CSCI 270 Homework #3

Due Date: Thursday, October 17th, 2pm

Submit in class or the dropbox (Box 3, first floor of SAL, opposite the Men's bathrooms).

1. Write your name, student ID Number, and which lecture you attend (morning or afternoon). Multi-page submissions must be stapled.

2. You are given a weighted directed acyclic graph $G = (V, E)$. You want to find the longest path in G.

   (a) Split the problem up into a list of ordered decisions. What is the first decision your algorithm must make?

   (b) Define $OPT(x)$ to be the length of the longest path in G that starts at node x. Give the recursive procedure to solve this problem (don't forget the base case!)

   (c) What order do you need to fill the array in to transform this into an iterative procedure?

   (d) Specify where the answer is stored in your array.

   (e) Analyze the running time of your iterative procedure.

   (f) Explain how to reconstruct the actual path, instead of just stating the length of the path.

3. Given a ribbon strip of length k, a tailor will cut it once, in any place you choose, for the price of k dollars. Suppose you are given a strip of length L, marked to be cut in n different locations labeled $1, 2, ..., n$. For simplicity, let indices 0 and $n + 1$ denote the left and right endpoints of the original ribbon strip of length L. Let the distance of mark i from the left end of the ribbon be d_i , and assume that $0 = d_0 < d_1 < d_2 < ... < d_n < d_{n+1} = L$.

   (a) Suppose you have a ribbon length 10, and you want to make two cuts at $d_0 = 6$ and $d_1 = 3$. What is the total cost if you make cut d_0 first? What is the total cost if you instead make cut d_1 first?

   (b) Give an efficient dynamic programming algorithm to determine the minimum payment required to make the specified cuts, and analyze the runtime of your algorithm.

4. Suppose a smuggler wants to sneak n units of contraband into a country by hiding them on k trucks that are driving across the border that day. The ith truck can hold c_i pieces of contraband. Each truck has a known probability p_i of being searched at the border. You need to decide which trucks to store contraband on. Give an efficient dynamic programming algorithm that determines the minimum possible probability that any of those trucks are searched. Analyze the runtime of your algorithm.

Continued on back.

5. The year is 20XX. After the downfall of mankind, the Earth is now ruled by robots (whom were all created by the genius Dr. Ight). TGO, or "Turing Game Online", is very popular amongst robots.

   Robots love powers of 5, that is, the numbers 1, 5, 25, 125, 625, and so on. In TGO, the player is given a bit string in binary. The ideal situation is when the string it represents is a power of 5 in binary, with no leading zeros. If that is not the case, the robot player tries to cut the given string into pieces, each piece being a binary representation of a power of 5, with no leading zeros. Of course, it may be the case that this is impossible. In that case, the robot goes into a murderous rampage. You, as one of the surviving humans, are in charge of checking the bit strings to prevent this from happening.

   You are given a String S that consists of characters '0' and '1' only. S represents the string given to a player of the Turing game. Return the smallest positive integer K such that it is possible to cut S into K pieces, each of them being a power of 5. Return -1 if there is no integer K that satisfies the constraints. Analyze the runtime of your algorithm.

If you would like some extra practice, you may do the following problems. Do not submit them, as they will not be graded. If you would like to check your answers, talk to the instructor or TA via email or office hours. All extra practice problems are from the Kleinberg and Tardos textbook.

Chapter 6: exercises 1, 4, 6, 16, 20, 27