PERMUTATION AND COMBINATION

Permutation and Combination

Permutation

If we are given a number of objects, we may arrange them in different ways. How many different orders can the objects be placed ?

Example

There are 5 routes for going from A to B, 3 routes for going from B to C and 7 routes for going from C to D. Find in how many different ways can a person go from A to D .

The number of different ways for going from A to D is

$$5\times 3\times 7 = 105 \quad ways$$

$$First \ ways \times Second \ ways \times Third \ ways$$

Three digits number formed from 1 to 9 digits

May be repeated

$$9 \times 9 \times 9 = 9^3$$

May not be repeated

$$9 \times 8 \times 7 = {}^{9}P_{3}$$

The symbol ${}^{n}P_{r}$ usually denotes the numbers of permutations of n things taken r at a time.

Fundamental Principle

If one operation can be performed in m ways, and then second can be performed in n ways, and a third in p ways, and so on

The number of ways of performing all operations in succession will be

$$m \times n \times p \times \ldots\ldots$$

Permutation in which the Quantities may be Repeated

The number of permutation of n things taken r at a time when each thing may occur any number of times is n^r

$$n \times n \times \ldots\ldots \times n \,(r \text{ times}) = n^r$$

Pg.

How many numbers between 3000 and 7000 can be formed by using the digits 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 each not more than once in each number ?

The named of digits are 1, 2 , 3 , 4 , 5 , 6 , 7 and 8.

The number of arrangement ways so that between 3000 and 7000 = $^4P_1 \times {}^7P_3$

(Not repeated) $\underline{\;^4P_1\;}$ / $\underline{\quad}$ $^7P_3$ $\underline{\quad}$ $\underline{\quad}$

(may be repeated) $\underline{\;^4P_1\;}$ / $\underline{\quad}$ 8^3 $\underline{\quad}$ $\underline{\quad}$

The number of arrangement ways so that between 300 and 60000 each digits not more than once =

$$\underline{\ ^6P_1\ }\ \underbrace{\_\ \_}_{}{}^{\ ^7P_2} + \underline{\ ^8P_1\ }\ \_\ \_\ \_\ {}^{7}P_3 + \underline{\ ^5P_1\ }\ \_\ \_\ \_\ \_\ {}^{7}P_4$$

$$(or)\ {}^{8}P_4$$

The number of arrangement ways so that between 300 and 60000 each digits may be repeated =

$$\underline{\ ^6P_1\ }\ \_\ \_\ 8^2 + \_\ \_\ \_\ \_\ 8^4 + \underline{\ ^5P_1\ }\ \_\ \_\ \_\ \_\ 8^4$$

The no. of even numbers so that between 100 and 100000 each digits not more than once =

The no. of odd numbers so that between 100 and 100000 each digits may be repeated =

How many numbers between 300 and 70000 can be formed by using the digits 0 , 2 , 3 , 4 , 5 , 6 , 7 , 8 each not more than once in each number ?

The name of digits are 0 , 2 , 3 , 4 , 5 , 6 , 7 and 8

The number of arrangement ways that between 300 and 70000 each digits not more than once =

_ _ _ + _ _ _ _ + _ _ _ _ _ $\left({}^6P_1 \times {}^7P_2 + {}^7P_1 \times {}^7P_3 + {}^5P_1 \times {}^7P_4 \right)$

The number of arrangement ways that between 300 and 70000 each digits may be repeated =

$$\left({}^6P_1 \times 8^2 + {}^7P_1 \times 8^3 + {}^5P_1 \times 8^4 \right) - 1$$

Pg.18 , No.6.

In how many ways can 3 different copper coins and 3 different silver coins be arranged in a line so that the silver coins may be in the odd place

S C S C S C $\quad {}^3P_3 \times {}^3P_3$

The no. of ways that silver coins are odd place

$$= {}^3p_3 \times {}^3p_3$$

The no. of ways that alternately $\quad = {}^3p_3 \times {}^3p_3 + {}^3p_3 \times {}^3p_3$

S C S C S C + C S C S C S

The no. of ways that silver coins are not adjacent = ? $\quad {}^3p_3 \times {}^4p_3$

Pg.18 , No.5.

How many new words can be formed by using the letters of the word – UNIVERSAL so that the central letter may be the vowel.

In the word UNIVERSAL ,there are 9 letters

The number of vowels = 4

The number of words can be formed such that central as a vowel = $^4P_1 \times {}^8P_8$

$$\_\_\ \_\_\ \_\_\ \_\_\ \underset{^4P_1}{\overset{V}{\_\_}}\ \_\_\ \_\_\ \_\_\ \_\_$$

The number of new words can be formed such that central as a vowel = $^4P_1 \times {}^8P_8 - 1$

The number of words can be formed such that begin and end with consonants =

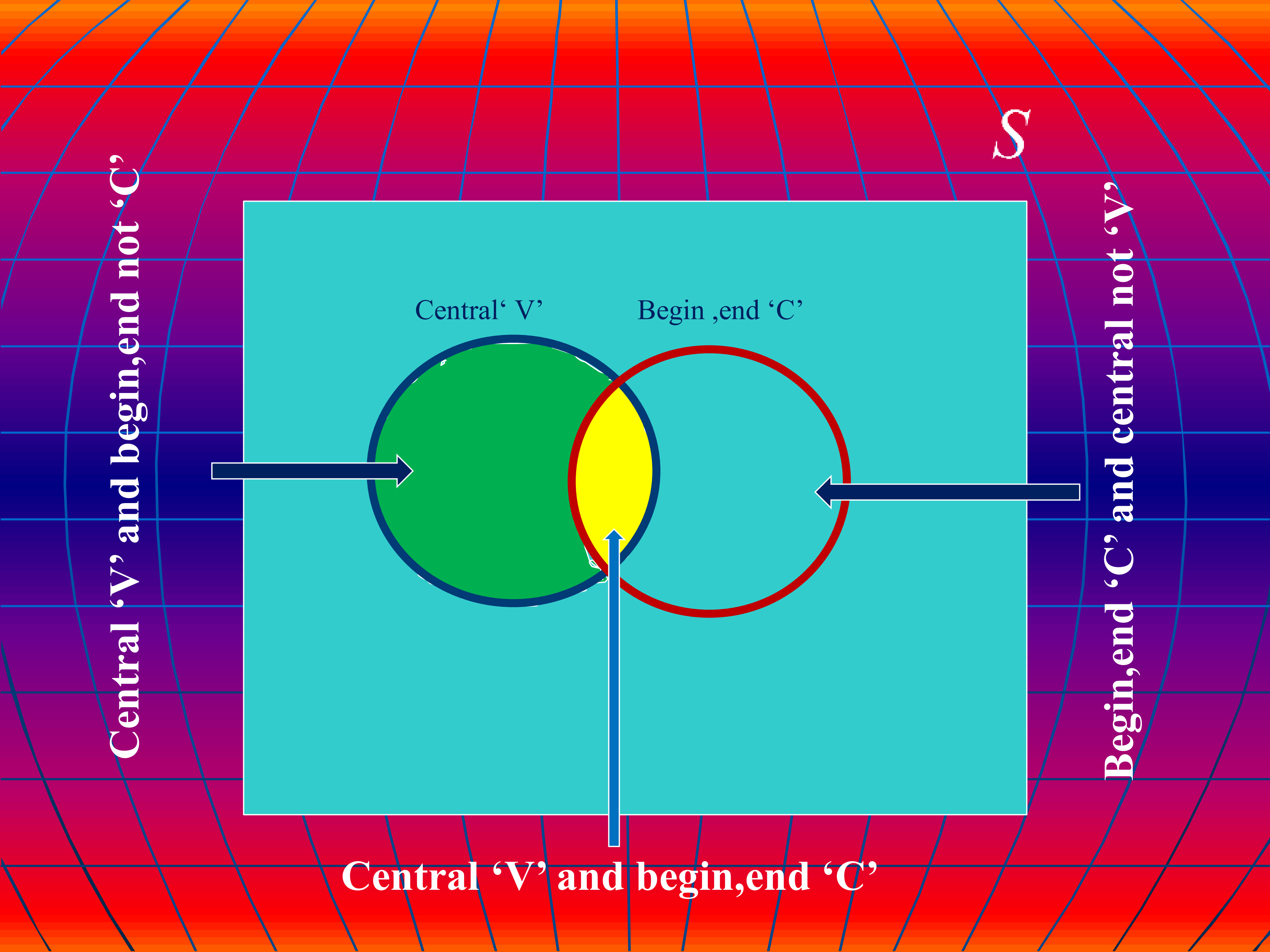
$${}^5P_2 \times {}^7P_7$$

$$\underline{C}\ \_\_\ \_\_\ \_\_\ \_\_\ \_\_\ \_\_\ \_\_\ \underline{C}$$

The number of words can be formed such that central letter may be vowel and begin and end with consonants =

$${}^4P_1 \times {}^5P_2 \times {}^6P_6$$

$$\underline{C}\ \_\_\ \_\_\ \_\_\ \underline{V}\ \_\_\ \_\_\ \_\_\ \underline{C}$$


S
Central‘ V’
Begin ,end ‘C’
Central ‘V’ and begin,end not ‘C’
Begin,end ‘C’ and central not ‘V’
Central ‘V’ and begin,end ‘C’

The number of words can be formed such that all vowels are together = $^6P_6 \times {}^4P_4$

V V V V   C C C C C

The number of words can be formed such that all vowels are separated = $^5P_5 \times {}^6P_4$

C C C C C

Pg26, No.17.

How many number of 5 digits can be formed with 0 , 1 , 2 , 3 , 4 , 5 and 6 , if each of these digits may be repeated? Of these how many are even and how many are divisible by 5?

The digits are 0 , 1 , 2 , 3 , 4 , 5 , 6

$^6P_1$ ___ ___ 7^4 ___ ___ ___

(i)The numbers of 5 digits can be formed $= {}^6P_1 \times 7^4$

(ii)The no. of 5 digits even number = ${}^6P_1 \times {}^4p_1 \times 7^3$

$^6P_1$ ___ ___ 7^3 ___ ___ $^4P_1$ ___

$^{6}P_{1}$ ___ 7^{3} ___ ___ $^{2}P_{1}$

(iii)The no. of 5 digits number which is divisible by 5 =

(iv)The no. of 5 digits number which is divisible by 25 =

$\_\ \_\ \_\ 0\ 0 + \_\ \_\ \_\ 2\ 5 + \_\ \_\ \_\ 5\ 0$

$^{6}P_{1}\times 7^{2}$ $\quad$ $^{6}P_{1}\times 7^{2}$ $\quad$ $^{6}P_{1}\times 7^{2}$

Consider, the no. of 5 digits number each digits not more than once.

The digits are 0 , 1 , 2 , 3 , 4 , 5 , 6(not repeated)

(i)The numbers of 5 digits can be formed $= {}^{6}P_{1} \times {}^{6}P_{4}$

${}^{6}P_{1}$ ___ ${}^{6}P_{4}$ ___ ___

(ii)The no. of 5 digits even number = ?

$${}^{6}P_{4} + {}^{3}P_{1} \times {}^{5}P_{1} \times {}^{5}P_{3}$$

_ _ _ _ 0 + _ _ _ _ not zero

(iii)The no. of 5 digits number which is divisible by 5 = ?

$${}^{6}P_{4} + {}^{5}P_{1} \times {}^{5}P_{3}$$

_ _ _ _ 0 + _ _ _ _ 5

(iv)The no. of 5 digits number which is divisible by 25 = ?

Permutation of n things not all different

Let n things be represented by letters, and p of them are alike, q of them are alike, of them are alike; and so on.

The required number of permutations is

$$\frac{n!}{p!\times q!\times r!\times \ldots\ldots\ldots}$$

A A A B B C C C C

A A A B B C C C C

A A A B B C C C C and so on

$$\frac{9!}{3!\times 2!\times 4!}$$

Pg.26,No.18.

In the word COMMITTEE, there are 9 letters, 1 C's , 1 O's , 2 M's , 1 I's , 2 T's and 2 E's .

The no. of total arrangement ways $= \dfrac{9!}{2!\times 2!\times 2!}$

The no. of ways that begin and end with consonants

$\underline{C}$ __ __ __ __ __ __ __ $\underline{C}$ $= \dfrac{{}^{5}P_{2} \times 7!}{2!\times 2!\times 2!}$

The no. of ways that central letters may be vowels and begin and end with consonants

$\underline{C}$ __ __ __ $\underline{V}$ __ __ __ $\underline{C}$ $= \dfrac{{}^{4}P_{1}\times {}^{5}P_{2} \times 6!}{2!\times 2!\times 2!}$

The no. of ways that all vowels are always come to appear together = ?

The no. of ways that all vowels are not come to appear together = ?

The no. of ways that all vowels are to be

separated = ?

In the word INFINTTESIMAL, there are 13 letters, 3 I's , 2 N's , 1 F's , 2 T's , 1 E's , 1 S's , 1 M's 1 A's and 1 L's .

The number of vowels = 5

The number of arrangement ways $= \dfrac{13!}{3!\times 2!\times 2!}$

The number of arrangement ways such that vowel as a central $= \dfrac{{}^{5}P_{1} \times 12!}{3! \times 2!\times 2!}$

The number of arrangement ways such that all vowel always come together

Pg.27, No.20.

In the word CONSONANTS, there are 10 letters, 1 C's , 2 O's , 3 N's , 2 S's , 1 A's and 1 T's .

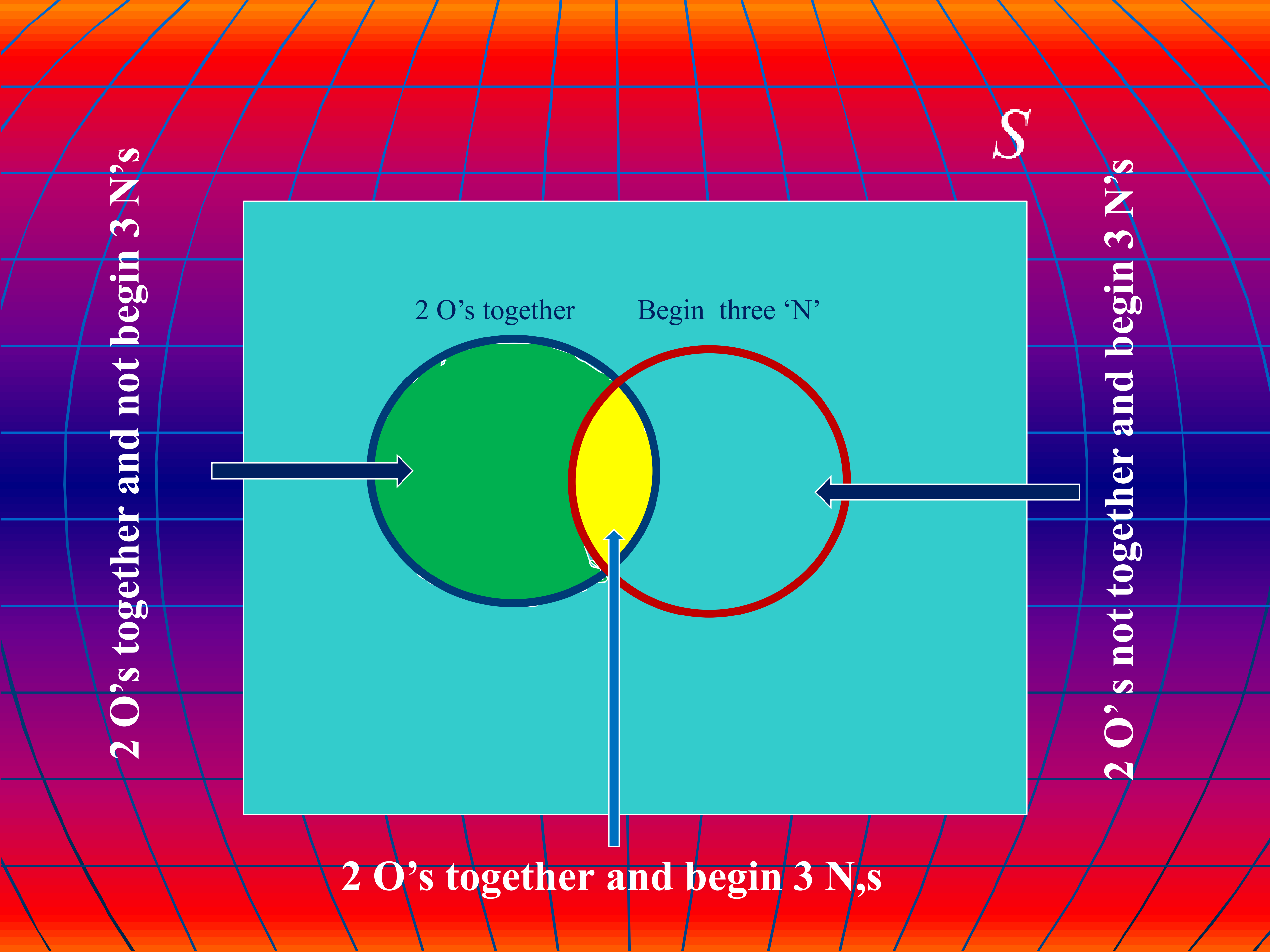
The number of arrangement ways $= \dfrac{10!}{2!\times 2!\times 3!}$

$(\underline{O}\ \underline{O})$ _ _ _ _ _ _ _ _

The number of arrangement ways such that the two O's always come together $= \dfrac{9!}{2!\times 3!}$

The number of arrangement ways such that begin with the three N's

$\underline{N}\ \underline{N}\ \underline{N}$ / _ _ _ _ _ _ _ $= \dfrac{7!}{2!\times 2!}$


S
2 O's together
Begin three 'N'
2 O's together and not begin 3 N's
2 O' s not together and begin 3 N's
2 O's together and begin 3 N,s

The number of arrangement ways such that the two O's always come together and begin with 3 N's $= \dfrac{6!}{2!}$

$\underline{N}\ \underline{N}\ \underline{N}\ /(\underline{O}\ \underline{O}\ )\ \_\_\ \_\_\ \_\_\ \_\_\ \_\_$

The number of arrangement ways such that the two O's always come together and do not begin with 3 N's

The number of arrangement ways such that the two O's never come together and begin with N's

Pg.27, No.21.

The maned of digits are 2 , 2 , 2 , 3 , 3 , 4 , 0 .

The no. of number over 2000000 $= \dfrac{{}^{6}P_{1} \times 6\,!}{3!\times 2!}$

$${}^{6}P_{1}\ \underline{\quad}\ \overset{6!}{\underline{\quad}}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}$$

The no. of even number over 2000000 numbers =

$$= \frac{6\,!}{3!\times 2!} + \frac{{}^{4}P_{1}\times {}^{5}P_{1}\times 5!}{3!\times 2!}$$

The no. of odd number over 2000000 numbers =

$$= \frac{{}^{5}P_{1}\times 5\,!}{3!\times 2!}$$

Pg.27, No.22.

In the word ENGINEERING, there are 11 letters, 3 E's , 3 N's , 2 G's , 2 I's and 1 R's .

The number of arrangement ways $= \dfrac{11!}{3!\times 3!\times 2!\times 2!}$

The number of arrangement ways such that the three E's always come together $= \dfrac{9!}{3!\times 2!\times 2!}$

The number of arrangement ways such that begin with the three E's and end N $= \dfrac{7!}{2!\times 2!\times 2!}$

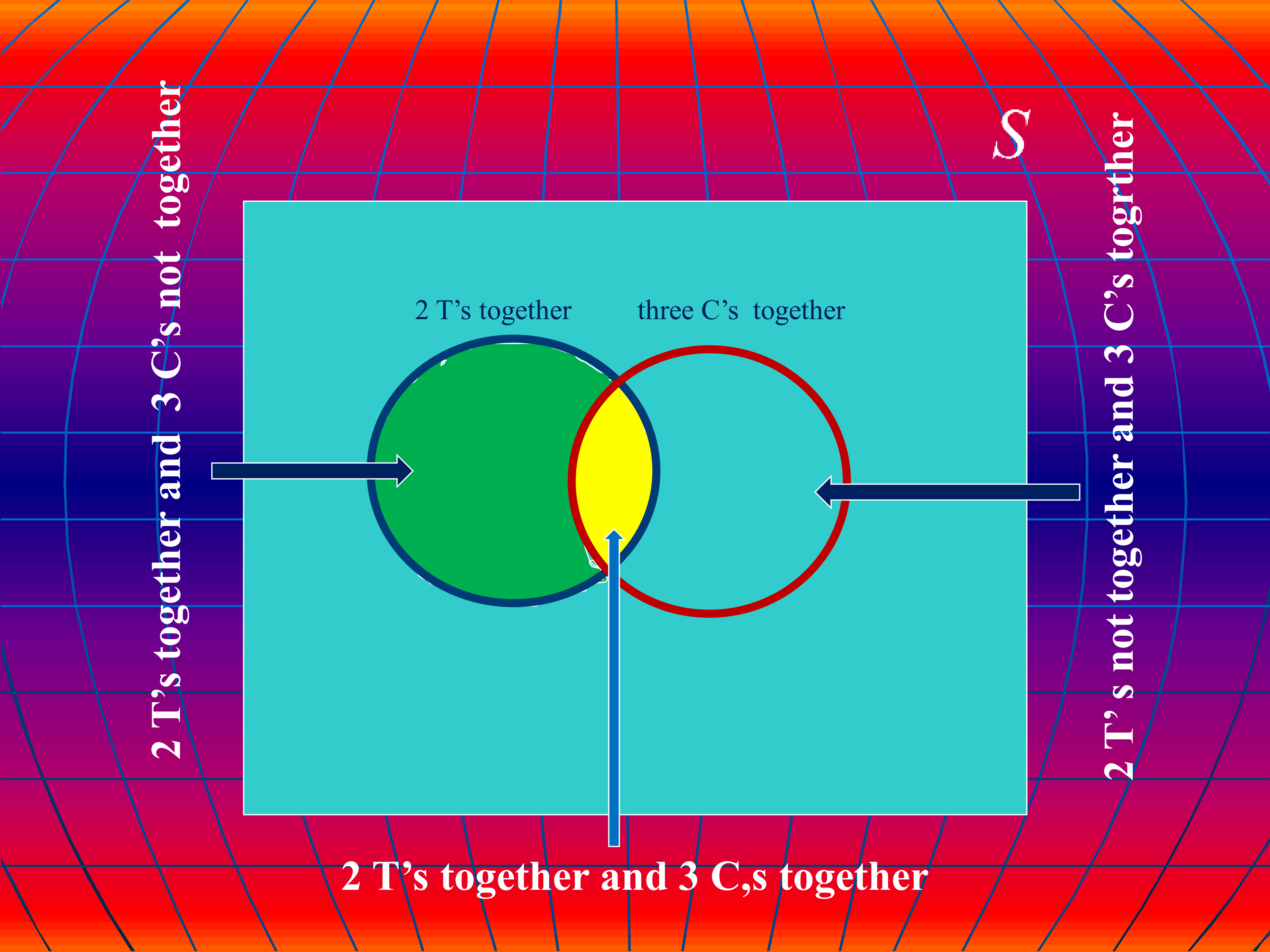
Pg.27, No.23

In the word CHARACTERISTICS, there are 15 letters, 3 C's , 1 H's , 2 A's , 2 R's , 2 T's , 1 E's ,2 I's and 2 S's .

The number of arrangement ways $= \dfrac{15!}{3!\times(2!)^5}$

The number of arrangement ways such that the two R's always come together $= \dfrac{14!}{3!\times(2!)^4}$

The number of arrangement ways such that two R's do not come together $= \dfrac{15!}{3!(2!)^5} - \dfrac{14!}{3!\times(2!)^4}$



S
2 T’s together
three C’s together
2 T’s together and 3 C’s not together
2 T’ s not together and 3 C’s togrther
2 T’s together and 3 C,s together

The number of arrangement ways such that the two T's always come together $= \dfrac{14!}{3!(2!)^4}$

The number of arrangement ways such that two T's come together and 3 C's come together $= \dfrac{12!}{(2!)^4}$

The number of arrangement ways such that two T's come together and 3 C's do not come together

$$= \frac{14!}{3!(2!)^4} - \frac{12!}{(2!)^4}$$

The number of ways that the two T's and 2 C's are always come together =

The number of ways that the two T's together and 2 C's together =

Combination

The number of combination of n different things taken r at time is

$$^{n}C_{r} = \frac{^{n}P_{r}}{r!} = \frac{n(n-1)......(n-r+1)}{r!}$$

$$= \frac{n!}{(n-r)!\,r!}$$

Pg.28 , No.31

In a decagon, there are 10 sides and 10 corners.

The number of triangles can be formed by joining the angular points $= {}^{10}C_3$

The number of straight line can be formed $= {}^{10}C_2$

The number of diagonals can be formed by joining the angular points $= {}^{10}C_2 - 10$

The name of diagonals can be formed by joining the angular points

$$= ({}^{10}C_2 - 10) \times 2! \quad or \quad {}^{10}P_2 - 20$$

Above problem consider in Polygon.

Combination Under Restrictions

The number of combination of n things taken r at time in which p particular things always occur is

$$^{n-p}C_{r-p}$$

The number of combination of n things taken r at time in which p particular things never occur is

$$^{n-p}C_{r}$$

Pg 29 , No.41 .

The number of boys = 25

The number of selected boys = 11

The number of ways that 6 of then being always exclude

$$= {}^{25-6}C_{11}$$

The number of ways that 5 of then being always include

$$= {}^{25-5}C_{11-5}$$

The number of ways that 6 of then being always exclude and 5 of then always include

$$= {}^{25-6-5}C_{11-5}$$

Pg.29 , No.36.

The number of persons in a committee = 5

The number of boys = 25

The number of girls = 10

25 boys	10 girls
5 4 3 2 1 0	0 1 2 3 4 5

$$^{35}C_5 = {}^{25}C_5 \times {}^{10}C_0 + {}^{25}C_4 \times {}^{10}C_1 + ------ + {}^{25}C_0 \times {}^{10}C_5$$

Pg 29 , No.36.

The number of persons in a committee = 5

The number of boys = 25

The number of girls = 10

The number of committees, so as to include at least one girls

$$= {}^{25}C_4 \times {}^{10}C_1 + {}^{25}C_3 \times {}^{10}C_2 + {}^{25}C_2 \times {}^{10}C_3 + {}^{25}C_1 \times {}^{10}C_4 + {}^{25}C_0 \times {}^{10}C_5$$

$$(\ or\ ) = {}^{35}C_5 - {}^{25}C_5 \times {}^{10}C_0$$

Pg.29 , No.42 .

The number of persons in a committee = 5

The number of boys = 6

The number of girls = 3

6 boys	3 girls
5 4 3 2	0 1 2 3

$$^{9}C_{5} = {}^{6}C_{5} \times {}^{3}C_{0} + {}^{6}C_{4} \times {}^{3}C_{1} + {}^{6}C_{3} \times {}^{3}C_{2} + {}^{6}C_{2} \times {}^{3}C_{3}$$

Pg.29 , No.42 .

The number of persons in a committee = 5

The number of boys = 6

The number of girls = 3

The number of committees, so as to include at least 3 boys $= {}^6C_3 \times {}^3C_2 + {}^6C_4 \times {}^3C_1 + {}^6C_5 \times {}^3C_0$

Pg.29 , No.43 .

The number of persons in a committee = 7

The number of boys = 10

The number of girls = 8

10 boys	8 girls
7	0
6	1
5	2
4	3
3	4
2	5
1	6
0	7

$$^{18}C_7 = {}^{10}C_7 \times {}^{8}C_0 + {}^{10}C_5 \times {}^{8}C_1 + - - - - - - + {}^{10}C_0 \times {}^{8}C_7$$

Pg.29 , No.37 .

The no. of engineers = 10

The no. of chemists = 5

The no. of mathematics = 7

The no. of committeess to contain 4 engineers ,2 chemists and 2 mathematics

$$= {}^{10}C_4 \times {}^{5}C_2 \times {}^{7}C_2$$

The no. of committeess to contain 4 engineers ,2 chemists and 2 mathematics such that particular 2 engineers, 1 chemists include and 1 mathematics exclude

$$= {}^{8}C_2 \times {}^{4}C_1 \times {}^{6}C_2$$

Permutation and Combination From two Sets

If m different things of one kind, and n different things of another kind are given the number of permutation which can be formed, containing r of the first and s of the second is

$$^{m}C_{r}\times {}^{n}C_{s}\times (r+s)!$$

The named of digits are 0 , 1 , 3 , 4 , 5 , 7 , 9.

The number of 6 digits number so as to contain the digit 0 ,4 , 5

$$= {}^{7-3}C_{6-3} \times {}^{5}P_{1} \times 5!$$

0,4,5, — , — , —

${}^{5}P_{1}$ $5!$

— — — — — —

The number between 400000 and 600000 digits number so as to contain the digit 0 , 4 , 5

$$= {}^{7-3}C_{6-3} \times {}^{2}P_{1} \times 5!$$

0,4,5, — , — , —

${}^{2}P_{1}$ $5!$

— — — — — —

The number of 6 digits number so as to contain the digit 0 ,4 , 5 which are divisible by 5

The number of 6 digits number so as to contain the digit 0 ,4 , 5 which are divisible by 5 = ?

$$= {}^{7-3}C_{6-3} \times \left(5! + {}^{4}P_{1} \times 4!\right)$$

0,4,5, — , — , —

$\_\ \_\ \_\ \_\ \_\ \mathbf{0} + \underline{\text{not zero}}\ \_\ \_\ \_\ \_\ \mathbf{5}$

The number of 6 digits even number so as to contain the digit 0 ,4 , 5 = ?

The number of 6 digits odd number so as to contain the digit 0 ,4 , 5 = ?

Pg .29 , N0.39.

There are 7 letters in the word FOMULA

The number of vowels = 3

The number of consonants = 4

4 consonants	3 vowels
3	0
2	1
1	2
0	3

$\times 3!$

$$^7P_3 = {}^7C_3 \times 3! = ({}^4C_3 \times {}^3C_0 + {}^4C_2 \times {}^3C_1 + {}^4C_1 \times {}^3C_2 + {}^4C_0 \times {}^3C_3) \times 3!$$

Pg 29 , No.39 .

In the word FORMULA, there are 7 letter

The number of consonants = 4

The number of vowels = 3

The number of 3 letter words, so as each words containing one vowel at least

$$= ({}^4C_2 \times {}^3C_1 + {}^4C_1 \times {}^3C_2 + {}^4C_0 \times {}^3C_3) \times 3!$$

The number of 3 letter words, so as each words containing at least one vowel and begin with vowel

$$= ({}^4C_2 \times {}^3C_1 \times {}^1P_1 \times 2! + {}^4C_1 \times {}^3C_2 \times {}^2P_1 \times 2! + {}^4C_0 \times {}^3C_3 \times {}^3P_1 \times 2!)$$

Pg 29 , No.44

The number of vowels = 5

The number of consonants = 15

The number of letters in a word =5

15 consonants	5 vowels
5	0
4	1
3	2
2	3
1	4
0	5

$\times 5!$

$$^{20}P_5 = {}^{20}C_5 \times 5! = ({}^{15}C_5 \times {}^5C_0 + {}^{15}C_4 \times {}^5C_1 + --- + {}^{15}C_0 \times {}^5C_5) \times 5!$$

pg.29 , No. 44

The number of vowels = 5

The number of consonants = 15

The number of 5 letters words so as to containing the 3 different vowels and 2 different consonants

$$= {}^{5}C_3 \times {}^{15}C_2 \times 5!$$

The number of 5 letters words so as to containing the at most 2 consonant

$$= ({}^{5}C_3 \times {}^{15}C_2 + {}^{5}C_4 \times {}^{15}C_1 + {}^{5}C_5 \times {}^{15}C_0) 5!$$

The number of 5 letters words so as to containing the 3 different vowels and 2 different consonants and centlal may be vowel

$$= {}^{5}C_3 \times {}^{15}C_2 \times {}^{3}P_1 \times 4!$$

$\times 4!$

(3 V , 2 C) $\_\_ \quad \_\_ \quad \underline{{}^{3}P_1} \quad \_\_ \quad \_\_$

The number of 5 letters words so as to containing the at most 2 consonant and begin with vowel

$$= {}^{15}C_2 \times {}^{5}C_3 \times {}^{3}P_1 \times 4! + {}^{15}C_1 \times {}^{5}C_4 \times {}^{4}P_1 \times 4! + {}^{15}C_0 \times {}^{5}C_5 \times {}^{5}P_1 \times 4!$$

The number of 5 letters words so as to containing the at most 2 consonant and begin with vowel

$$= {}^{15}C_2 \times {}^{5}C_3 \times {}^{3}P_1 \times 4! + {}^{15}C_1 \times {}^{5}C_3 \times {}^{4}P_1 \times 4! + {}^{15}C_5 \times {}^{5}C_0 \times {}^{5}P_1 \times 4!$$

$(2\,C, 3V)$ $\quad$ $(1\,C, 4V)$ $\quad$ $(,5V)$

$\underline{V}$ _ _ _ _ $\quad$ $\underline{V}$ _ _ _ _ $\quad$ $\underline{V}$ _ _ _ _

pg 30 , No 45 .

The number of vowels = 5

The number of consonants = 10

First, we take the the vowel " a "

10 consonants	4 Vowels + a
4 3 2 1 0	0 1 2 3 4

$\times 5!$

$$= ({}^{10}C_4 \times {}^4C_0 + {}^{10}C_3 \times {}^4C_1 + - - - - + {}^{15}C_0 \times {}^4C_4) \times 5!$$

Pg30 , No.45.

The number of consonants = 10

The number of vowels = 5

The number of 5 letters words so as ' a ' is always include and the words is to contain at least 2 consonants

$$= ({}^{10}C_2 \times {}^4C_2 + {}^{10}C_3 \times {}^4C_1 + {}^{10}C_4 \times {}^4C_0) \times 5!$$

The number of 5 letters words so as ' a ' is always include and the words is to contain at least 2 consonants and begin with “a”

$$= ({}^{10}C_2 \times {}^4C_2 + {}^{10}C_3 \times {}^4C_1 + {}^{10}C_4 \times {}^4C_0) \times 4!$$

The number of 5 letters words so as ' a ' is always include and the words is to contain at least 2 consonants and begin with vowel

$$= ({}^{10}C_2 \times {}^{4}C_2 \times {}^{3}P_1 \times 4! + {}^{10}C_3 \times {}^{4}C_1 \times {}^{2}P_1 \times 4!$$

$$+ {}^{10}C_4 \times {}^{4}C_0 \times {}^{1}P_1 \times 4!)$$

The number of 5 letters words so as ' a ' is always include and the words is to contain at least 3 consonants and vowels are separated

$$= {}^{10}C_3 \times {}^{4}C_1 \times 3! \times {}^{4}P_2 + {}^{10}C_4 \times {}^{4}C_0 \times 4! \times {}^{5}P_1$$

$(3\,C, 2V)$ $(4\,C, 1V)$

$\_\ \underline{C}\ \_\ \underline{C}\ \_\ \underline{C}\ \_$ $\_\ \underline{C}\ \_\ \underline{C}\ \_\ \underline{C}\ \_\ \underline{C}\ \_$