Cavity-Induced Atom Cooling in the Strong Coupling Regime

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We investigate the possibility of all optical trapping and cooling a single atom at the antinodes of a high Q optical cavity mode to which the atom is strongly coupled. For properly chosen parameters the dynamics of the cavity field introduces a novel Sisyphus type cooling mechanism yielding final temperatures much below the Doppler limit and allowing for long trapping times, avoiding the problems induced by spontaneous emission. [S0031-9007(97)04824-2]

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Using laser cooling and trapping very low steady-state temperatures of a trapped cloud of neutral atoms have been achieved leading to new phenomena as Bose-Einstein condensation [1] and interesting possible applications as atom lithography or quantum logic gates [2–4]. In this context the interaction between a single cooled atom and a high finesse optical cavity mode has started to raise much theoretical [3,5–8] and experimental interest [9–12]. One example is the continuous monitoring of a single atom traversing a high finesse optical cavity via the cavity output [13], showing the change in the cavity transmission related to the atom's motion in the cavity field. In a recent theoretical paper Doherty *et al.* [14] have investigated the mechanical effect of the optical potential of the cavity field on the atomic motion in the limit of a resonantly driven cavity [13]. In close analogy to Doppler cooling in a standing wave field [15] they found heating mechanisms (e.g., dipole heating), which lead to high kinetic energies and short confinement times of the atom.

In this work we investigate a novel cooling mechanism mediated by the combined cavity-atom dynamics. It works best in the strong coupling regime, when the atom-cavity coupling constant g exceeds the optical linewidth of the atom Γ and the cavity κ. We find a Sisyphus type cooling mechanism through the position dependent change of the intracavity photon number, which allows for sub-Doppler temperatures and long confinement times.

Let us first present a simple classical model to demonstrate the key physical principle of the proposed cooling and trapping scheme. We consider a pointlike particle at position x moving with momentum p inside a driven high finesse optical cavity as depicted in Fig. 1. Because of the large atom-field coupling the resonance frequency of the cavity is significantly shifted depending on the particle position x, i.e., the single particle induces a position (and time) dependent index of refraction in the cavity [2,3,9].

The system dynamics in this limit is given by the following coupled equations for the cavity field, the particle momentum, and its position:

$$\dot{E} = [-\kappa - \gamma(x) + i\Delta_c - iU(x)]E - \alpha\,, \tag{1a}$$

$$\dot{p} = -|E|^2 \frac{d}{dx} U(x)\,, \tag{1b}$$

$$\dot{x} = p/m\,. \tag{1c}$$

Here $\gamma(x) = \gamma_0 \cos^2(kx)$ is the rate at which the atom scatters light, $\Delta_c = \omega_p - \omega_c$ is the detuning of the empty cavity relative to the pump frequency, $U(x) = U_0 \cos^2(kx)$ is the frequency shift of the cavity due to the interaction with the particle, and α describes the external pump. The position dependence of $\gamma(x)$ and $U(x)$ derives from the cavity mode function, which is assumed sinusoidal for simplicity.

Equation (1a) contains the influence of the particle's position on the cavity field: the field decay is enhanced by spontaneous photon scattering at a rate $\gamma(x)$ and the mode frequency is shifted by $U(x)$. If $\gamma_0 \ll \kappa < |\Delta_c|$ and $U_0 \approx \Delta_c$ the field amplitude will change significantly with x. In order to find cooling we will assume $\Delta_c < 0$ and $U_0 < 0$. In this case the maximum field amplitude and the minimum cavity frequency is obtained when the particle sits at an antinode of the standing wave [see Fig. 2(a)]. For a moving particle we have to consider the whole system dynamics according to Eqs. (1). Because of the finite cavity response time the maximum field intensity will be attained *after* the particle has passed the potential minimum [dashed curve in Fig. 2(a)]. Hence on average the particle climbs the potential wells at times of higher intracavity field intensity and runs down the potential wells at times of lower intensity, as is indicated by the dotted vertical lines in the figure. Hence the particle loses kinetic energy and is slowed down until it is trapped within a single potential well. A typical trajectory showing this behavior obtained from a numerical integration of Eqs. (1) is plotted in Fig. 2(b).

The classical model predicts the particle to come to a complete stop at some antinode of the mode. In a more

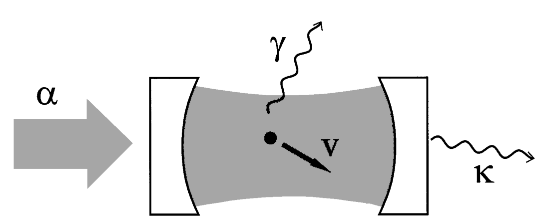

FIG. 1. Schematic representation of the system consisting of a single particle inside a driven optical cavity.

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refined quantum description field fluctuations due to the quantum nature of the cavity mode and the momentum diffusion through atomic spontaneous emission of the particle will introduce heating mechanisms yielding a nonzero final temperature.

In a quantum model of our system the particle is modeled by a single two-level atom with ground state $|0\rangle$ and excited state $|1\rangle$. Using dipole and rotating wave approximation the Hamiltonian for the compound atom-cavity system reads

$$H = -\Delta\sigma_{11} - \Delta_c a^\dagger a + i\Omega(\hat{x})(\sigma_{01}a^\dagger - a\sigma_{10}) + i\alpha(a - a^\dagger) + \frac{\hat{p}^2}{2m}, \quad (2)$$

where $\Delta = \omega_p - \omega_{01}$ is the atom-pump detuning, $a(a^\dagger)$ is the photon annihilation (creation) operator, $\Omega(\hat{x}) = \Omega_0 \cos(k\hat{x})$ is the atom-cavity coupling constant, $\hat{x}$ and $\hat{p}$ are the atomic position and momentum operators, and the atomic operators are given by $\sigma_{ij} = |i\rangle\langle j|$. The complete dynamics for the full density operator ρ is described by the master equation

$$\dot{\rho} = -i[H,\rho] + \gamma\left[-\sigma_{11}\rho - \rho\sigma_{11} + 2\int_{-1}^{1} du\, N(u)\sigma_{01}e^{-ik\hat{x}u}\rho e^{ik\hat{x}u}\sigma_{10}\right] + \kappa(2a\rho a^\dagger - \rho a^\dagger a - a^\dagger a\rho), \quad (3)$$

where the angular distribution $N(u)$ and the recoil kick of spontaneously emitted photons have been included.

To recover an interpretation of the cooling scheme related to our previous discussions we first calculate the eigenstates of the compound atom-cavity system in the semiclassical limit, where the atomic momentum and position operators are replaced by their expectation values. The eigenenergies of the Hamiltonian (2) for vanishing pump field ($\alpha = 0$) are given by

$$E_0 = 0,$$
$$E_{n,\pm}(x) = -\Delta_c n - \delta/2 \pm \sqrt{\delta^2/4 + \Omega(x)^2 n}, \quad n \geq 1, \quad (4)$$

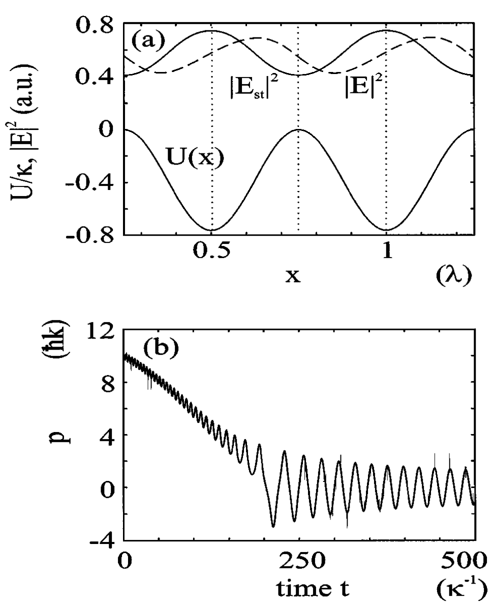

FIG. 2. (a) Potential $U(x)$ and cavity field intensity vs particle position x. $|E_{st}|^2$ is the field intensity for fixed particle position, whereas $|E|^2$ (dashed curve) is the field intensity for an atom with constant velocity. (b) Time evolution of the particle momentum. The particle gets cooled and finally trapped inside a single potential well. The parameters are $U_0 = 0.76$, $\gamma_0 = 0.07$, and $\Delta_c = 1.2$ in units of κ.

where $\delta = \Delta - \Delta_c$. A resonance with the driving field occurs, if one of the energies $E_{n,\pm}(x)$, $n > 0$, vanishes. As an example, let us consider the case $E_{1,-}(x = 0) = 0$, i.e., the lowest excited eigenstate of the system is resonant to the driving field at atomic positions $x = n\lambda/2$ for any integer n as shown in Fig. 3. An atom initially in the lowest energy eigenstate and moving with a velocity v small enough to follow adiabatically the energy eigenstates will then stay in the ground state until it is excited to the first energy level $E_{1,-}$ by the driving laser. Preferentially this transition will occur near the potential minima of $E_{1,-}$, where we have assumed resonance. Subsequently, the particle evolves in this energy eigenstate transferring kinetic energy into internal energy until spontaneous atomic decay or cavity decay brings it back to the ground state. The emitted photon then carries away the internal energy of the system. This leads to a diminishing kinetic energy of the atom.

Although the system closely resembles the Doppler cooling scheme in a classical standing-wave light field the cooling mechanism here is of different physical origin. Here the Sisyphus effect is due to the change of the intracavity photon number correlated with the

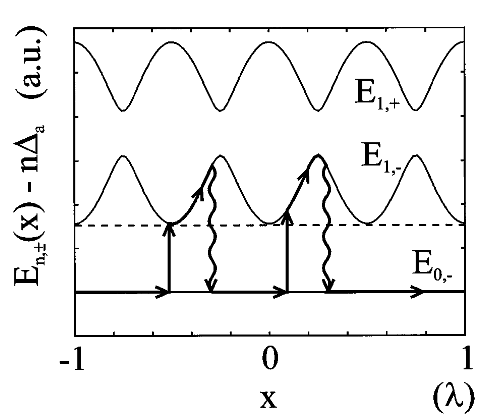

FIG. 3. Diagrammatic representation of the Sisyphus type cooling mechanism.

atomic position, whereas in standard Doppler cooling the intensity of the cooling fields is fixed. Because of its different physical origin the cavity-mediated cooling works well for small velocities, where the Doppler shift is negligible. Using large detunings the atom hardly ever gets excited to the upper internal state and compared to Doppler cooling the diffusion of the atomic momentum due to the arbitrary direction of the spontaneously emitted photons can be strongly suppressed. One therefore can expect must lower steady-state temperatures.

Unfortunately, an exact solution of Eq. (3) for the steady state seems impossible. To lowest order in the atomic velocity and in the driving field strength α an approximate result for the steady-state temperature can, however, be obtained from the semiclassical force and diffusion coefficient as presented in Ref. [16]. Here the calculations can be restricted to the lowest three eigenstates of the atom-cavity system with energies E_0 and $E_{1,\pm}$, cf. Eq. (4) and Fig. 3.

As in Ref. [16] we are able to derive analytic expressions for the friction coefficient β, which gives the semiclassical force $f = -\beta v$ acting on the atom to lowest order in v. Similarly, we find the diffusion coefficient $D = d/dt(\langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2)$:

$$D = \gamma \hbar^2 k^2 \frac{\alpha^2}{C} \left\{ \frac{2}{5}\, \Omega(x)^2 + \frac{[\nabla \Omega(x)]^2}{k^2} (1 + d_{\mathrm{cav}}) \right\}, \tag{5}$$

where

$$C = (\gamma\kappa + \Omega(x)^2 - \Delta\Delta_c)^2 + (\Delta\kappa + \Delta_c\gamma)^2,$$
$$d_{\mathrm{cav}} = \frac{4\Delta\Omega(x)^2}{\gamma} \frac{\Delta_c\gamma + \Delta\kappa}{C}. \tag{6}$$

Apart from the term d_{cav}, Eq. (5) corresponds exactly to the diffusion coefficient of an atom at position x in an external standing wave in the weak field limit, i.e., we have rederived the well-known diffusion coefficient of ordinary Doppler cooling [16]. The additional term d_{cav} is due to the cavity field dynamics and turns out to be the *dominating* term for the parameters discussed in this work.

Similarly, the friction coefficient can be split into the usual Doppler force β_D and the additional cavity-mediated force β_C, respectively. However, since the analytic expressions for β_D and β_C are rather unwieldy, we will not give the exact results here and restrict ourselves to the discussion of a numerical example. β, β_c, and β_D as well as the approximate steady-state temperature T are shown in Fig. 4.

In Fig. 4(a) we show the friction coefficient β (β_D, β_C) versus the detuning of the driving laser with respect to the cavity resonance frequency where the cavity-atom detuning δ is kept constant, i.e., the dressed levels remain the same for all parameters but the driving field frequency varies. As expected one finds a positive friction coefficient, which implies a slowing of the atom, where the driving field is approximately in resonance with the minima of the first excited dressed level $E_{1,-}$, see Eq. (4), so that the Sisyphus mechanism works efficiently yielding a low steady state temperature. For higher pump frequencies close to resonance with the maxima of $E_{1,-}$ (i.e., $\Delta_c \approx 0$), the Sisyphus mechanism is reversed and gives rise to heating of the atom with a negative friction coefficient. Increasing the pump frequency further, other cooling and heating peaks occur in the friction coefficient, which correspond essentially to the usual Doppler cooling force with a maximum value for atom detunings $\Delta \approx \pm\gamma$. Only a slight modification due to the spatially dependent excitation of the *second* excited dressed level $E_{1,+}$ occurs for these parameters.

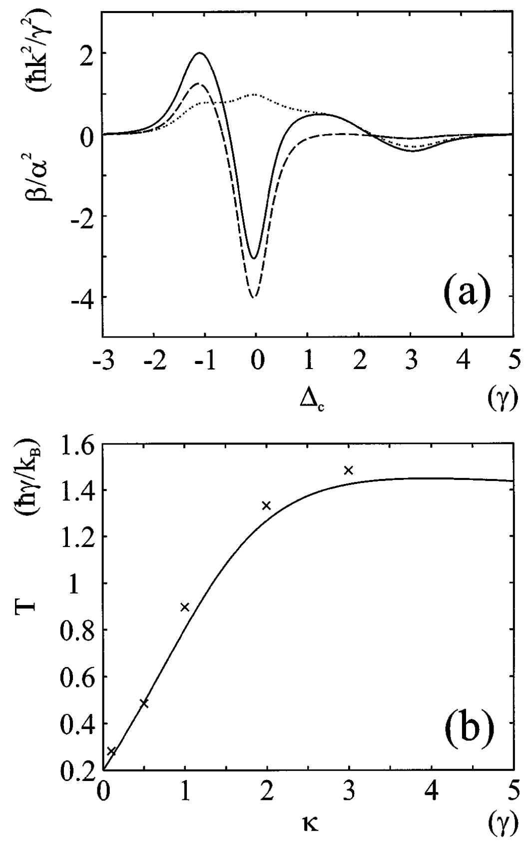

FIG. 4. Results of the semiclassical model in the weak pumping limit. (a) Total friction coefficient β (solid curve) and its decomposition into the Doppler part β_D (dotted) and the cavity part β_C (dashed) vs cavity detuning Δ_c for $\kappa = \gamma/2$. (b) Steady-state temperature vs κ for $\Delta_c = -1.3\gamma$. The other parameters are $\delta = -1.9\gamma$ and $\Omega_0 = 2\gamma$. The crosses are obtained from fully quantum Monte Carlo simulations.

Figure 4(b) shows the steady-state temperature $T = D/\beta k_B$ of the atom as a function of the cavity decay rate κ. The driving field frequency is chosen close to the point of optimum cooling [see Fig. 4(a)]. For small κ we, find a linear dependence of the temperature on κ, connected to the effective linewidth of the involved dressed level $E_{1,-}$. Hence the cavity decay rate κ plays essentially the same role in this system as does the atomic decay rate γ in usual Doppler cooling. This gives rise to a final temperature of the order of $k_B T = \hbar\kappa$. For κ larger than the spatial modulation of the dressed energy $E_{1,-}$ the position dependence of $E_{1,-}$ gets washed out and the effect of the Sisyphus cooling decreases significantly. Here the Doppler cooling force becomes predominant and the steady-state temperature approaches the value for weak classical standing wave field of fixed amplitude. Note that this temperature differs from the conventional Doppler limit $k_B T = \hbar\gamma$ because the detuning Δ is chosen to optimize the new cavity-mediated cooling.

In a final generalization we now relax the assumption of a classical point particle and include the external atomic degrees of freedom. This requires a full quantum treatment including the internal and the cavity dynamics. Because of the complexity of the model we are restricted to a pure numerical treatment based on quantum Monte Carlo wave-function simulations [17]. The results of these calculations for the steady-state temperature are indicated by the crosses in Fig. 4(b). As one can see, we find excellent agreement with the semiclassical calculations performed previously. Hence quantum effects of the external motion like the spreading of the atomic wave packet, the zero point energy in the well, or tunneling between wells essentially seem to play no role in the dynamics.

We have shown that in a high-finesse optical cavity new mechanical light effects on single atoms appear. In particular, we found a Sisyphus-type cooling mechanism based on the change of the resonance frequency of the cavity induced by the atomic position, which does not require atomic spontaneous emission but works through the cavity decay. The achievable steady-state temperatures are of the order of $k_B T = \hbar\kappa$ which, depending on the experimental parameters, can be significantly below the usual Doppler cooling limit in a standing wave and provide for long trapping times of the atoms at exactly the antinodes of the field, where the atom field coupling is maximal. These results could have significant impact on ongoing experiments with laser-cooled atoms in high-finesse cavities [9,13]. As the only requirement on the particle is a strong coupling to the cavity mode, the results should also apply to small molecules or other more complex objects (as, e.g., a Bose condensate) with a sufficiently large dipole moment.

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