E4 Wave Transmission and Reflection

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This experiment studies electromagnetic radiation by determining their wavelengths and speeds in air through in a coaxial cable. Further investigation was performed by attaching an open and short circuit to the cable's ends, creating a hard reflection and allowing for a standing wave analysis. In a second experiment, the cable was left as an "open circuit", resulting in a different reflection. The influence of various dielectric materials, such as Acrylic, PTFE, and Glass, on the refractive index was also calculated by placing them between the strip line and the ground plane. This report analyzes the collected data and address any potential sources of error and uncertainty.

Measurement of Wavelength and Phase Velocity

The first part of the experiment involved working out the wave velocity by working out wavelengths at different frequencies using a short circuit load on the right-hand side of the slotted line. The resulting resonance pattern was used to work out the wavelength, by using this formula:

$$\lambda = \frac{2}{N}(Q_1 - P_1)$$

To take uncertainties into account, the analysis is conducted in the `Julia` programming language using the `Measurements.jl` module. The module manages uncertainties in a comprehensive manner and has functional correlation built in, so it understands that `sec²(x) - tan²(x) ≈ 1 ± 0`. For example, we can calculate the velocity of a wave like this:

$$2.988e8 \pm 1.8e6$$

```
begin
    λ  = (0.083 ± 0.0005)    # Wavelength
    f  = (3.6e9 ± 1e6)       # Frequency
    v  = f * λ               # Speed
end
```

We can repeat this calculation for all of the datapoints like this:

	Frequency	Q1 - P1	N	λ	Speed	% Uncertainty
1	3.6e9 Hz	458.0±0.71 mm	11	83.27±0.13 mm	2.99782e8 ms^-1	0.15439
2	3.0e9 Hz	446.0±0.71 mm	9	99.11±0.16 mm	2.97333e8 ms^-1	0.158544
3	2.4e9 Hz	450.0±0.71 mm	7	128.57±0.2 mm	3.08571e8 ms^-1	0.157135

The average velocity worked out was 3.018 with an uncertainty of less than 1%, which is remarkably close to the actual value of the speed of light.

Amplitude reflection coefficients measured with the slotted line

In the subsequent phase of the experiment, an open circuit was attached to the slotted line, resulting in nearly complete wave reflection. The change in maxima and minima was observed, which permitted us to calculate the ratio of the shift to the wavelength. The measurements are detailed below.

	f	P2 - P1	Q2 - Q1	λ	Δ	Δ/λ
1	2.4e9 Hz	48.0±0.71 mm	26.0±0.71 mm	128.57±0.2 mm	37.0±0.5 mm	0.2878 ± 0.0039

We expect the the wave to reflect with a reflection coefficient of ρ = 1, corresponding to a shift of half a wavelength. This would, in turn, be expected to cause a shift of the standing wave by $\frac{\lambda}{2}/2$, so we expect $\Delta/\lambda \approx 0.25$.

The actual value is slightly higher, at 0.28. This is likely due to the standing wave being slightly different in each configuration.

Velocity of Microwaves in different materials

Results and Calculations

	Material	M	R	R - M	n
1	"PTFE"	314.0±0.5 mm	330.0±0.5 mm	16.0±0.71 mm	1.267 ± 0.012
2	"Glass"	314.0±0.5 mm	322.0±0.5 mm	8.0±0.71 mm	1.0667 ± 0.0059
3	"Acrylic"	314.0±0.5 mm	337.0±0.5 mm	23.0±0.71 mm	1.2556 ± 0.008
4	"Acrylic"	314.0±0.5 mm	343.0±0.5 mm	29.0±0.71 mm	1.2417 ± 0.006

Comments and Uncertainites

The readings appear to be very internally consistent, with Acrylic having almost the same refractive index across two measurements. However, they do not reference values of n, possibly due to experimental error.

Transmission through a resonator

For this experiment, the voltage was measured at the far end of the split line across a range of frequencies from 2 to 4 GHz in 2 MHz intervals. Two brass bolts were inserted across the split line 120 mm apart in order to set up resonance between them. The results are plotted below:

The peak voltage was at a frequency of $f \approx 2.38$ GHz, corresponding to a wavelength of $\lambda = u/f \approx 125.1$ mm, about 5% more than the distance between the brass bolts.

Conclusions

In conclusion, our readings produced results that were largely accurate, but were impacted by instrumental inaccuracies, approximations, and numerical rounding errors. This experiment demonstrated that the speed of light can be estimated with reasonable precision using straightforward yet precise equipment. Our prediction of the wave shift produced by an open circuit was confirmed. We were also not able to determine the refractive indices of different materials, obtaining results that were incorrect despite having low error bounds. Lastly, we were able to calculate the wavelength of a wave at resonance.