Implementations of dynamic chaos in different optical systems

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Goals

Globally we are to observe different realization of dynamic chaos and implement some on our own.

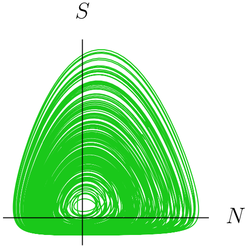
Here we will consider the following steps:

- study the dynamical chaos on its own;
- implement laser theory to create chaos;
- model laser behavior and attempt to build a circuit;
- study light branches chaoticly penetrating thin layer;
- model and implement this model.

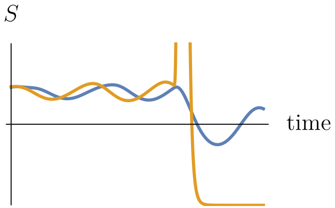
Definition of dynamic chaos

Map[1] f is **chaotic**, if

- f **sensitive** to the initial conditions.
- periodic orbits are dense everywhere;
- orbits are mixed;



Dense mixed orbits example



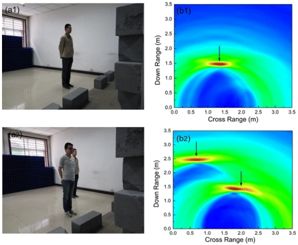
Sensitivity example

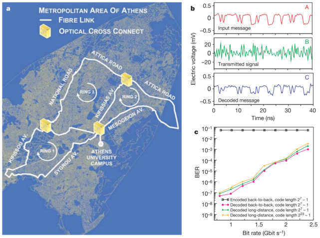
[1] W. Hirsch, S. Smale, Introduction to Chaos.

Applications of dynamic chaos

Possible applications:

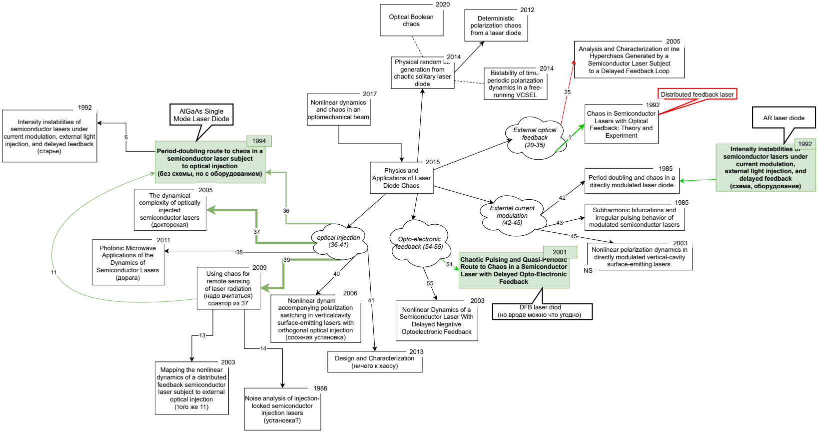
- optical reflectometry;
- chaos radar;
- random numbers generation;
- signal encryption.





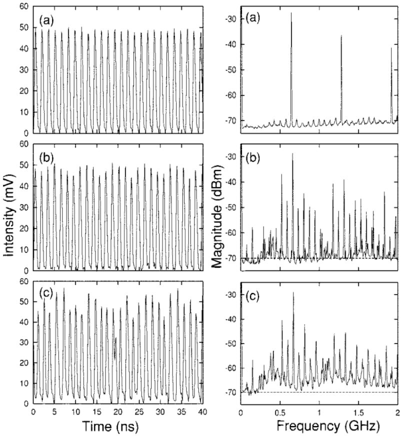
[1]Chaos Through-Wall Imaging Radar by Hang Xu et al. (2009)
[2]Chaos-based communications by Apostolos Argyris et al. (2005)

From the root article a tree has grown



[1] Physics and Applications of Laser Diode Chaos by M. Sciamanna et al.

The decided variant



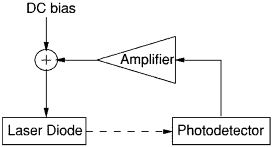
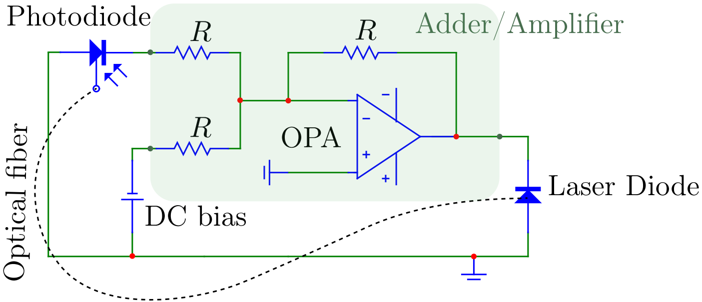


Figure 1: Experimental results of the time series and power spectra of different pulsing states at different delay times. On a circuit dashed line represents optical path.

[1] Chaotic Pulsing and Quasi-Periodic Route to Chaos in a Semiconductor Laser with Delayed Opto-Electronic Feedback S. Tang and J. M. Liu (2001)

Scheme

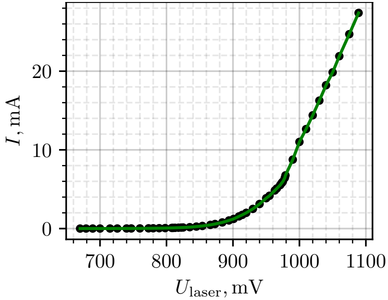
After several experiments came to this scheme with the summing amplifier:

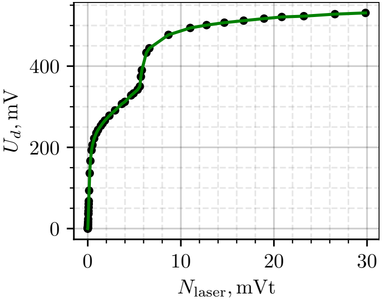


Photodiode power is enough to not use an additional amplifier.

I–V curve

Makes sense to be in the most sensitive range, it was measured:
- *I-V* curve for a laser
- the dependence of the ph. diode voltage on the laser power.

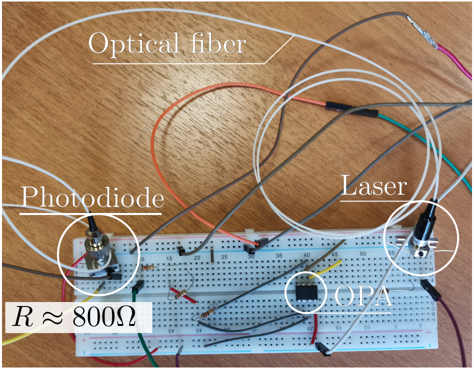


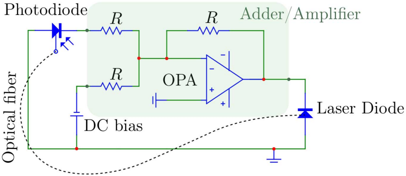


So, laser voltage range of 0.85V selected.

Realization

For testing, the assembly was carried out on the dumping board.

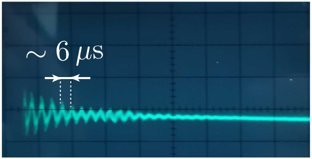




Thus, a scheme with positive feedback was implemented.
However, no desired oscillations were observed.

Problems

With used amplifiers, the following oscillations at the amplifier output with DC power can be observed:



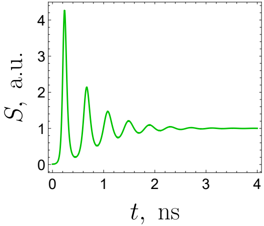
This is due to the instability of the amplifier.

The main problem is that desired oscillations ~ 10 ns.

We proceeded to experiments with faster amplifiers, but it is usefull to understand results of such delays.
oscillations megahertz

Laser parameters.

Unmodulated laser ($\xi = 0$):



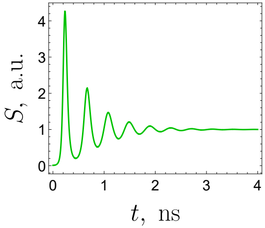
Laser equations:

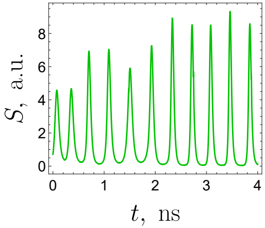
$$\frac{dS}{dt} = -\gamma_c S + \Gamma g S$$
$$\frac{dN}{dt} = \frac{J}{ed}\left[1 + \frac{\xi S(t-\tau)}{S_0}\right] - \gamma_s N - gS$$

Systems parameters:

- f_r – laser relaxation frequency. For our laser: $f_r = 1.4 \div 3.2$ GHz.
- τ – delay time. $\tau \sim f_r^{-1} \sim 1$ns.
- $\xi = 0.1$ feedback parameter.

Chaos parameters.





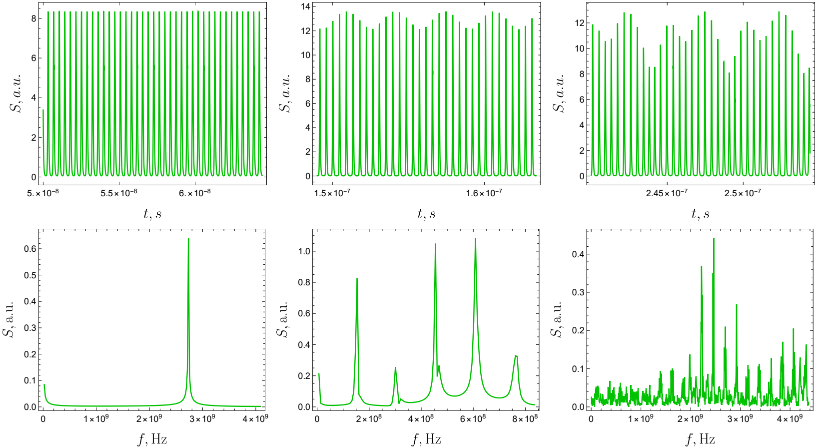
Chaos features:

- Unperiodic behaviour
- Frequency doubling scenario $f \rightarrow 2f \rightarrow 4f \rightarrow \cdots \rightarrow \infty f$

Chaos characteristics:

- λ – (Lyapunov's exponent). Exponential divergence on time for close i.c.: $\Delta S(t) \approx \Delta S(0)e^{\lambda t}$, where $\Delta S(0) \ll S_0$.
- Embedded dimensions
- Shannon entropy

Chaos modelling. Different regimes.

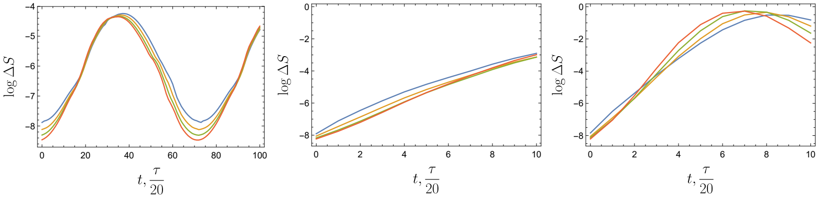


Delay time τ: $2\ T_r$ | $7.5\ T_r$ | $12\ T_r$

Chaos modelling.

Lyapunov exponents calculation for different points:

$$\Delta S \sim \exp(\lambda t) \;\Rightarrow\; \log \Delta S = \lambda t + \mathrm{const}$$



τ	$2\ T_r$	$7.5\ T_r$	$12\ T_r$
λ	0.0	$1.62\ f_r$	$1.84\ f_r$

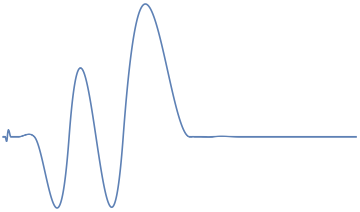
Chaos is possible!.

Thus, length of the fiber from the numerical analysis. $L \sim 1$ m

Real system

In real system[2] feedback is cumulative $\int_0^\infty f(\eta)S(t-\eta)d\eta$ instead of $S(t-\tau)$. Oscillations are reduced due to integrating over large time $\tau_A \gg \tau$. Only amplifier oscillations may be observed.

So, numerical integration gives:



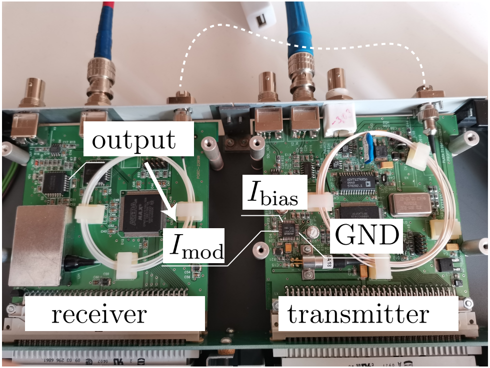
What about LF instead of HF?..

- For semi-conductors laser $f_r \sim 1$ GHz;
- For erbium-doped fiber ring laser (EDFRl) $f_r \sim 10$kHz;
- However, its hard to work with it.

[2] S. Tang, J. M. Liu, «Chaotic pulsing and quasi-periodic route to chaos in a semiconductor laser with delayed opto-electronic feedback», 2001.

Alternative implementations

We tried to use partially finished decision for video transmission:



$\tau_{\text{i}}/\tau_{\text{r}}$	$\lambda\tau_{\text{r}}$
0	1.62
2	0.71
6	0.11
10	$S \to \text{const}$

Observed the constant behavior.

The system is designed for oscillations < 20 MGz, so

$$S(t-\tau) \to \int_0^{\tau_{\text{i}}} f(\eta) S(t-\eta)\, d\eta, \quad \tau_{\text{i}} \approx 50\tau_{\text{r}} \gg 10\tau_{\text{r}}, \Rightarrow \lim_{t\to\infty} S(t-\tau) = \text{const}$$

Results in the modeling

Modeled good numerical solution. The existence of chaos is shown. The maximum «slowness» of the system is estimated.

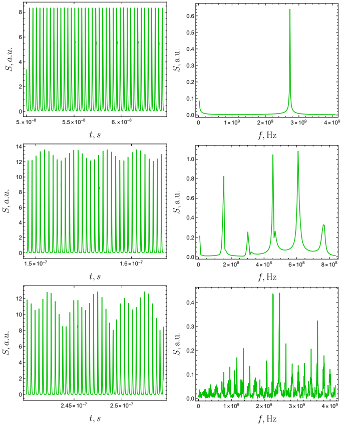


Figure 2: Modeling

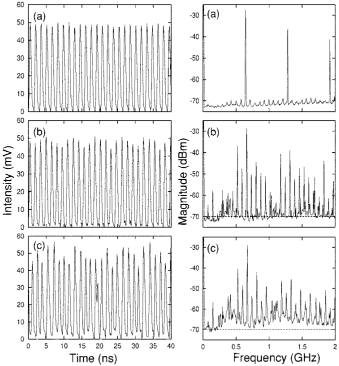
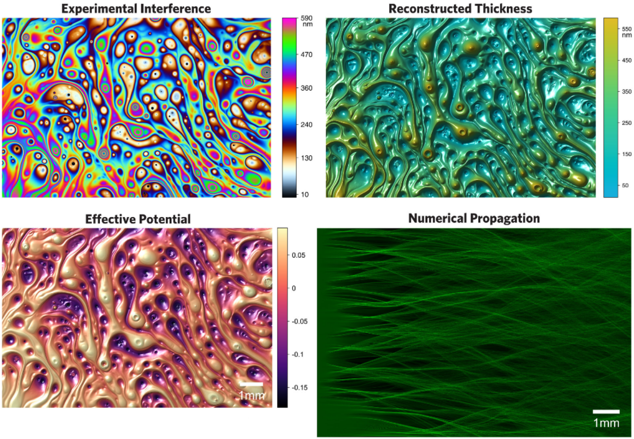


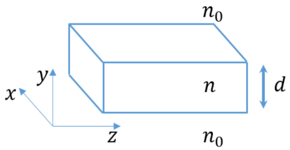
Figure 3: Article experiment

The other idea in nature



Patsyk, A., Sivan, U., Segev, M. et al. "Observation of branched flow of light". Nature 583, 60–65 (2020).

Theory of refraction index



The Helmholtz equation again

$$\Delta E + k_0^2 n^2(y) E = 0$$

while solving like $E = \psi(x,z)G(y)$ gives a solutions:

$$\partial_{yy} G + k_0^2 n^2(y) G = k_0^2 n_{\text{eff}}^2 G, \qquad \nabla_\perp^2 \psi + k_0^2 n_{\text{eff}}^2 \psi = 0.$$

And solving the left one we obtain

$$k_0^2 y \sqrt{n_{\text{soap}}^2 - n_{\text{eff}}^2} + 2 \arctan\left(\frac{\sqrt{n_{\text{soap}}^2 - n_{\text{eff}}^2}}{\sqrt{n_{\text{eff}}^2 - n_{\text{air}}^2}} \right) - \pi(m+1) = 0.$$

Having the eikonal equation

$$f = a\exp[i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t + \alpha)] = a\exp(i\psi) \qquad \Rightarrow \qquad k_i k^i = \partial_i\psi\partial^i\psi = 0.$$

Then we can find the light penetrating a layer with different refraction dedistribution:

$$\begin{cases} \dot{\boldsymbol{p}} = - & \partial_{\boldsymbol{r}} H \\ \dot{\boldsymbol{r}} = & \partial_{\boldsymbol{p}} H \end{cases} \qquad\qquad \delta S = \delta \int \boldsymbol{p}\cdot d\boldsymbol{l} = 0$$

$$\Rightarrow$$

$$\begin{cases} \dot{\boldsymbol{k}} = - & \partial_{\boldsymbol{r}} \omega \\ \dot{\boldsymbol{r}} = & \partial_{\boldsymbol{k}} \omega \end{cases} \qquad\qquad \delta \psi = \delta \int \boldsymbol{k}\cdot d\boldsymbol{l} = 0$$

Now we obtain the Fermat principle because $k^i = \mathrm{grad}\psi$:

$$\oint \mathrm{grad}\psi \, d\boldsymbol{l} = \oint n\boldsymbol{s} \, d\boldsymbol{l} = 0 \qquad \Leftrightarrow \qquad \delta \int n \, dl = 0.$$

We will consider that

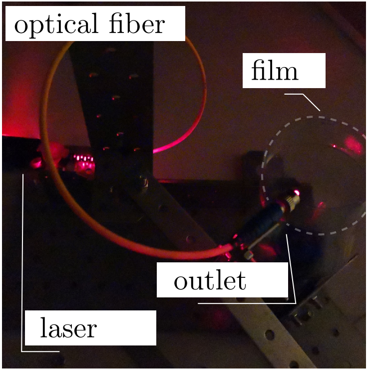
$$dl = \sqrt{dx^2 + dy^2} = \sqrt{1 + (y'_x)^2} \, dx \qquad \Rightarrow \qquad \delta \int n\sqrt{1 + (y'_x)^2} \, dx = 0.$$

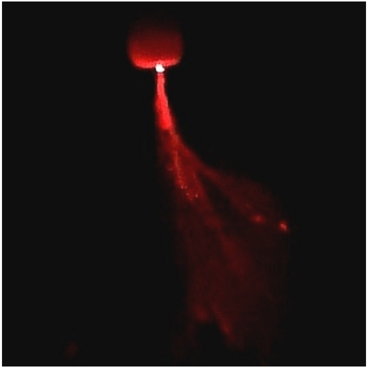
So it is Euler-Lagrange equation and the path of light is well known

$$\frac{d}{dx}\frac{\partial L}{\partial y'_x} - \frac{\partial L}{\partial y} = 0, \qquad \text{for } L = n\sqrt{1 + (y'_x)^2}.$$

Experimental setup

The beam goes to the film through the optical fiber:

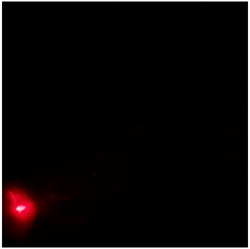




Light scattering occurs most likely on micro bubbles and other inhomogeneities inside the solution.

Film dynamics

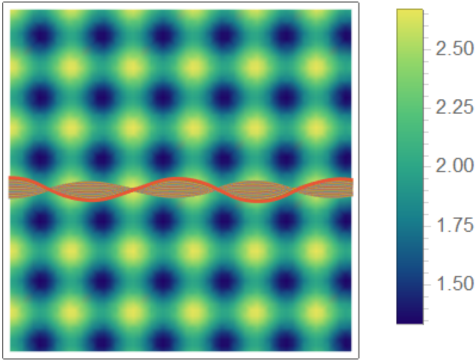
After outlet of the optical fiber ($d_{\text{core}} \sim 50\,\mu$m) light really starts branching.

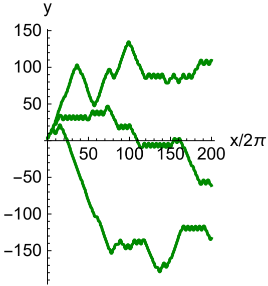


The effect of the thickness of the film on the system dynamics is noticeable. After the time expires, the film becomes thinner, the rays are actively branched.

Modeling

To demonstrate branching, consider the movement of light in the environment with $n(x, y) = \frac{1}{3}(\cos x + \cos y) + 2$.

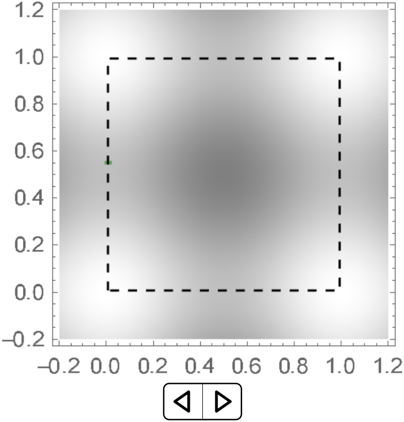




This happens random walk across heterogeneous media.

Sinai billiard

Walk in periodic media can be reduced to the billiard table.



For almost all trajectories:

- ✓ ergodic behaviour;
- ✓ positive Lyapunov exponent;

so we have pretty example of dynamical chaotic system.

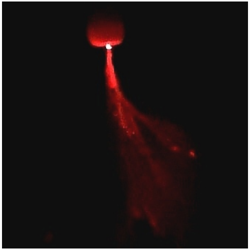
That can be considered as a smoothed version of Lorentz gas or Sinai billiard.

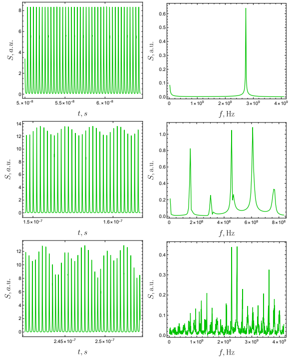
Conclusion

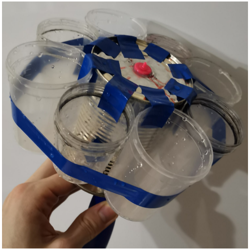
As a result of the project:

- According to the equations of the laser's evolution with a feedback, the **numerical model was built**, compliant observations from the article , in the ideal and non ideal case.
- The another system was observed to show that chaotic systems is all around us. The branches of light were observed and modelled. Also we found an analogy with billiard theory.
- It is shown that **chaos is possible in the optical systems**. Chaos parameters are estimated. The frequency limitation were evaluated for the system.

Chaos is everywhere!







Literature

- W. Hirsch, S. Smale, *"Introduction to Chaos"*;
- "Chaos Through-Wall Imaging Radar" by Hang Xu et al. (2009)
- *"Chaos-based communications"* by Apostolos Argyris et al. (2005);
- M. Pecora, L. Carroll, *"Synchronization in Chaotic Systems"* (1990);
- *"Physics and Applications of Laser Diode Chaos"* by M. Sciamanna, and K. A. Shore (2015);
- *"Chaotic Pulsing and Quasi-Periodic Route to Chaos in a Semiconductor Laser with Delayed Opto-Electronic Feedback"* by S. Tang and J. M. Liu (2001)
- *"Semiconductor Lasers I Fundamentals"* Chapter I by Ch.I by Bin Zhao, Amnon Yariv (1998)