1 Non-equilibrium and Transport in Integrable Models

Consider evolution

$$|\psi_0\rangle \to e^{-it\hat{H}}\,|\psi_0\rangle = |\psi_t\rangle\,.$$

But we expect ti thermal state be with

$$\hat{\rho} = \frac{1}{Z}e^{-\beta\hat{H}},$$

what is not the same to

$$|\psi_t\rangle\langle\psi_t| = \sum_{n,m} c_n \bar{c}_m e^{-it(E_n - E_m)}|n\rangle\langle m|.$$

But there is thermalization, at least with local observables

$$\langle\psi_t|\hat{O}|\psi_t\rangle = \frac{1}{Z}\,\mathrm{tr}\left(e^{-\beta\hat{H}}\hat{O}\right).$$

So, we could define something as local (extensive) quantity if

$$\partial_r Q = 0, \quad [Q, H] = 0, \qquad \hat{Q} = \sum_j \hat{q}_j,$$

with $\hat{q}_j$ a local operators. Cluster property

$$\lim_{|x-y|\to\infty}\langle Q_x Q_y\rangle = \langle Q_x\rangle\langle Q_y\rangle,$$

which is actually the same as extensivity.