CSCI 270 Homework #6

Due Date: Thursday, December 5th, 2pm

Submit in class or the dropbox (Box 3, first floor of SAL, opposite the Men's bathrooms).
Solutions will be posted Monday evening. You may only use 2 late days on this assignment.

1. Write your name, student ID Number, and which lecture you attend (morning or afternoon). Multi-page submissions must be stapled.

2. Please fill out the online evaluations for this course. I'm looking for feedback and am constantly changing how the course is taught. If you have suggestions, no matter how large or small, I'm interested in seeing them.

3. In the Half-Cover problem, we are given m sets $S_1, S_2, ..., S_m$, each of which contains a subset of the integers $1, 2, ..., n$. Our goal is to determine whether there exists a collection of k sets whose union has size at least $\frac{n}{2}$.

   (a) Suppose we prove that Half-Cover is NP-complete, and that we find an $O(n^4)$ algorithm for Half-Cover. Does this imply that there is a polynomial algorithm for 3-SAT? Does this imply that there is an $O(n^4)$ algorithm for 3-SAT? Explain your reasoning.

   (b) Show that Half-Cover is $\in NP$.

   (c) Suppose we find a polynomial-time reduction from Half-Cover to Set Cover. Would this reduction prove that Half-Cover is NP-complete? Explain your reasoning.

   (d) Give a polynomial-time reduction from Set Cover to Half-Cover. Make sure to mention how to convert your answer for Half-Cover back into an answer for Set Cover.

   (e) Briefly justify why your reduction is correct. That is, explain why your Half-Cover reduction has a solution if and only if Set Cover has a solution.

4. Consider the following modification of the 2-SAT problem: we are given a 2-SAT formula with n variables and m clauses (each clause is a disjunction of exactly 2 variables or their negation), and a positive integer $k \leq m$. We want to determine if there is an assignment of true/false values to the variables such that **at least** k clauses evaluate to true. Prove that this problem is NP-complete. **Hint: you will have more success with this problem if you do not attempt to reduce from a constraint-satisfaction problem.**

5. In the **Bipartite** Directed Hamiltonian Cycle (BDHC) problem, we are given a bipartite directed graph $G = (V, E)$. We want to determine if there is a cycle which visits every node exactly once. Either prove that BDHC is NP-complete, or give a polynomial-time algorithm to solve it.

6. Suppose we want to solve Independent Set on a **tree**. We are given a tree $T = (V, E)$, an integer k, and we want to determine if there is an independent set of size $\geq k$. Either prove that this problem is NP-Complete, or give a polynomial-time algorithm to solve it.

Practice Problems on back.

If you would like some extra practice, you may do the following problems. Do not submit them, as they will not be graded. If you would like to check your answers, talk to the instructor or TA via email or office hours. All extra practice problems are from the Kleinberg and Tardos textbook.

Chapter 8: exercises 6, 7, 12, 17, 26, 28