

FERMIONIC STATE PREPARATION AND IMAGING IN OPTICAL TWEEZER ARRAY

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Quantum simulation with ultracold atoms requires both high-fidelity preparation of the initial many-body state and site-resolved measurement of the final state. This thesis presents the development of experimental techniques for spin-resolved free-space imaging and deterministic, spin-selective preparation of ultracold fermionic ${}^6\text{Li}$ atoms in a two-dimensional optical tweezer array. The array is generated using crossed acousto-optic deflectors, with precise control achieved through a combination of direct camera-based calibration and atom-based feedback. A novel spilling method enables the preparation of arbitrary spin- and site-resolved occupation patterns. The thesis also introduces numerical tools for simulating Fermi-Hubbard dynamics in small systems, laying the groundwork for future out-of-equilibrium quantum simulation experiments.

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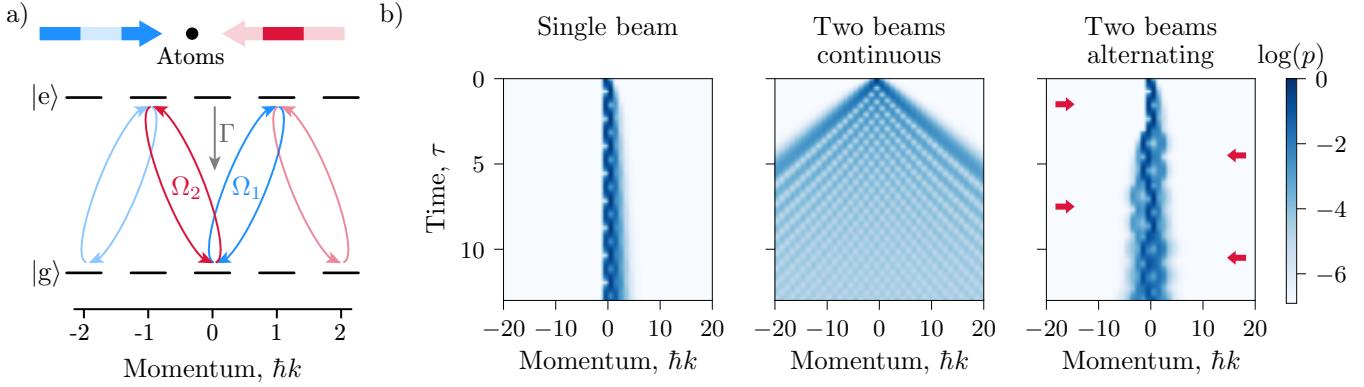


Figure 1: **Momentum-space dynamics in the SSH model.** a) Atoms undergo momentum-changing transitions via couplings Ω_1 and Ω_2 , realizing a SSH-like quantum walk. b) Momentum distributions over time for different beam configurations: single beam (left) shows small shift; two continuous beams (middle) result in fast spreading; alternating beams (right) suppress spread.

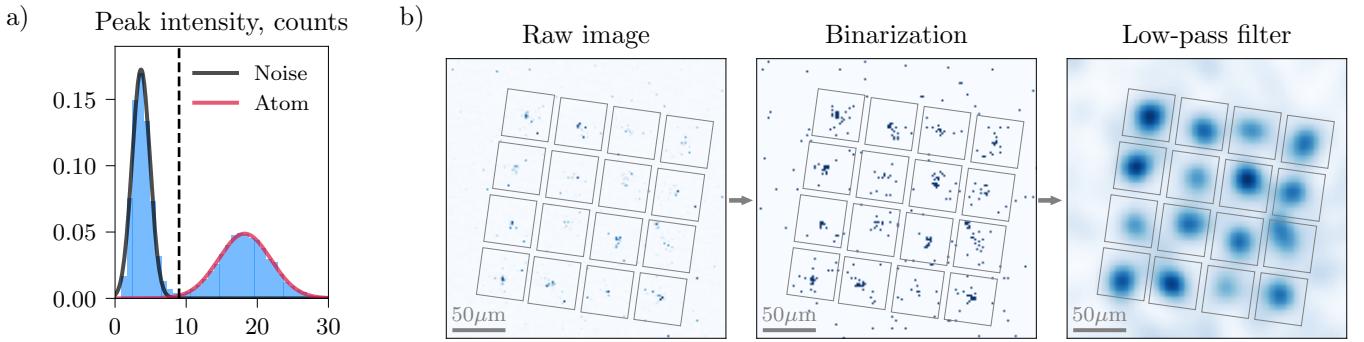


Figure 2: **Single-atom identification and image processing.** a) Histogram of peak intensities extracted from binarized and low-pass filtered images shows a bimodal distribution: the first peak corresponds to camera noise (black), the second corresponds to single atoms (red). The dashed line indicates the threshold used for atom identification. b) Image processing pipeline: Raw fluorescence image (left), binarization by intensity thresholding (center), and application of a low-pass filter (right) to reveal spatially localized atomic signals.

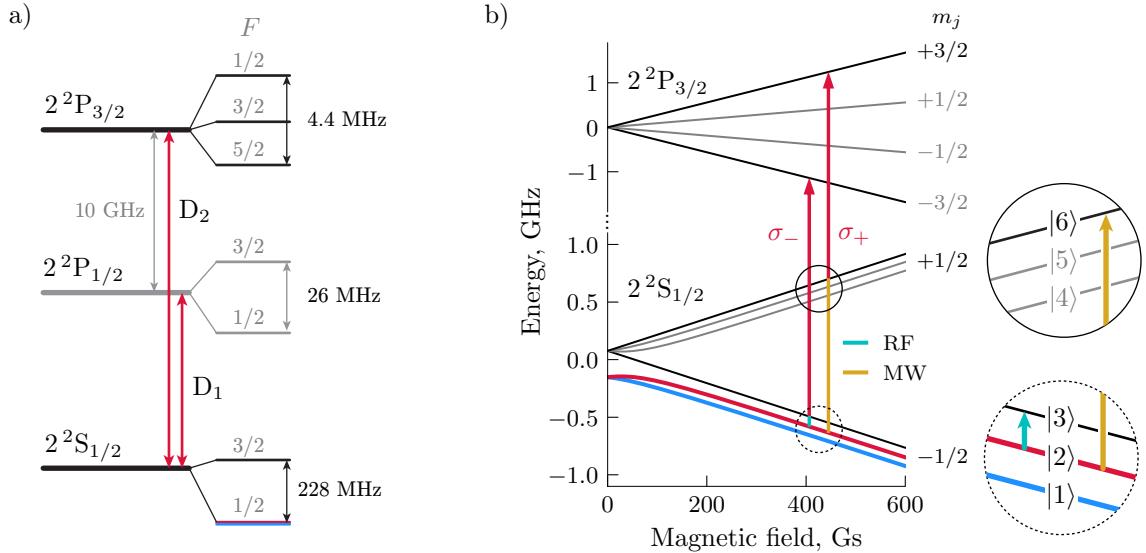


Figure 3: ${}^6\text{Li}$ energy levels. a) Level diagram of the ground and excited states of ${}^6\text{Li}$ [6], including the D₁ and D₂ transitions around $\lambda = 671$ nm. b) Zeeman splitting of the hyperfine levels of the $2^2\text{S}_{1/2}$ and $2^2\text{P}_{2/2}$ in ${}^6\text{Li}$ [12, 15]. As different spin states for physics we consider state $|1\rangle$ and $|2\rangle$, but for imaging it is worth to flip them to stretched states $|6\rangle$, $|3\rangle$. Colored lines indicate transitions driven by radiofrequency (RF) and microwave (MW) fields.

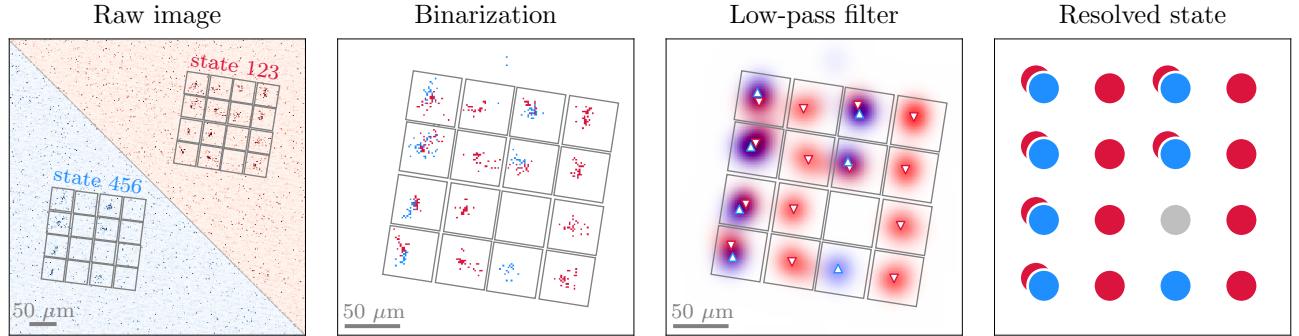


Figure 4: Spin-resolved single-atom imaging. Spatially separated σ_+ and σ_- fluorescence is imaged onto two distinct regions of the camera. The binarization step identifies photon counts above a threshold, followed by a low-pass filter to extract spatially localized signals. Final spin states are assigned based on relative signal strength in each channel: ● – $|1\rangle$, ● – $|2\rangle$, ● – no atom.

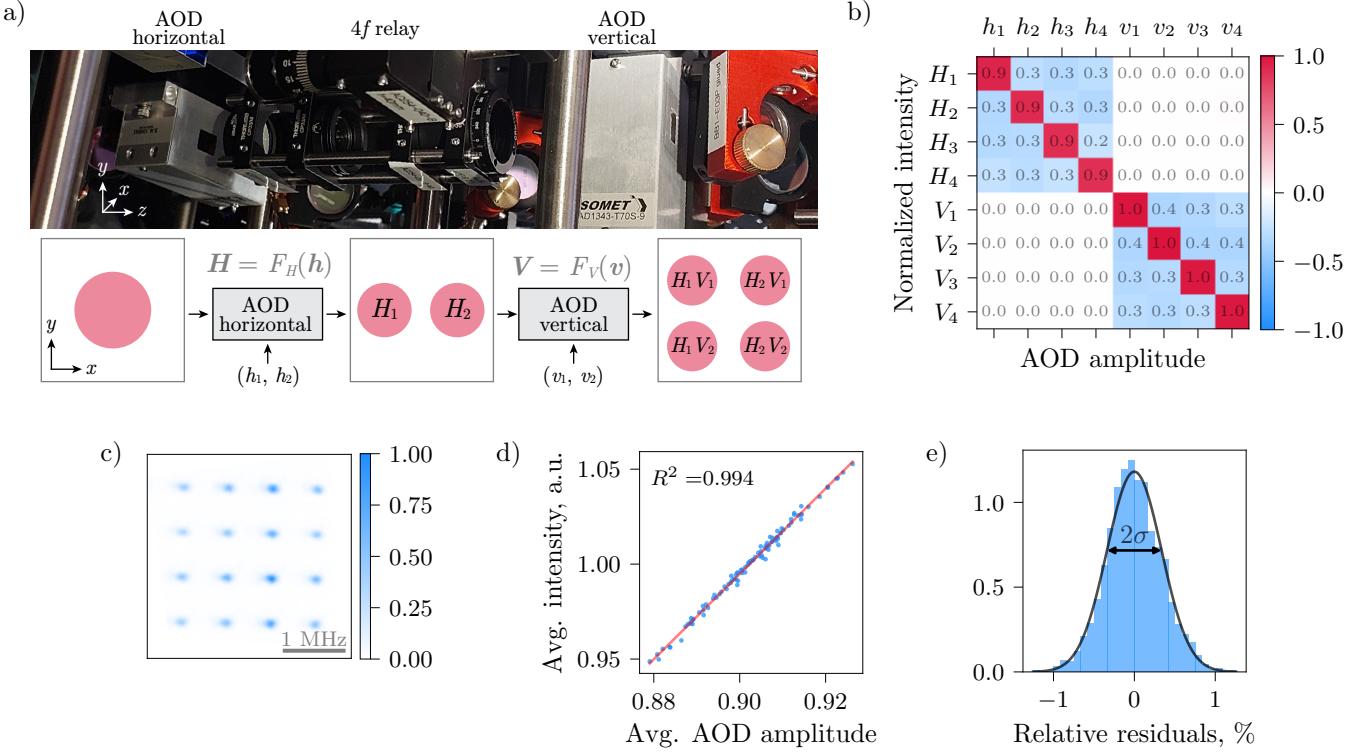


Figure 5: **Tweezer array control using orthogonal AODs.** (a) Experimental setup: two orthogonal AODs generate a 2D tweezer array. The applied harmonic amplitudes h_i , v_j define the output intensities $H_i = F_H(h)$ and $V_j = F_V(v)$ in horizontal and vertical directions, respectively. (b) Crosstalk matrix F' reconstructed via linear regression from camera images, showing how modulation of one harmonic affects others. (c) Example of measured intensity distribution at uniform input amplitudes ($h_i = v_j = 0.9$), illustrating imbalance in the resulting pattern. (d) The total intensity $\Lambda = \sum_{ij} H_i V_j$ scales linearly with the mean input amplitude. (e) Residuals of the linear model (fitted in the range [0.85, 0.95]) are normally distributed. All data were obtained in this work using direct camera-based measurements.

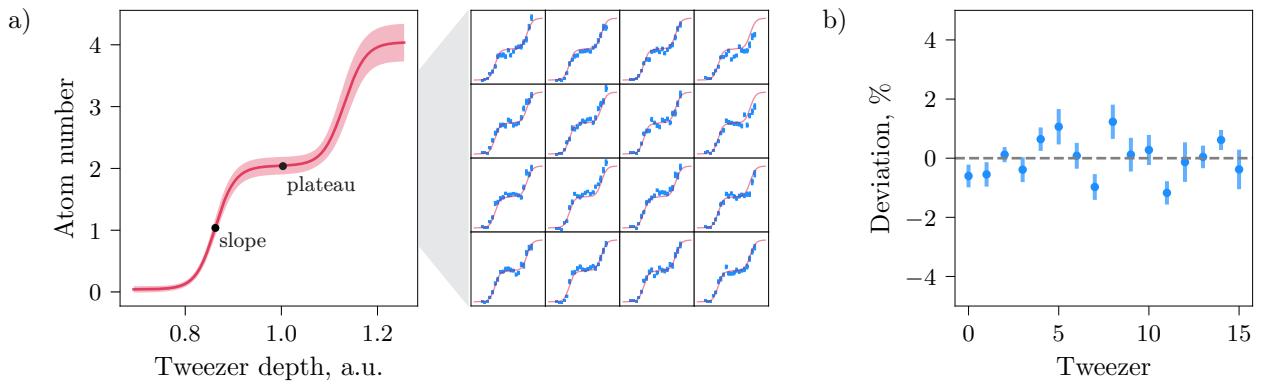


Figure 6: **Step plot.** (a) Atom number as a function of tweezer depth during the spilling sequence. Step plots for each tweezer in the 4×4 array are shown on the right. The average fit is shown as a solid red line, with standard deviation across sites indicated by the shaded area. (b) Relative deviation of the fitted sigmoid centers for each tweezer after SVF balancing. The standard deviation is $0.7(2)\%$, which is well within the plateau width ($\pm 5\%$), ensuring sufficient uniformity for array-wide spilling. $\chi^2?$

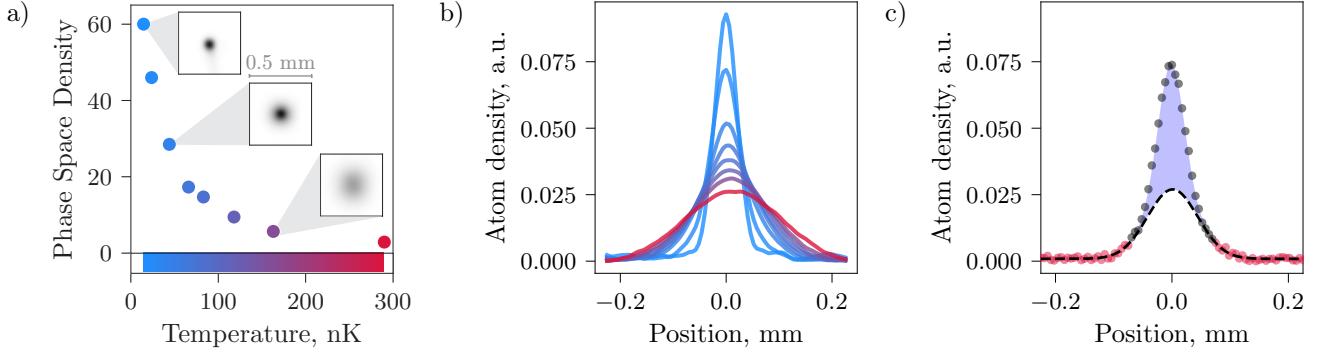


Figure 7: **Molecular Bose-Einstein condensate data.** (a) Phase space density (PSD) increases as temperature decreases via evaporative cooling, indicating condensation onset. (b) Atom density profiles normalized to unit area; color encodes temperature as in (a). (c) At low temperature, the profile shows a bimodal shape: a Gaussian fit to thermal wings (red dots) underestimates the central peak, revealing the mBEC component (blue area).

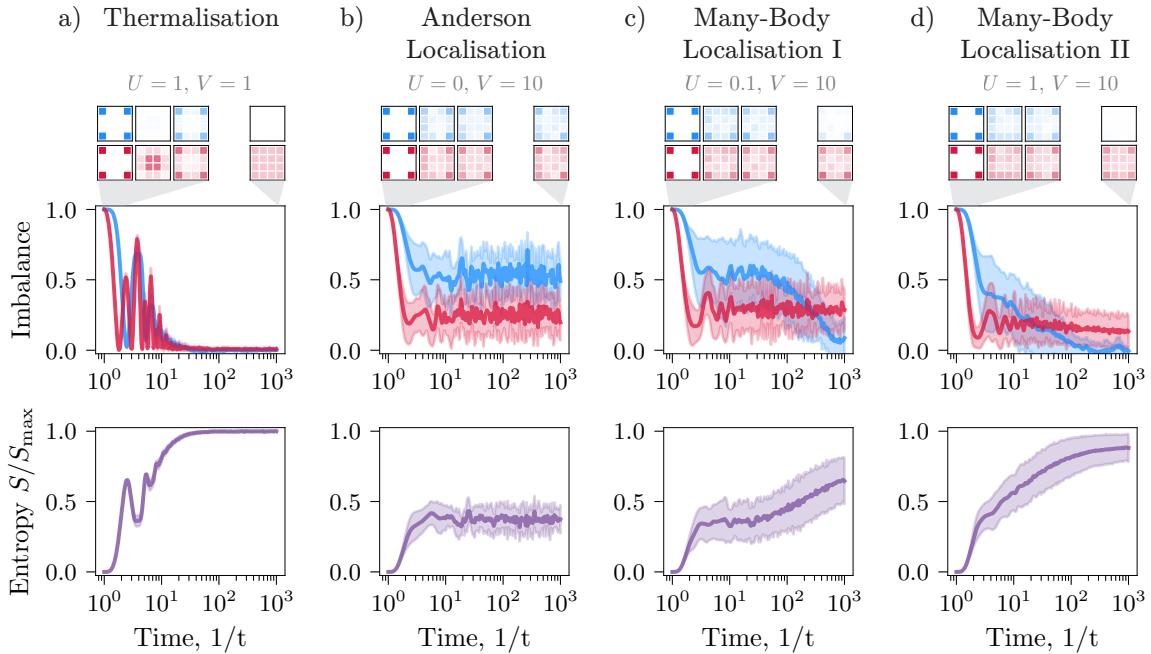


Figure 8: **Dynamical phases in a 2D Fermi-Hubbard system.** *Top:* Snapshots of particle density (red) and magnetization magnitude (blue). *Middle:* Time evolution of density imbalance (red) between corners and bulk, and subsystem magnetization (blue). *Bottom:* Normalized entanglement entropy evolution. All results are averaged over 10 noise realizations; shaded areas indicate standard deviation across realizations. Data were obtained using ED.

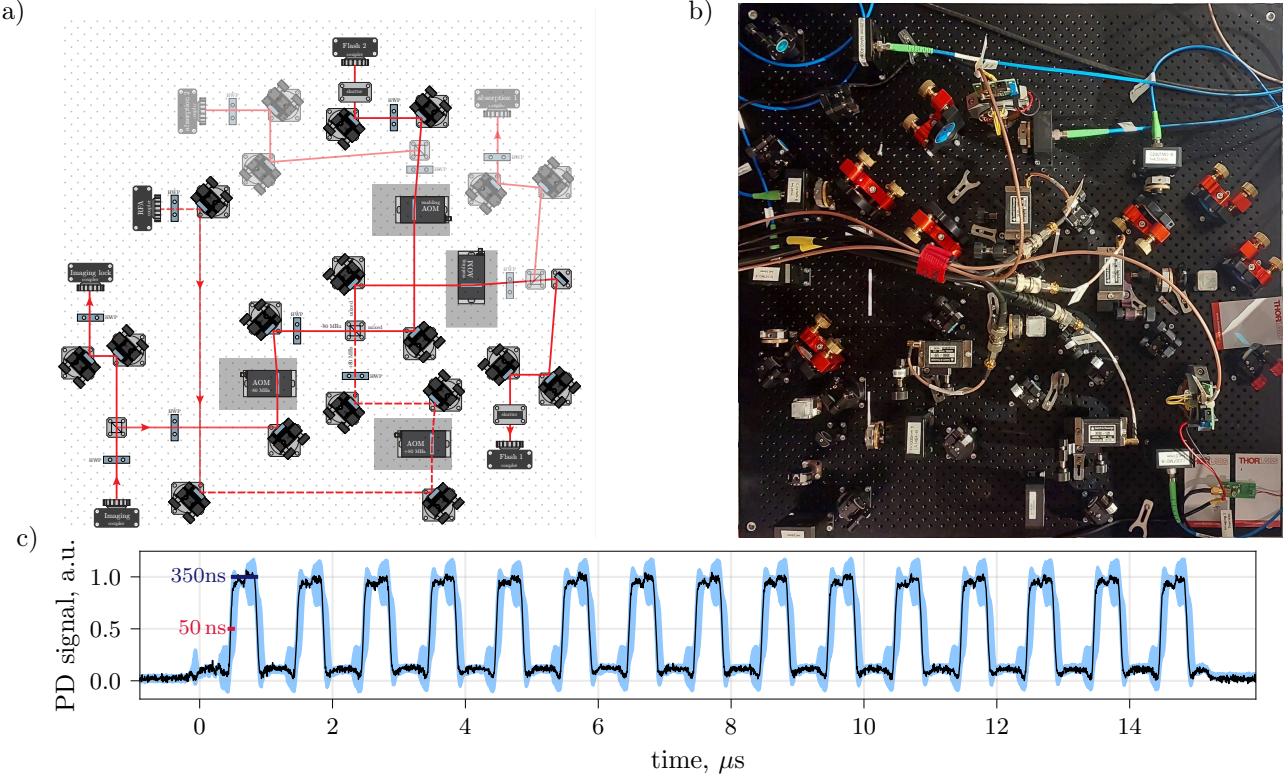


Figure 9: **Distribution board for flashing.** a) Optical layout of the board used to combine and control light for free-space imaging states $|3\rangle$ and $|6\rangle$. b) Experimental implementation. c) PD signal of the flashing measured on an oscilloscope (black – a single experimental run, blue – the standard deviation over 20 runs, red – rise time).

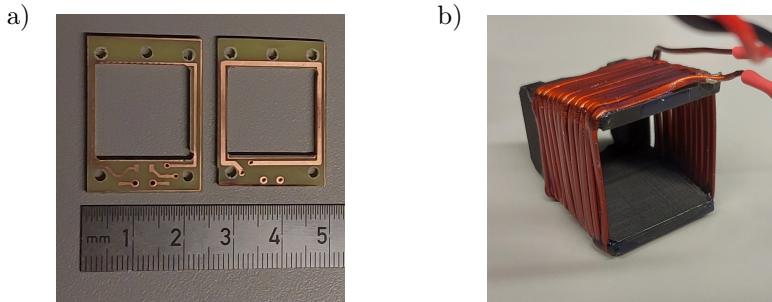


Figure 10: **RF and MW antennas used for spin control.** (a) PCB-based radiofrequency (RF) antenna used for driving spin transitions at MHz frequencies. (b) Microwave (MW) loop antenna, designed to efficiently couple to hyperfine transitions in ${}^6\text{Li}$. These antennas are used for coherent spin manipulation.

1 Introduction

1.1 Quantum Simulation with Fermionic Tweezer Arrays

The simulation of strongly correlated quantum systems remains one of the central challenges in modern physics. While exact numerical methods have provided deep insights in one-dimensional settings, the computational cost of simulating many-body dynamics in higher dimensions grows exponentially with system size, rendering classical approaches impractical. As first envisioned by Feynman, this motivates the development of physical quantum simulators that emulate target Hamiltonians using intrinsically quantum mechanical systems. Among several available platforms, ultracold atoms in optical potentials offer an exceptionally clean and versatile environment for realizing a broad range of many-body models, including the Fermi-Hubbard model relevant for high-temperature superconductivity [5, 7].

In particular, fermionic atoms loaded into optical lattices have enabled the realization of the two-dimensional Fermi-Hubbard model, with site-resolved imaging revealing spin correlations and signatures of antiferromagnetic ordering [11, 1]. These advances highlight the power of quantum gas microscopy in exploring equilibrium properties of lattice fermions. However, conventional approaches rely on thermal loading of large ensembles into periodic potentials, which often results in uncontrolled entropy and random filling defects. As a consequence, the system is typically initialized in a thermal ensemble, and the preparation of arbitrary low-entropy many-body states remains difficult.

Optical tweezer arrays offer an alternative, bottom-up approach. By providing single-site control, they allow deterministic preparation of initial states, flexible geometries, and site-selective addressing. While initially developed in the context of Rydberg atom arrays [2], these platforms have recently been extended to degenerate fermions, enabling programmable few-body Fermi-Hubbard dynamics [13, 14]. Such results position tweezer arrays as a promising architecture for scalable fermionic quantum simulators.

This thesis contributes to the development of a quantum simulation platform based on ultracold fermionic ${}^6\text{Li}$ atoms in a two-dimensional optical tweezer array. In this approach, the array is used for high-fidelity state preparation and control, while the optical lattice serves as the environment for Hamiltonian evolution. Compared to direct loading into a lattice, this separation of initialization and dynamics enables more efficient cooling, deterministic control over occupation patterns, and reduced cycle times. To support this workflow, we develop methods for spin-resolved free-space imaging, arbitrary pattern initialization via spin-selective spilling, and precise tweezer depth balancing.

Looking ahead, such a platform opens the door to nonequilibrium quantum dynamics. For instance, by performing randomized local operations followed by spin-resolved measurements, one can access entanglement entropy via measurement statistics [3]. These protocols offer a practical way to characterize entanglement growth and scrambling, even in regimes where full state tomography is infeasible. Extending such techniques to fermionic systems will provide new insights into thermalization, localization, and quantum information dynamics in strongly correlated matter.

In summary, this work supports the realization of a bottom-up fermionic quantum simulator by combining deterministic state preparation with single-atom, spin-resolved readout. These tools provide a foundation for studying both static and dynamical aspects of the Fermi-Hubbard model in a highly controlled setting.

1.2 Thesis Outline

This thesis describes the development of experimental and computational tools for the preparation and probing of fermionic many-body states in a programmable optical tweezer array. The overarching goal is to enable bottom-up quantum simulation of lattice models, with precise control over initial conditions and single-atom, spin-resolved readout.

Sec. 2 presents the implementation of spin-resolved single-atom imaging of ${}^6\text{Li}$ in free space. The section describes the optical layout, the image processing pipeline, and introduces the Su-Schrieffer-Heeger model as a conceptual framework for understanding spin-dependent imaging dynamics.

Sec. 3 focuses on the creation and control of two-dimensional tweezer arrays. The section begins with the optical setup and AOD control, followed by a detailed discussion of calibration procedures and tweezer depth balancing using both camera-based and atom-based feedback. A key result is the development of a spin-selective spilling technique, enabling the preparation of spin- and site-resolved occupation patterns. Arbitrary configurations are realized through iterative removal steps, formalized via boolean matrix factorization.

Sec. 4 introduces the concept of a matter-wave magnifier—a lensing scheme designed to enhance spatial resolution for future lattice imaging. Although not yet implemented experimentally, fast simulations of wavefunction propagation and Monte Carlo sampling are presented to validate the scheme.

Finally, Sec. 5 outlines numerical approaches for simulating Fermi-Hubbard dynamics on small lattices. The computational framework combines exact diagonalization and Krylov-based time evolution, accelerated on GPU hardware. These tools enable simulations of dynamics in the presence of noise and disorder, and serve as a theoretical reference for upcoming experimental investigations.

2 Single-atom spin-resolved free-space imaging

2.1 Motivation and background

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2.2 Experimental setup

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2.3 Image processing

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2.4 Spin-resolved detection

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3 Tweezer array system

3.1 Motivation and concept

Optical tweezer arrays provide a flexible platform for preparing ultracold atomic systems with site-resolved control. By trapping individual atoms in focused laser beams, it becomes possible to initialize many-body states with controlled geometry, low entropy, and tunable local parameters.

In our setup, a two-dimensional array of tweezers is created using two orthogonal acousto-optic deflectors (AODs). Each AOD diffracts multiple beams along one axis, and their intersection forms the full array. The resulting intensity at site (i, j) factorizes as $P_{ij} = H_i V_j$, where H_i and V_j are set by the drive amplitudes applied to each AOD. This structure simplifies calibration and allows fast control of the entire array using a small number of parameters.

Compared to holographic or lattice-based approaches, the AOD system offers rapid reconfigurability and independent control of individual traps. This enables preparation of custom spin and density patterns, as well as dynamic manipulation during the experimental sequence. Such capabilities are useful, for example, for initializing specific configurations, removing defects, or performing spatially selective operations before loading atoms into an optical lattice for further evolution.

AOD operation. Each AOD consists of a crystal driven by a piezoelectric transducer. An incoming laser beam $(\mathbf{k}_{\text{in}}, \omega_{\text{in}})$ interacts with the induced acoustic wave (\mathbf{q}, Ω) via Bragg diffraction, producing an outgoing beam $(\mathbf{k}_{\text{out}}, \omega_{\text{out}})$:

$$\mathbf{k}_{\text{out}} = \mathbf{k}_{\text{in}} + \mathbf{q}, \quad \omega_{\text{out}} = \omega_{\text{in}} + \Omega.$$

The frequency Ω determines the deflection angle θ through the Bragg condition, while the amplitude of the RF signal controls the diffracted optical power. Each RF tone can be described by a triple $(\Omega_j, a_j, \varphi_j)$, corresponding to its frequency, amplitude, and phase. Applying a set of such tones to an AOD results in a superposition of multiple diffracted beams, with the amplitude a_j determining the power in each beam and the phase φ_j influencing their relative coherence.

Factorized intensity distribution. To create 2D arrays, we combine two AODs oriented along orthogonal axes (fig. 5a), as described in [4]. Each axis is driven by a set of RF tones. In the paraxial approximation, the resulting 2D intensity pattern can be written as a rank-1 product of two vectors:

$$P_{ij} = H_i V_j,$$

where H_i and V_j correspond to the powers of individual beams generated by the horizontal and vertical AOD, respectively. The factorization of the output power can be verified via:

$$P \stackrel{\text{SVD}}{=} \sum_r \Lambda_r \mathbf{H}_r \mathbf{V}_r^T, \quad \text{factorisability} = \Lambda_0 / \sum_r \Lambda_r \quad (1)$$

which provides a natural measure of factorization. For arrays ranging from 2×2 to 10×10 , the factorisability measure $\Lambda_0 / \sum_r \Lambda_r$ is consistently above 0.99. For the 4×4 array used in most of our experiments, we obtain a typical value of 0.997(1).

Tweezer array control. The tweezer output beam powers are nonlinear functions of the input amplitudes \mathbf{a} :

$$P_j = F_j(\mathbf{a}) = \cancel{F_j(\mathbf{0})} + F'_{ji} a_i + \frac{1}{2} F''_{jii_2} a_{i_1} a_{i_2} + \dots \quad (2)$$

The goal is to control¹ the full matrix P_{ij} using only two sets of parameters: horizontal amplitudes \mathbf{h} and vertical amplitudes \mathbf{v} .

It will later be necessary to reconstruct (H_i, V_j) from measured intensity distribution P_{ij} , so it is convenient to choose a factorized model $P_{ij} = \Lambda H_i V_j$, with the normalization:

$$\sum_i H_i = \sum_j V_j = 1, \quad \sum_{ij} P_{ij} = \Lambda.$$

This allows for an explicit decomposition:

$$\frac{1}{\Lambda} \sum_j P_{ij} = H_i \sum_j V_j = H_i, \quad \frac{1}{\Lambda} \sum_i P_{ij} = V_j \sum_i H_i = V_j, \quad \sum_{ij} P_{ij} = \Lambda, \quad (3)$$

which is fully equivalent to a rank-1 SVD of P_{ij} .

It is worth noting that, as shown in Fig. 5d, the total scale parameter Λ defined in this way is proportional to the total input amplitude, $a_{\text{sum}} = \sum_j h_j + \sum_j v_j$. This effectively decouples local balancing from global power constraints, simplifying the control problem.

3.2 Optical setup

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¹For amplitudes in the range $a_i \in [0.7, 1.0]$, we find that a linear or quadratic approximation suffices. In practice, we reconstruct the Jacobian matrix F'_{ji} using camera-based calibration, as discussed in Sec. 3.4.

3.3 State preparation

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3.4 Control

Frequency to position mapping. To extract the local intensities P_{ij} from camera images, we need to determine which pixels correspond to which tweezer sites. For this purpose, we define an affine transformation from the drive frequency space ($\omega_{\text{hor}}, \omega_{\text{ver}}$) to image plane coordinates (x, y) :

$$\mathbf{r} = H\boldsymbol{\omega}, \quad \Leftrightarrow \quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \end{pmatrix} \begin{pmatrix} \omega_{\text{hor}} \\ \omega_{\text{ver}} \\ 1 \end{pmatrix}.$$

Here, H is a 2×3 matrix calibrated from a set of measured spot positions. For example, one can measure \mathbf{r}_j for random frequency vectors $\boldsymbol{\omega}_j \in [\omega_{\min}, \omega_{\max}]$, construct the matrices ω_{ij} with $i \in \{\text{hor}, \text{ver}\}$ and r_{ij} with $i \in \{x, y\}$, and solve the least-squares problem:

$$r = H\boldsymbol{\omega}, \quad \Rightarrow \quad r\boldsymbol{\omega}^T = H\boldsymbol{\omega}\boldsymbol{\omega}^T \quad \Rightarrow \quad r\boldsymbol{\omega}^T (\boldsymbol{\omega}\boldsymbol{\omega}^T)^{-1} = H. \quad (4)$$

This transformation defines a region of interest around each tweezer, within which we compute the integrated pixel intensity after background subtraction. The resulting values are proportional to the optical powers P_{ij} .

Linear reconstruction. The mapping from input amplitudes \mathbf{a} to optical power is approximated by Eq. (2). In the regime $a_i \in [0.85, 0.95]$, a linear approximation is sufficient². We construct the Jacobian matrix F_{ji} by fitting a linear regression model to a dataset of amplitude–intensity pairs. The resulting crosstalk matrix is shown in Fig. 5b. It is approximately diagonal, with comparable diagonal entries and off-diagonal elements typically reaching up to 30% in magnitude relative to the diagonal, due to power redistribution between neighboring tones. Crosstalk between the horizontal and vertical AODs remains negligible.

The quality of the linear fit for the 4×4 array is illustrated in Fig. 5e. The total intensity (Fig. 5d) scales linearly with the average input amplitude, yielding $R^2 > 0.99$. Relative residuals are normally distributed with width 0.3%, confirming the applicability of the model in this range.

Power-aware optimization. In the presence of limited laser power and finite AOM diffraction efficiency, we prefer solutions where all amplitudes remain close to 1. This preference can be incorporated into the optimization objective. In addition to minimizing intensity imbalance, we penalize deviations of the average amplitudes from a target value (e.g., 0.9).

3.5 Tweezer depth balancing

Precise control over the depth of each optical tweezer is essential for preparing few-fermion systems via spilling techniques. In our setup, each tweezer is initially loaded with approximately 100 atoms, which are then selectively removed by ramping down the potential depth. The number of remaining atoms as a function of spill power x_{sp} exhibits a quantized staircase structure, reflecting the discrete energy levels of the 1D harmonic oscillator. This behavior can be characterized by a step plot [9].

Step plot. To characterize this behavior in our tweezer array, we measure step plots for all sites simultaneously. Figure 6a shows the result for a 4×4 array. For each value of x_{sp} , we acquire 70 experimental realizations and compute the average photon signal per site. This signal serves as a robust proxy for atom number. In contrast to single-atom counting, this approach is parameter-free and effective even for large initial occupancies.

Uniformity characterization. To quantify depth inhomogeneity across the array, we fit each step trace with a sigmoid function:

$$\text{sigmoid}(x) = \frac{A_j}{1 + \exp(-(x - x_j)/\sigma_j)},$$

where x_j denotes the center of the step and σ_j its width for tweezer j . We define a relative uniformity metric as $\text{std}(x_j)/\langle x_j \rangle$. After camera-based balancing (Sec. 3.4), this metric typically yields $\sim 3\%$, which is insufficient for deterministic preparation across the array. A more precise balancing procedure is therefore required.

Single-value feedback. To further improve uniformity, we apply an iterative atom-based feedback scheme. Rather than fitting full step plots, we operate at a single point on the slope of the transition, near the half-filling level $A_j/2$. At this point, the sigmoid can be approximated by a linear response:

$$\text{sigmoid}(x) \approx \frac{A_j}{4\sigma_j}x - \frac{A_j x_j}{4\sigma_j},$$

²For wider amplitude ranges, higher-order terms can be added to the model. However, this is unnecessary in the present context.

assuming $x_j \gg \sigma_j$. In the feedback loop, we do not use the fitted sigmoid parameters directly. Instead, we measure a single photon-count matrix M_{ij} and treat it as a linear proxy for the power matrix P_{ij} . Since the sigmoid offset shift = $A_j x_j / (4\sigma_j)$ is known from the fits, we approximate:

$$M_{ij} + \text{shift} \propto P_{ij} = \Lambda H_i V_j.$$

We then factorize this matrix using the method introduced in (3) and update the amplitudes according to:

$$h \rightarrow h + \gamma(H - H_0), \quad v \rightarrow v + \gamma(V - V_0),$$

where (H_0, V_0) is the target point, and γ is the feedback rate. This model-free procedure avoids full sigmoid fitting and operates directly on experimental measurements.

Figure 6b shows the result of applying this single-value feedback (SVF) protocol to a 4×4 array. After five iterations, the relative deviation of the fitted step centers is reduced to 0.7(2)%, well within the plateau width (typically $\pm 5\%$), enabling deterministic state preparation across the full array. [Add fig. with single value feedback progress.](#)

3.6 Spin-selective spilling

After balancing the tweezer depths and performing standard spilling to prepare unit filling (one atom in state $|1\rangle$ and one in $|2\rangle$ per site), atoms in a selected spin state can be selectively removed while leaving the other unaffected. This enables the preparation of arbitrary spin-resolved configurations, a crucial ingredient for bottom-up simulation of spinful many-body systems.

Magnetic-field dependence. The key idea relies on the difference in magnetic moments between the hyperfine ground states. As shown in Fig. 3b, the energy of state $|2\rangle$ exhibits a maximum near 27 G, where its magnetic moment vanishes: $\mu_{|2\rangle} = \partial E / \partial B = 0$. In contrast, state $|1\rangle$ has a sizable negative magnetic moment at this field ([write down value](#)). As a result, when a magnetic field gradient is applied at $B = 27$ G, only atoms in state $|1\rangle$ experience a significant force and are spilled from the traps.

Spin-selective removal. The experiment starts with a $|1\rangle-|2\rangle$ spin mixture at unit filling in each tweezer. The magnetic field is ramped to 27 G, and a field gradient is applied. This results in spin-selective spilling: atoms in state $|1\rangle$ are removed, while those in $|2\rangle$ remain confined.

The crossed AOD configuration enables control over the local optical power P_{ij} through factorized amplitudes, such that $P_{ij} = H_i V_j$. This allows us to define arbitrary rank-1 intensity masks and thus selectively apply spilling to specific subsets of sites.

This protocol enables single-shot removal of state $|1\rangle$ atoms without perturbing state $|2\rangle$, providing a flexible method for initializing spin-imbalanced or spatially patterned states. The performance of the method in terms of selectivity and overall fidelity is summarized in [Fig. ?](#). Sequential applications of such steps to prepare arbitrary configurations are discussed in Sec. 3.7.

3.7 Arbitrary occupation loading

The ability to prepare arbitrary atom configurations is a key ingredient for bottom-up quantum simulation. After obtaining unit filling for both spin states (as discussed in Sec. 3.5 and 3.6), we implement a multi-stage spilling sequence that enables spin- and site-resolved initialization of arbitrary patterns.

The loading sequence proceeds in several steps:

1. Prepare a $|1\rangle-|2\rangle$ spin mixture with unit filling across the tweezer array.
2. Perform global spilling steps to remove atoms from factorized intensity patterns P_{ij} , affecting both spin states.
3. Apply spin-selective spilling steps to remove atoms in state $|1\rangle$ from additional factorized subsets of sites.
4. Flip the remaining atoms $|1\rangle \leftrightarrow |2\rangle$ using a microwave π -pulse.
5. Repeat spin-selective spilling to further refine the configuration

Factorized removal. Each spilling step removes atoms from sites where the local tweezer depth P_{ij} falls below a certain threshold. Since we can impose any rank-1 intensity mask $P_{ij} = H_i V_j$, it is possible to tailor the removal region to arbitrary product forms. To remove a single atom at site (i', j') , for example, we reduce $H_{i'}$ and $V_{j'}$ by a factor $\eta < 1$ and simultaneously increase the global power by η . This yields a relative intensity of $1/\eta$ at the intersection, while leaving all other sites unchanged or increased in depth. In this way, we can reliably isolate and remove atoms from any desired factorized subset.

Boolean decomposition. We represent the cumulative removal pattern as a binary matrix W_{ij} , where $W_{ij} = 1$ indicates that the atom at site (i, j) has been removed. Each spilling operation adds a binary outer product $u_i^\lambda v_j^\lambda$ via

Boolean logic ($1 + 1 = 1$). An arbitrary target pattern can therefore be reached through a sequence of such operations:

$$W_{ij} = \bigvee_{\lambda=1}^r u_i^\lambda v_j^\lambda, \quad (5)$$

which defines the exact Boolean matrix factorization (EBMF) of the removal matrix. In the worst case, any binary $n \times n$ matrix admits such a decomposition using at most n steps.

Optimal EBMF. While a naive strategy—such as row- or column-wise removal—may require up to n iterations, we find that optimal EBMF often reduces this number. The problem of finding an exact Boolean matrix factorization with minimal rank is known to be NP-complete [10] and NP-hard to approximate [8]. Nevertheless, for arrays up to 10×10 , optimal decompositions can be computed in a few seconds using a SAT solver. These improvements are particularly useful for minimizing experimental cycle time and improving overall sequence fidelity. A full discussion of the EBMF algorithm and its implementation is presented in Appendix.

4 Matter-wave magnifier

4.1 Motivation and overview

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4.2 Numerical modeling

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5 Fermi-Hubbard model: numerical approaches and simulations

5.1 Motivation and experimental relevance

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5.2 Theoretical and numerical methods

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5.3 GPU-accelerated implementation

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5.4 Simulations of noisy dynamics

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6 Appendix

6.1 Boolean matrix factorization

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