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# Optical chaos based on a laser diode with positive feedback

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# Goals

*Globally* we would like to transmit a high-frequency signal in encrypted form.

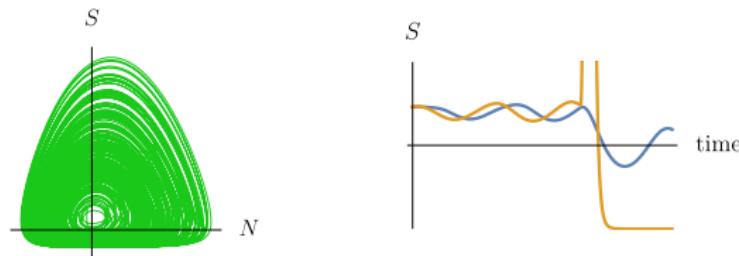
Here, we will consider the following steps towards this goal:

- dynamical chaos and synchronization  
(to encrypt and decrypt signal);
- theory of the laser evolution and its adaptation under our needs;
- realization of the positive feedback in laser:  
theory, modeling and experiment.

# Definition of dynamic chaos and applications

Map<sup>1</sup>  $f$  is **chaotic**, if

- periodic orbits are dense everywhere;
- orbits are mixed;
- $f$  sensitive to the initial conditions.



Possible applications:

- random numbers generation;
- signal encryption.

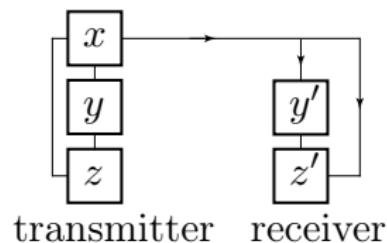
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<sup>1</sup>W. Hirsch, S. Smale, Introduction to Chaos.

# Synchronization

Possible<sup>2</sup> synchronization of chaotic systems:

*enough* to transmit  
part of the signal;  
configure system parameters.



The use of optics to transmit the signal allows to achieve a greater bandwidth of the channel.

UHFO (ultrahigh frequency oscillations) is a characteristic to optic systems.

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<sup>2</sup>M. Pecora, L. Carroll, Synchronization in Chaotic Systems, 1990.

# The concept of a semiconductor laser

To start the laser idea we need to obtain:

- Solution of the Schrödinger equation in a semiconductor medium for the wavefunction of an electron;
- Induced polarization for distribution of holes and electrons in a semiconductor;
- Interaction of electrons in a semiconductor with an wave equation and outer electric field.

# Electronic states in a semiconductor

We will need the Schrödinger equation:

$$H_{\text{crystal}} \Psi_n(\mathbf{r}) = \left[ \frac{\mathbf{p}^2}{2m_0} + U_p(\mathbf{r}) \right] \Psi_n(\mathbf{r})$$

where  $\mathbf{p} = -i\hbar\nabla$  is the momentum operator,  $m_0$  is the free electron mass,  $U_p(\mathbf{r})$  is the periodic potential of the bulk semiconductor.

The solution is the Bloch function:

$$\Psi_{n,\mathbf{k}}(\mathbf{r}) = u_{n,\mathbf{k}}(\mathbf{r}) \frac{1}{\sqrt{V}} e^{i\mathbf{k}\cdot\mathbf{r}}.$$

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# Hamilton in an outer field

In case of an optical field the Hamiltonian changes to

$$H = \frac{\mathbf{p} + e\mathbf{A}[\mathbf{r}, t]^2}{2m_0} + U_p(\mathbf{r}) = H_{\text{crystal}} + H'$$

$\mathbf{A}(\mathbf{r}, t)$  is the vector potential of the optical field. So the interaction Hamiltonian

$$H' \approx \frac{e}{m_0} \mathbf{A}(\mathbf{r}, t) \cdot \mathbf{p}.$$

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# Hamilton in an outer field

For y-propagating field:  $E(\mathbf{r}, t) = \frac{1}{2}E_0e^{i(\beta y - \omega t)} + \text{const}$  the Hamiltonian is

$$H' = \frac{1}{2}\mu(k_q, x, z)[E_0e^{i(\beta y - \omega t)} + \text{const}]$$

where  $\mu$  is the transition matrix that describes the semiconductor,  $k_q$  – quantized wavevector of the electron.

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# Induced polarization in an outer field

As one can obtain after rewritten density matrix in terms of carrier distributions, and in order avoid irrelevantly enormous formulas we get polarization as:

$$\begin{aligned}\mathcal{P}_{in}(\mathbf{r}, t) &= \frac{1}{2} \mathcal{P}_{in,0} e^{i(\beta y - \omega t)} + \text{const} = \\ &= - \sum \frac{\xi(\mathbf{r}, k_q, x, z)}{V(k_q, x, z)} [\rho_{eh}(k_q, x, z) \mu(k_q, x, z) + \text{const}]\end{aligned}$$

where  $V(k_q, x, z)$  is the confinement volume of electrons and holes and

$$\xi(\mathbf{r}, k_q, x, z) = \begin{cases} 1, & \mathbf{r} \text{ inside V} \\ 0, & \mathbf{r} \text{ outside V} \end{cases}$$

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# Injection the light

For the wave propagating along the y direction in active layer the equation is:

$$\Delta E(\mathbf{r}, t) - \mu_0 \varepsilon(\mathbf{r}) \frac{\partial^2}{\partial t^2} E(\mathbf{r}, t) = \mu_0 \frac{\partial^2}{\partial t^2} P_{in}(\mathbf{r}, t) \quad (1)$$

The partial solution for this equation we will be searching in a form of

$$E(\mathbf{r}, t) = A_0(t) E_{\text{eig}}(x, z) = \frac{1}{2} E_0 e^{i(\beta y - \omega t)} + \text{const.}$$

## Partial solution part

Substituting in wave equation(1) the obtained polarization and solution for  $E(\mathbf{r}, t)$  and assuming that  $A(t)$  changes slowly we get

$$\frac{dA_0}{dt} = \frac{i\omega}{2\varepsilon_0 n_r^2} A_0 \sum_{\alpha} \Gamma(0) \frac{1}{V} |\mu(k_q, x, z)|^2 [\rho_{ee} + \rho_{hh} - 1] \quad (2)$$

where

$$\Gamma(0) = \frac{\varepsilon(x, z) |E_0(x, z)|^2}{\iint \varepsilon(x, z) |E_0(x, z)|^2 dx dz}$$

is a *dimensional coupling factor*.

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# Getting the solution

Now we can rewrite

$$\frac{dA_0}{dt} = \frac{1}{2}v_g[\Gamma(0)G - i\Gamma(0)N_r]A_0, \quad (3)$$

where  $v_g = c/n_r$  for  $c$  – the speed of light in vacuum, and  $n_r$  – refraction coefficient. We will call the *gain coefficient*  $g = \Gamma(0)G$ . From Schrödinger equation we obtain the photon density as:

$$S = \frac{1}{2} \frac{\varepsilon_0 n_r^2 |A_0 E(0)|^2}{E_0}.$$

# The laser equations

Using the equation for  $A_0$  we obtain for the photon density (real part):

$$\frac{dS}{dt} = v_g \Gamma(0) GS. \quad (4)$$

and for electrons (holes) density (complex part):

$$\frac{dN}{dt} = -v_g GS. \quad (5)$$

And the dimensional gain coefficient is approximately:

$$G = G_0 - G_1 \frac{S}{S_s}. \quad (6)$$

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# Carrier density in an outer field

Irrelevantly enormous formulas if someone really need it:

$$\frac{d}{dt}[\rho_{ee}(k_q, x, z)] = \frac{i}{\hbar}[H' \rho_{eh}(k_q, x, z) - \text{const}] - \frac{\rho_{ee}(k_q, x, z) - f_e}{\tau_e},$$

$$\frac{d}{dt}[\rho_{hh}(k_q, x, z)] = \frac{i}{\hbar}[H' \rho_{eh}(k_q, x, z) - \text{const}] - \frac{\rho_{hh}(k_q, x, z) - f_e}{\tau_e},$$

$$\frac{d}{dt}[\rho_{eh}(k_q, x, z)] = \frac{i}{\hbar} H' [\rho_{ee} + \rho_{hh} - 1] - \frac{i}{\hbar} E_{tr} \rho_{eh} - \frac{\rho_{eh}}{T_{deph}}.$$

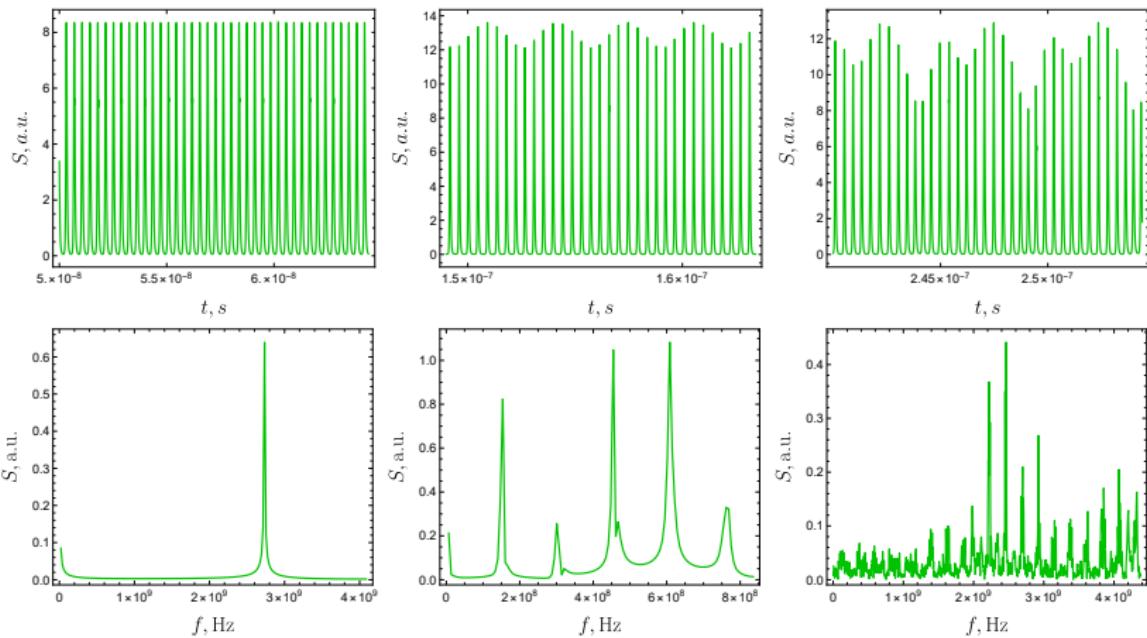
# The laser equations

And from the previous equation we can describe the quantum well laser behavior

$$\frac{dS}{dt} = \Gamma(0)G_0v_gS - \frac{S}{\tau_p}.$$

$$\frac{dN}{dt} = \frac{J}{e} - \frac{N}{\tau_n} - \Gamma G_0 v_g S.$$

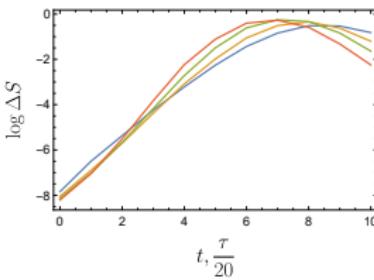
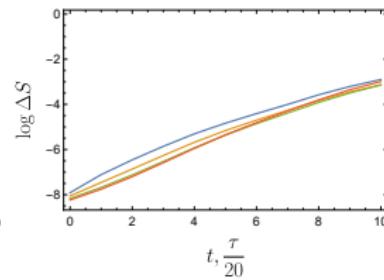
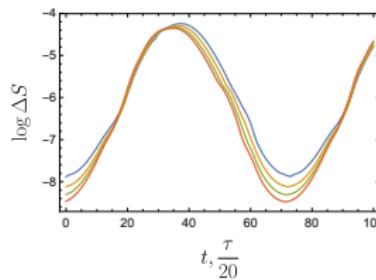
Chaos modelling. Different regimes.



Delay time  $\tau$ : 2  $T_r$  | 7.5  $T_r$  | 12  $T_r$

## Chaos modelling.

Lyapunov exponents calculation :

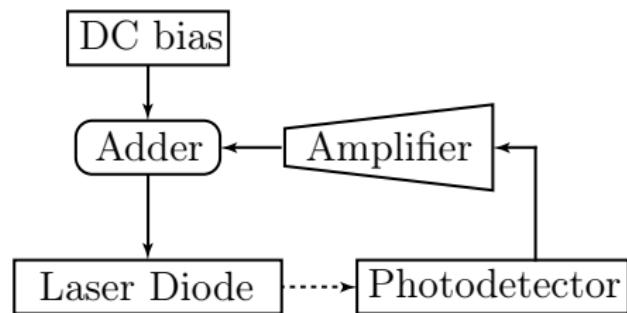


$\frac{\tau}{\lambda}$	$2 T_r$	$7.5 T_r$	$12 T_r$	$\Delta S \sim \exp(\lambda t) \Rightarrow \log \Delta S = \lambda t + \text{const}$
	0.0	$1.62 f_r$	$1.84 f_r$	

# Concept

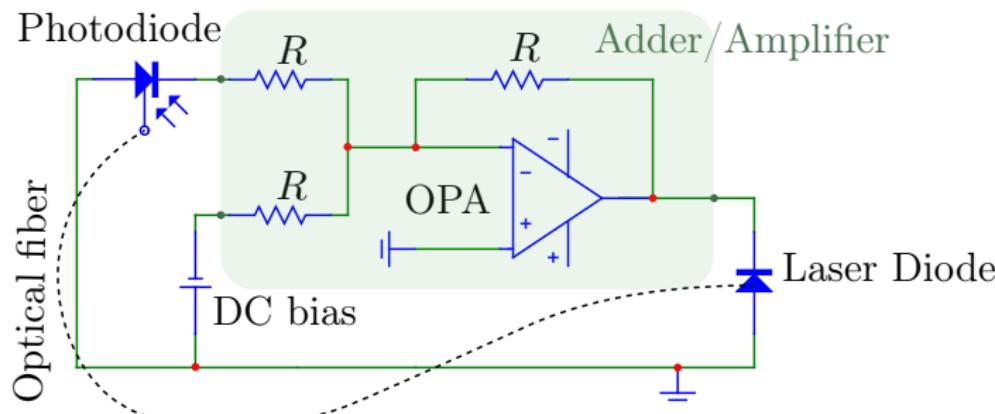
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# Scheme

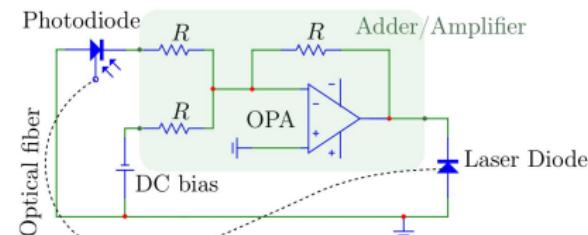
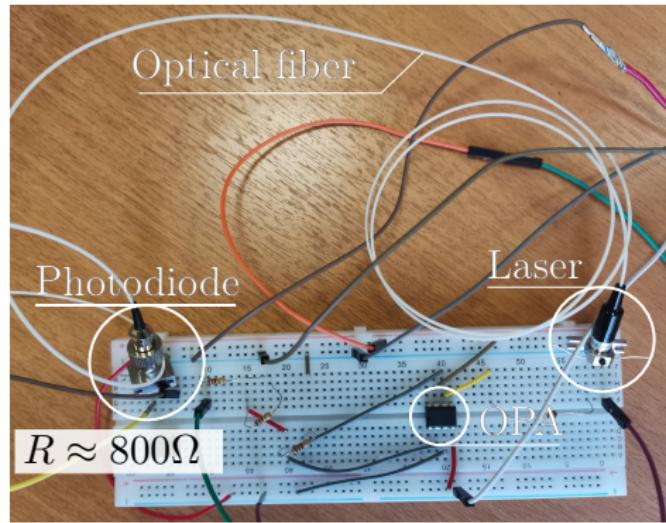
After several experiments came to this scheme with the summing amplifier:



Photodiode power is enough to not use an additional amplifier.

# Realization

For testing, the assembly was carried out on the dumping board.

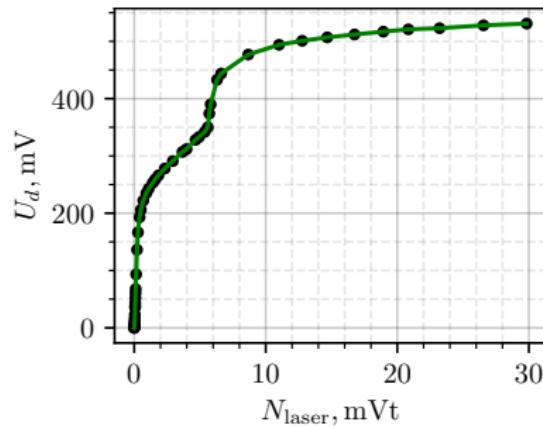
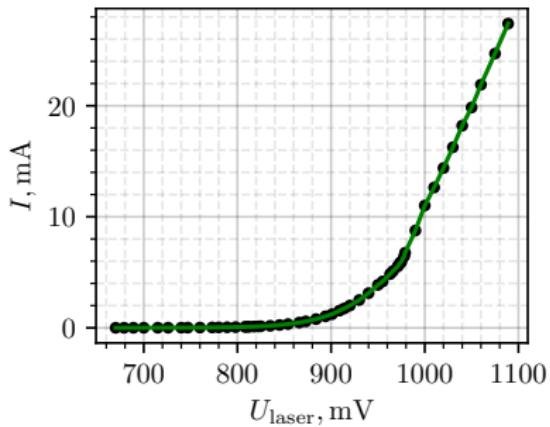


Thus, a scheme with positive feedback was implemented.  
However, no desired oscillations were observed.

# I-V curve

Makes sense to be in the most sensitive range, it was measured:

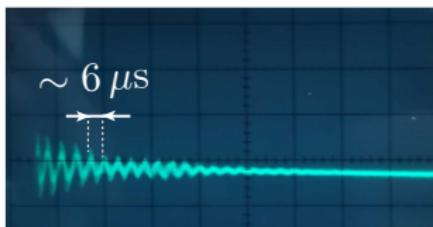
- $I$ - $V$  curve for a laser
- the dependence of the ph. diode voltage on the laser power.



So, laser voltage range of  $0.95 - 1.00$  mV selected.

# Problems

With used amplifiers, the following oscillations at the amplifier output with DC power can be observed:



This is due to the instability of the amplifier.

The main problem is that desired oscillations  $\sim 10$  ns.

We proceeded to experiments with faster amplifiers, but it is useful to understand results of such delays.

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## Real system

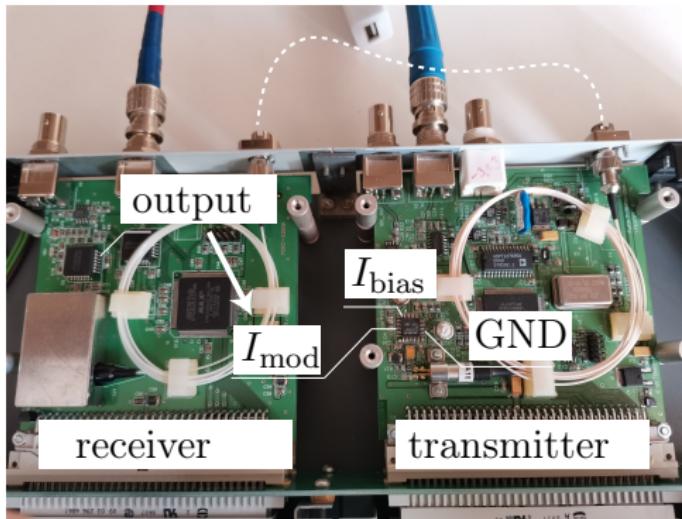
In real system<sup>3</sup>  $S_0^{-1} \int_0^\infty f(\eta)S(t - \eta)d\eta$  instead of  $S(t - \tau)$ .  
It reduces oscillations.

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<sup>3</sup>S. Tang, J. M. Liu, «Chaotic pulsing and quasi-periodic route to chaos in a semiconductor laser with delayed opto-electronic feedback», 2001.

# Alternative implementations

It is possible to easily use partially finished decision:



In theory, it is enough to turn around (or navigate through the amplifier) several clems, what is planned to be implemented after more thorough preparation.

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# Conclusion