Optical chaos based on a laser diode with positive feedback

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Goals

Globally we would like to transmit a high-frequency signal in encrypted form.

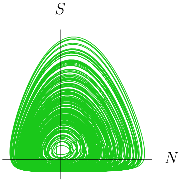
Here, we will consider the following steps towards this goal:

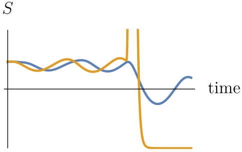
- dynamical chaos and synchronization (to encrypt and decrypt signal);
- theory of the laser evolution and its adaptation under our needs;
- realization of the positive feedback in laser: theory, modeling and experiment.

Definition of dynamic chaos and applications

Map[1] f is **chaotic**, if

- periodic orbits are dense everywhere;
- orbits are mixed;
- f sensitive to the initial conditions.





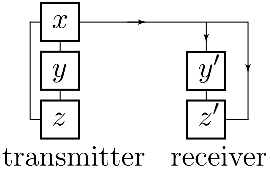
Possible applications:

- random numbers generation;
- signal encryption.

[1] W. Hirsch, S. Smale, Introduction to Chaos.

Synchronization

Possible[2] synchronization of chaotic systems:



enough to transmit
part of the signal;
configure system parameters.

The use of optics to transmit the signal allows to achieve a greater bandwidth of the channel.

UHFO (ultrahight frequency oscillations) is a characteristic to optic systems.

[2] M. Pecora, L. Carroll, Synchronization in Chaotic Systems, 1990.

The concept of a semiconductor laser

To start the laser idea we need to obtain:

- Solution of the Schrödinger equation in a semiconductor medium for the wavefunction of an electron;
- Induced polarization for distribution of holes and electrons in a semiconductor;
- Interaction of electrons in a semiconductor with an wave equation and outer electric field.

Electronic states in a semiconductor

We will need the Schrödinger equation:

$$H_{\text{crystal}}\Psi_n(\boldsymbol{r}) = \left[\frac{\boldsymbol{p}^2}{2m_0} + U_p(\boldsymbol{r})\right]\Psi_n(\boldsymbol{r})$$

where $\boldsymbol{p} = -i\hbar\nabla$ is the momentum operator, m_0 is the free electron mass, $U_p(\boldsymbol{r})$ is th periodic potential of the bulk semiconductor.
The solution is the Bloch function:

$$\Psi_{n,\boldsymbol{k}}(\boldsymbol{r}) = u_{n,\boldsymbol{k}}(\boldsymbol{r})\frac{1}{\sqrt{V}}e^{i\boldsymbol{k}\cdot\boldsymbol{r}}.$$

Hamilton in an outer field

In case of an optical field the Hamiltonian changes to

$$H = \frac{\boldsymbol{p} + e\boldsymbol{A}[\boldsymbol{r}, t]^2}{2m_0} + U_p(\boldsymbol{r}) = H_{\text{crystal}} + H'$$

$\boldsymbol{A}(\boldsymbol{r}, t)$ is the vector potential of the optical field. So the interaction Hamiltonian

$$H' \approx \frac{e}{m_0}\boldsymbol{A}(\boldsymbol{r}, t) \cdot \boldsymbol{p}.$$

Hamilton in an outer field

For y-propagating field: $E(\boldsymbol{r}, t) = \frac{1}{2} E_0 e^{i(\beta y - \omega t)} + \text{const}$ the Hamiltonian is

$$H' = \frac{1}{2} \mu(k_q, x, z)[E_0 e^{i(\beta y - \omega t)} + \text{const}]$$

where μ is the transition matrix that describes the semiconductor, k_q – quantized wavevector of the electron.

Induced polarization in an outer field

As one can obtain after rewritten density matrix in terms of carrier distributions, and in order avoid irrelevantly enormous formulas we get polarization as:

$$\mathcal{P}_{in}(\boldsymbol{r}, t) = \frac{1}{2}\mathcal{P}_{in,0}e^{i(\beta y - \omega t)} + \text{const} =$$

$$= -\sum \frac{\xi(\boldsymbol{r}, k_q, x, z)}{V(k_q, x, z)}[\rho_{eh}(k_q, x, z)\mu(k_q, x, z) + \text{const}]$$

where $V(k_q, x, z)$ is the confinement volume of electrons and holes and

$$\xi(\boldsymbol{r}, k_q, x, z) = \begin{cases} 1, & \boldsymbol{r} \text{ inside V} \\ 0, & \boldsymbol{r} \text{ outside V} \end{cases}$$

Injection the light

For the wave porpagating along the y direction in active layer the equation is:

$$\triangle E(\boldsymbol{r},t) - \mu_0 \varepsilon(\boldsymbol{r}) \frac{\partial^2}{\partial t^2} E(\boldsymbol{r},t) = \mu_0 \frac{\partial^2}{\partial t^2} P_{in}(\boldsymbol{r},t) \tag{1}$$

The partial solution for this equation we will be searching in a form of

$$E(\boldsymbol{r},t) = A_0(t) E_{\text{eig}}(x,z) = \frac{1}{2} E_0 e^{i(\beta y - \omega t)} + \text{const.}$$

Partial solution part

Substituting in wave equation(1) the obtained polarization and solution for $E(\boldsymbol{r}, t)$ and assuming that $A(t)$ changes slowly we get

$$\frac{dA_0}{dt} = \frac{i\omega}{2\varepsilon_0 n_r^2} A_0 \sum_\alpha \Gamma(0) \frac{1}{V} |\mu(k_q, x, z)|^2 [\rho_{ee} + \rho_{hh} - 1] \qquad (2)$$

where

$$\Gamma(0) = \frac{\varepsilon(x,z)|E_0(x,z)|^2}{\iint \varepsilon(x,z)|E_0(x,z)|^2 dxdz}$$

is a *dimensional coupling factor.*

Getting the solution

Now we can rewrite

$$\frac{dA_0}{dt} = \frac{1}{2} v_g [\Gamma(0) G - i\Gamma(0) N_r] A_0, \tag{3}$$

where $v_g = c/n_r$ for c – the speed of light in vacuum, and n_r – refraction coefficient. We will call the *gain coefficient* $g = \Gamma(0)G$. From Schrödinger equation we obtain the photon density as:

$$S = \frac{1}{2} \frac{\varepsilon_0 n_r^2 |A_0 E(0)|^2}{E_0}.$$

The laser equations

Using the equation for A_0 we obtain for the photon density (real part):

$$\frac{dS}{dt} = v_g \Gamma(0) G S. \tag{4}$$

and for electrons (holes) density (complex part):

$$\frac{dN}{dt} = -v_g G S. \tag{5}$$

And the demensional gain coefficient is approximately:

$$G = G_0 - G_1 \frac{S}{S_s}. \tag{6}$$

Carrier density in an outer field

Irrelevantly enormous formulas if someone really need it:

$$\frac{d}{dt}[\rho_{ee}(k_q, x, z)] = \frac{i}{\hbar}[H'\rho_{eh}(k_q, x, z) - \mathrm{const}] - \frac{\rho_{ee}(k_q, x, z) - f_e}{\tau_e},$$

$$\frac{d}{dt}[\rho_{hh}(k_q, x, z)] = \frac{i}{\hbar}[H'\rho_{eh}(k_q, x, z) - \mathrm{const}] - \frac{\rho_{hh}(k_q, x, z) - f_e}{\tau_e},$$

$$\frac{d}{dt}[\rho_{eh}(k_q, x, z)] = \frac{i}{\hbar}H'[\rho_{ee} + \rho_{hh} - 1] - \frac{i}{\hbar}E_{tr}\rho_{eh} - \frac{\rho_{eh}}{T_{deph}}.$$

The laser equations

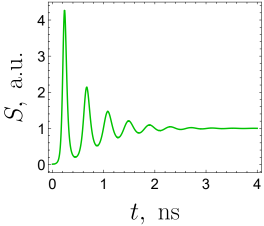
And from the previous equation we can describe the quantum well laser behavior

$$\frac{dS}{dt} = \Gamma(0)G_0 v_g S - \frac{S}{\tau_p}.$$

$$\frac{dN}{dt} = \frac{J}{e} - \frac{N}{\tau_n} - \Gamma G_0 v_g S.$$

Laser parameters.

Unmodulated laser ($\xi = 0$):



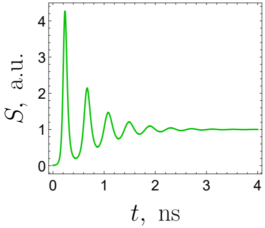
Laser equations:

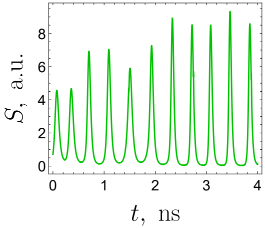
$$\frac{dS}{dt} = -\gamma_c S + \Gamma g S$$
$$\frac{dN}{dt} = \frac{J}{ed}\left[1 + \frac{\xi S(t-\tau)}{S_0}\right] - \gamma_s N - gS$$

Systems parameters:

- f_r - laser relaxation frequency. For our laser: $f_r = 1.4 \div 3.2$ GHz.
- τ - delay time. $\tau \sim f_r^{-1}$.
- $\xi = 0.1$ feedback parameter.

Chaos parameters.





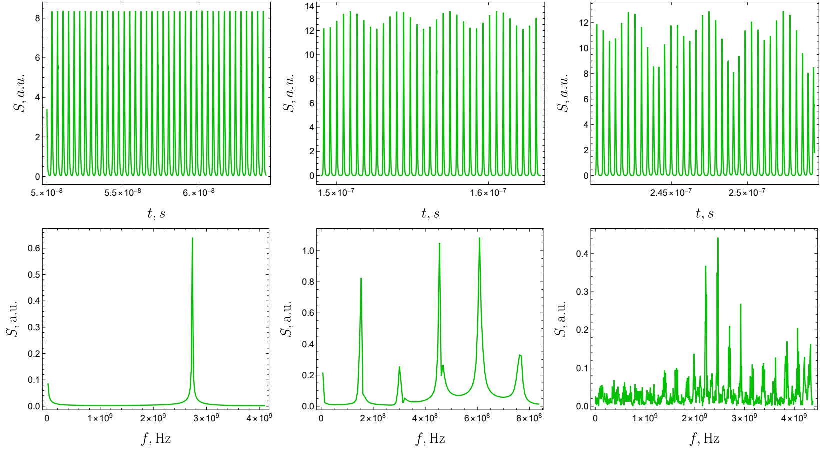
Chaos features:

- Unperiodic behaviour
- Frequency doubling scenario
  $f \ \to \ 2f \to \ 4f \to \cdots \to \infty f$

Chaos characteristics:

- λ – (Lyapunov's exponent). Exponential divergence on time for close i.c.: $\Delta S(t) \approx \Delta S(0) e^{\lambda t}$, where $\Delta S(0) \ll S_0$.
- Embedded dimensions
- Shannon entropy

Chaos modelling. Different regimes.

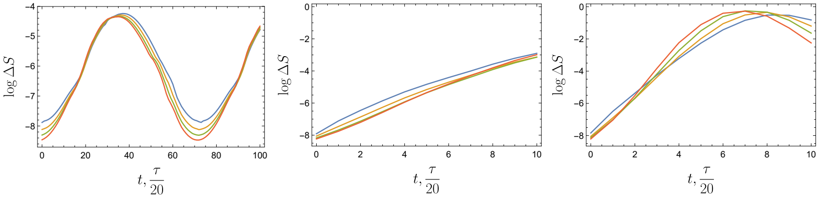


Delay time τ: $2\ T_r \mid 7.5\ T_r \mid 12\ T_r$

Chaos modelling.

Lyapunov exponents calculation for different points:

$$\Delta S \sim \exp(\lambda t) \;\Rightarrow\; \log \Delta S = \lambda t + \mathrm{const}$$



τ	$2\ T_r$	$7.5\ T_r$	$12\ T_r$
λ	0.0	$1.62\ f_r$	$1.84\ f_r$

Chaos is possible!.

Thus, length of fiber from numerical analysis. $L \sim 1$ m

Concept

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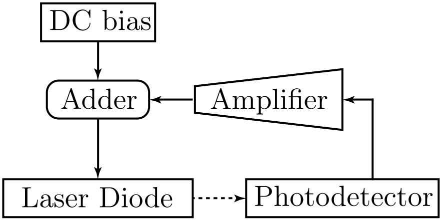
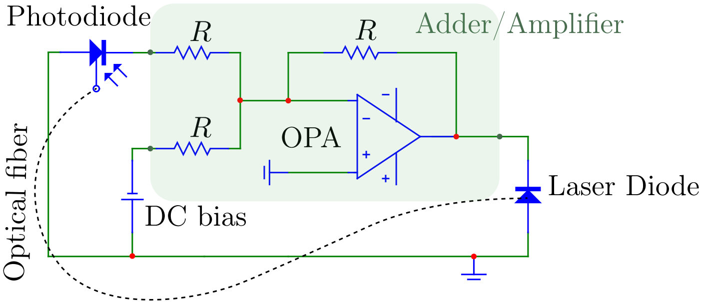


Scheme

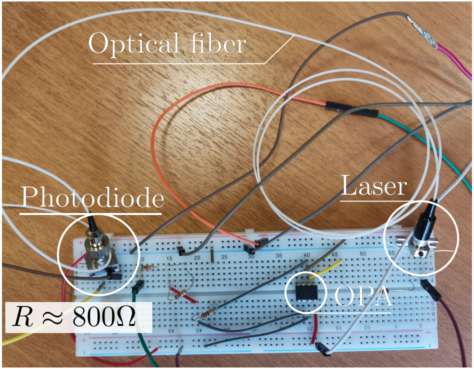
After several experiments came to this scheme with the summing amplifier:

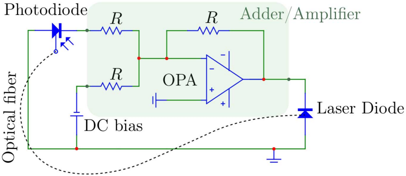


Photodiode power is enough to not use an additional amplifier.

Realization

For testing, the assembly was carried out on the dumping board.

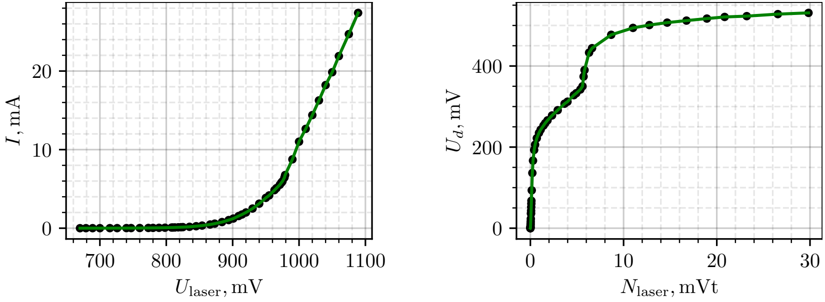




Thus, a scheme with positive feedback was implemented.
However, no desired oscillations were observed.

I–V curve

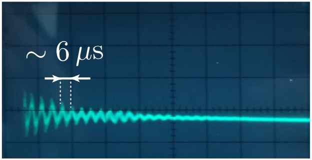
Makes sense to be in the most sensitive range, it was measured:
- *I-V* curve for a laser
- the dependence of the ph. diode voltage on the laser power.



So, laser voltage range of $0.95 - 1.00$ mV selected.

Problems

With used amplifiers, the following oscillations at the amplifier output with DC power can be observed:



This is due to the instability of the amplifier.

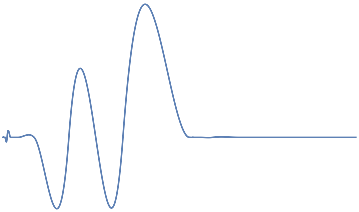
The main problem is that desired oscillations ~ 10 ns.

We proceeded to experiments with faster amplifiers, but it is usefull to understnd results of such delays.

Real system

In real system[3] feedback is cumulative $\int_0^\infty f(\eta)S(t-\eta)d\eta$ instead of $S(t-\tau)$. Oscillations are reduced due to integrating over large time $\tau_A \gg \tau$. Only amplifier oscillations may be observed.

So, numerical integration gives:



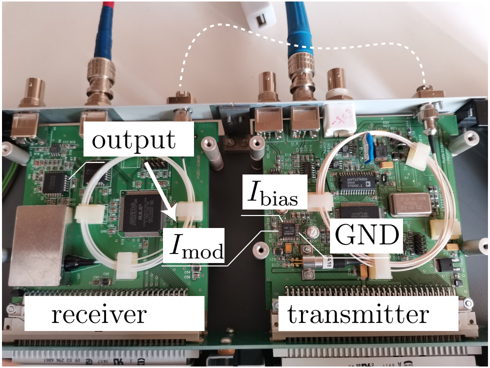
What about LF instead of HF?..

- For semi-conductors laser $f_r \sim 1$ GHz;
- For erbium-doped fiber ring laser (EDFRl) $f_r \sim 10$KHz;
- However, its hard to work with it.

[3] S. Tang, J. M. Liu, «Chaotic pulsing and quasi-periodic route to chaos in a semiconductor laser with delayed opto-electronic feedback», 2001.

Alternative implementations

It is possible to easily use partially finished decision:



In theory, it is enough to turn around (or navigate through the amplifier) several clems, what is planned to be implemented after more thorough preparation.

Conclusion

As a result of the project:

- According to the equations of moving the laser with feedback, a **numerical model was built** in the ideal and nonideal case.
- It is shown that **chaos is possible in the system**. Chaos parameters are estimated. The limitation of the frequencies of oscillations possible in the system is shown.
- A scheme of positive feedback has been developed and collected. The optimal parameters for the scheme are selected.
- Prepared for use industrial setup.