UNIVERSITY OF MORATUWA

FACULTY OF ENGINEERING

DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING

BSc Engineering Honours Degree
Semester 5 Examination (2017 Intake)

CS 3062 THEORY OF COMPUTING

Time allowed: 2 Hours September 2020

ADDITIONAL MATERIAL: *None*

INSTRUCTIONS TO CANDIDATES:

1. This paper consists of **5** questions in **5** pages, including this page.
2. Answer **ALL** questions.
3. The maximum attainable mark for each question/part is given in brackets.
4. This examination accounts for 70% of the module assessment.
5. This is a closed book examination.

   NB: It is an offence to be in possession of unauthorized material during the examination.
6. Only calculators approved and labeled by the Faculty of Engineering are permitted.
7. Assume reasonable values for any data not given in or with the examination paper. Clearly state such assumptions made on the script.
8. In case of any doubt as to the interpretation of the wording of a question, make suitable assumptions and clearly state them on the script.
9. This paper should be answered only in English.
10. Abbreviations and Notations
    - DFA – Deterministic Finite Automaton
    - NFA – Non-deterministic Finite Automaton
    - NFA-Λ – Non-deterministic Finite Automaton with Λ-transitions
    - CFG – Context-free Grammar
    - CFL – Context-free Language
    - PDA – Push-Down Automaton
    - In a CFG, non-terminals are denoted by upper-case letters and terminals are denoted by lower-case letters and/or digits; non-terminal S is usually the start symbol and Λ represents the null string
    - A DFA or NFA is defined as a 5-tuple $(Q, \Sigma, q_0, A, \delta)$, where each component has the usual meaning.
    - A PDA is expressed as a 7-tuple $(Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$ where each component has the usual meaning.

Question 1 [20 marks]

In this question, there are 10 statements, (a) – (j); for each, you have to state whether it is either **True** or **False**. A correct choice will result in 2 marks and an incorrect choice will have a penalty of -1 mark. If not answered, it will result in 0 marks (no penalty). The minimum total marks possible for Q1 is 0.

(a) A string *w* is not accepted by an NFA if there exists a path corresponding to *w* ending at a non-accepting state.

(b) The alphabet of a formal language can be infinite.

(c) Even if the number of states in two finite automata are not equal, they can still be equivalent.

(d) The regular expression $(a \mid ab)^*$ generates strings with no consecutive *b*'s.

(e) The set of all strings of the form $w=xx$ where $x=\{a, b\}^*$ is not a regular language.

(f) If a CFG is ambiguous, then we can make it unambiguous by eliminating unit productions from it.

(g) The CFG corresponding to the language $L = \{0^k1^k \mid k \geq 1\}$ is $S \rightarrow 0S1 \mid 01 \mid \Lambda$.

(h) The CFLs and regular languages are both closed under the concatenation operation.

(i) If a Turing machine T decides a language L, then for a string *x* not in L, T may loop forever.

(j) A Universal Turing machine can solve all intractable (computationally hard) problems.

Question 2 [20 marks]

(a) Suppose L is the language over $\{a, b\}$ such that, for every string in L, if it starts with *a* then it ends with *b* and if it starts with *b* then it ends with *a*.

(i) Give a regular expression that represents the language L.

[2 marks]

(ii) Design a DFA that accepts the language L.

[5 marks]

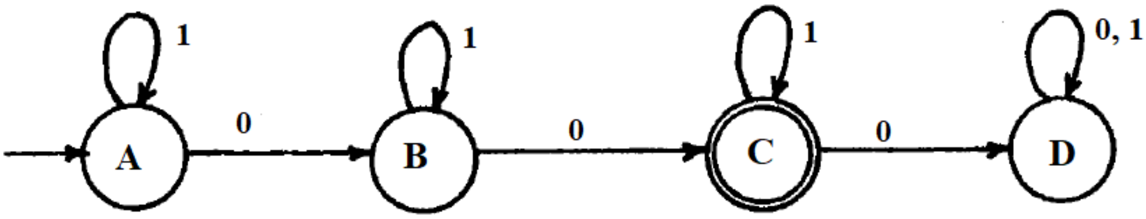
(b) (i) State the pumping lemma for regular languages.

[2 marks]

(ii) Use the pumping lemma to show that $L = \{0^k\}$ is not regular. Here $k \geq 1$ and *k* is a perfect square of the form $k = i^2$ for *i*=1, 2, 3,…

[6 marks]

(c) Describe either in words or by writing a regular expression, the strings that causes the following DFA to be in *each state*. What is the language accepted by this DFA?



[5 marks]

Question 3 [20 marks]

(a) Consider the DFA, M_1=({1, 2, 3, 4, 5, 6,7}, {*a*, *b*}, 1, {4, 6, 7}, δ), whose δ is specified below.

Current State	Next State			Current State	Next State	
	Input *a*	Input *b*			Input *a*	Input *b*
1	2	3		5	6	7
2	4	5		6	4	5
3	6	7		7	6	7
4	4	5				

(i) Identify the *equivalence classes* of the given set of states. Show your work.

[5 marks]

(ii) Show the transition diagram of the equivalent *minimum-state* DFA.

[2 marks]

(b) Suppose the NFA, M_2 = ({1, 2, 3}, {*a*, *b*}, 1, {3}, δ) is given, where the transitions are specified as follows.

Current State	Next State(s)	
	Input *a*	Input *b*
1	1, 2	1
2		3
3		

Construct an equivalent DFA and show its transition diagram. Show your work.

[7 marks]

(c) Suppose the NFA-Λ, $M_3 = (\{1, 2, 3, 4\}, \{a, b\}, 1, \{4\}, \delta)$ is given, where the transitions are specified as follows.

Current State	**Next State(s)**		
	Input a	**Input** b	**Input** Λ
1	2, 3		
2	1		
3		4	
4	4		2

Construct an equivalent NFA and show its transition diagram. Show your work.

[6 marks]

<u>Question 4</u> [20 marks]

(a) Construct a CFG that generates the language $L = \{0^n1^{2n} \mid n \geq 0\}$.

[4 marks]

(b) Describe what language is generated by each of the CFGs indicated by the following productions:

(i). $S \rightarrow SaS \mid b$

(ii). $S \rightarrow SS \mid bS \mid a$

[2×2=4 marks]

(c) Is the CFG given below *ambiguous*? Justify your answer with an example.

$$S \rightarrow aSb \mid aaSb \mid \Lambda$$

[3 marks]

(d) CFLs have interesting closure properties. One of them is string reversal. The formal way to state this is: "CFLs are closed over the operation of string reversal". Using this property, solve the following problem.

A CFL, $L_1(G_1)$, is given with $G_1 = (V_1, \sum, S, P_1)$, where $V_1 = \{S,A,B\}$, $\sum=\{a,b\}$ and the set P_1 of productions as follows:

$S \rightarrow AB$

$A \rightarrow aAb$

$B \rightarrow bbBab \mid ab$

Construct a CFG, $G_2 = (V_2, \{a, b\}, S, P_2)$, whose corresponding language L_2 contains only the strings that are the reverse of the strings in L_1. Basically, you have to specify V_2 and P_2.

[4 marks]

(e) Construct a push-down automaton (PDA) to accept the language $L = \{a^nba^n \mid n \geq 1\}$. Briefly describe how it works.

[5 marks]

Question 5 [20 marks]

(a) Given below is a transition table for a Turing machine (TM) with input alphabet {*a*, *b*}, where *q* is the current state and σ is the current input symbol.

q	σ	$\delta(q, \sigma)$	q	σ	$\delta(q, \sigma)$	q	σ	$\delta(q, \sigma)$
q_0	Δ	(q_1, Δ, R)	q_2	Δ	(h_a, Δ, R)	q_6	a	(q_6, a, R)
q_1	a	(q_1, a, R)	q_3	Δ	(q_4, a, R)	q_6	b	(q_6, b, R)
q_1	b	(q_1, b, R)	q_4	a	(q_4, a, R)	q_6	Δ	(q_7, b, L)
q_1	Δ	(q_2, Δ, L)	q_4	b	(q_4, b, R)	q_7	a	(q_7, a, L)
q_2	a	(q_3, Δ, R)	q_4	Δ	(q_7, a, L)	q_7	b	(q_7, b, L)
q_2	b	(q_5, Δ, R)	q_5	Δ	(q_6, b, R)	q_7	Δ	(q_2, Δ, L)

What is the final configuration if the TM starts with an arbitrary input string *x* in {*a*, *b*}*?

[8 marks]

(b) We do not define Λ-transitions for a Turing Machine. Why not? What features of a TM make it unnecessary or inappropriate to talk about Λ-transitions?

[4 marks]

(c) Briefly describe and distinguish the following terms with respect to computational problems, giving one example problem for each.

(i). Decidable
(ii). Unsolvable
(iii). Tractable
(iv). Intractable

[4 marks]

(d) State whether each statement below is True or False. (TM = Turing Machine)

(i). If a TM accepts a language L, then it will halt for every string in L.
(ii). A non-deterministic TM with multiple tapes has more computational power than a deterministic TM with a single tape.
(iii). Every context sensitive language is recursive.
(iv). Hilbert's 10^{th} problem is solvable, though initially thought to be unsolvable.

[4 marks]

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