Continuous Probability Distribution

If X is a continuous random variable with probability density function f (x) valid over the range

$$a \le x \le b \ \ then$$

$$P(a \le x \le b) = \int_a^b f(x)\,dx = 1$$

A continuous random variable X has probability density function f (x) where f (x) = k x , $0 \le x \le 4$ *then*

(a)find the value of the constant k

(b)sketch y = f (x)

(c)find $P(a \le x \le 2\frac{1}{2})$

$$P(0 \le x \le 4) = \int_0^4 f(x)\,dx = 1$$

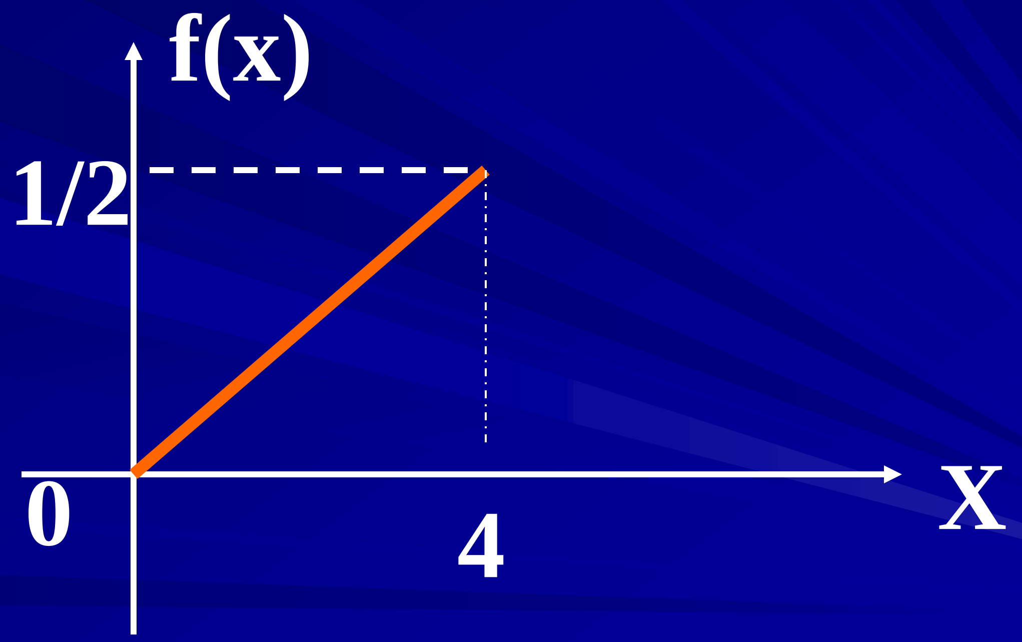
$$\int_0^4 kx\,dx = 1$$

$$\int_0^4 kx\,dx = 1$$

$$f(x) = \frac{1}{8}x \quad , 0 \le x \le 4$$

$$\left[k\frac{x^2}{2}\right]_0^4 = 1$$

$$f(0) = 0 \qquad f(4) = \frac{1}{2}$$



$$8k = 1$$

$$k = \frac{1}{8}$$

$$P(0 \leq x \leq 2\tfrac{1}{2}) = \int_{0}^{\frac{5}{2}} \frac{1}{8}\, x \, dx$$

$$= \frac{1}{16}\left[\, x^{2} \,\right]_{0}^{\frac{5}{2}}$$

$$= \frac{1}{16}\left[\, \frac{25}{4} - 0 \,\right]$$

$$= \frac{25}{64}$$

Mathematical Expectation

$$\mu = E(X) = \int_a^b f(x).\, x \; dx$$

$$E(X^2) = \int_a^b f(x).\, x^2 \; dx$$

Some Results 0f Expectation

$$a \text{ and } b \text{ are constant}$$

$$\text{(i)} \quad E(a) = a$$

$$\text{(ii)} \quad E[ax] = aE[x]$$

$$\text{(iii)} \quad E[ax+b] = aE[x]+b$$

$$\text{(iv)} \quad E[f(x)+g(x)] = E[f(x)]+E[g(x)]$$

$$E(X)=\int_0^4 \frac{1}{8}x.x\,dx$$

$$E(X)=\frac{1}{8}\int_0^4 x^2\,dx$$

$$E(X)=\frac{1}{24}\left[x^3\right]_0^4$$

$$E(X)=\frac{1}{24}[4^3-0]$$

$$E(X)=\frac{8}{3}$$

Variance

$$Var(X) = E(x^2) - \{E(x)\}^2$$

Standard deviation

$$\sigma = \sqrt{Var(x)}$$

$$\sigma = \sqrt{E(x^2) - [E(x)]^2}$$

Some Results 0f Variance

$$a \text{ and } b \text{ are constant}$$

$$\text{(i)} \quad Var(a) = 0$$

$$\text{(ii)} \quad Var[ax] = a^2 \, Var[x]$$

$$\text{(iii)} \quad Var[ax+b] = a^2 \, Var[x]$$

$$\text{(iv)} \quad Var[f(x)+g(x)] = Var[f(x)] + Var[g(x)]$$

$$E(X^2) = \int_0^4 \frac{1}{8} x . x^2 \quad dx$$

$$E(X^2) = \frac{1}{8} \int_0^4 x^3 \quad dx$$

$$E(X^2) = \frac{1}{32} . \left[x^4 \right]_0^4 = \frac{1}{32} \left[4^4 - 0 \right]$$

$$E(X^2) = 8$$

$$Var(X) = E[X^2] - \{E[x]\}^2$$

$$\sigma = \sqrt{Var(X)}$$

$$E(2X + 5) = 2 E(X) + 5$$

$$Var(2X + 5) = 4 Var(X)$$

Verify that $E(2X+5) = 2\,E(X)+5$

$$E(2X+5) = \int_0^4 \frac{1}{8} .\, x \,.\, (2x+5)\ dx$$

$$= \frac{1}{8}\int_0^4 (2x^2+5x)\ dx = \frac{1}{8}\left[\left(\frac{2x^3}{3}+\frac{5x^2}{2}\right)\right]_0^4$$

$$= \frac{1}{8}\left[\left(\frac{2\times 4^3}{3}+\frac{5\times 4^2}{2}\right) - 0\right] = \frac{1}{8}\left\{\frac{128}{3}+40\right\}$$

$$= \frac{1}{8}\times\frac{248}{3} = \frac{31}{3}$$

$$2\,E(X)+5 = 2\times\frac{8}{3}+5 = \frac{31}{3}$$

Cumulative Distribution Function

If X is a continuous random variable with probability density function f (x) define for a < x < b then the cumulative distribution function is given by F (t) where

$$F(t) = P(a \le x \le t) = \int_a^t f(x)\,dx$$

$$F(x) = P(a \le x \le x) = \int_a^x f(x)\,dx$$

Median

The median splits the area under the curve y = f (x) into two halves. So if the value of the median is m ,

$$P(a \le x \le m) = \int_a^m f(x)\,dx = \frac{1}{2} = 0.5$$

$$i.e, \quad F(m) = \frac{1}{2}$$

Median

$$P(a \le x \le m) = \int_a^m f(x)\,dx = \frac{1}{2} = 0.5$$

$$P(0 \le x \le m) = \int_0^m \frac{1}{8} x\,dx = \frac{1}{2}$$

$$\left[\frac{1}{16} x^2\right]_0^m = \frac{1}{2}$$

$$\left[\frac{1}{16} x^2\right]_0^m = \frac{1}{2}$$

$$m^2 - 0 = 8$$

$$m = \pm 2\sqrt{2}$$

$$m = 2\sqrt{2}$$

Cumulative Distribution Function

For $0 \leq x \leq 4$

$$F(x) = \int_0^x \frac{1}{8} x \, dx$$

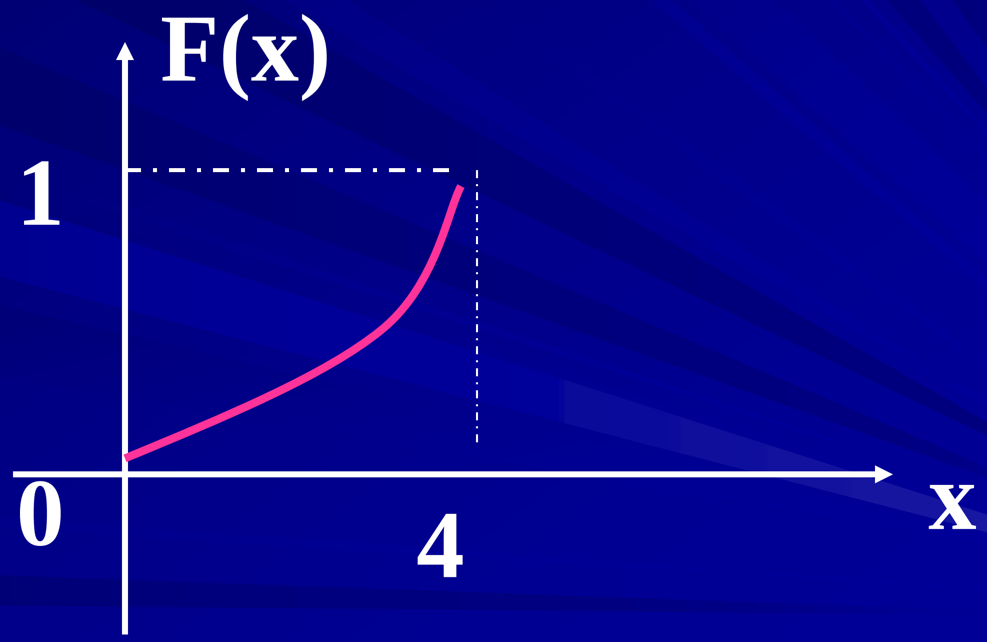
$$= \frac{1}{16} [x^2]_0^x$$

$$= \frac{1}{16} [x^2 - 0]$$

$$= \frac{1}{16} x^2$$

$$F(x) = \frac{1}{16}x^2 \qquad , 0 \le x \le 4$$

$$F(0) = 0 \qquad F(4) = 1$$



Median

$$F(m) = \frac{1}{2}$$

$$\frac{1}{16}m^2 = \frac{1}{2}$$

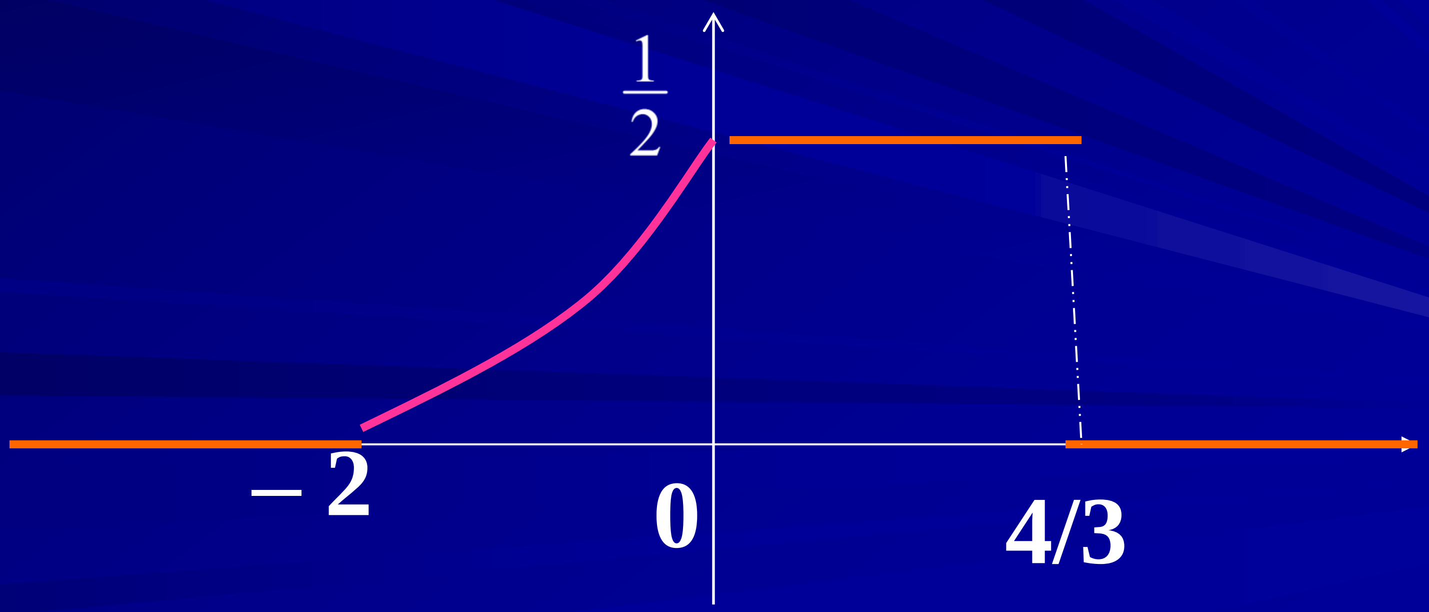
$$m^2 = 8$$

$$m = \pm 2\sqrt{2}$$

$$m = 2\sqrt{2}$$

$$f(x)=\begin{cases}\frac{1}{8}(x+2)^2 & -2\le x<0\\ \frac{1}{2} & 0\le x\le 1\frac{1}{3}\\ 0 & otherwise\end{cases}$$

$$f(-2)=0 \quad f(0)=\frac{1}{2} \quad f(0)=\frac{1}{2} \quad \mathrm{f}(1\tfrac{1}{3})=\frac{1}{2}$$



$$E(X)=\int_{-2}^{0}\frac{1}{8}(x+2)^2 x\,dx+\int_{0}^{\frac{4}{3}}\frac{1}{2}x\,dx$$

$$=\frac{1}{8}\int_{-2}^{0}(x^3+4x^2+4x)\,dx+\frac{1}{2}\int_{0}^{\frac{4}{3}}x\,dx$$

$$=\frac{1}{8}\left[\frac{x^4}{4}+\frac{4x^3}{3}+2x^2\right]_{-2}^{0}+\frac{1}{2}\left[\frac{x^2}{2}\right]_{0}^{4/3}$$

$$=\frac{1}{8}\left[0-(4-\frac{32}{3}+8)\right]+\frac{1}{4}\left[\frac{16}{9}-0\right]$$

$$=-\frac{1}{6}+\frac{4}{9}$$

$$=\frac{5}{18}$$

Cumulative Distribution Function

For $(-\infty < x < -2)$

$$F(x) = P(-\infty < x \leq x) = 0$$

For $(-2 \leq \mathrm{x} < 0)$

$$F(x) = F(-2) + \int_{-2}^{x} f(x)\,dx$$

$$F(x) = 0 + \int_{-2}^{x} \frac{1}{8}(x+2)^2\,dx$$

$$F(x) = 0 + \int_{-2}^{x} \frac{1}{8}(x+2)^2 \, dx$$

$$F(x) = \frac{1}{24}\Big[(x+2)^3\Big]_{-2}^{x} = \frac{1}{24}\Big[(x+2)^3 - 0\Big]$$

$$= \frac{1}{24}(x+2)^3$$

$For\,(0 \le x \le 1\frac{1}{3})$

$$F(x) = F(0) + \int_{0}^{x} \frac{1}{2} dx$$

For $(0 \leq x \leq 1\frac{1}{3})$

$$F(x) = F(0) + \int_{0}^{x} \frac{1}{2} dx$$

$$F(x) = \frac{1}{24}(0+2)^3 + \frac{1}{2}[x]_0^x = \frac{1}{3} + \frac{1}{2}[x-0] = \frac{1}{2}x + \frac{1}{3}$$

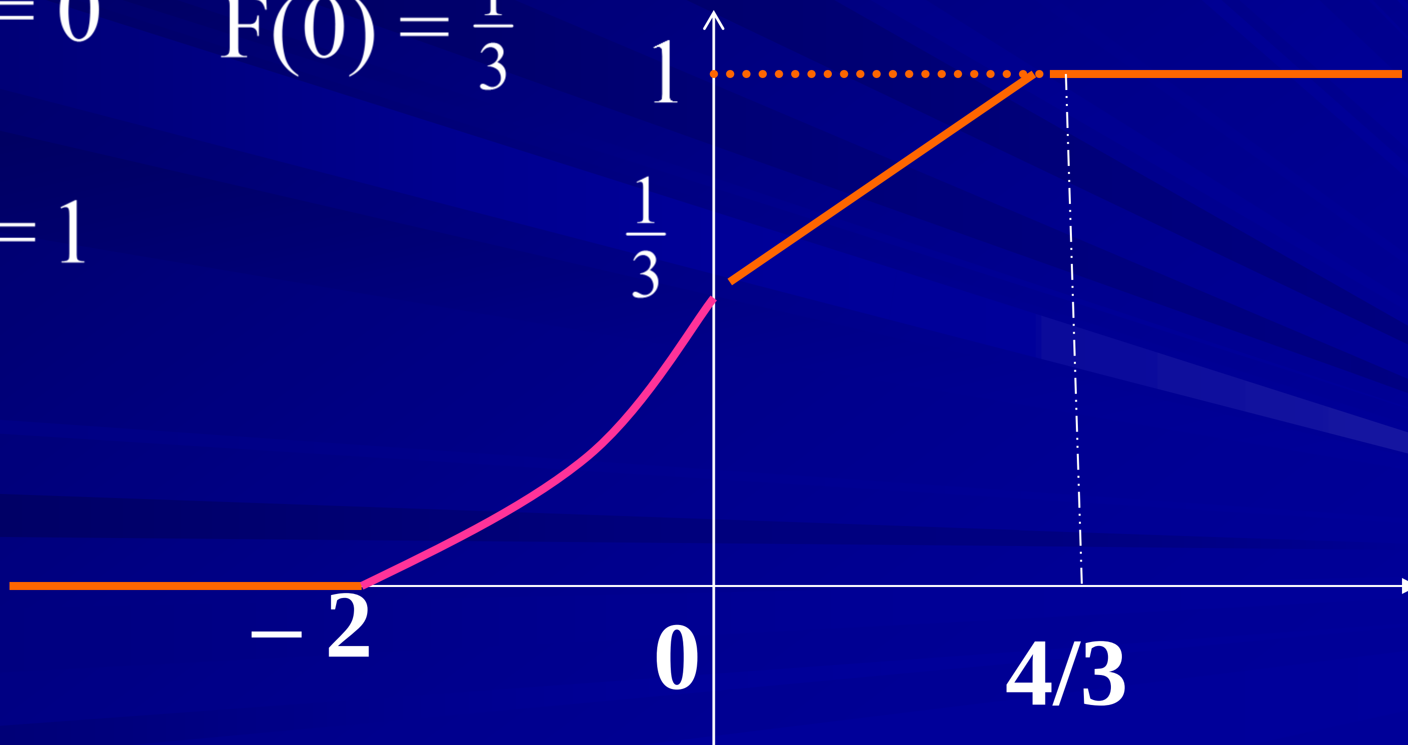
For $(1\frac{1}{3} < x < \infty)$

$$F(x) = F(1\frac{1}{3}) + 0 = \frac{1}{2} \times \frac{4}{3} + \frac{1}{3} = 1$$

$$
F(x) = \begin{cases} o & ,-\infty < x < -2 \\ \frac{1}{24}(x+2)^3 & , \quad -2 \le x < 0 \\ \frac{1}{2}x + \frac{1}{3} & , \ 0 \le x \le 1\frac{1}{3} \\ 1 & , 1\frac{1}{3} < x < \infty \end{cases}
$$

$F(-2) = 0$ $\qquad$ $F(0) = \frac{1}{3}$

$F(1\frac{1}{3}) = 1$



Median

$$\int_{-2}^{0} \frac{1}{8}(x+2)^2\,dx + \int_{0}^{m} \frac{1}{2}\,dx = \frac{1}{2}$$

$$\frac{1}{24}\left[(x+2)^3\right]_{-2}^{0} + \frac{1}{2}\left[\,x\,\right]_{0}^{m} = \frac{1}{2}$$

$$\frac{1}{24}[8 - 0] + \frac{1}{2}[\,m - 0\,] = \frac{1}{2}$$

$$\frac{1}{2}\,m = \frac{1}{6}$$

$$m = \frac{1}{3}$$

Median

$$F(m) = \frac{1}{2}$$

$$\frac{1}{2}m + \frac{1}{3} = \frac{1}{2}$$

$$\frac{1}{2}m = \frac{1}{6}$$

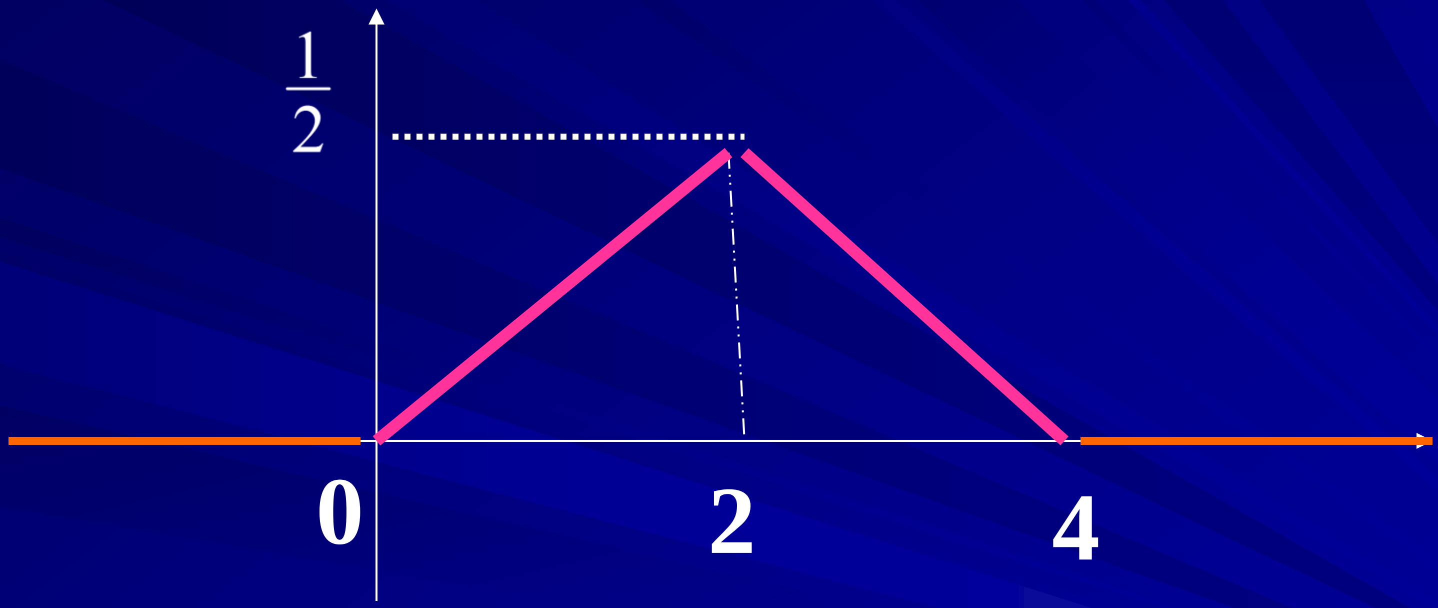
$$m = \frac{1}{3}$$

$$f(x)=\begin{cases}\frac{1}{4}x & 0\le x<2\\ \frac{1}{4}(4-x) & 2\le x\le 4\\ 0 & otherwise\end{cases}$$

$$f(0)=0 \qquad f(2)=\frac{1}{2} \qquad f(2)=\frac{1}{2} \qquad f(4)=0$$

$$(0,0) \qquad (2,\frac{1}{2}) \qquad (2,\frac{1}{2}) \qquad (4,0)$$

$(0,0)$ $(2,\frac{1}{2})$ $(2,\frac{1}{2})$ $(4,0)$



$$E(X)=\int_{0}^{2}\frac{1}{4}.\,x\,.\,x\;dx+\int_{2}^{4}\frac{1}{4}(4-x)x\;dx$$

$$=\frac{1}{4}\int_{0}^{2}x^{2}\,dx+\frac{1}{4}\int_{2}^{4}(4x-x^{2})\;dx$$

$$=\frac{1}{12}\,[x^{3}]_{0}^{2}+\frac{1}{4}\,[(2x^{2}-\frac{x^{3}}{3})]_{2}^{4}$$

$$=\frac{1}{12}\,[8-0]+\frac{1}{4}\,[(32-\frac{64}{3})-(8-\frac{8}{3})]$$

$$=\frac{2}{3}+8-\frac{16}{3}-2+\frac{2}{3}$$

$$=2$$

$$E(X^2) = \int_0^2 \frac{1}{4} . x . x^2 \, dx + \int_2^4 \frac{1}{4}(4 - x) x^2 \, dx$$

$$= \frac{1}{4} \int_0^2 x^3 \, dx + \frac{1}{4} \int_2^4 (4x^2 - x^3) \, dx$$

$$= \frac{1}{16} [x^4]_0^2 + \frac{1}{4} [(\frac{4x^3}{3} - \frac{x^4}{4})]_2^4$$

$$= \frac{1}{16} [16 - 0] + [(\frac{x^3}{3} - \frac{x^4}{16})]_2^4$$

$$= 1 + (\frac{64}{3} - 16) - (\frac{8}{3} - 1)$$

$$= 1 + \frac{64}{3} - 16 - \frac{8}{3} + 1$$

$$= \frac{14}{3}$$

$$\mathrm{Var}\,(X) = E\,[\,X^2\,] - \{E[\,x\,]\}^2$$

$$\sigma = \sqrt{\mathrm{Var}(X)}$$

$$E(2X + 5) = 2\,E(X) + 5$$

$$\mathrm{Var}(2X + 5) = 4\,\mathrm{Var}(X)$$

Cumulative Distribution Function

For $(-\infty < x < 0)$

$$F(x) = P(-\infty < x < x) = 0$$

For $(0 \leq x < 2)$

$$F(x) = F(0) + \int_{0}^{x} f(x)\,dx$$

$$F(x) = 0 + \int_{0}^{x} \frac{1}{4}\,x\,dx \quad = \frac{1}{8}\left[x^2\right]_0^x$$

$$= \frac{1}{8}\left[x^2 - 0\right] = \frac{1}{8}x^2$$

For $(2 \leq x \leq 4)$

$$F(x) = F(2) + \int_{2}^{x} \frac{1}{4}(4 - x)\,dx$$

$$F(x) = \frac{1}{2} + \frac{1}{4}\left[4x - \frac{x^2}{2}\right]_2^x = \frac{1}{2} + \left[x - \tfrac{1}{8}x^2\right]_2^x$$

$$= \frac{1}{2} + \left[(x - \tfrac{1}{8}x^2) - (2 - \tfrac{1}{2})\right]$$

$$= \frac{1}{2} + x - \frac{1}{8}x^2 - \frac{3}{2}$$

$$= -\frac{1}{8}x^2 + x - 1$$

For $(4 < x < \infty)$

$$F(x) = F(4) + 0$$

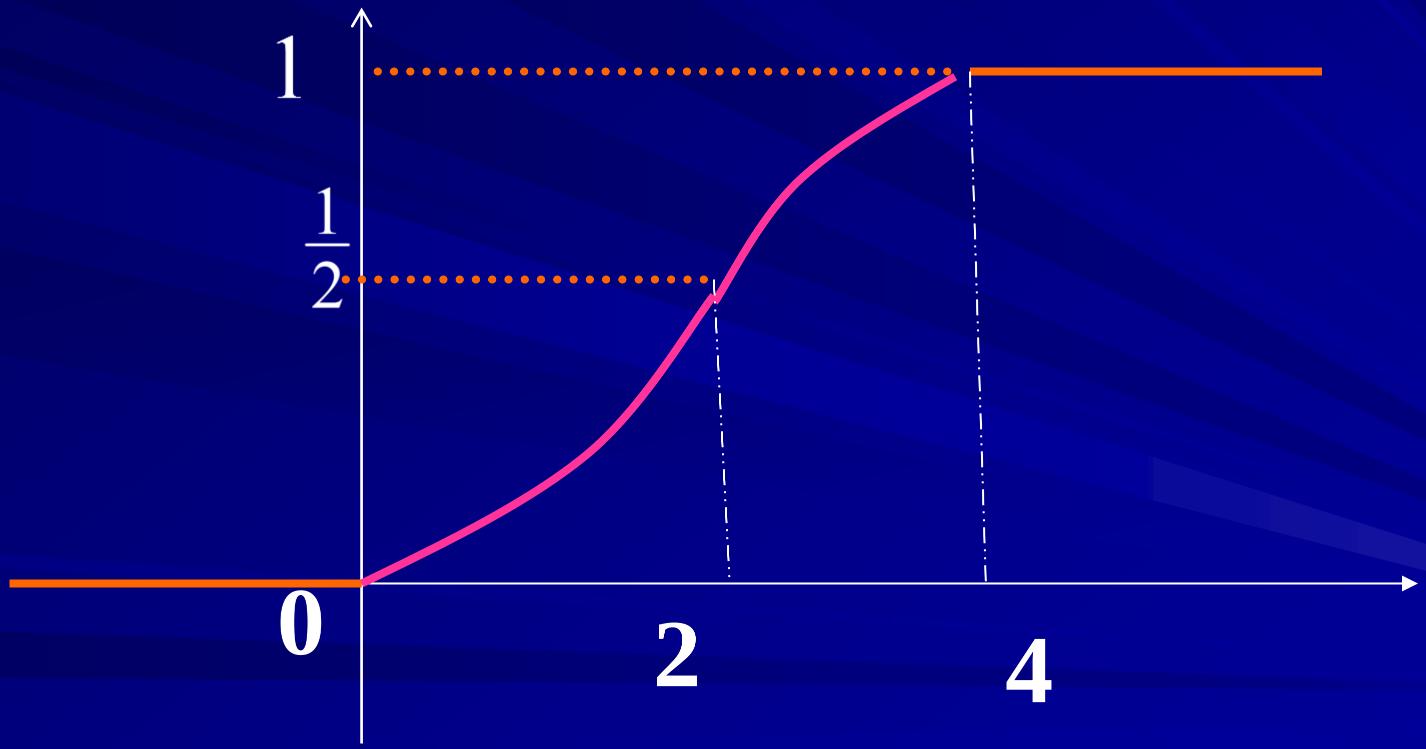
$$= -\frac{1}{8} \times 16 + 4 - 1$$

$$= 1$$

$$F(x) = \begin{cases} 0 & , -\infty < x < 0 \\ \frac{1}{8}x^2 & , \quad 0 \le x < 2 \\ -\frac{1}{8}x^2 + x - 1 & , \; 2 \le x \le 4 \\ 1 & , \; 4 < x < \infty \end{cases}$$

$$F(0) = 0 \qquad F(2) = \frac{1}{2} \qquad F(4) = 1$$

$F(0) = 0 \qquad F(2) = \frac{1}{2} \qquad F(4) = 1$



Median

$$\int_0^2 \frac{1}{4} x\,dx + \int_2^m \frac{1}{4}(4 - x)\,dx = \frac{1}{2}$$

$$\frac{1}{8}[x^2]_0^2 - \frac{1}{8}[(4 - x)^2]_2^m = \frac{1}{2}$$

$$\frac{1}{8}[4 - 0] - \frac{1}{8}[(4 - m)^2 - (2)^2] = \frac{1}{2}$$

$$16 - 8m + m^2 - 4 = 0$$

$$m^2 - 8m + 12 = 0$$

$$(m - 2)(m - 6) = 0$$

$m = 2$ or $m = 6$ (impossible)

$$m = 2$$

Median

$$F(m) = \frac{1}{2}$$

$$-\frac{1}{8}m^2 + m - 1 = \frac{1}{2}$$

$$m^2 - 8m + 12 = 0$$

$$(m-2)(m-6) = 0$$

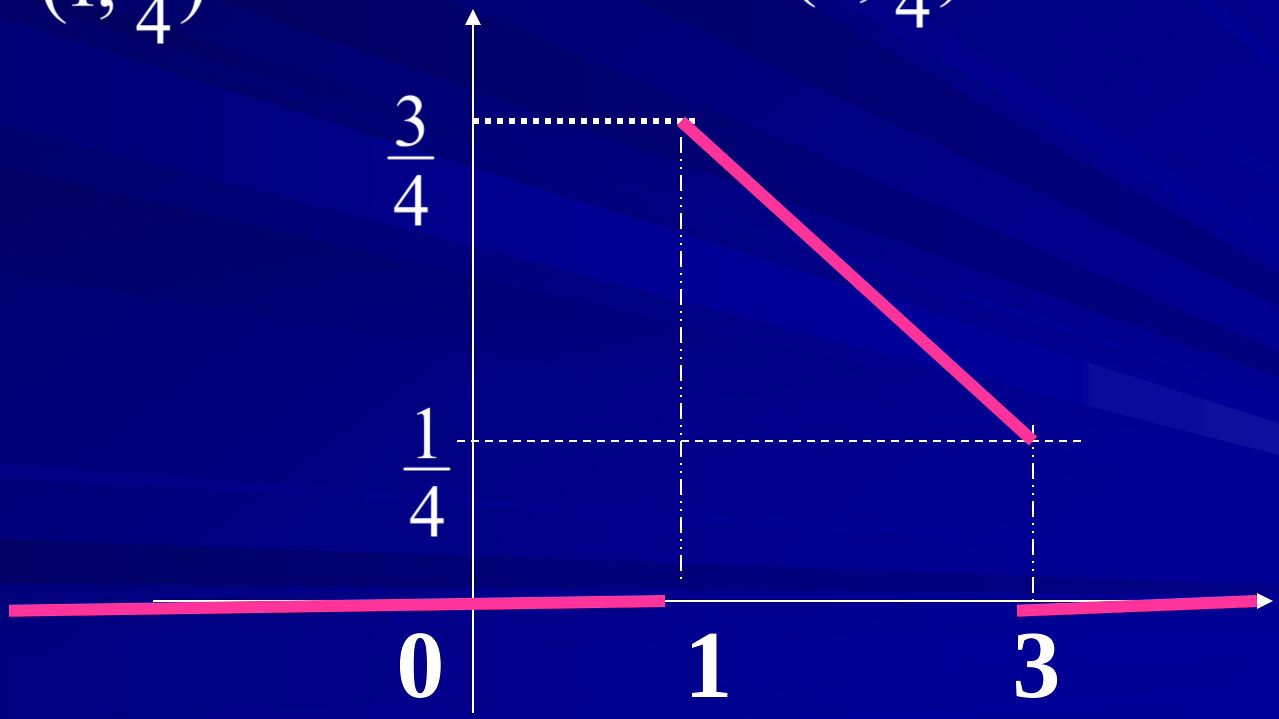
$m = 2$ (or) $m = 6$ (impossible)

$$m = 2$$

$$f(x)=\begin{cases}\frac{1}{4}(4-x)\,, & 1\le x\le 3\\ 0 & otherwise\end{cases}$$

$$f(1)=\frac{3}{4} \qquad f(3)=\frac{1}{4}$$

$$(1,\frac{3}{4}) \qquad (3,\frac{1}{4})$$



$$E(X) = \int_1^3 \frac{1}{4}(4 - x)\,x\;dx$$

$$E(X) = \frac{1}{4}\int_1^3 (4x - x^2)\;dx$$

$$E(X) = \frac{1}{4}\left[\,2x^2 - \frac{1}{3}x^3\,\right]_1^3$$

$$E(X) = \frac{1}{4}\left[(18 - 9) - \left(2 - \frac{1}{3}\right)\right]$$

$$E(X) = \frac{11}{6}$$

Cumulative Distribution Function

$For \quad -\alpha < x < 1$

$$F(x) = 0$$

Cumulative Distribution Function

For $1 \leq x \leq 3$

$$F(x) = F(1) + \int_{1}^{x} \frac{1}{4}(4 - x)\,dx$$

$$= 0 - \frac{1}{8}\left[(4 - x)^2\right]_1^x$$

$$= -\frac{1}{8}\left[(4 - x)^2 - (4 - 1)^2\right]$$

$$= -\frac{1}{8}x^2 - x + \frac{7}{8}$$

Cumulative Distribution Function

$For \quad 3 < x < \infty$

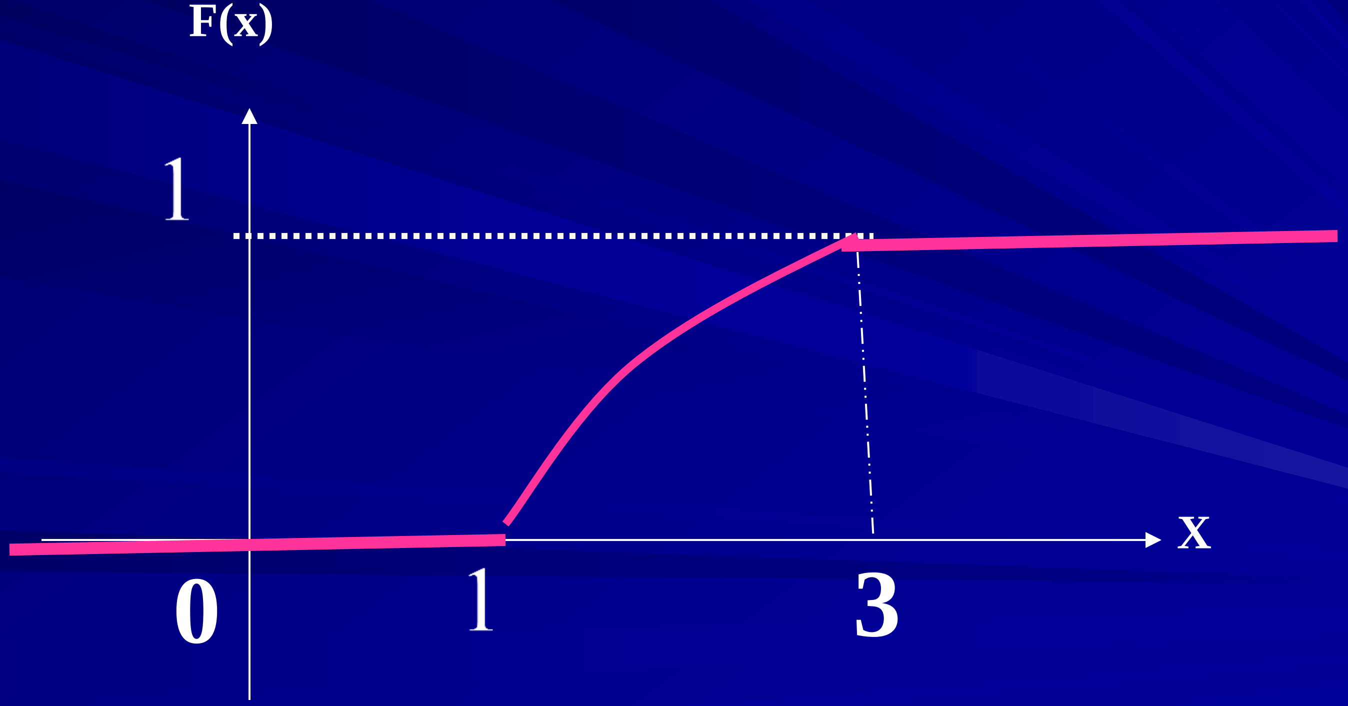
$$F(x) = F(3) + 0$$

$$F(x) = 1$$

$$f(x) = \begin{cases} 0 & , \quad -\infty < x < 1 \\ -\frac{1}{8}x^2 - x + \frac{7}{8} & , 1 \le x \le 3 \\ 1 & , \ 3 < x < \infty \end{cases}$$

$F(1) = 0$ $\qquad$ $F(3) = 1$

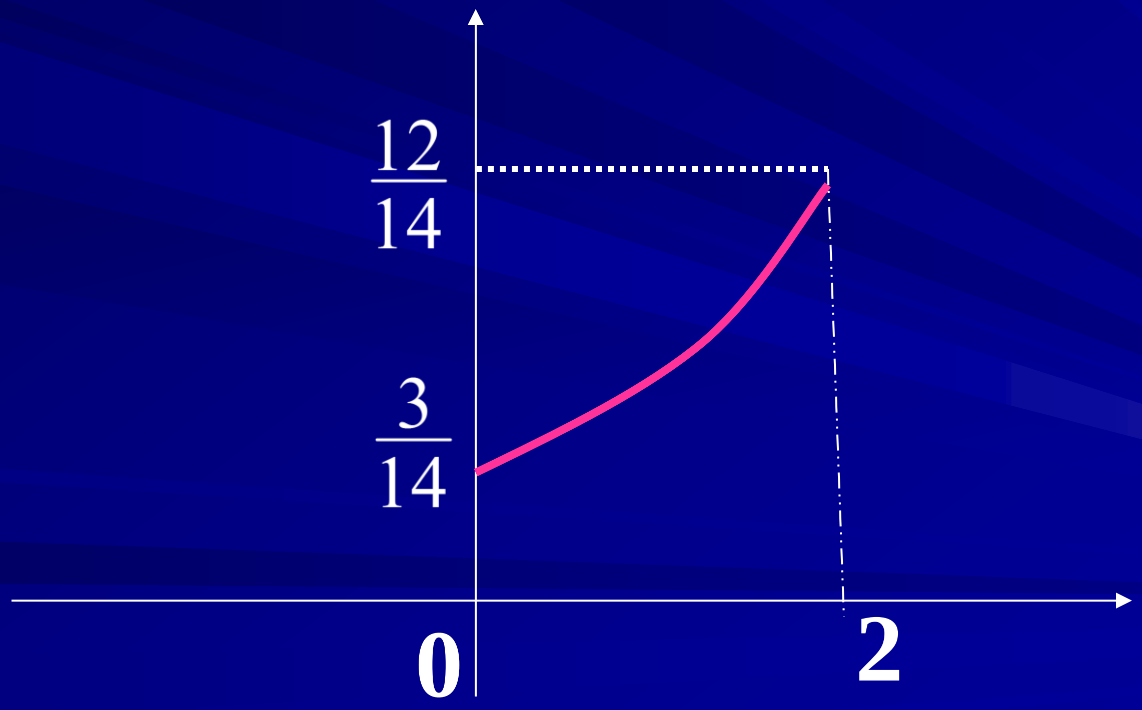
$(1,0)$ $\qquad$ $(3,1)$



$$f(x)=\frac{3}{56}(x+2)^2 \quad , 0\le x\le 2$$

$$f(0)=\frac{3}{14} \qquad f(2)=\frac{12}{14}$$

$$(0,\frac{3}{14}) \qquad (2,\frac{12}{14})$$



Cumulative Distribution Function

For $0 \le x \le 2$

$$F(x) = \int_0^x \frac{3}{56}(x+2)^2 \, dx$$

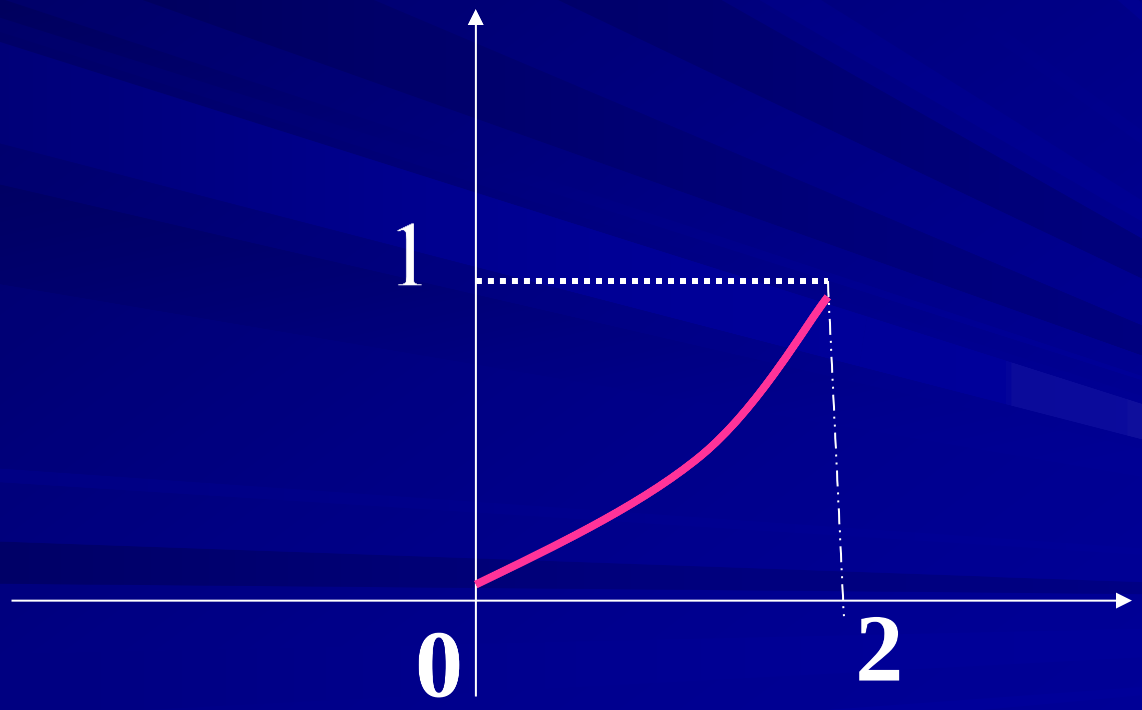
$$= \frac{1}{56}\left[(x+2)^3\right]_0^x$$

$$= \frac{1}{56}\left[(x+2)^3 - 8\right]$$

$$F(x) = \frac{1}{56}[(x+2)^3 - 8] \quad , 0 \leq x \leq 2$$

$$F(0) = 0 \qquad\qquad F(2) = 1$$

$$(0,0) \qquad\qquad (2,1)$$



A continuous random variable X has probability density function f (x) where

$$f(x)=\begin{cases}\frac{6}{7}x & 0\le x<1\\ \frac{6}{7}x(2-x) & 1\le x\le 2\\ 0 & otherwise\end{cases}$$

(a)find E (X)

(b)find E (X^2)

$$E(X)=\int_0^1 \frac{6}{7}.x.x\,dx+\int_1^2 \frac{6}{7}x(2-x)x\,dx$$

$$=\frac{6}{7}\int_0^1 x^2\,dx+\frac{6}{7}\int_1^2 (2x^2-x^3)\,dx$$

$$=\frac{2}{7}\,[x^3]_0^1+\frac{6}{7}\,[(\frac{2}{3}x^3-\frac{x^4}{4})]_1^2$$

$$=\frac{2}{7}\,[1-0]+\frac{6}{7}\,[(\frac{16}{3}-4)-(\frac{2}{3}-\frac{1}{4})]$$

$$=\frac{2}{7}+\frac{6}{7}\,[\frac{4}{3}-\frac{2}{3}+\frac{1}{4}]$$

$$=\frac{2}{7}+\frac{6}{7}\times\frac{11}{12}$$

$$=\frac{15}{14}$$

$$E(X^2)=\int_0^1 \frac{6}{7}.x.x^2\ dx+\int_1^2 \frac{6}{7}x(2-x)x^2\ dx$$

$$=\frac{6}{7}\int_0^1 x^3\,dx+\frac{6}{7}\int_1^2 (2x^3-x^4)\ dx$$

$$=\frac{3}{14}\left[x^4\right]_0^1+\frac{6}{7}\left[\left(\tfrac{1}{2}x^4-\frac{x^5}{5}\right)\right]_1^2$$

$$=\frac{3}{14}[1-0]+\frac{6}{7}\left[\left(8-\frac{32}{5}\right)-\left(\frac{1}{2}-\frac{1}{5}\right)\right]$$

$$=\frac{3}{14}+\frac{6}{7}\left[\frac{8}{5}-\frac{3}{10}\right]\quad=\frac{3}{14}+\frac{6}{7}\times\frac{13}{10}$$

$$=\frac{3}{14}+\frac{6}{7}\times\frac{13}{10}\quad=\frac{3}{14}+\frac{39}{35}\quad=\frac{93}{70}$$

A continuous random variable X has probability density function f (x) where

$$f(x)=\begin{cases} \dfrac{x}{3} & 0\le x<2 \\ -\dfrac{2x}{3}+2 & 2\le x\le 3 \\ 0 & otherwise \end{cases}$$

(a)sketch y = f (x) (b)sketch y = F (x)

(c)find $P(1\le x\le 2.5)$

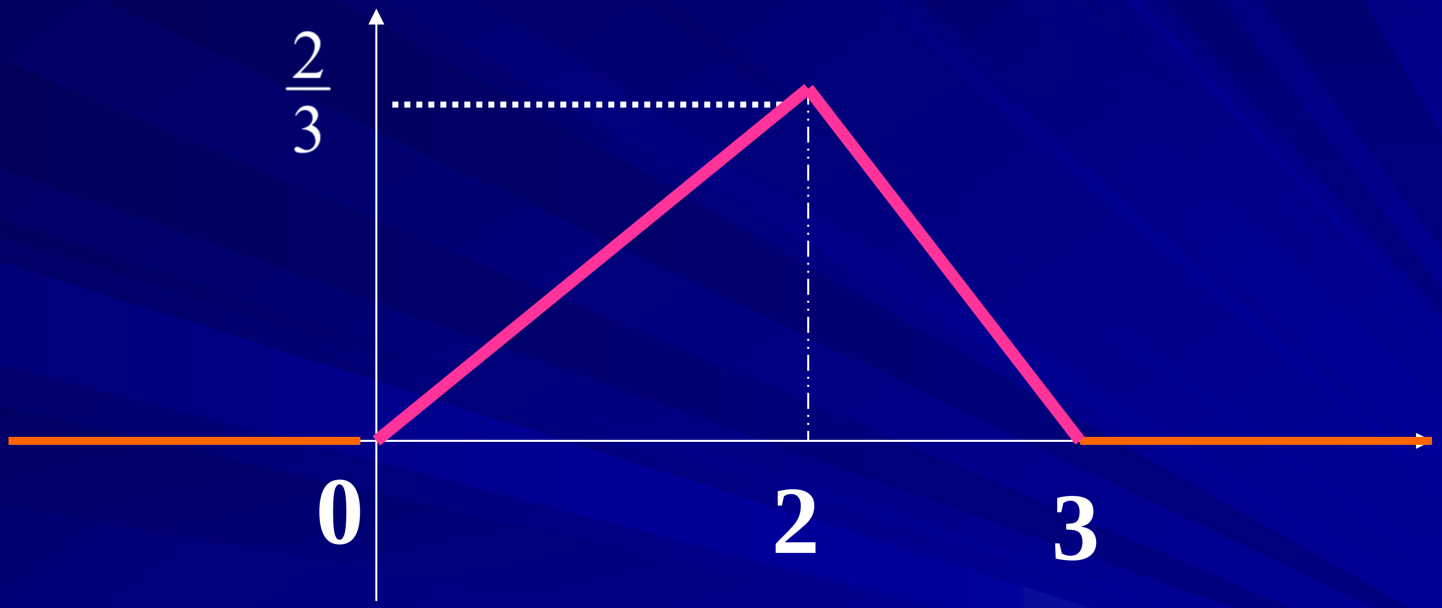
(d)find median

$$f(x)=\begin{cases}\frac{1}{3}x & 0\le x<2\\ -\frac{2}{3}x+2 & 2\le x\le 3\\ 0 & otherwise\end{cases}$$

$f(0)=0$ $\quad$ $f(2)=\frac{2}{3}$ $\quad$ $f(3)=0$

$(0,0)$ $\quad$ $(2,\frac{2}{3})$ $\quad$ $(3,0)$

$(0,0)$ $\left(2,\frac{2}{3}\right)$ $(3,0)$



$$E(X)=\int_0^2 \frac{1}{3}.x.x\ dx+\int_2^3 \left(-\frac{2}{3}x+2\right)x\ dx$$

$$=\frac{1}{3}\int_0^2 x^2\ dx+\int_2^3 \left(-\frac{2}{3}x^2+2x\right)\ dx$$

$$=\frac{1}{9}\ [x^3]_0^2+\left[\left(-\frac{2}{3}\ \frac{x^3}{3}+x^2\right)\right]_2^3$$

$$=\frac{1}{9}\ [8-0]\ +[(-6+9)-(-\frac{16}{9}-4)]$$

$$=\frac{8}{9}+3+\frac{16}{9}-4$$

$$=\frac{24-9}{9}$$

$$=\frac{5}{3}$$

Cumulative Distribution Function

$For\ (-\infty < x < 0)$

$$F(x) = P(-\infty < x < x) = 0$$

$For\ (0 \le x < 2)$

$$F(x) = F(0) + \int_0^x f(x)\,dx$$

$$F(x) = 0 + \int_0^x \frac{1}{3}\,x\,dx = \frac{1}{6}\left[x^2\right]_0^x = \frac{1}{6}x^2$$

For $(2 \le x \le 3)$

$$F(x) = F(2) + \int_2^x \left(-\frac{2}{3}x + 2\right)dx$$

$$F(x) = \frac{2}{3} + \left[-\frac{1}{3}x^2 + 2x\right]_2^x$$

$$= \frac{2}{3} + \left[\left(-\frac{1}{3}x^2 + 2x\right) - \left(-\frac{4}{3} + 4\right)\right]$$

$$= \frac{2}{3} - \frac{1}{3}x^2 + 2x + \frac{4}{3} - 4$$

$$= -\frac{1}{3}x^2 + 2x - 2$$

$$For\,(3 < x < \infty)$$

$$F(x) = F(3) + 0$$

$$= -\frac{1}{3} \times 3^2 + 2 \times 3 - 2$$

$$= 1$$

$$F(x) = \begin{cases} 0 & , -\infty < x < 0 \\ \frac{1}{6}\, x^2 & , \quad 0 \le x < 2 \\ -\frac{1}{3}x^2 + 2x - 2 & , 2 \le x \le 3 \\ 1 & , 3 < x < \infty \end{cases}$$

$$F(0) = 0 \qquad F(2) = \frac{2}{3} \qquad F(3) = 1$$

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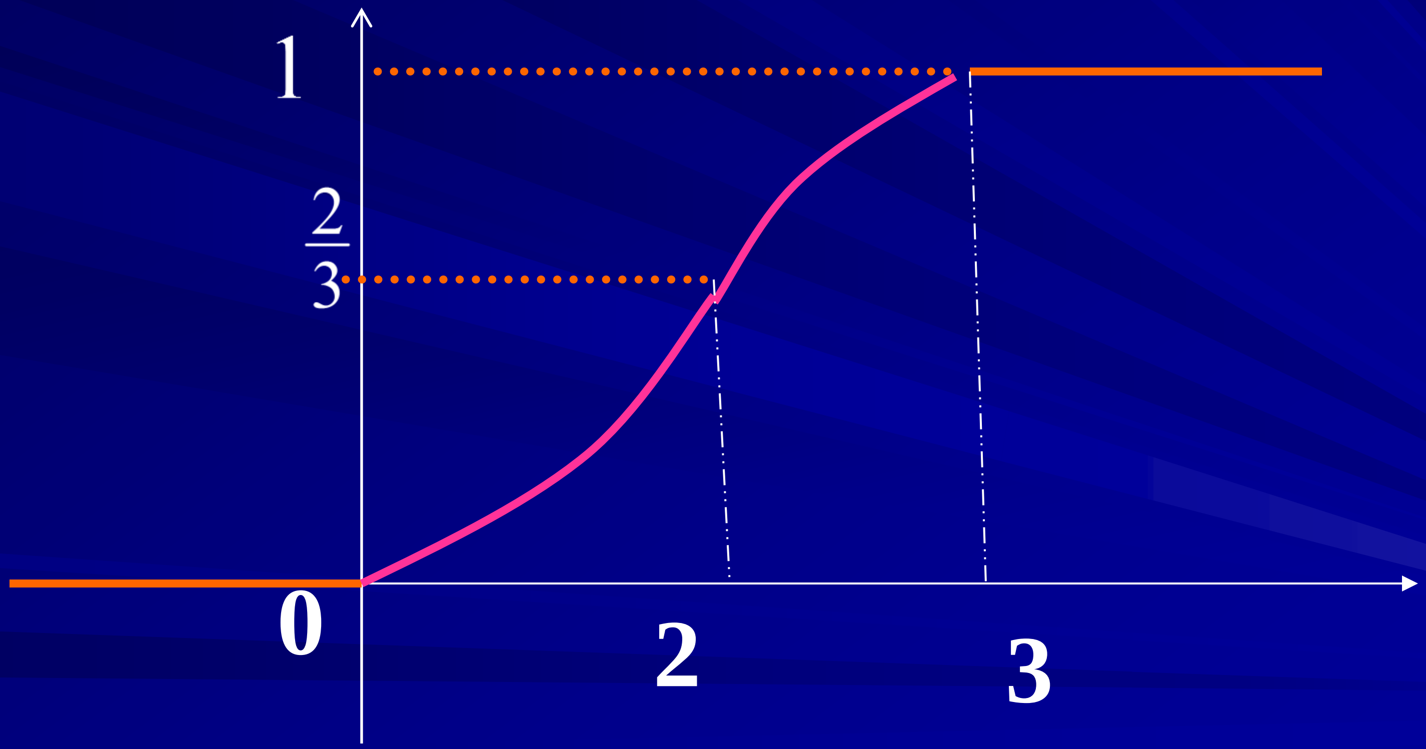


Median

$$P(0 \le x \le m) = \int_0^m \frac{1}{3} x \, dx = \frac{1}{2}$$

$$\frac{1}{6}[\ x^2\ ]_0^m = \frac{1}{2}$$

$$m^2 = 3$$

$$m = \pm\sqrt{3}$$

$$m = \sqrt{3}$$

Median

$$F(m) = \frac{1}{2}$$

$$\frac{1}{6}m^2 = \frac{1}{2}$$

$$m^2 = 3$$

$$m = \pm\sqrt{3}$$

$$m = \sqrt{3}$$

A continuous random variable X has probability density function f (x) where

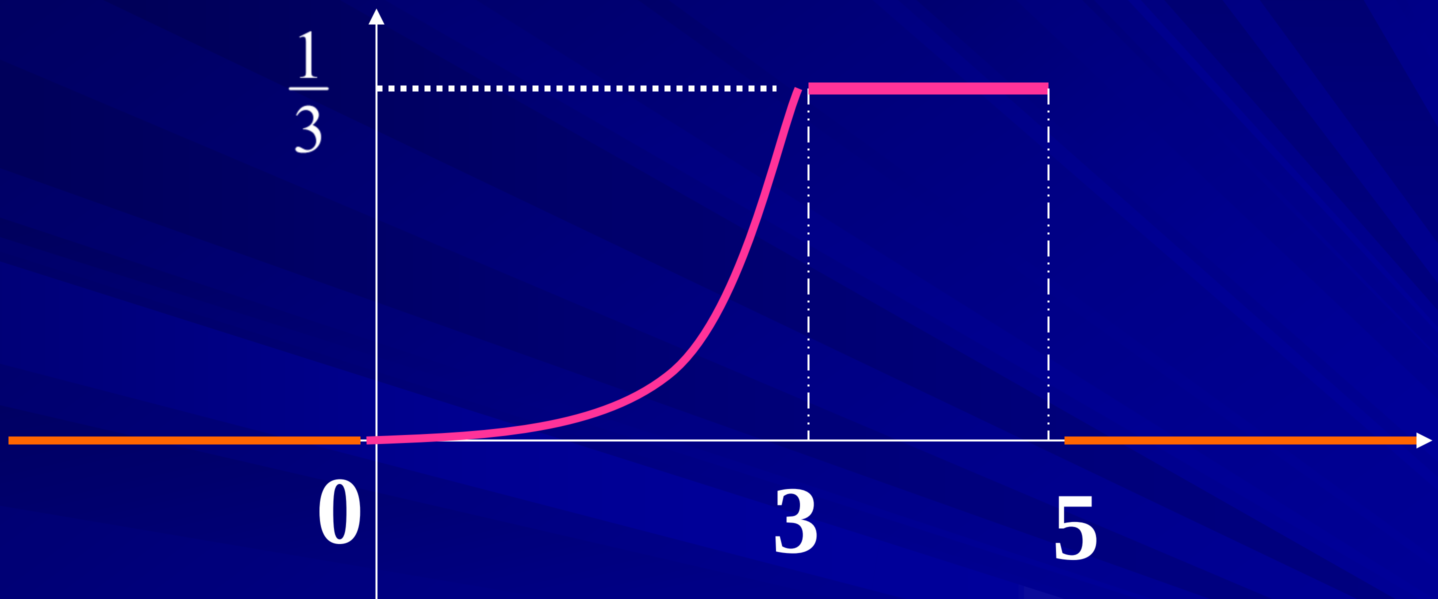
$$f(x)=\begin{cases}\dfrac{1}{27}x^2 & 0\le x<3\\ \dfrac{1}{3} & 3\le x\le 5\\ 0 & otherwise\end{cases}$$

(a) Sketch y = f (x) (b) Find $E(X)$

(c) Find $E(X^2)$

(d) Find standard deviation

$(0,0)$ $\quad (3,\frac{1}{3})$ $\quad (5,\frac{1}{3})$



$$E(X)=\int_0^3 \frac{1}{27}.x^2 .x \, dx+\int_3^5 \frac{1}{3}. x \, dx$$

$$E(X)=\int_0^3 \frac{1}{27}.x^3 \, dx+\int_3^5 \frac{1}{3} x \, dx$$

$$E(X)=\frac{1}{108}[x^4]_0^3+\frac{1}{6}[x^2]_3^5$$

$$\mathrm{E(X)}=\frac{1}{108}[3^4-0]+\frac{1}{6}[25-9]$$

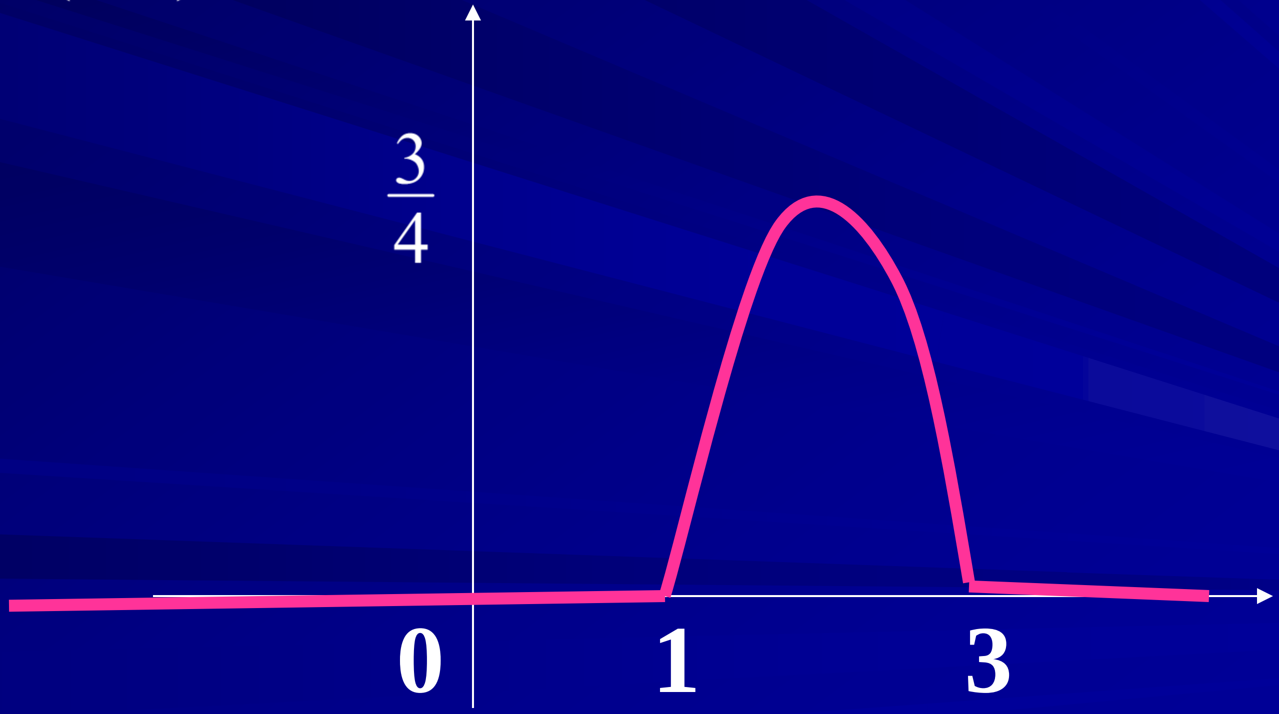
$$\mathrm{E(X)}=\frac{3}{4}+\frac{8}{3}$$

$$\mathrm{E(X)}=\frac{41}{12}$$

$$f(x)=\begin{cases}\frac{3}{4}(1-(2-x)^2),\ 1\le x\le 3\\ 0 \qquad\qquad otherwise\end{cases}$$

$f(1)=0$ $\qquad$ $f(3)=0$

$(1,0)$ $\qquad$ $(3,0)$



$$E(X)=\int_{1}^{3}\frac{3}{4}(1-(2-x)^2)x\,dx$$

$$=\frac{3}{4}\int_{1}^{3}(-x^3+4x^2-3x)\,dx$$

$$=\frac{3}{4}\left[-\frac{x^4}{4}+\frac{4x^3}{3}-\frac{3x^2}{2}\right]_1^3$$

$$=\frac{3}{4}\left[(-\frac{81}{4}+36-\frac{27}{2})-(-\frac{1}{4}+\frac{4}{3}-\frac{3}{2})\right]$$

$$=\frac{3}{4}\left[-\frac{81}{4}+36-\frac{27}{2}+\frac{1}{4}-\frac{4}{3}+\frac{3}{2})\right]$$

$$=\frac{3}{4}\left[-20+36-12-1\frac{1}{3}\right]$$

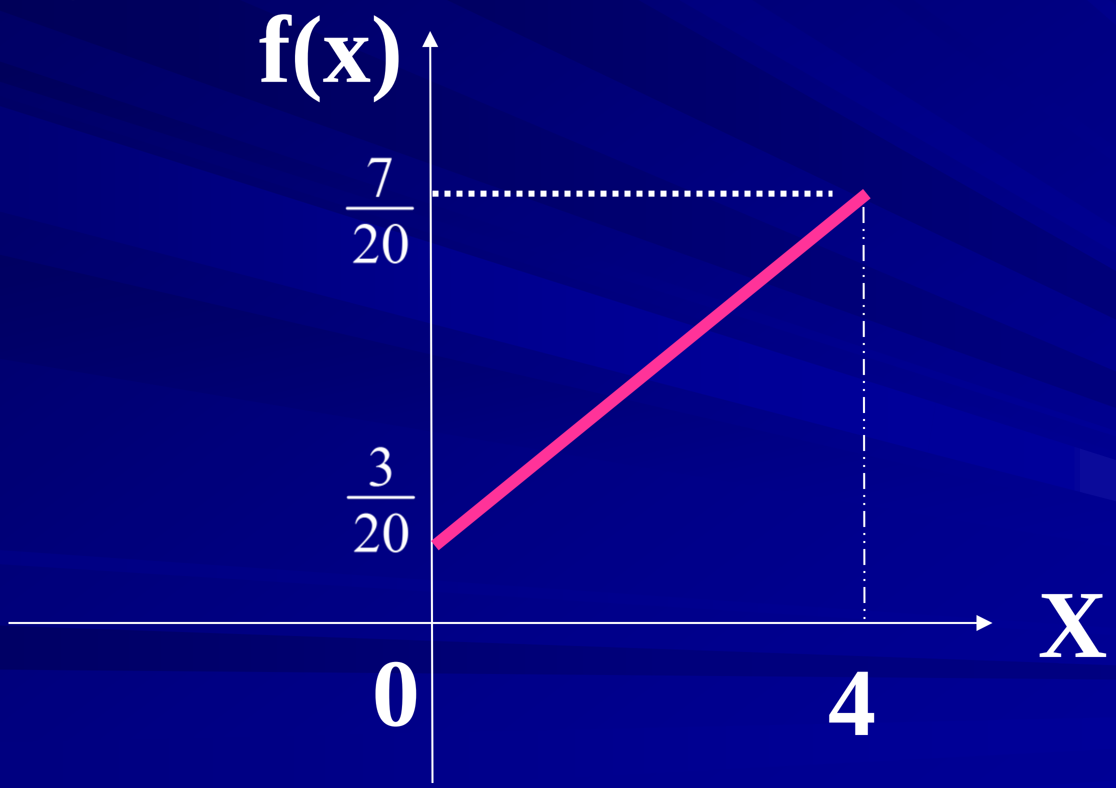
$$=\frac{3}{4}\times\frac{8}{3}$$

$$=2$$

$$f(x)=\frac{1}{20}(x+3) \qquad , 0 \le x \le 4$$

$$f(0)=\frac{3}{20} \qquad , f(4) = \frac{7}{20}$$

$(0, \frac{3}{20})$ $\qquad$ $(0, \frac{7}{20})$



$$E(X) = \int_0^4 \frac{1}{20}(x+3)x\,dx$$

$$= \int_0^4 \frac{1}{20}(x^2+3x)\,dx$$

$$= \frac{1}{20}\left[\frac{x^3}{3} + \frac{3x^2}{2}\right]_0^4$$

$$= \frac{1}{20}\left[\left(\frac{4^3}{3} + 24\right) - 0\right]$$

$$= \frac{1}{5}\left[\frac{16}{3} + 6\right]$$

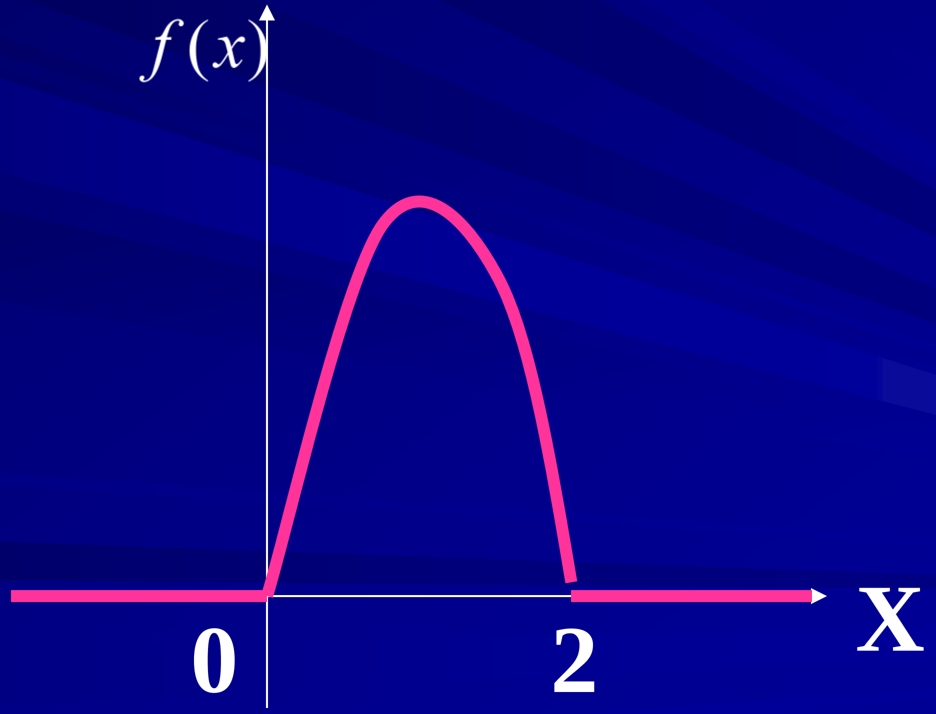
$$= \frac{1}{5} \times \frac{34}{3}$$

$$= \frac{34}{15}$$

$$f(x)=\begin{cases}\frac{3}{4}\,x\,(2-x)\,, & 1\le x\le 2\\ 0 & otherwise\end{cases}$$

$f(0)=0$ $\qquad\qquad$ $f(2)=0$

$(0,0)$ $\qquad\qquad$ $(2,0)$



$$E(X) = \frac{3}{4}\int_0^2 x(2-x)x\,dx = \frac{3}{4}\int_0^2 (2x^2 - x^3)\,dx$$

$$= \frac{3}{4}\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^2 = \frac{3}{4}\left[\left(\frac{16}{3} - 4\right) - 0\right]$$

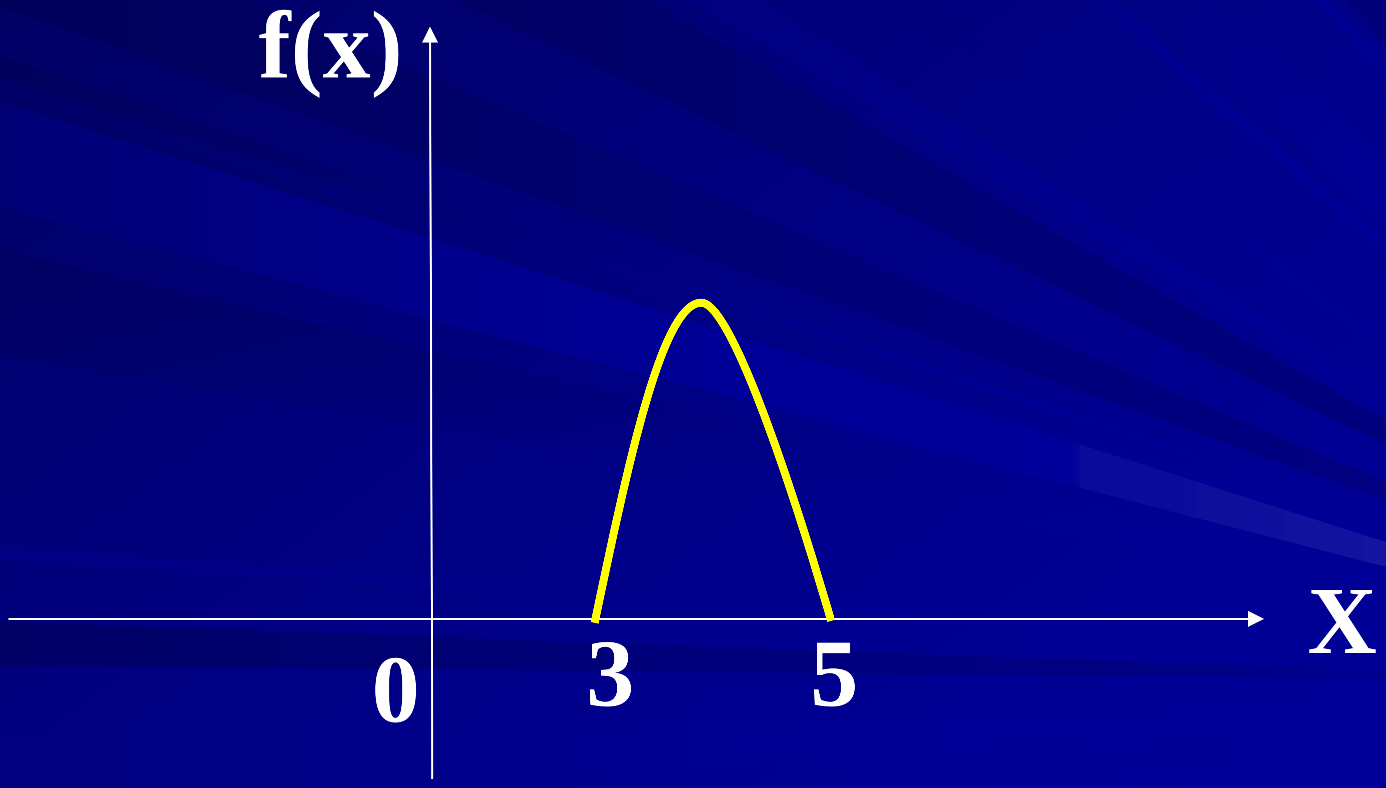
$$= \frac{3}{4} \times \frac{4}{3}$$

$$= 1$$

$$f(x)=\frac{3}{4}(3-x)(x-5) \qquad , 3 \le x \le 5$$

$$f(3)=0 \qquad , f(5)=0$$

$(3,0)$ $(5,0)$



$$E(X)=\int_3^5 \frac{3}{4}(3-x)(x-5)x\,dx \quad = \frac{3}{4}\int_3^5 (-x^3+8x^2-15x)\,dx$$

$$=\frac{3}{4}\left[-\frac{x^4}{4}+\frac{8x^3}{3}-\frac{15x^2}{2}\right]_3^5$$

$$=\frac{3}{4}\left[(-\frac{5^4}{4}+\frac{8\times5^3}{3}-\frac{3\times5^5}{2})-(-\frac{3^4}{4}+\frac{8\times3^3}{3}-\frac{5\times3^3}{2}\right]$$

$$=\frac{3}{4}\left[5^3(-\frac{5}{4}+\frac{8}{3}-\frac{3}{2})-3^3(-\frac{3}{4}+\frac{8}{3}-\frac{5}{2})\right]$$

$$=\frac{3}{4}\left[125\,(\frac{-15+32-18}{12})-27\,(\frac{-9+32-30}{12})\right]$$

$$=\frac{3}{4}\left[125\,(-\frac{1}{12})+27\times\frac{7}{12}\right]$$

$$=\frac{3}{4}\times\frac{64}{12}$$

$$=4$$