Theory background on topic «Chaos optical communication»

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05.03.2021

And God said let there be light, and there was light

$$\triangle E(\boldsymbol{r},t) - \mu_0 \varepsilon(\boldsymbol{r}) \frac{\partial^2}{\partial t^2} E(\boldsymbol{r},t) = \mu_0 \frac{\partial^2}{\partial t^2} P_{in}(\boldsymbol{r},t) \tag{1}$$

Which is solved as

$$E(\boldsymbol{r},t) = \frac{1}{2} E_0 e^{i(\beta y - \omega t)} + \mathrm{const}$$

And we assume:

$$E(\boldsymbol{r},t) = A(t) E(s').$$

$$E(\boldsymbol{r}, t) = A(t)E(s'), \tag{2}$$

where A states for for both amplitude and phase. And E(s') eigenmode function.

For most semiconductor lasers waveguides, the lasing action occurs in eigenmode. Also we use slow wave variation.

$$|\frac{d^2A}{dt^2}| \ll \omega|\frac{dA}{dt}|. \tag{3}$$

$$\frac{dA}{dt} = \frac{i\omega}{2} A \sum_{\alpha} \frac{1}{V(\alpha)} \frac{\int \zeta(\boldsymbol{r},\alpha)|E(s)|^2 ds}{\int \varepsilon(s)|E(s)|^2 ds} \times |\mu(\alpha)|^2 \frac{[\rho_{ee} + \rho_{hh}(\alpha) - 1]}{(E - E_{\alpha}) + iE_{T^2}}, \tag{4}$$

where ρ_{ee} – distribution of electrons, and ρ_{hh} – distribution of holes. It's time to oversimplify this monster:

$$\frac{dA}{dt} = \frac{i\omega}{2\varepsilon n_r^2} A \sum_{\alpha} \Gamma_{MD} \frac{1}{V_{MD}} |\mu(\alpha)|^2 [\rho_{ee} + \rho_{hh} - 1], \tag{5}$$

where n_r – refractive index of the active region. And Γ_{MD} – dimensional coupling factor that shows how injected carriers interact with photons.

For the most beauty we assume that well is 1D, and the equastion:

$$\frac{dA}{dt} = \frac{1}{2} v_g (\Gamma_{MD} G - i\Gamma_{MD} N_r) A, \tag{6}$$

where $v_g = c/n_r$ is the speed of light in vacuum. And $\Gamma_{MD} G = g$ is so called gain coefficient.

If we obtain photon density inside active region as:

$$P = \frac{1}{2}\varepsilon n_r^2 |AE(0)|^2 / E.$$

And from previous equation we get

$$\frac{dP}{dt} = v_g g p P = v_g \Gamma_{MDGP} = -\gamma_C P + \Gamma g P. \tag{7}$$

And in the same way:

$$\frac{dN}{dt} = -v_g G(E) P = \frac{J}{ed}\left(1 + \frac{\xi P(t-\tau)}{P_0}\right) - \gamma_s N - gP. \tag{8}$$

So we have two main equations to describe our model:

$$\frac{dN}{dt} = -v_g G(E) P = \frac{J}{ed}\left(1 + \frac{\xi P(t-\tau)}{P_0}\right) - \gamma_s N - gP. \tag{9}$$

$$\frac{dP}{dt} = v_g g p P = v_g \Gamma_{MDGP} = -\gamma_C P + \Gamma g P. \tag{10}$$

where P – intracavity photon density, N – carrier density, τ – feedback delay time.

And nonlinearities lies in the optical gain function, wich, due to several articles can be explained as:

$$g \simeq= g_0 + g_n(N - N_0) + g_P(P - P_0). \tag{11}$$

All theory gathered from book: "Semiconductor Lasers I – Fundamentals" by Eli Kapon, Institute of Micro and Optoelectronics Department of Physics, Swiss (1999).

Laser equations with feedback

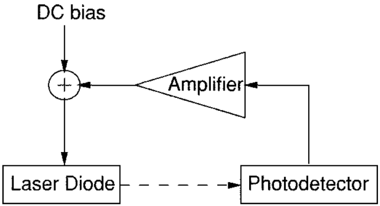

Figure 1: Schematic experimental setup

Laser – single-mode DFB laser diode (1300 nm);
Light output detector – high-speed InGaAs photodetector;

Important: it is suitable for any other semiconductor laser with an active medium.

Laser equations with feedback

The nonlinear behavior of a semiconductor laser with delayed opto-electronic feedback:

$$\frac{dS}{dt} = -\gamma_C S + \Gamma g\, S$$
$$\frac{dN}{dt} = \frac{J}{ed} - \gamma_s N - g\, S + \frac{J}{ed}\frac{\xi S[t-\tau]}{S_0}$$
$$g = g_0 + g_n(N - N_0) + g_p(S - S_0)$$

S – photon density;
N – carrier density;
τ – feedback delay;
g – opt. gain coeff.

positive opto-electronic feedback – $\xi > 0$.

Optical gain – measure of how well a medium amplifies photons by stimulated emission.

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Laser equations with feedback

Some transformations we can reduce this system to the

$$\frac{ds}{dt} = c_1 n(s+1) - c_2 s(s+1)$$
$$\frac{dn}{dt} = c_3 + c_4 s[t-\tau] + (c_5 s - c_8 n)(1+s) - c_6 n - c_7 s,$$

where $s = (S - S_0)/S_0$, $n = (N - N_0)/N_0$, which can be numerically integrated.

Numerical solution

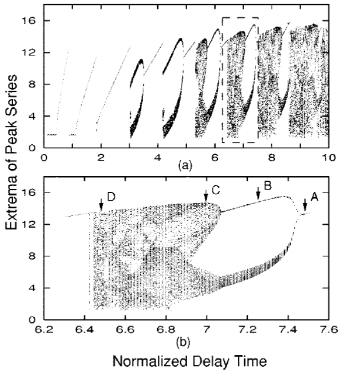

Figure 2: Bifurcation diagram of the extrema of the peak series

Here $\hat{\tau} = \tau f_r$ – demensionless delay.

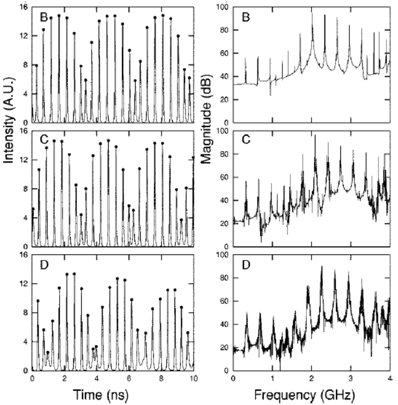

Figure 3: Time series and power spectra

Reconstruction of the attractor

With part of system variables, it is possible to restore the general view of the attractor.

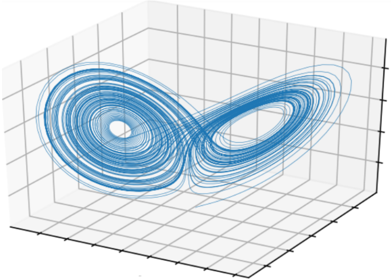

Figure 4: Original system attractor

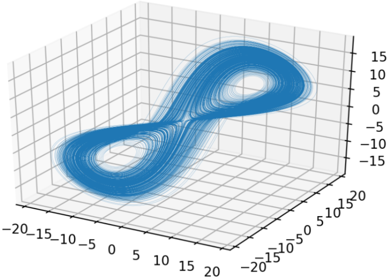

Figure 5: Reconstructed by one coordinate system attractor

Numerical features

- **Lyapunov exponent**:

$$\lambda(x_0) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln |f'(x_i)|,$$

where $\lambda > 0$ – characteristic for chaos.

- **Shannon Entropy**:

$$\mathrm{H}(X) = -\sum_{i=1}^{n} \mathrm{P}(x_i) \log \mathrm{P}(x_i),$$

where higher rates correspond to more random sources.

Applications

- **Random number generation.**
  Random bits produced at a much higher rate than other physical sources of entropy including quantum RNG.
- **Chaos computing.**
  It is possible to create "NOR" gate
  "NOR" → "AND" , "OR", "XOR", "NOT", ... → computer
  ∗ double-scrolled chaotic attractor and threshold