UNIVERSITY OF MORATUWA

FACULTY OF ENGINEERING

DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING

BSc Engineering Honours Degree
Semester 4 Examination (2020 Intake)

CS 3063 THEORY OF COMPUTING

Time allowed: 2 Hours July 2023

ADDITIONAL MATERIAL: *None*

INSTRUCTIONS TO CANDIDATES:

1. This paper consists of **5** questions in **6** pages, including this page.
2. Answer **ALL** questions.
3. The maximum attainable mark for each question/part is given in brackets.
4. This examination accounts for 70% of the module assessment.
5. This is a closed book examination.

   NB: It is an offence to be in possession of unauthorized material during the examination.
6. Only calculators approved and labeled by the Faculty of Engineering are permitted.
7. Assume reasonable values for any data not given in or with the examination paper. Clearly state such assumptions made on the script.
8. In case of any doubt as to the interpretation of the wording of a question, make suitable assumptions and clearly state them on the script.
9. This paper should be answered only in English.
10. Abbreviations and Notations
    - DFA – Deterministic Finite Automaton
    - NFA – Non-deterministic Finite Automaton
    - NFA-Λ – Non-deterministic Finite Automaton with Λ-transitions
    - CFG – Context-free Grammar
    - CFL – Context-free Language
    - PDA – Push-Down Automaton
    - TM – Turing Machine
    - In a CFG, non-terminals are denoted by upper-case letters and terminals are denoted by lower-case letters and/or digits; non-terminal S is usually the start symbol and Λ represents the null string
    - A DFA or NFA is defined as a 5-tuple (Q, Σ, q_0, A, δ), where each component has the usual meaning.
    - A PDA is expressed as a 7-tuple (Q, Σ, Γ, q_0, Z_0, A, δ) where each component has the usual meaning.

<u>Question 1</u> [20 marks]
In this question, there are 10 statements, (a) – (j); for each, you have to state whether it is either **True** or **False**. A correct choice will result in 2 marks and an incorrect choice will have a penalty of -1 mark. If not answered, it will result in 0 marks (no penalty). The minimum total marks possible for Q1 is 0.

(a) A string *w* is accepted by an NFA if all paths corresponding to *w* lead to an accepting state.

(b) A finite automaton can accept a null string only if it is an NFA.

(c) Two finite automata are not equivalent if the number of states in them are not equal.

(d) The regular expression 0(10)*(10|1) | 1(01)*(01|0) generates the alternating sequence of 0's and 1's.

(e) If a Mealy machine has *p* number of states and *q* number of distinct (unique) output symbols, then the maximum number of possible states in the equivalent Moore machine is *pq*.

(f) The CFG $S \rightarrow aS \mid bS \mid a \mid b$ corresponds to the regular expression $(a \mid b)^*$.

(g) CFLs and regular languages are both closed under the union operation.

(h) The language $L=\{a^nb^n \mid n \geq 1\}$ can be accepted by a PDA based on both empty stack and by final state.

(i) We can construct a Turing machine to solve any NP-Complete problem.

(j) For some non-deterministic Turing machines, an equivalent deterministic Turing machine cannot be found.

Question 2 [20 marks]

(a) Construct a DFA that accepts strings over the 4-symbol alphabet ∑={R, 0, 1, 2} and keeps a running count of the sum of the numerical input symbols it reads, modulo 3. When it receives the R symbol as input, it resets the count to 0. The DFA accepts if the sum is 0 modulo 3 (in other words, if the sum is a multiple of 3). Show the transition diagram of the DFA.

[6 marks]

(b) (i) State the pumping lemma for regular languages.

[2 marks]

(ii) Use the pumping lemma to show that the language of palindromes over {0,1} is not regular.

[6 marks]

(c) Consider the language of all strings containing at least two 0's, where the alphabet is {0,1}

(i) Find a regular expression for the above language.

[2 marks]

(ii) Show a DFA that recognizes the language.

[4 marks]

<u>Question 3</u> [20 marks]

(a) Consider the DFA, M_1=({1, 2, 3, 4, 5, 6,7}, {*a*, *b*}, 1, {2, 6}, δ), whose δ is specified below.

Current State	Next State			Current State	Next State	
	Input *a*	Input *b*			Input *a*	Input *b*
1	2	3		5	6	7
2	4	5		6	4	5
3	6	7		7	6	7
4	4	5				

(i) Identify the *equivalence classes* of the given set of states. Show your work.

[5 marks]

(ii) Show the transition diagram of the equivalent *minimum-state* DFA.

[2 marks]

(b) Suppose the NFA, M_2 = ({1,2,3}, {*a*, *b*}, 1, {3}, δ) is given, where the transitions are specified as follows.

Current State	Next State(s)	
	Input *a*	Input *b*
1	1, 2	1
2		3
3	3	3

Obtain the DFA equivalent to the given NFA. Show your work and the transition diagram of the DFA.

[4 marks]

(c) Suppose the NFA-Λ, M_3 = ({1,2,3,4,5}, {*a*, *b*}, 1, {1}, δ) is given, where the transitions are specified as follows.

Current State	Next State(s)		
	Input *a*	Input *b*	Input Λ
1	2		
2	5	3	
3	4		1
4		3	
5			1

Obtain an NFA accepting the same language as the given NFA-Λ. Show your work and the transition diagram of the NFA.

[4 marks]

(d) Suppose M_4 = (Q, Σ, q_0, A, δ) is an NFA-Λ recognizing the language *L*. Let M_5 be the NFA-Λ obtained from M_4 by adding Λ-transitions from each element of A to q_0. Describe L(M_5), the language recognized by M_5, in terms of *L*.

[5 marks]

Question 4 [20 marks]

(a) Describe what language is generated by each of the CFGs indicated by the following productions where $\{a, b\}$ are terminals and S is the non-terminal start symbol:

(i). $S \rightarrow aSa \mid bSb \mid \Lambda$

(ii). $S \rightarrow aSb \mid bSa \mid \Lambda$

[2×2=4 marks]

(b) Construct a CFG that generates the language $L = \{a^i b^j c^k \mid i = j + k\}$,

[4 marks]

(c) Is the CFG given below *ambiguous*? Justify your answer.

$S \rightarrow ABA$
$A \rightarrow aA \mid \Lambda$
$B \rightarrow bB \mid \Lambda$

[4 marks]

(d) Consider the transition table below for a PDA with initial state q_0 and accepting state q_2.

Move Number	State	Input	Stack symbol	Move(s)
1	q_0	a	Z_0	(q_0, XZ_0)
2	q_0	b	Z_0	(q_0, XZ_0)
3	q_0	a	X	(q_0, XX)
4	q_0	b	X	(q_0, XX)
5	q_0	c	X	(q_1, X)
6	q_0	c	Z_0	(q_1, Z_0)
7	q_1	a	X	(q_1, Λ)
8	q_1	b	X	(q_1, Λ)
9	q_1	Λ	Z_0	(q_2, Z_0)
(all other combinations)				None

Describe the language that is accepted by this PDA.

[4 marks]

(e) Show that if there strings x and y in the language L so that x is a prefix of y and $x \neq y$, then no deterministic PDA can accept L by empty stack.

[4 marks]

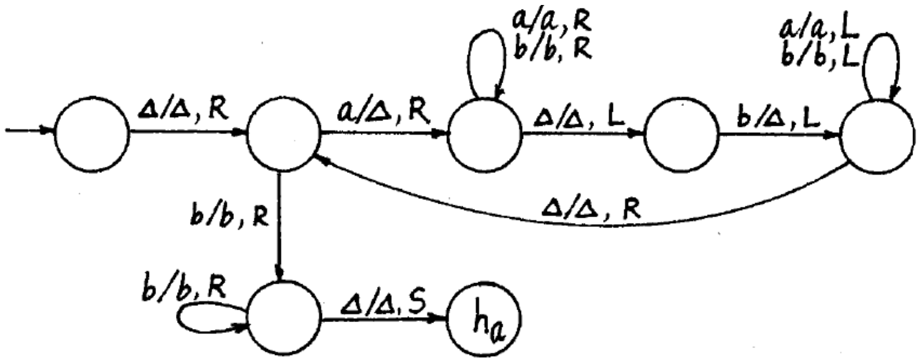
Question 5 [20 marks]

(a) Construct a Turing machine (TM) called **`Insert`**(σ) which changes the tape contents from $y\underline{z}$ to $y\underline{\sigma}z$ by inserting the symbol σ. Here $y \in (\Sigma \cup \{\Delta\})^*$, $\sigma \in \Sigma \cup \{\Delta\}$, and $z \in \Sigma^*$. You may assume that $\Sigma = \{a, b\}$ and that the initial configuration of the TM is $(q_0, y\underline{z})$, i.e., you need to start processing from the start of z. The final configuration of the TM should be $(h_a, y\underline{\sigma}z)$.

Describe how it works and show its transition diagram.

[7 marks]

(b) Consider the TM whose transition function is shown below.



Describe the language accepted by this TM.

[4 marks]

(c) Suppose T is a TM accepting a language L. Describe how you would modify T to obtain another TM accepting L that never halts in the reject state h_r. You may ignore the case where T can end up in h_r while trying to move its head off the left end of the tape.

[5 marks]

(d) We discussed the Post Correspondence Problem (PCP). Consider the correspondence system described by the following set of dominos.

100	101	110
10	01	1010

Does the above have a solution (a match)? If yes, show the answer. If no, explain why.

[4 marks]

------ End of Paper ------