the different technologies as well as the actual utilisation in each mode of our reconstructed load curve.

4.1 Base Model

The EV solution of our base model is shown in Figure 2, where Figure 2a is the optimal initial investment and Figure 2b is the usage in each mode.

In the result, it appears that an investment in all four of the technologies is necessary for the cost minimization problem. The most units purchased are of technology 4 due to its lower initial investment cost. Each of the technology plays a role in the production phase as well. A combination of technologies 1 and 3 is necessary to satisfy demand in the larger first mode, given their lower production cost. Technology 2 is used exclusively in mode 2 as it sits in the middle of the investment-production cost trade-off. Nevertheless, any usage combination of technology 1 and 3 is an optimum in mode 1. Technology 4 is used exclusively for the last mode, given its high production cost, its relatively cheap extra capacity is particularly usefully in the time of peak demand.

The expectation of the expected value solution is shown in Figure 3. By construction, the initial investment is the same of Figure 2a, however, the expected average use of the technologies in each mode differs as shown in Figure 3.



(a) Initial Investment

(b) Actual Utilisation

Figure 2: EV Solution of the Base Model

The optimization over all possible realization of the ξ_i's makes for a more complex electricity mix utilisation. Sometimes technology 1 is used in the second mode as well, while the characteristics of technology 2 are often beneficial in all 3 modes. The use of technology 3 is the same and restricted to the first mode, while in some cases, technology 4 is necessary to cover demand in mode 2. It is important to notice, nonetheless, that the presented use does not necessarily materialize in any of the scenarios, it represents a weighted use over the realisations. It has the scope of illustrating the influence of some of the outcomes have on the energy mixture.

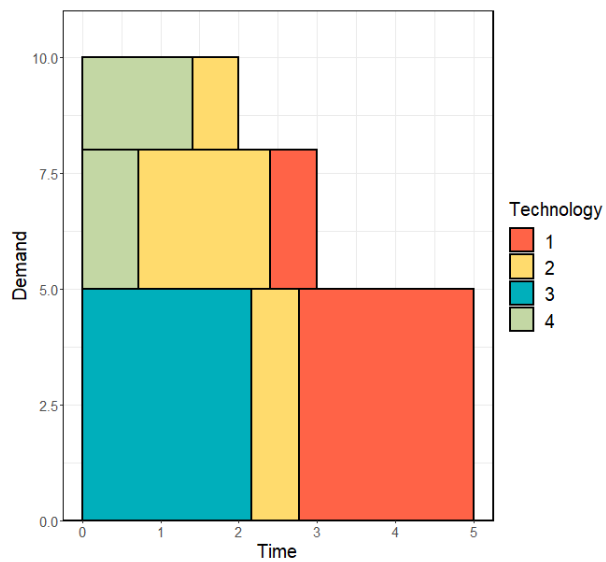


Figure 3: Average Utilisation in the EEV Solution of the Base Model

The TS investment solution is shown in Figure 4. In this case, there is a decrease of investment in technologies 3 and 4 in favor of 1 and 2, which suggest a higher utilisation of these technologies in the later modes. The investment WS solution is depicted in Figure 5, The purchase of technology 4 has returned to the level seen for the case of EV and EEV mostly at the expense of technology 2 which is now at an even lower volume. There have been some minor adjustment to the investments in 1 and 3 compared the TS solution. The structure of the LSDE formulation and the WS solution, however, makes the plot of the actual use non informative.

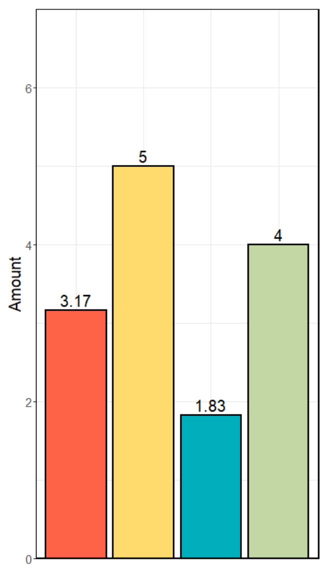


Figure 4: Initial Investment in the TS Solution of the Base Model

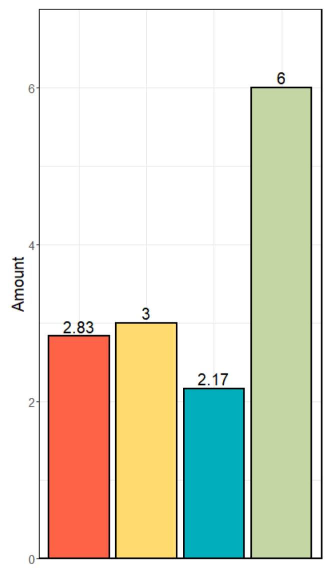


Figure 5: Initial Investment in the WS Solution of the Base Model

The results of the base model and the overall expected costs of the problem are summarized in Table 4.

	Value	Initial Investment
EV	**394.67**	$[2.83, 3, 2.17, 6]$
EEV	**399.59**	$[2.83, 3, 2.17, 6]$
TS	**397.75**	$[3.17, 5, 1.83, 4]$
WS	**394.97**	$[2.83, 3, 2.17, 6]$

Table 4: Results of the Base Model

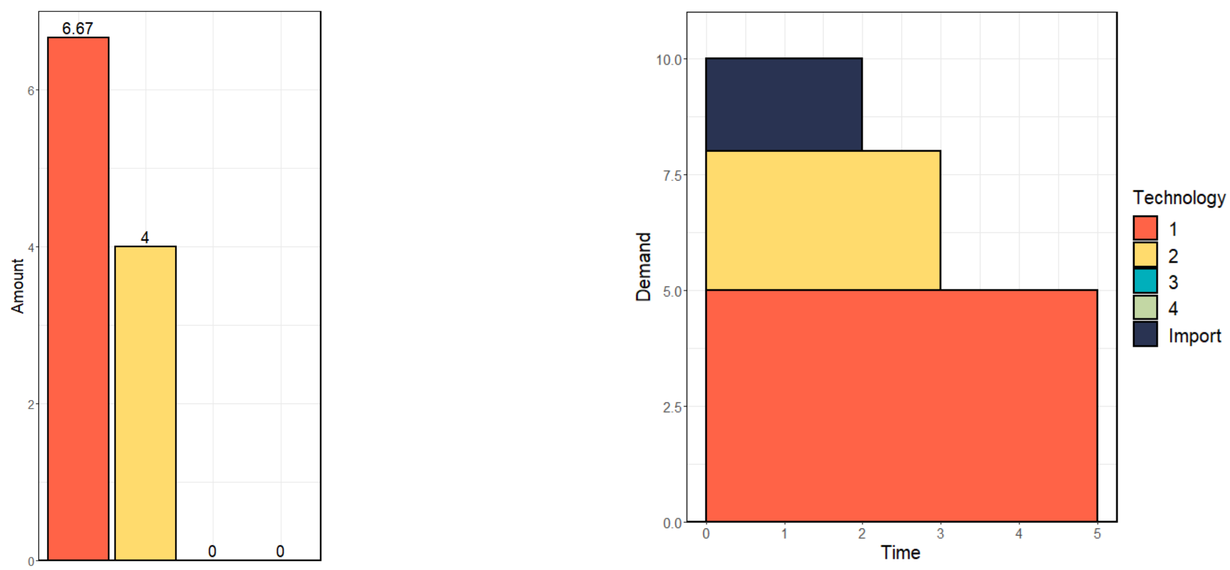
VSS = EEV - TS = 399.59 - 397.75 = **1.84**

The relative VSS is about 0.5% here, thus relatively low. As there is not much variance in the model so far, and the maximum demand constraint has to be satisfied, there is minimal value in exploring the stochastic solution here.
EVPI = TS - WS = 397.75 - 394.97 = **2.78**

Similarly to the VSS, the relative EVPI is low too. Although the investment can be adjusted towards the known realizations, it still costly to satisfy the need to always satisfy the maximum possible demand. If the constraint regarding maximum demand would be relaxed, a much larger EVPI is expected.

4.2 Random Technology Availability and Energy Import

The EV solution to the first extension in our model is shown in Figure 6, where, as usual, Figure 6a is the optimal initial investment and Figure 6b is the usage in each mode.



(a) Initial Investment

(b) Actual Utilisation

Figure 6: EV Solution of the Extended Model

The increase in uncertainty arising from effective output of the different production processes, and the possibility of importing electricity has been playing in favour of technology 1 and 2, which have substantially increased in their initial investment. Technology 1 has replaced technology 3 all together and so did the imported electricity for technology 4. They are now both obsolete. The main reason that technology 3 is now redundant is that it is relatively unreliable, but the high investment costs makes it expensive to compensate for. At mean performance, it is actually more expensive to use technology 4 for the expected demand of 2 in mode 3, hence for the EV solution, importing energy

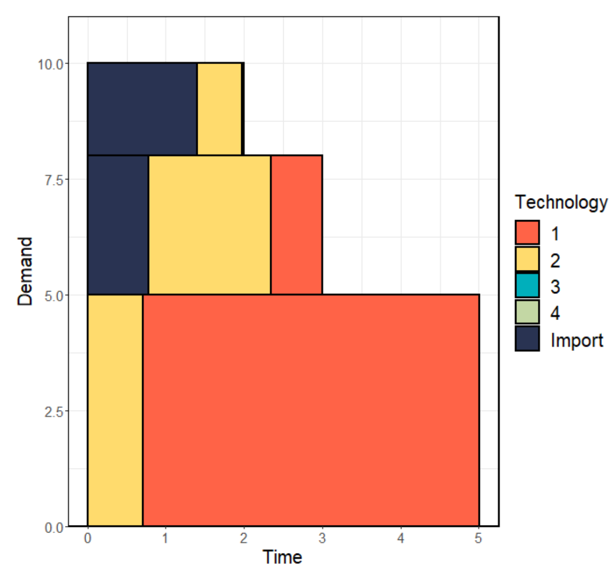


Figure 7: Average Utilisation in the EEV Solution of the Extended Model

is preferred. In this formulation, the two technologies left, suffice to cover demand in mode 1 and 2, while imports are used only in the last mode.

In the case of the EEV utilisation, we observe similar results to the base model. Technology 2 is useful in all of the modes of the production process, while the imported electricity has taken the place of technology 3 in the last two modes. Technology 1 is preferred over the 3rd one in the first mode. It has increased its output and in some cases it is utilized in the third mode as well. Similarly to the previous results, this effective utilisation is an average over all the realised scenarios and does not necessarily represent a feasible solution to the problem.

For the TS and WS solutions, the initial investments are shown in Figure 8 and Figure 9. The acquisition of the first two technologies remains high and the imported electricity plays an important role in the optimization process. However, we observe traces of investment in technology 4 and 3 in the TS and WS solutions, respectively. The reinvestment in technology 4 can be explained by technology 4 being much cheaper when utilized in mode 2 compared to importing energy. Hence, technology 4 becomes more attractive as it is more flexible to cover up for uncertainty. In scenarios of high peak demand, the high prices of importing causes a preference of these sources.

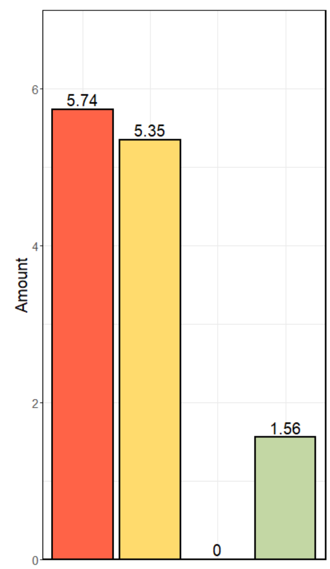


Figure 8: Initial Investment in the TS Solution of the Extended Model

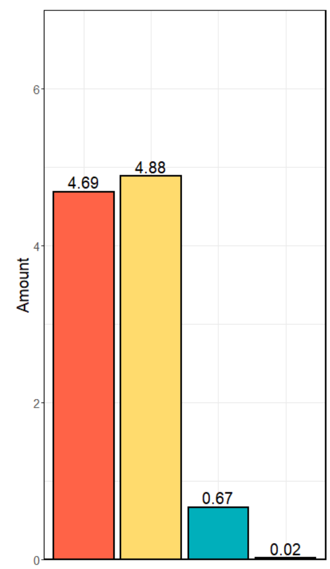


Figure 9: Initial Investment in the WS Solution of the Base Model

The results of the extended model and the overall expected costs of the problem are summarized in Table 5, while the cost distribution of the sampled realisations is shown in Figure 10 in Section A.4 of the Appendix.

	Value	Initial Investment
EV	**395.67**	$[6.67, 4, 0, 0]$
EEV	**419.73**	$[6.67, 4, 0, 0]$
TS	**411.23**	$[5.74, 5.33, 0, 1.56]$
WS	**394.19**	$[4.84, 4.83, 0.62, 0.01]$

Table 5: Results of the Extended Model

VSS = EEV - TS = 419.73 - 411.23 = **8.5**

Compared to the problem with fixed technological availability there is already a significant increase in the VSS. The introduction of more stochastic elements and the additional capacity required to satisfy maximum demand increases the potential value of accounting for this variability.

EVPI = TS - WS = 411.23 - 394.19 = **17.04**

For the EVPI the same holds. Knowing which technologies will remain more efficient can be key in selecting an efficient solution. This makes the increase in the EVPI even larger than the increase in the VSS.

4.3 Random Mode Duration

For this model, we allow the modes to have random duration. This could cause a change in the usage of technologies over the different modes. However, $T_1 > \tau_2 > \tau_3$, ensures that the lengths of the modes are still decreasing. In turn, it guarantees that lower production costs remain more important for the initial modes, but the trade-off between investment and production costs does vary with change in duration.

The results are available in Table 6. As discussed in the methodology, the EV solution does not change compared to Section 4.2, since it boils down to exactly the same model. However, the EEV does change, as the recourse actions now do get affected by the realizations of the durations. The EEV for the scenario with stochastic durations is slightly lower than that with fixed mode lengths. It appears that good outcomes of the random durations are as rewarding as the penalty of a longer duration is. This could make sense, as in poor outcomes all “safety stock” gets used, and the rest can be dealt with by importing energy. If there is a shorter outcome, a similar amount is saved on the import of energy.

The TS solution also has a lower value. This could also be caused by longer durations being less penalizing than the reward coming from shorter ones. The expected value of the length might be equal to the original fixed T’s, but there is still a lower probability of a higher duration. Although the higher duration is a larger deviation, the reward of more likely small realizations could compensate for that, as the rewards are marginally reducing, as the deviation grows. This is because saving on energy imported is a larger reward than idling technology in which already was invested. The selected technologies also show a preference in technology 2, which is slightly better for longer durations than technology 1, as the production cost is lower and the variance of availability is too. Furthermore, the overall investment decreased, probably because of the likelihood of shorter durations, where having to import energy is preferred over idling inventory. Overall, the different technologies are used in a very similar fashion as in Section 4.2.

For the WS solution there is also a significant decrease. Having knowledge about the duration lengths could be particularly useful, as longer durations incentivize to invest more and shorter durations less. Because of the increase in variance, the VSS also increases compared to the previous model, but with a smaller extend. Furthermore, the EVPI increases as we expected, there would be more information to exploit in the initial investment decision. The information on the mode duration is not as exploitable as the information on technology availability though, as the efficiency of different technologies does not vary much within these mode durations. The cost distribution of the sample used in this section is shown in Figure 11 of the Appendix.

4.4 Sensitivity Analysis

Solving the model for different values of the energy price with fixed x, yields the results in Table 7. Here, the first column is the relative factor of the energy prices, with the new energy price given by $q_I \cdot factor$. In the results, the relative change in the objective value is also displayed. The effect of the price of imported energy appears to have a marginal effect, a 25% decrease in price only yields a 2.33% drop in the objective value. Similarly, for a rise of 25%, the objective value only increases by 2%. In conclusion, it appears that the problem is relatively robust towards price changes of imported energy.

Factor	Obj. Value	Rel. Change
0.75	395.80	-2.33%
0.90	398.69	-1.61%
1	**405.23**	-
1.10	410.54	1.31%
1.25	413.34	2.00%

Table 7: Sensitivity Analysis

Model	Base Model		Stochastic Availability		Stochastic Mode Duration	
	Obj. Val.	x	Obj. Val.	x	Obj. Val.	x
EV	394.67	[2.83, 3.00, 2.17, 6.00]	395.67	[6.67. 4.00, 0.00, 0.00]	395.67	[6.67. 4.00, 0.00, 0.00]
EEV	399.59	-	419.73	-	418.40	-
TS	397.75	[3.17, 5.00, 1.83, 4.00]	411.23	[5.74, 5.35, 0.00, 1.56]	405.23	[5.67, 5.28, 0.00, 1.52]
WS	394.97	-	394.19	-	385.73	-
VSS	1.84		8.50		13.17	
EVPI	2.78		17.04		19.51	

Table 6: Complete Results

A Recourse Model Matrices

The matrices in the following sections are constructed in such a sense that all equations related to the constraints are written in a "less than equal" form. Some parameters have been shifted to the left side and a few equations have been multiplied by -1.

A.1 Random Demands

$$A = \begin{bmatrix} -1 & -1 & -1 & -1 \\ -c_1 & -c_2 & -c_3 & -c_4 \end{bmatrix} \tag{21}$$

$$T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{22}$$

$$W = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 \end{bmatrix} \tag{23}$$

$$h(\omega) = \begin{bmatrix} 0 & 0 & 0 & 0 & \xi_1 & \xi_2 & \xi_3 \end{bmatrix}' \tag{24}$$

$$b = \begin{bmatrix} -\sum_{j=1}^{k} \xi_j^{\max} \\ c_{\max} \end{bmatrix} \tag{25}$$

$$x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}' \tag{26}$$

$$y = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{21} & y_{22} & y_{23} & y_{31} & y_{32} & y_{33} & y_{41} & y_{42} & y_{43} \end{bmatrix}' \tag{27}$$

$$c = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \end{bmatrix} \tag{28}$$

$$q = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix} \otimes \begin{bmatrix} T_1 & T_2 & T_3 \end{bmatrix} \tag{29}$$

Then the problem is given by

$$\begin{aligned} &\min_{x\geq 0, y\geq 0} cx + qy \\ &\text{s.t.} \begin{bmatrix} A & O \\ T & W \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \leq \begin{bmatrix} b \\ h(\xi) \end{bmatrix} \end{aligned} \tag{30}$$

A.2 Importing Energy and Random Technology Availability

$$A = \begin{bmatrix} -c_1 & -c_2 & -c_3 & -c_4 \end{bmatrix} \tag{31}$$

$$T(\alpha) = \begin{bmatrix} -\alpha_1 & 0 & 0 & 0 \\ 0 & -\alpha_2 & 0 & 0 \\ 0 & 0 & -\alpha_3 & 0 \\ 0 & 0 & 0 & -\alpha_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{32}$$

$$W = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 \end{bmatrix} \tag{33}$$

$$h(\xi) = \begin{bmatrix} 0 & 0 & 0 & 0 & \xi_1 & \xi_2 & \xi_3 \end{bmatrix}' \tag{34}$$

$$b = \begin{bmatrix} c_{\text{max}} \end{bmatrix} \tag{35}$$

$$x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}' \tag{36}$$

$$y = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{21} & y_{22} & y_{23} & y_{31} & y_{32} & y_{33} & y_{41} & y_{42} & y_{43} & I_1 & I_2 & I_3 \end{bmatrix}' \tag{37}$$

$$c = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \end{bmatrix} \tag{38}$$

$$q = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_I \end{bmatrix} \otimes \begin{bmatrix} T_1 & T_2 & T_3 \end{bmatrix} \tag{39}$$

Then the problem is given by

$$\begin{aligned} &\min_{x\geq 0, y\geq 0} cx + qy \\ \text{s.t.} &\begin{bmatrix} A & O \\ T(\alpha) & W \end{bmatrix} \leq \begin{bmatrix} b \\ h(\xi) \end{bmatrix} \end{aligned} \tag{40}$$

A.3 Random Mode Durations

For the stochastic mode durations the same model can be used as for the random technology availability with one small adjustment. All that has to be changed is replacing Equation 39 by

$$q(\tau) = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & c_I \end{bmatrix} \otimes \begin{bmatrix} T_1 & \tau_2 & \tau_3 \end{bmatrix}. \tag{41}$$

A.4 Distribution Plots

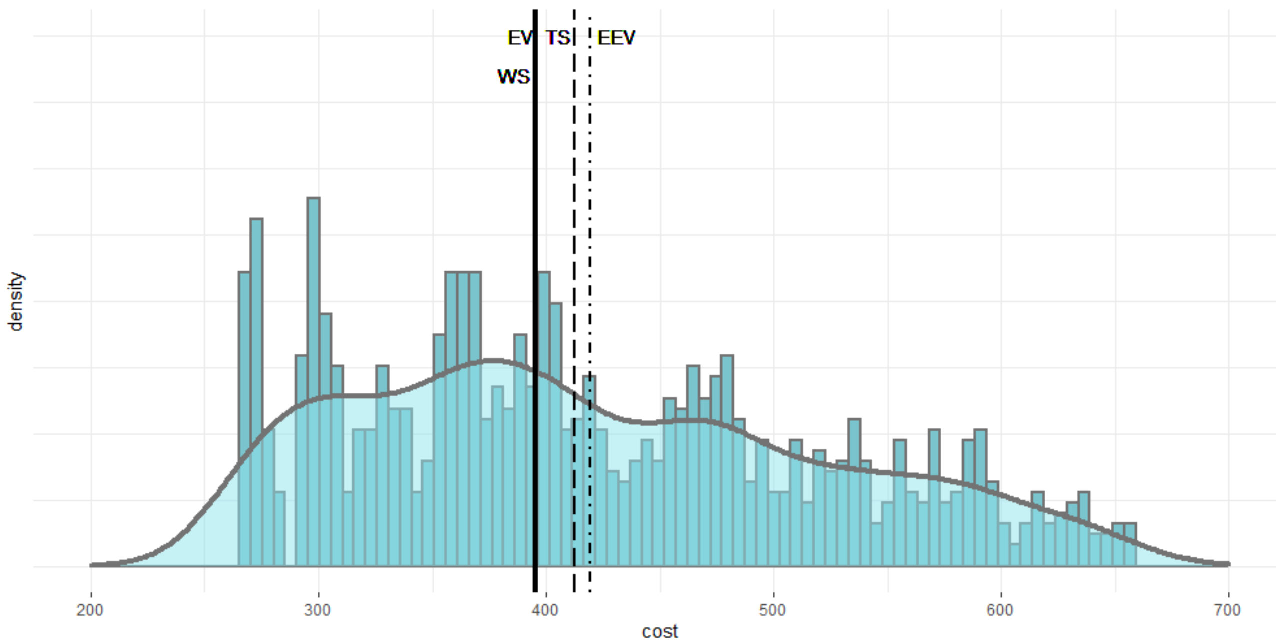


Figure 10: Distribution Plot of the Expected Costs from the Sample Drawn in the Extended Model

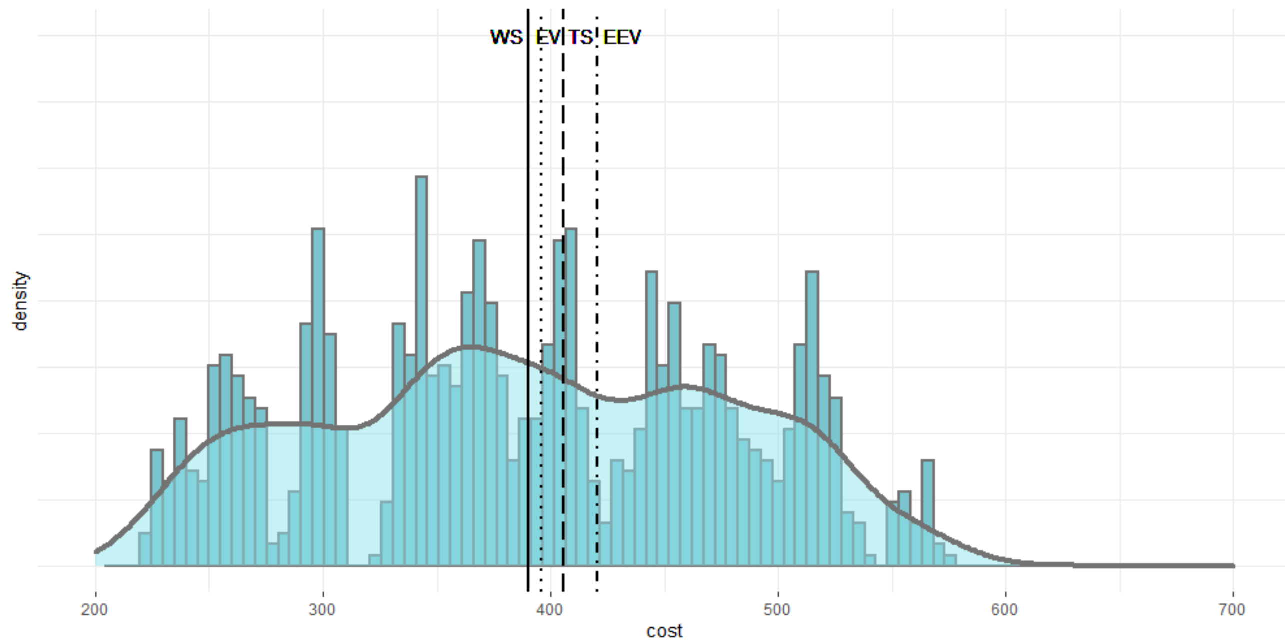


Figure 11: Distribution Plot of the Expected Costs from the Sample Drawn in the Full Model

References

[1] François V. Louveaux and Yves Smeers. "Optimal Investments for Electricity Generation: A Stochastic Model and a Test-Problem". In: 1988.