



Fig. 10. The maximum value of $|\overrightarrow{M_0P_i}|$ for a given incident angle and the ratio curve of $G_3(\theta_{M0})/G_3(\theta_{Pi})$. (a) The maximum value of $|\overrightarrow{M_0P_i}|$ for a given incident angle θ_{M0} . (b) The ratio $G_3(\theta_{M0})/G_3(\theta_{Pi})$ for projection model r_3 .

Conclusion: Without loss of generality, suppose $0 \leq \theta_{M0} \leq \theta_{Pi} < 90^\circ$. For the fish-eye cameras, if $|\overrightarrow{M_0M_i}| \leq 0.1Z_0$, $G_j(\theta_{M0})$ and $G_j(\theta_{Pi})$ can be regarded as the same i.e. $G_j(\theta_{M0}) \approx G_j(\theta_{Pi})$.

Illustration: We explain how the condition $|\overrightarrow{M_0M_i}| \leq 0.1Z_0$ is determined in the following. Suppose that if $G_j(\theta_{Pi}) \geq 0.95G_j(\theta_{M0})$ then $G_j(\theta_{Pi}) \approx G_j(\theta_{M0})$. According to Fig. 1(a), it can be seen that $\theta_{M0} = \angle HOM_0$ and $\theta_{Pi} = \angle HOP_i$, and let $\theta_{Pi} = \theta_{M0} + \Delta\theta$.

For the pinhole camera, $G(\theta) = 1$ according to the ratio of r_1/r_1 in Table I, and therefore, $G(\theta_{Pi}) \equiv G(\theta_{M0})$.

For the fish-eye cameras, $G(\theta)$ is a monotonically decreasing function. Therefore, according to Fig. 1(a), for a given $\theta_{M0} \in [0, 90^\circ)$, $\Delta\theta$ reaches the maximum value when $G(\theta_{Pi}) = 0.95G(\theta_{M0})$ and points H , M_0 and P_i are collinear, where the corresponding value of $|\overrightarrow{M_0P_i}|$ can be calculated by

$$|\overrightarrow{M_0P_i}| = Z_0 \tan(\theta_{M0} + \Delta\theta) - Z_0 \tan \theta_{M0} \quad (29)$$

Based on the above analysis, the values of $|\overrightarrow{M_0P_i}|$ are shown in Fig. 10(a) except for the camera expressed by r_3 . From Fig. 10(a), $|\overrightarrow{M_0P_i}| = 0.1Z_0$ is the maximum value that can make $G(\theta_{Pi}) = 0.95G(\theta_{M0})$ for these three fish-eye cameras at every point on the interval $[0, 90^\circ)$. According to (29) and $|\overrightarrow{M_0P_i}| = 0.1Z_0$, we can get the corresponding value of $\Delta\theta$ for a given θ_{M0} . Therefore, the ratio $G_j(\theta_{Pi})/G_j(\theta_{M0})$ of the camera expressed by r_3 can be obtained and is shown in Fig. 10(b). From Fig. 10(b), the minimum value is 0.965, therefore, $G_j(\theta_{Pi}) \geq 0.95G_j(\theta_{M0})$ is also satisfied for the camera expressed by r_3 when $|\overrightarrow{M_0P_i}| = 0.1Z_0$.

From the above analysis, we can conclude that if $|\overrightarrow{M_0P_i}| \leq 0.1Z_0$ then $G_j(\theta_{M0}) \approx G_j(\theta_{Pi})$. Since $\overrightarrow{P_iM_i}$ is perpendicular to the plane K , therefore, $|\overrightarrow{M_0P_i}| \leq |\overrightarrow{M_0M_i}|$, and if $|\overrightarrow{M_0M_i}| \leq 0.1Z_0$, we can conclude $G_j(\theta_{M0}) \approx G_j(\theta_{Pi})$. ■