CSCI 270 Homework #4

Due Date: Thursday, October 31st, 2pm

Submit in class or the dropbox (Box 3, first floor of SAL, opposite the Men's bathrooms). Solutions will be posted Friday evening. You may only use one late day on this assignment!

1. Write your name, student ID Number, and which lecture you attend (morning or afternoon). Multi-page submissions must be stapled.

2. You are driving across the country on a road trip, and are given a sorted list of integers $i_0 < i_1 < ... < i_{n-1}$ that correspond to the distance of each gas station from your starting location, and i_n being your destination. You can travel M miles on a full tank of gas. Write a greedy algorithm that minimizes the number of gas stations you stop at. Prove the correctness of your algorithm using a simplified exchange argument.

3. You are again driving across the country on a road trip. The gas stations are at distances $i_0 < i_1 < ... < i_{n-1}$, with i_n being your destination. You can travel M miles on a full tank of gas. However, each gas station $i : 0 \leq i \leq n-1$ has a service cost s_i. Your goal is to now reach your destination while minimizing the sum of service costs you incur.

   (a) Propose 2 reasonable greedy algorithms to solve this problem.

   (b) Prove that both of your algorithms presented in part (a) are **incorrect**.

   (c) Give a dynamic programming solution to this problem, and analyze the running time.

4. With political tempers running high, the dating site PolitiMatch has seen a boom in business amongst its Washington D.C. clientele. There are n male users who each specify their political ideology m_i on a scale from 0 to 10 (0 indicating extremely liberal, and 10 indicating extremely conservative). There are n female users, whose political ideology is f_i. Fractions are allowed, so a user could specify 3.14 as their ideology number. PolitiMatch attempts to minimize the difference of the ideology of those matched. More rigorously, if we match m_i, f_j, the value of the match v_k is $|m_i - f_j|$. Minimize the sum of the value of all pairings $\sum v_k$.

   Write a greedy algorithm to solve this problem, and prove the correctness of your algorithm using an exchange argument.

5. Professor Slacker has a stack of final exams to be graded, and he hates grading! Therefore, he just assigned a random grade out of $10n$ to every student in the class. Slacker's laziness is coming back to haunt him however, as a student has requested to see their final exam! There are n problems, each worth 10 points. Slacker gave this student G total points, and now has to justify the grade to the student. Looking over the exam, Slacker determines that, if he had graded properly, he should have assigned g_i points to problem i, but $\sum_i g_i > G$. Your goal is to come up with a point distribution h_i for each problem such that $\sum_i h_i = G$, and the inappopriateness of the grading is minimized. The inappropriateness of a single problem i is $(g_i - h_i)^2$, and the inappropriateness of the grading is $\sum_i (g_i - h_i)^2$. You are allowed to assign fractions such as 3.14 for grades.

   Write a greedy algorithm to solve this problem, and prove the correctness of your algorithm using an exchange argument.

   Practice Problems on back.

If you would like some extra practice, you may do the following problems. Do not submit them, as they will not be graded. If you would like to check your answers, talk to the instructor or TA via email or office hours. All extra practice problems are from the Kleinberg and Tardos textbook.

Chapter 4: exercises 5, 7, 9, 19, 29