

```
In [ ]: import numpy as np
import matplotlib.pyplot as plt
```

inital

```
In [ ]: a = 2
b = 3
x0 = (a, 1)
epsilon = 1e-15
```

```
In [ ]: def f(x):
    x1 = x[0]
    x2 = x[1]
    return 0.5*(x1**2 + (a * x2**2)) - np.cos(b * (x1 + x2))
```

```
In [ ]: def f_gradient(x):
    x1 = x[0]
    x2 = x[1]
    return np.array([(x1 + np.sinc(b)),
                    (a*x2 + np.sinc(b))])
```

steepest-descent method

```
In [ ]: def steepest_descent(x, max_iterations, t):
    x_hist = []
    x_hist.append(x)
    results = []
    results.append(f(x))
    k = 1
    while True:
        x = x + (t * -f_gradient(x))
        x_hist.append(x)
        results.append(f(x))
        if k == max_iterations or abs(results[-1] - results[-2]) <= epsilon:
            break
        else:
            k += 1
    return x_hist, results
```

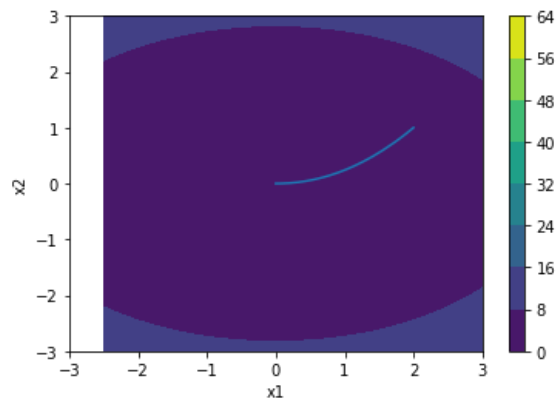
```
In [ ]: def plot_result(x):
    x1_plot = [float(x[0]) for x in x]
    x2_plot = [float(x[1]) for x in x]

    xlist = np.linspace(-2.5, 10.0, 100)
    ylist = np.linspace(-3.5, 3.5, 100)
    X, Y = np.meshgrid(xlist, ylist)
    Z = 1/2 * (X**2 + (a * Y**2))
    fig, ax = plt.subplots(1,1)
    cp = ax.contourf(X, Y, Z)
    fig.colorbar(cp)
    ax.set_xlabel('x1')
    ax.set_ylabel('x2')
    ax.set_xlim(-3,3)
    ax.set_ylim(-3,3)
    ax.plot(x1_plot, x2_plot)
    plt.show()
```

find x, p by steepest-descent method

```
In [ ]: t = 0.05
max_iterations = -1
x_hist, f_SD_t1 = steepest_descent(x0, max_iterations, t)
p_opt = min(f_SD_t1)
print('p* =', p_opt)
print('x_k', x_hist[:5])
print('k iterations:', len(x_hist))
plot_result(x_hist)
```

```
p* = -0.9999999999999995
x_k [(2, 1), array([1.9, 0.9]), array([1.805, 0.81 ]), array([1.71475, 0.729 ]), array([1.6290125, 0.6561 ])]
k iterations: 345
```



In []:

```
t = 0.15
max_iterations = -1
x_hist, f_SD_t1 = steepest_descent(x0, max_iterations, t)
p_opt = min(f_SD_t1)
print('p* =', p_opt)
print('x_k', x_hist[:5])
print('k iterations:', len(x_hist))
plot_result(x_hist)
```

p* = -0.99999999999999978

x_k [(2, 1), array([1.7, 0.7]), array([1.445, 0.49]), array([1.22825, 0.343]), array([1.0440125, 0.2401])]

k iterations: 114

