Implementations of dynamic chaos in different optical systems

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Goals

Globally we are to observe different realization of dynamic chaos and implement some on our own.

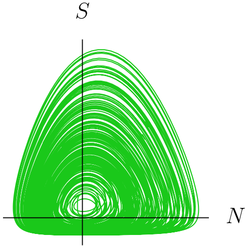
Here we will consider the following steps:

- study the dynamical chaos on its own;
- implement laser theory to create chaos;
- model laser behavior and attempt to build a circuit;
- study light branches chaoticly penetrating thin layer;
- model and implement this model.

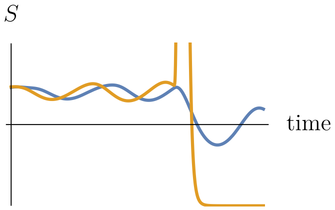
Definition of dynamic chaos

Map[1] f is **chaotic**, if

- f **sensitive** to the initial conditions.
- periodic orbits are dense everywhere;
- orbits are mixed;



Dense mixed orbits example



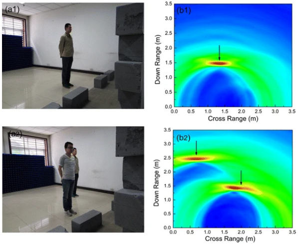
Sensitivity example

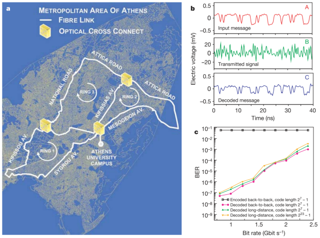
[1] W. Hirsch, S. Smale, Introduction to Chaos.

Applications of dynamic chaos

Possible applications:

- optical reflectometry;
- chaos radar;
- random numbers generation;
- signal encryption.





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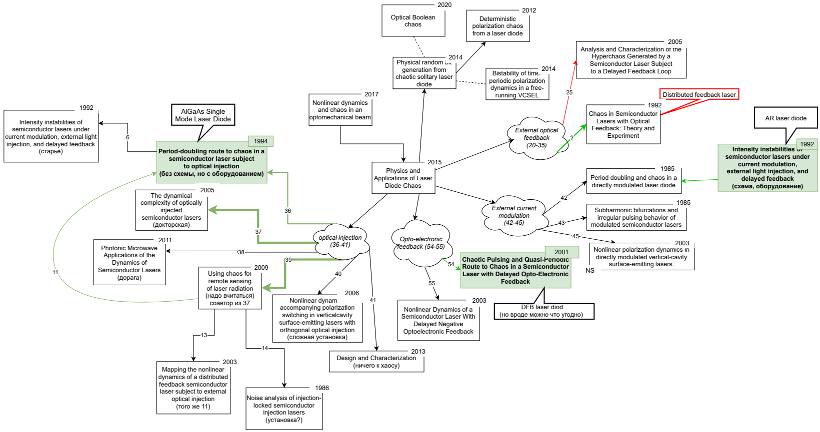
[1] Chaos Through-Wall Imaging Radar by Hang Xu et al. (2009)
[2] Chaos-based communications by Apostolos Argyris et al. (2005)

From the root article a tree has grown



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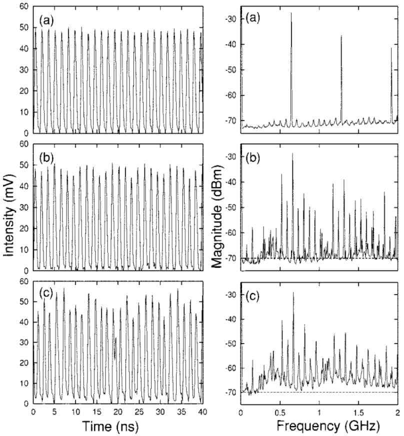
[1] Physics and Applications of Laser Diode Chaos by M. Sciamanna et al.

The decided variant



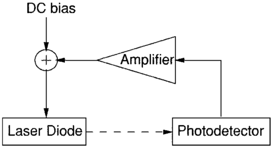


Figure 1: Experimental results of the time series and power spectra of different pulsing states at different delay times. On a circuit dashed line represents optical path.

[1] Chaotic Pulsing and Quasi-Periodic Route to Chaos in a Semiconductor Laser with Delayed Opto-Electronic Feedback S. Tang and J. M. Liu (2001)

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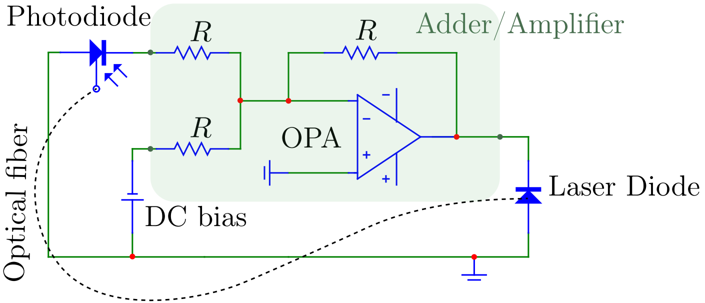
Scheme

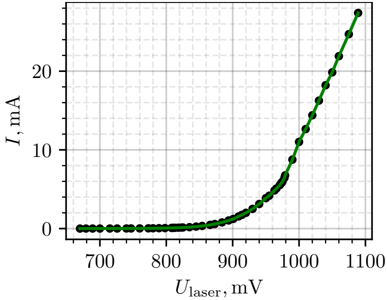
After several experiments came to this scheme with the summing amplifier:

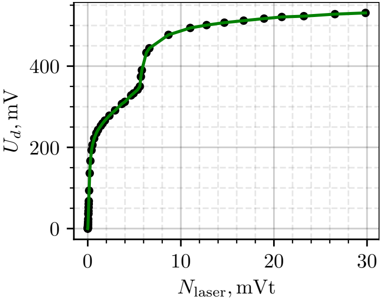


Photodiode power is enough to not use an additional amplifier.

I–V curve

Makes sense to be in the most sensitive range, it was measured:
- *I-V* curve for a laser
- the dependence of the ph. diode voltage on the laser power.

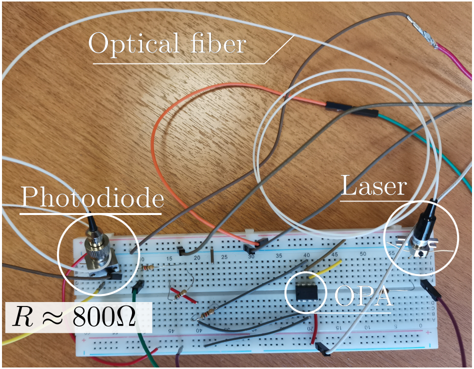


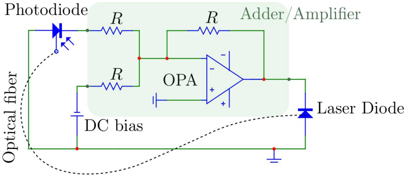


So, laser voltage range of 0.85V selected.

Realization

For testing, the assembly was carried out on the dumping board.





Thus, a scheme with positive feedback was implemented.
However, no desired oscillations were observed.

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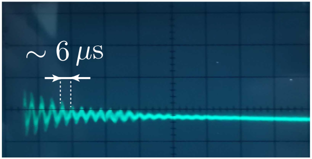
Problems

With used amplifiers, the following oscillations at the amplifier output with DC power can be observed:



This is due to the instability of the amplifier.

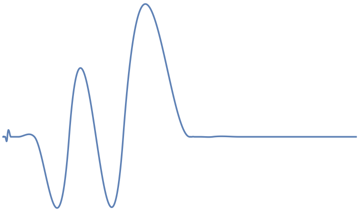
The main problem is that desired oscillations ~ 10 ns.

We proceeded to experiments with faster amplifiers, but it is usefull to understand results of such delays.
oscillations megahertz

Real system

In real system[2] feedback is cumulative $\int_0^\infty f(\eta)S(t-\eta)d\eta$ instead of $S(t-\tau)$. Oscillations are reduced due to integrating over large time $\tau_A \gg \tau$. Only amplifier oscillations may be observed.

So, numerical integration gives:



What about LF instead of HF?..

- For semi-conductors laser $f_r \sim 1$ GHz;
- For erbium-doped fiber ring laser (EDFRl) $f_r \sim 10$kHz;
- However, its hard to work with it.

[2] S. Tang, J. M. Liu, «Chaotic pulsing and quasi-periodic route to chaos in a semiconductor laser with delayed opto-electronic feedback», 2001.

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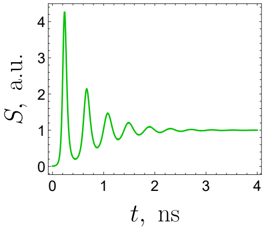
Laser parameters.

Unmodulated laser
($\xi = 0$):



Laser equations:

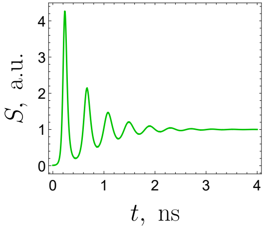
$$\frac{dS}{dt} = -\gamma_c S + \Gamma g S$$

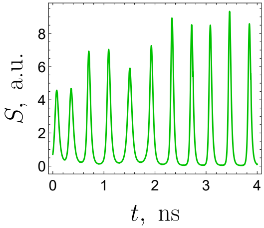
$$\frac{dN}{dt} = \frac{J}{ed}\left[1 + \frac{\xi S(t-\tau)}{S_0}\right] - \gamma_s N - gS$$

Systems parameters:

- f_r – laser relaxation frequency. For our laser: $f_r = 1.4 \div 3.2$ GHz.
- τ – delay time. $\tau \sim f_r^{-1} \sim 1$ns.
- $\xi = 0.1$ feedback parameter

Chaos parameters.





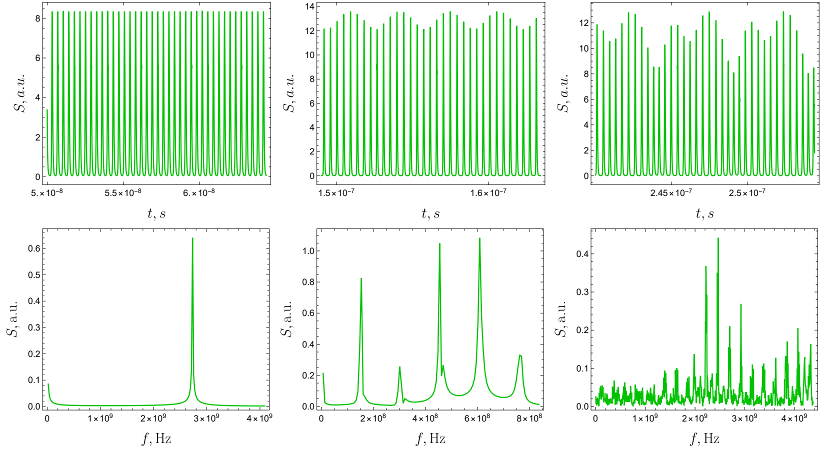
Chaos features:

- Unperiodic behaviour
- Frequency doubling scenario $f \to 2f \to 4f \to \cdots \to \infty f$

Chaos characteristics:

- λ – (Lyapunov's exponent). Exponential divergence on time for close i.c.: $\Delta S(t) \approx \Delta S(0) e^{\lambda t}$, where $\Delta S(0) \ll S_0$.
- Embedded dimensions
- Shannon entropy

Chaos modelling. Different regimes.

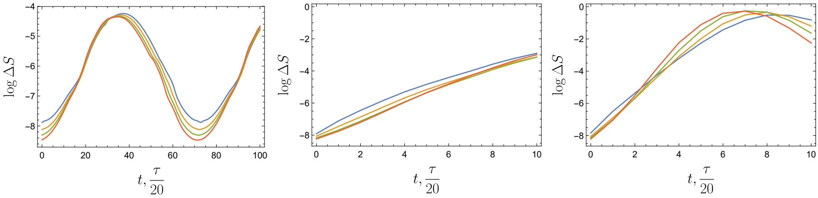


Delay time τ: $2\ T_r \mid 7.5\ T_r \mid 12\ T_r$

Chaos modelling.

Lyapunov exponents calculation for different points:

$$\Delta S \sim \exp(\lambda t) \;\Rightarrow\; \log \Delta S = \lambda t + \text{const}$$



τ	$2\ T_r$	$7.5\ T_r$	$12\ T_r$
λ	0.0	$1.62\ f_r$	$1.84\ f_r$

Chaos is possible!.

Thus, length of the fiber from the numerical analysis. $L \sim 1$ m

Results in the modeling

Modeled good numerical solution. The existence of chaos is shown. The maximum «slowness» of the system is estimated.

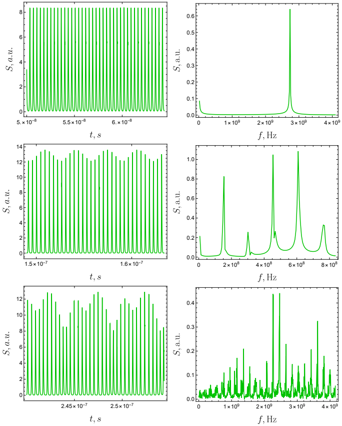


Figure 2: Modeling

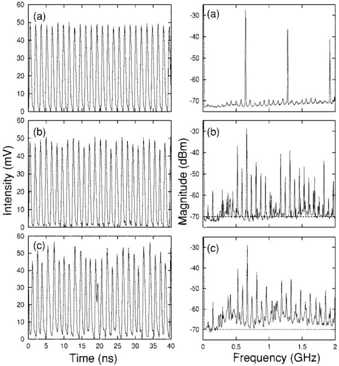


Figure 3: Article experiment

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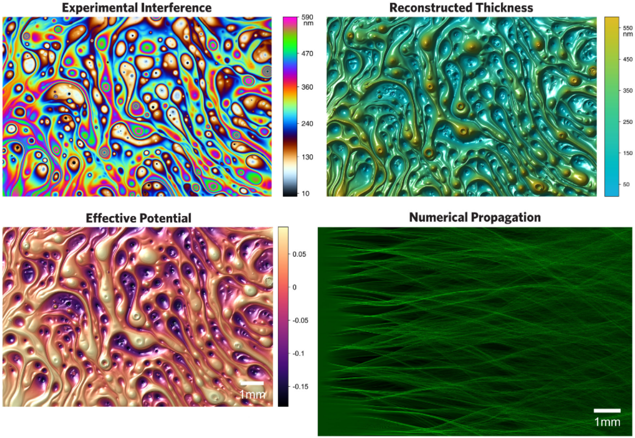
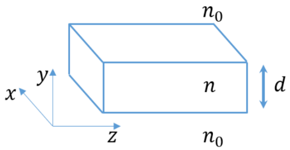
The idea in nature



Patsyk, A., Sivan, U., Segev, M. et al. "Observation of branched flow of light". Nature 583, 60–65 (2020).

Theory of refraction index



The Helmholtz equation again

$$\Delta E + k_0^2 n^2(y) E = 0$$

while solving like $E = \psi(x, z) G(y)$ gives a solutions:

$$\partial_{yy} + k_0^2 n^2(y) G = k_0^2 n_{\text{eff}}^2 G, \qquad \nabla_\perp^2 \psi + k_0^2 n_{\text{eff}}^2 \psi = 0.$$

And solving the left one we obtain

$$k_0^2 y \sqrt{n_{\text{soap}}^2 - n_{\text{eff}}^2} + 2 \arctan\left(\frac{\sqrt{n_{\text{soap}}^2 - n_{\text{eff}}^2}}{\sqrt{n_{\text{eff}}^2 - n_{\text{air}}^2}}\right) - \pi(m+1) = 0.$$

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Literature

- W. Hirsch, S. Smale, *"Introduction to Chaos"*;
- "Chaos Through-Wall Imaging Radar" by Hang Xu et al. (2009)
- *"Chaos-based communications"* by Apostolos Argyris et al. (2005);
- M. Pecora, L. Carroll, *"Synchronization in Chaotic Systems"* (1990);
- *"Physics and Applications of Laser Diode Chaos"* by M. Sciamanna, and K. A. Shore (2015);
- *"Chaotic Pulsing and Quasi-Periodic Route to Chaos in a Semiconductor Laser with Delayed Opto-Electronic Feedback"* by S. Tang and J. M. Liu (2001)
- *"Semiconductor Lasers I Fundamentals"* Chapter I by Ch.I

A new experimental set up came

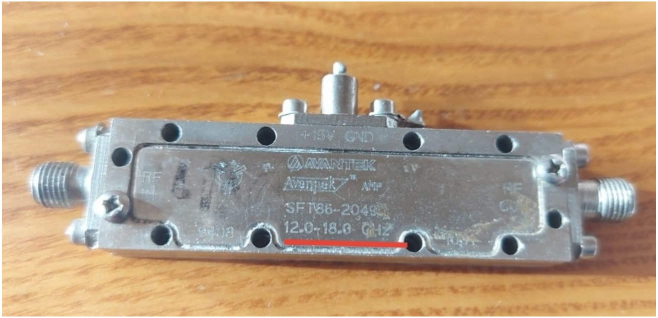
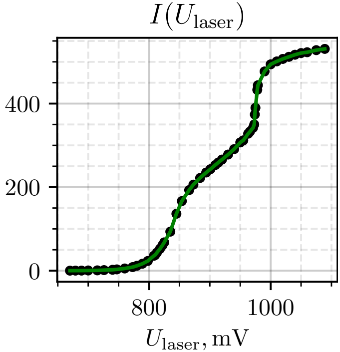
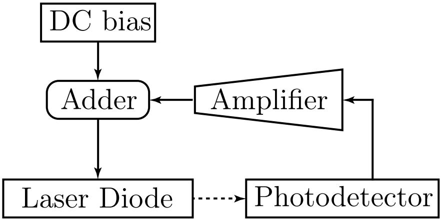


Figure 4: Exactly from the article that we were inspired by...

Range





It is interesting the burning of the laser, allowing an increase after feedback, therefore interests the value of order 0.85 V.

Other problems

However, it was not possible to move to the chaotic regime in the laser. Possible cause of the problem may be

- parasitic capacity and inductance
- We've measured it for our scheme and for $f \sim 1$ GHz parasitic $L \sim 1$ nH $R_L \sim 1\Omega$
- skin-effect for $f \sim 1$ GHz for Cu at $t = 20^\circ$ C

In terms of solutions – neatly soldered scheme.

The concept of a semiconductor laser

To start the laser idea we need to obtain:

- Solution of the Schrödinger equation in a semiconductor medium for the wavefunction of an electron;
- Induced polarization for distribution of holes and electrons in a semiconductor;
- Interaction of electrons in a semiconductor with an wave equation and outer electric field.

Electronic states in a semiconductor

We will need the Schrödinger equation:

$$H_{\text{crystal}}\Psi_n(\boldsymbol{r}) = \left[\frac{\boldsymbol{p}^2}{2m_0} + U_p(\boldsymbol{r})\right]\Psi_n(\boldsymbol{r})$$

where $\boldsymbol{p} = -i\hbar\nabla$ is the momentum operator, m_0 is the free electron mass, $U_p(\boldsymbol{r})$ is th periodic potential of the bulk semiconductor.
The solution is the Bloch function:

$$\Psi_{n,\boldsymbol{k}}(\boldsymbol{r}) = u_{n,\boldsymbol{k}}(\boldsymbol{r})\frac{1}{\sqrt{V}}e^{i\boldsymbol{k}\cdot\boldsymbol{r}}.$$

Hamilton in an outer field

In case of an optical field the Hamiltonian changes to

$$H = \frac{[\boldsymbol{p} + e\boldsymbol{A}(\boldsymbol{r}, t)]^2}{2m_0} + U_p(\boldsymbol{r}) = H_{\text{crystal}} + H'$$

$\boldsymbol{A}(\boldsymbol{r}, t)$ is the vector potential of the optical field. So the interaction Hamiltonian

$$H' \approx \frac{e}{m_0}\boldsymbol{A}(\boldsymbol{r}, t) \cdot \boldsymbol{p}.$$

Hamilton in an outer field

For y-propagating field: $E(\boldsymbol{r}, t) = \frac{1}{2}E_0 e^{i(\beta y - \omega t)} + \text{const}$ the Hamiltonian is

$$H' = \frac{1}{2}\mu(k_q, x, z)[E_0 e^{i(\beta y - \omega t)} + \text{const}]$$

where μ is the transition matrix that describes the semiconductor, k_q – quantized wavevector of the electron.

Induced polarization in an outer field

As one can obtain after rewritten density matrix in terms of carrier distributions, and in order avoid irrelevantly enormous formulas we get polarization as:

$$\mathcal{P}_{in}(\boldsymbol{r}, t) = \frac{1}{2}\mathcal{P}_{in,0}e^{i(\beta y - \omega t)} + \text{const} =$$

$$= -\sum \frac{\xi(\boldsymbol{r}, k_q, x, z)}{V(k_q, x, z)}[\rho_{eh}(k_q, x, z)\mu(k_q, x, z) + \text{const}]$$

where $V(k_q, x, z)$ is the confinement volume of electrons and holes and

$$\xi(\boldsymbol{r}, k_q, x, z) = \begin{cases} 1, & \boldsymbol{r} \text{ inside V} \\ 0, & \boldsymbol{r} \text{ outside V} \end{cases}$$

Injection the light

For the wave porpagating along the y direction in active layer the equation is:

$$\triangle E(\boldsymbol{r},t) - \mu_0 \varepsilon(\boldsymbol{r}) \frac{\partial^2}{\partial t^2} E(\boldsymbol{r},t) = \mu_0 \frac{\partial^2}{\partial t^2} P_{in}(\boldsymbol{r},t) \tag{1}$$

The partial solution for this equation we will be searching in a form of

$$E(\boldsymbol{r},t) = A_0(t) E_{\text{eig}}(x,z) = \frac{1}{2} E_0 e^{i(\beta y - \omega t)} + \text{const.}$$

Partial solution part

Substituting in wave equation(1) the obtained polarization and solution for $E(\boldsymbol{r},t)$ and assuming that $A(t)$ changes slowly we get

$$\frac{dA_0}{dt} = \frac{i\omega}{2\varepsilon_0 n_r^2} A_0 \sum_{k_q,x,z} \Gamma_{MD} \frac{1}{V_{MD}} |\mu(k_q,x,z)|^2 [\rho_{ee} + \rho_{hh} - 1] \quad (2)$$

where

$$\Gamma_{MD} = \left. \frac{\varepsilon(x,z)|E_0(x,z)|^2}{\iint \varepsilon(x,z)|E_0(x,z)|^2 dxdz} \right|_{x,z=0,0}$$

is a *dimensional coupling factor.*

Getting the solution

Now we can rewrite

$$\frac{dA_0}{dt} = \frac{1}{2} v_g [\Gamma_{MD} G - i\Gamma_{MD} N_r] A_0, \tag{3}$$

where $v_g = c/n_r$ for c – the speed of light in vacuum, and n_r – refraction coefficient. We will call the *gain coefficient* $g = \Gamma_{MD} G$. From Schrödinger equation we obtain the photon density as:

$$S = \frac{1}{2} \frac{\varepsilon_0 n_r^2 |A_0 E(0)|^2}{E_0}.$$

The laser equations

Using the equation for A_0 we obtain for the photon density (real part):

$$\frac{dS}{dt} = v_g \Gamma_{MD} G(E) S. \tag{4}$$

Total optical power in the active region:

$$-\int E(\boldsymbol{r}, t) \frac{d\mathcal{P}_{in}(\boldsymbol{r}, t)}{dt} d\boldsymbol{r} = -\hbar\omega V_{MD} \frac{dN_{MD}}{dt},$$

which leads to electrons (holes) density (complex part):

$$\frac{dN_{MD}}{dt} = -v_g G(E) S. \tag{5}$$

Carrier density in an outer field

Irrelevantly enormous formulas if someone really need it:

$$\frac{d}{dt}[\rho_{ee}(k_q,x,z)] = \frac{i}{\hbar}[H'\rho_{eh}(k_q,x,z) - \mathrm{const}] - \frac{\rho_{ee}(k_q,x,z) - f_e}{\tau_e},$$

$$\frac{d}{dt}[\rho_{hh}(k_q,x,z)] = \frac{i}{\hbar}[H'\rho_{eh}(k_q,x,z) - \mathrm{const}] - \frac{\rho_{hh}(k_q,x,z) - f_e}{\tau_e},$$

$$\frac{d}{dt}[\rho_{eh}(k_q,x,z)] = \frac{i}{\hbar}H'[\rho_{ee} + \rho_{hh} - 1] - \frac{i}{\hbar}E_{tr}\rho_{eh} - \frac{\rho_{eh}}{T_{deph}}.$$

The laser equations

And the demensional gain coefficient is approximately:

$$G \approx G_0 = \frac{1}{V_{MD}} \sum_{k_q,x,z} \frac{E}{E_0} \frac{|\mu(k_q,x,z)|^2}{\hbar c \varepsilon_0 n_r} [\rho_{ee} + \rho_{hh} - 1](E - E_0) \quad (6)$$

And from the previous equation we can describe the quantum well laser behavior[3]

$$\frac{dS}{dt} = \Gamma_{MD} G_0 v_g S - \frac{S}{\tau_p} = v_g g S(t) - \gamma_p S.$$

$$\frac{dN}{dt} = \frac{J}{e} - \frac{N}{\tau_n} - \Gamma_{MD} G_0 v_g S = \frac{J}{ed} - \gamma_s - gS.$$

for previously mentioned optical gain coefficient:

$$g = g_0 + g_n(N - N_0) + g_p(S - S_0).$$

[3] Semiconductor Lasers I Fundamentals Ch.I by Bin Zhao, Amnon Yariv.

Poincaré-Bendixson theorem

Thr. If there is no stationary points on the enclosed 2D region G and some trajectory exists $\gamma \subset G$, then γ is a closed loop path or tends to the closed one.

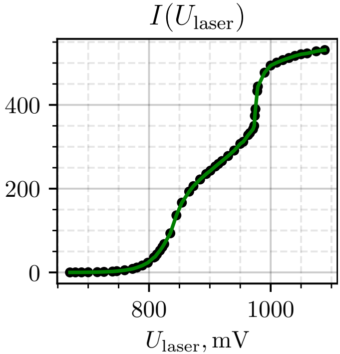
But there is still a hope for a chaos!

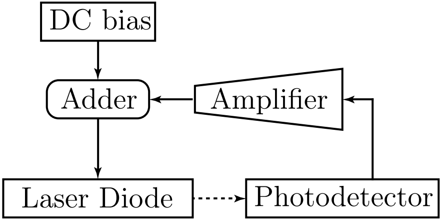
We add positive optoelectronic feedback in order to raise to 3D our equation

$$\frac{dS}{dt} = v_g S g(S, N) - \gamma_p S, \tag{7}$$

$$\frac{dN}{dt} = \frac{J}{ed}[1 + \frac{\xi S(t-\tau)}{S_0}] - \gamma_n N - S g(S, N). \tag{8}$$

Range





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