RIBBONS [SELECTED PROOFS]

Proposition: $\neg[f < g] \wedge \neg[g < f] \implies f \approx g$.

Let $\lceil a, b \rceil$ be the maximum and $\lfloor a, b \rfloor$ be the minimum of a and b. Then $z > 0 \implies \lceil f(z), g(z) \rceil < \lfloor f(-z), g(-z) \rfloor$.

Define $h(z) = \lfloor f(z), g(z) \rfloor$ for $z < 0$, $h(z) = \lceil f(z), g(z) \rceil$ for $z > 0$. Then $h \sqsubset f$ and $h \sqsubset g$, so $f \sim g$.

Proposition: $f \in \mathbb{I} \implies f^* \in \mathbb{I}$.

Note that $0 < f(1) \leq f(n) < f(-n)$, so that $[f(-n)]^{-1} < [f(n)]^{-1} \leq [f(1)]^{-1}$, and $f^*(-n) - f^*(n) = f(n)^{-1} - f(-n)^{-1} = [f(-n) - f(n)][f(-n)^{-1}f(n)^{-1}] < [f(-n) - f(n)][f(1)]^{-2} \to 0$. So $f^* \in I$.

1 A limit

Let $f_{n+1} \sqsubset f_n$ for all $n \in \mathbb{N}$. Then this sequence has a **center**, which is something like a limit in our realm without a distance function (because we don't have subtraction.)

Define $\mathring{f}(z) = f_{|z|}(z)$. Then $\forall n\ \mathring{f} \sqsubset f_n$, and any other ribbon that manages this is equivalent to $\mathring{f}$.

2 Identity

We check that $ff^* \in \mathbb{I}$. Let $f \in \mathbb{I}$. Then $n > 0 \implies (ff^*)(n) = \frac{f(n)}{f(-n)}$.

Also $f(n) < f(n+1)$ and $f(-n-1) < f(-n) \implies f(n)f(-n-1) < f(n+1)f(-n) \implies \frac{f(n)}{f(-n)} < \frac{f(n+1)}{f(-(n+1))}$. So ff^* is increasing on $\mathbb{N}$.

Note that $z < 0 \implies z = -n \implies (ff^*)(-n) = \frac{f(-n)}{f(n)}$.

Then $f(-n) > f(-(n+1))$ and $f(n+1) > f(n)$ give $f(-n)f(n+1) >$

$f(-(n+1))f(n)$. So $\frac{f(-n)}{f(n)} > \frac{f(-(n+1)}{f(n+1)}$, and ff^* is decreasing on $-\mathbb{N}$.

Also $\frac{f(-n)}{f(n)} - \frac{f(n)}{f(-n)} = \frac{(f(-n)-f(n))(f(-n)+f(n))}{f(n)f(-n)} \to 0$, since f is bounded and $f \in \mathbb{I}$.