# UE Artificial Intelligence

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► MGU(Student(x), Student(y))

▶  $MGU(Student(x), Student(y)) = \{x/y\}$ 

- ►  $MGU(Student(x), Student(y)) = \{x/y\}$
- ► MGU(Student(passed(w), y), Student(z, grade(passed(u), v)))

- ►  $MGU(Student(x), Student(y)) = \{x/y\}$
- ▶ MGU(Student(passed(w), y), Student(z, grade(passed(u), v)))
  = {z/passed(w), y/grade(passed(u), v)}

- ►  $MGU(Student(x), Student(y)) = \{x/y\}$
- ▶ MGU(Student(passed(w), y), Student(z, grade(passed(u), v)))
  = {z/passed(w), y/grade(passed(u), v)}
- MGU(Student(enthusiastic(x), passed(x), owns(y)), Student(x, y, z, owns(z))}

- ►  $MGU(Student(x), Student(y)) = \{x/y\}$
- ▶ MGU(Student(passed(w), y), Student(z, grade(passed(u), v)))
  = {z/passed(w), y/grade(passed(u), v)}
- MGU(Student(enthusiastic(x), passed(x), owns(y)), Student(x, y, z, owns(z))} FAIL: different arity.

- ►  $MGU(Student(x), Student(y)) = \{x/y\}$
- MGU(Student(passed(w), y), Student(z, grade(passed(u), v)))
  = {z/passed(w), y/grade(passed(u), v)}
- MGU(Student(enthusiastic(x), passed(x), owns(y)), Student(x, y, z, owns(z))} FAIL: different arity.
- MGU(Student(y, enthusiastic(x), MATH), Student(AICOURSE, enthusiastic(x), y))

- ►  $MGU(Student(x), Student(y)) = \{x/y\}$
- MGU(Student(passed(w), y), Student(z, grade(passed(u), v)))
  = {z/passed(w), y/grade(passed(u), v)}
- MGU(Student(enthusiastic(x), passed(x), owns(y)), Student(x, y, z, owns(z))} FAIL: different arity.
- MGU(Student(y, enthusiastic(x), MATH),
   Student(AICOURSE, enthusiastic(x), y))
   = MGU(Student(y<sub>1</sub>, enthusiastic(x<sub>1</sub>), MATH),
   Student(AICOURSE, enthusiastic(x<sub>2</sub>), y<sub>2</sub>))

- ►  $MGU(Student(x), Student(y)) = \{x/y\}$
- MGU(Student(passed(w), y), Student(z, grade(passed(u), v)))
  = {z/passed(w), y/grade(passed(u), v)}
- MGU(Student(enthusiastic(x), passed(x), owns(y)), Student(x, y, z, owns(z))} FAIL: different arity.
- ► MGU(Student(y, enthusiastic(x), MATH), Student(AICOURSE, enthusiastic(x), y)) = MGU(Student(y<sub>1</sub>, enthusiastic(x<sub>1</sub>), MATH), Student(AICOURSE, enthusiastic(x<sub>2</sub>), y<sub>2</sub>)) =  $\{x_1/x_2, y_1/AICOURSE, y_2/MATH\}$

MGU(Student(grade(x, AICOURSE), x), Student(grade(z, u), genius(150, v)), Student(y, genius(v, MATH))}

- MGU(Student(grade(x, AICOURSE), x), Student(grade(z, u), genius(150, v)), Student(y, genius(v, MATH))} Unify with 2 steps:
  - MGU(Student(grade(x, AICOURSE), x), Student(grade(z, u), genius(150, v)),

- MGU(Student(grade(x, AICOURSE), x), Student(grade(z, u), genius(150, v)), Student(y, genius(v, MATH))} Unify with 2 steps:
  - MGU(Student(grade(x, AICOURSE), x),
     Student(grade(z, u), genius(150, v)),
     = {x/z, u/AICOURSE, z/genius(150, v)}

MGU(Student(grade(x, AICOURSE), x), Student(grade(z, u), genius(150, v)),Student(y, genius(v, MATH))} Unify with 2 steps:

- $\bigcirc$  MGU(Student(grade(x, AICOURSE), x), Student(qrade(z, u), qenius(150, v)), $= \{x/z, u/AICOURSE, z/genius(150, v)\}$
- 2 MGU(Student(grade(genius(150, v), AICOURSE), genius(150, v)),Student(v, genius(v, MATH)))

MGU(Student(grade(x, AICOURSE), x), Student(grade(z, u), genius(150, v)), Student(y, genius(v, MATH))} Unify with 2 steps:

#### MGU(Student(grade(x, AICOURSE), x), Student(grade(z, u), genius(150, v)),

```
= \{x/z, u/AICOURSE, z/genius(150, v)\}\
```

```
2
```

```
MGU(Student(grade(genius(150, v), AICOURSE), genius(150, v)), Student(y, genius(v, MATH))) \\ MGU(Student(grade(genius(150, v<sub>1</sub>), AICOURSE), genius(150, v<sub>1</sub>)), Student(y, genius(v<sub>2</sub>, MATH)))
```

MGU(Student(grade(x, AICOURSE), x), Student(grade(z, u), genius(150, v)), Student(y, genius(v, MATH))} Unify with 2 steps:

```
MGU(Student(grade(x, AICOURSE), x),
Student(grade(z, u), genius(150, v)),
```

```
= \{x/z, u/AICOURSE, z/genius(150, v)\}
```

```
MGU(Student(grade(genius(150, v), AICOURSE), genius(150, v)),
Student(y, genius(v, MATH)))
MGU(Student(grade(genius(150, v<sub>1</sub>), AICOURSE), genius(150, v<sub>1</sub>)),
Student(y, genius(v<sub>2</sub>, MATH)))
= {y/grade(genius(150, v<sub>1</sub>)), v<sub>1</sub>/MATH, v<sub>2</sub>/150}
```

► MGU(Student(grade(x,y), genius(x)), Student(grade(enthusiastic(z),y), genius(enthusiastic(z)))}

MGU(Student(grade(x,y), genius(x)), Student(grade(enthusiastic(z),y), genius(enthusiastic(z)))) = MGU(Student(grade(x,y1), genius(x)), Student(grade(enthusiastic(z),y2), genius(enthusiastic(z))))

```
MGU(Student(grade(x,y), genius(x)),
Student(grade(enthusiastic(z), y), genius(enthusiastic(z))))
= MGU(Student(grade(x, y<sub>1</sub>), genius(x)),
Student(grade(enthusiastic(z), y<sub>2</sub>), genius(enthusiastic(z))))
= {x/enthusiastic(z), y<sub>1</sub>/y<sub>2</sub>}
```

```
MGU(Student(grade(x,y), genius(x)),
Student(grade(enthusiastic(z),y), genius(enthusiastic(z))))
= MGU(Student(grade(x,y<sub>1</sub>), genius(x)),
Student(grade(enthusiastic(z),y<sub>2</sub>), genius(enthusiastic(z))))
= {x/enthusiastic(z),y<sub>1</sub>/y<sub>2</sub>}
```

MGU(Student(y, passed(x), MATH, z), Professor(x, AICOURSE, enthusiastic(x), y)}

- MGU(Student(grade(x,y), genius(x)), Student(grade(enthusiastic(z), y), genius(enthusiastic(z)))) = MGU(Student(grade(x, y<sub>1</sub>), genius(x)), Student(grade(enthusiastic(z), y<sub>2</sub>), genius(enthusiastic(z)))) = {x/enthusiastic(z), y<sub>1</sub>/y<sub>2</sub>}
- MGU(Student(y, passed(x), MATH, z), Professor(x, AICOURSE, enthusiastic(x), y)) FAIL: heads of terms are different!

- MGU(Student(grade(x,y), genius(x)), Student(grade(enthusiastic(z), y), genius(enthusiastic(z)))) = MGU(Student(grade(x, y<sub>1</sub>), genius(x)), Student(grade(enthusiastic(z), y<sub>2</sub>), genius(enthusiastic(z)))) = {x/enthusiastic(z), y<sub>1</sub>/y<sub>2</sub>}
- MGU(Student(y, passed(x), MATH, z), Professor(x, AICOURSE, enthusiastic(x), y)) FAIL: heads of terms are different!
- MGU(Student(passed(x), grade(passed(u), v)), Student(passed(grade(w, v)), grade(z, w))}

```
MGU(Student(grade(x,y), genius(x)),
Student(grade(enthusiastic(z), y), genius(enthusiastic(z))))
= MGU(Student(grade(x, y<sub>1</sub>), genius(x)),
Student(grade(enthusiastic(z), y<sub>2</sub>), genius(enthusiastic(z))))
= {x/enthusiastic(z), y<sub>1</sub>/y<sub>2</sub>}
```

- MGU(Student(y, passed(x), MATH, z), Professor(x, AICOURSE, enthusiastic(x), y)) FAIL: heads of terms are different!
- MGU(Student(passed(x), grade(passed(u), v)), Student(passed(grade(w, v)), grade(z, w))} = MGU(Student(passed(x), grade(passed(u), v<sub>1</sub>)), Student(passed(grade(w, v<sub>2</sub>)), grade(z, w))}

```
MGU(Student(grade(x,y), genius(x)),
Student(grade(enthusiastic(z), y), genius(enthusiastic(z))))
= MGU(Student(grade(x, y<sub>1</sub>), genius(x)),
Student(grade(enthusiastic(z), y<sub>2</sub>), genius(enthusiastic(z))))
= {x/enthusiastic(z), y<sub>1</sub>/y<sub>2</sub>}
```

- MGU(Student(y, passed(x), MATH, z), Professor(x, AICOURSE, enthusiastic(x), y)) FAIL: heads of terms are different!
- ► MGU(Student(passed(x), grade(passed(u), v)), Student(passed(grade(w, v)), grade(z, w))} = MGU(Student(passed(x), grade(passed(u), v₁)), Student(passed(grade(w, v₂)), grade(z, w))} = {x/grade(w, v₂), z/passed(u), v₁/w}

- P(hungry)
- 2  $P(\neg cold \mid hungry \land cold)$
- $\bigcirc$  P(excited  $\lor \neg$ excited)
- **4**  $P(hungry \land cold \mid crying)$
- **5**  $P(\neg crying)$
- 6 P(cold | hungry)
- 8 P(crying ∨ excited)
- $\bigcirc$   $P(excited \land \neg hungry)$
- $\bigcirc$   $P((excited \land cold) \lor (\neg crying \land hungry))$

		crying		¬ c	rying
		cold ¬ cold		cold	$\neg \operatorname{cold}$
excited	hungry	0.02	0.01	0.02	0.06
	eg hungry	0.01	0.01	0.05	0.12
¬ excited	hungry	0.05	0.03	0.06	0.14
	eg hungry	0.03	0.01	0.1	0.28

#### Exercise 2 - Notation

$$P(a,b) \triangleq P(a \wedge b)$$
  
 $\sum_{A} P(A,b) \triangleq P(a,b) + P(\neg a,b)$ 

#### Exercise 2 – Rules

Marginalisation:

$$P(a) = \sum_{B,C} P(a,B,C)$$

Conditional Probability:

$$P(a \mid b) = \frac{P(a,b)}{P(b)}$$

"Or"-Rule:

$$P(a \lor b) = P(a) + P(b) - P(a,b)$$

		cr		¬	cr
		со	$\neg$ co	со	$\neg$ co
е	h	0.02	0.01	0.02	0.06
	$\negh$	0.01	0.01	0.05	0.12
¬ e	h	0.05	0.03	0.06	0.14
	¬ h	0.03	0.01	0.1	0.28

$$P(h) =$$

		cr		_ ¬	cr
		со	$\neg$ co	со	$\neg$ co
6	h	0.02	0.01	0.02	0.06
	$\neg h$	0.01	0.01	0.05	0.12
¬ e	h	0.05	0.03	0.06	0.14
	¬ h	0.03	0.01	0.1	0.28

$$P(h) = \sum_{E,Co,Cr} P(E,Co,Cr,h) = 0.39$$

		cr		_ ¬	cr
		со	$\neg$ co	со	$\neg$ co
	h	0.02	0.01	0.02	0.06
е	$\negh$	0.01	0.01	0.05	0.12
¬ e	h	0.05	0.03	0.06	0.14
	$\neg$ h	0.03	0.01	0.1	0.28

$$P(\neg c \mid h, c) =$$

		cr		_ ¬	cr
		со	$\neg$ co	со	$\neg$ co
е	h	0.02	0.01	0.02	0.06
	$\neg h$	0.01	0.01	0.05	0.12
¬ e	h	0.05	0.03	0.06	0.14
	$\neg h$	0.03	0.01	0.1	0.28

$$P(\neg c \mid h, c) = 0$$

		cr		_ ¬	cr
		со	$\neg$ co	со	$\neg$ co
	h	0.02	0.01	0.02	0.06
е	$\neg h$	0.01	0.01	0.05	0.12
¬e	h	0.05	0.03	0.06	0.14
	¬ h	0.03	0.01	0.1	0.28

$$P(e \lor \neg e) =$$

		cr		_ ¬	cr
		со	$\neg$ co	со	$\neg$ co
	h	0.02	0.01	0.02	0.06
е	$\negh$	0.01	0.01	0.05	0.12
¬e	h	0.05	0.03	0.06	0.14
	$\negh$	0.03	0.01	0.1	0.28

$$P(e \lor \neg e) = 1$$

		cr			cr
		со	$\neg$ co	со	$\neg$ co
	h	0.02	0.01	0.02	0.06
е	$\neg h$	0.01	0.01	0.05	0.12
¬e	h	0.05	0.03	0.06	0.14
	¬ h	0.03	0.01	0.1	0.28

$$P(h, co \mid cr) =$$

		cr		_ ¬	cr
		со	$\neg$ co	со	$\neg$ co
е	h	0.02	0.01	0.02	0.06
	$\neg h$	0.01	0.01	0.05	0.12
¬ e	h	0.05	0.03	0.06	0.14
	$\neg h$	0.03	0.01	0.1	0.28

$$P(h, co \mid cr) = \frac{P(h, co, cr)}{P(cr)} =$$

$$P(h,co \mid cr) = \frac{P(h,co,cr)}{P(cr)} = \frac{\sum_{E} P(E,h,co,cr)}{\sum_{E,H,Co} P(E,H,Co,cr)} \approx 0.41$$

		C	r	_ ¬	cr
		со	$\neg$ co	со	$\neg$ co
	h	0.02	0.01	0.02	0.06
е	$\negh$	0.01	0.01	0.05	0.12
	h	0.05	0.03	0.06	0.14
¬ e	$\neg$ h	0.03	0.01	0.1	0.28

$$P(\neg cr) =$$

		C	r		cr
		со	$\neg$ co	со	$\neg$ co
	h	0.02	0.01	0.02	0.06
е	$\negh$	0.01	0.01	0.05	0.12
	h	0.05	0.03		0.14
¬ e	¬ h	0.03	0.01	0.1	0.28

$$P(\neg cr) = \sum_{E,H,Co} P(E,H,Co,\neg cr) = 0.83$$

		C	r	¬ cr	
		со	$\neg$ co	со	$\neg$ co
	h	0.02	0.01	0.02	0.06
е	$\negh$	0.01	0.01	0.05	0.12
	h	0.05	0.03	0.02	0.14
¬ e	$\neg$ h	0.03	0.01	0.1	0.28

$$P(co \mid h) =$$

		C	r	_ ¬	cr
		со	$\neg$ co	со	$\neg$ co
	h	0.02	0.01	0.02	0.06
е	$\neg h$	0.01	0.01	0.05	0.12
	h	0.05	0.03	0.06	0.14
¬ e	¬ h	0.03	0.01	0.1	0.28

$$P(co \mid h) = \frac{P(co, h)}{P(h)} =$$

		C	r	¬ cr	
		со	$\neg$ co	со	$\neg$ co
	h	0.02	0.01	0.02	0.06
е	$\neg h$	0.01	0.01	0.05	0.12
	h	0.05	0.03		0.14
¬ e	$\neg h$	0.03	0.01	0.1	0.28

$$P(co \mid h) = \frac{P(co, h)}{P(h)} = \frac{\sum_{E,Cr} P(E, Cr, co, h)}{\sum_{E,Cr,Co} P(H, Cr, Co, h)} \approx 0.38$$

		C	r	¬ cr	
		со	$\neg$ co	со	$\neg$ co
	h	0.02	0.01	0.02	0.06
е	$\neg h$	0.01	0.01	0.05	0.12
	h	0.05	0.03	0.02	0.14
¬ e	$\neg h$	0.03	0.01	0.1	0.28

$$P(co \mid e, \neg h) =$$

		C	r	¬ cr	
		со	$\neg$ co	со	$\neg$ co
	h	0.02	0.01	0.02	0.06
е	$\neg h$	0.01	0.01	0.05	0.12
	h	0.05	0.03	0.06	0.14
¬ e	$\neg h$	0.03	0.01	0.1	0.28

$$P(co \mid e, \neg h) = \frac{P(co, e, \neg h)}{P(e, \neg h)} =$$

		C	r	¬	cr
		со	$\neg$ co	со	$\neg$ co
	h	0.02	0.01	0.02	0.06
е	$\neg h$	0.01	0.01	0.05	0.12
	h	0.05	0.03		0.14
¬ e	¬ h	0.03	0.01	0.1	0.28

$$P(co \mid e, \neg h) = \frac{P(co, e, \neg h)}{P(e, \neg h)} = \frac{\sum_{Cr} P(Cr, co, e, \neg h)}{\sum_{Cr, Co} P(Cr, Co, e, \neg h)} \approx 0.32$$

		C	r		cr
		со	$\neg$ co	со	$\neg$ co
	h	0.02	0.01	0.02	0.06
е	$\negh$	0.01	0.01	0.05	0.12
	h	0.05	0.03		0.14
¬ e	¬ h	0.03	0.01	0.1	0.28

$$P(cr \lor e) =$$

		C	r	¬ cr	
		со	$\neg$ co	со	$\neg$ co
	h	0.02	0.01	0.02	0.06
е	$\neg h$	0.01	0.01	0.05	0.12
	h	0.05	0.03	0.06	0.14
¬ e	¬ h	0.03	0.01	0.1	0.28

$$P(cr \lor e) = P(cr) + P(e) - P(cr, e) =$$

		C	r	¬ cr	
		со	$\neg$ co	со	$\neg$ co
	h	0.02	0.01	0.02	0.06
е	$\negh$	0.01	0.01	0.05	0.12
	h	0.05	0.03	0.02	0.14
¬ e	$\neg$ h	0.03	0.01	0.1	0.28

$$P(cr \lor e) = P(cr) + P(e) - P(cr, e) = 0.42$$

		C	r	¬ cr	
		со	$\neg$ co	со	$\neg$ co
	h	0.02	0.01	0.02	0.06
е	$\neg h$	0.01	0.01	0.05	0.12
	h	0.05	0.03	0.06	0.14
¬ e	¬ h	0.03	0.01	0.1	0.28

$$P(e,\neg h) = \sum_{Cr,Co} P(Cr,Co,e,\neg h) = 0.19$$

		cr		¬ cr	
		со	$\neg$ co	со	$\neg$ co
е	h	0.02	0.01	0.02	0.06
	$\neg h$	0.01	0.01	0.05	0.12
¬е	h	0.05	0.03	0.06	0.14
	¬ h	0.03	0.01	0.1	0.28

$$P((e \land co) \lor (\neg cr \land h)) =$$

		cr		¬ cr	
		со	$\neg$ co	со	$\neg$ co
е	h	0.02	0.01	0.02	0.06
	$\negh$	0.01	0.01	0.05	0.12
¬ e	h	0.05	0.03	0.06	0.14
	¬ h	0.03	0.01	0.1	0.28

$$P((e \land co) \lor (\neg cr \land h)) = P(e, co) + P(\neg cr, h) - P(e, co, \neg cr, h) = 0.36$$

- ► Unicorns, jackalopes and hippocamps are mammals.
- An offspring of a unicorn is a unicorn.
- HappyRainbowDancer is a unicorn.
- HappyRainbowDancer is Greenyboony's parent.
- Offspring and parent are inverse relations.

- ▶  $Unicorn(x) \rightarrow Mammal(x)$
- ▶  $Jackalope(x) \rightarrow Mammal(x)$
- ▶  $Hippocamp(x) \rightarrow Mammal(x)$
- An offspring of a unicorn is a unicorn.
- HappyRainbowDancer is a unicorn.
- HappyRainbowDancer is Greenyboony's parent.
- Offspring and parent are inverse relations.

- ▶  $Unicorn(x) \rightarrow Mammal(x)$
- ▶  $Jackalope(x) \rightarrow Mammal(x)$
- ▶  $Hippocamp(x) \rightarrow Mammal(x)$
- ▶  $Unicorn(y) \land Offspring(x,y) \rightarrow Unicorn(x)$
- HappyRainbowDancer is a unicorn.
- HappyRainbowDancer is Greenyboony's parent.
- Offspring and parent are inverse relations.

- ▶  $Unicorn(x) \rightarrow Mammal(x)$
- ▶  $Jackalope(x) \rightarrow Mammal(x)$
- ▶  $Hippocamp(x) \rightarrow Mammal(x)$
- ▶  $Unicorn(y) \land Offspring(x,y) \rightarrow Unicorn(x)$
- Unicorn(HappyRainbowDancer)
- HappyRainbowDancer is Greenyboony's parent.
- Offspring and parent are inverse relations.

- ▶  $Unicorn(x) \rightarrow Mammal(x)$
- ▶  $Jackalope(x) \rightarrow Mammal(x)$
- ▶  $Hippocamp(x) \rightarrow Mammal(x)$
- ▶  $Unicorn(y) \land Offspring(x,y) \rightarrow Unicorn(x)$
- Unicorn(HappyRainbowDancer)
- Parent(HappyRainbowDancer, Greenyboony)
- Offspring and parent are inverse relations.

- ▶  $Unicorn(x) \rightarrow Mammal(x)$
- ▶  $Jackalope(x) \rightarrow Mammal(x)$
- ▶  $Hippocamp(x) \rightarrow Mammal(x)$
- ▶  $Unicorn(y) \land Offspring(x,y) \rightarrow Unicorn(x)$
- Unicorn(HappyRainbowDancer)
- Parent(HappyRainbowDancer, Greenyboony)
- ▶ Offspring(x,y) → Parent(y,x)
- ▶  $Parent(x,y) \rightarrow Offspring(y,x)$

## Exercise 3 – Standardising Apart

- $2 J(x) \to M(x)$
- $3 H(x) \rightarrow M(x)$
- **5** *U*(*HRD*)
- **6** *P*(*HRD*, *GB*)
- $O(x,y) \to P(y,x)$
- $P(x,y) \to O(y,x)$

## Exercise 3 – Standardising Apart

- $2 J(b) \rightarrow M(b)$
- $3 H(c) \rightarrow M(c)$
- **5** *U*(*HRD*)
- **6** *P*(*HRD*, *GB*)
- $O(f,g) \to P(g,f)$
- 8  $P(h,i) \rightarrow O(i,h)$

- $2 J(b) \to M(b)$

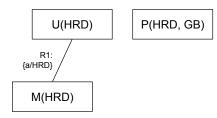
- U(HRD)
- 6 P(HRD, GB)
- $8 P(h,i) \to O(i,h)$

U(HRD)

P(HRD, GB)

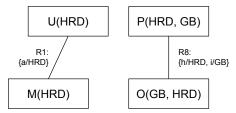
- $2 J(b) \to M(b)$

- 5 U(HRD)
- 6 P(HRD, GB)
- $8 P(h,i) \to O(i,h)$



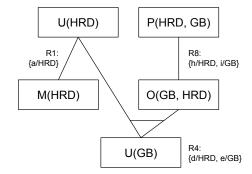
- 1  $U(a) \rightarrow M(a)$
- $2 J(b) \to M(b)$

- U(HRD)
- 6 P(HRD, GB)
- $8 P(h,i) \to O(i,h)$



- 1  $U(a) \rightarrow M(a)$
- $2 J(b) \to M(b)$

- 5 U(HRD)
- 6 P(HRD, GB)
- $0(f,g) \rightarrow P(g,f)$
- 8  $P(h,i) \rightarrow O(i,h)$



- $2 J(b) \to M(b)$

- 5 U(HRD)
- 6 P(HRD, GB)
- $8 P(h,i) \to O(i,h)$

U(GB)

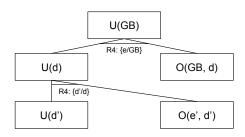
- $2 J(b) \to M(b)$

- 5 U(HRD)
- 6 P(HRD, GB)
- $8 P(h,i) \to O(i,h)$



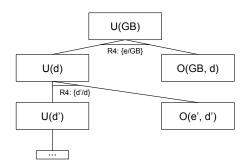
- $2 J(b) \to M(b)$

- 5 U(HRD)
- 6 P(HRD, GB)
- $8 P(h,i) \to O(i,h)$



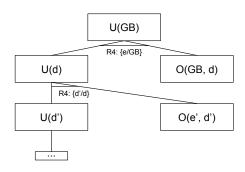
- $2 J(b) \to M(b)$

- 5 U(HRD)
- 6 P(HRD, GB)
- $0(f,g) \to P(g,f)$
- $8 P(h,i) \to O(i,h)$



- 1  $U(a) \rightarrow M(a)$
- $2 J(b) \to M(b)$

- 5 U(HRD)
- 6 P(HRD, GB)
- $0(f,g) \to P(g,f)$
- $8 P(h,i) \to O(i,h)$



To prevent the loop, just switch rule 4 and 5.

## Exercise 4 – FWD/BCK Chaining

- 1 Horse(x)  $\land$  Fish(x)  $\rightarrow$  Hippocamp(x)
- **2** Horse(x)  $\land$  Horned(x)  $\rightarrow$  Unicorn(x)
- 3  $Rabbit(z) \land Horned(z) \rightarrow Jackalope(z)$
- **4** Unicorn(x)  $\land$  Jackalope(y)  $\rightarrow$  Friendlier(x,y)
- **5**  $Jackalope(y) \land Hippocamp(z) \rightarrow Friendlier(y, z)$
- **6** Friendlier $(x,y) \land$  Friendlier $(y,z) \rightarrow$  Friendlier(x,z)
- 7 Horse(Steve)
- 8 Horse(Greenyboony)
- Horned(Greenyboony)
- 10 Jackalope(Bob)
- fish(Steve)

# Exercise 4 – FWD/BCK Chaining

- 2  $Hs(x) \wedge Hn(x) \rightarrow U(x)$
- 3  $R(z) \wedge Hn(z) \rightarrow J(z)$
- $4 U(x) \wedge J(y) \rightarrow F(x,y)$
- **5**  $J(y) \wedge Hi(z) \rightarrow F(y,z)$
- 6  $F(x,y) \wedge F(y,z) \rightarrow F(x,z)$
- Hs(S)
- 8 Hs(G)
- 9 Hn(G)
- 10 J(B)
- $\mathbf{1}$  F(S)

# Exercise 4 – FWD/BCK Chaining

- 2  $Hs(b) \wedge Hn(b) \rightarrow U(b)$
- $3 R(c) \wedge Hn(c) \rightarrow J(c)$
- $4 U(d) \wedge J(e) \rightarrow F(d,e)$
- **6**  $F(h,i) \wedge F(i,j) \rightarrow F(h,j)$
- Hs(S)
- 8 Hs(G)
- 9 Hn(G)
- 10 J(B)
- $\mathbf{1}$  F(S)

## Exercise 4 – Forward Chaining F(G, S)

- 1  $Hs(a) \wedge F(a) \rightarrow Hi(a)$
- 2  $Hs(b) \wedge Hn(b) \rightarrow U(b)$
- 4  $U(d) \wedge J(e) \rightarrow F(d, e)$
- 6  $F(h,i) \wedge F(i,j) \rightarrow F(h,j)$
- $F(n,i) \wedge F(i,j) \to F(n,j)$
- 7 Hs(S)
- 8 Hs(G)
- 9 Hn(G)
- 10 J(B)
- $\mathbf{1}$  F(S)

Hs(S) F

F(S)

J(B)

Hs(G)

Hn(G)

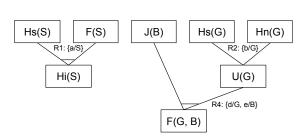
- 1  $Hs(a) \wedge F(a) \rightarrow Hi(a)$
- 2  $Hs(b) \wedge Hn(b) \rightarrow U(b)$
- $4 \quad U(d) \wedge J(e) \rightarrow F(d,e)$
- 6  $F(h,i) \wedge F(i,j) \rightarrow F(h,j)$
- 7 Hs(S)
- 8 Hs(G)
- 9 Hn(G)
- 10 J(B)
- $\mathbf{1}$  F(S)



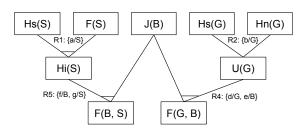
- 1  $Hs(a) \wedge F(a) \rightarrow Hi(a)$
- 2  $Hs(b) \wedge Hn(b) \rightarrow U(b)$
- $4 \quad U(d) \wedge J(e) \rightarrow F(d,e)$
- $5 J(f) \wedge Hi(g) \rightarrow F(f,g)$
- 6  $F(h,i) \wedge F(i,j) \rightarrow F(h,j)$
- 7 Hs(S)
- 8 Hs(G)
- 9 Hn(G)
- 10 J(B)
- $\mathbf{1}$  F(S)



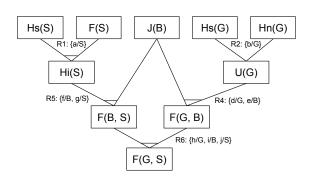
- 1  $Hs(a) \wedge F(a) \rightarrow Hi(a)$
- 2  $Hs(b) \wedge Hn(b) \rightarrow U(b)$
- $4 \quad U(d) \wedge J(e) \rightarrow F(d,e)$
- 6  $F(h,i) \wedge F(i,j) \rightarrow F(h,j)$
- 7 Hs(S)
- 8 Hs(G)
- 9 Hn(G)
- 10 J(B)
- $\mathbf{1}$  F(S)



- 1  $Hs(a) \wedge F(a) \rightarrow Hi(a)$
- 2  $Hs(b) \wedge Hn(b) \rightarrow U(b)$
- $4 \quad U(d) \wedge J(e) \rightarrow F(d,e)$
- 6  $F(h,i) \wedge F(i,j) \rightarrow F(h,j)$
- 7 Hs(S)
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- 9 Hn(G)
- 10 J(B)
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- 1  $Hs(a) \wedge F(a) \rightarrow Hi(a)$
- 2  $Hs(b) \wedge Hn(b) \rightarrow U(b)$
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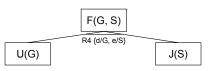


- 1  $Hs(a) \wedge F(a) \rightarrow Hi(a)$
- 2  $Hs(b) \wedge Hn(b) \rightarrow U(b)$
- $4 \quad U(d) \wedge J(e) \rightarrow F(d,e)$
- 6  $F(h,i) \wedge F(i,j) \rightarrow F(h,j)$
- 7 Hs(S)
- 8 Hs(G)
- 9 Hn(G)
- 10 J(B)
- $\mathbf{1}$  F(S)

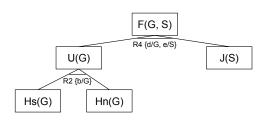
F(G, S)

- 1  $Hs(a) \wedge F(a) \rightarrow Hi(a)$
- 2  $Hs(b) \wedge Hn(b) \rightarrow U(b)$

- 6  $F(h,i) \wedge F(i,j) \rightarrow F(h,j)$
- 7 Hs(S)
- 8 Hs(G)
- 9 Hn(G)
- 10 J(B)
- $\mathbf{1}$  F(S)

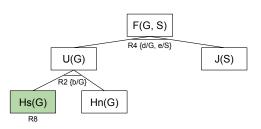


- 1  $Hs(a) \wedge F(a) \rightarrow Hi(a)$
- 2  $Hs(b) \wedge Hn(b) \rightarrow U(b)$
- $4 \quad U(d) \wedge J(e) \rightarrow F(d,e)$
- 6  $F(h,i) \wedge F(i,j) \rightarrow F(h,j)$
- 7 Hs(S)
- 8 Hs(G)
- 9 Hn(G)
- 10 J(B)
- $\mathbf{1}$  F(S)



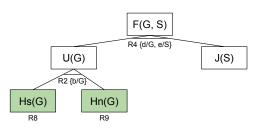
- 1  $Hs(a) \wedge F(a) \rightarrow Hi(a)$
- 2  $Hs(b) \wedge Hn(b) \rightarrow U(b)$

- 6  $F(h,i) \wedge F(i,j) \rightarrow F(h,j)$
- 7 Hs(S)
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- 9 Hn(G)
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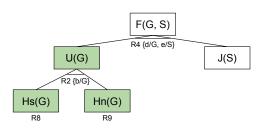
- 1  $Hs(a) \wedge F(a) \rightarrow Hi(a)$
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- 6  $F(h,i) \wedge F(i,j) \rightarrow F(h,j)$
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- 9 Hn(G)
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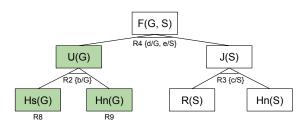


- 1  $Hs(a) \wedge F(a) \rightarrow Hi(a)$
- 2  $Hs(b) \wedge Hn(b) \rightarrow U(b)$

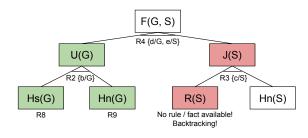
- 6  $F(h,i) \wedge F(i,j) \rightarrow F(h,j)$
- 7 Hs(S)
- 8 Hs(G)
- 9 Hn(G)
- 10 J(B)
- $\mathbf{1}$  F(S)



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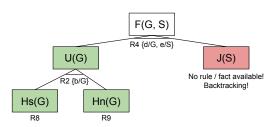


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- 2  $Hs(b) \wedge Hn(b) \rightarrow U(b)$
- $4 \quad U(d) \wedge J(e) \rightarrow F(d,e)$
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- $6 F(h,i) \wedge F(i,j) \to F(h,j)$
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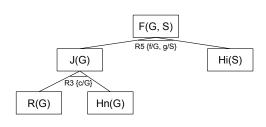
F(G, S)

- 2  $Hs(b) \wedge Hn(b) \rightarrow U(b)$

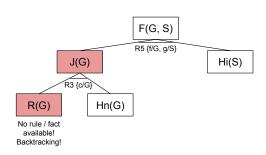
- 6  $F(h,i) \wedge F(i,j) \rightarrow F(h,j)$
- 7 Hs(S)
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- 9 Hn(G)
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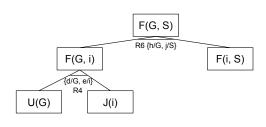
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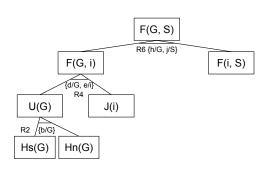


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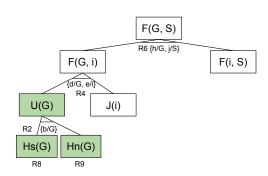
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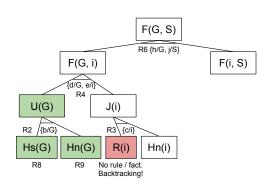
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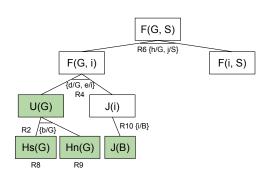
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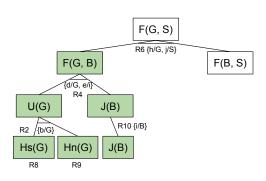


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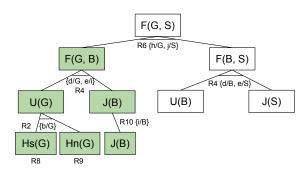


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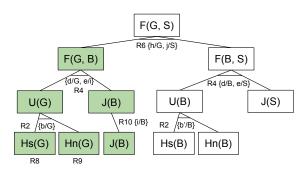
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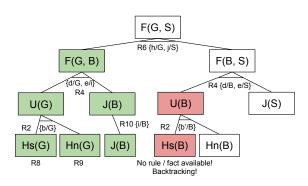
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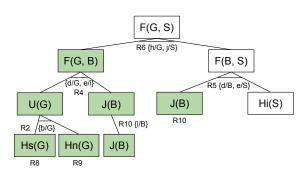
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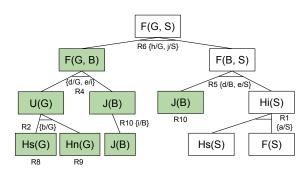
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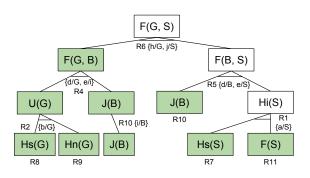
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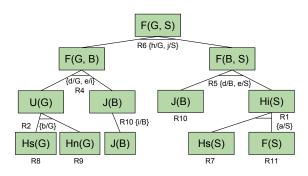


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#### Exercise 4 - Question

Imagine many descriptions of individuals in the KB, like Rabbit(Bugs), Horse(FuManchu). Which algorithm would you use?

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- Until now, Backward Chaining seemed wasteful: Runs into many dead ends.
- Forward Chaining first determines the "species" of each individual, then establishes relations.
- If we have many individuals, FC would unnecessarily expand all of them, even if they are unrelated to the query.
- Backward Chaining would focus the search!

## Questions



# Assignment 4 – Bayesian Nets, Reinforcement Learning

#### **Topics**

- Construct a Bayesian Net from a textual description of a domain.
- ▶ Write an algorithm that **learns** to play a (simple) game.

#### Deadline

- 25th of January 2016, 23:55
- You can even submit after the exam!