CSCI 270 Homework #1

Due Date: Thursday, September 12th, 2pm

Submit in class or the dropbox (Box 3, first floor of SAL, opposite the Men's bathrooms).

1. Let T be a binary tree with n nodes. Each node will either have 2 children, 1 child, or it will be a leaf node. T has L leaf nodes, P nodes with 2 children, and $(n - L - P)$ nodes with 1 child. Use induction on n to prove that $L = P + 1$.

2. Suppose $f(n) = O(s(n))$, and $g(n) = O(r(n))$. Prove or disprove (by giving a counterexample) the following claims:

   (a) $f(n) - g(n) = O(f(n) - r(n))$

   (b) $\frac{f(n)}{g(n)} = O(\frac{s(n)}{g(n)})$

   (c) if $f(n) = O(g(n))$, then $f(n) + g(n) = O(s(n))$

   (d) if $s(n) = O(r(n))$, then $f(n) = O(g(n))$

3. Order the following functions from smallest asymptotic running time to greatest. Additionally, identify the two functions i and j where $f_i(n) = \theta(f_j(n))$. Justify your answers. **In this class, when I ask for a justification, it is sufficient to give an intuitive explanation.**

   (a) $f_a(n) = \frac{n^2}{\log n}$

   (b) $f_b(n) = n^2$

   (c) $f_c(n) = \log^2 n$

   (d) $f_d(n) = \sum_{i=1}^{n} i$

   (e) $f_e(n) = \log_{1.5} n^2$

   (f) $f_f(n) = 2^{\log n}$

   (g) $f_g(n) = n^{\frac{8}{9}}$

   (h) $f_h(n) = 1.001^n$

4. You are given a unweighted directed acyclic graph G, and a topological ordering for G. The topological ordering is represented as a function f which takes as input an integer between 1 and n and outputs the node with that index in the topological ordering in constant time. Give an efficient algorithm to determine the length of the longest path in G. You should aim for a linear run-time algorithm.

If you would like some extra practice, you may do the following problems. Do not submit them, as they will not be graded. If you would like to check your answers, talk to the instructor or TA via email or office hours. All extra practice problems are from the Kleinberg and Tardos textbook.

Chapter 2: exercises 3, 4, 5, 6

Chapter 3: exercises 2, 4, 5, 7, 9, 12