

Home Assignment #1.

3.1. Big O notation

1. $\frac{n^2}{3} - 3n = O(n^2)$

$$\frac{n^2}{3} - 3n \leq C \cdot n^2, \text{ for some } n \geq n_0$$

$$\frac{1}{3} - \frac{3}{n} \leq C$$

For $n_0 = 1$ and $C = \frac{1}{3} \rightarrow \frac{1}{3} - \frac{3}{n} \leq C$

2. $k_1 n^2 + k_2 n + k_3 = O(n^2)$

$$k_1 n^2 + k_2 n + k_3 \leq C n^2, \text{ for some } n \geq n_0$$

$$k_1 + \frac{k_2}{n} + \frac{k_3}{n^2} \leq C$$

For $n_0 = 1$ and $C = k_1 + k_2 + k_3 \rightarrow k_1 + \frac{k_2}{n} + \frac{k_3}{n^2} \leq C$

3. $3^n = O(2^n)$

$$3^n \leq C \cdot 2^n, \text{ for}$$

$$\left(\frac{3}{2}\right)^n \leq C$$

Therefore, the Big-Oh condition cannot hold (the left side of the latter inequality is growing infinitely, so that there is no such constant factor C).

4. $0.001n \cdot \log n - 2000n + 6 = O(n \log n)$

$0.001n \cdot \log n - 2000n + 6 \leq c \cdot n \cdot \log n$, for some $n \geq n_0$

$0.001 - \frac{2000}{\log n} + \frac{6}{n \log n} \leq c$

For $n_0 = 2$ and $c = 7 \rightarrow 0.001 - \frac{2000}{\log n} + \frac{6}{n \log n} \leq c$

3.2 Hashing

