

# Optical chaos based on a laser diode with positive feedback

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# Goals

*Globally* we would like to transmit a high-frequency signal in encrypted form.

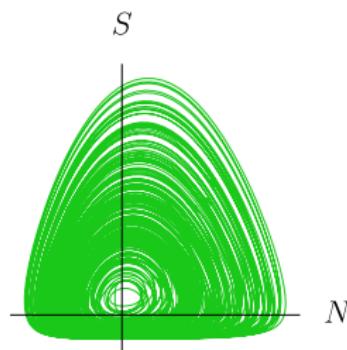
Here we will consider the following steps towards the goal:

- dynamical chaos and synchronization  
(to encrypt and decrypt signal);
- theory of the laser evolution and its adaptation under our needs;
- modeling theoretical equations and investigating chaos parameters;
- realization of the positive feedback in laser:  
theory, modeling and the experiment.

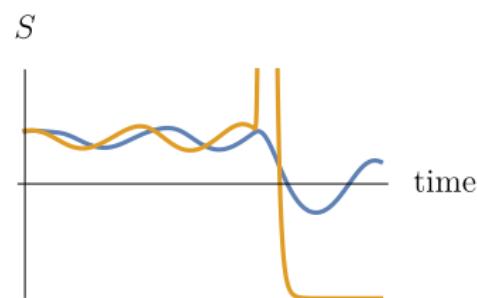
# Definition of dynamic chaos

Map<sup>1</sup>  $f$  is **chaotic**, if

- $f$  **sensitive** to the initial conditions.
- periodic orbits are dense everywhere;
- orbits are mixed;



Dense mixed orbits example



Sensitivity example

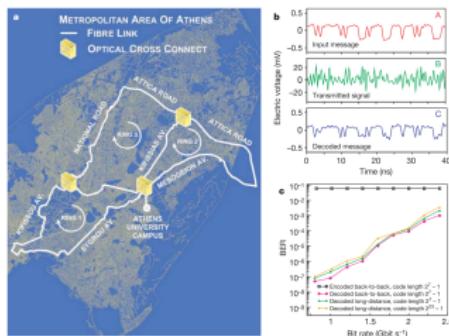
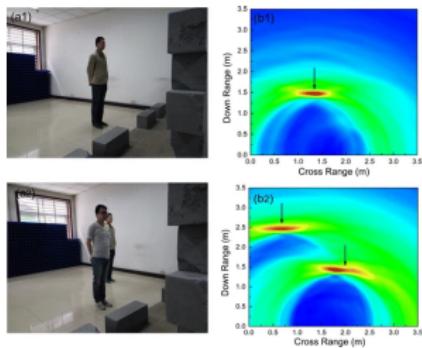
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<sup>1</sup>W. Hirsch, S. Smale, Introduction to Chaos.

# Applications of dynamic chaos

Possible applications:

- optical reflectometry;
- chaos radar;
- random numbers generation;
- signal encryption.



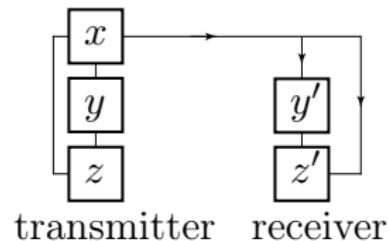
<sup>1</sup>Chaos Through-Wall Imaging Radar by Hang Xu et al. (2009)

<sup>2</sup>Chaos-based communications by Apostolos Argyris et al. (2005)

# Synchronization

Possible<sup>2</sup> synchronization of chaotic systems:

*enough* to transmit  
part of the signal;  
configure system parameters.



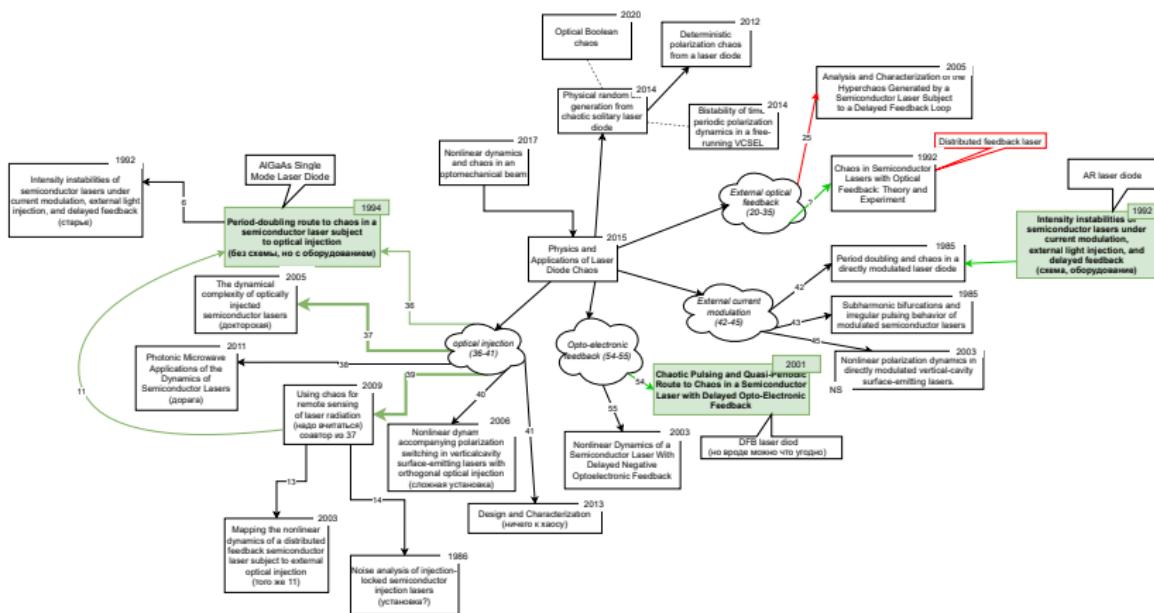
The use of optics to transmit the signal allows to achieve a greater bandwidth of the channel.

UHFO (ultrahight frequency oscillations) is a characteristic to optic systems.

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<sup>2</sup>M. Pecora, L. Carroll, Synchronization in Chaotic Systems, 1990.

From the root article a tree has grown



<sup>2</sup>Physics and Applications of Laser Diode Chaos by M. Sciamanna et al.

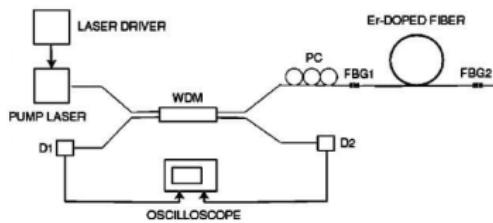
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Optical chaos

# Problems of implementations

## Pure Optical Feedback

- Frequency depends on laser
- Coupler and fibre is needed
- Precise instrumentations

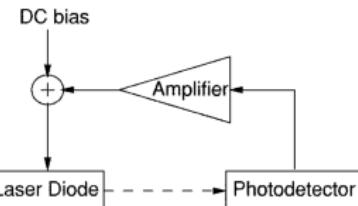
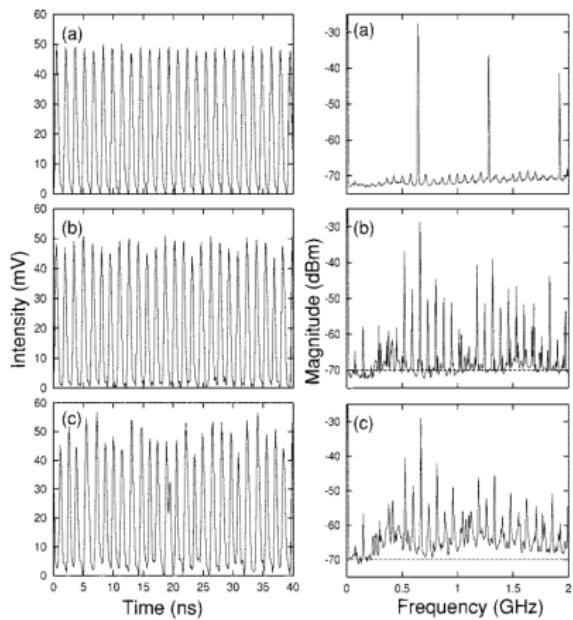


## Electro-Optical Feedback

- All relaxations must correlate
- Electrical amplifier and fibre is needed
- Well stabilization is desired

← "Optomechanically induced stochastic resonance and chaos transfer between optical fields" by Faraz Monifi

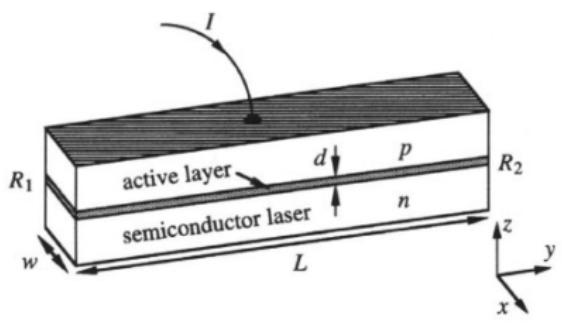
# The decided variant



**Figure 1:** Experimental results of the time series and power spectra of different pulsing states at different delay times. On a circuit dashed line represents optical path.

<sup>2</sup>Chaotic Pulsing and Quasi-Periodic Route to Chaos in a Semiconductor Laser with Delayed Opto-Electronic Feedback S. Tang and J. M. Liu (2001)

# The equations and ideas behind



Photon density

$$\frac{dS}{dt} = v_g \Gamma_{MD} G(E) S. \quad (1)$$

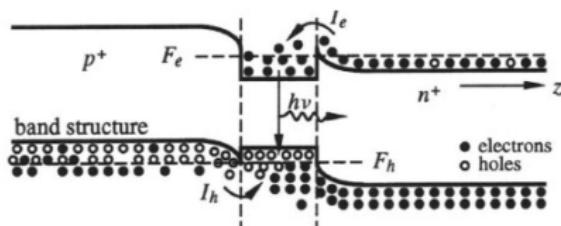
And also carrier density

$$\frac{dN_{MD}}{dt} = -v_g G(E) S. \quad (2)$$

Y-directed wave in active layer obeys the equation:

$$\Delta E(\mathbf{r}, t) - \mu_0 \varepsilon(\mathbf{r}) \frac{\partial^2}{\partial t^2} E(\mathbf{r}, t) = \mu_0 \frac{\partial^2}{\partial t^2} P_{in}(\mathbf{r}, t) \quad (3)$$

# Laser device geometry and induced polarization



In the active layer we obtain the Schrödinger equation:

$$H_{\text{crystal}} \Psi_n(\mathbf{r}) = \left[ \frac{[\mathbf{p} + e\mathbf{A}(\mathbf{r}, t)]^2}{2m_0} + U_p(\mathbf{r}) \right] \Psi_n(\mathbf{r}) \quad (4)$$

which will help us to state the induced polarization in an active region:

$$\mathcal{P}_{in}(\mathbf{r}, t) = - \sum \frac{\xi(\mathbf{r}, k_q, x, z)}{V(k_q, x, z)} [\rho_{eh}(k_q, x, z) \mu(k_q, x, z) + \text{const}] \quad (5)$$

# The concluding formulas

And from the previous equation we can describe the quantum well laser behavior<sup>3</sup>

$$\frac{dS}{dt} = \Gamma_{MD} G_0 v_g S - \frac{S}{\tau_p} \Rightarrow \frac{dS}{dt} = v_g g S(t) - \gamma_p S. \quad (6)$$

$$\frac{dN}{dt} = \frac{J}{e} - \frac{N}{\tau_n} - \Gamma_{MD} G_0 v_g S \Rightarrow \frac{dN}{dt} = \frac{J}{ed} - \gamma_s - gS. \quad (7)$$

for previously mentioned optical gain coefficient:

$$g = g_0 + g_n(N - N_0) + g_p(S - S_0). \quad (8)$$

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<sup>3</sup>Semiconductor Lasers I Fundamentals Ch.I by Bin Zhao, Amnon Yariv.

# Poincaré-Bendixson theorem

**Thr.** If there is no stationary points on the enclosed 2D region  $G$  and some trajectory exists  $\gamma \subset G$ , then  $\gamma$  is a closed loop path or tends to the closed one.

But there is still a hope for a chaos!

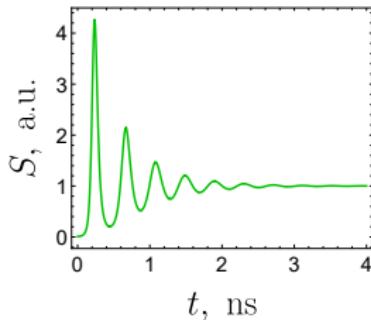
We add positive optoelectronic feedback in order to raise to 3D our equation

$$\frac{dS}{dt} = v_g S g(S, N) - \gamma_p S, \quad (9)$$

$$\frac{dN}{dt} = \frac{J}{ed} \left[ 1 + \frac{\xi S(t - \tau)}{S_0} \right] - \gamma_n N - S g(S, N). \quad (10)$$

# Laser parameters.

Unmodulated laser  
 $(\xi = 0)$ :



Laser equations:

$$\frac{dS}{dt} = -\gamma_c S + \Gamma g S$$
$$\frac{dN}{dt} = \frac{J}{ed} \left[ 1 + \frac{\xi S(t - \tau)}{S_0} \right] - \gamma_s N - g S$$

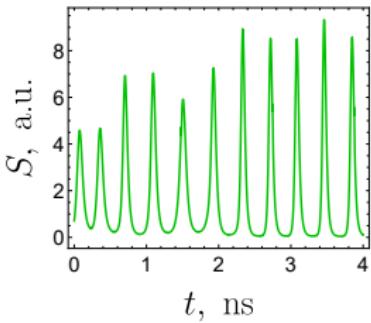
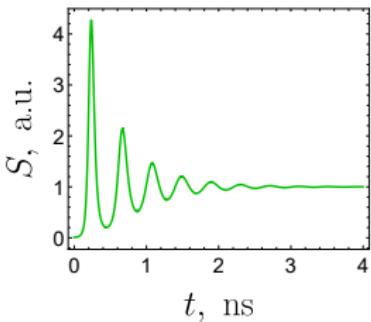
Systems parameters:

- $f_r$  – laser relaxation frequency. For our laser:

$$f_r = 1.4 \div 3.2 \text{ GHz.}$$

- $\tau$  – delay time.  $\tau \sim f_r^{-1} \sim 1\text{ns}$ .
- $\xi = 0.1$  feedback parameter.

# Chaos parameters.



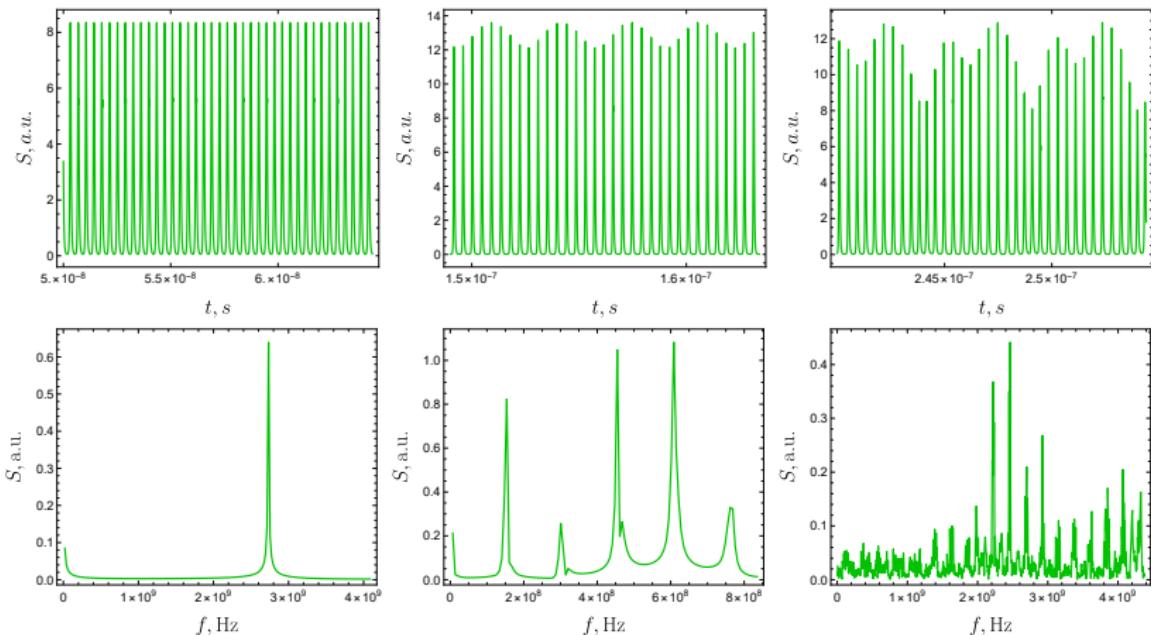
Chaos features:

- Unperiodic behaviour
- Frequency doubling scenario  
 $f \rightarrow 2f \rightarrow 4f \rightarrow \dots \rightarrow \infty f$

Chaos characteristics:

- $\lambda$  – (Lyapunov's exponent).  
Exponential divergence on time  
for close i.c.:  $\Delta S(t) \approx \Delta S(0)e^{\lambda t}$ ,  
where  $\Delta S(0) \ll S_0$ .
- Embedded dimensions
- Shannon entropy

# Chaos modelling. Different regimes.

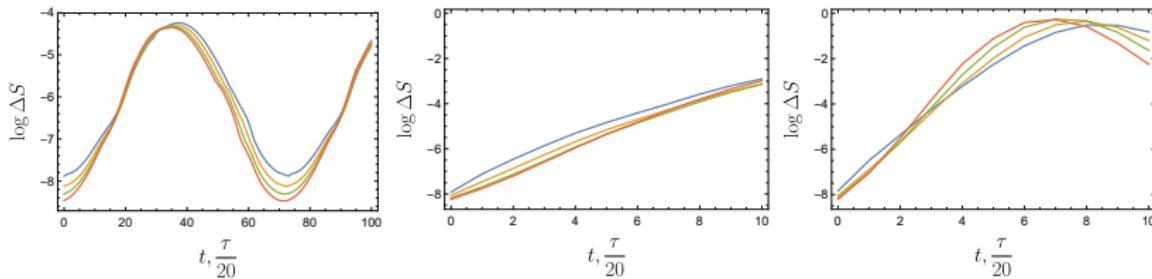


Delay time  $\tau$ :  $2 T_r \mid 7.5 T_r \mid 12 T_r$

# Chaos modelling.

Lyapunov exponents calculation for different points:

$$\Delta S \sim \exp(\lambda t) \Rightarrow \log \Delta S = \lambda t + \text{const}$$



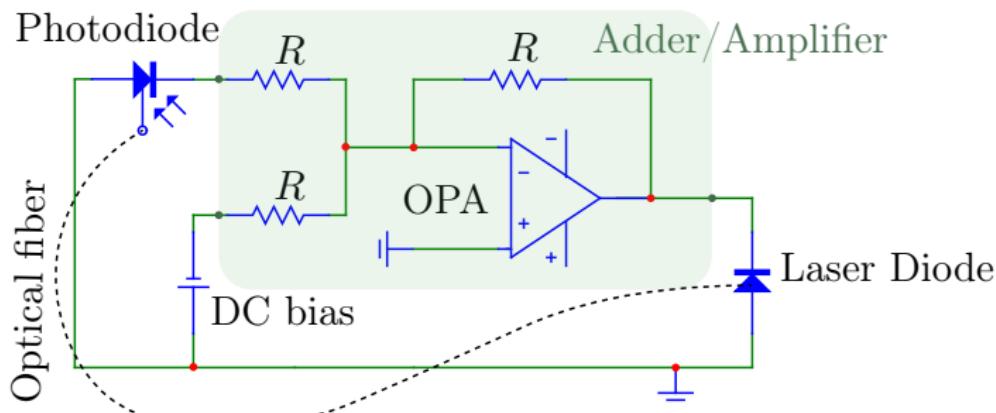
$\tau$	$2 T_r$	$7.5 T_r$	$12 T_r$
$\lambda$	$0.0$	$1.62 f_r$	$1.84 f_r$

**Chaos is possible!**

Thus, length of the fiber from the numerical analysis.  $L \sim 1 \text{ m}$

# Scheme

After several experiments came to this scheme with the summing amplifier:

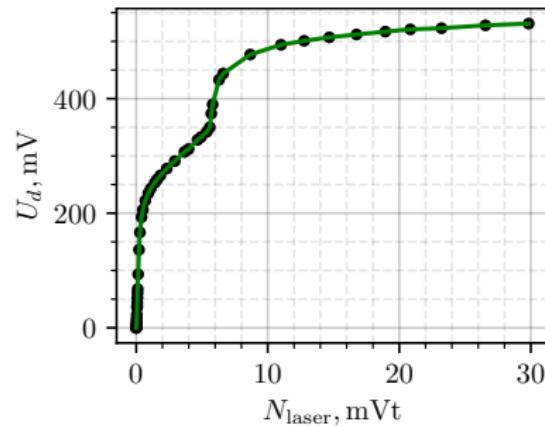
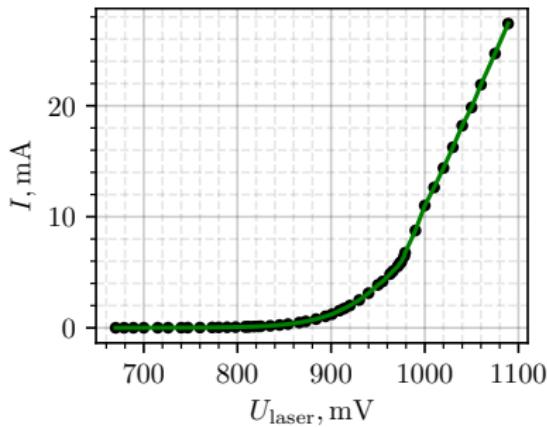


Photodiode power is enough to not use an additional amplifier.

# I-V curve

Makes sense to be in the most sensitive range, it was measured:

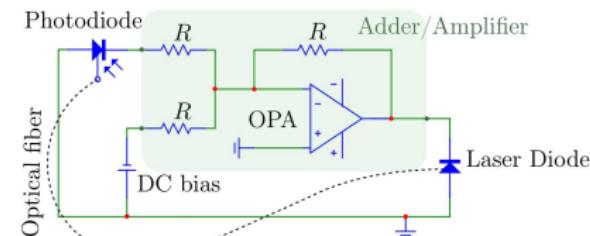
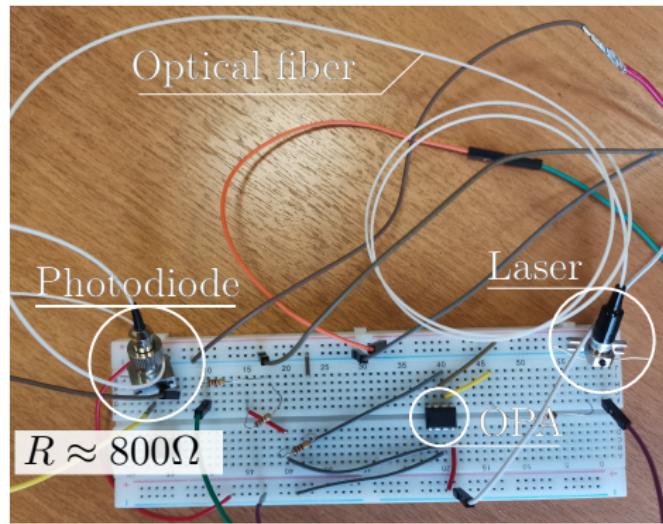
- $I$ - $V$  curve for a laser
- the dependence of the ph. diode voltage on the laser power.



So, laser voltage range of 0.85V selected.

# Realization

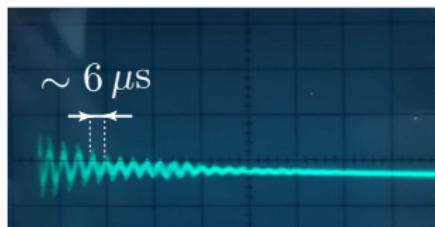
For testing, the assembly was carried out on the dumping board.



Thus, a scheme with positive feedback was implemented.  
However, no desired oscillations were observed.

# Problems

With used amplifiers, the following oscillations at the amplifier output with DC power can be observed:



This is due to the instability of the amplifier.

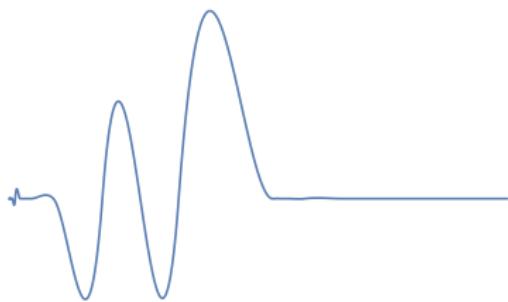
The main problem is that desired oscillations  $\sim 10$  ns.

We proceeded to experiments with faster amplifiers, but it is useful to understand results of such delays.

# Real system

In real system<sup>4</sup> feedback is cumulative  $\int_0^\infty f(\eta)S(t - \eta)d\eta$  instead of  $S(t - \tau)$ . Oscillations are reduced due to integrating over large time  $\tau_A \gg \tau$ . Only amplifier oscillations may be observed.

So, numerical integration gives:



What about LF instead of HF?..

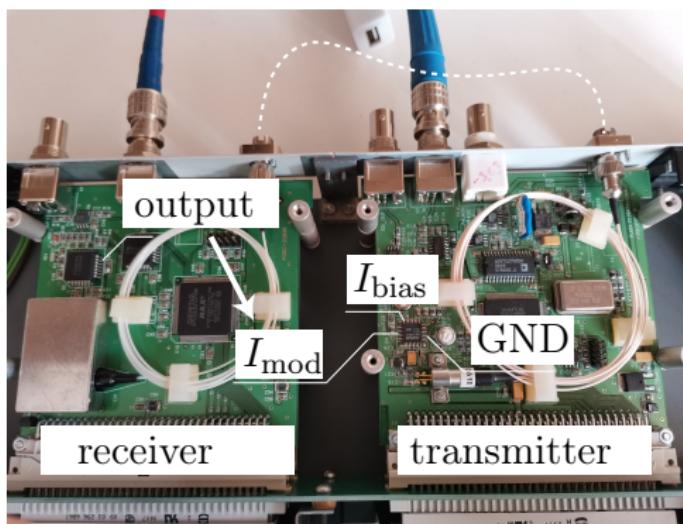
- For semi-conductors laser  $f_r \sim 1$  GHz;
- For erbium-doped fiber ring laser (EDFRL)  $f_r \sim 10$  KHz;
- However, its hard to work with it.

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<sup>4</sup>S. Tang, J. M. Liu, «Chaotic pulsing and quasi-periodic route to chaos in a semiconductor laser with delayed opto-electronic feedback», 2001.

# Alternative implementations

We tried to use partially finished decision for video transmission:



$\tau_i/\tau_r$	$\lambda\tau_r$
0	1.62
2	0.71
6	0.11
10	$\rightarrow \text{const}$

*Observed* the constant behavior.

The system is designed for oscillations  $< 20$  MGz, so

$$S(t-\tau) \rightarrow \int_0^{\tau_i} f(\eta) S(t-\eta) d\eta, \quad \tau_i \approx 50\tau_r > 5\tau_r, \Rightarrow \lim_{t \rightarrow \infty} S(t-\tau) = \text{const}$$

# Results in the modeling

Modeled good numerical solution. The existence of chaos is shown. The maximum «slowness» of the system is estimated.

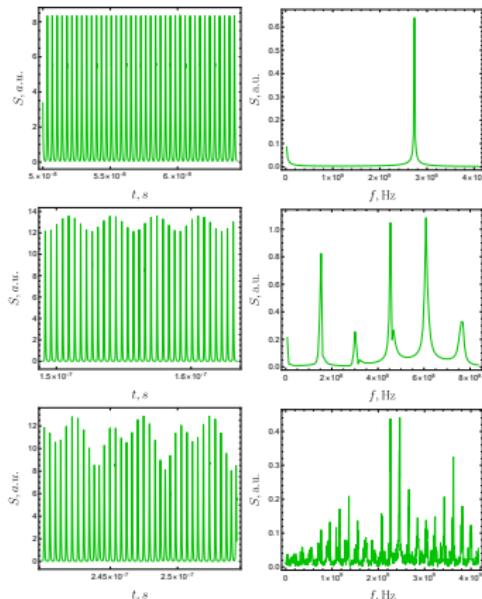


Figure 2: Modeling

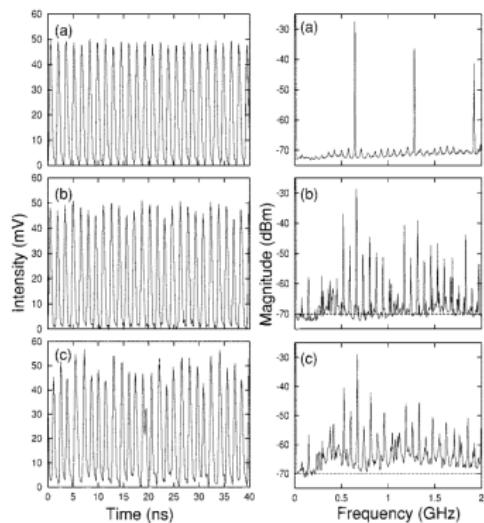


Figure 3: Article experiment

# Results in the practice work

Principal restrictions in implementation:

- High-frequency oscillations, so, as we found, equipment for working at a frequency  $\geq (10\tau_r)^{-1} \approx 200$  MGz is necessary.
- No possibility for a specific laser change the frequency of oscillations.

Successes on the way to implementation:

- Optimal values for  $\tau_{\text{delay}}$  and laser range were found selected; stabilized operational amplifiers;
- A scheme has been developed, a slow positive feedback is implemented.
- A video signal transmission system is adjusted, also observed the constant behavior with  $\tau_i \approx 50\tau_r$ .

# Conclusion

As a result of the project:

- According to the equations of the laser's evolution with a feedback, the **numerical model was built**, compliant observations from the article , in the ideal and non ideal case.
- It is shown that **chaos is possible in the system**. Chaos parameters are estimated. The frequency limitation were evaluated for the system.
- Two schemes were implemented: 1) our a scheme of positive feedback has been developed and built; 2) industrial signal transmission system was adjusted.  
⇒ In harmony with theory, there was a constant behavior.

# Literature

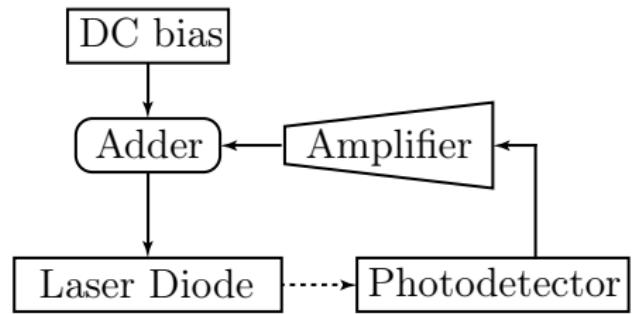
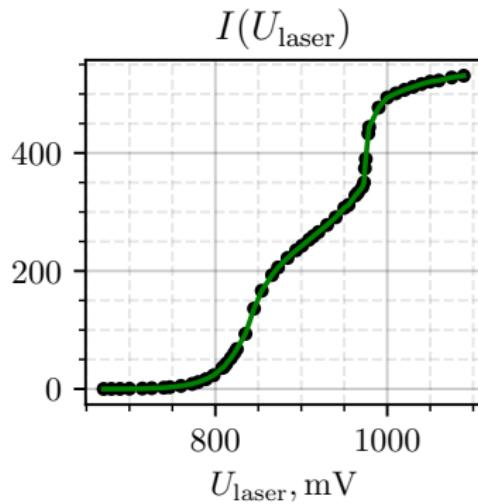
- W. Hirsch, S. Smale, "*Introduction to Chaos*";
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- "*Chaos-based communications*" by Apostolos Argyris et al. (2005);
- M. Pecora, L. Carroll, "*Synchronization in Chaotic Systems*" (1990);
- "*Physics and Applications of Laser Diode Chaos*" by M. Sciamanna, and K. A. Shore (2015);
- "*Chaotic Pulsing and Quasi-Periodic Route to Chaos in a Semiconductor Laser with Delayed Opto-Electronic Feedback*" by S. Tang and J. M. Liu (2001)
- "*Semiconductor Lasers I Fundamentals*" Chapter I by Ch.I by Bin Zhao, Amnon Yariv (1998)

# A new experimental set up came



Figure 4: Exactly from the article that we were inspired by...

# Range



It is interesting the burning of the laser, allowing an increase after feedback, therefore interests the value of order 0.85 V.

# The concept of a semiconductor laser

To start the laser idea we need to obtain:

- Solution of the Schrödinger equation in a semiconductor medium for the wavefunction of an electron;
- Induced polarization for distribution of holes and electrons in a semiconductor;
- Interaction of electrons in a semiconductor with an wave equation and outer electric field.

# Electronic states in a semiconductor

We will need the Schrödinger equation:

$$H_{\text{crystal}} \Psi_n(\mathbf{r}) = \left[ \frac{\mathbf{p}^2}{2m_0} + U_p(\mathbf{r}) \right] \Psi_n(\mathbf{r})$$

where  $\mathbf{p} = -i\hbar\nabla$  is the momentum operator,  $m_0$  is the free electron mass,  $U_p(\mathbf{r})$  is the periodic potential of the bulk semiconductor.

The solution is the Bloch function:

$$\Psi_{n,\mathbf{k}}(\mathbf{r}) = u_{n,\mathbf{k}}(\mathbf{r}) \frac{1}{\sqrt{V}} e^{i\mathbf{k}\cdot\mathbf{r}}.$$

# Hamilton in an outer field

In case of an optical field the Hamiltonian changes to

$$H = \frac{[\mathbf{p} + e\mathbf{A}(\mathbf{r}, t)]^2}{2m_0} + U_p(\mathbf{r}) = H_{\text{crystal}} + H'$$

$\mathbf{A}(\mathbf{r}, t)$  is the vector potential of the optical field. So the interaction Hamiltonian

$$H' \approx \frac{e}{m_0} \mathbf{A}(\mathbf{r}, t) \cdot \mathbf{p}.$$

# Hamilton in an outer field

For y-propagating field:  $E(\mathbf{r}, t) = \frac{1}{2}E_0e^{i(\beta y - \omega t)} + \text{const}$  the Hamiltonian is

$$H' = \frac{1}{2}\mu(k_q, x, z)[E_0e^{i(\beta y - \omega t)} + \text{const}]$$

where  $\mu$  is the transition matrix that describes the semiconductor,  $k_q$  – quantized wavevector of the electron.

# Induced polarization in an outer field

As one can obtain after rewritten density matrix in terms of carrier distributions, and in order avoid irrelevantly enormous formulas we get polarization as:

$$\begin{aligned}\mathcal{P}_{in}(\mathbf{r}, t) &= \frac{1}{2} \mathcal{P}_{in,0} e^{i(\beta y - \omega t)} + \text{const} = \\ &= - \sum \frac{\xi(\mathbf{r}, k_q, x, z)}{V(k_q, x, z)} [\rho_{eh}(k_q, x, z) \mu(k_q, x, z) + \text{const}]\end{aligned}$$

where  $V(k_q, x, z)$  is the confinement volume of electrons and holes and

$$\xi(\mathbf{r}, k_q, x, z) = \begin{cases} 1, & \mathbf{r} \text{ inside V} \\ 0, & \mathbf{r} \text{ outside V} \end{cases}$$

# Injection the light

For the wave propagating along the y direction in active layer the equation is:

$$\Delta E(\mathbf{r}, t) - \mu_0 \varepsilon(\mathbf{r}) \frac{\partial^2}{\partial t^2} E(\mathbf{r}, t) = \mu_0 \frac{\partial^2}{\partial t^2} P_{in}(\mathbf{r}, t) \quad (11)$$

The partial solution for this equation we will be searching in a form of

$$E(\mathbf{r}, t) = A_0(t) E_{\text{eig}}(x, z) = \frac{1}{2} E_0 e^{i(\beta y - \omega t)} + \text{const.}$$

## Partial solution part

Substituting in wave equation(11) the obtained polarization and solution for  $E(\mathbf{r}, t)$  and assuming that  $A(t)$  changes slowly we get

$$\frac{dA_0}{dt} = \frac{i\omega}{2\varepsilon_0 n_r^2} A_0 \sum_{k_q, x, z} \Gamma_{MD} \frac{1}{V_{MD}} |\mu(k_q, x, z)|^2 [\rho_{ee} + \rho_{hh} - 1] \quad (12)$$

where

$$\Gamma_{MD} = \left. \frac{\varepsilon(x, z) |E_0(x, z)|^2}{\iint \varepsilon(x, z) |E_0(x, z)|^2 dx dz} \right|_{x, z=0, 0}$$

is a *dimensional coupling factor*.

# Getting the solution

Now we can rewrite

$$\frac{dA_0}{dt} = \frac{1}{2}v_g[\Gamma_{MD}G - i\Gamma_{MD}N_r]A_0, \quad (13)$$

where  $v_g = c/n_r$  for  $c$  – the speed of light in vacuum, and  $n_r$  – refraction coefficient. We will call the *gain coefficient*  $g = \Gamma_{MD}G$ . From Schrödinger equation we obtain the photon density as:

$$S = \frac{1}{2} \frac{\varepsilon_0 n_r^2 |A_0 E(0)|^2}{E_0}.$$

# The laser equations

Using the equation for  $A_0$  we obtain for the photon density (real part):

$$\frac{dS}{dt} = v_g \Gamma_{MD} G(E) S. \quad (14)$$

Total optical power in the active region:

$$-\int E(\mathbf{r}, t) \frac{d\mathcal{P}_{in}(\mathbf{r}, t)}{dt} d\mathbf{r} = -\hbar\omega V_{MD} \frac{dN_{MD}}{dt},$$

which leads to electrons (holes) density (complex part):

$$\frac{dN_{MD}}{dt} = -v_g G(E) S. \quad (15)$$

# Carrier density in an outer field

Irrelevantly enormous formulas if someone really need it:

$$\frac{d}{dt}[\rho_{ee}(k_q, x, z)] = \frac{i}{\hbar}[H' \rho_{eh}(k_q, x, z) - \text{const}] - \frac{\rho_{ee}(k_q, x, z) - f_e}{\tau_e},$$

$$\frac{d}{dt}[\rho_{hh}(k_q, x, z)] = \frac{i}{\hbar}[H' \rho_{eh}(k_q, x, z) - \text{const}] - \frac{\rho_{hh}(k_q, x, z) - f_e}{\tau_e},$$

$$\frac{d}{dt}[\rho_{eh}(k_q, x, z)] = \frac{i}{\hbar}H'[\rho_{ee} + \rho_{hh} - 1] - \frac{i}{\hbar}E_{tr}\rho_{eh} - \frac{\rho_{eh}}{T_{deph}}.$$

# The laser equations

And the demensional gain coefficient is approximately:

$$G \approx G_0 = \frac{1}{V_{MD}} \sum_{k_q, x, z} \frac{E}{E_0} \frac{|\mu(k_q, x, z)|^2}{\hbar c \varepsilon_0 n_r} [\rho_{ee} + \rho_{hh} - 1] (E - E_0) \quad (16)$$

And from the previous equation we can describe the quantum well laser behavior<sup>5</sup>

$$\frac{dS}{dt} = \Gamma_{MD} G_0 v_g S - \frac{S}{\tau_p} = v_g g S(t) - \gamma_p S.$$

$$\frac{dN}{dt} = \frac{J}{e} - \frac{N}{\tau_n} - \Gamma_{MD} G_0 v_g S = \frac{J}{ed} - \gamma_s - g S.$$

for previously mentioned optical gain coefficient:

$$g = g_0 + g_n(N - N_0) + g_p(S - S_0).$$

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<sup>5</sup>Semiconductor Lasers I Fundamentals Ch.I by Bin Zhao, Amnon Yariv.

# Poincaré-Bendixson theorem

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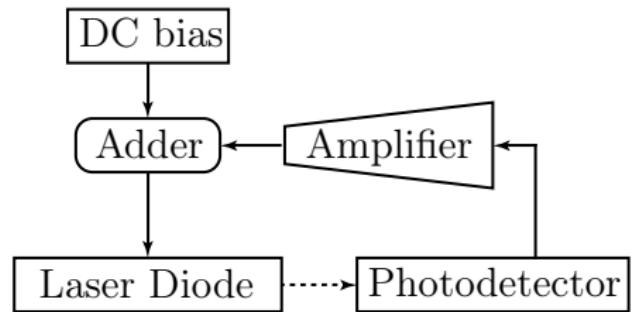
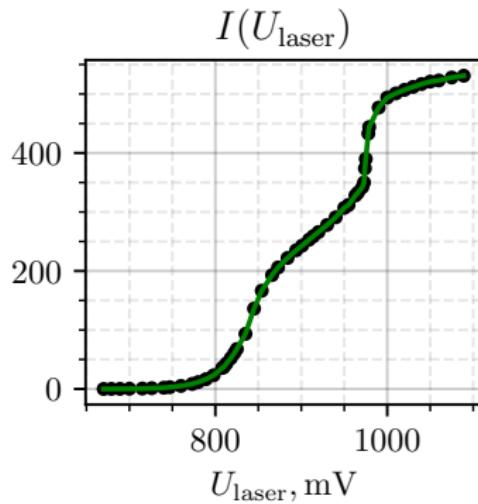
But there is still a hope for a chaos!

We add positive optoelectronic feedback in order to raise to 3D our equation

$$\frac{dS}{dt} = v_g S g(S, N) - \gamma_p S, \quad (17)$$

$$\frac{dN}{dt} = \frac{J}{ed} \left[ 1 + \frac{\xi S(t - \tau)}{S_0} \right] - \gamma_n N - S g(S, N). \quad (18)$$

## Range



It is interesting the burning of the laser, allowing an increase after feedback, therefore interests the value of order 0.85 V.