ARMSTRONG’S AXIOMS

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ABSTRACT: This essay describes the closure of a functional dependency, attribute sets and its relation with the ARMSTRONG’s AXIOMS with the help of an example.

CLOSURE OF FUNCTIONAL DEPENDENCY:

The closure set of a set F of Functional Dependencies (FDs) is the set of all FDs implied by F. This closure set is denoted by F+ [3]. Computing the closure of functional dependency in a schema plays a major and important role for the design of a good database. Practically, computing closures for a large set of FD’s is a very expensive process. In order to infer all the non-trivial and trivial FDs, ARMSTRONG’S AXIOMS are applied.

ARMSTRONG’S AXIOMS:

A set of rules that can be applied repeatedly to infer *all* the Functional Dependencies (FDs) implied by a set F of FDs are called ARMSTRONG’S AXIOMS. Namely, reflexivity, augmentation and transitivity. Using, these axioms, additional rules namely union, decomposition can be created. The above three rules of inference satisfy the soundness and completeness property [1].

Consider a relational schema over R, where X, Y, Z denote the set of attributes

Reflexivity: If X ⊇ Y (i.e. if X is a superset of Y), then X -> Y

Augmentation: If X -> Y, then XZ -> YZ for any Z.

Transitivity: If X -> Y and Y -> Z, then X -> Z

Union: If X -> Y and X -> Z then X -> YZ

Decomposition: If X -> YZ, then X -> Y and X -> Z

Example: With the help of these rules, computing closure of a functional dependency {A->B,

B->C} over the set {A, B, C, D}

We get,

F+ = {A → A, B → B, C → C, D → D, (all the four from reflexivity)

A → B, B → C, (given)

$A \rightarrow C$, (transitivity)

$AB \rightarrow A$, $AB \rightarrow B$, $AB \rightarrow C$, $AC \rightarrow A$, $AC \rightarrow B$, $AC \rightarrow C$, $AD \rightarrow A$, $AD \rightarrow B$, $AD \rightarrow C$, $AD \rightarrow D$, $BC \rightarrow B$, $BC \rightarrow C$, $BD \rightarrow B$, $BD \rightarrow C$, $BD \rightarrow D$, $CD \rightarrow C$, $CD \rightarrow D$, $ABC \rightarrow A$, $ABC \rightarrow B$, $ABC \rightarrow C$, $ABD \rightarrow A$, $ABD \rightarrow B$, $ABD \rightarrow C$, $ABD \rightarrow D$, $BCD \rightarrow B$, $BCD \rightarrow C$, $BCD \rightarrow D$, $ABCD \rightarrow A$, $ABCD \rightarrow B$, $ABCD \rightarrow C$, $ABCD \rightarrow D$.}

CLOSURE OF ATTRIBUTE SET:

As mentioned earlier that computing closures for large set of functional dependencies can be very expensive, we tend to compute closure set of attribute. For the same example, on computation of individual attribute closure using Armstrong's axioms we have,

$A+ = AB+ = AC+ = ABC+ = \{A, B, C\}$

$B+ = BC+ = \{B, C\}$

$C+ = \{C\}$

$D+ = \{D\}$

$AD+ = \{A, B, C\ D\}$

$BC+ = \{B, C\}$

$BD+ = BCD+ = \{B, C, D\}$

$ABD+ = ABCD+ = \{A, B, C, D\}$

$ACD+ = \{A, C, D\}$

With the help of above closures, it is easier to generate the FD in F+. Also, based on this, proper candidate key selection is possible for a good database design. A good database is the one which doesn't result in any anomalies on insertion, deletion or updation. This candidate key is chosen such that, on computation of closure of attribute set, it includes all the attributes on which the schema is defined.[2] From the above given equations, AD+ contains the complete set of attributes of the relation. Hence, it is the candidate key.

References:

1. http://wwwis.win.tue.nl/~debra/2M400/colstructie5

2. http://www.cse.cuhk.edu.hk/~taoyf/course/bmeg3120/notes/fd2.pdf

3.https://www.cs.colostate.edu/~cs430dl/yr2016su/more_examples/Ch8/Closure%20Sets%20and%20Armstrong's%20Axioms.pdf