Quantum Hardware
Problem Set No. 9

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1 Qubit Drive and JC Hamiltonian for Circuits

The Hamiltonian of a Cooper-pair box approximated to two levels is given by

$$H_2 = -2E_C(1-2n_g)\sigma_z - \frac{E_J}{2}\sigma_x$$

This is considered a Cooper-pair box qubit, where charging energy E_C and Josephson energy E_J are determined by the design of the qubit, and the offset charge n_g is controllable via some gate voltage. The transition energy of the qubit can be modified by changing n_g with the gate voltage. In this exercise, we will explore how the offset charge can be used to drive the qubit, and how a resonator can be coupled to the qubit through the offset charge to recover the Jaynes-Cummings Hamiltonian.

(a) Add an oscillating drive to the gate voltage such that

$$n_g \to n_g + \eta\cos\omega t.$$

Choose a charge offset and perform a change of basis so that the $|0\rangle$ and $|1\rangle$ states are eigenstates of the Hamiltonian when the drive is turned off. What is the transition energy of the qubit?

(b) The n_g drive results in a rotation on the qubit. In the case that the drive is resonant with the qubit transition, how long should the drive be turned on to implement a π rotation?

Hint: The transformed Hamiltonian should look familiar (recall Problem Set 2, Exercise 2). What is the Rabi frequency of the rotation induced by this drive?

(c) Our Cooper-pair box qubit is capacitively coupled to an LC oscillator as shown in the diagram. The Hamiltonian of the combined system is now

$$H_2 = -2E_C(1-2n_g)\sigma_z - \frac{E_J}{2}\sigma_x + \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right)$$

It is not immediately obvious how the qubit and resonator are coupled together, until we consider breaking n_g into a classical part and a quantum part

$$n_g \to n_g^{clas.} + \hat{n}_g^{q.m.}$$

This quantum part of the offset charge is due to the zero point fluctuations in the resonator. Show that

$$\hat{n}_g^{q.m.} = \frac{C_g}{2e}\sqrt{\frac{\hbar\omega_r}{2C}}(\hat{a}^\dagger + \hat{a}).$$

Hint: How does voltage across the resonator effect the charge on the CPB island? What are the voltage zero point fluctuations in the resonator?

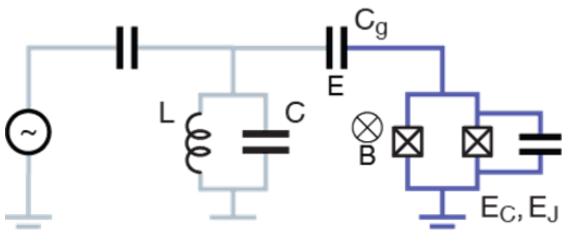

Figure 1: CPB coupled capacitively to a LC resonator.

(d) Use the same arguments as part (a) to put the coupled resonator-qubit Hamiltonian into the form of the Jaynes-Cummings Hamiltonian. Calculate the coupling between the resonator and the qubit. *Hint: Recall Problem Set 5, Exercise 1*

2 Transmon Simulations with DRAG

Use the Jupyter notebook provided for this problem, which will guide you step by step through the following problems.

(a) Simulate the dynamics of a two level system, i.e. a qubit, both in the lab frame and in the qubit frame.

(b) Repeat this simulation, but for a transmon qubit with 3 levels and small anharmonicity.

(c) Implement a Gaussian-drag pulse to reduce the effect of leakage.

(d) Apply the GRAPE algorithm to find optimal control solutions.

3 CPhase Gate

In this exercise, the implementation of non-adiabatic and adiabatic C-Phase gates for a system of two coupled flux tunable transmon qubits is considered. The Hamiltonian which describes the coupling between $|11\rangle$ and $|20\rangle$ state at the $|11\rangle \leftrightarrow |20\rangle$ avoided crossing is given by:

$$\mathcal{H}_2 = J_2 \left(|11\rangle \langle 02| + |02\rangle \langle 11| \right), \tag{1}$$

where J_2 is the coupling strength between $|11\rangle$ and $|02\rangle$ states.

(a) Show that a π phase shift can be imposed on the $|11\rangle$ state (i.e. a C-Phase gate) when a sudden excursion into the $|11\rangle \leftrightarrow |20\rangle$ avoided crossing is made and the system evolves under the Hamiltonian in Eq. (1). Find the evolution time t in terms of J_2, such that the time evolution of the system results in a C-Phase gate.

(b) Now, consider the situation when one of the qubits is adiabatically moved to the $|11\rangle \leftrightarrow |20\rangle$ avoided crossing. This is followed by a waiting time of τ at the avoided crossing. Calculate the phase acquired by the $|01\rangle$, $|10\rangle$ (single-qubit phases) and $|11\rangle$ state (two-qubit phase) and write down the resulting time evolution unitary. Show that the phase acquired by the $|11\rangle$ state is a sum of single-qubit phases and a two-qubit phase. Determine the condition involving the single and two-qubit phases such that the time evolution unitary at a time τ represents a C-Phase gate. The unitary operator for this protocol in the computational subspace can be written as

$$U_{ad} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\theta_{01}(l)} & 0 & 0 \\ 0 & 0 & e^{i\theta_{10}(l)} & 0 \\ 0 & 0 & 0 & e^{i\theta_{11}(l)} \end{bmatrix}$$

where $l(t)$ is the path the transition takes in time, which results in a phase

$$\theta_{ij}(l(\tau)) = \int_0^\tau dt\, \omega_{ij}(l(t))$$

Hint: the equation above can be applied linearly to a sum of frequencies. What is the phase acquired by the $|11\rangle$ state relative to the $|01\rangle$ and $|10\rangle$ states? How can these extra single qubit phases be undone?