## A brief intro into Luthinger liquids

Let's look at the density correlation functions

$$D(\boldsymbol{q},\omega)=\int\frac{d^Dp}{d(2\pi)^D}\frac{n_p-n_{p+q}}{\omega-\varepsilon(p+q)+\varepsilon(p)+i\varepsilon}=\frac{q}{2\pi}\left(\frac{1}{\omega-qv_F+i\varepsilon}-\frac{1}{\omega+qv_F+i\varepsilon}\right).$$

The spinless Tomonaga0Luthinger model

$$H=H_0+H_{int}=\sum_k\varepsilon_kc_k^\dagger c_k+\frac{1}{2L}\sum_{k,k',q}V(q)c_k^\dagger c_{k'}^\dagger c_{k'-q}c_{k+q}.$$

We could work with the linear dispersion for low energy

$$\xi_k=\varepsilon_k-\mu\approx(|k|-k_F)v_F.$$

Next we separate rightmovers and leftmovers

$$c_k=c_{kR}\theta(k)+c_{kL}\theta(-k),$$

and than

$$H_0=v_F\sum_{k>0}\left(kc_{kR}^\dagger c_{kR}-kc_{kL}^\dagger c_{kL}\right)-(N_R+N_L)\,k_Fv_F.$$

There are two interactions

$$H_{int}^{(1)}=\frac{1}{2L}\sum_{k>0,q,k'<0}V(q)\left(c_{kR}^\dagger c_{k'L}^\dagger c_{k'-q,L}c_{k+q,R}+c_{kR}^\dagger c_{k'L}^\dagger c_{k'-q,R}c_{k+q,L}\right)+(R\leftrightarrow L).$$

Note that $k\approx k_F$, $q\approx\pm-2k_F$. Than we could rewrite it in form

$$H_{int}^{(1)}=\frac{1}{2L}\sum\left(V(q)c_{kR}|^\dagger c_{k+q,R}c_{k'L}^\dagger c_{k'-q,L}-V(-q+k'-k)c_{kR}^\dagger c_{k+q,R}c_{k'c}^\dagger c_{k'-q,L}\right).$$

Then we can introduce left/right moving density operators

$$\rho_R(q)=\sum_{k>0}c_k^\dagger c_{k+q}\approx\sum_{k>0}c_{kR}^\dagger c_{k+qR},$$

and rewrite

$$H_{int}^{(1)}\approx\frac{1}{2L}\sum_q\left(V(0)-V(2k_F)\right)\rho_R(q)\rho_L(-q)+(R\leftrightarrow L),$$

and finally we have

$$H_{int}=\frac{1}{2L}\sum_{q\neq0}V_1\left(\rho_L(q)+\rho_R(q)\right)\left(\rho_L(-q)+\rho_R(-q)\right).$$