RIBBONS

1 Definition

Let $\mathbb{M}$ (for mirror) be the set of all nonzero integers.

A function $f : \mathbb{M} \to \mathbb{Q}$ is a **ribbon** when

$0 < f(1) < f(2) < f(3) < ... < f(-3) < f(-2) < f(-1)$

and

$f(-n) - f(n) \longrightarrow 0$ as $n \longrightarrow \infty$.

Denote the set of all such ribbons by $\mathbb{I}$.

2 An Example

For $n \in \mathbb{N}$, define $I(n) = \frac{n}{n+1}$ and $I(-n) = [I(n)]^{-1}$.

Then $I = ...\frac{6}{5}, \frac{5}{4}, \frac{4}{3}, \frac{3}{2}, \frac{2}{1}, , \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, ...$ is a ribbon.

Note the repeated commas that mark the absent center, since I is not defined at zero.

Note also that I is (will turn out to be) a multiplicative identity.

3 Order

Define $f \sqsubset g$ if $n > 0 \implies g(n) \leq f(n) < f(-n) \leq g(-n)$.

Then define $f \sim g$ if $\exists h \in \mathbb{I}$ such that $h \sqsubset f$ and $h \sqsubset g$.

In other words, f and g are equivalent if there is some h that "forks" or "pierces" them both.

Also define $f < g$ if there $\exists n > 0$ such that $f(-n) < g(n)$.

This gives us $f < g, f > g$, or $f \sim g$ for any $f, g \in \mathbb{I}$

4 Addition and Multiplication

Define $f+g$ by $(f+g)(z)=f(z)+g(z)$.
Define fg by $(fg)(z)=f(z)g(z)$.
Then $f\sim f', g\sim g' \implies f+g\sim f'+g'$.
Also $f\sim f', g\sim g' \implies fg\sim f'g'$.

5 A Multiplicative Inverse

For $f\in\mathbb{I}$, define f^* by $f^*(z)=[f(-z)]^{-1}$. Then $f^*\in\mathbb{I}$ and $ff^*\sim I$.

6 Injecting Positive Rationals

We can inject any positive $q\in\mathbb{Q}$ into the ribbons by scaling I.
Define $[q]\in\mathbb{I}$ by $[q](n)=\frac{n}{n+1}q$ and $[q](-n)=[q(n)]^{-1}$.