UNIVERSITY OF MORATUWA

FACULTY OF ENGINEERING

DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING

BSc Engineering Honours Degree
Semester 5 Examination (2019 Intake)

CS 3062 THEORY OF COMPUTING

Time allowed: 2 Hours December 2022

ADDITIONAL MATERIAL: *None*

INSTRUCTIONS TO CANDIDATES:

1. This paper consists of **5** questions in **4** pages, including this page.
2. Answer **ALL** questions.
3. The maximum attainable mark for each question/part is given in brackets.
4. This examination accounts for 70% of the module assessment.
5. This is a closed book examination.

   NB: It is an offence to be in possession of unauthorized material during the examination.
6. Only calculators approved and labeled by the Faculty of Engineering are permitted.
7. Assume reasonable values for any data not given in or with the examination paper. Clearly state such assumptions made on the script.
8. In case of any doubt as to the interpretation of the wording of a question, make suitable assumptions and clearly state them on the script.
9. This paper should be answered only in English.
10. Abbreviations and Notations
    - DFA – Deterministic Finite Automaton
    - NFA – Non-deterministic Finite Automaton
    - NFA-Λ – Non-deterministic Finite Automaton with Λ-transitions
    - CFG – Context-free Grammar
    - CFL – Context-free Language
    - PDA – Push-Down Automaton
    - TM – Turing Machine
    - In a CFG, non-terminals are denoted by upper-case letters and terminals are denoted by lower-case letters and/or digits; non-terminal S is usually the start symbol and Λ represents the null string
    - A DFA or NFA is defined as a 5-tuple (Q, Σ, q_0, A, δ), where each component has the usual meaning.
    - A PDA is expressed as a 7-tuple (Q, Σ, Γ, q_0, Z_0, A, δ) where each component has the usual meaning.

<u>Question 1</u> [20 marks]

In this question, there are 10 statements, (a) – (j); for each, you have to state whether it is either **True** or **False**. A correct choice will result in 2 marks and an incorrect choice will have a penalty of -1 mark. If not answered, it will result in 0 marks (no penalty). The minimum total marks possible for Q1 is 0.

(a) A string *w* is accepted by a NFA if none of the paths corresponding to *w* ends at a non-accepting state.

(b) If A and B are regular languages over alphabet Σ, then the strings formed by concatenating a string from B with a string from A is not a regular language.

(c) If a DFA M has *n* number of states, for any string longer than *n* and accepted by M, there exists a string *x* such that $|x| \leq n$ and *x* is accepted by M.

(d) The regular expression (0 | 010 | 01)* generates strings with no consecutive 1's.

(e) The language $L = \{1^{2n+1} \mid n \geq 0\}$ is represented by the regular expression 1(11)*.

(f) If a CFG is ambiguous, then we can make it unambiguous by transforming it into Chomsky Normal Form.

(g) The CFG corresponding to the language $L = \{0^k 1^k \mid k \geq 0\}$ is $S \rightarrow 0S1 \mid 01$.

(h) The "reduce" move in shift-reduce parsing involves popping the stack as well as pushing onto the stack.

(i) The language $\{xx \mid x \text{ is in } \{a, b\}^*\}$ can be accepted by a nondeterministic PDA.

(j) If a Turing machine T decides a language L, then for a string *x* not in L, T will halt.

<u>Question 2</u> [20 marks]

(a) Construct a Mealy machine whose input alphabet is {0,1} and which outputs 0 or 1 depending on whether the total number of 1's in the input string is odd or even, respectively. Assume that 0 is even.

[6 marks]

(b) (i) State the pumping lemma for regular languages.

[2 marks]

(ii) Use the pumping lemma to show that $L = \{x \mid x \text{ has an equal number of 0's and 1's}\}$ is not regular.

[6 marks]

(c) Consider the language of all strings containing exactly two 0's, where the alphabet is {0,1}.

(i) Find a regular expression for the above language.

[2 marks]

(ii) Show a DFA that recognizes the language.

[4 marks]

<u>Question 3</u> [20 marks]

(a) Consider the DFA, M_1=({0,1, 2, 3, 4, 5, 6,7}, {a, b}, 0, {3}, δ), whose δ is specified below.

Current State	Next State			Current State	Next State	
	Input *a*	Input *b*			Input *a*	Input *b*
0	1	0		4	3	5
1	0	2		5	6	4
2	3	1		6	5	6
3	3	0		7	6	3

(i) Identify the *equivalence classes* of the given set of states. Show your work.

[7 marks]

(ii) Show the transition diagram of the equivalent *minimum-state* DFA.

[3 marks]

(b) Suppose the NFA-Λ, M_2 = ({S,A,B,C}, {0, 1}, S, {S, A, C}, δ) is given, where the transitions are specified as follows.

Current State	Next State(s)		
	Input 0	Input 1	Input Λ
S		S	A
A		B	C
B	A		
C		C	

(i) Obtain the NFA equivalent to the given NFA-Λ. Show your work.

[5 marks]

(ii) Using the NFA obtained in (i) above, construct an equivalent DFA and show its transition diagram.

[5 marks]

<u>Question 4</u> [20 marks]

(a) Construct a CFG that generates the language L = {x in {0,1}* | $n_0(x)=n_1(x)$}, where $n_p(x)$ is the number of p's in the string x.

[5 marks]

(b) Is the CFG given below *ambiguous*? Justify your answer.

$S \rightarrow AB$
$A \rightarrow aA \mid \Lambda$
$B \rightarrow ab \mid bB \mid \Lambda$

[5 marks]

(c) Consider the transition table below for a PDA with initial state q_0 and accepting state q_2.

Move Number	State	Input	Stack symbol	Move(s)
1	q_0	a	Z_0	(q_1, aZ_0)
2	q_0	b	Z_0	(q_1, bZ_0)
3	q_1	a	a	$(q_1, a), (q_2, a)$
4	q_1	b	a	(q_1, a)
5	q_1	a	b	(q_1, b)
6	q_1	b	b	$(q_1, b), (q_2, b)$
(all other combinations)				None

Describe the language that is accepted by this PDA.

[5 marks]

(d) Suppose M_1 and M_2 are PDAs accepting languages L_1 and L_2, respectively.

Describe how to construct a PDA accepting the language $L_1 \cup L_2$. Note that nondeterminism would be necessary and that the stack alphabets of M_1 and M_2 are independent. State clearly how the new machine works.

[5 marks]

<u>Question 5</u> [20 marks]

(a) Construct a Turing machine (TM) to accept the language $\{a^i b^j \mid i > j\}$. Describe how it works and show its transition diagram.

[10 marks]

(b) A variation of Turing Machines is a more restricted one in which the tape head must move either to the left (L) or to the right (R) on each move. That is, the tape head being stationary (S) at the same cell is not allowed. An ordinary TM can be simulated by such a restricted TM. Explain how the move $\delta(p, a) = (q, b, S)$ could be simulated by such a restricted TM.

[5 marks]

(c) State whether each statement below is True or False.

(i) For any computational problem, there exists a TM that solves it.
(ii) For any non-deterministic TM, an equivalent deterministic TM can be found.
(iii) A multi-track TM (consisting of multiple tracks on a single tape where the read/write head can read and write from all cells which lie on the same column simultaneously) is more computationally powerful than a TM that has a single-track tape.
(iv) A linear bounded automaton is computationally more powerful than a PDA.
(v) A universal TM can solve problems not solvable by any special-purpose TM.

[5 marks]

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