Theory background on topic «Chaos optical communication»

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And God said let there be light, and there was light

$$\triangle E(\boldsymbol{r},t) - \mu_0 \varepsilon(\boldsymbol{r}) \frac{\partial^2}{\partial t^2} E(\boldsymbol{r},t) = \mu_0 \frac{\partial^2}{\partial t^2} P_{in}(\boldsymbol{r},t) \tag{1}$$

Which is solved as

$$E(\boldsymbol{r},t) = \frac{1}{2} E_0 e^{i(\beta y - \omega t)} + \mathrm{const}$$

And we assume:

$$E(\boldsymbol{r},t) = A(t)E(s').$$

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where A states for for both amplitude and phase. And E(s') eigenmode function.
For most semiconductor lasers waveguides, the lasing action occurs in eigenmode. Also we use slow wave variation.

$$|\frac{d^2 A}{dt^2}| \ll \omega |\frac{dA}{dt}|. \tag{3}$$

$$\frac{dA}{dt} = \frac{i\omega}{2} A \sum_{\alpha} \frac{1}{V(\alpha)} \frac{\int \zeta(\boldsymbol{r}, \alpha)|E(s)|^2 ds}{\int \varepsilon(s)|E(s)|^2 ds} \times |\mu(\alpha)|^2 \frac{[\rho_{ee} + \rho_{hh}(\alpha) - 1]}{(E - E_\alpha) + iE_{T^2}}, \tag{4}$$

where ρ_{ee} – distribution of electrons, and ρ_{hh} – distribution of holes. It's time to oversimplify this monster:

$$\frac{dA}{dt} = \frac{i\omega}{2\varepsilon n_r^2} A \sum_{\alpha} \Gamma_{MD} \frac{1}{V_{MD}} |\mu(\alpha)|^2 [\rho_{ee} + \rho_{hh} - 1], \tag{5}$$

where n_r – refractive index of the active region. And Γ_{MD} – dimensional coupling factor that shows how injected carriers interact with photons.

For the most beauty we assume that well is 1D, and the equastion:

$$\frac{dA}{dt} = \frac{1}{2} v_g (\Gamma_{MD} G - i \Gamma_{MD} N_r) A, \tag{6}$$

where $v_g = c/n_r$ is the speed of light in vacuum. And $\Gamma_{MD} G = g$ is so called gain coefficient.

If we obtain photon density inside active region as:

$$P = \frac{1}{2}\varepsilon n_r^2 |AE(0)|^2/E.$$

And from previous equation we get

$$\frac{dP}{dt} = v_g g p P = v_g \Gamma_{MDGP} = -\gamma_C P + \Gamma g P. \tag{7}$$

And in the same way:

$$\frac{dN}{dt} = -v_g G(E) P = \frac{J}{ed}\left(1 + \frac{\xi P(t-\tau)}{P_0}\right) - \gamma_s N - gP. \tag{8}$$

So we have two main equations to describe our model:

$$\frac{dN}{dt} = -v_g G(E) P = \frac{J}{ed}\left(1 + \frac{\xi P(t-\tau)}{P_0}\right) - \gamma_s N - gP. \tag{9}$$

$$\frac{dP}{dt} = v_g gpP = v_g \Gamma_{MDGP} = -\gamma_C P + \Gamma gP. \tag{10}$$

where P – intracavity photon density, N – carrier density, τ – feedback delay time.

And nonlinearities lies in the optical gain function, wich, due to several articles can be explained as:

$$g \simeq= g_0 + g_n(N - N_0) + g_P(P - P_0). \tag{11}$$

All theory gathered from book: "Semiconductor Lasers I – Fundamentals" by Eli Kapon, Institute of Micro and Optoelectronics Department of Physics, Swiss (1999).

Laser equations with feedback

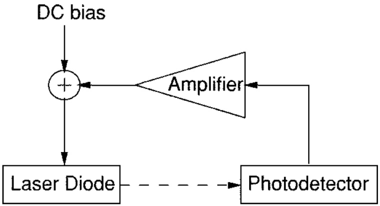

Figure 1: Schematic experimental setup

Laser – single-mode DFB laser diode (1300 nm);
Light output detector – high-speed InGaAs photodetector;

Important: it is suitable for any other semiconductor laser with an active medium.

Laser equations with feedback

The nonlinear behavior of a semiconductor laser with delayed opto-electronic feedback:

$$\frac{dS}{dt} = -\gamma_C S + \Gamma g\, S$$

$$\frac{dN}{dt} = \frac{J}{ed} - \gamma_s N - g\, S + \frac{J}{ed}\frac{\xi S[t-\tau]}{S_0}$$

$$g = g_0 + g_n(N - N_0) + g_p(S - S_0)$$

S – photon density;
N – carrier density;
τ – feedback delay;
g – opt. gain coeff.

positive opto-electronic feedback – $\xi > 0$.

Optical gain – measure of how well a medium amplifies photons by stimulated emission.

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Laser equations with feedback

Some transformations we can reduce this system to the

$$\frac{ds}{dt} = c_1 n(s+1) - c_2 s(s+1)$$
$$\frac{dn}{dt} = c_3 + c_4 s[t-\tau] + (c_5 s - c_8 n)(1+s) - c_6 n - c_7 s,$$

where $s = (S - S_0)/S_0$, $n = (N - N_0)/N_0$, which can be numerically integrated.

Numerical solution

Here $\hat{\tau} = \tau f_r$ – demensionless delay.

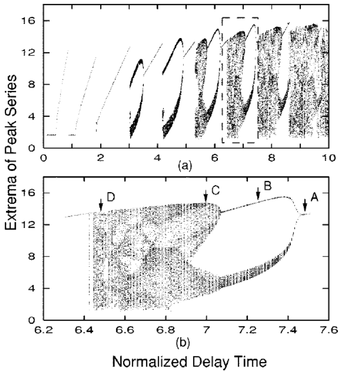

Figure 2: Bifurcation diagram of the extrema of the peak series

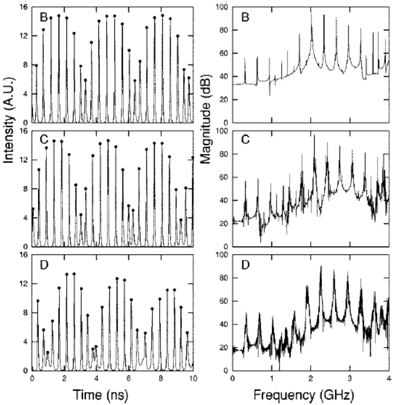

Figure 3: Time series and power spectra

Conclusions and thoughts

During this week of our research we have

- found the theory and learned origins of the equations used;
- investigated the parameters of gathered equations;
- searched applications of the setup used in article;
- agreed that this setup is most stable and accessable for the following work.

Reconstruction of the attractor

With part of system variables, it is possible to restore the general view of the attractor.

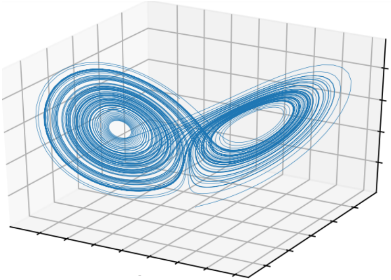

Figure 4: Original system attractor

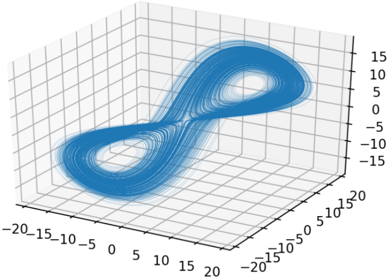

Figure 5: Reconstructed by one coordinate system attractor

Numerical features

- **Lyapunov exponent**:

$$\lambda(x_0) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln|f'(x_i)|,$$

where $\lambda > 0$ – characteristic for chaos.

- **Shannon Entropy**:

$$\mathrm{H}(X) = -\sum_{i=1}^{n} \mathrm{P}(x_i) \log \mathrm{P}(x_i),$$

where higher rates correspond to more random sources.

Applications

- **Random number generation.**
  Random bits produced at a much higher rate than other physical sources of entropy including quantum RNG.
- **Chaos computing.**
  It is possible to create "NOR" gate
  $*$ double-scrolled chaotic attractor and threshold function.