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Instantaneous Interaction With Time

Relativity theory is based on a single premise: nothing can go faster than the speed of light. As the brilliant Leonard Susskind says in his book *General Relativity*, "a light ray is of course the fastest thing that can exist; indeed it moves with the speed of light" (Susskind 194). The concept is easy to comprehend but make no mistake – this simple sentence has caused physicists many headaches in the recent past. One such headache was the introduction of spacetime (Minkowski space), a four dimensional view of the universe which merges two concepts previously seen as separate into a single four-dimensional manifold. Understandably, this comes with some interesting phenomena. In spacetime, it is found that the light from far away stars becomes red-shifted because its wavelength is literally being stretched. Similarly, it is found that as objects move faster, outside observers see them as smaller. What can be taken away from the concept of spacetime is that stretches and squishes of space, length contractions, also come with stretches and squishes of time, time dilation, to create another combined concept called Lorentz transformations. These transformations change the axes (spatial and temporal) in the frame being observed – the primed frame – to show how the axes of the rest frame look in comparison. Often, this change is negligible – the speed of light in comparison to speeds regularly experienced is too massive to allow much of a difference. Though when the velocity of the reference becomes "c" an interesting question presents itself: how would a frame moving at the speed of light relative to the universe experience time? Honestly, the answer is fairly simple.

A reference frame moving at the speed of light relative to the universe would experience time exactly the same. From that frame, it looks as though the universe is moving around it. But what would be seen? Or more accurately – how would a particle moving at the speed of light perceive the universe around it? <u>A particle moving at the speed of light would perceive the universe from a single point in time.</u>

To understand a particle, it is important to first understand what contains it. Spacetime is a somewhat new concept, originally thought of by Albert Einstein in his 1905 paper *On the Electrodynamics of Moving Bodies* (EoMB) and later reimagined in 1908 by Hermann Minkowski's *The Fundamental Equations for Electromagnetic Processes* (FEEP). Technically, spacetime could include any number of spatial and temporal dimensions but that which these two describe is characterized by a familiar three spatial dimensions and one temporal. The properties of this manifold were described by Minkowski similar to a four-dimensional space without time: just as a three-dimensional space has infinite two-dimensional planes, so does this four-dimensional Minkowski spacetime have infinite three-dimensional geometries. Though properties of three-dimensional geometries have majorly been theorized by three mathematicians: Euclid, Riemann, and Minkowski – Minkowski's take is used because of its use of invariant[1] scalars and negatives in the definition of distance[2].

On the Electrodynamics of Moving Bodies talks of the topic of special relativity, one of the two major sections of relativity theory. Special relativity is particularly useful because its specific focus is on very high velocities. With so many large quantities being slightly varied in each frame of reference, notation becomes very important. Because of this, Einstein opted instead of a variable-oriented format to use the tensor/matrix format for EoMB. This allows

[1] Saying "invariant" here is redundant. Scalars are often used because they are always invariant.
[2] Leo Cory goes into this extensively in "Hermann Minkowski and the Postulate of Relativity"

vectors not only to be shown in a single character but also allows them to be shown in their covariant and contravariant variations. This later showed itself to be another genius move by Einstein when the introduction of the Einstein summation convention, which showed its full use while building relativistic Lagrangians[3]. Conveniently, it is these Lagrangians that give a theoretical understanding of how an object moving at the speed of light would experience the cosmos.

What would an object moving at the speed of light and its properties look like? The most famous equation of all time, $E = mc^2$, gives the answer. However, Einstein's masterpiece as most know it is incomplete. As it is, the above equation describes a stationary object. Its energy is entirely held in its mass. The full equation is $E^2 = (mc^2)^2 + (pc)^2$. This describes the total energy of an object that is both moving and has mass. This simplifies energy into a function of mass and velocity. Though studying the equation closer brings an interesting parallel: Einstein's equation looks just like a pythagorean triple, where the hypotenuse is described by "E^2." This means the energy can be seen as a triangle with the momentum component as the base and the mass as the side. The only side that describes the velocity is the base but the function that calculates the energy using the velocity involves mass because $p = mv$.[4] Therefore, no matter how high the velocity is, the mass will always hold a portion of its energy, both in its term as well as in its own. This means an object moving at the speed of light must have no mass. It must be a mass-less particle moving at the speed of light. This description matches that of a photon perfectly. This is also why the constant "c" is called the speed of light: it is the speed light moves in a vacuum.

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[3] Lagrangian mechanics is used in relativity while Hamiltonian is used in quantum physics.

[4] This is technically incorrect but this generalization is a good description of how momentum is dependent upon an object's mass and velocity.

When talking about particle interactions in spacetime, a fundamental concept is the "light cone." Imagine a field of spacetime governed by (2+1), or two dimensions of space and one of time. In this field, a light flashes, moving in every direction to make a ring of influence around the original point. When this ring is graphed, using the x and y axes to the size of the ring and the z axis to show time passed, the result is a spacetime diagram. Movement along the time axis shows the growth of the ring, making a cone. Using this cone, the amount of time that needs to pass for a separate point to interact with the observed point can be found by observing the boundaries of the cone. This is the boundary of separation. At first this definition may seem confusing but as Leonard Susskind explains in SRCFT, "if two points in spacetime have a separation of zero, this does not mean they have to be the same point. Zero separation simply means that the two points are related by the possibility of a light ray going from one of them to the other" (Susskind, 71)

If this observed point is moving, the light cone becomes stretched. The light moving opposite to the point is moving at the speed of light relative to the point but an outside observer would think the light was moving a little less than the speed of light. As the point moves closer to the speed of light relative to an outside observer, the light cone shifts closer to the speed of light, so too does the cone shift closer to being completely vertical. Following this line of thinking, Einstein thought of watching a clock. If suddenly, he started moving at the speed of light away from the clock, he would think the clock had stopped ticking. The light from it, moving at the same speed as him, would not be able to reach his eyes to show him that it was moving. He would be experiencing time the same but it would appear as though the time on earth relative to him had completely stopped.

This is the easy part of the argument. Coming back to the moving particle – if the x-axis is oriented along the particle's line of movement, any objects with $y \leq 0$ or $z \leq 0$ would seem as though they had completely frozen in time. Though an observer on the object could both experience time normally and watch the particle move by, from the frame of reference of the particle, time has completely stopped for that frame. The plane of separation between what is frozen and what has not been discussed is called the particle's "hypersurface of the present." Notice this title comes with descriptions of both space and time. Space comes with the outside objects' positions. The time component describes the particle's present, which means that objects beyond that plane have entered the particle's past. In essence, this is how a particle can see the universe as "instantaneous." Spacetime allows for local movement through space to change shown global movement through time.

Deviation of opinion occurs in relation to objects with y and z values that are both positive. Some say that it is impossible to see time instantaneously but not only is it theoretically and mathematically possible but particles moving at the speed of light do experience time in an instant. To reiterate, from every point of reference, time is moving at a standard, constant pace. What changes is how that frame of reference sees other points moving through time, just how observers from different frames of reference can see different rates of change of position, so can they see different rates of how objects travel through time.

To see that experiencing time in an instant happens, it must first be possible. The easiest way to see this is a mildly unexpected approach – the introduction of gravity. A little over a decade after EoMB, Einstein published his general theory of relativity. Its goal was to combine Newton's theory of gravity and accelerated frames of reference, an issue that, at the time, was generating confusion for many of his peers. Per the norm, Einstein solved this with a concept even an adept third grader could understand: the equivalence principle, which states that on a very small scale[5], gravity and acceleration are indistinguishable. In a more personal sense – if someone was standing in a box, unable to see their surroundings, and suddenly they felt weight on their body – they would not be able to tell if the box was being moved or if they were standing on the surface of a planet.

Understanding this parallel shows where the physical universe deviates from Euclidean geometry. To someone in an elevator moving perpendicular to the movement of other objects would see curved trajectories where stationary observers would see straight ones. Leonard Susskind explains in SRCFT that, "Spacetime tells matter how to move; matter tells spacetime how to curve" (Leonard 155). A spacetime diagram of outside objects' trajectories would show that spacetime is curved so external frames would need to accelerate perpendicular to the direction of the curvature of spacetime, similar to the reference frame. According to the equivalence principle, this means objects that are stationary relative to a gravitational field also see that spacetime is curved. Per the general theme of relativity, all changes in spacetime come with both intelligible changes in space and counterintuitive changes in time. It is a common fact that in space, the shortest distance between two points is a straight line[6]. This is true of time as well. As time bends in gravity, the rate of passage of time slows. To an outside observer, it seems as though it takes longer for the frame under gravity to reach the same point in time.

The problem with humans is scale. Life is small, which makes sense – the bigger the organism the more resources it needs. But it is this small scale that gives rise to concepts akin to Newtonian physics that make sense for human-like distances and forces, though these mechanics

[5] The scale used was that of humans because it was both familiar and allowed Einstein to initially ignore special relativity and solely focus on gravity using classical mechanics

[6] This does not apply in curvilinear coordinate systems – those used in general relativity but this description is purely for comparison

break down in bigger sizes. Just like "zooming out" to see that gravitational fields are vector fields rather than constant rates, to see major changes in time, a more extreme example is necessary. In 1974, Stephen Hawking thought of the universe's silent killers: mass packed so densely that it forms a singularity. Gravity so strong nothing in existence can withstand its might.

Black holes are formed in the deaths of massive stars. In less than a quarter of a second, the gravity of the star overcomes fusion and compresses the mass of (at least) eight suns into about the size of a large city. As its radius decreases, its rotational speed sharply increases like a cosmic figure skater pulling its arms in as it spins faster and faster. Not only does the gravity of this singularity infinitely bend spacetime toward itself but also the stupendous spin pulls spacetime along the direction of rotation. Rather than objects falling into an infinitely steep, inescapable hole – they instead swirl rapidly into this celestial toilet bowl.

The claim of most who see it impossible to experience time instantly profess that it requires physics to be broken in order for the time axis to entirely change its orientation. While this at first seems true, it turns out to be marginally untrue but physics needs to be at its absolute limits. The macabre beauty of black holes is that they are the barrier between known and unknown. To better see this relationship, one must first understand what a black hole even is. Consider a Schwarzschild radius: a metric that most consider only a descriptor of black holes even though all objects with mass have their own Swarzschild radius[7], dependent on the mass of the gravitating object. Thus black holes can be explained simply. As Susskind puts it in *General Relativity*, "a black hole is a body whose mass is contained within its Swarzschild radius" (Susskind, 195). Beyond this horizon, the current understanding of existence breaks down[8]. To a

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[7] Any object's Swarzschild radius is found to be its mass times the gravitational constant doubled and divided by the speed of light squared.

[8] Though technically, this is a definition change of what "breaks down" refers to, it is probably best to avoid these breaking points altogether.

frame watching an entity fall into a black hole, it would appear that the entity slows continually until – suddenly, time stops. It is held on the two-dimensional geometry of the sphere of unknown. The question remains though: is this possible within the realm of understanding? The event horizon is the breaking point of understanding. Can this even happen in an understandable frame of reference?

Though "understandable" may not be the first word to come to mind when talking about the ergosphere, at the very least it does obey standard physics. If a black hole is a celestial toilet bowl, the ergosphere is the cosmic water flowing into it. The ergosphere is the area of extreme warping of spacetime surrounding a black hole. It is possible to escape this region by moving with its current however, even to stay static in this galactic flow, one would need to move at or above the speed of light[9]. Unfortunately, the ergosphere is a consequence of gravity with its rotation and not part of the field itself. This means that rather than being an added field with gravity, they are changes to gravity's vector field itself. Though this may appear to bring time closer to instantaneous, it merely approaches infinity.

As it turns out, stopping time is hard. Using gravity to fully change time's orientation is (probably) impossible. Gravity affects everything the same[10], whether or not the particle has mass. Ultimately this constant effect is the reason doing this is even impossible – because of this fact, the point time is oriented perpendicular to its rest state happens to be the exact spot where physics breaks. It is this fact that makes it possible for time to be locally stopped at all. Gravity's effect in the case of black holes is universal. There is not a reference frame that can see anything at the black hole except for all things falling past the event horizon; however the "personal" view

[9] Which, of course, is not possible

[10] If the conditions are the same and the two objects' centers of mass are at the same point, the acceleration will be the same.

of time is entirely separate: gravity proves there is no field that can fully vary the orientation of time – there will always be a velocity component.

Again, Einstein's genius shows itself: special relativity is the start because it is the core of how the universe works. It is based on a single simple fact because everything added to it will always, in some form, be irrelevant. Einstein started where all arguments effectively lead to. Therefore following his logic, all that needs to be focused upon is the particle itself and its frame. At this point however, a line must be drawn between thought and practice. Now that fields are excluded, simply imagining what a particle experiences will not be enough, which calls for the construction of a relativistic Lagrangian.

Lagrangian mechanics is an extension of classical mechanics that offers new paths to reach solutions of problems that would otherwise be fairly tedious. Its foundation stands in the action principle – one of the four principles that govern all physical laws[11] that states the path taken by a particle will be that of least action shown by an integral over time of incremental steps of the particle. The action principle, also called the stationary action principle or the minimum action principle, is the source from which all conservation laws are derived. Thus, keeping with the conservation of energy, the Lagrangian is a value equal to the potential energy subtracted from the kinetic energy, or $L = T - V$. Use cases for Lagrangian mechanics often include more than two dimensions, which can easily become confusing so notation is important. Again following in Einstein's path, the optimal coordinate system is a tensor-based coordinate system where the position of the particle in the reference frame is $X^{\mu} = (X^0, X^1, X^2, X^3)$. A standard is to use the first component to represent the temporal dimension. Because of this notation, there are three steps to making a relativistic lagrangian: the 4-vector, creating a scalar, and a personal

[11] Those being the action principle, locality, Lorentz invariance, and Gauge invariance

favorite (which will definitely be underutilized) – adding spice. By following this process, it will be easy to see how time interacts with particle motion similarly to its spatial counterparts.

In the frame of the particle, there are no fields. This means spacetime can be described by a scalar field ϕ and, as Einstein put it in *The Meaning of Relativity*, study spacetime "irrespective of the assumption as to the validity of the Euclidean geometry… [where] all configurations of Cartesian systems of co-ordinates are physically equivalent" (Einstein 24). To make a 4-vector to describe the particle's movement, all that is necessary is to add dimension to the scalar field using the particle's position[12]. $\frac{\partial\phi}{\partial X^{\mu}}$ is the four-vector which can also be written as

$\frac{\partial\phi}{\partial X^{0}}, \frac{\partial\phi}{\partial X^{1}}, \frac{\partial\phi}{\partial X^{2}}, \frac{\partial\phi}{\partial X^{3}}$ using the aforementioned notation. Looking at other usable scalars in relativity, formatting the new scalar is already somewhat layed out. Time will later be the focus of this Lagrangian so the scalar to follow is proper time, $\tau = (X^{0})^{2} - \sum_{n=1}^{3}(X^{n})^{2}$ which already shows the relationship between time and space. As all spatial dimensions transform in one "direction" the temporal transforms by the same factor but to the opposite "direction. To follow proper time, the terms with their corresponding dimensions will take the place of those terms – $(\frac{\partial\phi}{\partial X^{0}})^{2} - \sum_{n=1}^{3}(\frac{\partial\phi}{\partial X^{n}})^{2}$ or otherwise written as $(\frac{\partial\phi}{\partial X^{0}})^{2} - (\frac{\partial\phi}{\partial X^{1}})^{2} - (\frac{\partial\phi}{\partial X^{2}})^{2} - (\frac{\partial\phi}{\partial X^{3}})^{2}$. Finally, the spice: numerical constants, unit changes, and scalar functions. The only useful one of these would be a scalar function to represent potential energy. Which finalizes the lagrangian to be $L = \frac{1}{2}\left[(\frac{\partial\phi}{\partial X^{0}})^{2} - (\frac{\partial\phi}{\partial X^{1}})^{2} - (\frac{\partial\phi}{\partial X^{2}})^{2} - (\frac{\partial\phi}{\partial X^{3}})^{2}\right] - \frac{\mu^{2}}{2}\phi^{2}$. The field phi is a scalar which means the function at the end must also be a scalar function.

[12] These are needed to describe both the particle and the field it moves through.

As much as has already been found through the creation of the Lagrangian, no journey through Lagrangian mechanics is complete without the Euler-Lagrange equation:

$\sum_{\mu} \frac{\partial}{\partial X^{\mu}} \frac{\partial L}{\partial(\frac{\partial \phi}{\partial X^{\mu}})} - \frac{\partial L}{\partial \phi} = 0$. Technically, this cycles through each term of X^{μ} but the only term that matters is the 0^{th} term – time. The purpose of the Euler-Lagrange equation is to find the trajectory of least action between two fixed end points though, this result does not pertain to the topic at hand. Therefore the final three indices of mu can be ignored. Interpreting time like other dimensions shows that the trajectory of least action through time is the time it takes to complete a trajectory of least action. A common tactic is to take the terms separately, starting at the partial of the Lagrangian[13], which becomes $\frac{\partial L}{\partial(\frac{\partial \phi}{\partial X^{0}})}$. It is situations like these that truly show the beauty of partial derivatives. This complex-seeming term simply becomes $\frac{\partial \phi}{\partial X^{0}}$. Deriving with respect to time brings $\frac{\partial^2 \phi}{\partial t^2}$ (quick notation change for ease of reading) and unfortunately, the implications of this term means the spatial dimensions must be reintroduced. The full term becomes $\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 \phi}{\partial z^2} = \frac{\partial L}{\partial \phi}$ where $\frac{\partial L}{\partial \phi} =- gP(x)$, a function of potential energy. In a plane of spacetime lacking fields, this essentially is nullified. Which means there is no effect on time.

Remember, time is unaffected in the particle's own frame of reference. For this Lagrangian to be effective, the particle's frame must be the primed frame. If, from what is now the reference frame, the particle is moving in the positive direction on the x-axis – the relativistic Lagrangian becomes $\frac{\partial^2 \phi}{\partial t'^2} - \frac{\partial^2 \phi}{\partial x'^2} - \frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 \phi}{\partial z} =- \mu^2 \phi$. Here, mu is just a constant to make an

[13] The term differentiating with respect to time speaks for itself

arbitrary function of phi (one thing of note is that the source term prevents the scenario where $\phi = 0$). Lorentz transformations are the final step in finding the result. Lorentz transformations bend the axes around the line $c = 1$ but for an object moving at the speed of light, the axis of time bends to be equal to that line. Because the time and spatial axes are bent into an equal line, in the direction of movement time and space become the same thing. All time there is to play out becomes shown in an instant as the particle moves.

The concept that time seems instantaneous is basically synonymous with saying nothing can go faster than the speed of light. Just as passing the event horizon of a black hole breaks physics, moving beyond this cosmic speed limit breaks the current physical understanding of the universe. The very foundation of relativity is based upon the fact that time can be instantaneous. Not only is it possible that light can see time instantaneously but it is imperative that it does for the rules of relativity to stand.

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