

# Theory background on topic «Optical chaos»

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And God said let there be light, and there was light

$$\Delta E(\mathbf{r}, t) - \mu_0 \varepsilon(\mathbf{r}) \frac{\partial^2}{\partial t^2} E(\mathbf{r}, t) = \mu_0 \frac{\partial^2}{\partial t^2} P_{in}(\mathbf{r}, t) \quad (1)$$

Which is solved as

$$E(\mathbf{r}, t) = \frac{1}{2} E_0 e^{i(\beta y - \omega t)} + \text{const}$$

And we assume:

$$E(\mathbf{r}, t) = A(t) E(s').$$

$$E(\mathbf{r}, t) = A(t)E(s'), \quad (2)$$

where  $A$  states for both amplitude and phase. And  $E(s')$  eigenmode function.

For most semiconductor lasers waveguides, the lasing action occurs in eigenmode. Also we use slow wave variation.

$$\left| \frac{d^2 A}{dt^2} \right| \ll \omega \left| \frac{dA}{dt} \right|. \quad (3)$$

$$\frac{dA}{dt} = \frac{i\omega}{2} A \sum_{\alpha} \frac{1}{V(\alpha)} \frac{\int \zeta(\mathbf{r}, \alpha) |E(s)|^2 ds}{\int \varepsilon(s) |E(s)|^2 ds} \times |\mu(\alpha)|^2 \frac{[\rho_{ee} + \rho_{hh}(\alpha) - 1]}{(E - E_{\alpha}) + iE_{T^2}}, \quad (4)$$

where  $\rho_{ee}$  – distribution of electrons, and  $\rho_{hh}$  – distribution of holes. It's time to oversimplify this monster:

$$\frac{dA}{dt} = \frac{i\omega}{2\varepsilon n_r^2} A \sum_{\alpha} \Gamma_{MD} \frac{1}{V_{MD}} |\mu(\alpha)|^2 [\rho_{ee} + \rho_{hh} - 1], \quad (5)$$

where  $n_r$  – refractive index of the active region. And  $\Gamma_{MD}$  – dimensional coupling factor that shows how injected carriers interact with photons.

For the most beauty we assume that well is 1D, and the equation:

$$\frac{dA}{dt} = \frac{1}{2}v_g(\Gamma_{MD}G - i\Gamma_{MD}N_r)A, \quad (6)$$

where  $v_g = c/n_r$  is the speed of light in vacuum. And  $\Gamma_{MD}G = g$  is so called gain coefficient.

If we obtain photon density inside active region as:

$$P = \frac{1}{2} \varepsilon n_r^2 |AE(0)|^2 / E.$$

And from previous equation we get

$$\frac{dP}{dt} = v_g g P = v_g \Gamma_{MDGP} = -\gamma_C P + \Gamma g P. \quad (7)$$

And in the same way:

$$\frac{dN}{dt} = -v_g G(E) P = \frac{J}{ed} \left( 1 + \frac{\xi P(t - \tau)}{P_0} \right) - \gamma_s N - g P. \quad (8)$$

So we have two main equations to describe our model:

$$\frac{dN}{dt} = -v_g G(E)P = \frac{J}{ed} \left( 1 + \frac{\xi P(t - \tau)}{P_0} \right) - \gamma_s N - gP. \quad (9)$$

$$\frac{dP}{dt} = v_g g p P = v_g \Gamma_{MDGP} = -\gamma_C P + \Gamma g P. \quad (10)$$

where  $P$  – intracavity photon density,  $N$  – carrier density,  $\tau$  – feedback delay time.

And nonlinearities lies in the optical gain function, wich, due to several articles can be explained as:

$$g \cong g_0 g_n (N - N_0) + g_P (P - P_0). \quad (11)$$



# Laser equations with feedback

Basic scheme<sup>1</sup>:

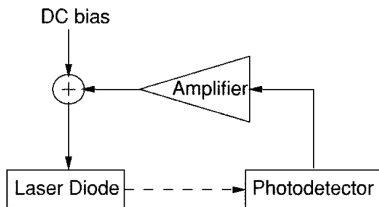


Figure 1: Schematic experimental setup

**Laser** – single-mode DFB laser diode (1300 nm);

**Light output detector** – high-speed InGaAs photodetector;

**Important:** it is suitable for any other semiconductor laser with an active medium.

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<sup>1</sup>S. Tang; J.M. Liu, Chaotic Pulsing and Quasi-Periodic Route to Chaos in a Semiconductor Laser with Delayed Opto-Electronic Feedback

# Laser equations with feedback

The nonlinear behavior of a semiconductor laser with delayed opto-electronic feedback:

$$\begin{aligned}\frac{dS}{dt} &= -\gamma_C S + \Gamma g S & S - \text{photon density;} \\ \frac{dN}{dt} &= \frac{J}{ed} - \gamma_s N - g S + \frac{J}{ed} \frac{\xi S[t - \tau]}{S_0} & N - \text{carrier density;} \\ & & \tau - \text{feedback delay;} \\ & & g - \text{opt. gain coeff.}\end{aligned}$$

$g = g_0 + g_n(N - N_0) + g_p(S - S_0)$   
positive opto-electronic feedback –  $\xi > 0$ .

*Optical gain* – measure of how well a medium amplifies photons by stimulated emission.

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# Laser equations with feedback

Some transformations we can reduce this system to the<sup>2</sup>

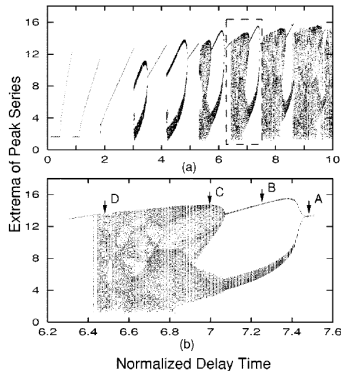
$$\begin{aligned}\frac{ds}{dt} &= c_1 n(s+1) - c_2 s(s+1) \\ \frac{dn}{dt} &= c_3 + c_4 s[t - \tau] + (c_5 s - c_8 n)(1+s) - c_6 n - c_7 s,\end{aligned}$$

where  $s = (S - S_0)/S_0$ ,  $n = (N - N_0)/N_0$ , which can be numerically integrated.

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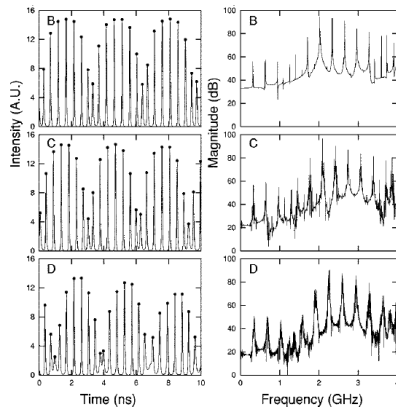
<sup>2</sup>S. Tang; J.M. Liu, Chaotic Pulsing and Quasi-Periodic Route to Chaos in a Semiconductor Laser with Delayed Opto-Electronic Feedback

# Numerical solution



**Figure 2:** Bifurcation diagram of the extrema of the peak series

Here  $\hat{\tau} = \tau f_r$  – demensionless delay.



**Figure 3:** Time series and power spectra

# Restoration of the attractor

With part of system variables, it is possible to restore the general view of the attractor.

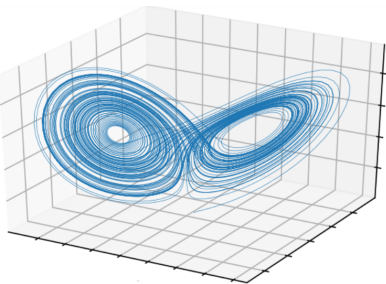


Figure 4: Original system attractor

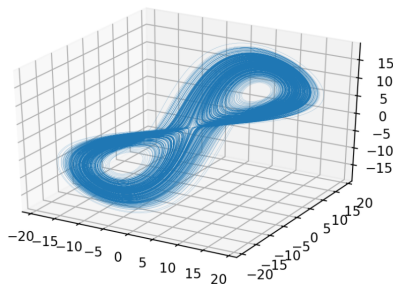


Figure 5: Reconstructed by one coordinate system attractor

# Numerical features

## ■ Lyapunov exponent:

$$\lambda(x_0) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln |f'(x_i)|,$$

where  $\lambda > 0$  – characteristic for chaos.

## ■ Shannon Entropy:

$$H(X) = - \sum_{i=1}^n P(x_i) \log P(x_i),$$

where higher rates correspond to more random sources.

# Applications

## ■ Random number generation.

Random bits produced at a much higher rate than other physical sources of entropy including quantum RNG.

## ■ Chaos computing.

It is possible to create "NOR" gate,  
 \* double-scrolled chaotic attractor leads to threshold function.

