The s.c. eq of motion

$$\hbar\dot{\boldsymbol{k}} = -\frac{e}{c}\boldsymbol{v}\times\boldsymbol{B} = -\frac{e}{\hbar^2 c}\nabla_{\boldsymbol{k}}\varepsilon_n(\boldsymbol{k})\times\boldsymbol{B},$$

hence we get

$$\frac{d\varepsilon)n(\boldsymbol{k})}{dt} = \boldsymbol{k}\left[\nabla_{\boldsymbol{k}},\varepsilon_n(\boldsymbol{k})\right] = 0,$$

thus $\varepsilon_n(\boldsymbol{k})$ is a c.o.m.

Consider 2D system with holes and electrons: $\boldsymbol{k}\cdot\boldsymbol{B} = 0$. What is corresponding real space trajectory? We could look at components

$$\boldsymbol{e}_B\times\hbar\dot{\boldsymbol{k}} = -\frac{e}{c}\boldsymbol{e}_B\times[\boldsymbol{v}\times\boldsymbol{B}] = -\frac{eB}{c}\left[\boldsymbol{v}_n(\boldsymbol{k}) - \boldsymbol{e}_B\cdot(\boldsymbol{e}_B\cdot\boldsymbol{v}_n(\boldsymbol{k}))\right] = -\frac{eB}{c}\boldsymbol{v}_\perp.$$

We could integrate over time

$$\boldsymbol{r}_\perp - \boldsymbol{r}_\perp(0) = -\frac{\hbar c}{eB}\boldsymbol{e}_B\times\left[\boldsymbol{k}(t) - \boldsymbol{k}(0)\right].$$

Thus $\boldsymbol{r}_\perp$ orbit is simpy the rotated momentum space orbit:

$$z(t) - z(0) = \int_0^t \frac{1}{\hbar}\partial_{k_z}\varepsilon_n(\boldsymbol{k}(z))\,dz \qquad \text{maybe}.$$

The period of an orbit

$$T = \int_{t_1}^{t_2} dt = \oint\frac{1}{|\dot{\boldsymbol{k}}|}\,d|\boldsymbol{k}| = \oint\frac{d|\boldsymbol{k}|}{|(\nabla_{\boldsymbol{k}}\cdot\varepsilon_n(\boldsymbol{k}))_\perp|}\frac{\hbar^2 c}{eB}.$$

Consider two trajectories with energy difference $\Delta\varepsilon$ and Δk in momentum space:

$$\Delta\varepsilon = \nabla_{\boldsymbol{k}}\varepsilon_n(\boldsymbol{k})\cdot\boldsymbol{\Delta}(\boldsymbol{k}) = |(\nabla_{\boldsymbol{k}}\varepsilon_n)_\perp|\cdot|\boldsymbol{\Delta}(\boldsymbol{k})|.$$

and back to the period

$$T = \frac{\hbar^2 c}{eB}\oint|d\boldsymbol{k}|\frac{|\boldsymbol{\Delta}(\boldsymbol{k})|}{\Delta\varepsilon} = \frac{\hbar^2 v}{eB}\frac{A(\varepsilon+\Delta\varepsilon) - A(\varepsilon)}{\Delta\varepsilon} = \frac{\hbar^2 c}{eB}\frac{\partial A}{\partial\varepsilon}\bigg|_{kz}.$$

For example with $\varepsilon = \frac{\hbar^2 k^2}{2m}$ we have $T = \frac{2\pi}{\omega_c}$ with $\omega_c = \frac{eB}{m^* c}$.

The Bohr-Sommerfeld quantization rule

It is true that

$$\Delta\varepsilon = \frac{2\pi\hbar}{T} = \frac{2\pi eB}{\hbar c}\left(\frac{\partial A(\varepsilon)}{\partial\varepsilon}\right)^{-1},$$

and that directly leads to a quantization of momentum space.

$$\left(\frac{\partial A}{\partial\varepsilon}\right)\Delta\varepsilon = \Delta A = \frac{2TeB}{\hbar c} \qquad\Rightarrow\qquad A_n = \frac{2\pi eB}{\hbar c}(n+\nu),$$

with ν is arbitrary. Energy thus

$$\varepsilon_n = \hbar\omega_c\left(n+\tfrac{1}{2}\right).$$