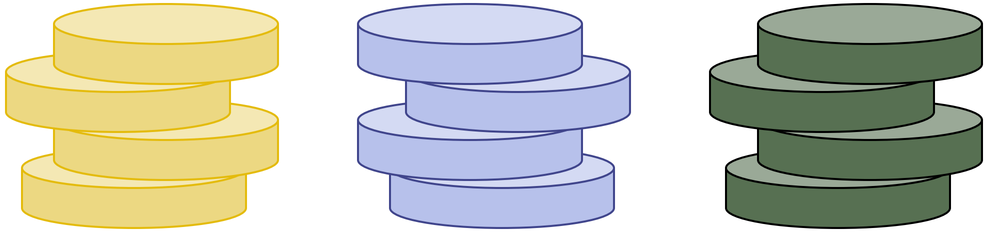
Stacks



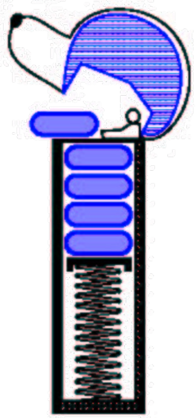
Abstract Data Types (ADTs)

- An abstract data type (ADT) is an abstraction of a data structure
- An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations

- Example: ADT modeling a simple stock trading system
  - The data stored are buy/sell orders
  - The operations supported are
    - order buy(stock, shares, price)
    - order sell(stock, shares, price)
    - void cancel(order)
  - Error conditions:
    - Buy/sell a nonexistent stock
    - Cancel a nonexistent order

The Stack ADT

- The Stack ADT stores arbitrary objects
- Insertions and deletions follow the last-in first-out scheme
- Think of a spring-loaded plate dispenser
- Main stack operations:
  - push(object): inserts an element
  - object pop(): removes and returns the last inserted element



- Auxiliary stack operations:
  - object top(): returns the last inserted element without removing it
  - integer len(): returns the number of elements stored
  - boolean is_empty(): indicates whether no elements are stored

Example

Operation	Return Value	Stack Contents
S.push(5)	–	[5]
S.push(3)	–	[5, 3]
len(S)	2	[5, 3]
S.pop()	3	[5]
S.is_empty()	False	[5]
S.pop()	5	[ ]
S.is_empty()	True	[ ]
S.pop()	“error”	[ ]
S.push(7)	–	[7]
S.push(9)	–	[7, 9]
S.top()	9	[7, 9]
S.push(4)	–	[7, 9, 4]
len(S)	3	[7, 9, 4]
S.pop()	4	[7, 9]
S.push(6)	–	[7, 9, 6]
S.push(8)	–	[7, 9, 6, 8]
S.pop()	8	[7, 9, 6]

Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in a language that supports recursion
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element

Array-based Stack (cont.)

- The array storing the stack elements may become full
- A push operation will then need to grow the array and copy all the elements over.

Performance and Limitations

- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$ (amortized in the case of a push)

Array-based Stack in Python

```
class ArrayStack:
  """LIFO Stack implementation using a Python list as underlying storage."""

  def __init__(self):
    """Create an empty stack."""
    self._data = [ ]                      # nonpublic list instance

  def __len__(self):
    """Return the number of elements in the stack."""
    return len(self._data)

  def is_empty(self):
    """Return True if the stack is empty."""
    return len(self._data) == 0

  def push(self, e):
    """Add element e to the top of the stack."""
    self._data.append(e)                  # new item stored at end of list

  def top(self):
    """Return (but do not remove) the element at the top of the stack.

    Raise Empty exception if the stack is empty.
    """
    if self.is_empty( ):
      raise Empty('Stack is empty')
    return self._data[−1]                 # the last item in the list

  def pop(self):
    """Remove and return the element from the top of the stack (i.e., LIFO).

    Raise Empty exception if the stack is empty.
    """
    if self.is_empty( ):
      raise Empty('Stack is empty')
    return self._data.pop( )              # remove last item from list
```

Parentheses Matching

- Each "(", "{", or "[" must be paired with a matching ")", "}", or "["
  - correct: ( )(( )){([( )])}
  - correct: ((( )(( )){([( )])}
  - incorrect:)(( )){([( )])}
  - incorrect: ({[ ])}
  - incorrect: (

Parentheses Matching Algorithm

Algorithm ParenMatch(X,n):
Input: An array X of n tokens, each of which is either a grouping symbol, a variable, an arithmetic operator, or a number
Output: **true** if and only if all the grouping symbols in X match
Let S be an empty stack
for i=0 to n-1 **do**
    if $X[i]$ is an opening grouping symbol **then**
        S.push($X[i]$)
    else if $X[i]$ is a closing grouping symbol **then**
        if S.is_empty() **then**
            return false {nothing to match with}
        if S.pop() does not match the type of $X[i]$ **then**
            return false {wrong type}
if S.isEmpty() **then**
    return true {every symbol matched}
else return false {some symbols were never matched}

HTML Tag Matching

- For fully-correct HTML, each <name> should pair with a matching </name>

```
<body>
<center>
<h1> The Little Boat </h1>
</center>
<p> The storm tossed the little
boat like a cheap sneaker in an
old washing machine. The three
drunken fishermen were used to
such treatment, of course, but
not the tree salesman, who even as
a stowaway now felt that he
had overpaid for the voyage. </p>
<ol>
<li> Will the salesman die? </li>
<li> What color is the boat? </li>
<li> And what about Naomi? </li>
</ol>
</body>
```

The Little Boat

The storm tossed the little boat like a cheap sneaker in an old washing machine. The three drunken fishermen were used to such treatment, of course, but not the tree salesman, who even as a stowaway now felt that he had overpaid for the voyage.

1. Will the salesman die?
2. What color is the boat?
3. And what about Naomi?

Tag Matching Algorithm in Python

```
def is_matched_html(raw):
  """Return True if all HTML tags are properly match; False otherwise."""
  S = ArrayStack()
  j = raw.find('<')                       # find first '<' character (if any)
  while j != −1:
    k = raw.find('>', j+1)                # find next '>' character
    if k == −1:
      return False                        # invalid tag
    tag = raw[j+1:k]                      # strip away < >
    if not tag.startswith('/'):           # this is opening tag
      S.push(tag)
    else:                                 # this is closing tag
      if S.is_empty():
        return False                      # nothing to match with
      if tag[1:] != S.pop():
        return False                      # mismatched delimiter
    j = raw.find('<', k+1)                # find next '<' character (if any)
  return S.is_empty()                     # were all opening tags matched?
```

Evaluating Arithmetic Expressions

Slide by Matt Stallmann included with permission.

$$14 - 3 * 2 + 7 = (14 - (3 * 2)) + 7$$

Operator precedence

* has precedence over +/–

Associativity

operators of the same precedence group evaluated from left to right

Example: $(x - y) + z$ rather than $x - (y + z)$

Idea: push each operator on the stack, but first pop and perform higher and *equal* precedence operations.

Algorithm for Evaluating Expressions

Slide by Matt Stallmann included with permission.

Two stacks:

- opStk holds operators
- valStk holds values
- Use $ as special "end of input" token with lowest precedence

Algorithm doOp()

```
x ← valStk.pop();
y ← valStk.pop();
op ← opStk.pop();
valStk.push( y op x )
```

Algorithm repeatOps(refOp):

```
while ( valStk.size() > 1 ∧
        prec(refOp) ≤ prec(opStk.top())
  doOp()
```

Algorithm EvalExp()

Input: a stream of tokens representing an arithmetic expression (with numbers)

Output: the value of the expression

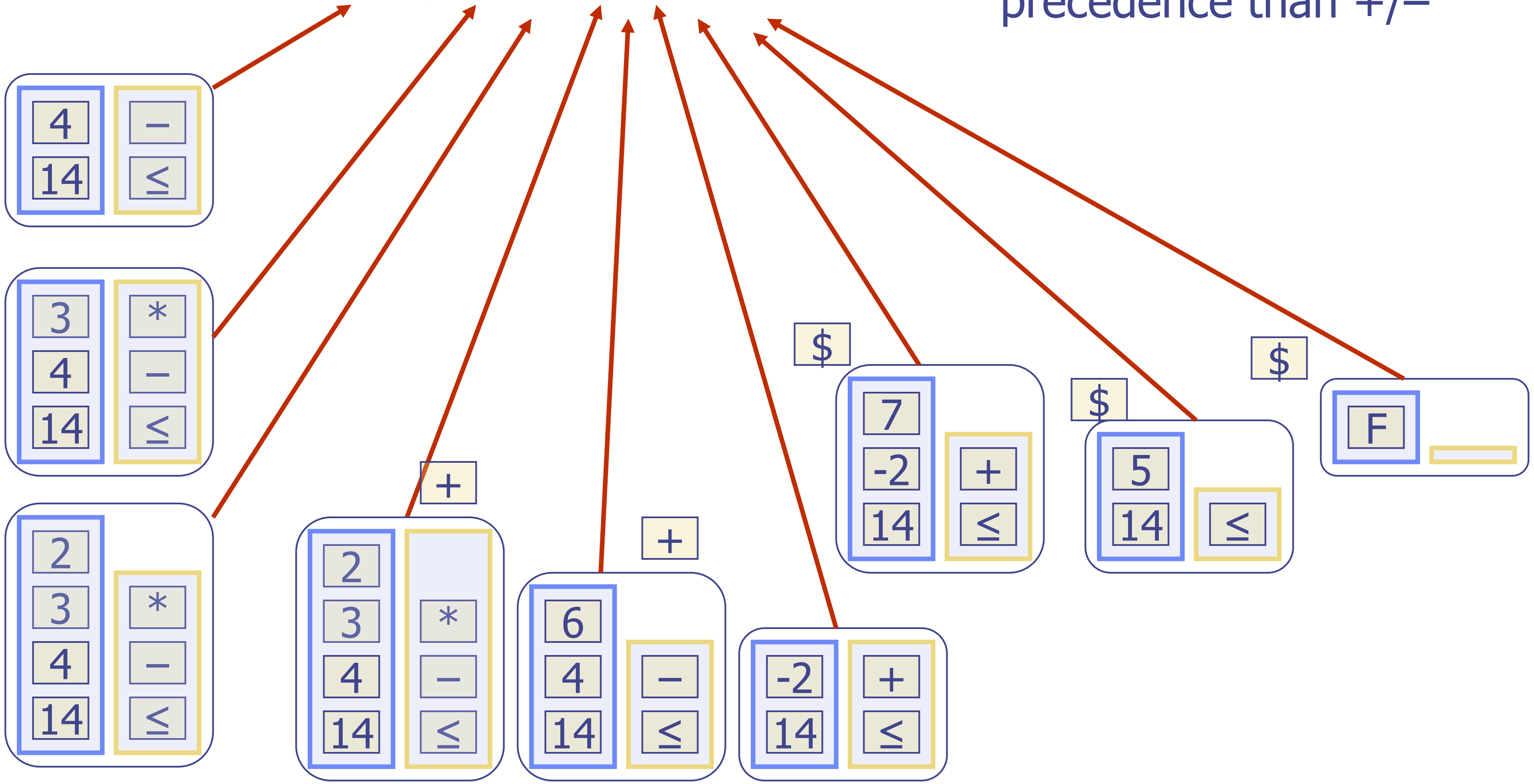
```
while there's another token z
   if isNumber(z) then
         valStk.push(z)
   else
         repeatOps(z);
         opStk.push(z)
repeatOps($);
return valStk.top()
```

Algorithm on an Example Expression

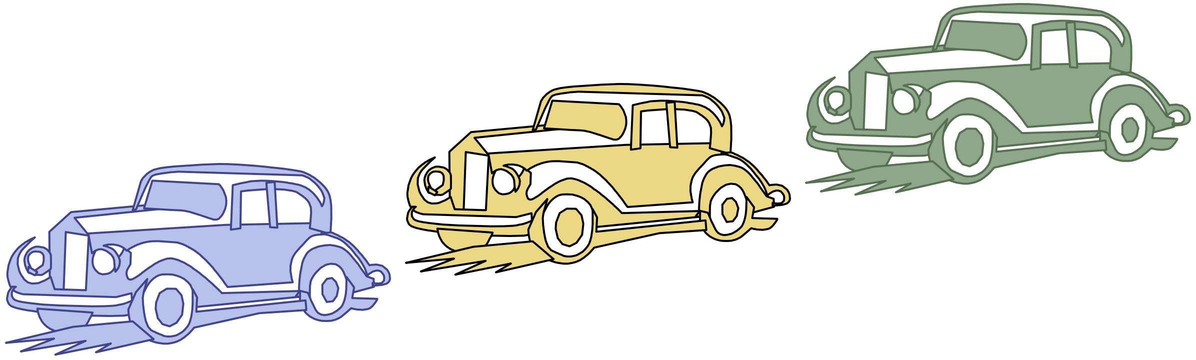
Slide by Matt Stallmann included with permission.

14 ≤ 4 – 3 * 2 + 7

Operator ≤ has lower precedence than +/–



Queues



The Queue ADT

- The Queue ADT stores arbitrary objects
- Insertions and deletions follow the first-in first-out scheme
- Insertions are at the rear of the queue and removals are at the front of the queue
- Main queue operations:
  - enqueue(object): inserts an element at the end of the queue
  - object dequeue(): removes and returns the element at the front of the queue
- Auxiliary queue operations:
  - object first(): returns the element at the front without removing it
  - integer len(): returns the number of elements stored
  - boolean is_empty(): indicates whether no elements are stored
- Exceptions
  - Attempting the execution of dequeue or front on an empty queue throws an EmptyQueueException

Example

Operation	Return Value	first ← Q ← last
Q.enqueue(5)	–	[5]
Q.enqueue(3)	–	[5, 3]
len(Q)	2	[5, 3]
Q.dequeue( )	5	[3]
Q.is_empty( )	False	[3]
Q.dequeue( )	3	[ ]
Q.is_empty( )	True	[ ]
Q.dequeue( )	"error"	[ ]
Q.enqueue(7)	–	[7]
Q.enqueue(9)	–	[7, 9]
Q.first( )	7	[7, 9]
Q.enqueue(4)	–	[7, 9, 4]
len(Q)	3	[7, 9, 4]
Q.dequeue( )	7	[9, 4]

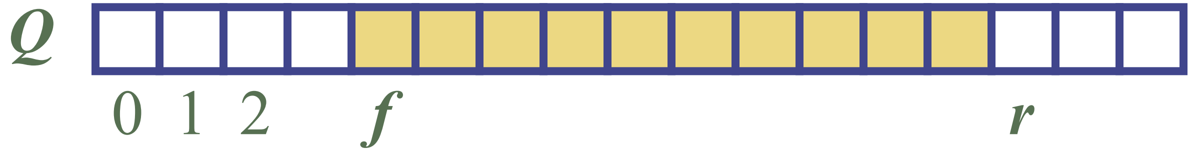
Applications of Queues

- Direct applications
  - Waiting lists, bureaucracy
  - Access to shared resources (e.g., printer)
  - Multiprogramming
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

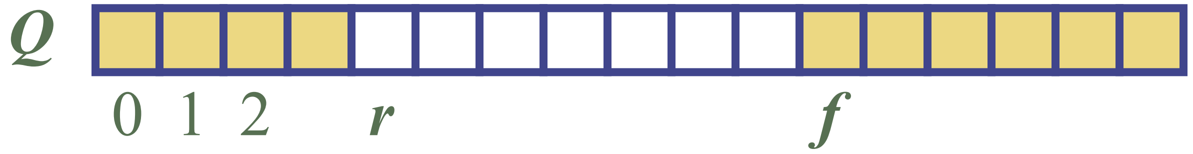
Array-based Queue

- Use an array of size N in a circular fashion
- Two variables keep track of the front and rear
  - f index of the front element
  - r index immediately past the rear element
- Array location r is kept empty

normal configuration



wrapped-around configuration

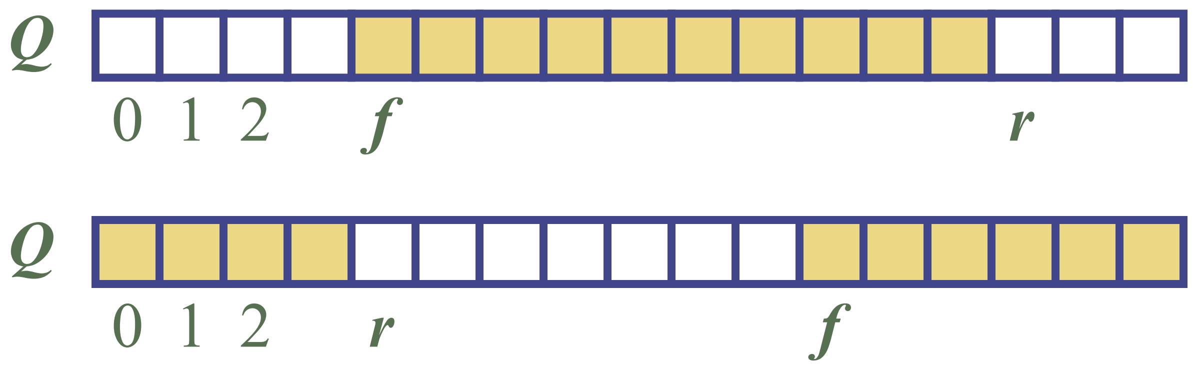


Queue Operations

- We use the modulo operator (remainder of division)

Algorithm *size*()
  return $(N - f + r) \bmod N$

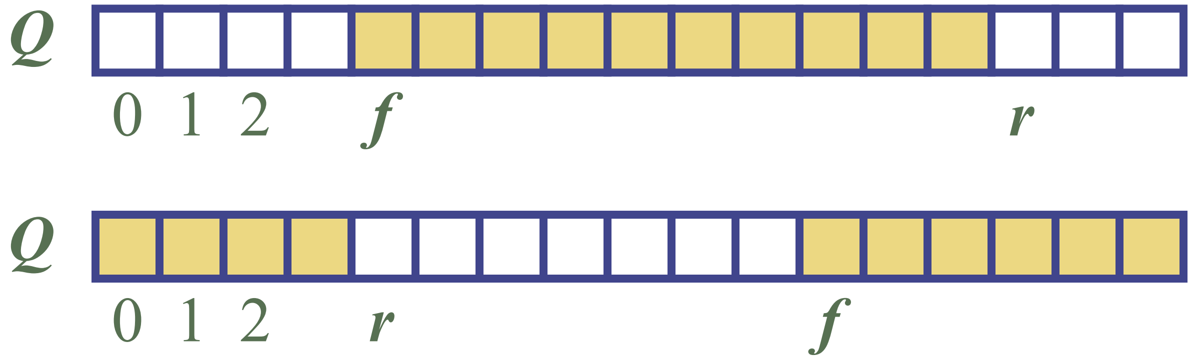
Algorithm *isEmpty*()
  return $(f = r)$



Queue Operations (cont.)

- Operation enqueue throws an exception if the array is full
- This exception is implementation-dependent

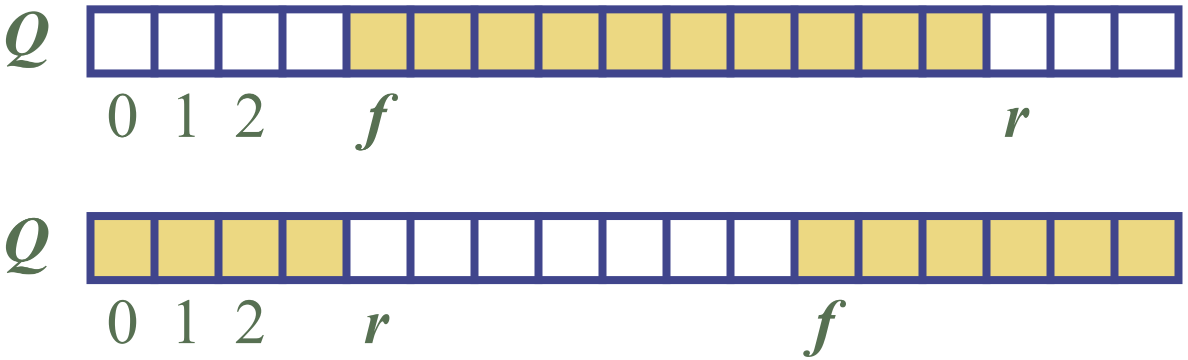
Algorithm *enqueue*(o)
  if $size() = N - 1$ **then**
    throw *FullQueueException*
  else
    $Q[r] \leftarrow o$
    $r \leftarrow (r + 1) \bmod N$



Queue Operations (cont.)

- Operation dequeue throws an exception if the queue is empty
- This exception is specified in the queue ADT

Algorithm *dequeue*()
  if *isEmpty*() **then**
    throw *EmptyQueueException*
  else
    $o \leftarrow Q[f]$
    $f \leftarrow (f + 1) \bmod N$
    return o



Queue in Python

- Use the following three instance variables:
  - _data: is a reference to a list instance with a fixed capacity.
  - _size: is an integer representing the current number of elements stored in the queue (as opposed to the length of the data list).
  - _front: is an integer that represents the index within data of the first element of the queue (assuming the queue is not empty).

Queue in Python, Beginning

```
class ArrayQueue:
  """FIFO queue implementation using a Python list as underlying storage."""
  DEFAULT_CAPACITY = 10          # moderate capacity for all new queues

  def __init__(self):
    """Create an empty queue."""
    self._data = [None] * ArrayQueue.DEFAULT_CAPACITY
    self._size = 0
    self._front = 0

  def __len__(self):
    """Return the number of elements in the queue."""
    return self._size

  def is_empty(self):
    """Return True if the queue is empty."""
    return self._size == 0

  def first(self):
    """Return (but do not remove) the element at the front of the queue.

    Raise Empty exception if the queue is empty.
    """
    if self.is_empty():
      raise Empty('Queue is empty')
    return self._data[self._front]

  def dequeue(self):
    """Remove and return the first element of the queue (i.e., FIFO).

    Raise Empty exception if the queue is empty.
    """
    if self.is_empty():
      raise Empty('Queue is empty')
    answer = self._data[self._front]
    self._data[self._front] = None                   # help garbage collection
    self._front = (self._front + 1) % len(self._data)
    self._size -= 1
    return answer
```

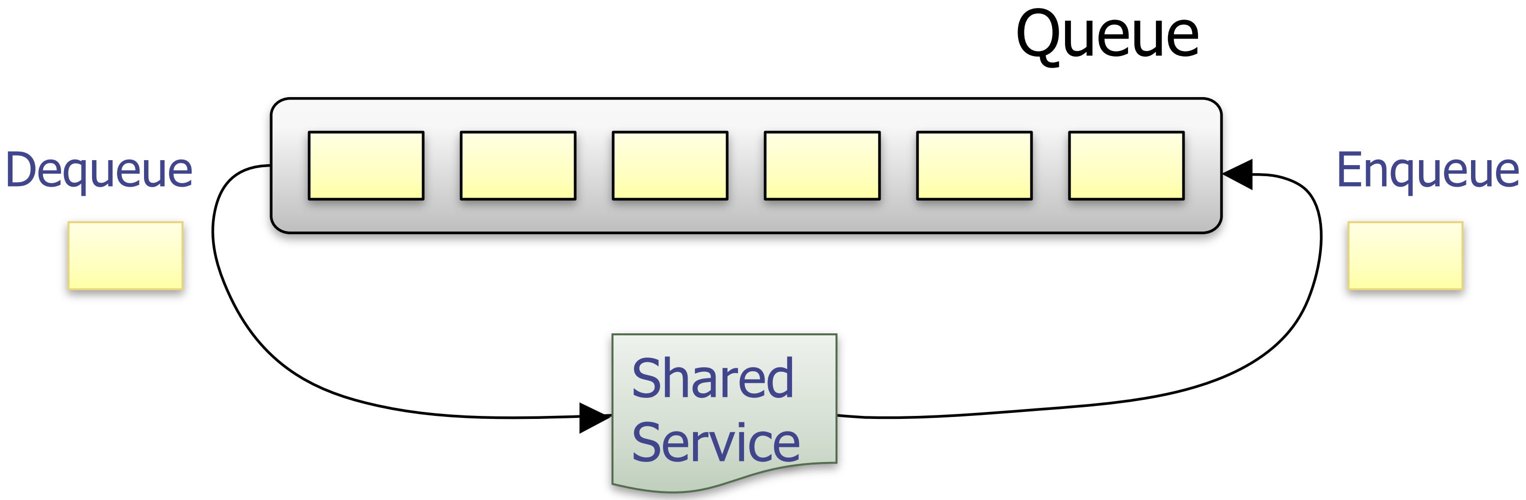
Queue in Python, Continued

```
    def enqueue(self, e):
      """Add an element to the back of queue."""
      if self._size == len(self._data):
        self._resize(2 * len(self.data))           # double the array size
      avail = (self._front + self._size) % len(self._data)
      self._data[avail] = e
      self._size += 1

    def _resize(self, cap):                        # we assume cap >= len(self)
      """Resize to a new list of capacity >= len(self)."""
      old = self._data                             # keep track of existing list
      self._data = [None] * cap                    # allocate list with new capacity
      walk = self._front
      for k in range(self._size):                  # only consider existing elements
        self._data[k] = old[walk]                  # intentionally shift indices
        walk = (1 + walk) % len(old)               # use old size as modulus
      self._front = 0                              # front has been realigned
```

Application: Round Robin Schedulers

- We can implement a round robin scheduler using a queue Q by repeatedly performing the following steps:
  1. e = Q.dequeue()
  2. Service element e
  3. Q.enqueue(e)



Priority Queue ADT

- A priority queue stores a collection of entries
- Each entry is a pair (key, value)
- Main methods of the Priority Queue ADT
  - insert(k, v)
    inserts an entry with key k and value v
  - removeMin()
    removes and returns the entry with smallest key, or null if the the priority queue is empty
- Additional methods
  - min()
    returns, but does not remove, an entry with smallest key, or null if the the priority queue is empty
  - size(), isEmpty()

Example

- A seque

Method	Return Value	Priority Queue Contents
insert(5,A)		{ (5,A) }
insert(9,C)		{ (5,A), (9,C) }
insert(3,B)		{ (3,B), (5,A), (9,C) }
min( )	(3,B)	{ (3,B), (5,A), (9,C) }
removeMin( )	(3,B)	{ (5,A), (9,C) }
insert(7,D)		{ (5,A), (7,D), (9,C) }
removeMin( )	(5,A)	{ (7,D), (9,C) }
removeMin( )	(7,D)	{ (9,C) }
removeMin( )	(9,C)	{ }
removeMin( )	null	{ }
isEmpty( )	true	{ }

Total Order Relations

- Keys in a priority queue can be arbitrary objects on which an order is defined
- Two distinct entries in a priority queue can have the same key
- Mathematical concept of total order relation $\leq$
  - Comparability property: either $x \leq y$ or $y \leq x$
  - Antisymmetric property: $x \leq y$ and $y \leq x \Rightarrow x = y$
  - Transitive property: $x \leq y$ and $y \leq z \Rightarrow x \leq z$

Entry ADT

- An **item** in a priority queue is simply a key-value pair
- Priority queues store entries to allow for efficient insertion and removal based on keys
- Methods:
  - getKey: returns the key for this entry
  - getValue: returns the value associated with this entry

```
class PriorityQueueBase:
  """Abstract base class for a priority queue."""

  class _Item:
    """Lightweight composite to store priority queue items."""
    __slots__ = '_key', '_value'

    def __init__(self, k, v):
      self._key = k
      self._value = v

    def __lt__(self, other):
      return self._key < other._key          # compare items based on their keys

  def is_empty(self):                        # concrete method assuming abstract len
    """Return True if the priority queue is empty."""
    return len(self) == 0
```

Comparator ADT

- A comparator encapsulates the action of comparing two objects according to a given total order relation
- The comparator is external to the keys being compared
- When the priority queue needs to compare two keys, it uses its comparator
- Example – 2D data point, lexicographic order

- Primary method of the Comparator ADT
- compare(x, y): returns an integer i such that
  - i < 0 if a < b,
  - i = 0 if a = b
  - i > 0 if a > b
  - An error occurs if a and b cannot be compared.

Sequence-based Priority Queue

- Implementation with an unsorted list

- Performance:
  - insert takes $O(1)$ time since we can insert the item at the beginning or end of the sequence
  - removeMin and min take $O(n)$ time since we have to traverse the entire sequence to find the smallest key

- Implementation with a sorted list

- Performance:
  - insert takes $O(n)$ time since we have to find the place where to insert the item
  - removeMin and min take $O(1)$ time, since the smallest key is at the beginning