

Noninteracting Fermions

Consider Hamiltonian

$$H = \sum_{n,\sigma} c_{k\sigma}^\dagger (\varepsilon_k - \mu) c_{k\sigma} = \sum_{k,\sigma} \xi_k c_{k\sigma}^\dagger c_{k\sigma},$$

with $T = 0$

$$|\text{gs}\rangle = \prod_{|k| < k_F} c_{k\sigma}^\dagger |0\rangle.$$

And we know very well single-particle excitations. Let's add a particle $k > k_F$

$$\delta E_k = \frac{1}{2m} (k^2 - k_F^2) = \frac{1}{2m} ((k_F + \delta k)^2 - k_F^2) \approx v_F \cdot \delta k.$$

with $\delta k = |k - k_F| \ll k_F$ and $v_F = k_F/m$. If we remove a particle $k < k_F$

$$\delta E_k = \frac{1}{2m} (k_F^2 - (k_F - \delta k)^2) = v_F \cdot \delta k.$$

We could define Wilson ratio as

$$R_W = \frac{\pi^2 k_B^2 \chi}{3\mu_B^2 \gamma} = 1,$$

for free fermions despite the band structure.

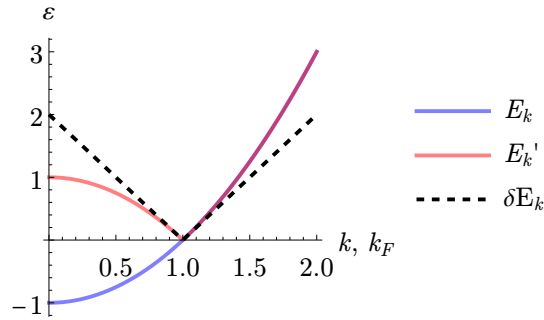


Рис. 1: Single-particle excitations